Title: Quantum Field Theory (PHYS 601) - Lecture 9

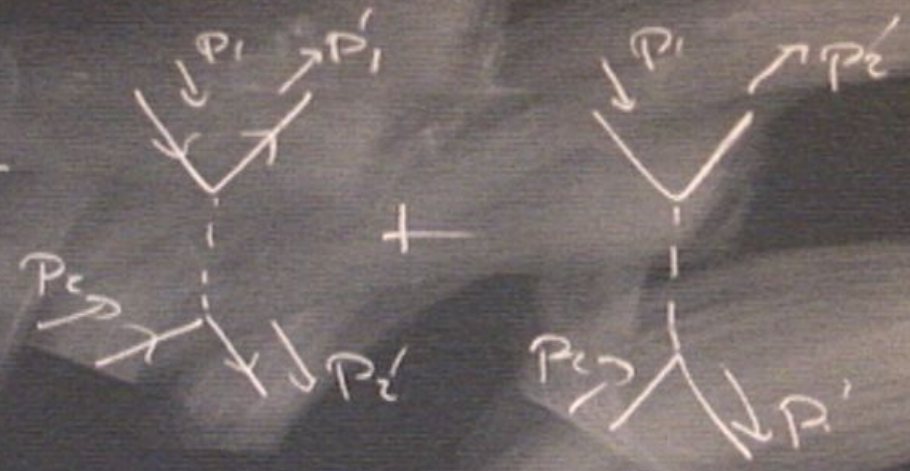
Date: Oct 08, 2009 09:00 AM

URL: http://pirsa.org/09100114

Abstract:

Recall: $\psi + \psi \to \psi + \psi$

$\langle f | S - 1 | i \rangle =$



Recall: $\psi + \psi \to \psi + \psi$

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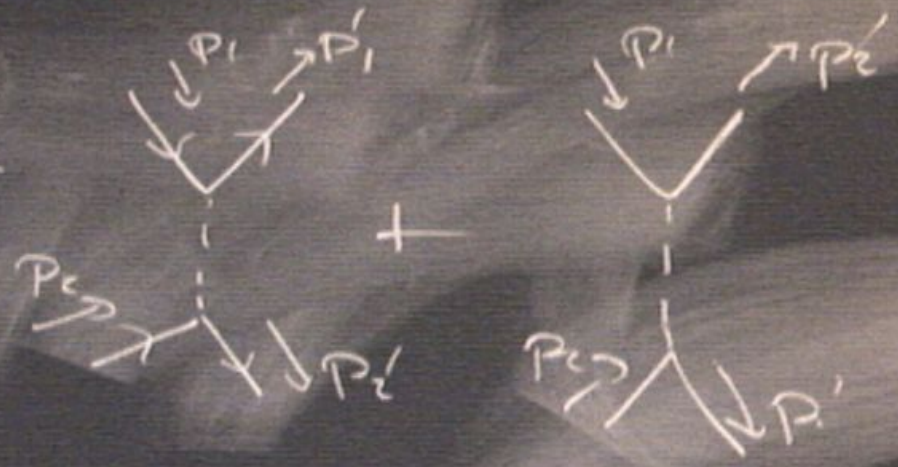

$$= (-ig)^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_1' - p_2')$$

$$\times \left(\frac{i}{(p_1' - p_1)^2 - m^2 + i\epsilon} + \frac{i}{(p_2' - p_1)^2 - m^2 + i\epsilon} \right)$$

We'll define the <u>amplitude</u>,

$$\langle f | S - 1 | i \rangle = i \mathcal{A}_{fi} \cdot (2\pi)^4 \cdot \delta^{(4)}(p_F - p_I)$$

Σ_i final momenta

Σ_i initial momenta

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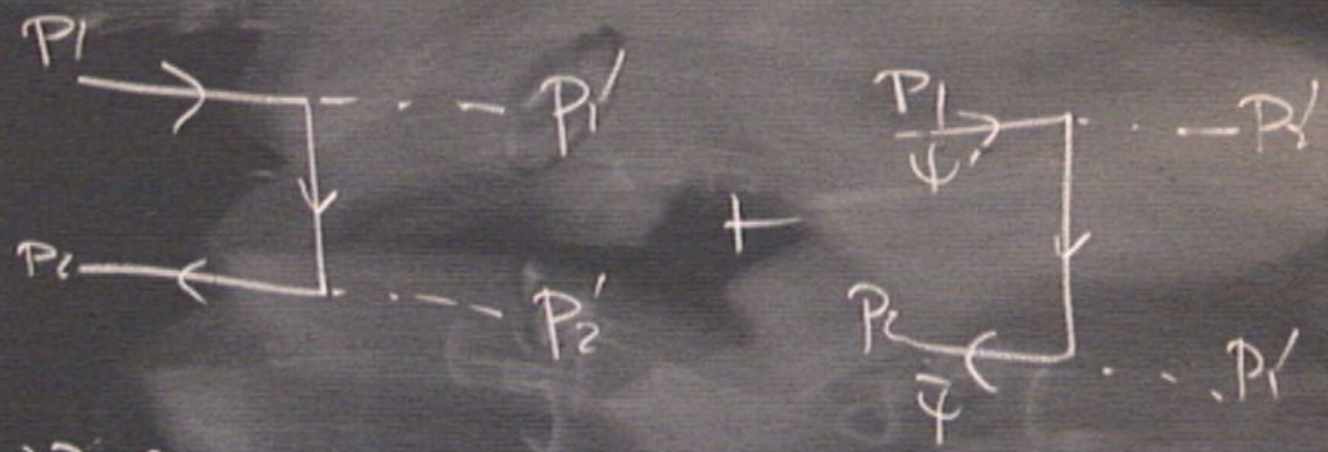
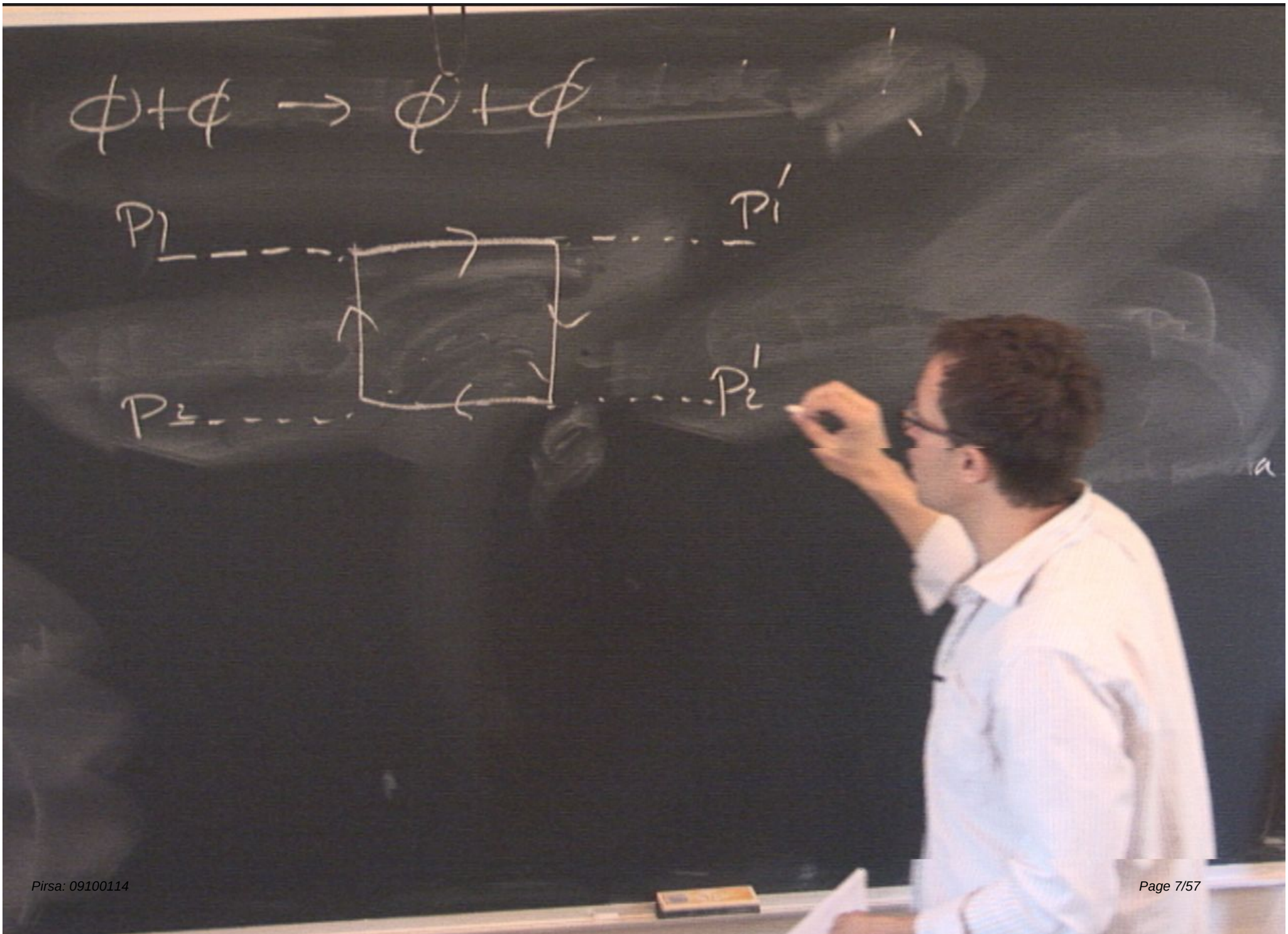
Other processes

$$\psi + \bar{\psi} \to \phi + \phi$$



$$\mathcal{A} = (-ig)^2 \left[\frac{i}{(p_1 - p_1')^2 - M^2} + \frac{i}{(p_1 - p_2')^2 - M^2} \right]$$


$\phi + \phi \to \phi + \phi$
P_1
P_1'
P_2
P_2'

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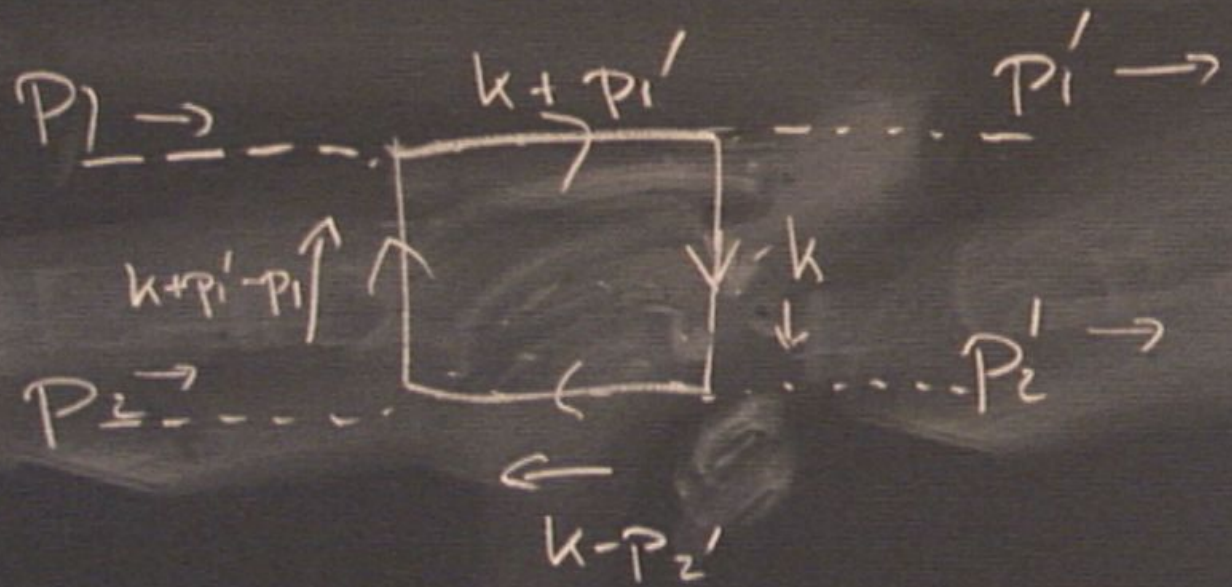

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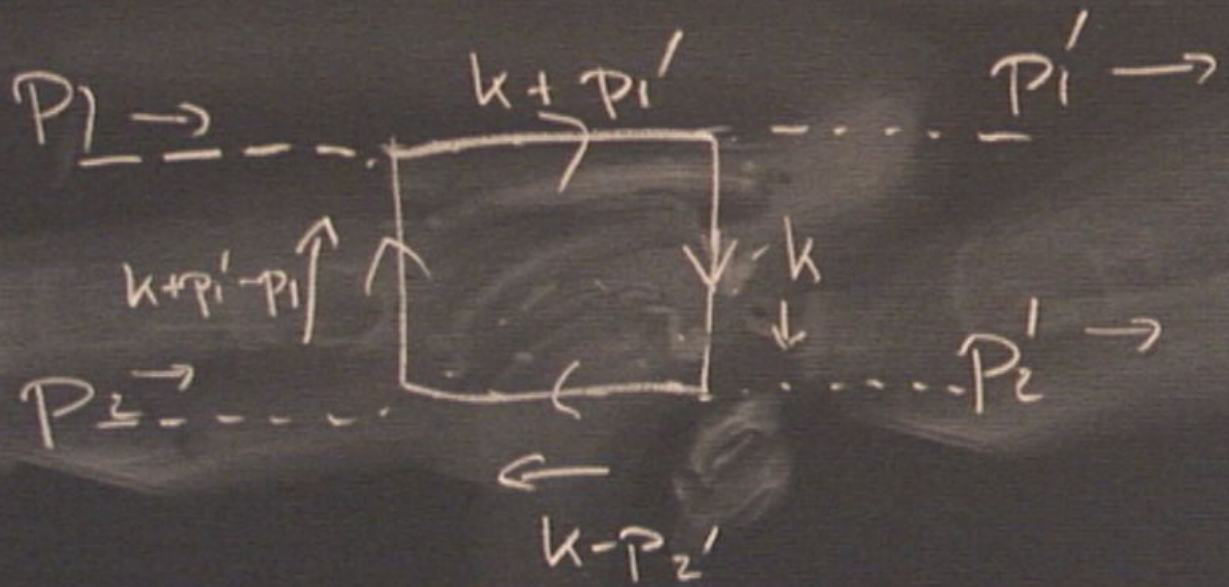

$$= (-ig)^4 \int \frac{d^4k}{(2\pi)^4} \frac{i^4}{(k^2 - M^2 + i\epsilon)((k+p_1')^2 - M^2 + i\epsilon)((k+p_1' - p_1)^2 - M^2 + i\epsilon) \times ((k - p_2') - M^2 + i\epsilon)}$$

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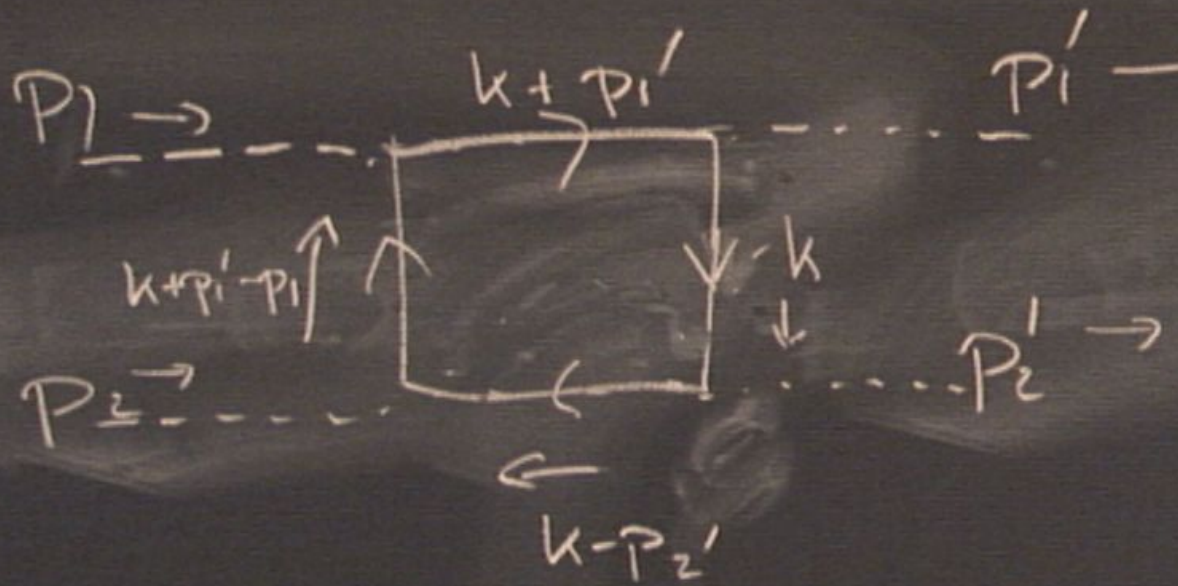

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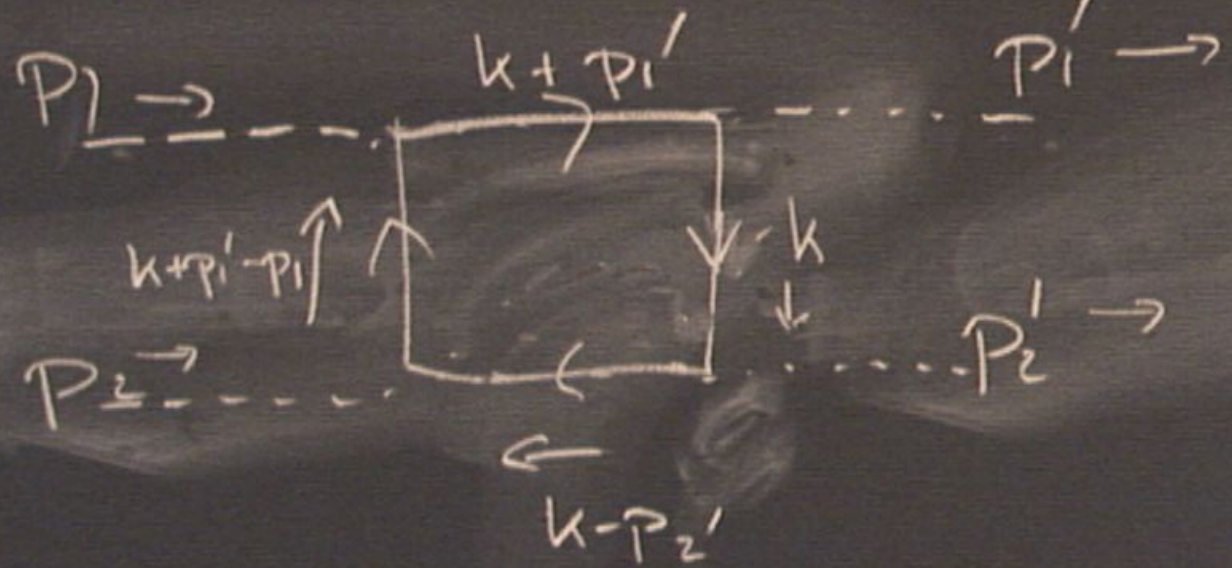

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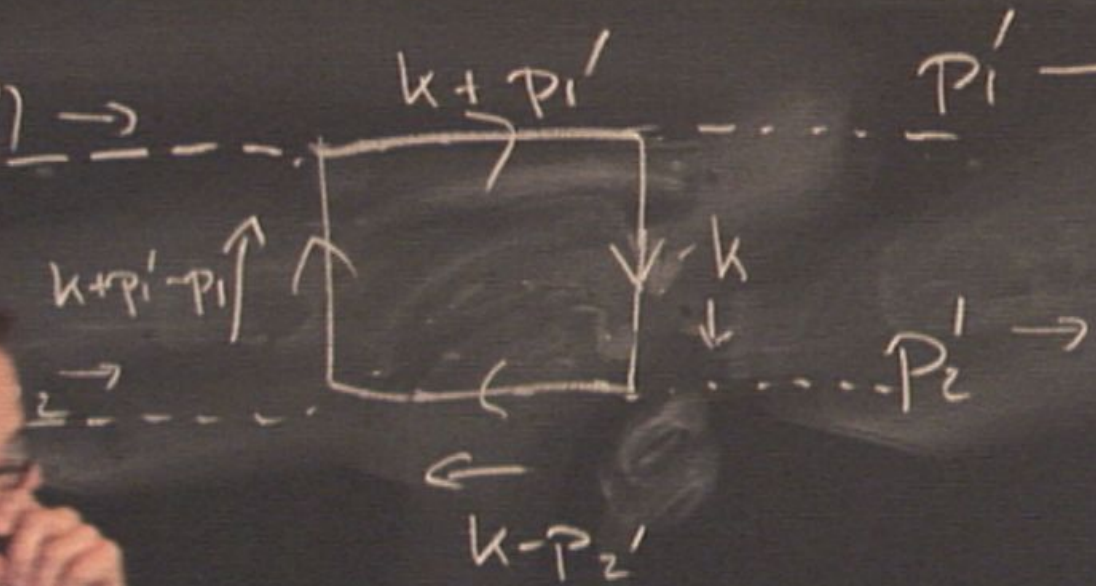

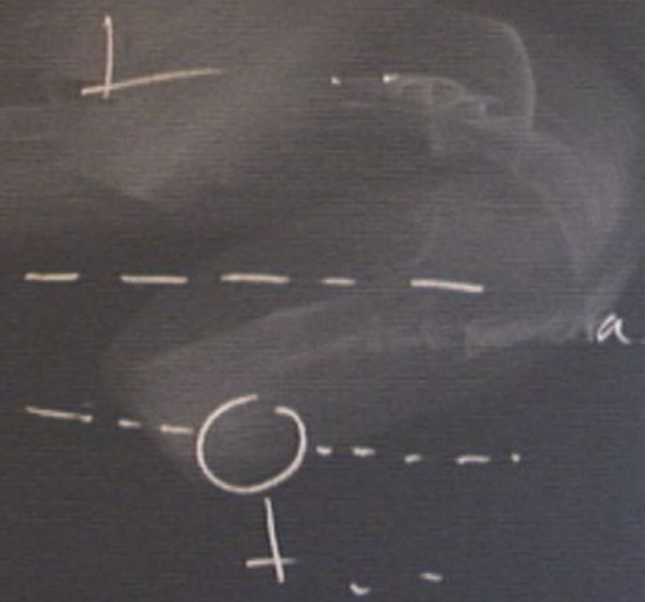

$$\int \frac{d^4k}{(2\pi)^4} \frac{i^4}{(k^2 - M^2 + i\epsilon)((k+p')^2 - M^2 + i\epsilon)((k + p_1' - p_1)^2 - M^2 + i\epsilon) \times ((k - p_2') - M^2 + i\epsilon)}$$

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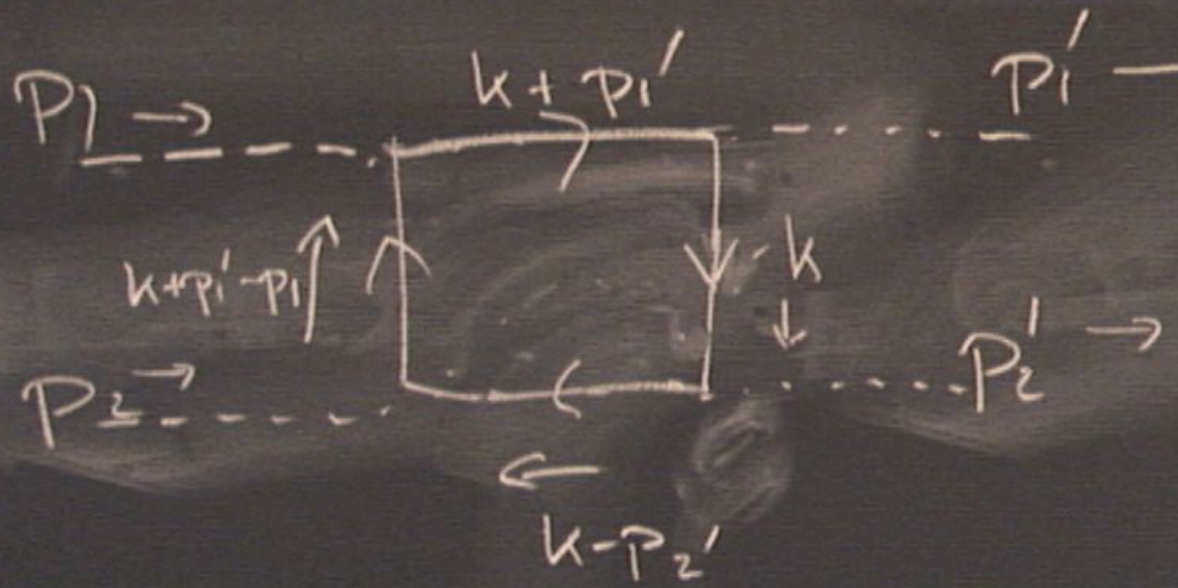

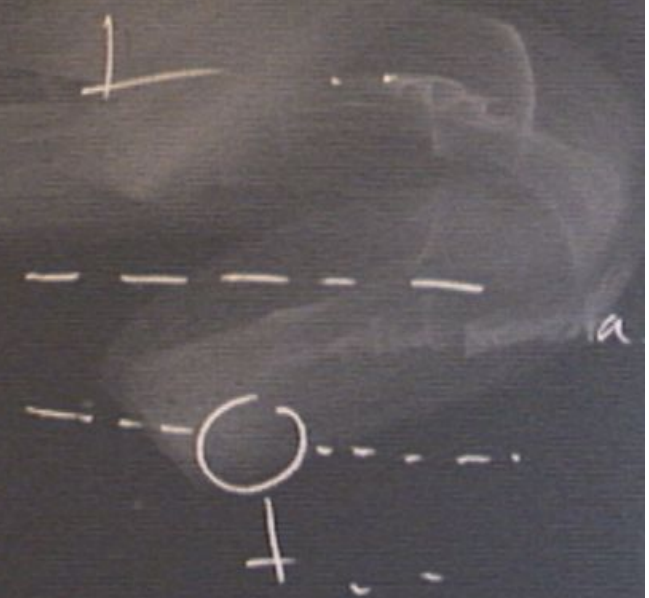

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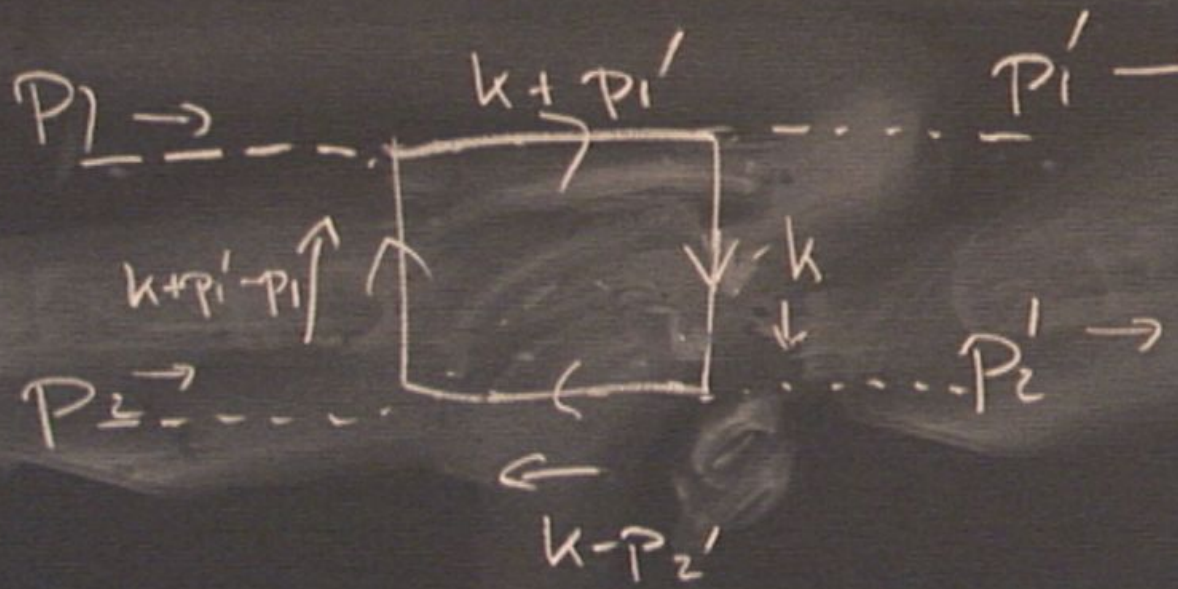

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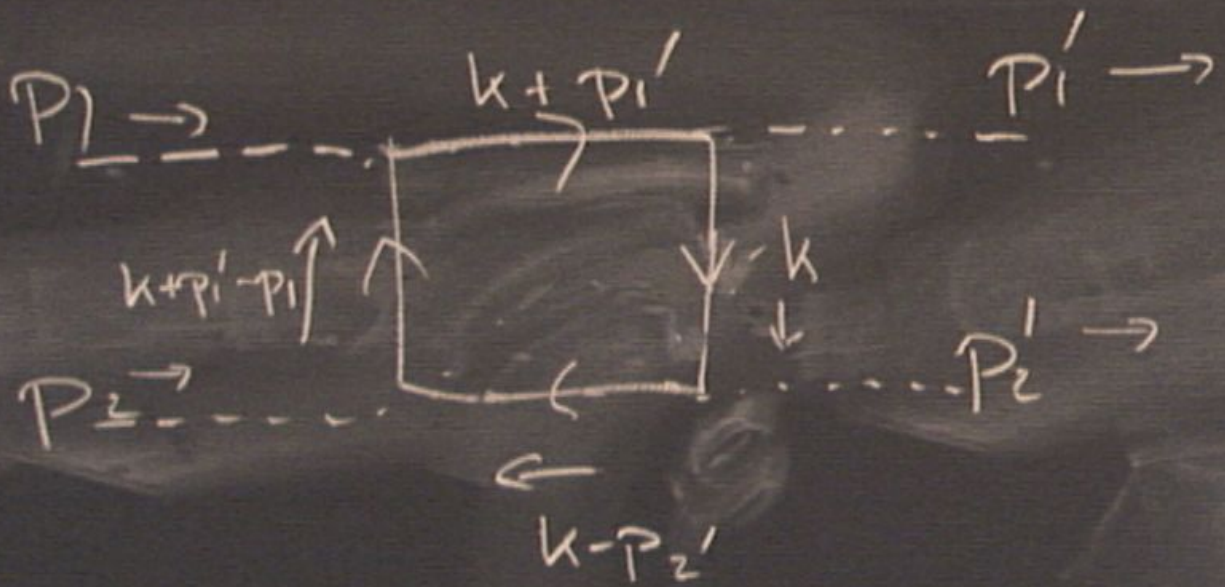

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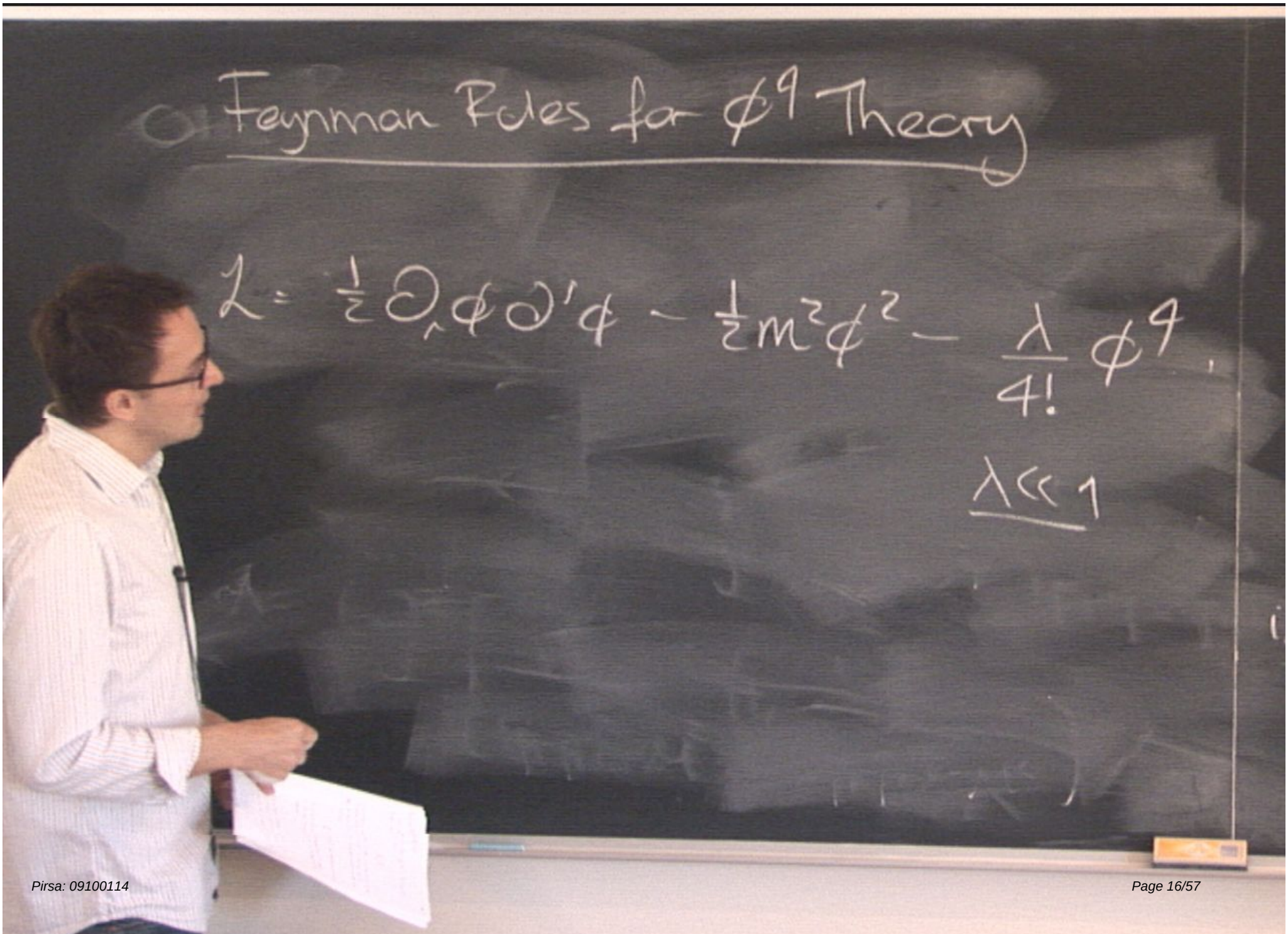
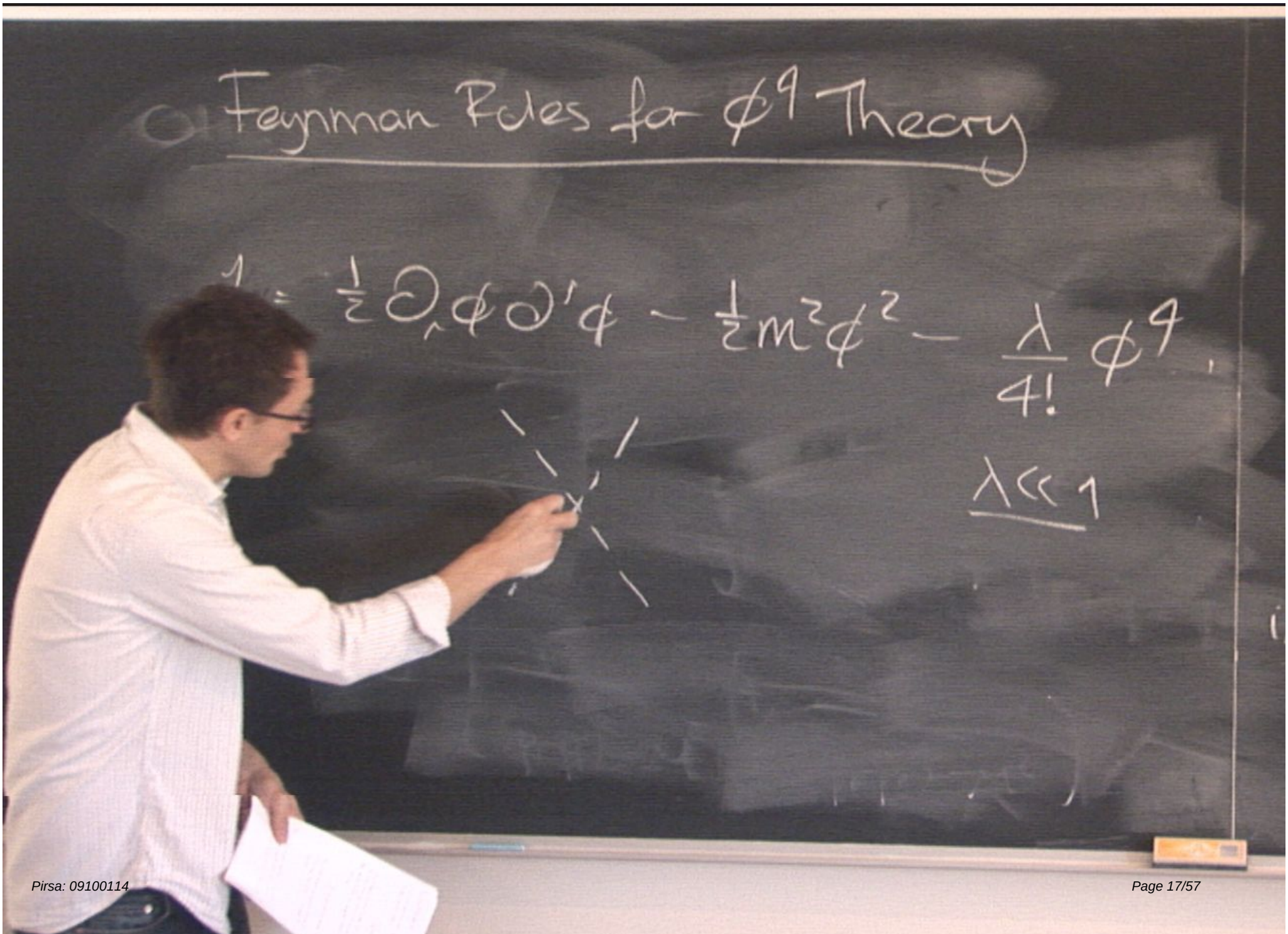

Feynman Rules for ϕ^4 Theory
$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$
$\lambda \ll 1$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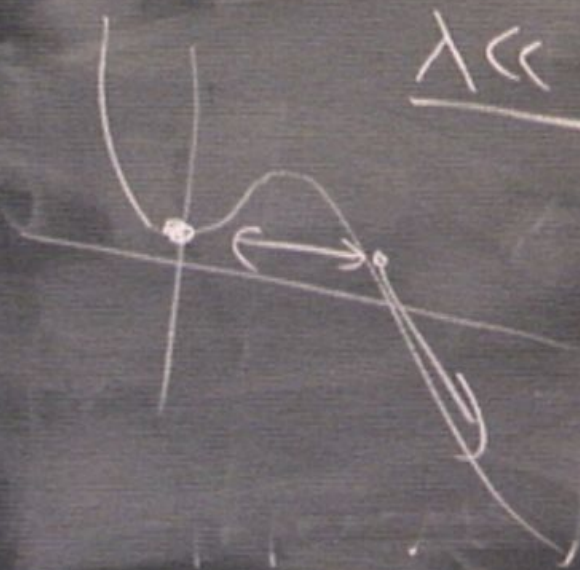

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$(-i\lambda)$

$\underline{\lambda \ll 1}$

Look $\phi + \phi \to \phi + \phi$ scattering

$$-\frac{i\lambda}{4!} \langle p_1', p_2' | :\phi(x)\phi(x)\phi(x)\phi(x): | p_1, p_2 \rangle$$

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There are 4! ways of picking creation/annihilation operators that cancels the 1/4! out front.

Things I'm not going to tell you.

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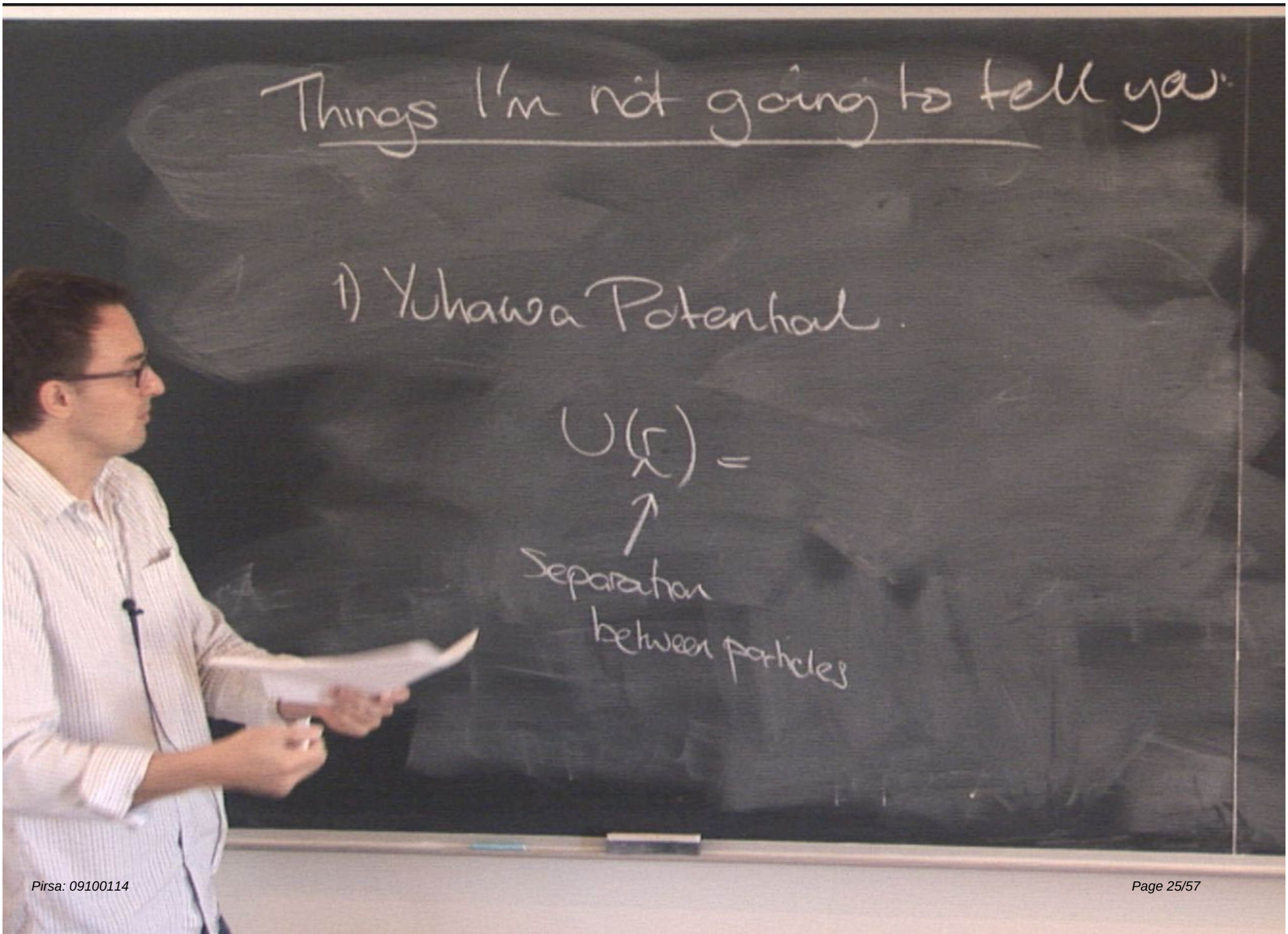
1) Yukawa Potential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2) Cross Sections + Decay Rates

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3) Correlation Functions (Green's Functions)

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$$G^{(n)}(x_1 \ldots x_n) = \langle \Omega | T \phi_H(x_1) \ldots \phi_H(x_n) | \Omega \rangle$$

Heisenberg picture

$$H|\Omega\rangle = 0$$

Vacuum of true theory..

§4 The Dirac Equation

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So far we've only considered scalar fields

$$\phi(x) \longrightarrow \phi'(x) = \phi(\Lambda^{-1}x)$$

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e.g $A^\mu(x) \longrightarrow A^{\mu\prime}(x) = \Lambda^\mu{}_\nu A^\nu(\Lambda^{-1}x)$

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$$\phi^a(x) \longrightarrow \underbrace{D^a{}_b(\Lambda)}\,\phi^b(\Lambda^{-1}x)$$

where $D^a{}_b(\Lambda)$ is a <u>representation</u> of the Lorentz group:

$$D(\Lambda_1)D(\Lambda_2) = D(\Lambda_1\Lambda_2)$$

$$D(\Lambda^{-1}) = D(\Lambda)^{-1}$$

and $D(\mathbb{1}) = \mathbb{1}$.

Typically

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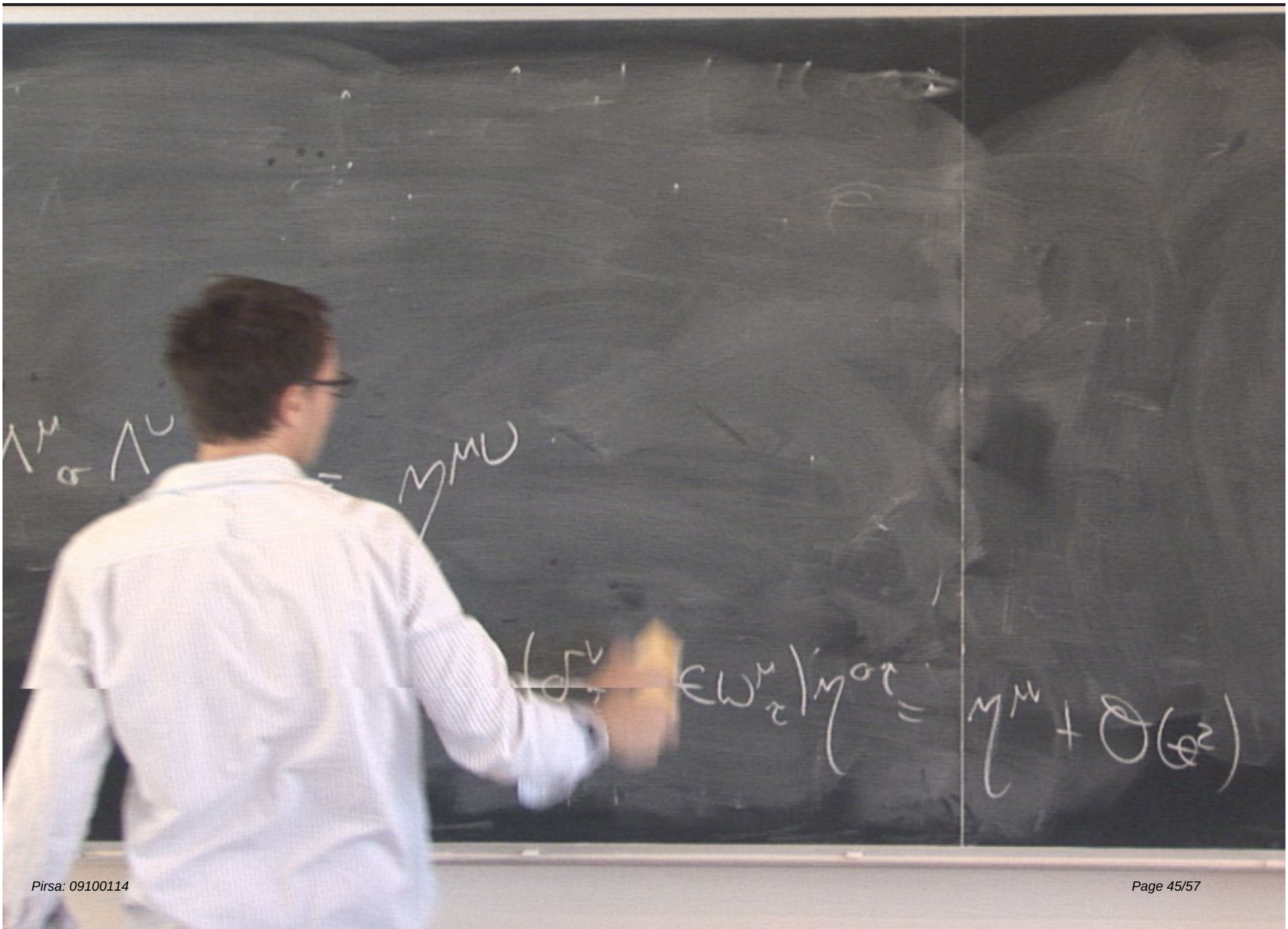
To find representations we look at the <u>Lie algebra</u> of infinitesimal Lorentz transformations.

$$\Lambda^{\mu}{}_{\nu} = \delta^{\mu}_{\nu} + \epsilon\,\omega^{\mu}{}_{\nu} + \mathcal{O}(\epsilon^2).$$

$$\Lambda^\mu{}_\nu = \delta^\mu_\nu + \epsilon\,\omega^\mu{}_\nu + \mathcal{O}(\epsilon^2).$$

$$\Lambda^\mu{}_\sigma \Lambda^\nu{}_\rho \eta^{\sigma\rho} = \eta^{\mu\nu}$$

$$\Rightarrow (\delta^\mu{}_\sigma + \epsilon\,\omega^\mu{}_\sigma)(\delta^\nu_\tau + \epsilon\,\omega^\nu{}_\tau)\eta^{\sigma\tau} = \eta^{\mu\nu} +$$


$\Lambda^\mu{}_\sigma \Lambda^\nu$
$= \eta^{\mu\nu}$
$\epsilon\omega^\mu{}_\tau)\eta^{\sigma\tau} = \eta^{\mu\nu} + \mathcal{O}(\epsilon^2)$

$$\Lambda^{\mu}{}_{\sigma}\Lambda^{\nu}{}_{\rho}\eta^{\sigma\rho} = \eta^{\mu\nu}$$

$$\Rightarrow (\delta^{\mu}{}_{\sigma} + \epsilon\omega^{\mu}{}_{\sigma})(\delta^{\nu}{}_{\tau} + \epsilon\omega^{\mu}{}_{\tau})\eta^{\sigma\tau} = \eta^{\mu\nu} + \mathcal{O}(\epsilon^2)$$

$$\Rightarrow \omega^{\mu\nu} + \omega^{\nu\mu} = 0$$ (from terms linear in ϵ).

i.e. antisymmetric matrices

$$\left(\frac{4 \times 3}{2} = 6 \quad \text{i.e. } 3 \text{ rotations}, \ 3 \text{ boosts} \right)$$

$+ O(\epsilon^2)$

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We'll introduce a basis of six 4x4 anti-symmetric matrices

$$(\mu\rho\sigma)^{\mu}{}_{\nu} = \eta^{\rho\mu}\delta^{\sigma}_{\nu} - \eta^{\sigma\mu}\delta^{\rho}_{\nu}$$

4x4 row-column labels

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labels which of six matrices.

$$(M^{\rho\sigma})^{\mu}{}_{\nu} = \eta^{\rho\mu}\delta^{\sigma}_{\nu} - \eta^{\sigma\mu}\delta^{\rho}_{\nu}$$

4x4 row-column labels

e.g

$$(M^{01})^{\mu}{}_{\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$(M^{12})^{\mu}{}_{\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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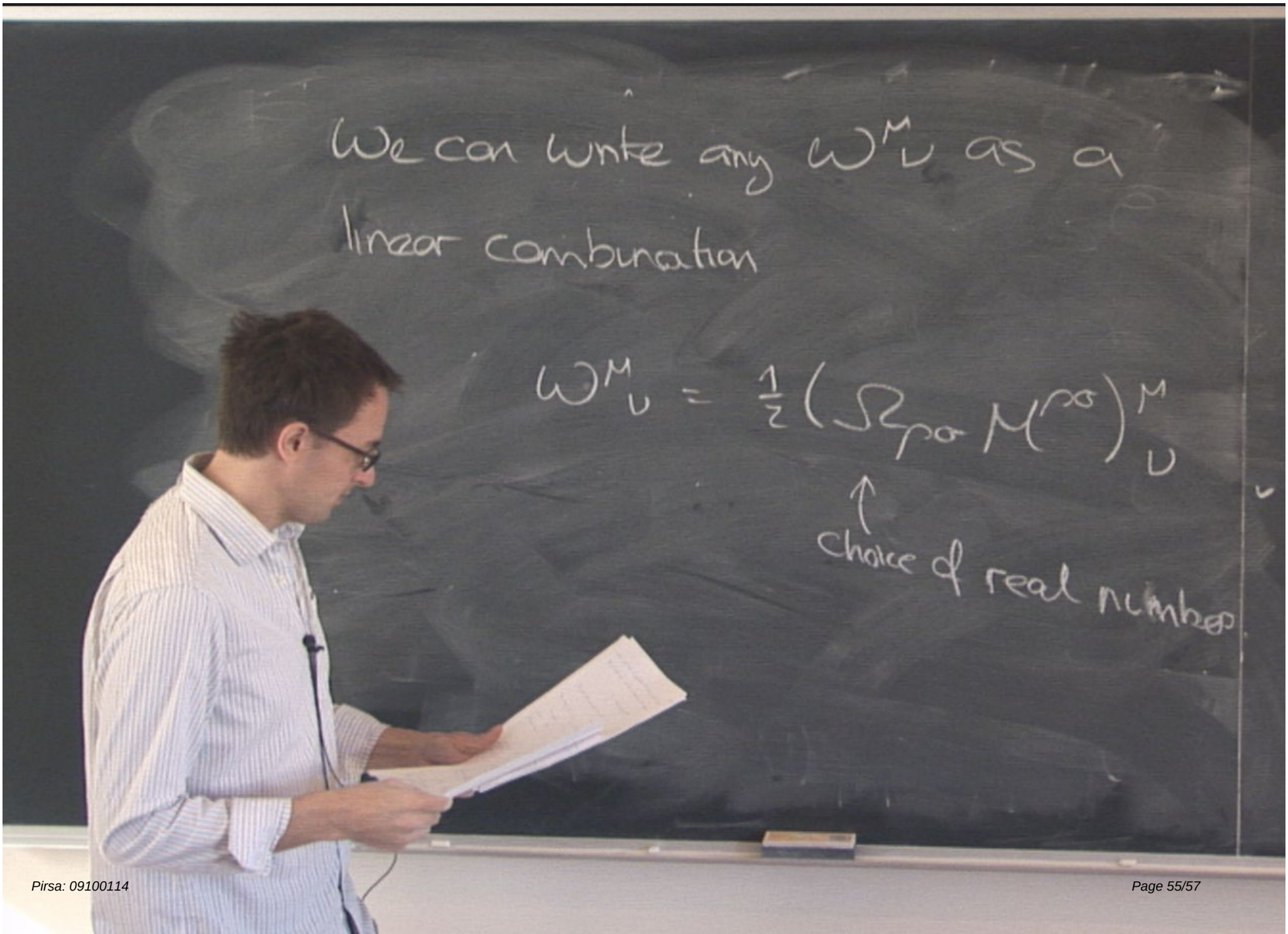
is infinitesimal boost in x-dir^{n}.

$$(M^{12})^{\mu}{}_{\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

is an infinitesimal rotation in z-direction


We can write any $\omega^\mu{}_\nu$ as a
linear combination
$\omega^\mu{}_\nu = \frac{1}{2}(\Omega_{\rho\sigma} M^{(\rho\sigma)})^\mu{}_\nu$
choice of real numbers

- Any finite Lorentz transformation can be written as

$$\Lambda = \exp\left(\frac{1}{2}\Omega_{\rho\sigma} M^{\rho\sigma}\right).$$

The generators obey the Lie algebra of Lorentz trasformations.

$$\boxed{[M^{\rho\sigma}, M^{\tau\nu}] = \eta^{\sigma\tau} M^{\rho\nu} - \eta^{\rho\tau} M^{\sigma\nu} + \eta^{\rho\nu} M^{\sigma\tau} - \eta^{\sigma\nu} M^{\rho\tau}}$$

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