Title: Quantum Field Theory (PHYS 601) - Lecture 6

Date: Oct 05, 2009 09:00 AM

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Abstract:

## Propagators

We could ask the following question:

# Propagators

We could ask the following question: disturb the field at point y. What is the amplitude.

$$\langle 0|\phi(x)\phi(y)|0\rangle$$

$$\langle 0|\phi(x)\,\phi(y)|0\rangle = \int \frac{d^3p\, d^3p'}{(2\pi)^6}\,\frac{1}{\sqrt{4E_k E_{k'}}}\,\langle 0|a_k a^\dagger_{k'}|0\rangle\, e^{-i(p\cdot x - p'\cdot y)}$$

d $\phi(x)\,\phi(y)|0\rangle = \int \frac{d^3p\, d^3p'}{(2\pi)^6} \frac{1}{\sqrt{4E_p E_{p'}}} \langle 0 | a_p a_{p'}^{\dagger} | 0 \rangle \, e^{-i(p\cdot x - p'\cdot y)}$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{-ip\cdot(x-y)}$$

$\equiv D(x-y)$ This is the <u>propogat</u>

For spacelike separation $(x-y)^2 < 0$

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$$D(x-y) \sim e^{-m|\mathbf{x}-\mathbf{y}|} \neq 0.$$

# Feynman Propagator

The important quantity for interacting field theories is the <u>Feynman propagator</u>.

$$\Delta_F(x-y) = \langle 0 | T\phi(x)\phi(y) | 0 \rangle$$

time ordering

$$\equiv \begin{cases} \langle 0|\phi(x)\phi(y)|0\rangle & x^0 > y^0 \\ \langle 0|\phi(y)\phi(x)|0\rangle & y^0 > x^0 \end{cases}$$

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Claim: $$\Delta_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2} e^{-ip\cdot(x-y)}$$

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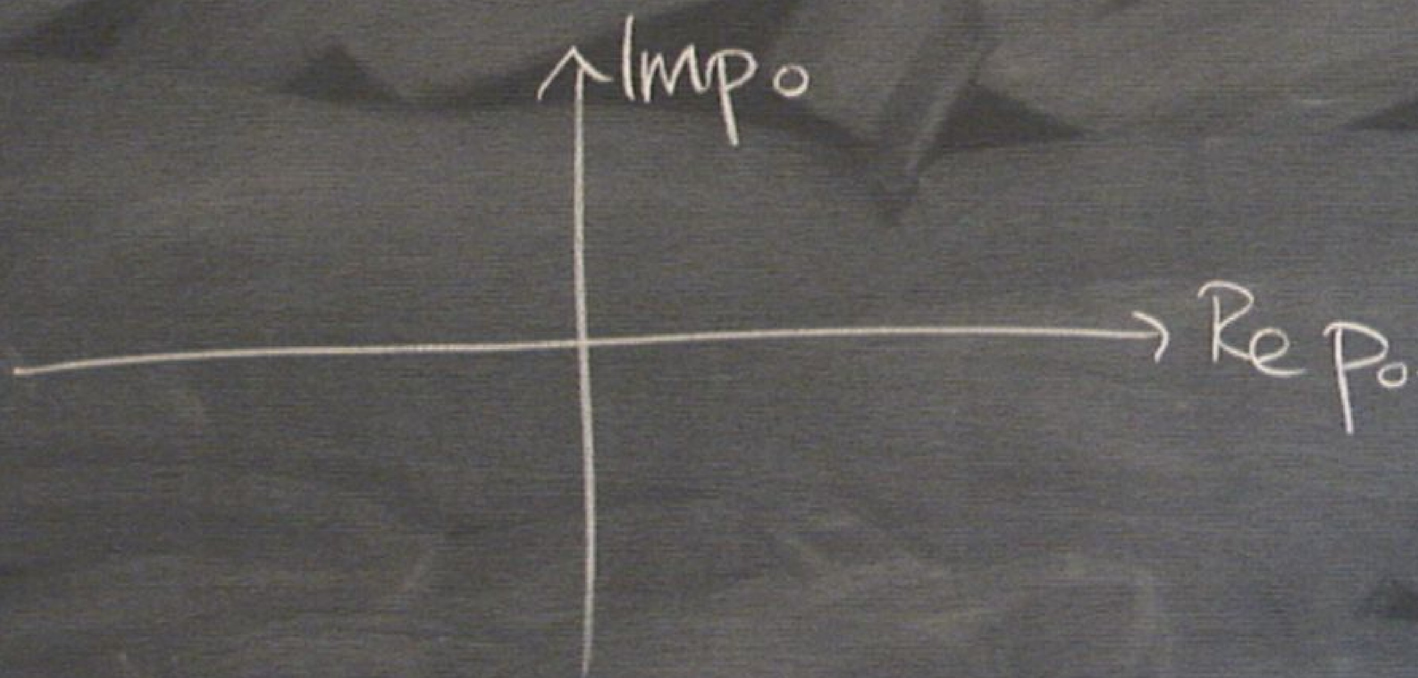
Claim: $\Delta_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2} e^{-ip\cdot(x-y)}$

This isn't yet well-defined because, for $\vec{p}$, the $\int dp_0$ has a pole when $p_0^2 = \vec{p}^2 + m^2$

We'll define the integral by the contour in the complex $p^0$ plane



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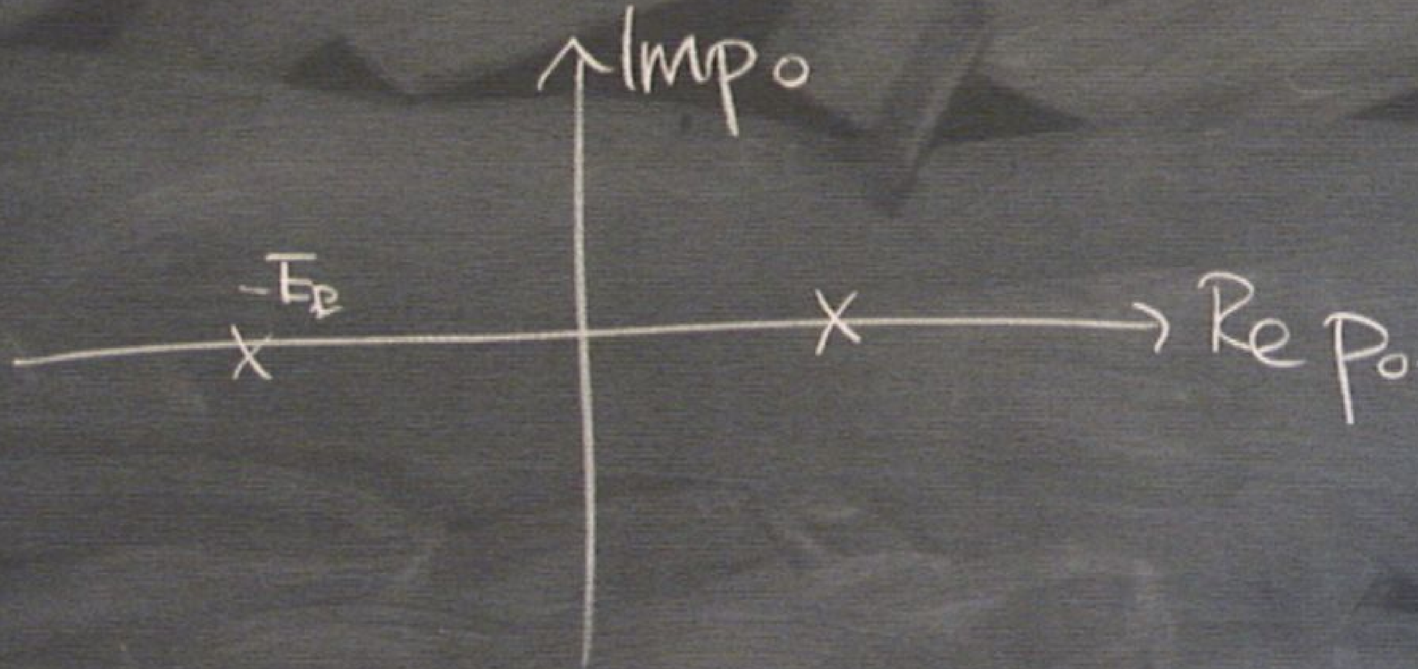

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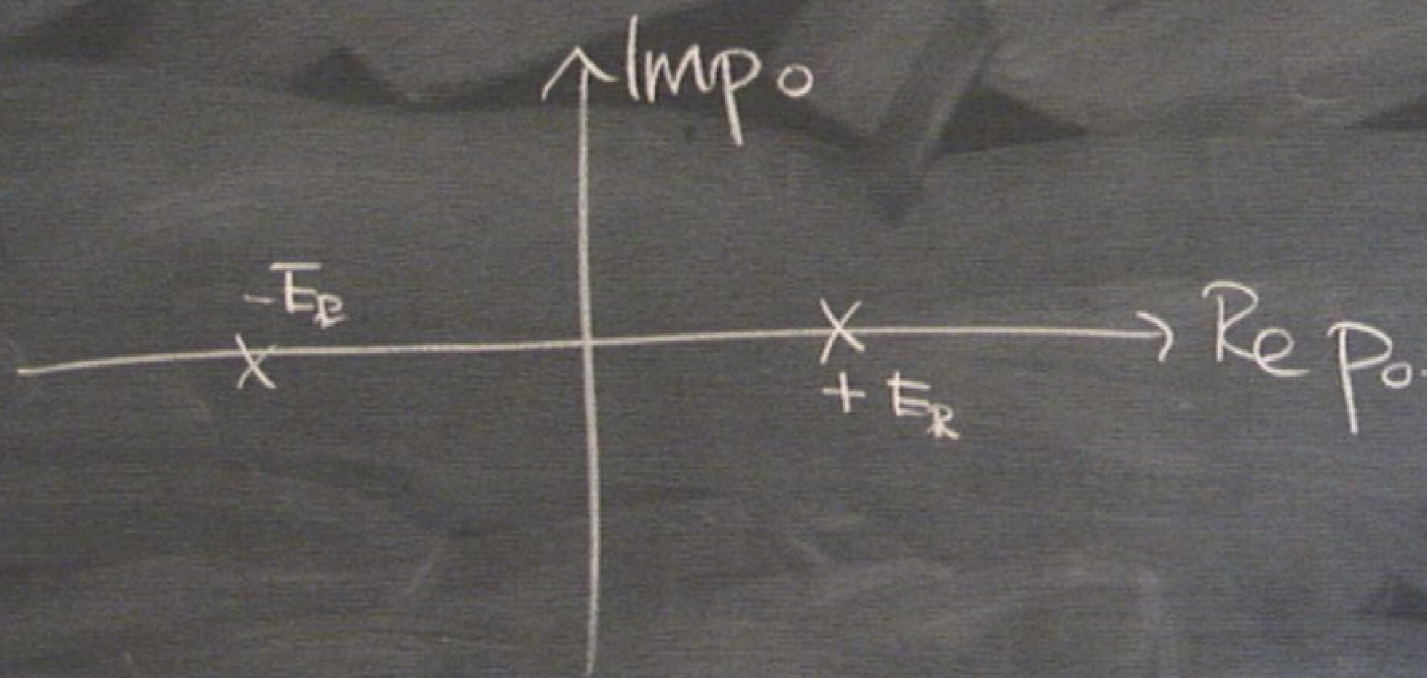

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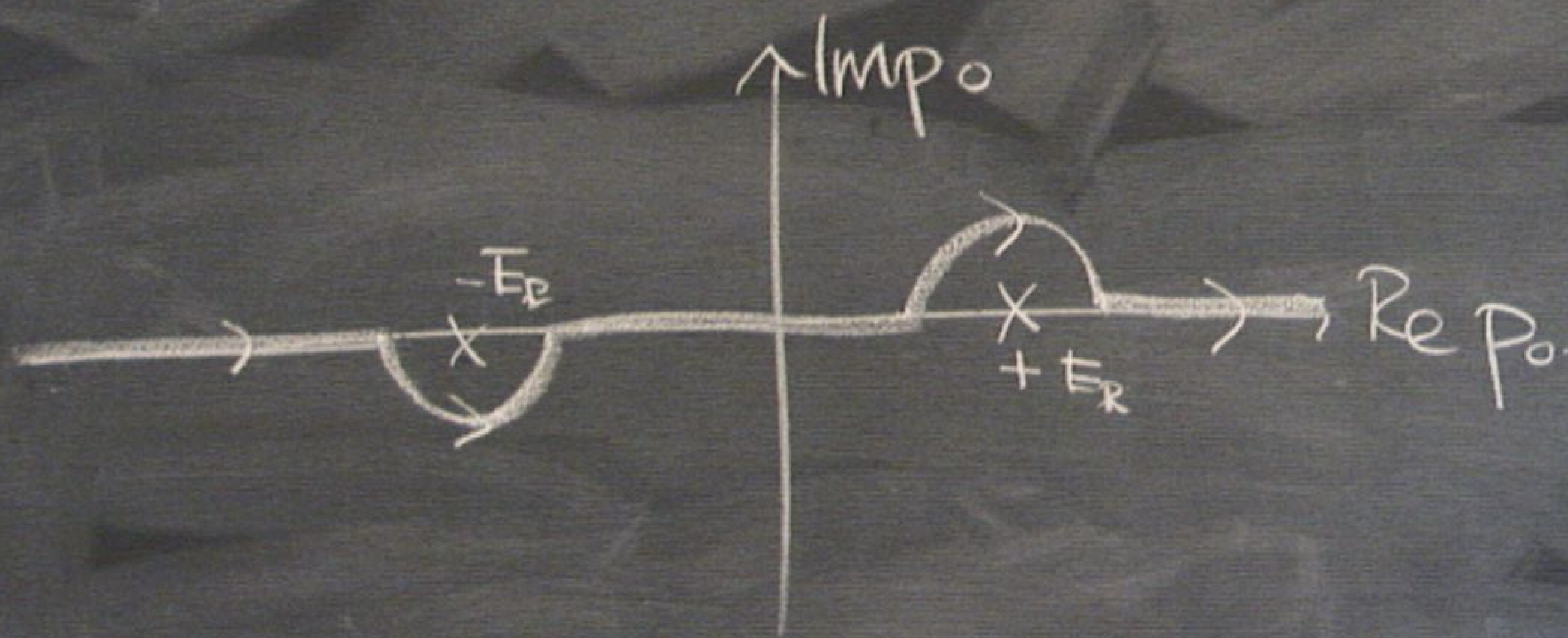

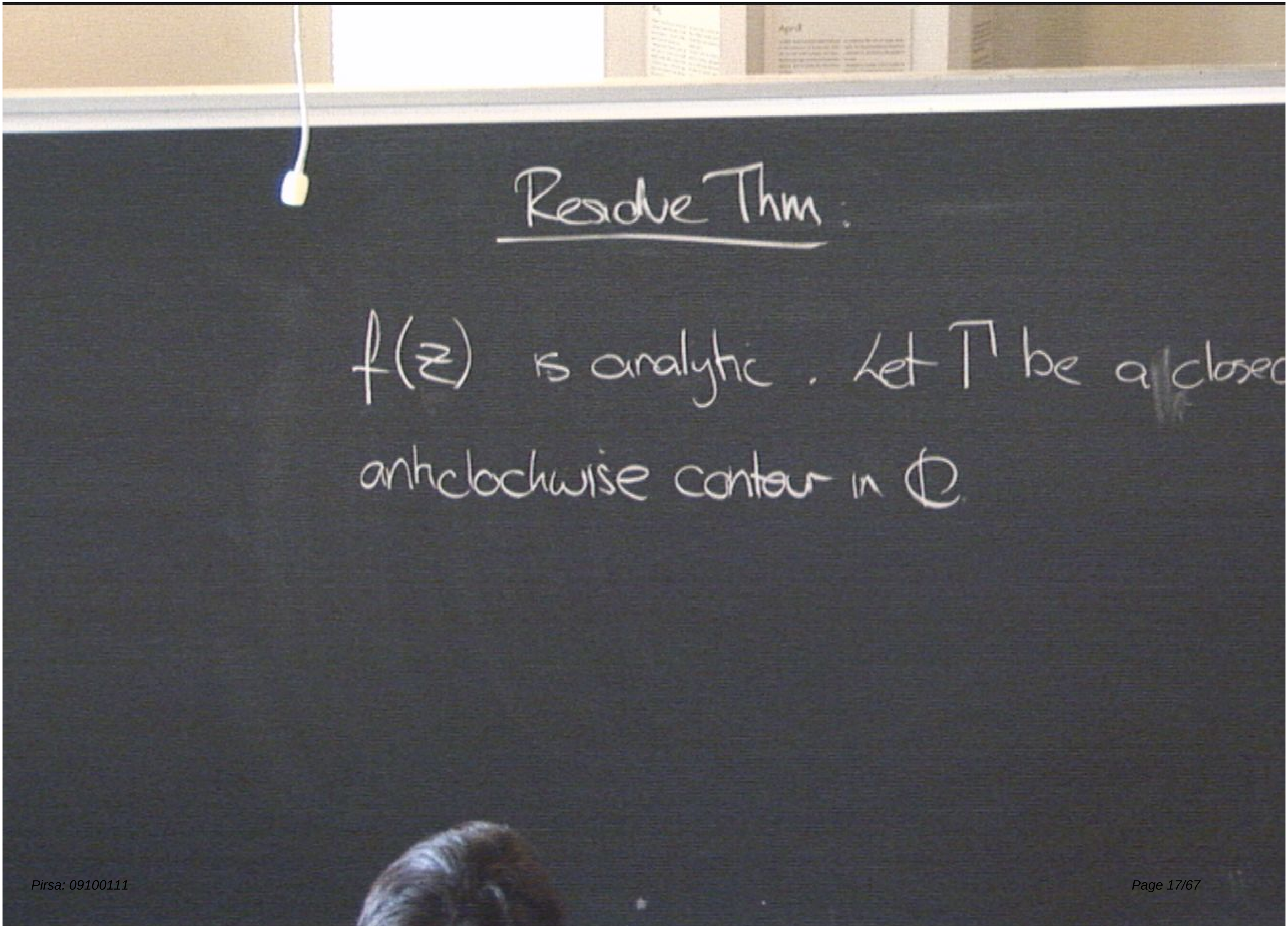
Residue Thm:
f(z) is analytic. Let Γ be a closed
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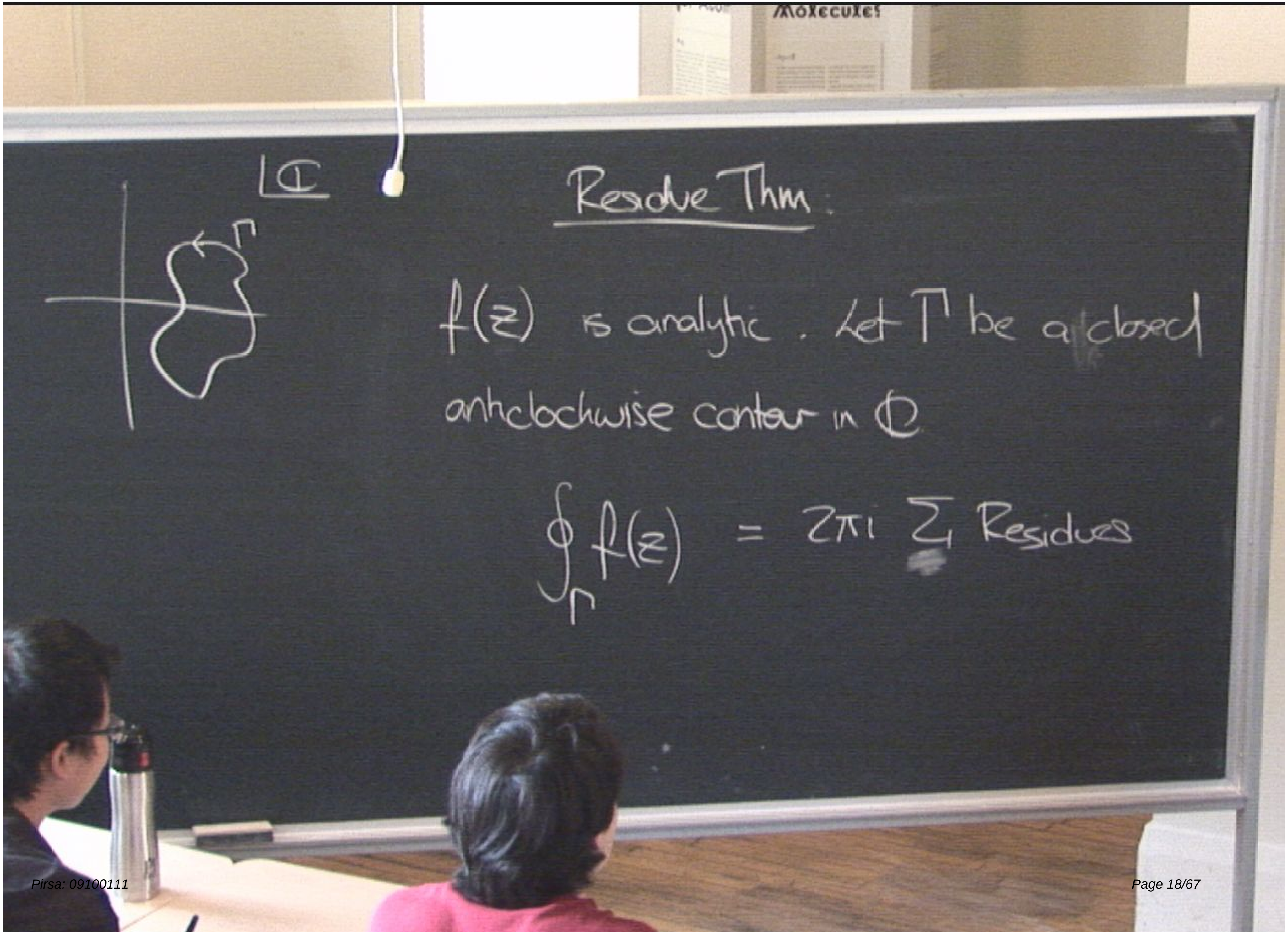

Molecules
$\mathbb{C}$
$\Gamma$
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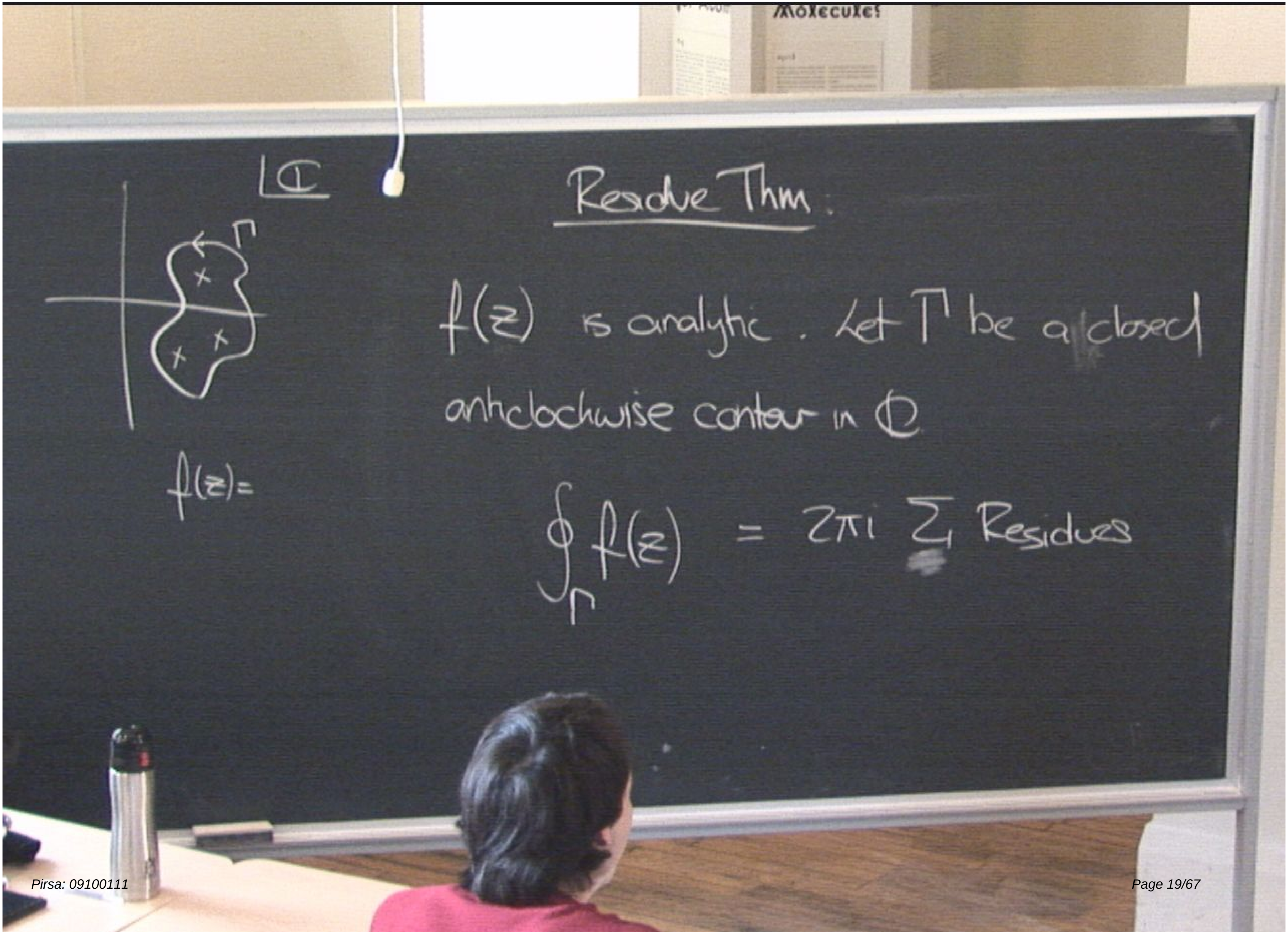
$\oint_\Gamma f(z) = 2\pi i \sum$ Residues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$\mathbb{C}$

## Residue Thm: something

$f(z)$ is ~~analytic~~ ~~holomorphic~~. Let $\Gamma$ be a closed anticlockwise contour in $\mathbb{C}$.

$$\oint_\Gamma f(z) = 2\pi i \sum \text{Residues}$$

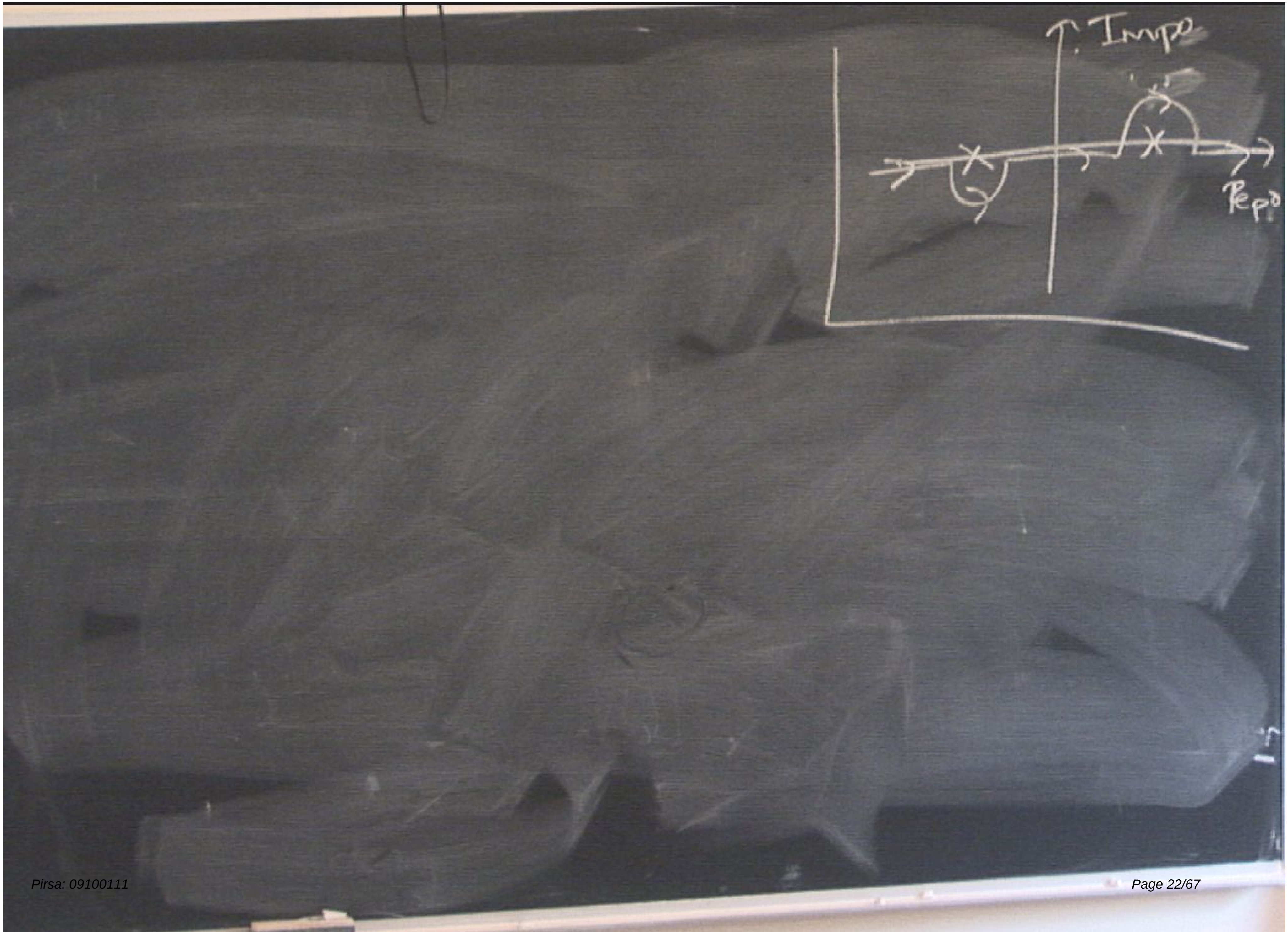
Near $z_0$

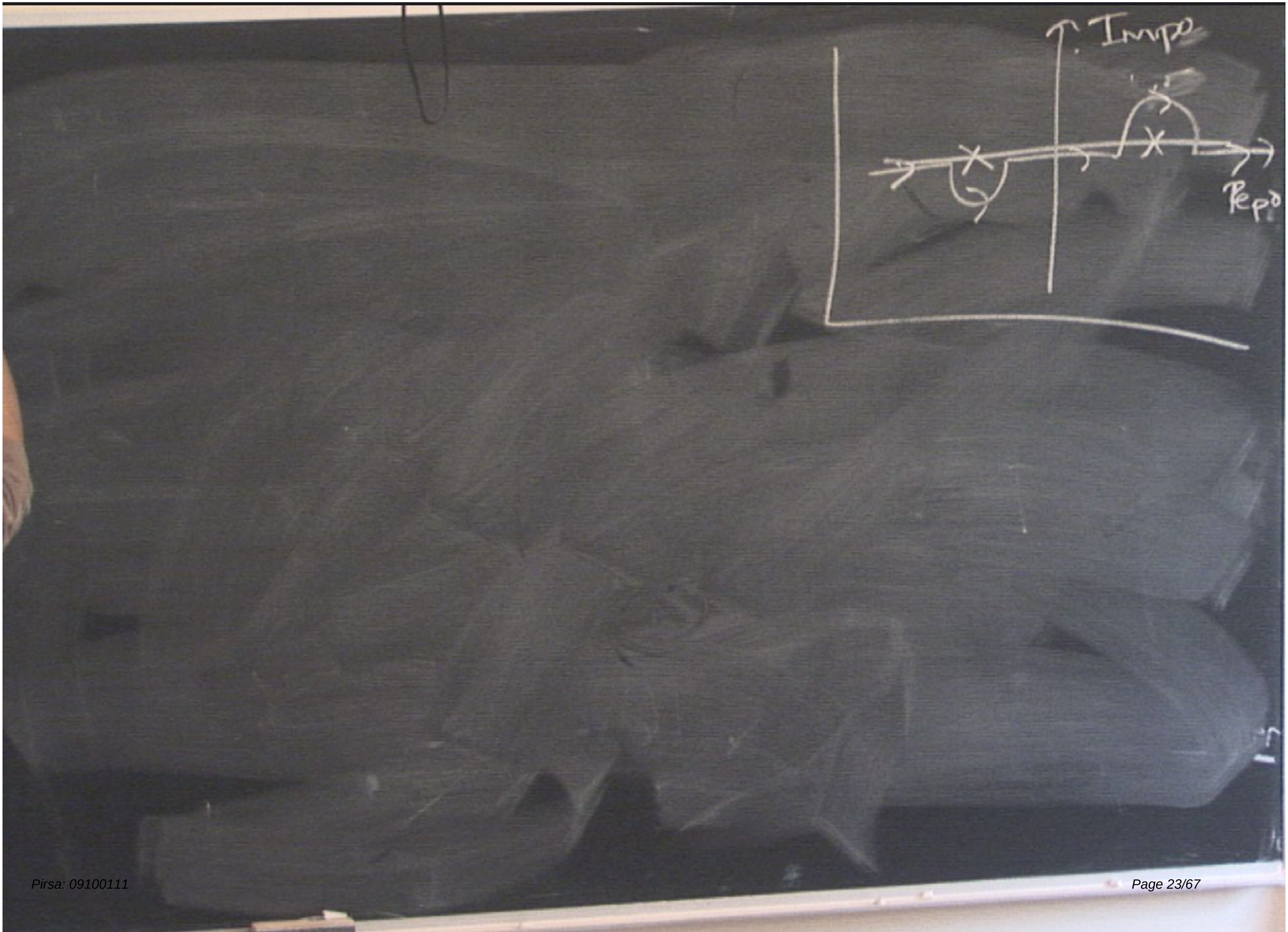
$$(z-z_0)^2 + (z-z_0) + \text{const} + \frac{\text{Res}}{(z-z_0)} + \frac{*}{(z-z_0)^2} + \dots$$

Proof: $\dfrac{1}{p^2 - m^2} = \dfrac{1}{(p^0)^2 - E_p^2}$

$$= \frac{1}{(p^0 - E_p)(p^0 + E_p)}$$

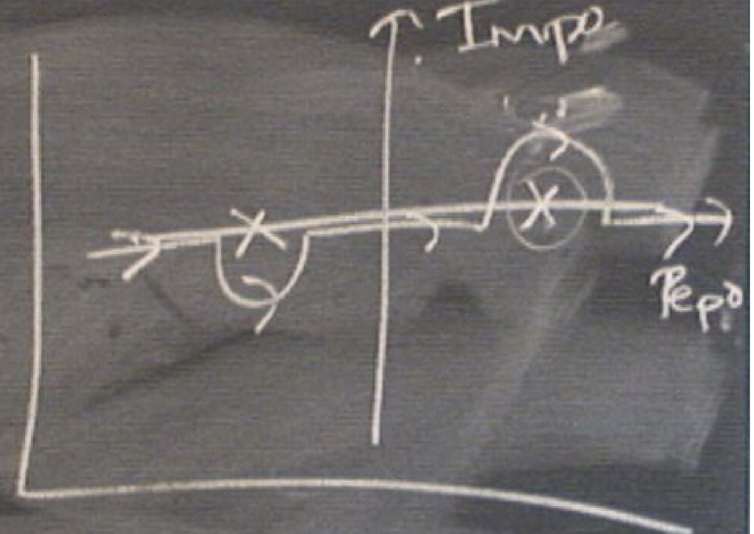
Residue at $p^0 = \pm E_p$ is $\pm \dfrac{1}{2E_p}$


Im $p^0$
Re $p^0$


Im p0
Re p0

en $x^0 > y^0$, we close the

tour in lower-half plane




$p_0 \to -i\infty$

$$\Rightarrow e^{-ip^0(x^0-y^0)} \to e^{-\infty} = 0$$

$\Rightarrow \int dp^0$ picks up residue at $E_k$

$$\int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-M^2} e^{-ip\cdot(x-y)}$$

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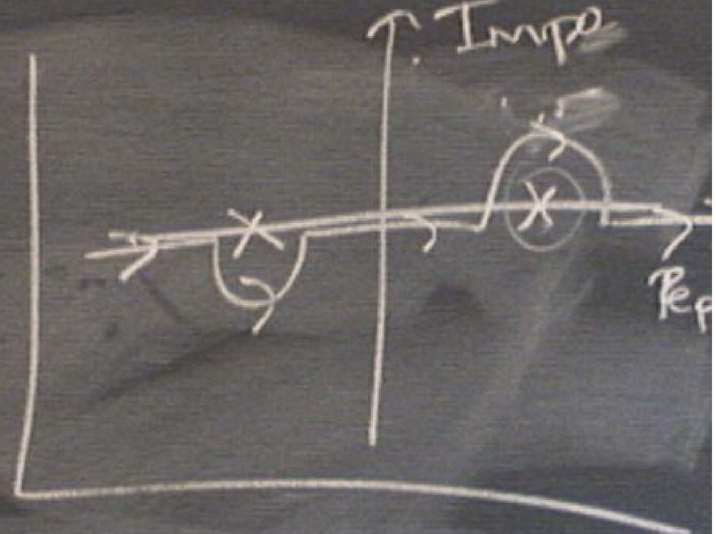


$p_0 \to -i\infty$

$\Rightarrow e^{-ip^0(x^0-y^0)}$

$\to e^{-\infty} = 0$

$\Rightarrow \int dp^0$ picks up residue at $E_p$

Residue is $+1/2E_p$

$\int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2} e^{-ip\cdot(x-\ldots}$

because clockwise contour

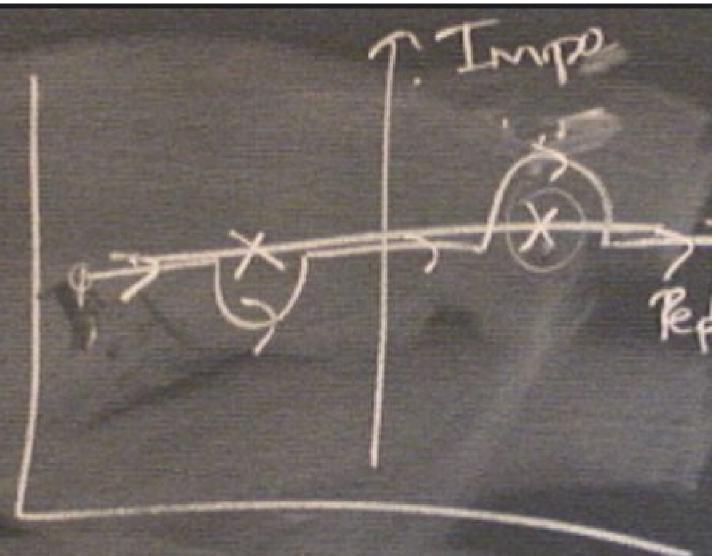
$$\Rightarrow \Delta(x-y) = \int \frac{d^3p}{(2\pi)^4} \frac{-(2\pi i)}{2E_p} \cdot i\, e^{-iE_p(x^0-y^0)} \times e^{i\underline{p}\cdot(\underline{x}-\underline{y})}$$

residue

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{-ip\cdot(x-y)}.$$

when $x^0 > y^0$

When $y^0 > x^0$, we close the contour in the upper half plane

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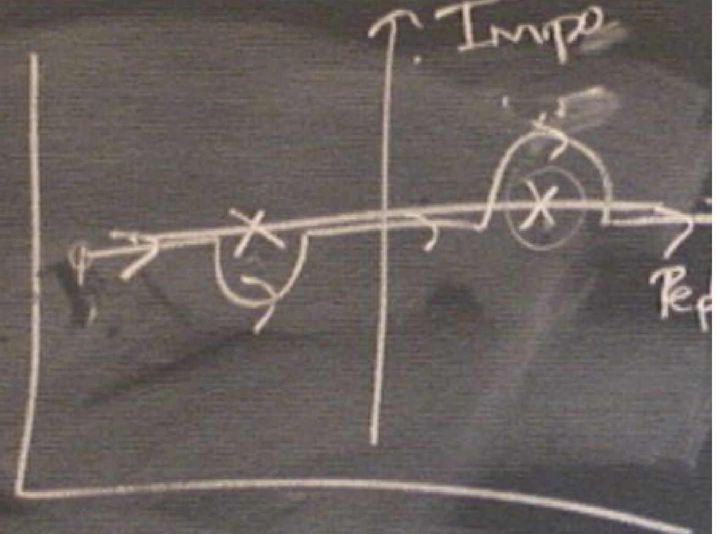


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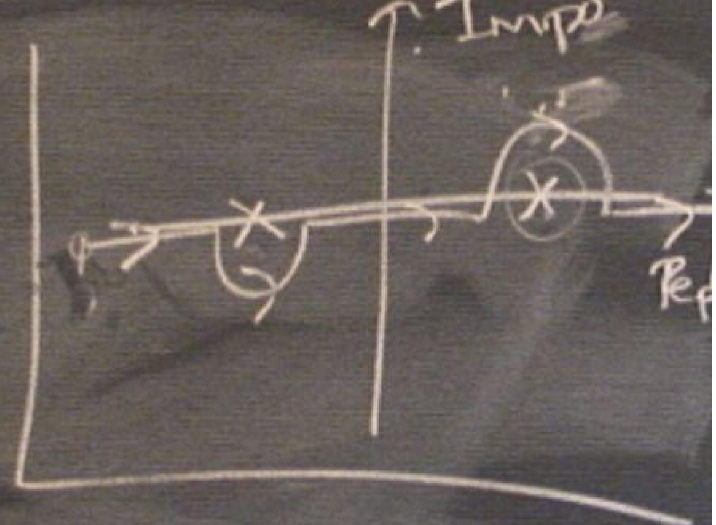
When $y^0 > x^0$, we close the contour in the upper half plane. Some calculation now gives

$$\Delta_F(x-y) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{-ip\cdot(y-x)},$$

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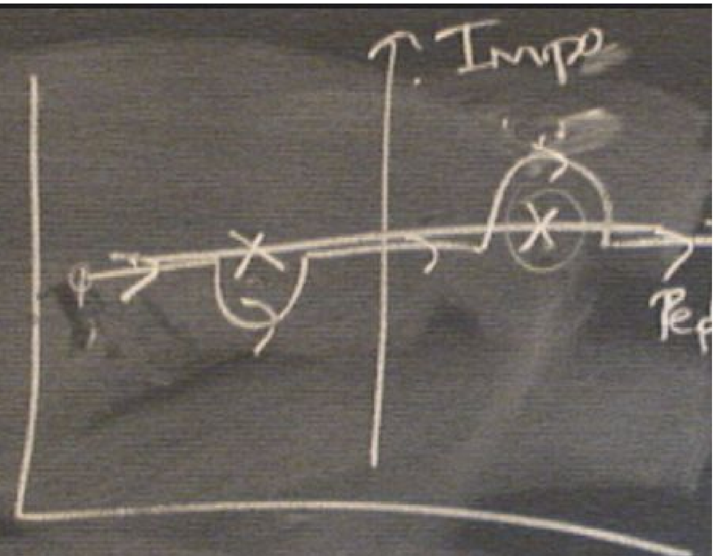


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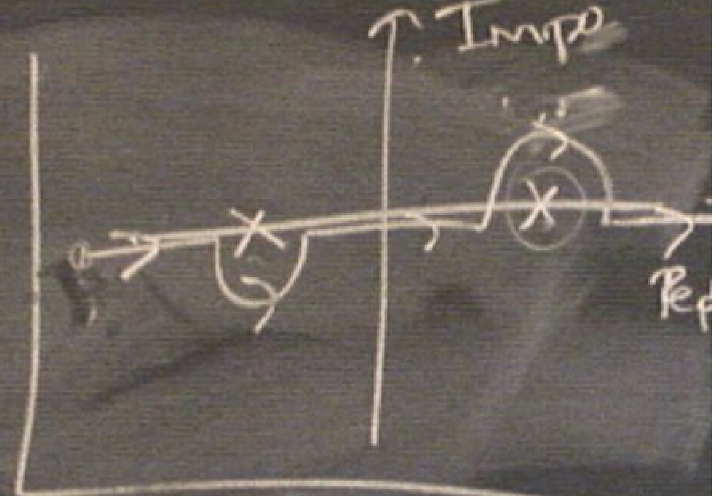
$\square$

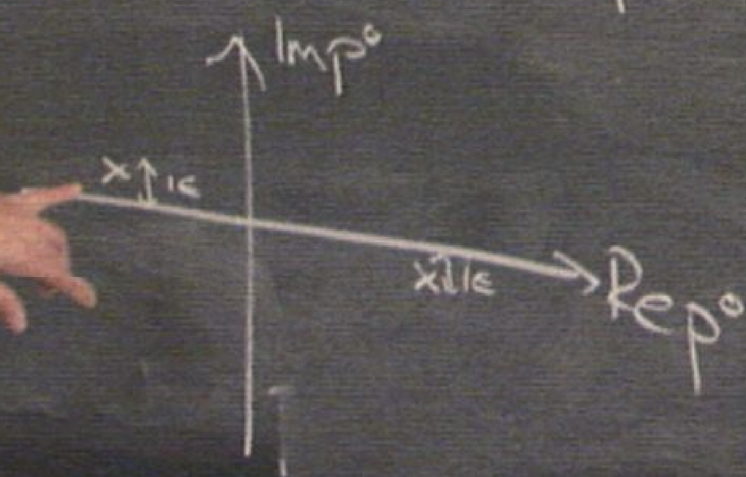



$$\int \frac{d^4p}{(2\pi)^4}\,\frac{i}{p^2-m^2}\, e^{-ip\cdot(x-}$$

Instead of specifying the contour, we may instead write

$$\Delta_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i e^{-ip\cdot(x-y)}}{p^2 - m^2 + i\epsilon}$$

$\epsilon > 0$



## Another Avatar

The propagator is a <u>Greens function</u> for the KG operator.

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$$\left(\partial_t^2 - \nabla^2 + m^2\right)\Delta_F(x-y)$$

# Another Avatar

The propagator is a Greens function for the KG operator.

$$\left(\partial_t^2 - \nabla^2 + M^2\right)\Delta_F(x-y) = \int \frac{d^4p}{(2\pi)^4}\,\frac{i}{p^2-M^2}\,(-p^2+M^2)\,e^{-ip(x-}$$

$$= -i \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)}$$

$$= -i\,\delta^{(4)}(x-y)$$

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$$= -i \int \frac{d^4 p}{(2\pi)^4} e^{-ip\cdot(x-y)}$$

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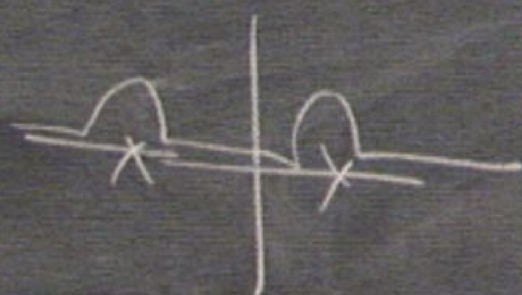
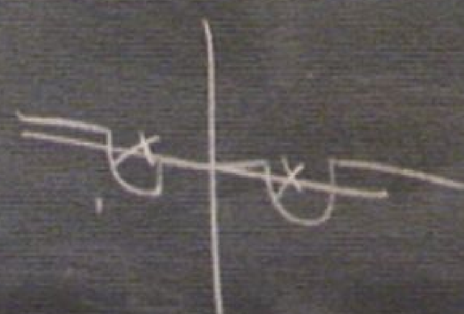
We didn't use the choice of contour here



or



both work fine

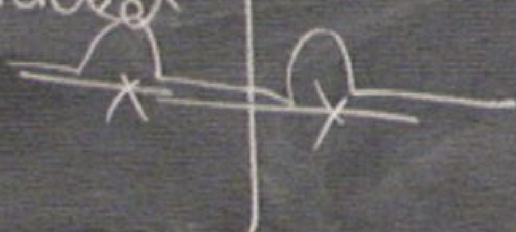
$$= -i\int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}$$

$$= -i\,\delta^{(4)}(x-y)$$

---

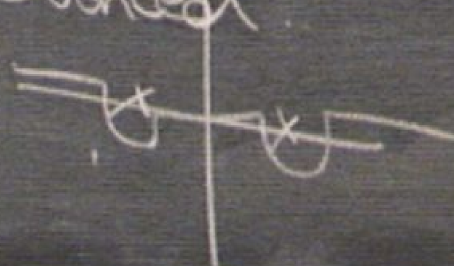
We didn't use the choice of contour here

retarded



or

advanced



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## § Interacting Theories

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## § Interacting Theories

Free field theories $\equiv$ Quadratic Lagrangians

$\Rightarrow$ linear equations of motion

$\Rightarrow$ exactly solvable

Consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \sum_{n=3}^{\infty}\frac{\lambda_n}{n!}\phi^n$$

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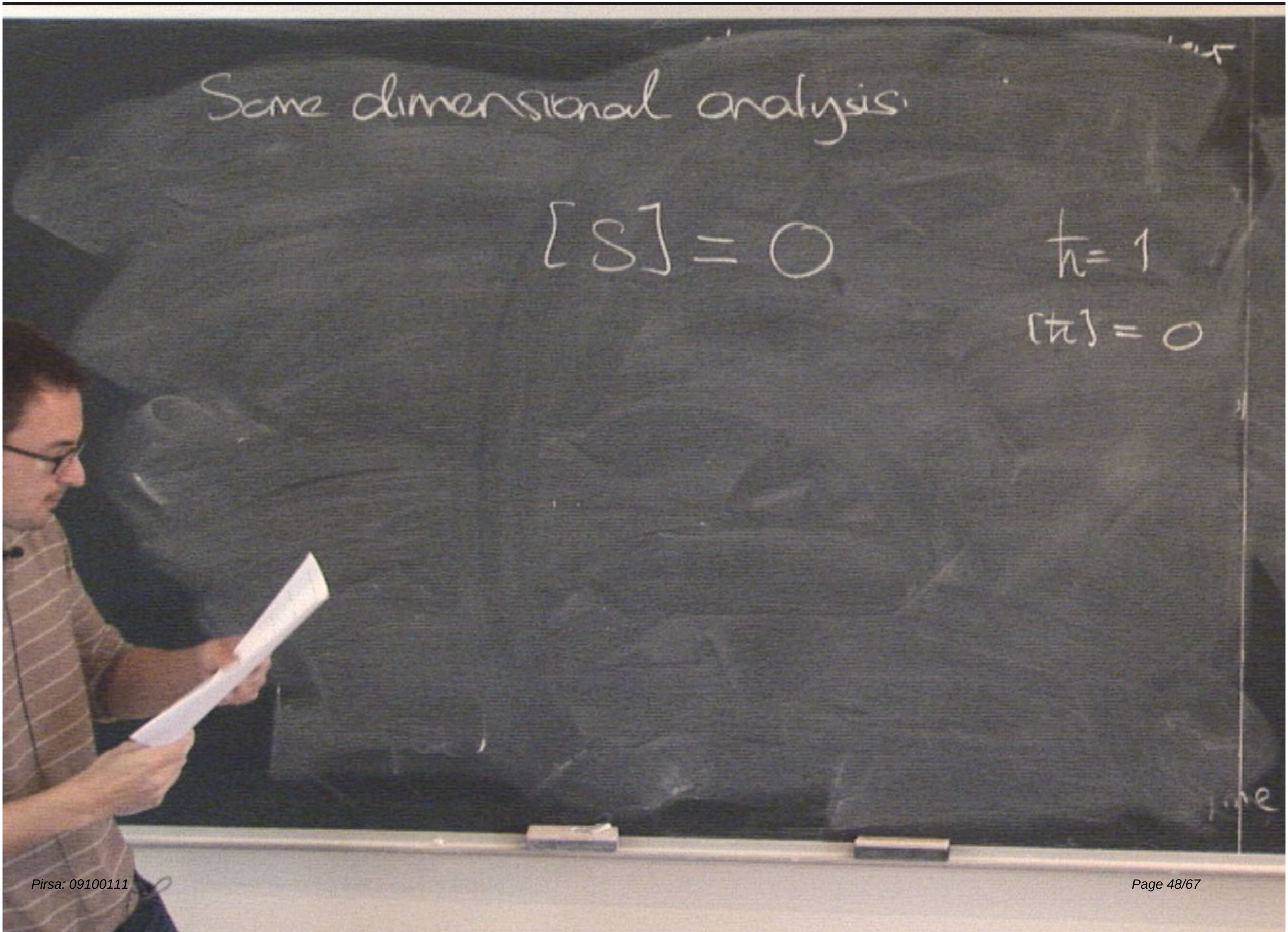
$\lambda_n$ are called <u>coupling constants</u>.

Some dimensional analysis:

$$[S]$$


Some dimensional analysis:
$[S] = 0$
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but $S = \int d^4x \, \mathcal{L} \Rightarrow [\mathcal{L}] = 4$

$[\ ] = -4$

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$$[\partial] = 1 \Rightarrow [\phi] = 1 \text{ and } [m] = 1$$

and $[\lambda_n] = 4 - n$

what does "$\lambda_n$ small" mean?

$[\lambda_3] = 1 \implies$ dimensionless parameter is... $\left(\lambda_3/E\right)$

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where $E$ is energy scale

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what does "$\lambda_n$ small" mean?

$[\lambda_3] = 1 \implies$ dimensionless parameter is… $\left(\lambda_3/E\right)$

where $E$ is energy scale of typical configuration or process that we're interested in

- small perturbation of $E \gg \lambda_3$
- large perturbation of $E \ll \lambda_3$

Perturbations of this type are relevant

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Perturbations of this type are <u>relevant</u>

2)

- small perturbation if $E \gg \lambda_3$
- large perturbation if $E \ll \lambda_3$

Perturbations of this type are <u>relevant</u>

For relativistic theories, $E > M \Rightarrow$ small perturbation $\lambda_3 \ll$

2) $\frac{\lambda_4 \phi^4}{4!}$ $\quad [\lambda_4] = 0 \Rightarrow \lambda_4 \ll 1$ is small perturbations

These perturbations are called <u>marginal</u>

3)

an $\lambda_3 \ll M$

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3) $\frac{\lambda_n \phi^n}{n!}$ : $[\lambda_n] < 0 \Rightarrow \left(\lambda E^{4-n}\right)$ is dimensionless

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