Title: Observable consequences of small field models of inflation

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Abstract: TBA

CMB Observables of Small Field Models of Inflation

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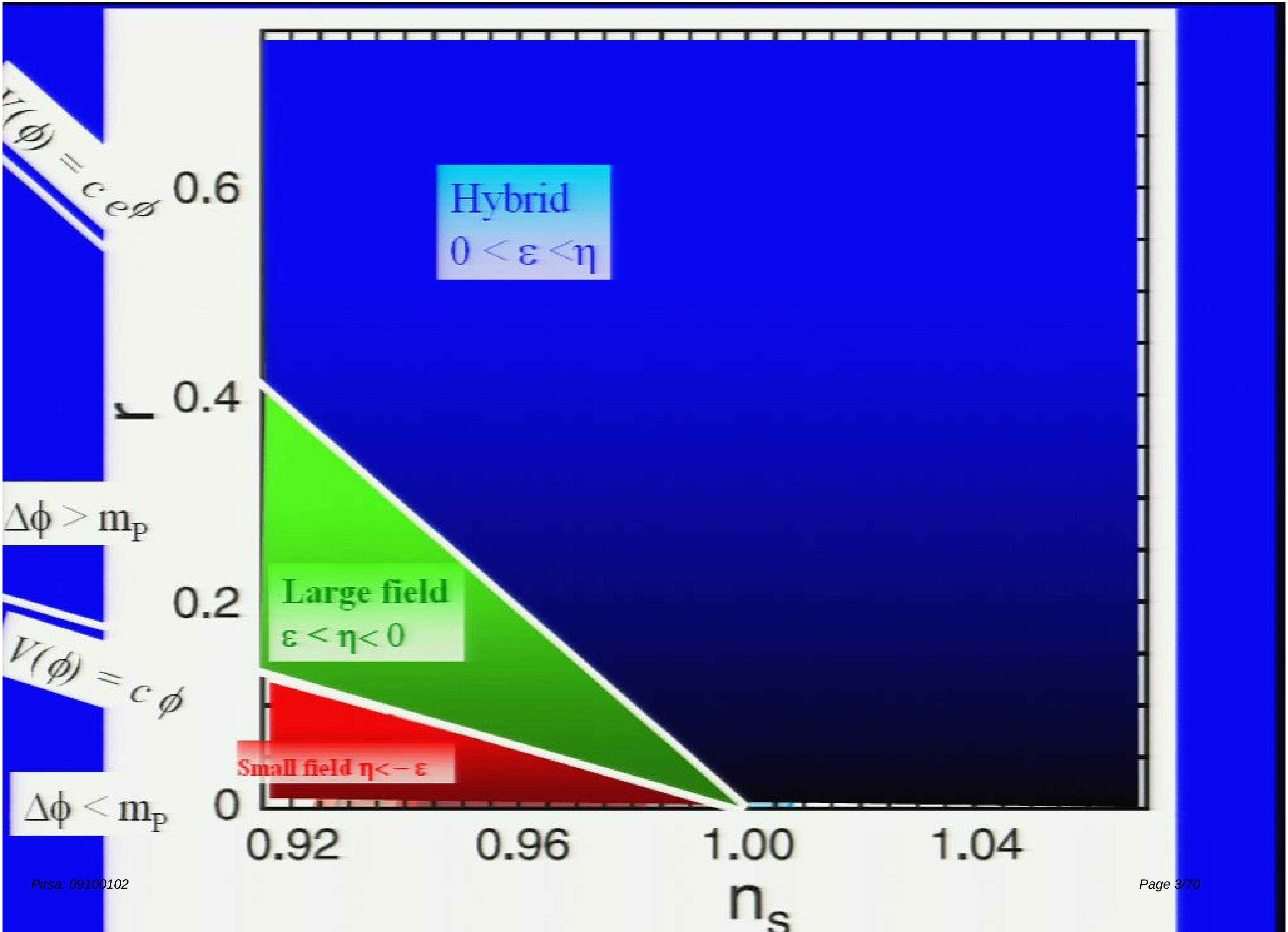
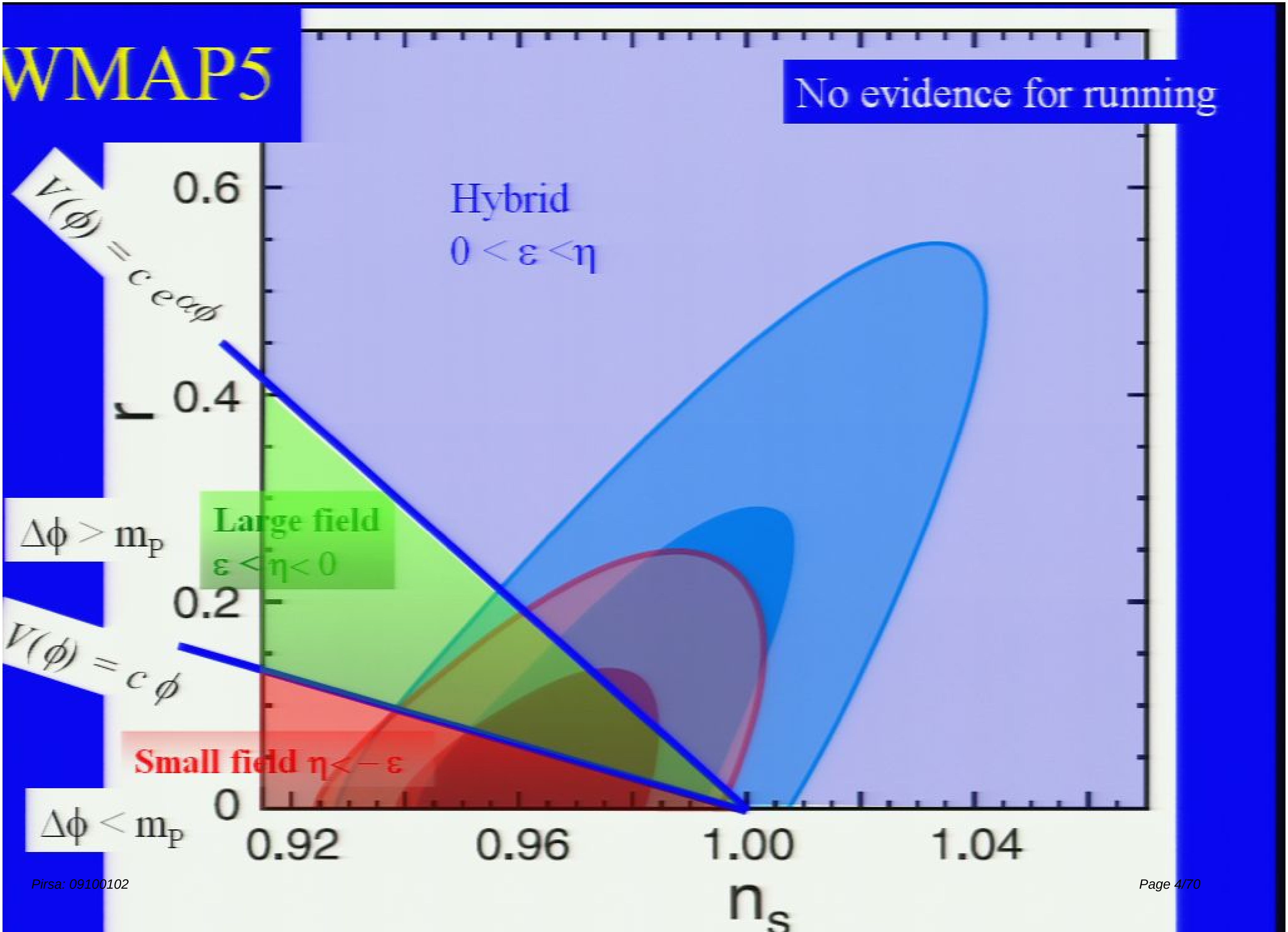
אוניברסיטת בן-גוריון

I. Ben-Dayan 0907.2384
+ in progress
I. Ben-Dayan, S. de Alwis 0802.3160

- Small field models of inflation
- Predictions for the CMB:
  - Simplest models: $n_S<1$, $r_{0.01}<<1$, $\alpha_{0.05}<<1$
  - New class: n_S , $r_{0.01}$, $\alpha_{0.05}$ all allowed values
- Designing small field SUGRA models
- Relevance to string theory



$V(\phi) = c\, e^{\phi}$
0.6
Hybrid
$0 < \varepsilon < \eta$
r
0.4
$\Delta\phi > m_P$
0.2
Large field
$\varepsilon < \eta < 0$
$V(\phi) = c\,\phi$
Small field $\eta < -\varepsilon$
$\Delta\phi < m_P$
0
0.92
0.96
1.00
1.04
n_s



WMAP5
No evidence for running
Hybrid
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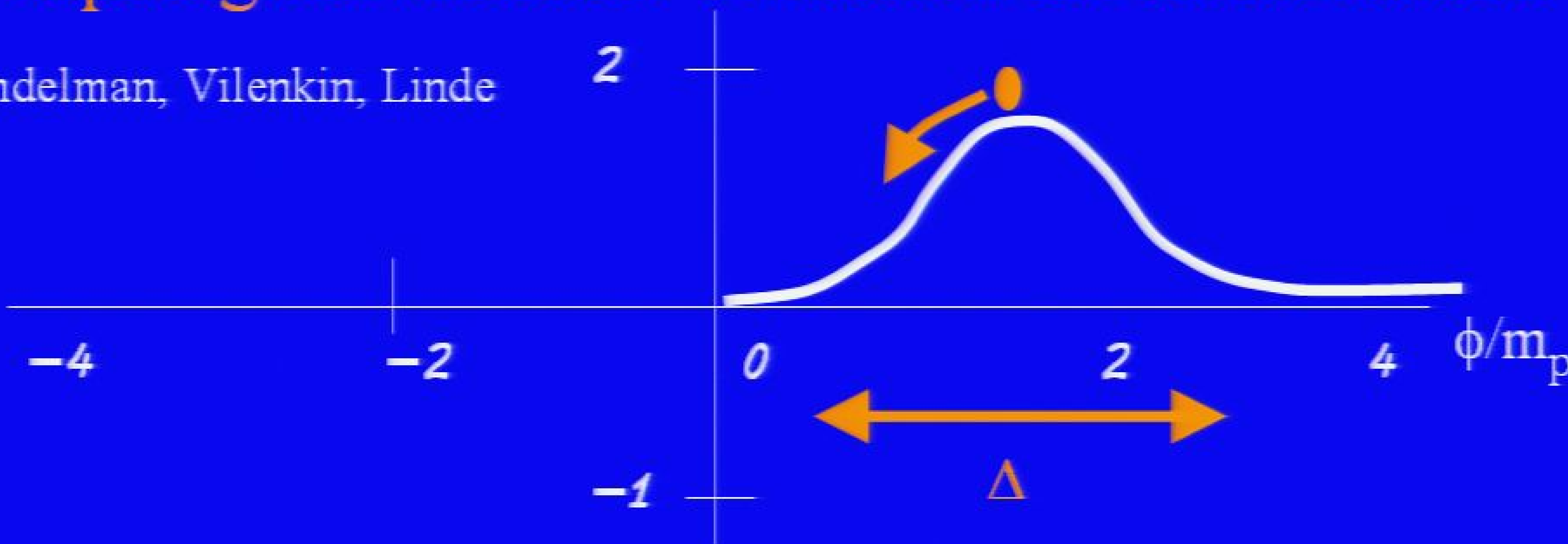
hep-th/0205042
hep-th/0212344
S. de Alwis, E. Novak

(My) preferred models of modular inflation: small field models

- "Topological inflation": inflation off a flat feature

Guendelman, Vilenkin, Linde



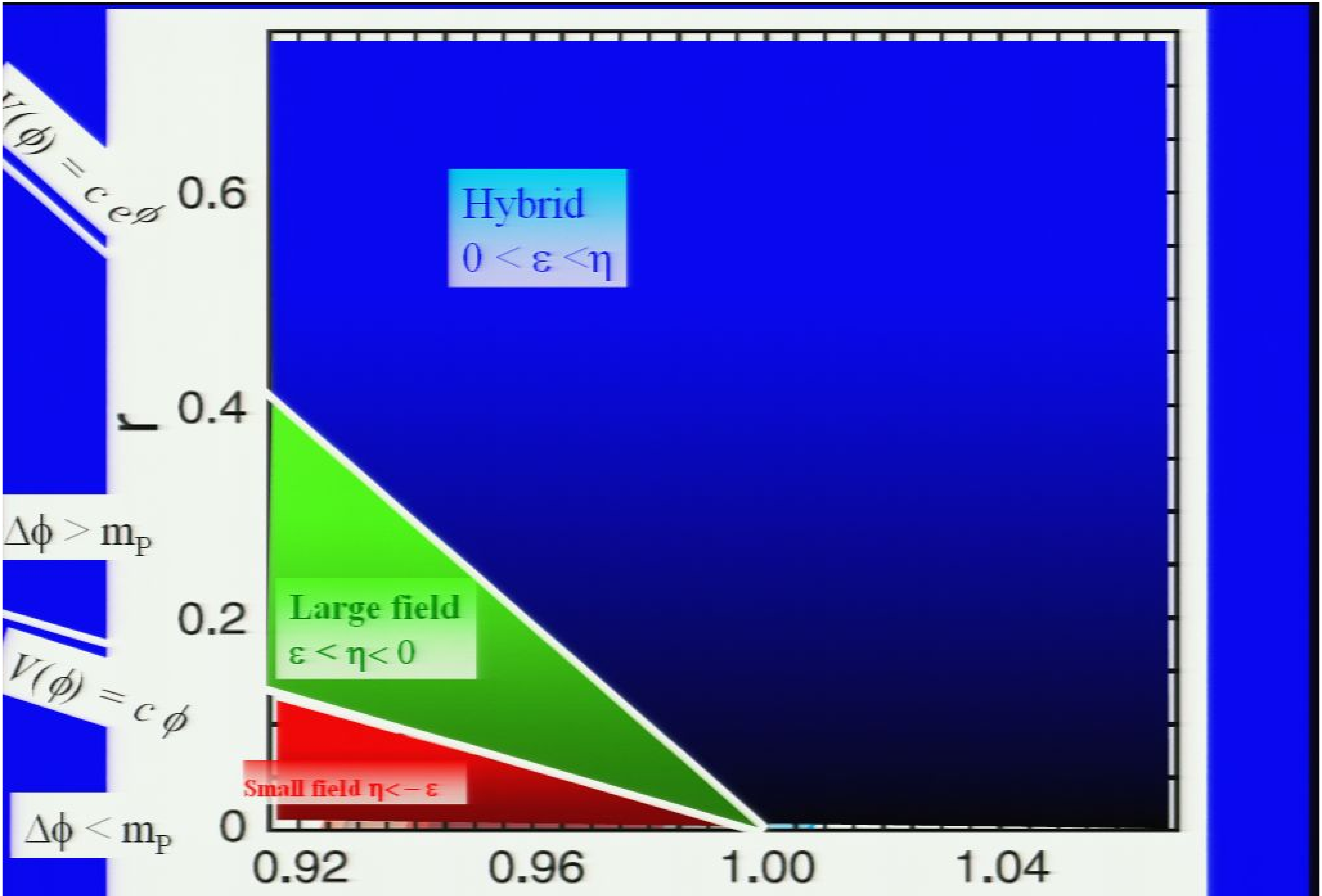
δ – wall thickness in space

$(\Delta/\delta)^2 \sim \Lambda^4$

$H^2 \sim 1/3\ \Lambda^4/m_p^{\ 2}$

Inflation ⇔ $\delta H > 1$ ⇔ $\Delta > m_p$

$m_p = \frac{1}{\ \ } = 2.4 \times 10^{18} GeV$



Hybrid
0 < ε < η
Large field
ε < η < 0
Small field η < − ε
$V(\phi) = c\, e^{\phi}$
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Δφ > m_P
Δφ < m_P
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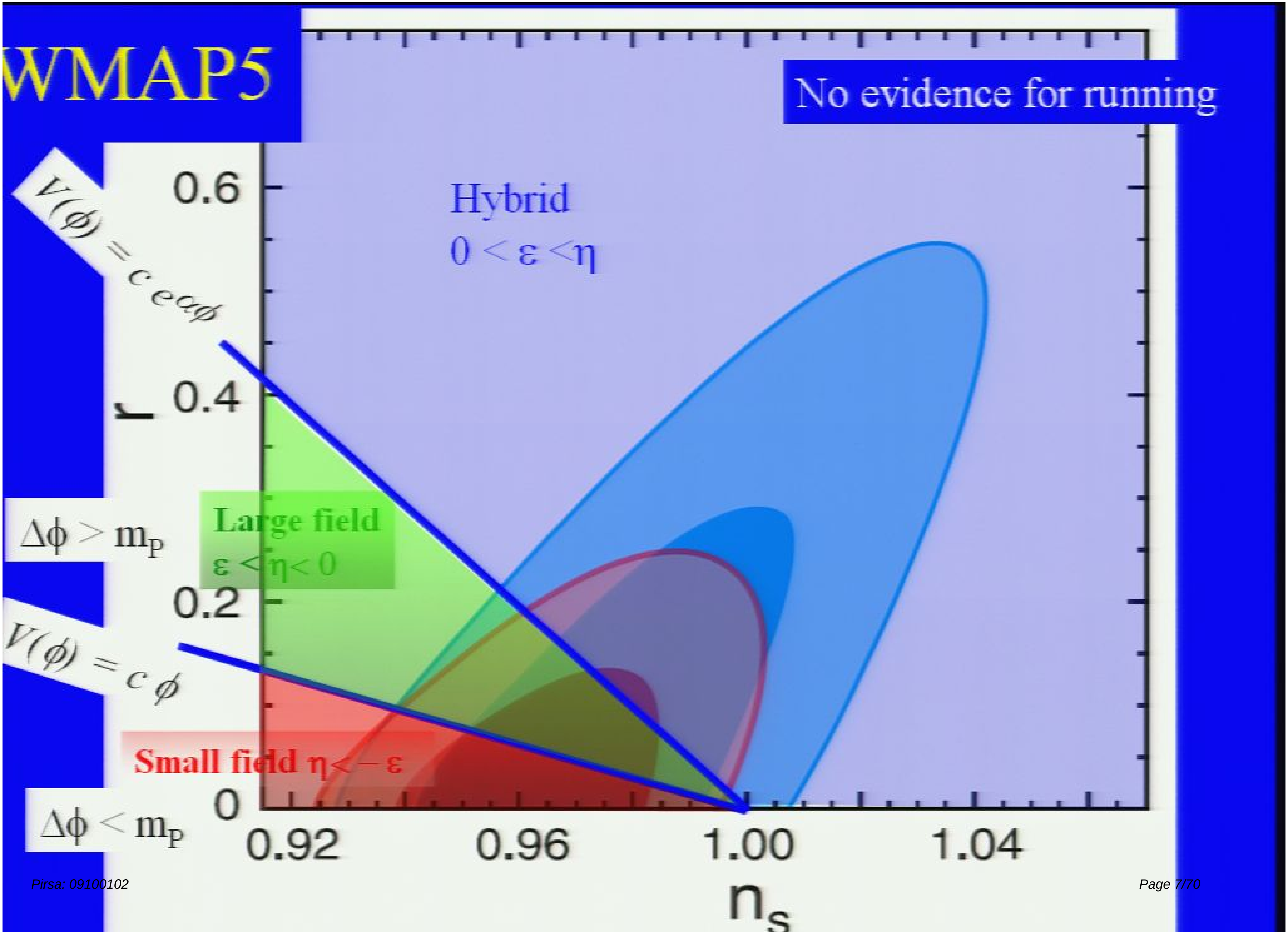

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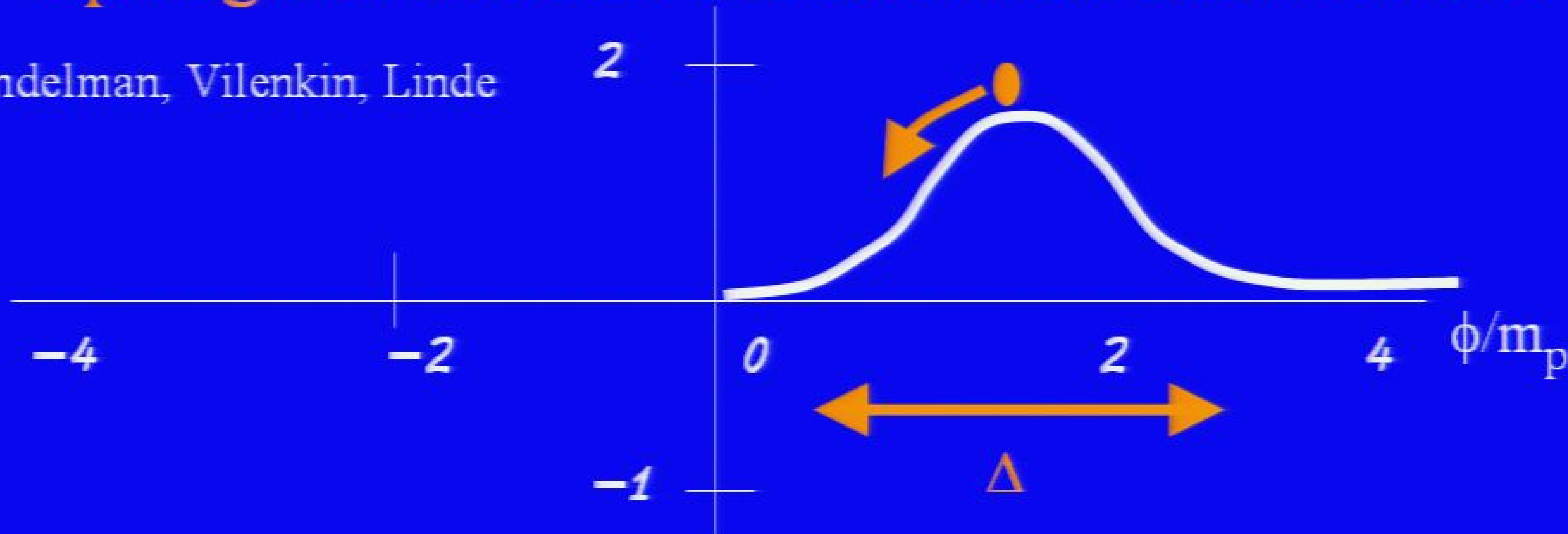

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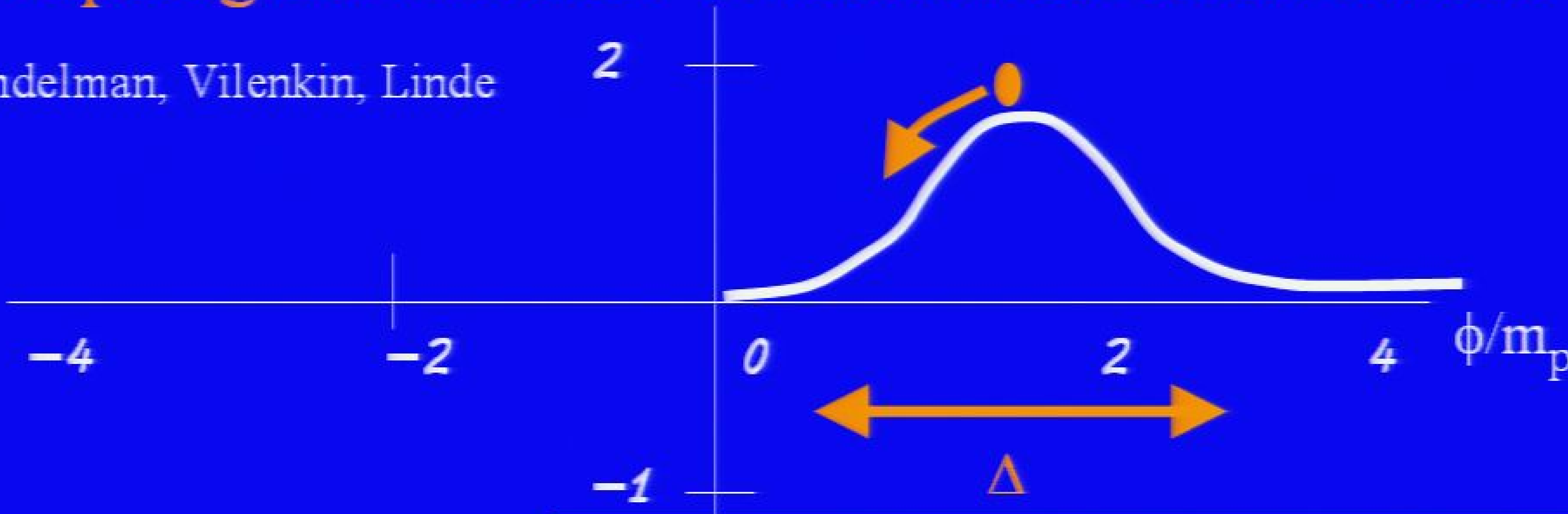

Enough inflation ⇔ *V''/V<1/50*

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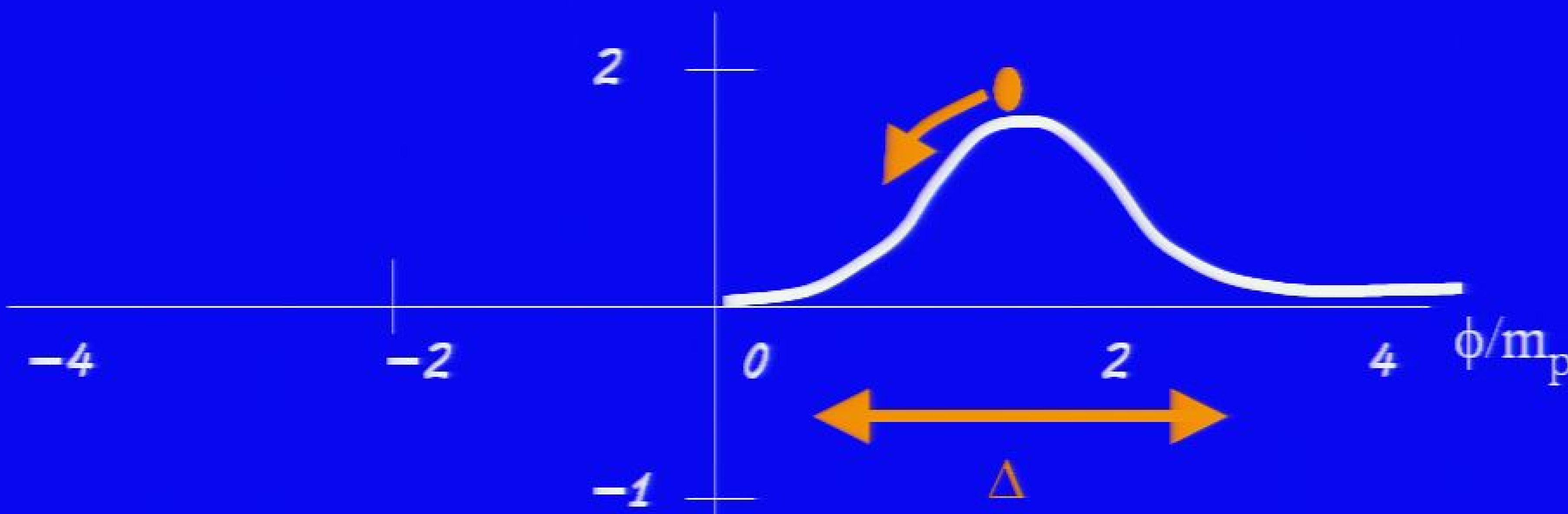
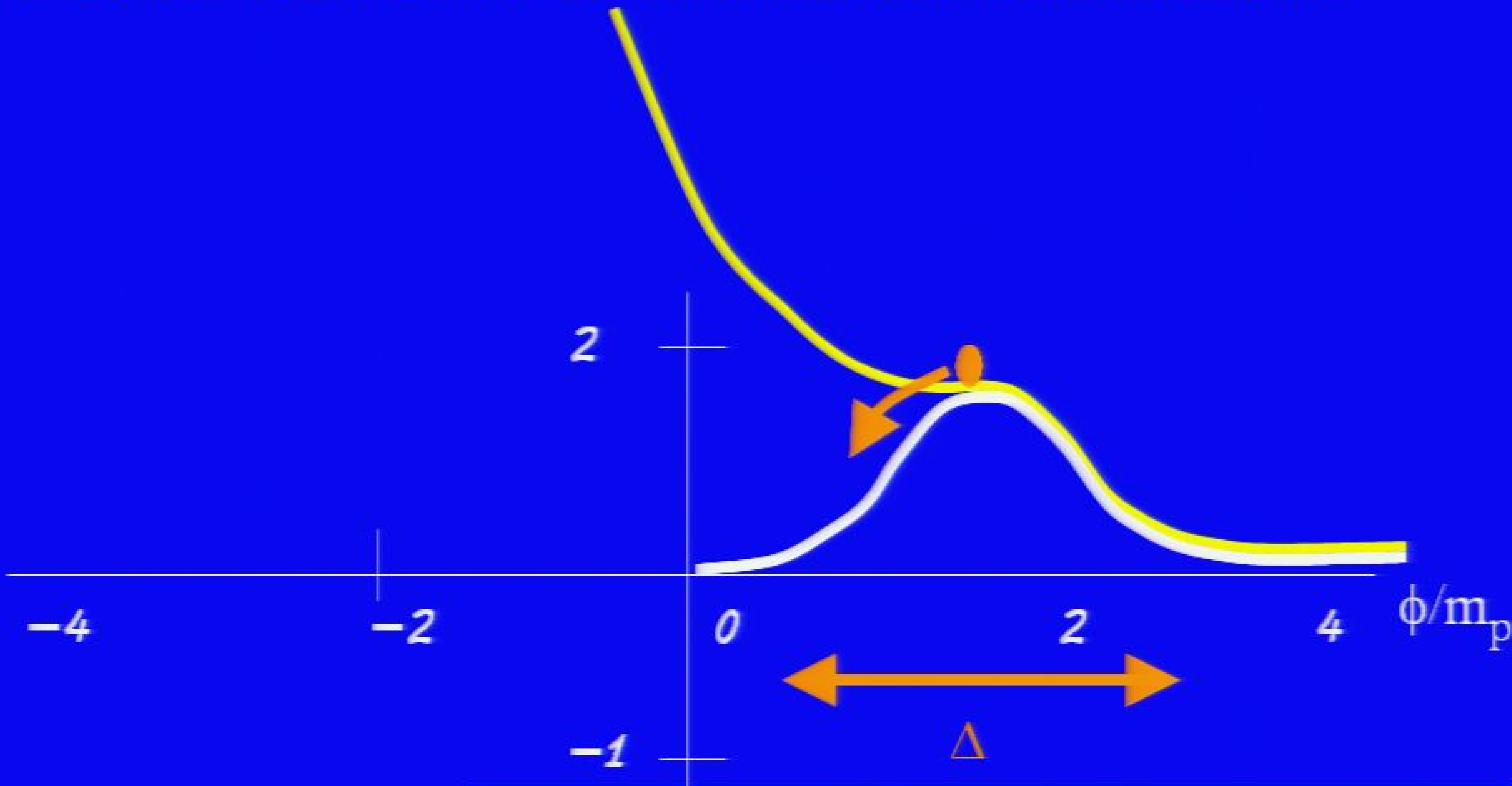


Enough inflation ⇔ $V''/V<1/50$

(My) preferred models of modular inflation: small field models

hep-th/0408160
S. de Alwis, P. Martens

- Another version of inflation off a flat feature



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Models of inflation: Background

de Sitter phase $\rho + p << \rho$ ➔ $H \sim const.$

Parametrize the deviation from constant H

by the value of the field $\varepsilon(\varphi) = \frac{m_p^{\ 2}}{2}\left(\frac{V'}{V}\right)^2$ $\eta(\varphi) = m_p^{\ 2}\frac{V''}{V}$

$\xi^2(\varphi) = m_p^{\ 2}\frac{V'''V'}{V^2}$

Or by the number of e-folds

$$N(\varphi) = \int_t^{t_{ei}} d\log a(t) = \int_t^{t_{ei}} H dt = \int_\varphi^{\varphi_{ei}} \frac{H}{\dot{\varphi}} d\varphi = \frac{1}{\sqrt{2}m_p}\int_{\varphi_{ei}}^{\varphi} \frac{1}{\sqrt{\varepsilon(\varphi)}} d\varphi$$

Inflation ends when ε = 1

Models of inflation:Perturbations

- Spectrum of scalar perturbations

$$P_{\Re}(k) = \frac{2}{\pi}\left(\frac{H}{m_p}\right)^2 \frac{1}{\varepsilon}\Big|_{k=aH}$$

$$n - 1 \equiv \frac{d\ln P_{\mathcal{R}}}{d\ln k}$$

$$\alpha = \frac{dn_S}{d\ln k}$$

- Spectrum of tensor perturbations

$$P_T(k) = \frac{2}{\pi}\left(\frac{H}{m_p}\right)^2 \Big|_{k=aH}$$

Spectral indices

$$n_S = 1 - 6\epsilon_{CMB} + 2\eta_{CMB}$$

$$n_T \simeq -2\epsilon = -2\frac{P_T}{P_{\mathcal{R}}}$$

$$\alpha = -16\epsilon\eta + 24\epsilon^2 + 2\xi^2$$

Tensor to scalar ratio (many definitions)
r is determined by P_T/P_R
("current canonical" $r = 16\,\varepsilon$)

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CMB observables determined by quantities ~ 60 efolds before the end of inflation

Small field models, standard lore: No Observable GW

$$N(\varphi)=\int_t^{t_{ei}} d\log a(t)=\int_t^{t_{ei}} H\,dt=\int_\varphi^{\varphi_{ei}} \frac{H}{\dot\varphi}\,d\varphi=\frac{1}{\sqrt{2}m_p}\int_{\varphi_{ei}}^{\varphi} \frac{1}{\sqrt{\varepsilon(\varphi)}}\,d\varphi$$

$r = 16\,\varepsilon \rightarrow \dfrac{dN}{d\phi} = \sqrt{\dfrac{8}{r}}$

If $\varepsilon \sim$ const. $\rightarrow \; r \simeq 8\left(\dfrac{\Delta\phi}{N_{CMB}}\right)^2$

"Lyth theorem" $\Delta\phi \sim 1 \rightarrow r_{0.01} > 1$

In practice need $\Delta\phi \sim 10$

Small field models, standard lore: No Observable SIR

Easther+Peiris, 06040214:

"Thus, a definitive observation of a large negative running would imply that any inflationary phase requires multiple fields or the breakdown of slow roll. Alternatively, if single field, slow roll inflation is sources the primordial fluctuations, we can expect the observed running to move much closer to zero as the CMB is measured more accurately at small angular scales."

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Simple example: $V(\phi) = \Lambda^4 (1 - a_p \phi^p)$ $\phi_{END} \lesssim 1$

I. Ben-Dayan, S. de Alwis 0802.3160

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$$(n_S)_{CMB} = 1 - \frac{2}{N_{CMB}} \frac{p-1}{p-2}$$

$$\eta_{CMB} = -\frac{1}{N_{CMB}} \frac{p-1}{p-2}$$

$$\epsilon_N = \frac{1}{2} \left(\frac{1}{p a_p} \right)^{\frac{2}{p-2}} \left(\frac{1}{(p-2)N} \right)^{2\frac{p-1}{p-2}}$$

$$\epsilon_{N,max} = e^{-2\frac{(p-1)^2}{p-2}}$$

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$$\alpha = -16\epsilon\eta + 24\epsilon^2 + 2\xi^2$$

$$\alpha_N = 2\frac{(p-1)}{(p-2)} \frac{1}{N^2}$$

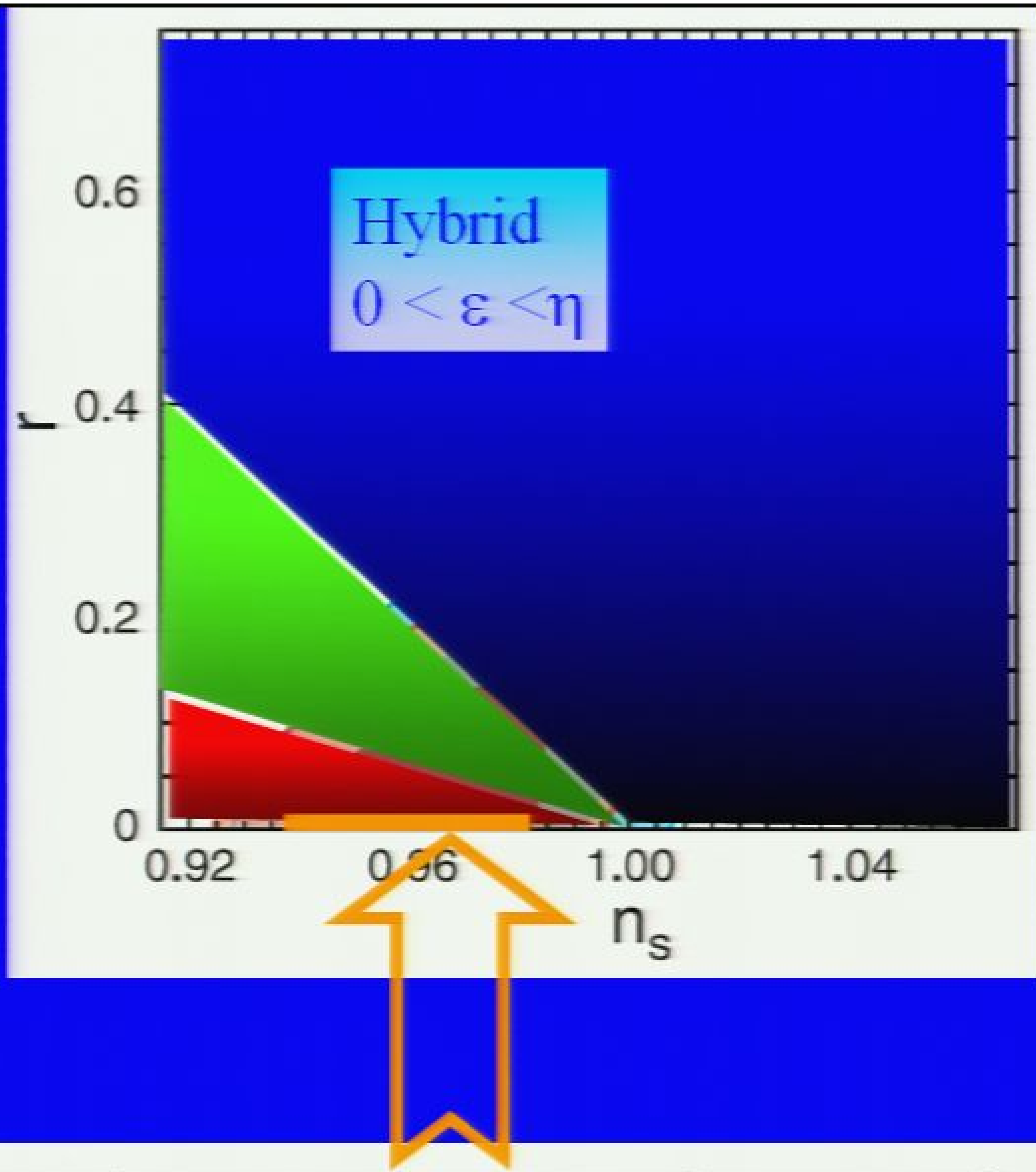
$$\alpha_{CMB} = 2.8 \times 10^{-4} \frac{(p-1)}{(p-2)} \left(\frac{60}{N_{CMB}} \right)^2$$

Simple example:

$$V(\phi) = \Lambda^4 (1 - a_p \phi^p)$$

$$\varepsilon_{CMB} \ll \eta_{CMB}^2$$

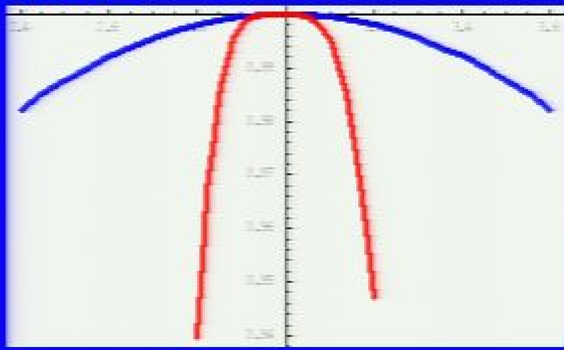
$$\alpha \approx \eta_{CMB}^2$$



p	3	4	5	7	10	$p \to \infty$
r	3.1×10^{-7}	3.3×10^{-6}	6.1×10^{-6}	7.9×10^{-6}	6.8×10^{-6}	0
α	5.6×10^{-4}	4.2×10^{-4}	3.7×10^{-4}	3.4×10^{-4}	3.4×10^{-4}	2.8×10^{-4}

- The "minimal" model:
  - Quadratic maximum
  - End of inflation determined by higher order terms

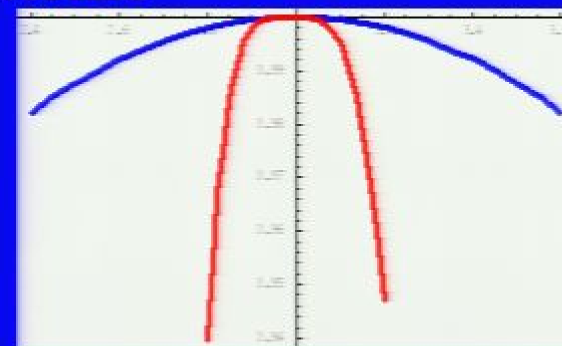
$$V(\phi) = \Lambda^4 \left(1 - a_2 \phi^2 - a_p \phi^p\right)$$



- The "minimal" model:

$$V(\phi) = \Lambda^4 \left(1 - a_2\phi^2 - a_p\phi^p\right)$$

  - Quadratic maximum
  - End of inflation determined by higher order terms
- Results:
  - Extra suppression of GW



$$r_{max} = 16 \left[\left(\frac{1}{60e\sqrt{2}} \right) \frac{p-1}{p-2} \right]^{2\frac{p-1}{p-2}} (\phi_{END})^{2\frac{p-1}{p-2}} \left(\frac{60}{N_{CMB}} \right)^{2\frac{p-1}{p-2}}$$

  - Extra suppression of running

$$\alpha_{max} = 3 \times 10^{-4} \, \frac{p-1}{p-2} \left(\frac{60}{N_{CMB}} \right)^2$$

$\phi_{END}=1$

p	3	4	5	7	10	$p \to \infty$
r_{max}	9.0×10^{-8}	4.4×10^{-6}	1.7×10^{-5}	5.3×10^{-5}	1.0×10^{-4}	3.0×10^{-4}
α	6.0×10^{-4}	3.7×10^{-4}	2.1×10^{-4}	6.0×10^{-5}	6.2×10^{-6}	0
α_{max}	6.0×10^{-4}	4.5×10^{-4}	4.0×10^{-4}	3.6×10^{-4}	3.4×10^{-4}	3.0×10^{-4}
r	9.0×10^{-8}	3.5×10^{-6}	1.0×10^{-5}	1.9×10^{-5}	2.0×10^{-5}	0

Observational consequences

- Observation of GW signal in the CMB ➔

  ~~small field models~~ ?

- Observation of SIR in the CMB ➔

  ~~small field models~~ ? ~~single field models~~ ?

SIR observations*

* A *hard* measurement

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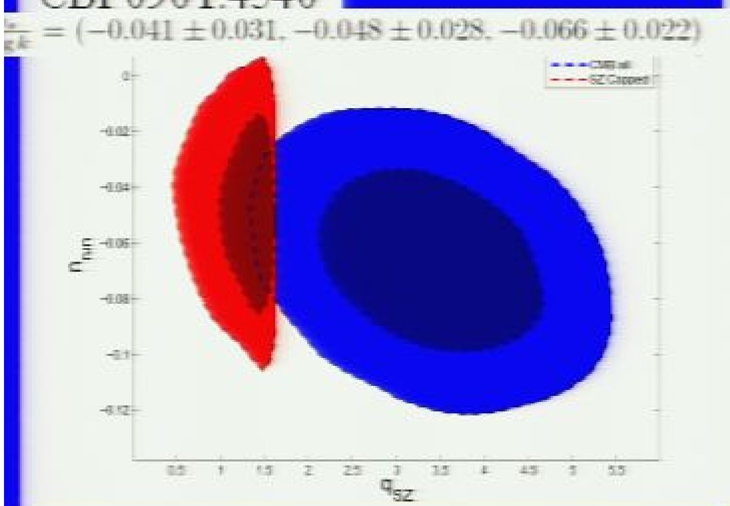

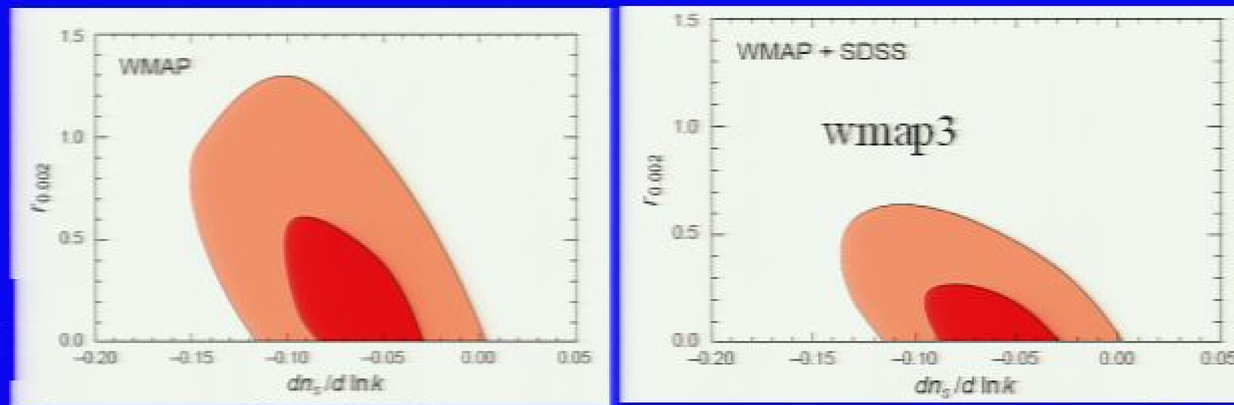

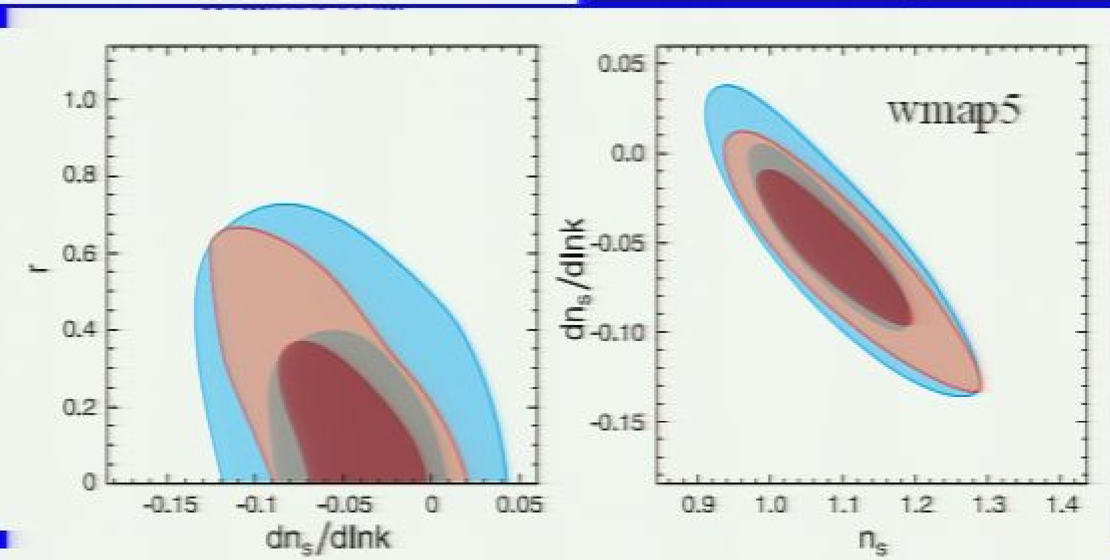

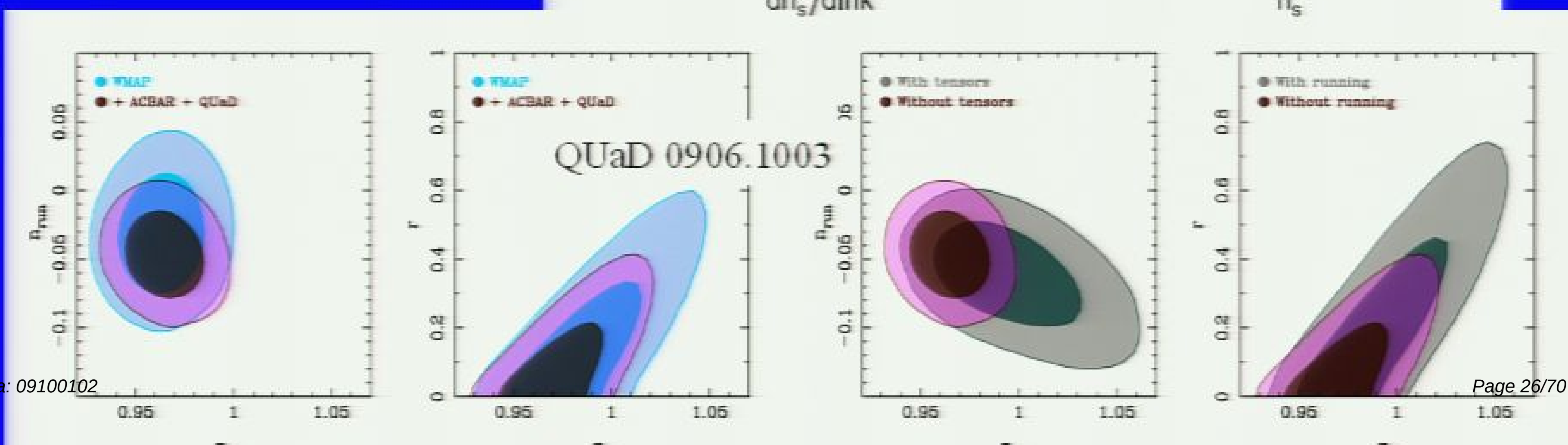

New class of small field models

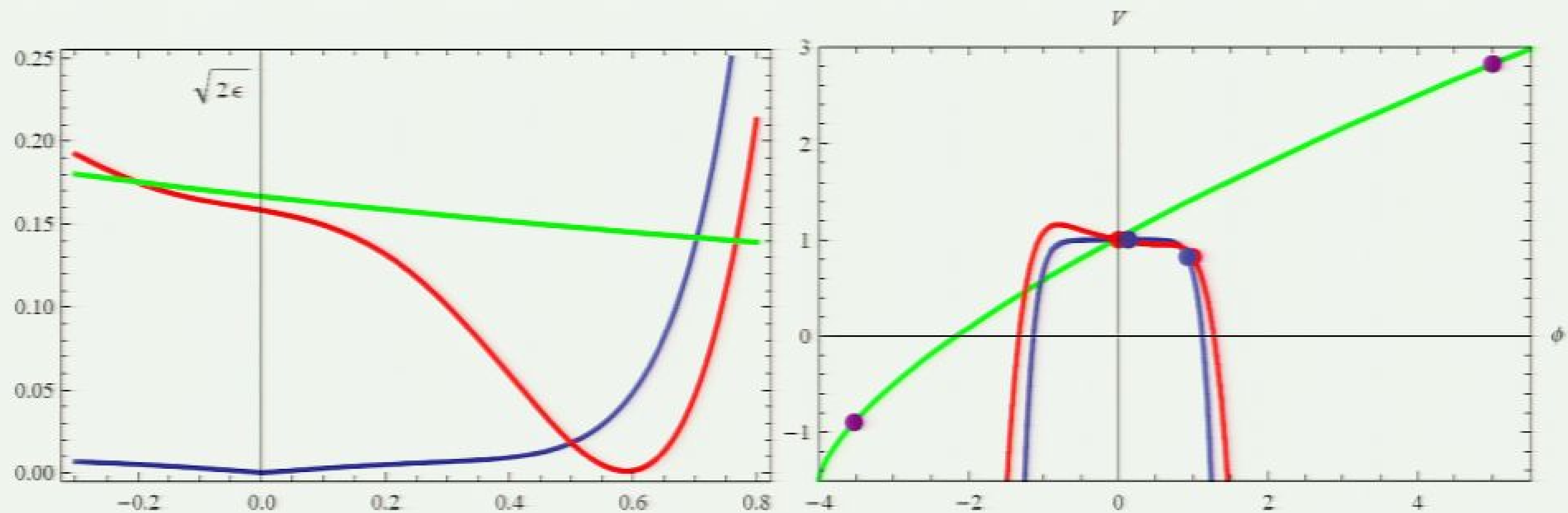

FIG. 1: Shown is a graph of $\sqrt{2\epsilon} = V'/V$ (left) and V (right) for a small field canonical SUGRA model (blue), a large field model (green) and a model of the new class with non-monotonic ϵ (red). The new model interpolates between the two others. For the small field model (blue) the CMB point is at $\phi_{CMB} = 0.13$ and inflation ends at $\phi_{END} = 0.93$. For the large field model (green) ($\phi_{CMB} = 5, \phi_{END} = -3.53$) and for the new model (red) ($\phi_{CMB} = 0, \phi_{END} = 1.0$). The large field model is offset $V \to V - 1.5$. Additionally, to demonstrate the similarity between the small field model(blue) and the new model (red) a symmetric example was chosen, i.e. $a_5 = 0, a_6 = 0.3911$. The CMB observables are $n_s = 1.03, r = 0.2, \alpha = -0.07$.

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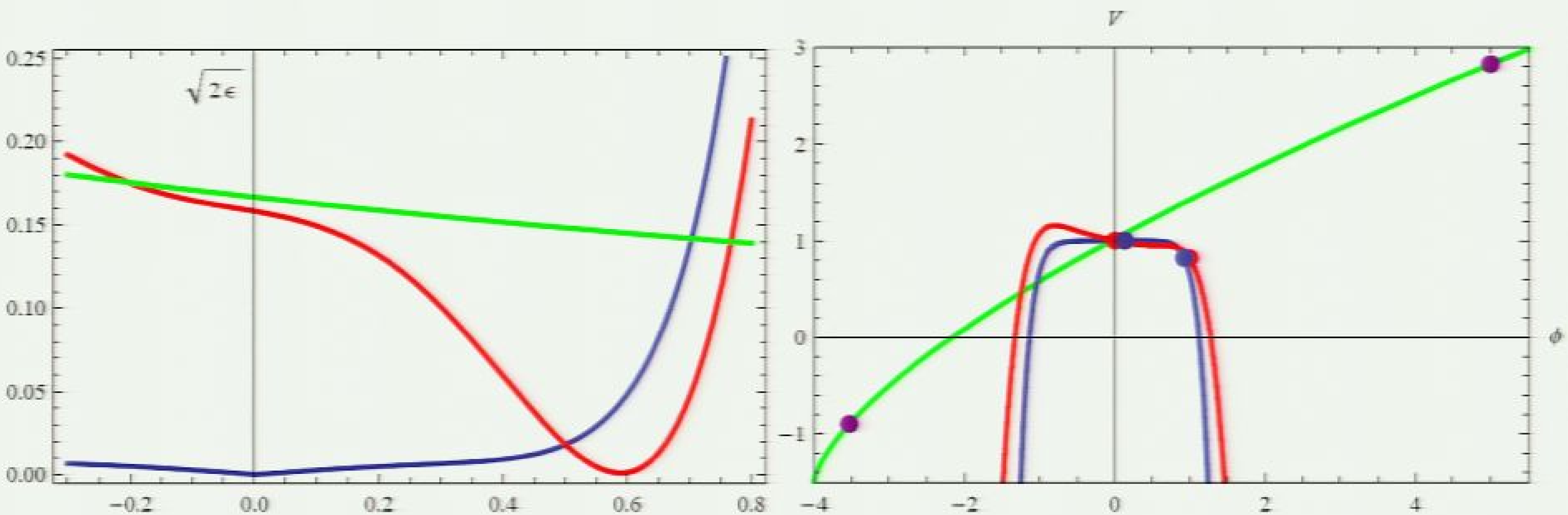

$$\sqrt{\varepsilon} \sim \frac{1}{N} + A(\phi - \phi_{\min})^2 \Rightarrow \frac{V}{V_0} \sim 1 + \frac{1}{N}\phi + B(\phi - \phi_{\min})^3$$

$$\frac{V(\phi)}{V(0)} = 1 - \sqrt{\frac{r_0}{8}}\phi + \frac{\eta_0}{2}\phi^2 - \frac{\alpha_0}{3\sqrt{2r_0}}\phi^3 - a_4\phi^4 - a_5\phi^5$$

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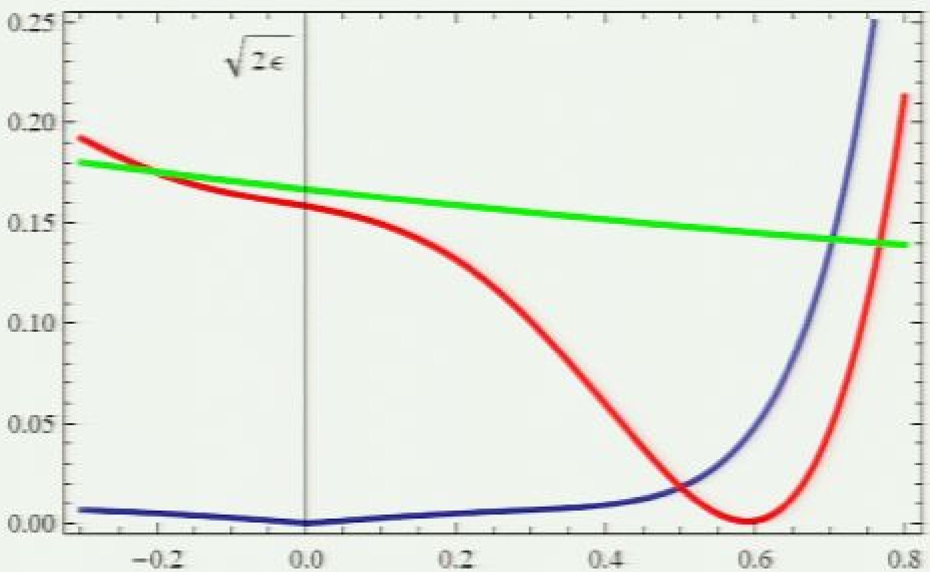
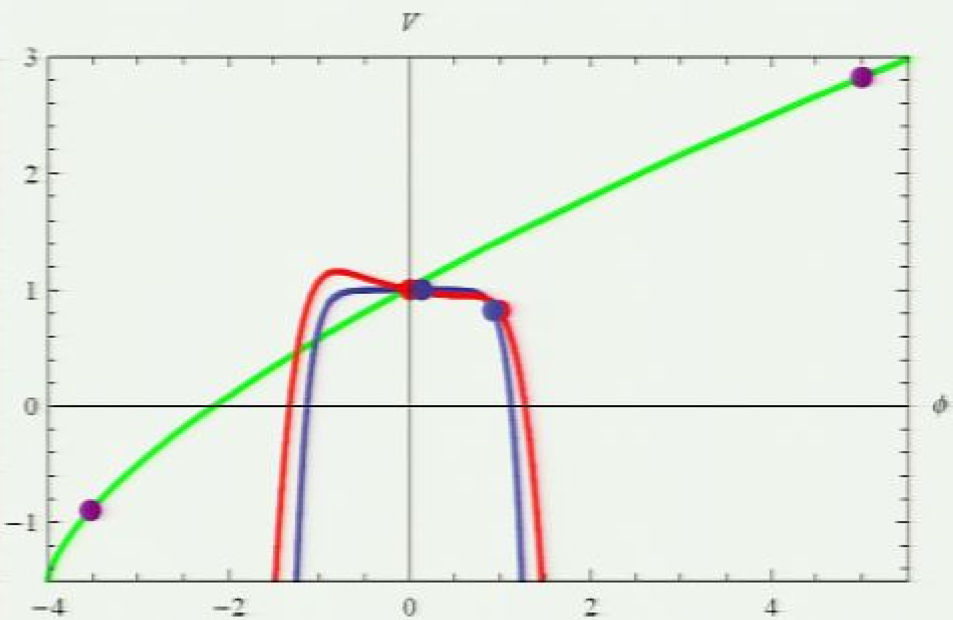
$$\frac{1}{2}\left(\frac{-\sqrt{\frac{r_0}{8}} + \eta_0\phi_{END} - \frac{\alpha_0}{\sqrt{2r_0}}\phi_{END}^2 - 4a_4\phi_{END}^3 - 5a_5\phi_{END}^4}{1 - \sqrt{\frac{r_0}{8}}\phi_{END} + \frac{\eta_0}{2}\phi_{END}^2 - \frac{\alpha_0}{3\sqrt{2r_0}}\phi_{END}^3 - a_4\phi_{END}^4 - a_5\phi_{END}^5}\right)^2 = 1$$

$$N_{CMB} = \int_0^{\phi_{END}} \frac{d\phi}{\sqrt{2\epsilon(\phi; a_5)}}$$

4 physical parameters* + 2 initial conditions, 5 equations, 1 non-linear *only derivatives of the potential are measured

New class of small field models





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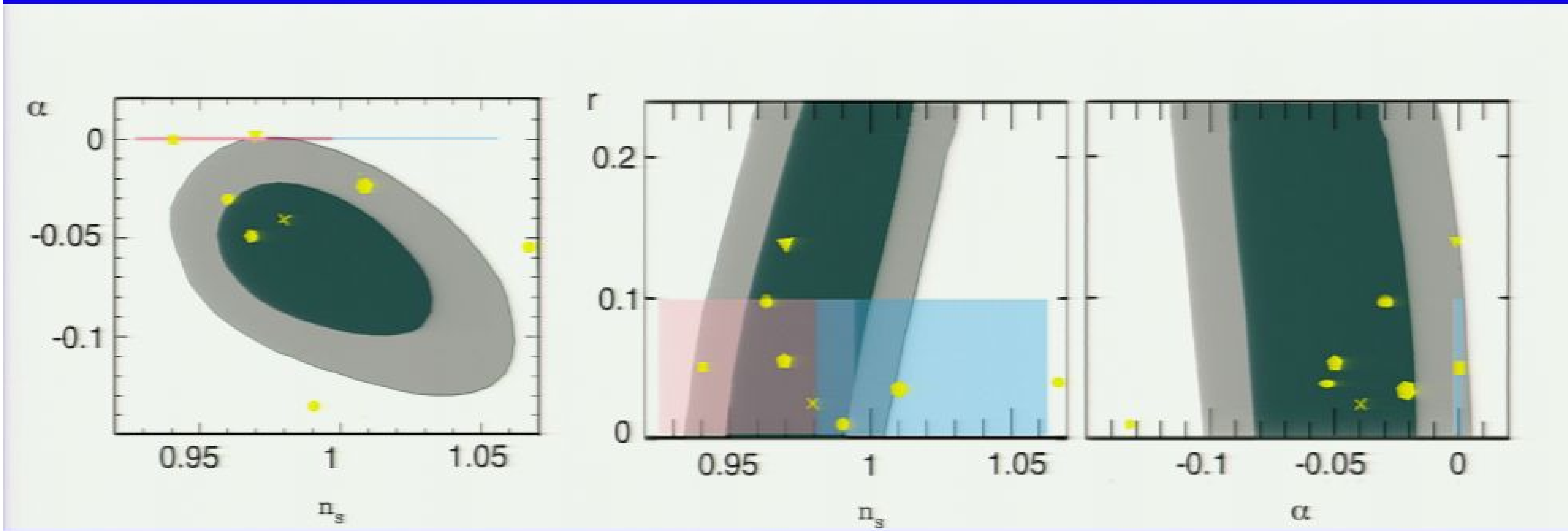
New class of models: "Predictions"

Potential parameters					Range		CMB observables		
r_0	η_0	α_0	a_4	a_5	$\Delta\phi_{50}$	$\Delta\phi_{60}$	n_s	r	α
$*$ 0.05	−0.02	−0.001	−0.1752	0.1314	0.855	1.5	0.94	0.05	0.0002
$*$ 0.10	0.015	−0.03	−0.6102	0.709	0.567	1.0	0.96	0.10	−0.031
$*$ 0.04	0.07	−0.05	−0.2739	0.48	0.5	1.0	1.07	0.04	−0.052
0.02	−0.02	0.001	−4.1355	10.505	0.245	0.5	0.95	0.02	0.001
0.20	0.09	−0.07	−0.6100	0.7253	0.58	1.0	1.02	0.20	−0.084
0.08	0.05	−0.05	−0.6982	0.9935	0.487	0.9	1.01	0.08	−0.053
$*$ 0.04	0.025	−0.02	−0.436	0.574	0.525	1.0	1.01	0.04	−0.021
$*$ 0.13	0.01	0.001	−0.4072	0.367	0.705	1.2	0.97	0.13	0.001
$*$ 0.05	0.02	−0.05	−0.425	0.591	0.53	1.0	0.97	0.05	−0.051
$*$ 0.02	0.015	−0.04	−0.691	1.33	0.39	0.8	0.98	0.02	−0.04
0.02	0.1144	0	0.0325	0	0.8	2	1.23	0.02	−0.0022
$*$ 0.01	0.065	−0.133	0.671	0	0.315	0.9	0.99	0.01	−0.134

New class of models: "Predictions"



New class of small field models: EFT considerations

$$V = \Lambda^4 \left(1 + \sum_{n=1} \lambda_n (\phi/m_P)^n \right)$$

$$\Lambda \simeq 1 \times 10^{16}\ \text{GeV}\ (r/.01)^{1/4}$$

$(E/\Lambda)^{+ve}$ $\quad \lambda_n \ll 1,\ n \geq 4$

Small scale-separation

$$\min(\sqrt{\varepsilon}, \sqrt{\eta}) \frac{\Lambda}{m_p} \Lambda = \min(\sqrt{\varepsilon}, \sqrt{\eta}) H < E < \Lambda$$

λ_i i=1,2,3, special. For example $\lambda_1 = -.035(r/.01)^{1/2}$

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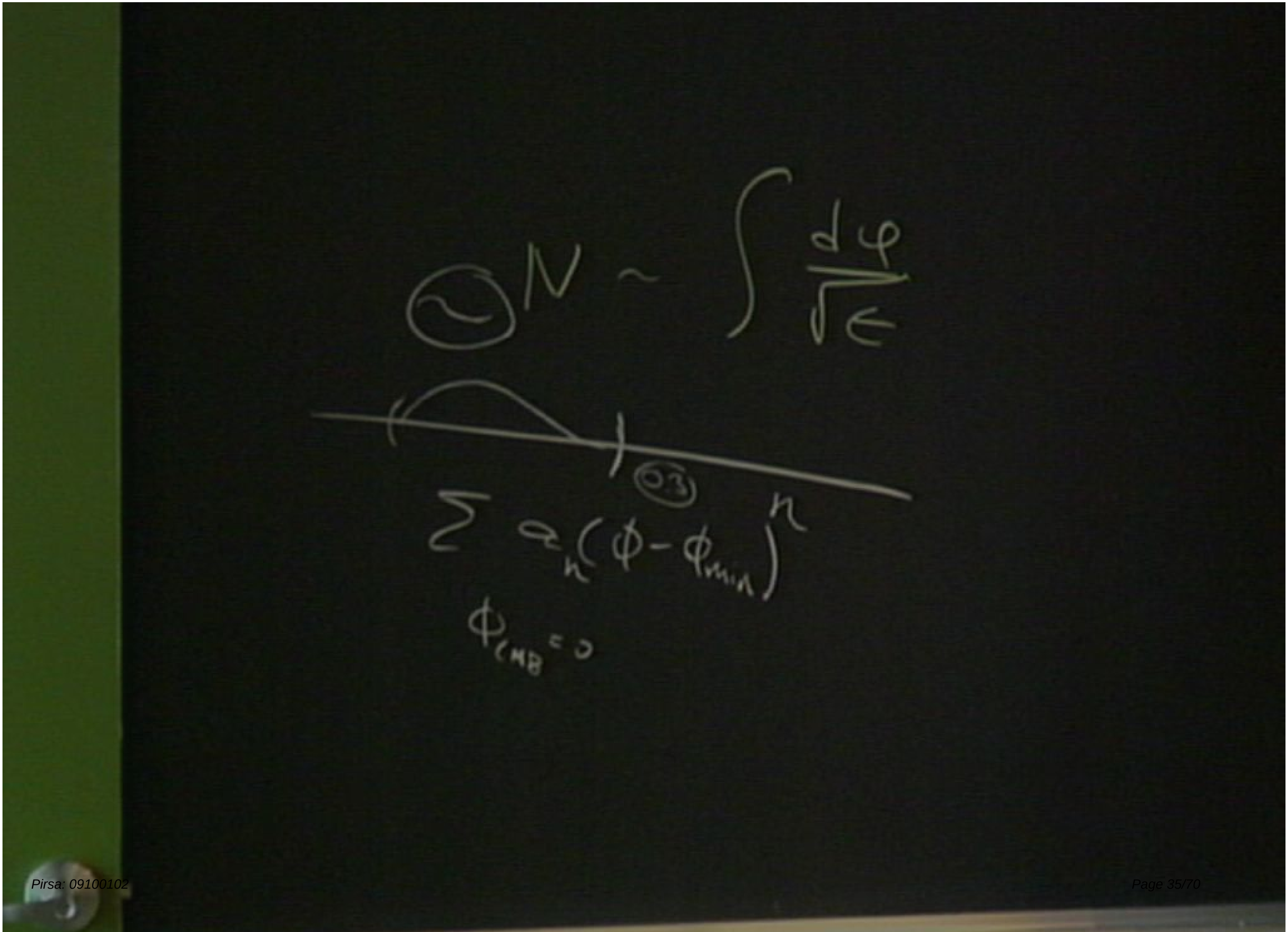
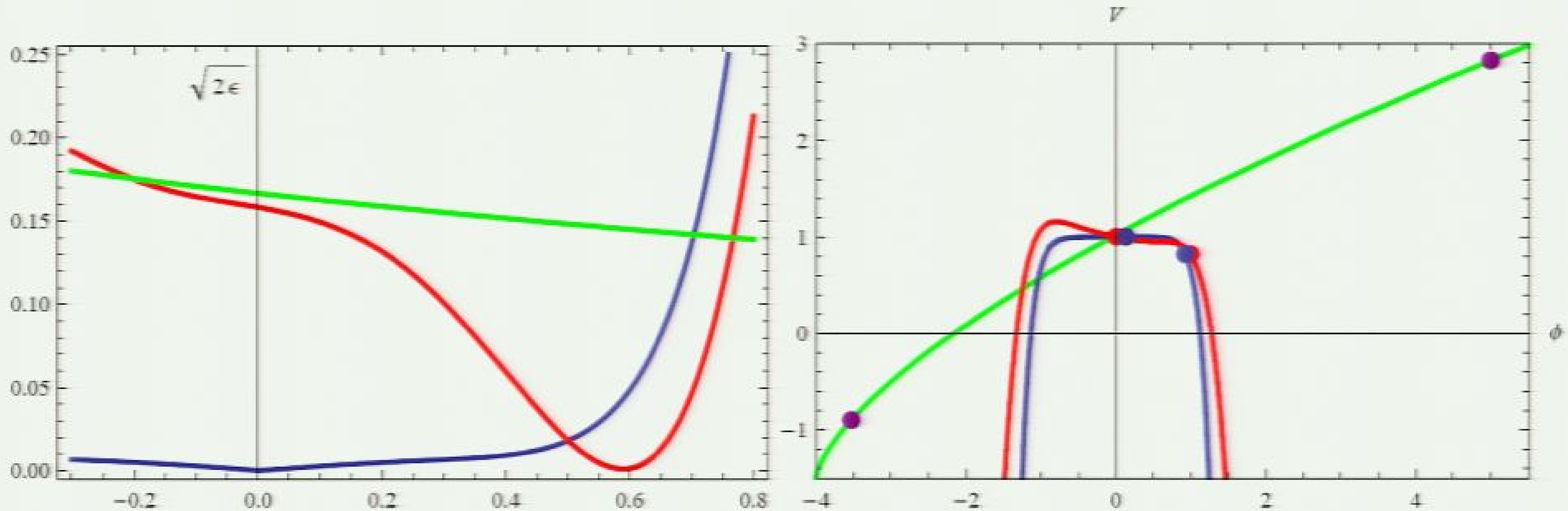

N ~ ∫ dφ/√ε
Σ a_n(φ - φ_min)^n
φ_CMB = 0

New class of small field models



$$\sqrt{\varepsilon} \sim \frac{1}{N} + A(\phi - \phi_{\min})^2 \Rightarrow \frac{V}{V_0} \sim 1 + \frac{1}{N}\phi + B(\phi - \phi_{\min})^3$$

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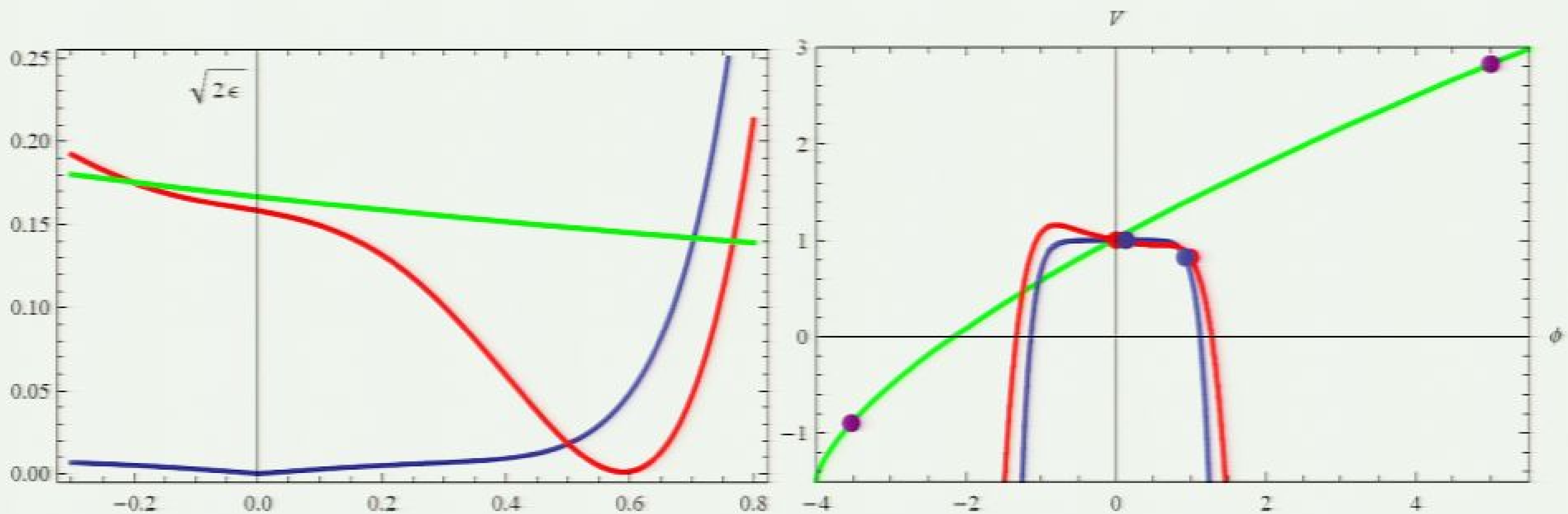

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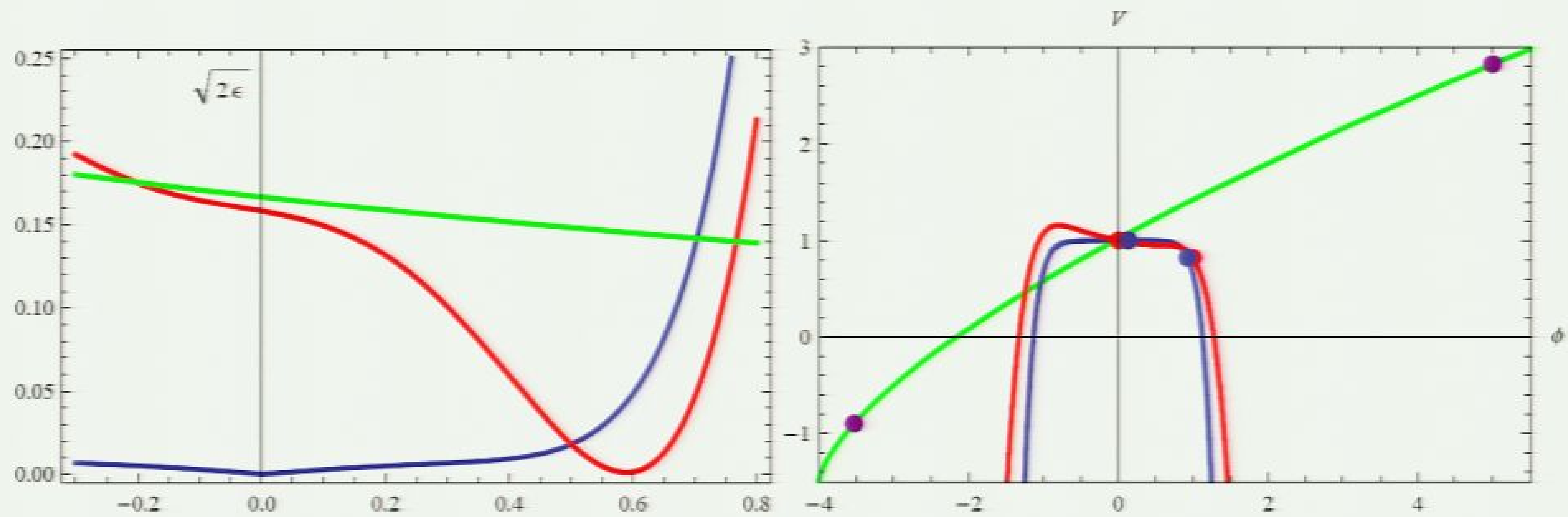

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$$\frac{1}{2}\left(\frac{-\sqrt{\frac{r_0}{8}} + \eta_0\phi_{END} - \frac{\alpha_0}{\sqrt{2r_0}}\phi_{END}^2 - 4a_4\phi_{END}^3 - 5a_5\phi_{END}^4}{1 - \sqrt{\frac{r_0}{8}}\phi_{END} + \frac{\eta_0}{2}\phi_{END}^2 - \frac{\alpha_0}{3\sqrt{2r_0}}\phi_{END}^3 - a_4\phi_{END}^4 - a_5\phi_{END}^5}\right)^2 = 1$$

$$N_{CMB} = \int_0^{\phi_{END}} \frac{d\phi}{\sqrt{2\epsilon(\phi; a_5)}}$$

4 physical parameters* + 2 initial conditions, 5 equations, 1 non-linear *only derivatives of the potential are measured

New class of models: "Predictions"

Potential parameters					Range		CMB observables		
r_0	η_0	α_0	a_4	a_5	$\Delta\phi_{50}$	$\Delta\phi_{60}$	n_s	r	α
* 0.05	−0.02	−0.001	−0.1752	0.1314	0.855	1.5	0.94	0.05	0.0002
* 0.10	0.015	−0.03	−0.6102	0.709	0.567	1.0	0.96	0.10	−0.031
* 0.04	0.07	−0.05	−0.2739	0.48	0.5	1.0	1.07	0.04	−0.052
0.02	−0.02	0.001	−4.1355	10.505	0.245	0.5	0.95	0.02	0.001
0.20	0.09	−0.07	−0.6100	0.7253	0.58	1.0	1.02	0.20	−0.084
0.08	0.05	−0.05	−0.6982	0.9935	0.487	0.9	1.01	0.08	−0.053
* 0.04	0.025	−0.02	−0.436	0.574	0.525	1.0	1.01	0.04	−0.021
* 0.13	0.01	0.001	−0.4072	0.367	0.705	1.2	0.97	0.13	0.001
* 0.05	0.02	−0.05	−0.425	0.591	0.53	1.0	0.97	0.05	−0.051
* 0.02	0.015	−0.04	−0.691	1.33	0.39	0.8	0.98	0.02	−0.04
0.02	0.1144	0	0.0325	0	0.8	2	1.23	0.02	−0.0022
* 0.01	0.065	−0.133	0.671	0	0.315	0.9	0.99	0.01	−0.134

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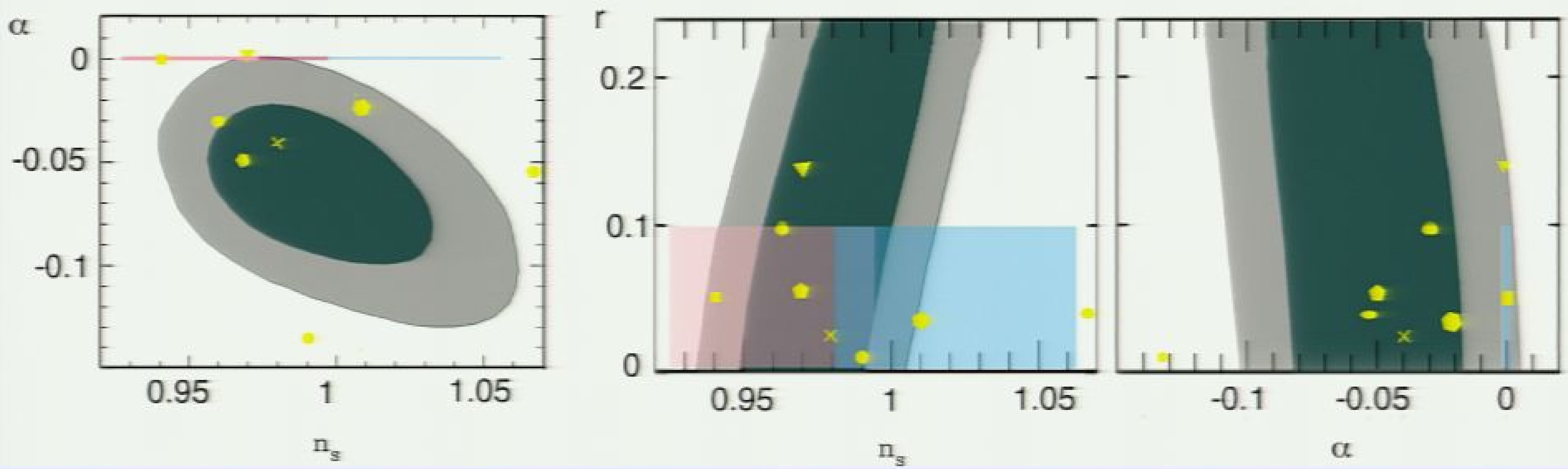

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$\alpha \simeq 2\xi^2 = 2m_p^4 \frac{V'''V'}{V^2}$ $m_p^3 \frac{V'''}{V} = 3!\ \lambda_3$

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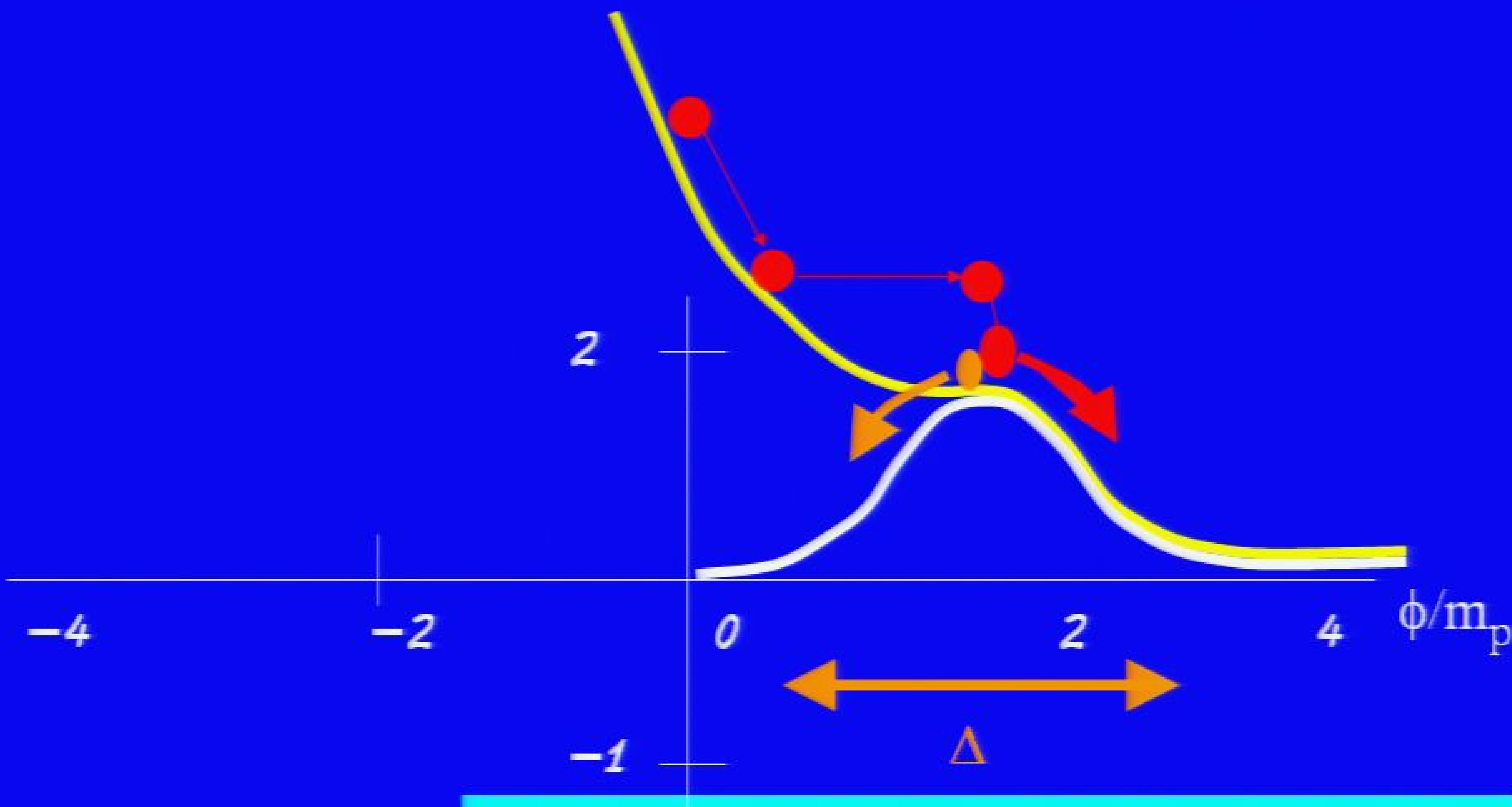
Designing flat features for single field SUGRA modular inflation



Enough inflation ⇔ $V''/V<1/50$

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$$V = e^K[K^{\phi\bar{\phi}}|D_\phi W|^2 - 3|W|^2]$$
$$\simeq (1+\phi\bar{\phi})[(\bar{\phi}W + W_\phi)(\phi\bar{W} + \bar{W}_{\bar{\phi}}) - 3W\bar{W}]$$

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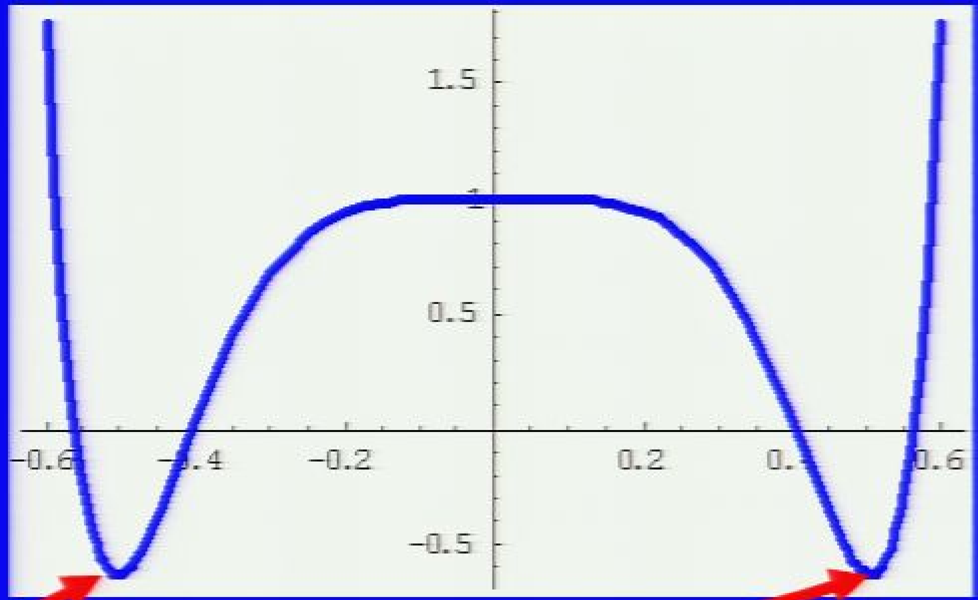
Example: real b_i

A numerical example:



The potential is not sensitive to small changes in coefficients Including adding small higher order terms, inflation is indeed 1/100 of tuning away

$b_2=0,\ b_4=0,\ b_1=1, b_3=\eta/6,$

$b_5\, y^4(y^2+5) + y^2+1=0$

$\eta = 6\, b_1\, b_3 - 2(b_0)^2$

Need 5 parameters:

$V'(0)=0, V(0)=1, V''/V=\eta$

$D_T W(-y),\ D_T W(+y) = 0$

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If one wishes to tune the CC @ min to be small enough replace D_T W=0 by V_T=0, V=0 (one more condition)

Relevance to string theory

- SIR: Extremely sensitive indicator for a high scale inflation
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$$g_s \lesssim 1, \ \ V_{compact} \gtrsim 1$$

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Some hope for stringy physics in anisotropies: explain!

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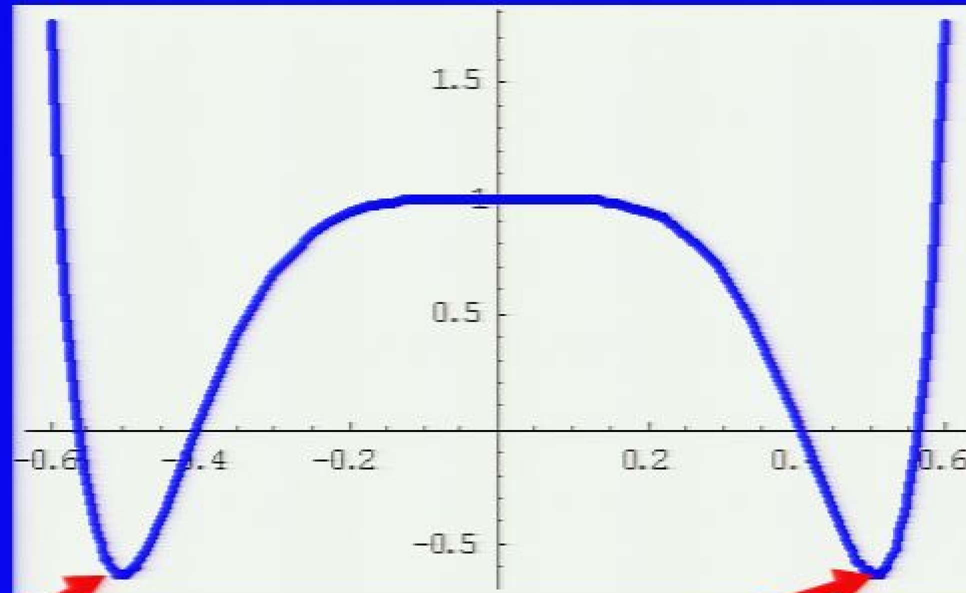

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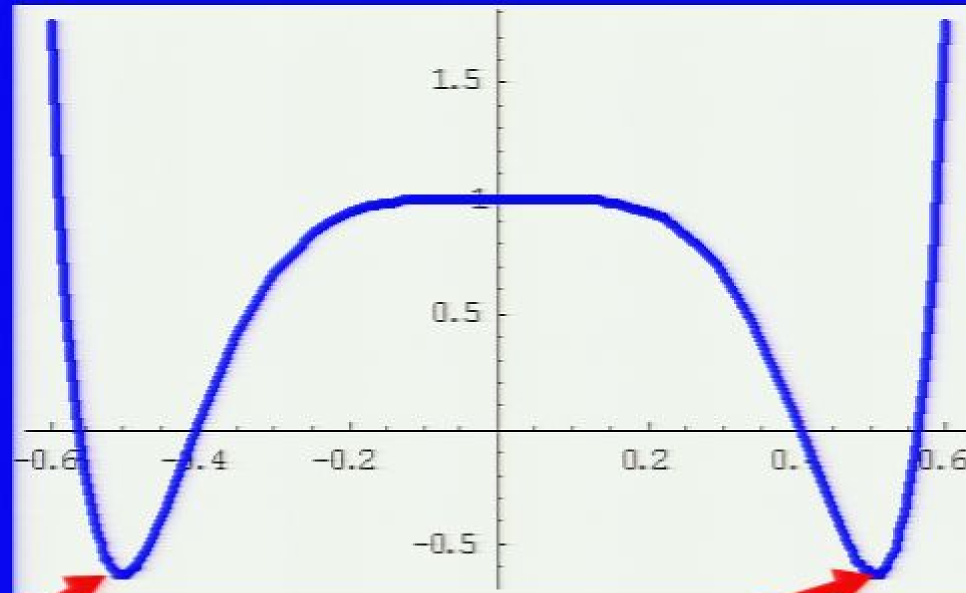

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$$D_T W(\pm y) = 0, \quad y \sim 1$$

A local equation

Not a local equation

$$\eta = \min\left\{\text{Eigenvalues}\left(\frac{g^{ac}\partial_c\partial_b V - g^{ac}\Gamma_{cb}{}^{d}\partial_d V}{V}\right)\right\}$$

$$\begin{aligned}\partial_{\bar{n}}\partial_m V \;&=\; e^K\Big[D_m D_i W K^{i\bar{j}} D_{\bar{n}} D_{\bar{j}}\overline{W} - K^{i\bar{j}} R_{m\bar{n}i}{}^{k} D_k W D_{\bar{j}}\overline{W} \\ &\quad + K_{m\bar{n}} K^{i\bar{j}} D_i W D_{\bar{j}}\overline{W} - D_m W D_{\bar{n}}\overline{W} - 2K_{m\bar{n}} W\overline{W}\Big]\end{aligned}$$

$$R_{i\bar{j}k\bar{l}} \;=\; K_{m\bar{l}}\partial_{\bar{j}}\Gamma_{ik}{}^{m}$$

$$\nabla_n\partial_m V \;=\; e^K\left[K^{i\bar{j}} D_n D_m D_i W D_{\bar{j}}\overline{W} - D_n D_m W\overline{W}\right]$$

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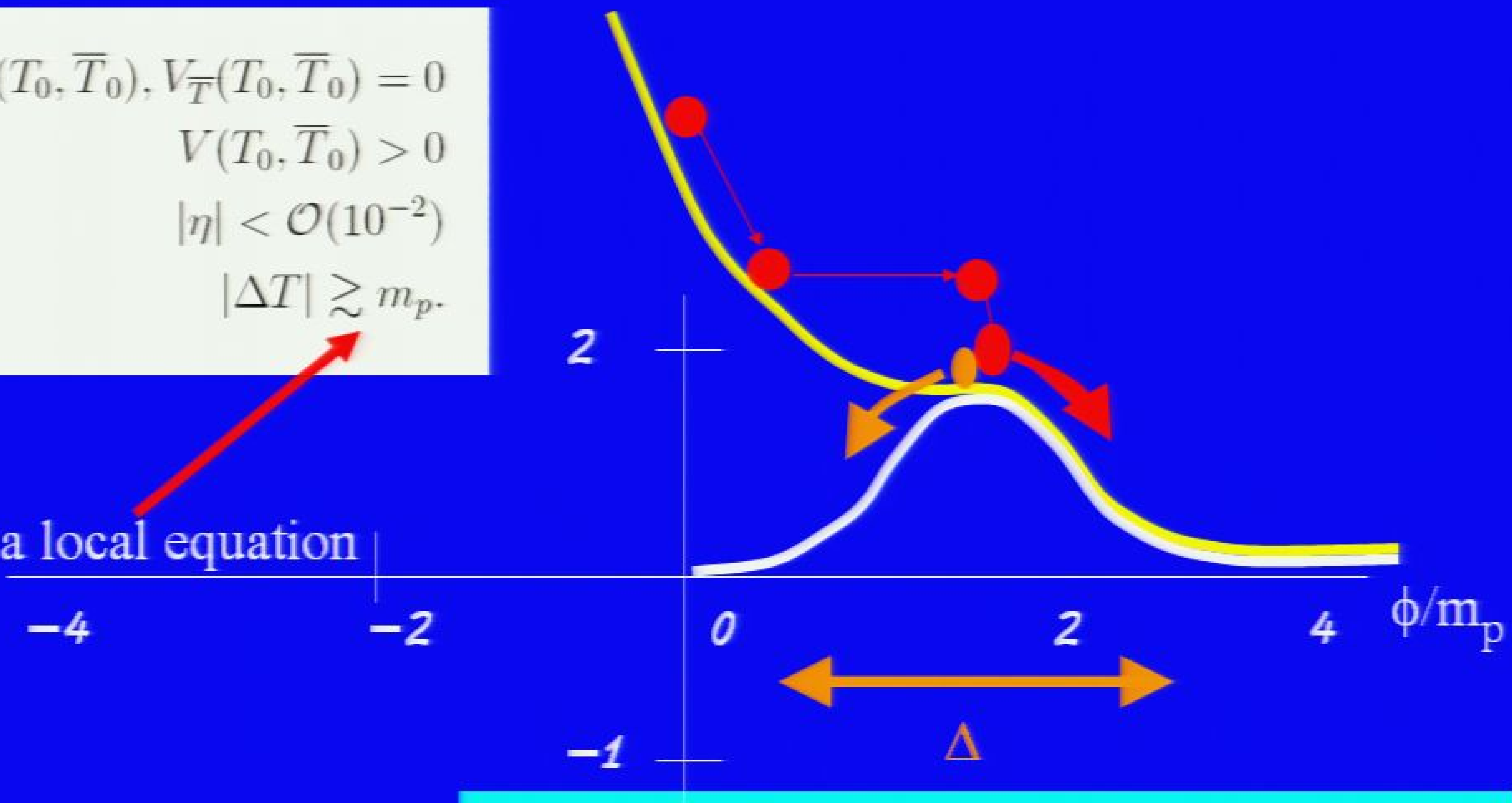

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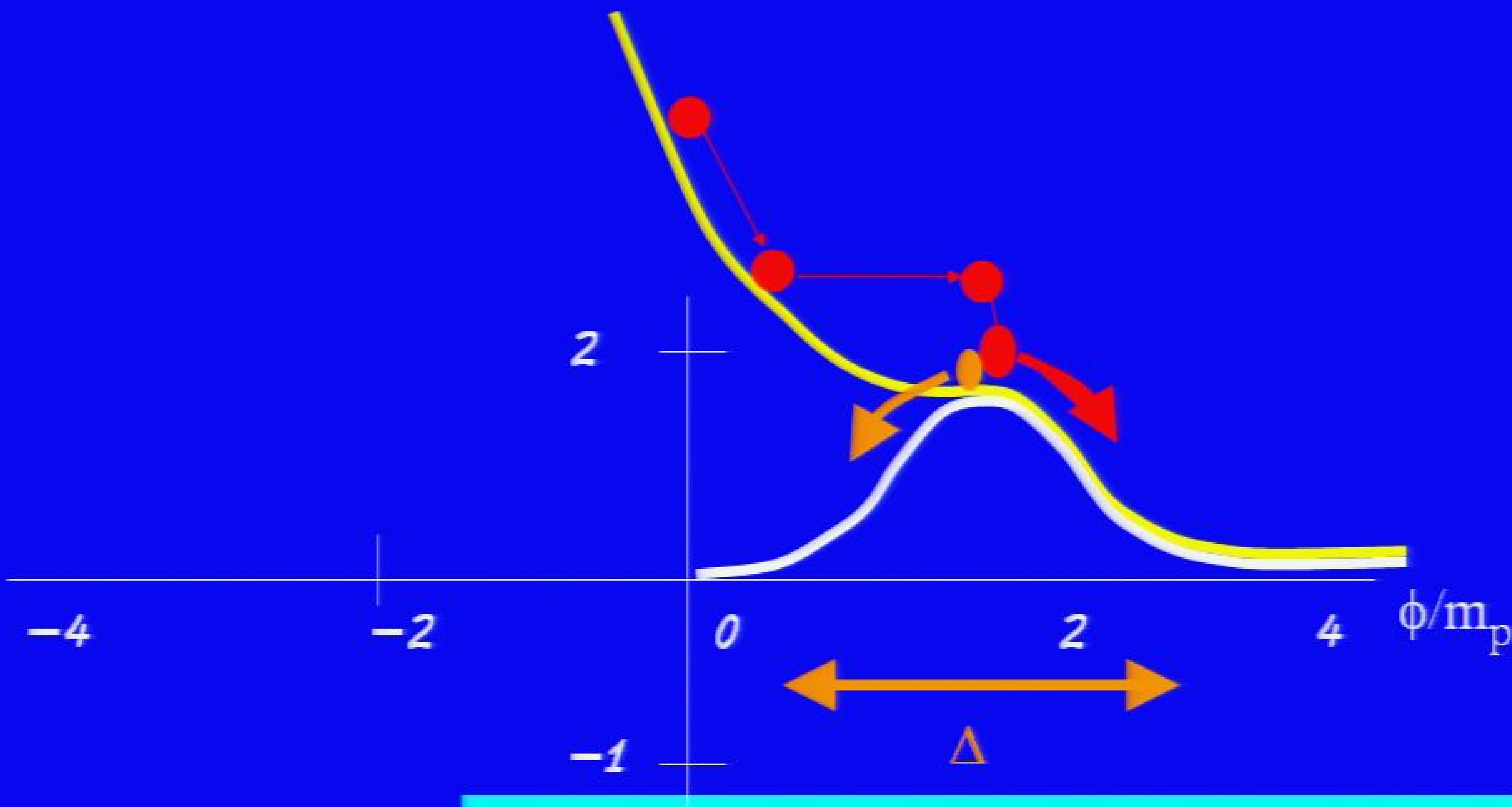

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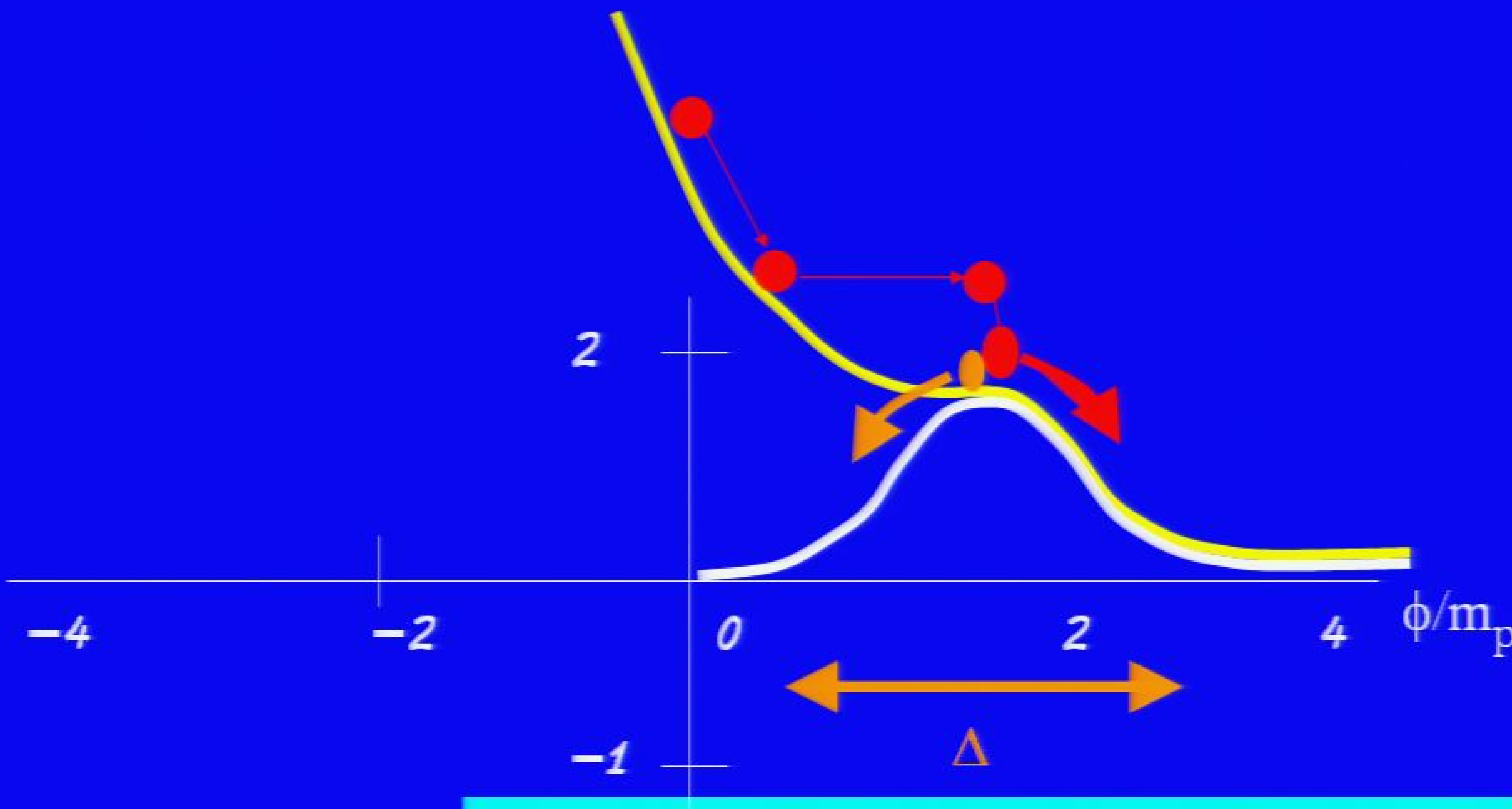

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Conclusions

- Small field models of inflation are interesting
- Predictions for the CMB:
  - Simplest models: $n_S<1$, $r_{0.01}<<1$, $\alpha_{0.05}<<1$
  - New class: n_S , $r_{0.01}$, $\alpha_{0.05}$ all allowed values
- SIR has a strong discriminating power among cosmological models, linked with high *r* in our models

Design a wide (symmetric) plateau with polynomial eqs.

$$D_\phi W(\pm y) = \bar{\phi} W + W_\phi|_{\pm y} = 0$$

In practice creates two minima @ +y,-y

$$y(b_0 + b_1 y + b_2 y^2 + b_3 y^3 + b_4 y^4 + b_5 y^5) + b_1 + 2b_2 y + 3b_3 y^2 + 4b_4 y^3 + 5b_5 y^4 = 0$$

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A simple solution: $b_2=0$, $b_4=0$, $b_1=1$, $b_3=\eta/6$, Design a wide (symmetric) plateau with polynomial eqs.

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Example: real b_i

$$b_1^2 - 3b_0^2 = 1 \Rightarrow b_1 \neq 0$$
$$2b_1(b_2 - b_0) = 0 \Rightarrow b_0 = b_2$$
$$\eta = \frac{V_{\phi\phi}(0)}{V(0)} = V_{\phi\phi}(0) = 6b_1b_3 - 2b_0^2$$

Needs to be tuned for inflation

$$V(0) = 1 = (1+\phi\bar\phi)[(\bar\phi W + W_\phi)(\phi\bar W + \bar W_{\bar\phi}) - 3W\bar W] = b_1^2 - 3b_0^2,$$

$$V_\phi(0) = 0 = \bar\phi[\cdots] + (1+\phi\bar\phi)\{(\bar\phi W_\phi + W_{\phi\phi})(\phi\bar W + \bar W_{\bar\phi}) + (\bar\phi W + W_\phi)\bar W - 3W_\phi\bar W\}$$
$$= W_{\phi\phi}\bar W_{\bar\phi} - 2W_\phi\bar W|_{\phi=0} = 2b_2b_1 - 2b_1b_0$$

$$V_{\phi\phi}(0) = 2\bar\phi\{\cdots\} + (1+\phi\bar\phi)[(\bar\phi W_{\phi\phi} + W_{\phi\phi\phi})(\phi\bar W + \bar W_{\bar\phi}) + (\bar\phi W_\phi + W_{\phi\phi})\bar W + \bar\phi W_\phi\bar W - 2W_{\phi\phi}\bar W]$$
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$$V(0) = 1$$
$$|\eta| < \mathcal{O}(10^{-2})$$
$$D_T W(\pm y) = 0, \quad y \sim 1$$

A local equation