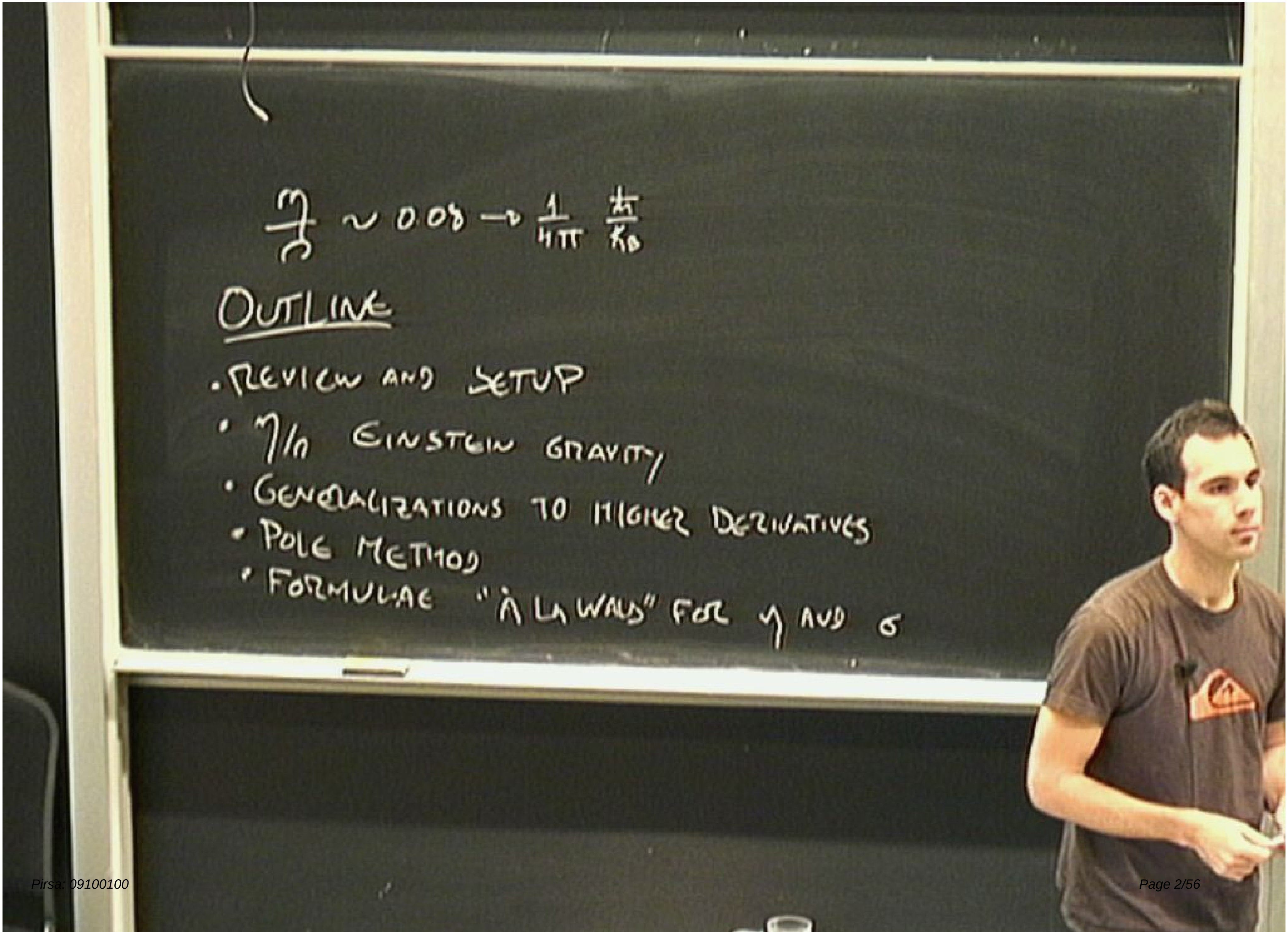
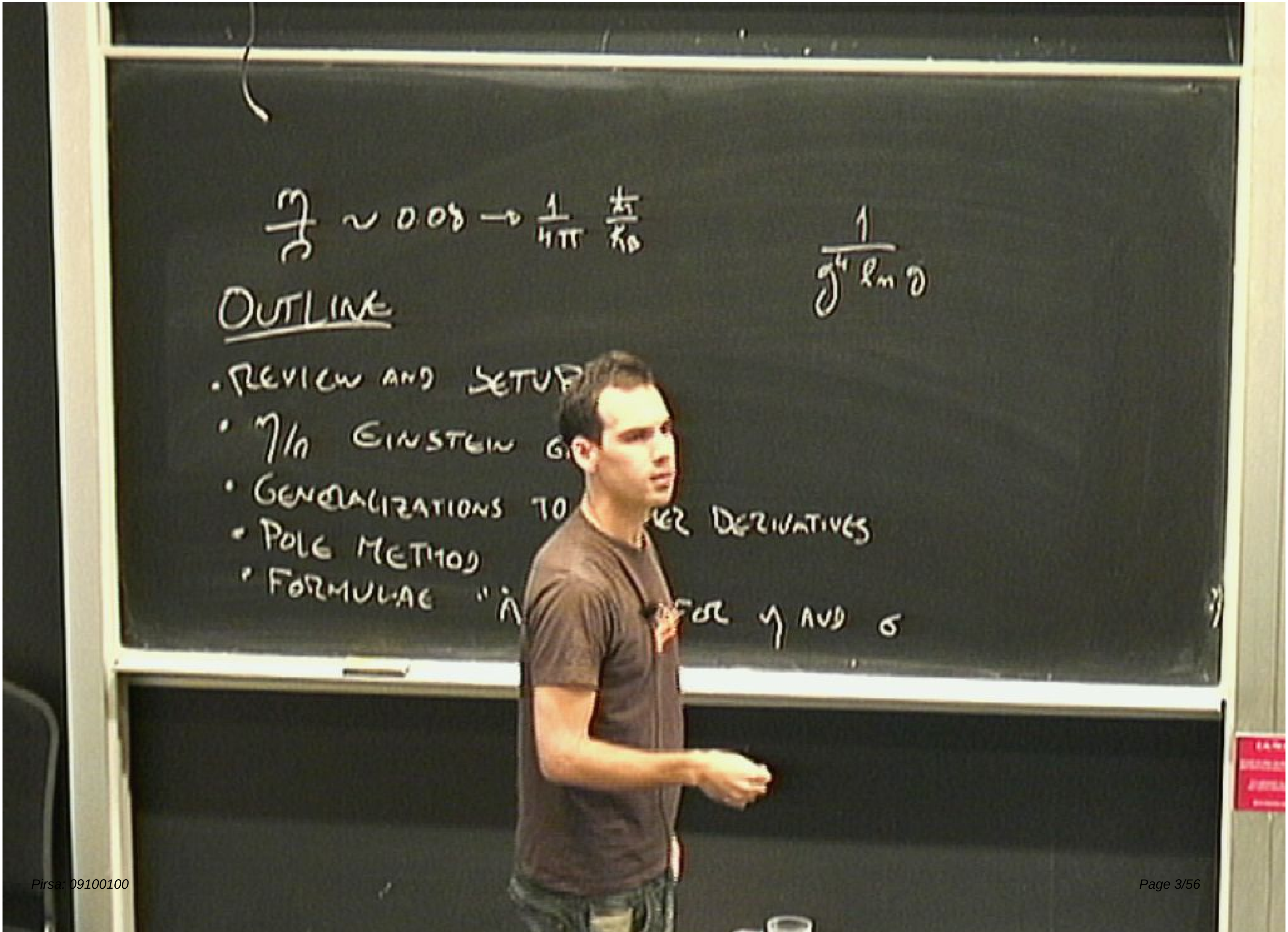
Title: Viscosity and conductivity in general theories of gravity

Date: Oct 20, 2009 11:00 AM

URL: http://pirsa.org/09100100

Abstract: Recently there has been great interest in calculating transport coefficients for field theories at large coupling, using AdS/CFT. In this talk I will discuss recent work showing how to use the membrane paradigm to easily compute the shear viscosity and conductivity in arbitrary gravity theories. In a certain sense these can be thought of as effective couplings at the black hole horizon dual to the field theory plasma. An explicit Wald-like formula for these couplings is given for a large class of generalized gravity theories.


$\frac{\eta}{s} \sim 0.08 \to \frac{1}{4\pi}\frac{\hbar}{k_B}$
OUTLINE
• REVIEW AND SETUP
• η/s EINSTEIN GRAVITY
• GENERALIZATIONS TO HIGHER DERIVATIVES
• POLE METHOD
• FORMULAE "À LA WALD" FOR η AND σ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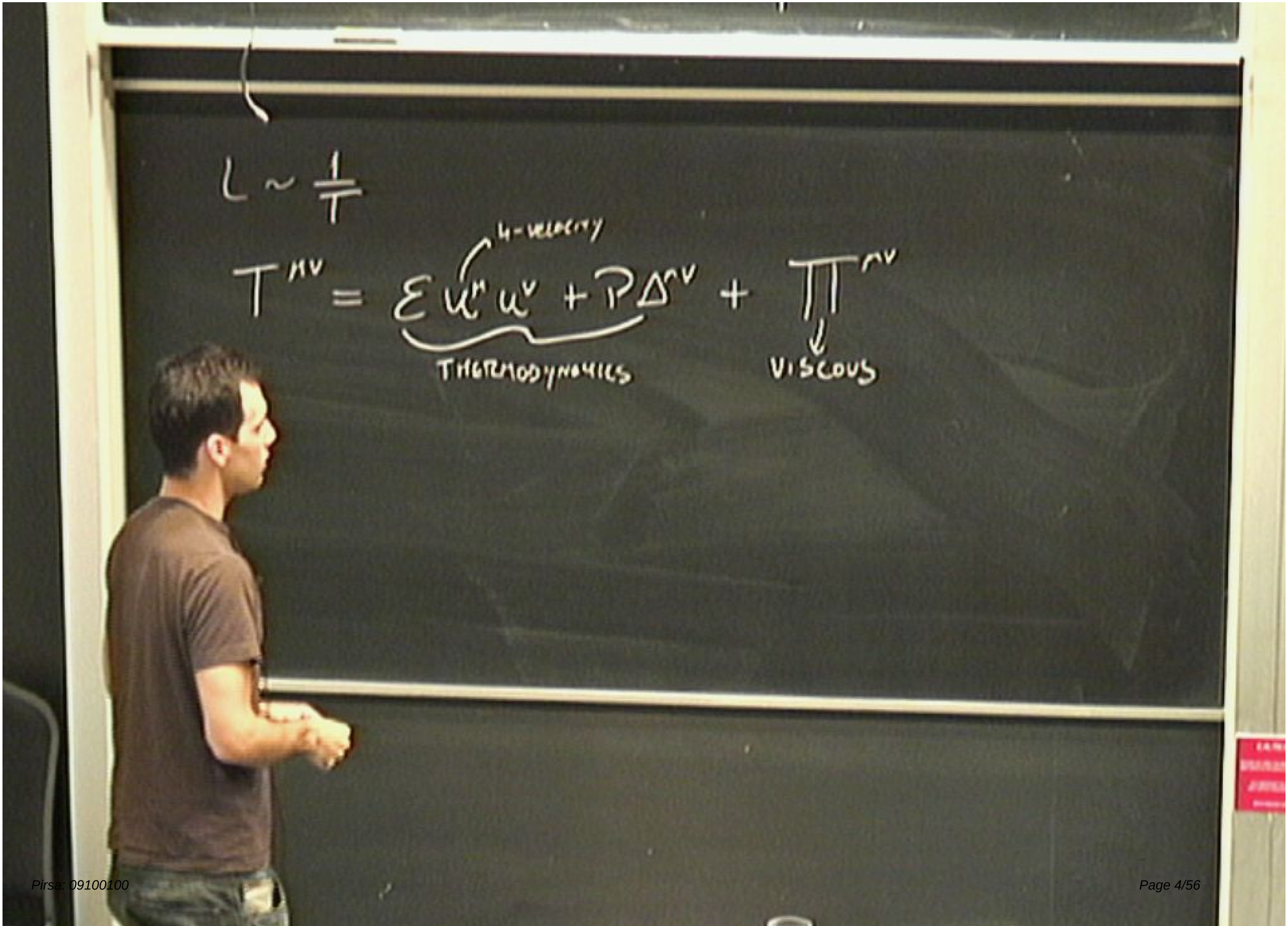
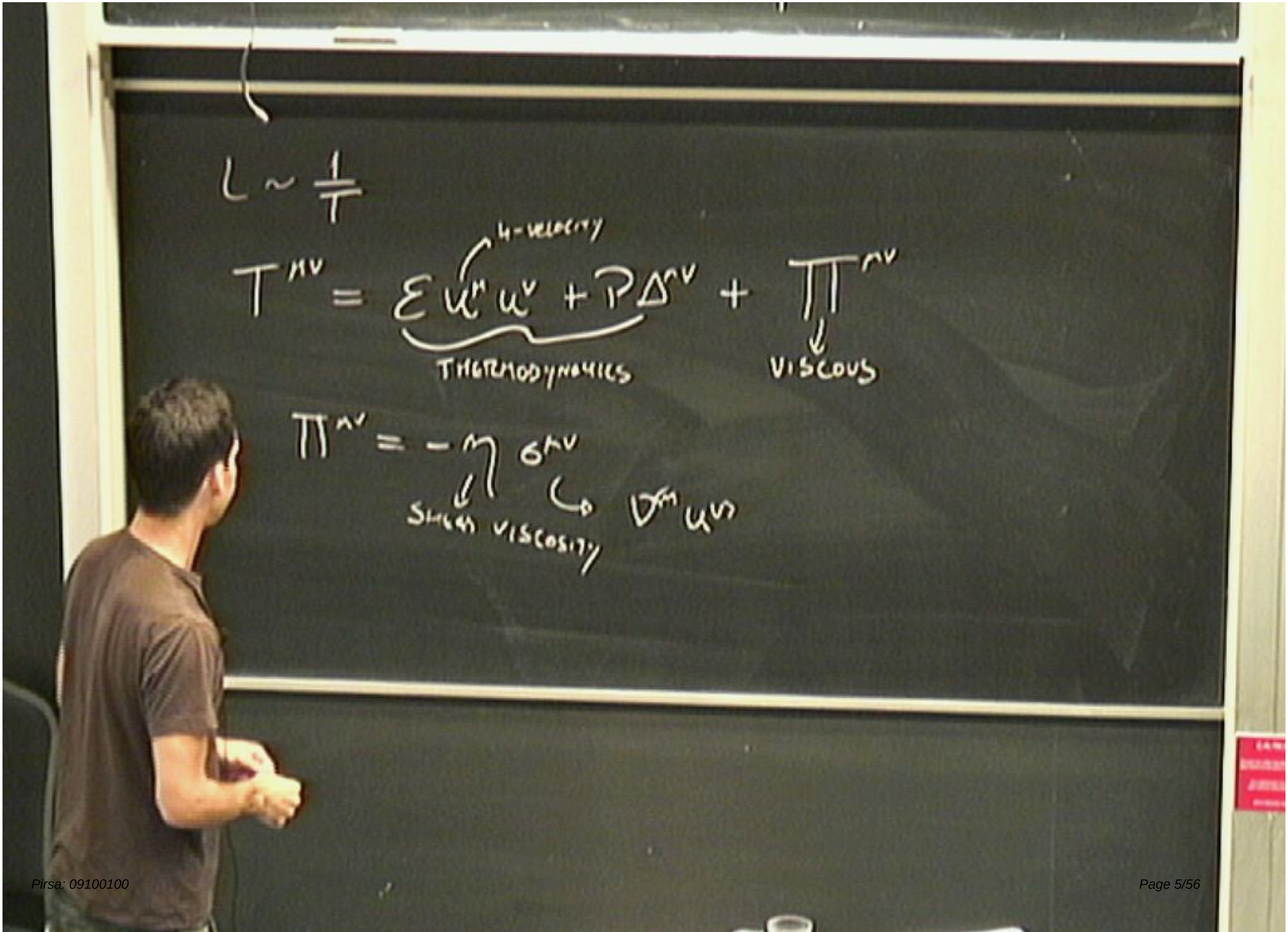
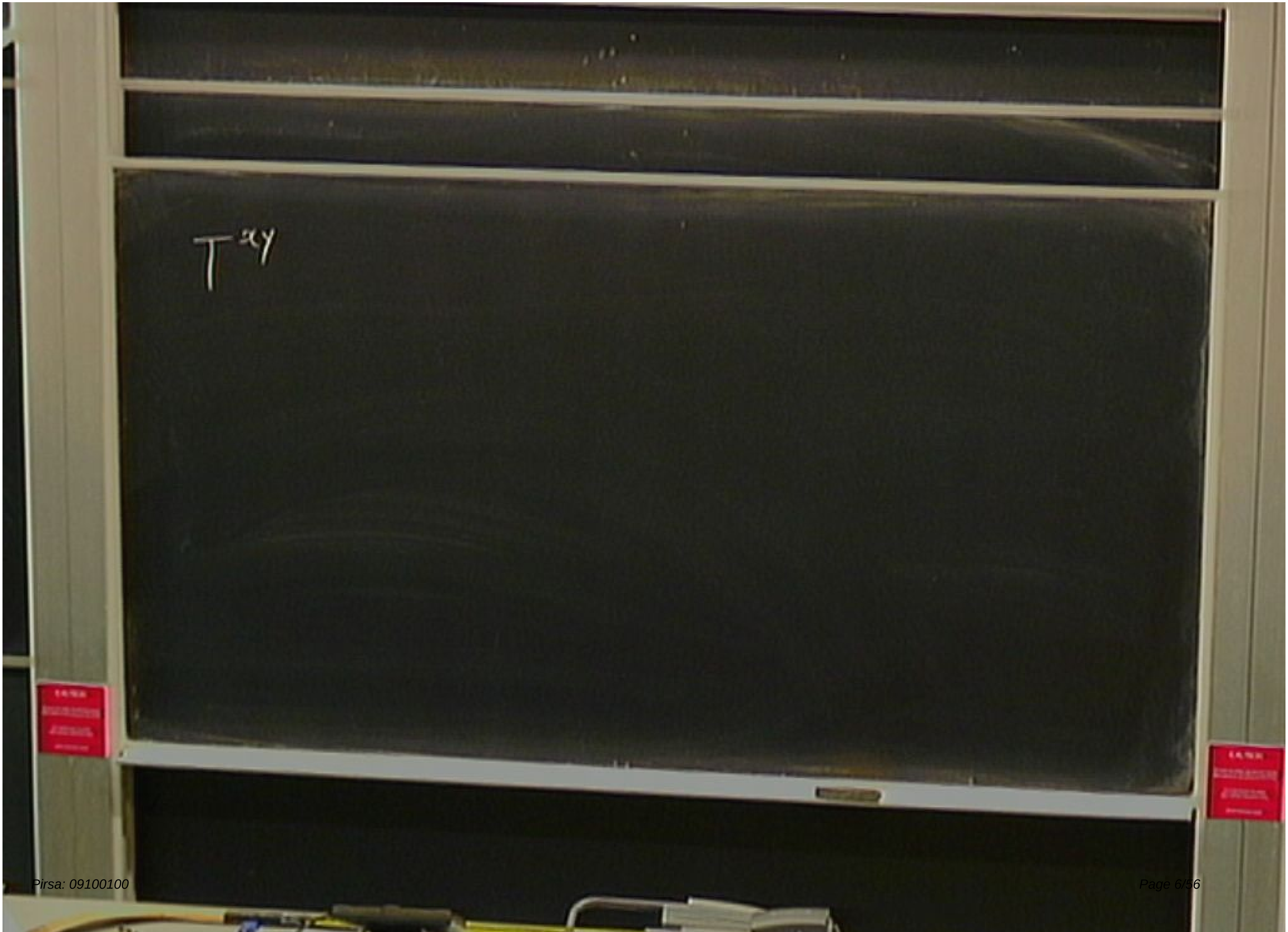

$L \sim \frac{1}{T}$
4-VELOCITY
$T^{\mu\nu} = \varepsilon u^\mu u^\nu + P\Delta^{\mu\nu} + \Pi^{\mu\nu}$
THERMODYNAMICS
VISCOUS


$L \sim \frac{1}{T}$
4-VELOCITY
$T^{\mu\nu} = \underbrace{\varepsilon u^\mu u^\nu + P\Delta^{\mu\nu}}_{\text{THERMODYNAMICS}} + \Pi^{\mu\nu}$
VISCOUS
$\Pi^{\mu\nu} = -\eta\, \sigma^{\mu\nu}$
SHEAR VISCOSITY
$\nabla^{\langle\mu} u^{\nu\rangle}$


T^{xy}

$$L \sim \frac{1}{T}$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^{\mu} u^{\nu}$$

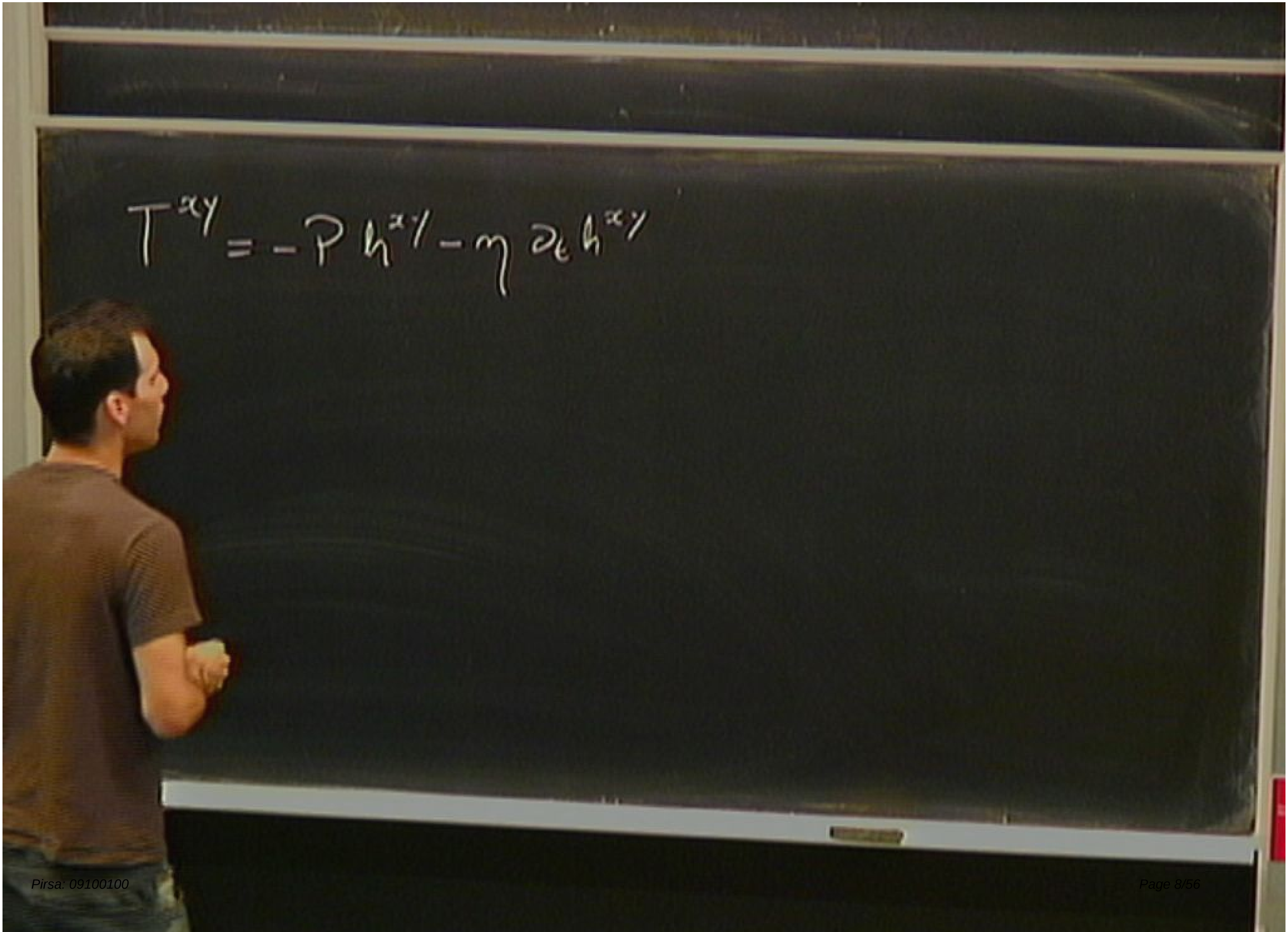
4-VELOCITY

$$T^{\mu\nu} = \underbrace{\varepsilon\, u^{\mu} u^{\nu} + P \Delta^{\mu\nu}}_{\text{THERMODYNAMICS}} + \underbrace{\Pi^{\mu\nu}}_{\text{VISCOUS}}$$

$$\Pi^{\mu\nu} = -\eta\, \sigma^{\mu\nu}$$

η: SHEAR VISCOSITY

$\sigma^{\mu\nu} \to \nabla^{\langle\mu} u^{\nu\rangle}$


$T^{xy} = -P\, h^{xy} - \eta\, \partial_t h^{xy}$

$$T^{xy} = -P\,h^{xy} - \eta\,\partial_t h^{xy}$$

$$= -P\,h^{xy}(\omega) - i\omega\eta\,h^{xy}(\omega)$$

$$\langle T^{xy}(\omega)\rangle = +\,G_R^{xy,}{}_{xy}\,h^{xy}$$

$$T^{xy} = -P h^{xy} - \eta \partial_t h^{xy}$$

$$= -P h^{xy}(\omega) - i\omega \eta h^{xy}(\omega) + \text{HIGHER ORDER}$$

$$\langle T^{xy}(\omega) \rangle = + G_R^{xy,xy} h_{xy}$$

$$\lim_{\omega \to 0} \frac{\operatorname{Im} G_R^{xy,xy}}{i\omega} = \eta$$

• AdS/CFT

gauge theory ⟷ gravity theory
(strings)

$\mathcal{N}=4$ SYM $SU(N)$ ⟷ Type IIB superstring

• AdS/CFT

gauge theory $\longleftrightarrow$ gravity (strings) theory

$\mathcal{N}=4$ SYM $SU(N)$ $\longleftrightarrow$ TYPE IIB SUPERSTRING ON $AdS_5 \times S_5$

$\lambda = g_{YM}^2 N$

$$\lambda \longleftrightarrow L^2/\alpha'^2$$

$$\frac{\lambda}{N} \longleftrightarrow g_s$$

• AdS/CFT

gauge theory $\longleftrightarrow$ gravity (strings) theory

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· AdS/CFT

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$\mathcal{N}=4$ SYM $SU(N)$ $\longleftrightarrow$ TYPE IIB SUPERSTRING
ON $AdS_5 \times S_5$

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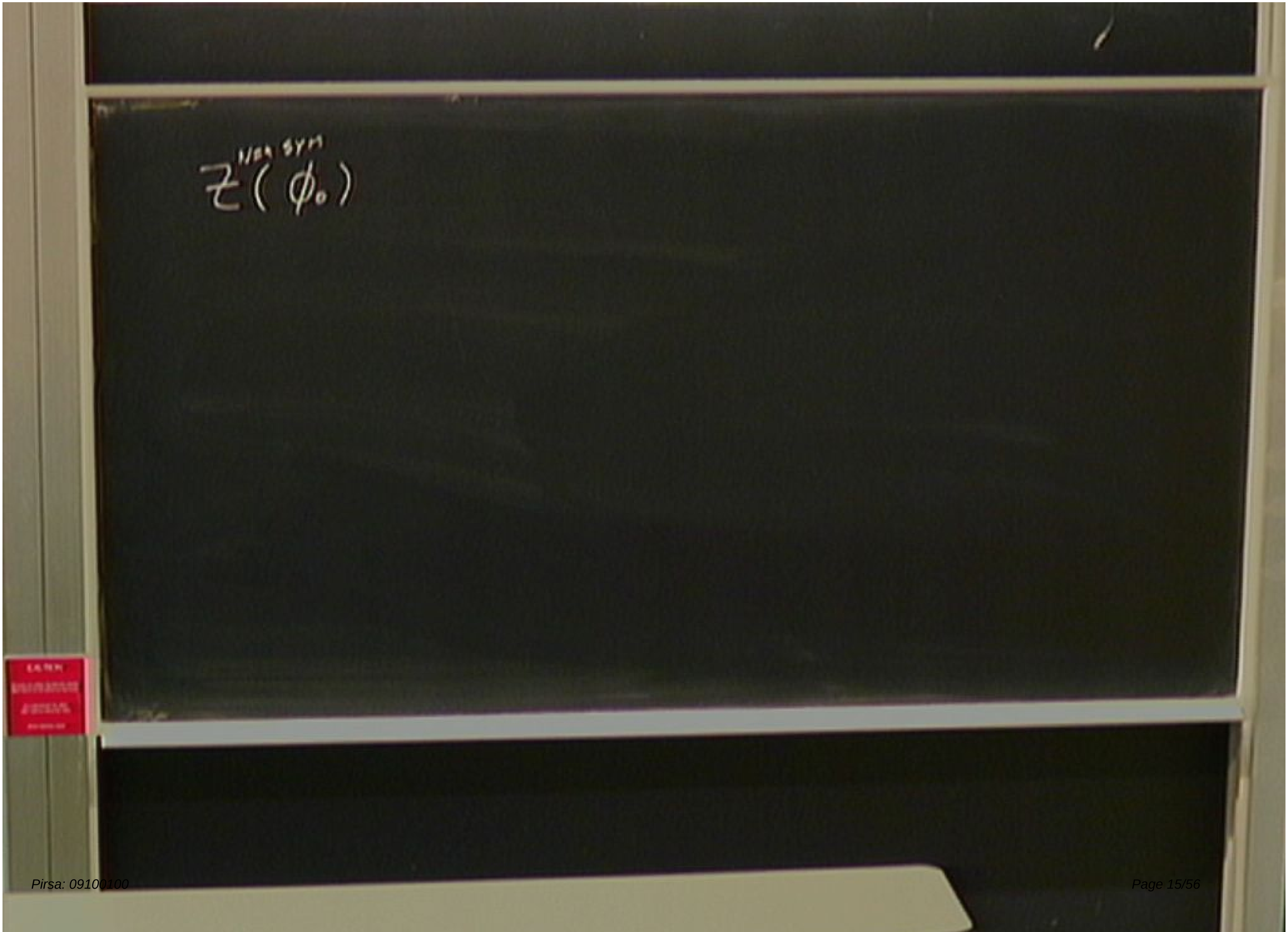
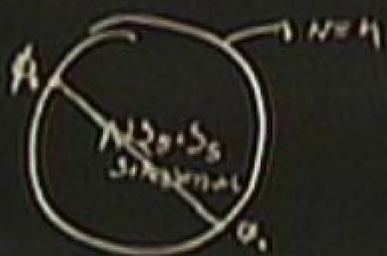
$\frac{\lambda}{N} \longleftrightarrow g_s$

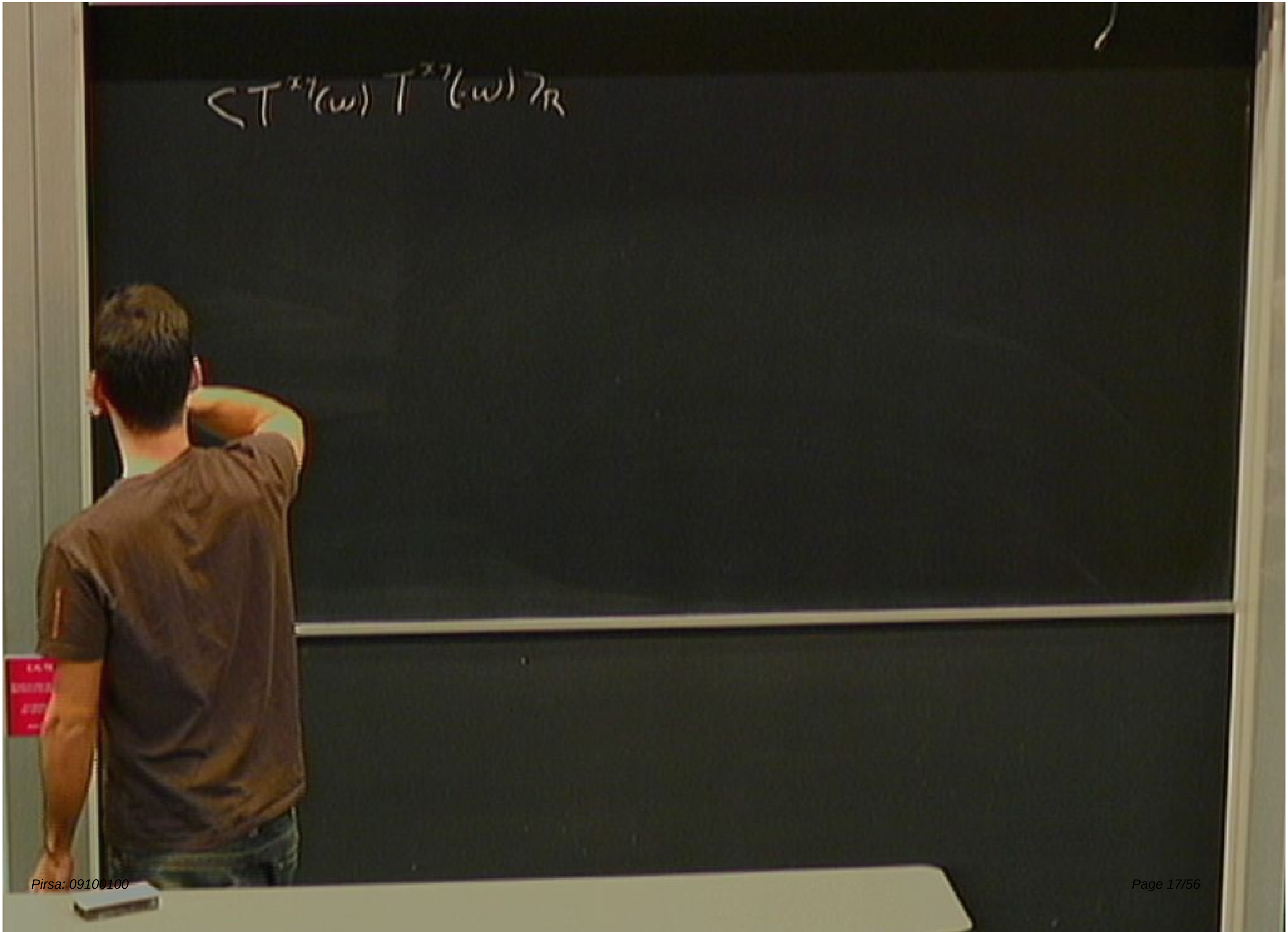
$R + \gamma R^4 + \dots$

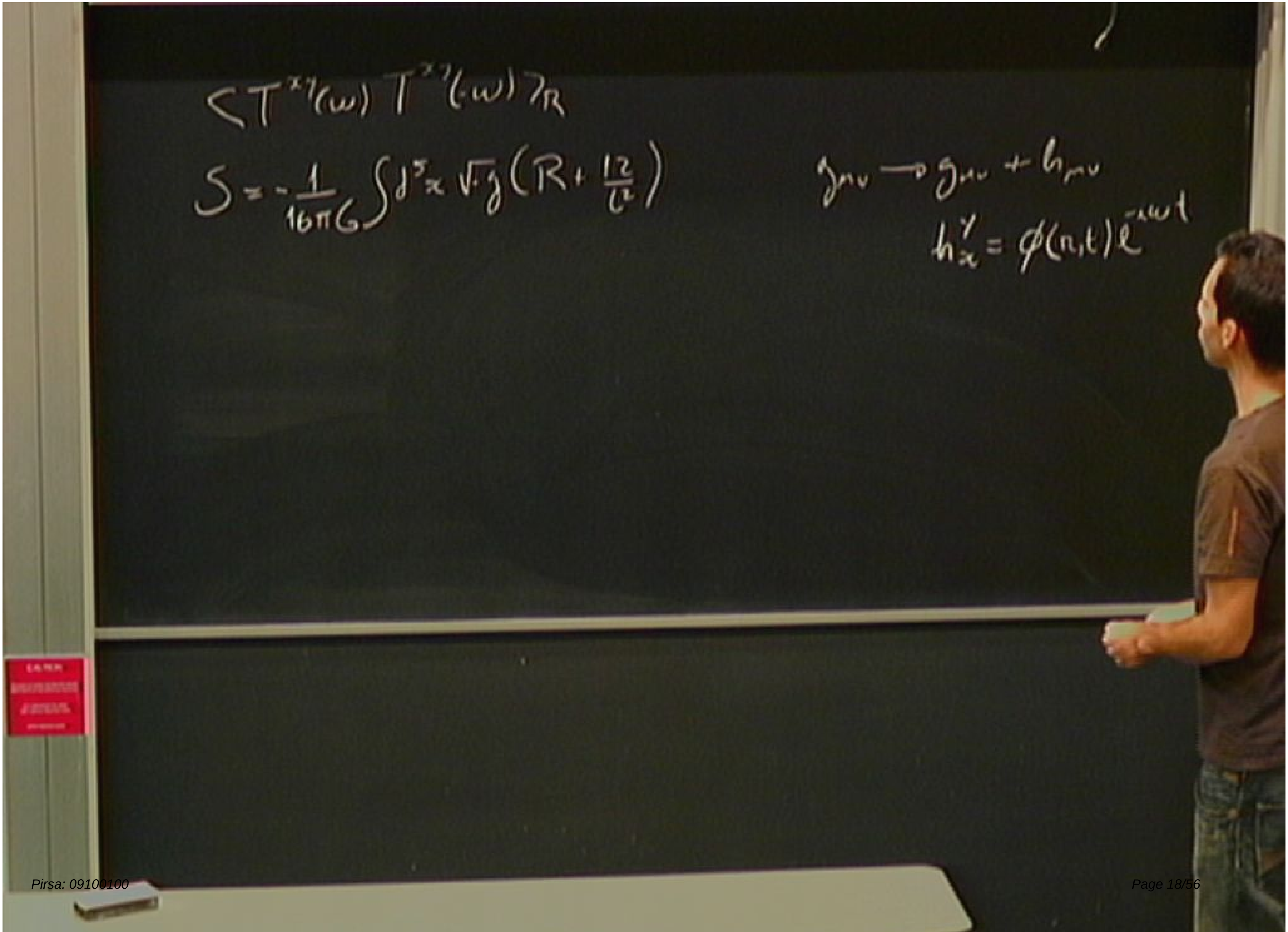
$1/\sqrt{\lambda}$

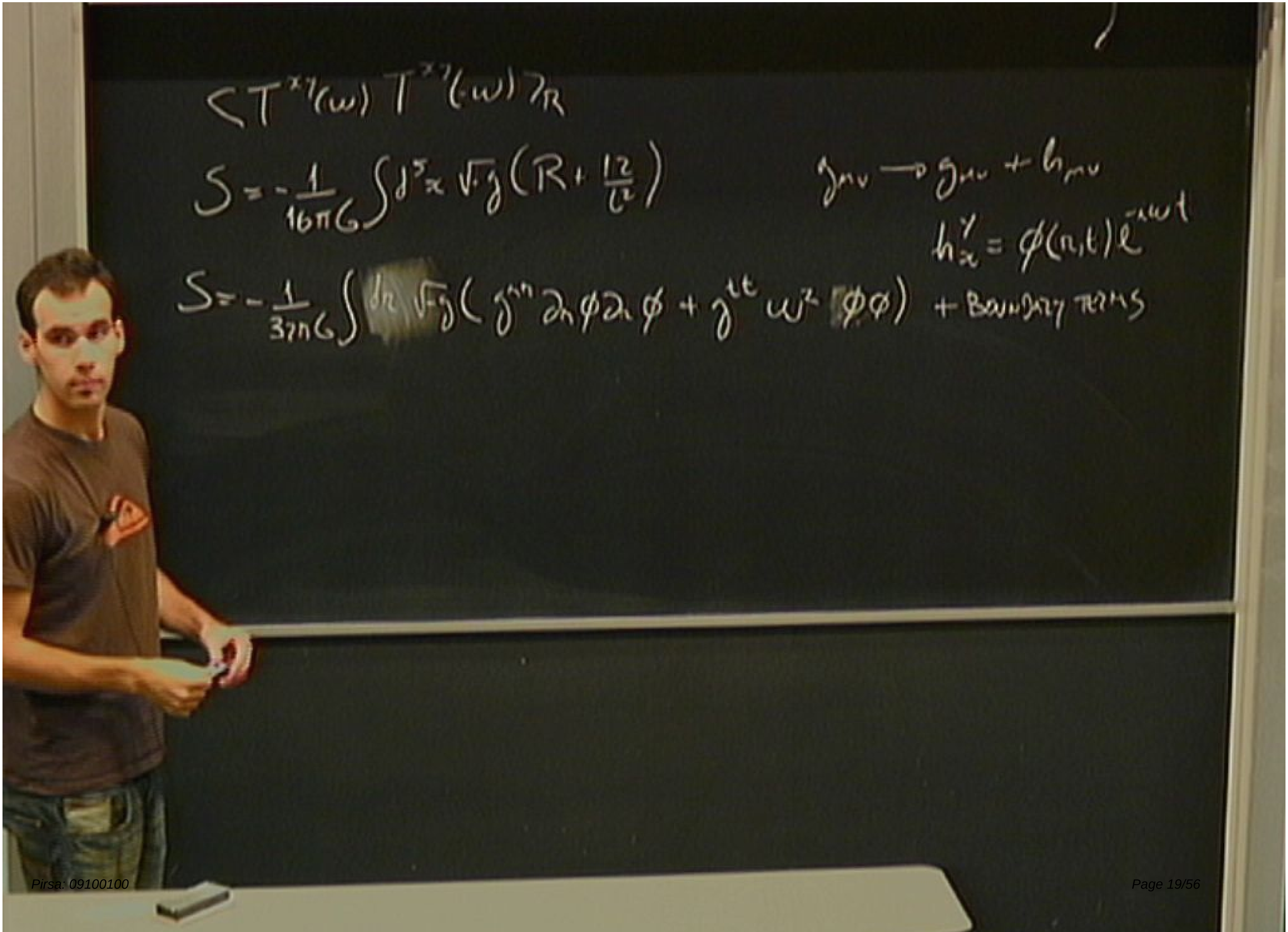

$Z^{\text{non sym}}(\phi_0)$

$$Z^{N=4\ SYM}(\phi_0) = e^{-S^{on\text{-}shell}_{GRAV}(\phi_0)}$$




$\langle T^{xy}(\omega)\, T^{xy}(-\omega) \rangle_R$


$\langle T^{xy}(\omega)\, T^{xy}(-\omega) \rangle_R$
$S = -\frac{1}{16\pi G} \int d^5x \sqrt{-g} \left(R + \frac{12}{l^2} \right)$
$g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu}$
$h^y_x = \phi(r,t)\, e^{-i\omega t}$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$S = -\frac{1}{16\pi G}\int d^5x \sqrt{-g}\left(R + \frac{12}{L^2}\right)$
$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$
$h^y_x = \phi(r,t)\, e^{-i\omega t}$
$S = -\frac{1}{32\pi G}\int dr \sqrt{-g}\left(g^{rr}\partial_r\phi\partial_r\phi + g^{tt}\omega^2\phi\phi\right)$ + BOUNDARY TERMS

$$\langle T^{xy}(\omega)\, T^{xy}(-\omega)\rangle_R$$

$$S = -\frac{1}{16\pi G}\int d^5x \sqrt{-g}\left(R + \frac{12}{L^2}\right)$$

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

$$h_x^y = \phi(r,t)\, e^{-i\omega t}$$

$$S = -\frac{1}{32\pi G}\int dr \sqrt{-g}\left(g^{rr}\partial_r\phi\,\partial_r\phi + g^{tt}\omega^2\phi\phi\right) + \text{Boundary terms}$$

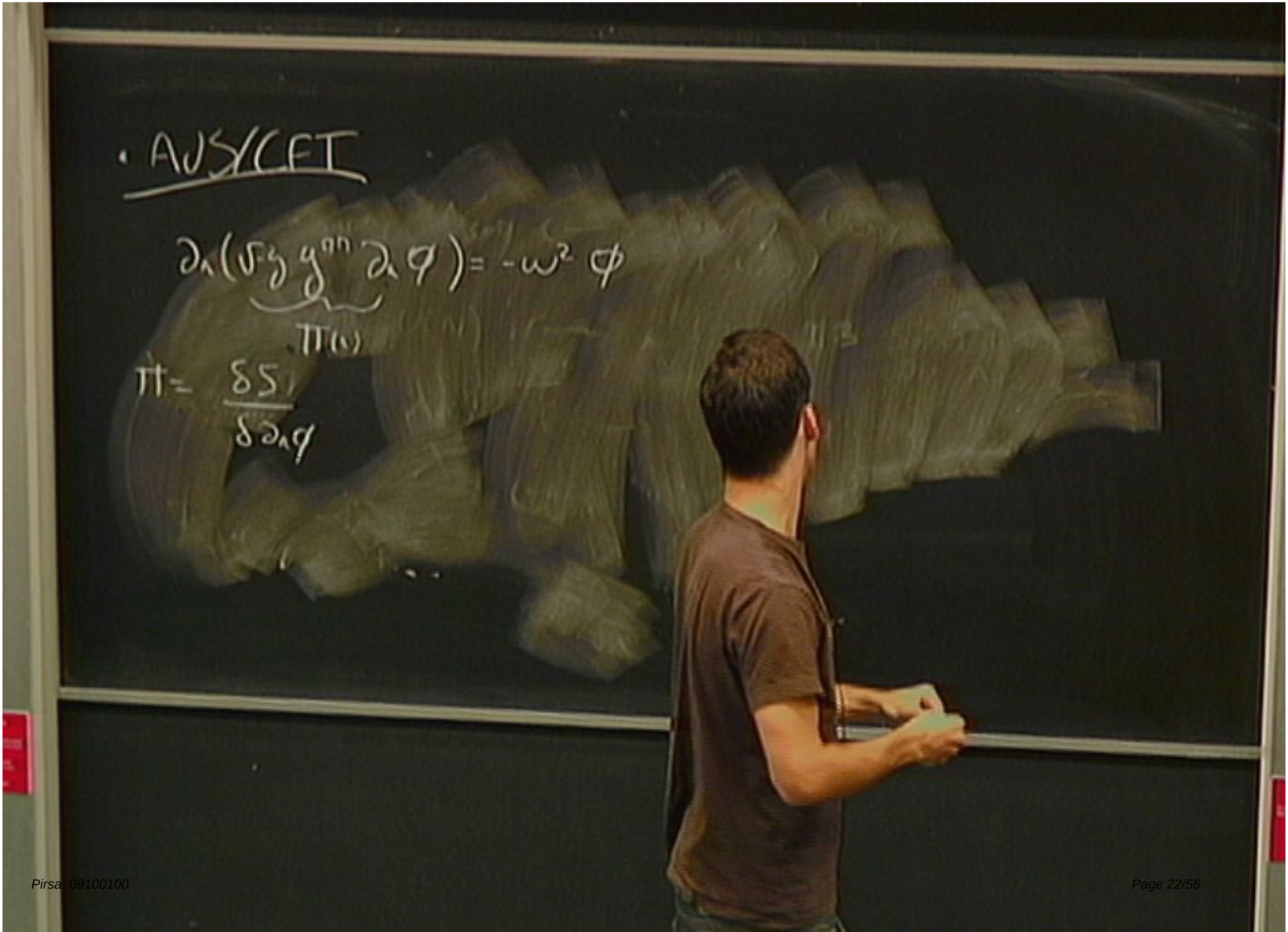
$$h_{r\lambda} = 0$$

$$+\, g_{tt}\,dt^2 + g_{rr}\,dr^2$$

$$+\, g_{xx}\underbrace{(dx^i)^2}_{SO(3)} \qquad dx, dy, dz$$

• AdS/CFT

$$\partial_n\left(\sqrt{-g}\, g^{nn}\, \partial_n \phi\right) = -\omega^2 \phi$$


AdS/CFT
∂n(√-g g^nn ∂n φ) = -ω² φ
Π(ε)
Π = δS / δ∂n φ

• AdS/CFT

$$\partial_n\left(\underbrace{\sqrt{-g}\, g^{nn}\,\partial_n\varphi}_{\Pi(n)}\right) = -\omega^2\phi$$

$$\Pi = \frac{\delta S}{\delta \partial_n \varphi}$$

• AdS/CFT

$$\partial_n\big(\underbrace{\sqrt{-g}\, g^{nn}\, \partial_n \varphi}_{\pi(n)}\big) = -\omega^2 \varphi$$

$$\pi = \frac{\delta S}{\delta \partial_n \varphi} \qquad \partial_n \pi(n) = 0$$

- AdS/CFT

$$\partial_r(\underbrace{\sqrt{-g}\, g^{rr}\partial_r \phi}_{\Pi(r)}) = -\omega^2 \phi$$

$$\Pi = \frac{\delta S}{\delta \partial_r \phi}$$

$$\partial_r \Pi(r) = 0$$

$$G_R^{xy,xy} = \lim_{\substack{\omega \to 0 \\ r \to +\infty}} \frac{\Pi(r)}{\phi(r)}$$

$$\langle T^{xy}(\omega)\, T^{xy}(-\omega)\rangle_R$$

$$S = -\frac{1}{16\pi G}\int d^5x \sqrt{-g}\left(R + \frac{12}{L^2}\right)$$

$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$

$h^y_x = \phi(r)\, e^{-i\omega t}$

$$S = -\frac{1}{32\pi G}\int dr \sqrt{-g}\left(g^{rr}\partial_r\phi\,\partial_r\phi + g^{tt}\,\underline{\omega^2\,\phi\phi}\right) + \text{BOUNDARY TERMS}$$

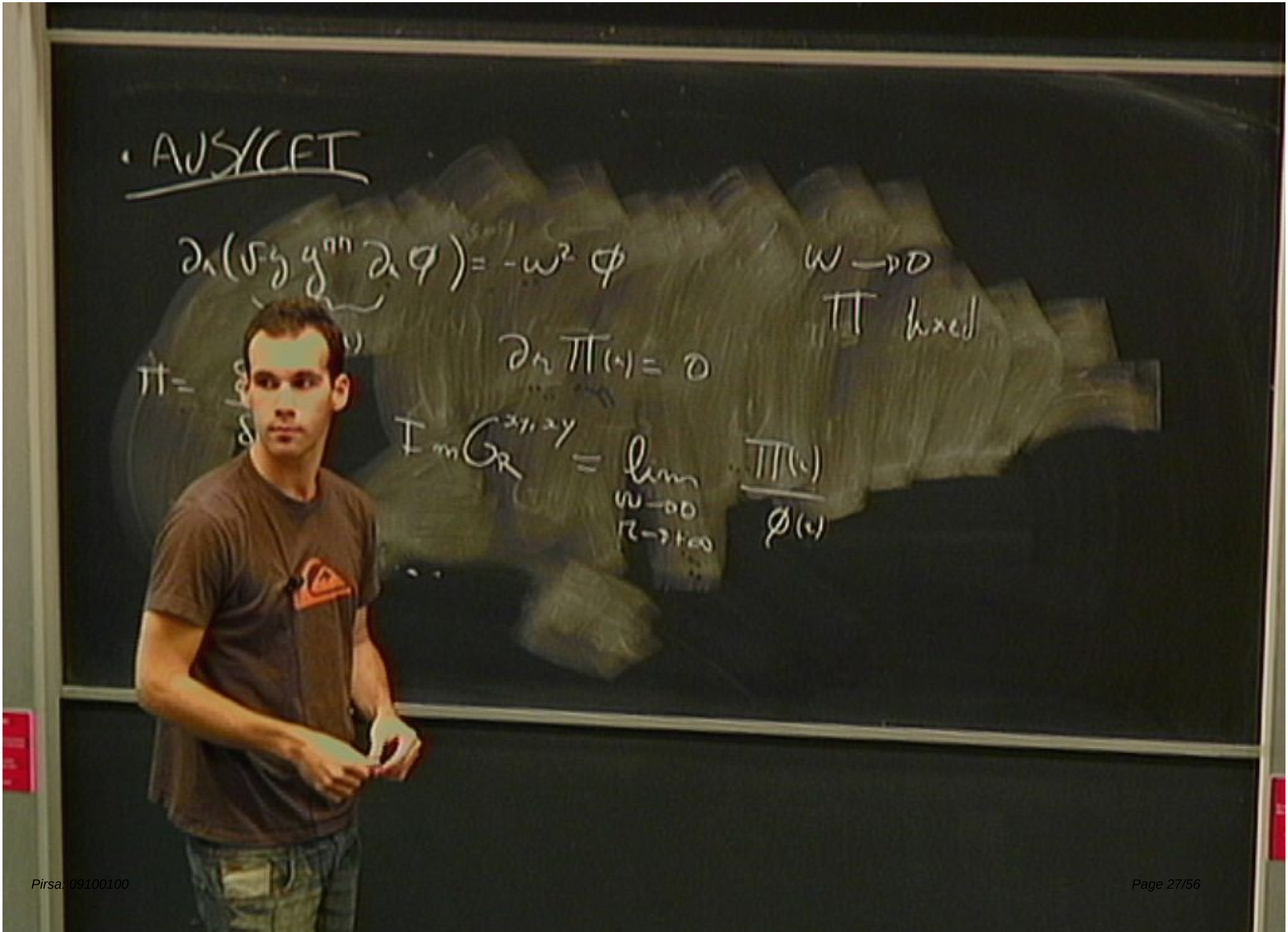
$h_{r\lambda} = 0$

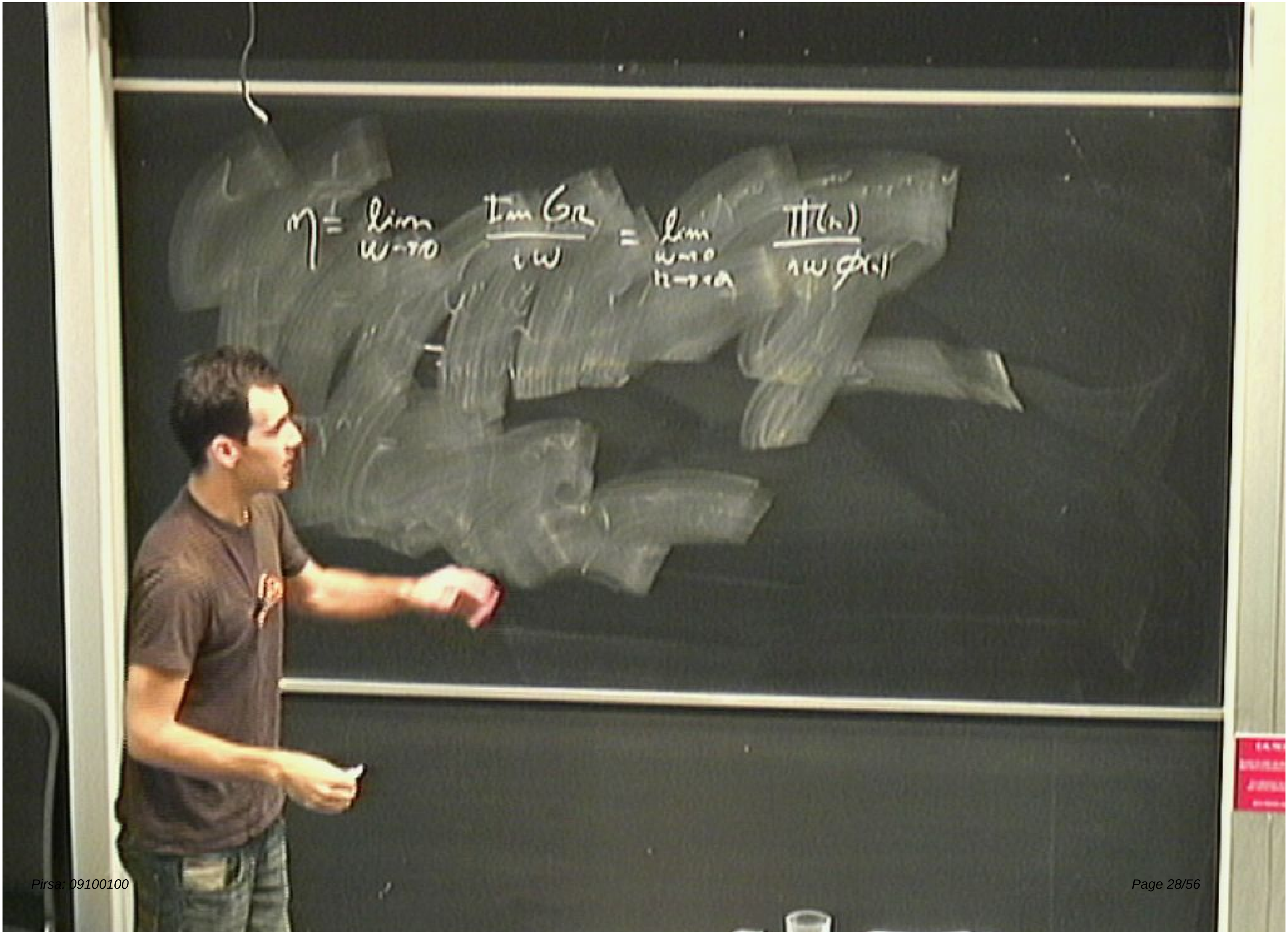
$$-\frac{1}{32\pi G}\sqrt{-g}\, g^{rr}\,\partial_r\phi\;\phi$$

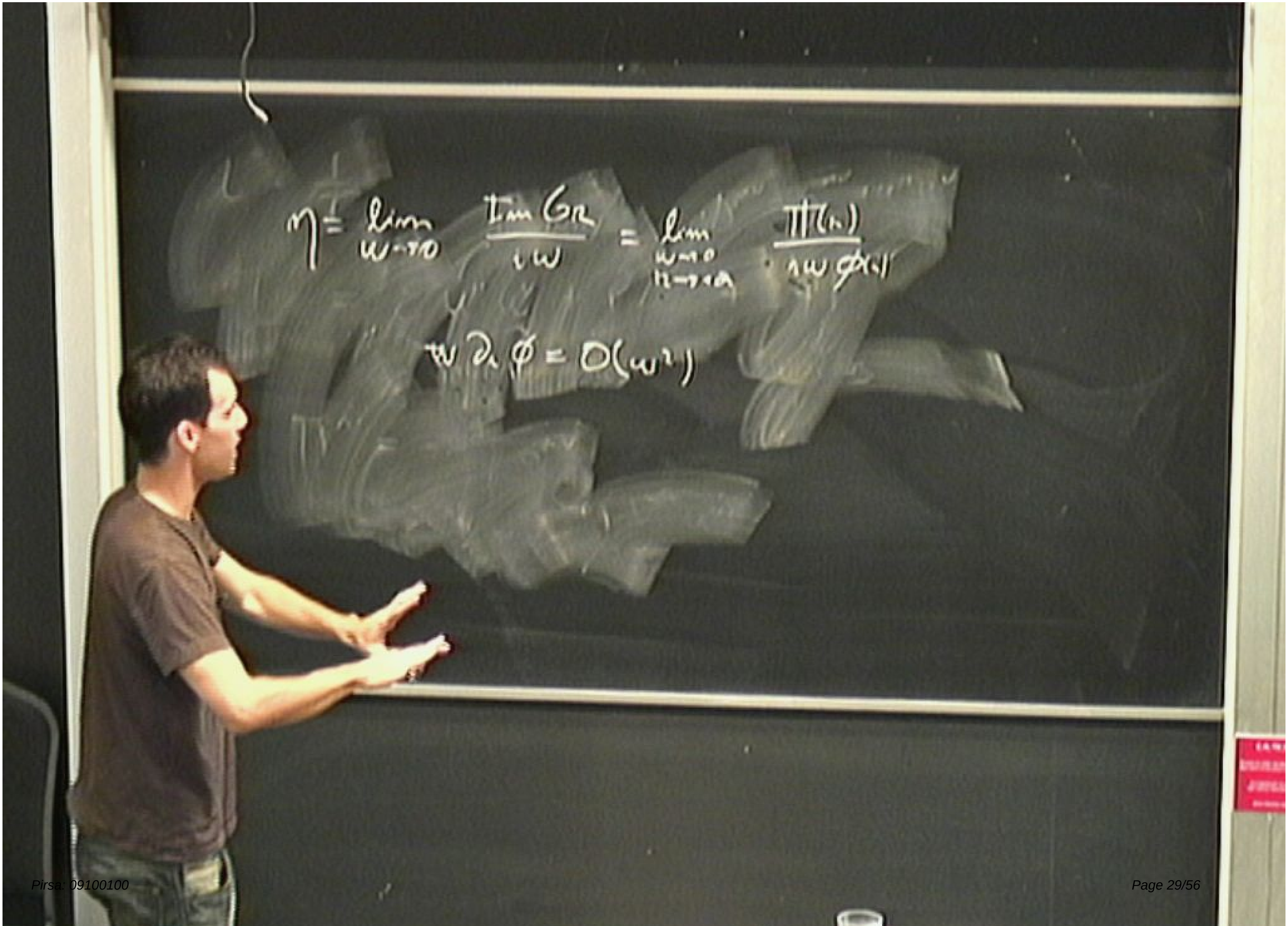
$+\, g_{tt}\,dt^2 + g_{rr}\,dr^2$

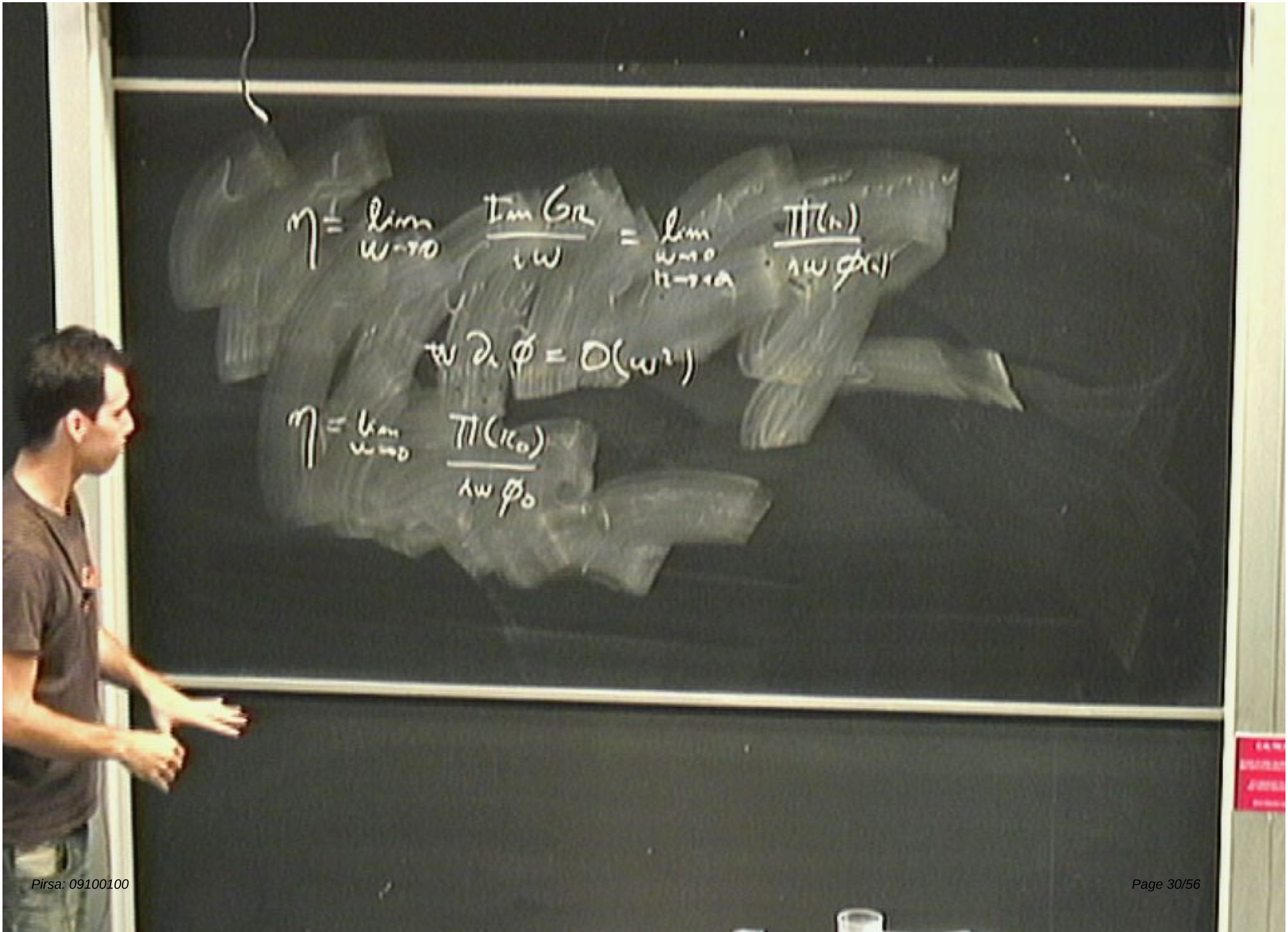
$+\, g_{xx}\underline{(dx^i)^2}$ $\quad dx, dy, dz$

$SO(3)$


AdS/CFT
ω → 0
Π fixed
∂r Π(r) = 0
lim
ω → 0
r → +∞


$\eta = \lim_{\omega \to 0} \frac{\mathrm{Im}\, G_R}{i\omega} = \lim_{\substack{\omega \to 0 \\ k \to 0}} \frac{\Pi(k)}{i\omega\, \phi(k)}$


$\eta = \lim_{\omega \to 0} \frac{\mathrm{Im}\, G_R}{i\omega} = \lim_{\substack{\omega \to 0 \\ r \to r_h}} \frac{\Pi(r)}{i\omega\, \phi(r)}$
$\omega\, \partial_r \phi = O(\omega^2)$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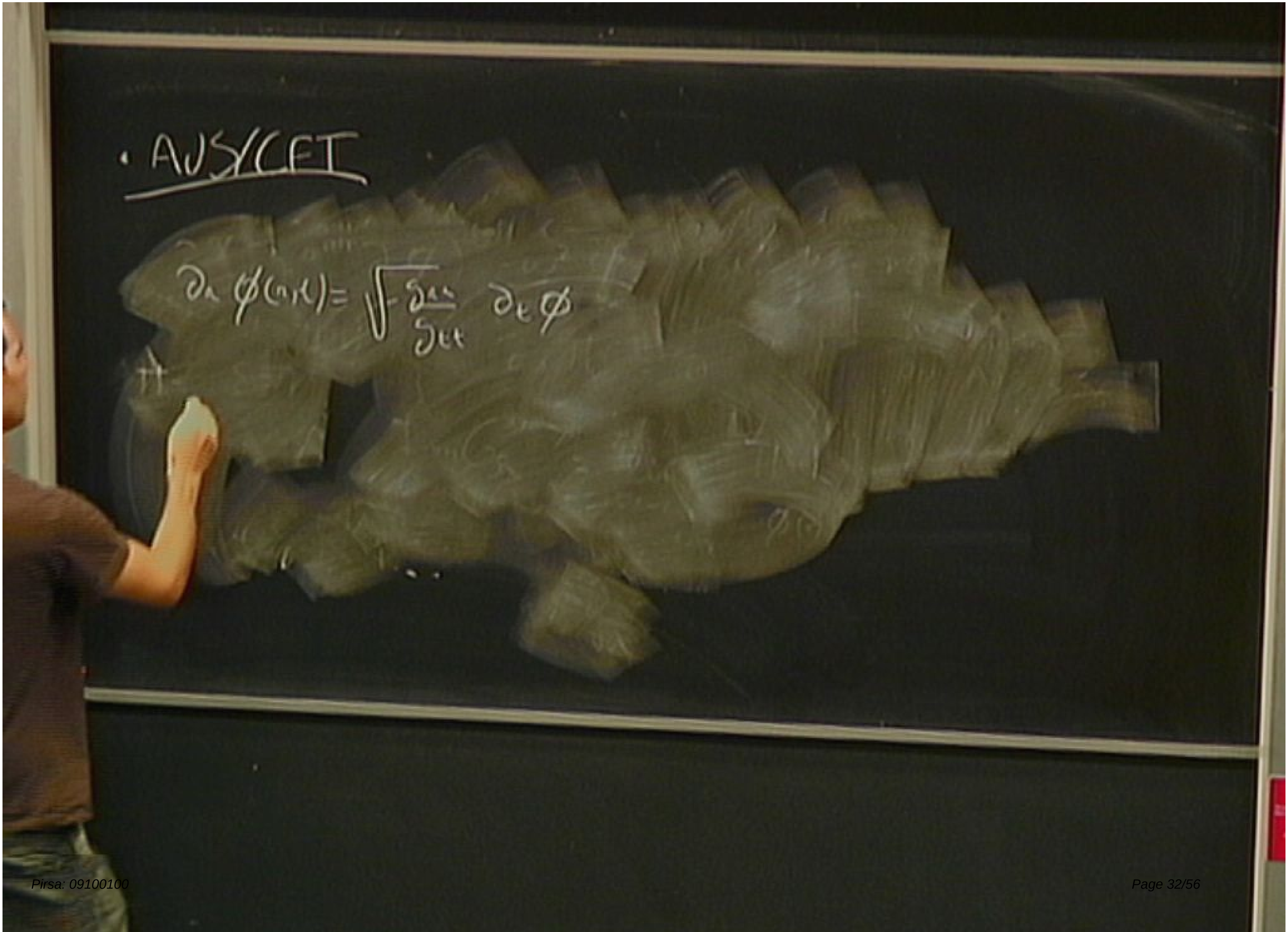
$\omega\, \partial_r \phi = O(\omega^2)$
$\eta = \lim_{\omega \to 0} \frac{\Pi(r_0)}{i\omega\, \phi_0}$

• AdS/CFT

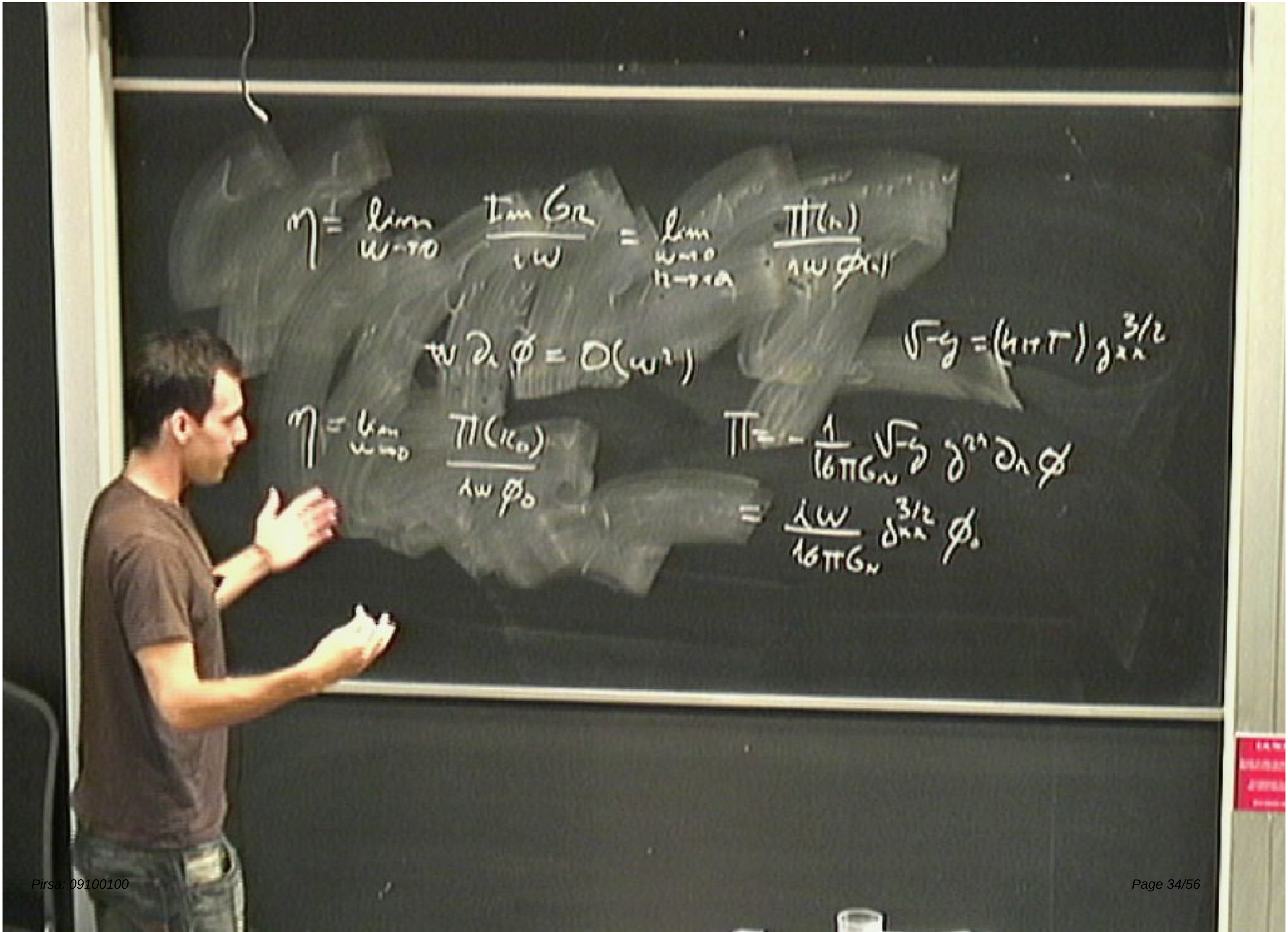
$$\partial_r \phi(r,t) = \sqrt{-\frac{g_{rr}}{g_{tt}}}\, \partial_t \phi$$

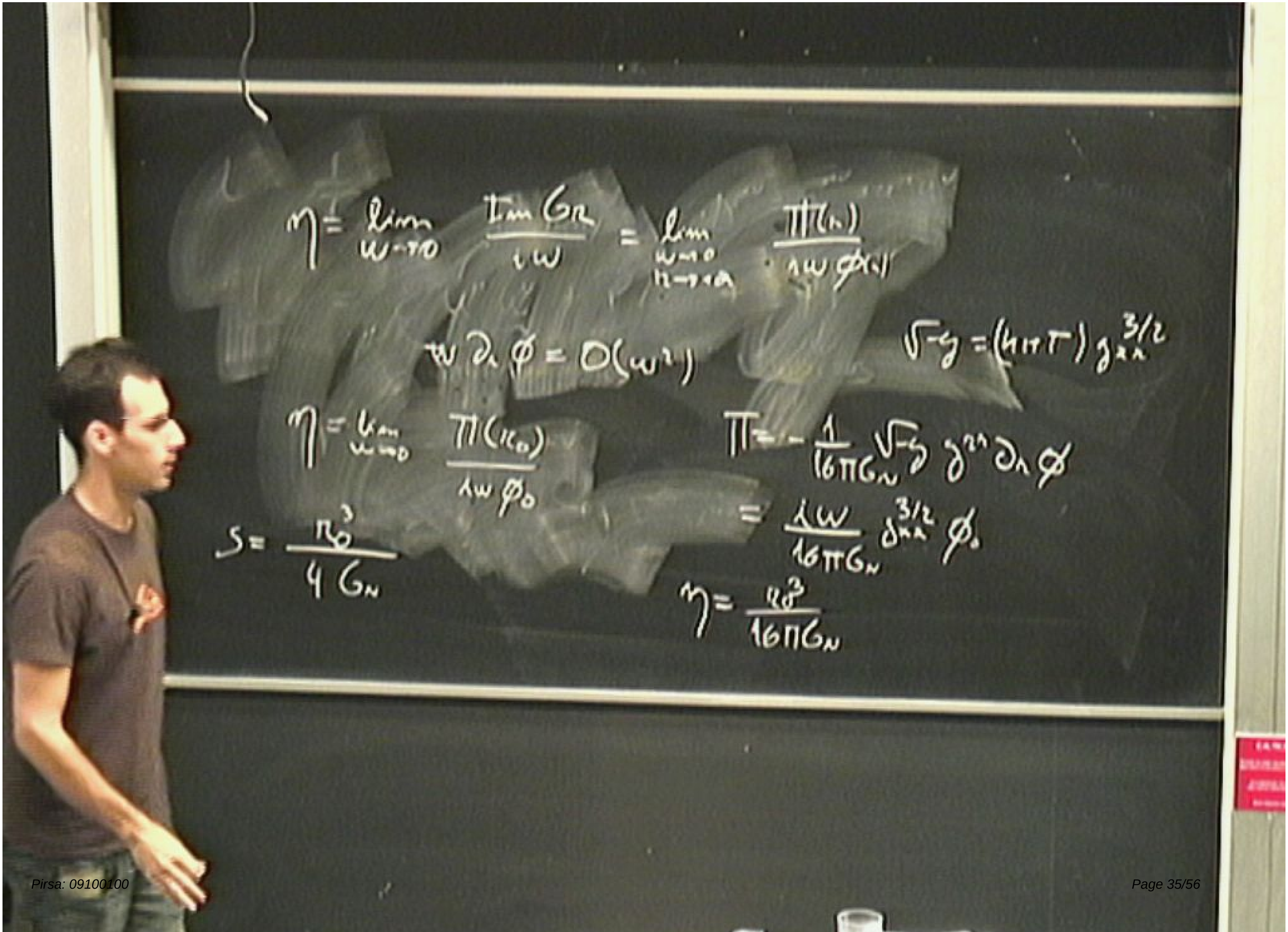

• AdS/CFT
$\partial_r \phi(r,x) = \sqrt{-\frac{g_{rr}}{g_{tt}}}\, \partial_t \phi$

• AdS/CFT

$$\partial_r \phi(r,t) = \sqrt{-\frac{g_{rr}}{g_{tt}}}\;\partial_t \phi$$

$$\partial_r \phi(r) = \pm \frac{i\omega}{4\pi T} \frac{\phi(r)}{(r-r_0)}$$


$\eta = \lim_{\omega \to 0} \frac{\mathrm{Im}\, G_R}{i\omega} = \lim_{\substack{\omega \to 0 \\ r \to r_0}} \frac{\Pi(r)}{i\omega\, \phi(r)}$
$\partial_r \phi = O(\omega^2)$
$\sqrt{-g} = (4\pi T)\, g_{xx}^{3/2}$
$\eta = \lim_{\omega \to 0} \frac{\Pi(r_0)}{i\omega\, \phi_0}$
$\Pi = -\frac{1}{16\pi G_N} \sqrt{-g}\, g^{rr} \partial_r \phi$
$= \frac{i\omega}{16\pi G_N} g_{xx}^{3/2} \phi_0$



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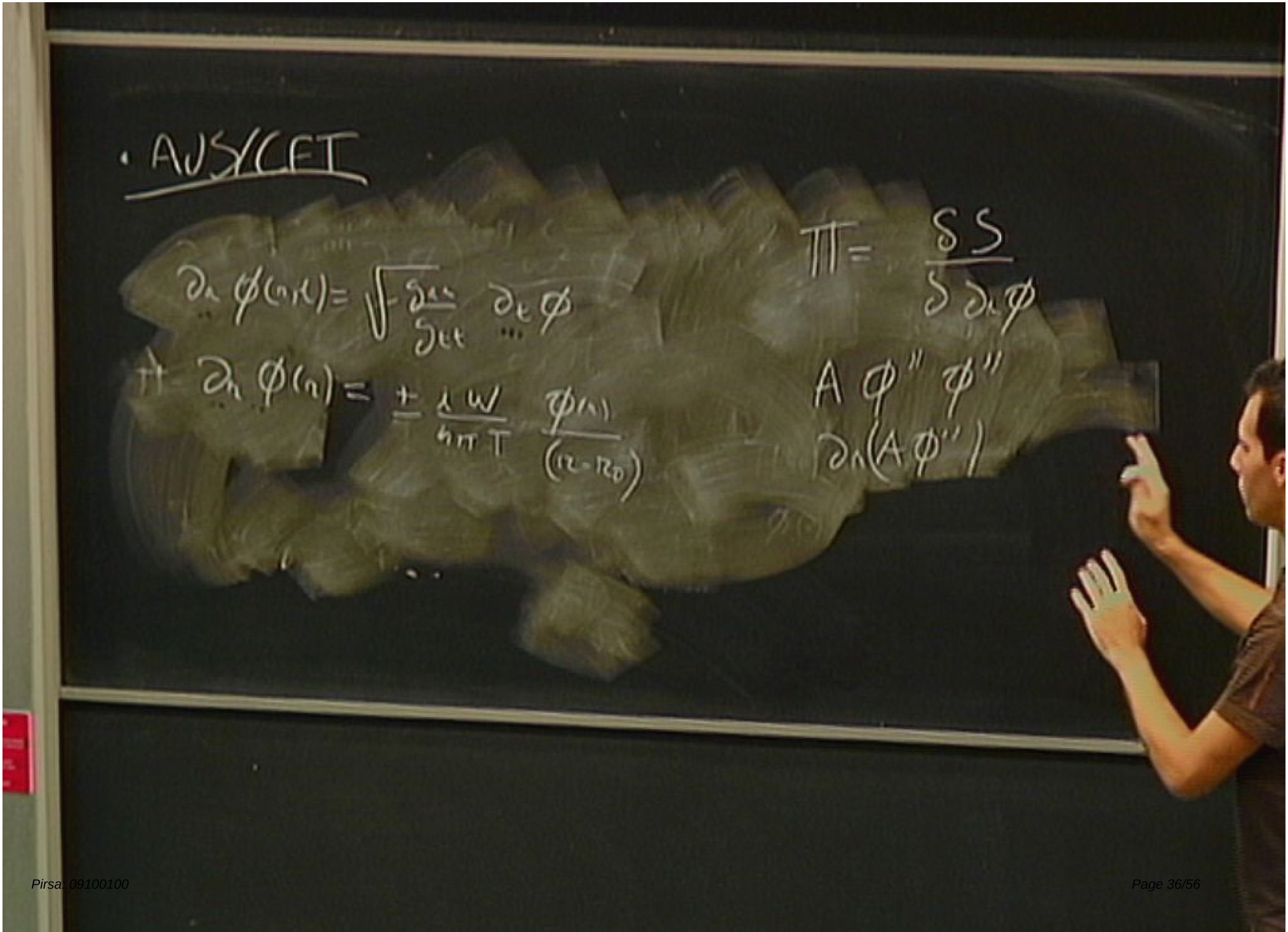
$\Pi = -\frac{1}{16\pi G_N} \sqrt{-g}\, g^{rr} \partial_r \phi$
$= \frac{i\omega}{16\pi G_N} g_{xx}^{3/2} \phi_0$
$S = \frac{r_0^3}{4 G_N}$
$\eta = \frac{r_0^3}{16\pi G_N}$


• AdS/CFT
$\partial_r \phi(r,t) = \sqrt{-\frac{g_{rr}}{g_{tt}}}\, \partial_t \phi$
$\Pi = \frac{\delta S}{\delta \partial_r \phi}$
$\partial_r \phi(r) = \pm \frac{i w}{4\pi T} \frac{\phi(r)}{(r - r_0)}$
$A \phi'' \phi''$
$\partial_r (A \phi'')$

• AdS/CFT

$$\partial_n \phi(n,t) = \sqrt{-\frac{g_{nn}}{g_{tt}}}\;\partial_t \phi$$

$$\partial_n \phi(n) = \pm \frac{i w}{4\pi T}\,\frac{\phi(n)}{(r-r_0)}$$

$$\Pi = \frac{\delta S}{\delta \partial_n \phi}$$

$$A\,\phi''\,\phi''$$

$$\partial_n (A\,\phi'')$$

$$\partial_n \Pi = O(w^2)$$

- AdS/CFT

$$\partial_r \phi(r,t) = \sqrt{-\frac{g_{rr}}{g_{tt}}}\,\partial_t \phi$$

$$\partial_r \phi(r) = \pm \frac{i\omega}{4\pi T}\,\frac{\phi(r)}{(r-r_0)}$$

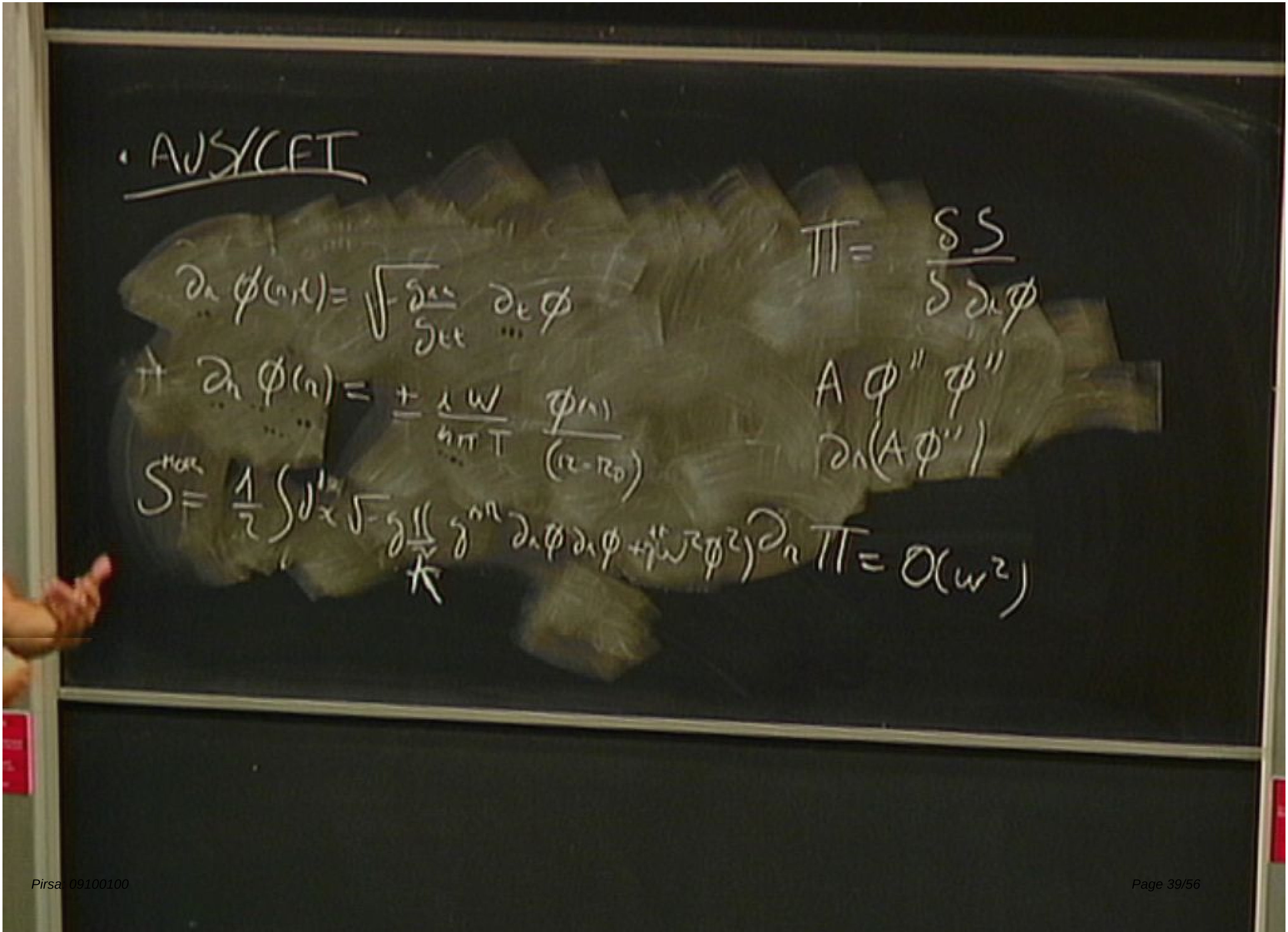
$$S = \frac{1}{2}\int d^4x \sqrt{-g}\left(g^{rr}\partial_r\phi\,\partial_r\phi + \dots\omega^2\phi^2\right)$$

$$\Pi = \frac{\delta S}{\delta \partial_r \phi}$$

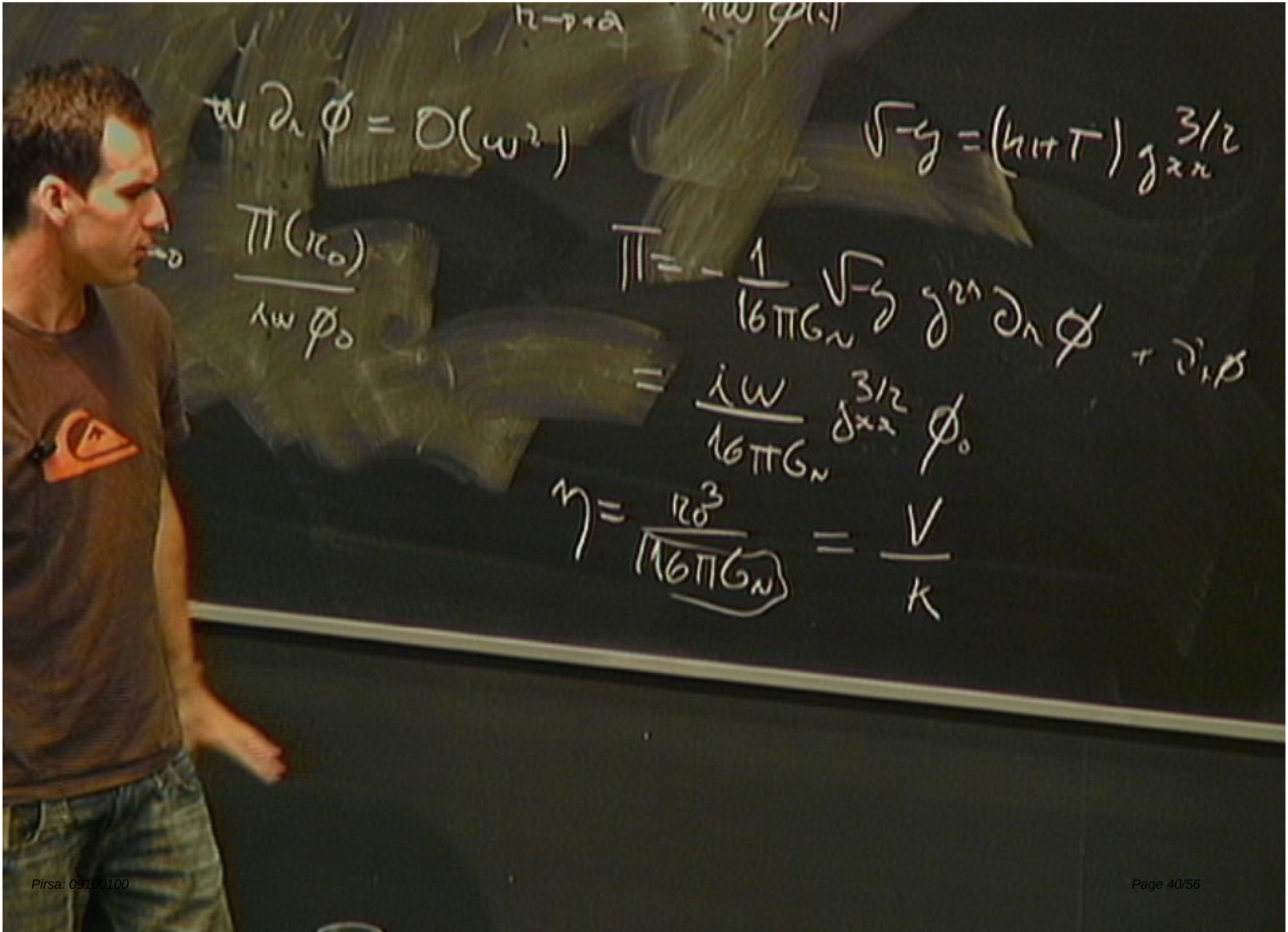
$$A\phi''\ \phi''$$

$$\partial_r(A\phi'')$$

$$\partial_r \Pi = O(\omega^2)$$



• AdS/CFT
$\Pi = \frac{\delta S}{\delta \partial_r \phi}$
$\partial_r \phi(r,t) = \sqrt{-\frac{g_{rr}}{g_{tt}}}\, \partial_t \phi$
$\partial_r \phi(r) = \pm \frac{i\omega}{4\pi T} \frac{\phi(r)}{(r-r_0)}$
$A\, \phi''\, \phi''$
$\partial_r(A \phi'')$
$S^{hor} = \frac{1}{2} \int d^4x \sqrt{-g}\, g^{rr} (g^{rr} \partial_r \phi \partial_r \phi + g^{tt} \omega^2 \phi^2)$
$\partial_r \Pi = O(\omega^2)$


$\omega\, \partial_r \phi = O(\omega^2)$
$\sqrt{-g} = (h_{rr} T)\, g_{xx}^{3/2}$
$\frac{\Pi(r_0)}{i\omega\, \phi_0}$
$\Pi = -\frac{1}{16\pi G_N}\sqrt{-g}\, g^{rr}\partial_r \phi$
$= \frac{i\omega}{16\pi G_N} g_{xx}^{3/2}\, \phi_0$
$\eta = \frac{r_0^3}{16\pi G_N} = \frac{V}{K}$

$$\phi = (r - r_0)^{\frac{i\omega}{4\pi T}}$$

$$\phi = (r - r_0)^{\frac{i\alpha}{4\pi T}}$$

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

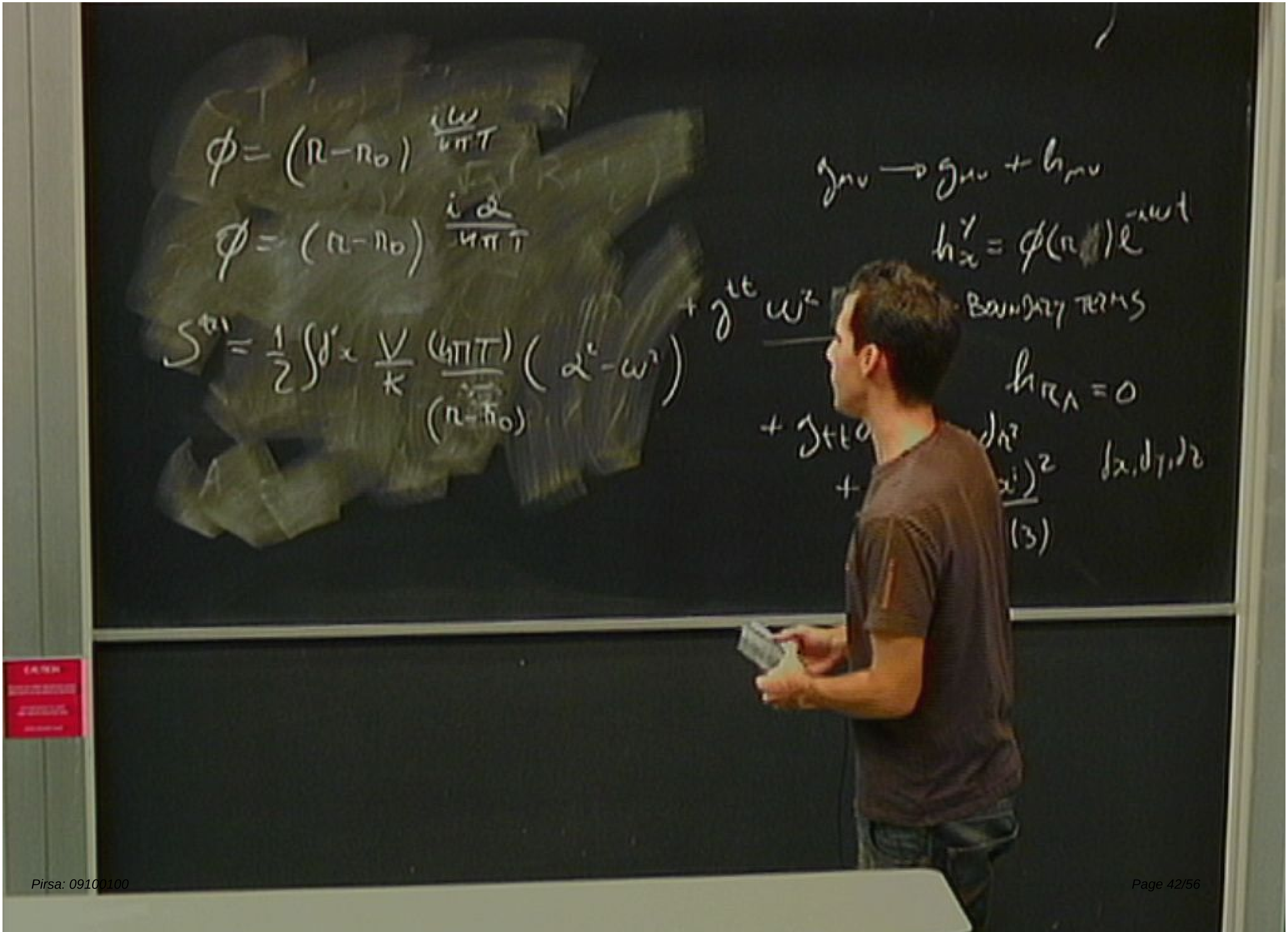
$$h^y_x = \phi(r)\, e^{-i\omega t}$$

$$+ g^{tt} \underline{\omega^2 \phi\phi)} + \text{BOUNDARY TERMS}$$

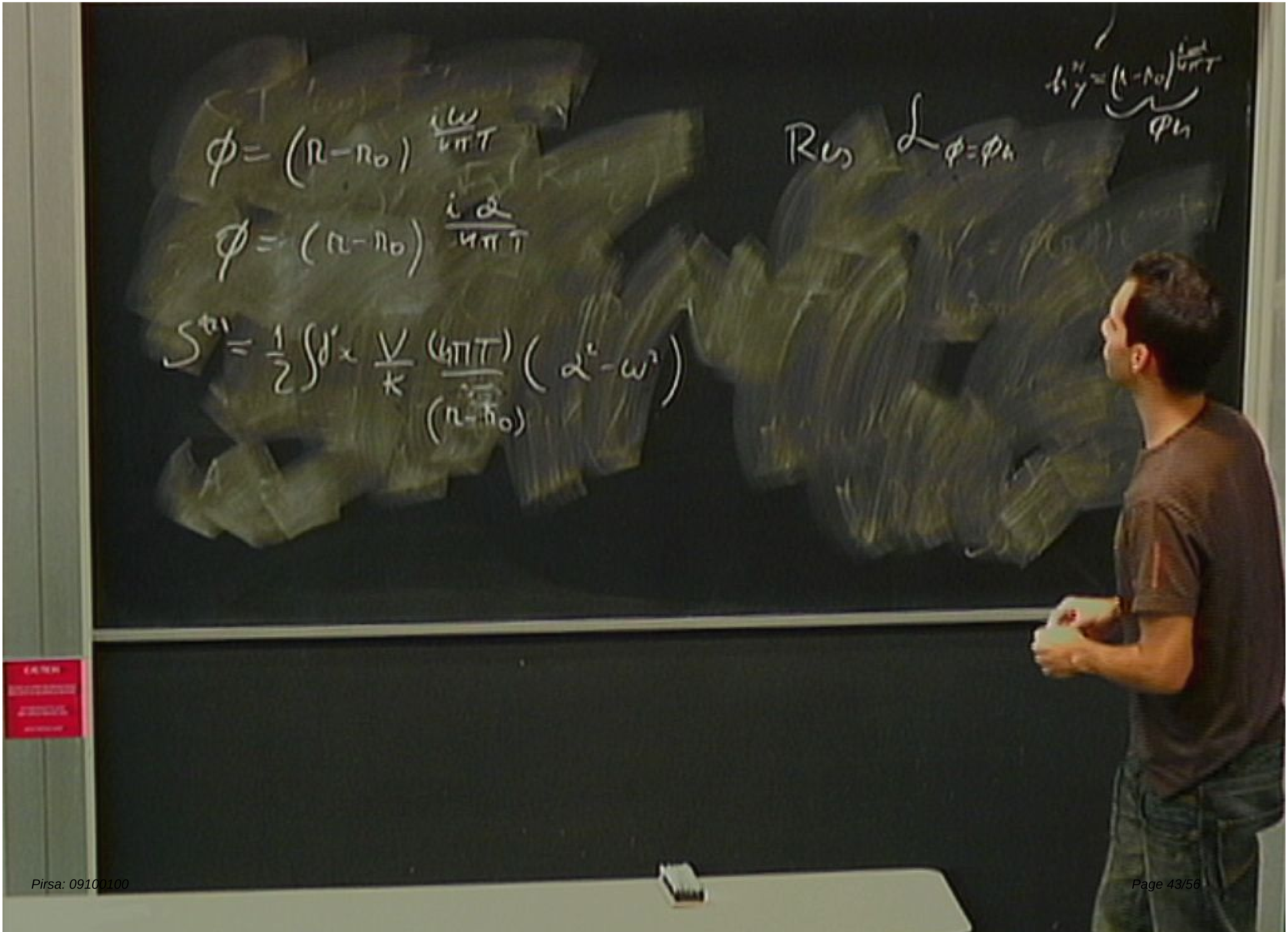
$$h_{r\lambda} = 0$$

$$+ g_{tt}\, dt^2 + g_{rr}\, dr^2$$

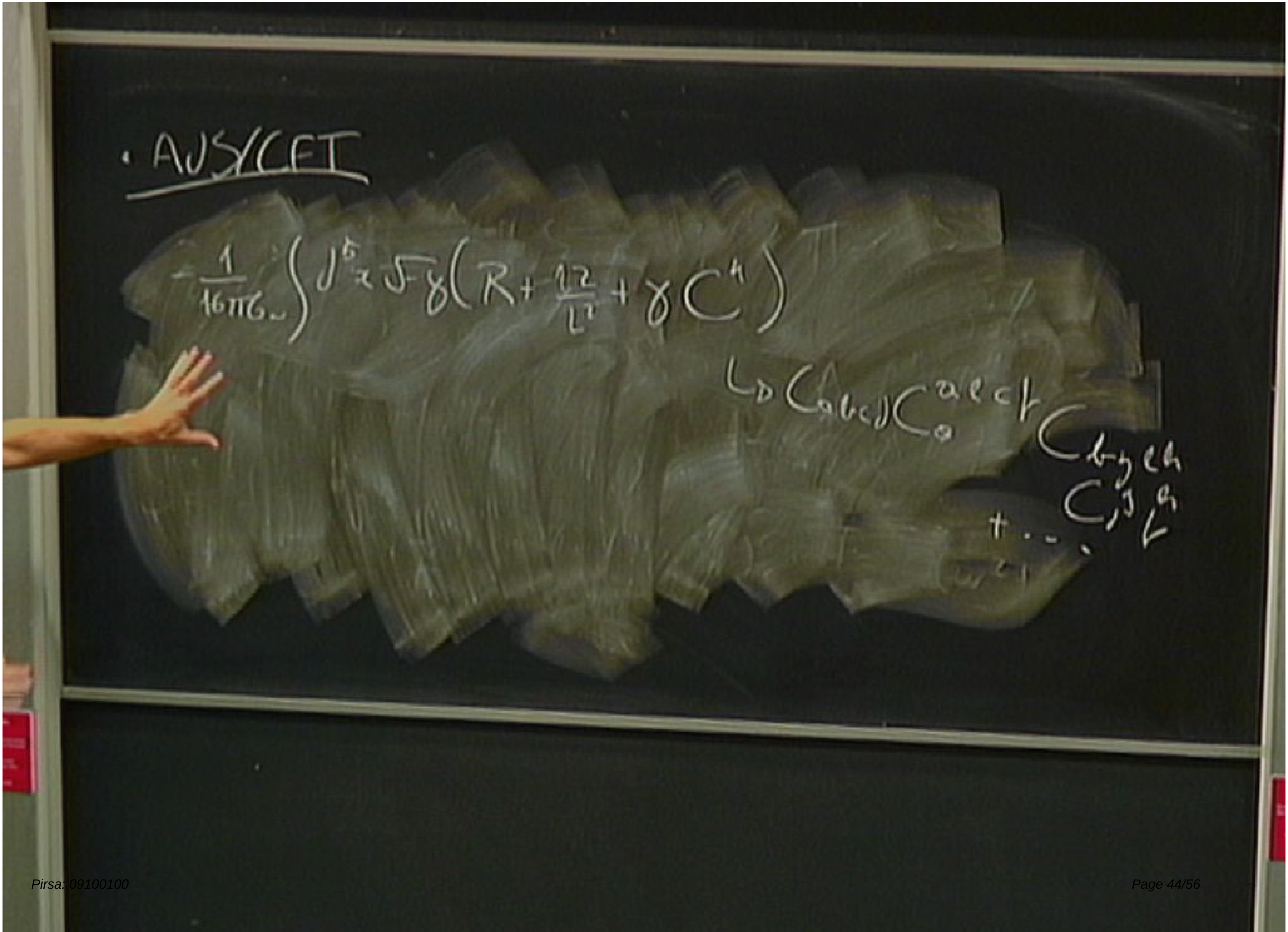
$$+ g_{xx} \underbrace{(dx^i)^2}_{SO(3)} \qquad dx, dy, dz$$

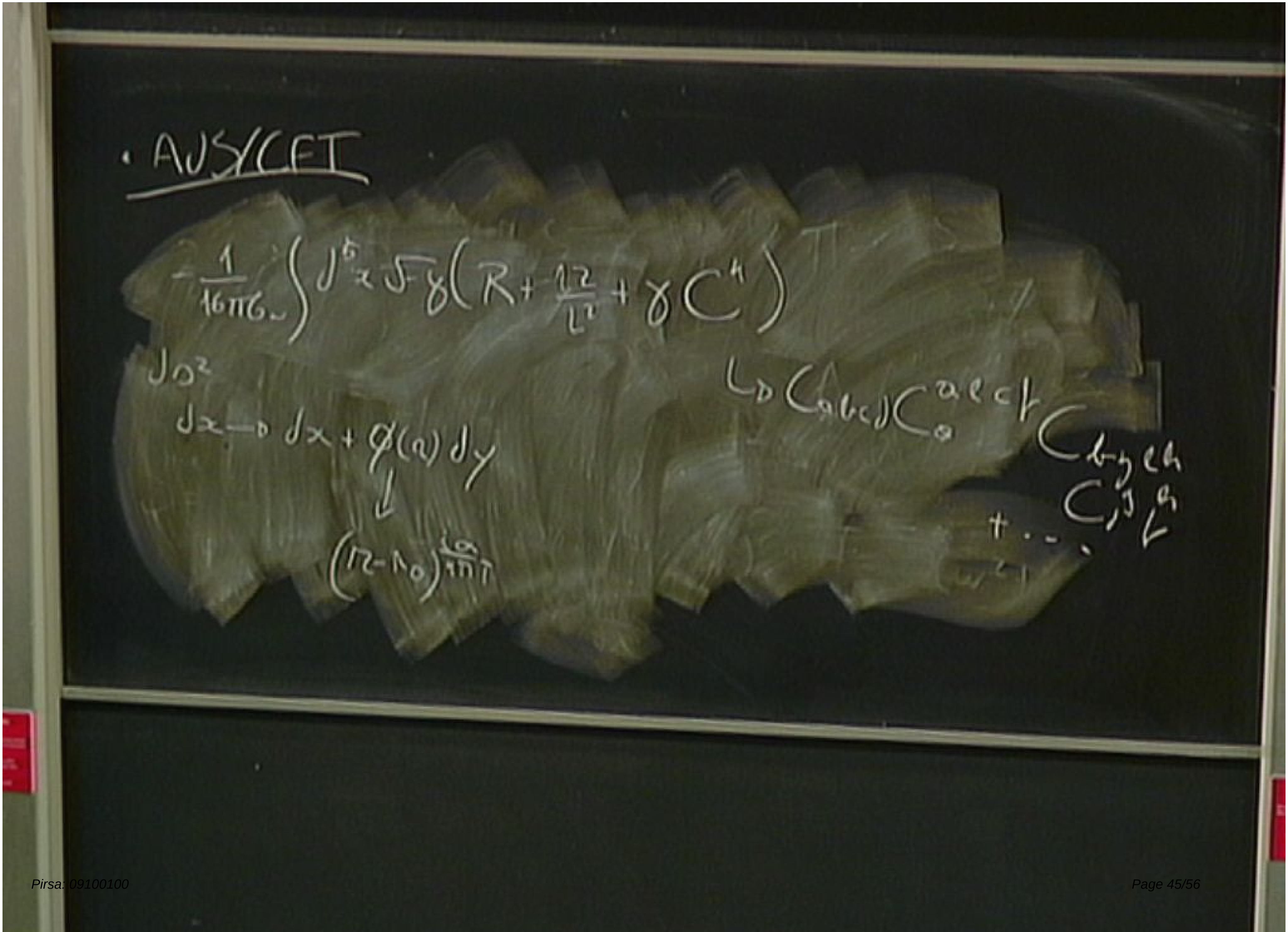


$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$
BOUNDARY TERMS
$h_{r\lambda} = 0$



φ = (r − r₀)^(iω/4πT)
φ = (r − r₀)^(iα/4πT)
S^(2) = 1/2 ∫d⁴x V/κ (4πT)/(r − r₀) (α² − ω²)
Res L_(φ=φ_h)


AdS/CFT
$-\frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left(R+\frac{12}{L^2}+\gamma C^4\right)$


• AdS/CFT
$-\frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left(R+\frac{12}{L^2}+\gamma C^4\right)$
ds^2
$dx \to dx + \phi(r)\,dy$
$(r-r_0)$
$C_{abcd}C^{aecf}C_{bgeh}C_f{}^{gh}$
$+\ldots$

• AdS/CFT

$$-\frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left(R+\frac{12}{L^2}+\gamma C^4\right)$$

ds^2

$dx \to dx + \phi(r)\,dy$

$\downarrow$

$(r-r_0)^{\frac{i\omega}{4\pi T}}$

$L_D \, C_{abcd}C^{abcd} + C_{bgeh} C \dots$

$$L_D = \frac{1}{16\pi G_N}\frac{V(4\pi T)}{(r-r_0)}(1+180\gamma)+\dots$$

$$+\,O\left(\frac{1}{(r-r_0)^2}\right) + \text{regular}$$

• AdS/CFT

$$-\frac{1}{4}F^2 + (DF)^2 + (DDF)^2 + F^4$$

$$-\frac{1}{16\pi G_N}\int d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} + \gamma C^4\right)$$

ds^2

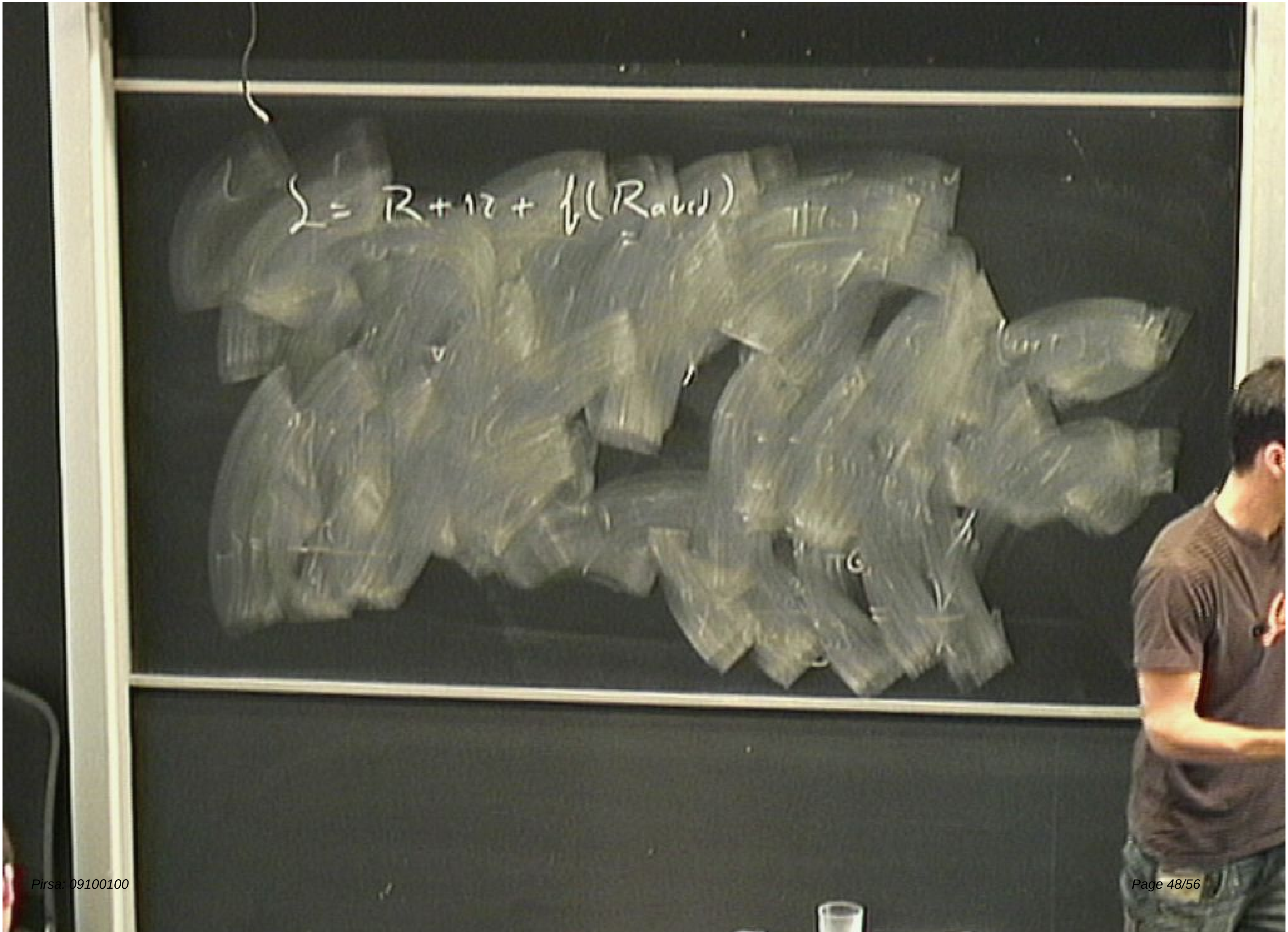
$$dx \to dx + \phi(r)\,dy$$

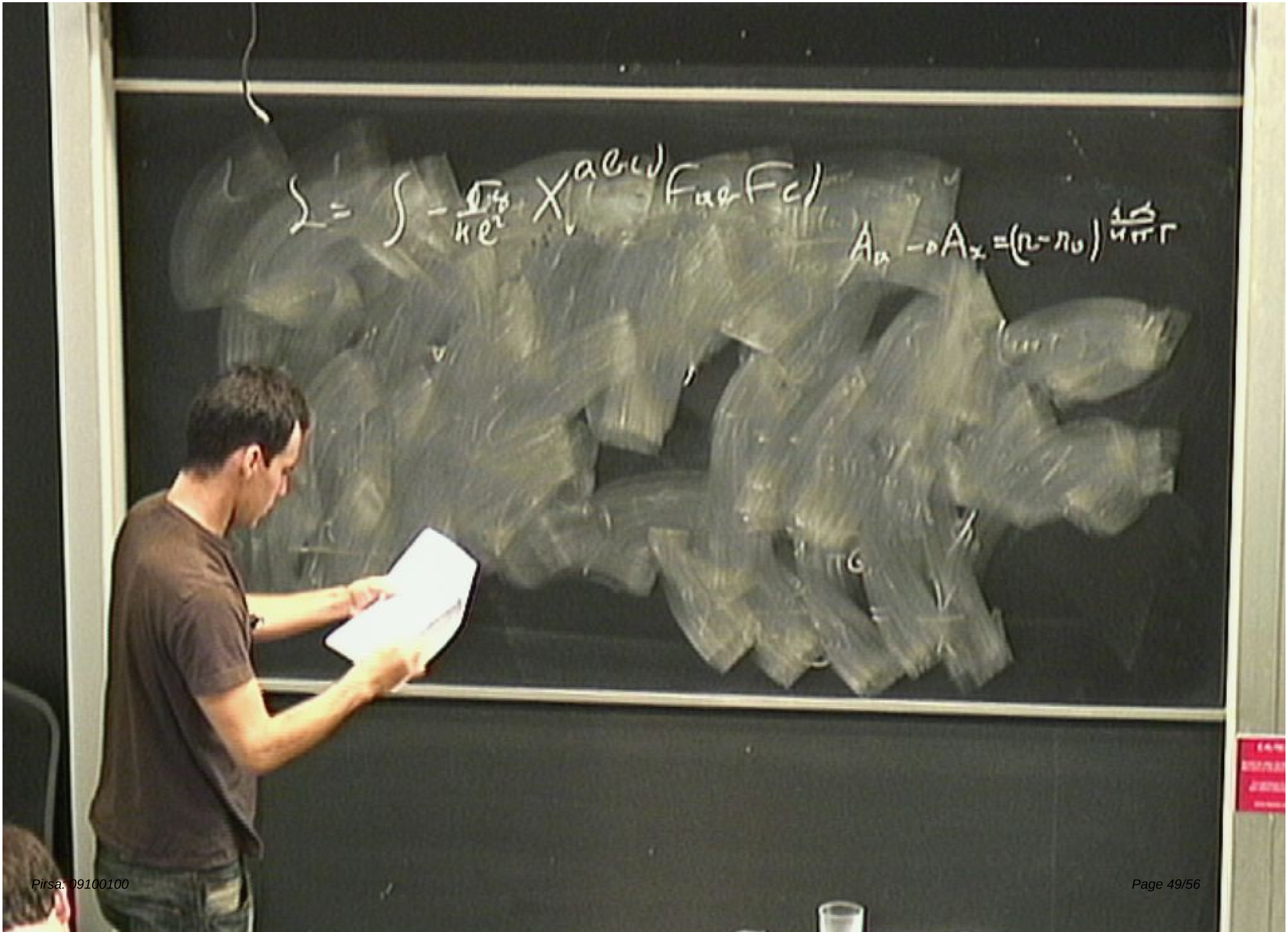
$$\downarrow$$

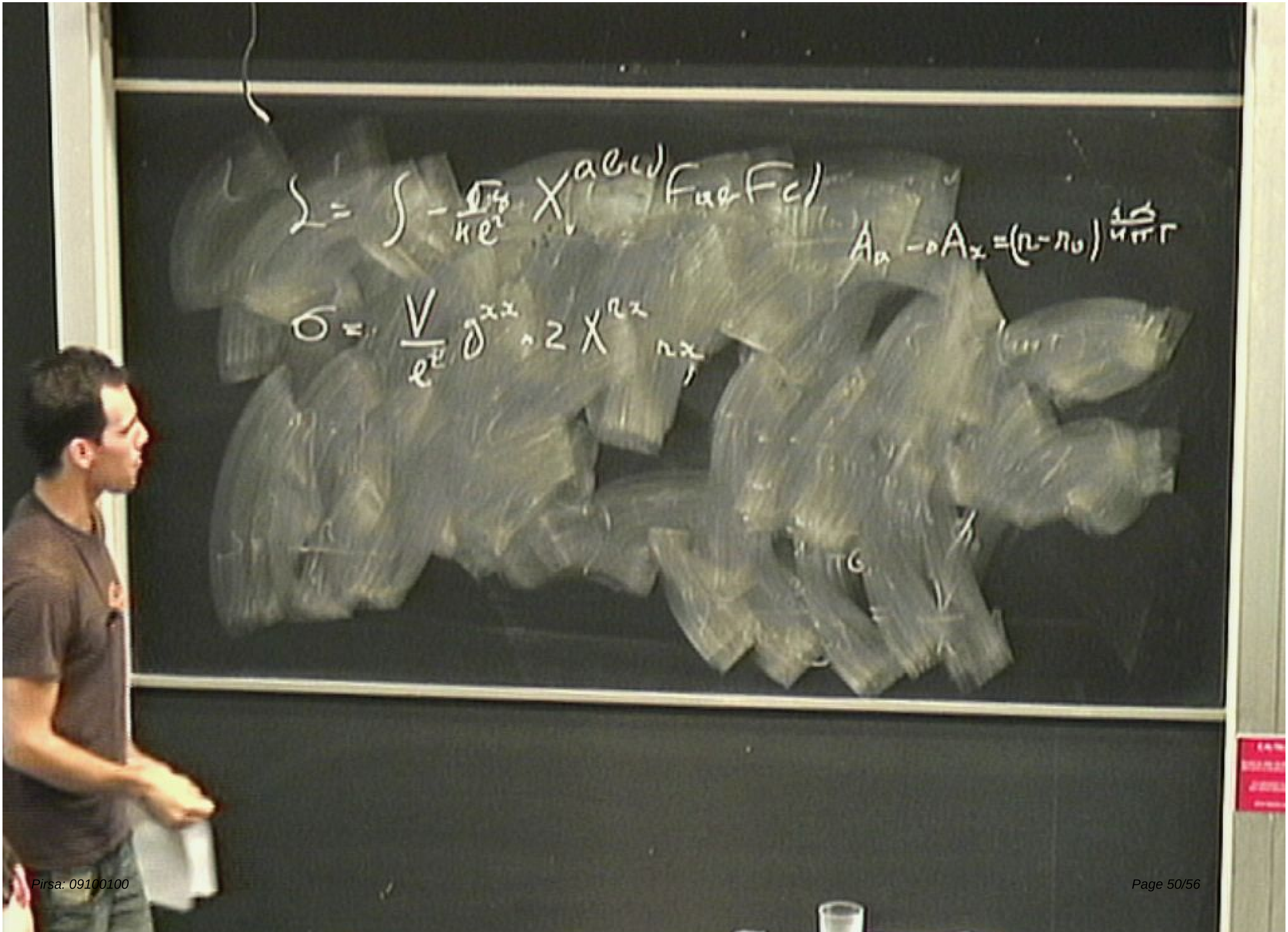
$$(r-r_0)^{\frac{i\omega}{4\pi T}}$$

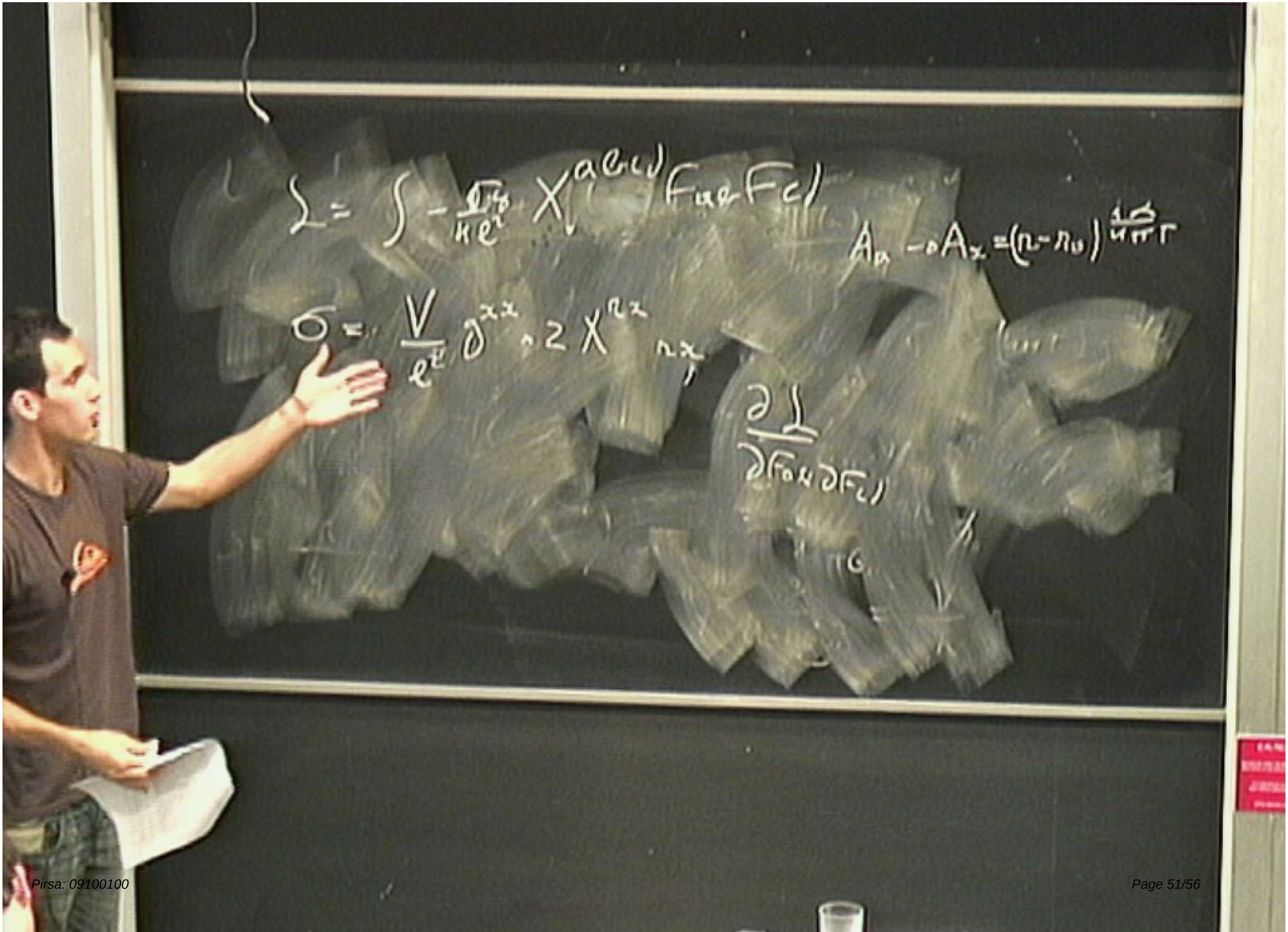
$$L_D \quad C_{abcd}\, C^{aecf}\, C_{bgeh}\, C_{f}^{\ g\ h}$$

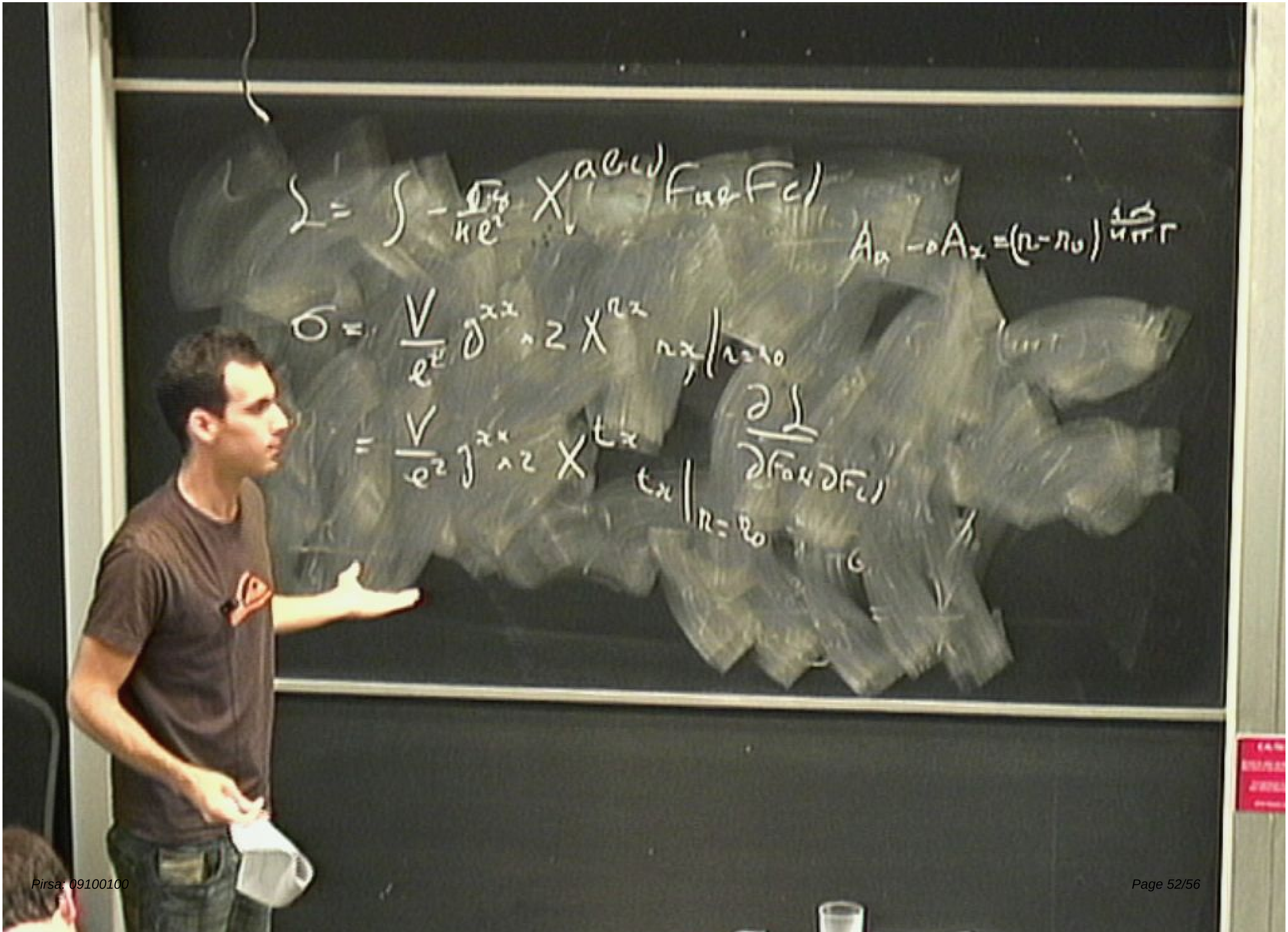
$$L_D = \frac{1}{16\pi G_N}\,\frac{V(4\pi T)}{(r-r_0)}\,(1+180\gamma) + \dots + O\left(\frac{1}{(r-r_0)^2}\right) + \text{regular}$$


L = R + 12 + f(Rabcd)


$\mathcal{L} = \int -\frac{\sqrt{-g}}{4e^2} X^{abcd} F_{ab} F_{cd}$
$A_x \to A_x = (n - n_0)\frac{1}{4\pi r}$

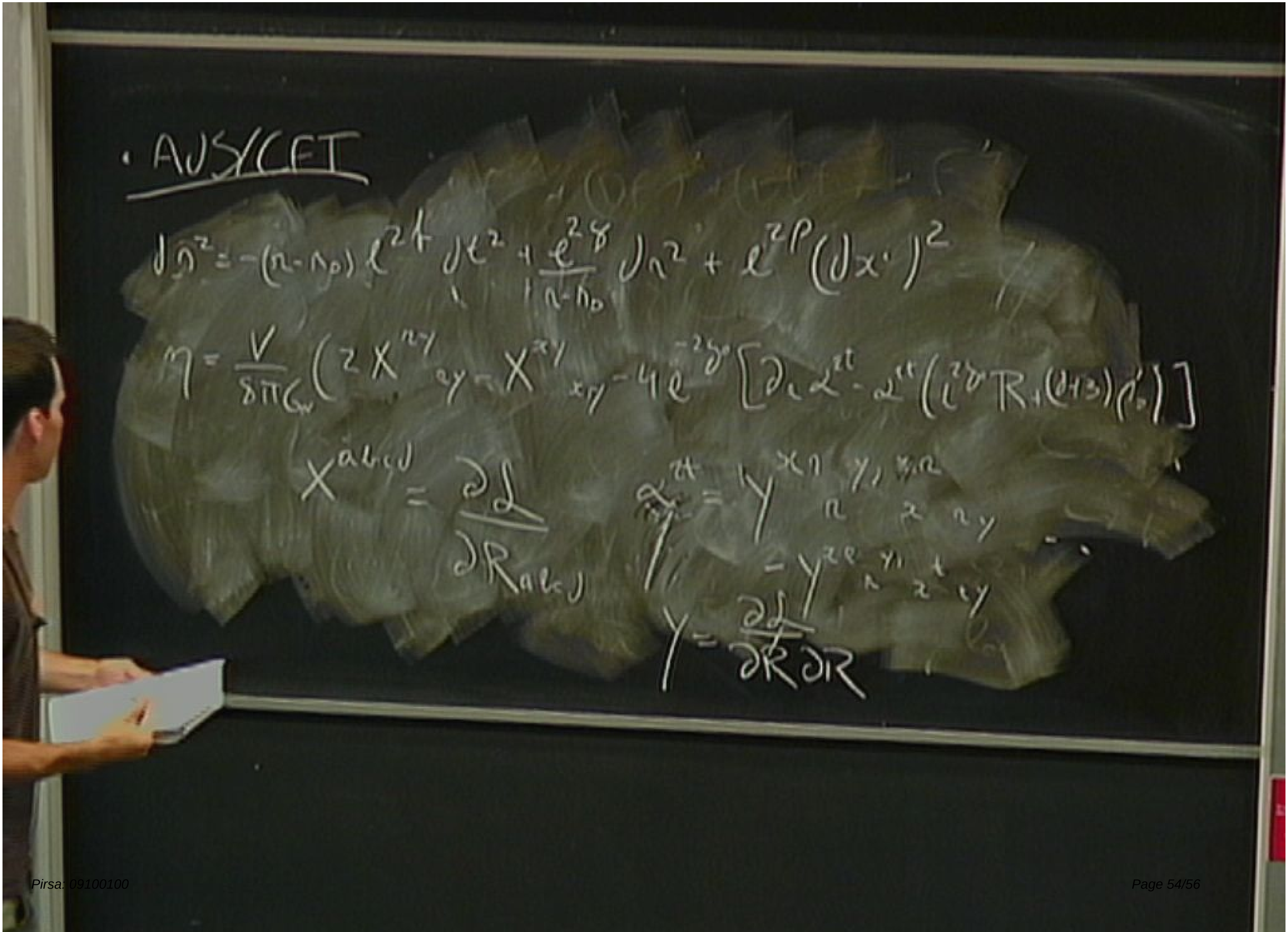

$\mathcal{L} = \int -\frac{\sqrt{-g}}{4e^2} X^{abcd} F_{ab} F_{cd}$
$A_x \to A_x = (r - r_0)\frac{1}{4\pi r}$
$\sigma = \frac{V}{e^2} g^{xx} \, 2 X^{rx}{}_{rx}$




L = ∫ -√-g/4e² X^{abcd} F_ab F_cd
A_x → A_x + (r - r_0)
σ = V/e² g^{xx} 2 X^{rx}_{rx}|_{r=r_0}
= V/e² g^{xx} 2 X^{tx}_{tx}|_{r=r_0}
∂L/∂F_ab ∂F_cd

• AdS/CFT

$$ds^2 = -(r-r_0)\, e^{2f}\, dt^2 + \frac{e^{2g}}{r-r_0}\, dr^2 + e^{2\rho}(dx^i)^2$$



• AdS/CFT
$ds^2 = -(r-r_0)e^{2f}dt^2 + \frac{e^{2g}}{r-r_0}dr^2 + e^{2p}(dx^i)^2$
$\eta = \frac{V}{8\pi G_N}\left(2X^{rxy}{}_{ry} - X^{xy}{}_{xy} - 4e^{-2g_0}\left[\partial_r \alpha^{2f} - \alpha^{2f}\left(e^{2g}R + (d+3)f_0'\right)\right]\right)$
$X^{abcd} = \frac{\partial \mathcal{L}}{\partial R_{abcd}}$
$Y = \frac{\partial^2 \mathcal{L}}{\partial R \partial R}$

• AdS/CFT

$$ds^2 = -(r-r_0)\,e^{2f}\,dt^2 + \frac{e^{2g}}{r-r_0}\,dr^2 + e^{2h}(dx^i)^2$$

$$\eta = \frac{V}{8\pi G_N}\left(2X^{rxry}{}_{ry} - X^{xy}{}_{xy} - 4e^{-2g_0}\left[\partial_r \alpha^{rt} - \alpha^{rt}\left(\ldots R + (d+3)f_0'\right)\right]\right)$$

$$S_{\partial V} X^{rt}{}_{rt} \qquad X^{abcd} = \frac{\partial \mathcal{L}}{\partial R_{abcd}} \qquad \alpha^{rt} = Y^{xr}{}_{r}{}^{yr}{}_{x}{}^{xr}{}_{ry} - Y^{xr}{}_{r}{}^{yr}{}_{x}{}^{t}{}_{ry}$$

$$Y = \frac{\partial^2 \mathcal{L}}{\partial R\,\partial R}$$

$$\delta h^x_y = \underbrace{(r-r_0)^{\frac{i\omega}{4\pi T}}}_{\phi_h}$$

$$\phi = (r-r_0)^{\frac{i\omega}{4\pi T}}$$

$$\eta = -(8\pi T)\frac{\mathrm{Res}\ \mathcal{L}_{\phi=\phi_h}}{\alpha^2}$$

$$\phi = (r-r_0)^{\frac{i\alpha}{4\pi T}}$$

$$\phi = e^{-i\omega t}$$

$$\partial_r \phi = \sqrt{-\frac{g_{rr}}{g_{tt}}}\,(-i\omega)\,\phi$$