Title: Quantum Field Theory (PHYS 601) - Lecture 11

Date: Oct 13, 2009 09:00 AM

URL: http://pirsa.org/09100119

Abstract:


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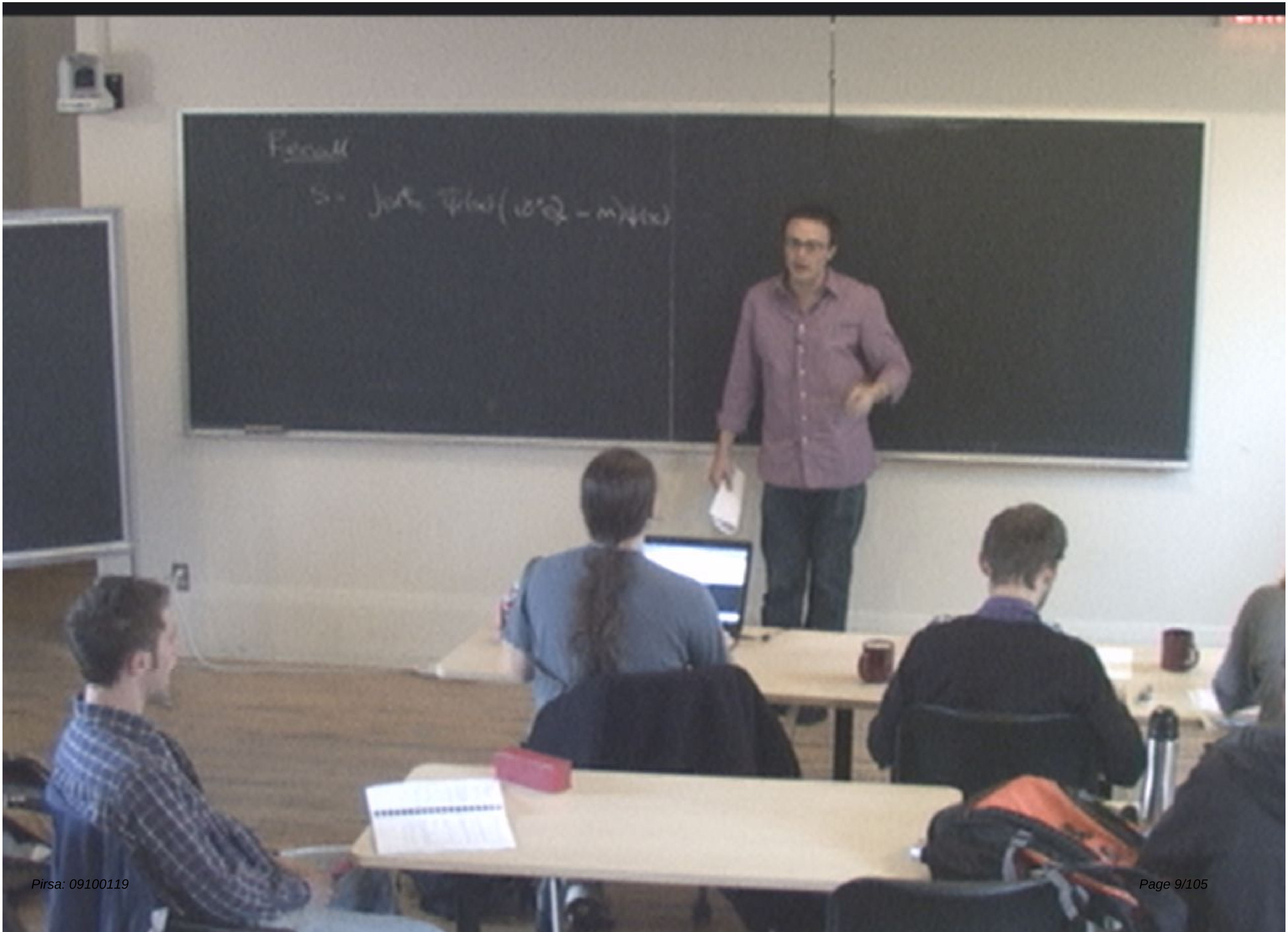
$S = \int d^4x\, \bar{\psi}(x)(i\gamma^\mu\partial_\mu - m)\psi(x)$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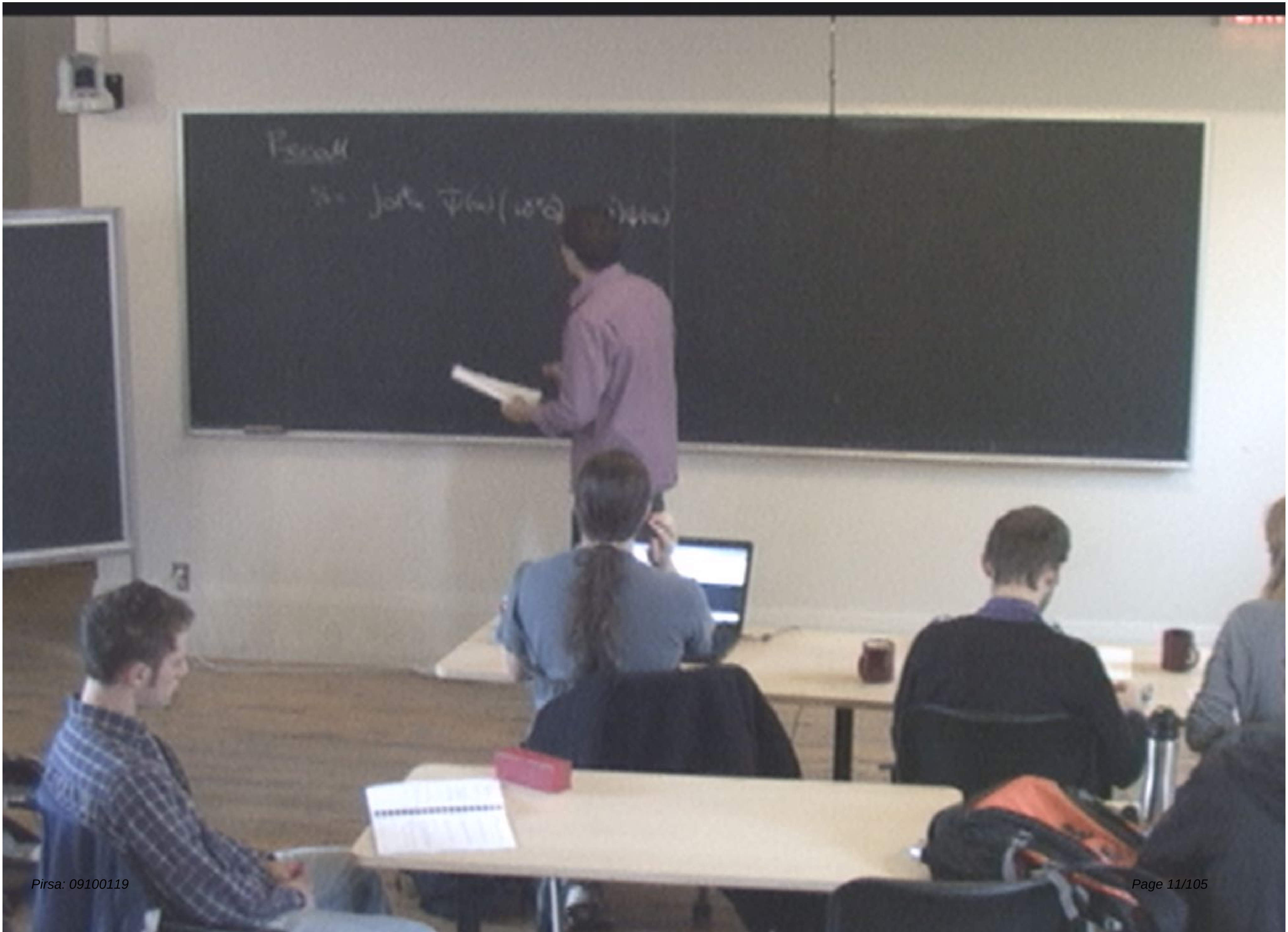

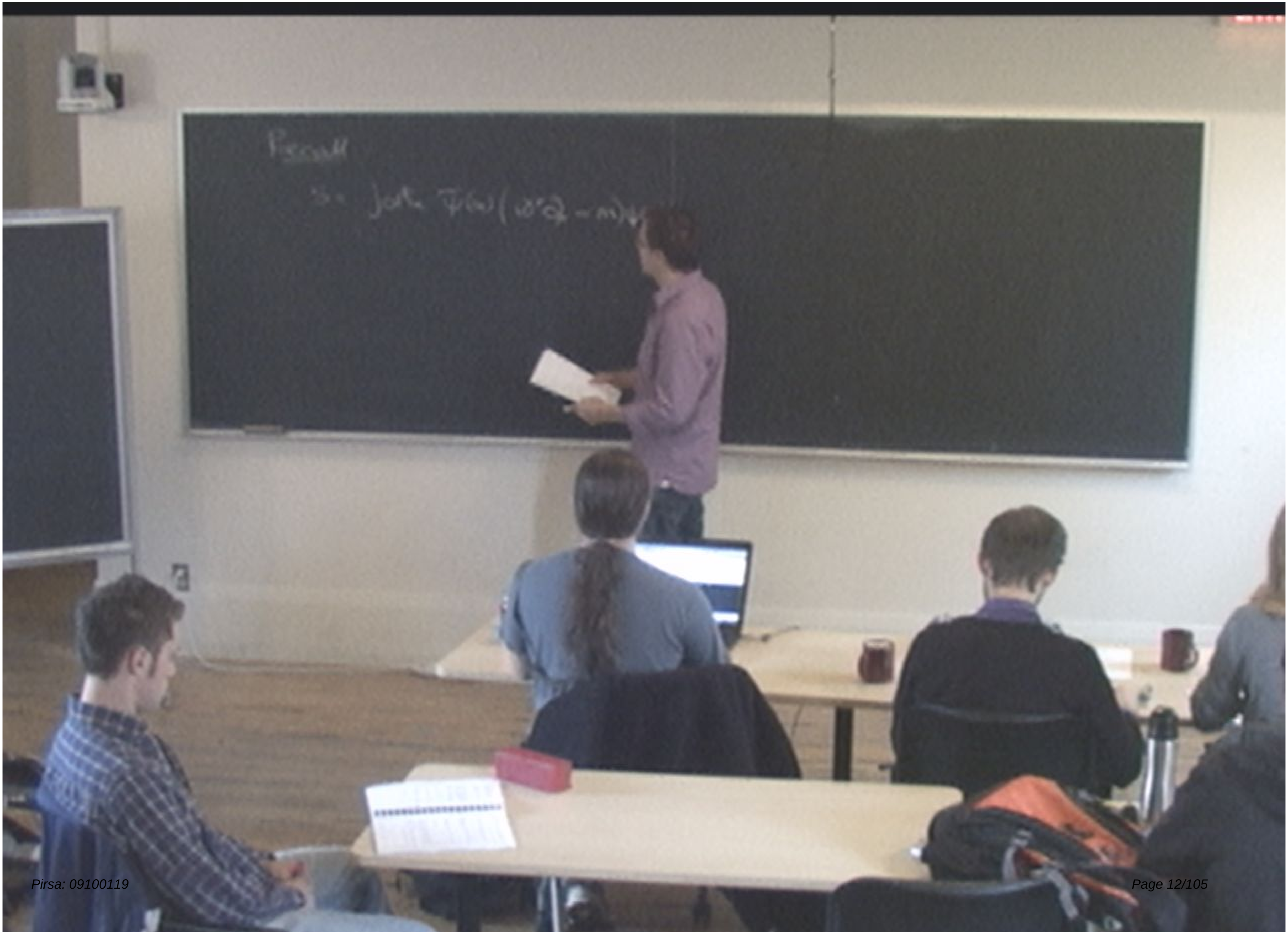

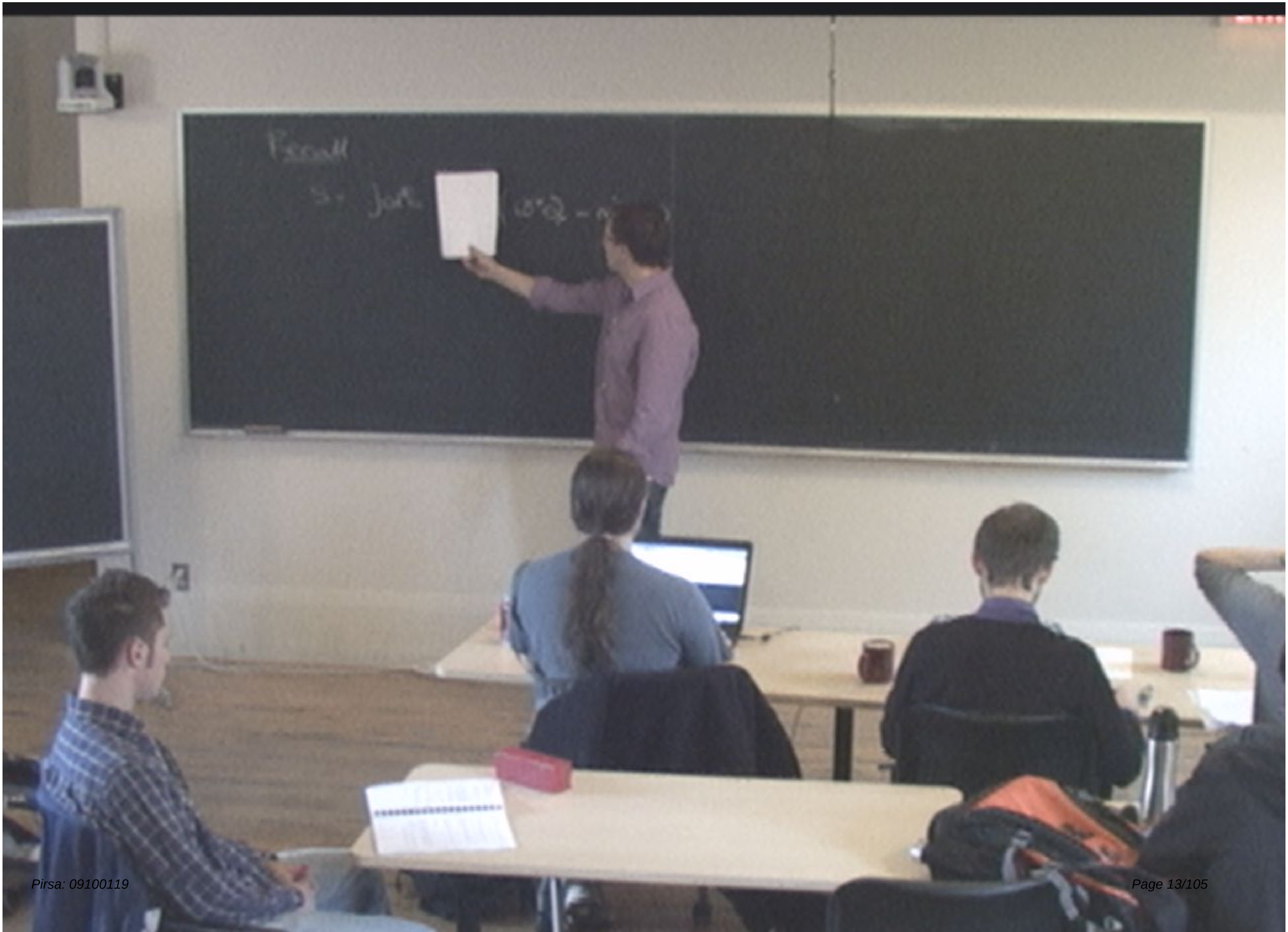
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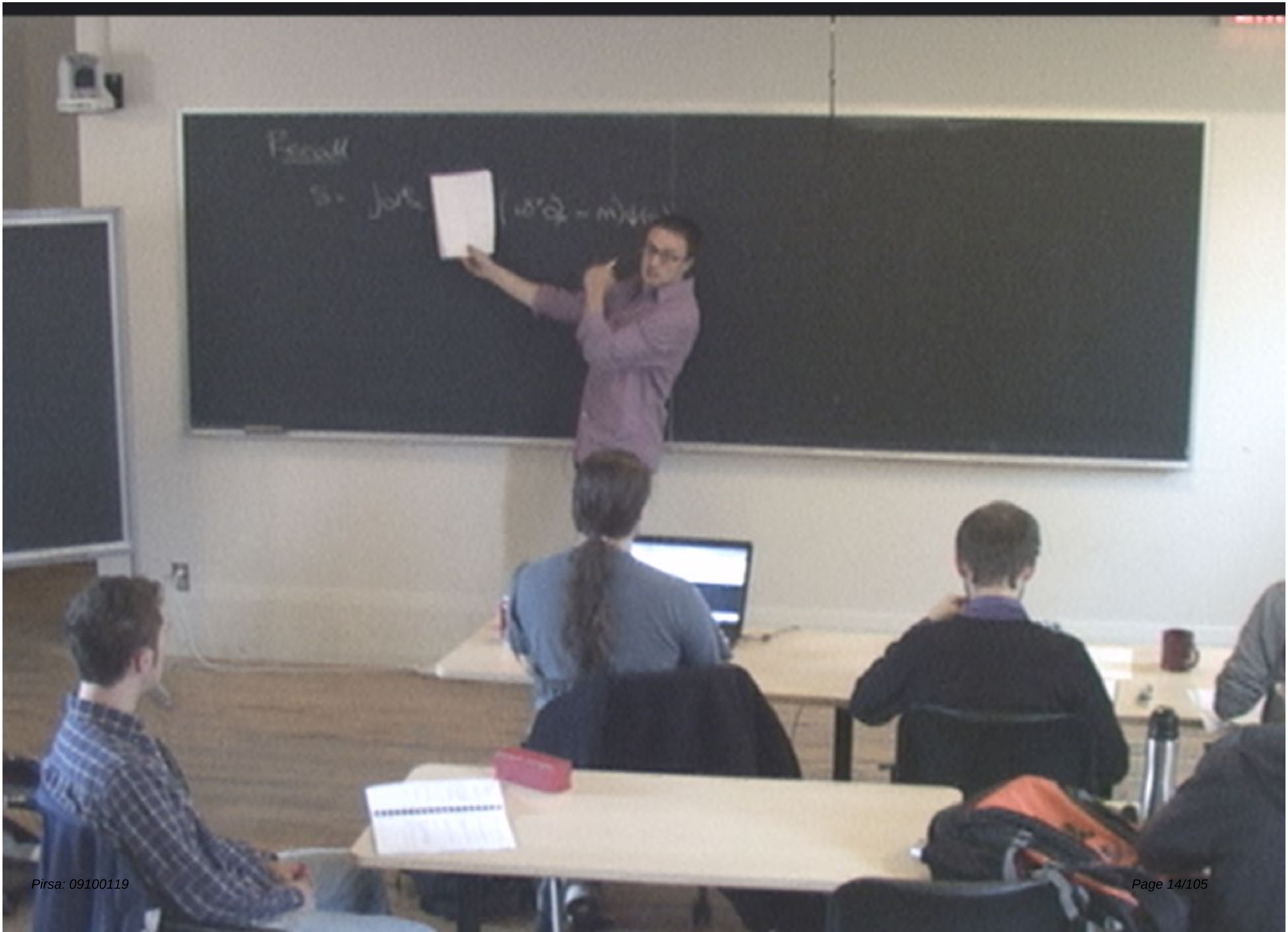
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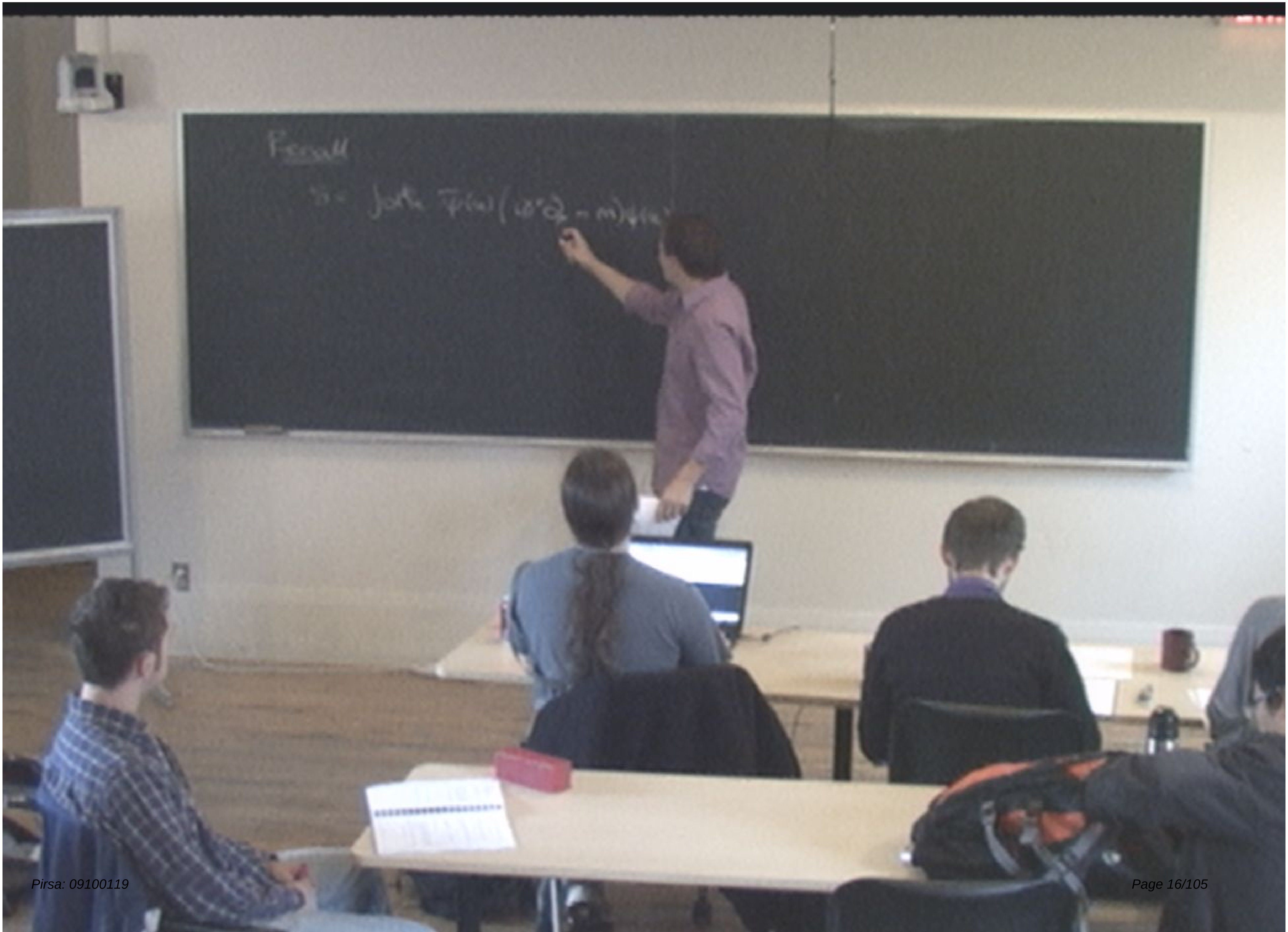
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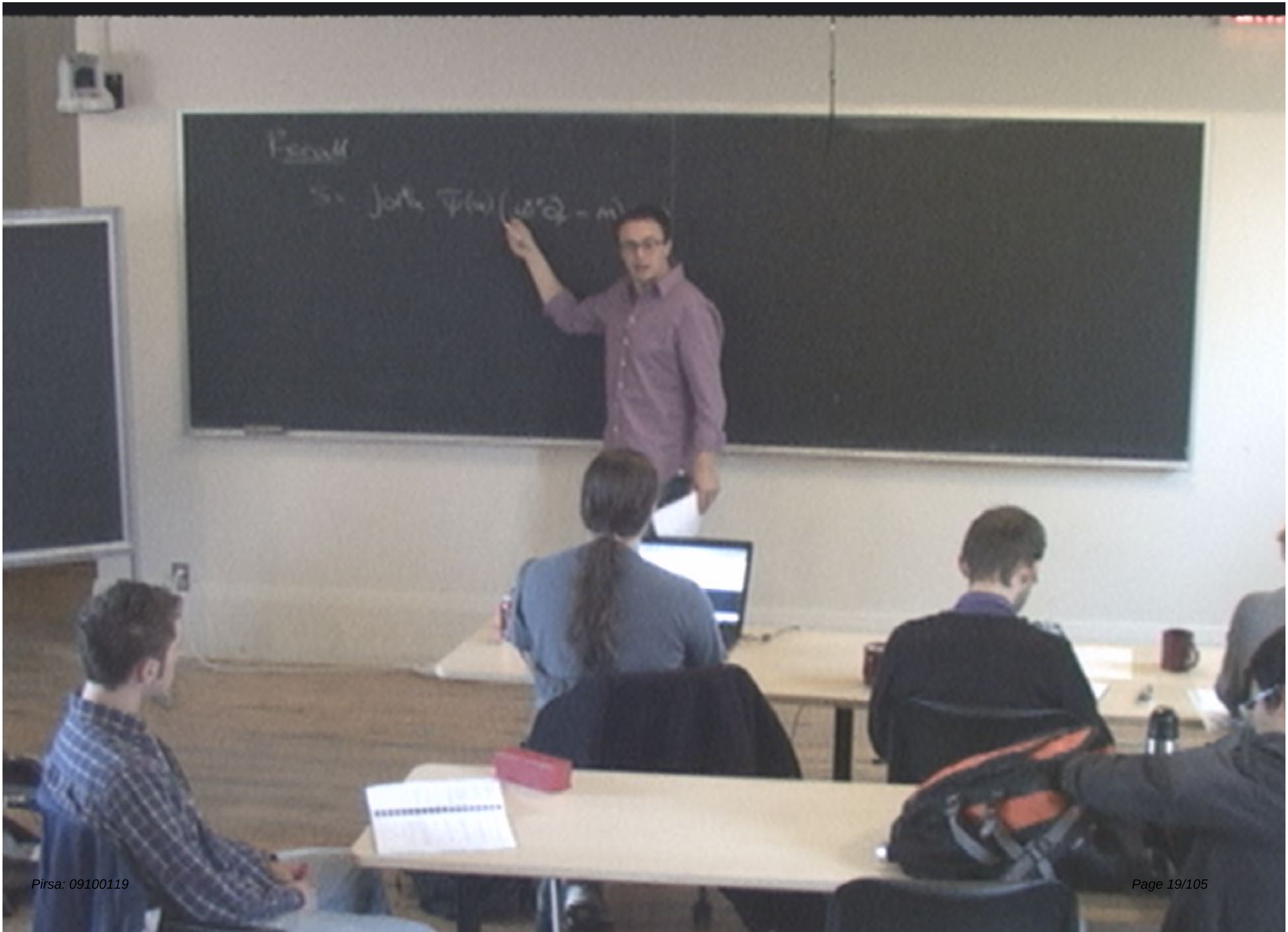

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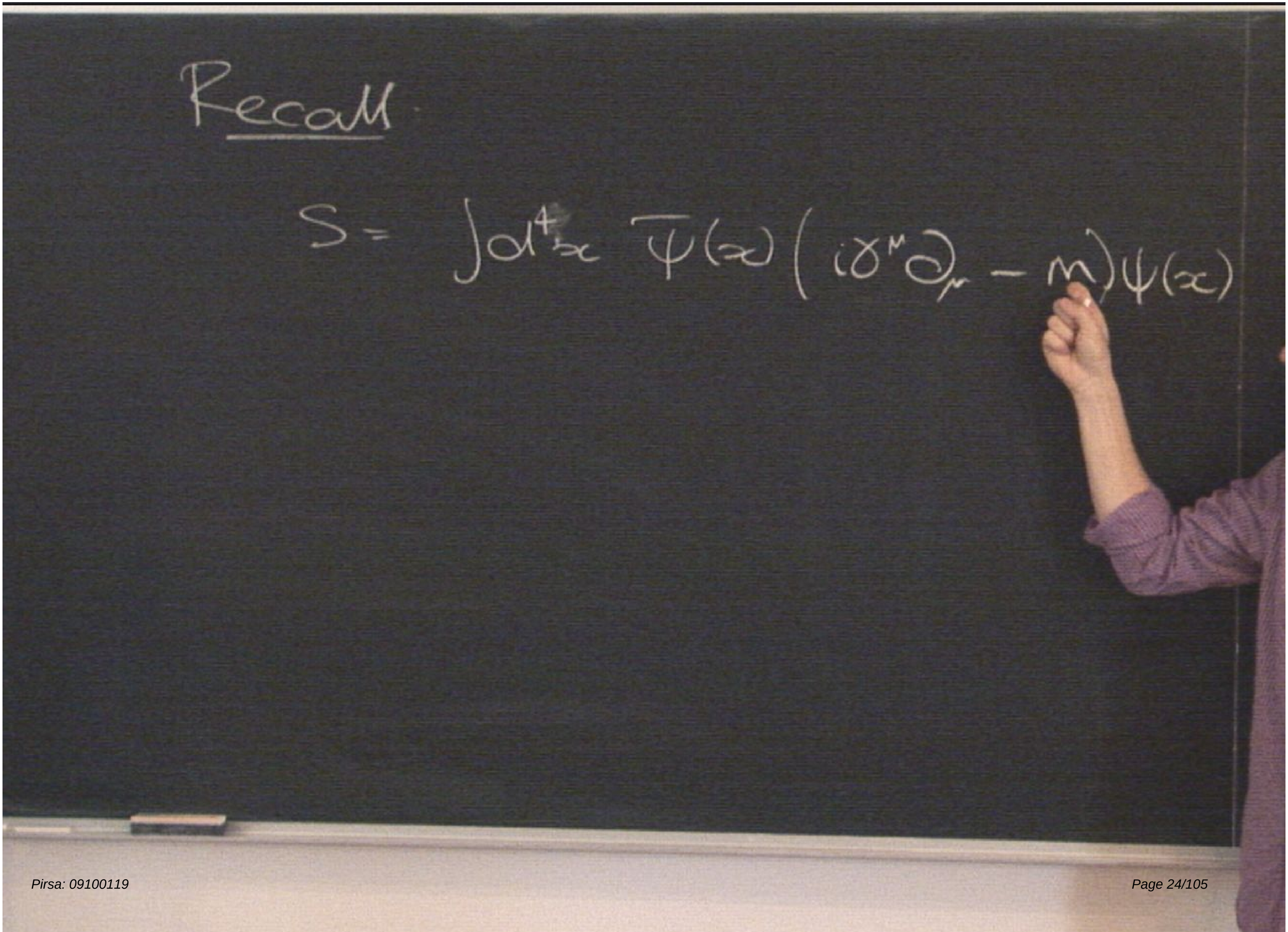
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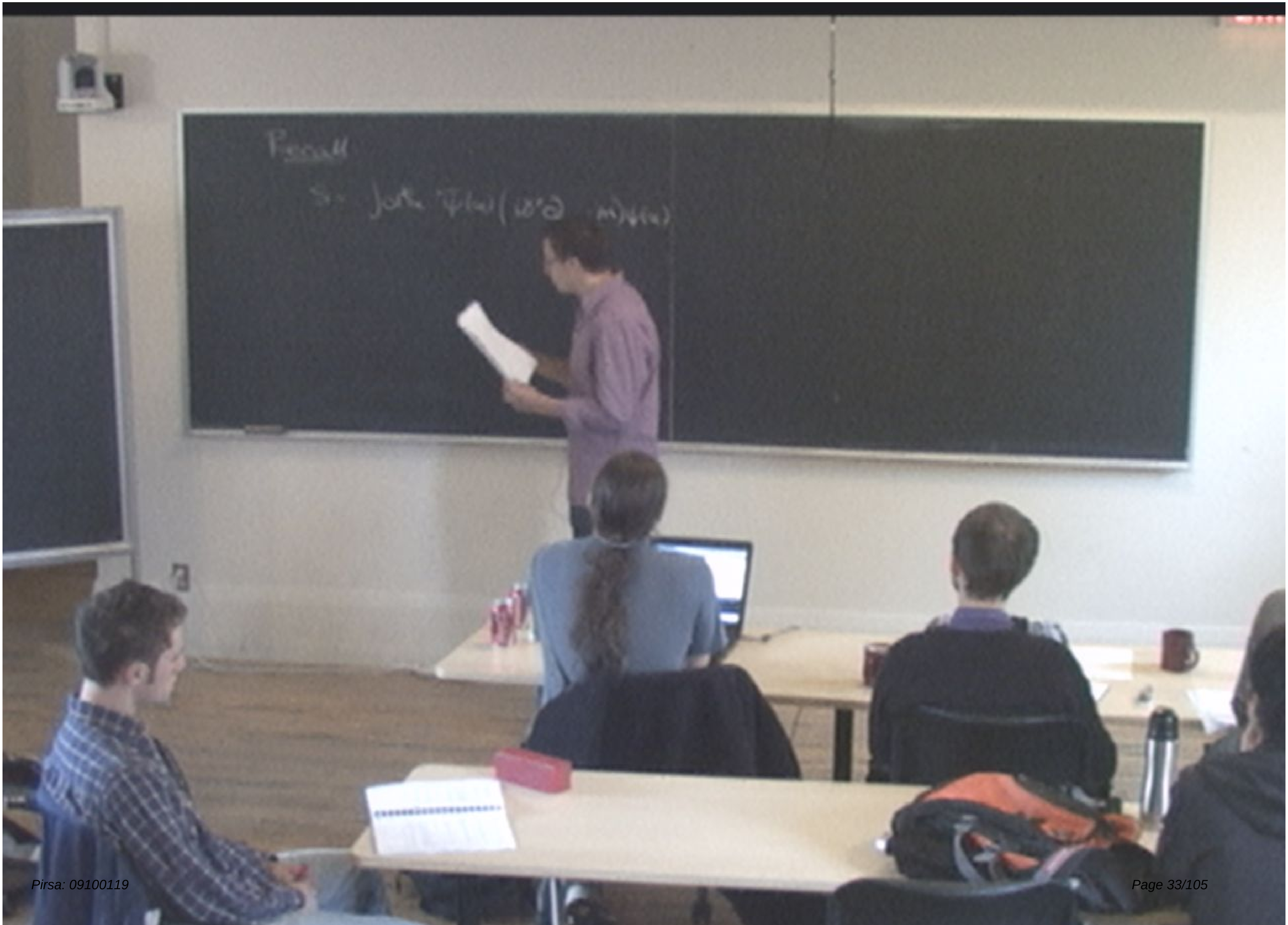
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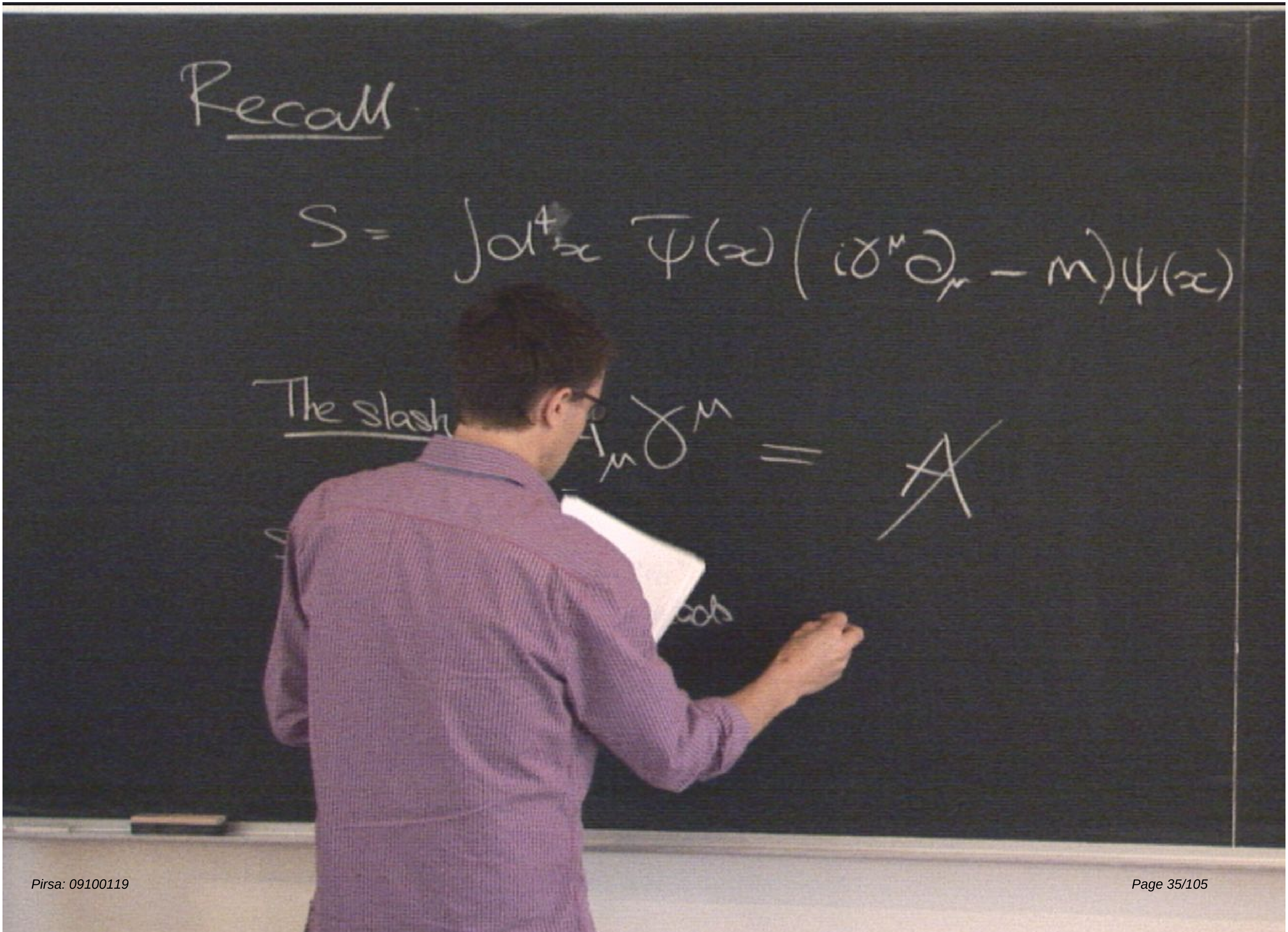
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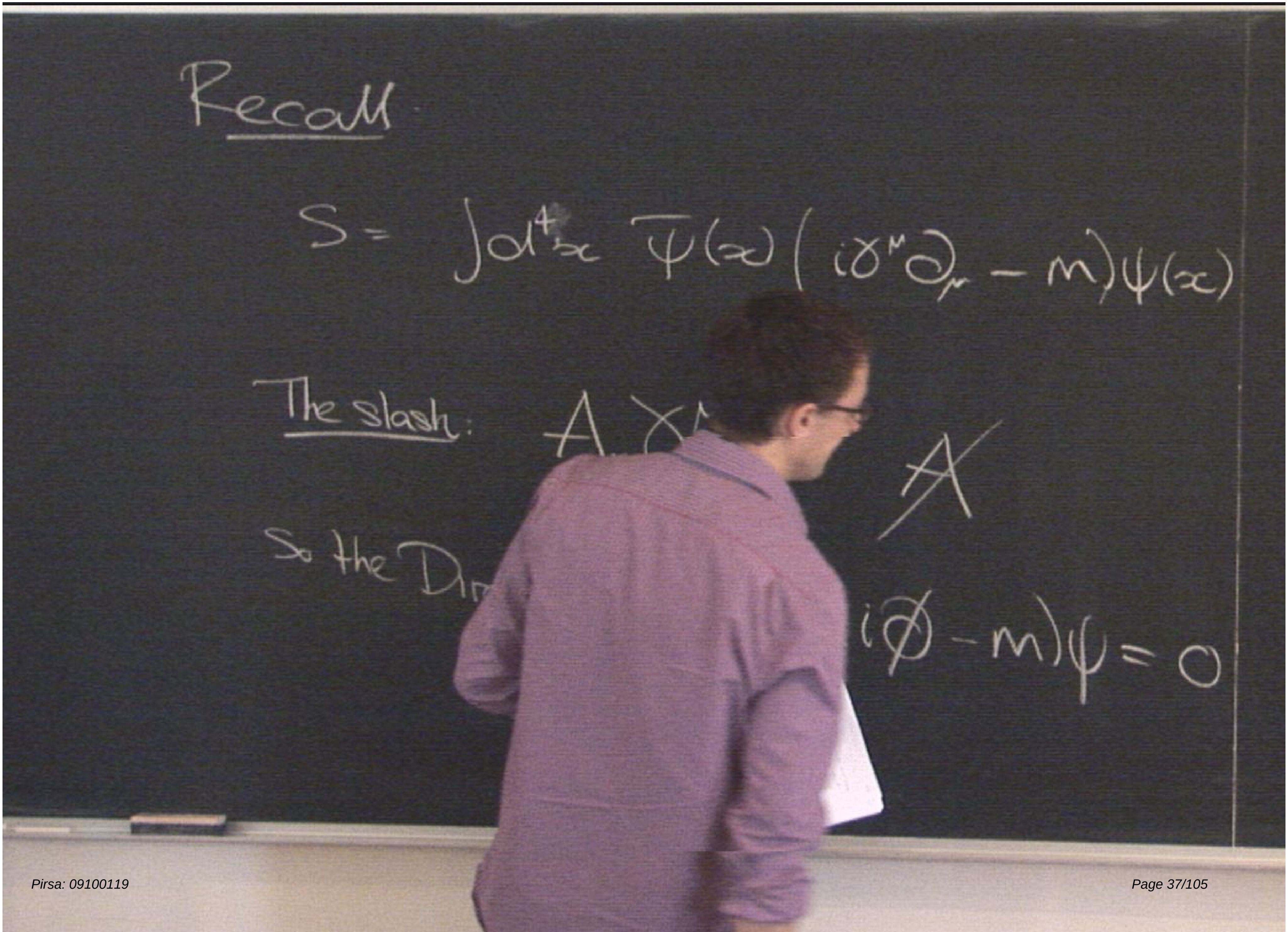
The slash
$A_\mu \gamma^\mu = \not{A}$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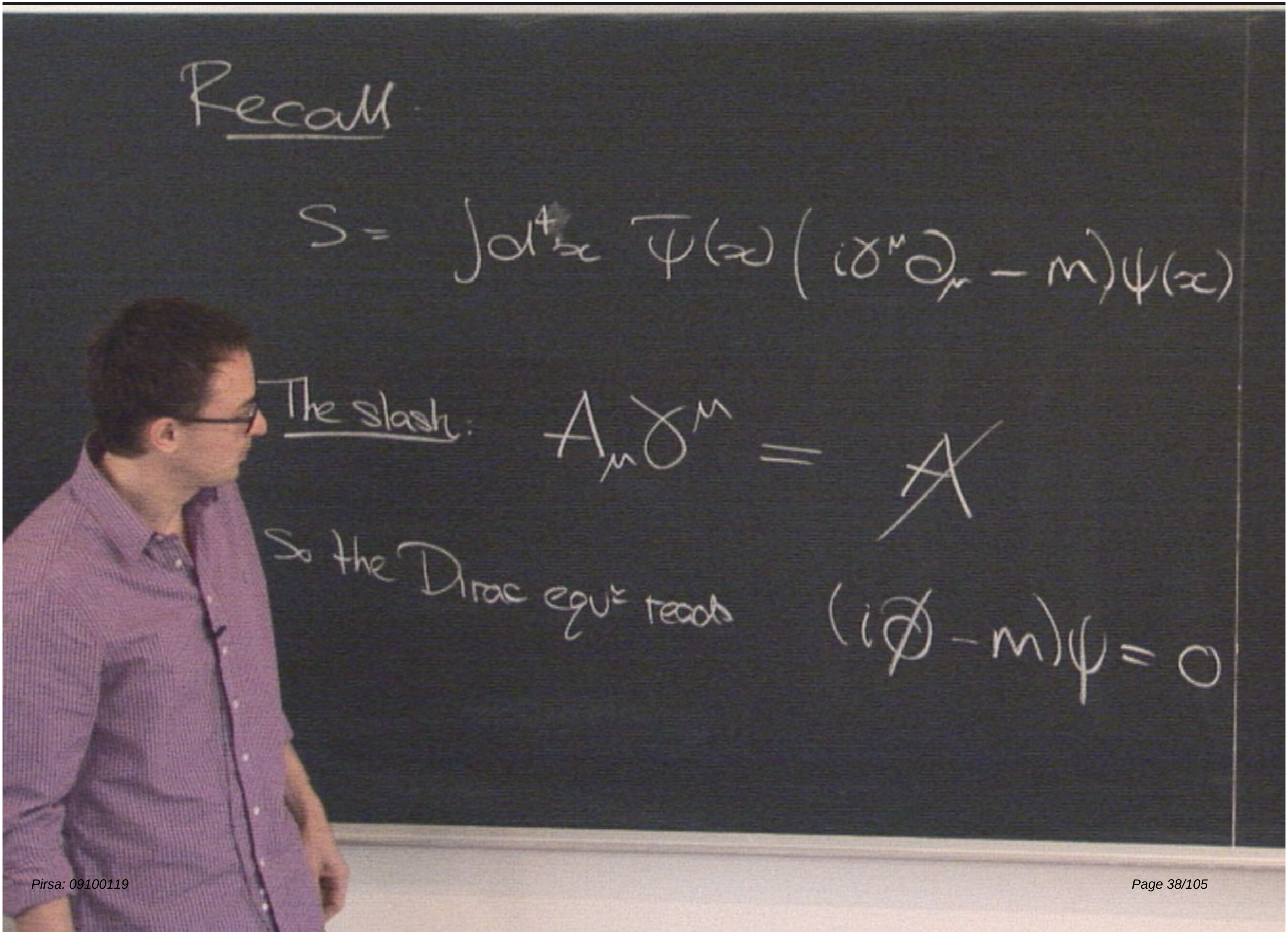
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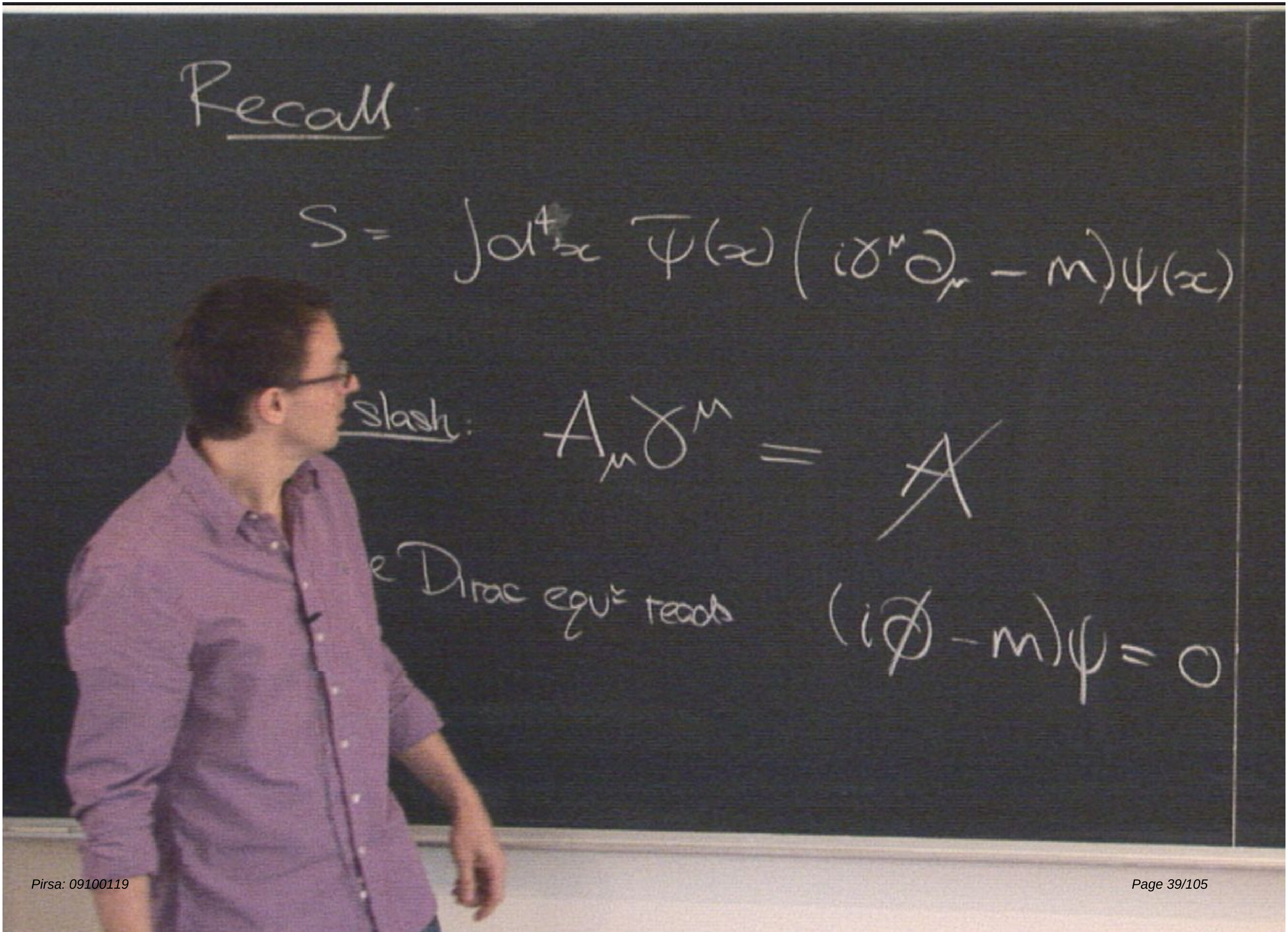
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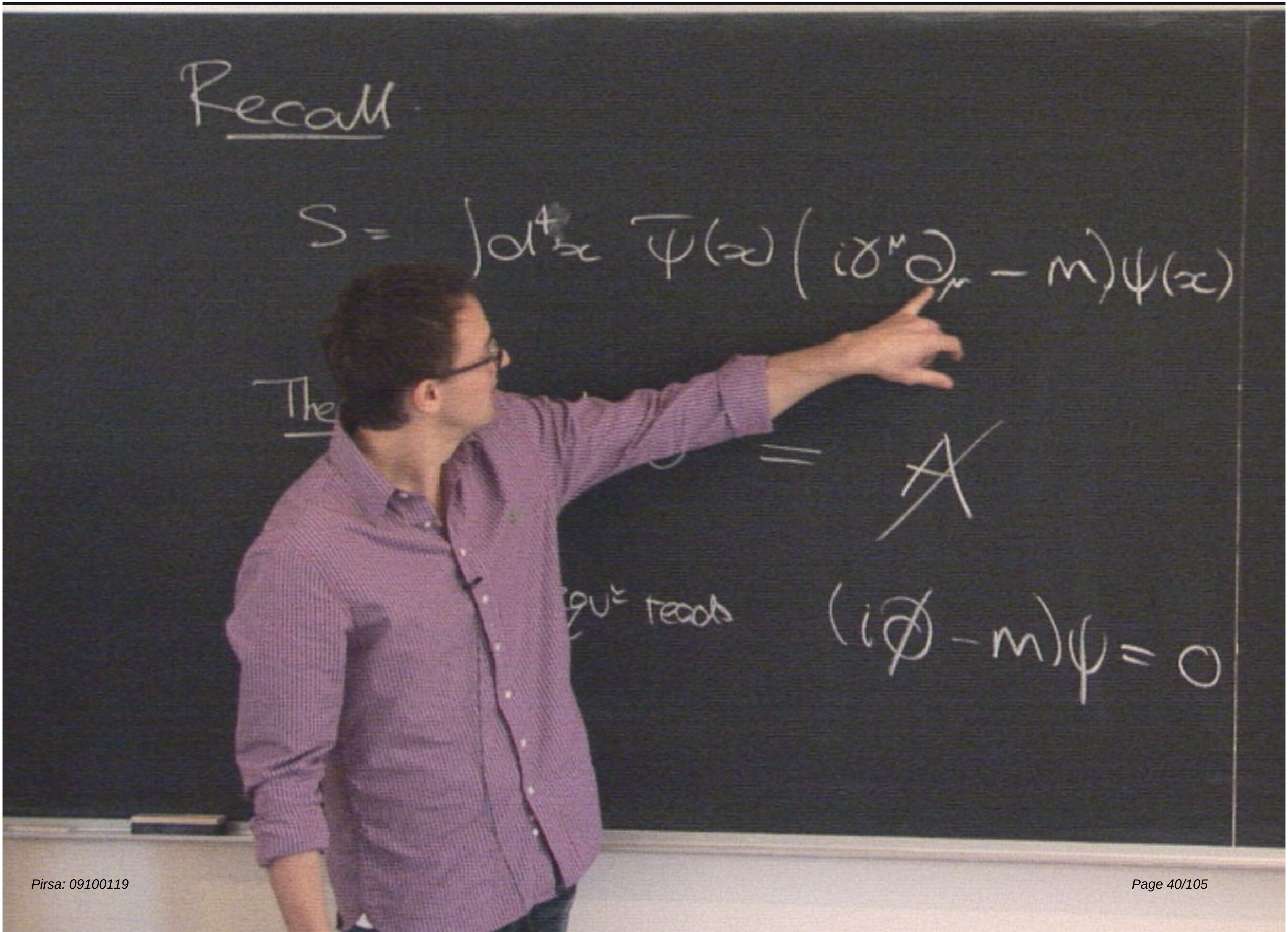
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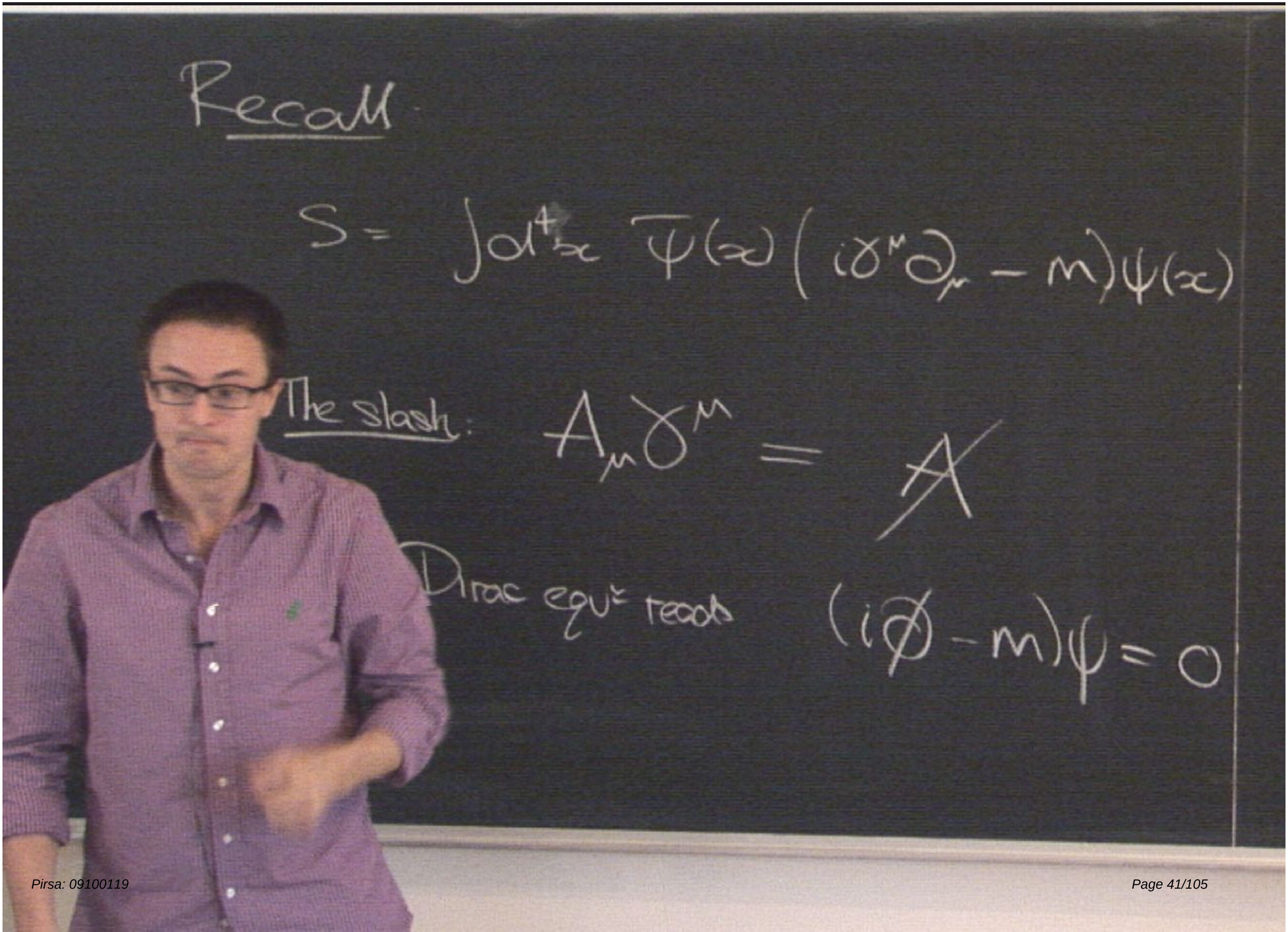
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Chiral Spinors

For our choice $\gamma^0 = \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix}$, $\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$

we found

$$S[\Lambda] = \begin{pmatrix} e^{+i/2\, \underline{\varphi}\cdot\underline{\sigma}} & 0 \\ 0 & e^{+i/2\, \underline{\varphi}\cdot\underline{\sigma}} \end{pmatrix}$$ for rotations

$$S[\Lambda] = \begin{pmatrix} e^{+\frac{1}{2}\underset{\sim}{\chi}\cdot\underset{\sim}{\sigma}} & 0 \\ 0 & e^{-\frac{1}{2}\underset{\sim}{\chi}\cdot\underset{\sim}{\sigma}} \end{pmatrix} \quad \text{for boosts}$$

This is block diagonal $\Rightarrow$ this is a reducible representation of Lorentz group.

It decomposes into irreducible representations

Write $\psi = \begin{pmatrix} u_+ \\ u_- \end{pmatrix}$ $u_\pm$ each 2-component spinor

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These are <u>Weyl spinors</u>.

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Under rotations, $u_\pm \rightarrow e^{+i/2\, \underline{\theta} \cdot \underline{\sigma}}\, u_\pm$

Under boosts

Write $\psi = \begin{pmatrix} u_+ \\ u_- \end{pmatrix}$ $\quad$ $u_\pm$ each 2-component spinor.

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Under rotations, $u_\pm \to e^{+i/2\, \underline{\theta} \cdot \underline{\sigma}}\, u_\pm$

Under boosts, $u_\pm \to e^{\pm 1/2\, \underline{\chi} \cdot \underline{\sigma}}\, u_\pm$

Look at Dirac action

Write $\psi = ($

$$\bar{\psi}(i\not{\partial}-m)\psi = (u_+^\dagger, u_-^\dagger)\begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}\left[i\begin{pmatrix} 0 & \partial_0+\sigma^i\partial_i \\ \partial_0-\sigma^i\partial_i & 0 \end{pmatrix} - m\right]\begin{pmatrix} u_+ \\ u_- \end{pmatrix}$$

$$= i u_-^\dagger \sigma^\mu \partial_\mu u_- + i u_+^\dagger \bar{\sigma}^\mu \partial_\mu u_+ - m(u_+^\dagger u_- + u_-^\dagger u_+)$$

$\sigma^\mu = (\mathbb{1}, \sigma^i)$

$\bar{\sigma}^\mu = (\mathbb{1}, -\sigma^i)$

Write $\psi = \begin{pmatrix} u_+ \\ u_- \end{pmatrix}$ $u_\pm$ each 2-component spinor

$\ldots_i\Big) - m\mathbb{1}_4\Big]\begin{pmatrix} u_+ \\ u_- \end{pmatrix}$

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$- m(u_+^\dagger u_- + u_-^\dagger u_+)$

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For massive fermions, we need both u_+ and u_-. But a massless fermion can be described by a single Weyl fermion, obeying the Weyl eqn:

$$\boxed{i\bar{\sigma}^\mu \partial_\mu u_+ = 0}$$

$i)$

γ^5

We introduce a "fifth" gamma matri…

$$\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$$

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$$\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$$

You can check that $\{\gamma^5, \gamma^\mu\} = 0$

and $(\gamma^5)^2 = +1$

Since $(\gamma^5)^2 = 1$, we can define a projection operator

$$P_\pm = \frac{1}{2}(1 \pm \gamma^5)$$

s.t. $P_\pm^2 = P_\pm$ and $P_+ P_- = 0$.

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e.g. in chiral rep^{n}, would find

$$\psi_+ = \begin{pmatrix} u_+ \\ 0 \end{pmatrix}$$

$$\psi_- = \begin{pmatrix} 0 \\ u_- \end{pmatrix}$$

Note: Can show that

$\bar{\psi}\gamma^5\psi$ is a Lorentz (pseudo)-scalar

$\bar{\psi}\gamma^5\gamma^\mu\psi$ is a Lorentz (axial/pseudo)-vector

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$$\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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Sin

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We defin

Symmetries

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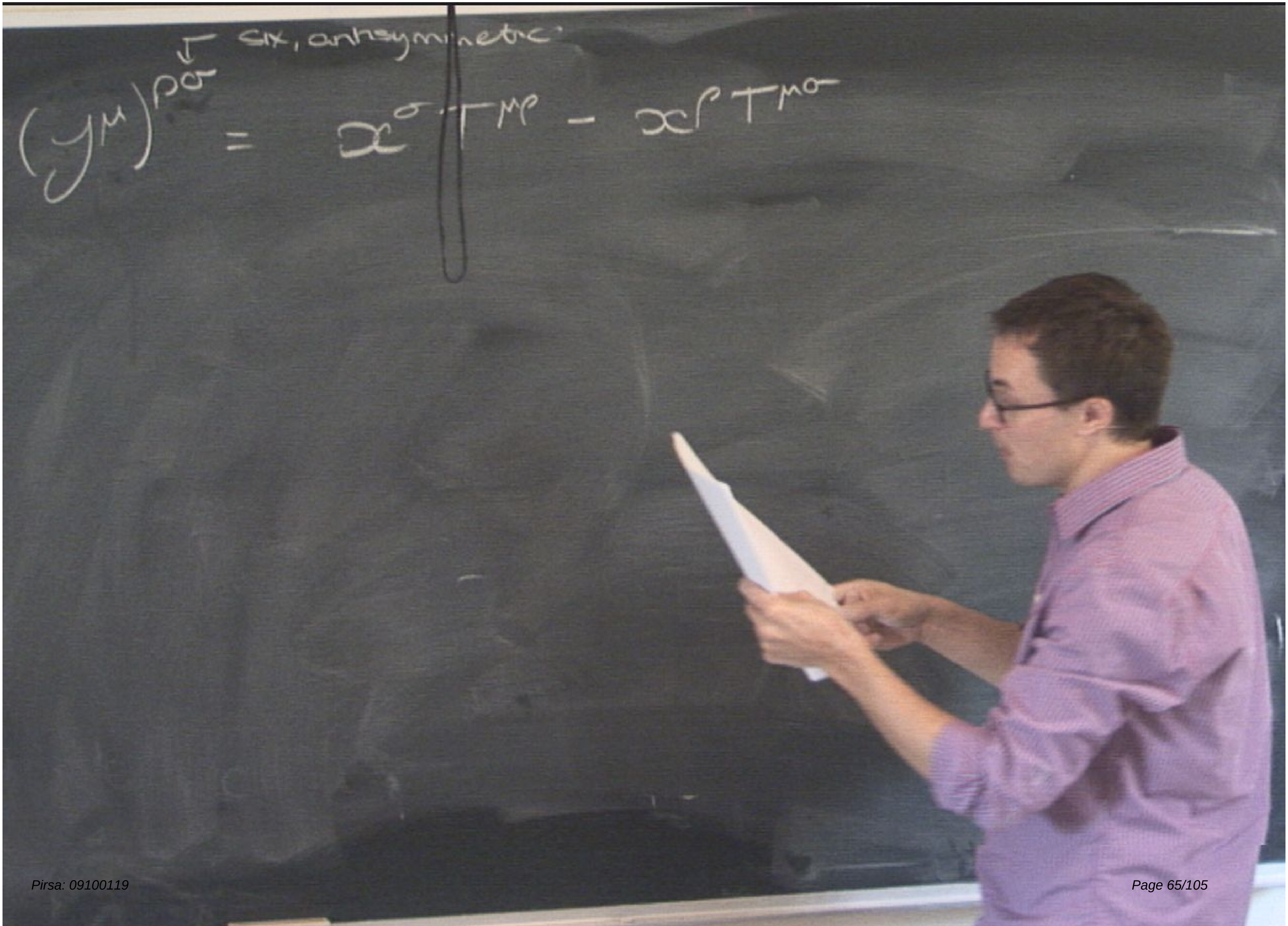
Lorentz Transformations.

$$\psi^\alpha \rightarrow S[\Lambda]^\alpha{}_\beta \psi^\beta(\Lambda^{-1}x)$$


six, antisymmetric
$(J^{\mu})^{\rho\sigma} = x^{\sigma} T^{\mu\rho} - x^{\rho} T^{\mu\sigma}$

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Internal symmetries

$$\psi \to e^{i\alpha}\psi$$

six, antisymmetric

$$(j^\mu)^{\rho\sigma} = x^\sigma T^{\mu\rho} - x^\rho T^{\mu\sigma} - i\bar{\psi}\gamma^\mu S^{\rho\sigma}\psi$$

Internal symmetries

$$\psi \to e^{i\alpha}\psi \implies j^\mu_V = \bar{\psi}\gamma^\mu\psi$$

vector

six, antisymmetric

$$(\mathcal{J}^{\mu})^{\rho\sigma} = x^{\sigma} T^{\mu\rho} - x^{\rho} T^{\mu\sigma} - i\bar{\psi}\gamma^{\mu} S^{\rho\sigma}\psi$$

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$(\mathcal{J}^\mu)^{\rho\sigma}$ six, antisymmetric

$$(\mathcal{J}^\mu)^{\rho\sigma} = x^\sigma T^{\mu\rho} - x^\rho T^{\mu\sigma} - i\bar{\psi}\gamma^\mu S^{\rho\sigma}\psi$$

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$$\psi \to e^{i\alpha}\psi \implies j^\mu_V = \bar{\psi}\gamma^\mu\psi$$

vector

$\not{M} = 0$

six, antisymmetric

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Internal symmetries

$$\psi \to e^{i\alpha}\psi \implies j^\mu_V = \bar{\psi}\gamma^\mu\psi$$

vector

If $M=0$, we have a new symmetry $\psi \to e^{i\alpha\gamma^5}\psi$

six, antisymmetric

$$(\mathcal{J}^\mu)^{\rho\sigma} = x^\sigma T^{\mu\rho} - x^\rho T^{\mu\sigma} - i\bar\psi \gamma^\mu S^{\rho\sigma} \psi$$

Internal symmetries

$$\psi \to e^{i\alpha}\psi \implies j^\mu_V = \bar\psi \gamma^\mu \psi$$

vector

If $m=0$, we have a new symmetry $\psi \to e^{i\alpha\gamma^5}\psi$

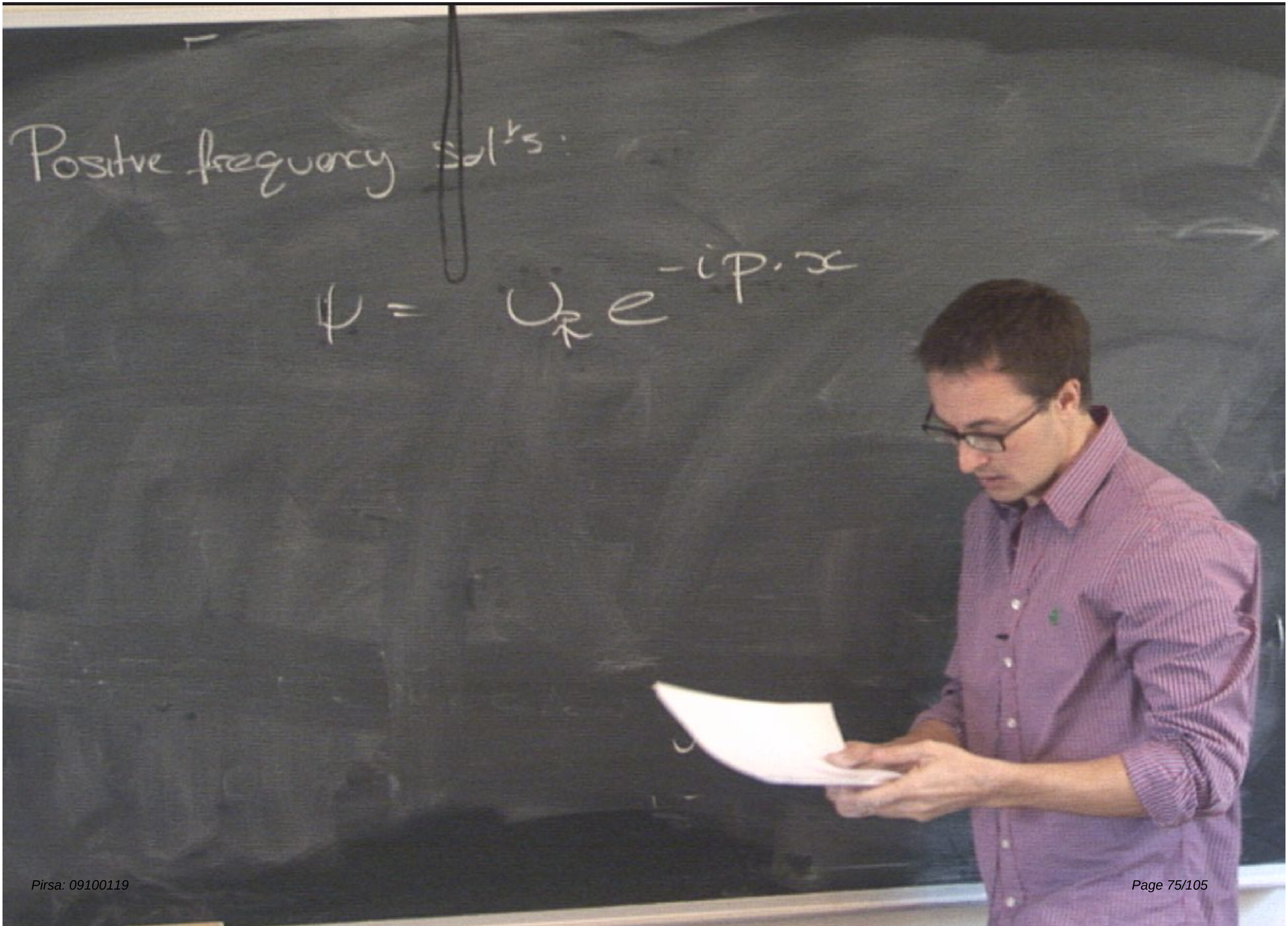
$$\Longrightarrow \quad (j^{\mu}_{A}) = \bar{\psi}\gamma^{5}\gamma^{\mu}\psi$$

Axial

Plane Wave Sol^{n}s

We want to solve

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$


Positive frequency sol'ns:
$\psi = u_k e^{-ip \cdot x}$

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$$\psi = u_{\vec{p}}\, e^{-ip\cdot x}$$

Claim: The sol^n is $$u_{\vec{p}} = \begin{pmatrix} \sqrt{p\cdot\sigma}\ \xi \\ \sqrt{p\cdot\bar{\sigma}}\ \xi \end{pmatrix}$$

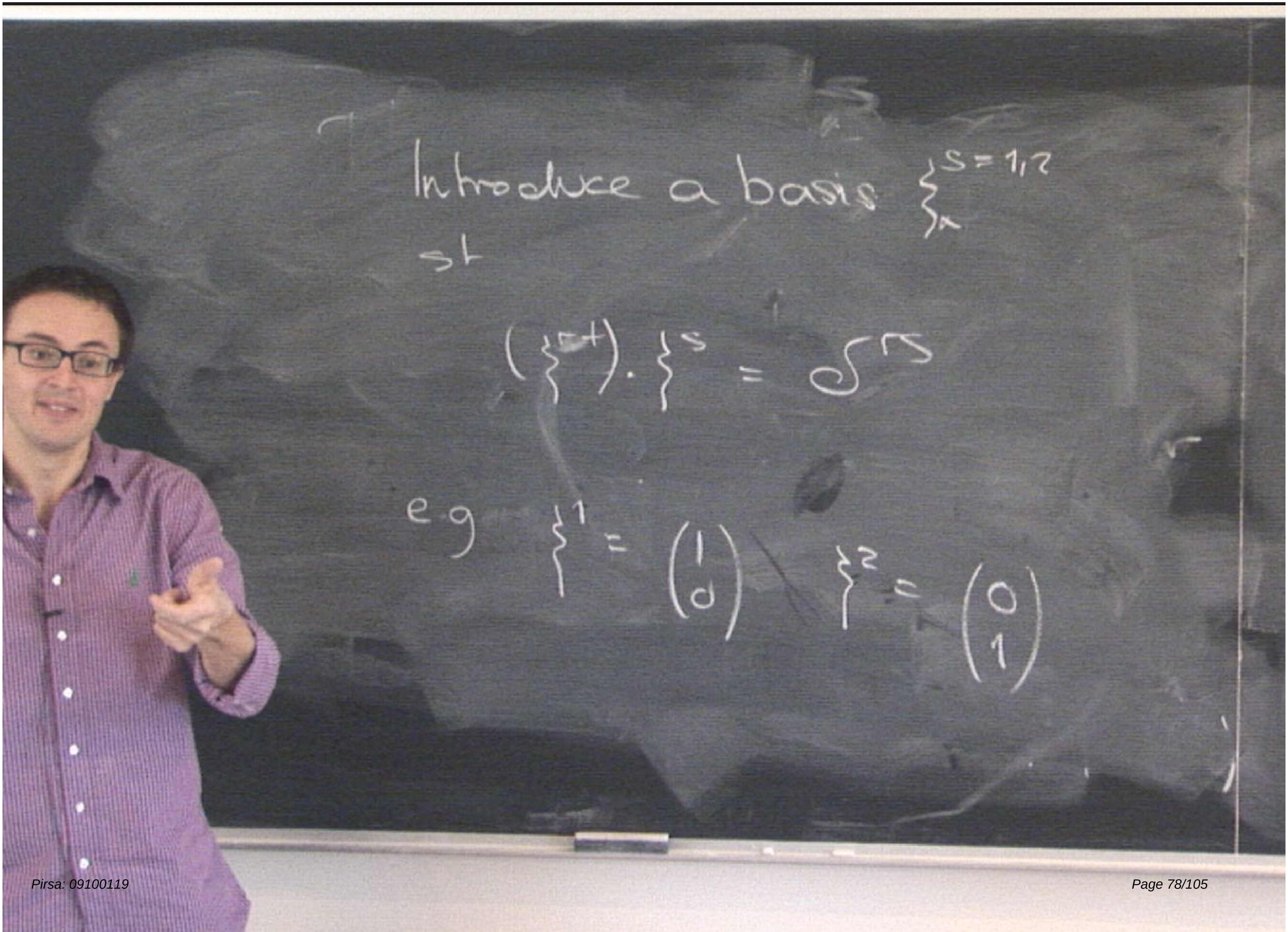
Positive frequency sol^n's:

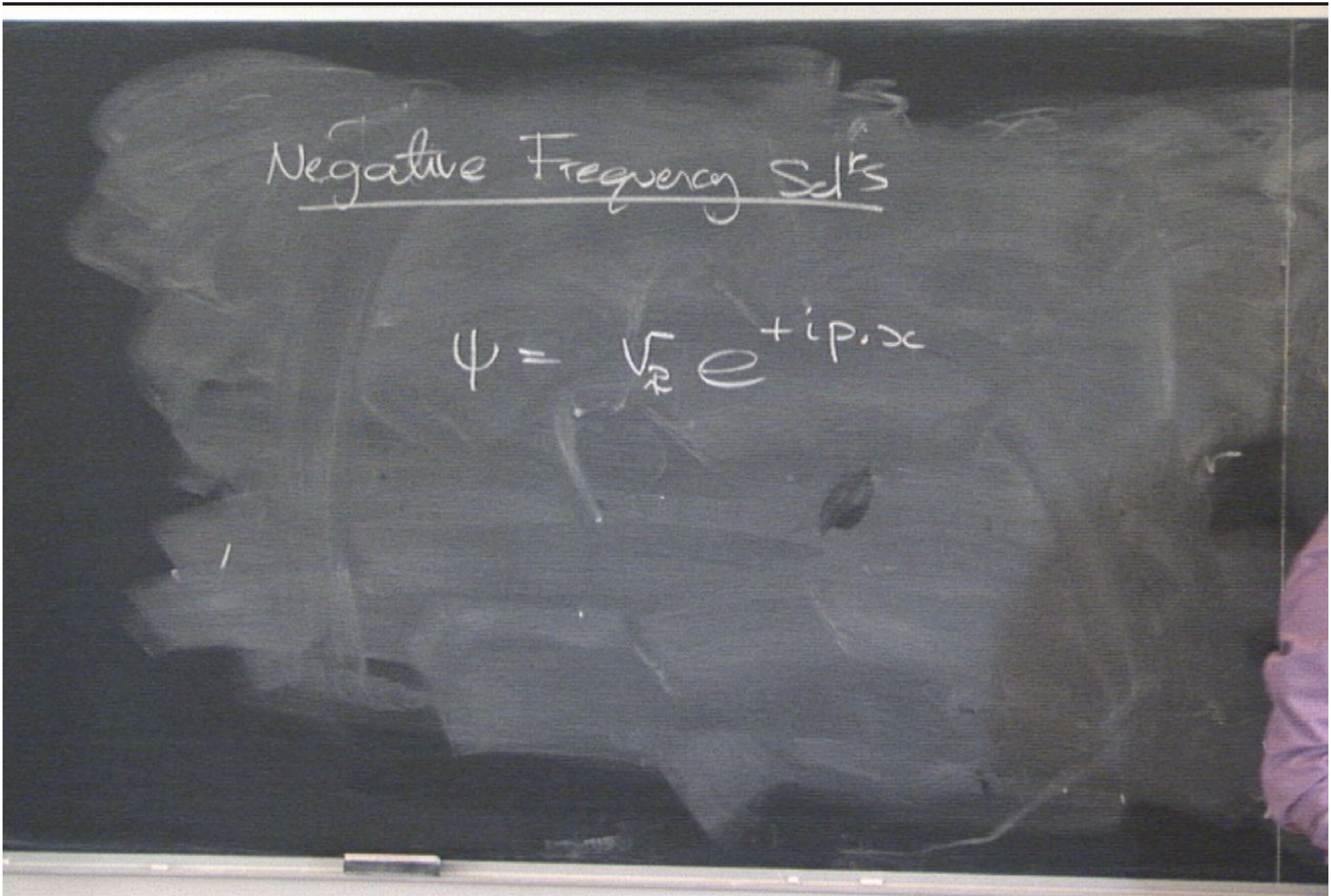
$$\psi = u_R \, e^{-ip\cdot x}$$

2-component spinor.

Claim: The sol^n is

$$u_R = \begin{pmatrix} \sqrt{p\cdot\sigma} \; \xi \\ \sqrt{p\cdot\bar{\sigma}} \; \xi \end{pmatrix}$$


Introduce a basis $\xi^{s=1,2}_{A}$
st
$(\xi^{r\dagger}) \cdot \xi^{s} = \delta^{rs}$
e.g. $\xi^{1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\xi^{2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$


Negative Frequency Sol^{ns}
$\psi = V_k e^{+ip.x}$

Positive frequency sol^{n}s: $p_0 = +\sqrt{\vec{k}^2 + m^2}$

$$\psi = u_k \, e^{-ip\cdot x}$$

in the chiral rep^n

2-component spinor.

Claim: The sol^n is $u_k = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi \\ \sqrt{p\cdot\bar{\sigma}}\,\xi \end{pmatrix}$

Proof: Exercise

Negative Frequency Sol^{ns}

$$\psi = v_{\vec{p}}\, e^{+ip\cdot x}$$

with $v_{\vec{p}} = \begin{pmatrix} \sqrt{p\cdot\sigma}\;\eta \\ -\sqrt{p\cdot\bar{\sigma}}\;\eta \end{pmatrix}$

any 2 component Spinor

Negative Frequency Sol$^{\text{ns}}$

$$p_0 = +\sqrt{\vec{k}^2 + m^2}$$

$$\psi = v_{\vec{k}}\, e^{+ip\cdot x}$$

with $$v_{\vec{k}} = \begin{pmatrix} \sqrt{p\cdot\sigma}\;\eta \\ \sqrt{p\cdot\bar{\sigma}}\;\eta \end{pmatrix}$$

any 2 component spinor

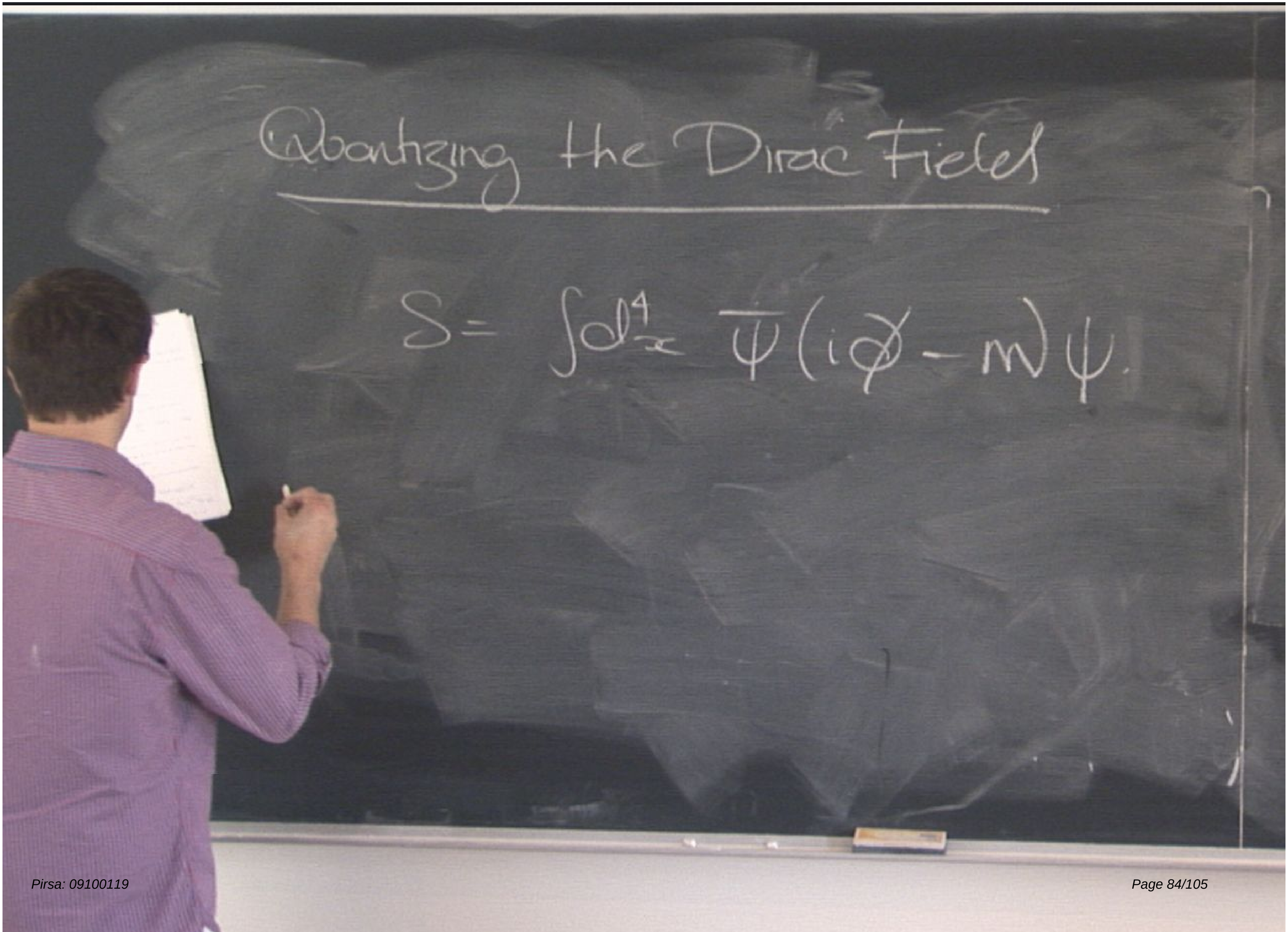
Negative Frequency Sol^rs

$$\psi = v_k\, e^{+ip\cdot x}$$

$p_0 = +\sqrt{k^2 + m^2}$

with $v_k = \begin{pmatrix} \sqrt{p\cdot\sigma}\;\eta \\ -\sqrt{p\cdot\bar{\sigma}}\;\eta \end{pmatrix}$

any 2 component Spinor.


Quantizing the Dirac Field
$S = \int d^4x \, \bar{\psi} (i \not{\partial} - m) \psi .$

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$$S = \int d^4x \; \bar{\psi}\,(i\not{\partial} - m)\,\psi.$$

The momentum is $\pi \equiv \dfrac{\partial \mathcal{L}}{\partial \dot{\psi}} = i\bar{\psi}\gamma^0 = i\psi^\dagger$

We'll proceed as we did for the scalar field (this will be the wrong thing to do).

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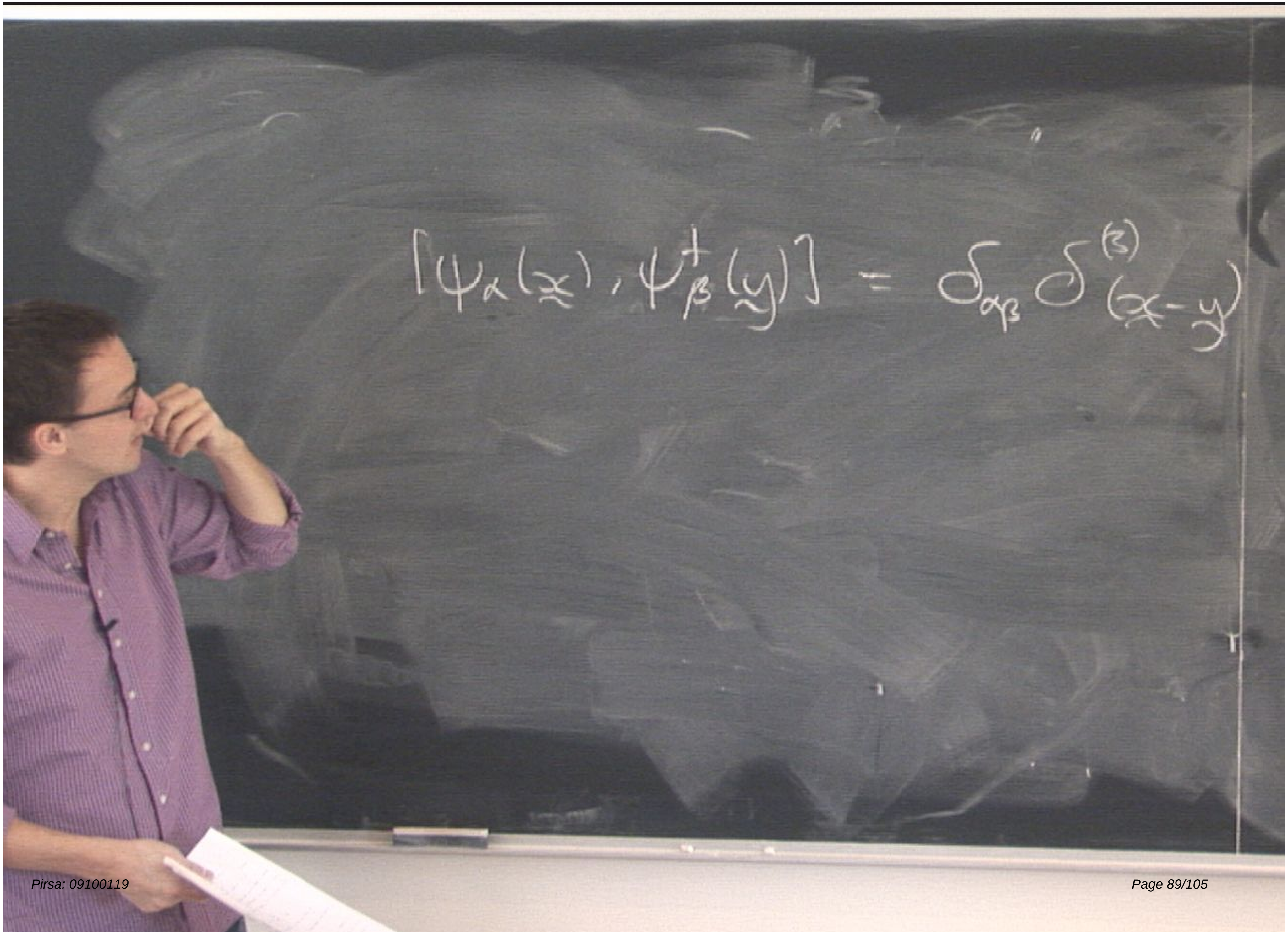
Schrödinger picture $\Rightarrow \underset{\sim}{x}$, not x

$$[\psi_\alpha(\underset{\sim}{x}), \psi_\beta(\underset{\sim}{y})] = [\psi^\dagger_\alpha(\underset{\sim}{x}), \psi^\dagger_\beta(\underset{\sim}{y})] = 0$$


$[\psi_\alpha(\underset{\sim}{x}), \psi^\dagger_\beta(\underset{\sim}{y})] = \delta_{\alpha\beta}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})$

$$[\psi_\alpha(\underset{\sim}{x}), \psi^\dagger_\beta(\underset{\sim}{y})] = \delta_{\alpha\beta}\,\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y})$$

We need a mode expansion

$$\psi(x) = \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[b_p^s\, u_p^s\, e^{+ip\cdot x} + c_p^{s*}\, v_p^s\, e^{-ip\cdot} \right.$$

$$\psi(\underset{\sim}{x}) = \sum_{s=1}^{2} \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[b_p^s u_p^s e^{+ip\cdot x} + c_p^{s+} v_p^s e^{-ip\cdot x} \right.$$

$$\psi^+(\underset{\sim}{x}) = \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[b_p^{s+} u_p^{s+} e^{-ip\cdot x} + c_p^s v_p^{s+} e^{+ip\cdot x} \right]$$

$$\psi(\underset{\sim}{x}) = \sum_{s=1}^{2}\int \frac{d^3p}{(2\pi)^3}\,\frac{1}{\sqrt{2E_p}}\left[\,b_p^s\, u_p^s\, e^{+ip\cdot x} + c_p^{s+}\, v_p^{s}\, e^{-ip\cdot x}\right.$$

operators

$$\psi^+(\underset{\sim}{x}) = \sum_{s=1}^{2}\int \frac{d^3p}{(2\pi)^3}\,\frac{1}{\sqrt{2E_p}}\left[\,b_p^{s+}\, u_p^{s+}\, e^{-ip\cdot x} + c_p^{s}\, v_p^{s+}\, e^{+ip\cdot \underset{\sim}{x}}\right]$$

Spinors

$$\psi(\underset{\sim}{x}) = \sum_{s=1}^{2}\int \frac{d^3\underset{\sim}{p}}{(2\pi)^3}\frac{1}{\sqrt{2E_{\underset{\sim}{p}}}}\left[\, b_{\underset{\sim}{p}}^{s}\, u_{\underset{\sim}{p}}^{s}\, e^{+i\underset{\sim}{p}\cdot\underset{\sim}{x}} + c_{\underset{\sim}{p}}^{s\dagger}\, v_{\underset{\sim}{p}}^{s}\, e^{-i\underset{\sim}{p}\cdots}\right.$$

operators

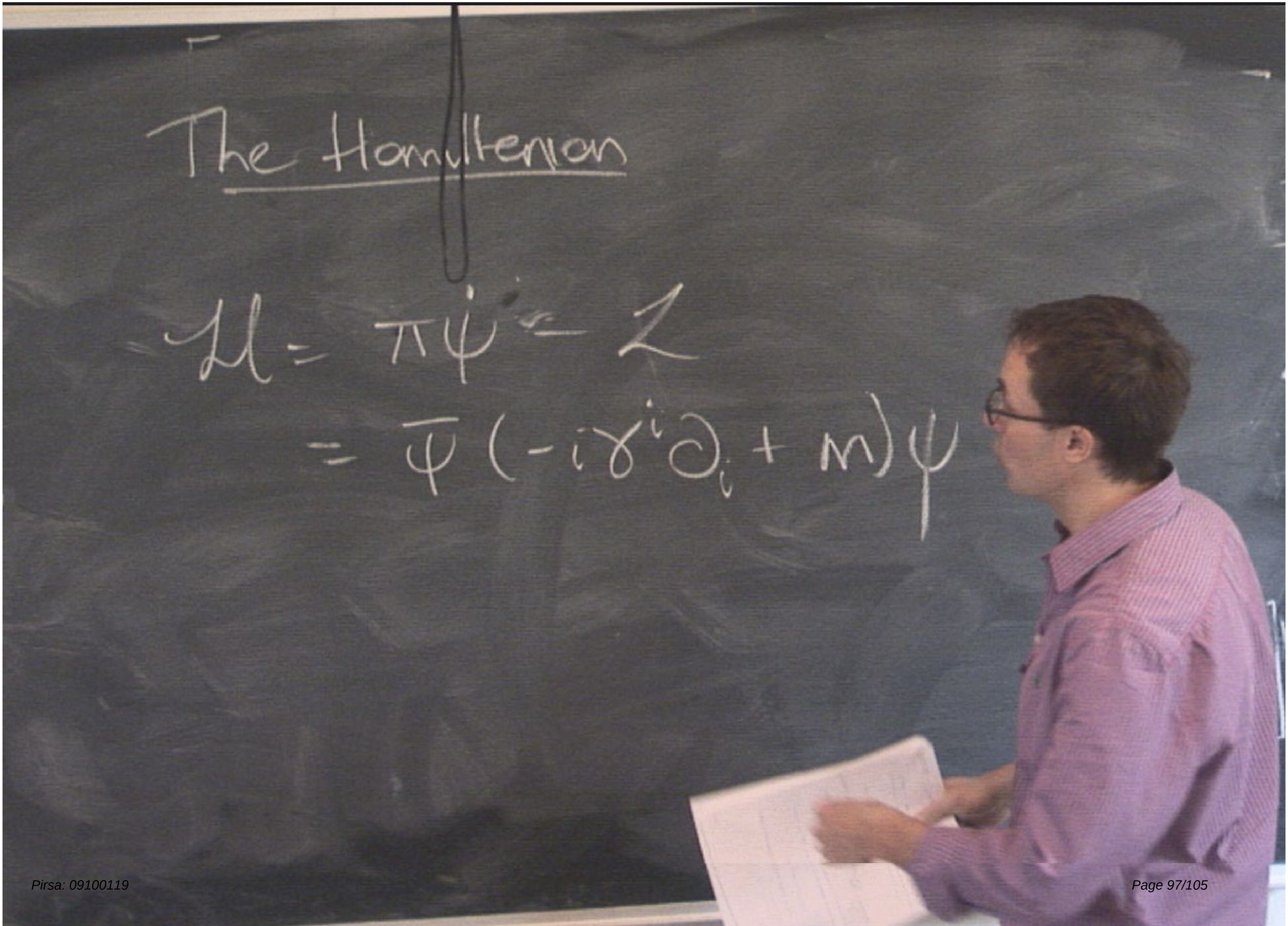
$$\psi^\dagger(\underset{\sim}{x}) = \sum_{s=1}^{2}\int \frac{d^3\underset{\sim}{p}}{(2\pi)^3}\frac{1}{\sqrt{2E_{\underset{\sim}{p}}}}\left[\, b_{\underset{\sim}{p}}^{s\dagger}\, u_{\underset{\sim}{p}}^{s\dagger}\, e^{-i\underset{\sim}{p}\cdot\underset{\sim}{x}} + c_{\underset{\sim}{p}}^{s}\, v_{\underset{\sim}{p}}^{s\dagger}\, e^{+i\underset{\sim}{p}\cdot\underset{\sim}{x}}\right]$$

e.g. $u_p^s = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi^s \\ \sqrt{p\cdot\bar{\sigma}}\,\xi^s \end{pmatrix}$

<u>Claim</u>

$$[b^r_{\mathbf{k}}, b^{s\dagger}_{\mathbf{q}}] = (2\pi)^3 \delta^{rs} \delta^{(3)}(\mathbf{k}-\mathbf{q})$$

$$[c^r_{\mathbf{k}}, c^{s\dagger}_{\mathbf{q}}] = -(2\pi)^3 \delta^{rs} \delta^{(3)}(\mathbf{k}-\mathbf{q}$$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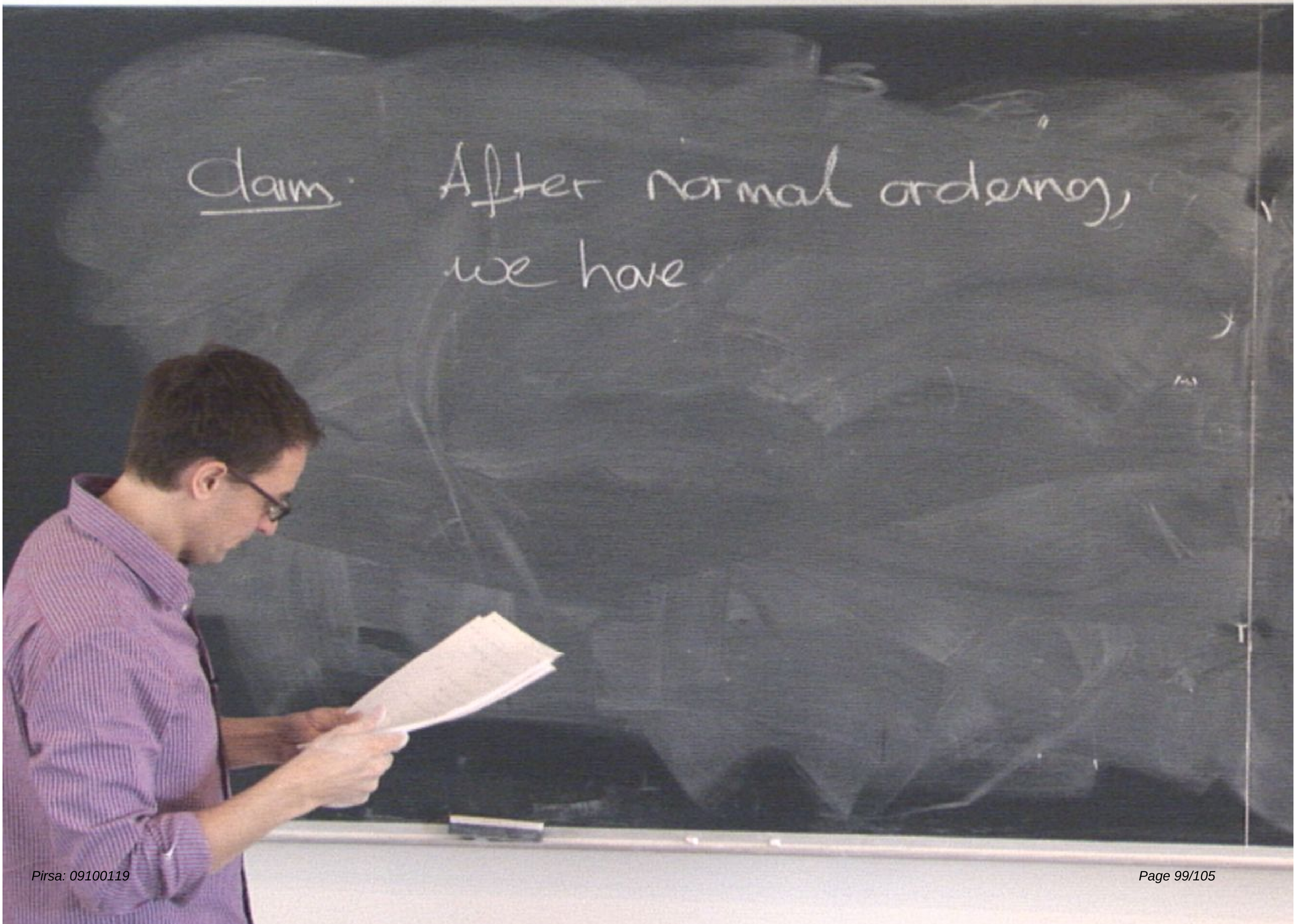
and $$H = \int d^3x \, \mathcal{H}$$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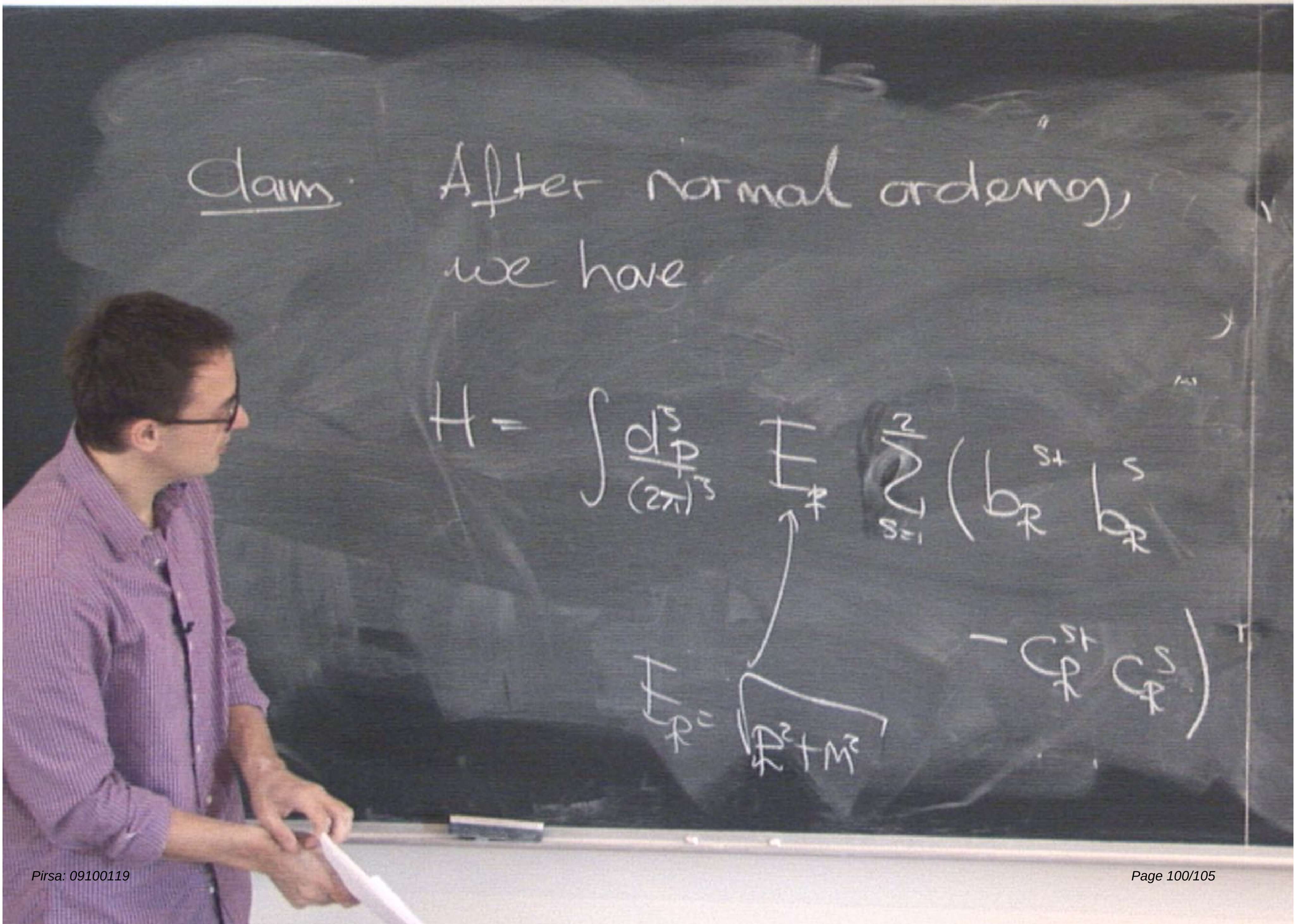
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$H = \int \frac{d^3p}{(2\pi)^3} E_p \sum_{s=1}^{2} \left(b_p^{s\dagger} b_p^s - c_p^{s\dagger} c_p^s \right)$
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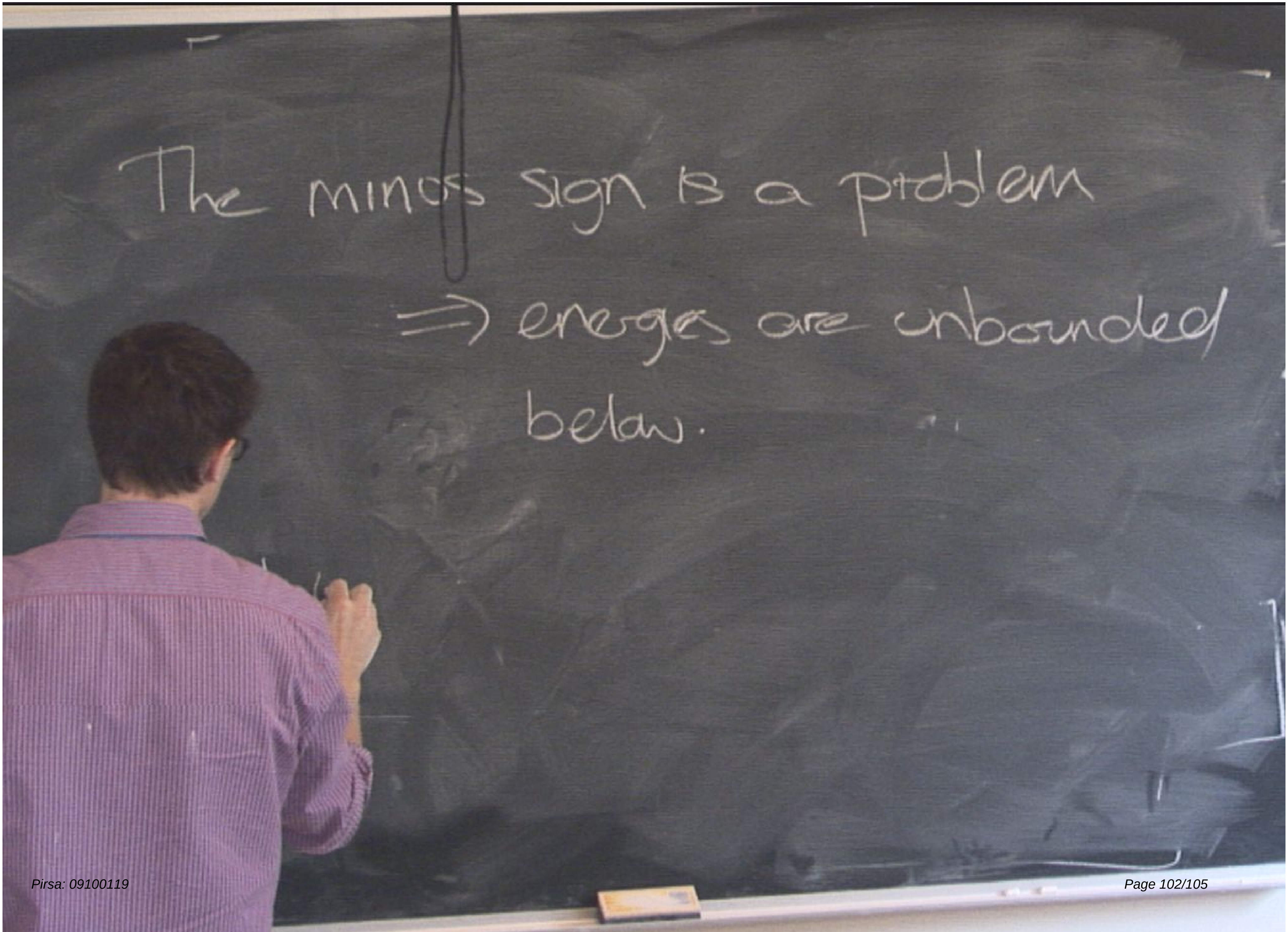
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In fact we can't make sense of spinor fields quantized in this way — we must treat them as <u>fermions</u>