Title: Stability Walls in Heterotic Theories

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Abstract: We study the sub-structure of heterotic Kahler moduli space due to the presence of non-Abelian internal gauge fields from the perspective of the four-dimensional effective theory. Internal gauge fields can be supersymmetric in some regions of Kahler moduli space but break supersymmetry in others. In the context of the four-dimensional theory, we investigate what happens when the Kahler moduli are changed from the supersymmetric to the non-supersymmetric region. Our results provide a low-energy description of supersymmetry breaking by internal gauge fields as well as a physical picture for the mathematical notion of bundle stability. Specifically, we find that at the transition between the two regions an additional anomalous U(1) symmetry appears under which some of the states in the low-energy theory acquire charges. We compute the associated D-term contribution to the four-dimensional potential which contains a Kahler modulus dependent Fayet-Iliopoulos term and contributions from the charged states. We show that this Dterm correctly reproduces the expected physics. Several mathematical conclusions concerning vector bundle stability are drawn from our arguments. We also discuss possible physical applications of our results to heterotic model building and moduli stabilisation.

Stability Walls in Heterotic Theories

James Gray - University of Oxford

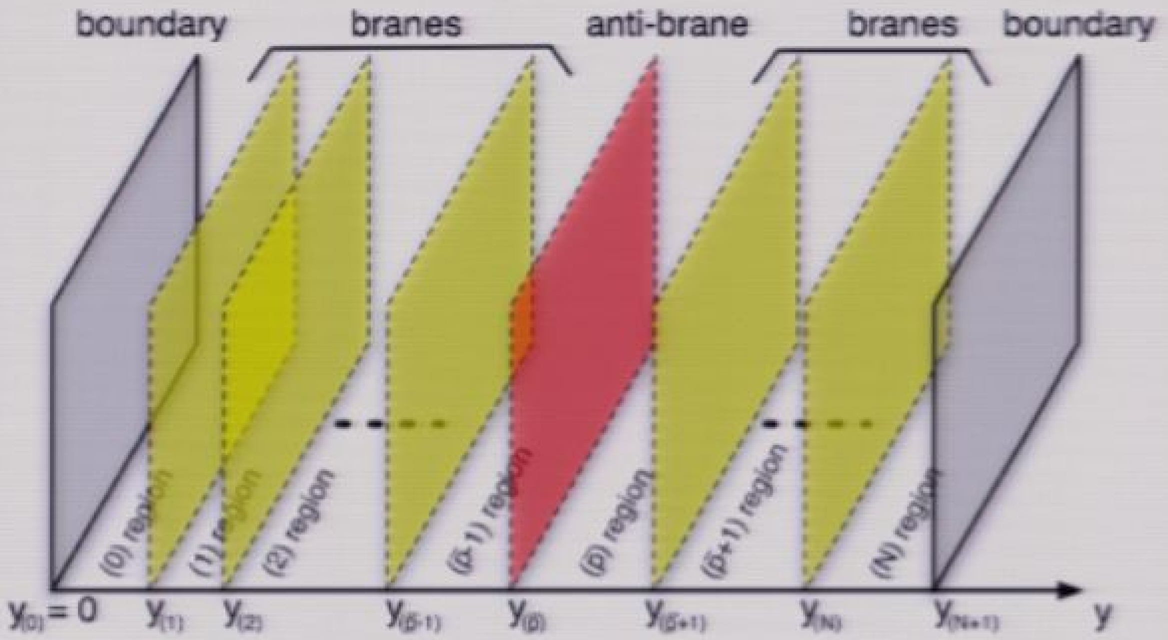
arXiv:0903.5088 [hep-th]
arXiv:0905.1748 [hep-th]

with Lara Anderson, Andre Lukas and Burt Ovrut.

Earlier work by Eric Sharpe: arXiv:hep-th/9810064

Introduction to Heterotic M-theory

Horava Witten - the strongly coupled limit of heterotic string



- Bulk is eleven dimensional supergravity.
- Boundaries support ten dimensional E8 SYM.
- M5 world volume actions for central branes.

- One dimension out of eleven is already compact. So that gets us down to ten dimensions.

- In order to get an $\mathcal{N} = 1$ supersymmetric 4D EFT (with perturbative Minkowski ground state) we compactify remaining six on a Calabi-Yau threefold.

Bianchi Identity in ten dimensions:

$$dH = -\frac{3\alpha'}{\sqrt{2}}\left(\mathrm{tr}F^{(1)} \wedge F^{(1)} + \mathrm{tr}F^{(2)} \wedge F^{(2)} - \mathrm{tr}R \wedge R\right)$$

integrate both sides over a non-trivial 4 cycle in the Calabi-Yau...

$$\int_{C_4} \mathrm{tr} F^{(1)} \wedge F^{(1)} + \int_{C_4} \mathrm{tr} F^{(2)} \wedge F^{(2)} = \int_{C_4} \mathrm{tr} R \wedge R$$

- The right hand side of this expression is non zero for a Calabi-Yau manifold.
- We see that we must have non-trivial gauge field vevs in the internal dimensions in our vacuum
- We want to pick these gauge field vevs such that they too preserve $\mathcal{N} = 1$ supersymmetry in the 4D EFT.

$\Longrightarrow$ Must solve the Hermitian Yang Mills eqns.

$$F_{ab} = F_{\bar{a}\bar{b}} = 0 \;,\; g^{a\bar{b}} F_{a\bar{b}} = 0$$

The four dimensional theory:

- If gauge field vevs on standard model orbifold fixed plane are valued in a group G then 4D visible sector gauge group is commutant of G in E8.
- Dimensional reduction of SYM fields gives 4D matter.
- Integration constants in solution for metric and gauge fields become 4D scalar fields - moduli.

The Calabi-Yau metric and gauge fields are not known explicitly

$\longrightarrow$ Use of algebraic geometry to describe the compactification

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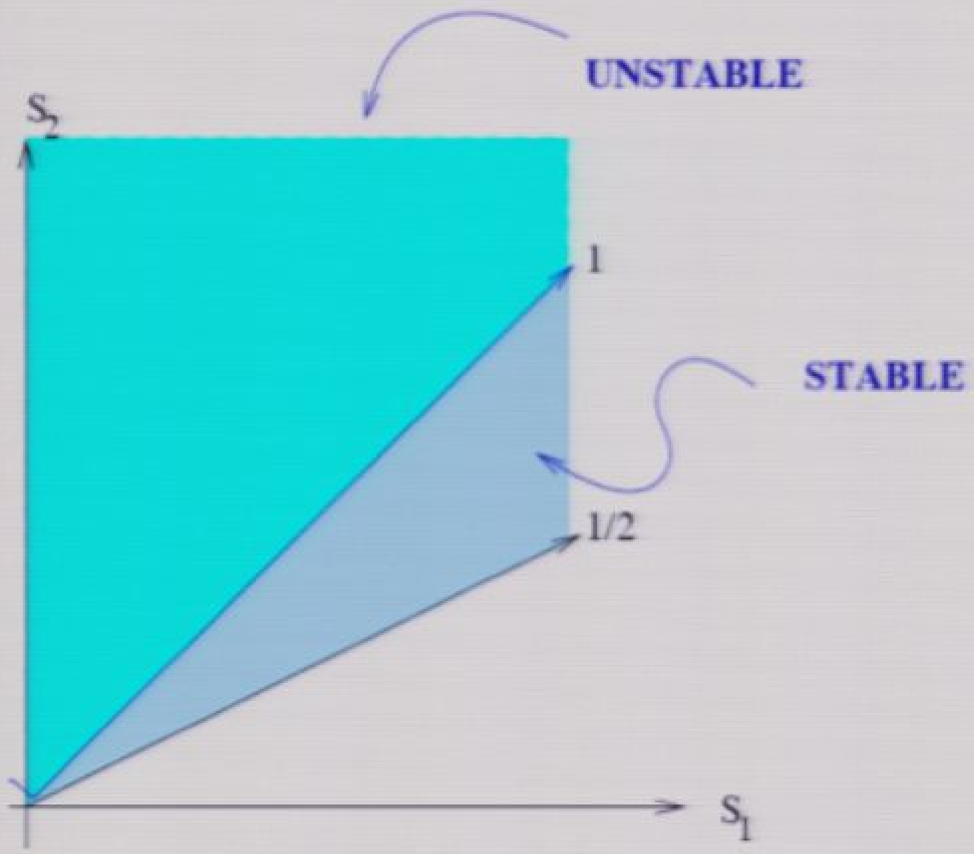

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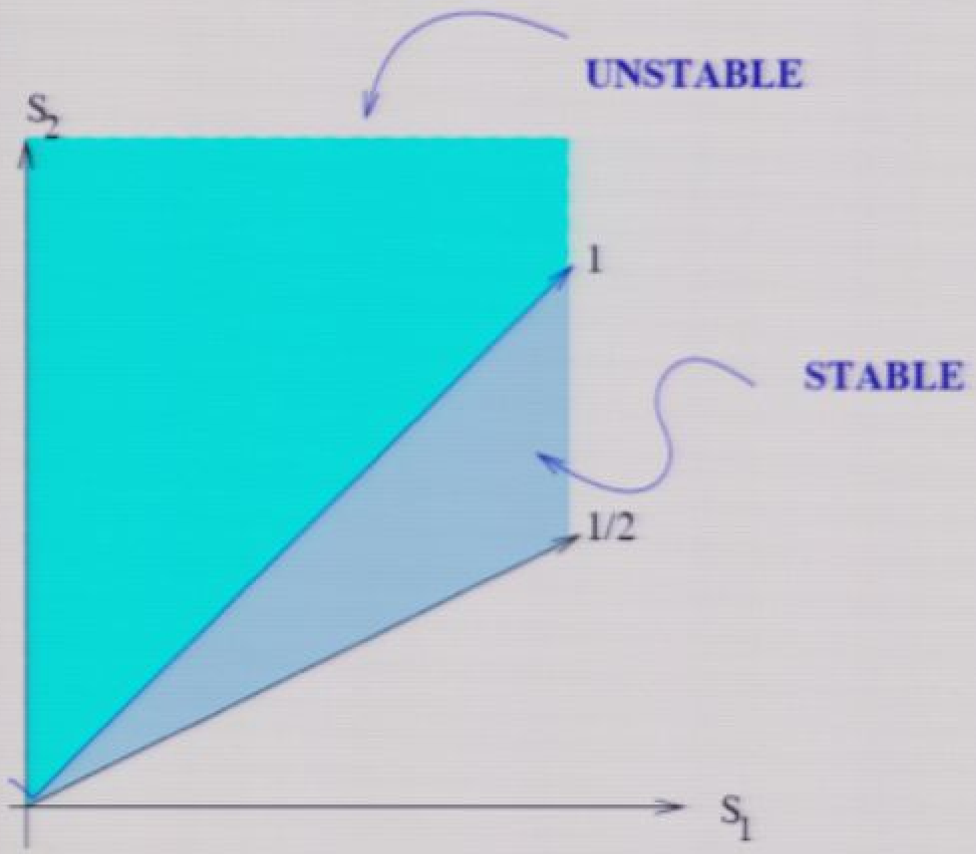

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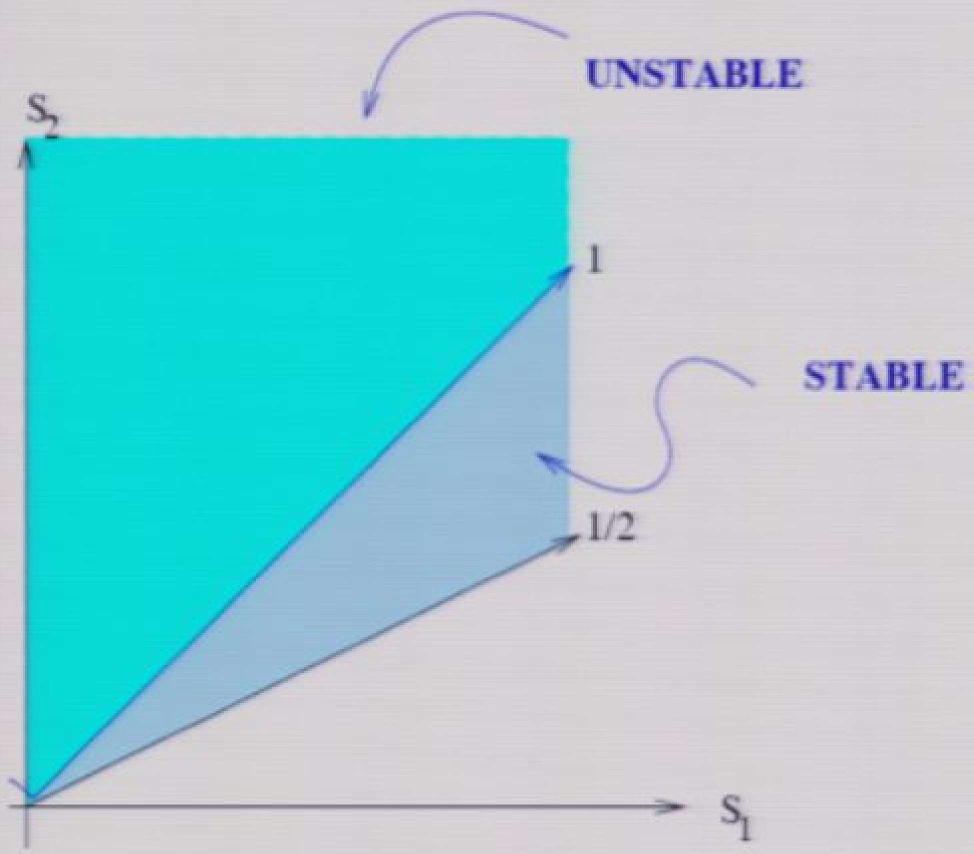

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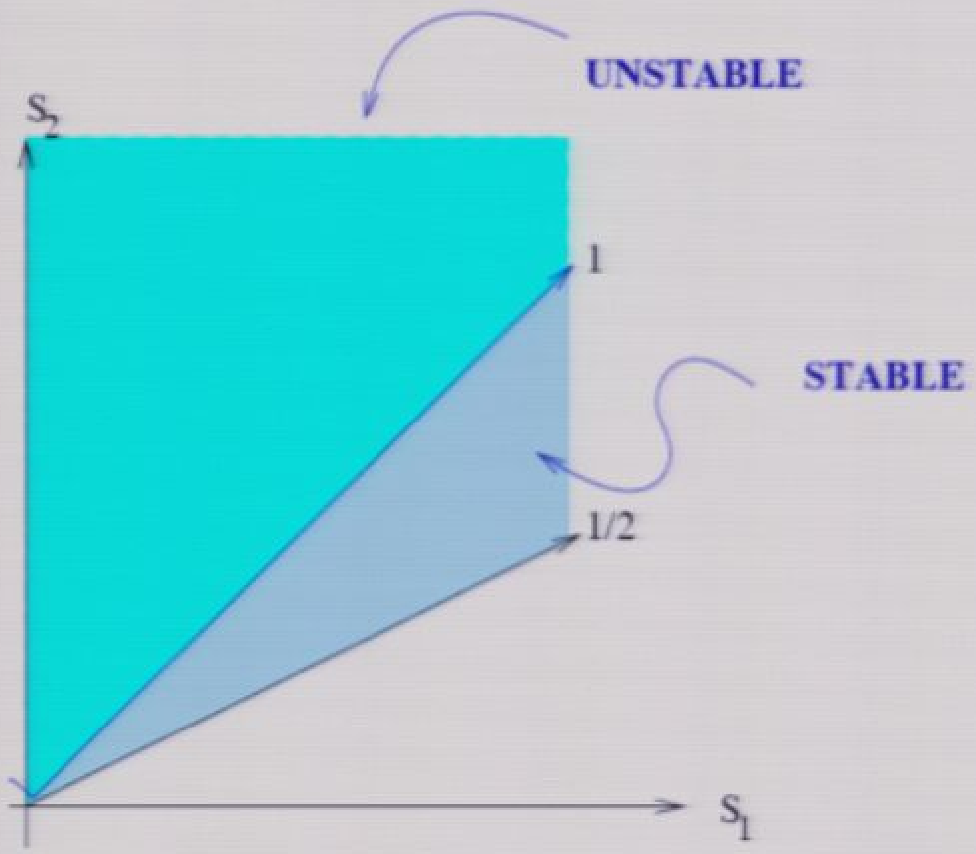

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In this work we calculate the exact form of this potential:

The Punch Line: it's a D-term potential.

- An extra U(1) appears at the boundary of the supersymmetric region.
- This extra U(1) has a D-term:

$$D^{U(1)} = A\frac{\mu(\mathcal{F})}{\mathcal{V}} - \sum_{M,\bar{N}} Q^M G_{M\bar{N}} C^M \bar{C}^{\bar{N}}$$

- The C's here are "matter" fields charged under the U(1).
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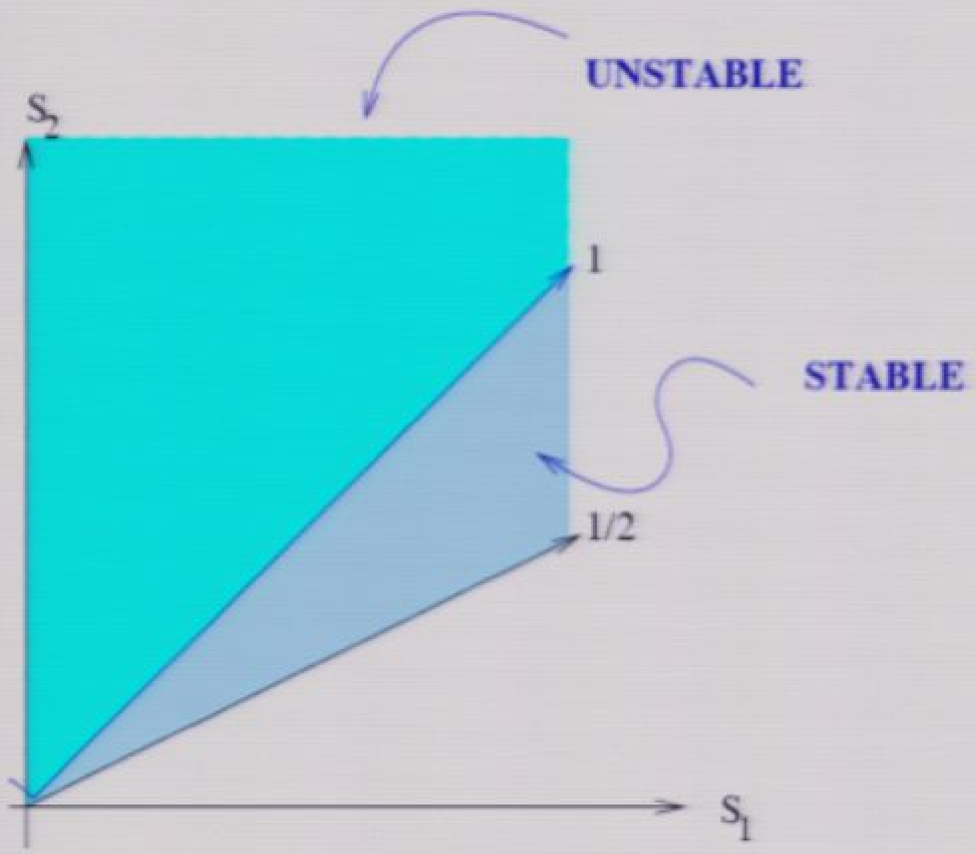

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Vector Bundle Stability

D.U.Y: For each poly-stable holomorphic vector bundle there exists a unique connection satisfying HYM.

Stability:

Define slope: $$\mu(V) \equiv \frac{1}{\mathrm{rk}(V)} \int_X c_1(V) \wedge J^{\dim(X)-1}$$

We say: $\mathcal{V}$ (semi) stable if $\forall$ sub-sheaves $\mathcal{F} \subset \mathcal{V}$

with $0 < \mathrm{rk}(\mathcal{F}) < \mathrm{rk}(\mathcal{V})$

$\mu(\mathcal{F}) < \mu(\mathcal{V})$ (resp. $\mu(\mathcal{F}) \leq \mu(\mathcal{V})$)

$\mathcal{V}$ is poly-stable if $\mathcal{V} = \oplus_n \mathcal{V}_n$ such that all $\mathcal{V}_n$

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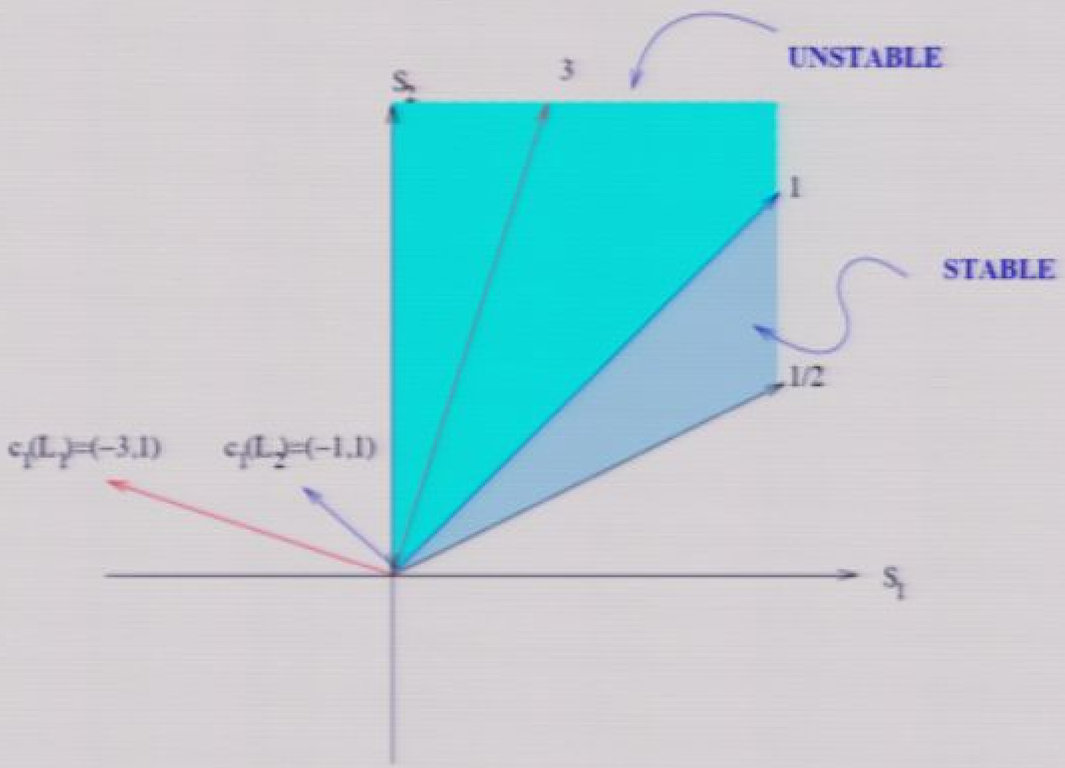
Specific Example:

Consider the following SU(3) monad bundle on the $\left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^3 & 4 \end{array}\right]$.

$$0 \to \mathcal{V} \to \mathcal{O}(1,0) \oplus \mathcal{O}(1,-1) \oplus \mathcal{O}(0,1)^{\oplus 2} \xrightarrow{f} \mathcal{O}(2,1) \to 0$$



Effective Field Theory

- On the line $\mathcal{F}$ injects into, and has same slope as, $\mathcal{V}$.
- System only supersymmetric if $\mathcal{V}$ poly-stable, so $\mathcal{V} = \mathcal{F} \oplus \mathcal{K}$.

$\mathcal{F}$ injects: $$0 \longrightarrow \mathcal{F} \longrightarrow \mathcal{V} \longrightarrow \mathcal{K} \longrightarrow 0$$

Theorem: If you have such a sequence then $\mathcal{V} = \mathcal{F} \oplus \mathcal{K}$ somewhere in moduli space.

Note: $$c_1(\mathcal{F}) = -c_1(\mathcal{K}) \neq 0$$

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Structure group of $\mathcal{V}$:-

$$SU(3) \longrightarrow S[U(2) \times U(1)] \sim SU(2) \times U(1)$$

Resulting visible sector gauge group:-

$$E6 \longrightarrow E6 \times U(1)$$

So on the boundary of the stable region in Kahler moduli space, and at the "split locus" in bundle moduli space, we pick up an extra low energy U(1).

We are going to work out the EFT for fluctuations about such a locus in moduli space - including this extra U(1).

Matter Content:

Matter descending from higher D gauge fields:

$$E8 \supset E6 \times SU(2) \times U(1)$$

$$\begin{aligned} \mathbf{248} \quad = \quad & (\mathbf{1},\mathbf{1})_0 + (\mathbf{1},\mathbf{2})_{-3} + (\mathbf{1},\mathbf{2})_3 + (\mathbf{1},\mathbf{3})_0 + (\mathbf{78},\mathbf{1})_0 \\ & +(\mathbf{27},\mathbf{1})_2 + (\mathbf{27},\mathbf{2})_{-1} + (\overline{\mathbf{27}},\mathbf{1})_{-2} + (\overline{\mathbf{27}},\mathbf{2})_1 \end{aligned}$$

As usual: $H^1 \longrightarrow \text{scalars}$ and $H^0 \longrightarrow \text{vectors}$

We have to be careful in relating these charges to the cohomologies we want to compute and remember that the U(1) here is not the structure group of our line bundle $\mathcal{K}$.

Representation	Cohomology	Physical $U(1)$charge	Dimension of Cohomology
$(\mathbf{1},\mathbf{2})_{-3}$	$H^1(X,\mathcal{F}\otimes\mathcal{K}^*)$	$-3/2$	16
$(\mathbf{1},\mathbf{2})_{3}$	$H^1(X,\mathcal{F}^*\otimes\mathcal{K})$	$3/2$	0
$(\mathbf{1},\mathbf{3})_{0}$	$H^1(X,\mathcal{F}\otimes\mathcal{F}^*)$	0	7
$(\mathbf{27},\mathbf{1})_{2}$	$H^1(X,\mathcal{K})$	1	0
$(\mathbf{27},\mathbf{2})_{-1}$	$H^1(X,\mathcal{F})$	$-1/2$	2
$(\overline{\mathbf{27}},\mathbf{1})_{-2}$	$H^1(X,\mathcal{K}^*)$	-1	0
$(\overline{\mathbf{27}},\mathbf{2})_{1}$	$H^1(X,\mathcal{F}^*)$	$1/2$	0

- $27\ (\overline{27})$ are unambiguously matter.
- $(1,3)_0$ are unambiguously bundle moduli.
- $(1,2)_{-3}$ are more interesting...

$(1,2)_{-3}$

- Strictly speaking these fields are matter (they are charged under the visible U(1)).
- But they can also be thought of as bundle moduli:

$$\mathcal{V} = \mathcal{F} \oplus \mathcal{K}$$

so: $\mathcal{V} \otimes \mathcal{V}^* = \mathcal{O} \oplus \mathcal{F} \otimes \mathcal{K}^* \oplus \mathcal{F}^* \otimes \mathcal{K} \oplus \mathcal{F}^* \otimes \mathcal{F}$

thus we have:-

$$H^1(\mathcal{V} \otimes \mathcal{V}^*) = H^1(\mathcal{F} \otimes \mathcal{F}^*) \oplus H^1(\mathcal{F}^* \otimes \mathcal{K}) \oplus H^1(\mathcal{F} \otimes \mathcal{K}^*)$$

In fact they are the bundle moduli which take us away from the split locus:

$$\langle C \rangle = 0 \longrightarrow \text{split}$$

$$\langle C \rangle \neq 0 \longrightarrow \text{mixed up again}$$

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- But they can also be thought of as bundle moduli:

$$\mathcal{V} = \mathcal{F} \oplus \mathcal{K}$$

so: $\mathcal{V} \otimes \mathcal{V}^* = \mathcal{O} \oplus \mathcal{F} \otimes \mathcal{K}^* \oplus \mathcal{F}^* \otimes \mathcal{K} \oplus \mathcal{F}^* \otimes \mathcal{F}$

thus we have:-

$$H^1(\mathcal{V} \otimes \mathcal{V}^*) = H^1(\mathcal{F} \otimes \mathcal{F}^*) \oplus H^1(\mathcal{F}^* \otimes \mathcal{K}) \oplus H^1(\mathcal{F} \otimes \mathcal{K}^*)$$

In fact they are the bundle moduli which take us away from the split locus:

$$\langle C \rangle = 0 \longrightarrow \text{split}$$

$$\langle C \rangle \neq 0 \longrightarrow \text{mixed up again}$$

Moduli Content:

Descending from higher dimensional gravitational sector

- We have all of the usual fields: $T^i,\ S,\ Z_{\hat{i}},\ \mathfrak{z}^a,$ etc.
- But some of them are charged under the U(1).

To lowest order we need only worry about the Kahler moduli, T^k:

$$\mathrm{Im}[T^k] = i2\chi^k \qquad\qquad C_{11a\bar{b}} = \chi^k \omega_{ka\bar{b}}$$

and it is well known that the three-form transforms under gauge transformations...

The axion transforms as:

$$\delta\chi^i = -\frac{3}{16}\epsilon_S\epsilon_R^2 c_1^i(\mathcal{F})\eta$$

under gauge transformations - the shift symmetry is gauged.

The potential:

- Our 4D theory is $\mathcal{N} = 1$ supersymmetric.
- The potential therefore has D-term and F-term contributions.
- F-terms turn out to be unimportant here (not true in more complicated cases).

D-terms:

$$D^{E6} \Longrightarrow \langle 27 \rangle = 0$$

$$D^{U(1)} = \frac{3}{16} \frac{\epsilon_S \epsilon_R^2}{\kappa_4^2} \frac{\mu(\mathcal{F})}{\mathcal{V}} - \sum_{L,\bar{M}} Q^L G_{L\bar{M}} C^L \bar{C}^{\bar{M}}$$

$$\mu(\mathcal{F}) = \frac{1}{2} d_{ijk} c_1^i(\mathcal{F}) t^j t^k$$

- "FI" term negative in stable region, zero on boundary, and positive in unstable region.
- The matter states C^L are all negatively charged.
- D=0 possible in region where bundle is stable
- D=0 not possible in region where bundle is unstable
- This reproduces the structure seen in the introduction.

Regaining the usual spectrum in the stable region:

- Extra U(1) is Higgsed (further) by the matter field vevs (in fact it is somewhat massive everywhere in moduli space).

- One of the C^L fields gains mass too:

write: $C^L = \langle C^L \rangle + \delta C^L$ etc...

$$D^{U(1)}_{\text{Stable Region}} = -\frac{3}{16}\frac{\epsilon_S \epsilon_R^2}{\kappa_4^2} G_{jk} c_1^j(\mathcal{F}) \delta t^k - \sum_{L,\bar{M}} Q^L G_{L\bar{M}} \left(\langle C^L \rangle \delta \bar{C}^{\bar{M}} + \delta C^L \left\langle \bar{C}^{\bar{M}} \right\rangle \right)$$

- This is just the corresponding Higgs field (can show it has the same mass as the gauge boson).

- Corresponding axions and phase combination is swallowed by the gauge boson.

So in the stable region we end up with:

- An E6 gauge group.
- 16-1=15 C^L fields uncharged under the E6.
- 7 bundle moduli uncharged under E6.
- 2 families of E6 in the 27 representation.

This precisely reproduces the results of a standard heterotic analysis in the supersymmetric region.

- That there is "only one sign" of C^L charge and "regaining the usual spectrum" etc can all be proved in complete generality (see big paper).

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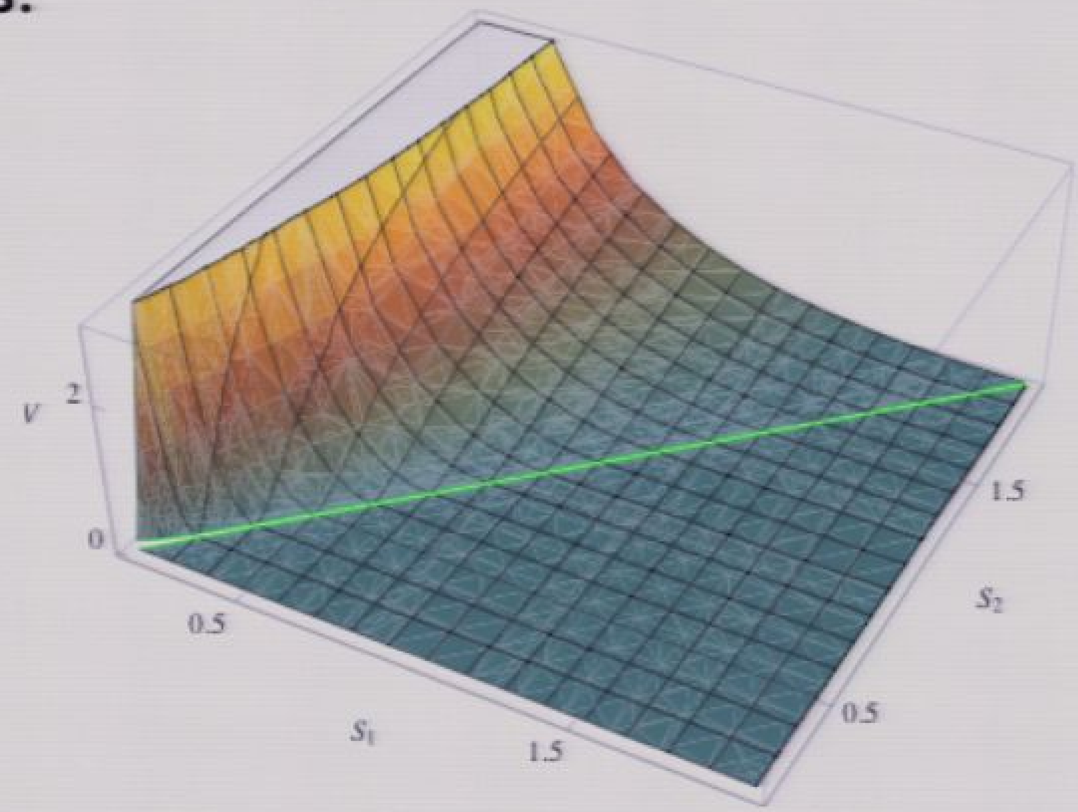
On the boundary line:

- "FI" term vanishes.
- D-term can be set to zero by $\langle C^L \rangle = 0$.
- From interpretation of these fields as bundle moduli we see that we regain the fact that we require a split bundle for supersymmetry on the line (semi-stability).
- Extra U(1) is light, as is an extra matter state. One linear combination of Kahler moduli heavy.

In the unstable region:

- D-term minimized at non-zero value by $\langle C^L \rangle = 0$. Light U(1) persists.
- Potential is non-vanishing.



- No stable perturbative vacuum - so I will not discuss masses further.

Higher order corrections

At next order in our expansions the dilaton and M5 position superfields also transform:

$$\delta\sigma = -\frac{3}{8}\pi\epsilon_S^2\epsilon_R^2 c_1^i(\mathcal{F})\beta_i\eta$$

From the following superfield definitions...

$$T^K = t^k + 2i\chi^k$$

$$Z^\alpha = \beta_i^\alpha \left(t^i z_\alpha + 2i\left(-n_\alpha^i \nu_\alpha + \chi^i z_\alpha\right)\right)$$

$$S = V_0 + \pi\epsilon_S \sum_{\alpha=1}^{N} \beta_i^\alpha t^i z_\alpha^2 + i\left(\sigma + 2\pi\epsilon_S \sum_{\alpha=1}^{N} \beta_i^\alpha \chi^i z_\alpha^2\right)$$

...we see that all three of these superfields transform.

The resulting D-term is:

$$D^{U(1)} = f - \sum_{L\bar{M}} Q^L G_{L\bar{M}} C^L \bar{C}^{\bar{M}}$$

$$\begin{aligned} f &= f^{(0)} + f^{(1)} \\ f^{(0)} &= \frac{3}{16}\frac{\epsilon_S \epsilon_R^2}{\kappa_4^2}\frac{\mu(\mathcal{F})}{\mathcal{V}} \\ f^{(1)} &= \frac{3\pi\epsilon_S^2\epsilon_R^2}{8\kappa_4^2}\frac{1}{S+\bar{S}}\left[\beta_i c_1^i(\mathcal{F}) + \pi\sum_{\alpha=1}^{N}\frac{(Z^\alpha+\bar{Z}^\alpha)^2}{(\beta_i^\alpha(T^i+\bar{T}^i))^2}\beta_i^\alpha c_1^i(\mathcal{F})\right] \end{aligned}$$

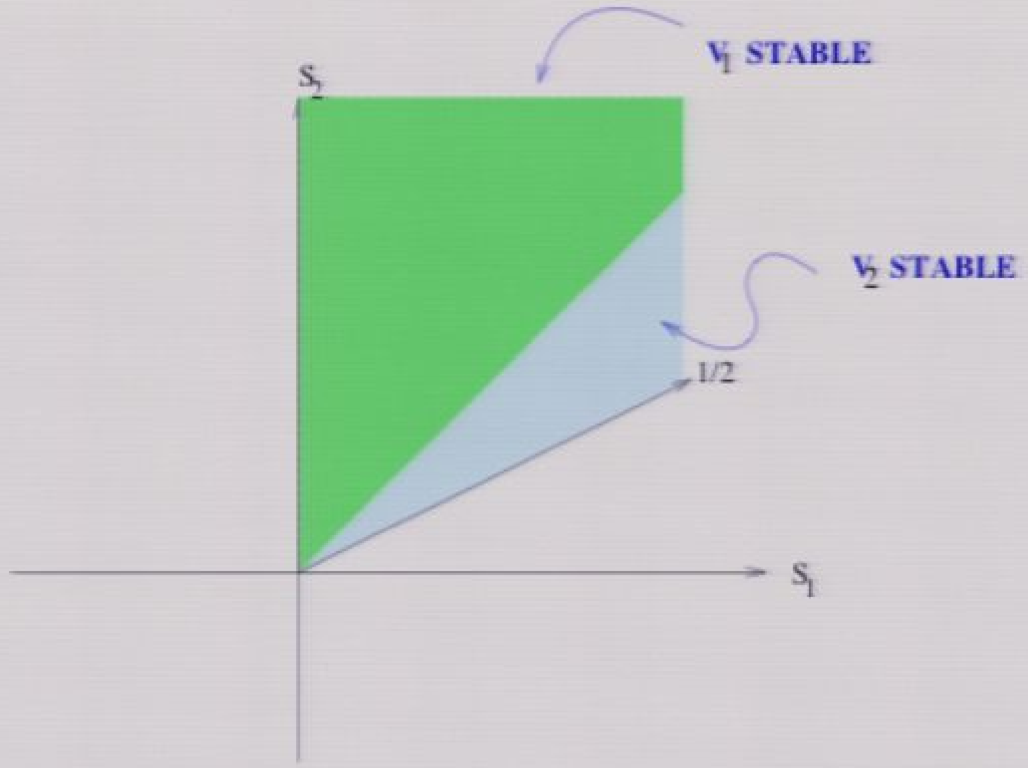
- Might think of this as "a correction to the mathematical notion of slope stability".
- Really, this is misleading. The corrections are just due to the difference between 4d moduli and what the gauge fields actually see....

Bundle Transitions

Consider the two bundles:

$$0 \to Q^* \to V_1 \to Q \to 0$$

$$0 \to Q \to V_2 \to Q^* \to 0$$



where $$0 \to Q \to \mathcal{O}(1,0) \oplus \mathcal{O}(0,1)^{\oplus 2} \to \mathcal{O}(2,1) \to 0$$

- These are non-isomorphic bundles...
- ... but they share the same split point.

Spectrum at the split point:

Charges	Name of Field	Number
16_1	f_1	2
$\overline{16}_1$	f_2	2
1_0	s_1	88
10_0	h_1	22
1_2	C_1	27
1_{-2}	C_2	1

$$D \sim \mu(Q^*)/\mathcal{V} - |f_1|^2 - |f_2|^2 - |C_1|^2 + |C_2|^2$$

- Now both signs of the FI term can be cancelled (by C_1 and C_2 for example).
- These two different branches in the vacuum space correspond to the two non-isomorphic bundles.

- Most general cubic superpotential consistent with massless field content and gauge invariance, etc.

$$W \sim f_1 f_2 C_2 + s_1 h_1 h_1 + s_1 C_1 C_2$$

- Gives a variety of terms in the potential, for example:

$$V \ni |f_2|^2 |C_2|^2 + |f_1|^2 |C_2|^2 \qquad V \ni |C_1|^2 |C_2|^2$$

- Reproduces expected physics of V_1 and V_2 precisely.

- You can describe smooth transitions between the bundles in a well controlled manner using this field theory.

Conclusions

- We now have a 4d effective description of Heterotic 'everywhere' in the Kahler cone.
- An extra U(1) appears at the boundary between the supersymmetric and non-supersymmetric regions of moduli space.
- There is a potential in the non-supersymmetric region associated to the D-term of this extra U(1).
- The effective field theory describing all this can be written down explicitly.
- Corrections to the D-term are due to warping and shouldn't be interpreted as a "correction to stability".

Further Work

- Conjectures on complex structure dependence of stability regions.
- More complicated branch structure studies and transitioning between bundles.
- Using potential for moduli stabilization (although...).
- Susy breaking and phenomenology in the unstable region.
- Parts of bundle stability just from the 4d EFT.
- Constraining Yukawa couplings with remnants of the U(1) structure.