Title: Viscosity and conductivity in general theories of gravity

Date: Oct 20, 2009 11:00 AM

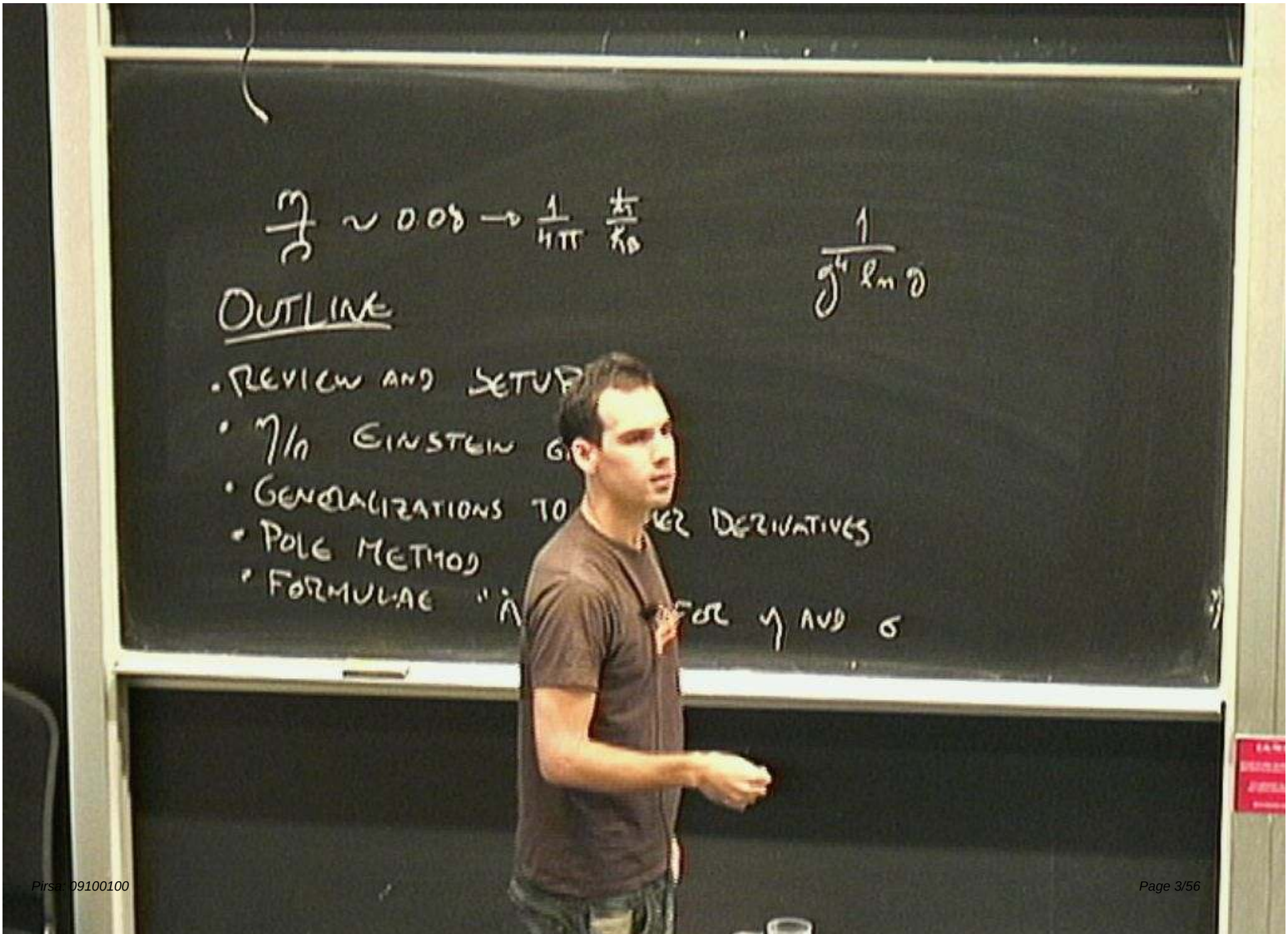
URL: http://pirsa.org/09100100

Abstract: Recently there has been great interest in calculating transport coefficients for field theories at large coupling, using AdS/CFT. In this talk I will discuss recent work showing how to use the membrane paradigm to easily compute the shear viscosity and conductivity in arbitrary gravity theories. In a certain sense these can be thought of as effective couplings at the black hole horizon dual to the field theory plasma. An explicit Wald-like formula for these couplings is given for a large class of generalized gravity theories.

$$\frac{\eta}{s} \sim 0.08 \to \frac{1}{4\pi}\frac{\hbar}{k_B}$$

## OUTLINE

- REVIEW AND SETUP
- $\eta/s$ EINSTEIN GRAVITY
- GENERALIZATIONS TO HIGHER DERIVATIVES
- POLE METHOD
- FORMULAE "À LA WALD" FOR $\eta$ AND $\sigma$


OUTLINE
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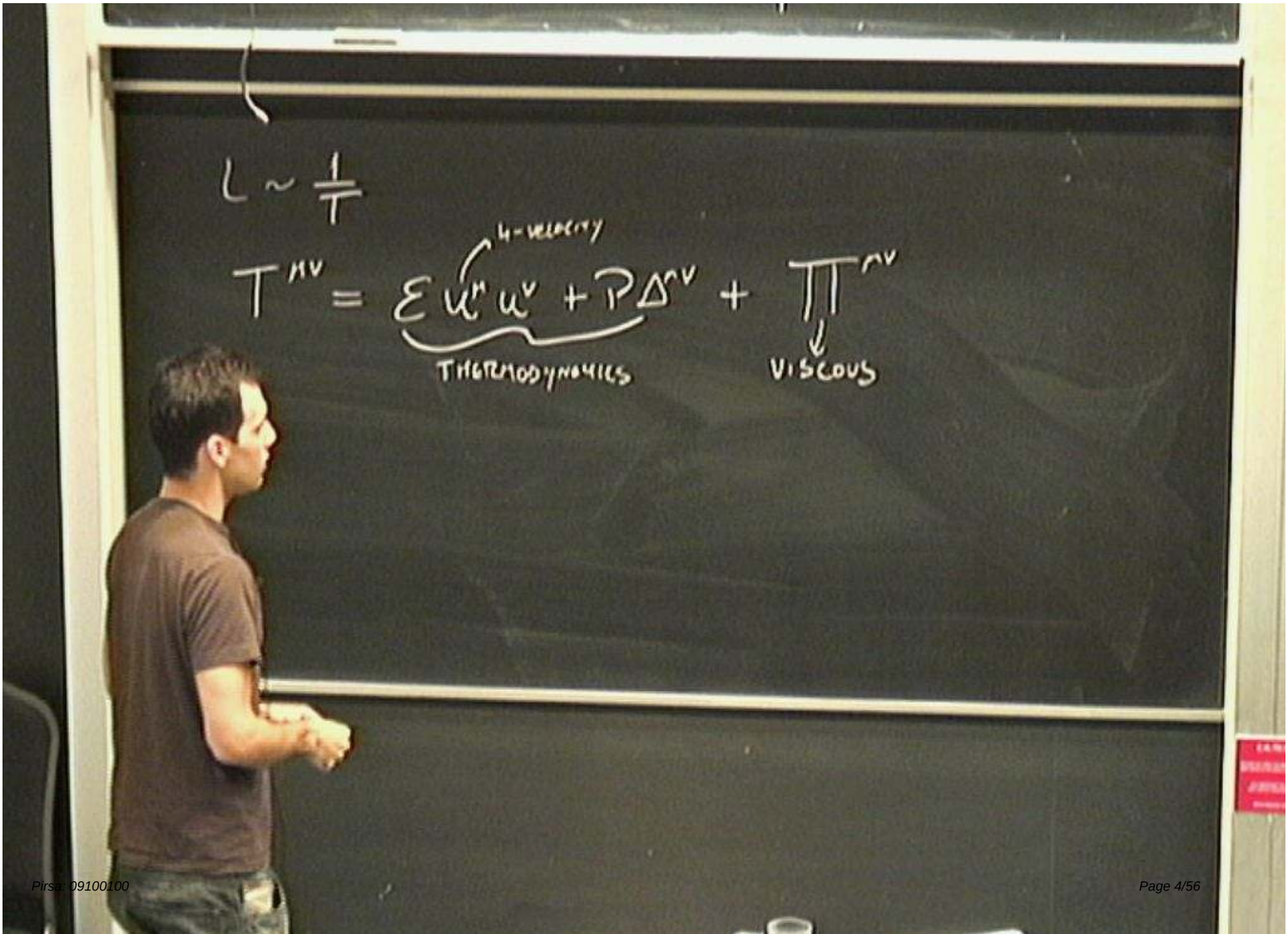
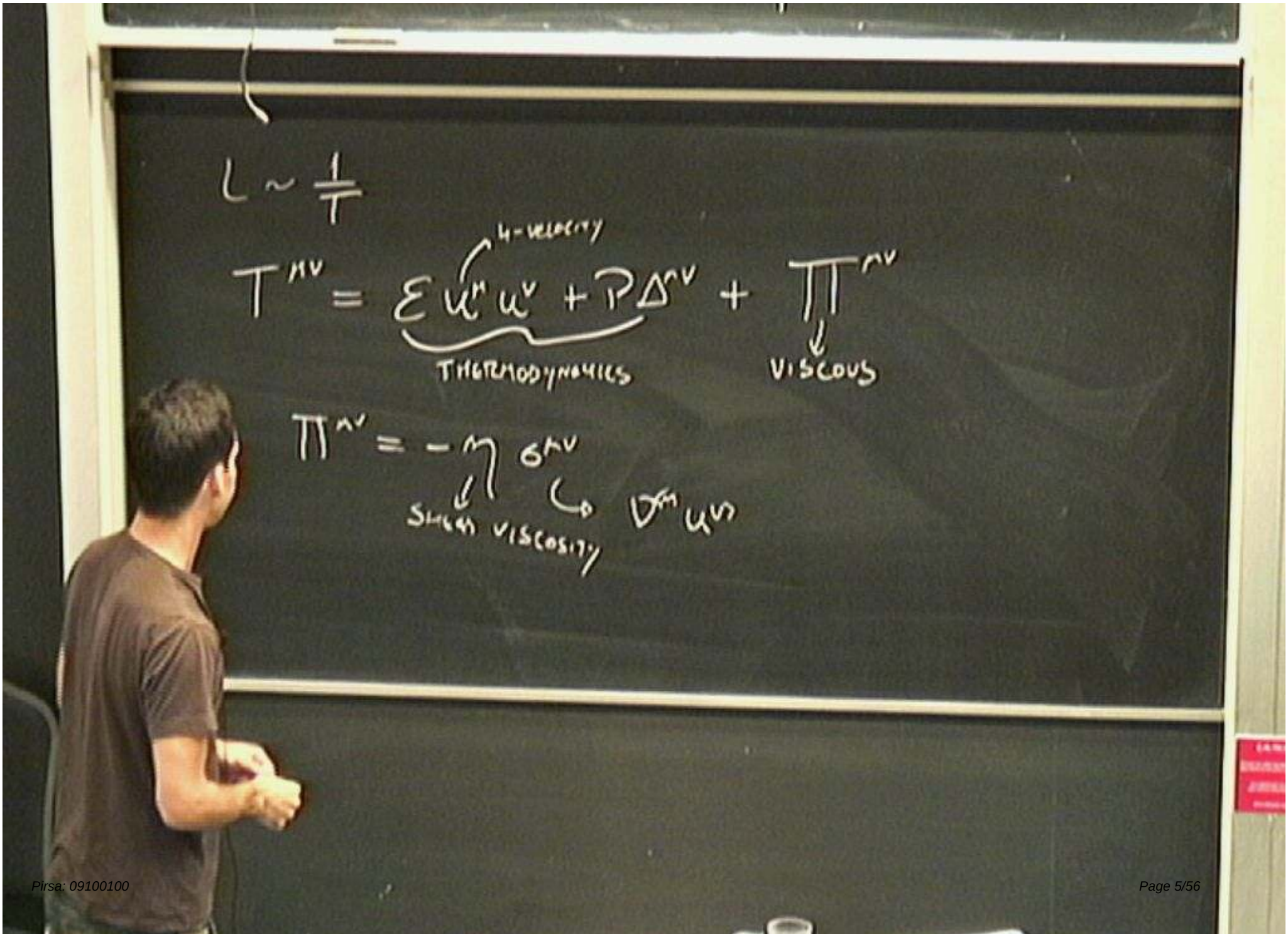

$L \sim \frac{1}{T}$
4-VELOCITY
$T^{\mu\nu} = \underbrace{\varepsilon u^\mu u^\nu + P\Delta^{\mu\nu}}_{\text{THERMODYNAMICS}} + \underbrace{\Pi^{\mu\nu}}_{\text{VISCOUS}}$


$l \sim \frac{1}{T}$
4-velocity
$T^{\mu\nu} = \underbrace{\varepsilon u^\mu u^\nu + P\Delta^{\mu\nu}}_{\text{THERMODYNAMICS}} + \Pi^{\mu\nu}$
VISCOUS
$\Pi^{\mu\nu} = -\eta\, \sigma^{\mu\nu}$
$\nabla^\mu u^\nu$
SHEAR VISCOSITY

$T^{\alpha\gamma}$

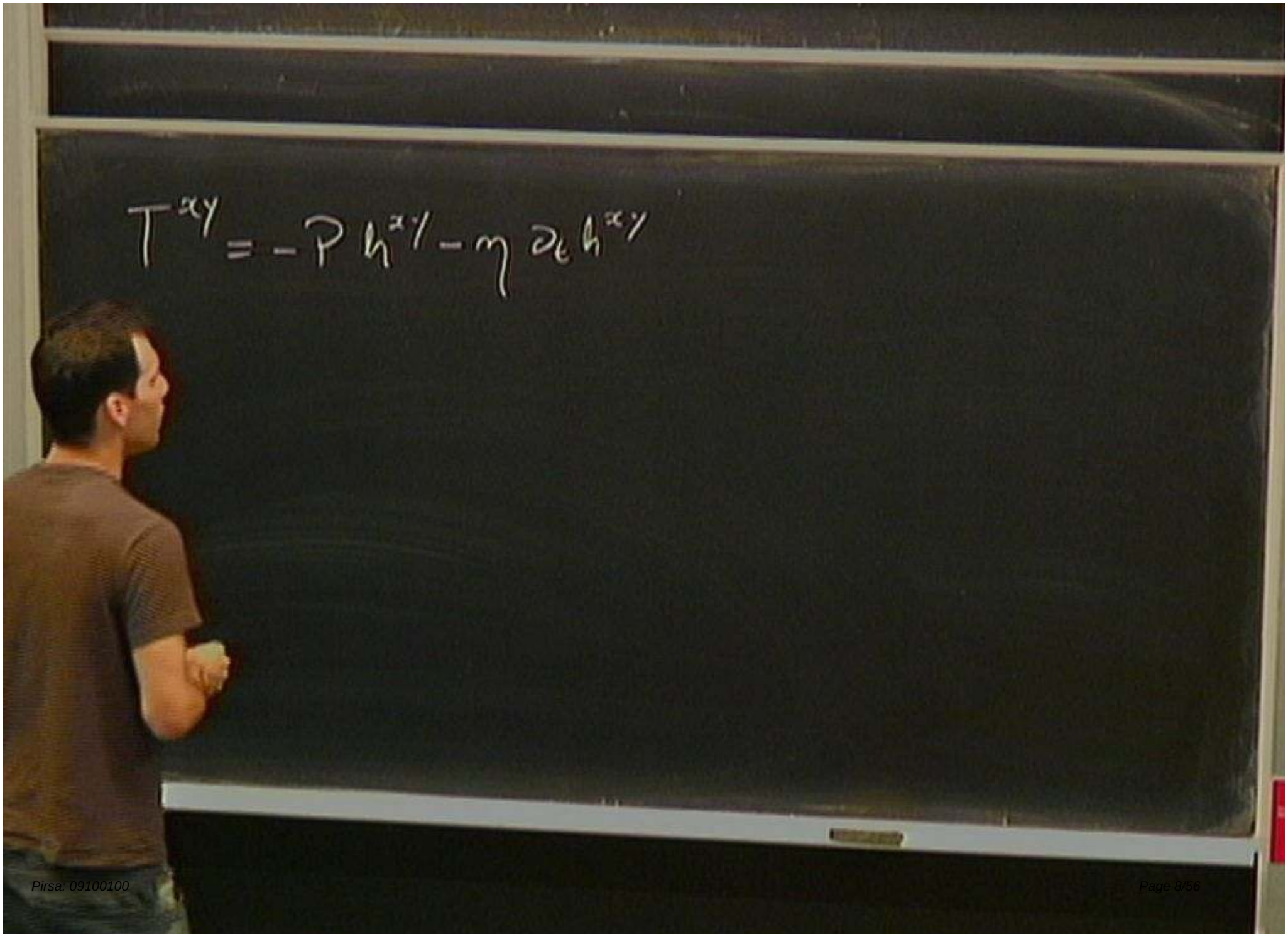
$$L \sim \frac{1}{T}$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$T^{\mu\nu} = \underbrace{\varepsilon \overset{\text{4-velocity}}{u^\mu} u^\nu + P\Delta^{\mu\nu}}_{\text{THERMODYNAMICS}} + \underbrace{\Pi^{\mu\nu}}_{\text{VISCOUS}}$$

$$\Pi^{\mu\nu} = -\underbrace{\eta}_{\text{SHEAR VISCOSITY}} \sigma^{\mu\nu}$$

$$\sigma^{\mu\nu} \hookrightarrow \nabla^{(\mu} u^{\nu)}$$


$T^{xy} = -P h^{xy} - \eta \partial_t h^{xy}$

$$T^{xy} = -P h^{xy} - \eta \partial_t h^{xy}$$

$$= -P h^{xy}(\omega) - i\omega \eta h^{xy}(\omega)$$

$$\langle T^{xy}(\omega) \rangle = + G_R^{xy,\,xy} h^{xy}$$

$$T^{xy} = -P h^{xy} - \eta \partial_t h^{xy}$$

$$= -P h^{xy}(\omega) - i\omega \eta h^{xy}(\omega) + \text{HIGHER ORDER}$$

$$\langle T^{xy}(\omega) \rangle = + G_R^{xy,xy} h^{xy}$$

$$\lim_{\omega \to 0} \frac{\mathrm{Im}\, G_R^{xy,xy}}{i\omega} = \eta$$

- AdS/CFT

gauge theory $\longleftrightarrow$ gravity theory (strings)

$N=4$ SYM $SU(N)$ $\longleftrightarrow$ TYPE IIB SUPERSTRING

## • AdS/CFT

gauge theory $\longleftrightarrow$ gravity theory (strings)

$N=4$ SYM $SU(N)$ $\longleftrightarrow$ TYPE IIB SUPERSTRING ON $AdS_5 \times S_5$

$\lambda = g_{YM}^2 N$

$$\lambda \longleftrightarrow L^2/\alpha'^2$$

$$\frac{\lambda}{N} \longleftrightarrow g_s$$

- AdS/CFT

gauge theory $\longleftrightarrow$ gravity theory (strings)

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gauge theory $\longleftrightarrow$ gravity theory (strings)

$N=4$ SYM $SU(N)$ $\longleftrightarrow$ TYPE IIB SUPERSTRING ON $AdS_5 \times S_5$

$\lambda = g_{YM}^2 N$

$\lambda \longleftrightarrow L^2/\alpha'$

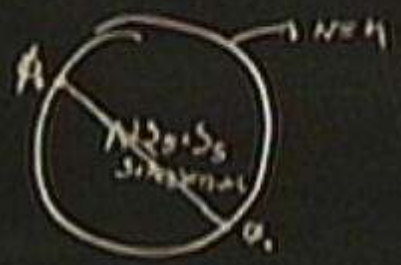
$\frac{\lambda}{N} \longleftrightarrow g_s$

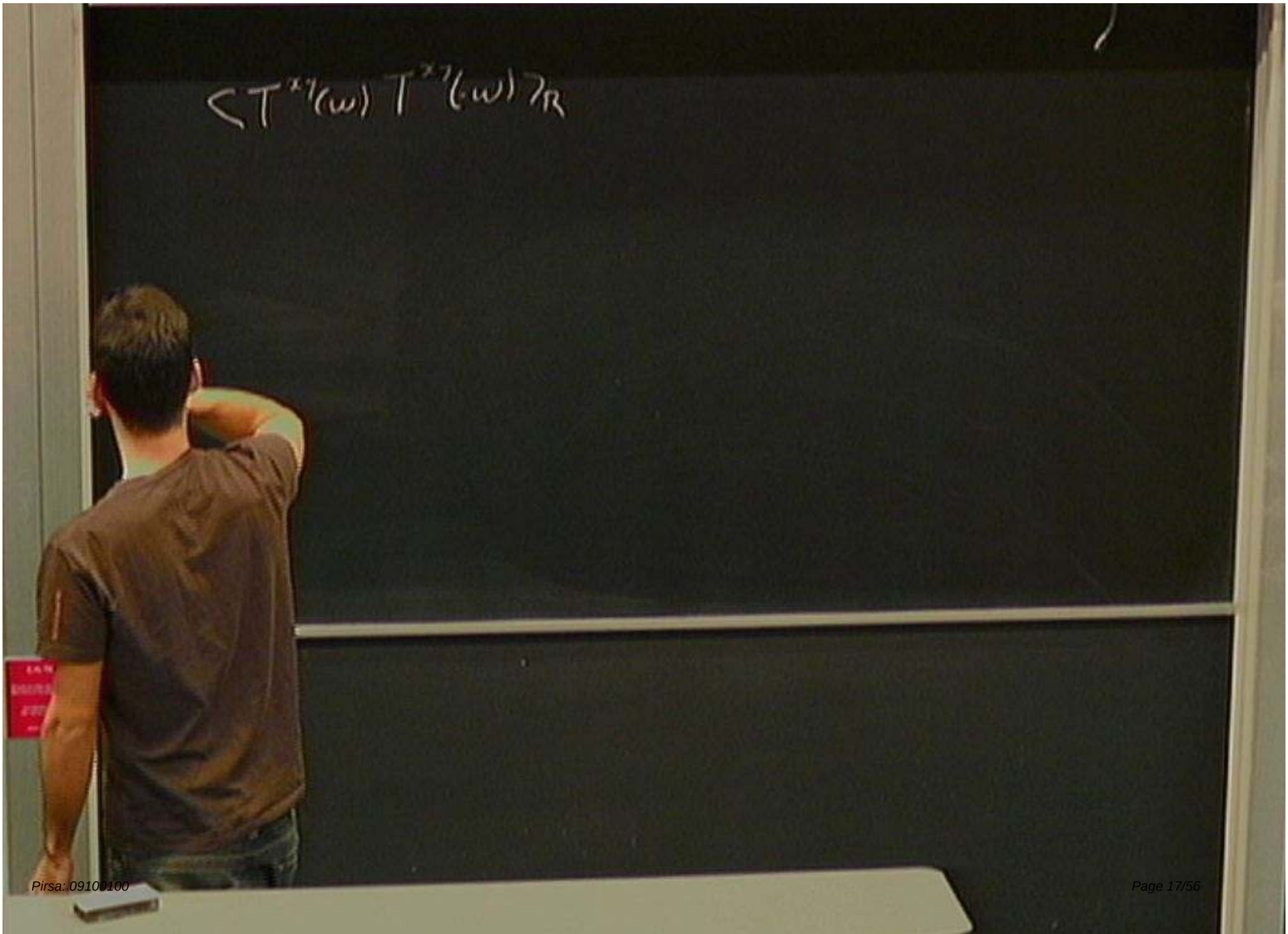
$R + \gamma R^4 + \ldots$

$1/\sqrt{\lambda}$

$$Z^{\text{non sym}}(\phi_0)$$

$$Z^{N=4\ SYM}(\phi_0) = e^{-S^{on\text{-}shell}_{SUGRA}(\phi_0)}$$



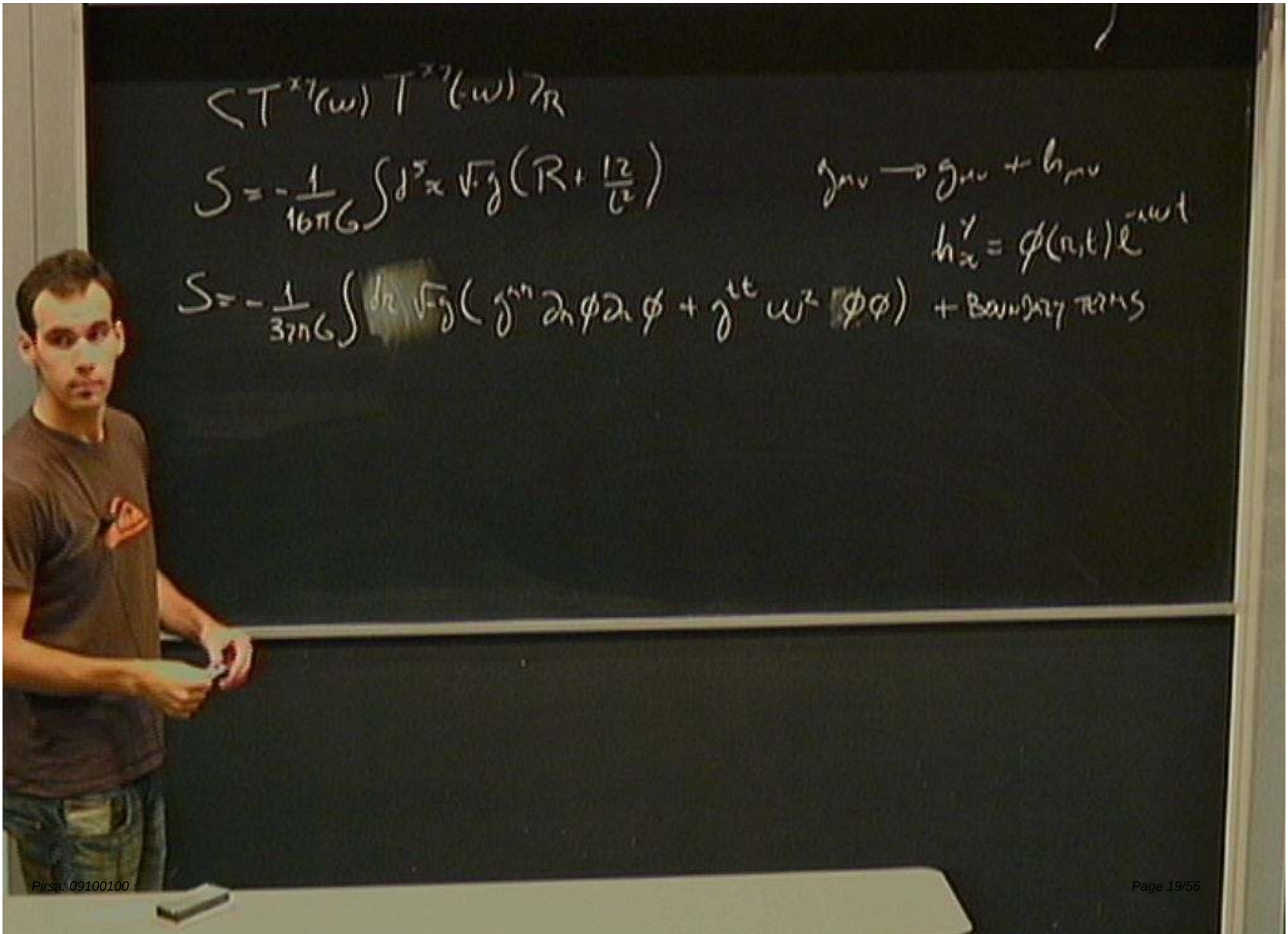

$\langle T^{xy}(\omega)\, T^{xy}(\omega) \rangle_R$

$$\langle T^{xy}(\omega)\, T^{xy}(-\omega)\rangle_R$$

$$S = -\frac{1}{16\pi G}\int d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2}\right)$$

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

$$h^{y}_{x} = \phi(r,t)\, e^{-i\omega t}$$


$\langle T^{xy}(\omega)\, T^{xy}(-\omega) \rangle_R$
$S = -\frac{1}{16\pi G}\int d^5x \sqrt{-g}\left(R + \frac{12}{L^2}\right)$
$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$
$h^y_x = \phi(r,t)\, e^{-i\omega t}$
$S = -\frac{1}{32\pi G}\int dr \sqrt{-g}\left(g^{rr}\partial_r\phi\,\partial_r\phi + g^{tt}\omega^2\phi\phi\right)$ + BOUNDARY TERMS

$$\langle T^{xy}(\omega)\, T^{xy}(-\omega)\rangle_R$$

$$S = -\frac{1}{16\pi G}\int d^5x\,\sqrt{g}\left(R + \frac{12}{L^2}\right)$$

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

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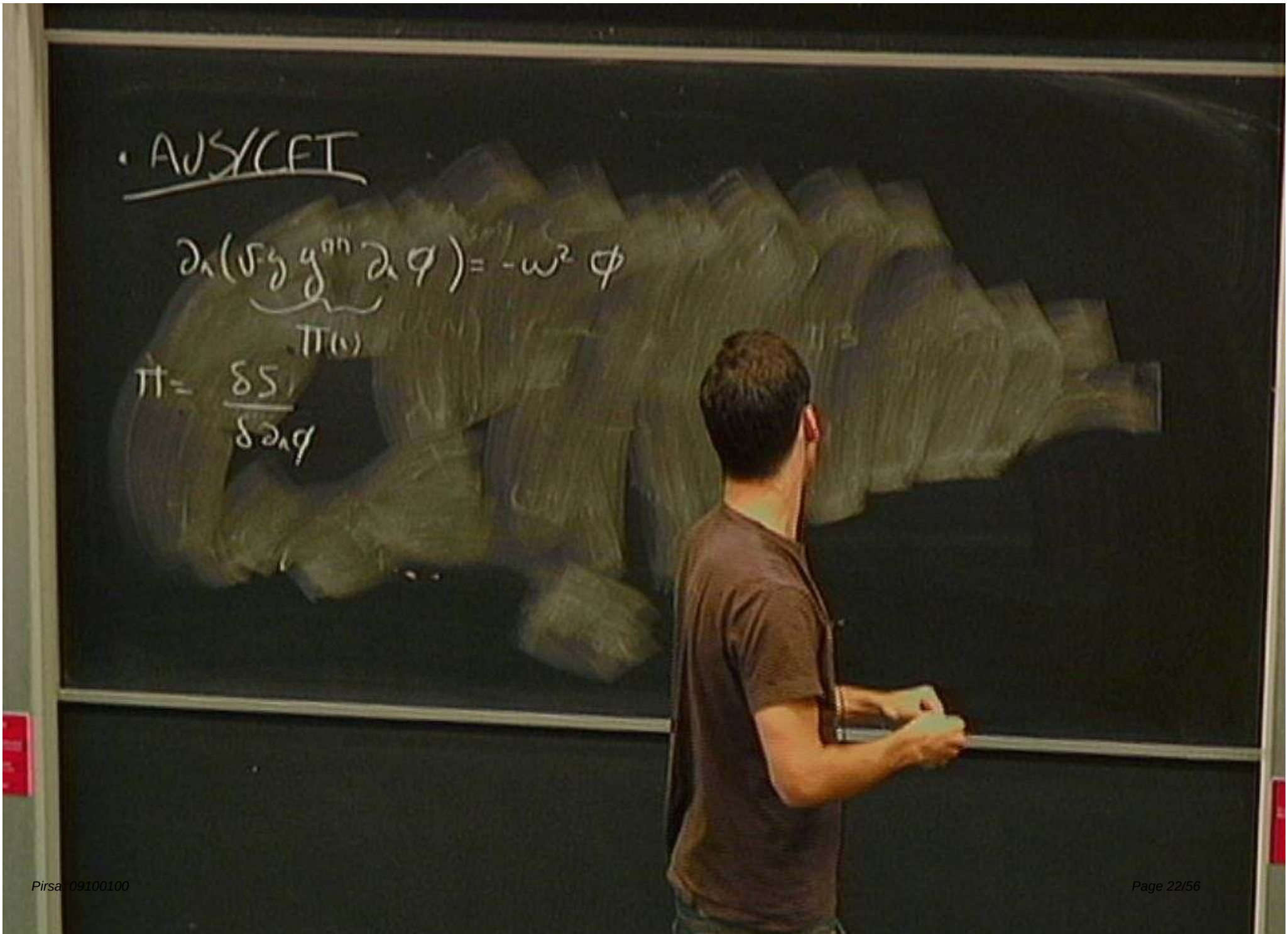
$$S = -\frac{1}{32\pi G}\int dr\,\sqrt{g}\left(g^{rr}\partial_r\phi\,\partial_r\phi + g^{tt}\omega^2\phi\phi\right) + \text{BOUNDARY TERMS}$$

$$h_{rr} = 0$$

$$+\, g_{tt}\,dt^2 + g_{rr}\,dr^2 + g_{xx}\frac{(dx^i)^2}{SO(3)} \qquad dx, dy, dz$$

• AdS/CFT

$$\partial_\mu(\sqrt{-g}\, g^{mn} \partial_\mu \phi) = -\omega^2 \phi$$


• AdS/CFT
$\partial_\mu(\sqrt{g}\, g^{\mu\nu}\partial_\mu\phi) = -\omega^2\phi$
$\Pi(\omega)$
$\Pi = \frac{\delta S}{\delta\partial_\mu\phi}$

- AdS/CFT

$$\partial_\lambda\big(\sqrt{-g}\,\underbrace{g^{mn}}_{\Pi(\lambda)}\,\partial_\lambda\varphi\big) = -\omega^2\varphi$$

$$\Pi = \frac{\delta S}{\delta\,\partial_\lambda\varphi}$$

## • AdS/CFT

$$\partial_\mu(\underbrace{\sqrt{g}\, g^{\mu\nu}\partial_\nu \varphi}_{\Pi(\mu)}) = -\omega^2 \varphi$$

$$\Pi = \frac{\delta S}{\delta \partial_\mu \varphi}$$

$$\partial_\mu \Pi(\mu) = 0$$

## • AdS/CFT

$$\partial_\mu(\underbrace{\sqrt{-g}\, g^{mn}\, \partial_\mu \varphi}_{\Pi(\mu)}) = -\omega^2 \phi$$

$$\Pi = \frac{\delta S}{\delta \partial_\mu \varphi}$$

$$\partial_\mu \Pi(\mu) = 0$$

$$G_R^{x_1, \ldots, x_y} = \lim_{\substack{\omega \to 0 \\ k \to +\infty}} \frac{\Pi(\mu)}{\phi(\mu)}$$

$$\langle T^{xy}(\omega)\, T^{xy}(-\omega)\rangle_R$$

$$S = -\frac{1}{16\pi G}\int d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2}\right)$$

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

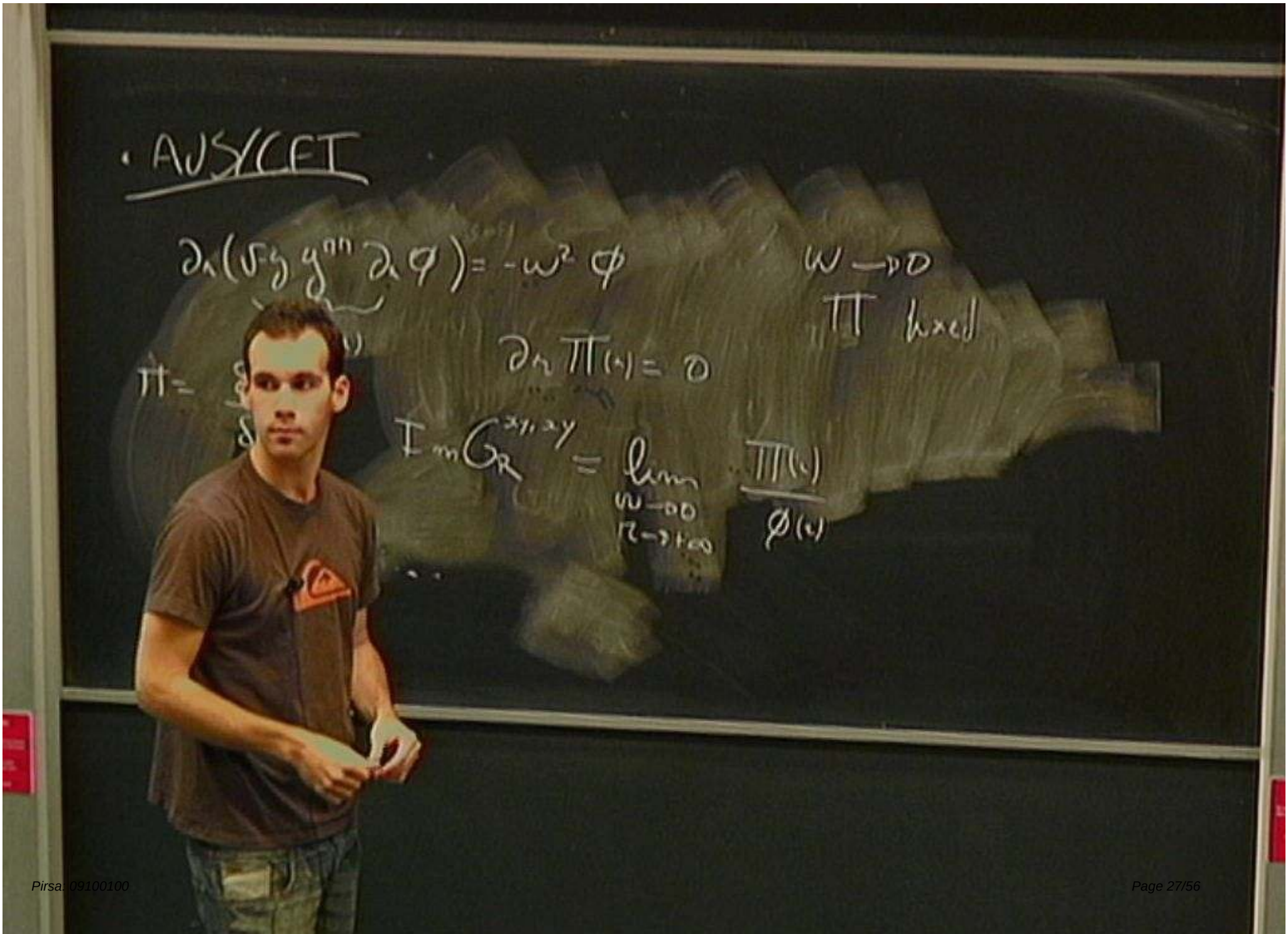
$$h^{y}_{x} = \phi(r)\, e^{-i\omega t}$$

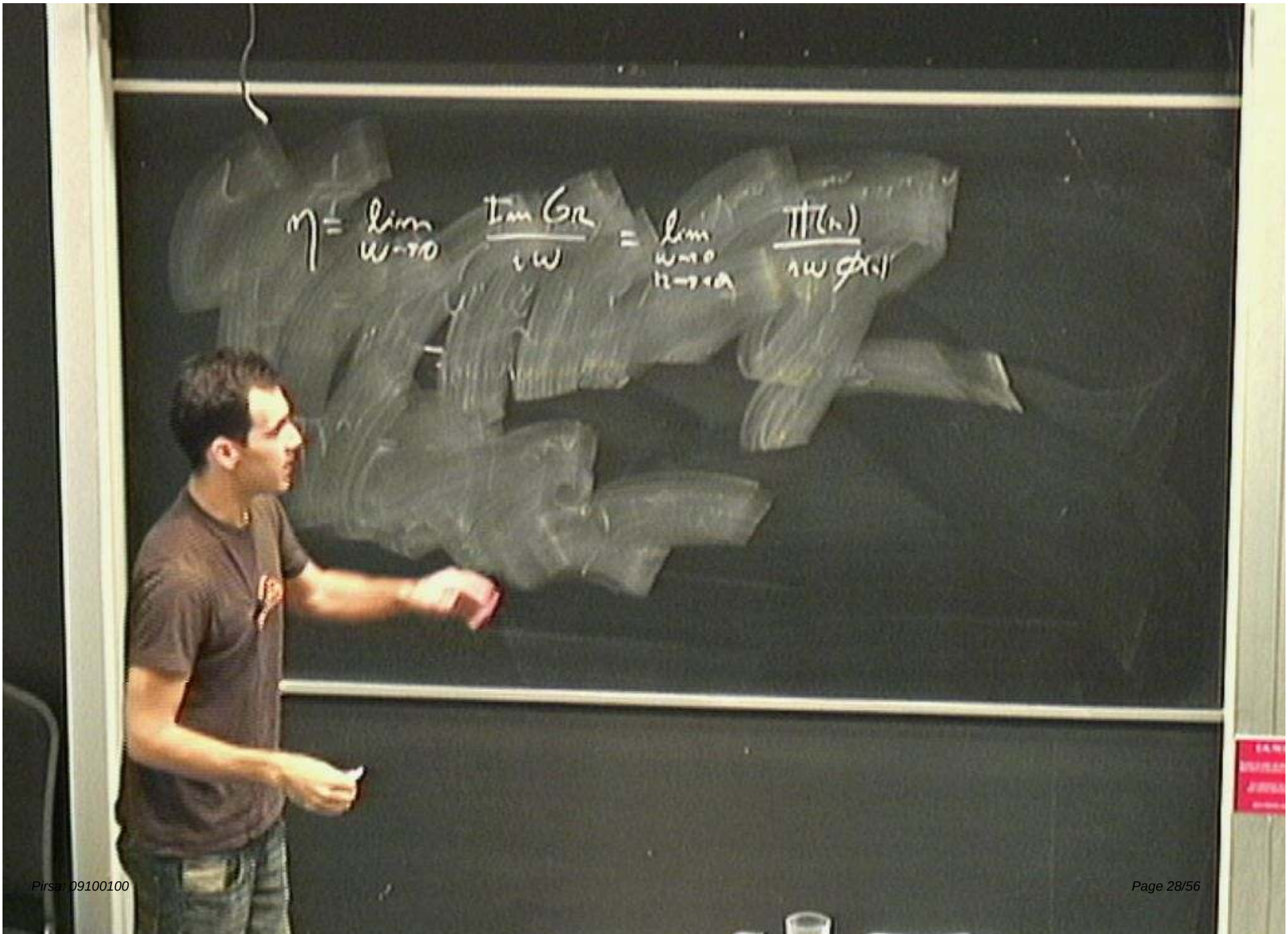
$$S = -\frac{1}{32\pi G}\int dr\,\sqrt{-g}\left(g^{rr}\partial_r\phi\,\partial_r\phi + g^{tt}\,\underline{\omega^2\,\phi\phi}\right) + \text{BOUNDARY TERMS}$$

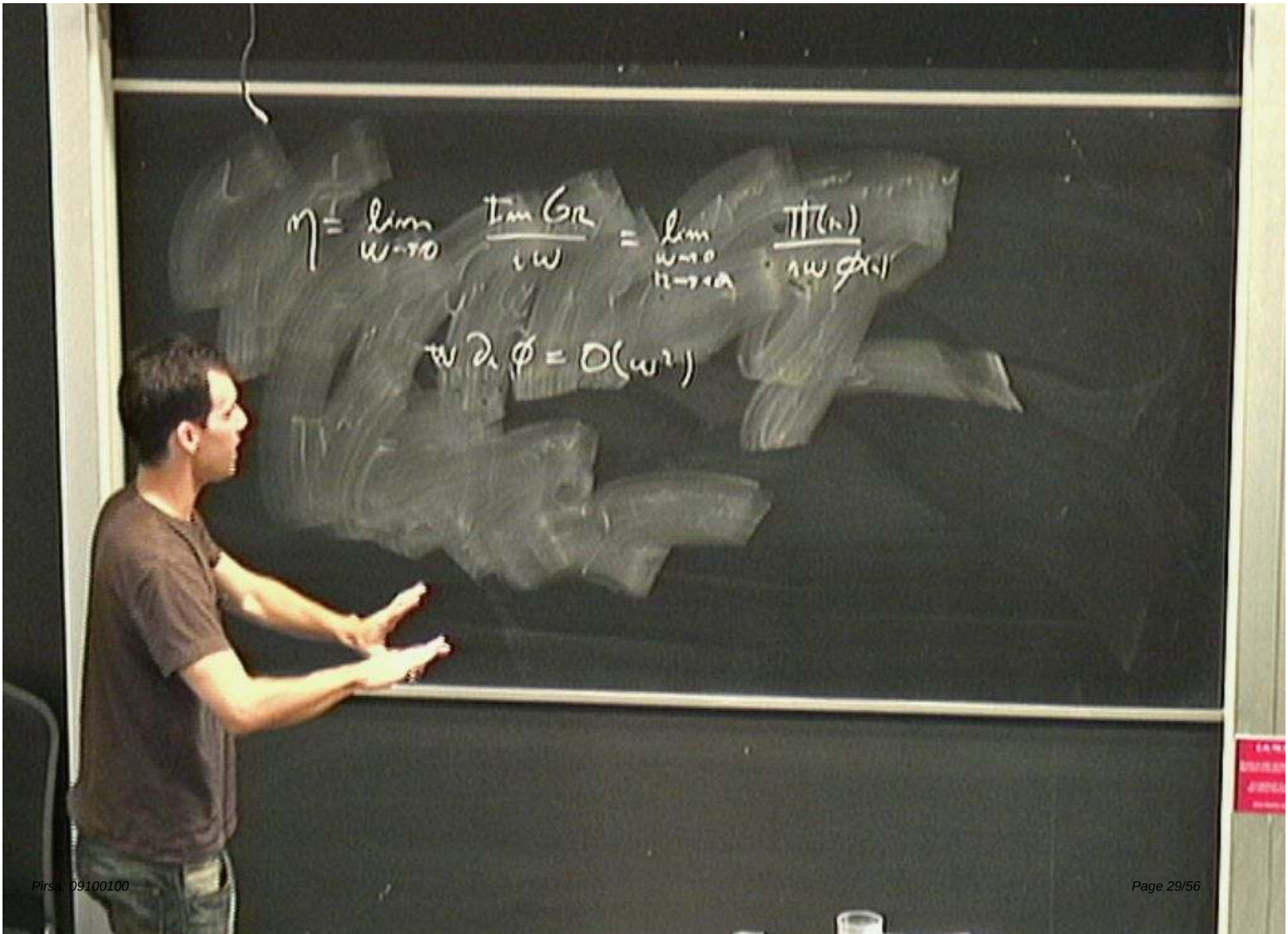
$$h_{rr} = 0$$

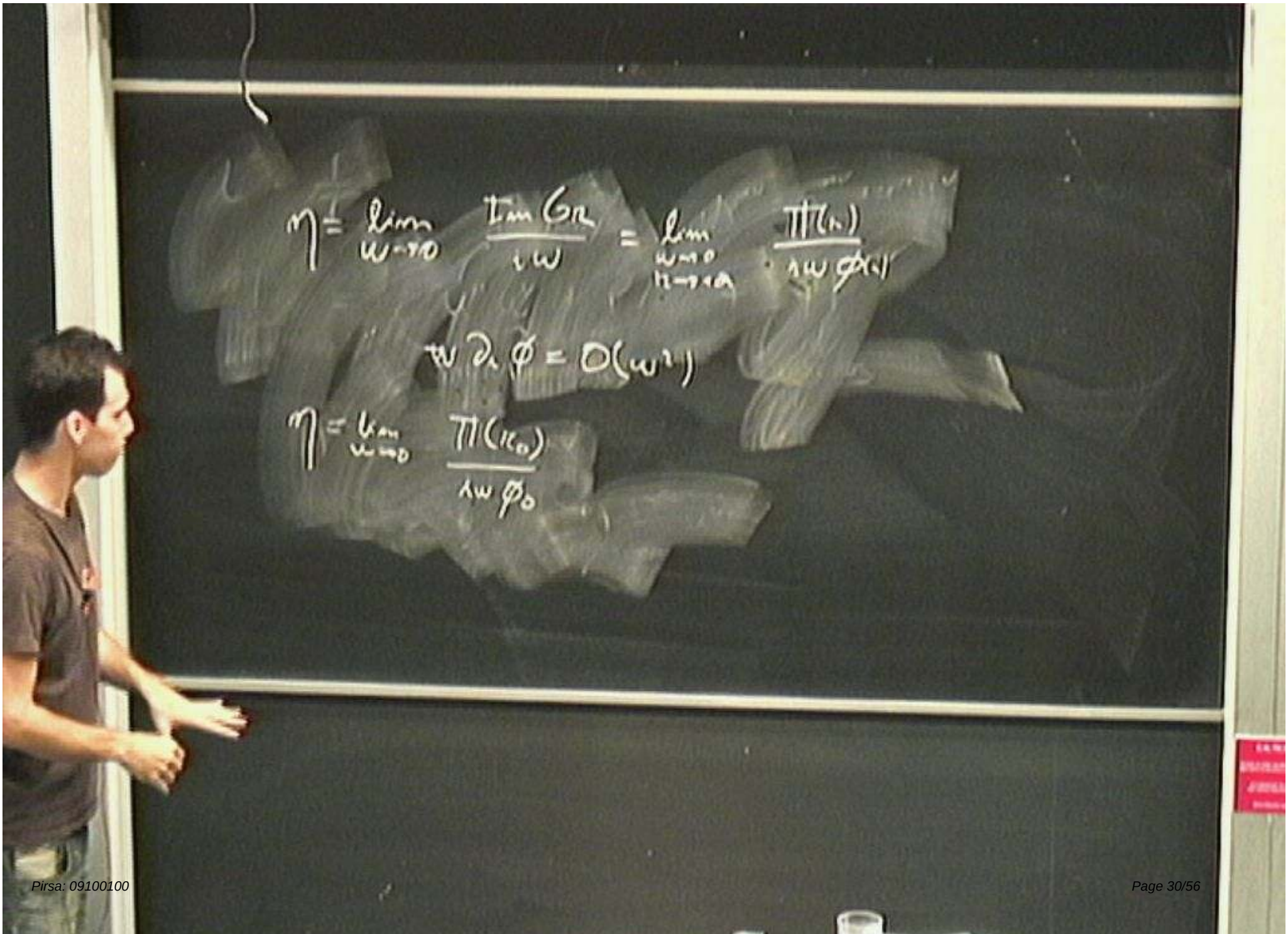
$$-\frac{1}{32\pi G}\sqrt{-g}\, g^{rr}\,\partial_r\phi\,\phi$$

$$+\, g_{tt}\,dt^2 + g_{rr}\,dr^2 + g_{xx}\,\underbrace{(dx^i)^2}_{SO(3)} \qquad dx, dy, dz$$


AdS/CFT
$\partial_\mu(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\phi) = -\omega^2\phi$
$\omega \to 0$
$\Pi$ fixed
$\partial_r \Pi(r) = 0$
$\mathrm{Im}\, G_R^{xy,xy} = \lim_{\substack{\omega\to 0\\ r\to+\infty}} \frac{\Pi(r)}{\phi(r)}$

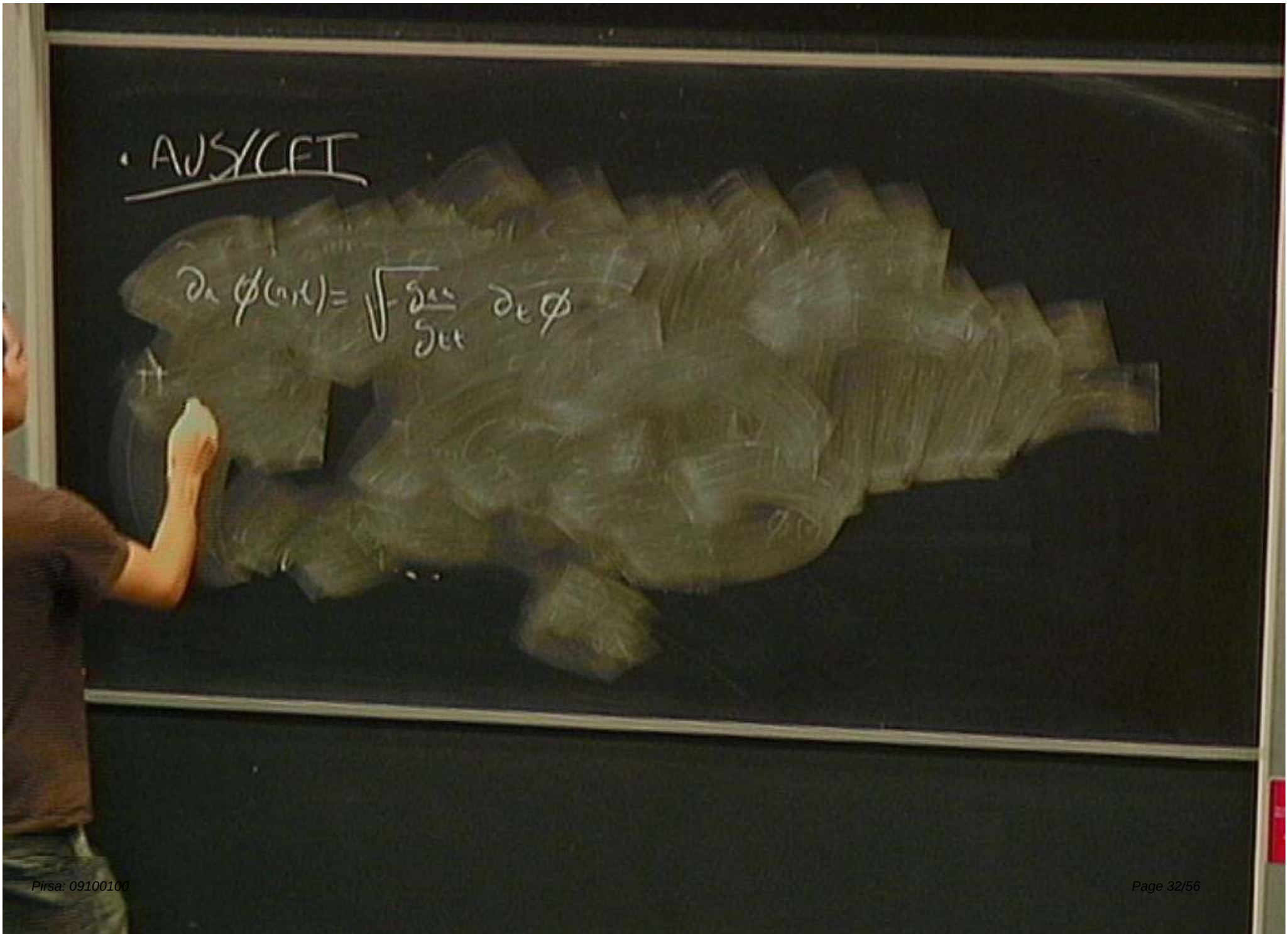

$\eta = \lim_{\omega \to 0} \frac{\mathrm{Im}\, G_R}{i\omega} = \lim_{\substack{\omega \to 0 \\ k \to 0}} \frac{\Pi(k)}{i\omega\, \phi(k)}$


$\eta = \lim_{\omega \to 0} \frac{\mathrm{Im}\, G_R}{i\omega} = \lim_{\substack{\omega \to 0 \\ k \to 0}} \frac{\Pi(k)}{i\omega\, \phi(k)}$
$\omega\, \partial_t \phi = O(\omega^2)$


$\eta = \lim_{\omega \to 0} \frac{\mathrm{Im}\, G_R}{i\omega} = \lim_{\substack{\omega \to 0 \\ k \to 0}} \frac{\Pi(k)}{i\omega\, \phi(k)}$
$\omega\, \partial_t \phi = O(\omega^2)$
$\eta = \lim_{\omega \to 0} \frac{\Pi(k_0)}{i\omega\, \phi_0}$

• AdS/CFT

$$\partial_x \phi(x,t) = \sqrt{-\frac{g_{xx}}{g_{tt}}}\, \partial_t \phi$$

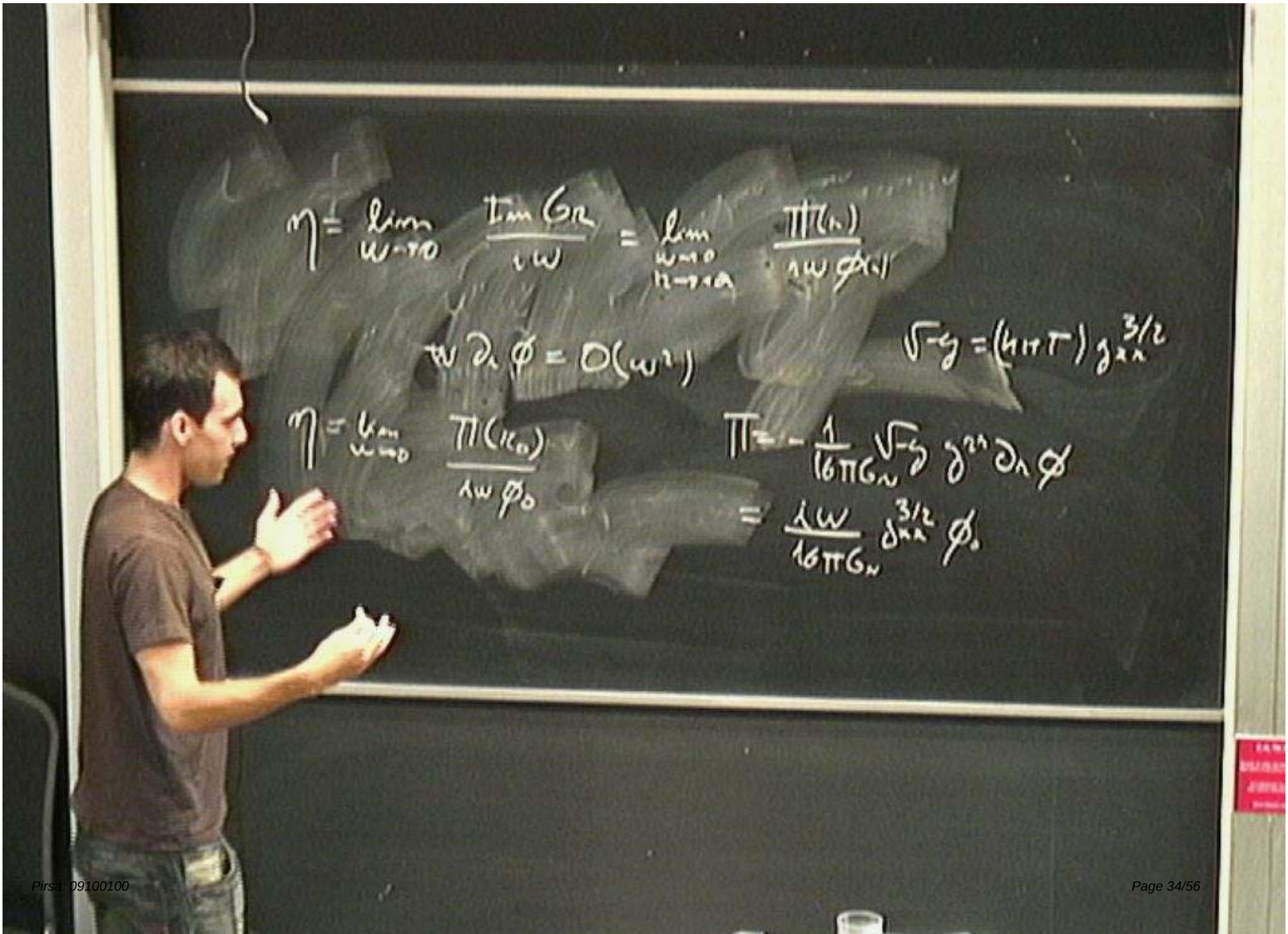

AdS/CFT
$\partial_a \phi(x,t) = \sqrt{-\frac{g_{aa}}{g_{tt}}}\, \partial_t \phi$

## • AdS/CFT

$$\partial_n \phi(n,t) = \sqrt{-\frac{g_{nn}}{g_{tt}}}\, \partial_t \phi$$

$$+ \quad \partial_n \phi(n) = \pm \frac{i\omega}{4\pi T} \frac{\phi(n)}{(r - r_0)}$$

...


$\eta = \lim_{\omega \to 0} \frac{\mathrm{Im}\, G_R}{i\omega} = \lim_{\substack{\omega \to 0 \\ k \to 0}} \frac{\Pi(k)}{i\omega\, \phi(k)}$
$i\omega\, \partial_\lambda \phi = O(\omega^2)$
$\sqrt{-g} = (4\pi T)\, g_{xx}^{3/2}$
$\eta = \lim_{\omega \to 0} \frac{\Pi(k_0)}{i\omega\, \phi_0}$
$\Pi = -\frac{1}{16\pi G_N} \sqrt{-g}\, g^{\lambda\lambda} \partial_\lambda \phi$
$= \frac{i\omega}{16\pi G_N} g_{xx}^{3/2} \phi_0$

$$\eta = \lim_{\omega \to 0} \frac{\mathrm{Im}\, G_R}{i\omega} = \lim_{\substack{\omega \to 0 \\ k \to 0}} \frac{\Pi(k)}{i\omega\, \phi(k)}$$

$$\omega\, \partial_\lambda \phi = O(\omega^2)$$

$$\sqrt{-g} = (4\pi r)\, g_{xx}^{3/2}$$

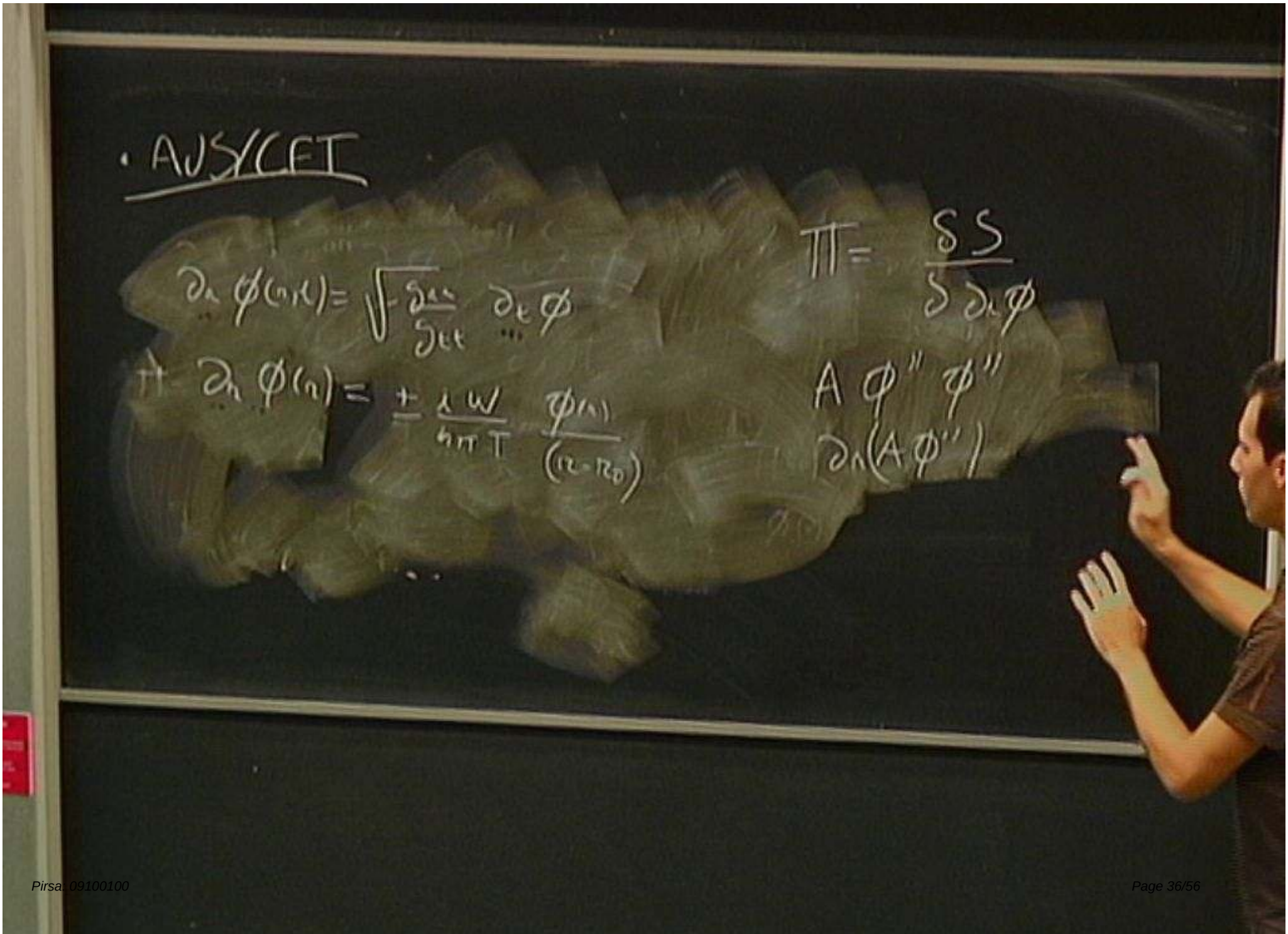
$$\eta = \lim_{\omega \to 0} \frac{\Pi(k_0)}{i\omega\, \phi_0}$$

$$\Pi = -\frac{1}{16\pi G_N} \sqrt{-g}\, g^{\lambda\lambda} \partial_\lambda \phi$$

$$= \frac{i\omega}{16\pi G_N} g_{xx}^{3/2} \phi_0$$

$$S = \frac{r_0^3}{4 G_N}$$

$$\eta = \frac{r_0^3}{16\pi G_N}$$


• AdS/CFT
$\partial_n \phi(n,t) = \sqrt{-\frac{g_{nn}}{g_{tt}}}\, \partial_t \phi$
$\partial_n \phi(n) = \pm \frac{i\omega}{4\pi T} \frac{\phi(n)}{(n-n_0)}$
$\Pi = \frac{\delta S}{\delta \partial_n \phi}$
$A \phi'' \phi''$
$\partial_n (A \phi'')$

• AdS/CFT

$$\partial_n \phi(n,t) = \sqrt{-\frac{g_{nn}}{g_{tt}}}\, \partial_t \phi$$

$$+ \quad \partial_n \phi(n) = \pm \frac{i\omega}{4\pi T} \frac{\phi(n)}{(r - r_D)}$$

$$\Pi = \frac{\delta S}{\delta \partial_n \phi}$$

$$A \phi'' \phi''$$

$$\partial_n (A \phi'')$$

$$\partial_n \Pi = O(\omega^2)$$

• AdS/CFT

$$\partial_n \phi(n,t) = \sqrt{-\frac{g_{nn}}{g_{tt}}}\, \partial_t \phi$$

$$\Pi = \frac{\delta S}{\delta \partial_n \phi}$$

$$\partial_n \phi(n) = \pm \frac{i\omega}{4\pi T} \frac{\phi(n)}{(r - r_0)}$$

$$A\, \phi'' \phi''$$

$$\partial_n (A \phi'')$$

$$S = \frac{1}{2} \int d^4x \sqrt{-g}\, (g^{nn} \partial_n \phi \partial_n \phi + m^2 \omega^2 \phi^2)$$

$$\partial_n \Pi = O(\omega^2)$$

## • AdS/CFT

$$\partial_n \phi(n,t) = \sqrt{-\frac{g_{nn}}{g_{tt}}}\, \partial_t \phi$$

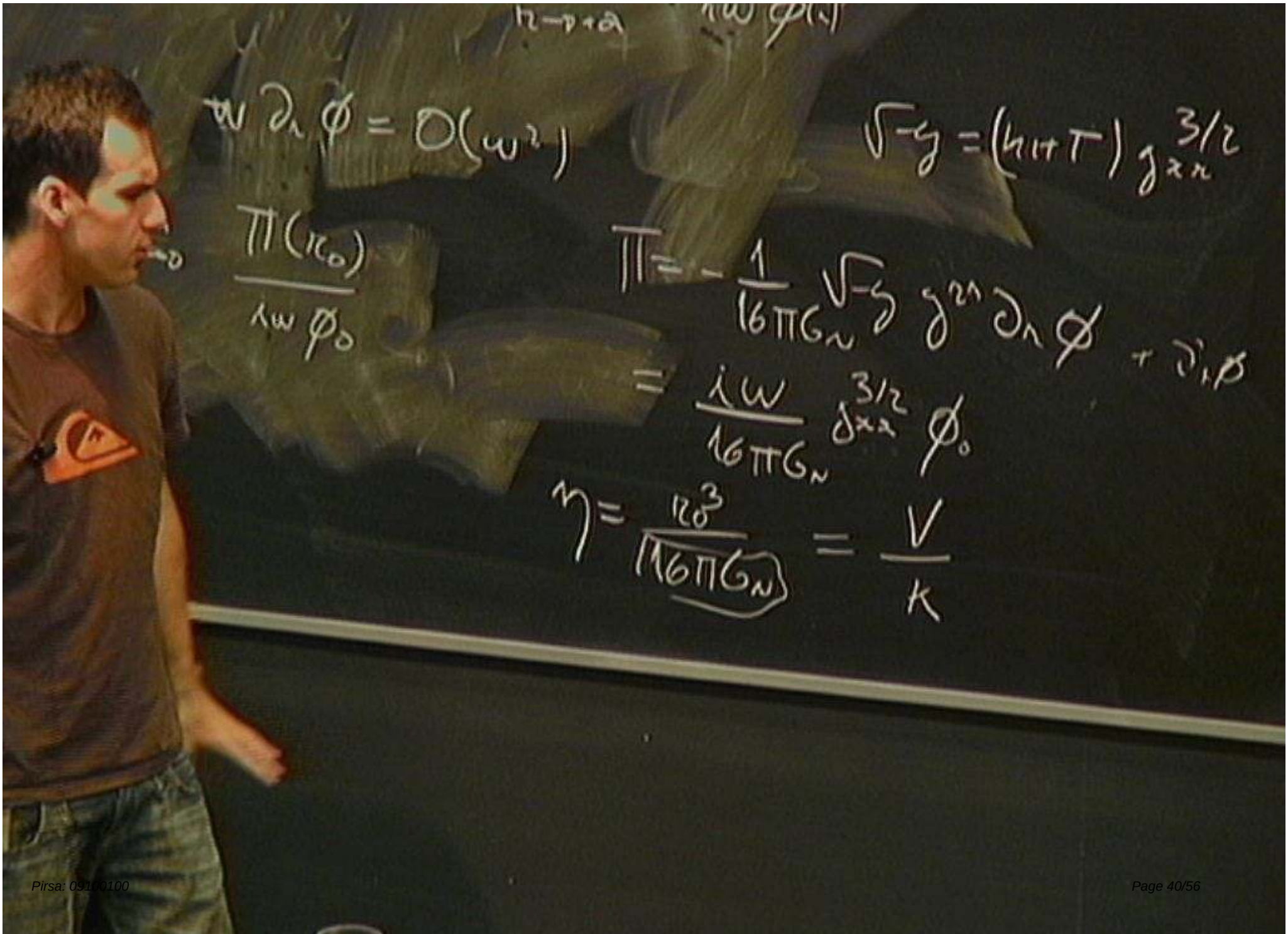
$$\pm\, \partial_n \phi(n) = \pm \frac{i\omega}{4\pi T} \frac{\phi(n)}{(r - r_0)}$$

$$S^{mass} = \frac{1}{2} \int d^4x \sqrt{-g}\, \frac{g_{11}}{\hbar} \left( g^{nn} \partial_n \phi \partial_n \phi + m^2 \omega^2 \phi^2 \right) \partial_n \Pi = O(\omega^2)$$

$$\Pi = \frac{\delta S}{\delta \partial_n \phi}$$

$$A\, \phi'' \phi''$$

$$\partial_n (A \phi'')$$


$\omega \partial_\lambda \phi = O(\omega^2)$
$\sqrt{-g} = (k_\mu + T)\, g^{x_n}$
$\frac{\Pi(k_0)}{i\omega \phi_0}$
$\Pi = -\frac{1}{16\pi G_N} \sqrt{-g}\, g^{2n} \partial_n \phi + \partial_i \beta$
$= \frac{i\omega}{16\pi G_N} g^{x_n} \phi_0$
$\eta = \frac{k^3}{16\pi G_N} = \frac{V}{K}$

$$\phi = (r - r_0)^{\frac{i\omega}{4\pi T}}$$

$$\phi = (r - r_0)^{\frac{i\alpha}{4\pi T}}$$

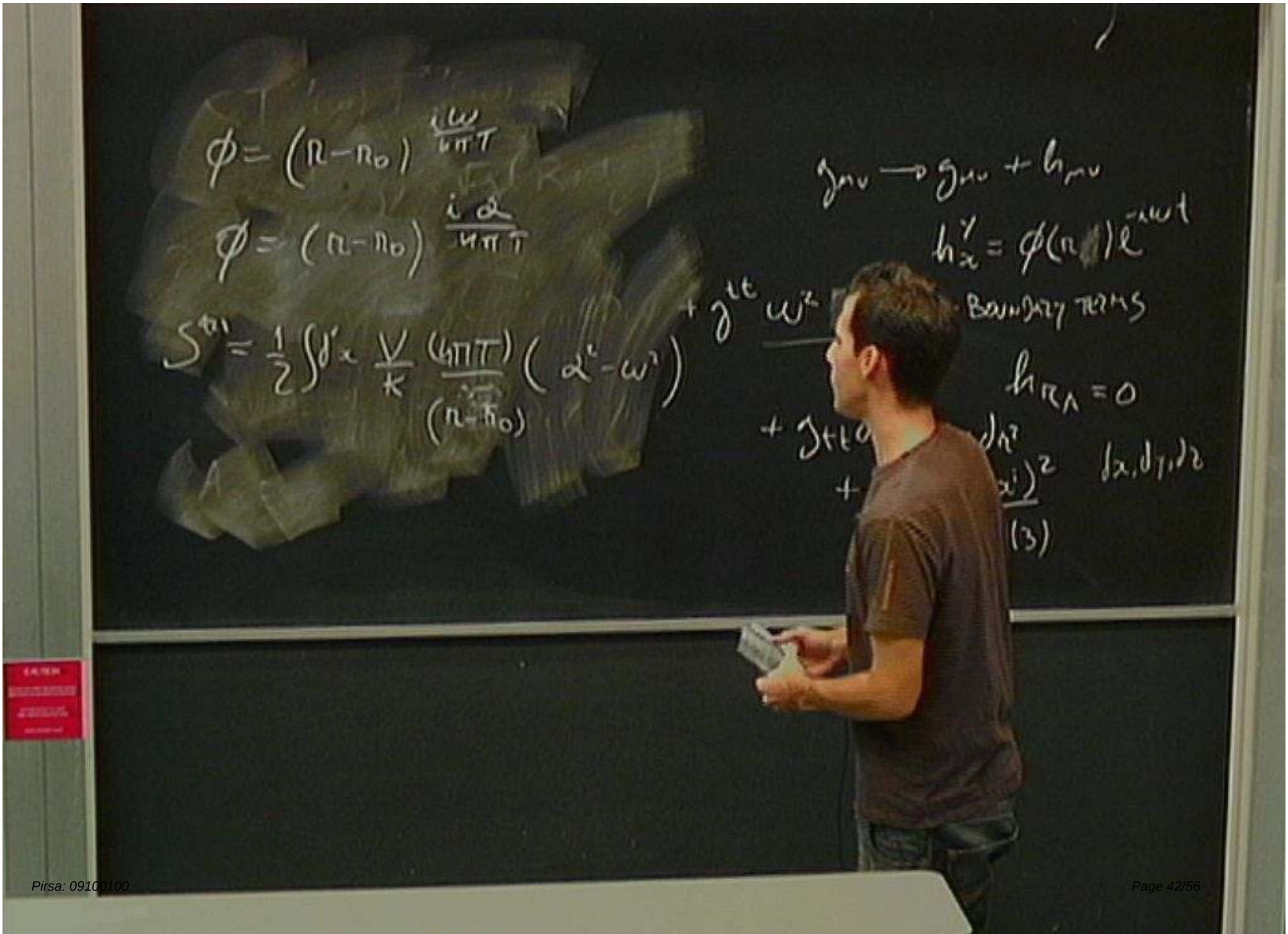
$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

$$h^{\gamma}_{x} = \phi(r)\, e^{-i\omega t}$$

$$+ g^{tt}\, \underline{\omega^2 (\phi\phi)} + \text{BOUNDARY TERMS}$$

$$h_{rr} = 0$$

$$+ g_{tt}\, dt^2 + g_{rr}\, dr^2 + g_{xx} \underbrace{(dx^i)^2}_{SO(3)} \qquad dx, dy, dz$$


$\phi = (r - r_0)^{\frac{i\omega}{4\pi T}}$
$\phi = (r - r_0)^{\frac{i\alpha}{4\pi T}}$
$S^{(1)} = \frac{1}{2}\int d^4x \frac{V}{K} \frac{(4\pi T)}{(r - r_0)} (\alpha^2 - \omega^2)$
$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$
$h_x^y = \phi(r)e^{-i\omega t}$
BOUNDARY TERMS
$h_{rA} = 0$
$dx, dy, dz$
(3)

$$\phi = (r - r_0)^{\frac{i\omega}{4\pi T}}$$

$$\phi = (r - r_0)^{\frac{i\alpha}{4\pi T}}$$

$$S^{(1)} = \frac{1}{2}\int d^4x \, \frac{V}{k} \frac{(4\pi T)}{(r - r_0)} (\alpha^2 - \omega^2)$$

Res $\mathcal{L}_{\phi = \phi_h}$

$$\ln y = \frac{(r - r_0)^{\frac{i\omega}{4\pi T}}}{\phi_h}$$

• AdS/CFT

$$-\frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left(R+\frac{12}{L^2}+\gamma C^4\right)$$

$\hookrightarrow C_{abcd}C^{abcd}$

$C_{abcd}$

$C_{ijkl}$

$+\ldots$

## • AdS/CFT

$$-\frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left(R+\frac{12}{L^2}+\gamma C^4\right)$$

$ds^2$

$dx \to dx + \phi(z)\,dy$

$\downarrow$

$(12-\Lambda_0)\frac{\phi}{4\pi T}$

$\hookrightarrow C_{abcd}C^{abcd}$

$C_{abcd}$

$C_{ijkl}$

$+\ldots$

## • AdS/CFT

$$-\frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left(R+\frac{12}{L^2}+\gamma C^4\right)$$

$$ds^2$$

$$dx \to dx + \varphi(r)\,dy$$

$$\downarrow$$

$$(r-r_0)^{\frac{\alpha}{4\pi T}}$$

$$\hookrightarrow C_{abcd}C^{abcd}$$

$$\hookrightarrow -\frac{1}{16\pi G}\frac{V(4\pi T)}{(r-r_0)}(1+180\gamma)+\dots$$

$$+\mathcal{O}\left(\frac{1}{(r-r_0)^2}\right)+\text{regular}$$

$$C_{abcd}$$

$$C_{ij}{}^{ij}$$

• AdS/CFT

$$-\frac{1}{4}F^2 + (DF)^2 + (DDF)^2 + F^4$$

$$-\frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left(R + \frac{12}{L^2} + \gamma C^4\right)$$

$$ds^2$$

$$dx \to dx + \phi(r)\,dy$$

$$\downarrow$$

$$(r-r_0)^{\frac{\alpha}{4\pi T}}$$

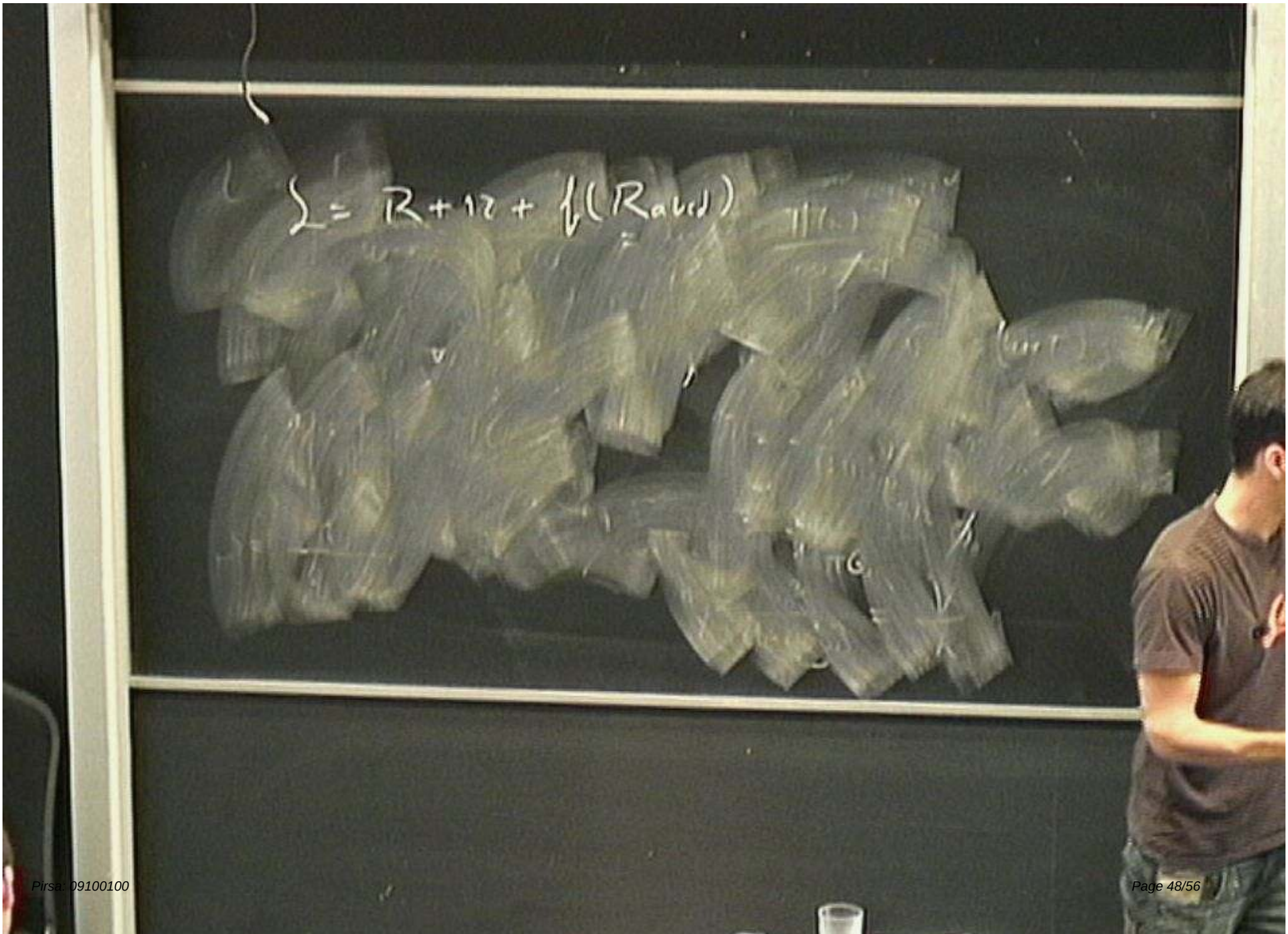
$$\hookrightarrow C_{abcd}C^{abcd}$$

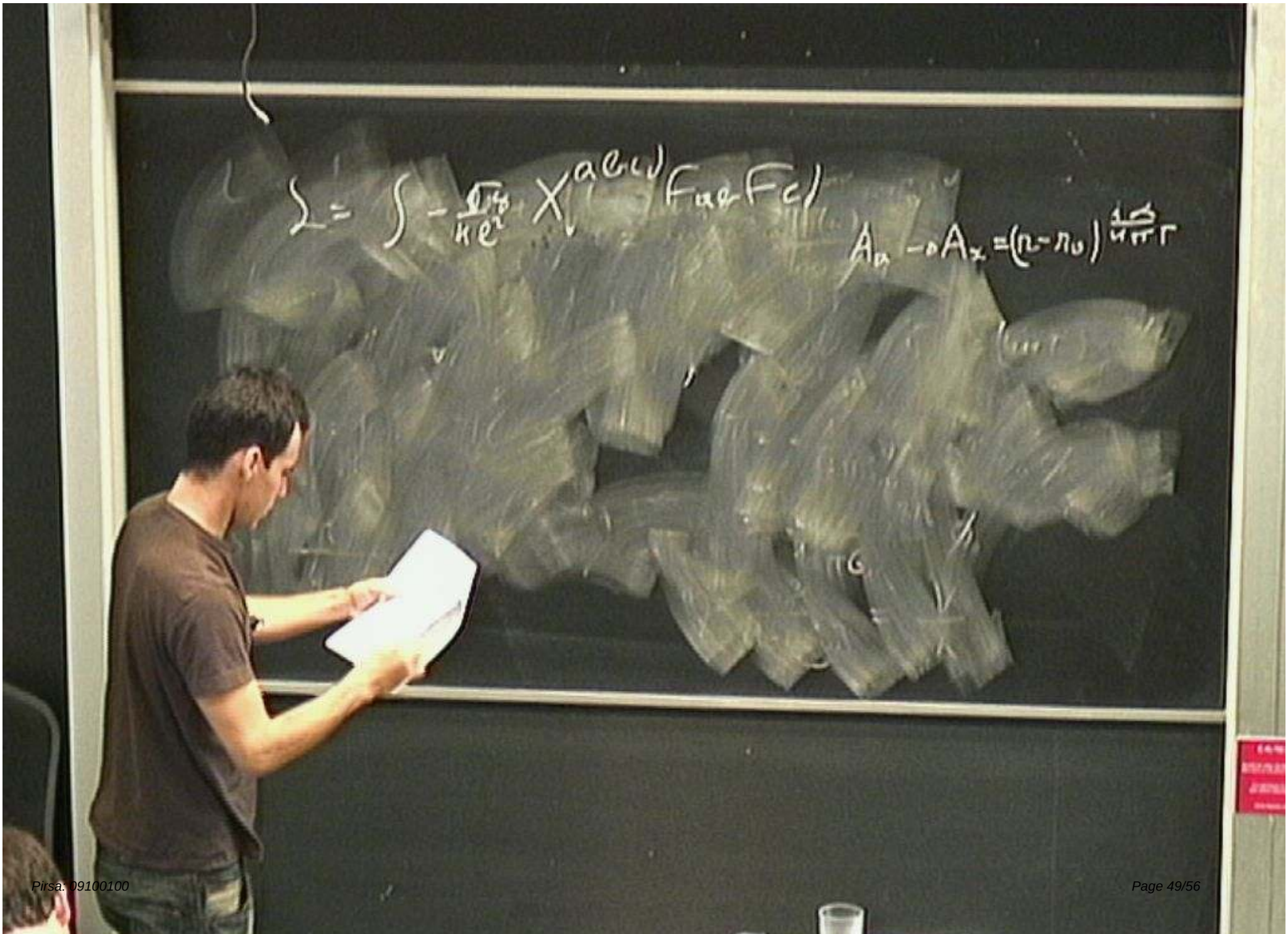
$$\hookrightarrow -\frac{1}{16\pi G}\frac{V(4\pi T)}{(r-r_0)}(1+180\gamma)+\dots$$

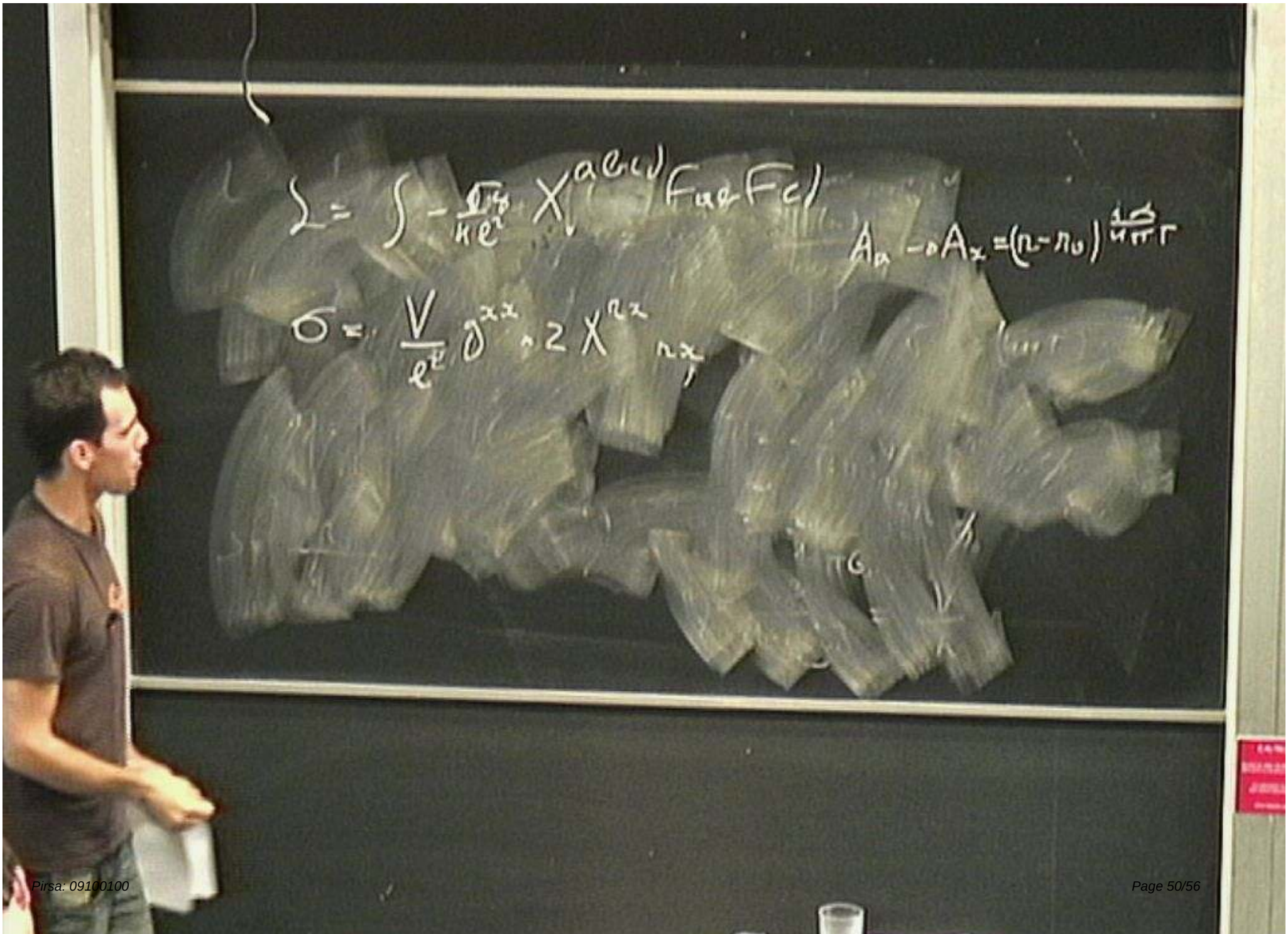
$$+\mathcal{O}\left(\frac{1}{(r-r_0)^2}\right) + \text{regular}$$

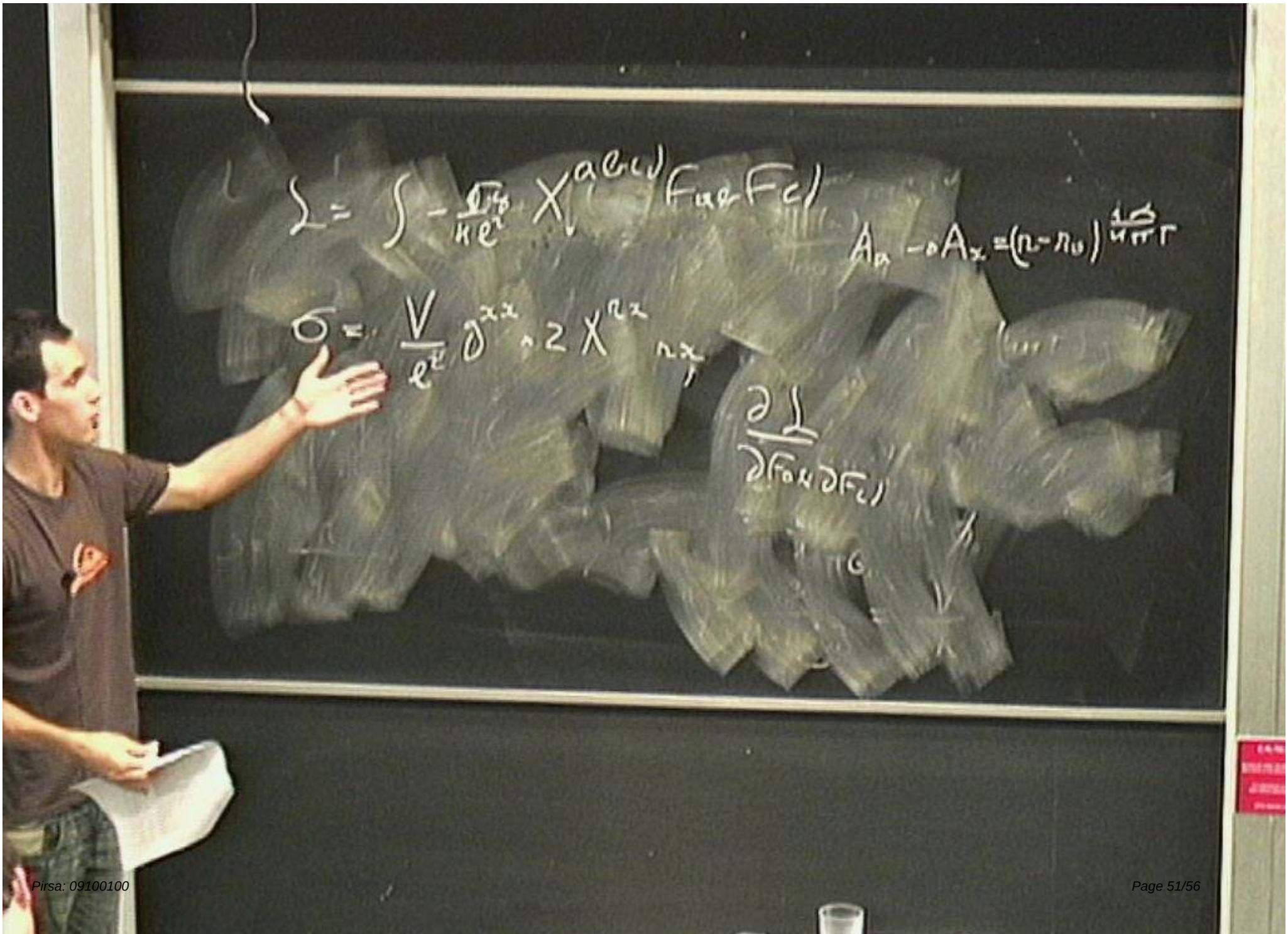
$$C_{abcd}$$

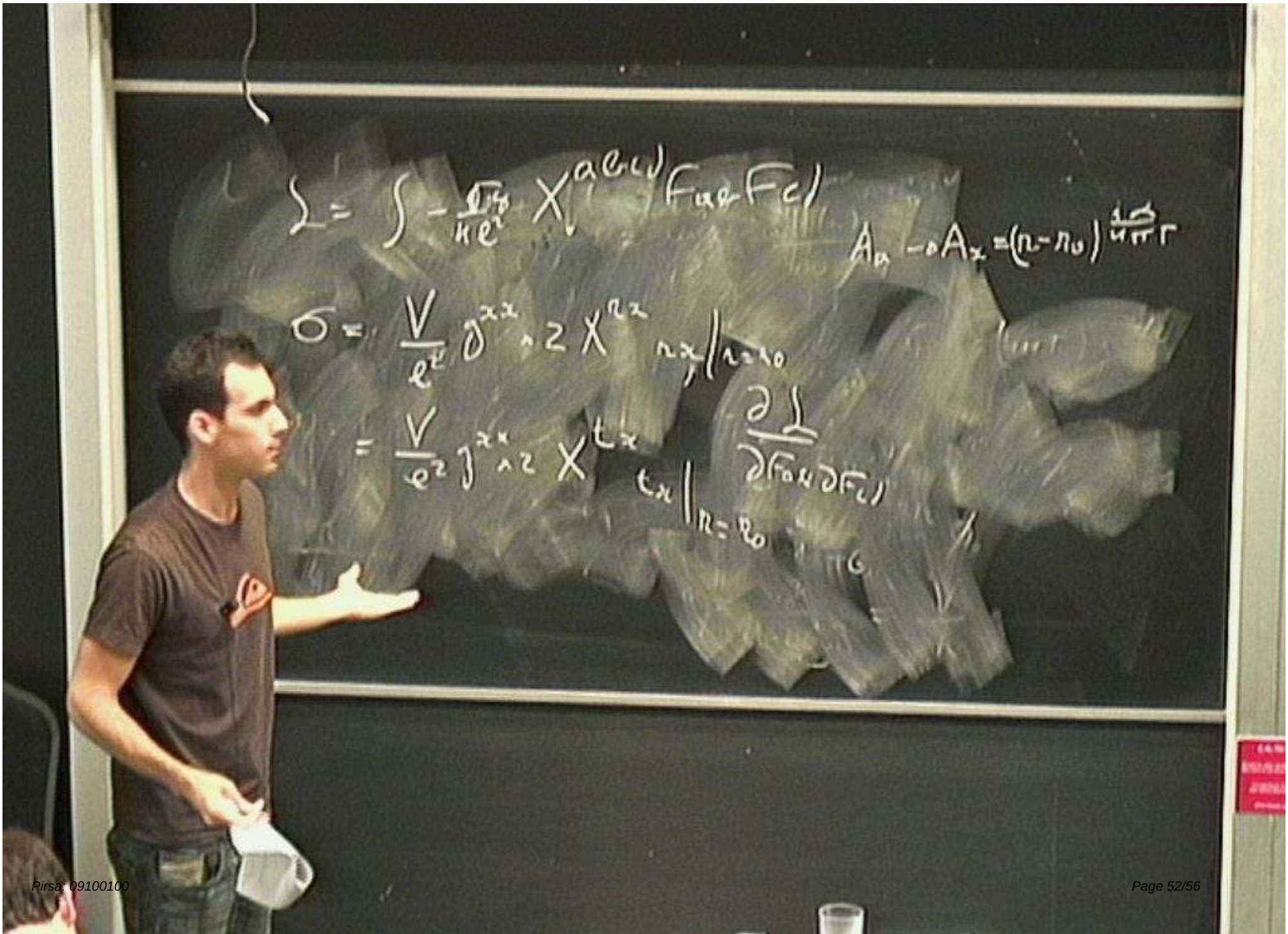
$$C_{ij}{}^{ij}$$


$\mathcal{L} = R + \mathcal{R} + f(R_{abcd})$


$\mathcal{L} = \int -\frac{\sqrt{-g}}{4e^2} X^{ab(cd)} F_{ab} F_{cd}$
$A_\mu \to A_x = (n - n_0) \frac{4\pi}{4\pi r}$

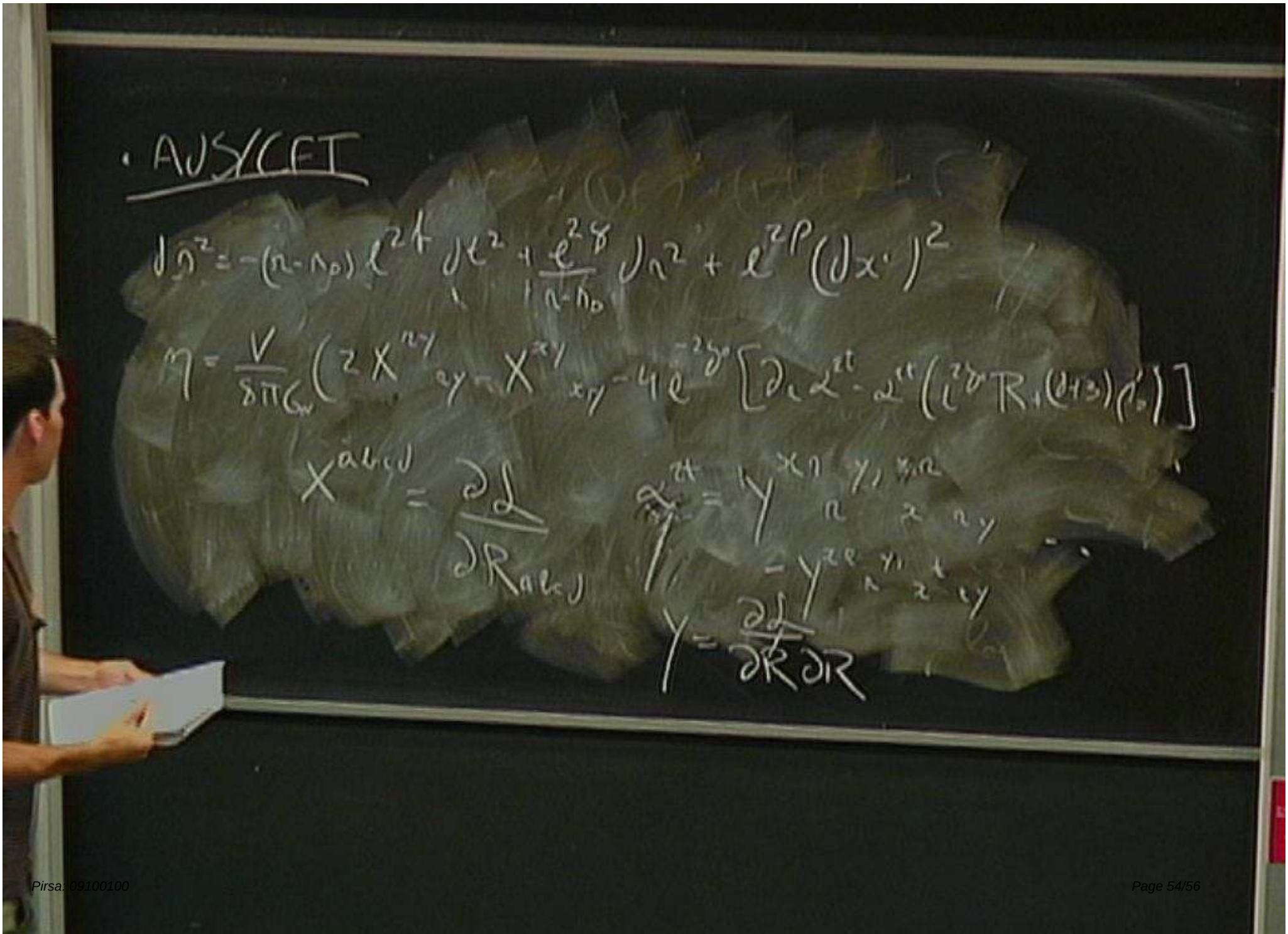

$S = \int -\frac{1}{4e^2} X^{ab(c)} F_{ab} F_{c}$
$A_n \to A_x = (n - n_0) \frac{4\pi}{4\pi r}$
$\sigma = \frac{V}{e^2} \delta^{xx} + 2 X^{nx}{}_{nx}$


$\mathcal{L} = \int -\frac{\Gamma_0}{4e^2} X^{ab(c)} F_{ab} F_{c)}$
$A_\theta \to A_x = (n - n_0) \frac{4\pi}{4\pi r}$
$\sigma = \frac{V}{e^2} \delta^{xx} + 2 X^{nx} n_x$
$\frac{\partial \mathcal{L}}{\partial F_{ab} \partial F_{c)}}$


$\mathcal{L} = \int -\frac{\sqrt{-g}}{4e^2} X^{ab(cd)} F_{ab} F_{cd}$
$A_\mu \to A_x = (r - r_0) \frac{\Delta\phi}{4\pi r}$
$\sigma = \frac{V}{e^2} \partial^{xx} \wedge 2 X^{rx}{}_{rx} \Big|_{r=r_0}$
$= \frac{V}{e^2} \partial^{xx} \wedge 2 X^{tx}{}_{tx} \Big|_{r=r_0} \frac{\partial \mathcal{L}}{\partial F_{ab} \partial F_{cd}}$

• AdS/CFT

$$ds^2 = -(r-r_0)\, e^{2f}\, dt^2 + \frac{e^{2\gamma}}{r-r_0}\, dr^2 + e^{2\rho}\,(dx^i)^2$$


• AdS/CFT
$ds^2 = -(r-r_0)e^{2f}dt^2 + \frac{e^{2g}}{(r-r_0)}dr^2 + e^{2p}(dx^i)^2$
$\eta = \frac{V}{8\pi G_N}(2X^{ny}{}_{ny} - X^{xy}{}_{xy} - 4e^{-2\phi}[\partial_\mu \alpha^\mu - \alpha^\mu(e^{2\phi}R_\mu(d+3)(r_0))]$
$X^{abcd} = \frac{\partial \mathcal{L}}{\partial R_{abcd}}$
$\alpha^{\mu} = Y^{xn}{}_{n}{}^{y}{}_{x}{}^{n}{}_{ny}$
$= Y^{xe}{}_{n}{}^{y}{}_{x}{}^{t}{}_{ey}$
$Y = \frac{\partial \mathcal{L}}{\partial R \partial R}$

## • AdS/CFT

$$ds^2 = -(r-r_0)\, e^{2f}\, dt^2 + \frac{e^{2g}}{(r-r_0)}\, dr^2 + e^{2p}\,(dx^i)^2$$

$$\eta = \frac{V}{8\pi G_N}\Big(2X^{ry}{}_{ry} - X^{xy}{}_{xy} - 4e^{-2g}\big[\partial_r \alpha^{rt} - \alpha^{rt}\big(e^{2g} R + (d+3)(p')\big)\big]$$

$$\delta_\omega X^{rt}{}_{rt} \qquad X^{abcd} = \frac{\partial \mathcal{L}}{\partial R_{abcd}} \qquad \alpha^{rt} = Y^{xr}{}_{r}{}^{y}{}_{x}{}^{ry} - Y^{xe}{}_{x}{}^{y}{}_{x}{}^{t}{}_{y}$$

$$Y = \frac{\partial \mathcal{L}}{\partial R\, \partial R}$$

$$\phi = (r - r_0)^{\frac{i\omega}{4\pi T}}$$

$$\phi = (r - r_0)^{\frac{i\alpha}{4\pi T}}$$

$$\partial_r \phi = \sqrt{-\frac{\Omega_H}{\Delta H}}\,(-i\omega)\,\phi$$

$$\eta = -(8\pi T)\,\frac{\mathrm{Res}\ \mathcal{L}_{\phi=\phi_n}}{\alpha^2}$$

$$\phi = e^{-i\omega t}$$

$$h^{xy} = \underbrace{(r - r_0)^{\frac{i\omega}{4\pi T}}}_{\phi_n}$$