Title: Beyond the Quantum

Date: Oct 07, 2009 02:00 PM

URL: http://pirsa.org/09100103

Abstract: According to hidden-variables theories, quantum physics is a special 'equilibrium' case of a much wider 'nonequilibrium' physics. We describe the search for that wider physics in a cosmological context. The hypothesis that the universe began in a state of quantum nonequilibrium is shown to have observable consequences. In de Broglie-Bohm theory on expanding space, relaxation to quantum equilibrium is shown to be suppressed for field modes whose quantum time evolution satisfies a certain inequality, resulting in a 'freezing' of early nonequilibrium for these particular modes. For an early radiation-dominated expansion, the inequality implies a corresponding physical wavelength that is larger than the (instantaneous) Hubble radius. These results make it possible, for the first time, to make quantitative predictions for deviations from quantum theory. We consider, in particular, corrections to inflationary predictions for the cosmic microwave background, and the possibility of finding relic cosmological particles that violate the laws of quantum mechanics. (Reference: De Broglie-Bohm Prediction of Quantum Violations for Cosmological Super-Hubble Modes, http://arxiv.org/abs/0804.4656.)

Relaxation in the short-wavelength limit

Roughly: short-wavelength limit, $\lambda_{\mathrm{phys}} << H^{-1}$,

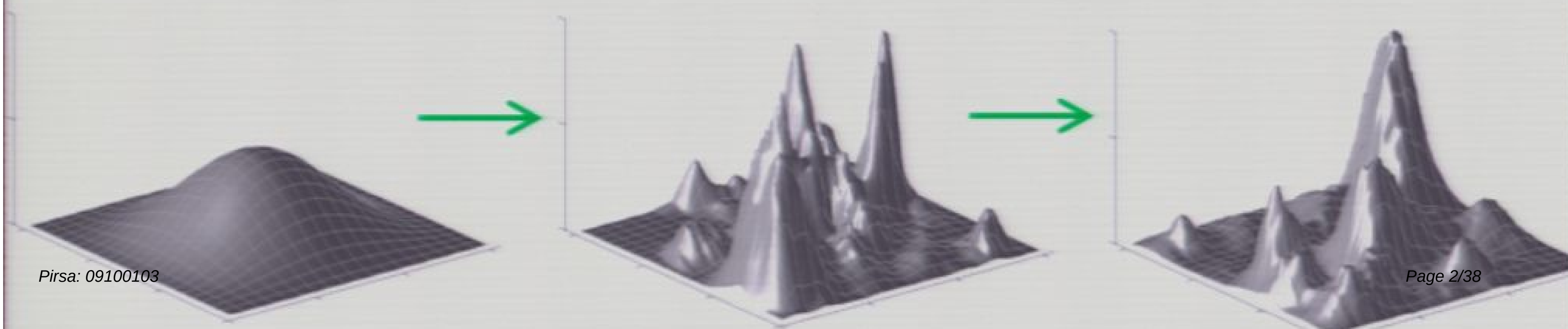
(strictly, $\lambda_{\mathrm{phys}} << \Delta n_{\mathbf{k}} \cdot H^{-1}$)

$\downarrow$

Minkowski

the timescale $\Delta t \propto \lambda_{\mathrm{phys}}$ over which $\psi_{\mathbf{k}} = \psi_{\mathbf{k}}(q_{\mathbf{k}1}, q_{\mathbf{k}2}, t)$ evolves will be much smaller than the expansion timescale $H^{-1} \equiv a/\dot{a}$

Obtain usual efficient relaxation



Freezing of the Wave Function for Super-Hubble Modes

long-wavelength limit, $\lambda_{\text{phys}} >> \Delta n_{\mathbf{k}} \cdot H^{-1}$, $\Delta t \equiv 1/\Delta E_{\mathbf{k}} >> H^{-}$

sudden perturbation,

$\psi_{\mathbf{k}}$ is approximately static — or 'frozen' — over timescales H^{-1}

Continuity equation $$\frac{\partial\left|\psi_{\mathbf{k}}\right|^2}{\partial t} + \frac{\partial}{\partial q_{\mathbf{k}1}}\left(\left|\psi_{\mathbf{k}}\right|^2 \dot{q}_{\mathbf{k}1}\right) + \frac{\partial}{\partial q_{\mathbf{k}2}}\left(\left|\psi_{\mathbf{k}}\right|^2 \dot{q}_{\mathbf{k}2}\right) = ($$

$$\downarrow$$

expect that the trajectories $(q_{\mathbf{k}1}(t), q_{\mathbf{k}2}(t))$ will also be frozen over timescales H^{-1}

Initial nonequilibrium will be frozen over timescales H^{-1}

Suggests we search for nonequilibrium at long wavelengths

But limited treatment:

- free, decoupled mode in a pure quantum state
- have not proved that trajectories are frozen
- $H^{-1} \to 0$ as $t \to 0$.

freezing over the timescale H^{-1} does not tell us very much

Will now derive a rigorous condition for nonequilibrium freezing:

- arbitrary time interval $[t_i, t_f]$
- any quantum state (entangled, mixed)
- interacting fields

Inequality for the freezing of quantum nonequilibrium

(Valentini 2008)

examine the behaviour of the trajectories themselves

First: general (entangled) pure quantum state of a free field
(Generalise later to mixed, interacting.)

collection of non-interacting one-dimensional harmonic oscillators

$$\hat{H} = \sum_{\mathbf{k}r} \hat{H}_{\mathbf{k}r}, \text{ with } \hat{H}_{\mathbf{k}r} = \frac{\hat{\pi}^2_{\mathbf{k}r}}{2a^3} + \frac{1}{2}a^3\omega^2\hat{q}^2_{\mathbf{k}r}$$

arbitrary wave functional $\Psi[q_{\mathbf{k}r}, t]$,

de Broglie velocity field is given by $\dfrac{dq_{\mathbf{k}r}}{dt} = \dfrac{1}{a^3}\dfrac{\partial S}{\partial q_{\mathbf{k}r}}$

evolution of an arbitrary ensemble distribution $P[q_{\mathbf{k}r}, t]$

$$\frac{\partial P}{\partial t} + \sum_{\mathbf{k}r} \frac{\partial}{\partial q_{\mathbf{k}r}}\left(P\frac{1}{a^3}\frac{\partial S}{\partial q_{\mathbf{k}r}}\right) = 0$$

Initial nonequilibrium distribution $P[q_{\mathbf{k}r}, t_i] \neq |\Psi[q_{\mathbf{k}r}, t_i]|^2$

Can relax (in general, on a coarse-grained level)

only if the trajectories move far enough

(Cf. gas molecules in a box)

Final displacement $\delta q_{\mathbf{k}r}(t_f) = \int_{t_i}^{t_f} dt\ \dot{q}_{\mathbf{k}r}(t)$

Simple condition for "freezing": $|\delta q_{\mathbf{k}r}(t_f)| << \Delta q_{\mathbf{k}r}(t_f)$

(magnitude of final displacement smaller than width of wave packet)

Too strong. Take weaker condition $\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\mathrm{eq}} < \Delta q_{\mathbf{k}r}(t_f)$

(equilibrium mean smaller than width of wave packet)

Implies that most of the ensemble cannot move

by much more than $\Delta q_{\mathbf{k}r}(t_f)$

If $\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\mathbf{eq}} < \Delta q_{\mathbf{k}r}(t_f)$

relaxation will in general be suppressed (for the mode $\mathbf{k}r$)

Now: $\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\mathbf{eq}} \leq \left\langle \int_{t_i}^{t_f} dt \ |\dot{q}_{\mathbf{k}r}(t)| \right\rangle_{\mathbf{eq}} = \int_{t_i}^{t_f} dt \ \langle|\dot{q}_{\mathbf{k}r}(t)|\rangle_{\mathbf{eq}}$.

(where $\langle|\dot{q}_{\mathbf{k}r}(t)|\rangle_{\mathbf{eq}} = \int dq \ |\Psi[q,t]|^2 |\dot{q}_{\mathbf{k}r}(q,t)|$)

Or

$$\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\mathbf{eq}} \leq \int_{t_i}^{t_f} dt \ \sqrt{\left\langle |\dot{q}_{\mathbf{k}r}(t)|^2 \right\rangle_{\mathbf{eq}}} .$$

We have

$$a^6 \left\langle |\dot{q}_{\mathbf{k}r}|^2 \right\rangle_{\mathbf{eq}} = \left\langle \left(\frac{\partial S}{\partial q_{\mathbf{k}r}} \right)^2 \right\rangle_{\mathbf{eq}} = \int dq \ |\Psi[q,t]|^2 \left(\frac{\partial S[q,t]}{\partial q_{\mathbf{k}r}} \right)^2$$

$$= \left\langle \hat{\pi}_{\mathbf{k}r}^2 \right\rangle - \int dq \ \left(\frac{\partial |\Psi[q,t]|}{\partial q_{\mathbf{k}r}} \right)^2$$

Thus, since $(\partial|\Psi|/\partial q_{\mathbf{k}r})^2 \geq 0$, we have

$$a^6 \left\langle |\dot{q}_{\mathbf{k}r}|^2 \right\rangle_{\mathbf{eq}} \leq \left\langle \hat{\pi}^2_{\mathbf{k}r} \right\rangle ,$$

and so

$$\langle |\delta q_{\mathbf{k}r}(t_f)| \rangle_{\mathbf{eq}} \leq \int_{t_i}^{t_f} dt \; \frac{1}{a^3} \sqrt{\langle \hat{\pi}^2_{\mathbf{k}r} \rangle}$$

Since $\left\langle \hat{q}^2_{\mathbf{k}r} \right\rangle > 0$, we also have

$$\left\langle \hat{\pi}^2_{\mathbf{k}r} \right\rangle < 2a^3 \left\langle \hat{H}_{\mathbf{k}r} \right\rangle .$$

and so

$$\langle |\delta q_{\mathbf{k}r}(t_f)| \rangle_{\mathbf{eq}} < \int_{t_i}^{t_f} dt \; \frac{1}{a^3} \sqrt{2a^3 \left\langle \hat{H}_{\mathbf{k}r} \right\rangle} .$$

Introducing the number operator $\hat{n}_{\mathbf{k}r}$,

$$\langle |\delta q_{\mathbf{k}r}(t_f)| \rangle_{\mathbf{eq}} < \int_{t_i}^{t_f} dt \; \frac{1}{a^2} \sqrt{2k(\langle \hat{n}_{\mathbf{k}r} \rangle + 1/2)} .$$

The mean $\langle |\delta q_{\mathbf{k}r}(t_f)| \rangle_{\mathbf{eq}}$ at time t_f is to be compared with the width $\Delta q_{\mathbf{k}r}(t_f)$

Using uncertainty relation $\Delta q_{\mathbf{k}r}\Delta\pi_{\mathbf{k}r} \geq \frac{1}{2}$ and $\Delta\pi_{\mathbf{k}r} \leq \sqrt{\langle\hat{\pi}^2_{\mathbf{k}r}\rangle}$,

we have

$$1/\Delta q_{\mathbf{k}r} < 2\sqrt{2a^3\left\langle \hat{H}_{\mathbf{k}r}\right\rangle} = 2a\sqrt{2k(\langle\hat{n}_{\mathbf{k}r}\rangle + 1/2)}\ .$$

Combine with previous

$$\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\mathrm{eq}} < \int_{t_i}^{t_f} dt\ \frac{1}{a^2}\sqrt{2k(\langle\hat{n}_{\mathbf{k}r}\rangle + 1/2)}\ .$$

Have upper bound for the ratio

$$\frac{\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\mathrm{eq}}}{\Delta q_{\mathbf{k}r}(t_f)} < 4ka_f\sqrt{\langle\hat{n}_{\mathbf{k}r}\rangle_f + 1/2}\int_{t_i}^{t_f} dt\ \frac{1}{a^2}\sqrt{\langle\hat{n}_{\mathbf{k}r}\rangle + 1/2}$$

(where $a_f \equiv a(t_f)$, and so on). Note that $\langle\hat{n}_{\mathbf{k}r}\rangle$ is in general a function of tim

True for arbitrary entangled state Ψ.

"Freezing inequality": right-hand side is less than one ….

Freezing inequality

$$\frac{1}{k} > 4a_f\sqrt{\langle \hat{n}_{\mathbf{k}r}\rangle_f + 1/2}\int_{t_i}^{t_f} dt\ \frac{1}{a^2}\sqrt{\langle \hat{n}_{\mathbf{k}r}\rangle + 1/2}\ .$$

"Frozen" nonequilibrium will exist at later times $t_f > t_i$

for modes satisfying the inequality

Mixed states:

- Statistical mixture of physically-real pilot waves
- Consider freezing inequality for each pure subensemble separately
- Might hold for some subensembles and not for others (or for all of them, or none)

Interacting fields:

- Finite models with a cutoff (ignore divergences)
- Scalar ϕ interacts with other fields
- Only difference is in time evolution of $\langle \hat{n}_{\mathbf{k}r}\rangle$

General implication of the freezing inequality

$$\frac{1}{k} > 4a_f\sqrt{\langle \hat{n}_{\mathbf{k}r}\rangle_f + 1/2}\int_{t_i}^{t_f} dt\ \frac{1}{a^2}\sqrt{\langle \hat{n}_{\mathbf{k}r}\rangle + 1/2}\ .$$

Satisfied? Depends on history of expansion, and on time evolution of quantum state

For a radiation-dominated expansion on $[t_i, t_f]$, with $a(t) = a_f(t/t_f)^{1/2}$,

the physical wavelength $\lambda_{\rm phys}(t_f) = a_f(2\pi/k)$ at time t_f

must be larger than

the Hubble radius H_f^{-1} at time t_f

Since $\langle \hat{n}_{\mathbf{k}r}\rangle \geq 0$,

$$\frac{1}{k} > 2a_f\int_{t_i}^{t_f} dt\ \frac{1}{a^2} = \frac{2t_f}{a_f}\ln(t_f/t_i)\ , \quad \text{or} \quad \lambda_{\rm phys}(t_f) > 2\pi H_f^{-1}\ln(t_f/t_i)$$

(where $H_f^{-1} = 2t_f$)

Freezing inequality

$$\frac{1}{k} > 4a_f\sqrt{\langle \hat{n}_{\mathbf{k}r}\rangle_f + 1/2}\int_{t_i}^{t_f} dt\ \frac{1}{a^2}\sqrt{\langle \hat{n}_{\mathbf{k}r}\rangle + 1/2}\ .$$

"Frozen" nonequilibrium will exist at later times $t_f > t_i$

for modes satisfying the inequality

Mixed states:

- Statistical mixture of physically-real pilot waves
- Consider freezing inequality for each pure subensemble separately
- Might hold for some subensembles and not for others (or for all of them, or none)

Interacting fields:

- Finite models with a cutoff (ignore divergences)
- Scalar ϕ interacts with other fields
- Only difference is in time evolution of $\langle \hat{n}_{\mathbf{k}r}\rangle$

General implication of the freezing inequality

$$\frac{1}{k} > 4a_f \sqrt{\langle \hat{n}_{\mathbf{k}r} \rangle_f + 1/2} \int_{t_i}^{t_f} dt \, \frac{1}{a^2} \sqrt{\langle \hat{n}_{\mathbf{k}r} \rangle + 1/2} \, .$$

Satisfied? Depends on history of expansion,
and on time evolution of quantum state

For a radiation-dominated expansion on $[t_i, t_f]$, with $a(t) = a_f (t/t_f)^{1/2}$,

the physical wavelength $\lambda_{\text{phys}}(t_f) = a_f(2\pi/k)$ at time t_f

must be larger than

the Hubble radius H_f^{-1} at time t_f

Since $\langle \hat{n}_{\mathbf{k}r} \rangle \geq 0$,

$$\frac{1}{k} > 2a_f \int_{t_i}^{t_f} dt \, \frac{1}{a^2} = \frac{2t_f}{a_f} \ln(t_f/t_i) \, , \text{ or } \lambda_{\text{phys}}(t_f) > 2\pi H_f^{-1} \ln(t_f/t_i)$$

(where $H_f^{-1} = 2t_f$)

General implication of the freezing inequality

$$\frac{1}{k} > 4a_f\sqrt{\langle \hat{n}_{\mathbf{k}r}\rangle_f + 1/2}\int_{t_i}^{t_f} dt\ \frac{1}{a^2}\sqrt{\langle \hat{n}_{\mathbf{k}r}\rangle + 1/2}\ .$$

Satisfied? Depends on history of expansion, and on time evolution of quantum state

For a radiation-dominated expansion on $[t_i, t_f]$, with $a(t) = a_f(t/t_f)^{1/2}$,

the physical wavelength $\lambda_{\mathrm{phys}}(t_f) = a_f(2\pi/k)$ at time t_f

must be larger than

the Hubble radius H_f^{-1} at time t_f

Since $\langle \hat{n}_{\mathbf{k}r}\rangle \geq 0$,

$$\frac{1}{k} > 2a_f\int_{t_i}^{t_f} dt\ \frac{1}{a^2} = \frac{2t_f}{a_f}\ln(t_f/t_i)\ ,\ \text{or}\ \ \lambda_{\mathrm{phys}}(t_f) > 2\pi H_f^{-1}\ln(t_f/t_i)$$

(where $H_f^{-1} = 2t_f$)

Possible consequences of early nonequilibrium freezing

Write $\lambda_{\rm phys}(t_f) > 2\pi H_f^{-1} \ln(t_f/t_i)$ as $\lambda_{\rm \mathbf{phys}}(t_f) > 4\pi H_f^{-1} \ln(T_i/T_f)$.

(A necessary but not sufficient condition.)

Points to where nonequilibrium *could* be found.

We should search for nonequilibrium above a specific critical wavelength

Two main areas of research (in progress)

Corrections to Inflationary Predictions for the CMB

Relic Nonequilibrium Particles

Corrections to Inflationary Predictions for the CMB

inflaton perturbation ϕ $\longrightarrow$ curvature perturbation $\mathcal{R}_{\mathbf{k}}$

curvature perturbation $\mathcal{R}_{\mathbf{k}}$ $\longrightarrow$ temperature anisotropy a_{lm}

$$\mathcal{R}_{\mathbf{k}} = -\left[\frac{H}{\dot{\phi}_0}\phi_{\mathbf{k}}\right]_{t=t_*(k)}$$

$$a_{lm} = \frac{i^l}{2\pi^2}\int d^3\mathbf{k}\, \mathcal{T}(k,l)\mathcal{R}_{\mathbf{k}} Y_{lm}(\hat{\mathbf{k}})$$

$$\frac{\Delta T(\theta,\phi)}{\bar{T}} = \sum_{l=2}^{\infty}\sum_{m=-l}^{+l} a_{lm} Y_{lm}(\theta,\phi)$$

Corrections to Inflationary Predictions for the CMB

inflaton perturbation ϕ $\longrightarrow$ curvature perturbation $\mathcal{R}_{\mathbf{k}}$

curvature perturbation $\mathcal{R}_{\mathbf{k}}$ $\longrightarrow$ temperature anisotropy a_{lm}

$$\mathcal{R}_{\mathbf{k}} = -\left[\frac{H}{\dot{\phi}_0}\phi_{\mathbf{k}}\right]_{t=t_*(k)}$$

$$a_{lm} = \frac{i^l}{2\pi^2}\int d^3\mathbf{k}\; \mathcal{T}(k,l)\mathcal{R}_{\mathbf{k}} Y_{lm}(\hat{\mathbf{k}})$$

$$\frac{\Delta T(\theta,\phi)}{\bar{T}} = \sum_{l=2}^{\infty}\sum_{m=-l}^{+l} a_{lm} Y_{lm}(\theta,\phi)$$

Possible consequences of early nonequilibrium freezing

Write $\lambda_{\rm phys}(t_f) > 2\pi H_f^{-1} \ln(t_f/t_i)$ as $\lambda_{\rm phys}(t_f) > 4\pi H_f^{-1} \ln(T_i/T_f)$.

(A necessary but not sufficient condition.)

Points to where nonequilibrium *could* be found.

We should search for nonequilibrium above a specific critical wavelength

Two main areas of research (in progress)

Corrections to Inflationary Predictions for the CMB

Relic Nonequilibrium Particles

Corrections to Inflationary Predictions for the CMB

inflaton perturbation ϕ $\longrightarrow$ curvature perturbation $\mathcal{R}_{\mathbf{k}}$

curvature perturbation $\mathcal{R}_{\mathbf{k}}$ $\longrightarrow$ temperature anisotropy a_{lm}

$$\mathcal{R}_{\mathbf{k}} = -\left[\frac{H}{\dot{\phi}_0}\phi_{\mathbf{k}}\right]_{t=t_*(k)}$$

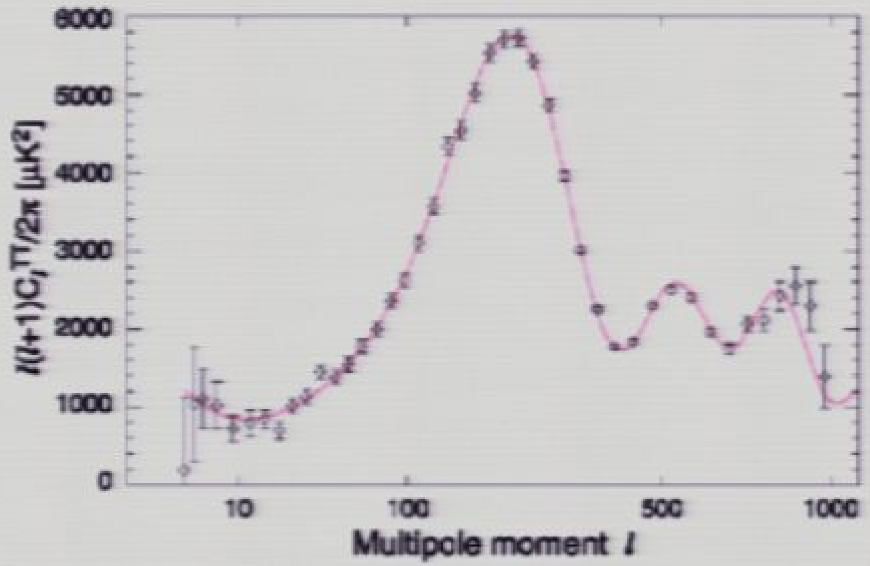
$$a_{lm} = \frac{i^l}{2\pi^2}\int d^3\mathbf{k}\ \mathcal{T}(k,l)\mathcal{R}_{\mathbf{k}}Y_{lm}(\hat{\mathbf{k}})$$

$$\frac{\Delta T(\theta,\phi)}{\bar{T}} = \sum_{l=2}^{\infty}\sum_{m=-l}^{+l} a_{lm}Y_{lm}(\theta,\phi)$$

Angular power spectrum $C_l \equiv \left\langle |a_{lm}|^2 \right\rangle$

$$C_l = \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k}\, \mathcal{T}^2(k,l) \mathcal{P}_{\mathcal{R}}(k)$$

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{4\pi k^3}{V} \left\langle |\mathcal{R}_{\mathbf{k}}|^2 \right\rangle$$



Quantum equilibrium

$$\left\langle |\phi_{\mathbf{k}}|^2 \right\rangle_{\mathrm{QT}} = \frac{V}{2(2\pi)^3} \frac{H^2}{k^3} \qquad \mathcal{P}_\phi^{\mathrm{QT}}(k) \equiv \frac{4\pi k^3}{V} \left\langle |\phi_{\mathbf{k}}|^2 \right\rangle_{\mathrm{QT}} = \frac{H^2}{4\pi^2}$$

Quantum nonequilibrium (no relaxation during inflation; product state)

$$\left\langle |\phi_{\mathbf{k}}|^2 \right\rangle = \left\langle |\phi_{\mathbf{k}}|^2 \right\rangle_{\mathrm{QT}} \xi(k) \qquad \mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R}}^{\mathrm{QT}}(k) \xi(k)$$

Can set empirical limits on $\xi(k)$ (Valentini 2007, 2008)

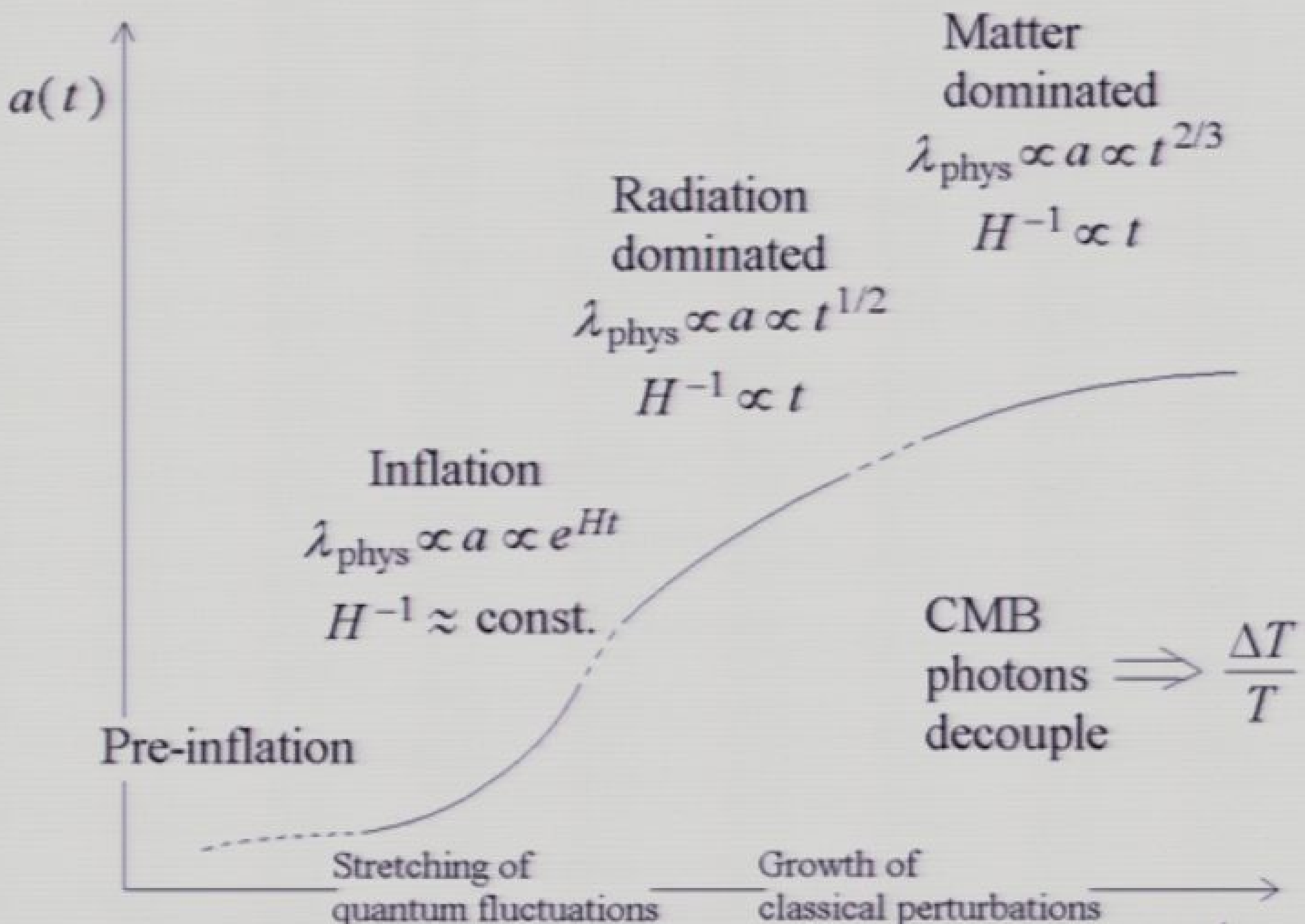
But can we *predict* something about $\xi(k)$?

Can we *predict* something about $\xi(k)$?

(Work in progress)

One possible strategy:

- Consider a pre-inflationary (radiation-dominated) era
- Derive constraints on relic nonequilibrium from that era



Plausible scenario:

- Pre-inflation: nonequilibrium at super-Hubble wavelengths (all $\lambda_{\rm phys} > H^{-1}$ at sufficiently early times)
- Some nonequilibrium modes enter the Hubble radius, and do not completely relax by the time inflation begins
- Larger wavelengths enter later, less likely to relax before inflation begins
- Nonequilibrium possible only for λ larger than infra-red cutoff $\lambda_{\rm c}$
- Try to predict $\lambda_{\rm c}$ (depends on cosmology)

There is some (weak) evidence for an infra-red cutoff
In the primordial power spectrum

Relic Nonequilibrium Particles

(to be detected today)

(Work in progress: rough scenario only)

Lower bound

$$\lambda_{\mathrm{phys}}(t_f) > 2\pi H_f^{-1}\ln(t_f/t_i) \quad \text{or} \quad \lambda_{\mathrm{phys}}(t_f) > 4\pi H_f^{-1}\ln(T_i/T_f) \; .$$

To maximise our chances, minimise the right-hand-side

Want t_f as small as possible (subject to the constraint that further relaxation can be neglected)

Take t_f to be time t_{dec} of decoupling

Nonequilibrium present at t_{dec} *might* persist until much later

Then have lower bound $\lambda_{\mathrm{phys}}(t_{\mathrm{dec}}) > 4\pi H_{\mathrm{dec}}^{-1}\ln(k_{\mathrm{B}}T_i/k_{\mathrm{B}}T_{\mathrm{dec}})$

Rough estimates using the lower bound

$$\lambda_{\text{phys}}(t_{\text{dec}}) > 4\pi H_{\text{dec}}^{-1} \ln(k_B T_i / k_B T_{\text{dec}})$$

$\lambda_{\text{phys}}(t_{\text{dec}}) = a_{\text{dec}}\lambda$, where $a_{\text{dec}} = T_0/T_{\text{dec}}$ (with $T_0 \simeq 2.7$ K

$H_{\text{dec}}^{-1} = 2t_{\text{dec}}$, where t_{dec} may be expressed in terms of T_{dec} using

$$t \sim (1\text{ s}) \left(\frac{1\text{ MeV}}{k_B T}\right)^2$$

$$\lambda \gtrsim 8\pi c (1\text{ s}) \left(\frac{1\text{ MeV}}{k_B T_{\text{dec}}}\right) \left(\frac{1\text{ MeV}}{k_B T_0}\right) \ln\left(\frac{k_B T_i}{k_B T_{\text{dec}}}\right)$$

or

$$\lambda \gtrsim (3.3 \times 10^{21}\text{ cm}) \left(\frac{1\text{ MeV}}{k_B T_{\text{dec}}}\right) \ln\left(\frac{k_B T_i}{k_B T_{\text{dec}}}\right)$$

Lower bound on wavelength *today*,
at which nonequilibrium *could* be found.

Relic Nonequilibrium Particles

(to be detected today)

(Work in progress: rough scenario only)

Lower bound

$\lambda_{\mathrm{phys}}(t_f) > 2\pi H_f^{-1} \ln(t_f/t_i)$ or $\lambda_{\mathrm{phys}}(t_f) > 4\pi H_f^{-1} \ln(T_i/T_f)$.

To maximise our chances, minimise the right-hand-side

Want t_f as small as possible (subject to the constraint that further relaxation can be neglected)

Take t_f to be time t_{dec} of decoupling

Nonequilibrium present at t_{dec} *might* persist until much later

Then have lower bound $\lambda_{\mathrm{phys}}(t_{\mathrm{dec}}) > 4\pi H_{\mathrm{dec}}^{-1} \ln(k_{\mathrm{B}} T_i / k_{\mathrm{B}} T_{\mathrm{dec}})$

standard Friedmann cosmology with no inflationary period

initial conditions at the Planck era $k_B T_i \sim k_B T_P \sim 10^{19}$ GeV

(Or: nonequilibrium relics could be produced by inflaton decay.)

Photons decouple from matter at $k_B (T_{dec})_\gamma \sim 0.3$ eV

$\lambda_\gamma \gtrsim 0.7 \times 10^{30}$ cm (Ridiculous)

neutrinos, which decouple at $k_B (T_{dec})_\nu \sim 1$ MeV

$\lambda_\nu \gtrsim 1.7 \times 10^{23}$ cm $\simeq 5.5 \times 10^4$ pc (or $\sim 10^5$ light years) (Hopeless)

Gravitons decouple at a temperature $(T_{dec})_g \lesssim T_P$

$k_B (T_{dec})_g \equiv x_g (k_B T_P) \simeq x_g (10^{19}$ GeV$)$ $\quad x_g \lesssim 1$

$\lambda_g \gtrsim (0.3$ cm$)(1/x_g) \ln(1/x_g)$ $\quad \lambda_{max}(1$ K$) \simeq 0.3$ cm

(But can't detect relic gravitons. Still hopeless)

Relic Nonequilibrium Particles

(to be detected today)

(Work in progress: rough scenario only)

Lower bound

$\lambda_{\mathrm{phys}}(t_f) > 2\pi H_f^{-1} \ln(t_f/t_i)$ or $\lambda_{\mathrm{phys}}(t_f) > 4\pi H_f^{-1} \ln(T_i/T_f)$.

To maximise our chances, minimise the right-hand-side

Want t_f as small as possible (subject to the constraint that further relaxation can be neglected)

Take t_f to be time t_{dec} of decoupling

Nonequilibrium present at t_{dec} *might* persist until much later

Then have lower bound $\lambda_{\mathrm{phys}}(t_{\mathrm{dec}}) > 4\pi H_{\mathrm{dec}}^{-1} \ln(k_{\mathrm{B}}T_i/k_{\mathrm{B}}T_{\mathrm{dec}})$

standard Friedmann cosmology with no inflationary period

initial conditions at the Planck era $k_B T_i \sim k_B T_P \sim 10^{19}$ GeV

(Or: nonequilibrium relics could be produced by inflaton decay.)

Photons decouple from matter at $k_B(T_{dec})_\gamma \sim 0.3$ eV

$\lambda_\gamma \gtrsim 0.7 \times 10^{30}$ cm (Ridiculous)

neutrinos, which decouple at $k_B(T_{dec})_\nu \sim 1$ MeV

$\lambda_\nu \gtrsim 1.7 \times 10^{23}$ cm $\simeq 5.5 \times 10^4$ pc (or $\sim 10^5$ light years) (Hopeless)

Gravitons decouple at a temperature $(T_{dec})_g \lesssim T_P$

$k_B(T_{dec})_g \equiv x_g(k_B T_P) \simeq x_g(10^{19}$ GeV$)$ $x_g \lesssim 1$

$\lambda_g \gtrsim (0.3$ cm$)(1/x_g) \ln(1/x_g)$ $\lambda_{max}(1$ K$) \simeq 0.3$ cm

(But can't detect relic gravitons. Still hopeless)

Relic Nonequilibrium Particles

(to be detected today)

(Work in progress: rough scenario only)

Lower bound

$\lambda_{\mathrm{phys}}(t_f) > 2\pi H_f^{-1} \ln(t_f/t_i)$ or $\lambda_{\mathrm{phys}}(t_f) > 4\pi H_f^{-1} \ln(T_i/T_f)$.

To maximise our chances, minimise the right-hand-side

Want t_f as small as possible (subject to the constraint that further relaxation can be neglected)

Take t_f to be time t_{dec} of decoupling

Nonequilibrium present at t_{dec} *might* persist until much later

Then have lower bound $\lambda_{\mathrm{phys}}(t_{\mathrm{dec}}) > 4\pi H_{\mathrm{dec}}^{-1} \ln(k_{\mathrm{B}}T_i/k_{\mathrm{B}}T_{\mathrm{dec}})$

Rough estimates using the lower bound

$$\lambda_{\mathbf{phys}}(t_{\mathbf{dec}}) > 4\pi H_{\mathbf{dec}}^{-1} \ln(k_{\mathrm{B}} T_i / k_{\mathrm{B}} T_{\mathbf{dec}})$$

$\lambda_{\mathrm{phys}}(t_{\mathrm{dec}}) = a_{\mathrm{dec}}\lambda$, where $a_{\mathrm{dec}} = T_0/T_{\mathrm{dec}}$ (with $T_0 \simeq 2.7\ \mathrm{K}$

$H_{\mathrm{dec}}^{-1} = 2t_{\mathrm{dec}}$, where t_{dec} may be expressed in terms of T_{dec} using

$$t \sim (1\ \mathrm{s})\left(\frac{1\ \mathrm{MeV}}{k_{\mathrm{B}}T}\right)^2$$

$$\lambda \gtrsim 8\pi c(1\ \mathrm{s})\left(\frac{1\ \mathrm{MeV}}{k_{\mathrm{B}}T_{\mathrm{dec}}}\right)\left(\frac{1\ \mathrm{MeV}}{k_{\mathrm{B}}T_0}\right)\ln\left(\frac{k_{\mathrm{B}}T_i}{k_{\mathrm{B}}T_{\mathrm{dec}}}\right)$$

or

$$\lambda \gtrsim (3.3 \times 10^{21}\ \mathrm{cm})\left(\frac{1\ \mathrm{MeV}}{k_{\mathrm{B}}T_{\mathrm{dec}}}\right)\ln\left(\frac{k_{\mathrm{B}}T_i}{k_{\mathrm{B}}T_{\mathrm{dec}}}\right)$$

Lower bound on wavelength *today*,
at which nonequilibrium *could* be found.

standard Friedmann cosmology with no inflationary period

initial conditions at the Planck era $k_B T_i \sim k_B T_P \sim 10^{19}$ GeV

(Or: nonequilibrium relics could be produced by inflaton decay.)

Photons decouple from matter at $k_B (T_{dec})_\gamma \sim 0.3$ eV

$\lambda_\gamma \gtrsim 0.7 \times 10^{30}$ cm (Ridiculous)

neutrinos, which decouple at $k_B (T_{dec})_\nu \sim 1$ MeV

$\lambda_\nu \gtrsim 1.7 \times 10^{23}$ cm $\simeq 5.5 \times 10^4$ pc (or $\sim 10^5$ light years) (Hopeless)

Gravitons decouple at a temperature $(T_{dec})_g \lesssim T_P$

$k_B (T_{dec})_g \equiv x_g (k_B T_P) \simeq x_g (10^{19}$ GeV$)$ $x_g \lesssim 1$

$\lambda_g \gtrsim (0.3$ cm$)(1/x_g) \ln(1/x_g)$ $\lambda_{max}(1$ K$) \simeq 0.3$ cm

(But can't detect relic gravitons. Still hopeless)

Rough estimates using the lower bound

$$\lambda_{\mathbf{phys}}(t_{\mathbf{dec}}) > 4\pi H_{\mathbf{dec}}^{-1} \ln(k_{\mathrm{B}} T_i / k_{\mathrm{B}} T_{\mathbf{dec}})$$

$\lambda_{\mathrm{phys}}(t_{\mathrm{dec}}) = a_{\mathrm{dec}}\lambda$, where $a_{\mathrm{dec}} = T_0/T_{\mathrm{dec}}$ (with $T_0 \simeq 2.7$ K

$H_{\mathrm{dec}}^{-1} = 2t_{\mathrm{dec}}$, where t_{dec} may be expressed in terms of T_{dec} using

$$t \sim (1\ \mathrm{s}) \left(\frac{1\ \mathrm{MeV}}{k_{\mathrm{B}} T}\right)^2$$

$$\lambda \gtrsim 8\pi c (1\ \mathrm{s}) \left(\frac{1\ \mathrm{MeV}}{k_{\mathrm{B}} T_{\mathrm{dec}}}\right) \left(\frac{1\ \mathrm{MeV}}{k_{\mathrm{B}} T_0}\right) \ln\left(\frac{k_{\mathrm{B}} T_i}{k_{\mathrm{B}} T_{\mathrm{dec}}}\right)$$

or

$$\lambda \gtrsim (3.3 \times 10^{21}\ \mathrm{cm}) \left(\frac{1\ \mathrm{MeV}}{k_{\mathrm{B}} T_{\mathrm{dec}}}\right) \ln\left(\frac{k_{\mathrm{B}} T_i}{k_{\mathrm{B}} T_{\mathrm{dec}}}\right)$$

Lower bound on wavelength *today,*
at which nonequilibrium *could* be found.

standard Friedmann cosmology with no inflationary period

initial conditions at the Planck era $k_{\rm B}T_i \sim k_{\rm B}T_{\rm P} \sim 10^{19}$ GeV

(Or: nonequilibrium relics could be produced by inflaton decay.)

Photons decouple from matter at $k_{\rm B}(T_{\rm dec})_\gamma \sim 0.3$ eV

$\lambda_\gamma \gtrsim 0.7 \times 10^{30}$ cm (Ridiculous)

neutrinos, which decouple at $k_{\rm B}(T_{\rm dec})_\nu \sim 1$ MeV

$\lambda_\nu \gtrsim 1.7 \times 10^{23}$ cm $\simeq 5.5 \times 10^4$ pc (or $\sim 10^5$ light years) (Hopeless)

Gravitons decouple at a temperature $(T_{\rm dec})_g \lesssim T_{\rm P}$

$k_{\rm B}(T_{\rm dec})_g \equiv x_g(k_{\rm B}T_{\rm P}) \simeq x_g(10^{19}\ {\rm GeV}) \quad x_g \lesssim 1$

$\lambda_g \gtrsim (0.3\ {\rm cm})(1/x_g)\ln(1/x_g) \quad \lambda_{\max}(1\ {\rm K}) \simeq 0.3$ cm

(But can't detect relic gravitons. Still hopeless)

Ray of hope:

unstable gravitino $\tilde{G}$,

Decouples very early, decay products (e.g. photons) might be observable today

$$k_\mathrm{B}(T_\mathrm{dec})_{\tilde{G}} \equiv x_{\tilde{G}}(k_\mathrm{B}T_\mathrm{P}) \approx (1\ \mathrm{TeV})\left(\frac{g_*}{230}\right)^{1/2}\left(\frac{m_{\tilde{G}}}{10\ \mathrm{keV}}\right)^2\left(\frac{1\ \mathrm{TeV}}{m_{gl}}\right)^2$$

g_* is the number of spin degrees of freedom at the temperature $(T_\mathrm{dec})_{\tilde{G}}$

m_{gl} is the gluino mass

$m_{\tilde{G}}$ is the gravitino mass

$$\lambda_{\tilde{G}} \gtrsim (0.3\ \mathrm{cm})(1/x_{\tilde{G}})\ln(1/x_{\tilde{G}})$$

Illustration: take $(g_*/230)^{1/2} \sim 1$ and $(1\ \mathrm{TeV}/m_{gl})^2 \sim 1$.

$$x_{\tilde{G}} \approx \left(\frac{m_{\tilde{G}}}{10^3\ \mathrm{GeV}}\right)^2$$

If, for example, $m_{\tilde{G}} \approx 100$ GeV, then $x_{\tilde{G}} \approx 10^{-2}$ and $\lambda_{\tilde{G}} \gtrsim 140$ cm.

Low energies, perhaps accessible (dark matter decay?)

Inflaton decay

'reheating' perturbative decay of the inflaton

Require:

particles are created at a temperature below their decoupling temperature

Gravitinos

copiously produced by inflaton decay

significant component of dark matter ?

Decay photons? Violations of Malus' law?

Ray of hope:

unstable gravitino $\tilde{G}$,

Decouples very early, decay products (e.g. photons) might be observable today

$$k_{\rm B}(T_{\rm dec})_{\tilde{G}} \equiv x_{\tilde{G}}(k_{\rm B}T_{\rm P}) \approx (1\ {\rm TeV})\left(\frac{g_*}{230}\right)^{1/2}\left(\frac{m_{\tilde{G}}}{10\ {\rm keV}}\right)^2\left(\frac{1\ {\rm TeV}}{m_{gl}}\right)^2$$

g_* is the number of spin degrees of freedom at the temperature $(T_{\rm dec})_{\tilde{G}}$

m_{gl} is the gluino mass

$m_{\tilde{G}}$ is the gravitino mass

$$\lambda_{\tilde{G}} \gtrsim (0.3\ {\rm cm})(1/x_{\tilde{G}})\ln(1/x_{\tilde{G}})$$

Illustration: take $(g_*/230)^{1/2} \sim 1$ and $(1\ {\rm TeV}/m_{gl})^2 \sim 1$.

$$x_{\tilde{G}} \approx \left(\frac{m_{\tilde{G}}}{10^3\ {\rm GeV}}\right)^2$$

If, for example, $m_{\tilde{G}} \approx 100$ GeV, then $x_{\tilde{G}} \approx 10^{-2}$ and $\lambda_{\tilde{G}} \gtrsim 140$ cm.

Low energies, perhaps accessible (dark matter decay?)

Inflaton decay

'reheating'. perturbative decay of the inflaton

Require:

particles are created at a temperature below their decoupling temperature

Gravitinos

copiously produced by inflaton decay

significant component of dark matter ?

Decay photons? Violations of Malus' law?

End of slide show, click to exit.