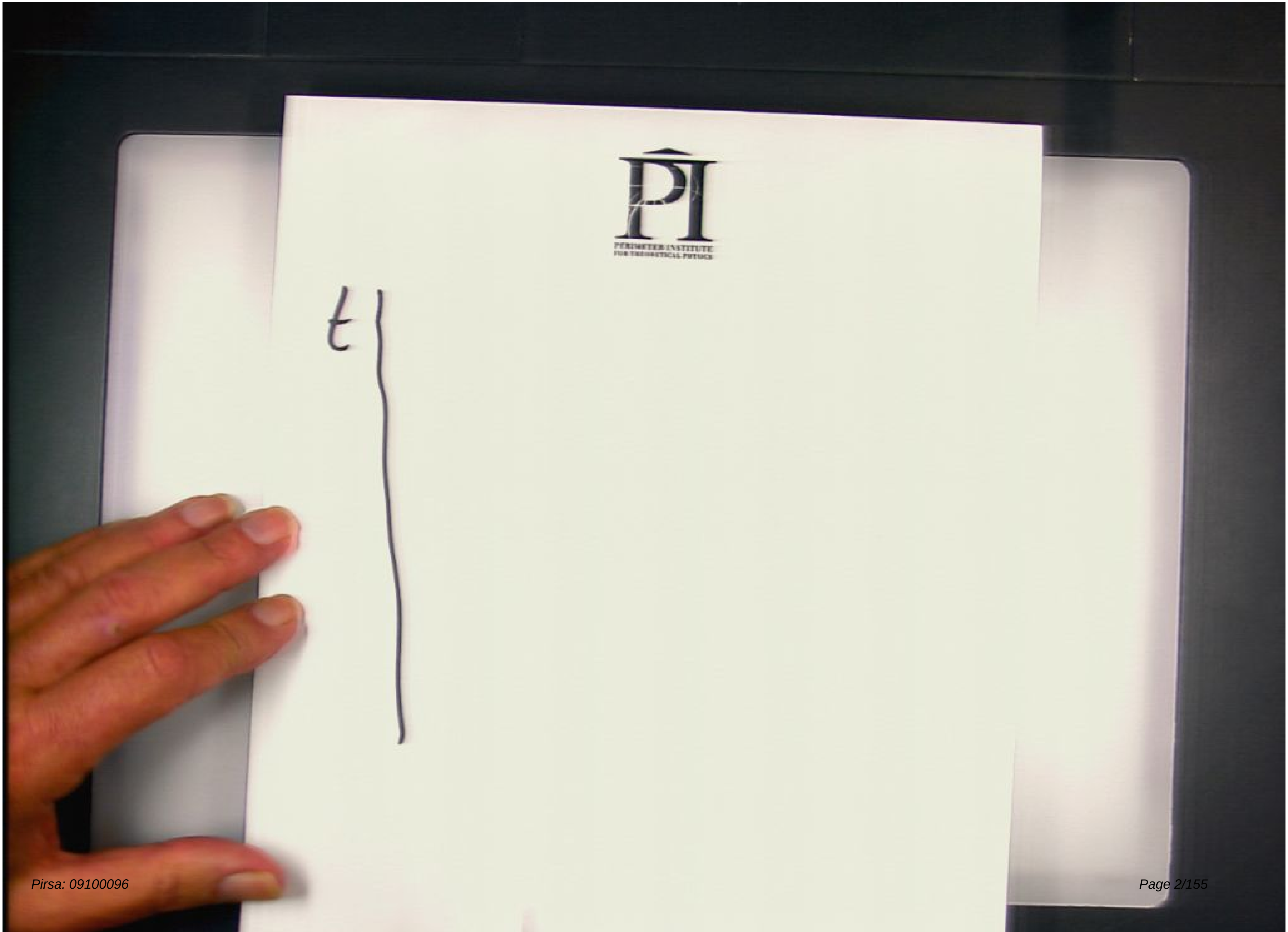
Title: On the 2-vector re-formulation of quantum mechanics

Date: Sep 30, 2009 02:00 PM

URL: http://pirsa.org/09100096

Abstract: I will discuss properties of pre- and post-selected ensembles in quantum mechanics. I will also discuss the proper way to observe these properties through the use of a new type of non-disturbing measurement which I call 'weak measurement'. A number of these new experiments have already been successfully performed and others are in the planning stage. These experiments have confirmed the unique property of pre- and post-selected ensembles that I call 'weak values.' Theoretical analysis of the outcomes of these experiments have produced several very rich results. First, it has shed new light on the most puzzling features of quantum mechanics, such as interference, entanglement, etc. Secondly, it has uncovered a host of new quantum phenomena, which were previously hidden."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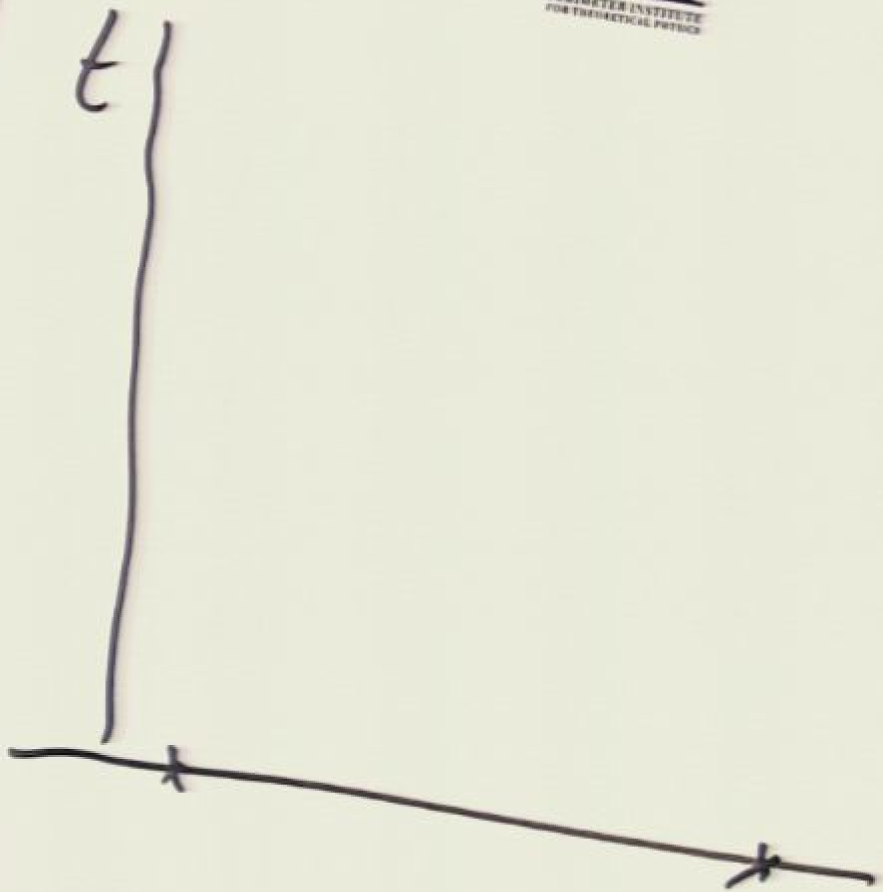
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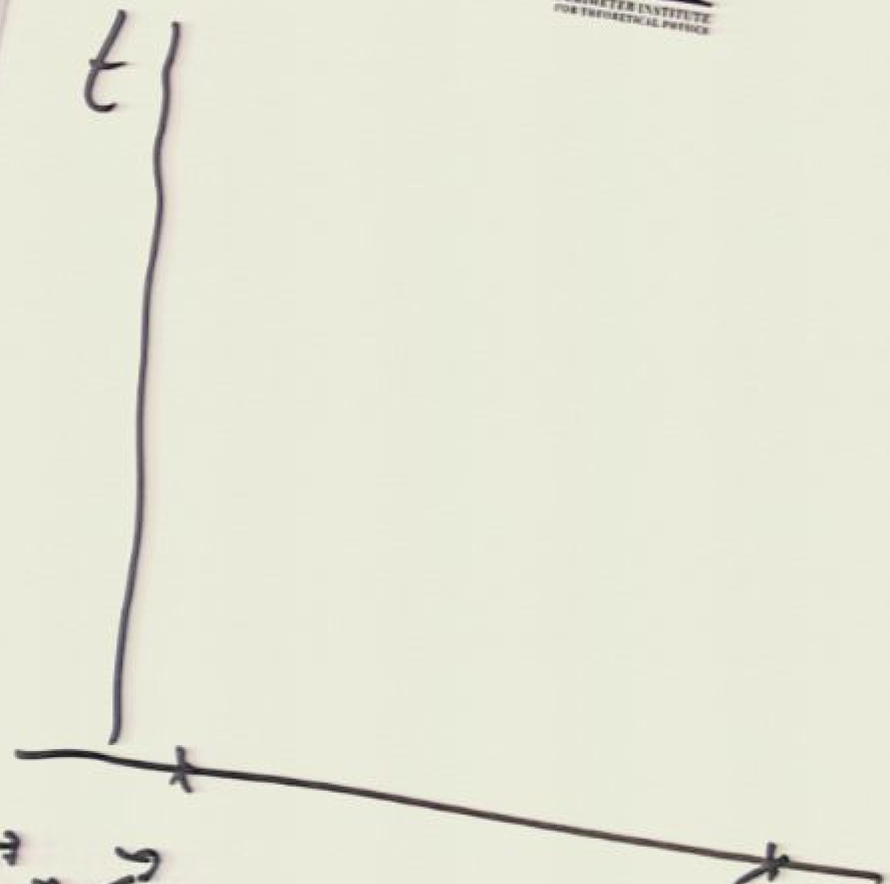
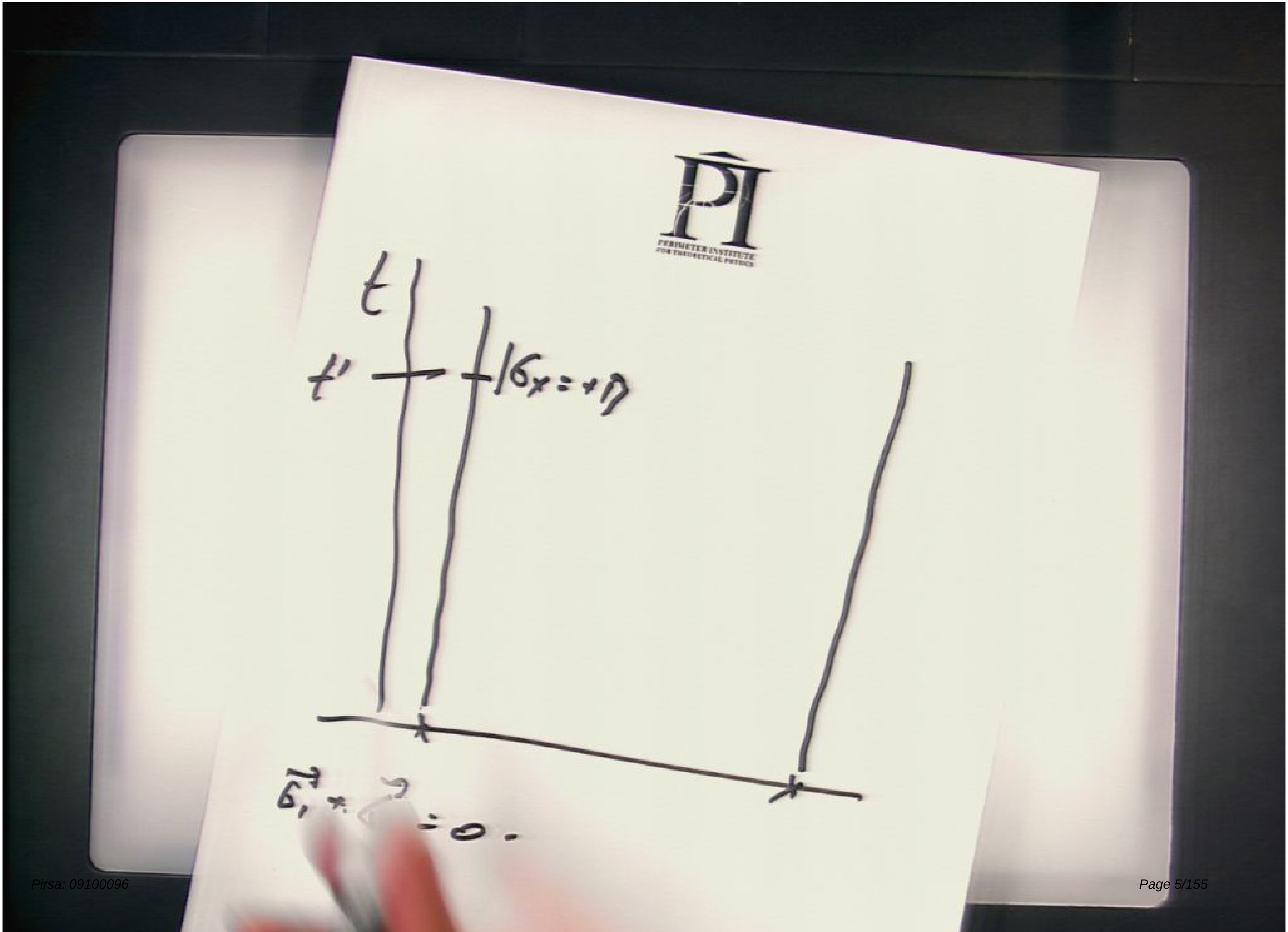
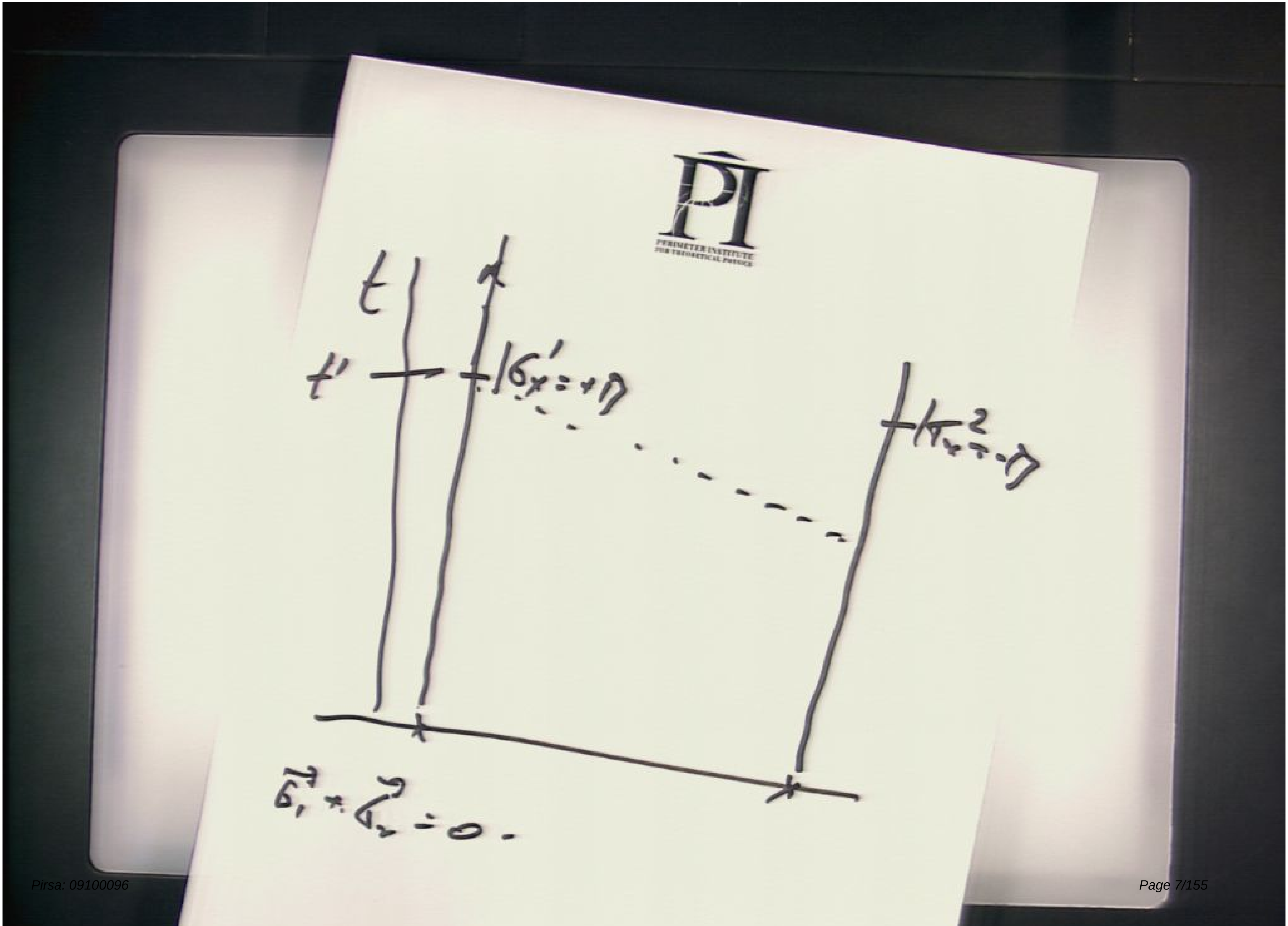


$\vec{b}_1 + \vec{b}_2 = 0.$


t
t'
σx = +1

$\vec{\sigma}_1 + \vec{\sigma}_2 = 0 .$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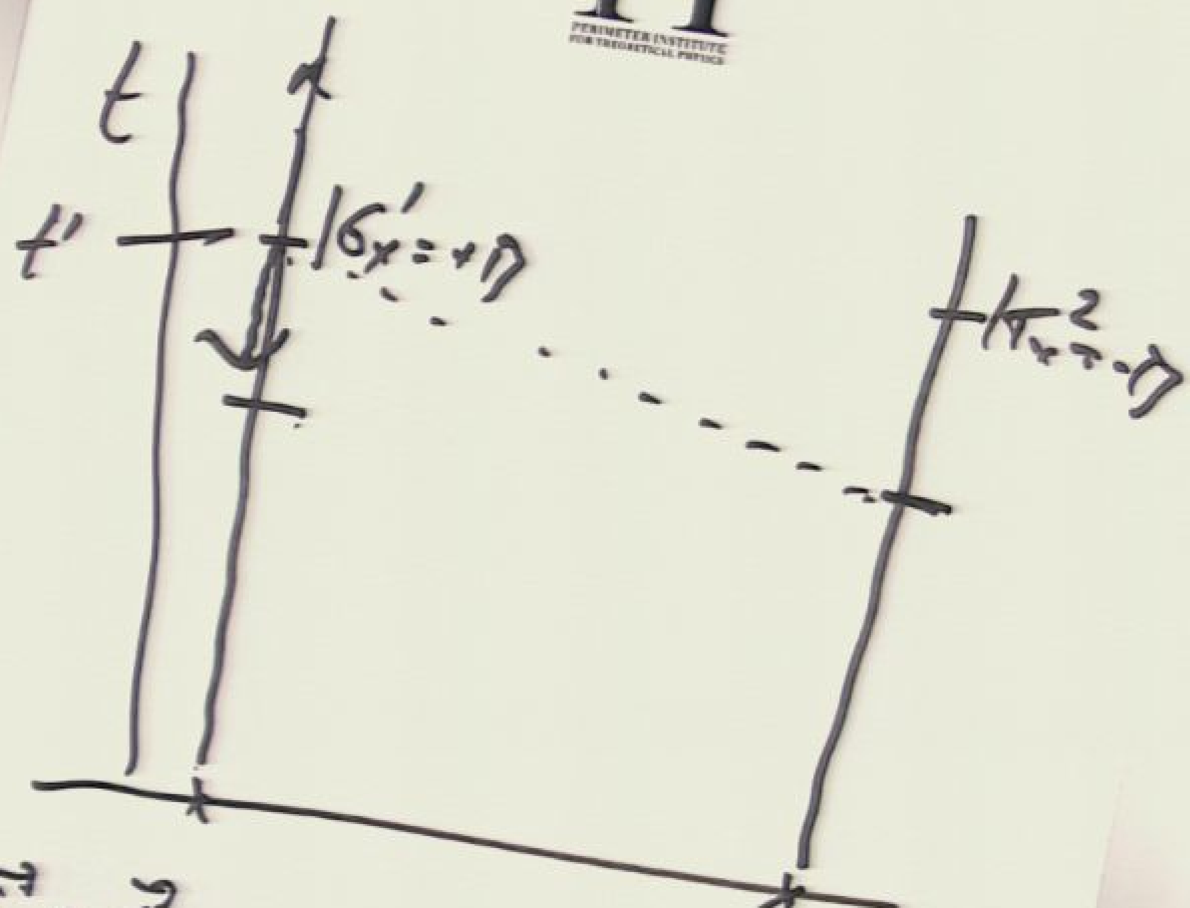
t
t'
|σ'_x = +1⟩
|σ_x = -1⟩
σ_1 + σ_2 = 0.



$\vec{\sigma}_1 + \vec{\sigma}_2 = 0$.

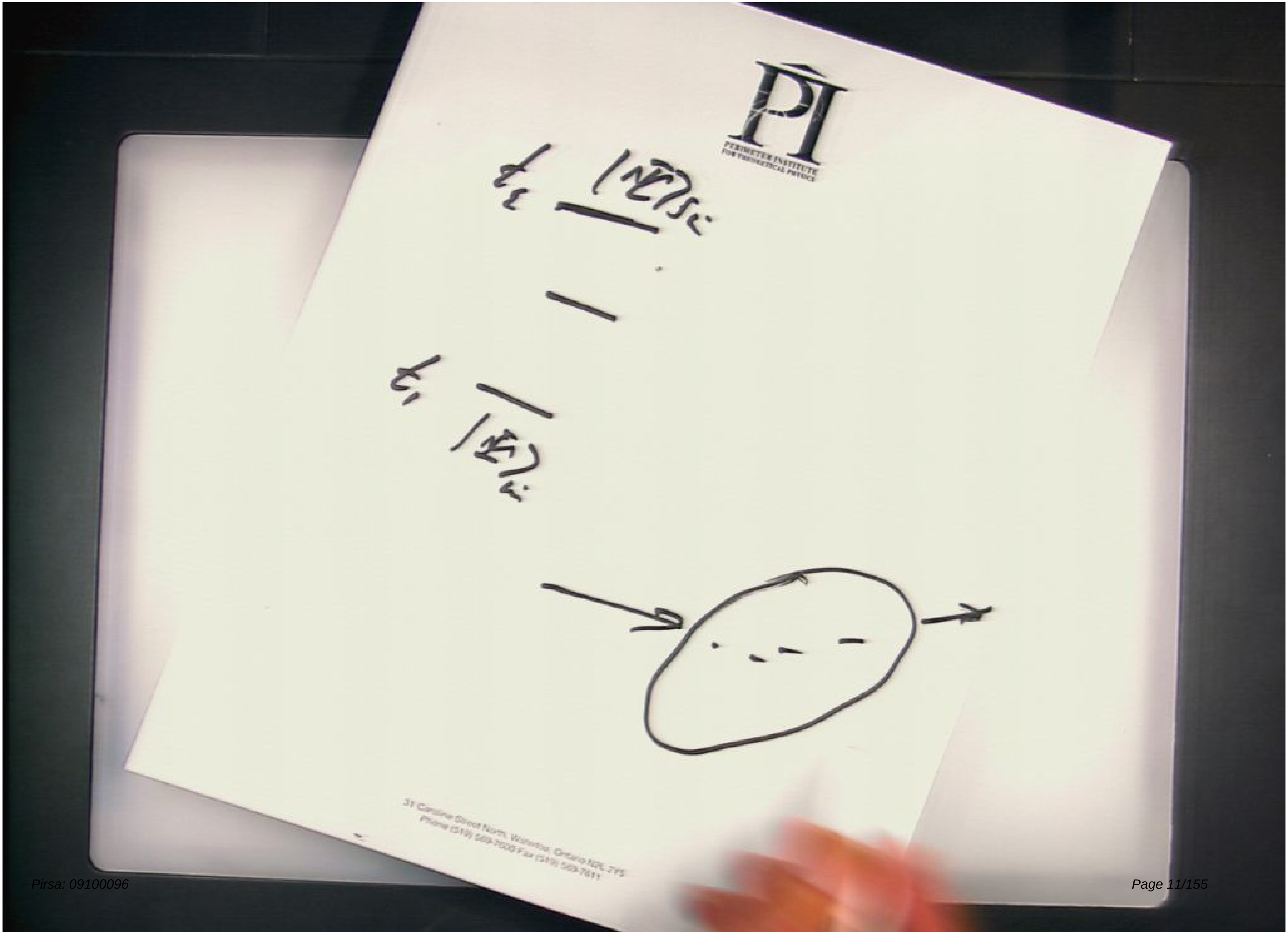
$t_1 \quad |E\rangle_i$

t_2 ——— $|\Psi\rangle_{Si}$

———

t_1 ———

$|\Psi\rangle_i$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t_2
t_1

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t_2 —— $|\tilde{N}\rangle_{s,i}$

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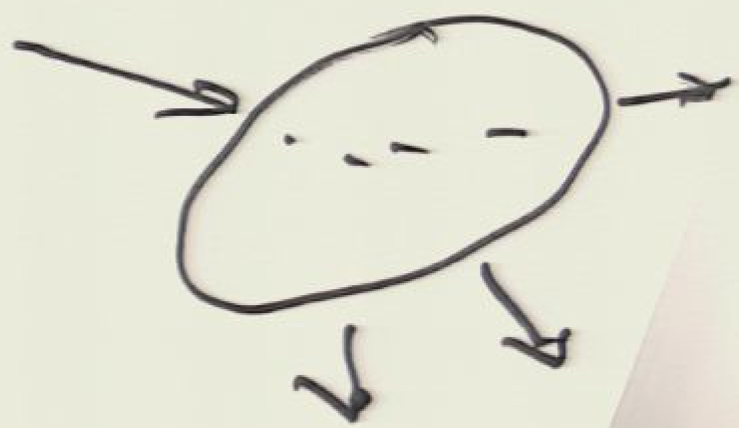
t_1 —— $|E\rangle_{s,i}$



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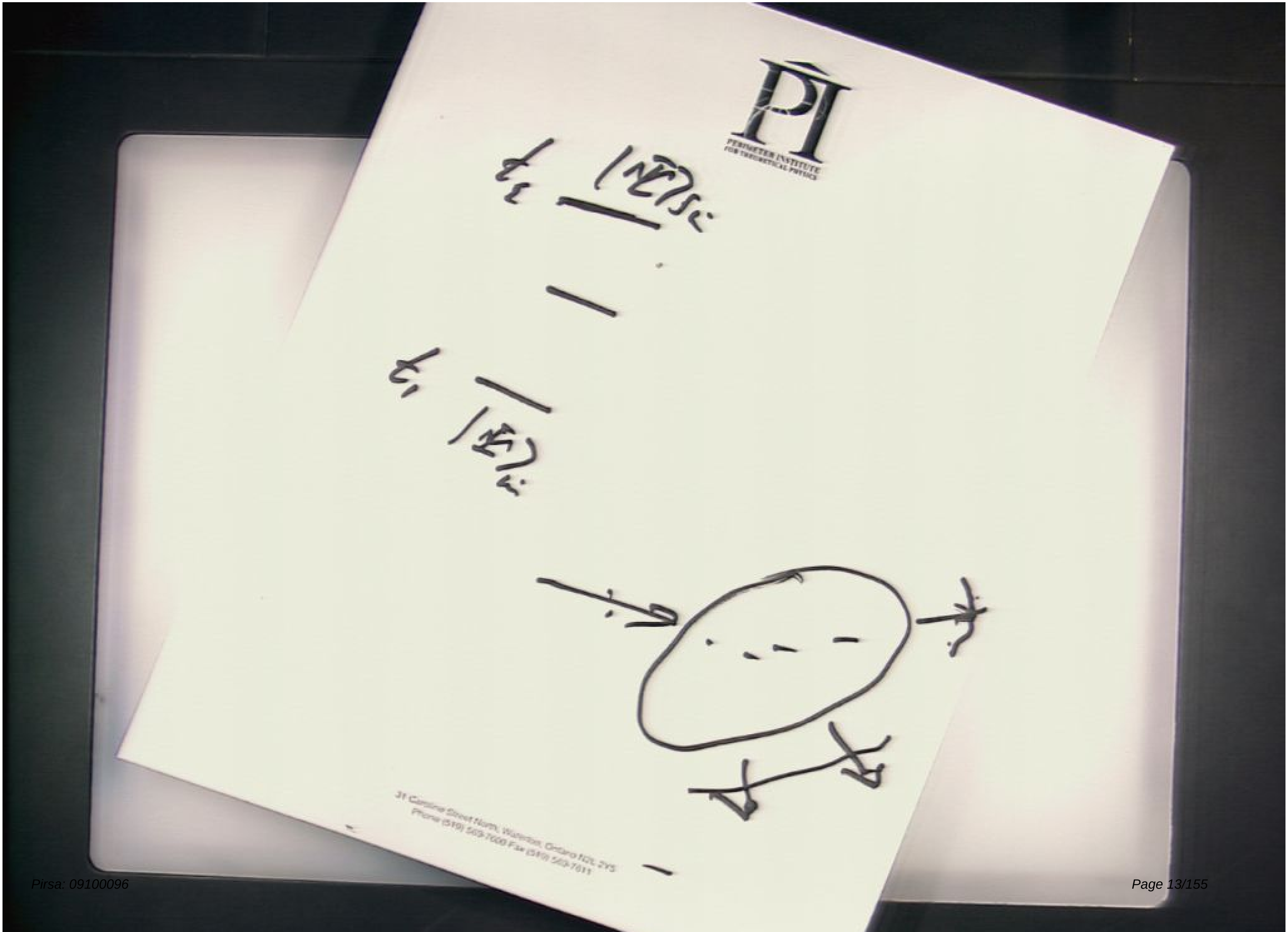

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t_1 ——————

$|\Sigma_1)$

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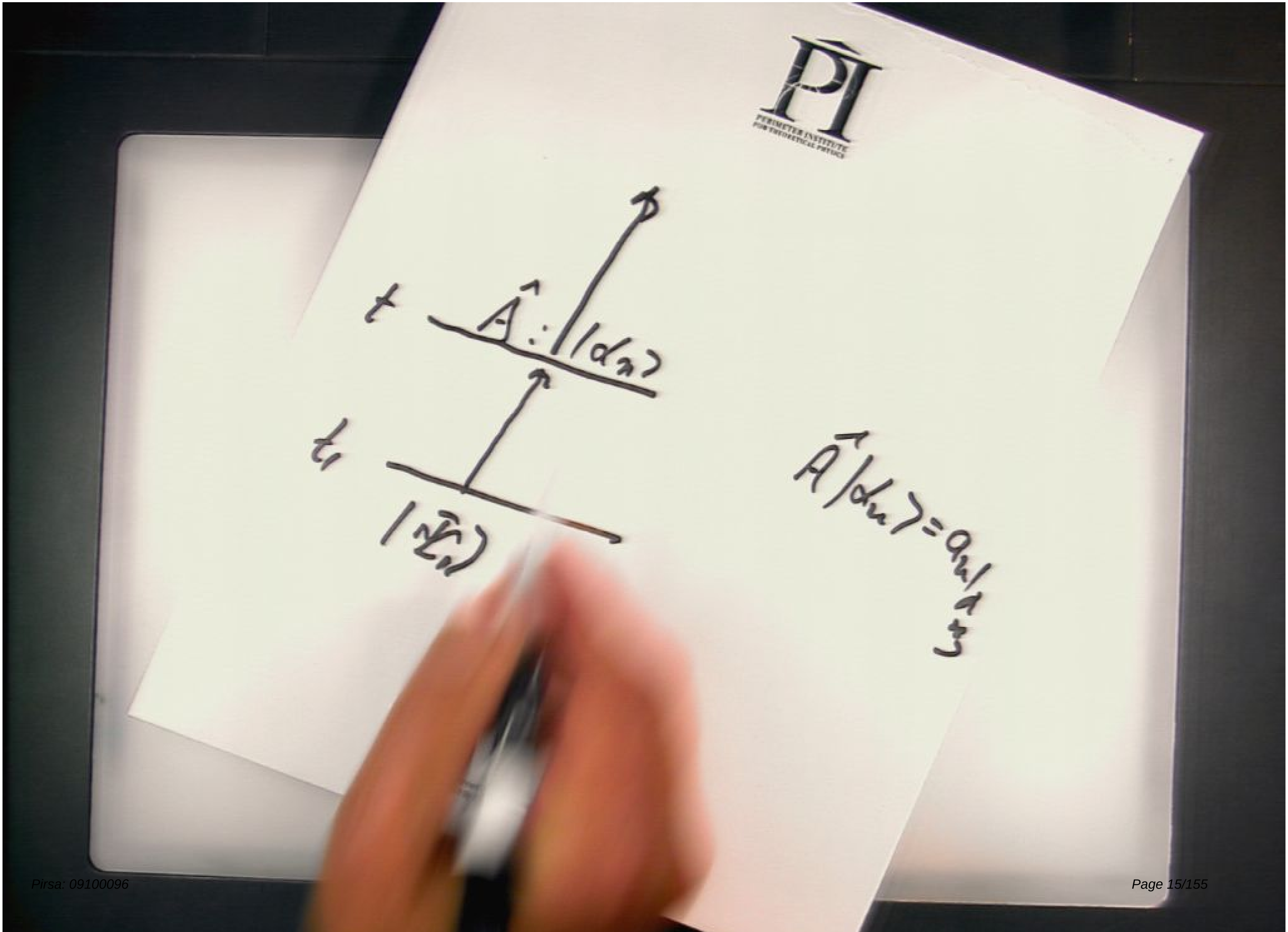
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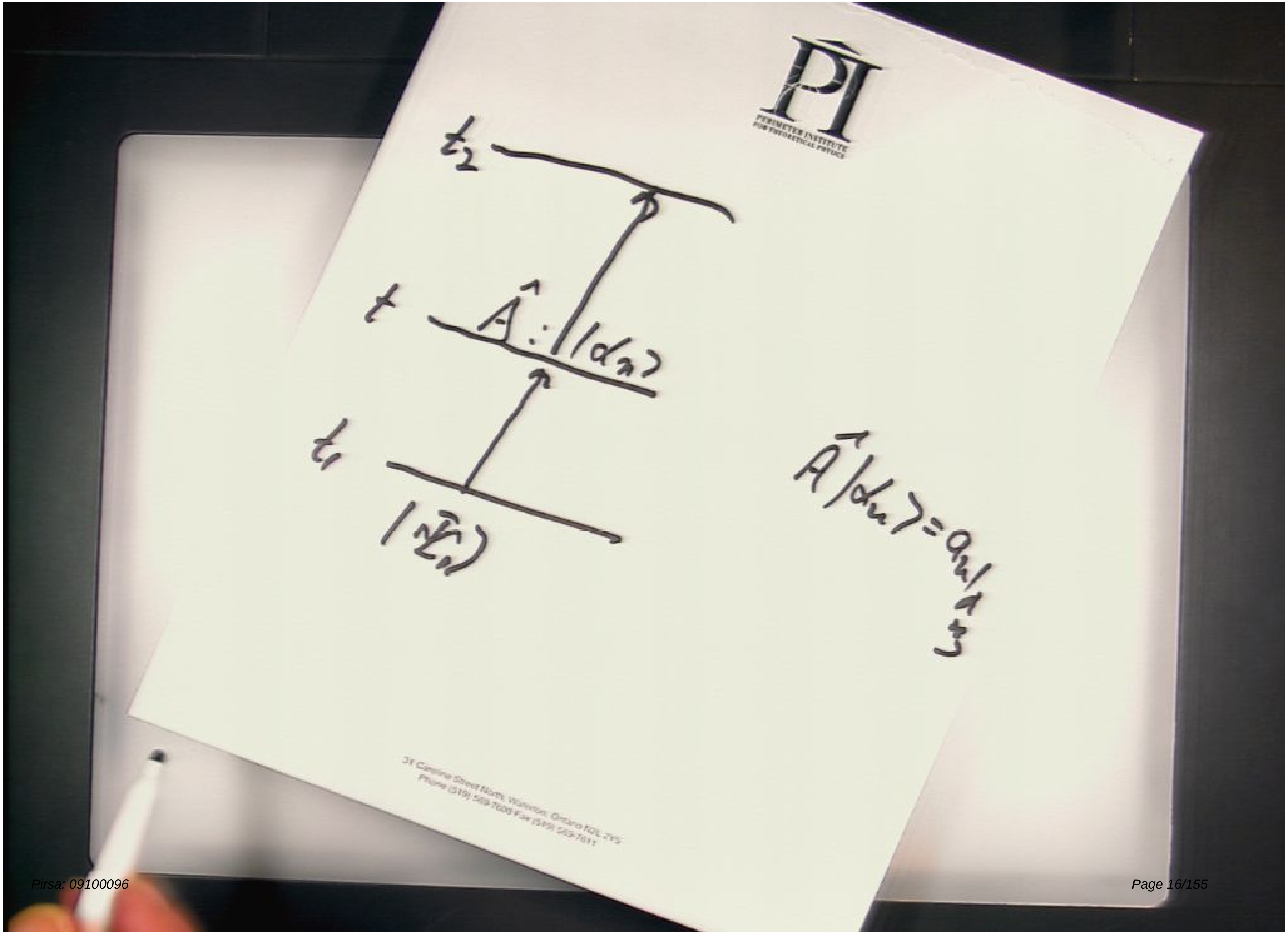
t
$\hat{A}$: $|\alpha_n\rangle$
t_1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t_2
t
Â : |α_n⟩
t_1
Â|α_n⟩ = a_n|α_n⟩
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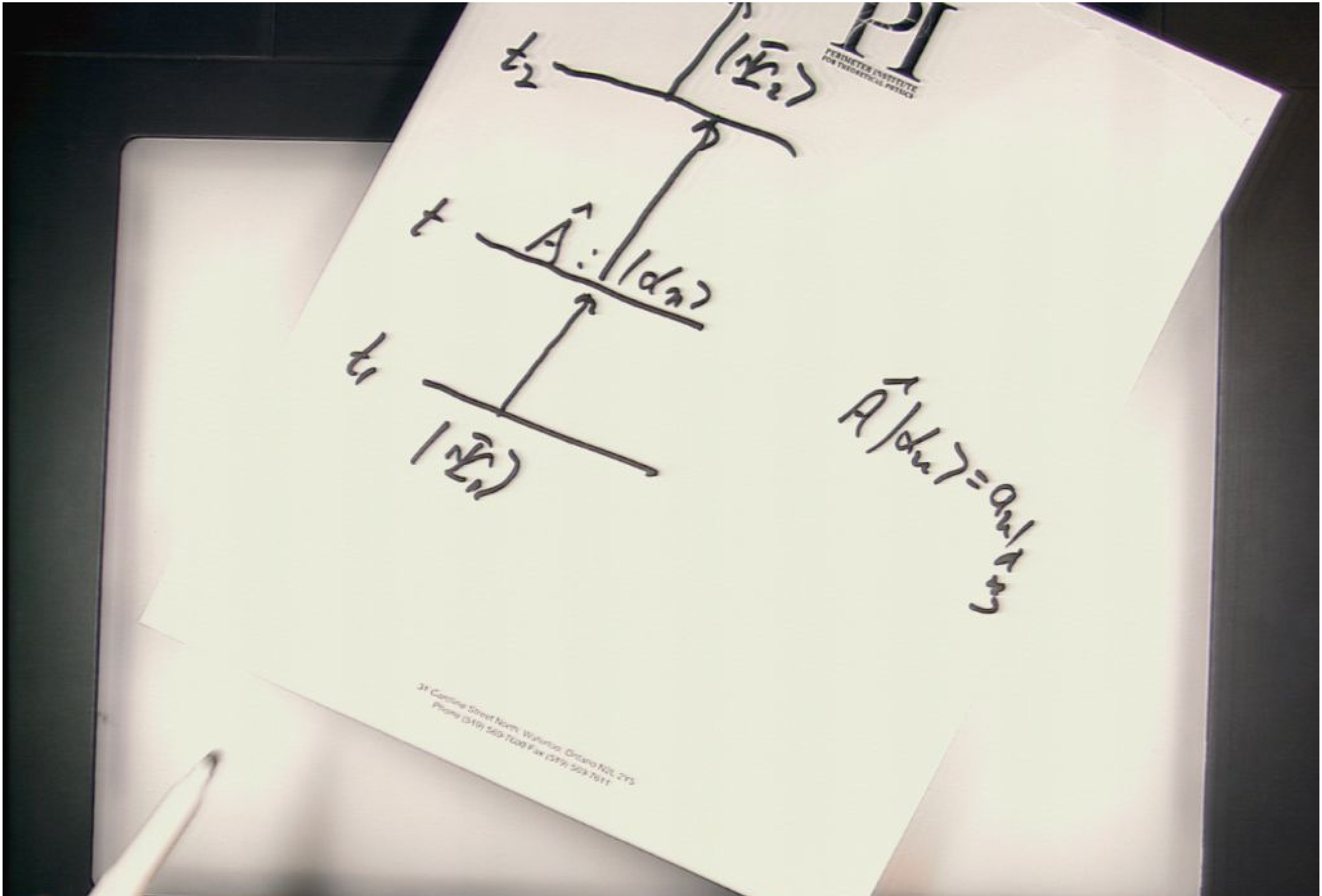

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t_2
$|\Sigma_2\rangle$
t
$\hat{A}: |\alpha_n\rangle$
t_1
$|\Sigma_1\rangle$
$\hat{A}|\alpha_n\rangle = a_n|\alpha_n\rangle$
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$U_{t t_i} |\Psi_{t_i}\rangle$

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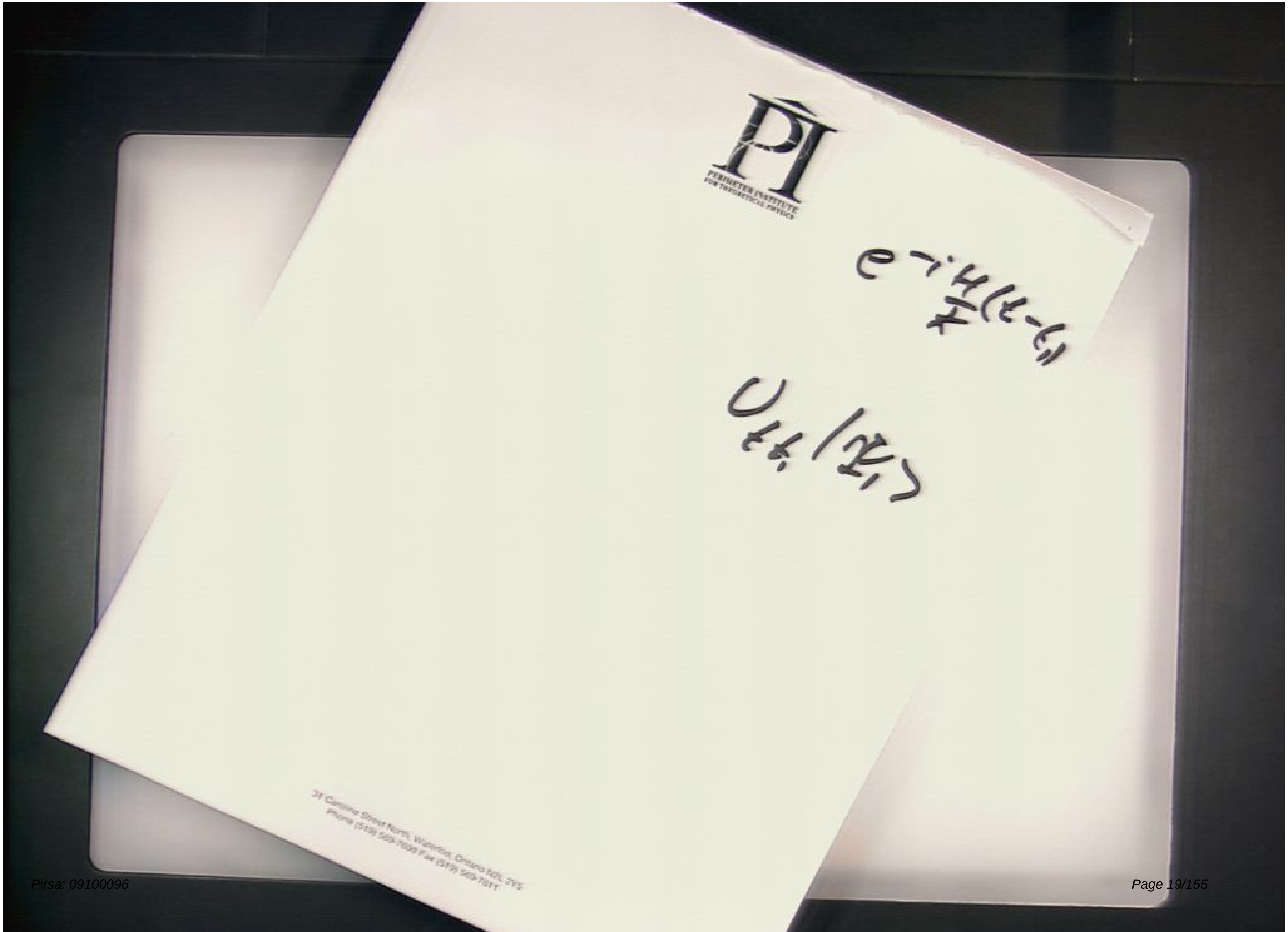
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$$e^{-iH(t-t')/\hbar}$$

$$|d_m\rangle\langle d_m|U_{tt'}|\Psi_i\rangle$$

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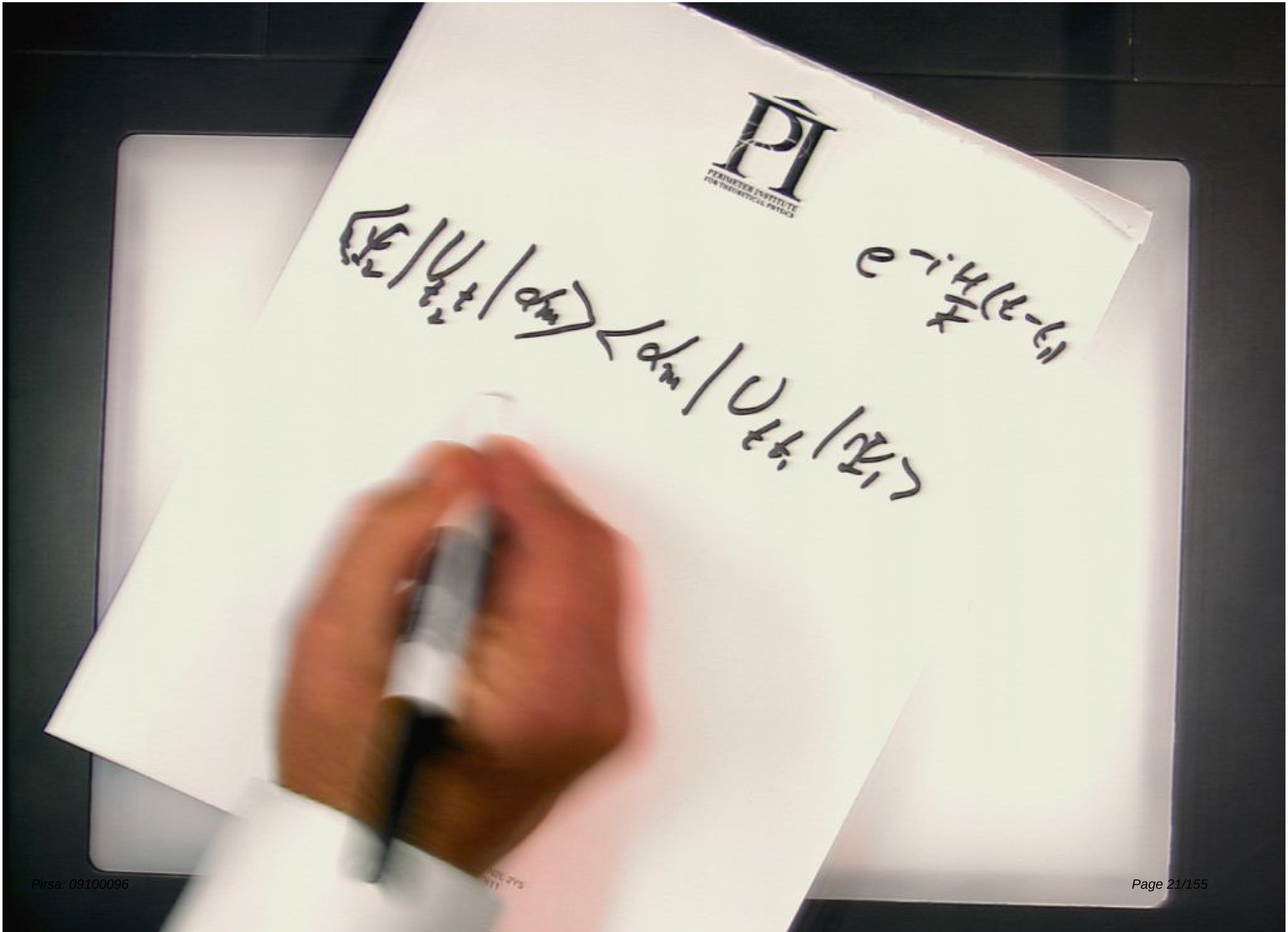
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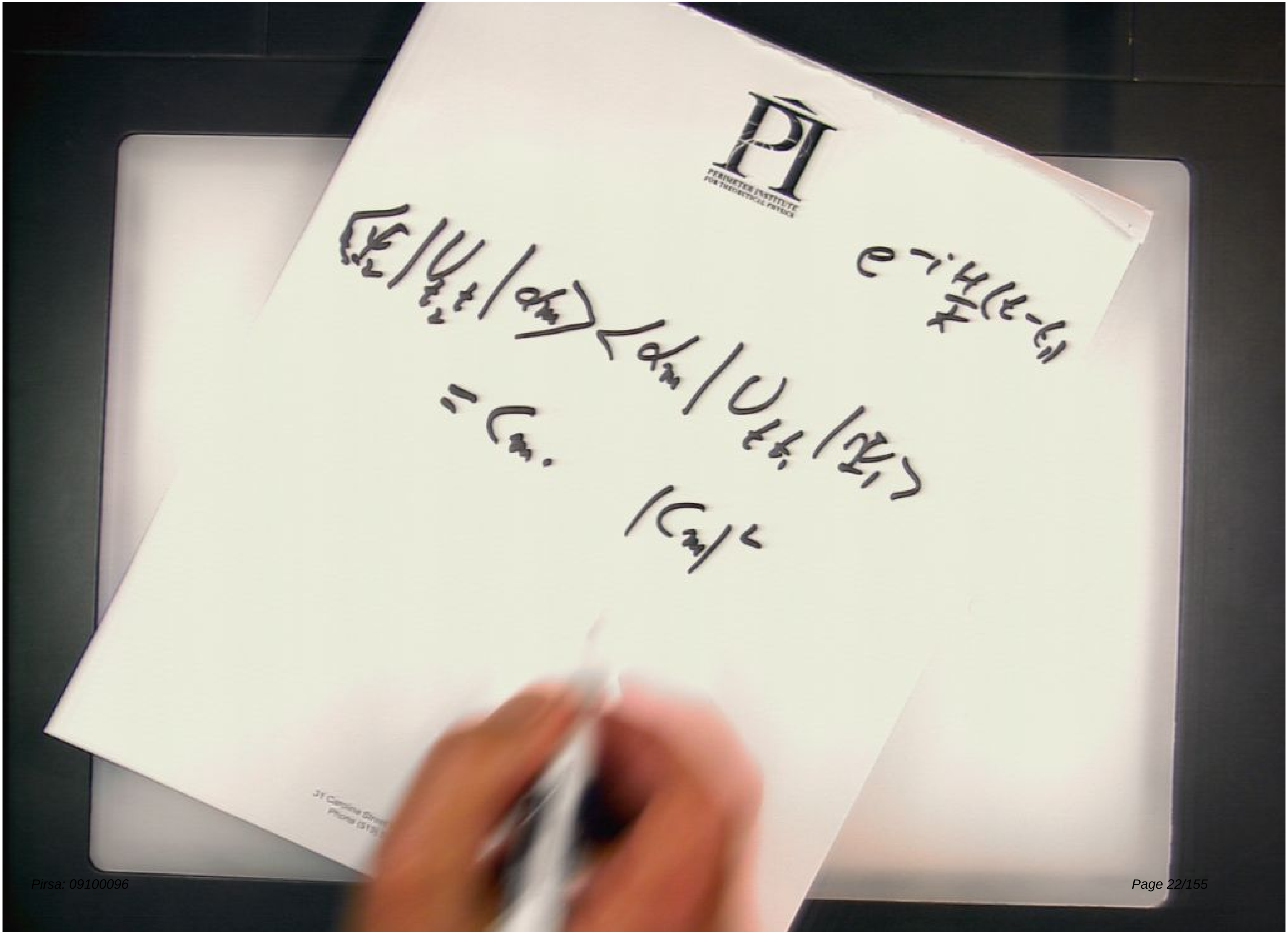
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⟨ψ₂|U_{t₂t}|α_m⟩⟨α_m|U_{tt₁}|ψ₁⟩
= c_m.
|c_m|²
e^{-iH(t-t₁)/ħ}

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$$\langle \Psi_2 | U_{t_2 t} | \alpha_m \rangle \langle \alpha_m | U_{t t_1} | \Psi_1 \rangle$$

$$e^{-\frac{i}{\hbar} H (t - t_1)}$$

$$= c_m \cdot$$

$$|c_m|^2$$

$$\Pr(\alpha_m)$$

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$$e^{-\frac{i H (t - t_1)}{\hbar}}$$

$$\langle \Psi_2 | U_{t_2 t} | \alpha_m \rangle \langle \alpha_m | U_{t t_1} | \Psi_1 \rangle$$

$$= \langle c_m \ldots \qquad |c_m|^2 \qquad \Pr(\alpha_m)$$

$$\langle U^{\dagger}_{t_2 t} \Psi_2 | \alpha_m \rangle \langle \alpha_m | U_{t t_1} | \Psi_1 \rangle = C_m$$


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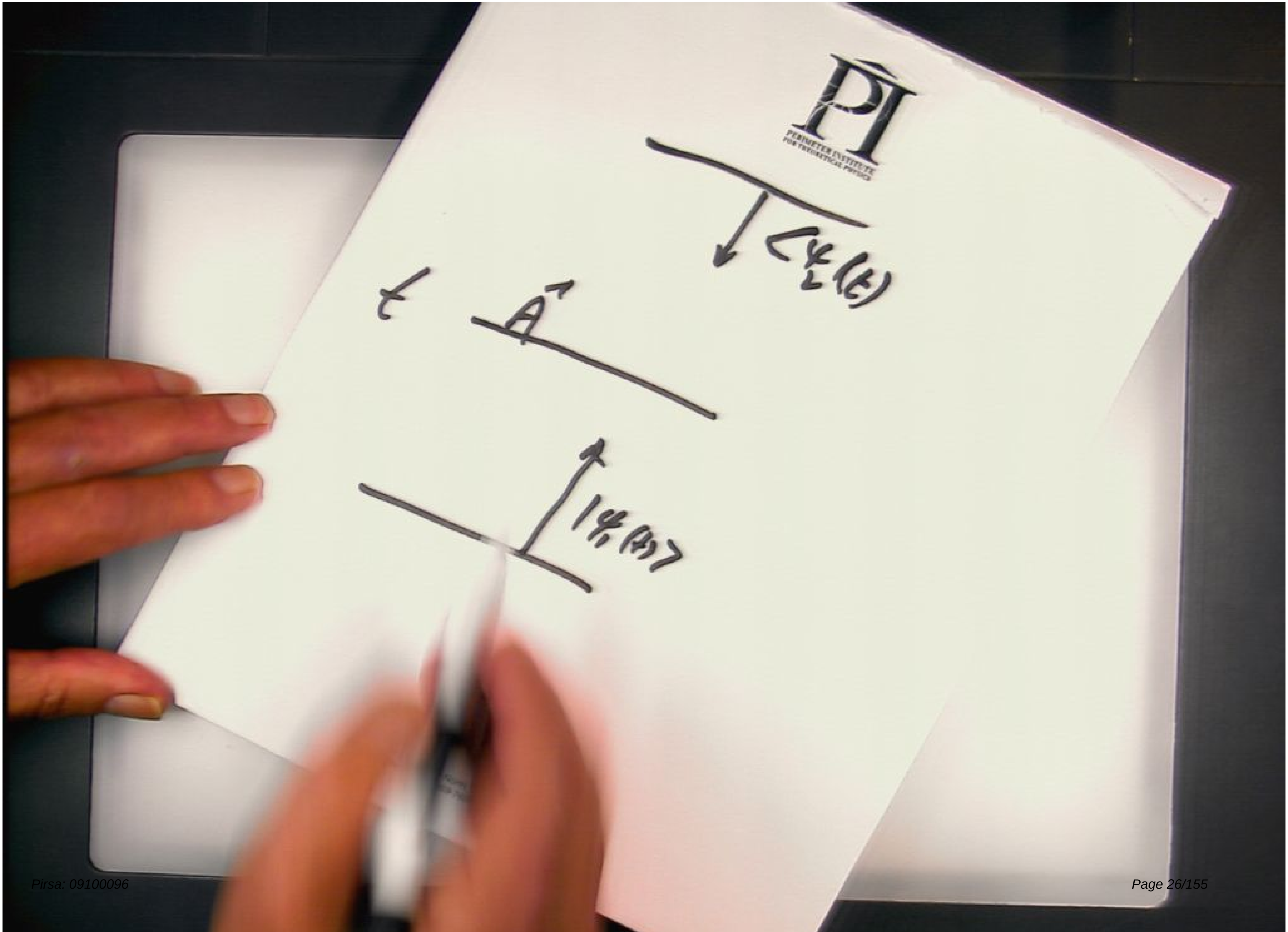
⟨ψ2(t)
t
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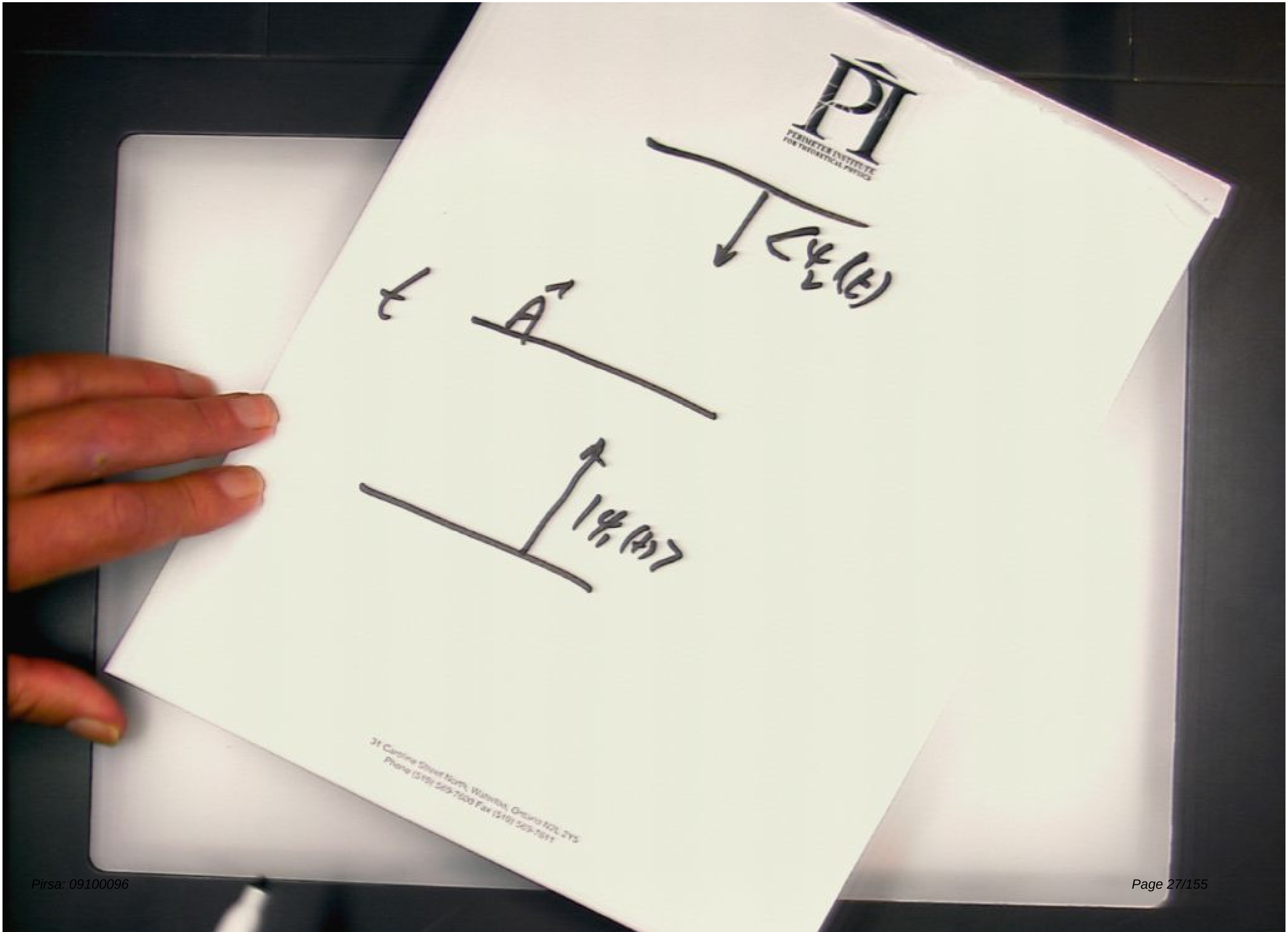
|ψ1(t)⟩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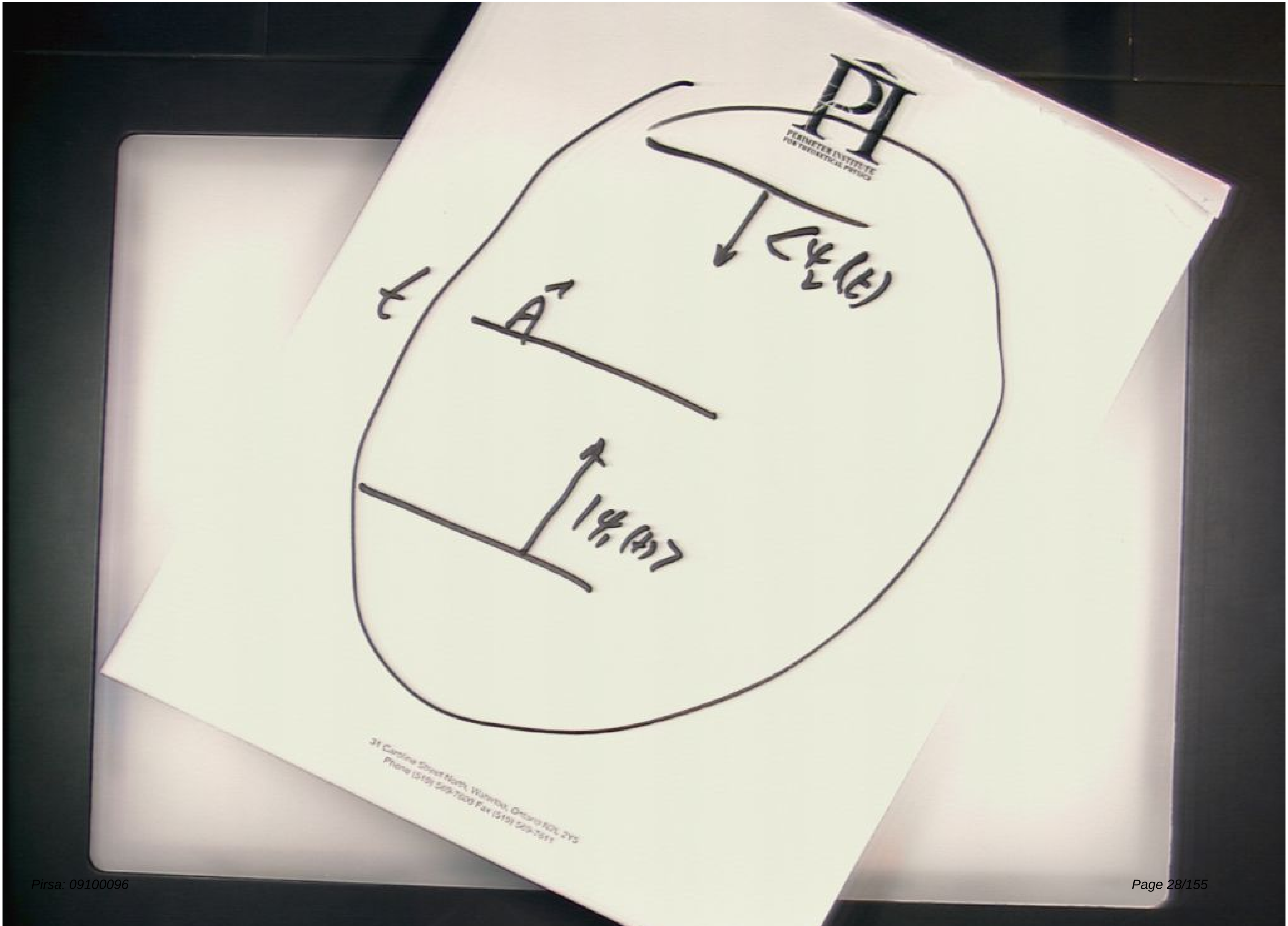
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⟨ψ_f(t)|
t
Â
|ψ_i(t)⟩
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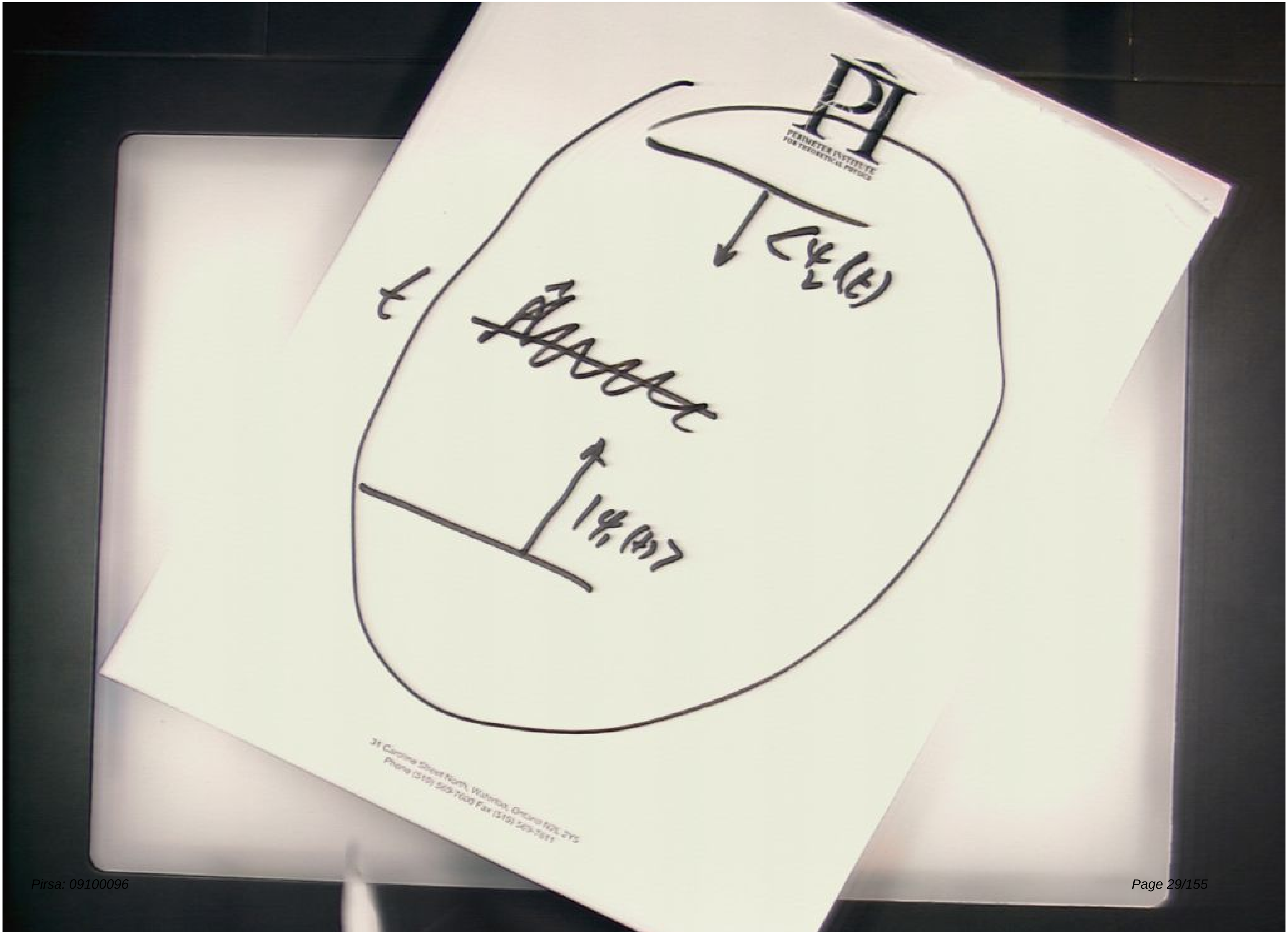
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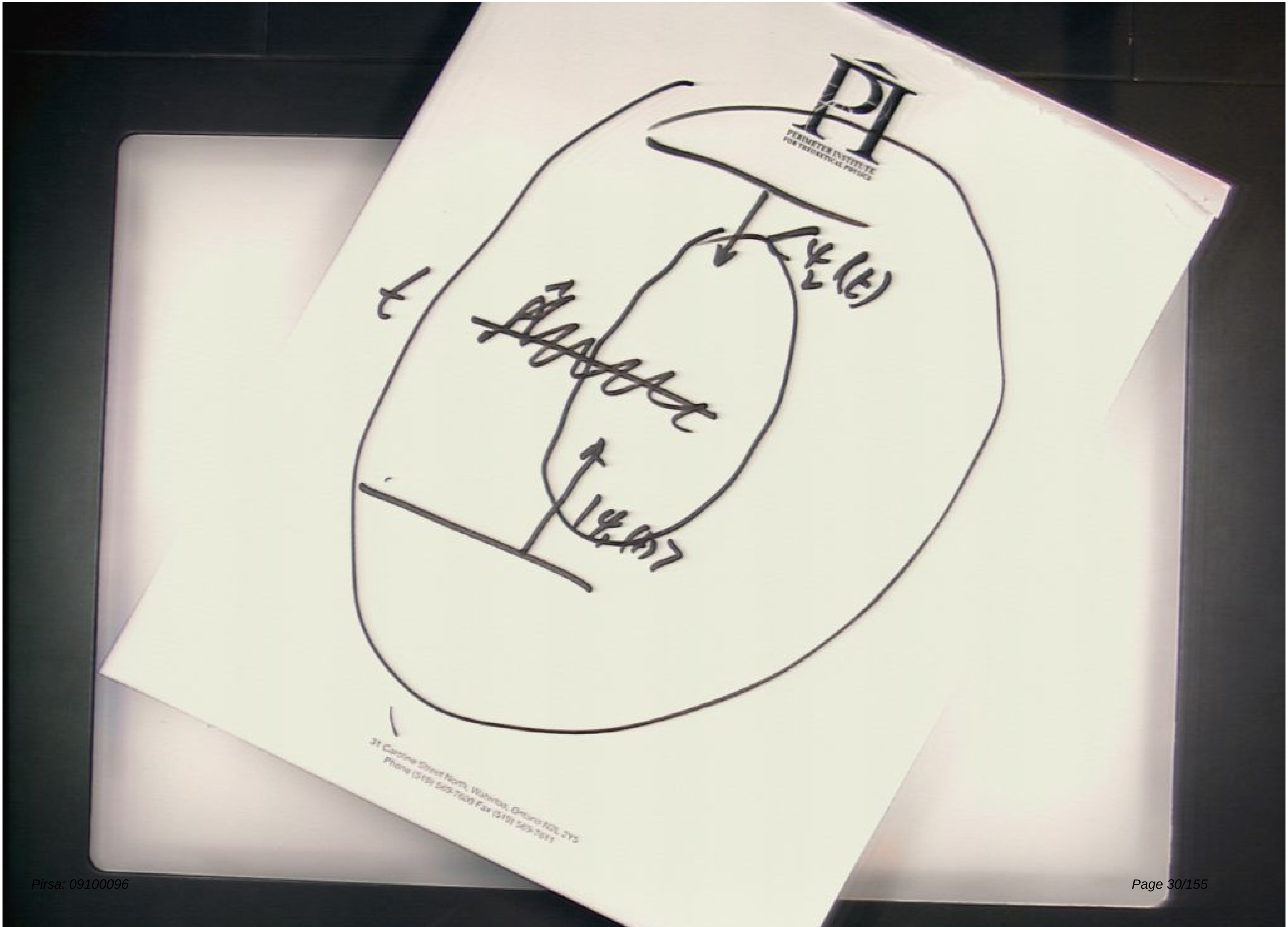
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$$e^{-i\frac{H}{\hbar}(t-t_1)}$$

$$\langle \Psi_2 | U_{t_2 t} | \alpha_m \rangle \langle \alpha_m | U_{t t_1} | \Psi_1 \rangle$$

$$= C_m . \qquad |C_m|^2$$

$$P_r(\alpha_m)$$

$$\langle U^\dagger_{t_2 t} \Psi_2 | \alpha_m \rangle \langle \alpha_m | U_{t t_1} | \Psi_1 \rangle =$$

$$\underline{C_m}$$

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$$e^{-i\frac{H}{\hbar}(t-t_1)}$$

$$\langle \Psi_2 | U_{t_2 t} | \alpha_m \rangle \langle \alpha_m | U_{t t_1} | \Psi_1 \rangle$$

$$= C_m . \qquad |C_m|^2$$

$$P_r(\alpha_m)$$

$$\langle U^{+}_{t_2 t} \Psi_2 | \alpha_m \rangle \langle \alpha_m | U_{t t_1} | \Psi_1 \rangle =$$

$$\underline{C_m}$$

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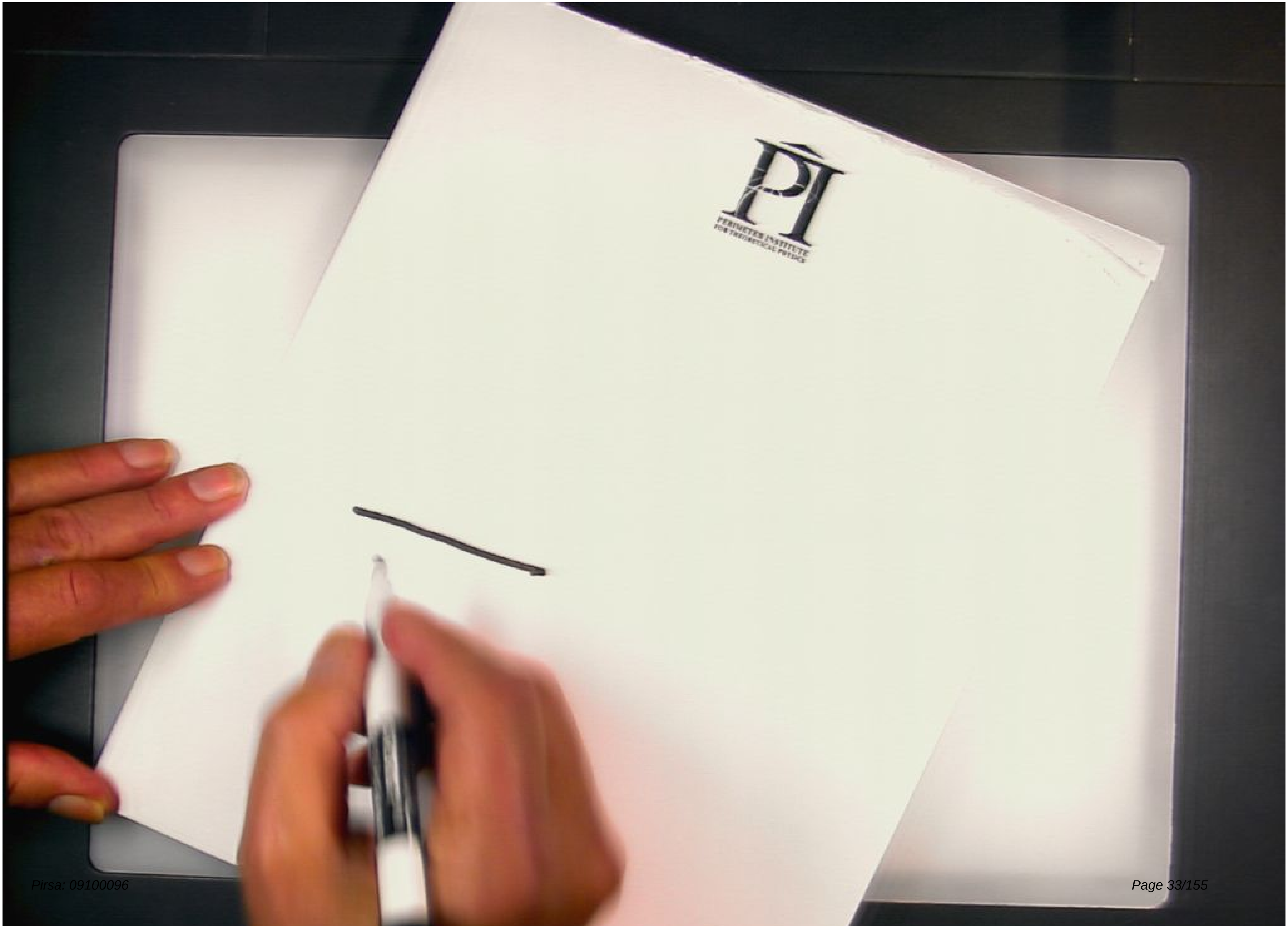
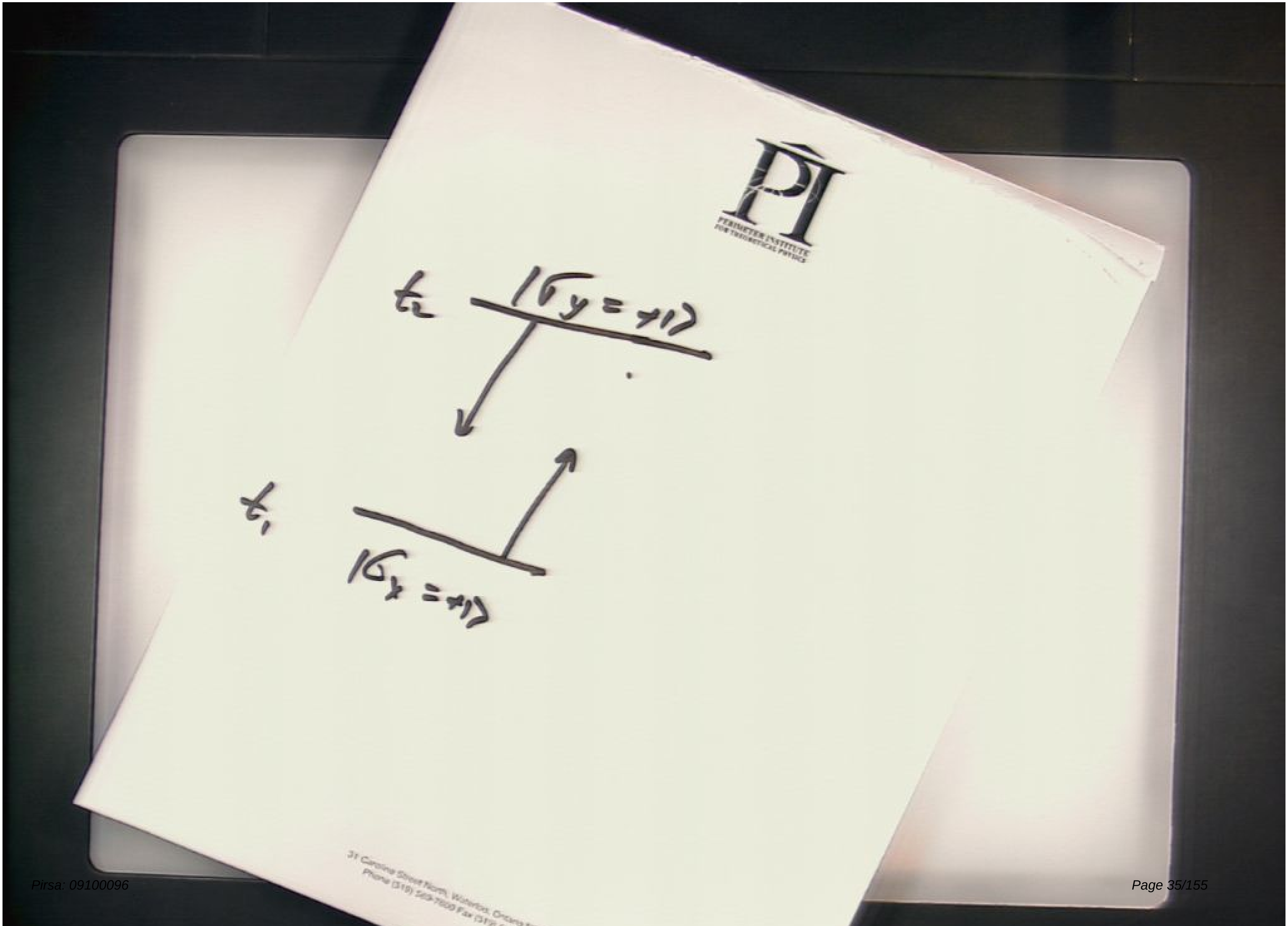


t_2 $|\sigma_y = +1\rangle$

t_1 $|\sigma_x = +1\rangle$



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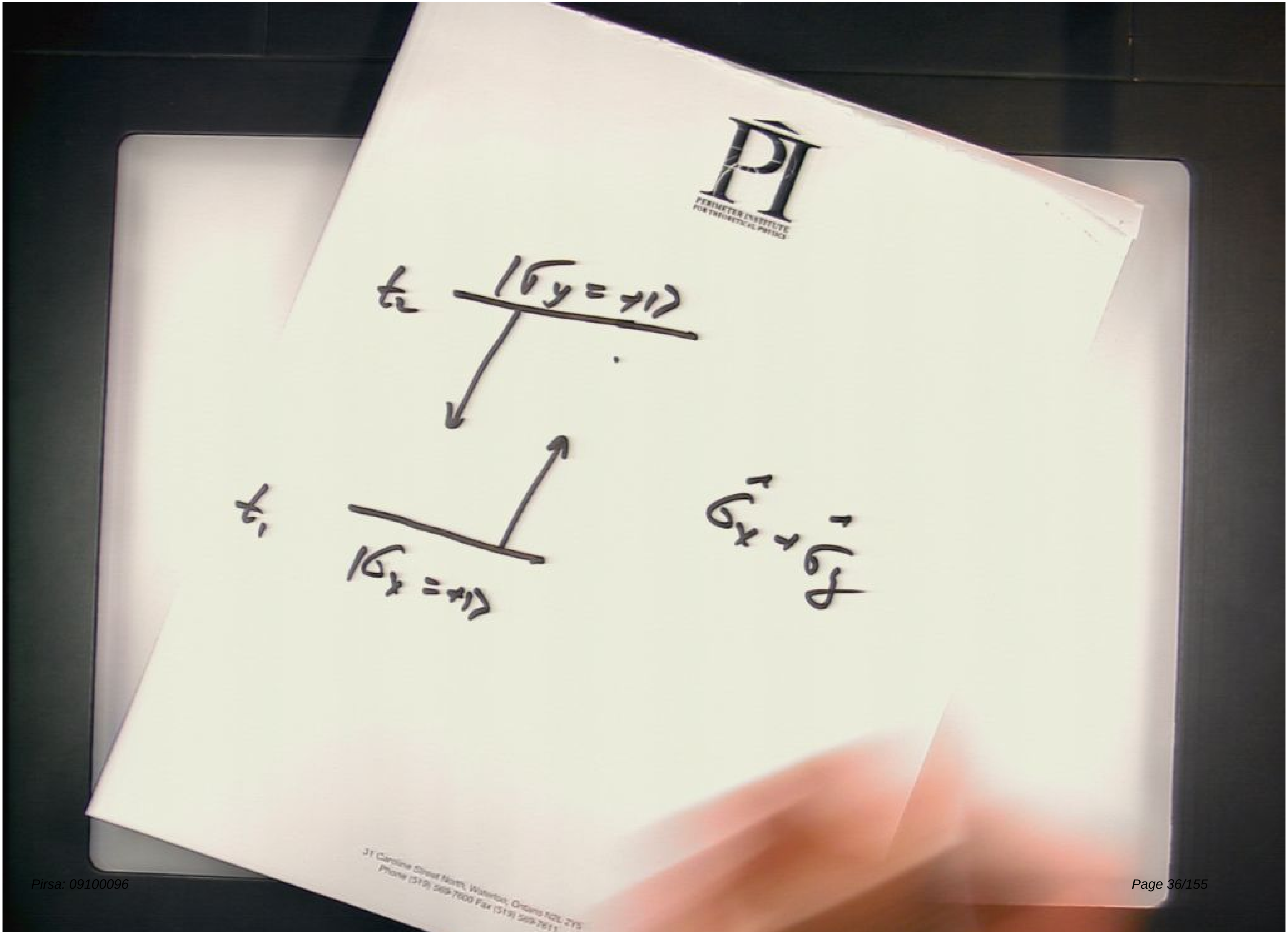
t_2
$|\sigma_y = +1\rangle$
t_1
$|\sigma_x = +1\rangle$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t_2
$|\sigma_y = +1\rangle$
t_1
$|\sigma_x = +1\rangle$
$\vec{\sigma}_x + \vec{\sigma}_y$
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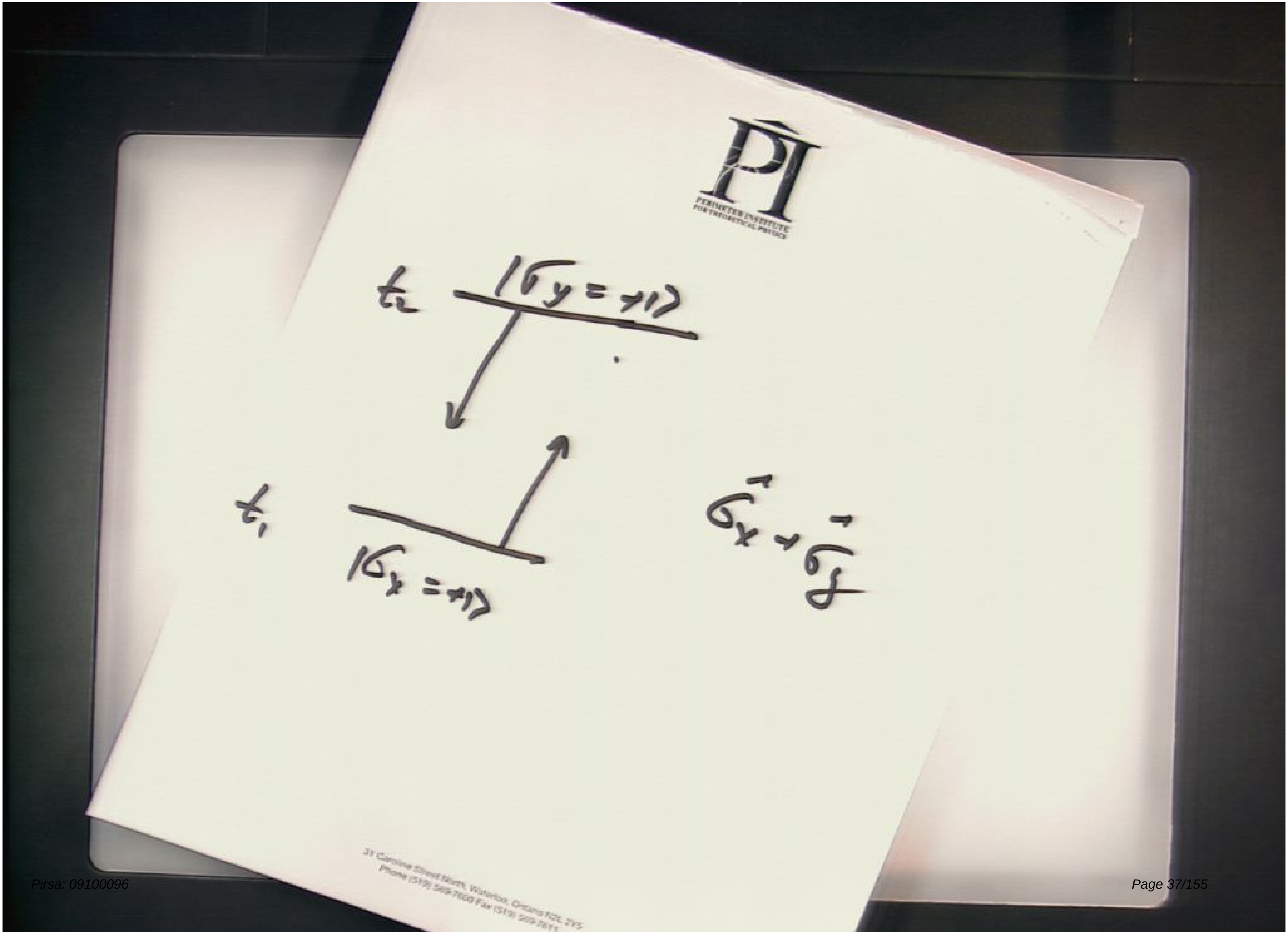
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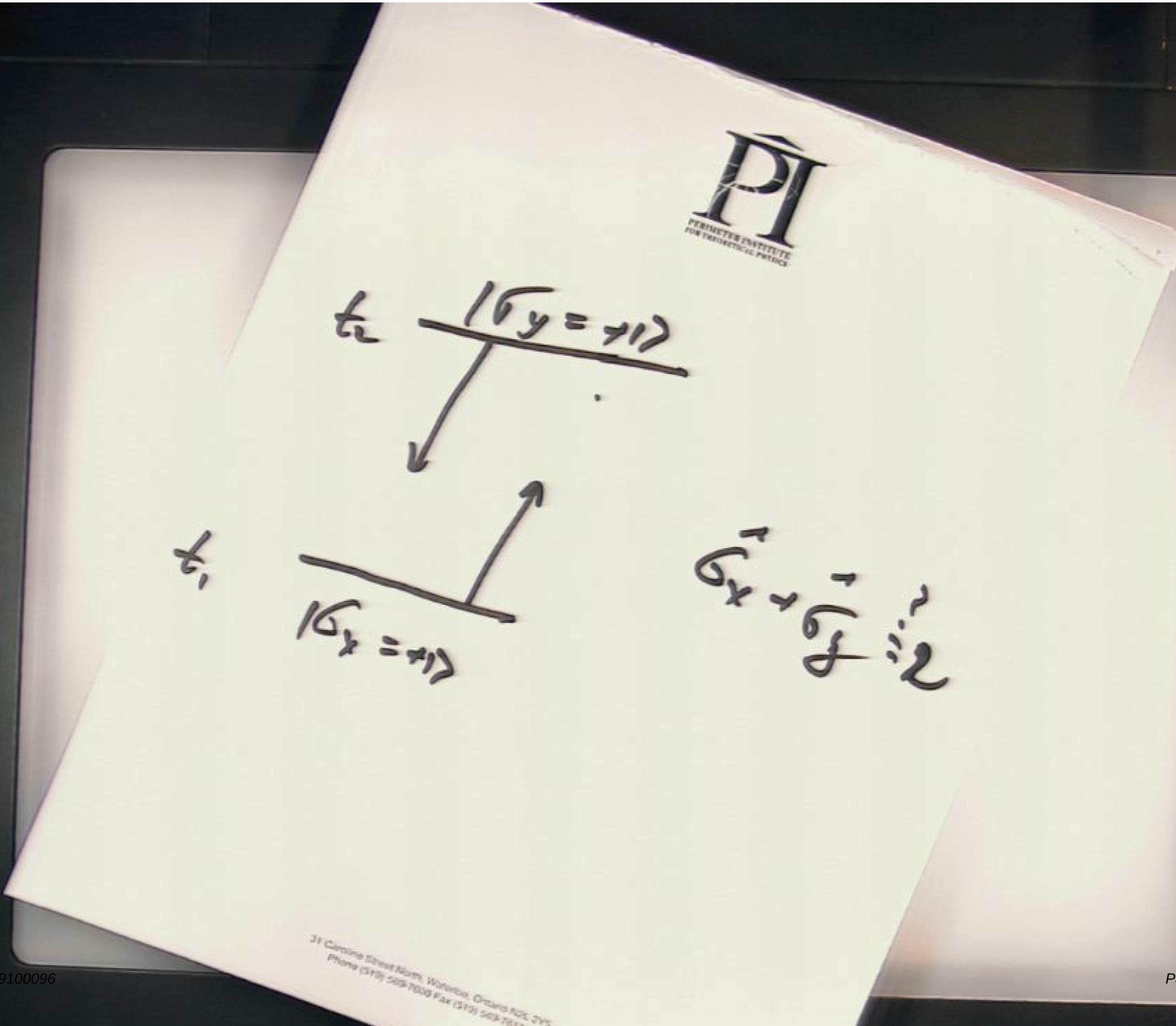
t₂
|σ_y = +1⟩
t₁
|σ_x = +1⟩
σ_x + σ_y



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t2
|σy = +1⟩
t1
|σx = +1⟩
σx → σy ?
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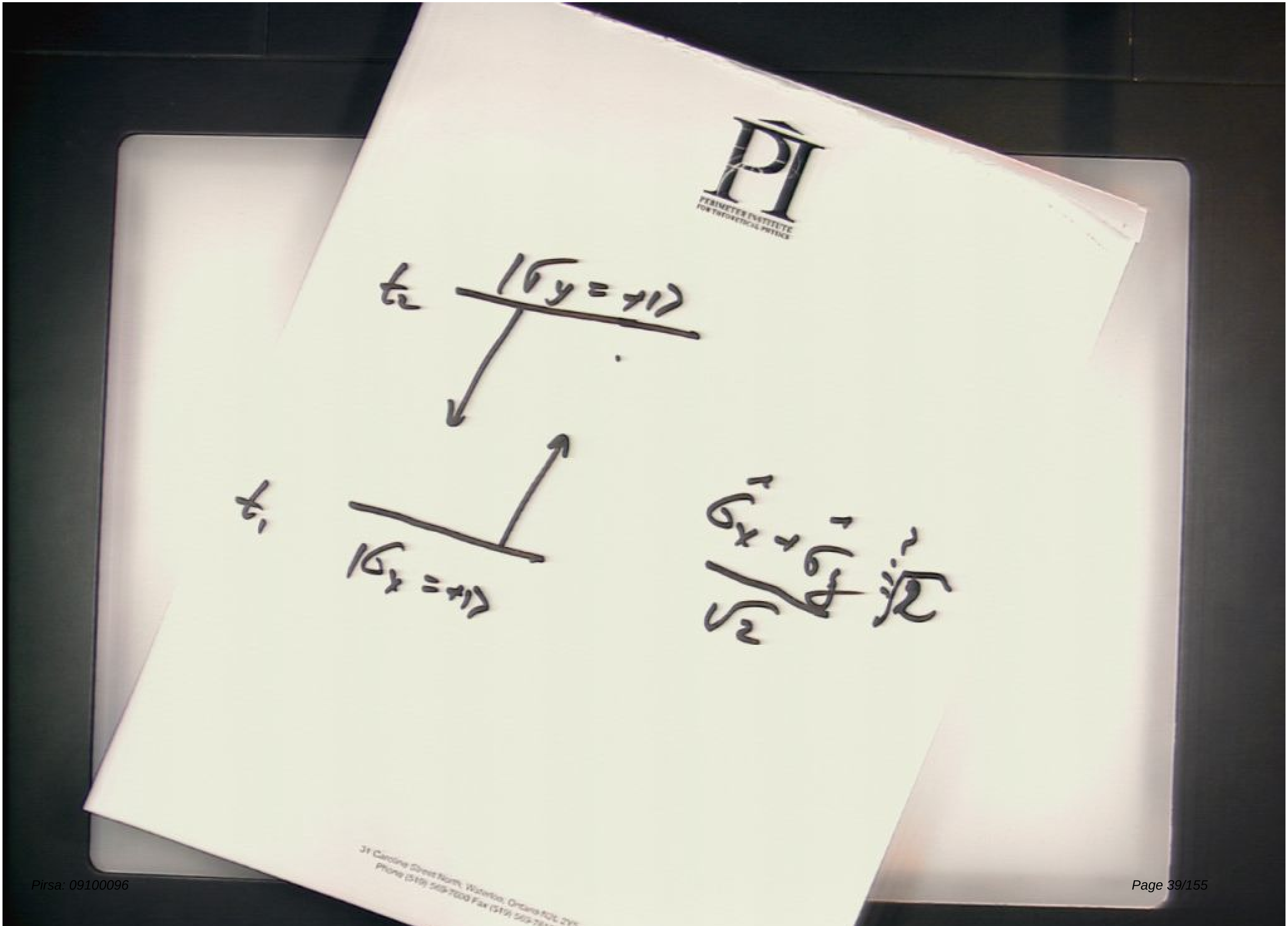
t_2
$|\sigma_y = +1\rangle$
t_1
$|\sigma_x = +1\rangle$
$\frac{\vec{\sigma}_x + \vec{\sigma}_y}{\sqrt{2}}$
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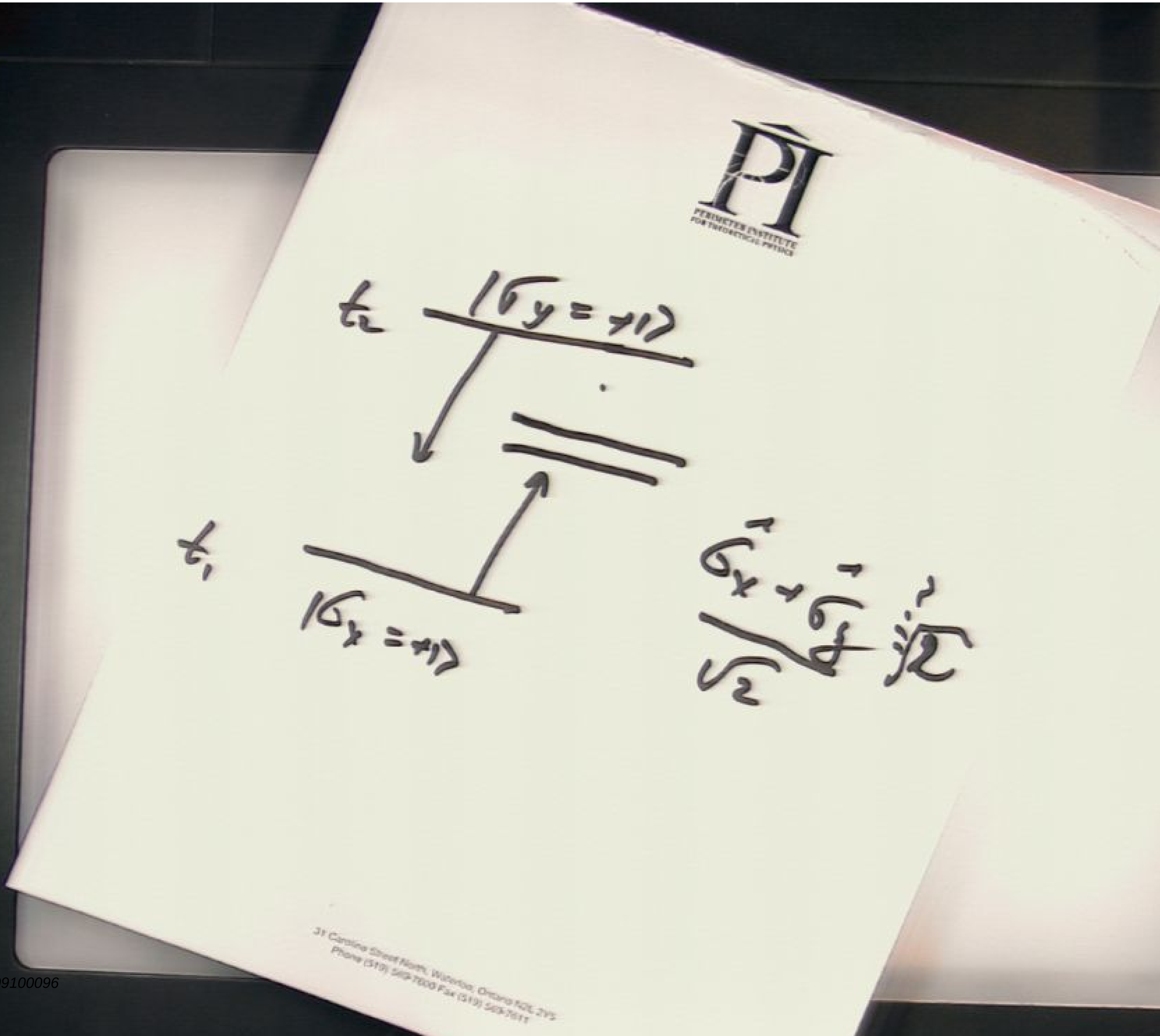
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t_2
$|\sigma_y = +1\rangle$
t_1
$|\sigma_x = +1\rangle$
$\frac{\vec{\sigma}_x + \vec{\sigma}_y}{\sqrt{2}}$
$\sqrt{2}$
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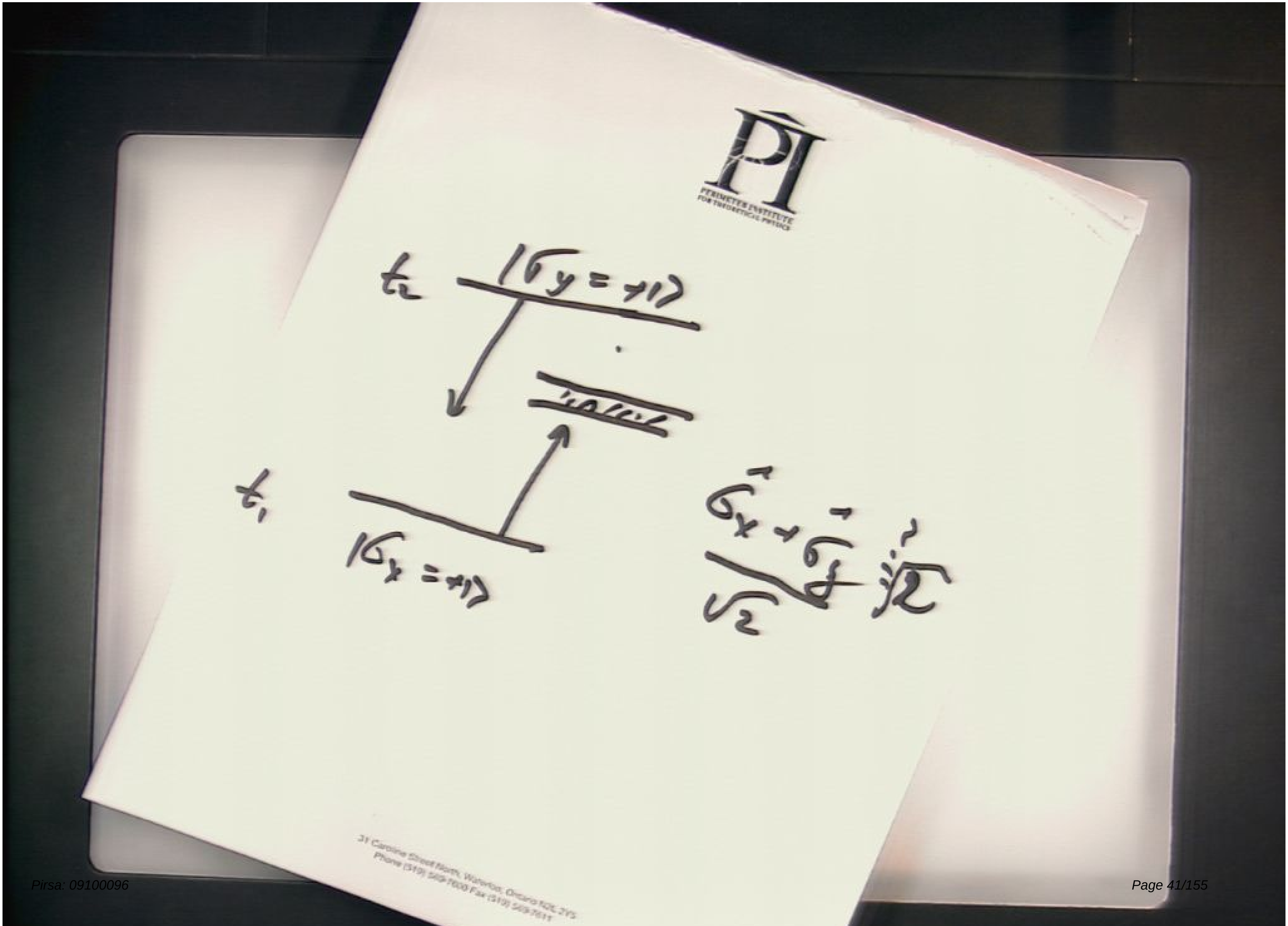
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t2
|σy = +1⟩
t1
|σx = +1⟩
(σx + σy)/√2
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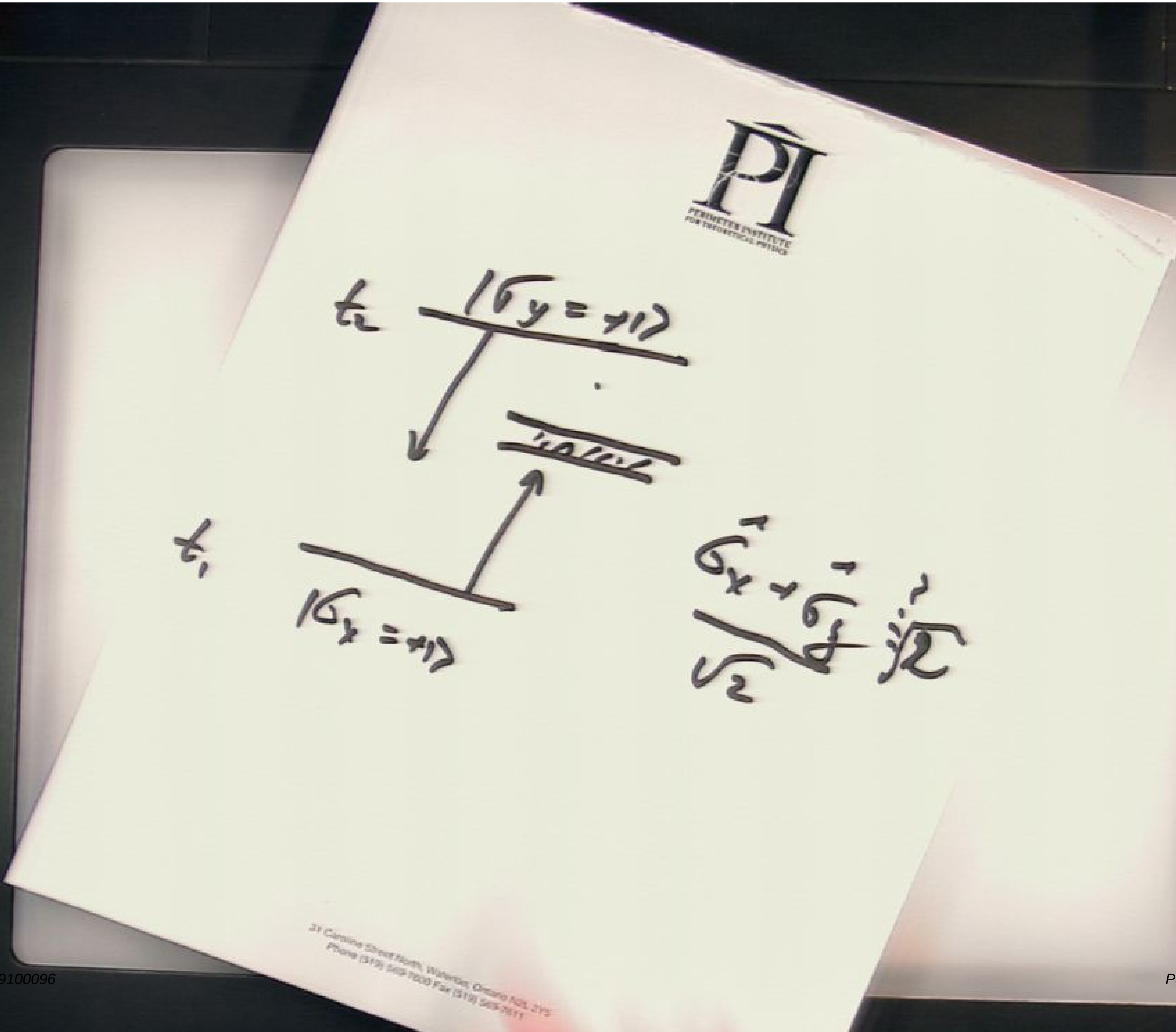
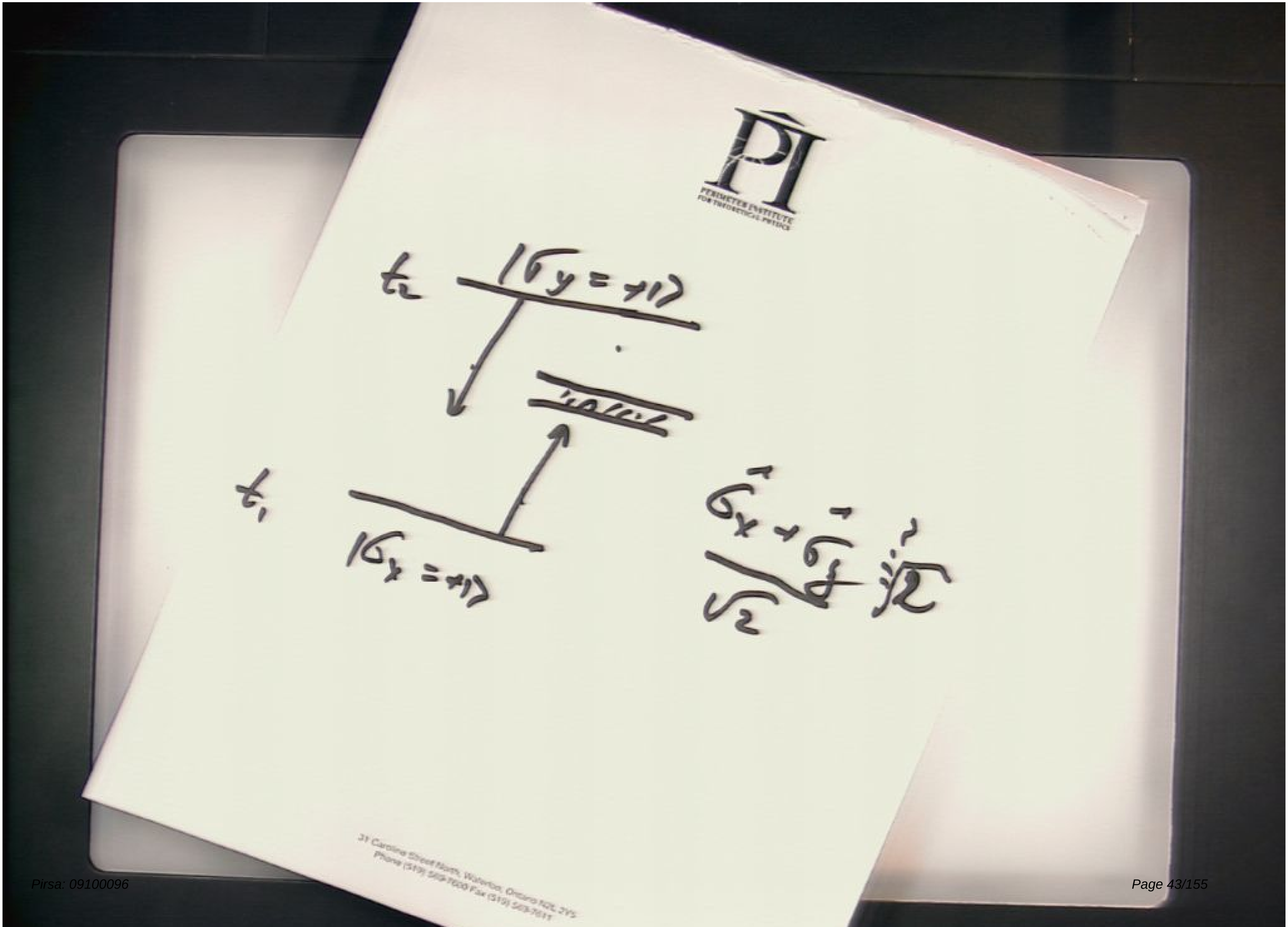

t_2
$|\sigma_y = +1\rangle$
t_1
$|\sigma_x = +1\rangle$
$\frac{\vec{\sigma}_x + \vec{\sigma}_y}{\sqrt{2}}$
$\sqrt{2}$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t2
|σy = +1⟩
t1
|σx = +1⟩
σx + σy
√2
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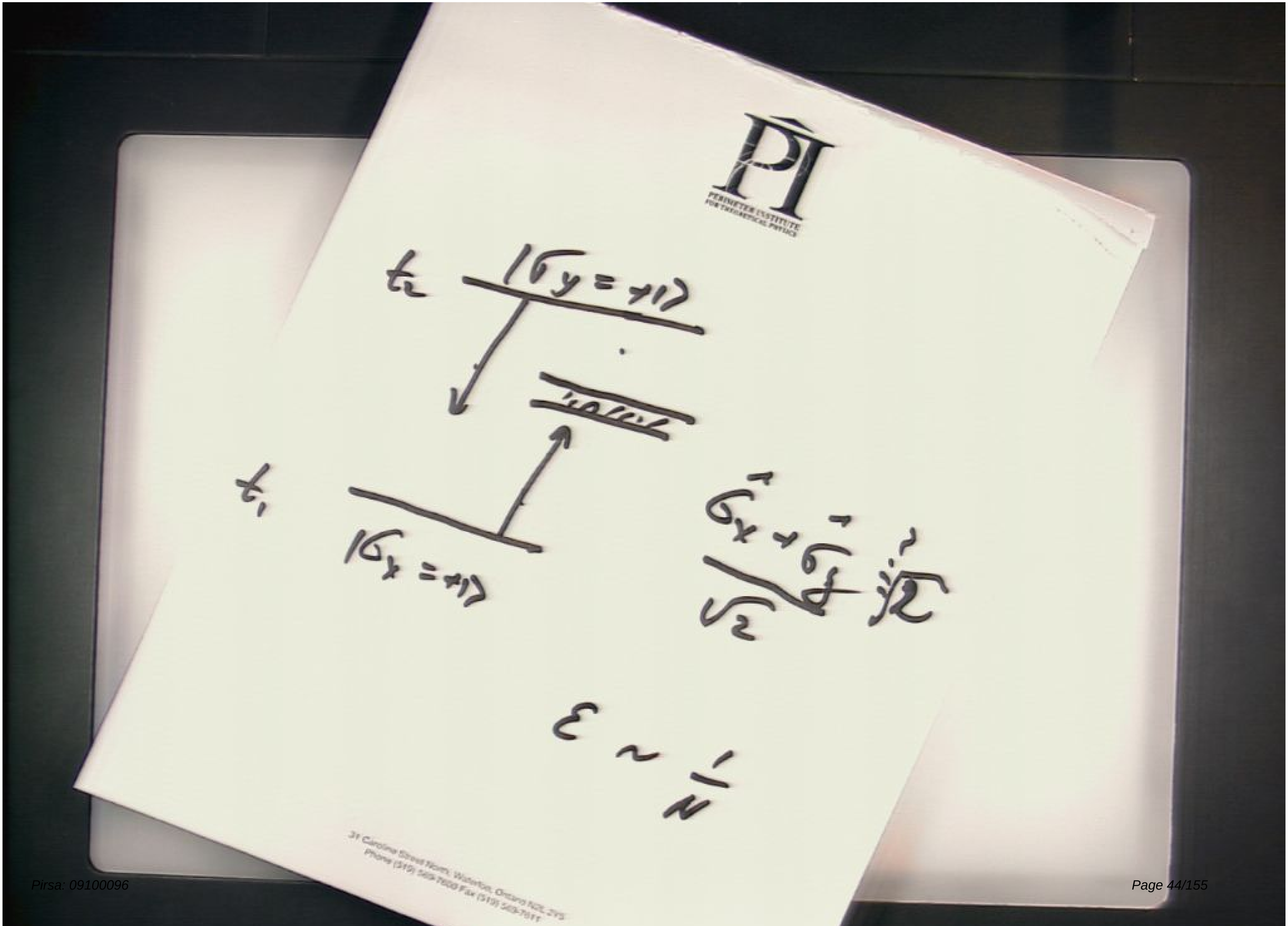
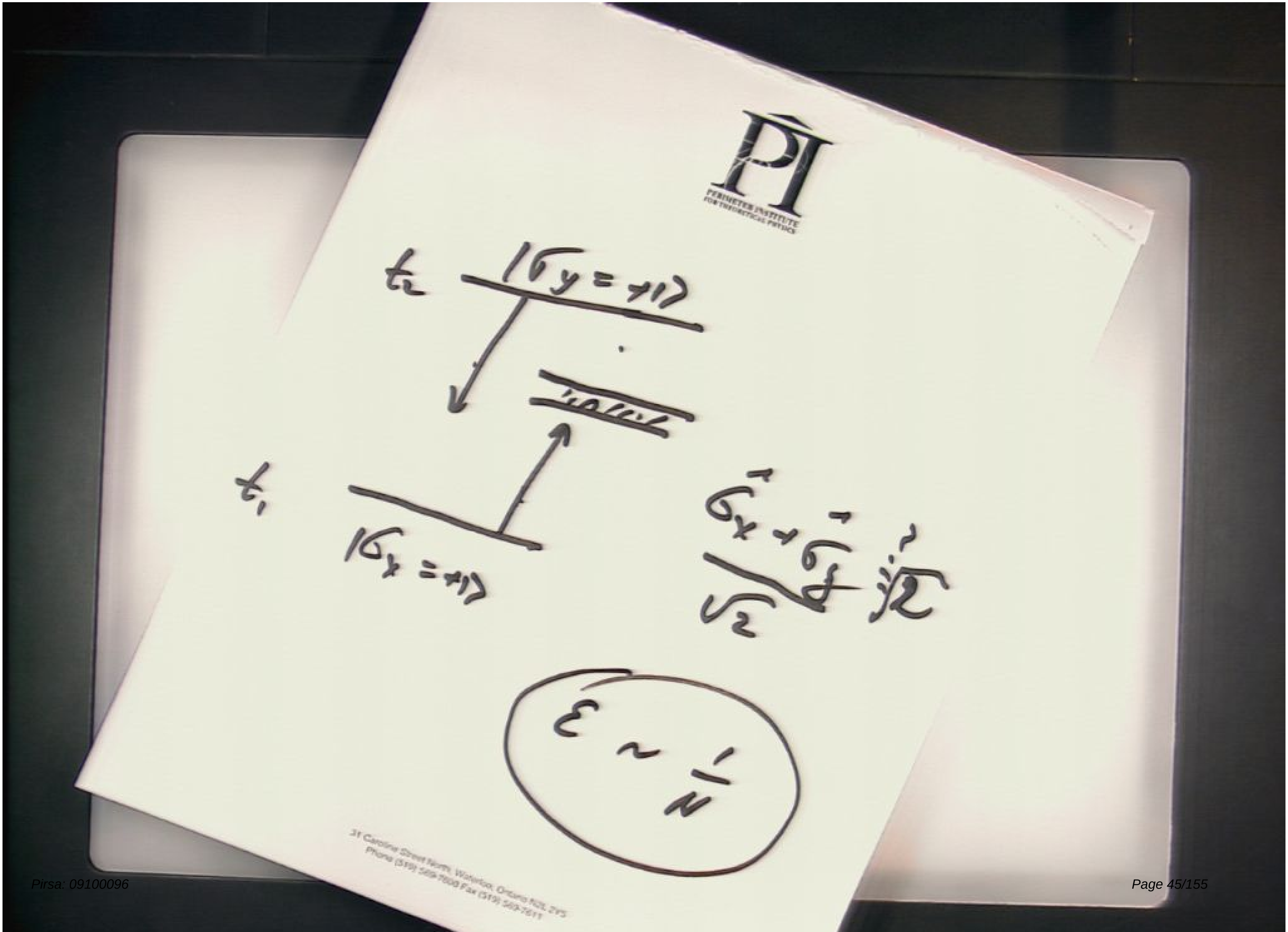

t2
|σy = +1⟩
t1
|σx = +1⟩
ε ~ 1/N


t2
|σy = +1⟩
t1
|σx = +1⟩
ε ~ 1/N
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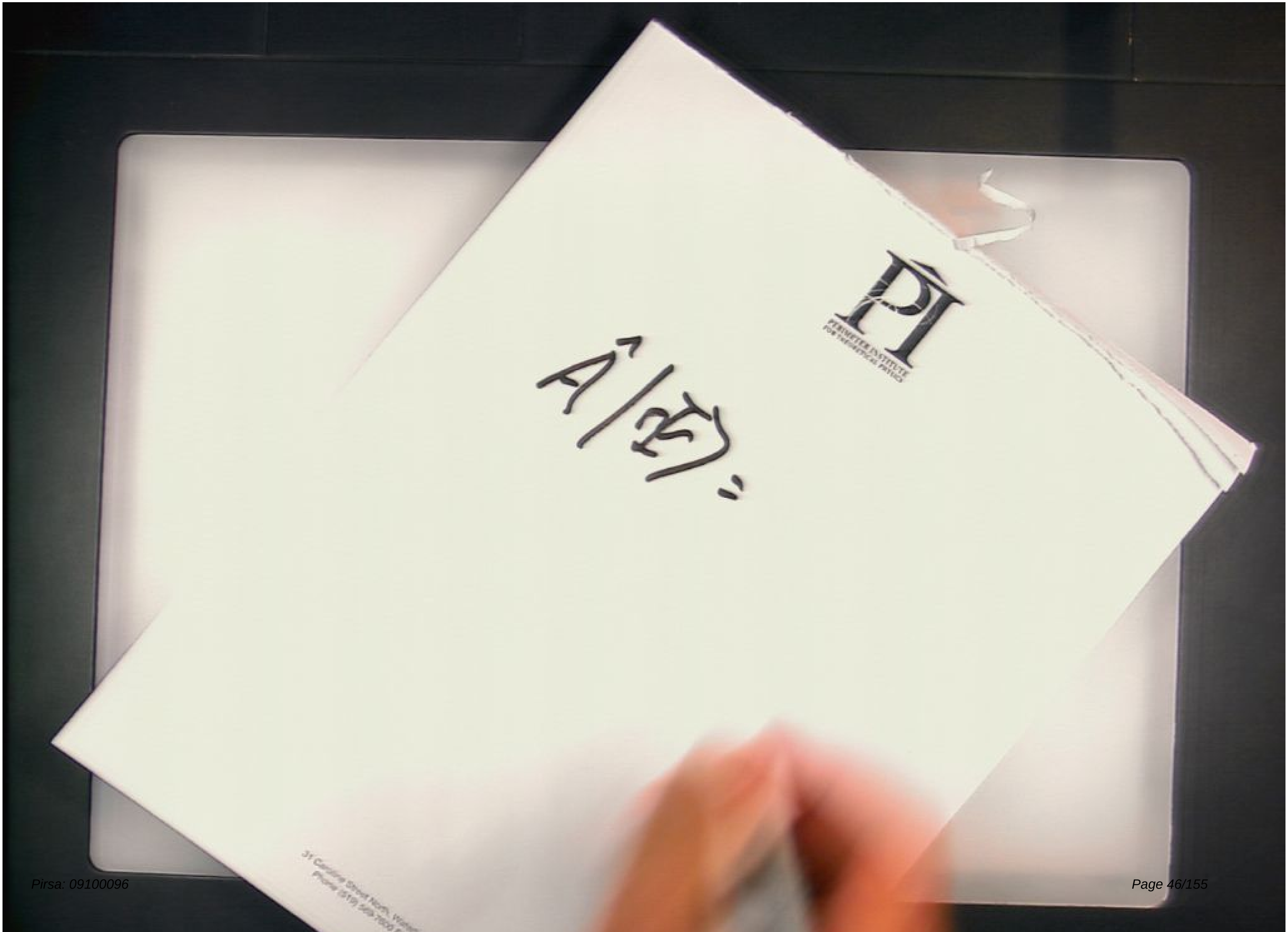
$\hat{A}|E\rangle =$
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$$\hat{A}|\xi\rangle = \xi|\xi\rangle$$

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$$\hat{A}|\Psi\rangle = |\Psi'\rangle = a|\Psi\rangle$$

$$= + \, b|\Psi_\perp\rangle$$

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$$\hat{A}|\xi\rangle = |\xi'\rangle = a|\xi\rangle + b|\xi_\perp\rangle$$

$$\langle\xi|\hat{A}|\xi\rangle = a = \bar{A}$$

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$$\hat{A}|\psi\rangle = |\psi'\rangle = a|\psi\rangle + b|\psi_\perp\rangle$$

$$\langle\psi|\hat{A}|\psi\rangle = a = \bar{A}$$

$$\langle\hat{A}\psi|\hat{A}\psi\rangle = a^2 + b^2$$

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$$\hat{A}|\Psi\rangle = |\Psi'\rangle = a|\Psi\rangle + b|\Psi_\perp\rangle$$

$$\langle\Psi|\hat{A}|\Psi\rangle = a = \bar{A}$$

$$\langle\hat{A}\Psi|\hat{A}\Psi\rangle = a^2 + b^2$$

$$= \overline{A^2}$$

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$$\hat{A}|\Psi\rangle = |\Psi'\rangle = a|\Psi\rangle + b|\Psi_\perp\rangle$$

$$\langle\Psi|\hat{A}|\Psi\rangle = a = \overline{A}$$

$$\langle\hat{A}\Psi|\hat{A}\Psi\rangle = a^2 + b^2 = \overline{A^2}$$

$$b^2 = \overline{\Delta A^2}$$

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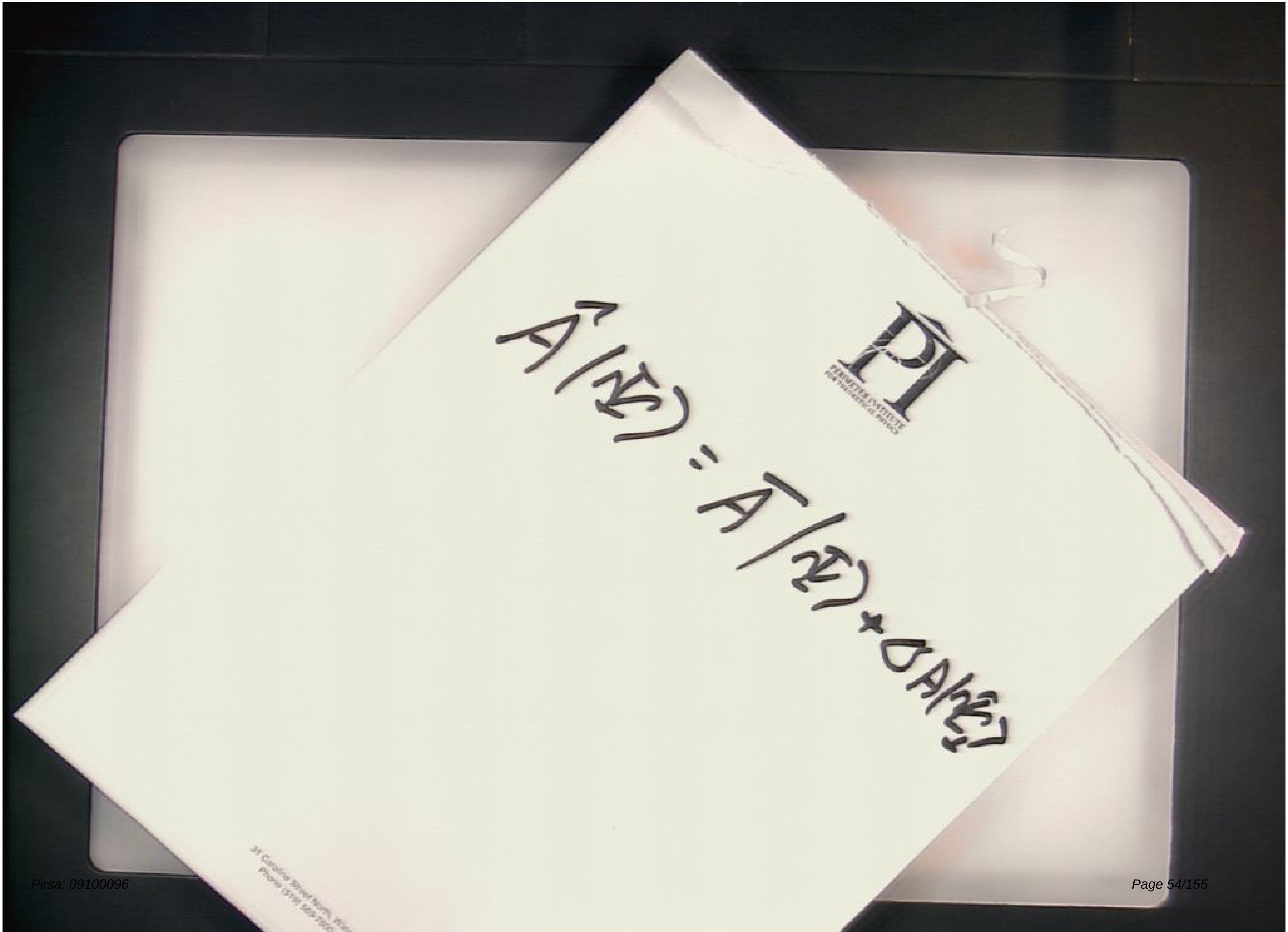
$\hat{A}(\vec{k})$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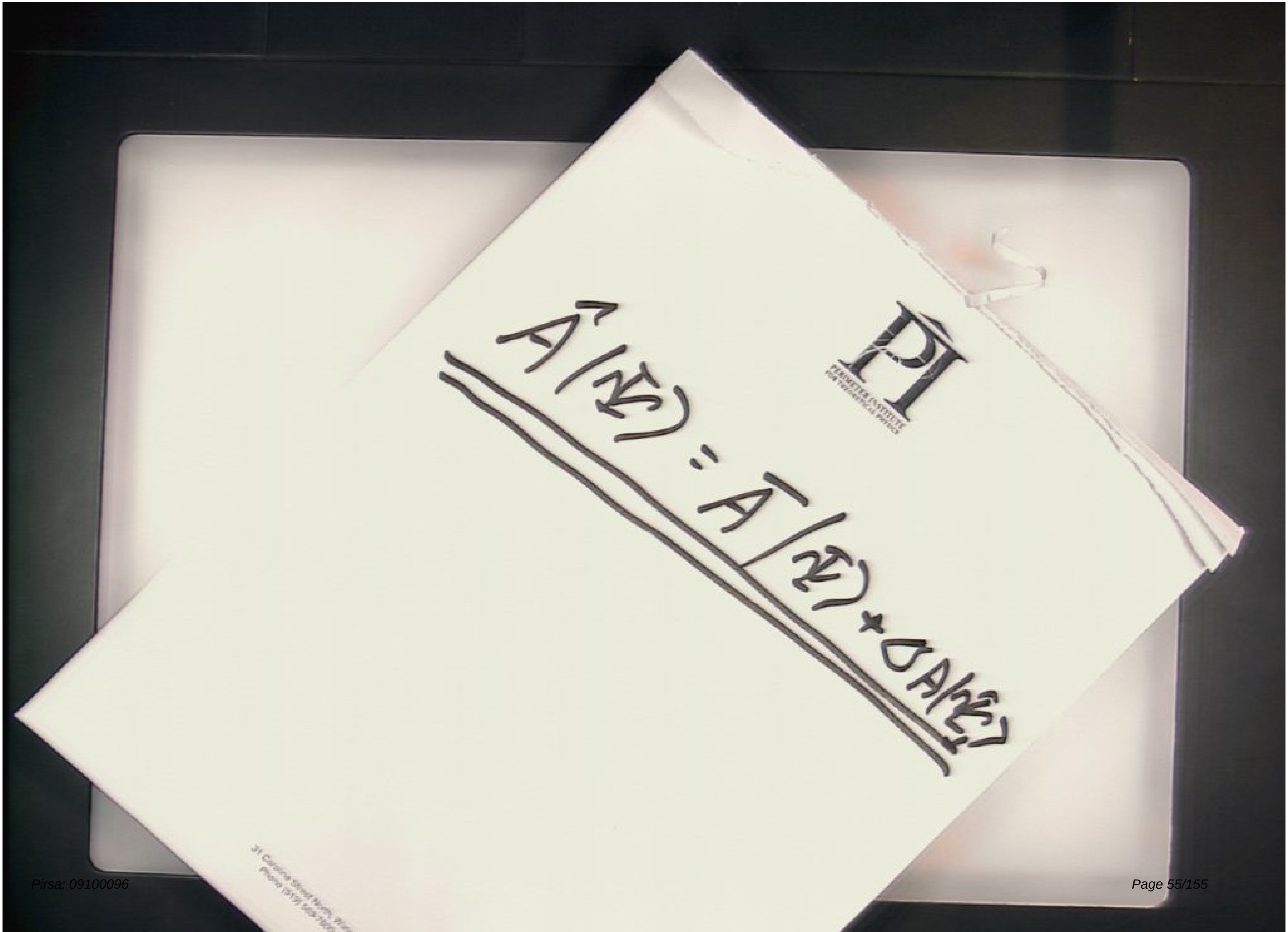
$\hat{A}(\vec{x}) = \bar{A}(\vec{x}) + \delta\hat{A}(\vec{x})$


Â(Σ) = Ā(Σ) + δA(Σ)
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$$\hat{A}(\hat{\Sigma}) = \bar{A}(\Sigma) + \delta A(\hat{\Sigma})$$

$$[\Sigma]_{t_0} = n$$

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$$\hat{A}|\Psi\rangle = \bar{A}|\Psi\rangle + \Delta A|\Psi_\perp\rangle$$

$$|\Psi\rangle_{tot} = \prod_i |\Psi_i\rangle$$

$$\hat{A}_{tot} = \sum_i \frac{\hat{A}_i}{N}.$$

$$\hat{A}|\Psi\rangle = \bar{A}|\Psi\rangle + \Delta A|\Psi_\perp\rangle$$

$$|\Psi\rangle_{tot} = n\,|\Psi\rangle$$

$$\hat{A}_{tot} \equiv \sum \hat{A}_i / N.$$

$$\hat{A}_{tot}|\Psi\rangle_{tot} = \bar{A}$$

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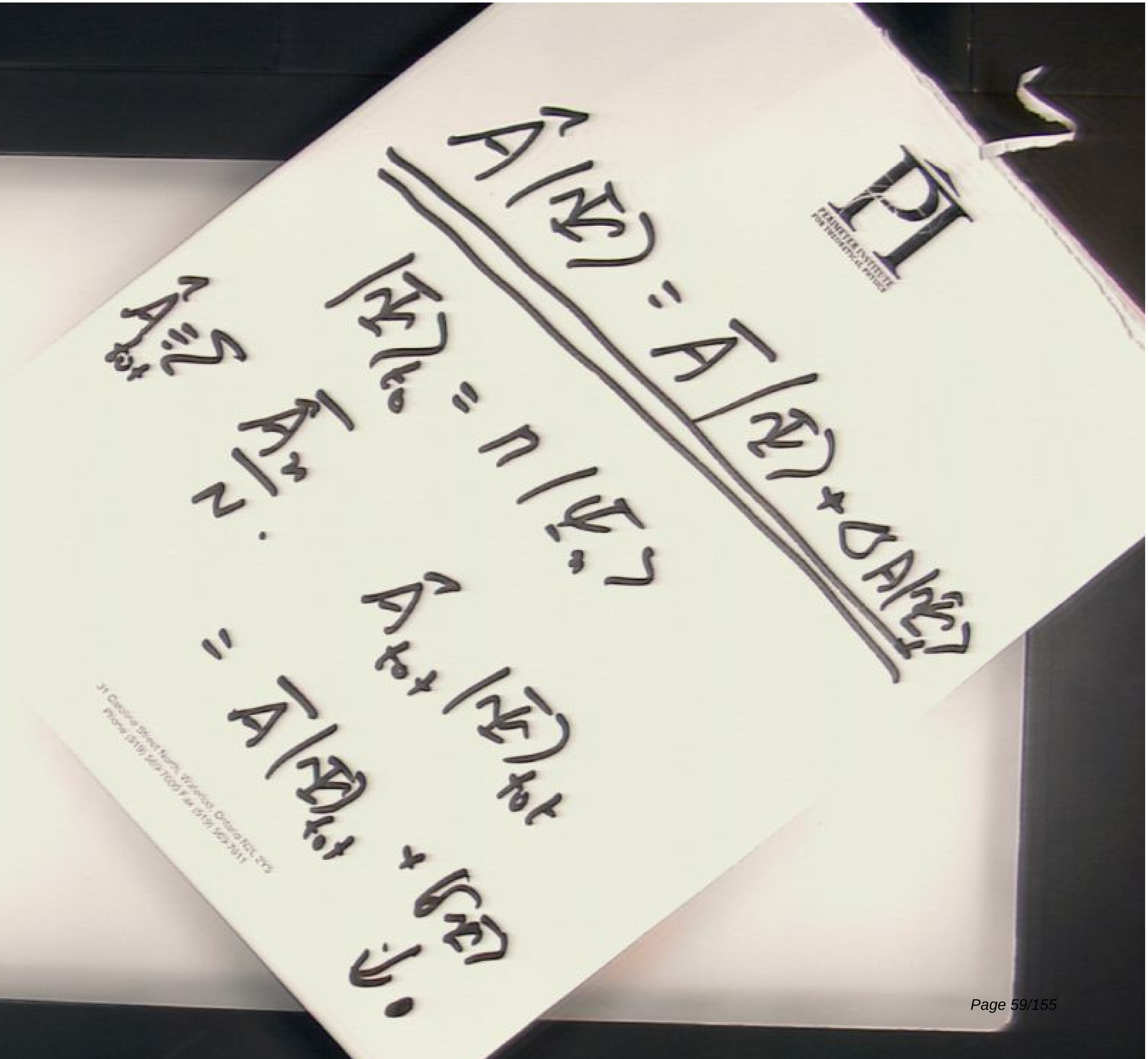
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$\hat{A}|\Psi\rangle = \bar{A}|\Psi\rangle + \Delta A|\Psi_\perp\rangle$
$|\Psi\rangle_{tot} = \Pi|\Psi_i\rangle$
$\hat{A}_{tot} = \sum_i \frac{\hat{A}_i}{N}$.
$\hat{A}_{tot}|\Psi\rangle_{tot} = \bar{A}|\Psi\rangle_{tot} +$
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$$\left(1-\frac{1}{N_y}\right)^N$$

$$\hat{A}|\Psi\rangle = \bar{A}|\Psi\rangle + \Delta A|\Psi_\perp\rangle$$

$$|\Psi\rangle_{tot} = \Pi |\psi_i\rangle$$

$$\hat{A}_{tot} = \sum \hat{A}_i$$

$$\bar{A} = \frac{\hat{A}_{tot}}{N}.$$

$$\hat{A}_{tot}|\Psi\rangle_{tot} = \bar{A}|\Psi\rangle_{tot} + |\Psi_\perp\rangle\sqrt{N}.$$

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$(1-\frac{1}{N\gamma})^N \to 1.$

$$\hat{A}|\Psi\rangle = \bar{A}|\Psi\rangle + \Delta A|\Psi_\perp\rangle$$

$|\Psi\rangle_{tot} = \Pi|\Psi_n\rangle$

$\hat{A}_{tot} = \sum \frac{\hat{A}_n}{N}$. $\quad \hat{A}_{tot}|\Psi\rangle_{tot}$

$= \bar{A}|\Psi\rangle_{tot} + |\Psi\rangle$

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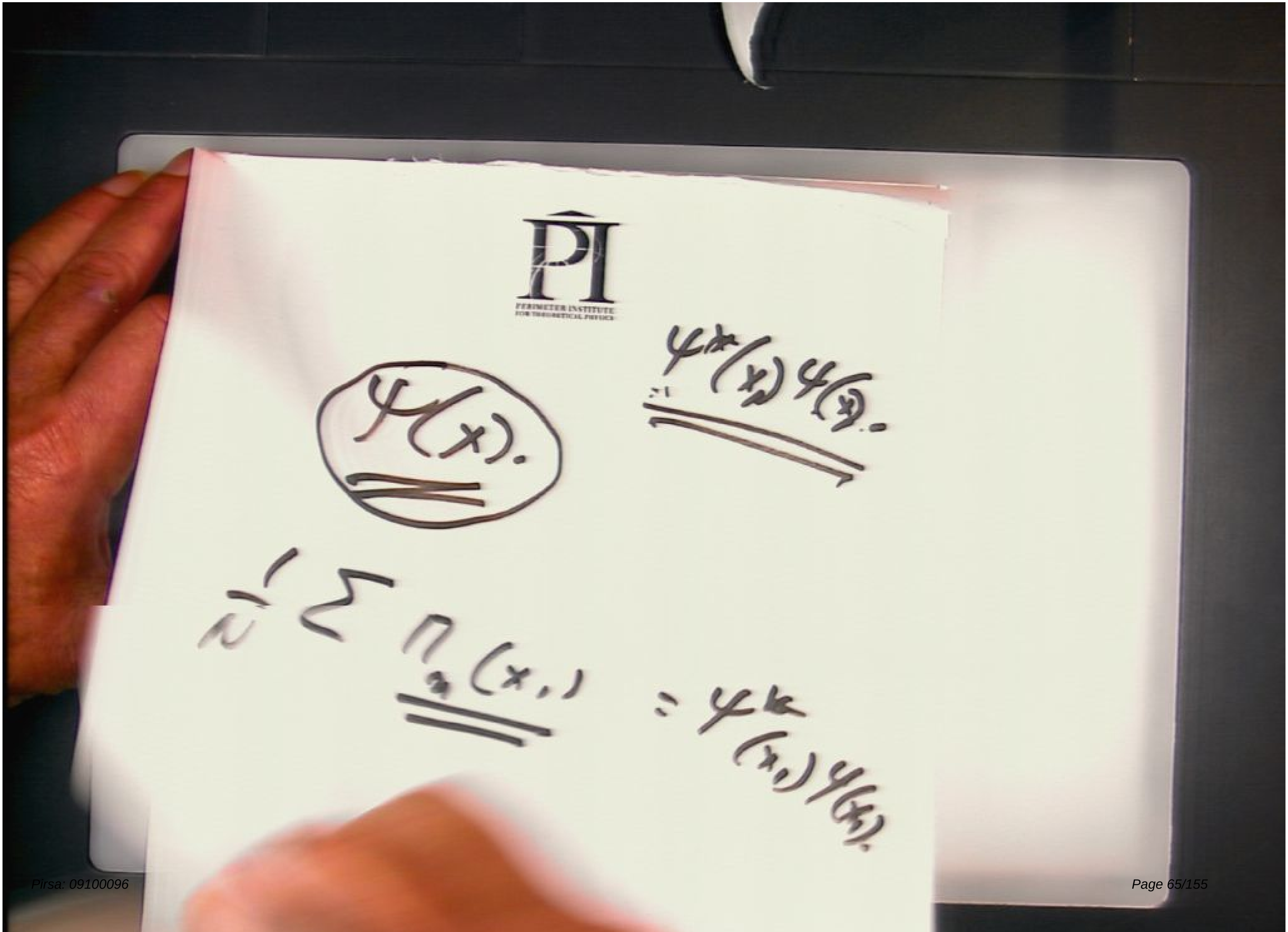
$\psi(x)$.

$\psi^*(x)\,\psi(x).$

$\psi(x).$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$$\frac{1}{N}\sum \hat{A}_n \,|\Sigma\rangle$$

$$\hat{A}_n \,|\hat{\Sigma}_{tot}\rangle = n$$

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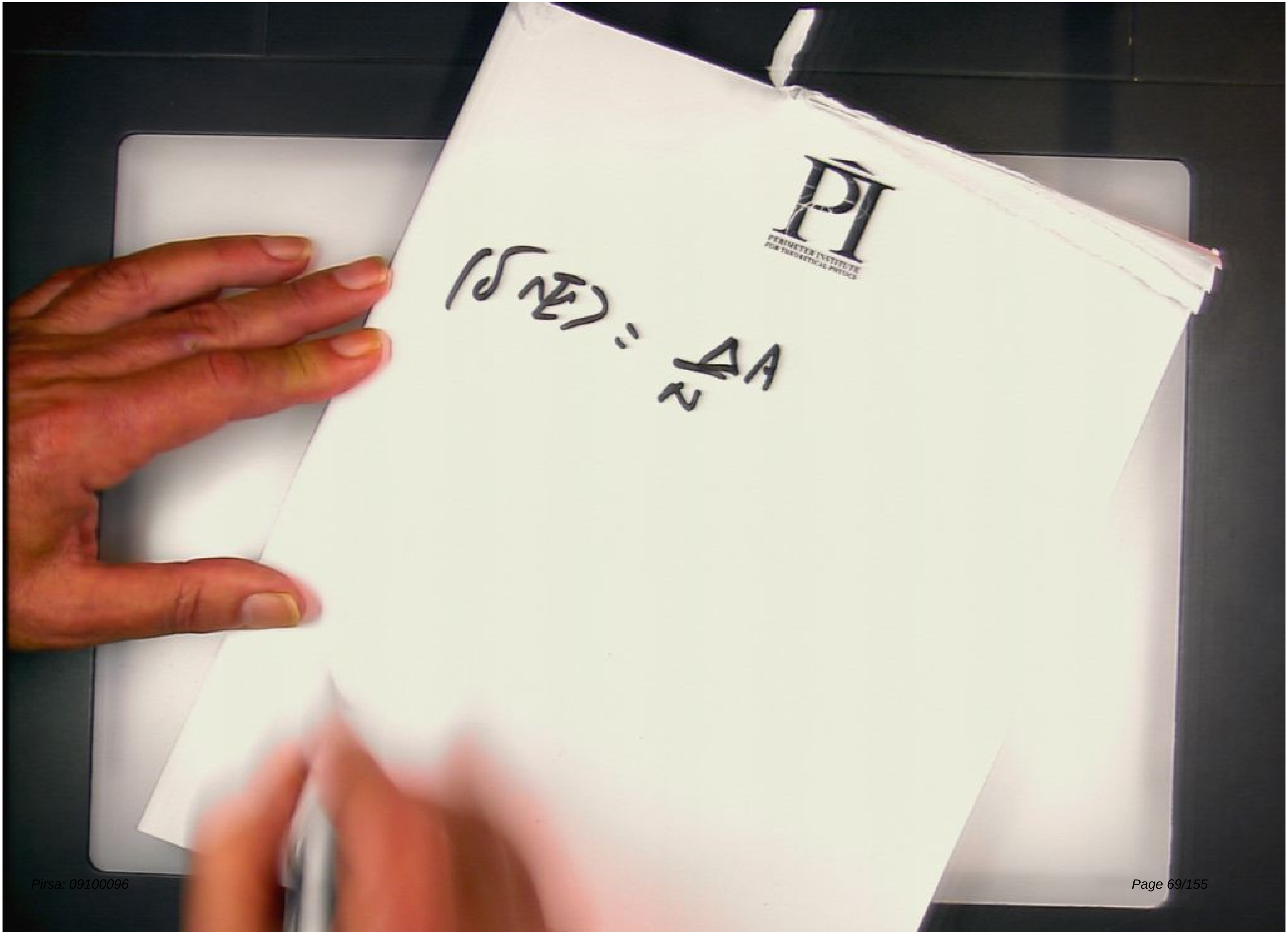
$$\frac{1}{N}\sum \hat{A}_n \,|\Psi\rangle$$

$$\hat{A}_n \,|\hat{\Psi}_{tot}\rangle = \prod_{m\neq n} \psi(x_m)[\;\;]$$

$$\frac{1}{N}\sum \hat{A}_n \,|\Psi\rangle$$

$$\hat{A}_m \,|\Psi_{tot}\rangle = \prod_{m \neq n} \Psi(x)[\;]$$

$$\hat{A}_n \,|\Psi(x)\rangle = \bar{A}\,|\Psi(x)\rangle + \Delta A\,|\Psi_n^{\perp}\rangle$$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$$\langle \delta \tilde{N} \rangle = \frac{\Delta A}{2} \sum_x \langle H_\perp(x) \rangle$$

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$$\langle \delta \Sigma \rangle = \frac{\Delta A}{2} \sum_{\lambda} \langle H_{\perp}(\lambda) \rangle$$

$$\langle \delta \Sigma \, \delta \Sigma \rangle =$$

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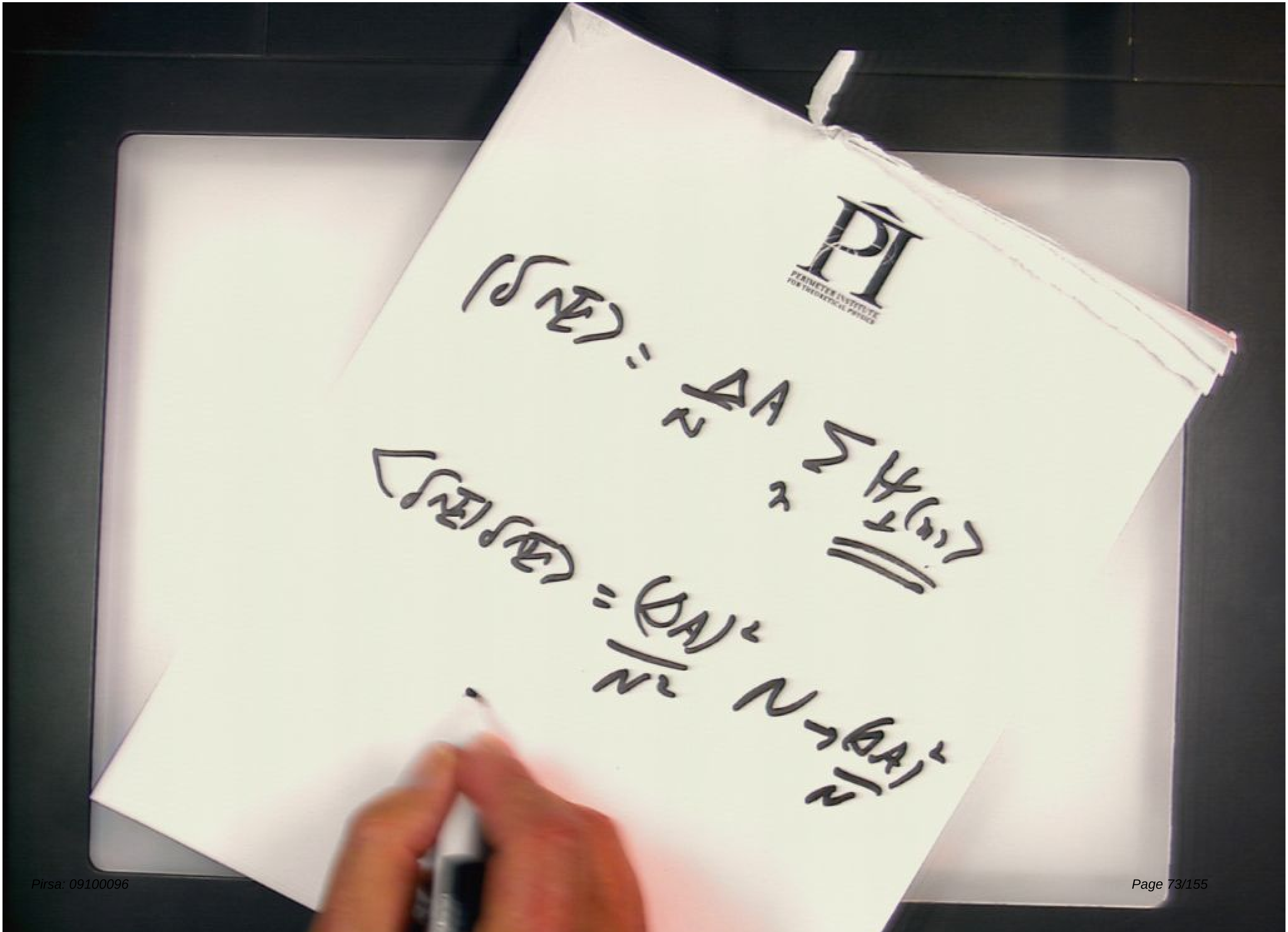
$$\langle \delta N_E \rangle = \frac{\Delta A}{2} \sum_x \langle \psi_x(1) \rangle$$

$$\langle \delta N_E \, \delta N_E \rangle = \frac{(\Delta A)^2}{N^2} N$$


⟨δ N(x)⟩ = ΔA/N Σ_x ⟨H_⊥(x)⟩
⟨δN(x) δN(x)⟩ = (ΔA)²/N² N → (ΔA)²/N

$$\langle \delta \Sigma_I \rangle = \frac{\Delta A}{N} \sum_i \langle H_\perp(i) \rangle$$

$$\langle \delta \Sigma_I \, \delta \Sigma_I \rangle = \frac{(\Delta A)^2}{N^2} N \to \frac{(\Delta A)^2}{N} \to 0. \quad N \to \infty$$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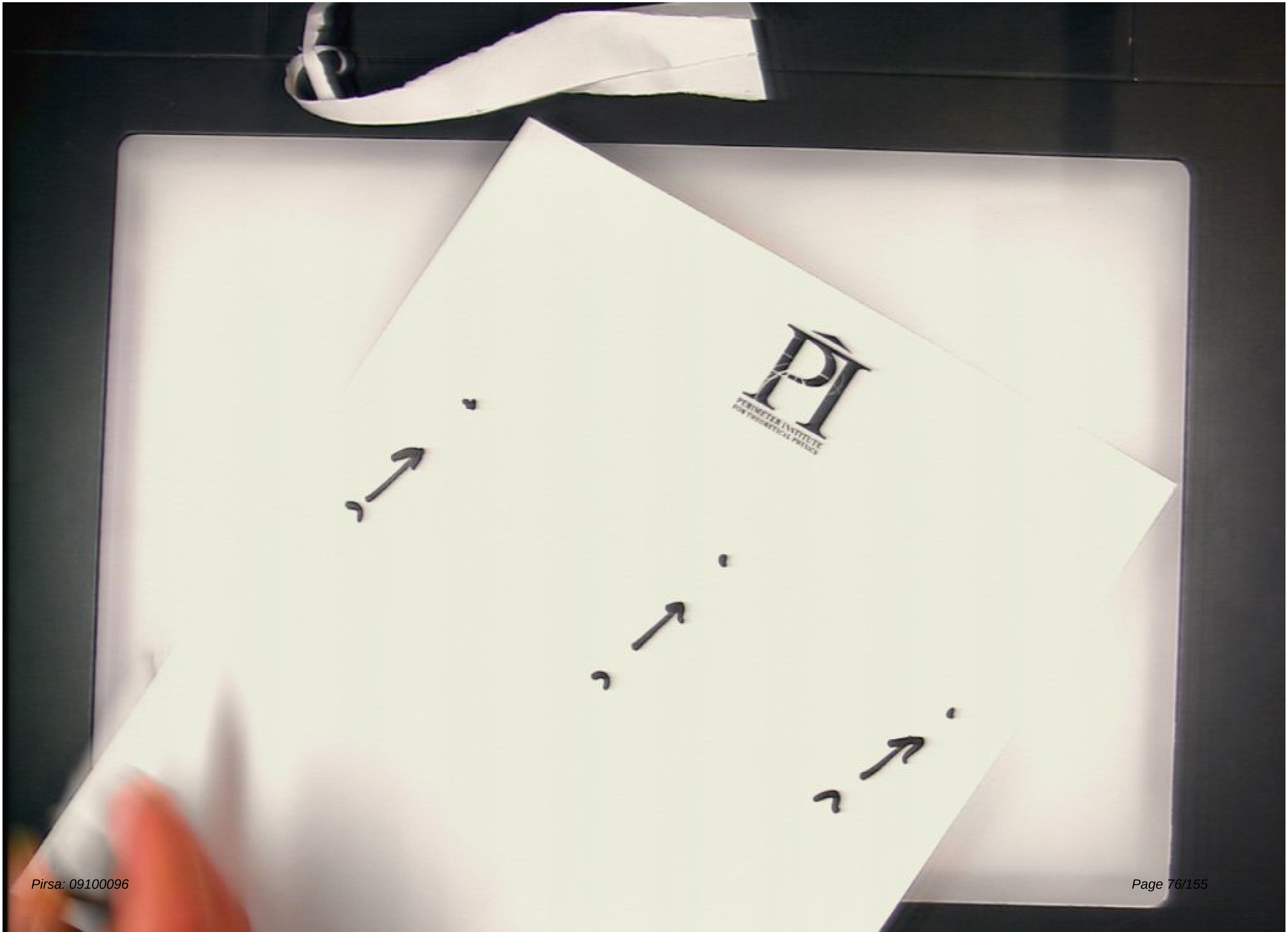


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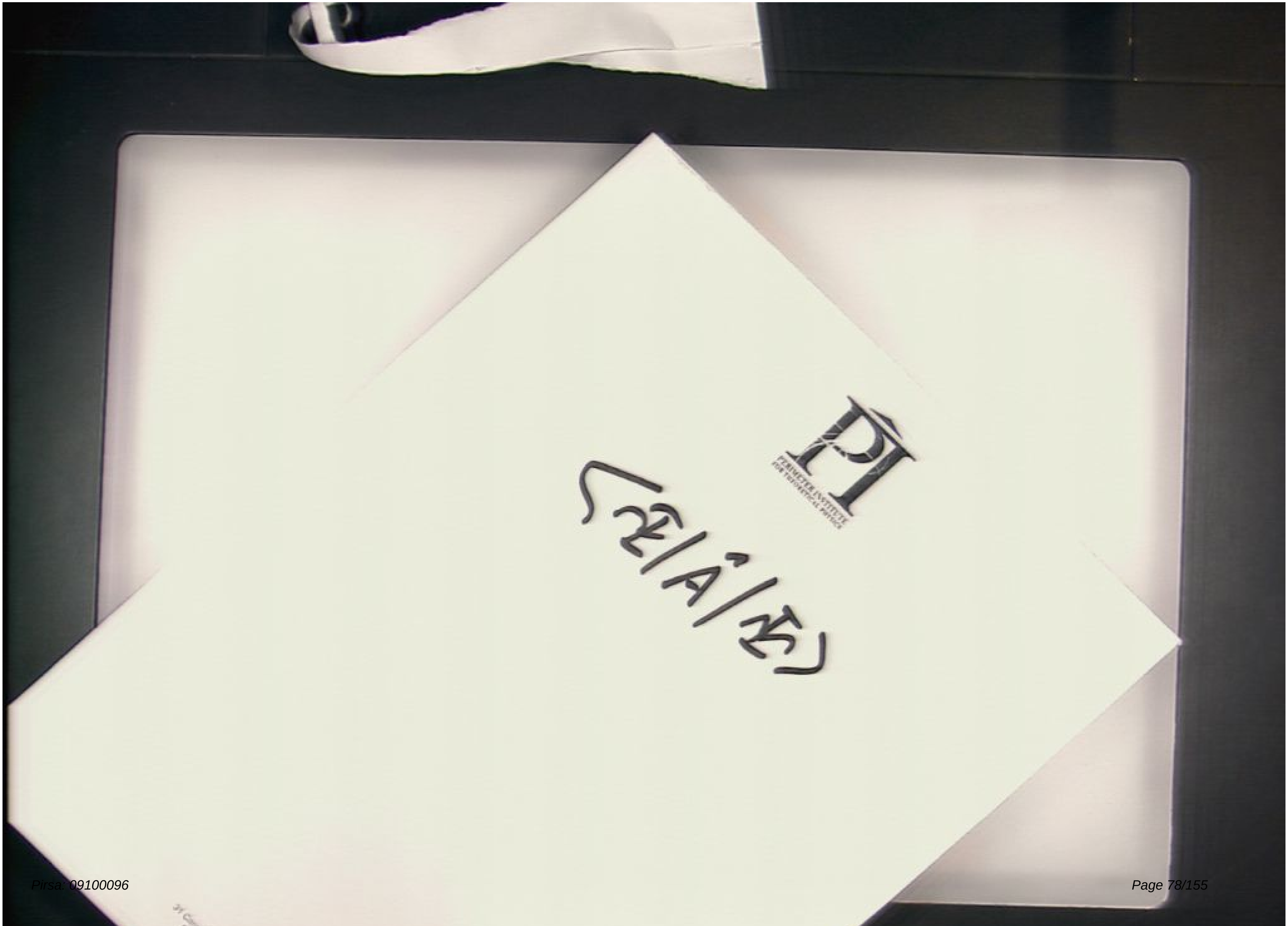

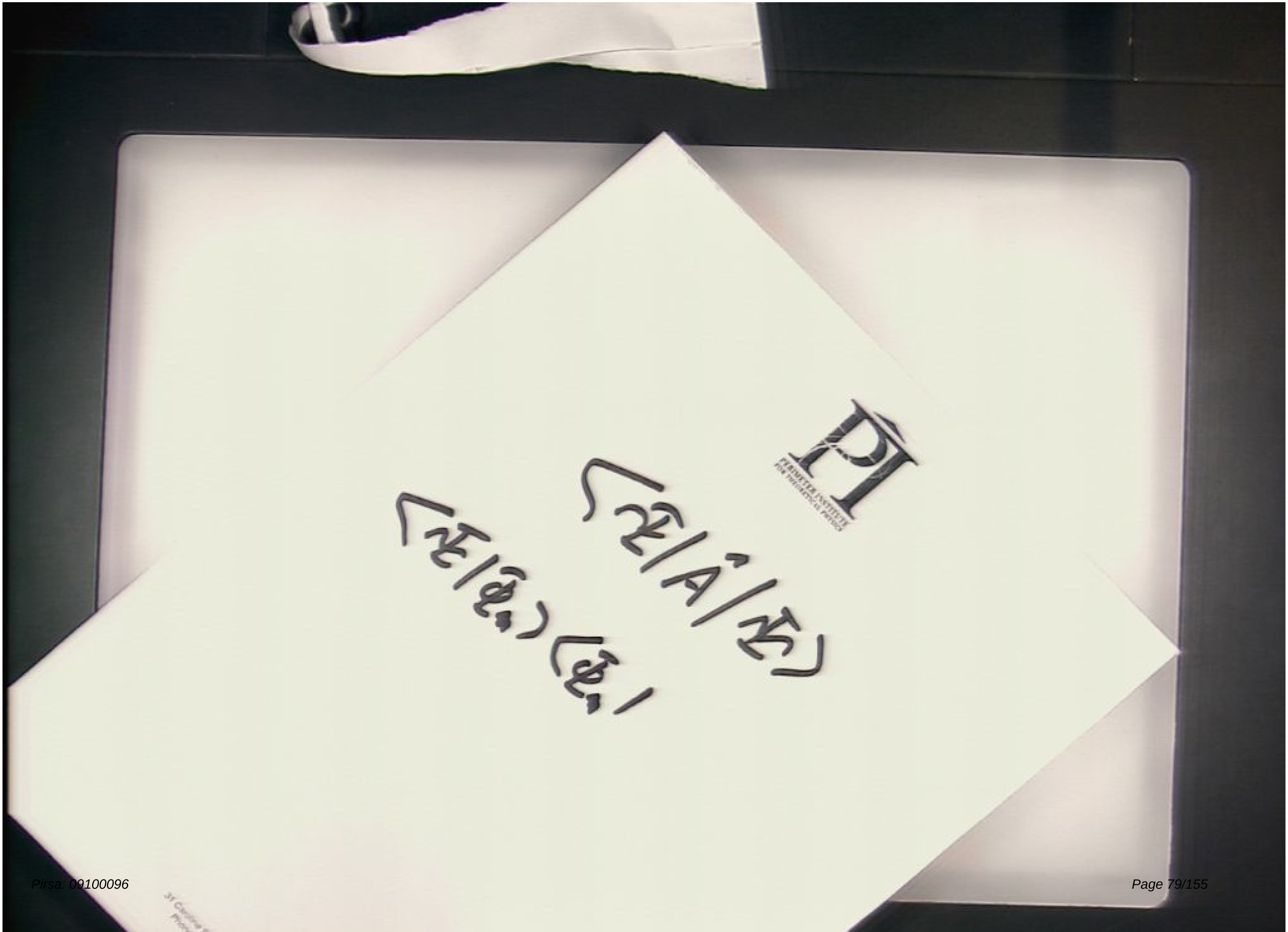
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⟨Ψ|Â|Ψ⟩
⟨Ψ|φ_n⟩⟨φ_n|

$$\langle \Psi | \hat{A} | \Psi \rangle$$

$$= \langle \Psi | \Phi_n \rangle \langle \Phi_n | \hat{A} | \Psi \rangle$$

$$= \langle \Psi | \Phi_n \rangle \langle \Phi_n | \Psi \rangle \frac{\langle \Phi_n | \hat{A} | \Psi \rangle}{\langle \Phi_n | \Psi \rangle}$$

$$\langle \tilde{\Psi} | \hat{A} | \Psi \rangle$$

$$= \sum_j \langle \tilde{\Psi} | \Phi_j \rangle \langle \Phi_j | \hat{A} | \Psi \rangle$$

$$= \sum_j \langle \psi | \Phi_j \rangle \langle \Phi_j | \Psi \rangle \; \frac{\langle \Phi_j | \hat{A} | \Psi \rangle}{\langle \Phi_j | \Psi \rangle}$$

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$$\langle \Psi | \hat{A} | \Psi \rangle$$

$$= \sum_j \langle \Psi | \Phi_j \rangle \langle \Phi_j | \hat{A} | \Psi \rangle$$

$$= \sum_j \underline{\underline{\langle \Psi | \Phi_j \rangle \langle \Phi_j | \Psi \rangle}} \frac{\langle \Phi_j | \hat{A} | \Psi \rangle}{\langle \Phi_j | \Psi \rangle}$$

$A\omega \stackrel{?}{=}$

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$$A_\omega \equiv \langle \Sigma_\omega | \Sigma \rangle$$

$$A_w \equiv \frac{\langle \psi_2 | \hat{A} | \psi_1 \rangle}{\langle \psi_2 | \psi_1 \rangle}.$$

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$$A_w = \frac{\langle \Phi_f | \hat{A} | \Psi_i \rangle}{\langle \Phi_f | \Psi_i \rangle}.$$

$$e^{iq\hat{A}} \, |\Psi\rangle |\Phi\rangle$$

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$$A_w = \frac{\langle \Phi_2 | \hat{A} | \Phi_1 \rangle}{\langle \Phi_2 | \Phi_1 \rangle}.$$

$$e^{iq\hat{A}} \cdots |\Psi\rangle |\Phi\rangle$$

$$e^{i\lambda q\hat{A}}$$

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$$A_w = \frac{\langle \Psi_2 | \hat{A} | \Psi_1 \rangle}{\langle \Psi_2 | \Psi_1 \rangle}.$$

$$e^{iq\hat{A}} \cdots |\Psi\rangle|\Phi\rangle$$

$$e^{i\lambda q\hat{A}} = 1 + i\lambda q\hat{A}$$

$$\langle \Phi_2 | e^{i\lambda q \hat{A}} | \Phi_1 \rangle | \Phi \rangle$$

$$A_w = \frac{\langle \Phi_2 | \hat{A} | \Phi_1 \rangle}{\langle \Phi_2 | \Phi_1 \rangle}.$$

$$e^{iq\hat{A}} \cdots | \Psi \rangle | \Phi \rangle$$

$$e^{i\lambda q \hat{A}} = 1 + i\lambda q \hat{A}$$

$\langle \Psi_2 | e^{i\lambda q \hat{A}} | \Psi_1 \rangle | \Phi \rangle$

$$A_w = \frac{\langle \Psi_2 | \hat{A} | \Psi_1 \rangle}{\langle \Psi_2 | \Psi_1 \rangle}.$$

$e^{iq\hat{A}} \ldots | \Psi \rangle | \Phi \rangle$

$e^{i\lambda q \hat{A}}$

$= 1 + i\lambda q \hat{A}$

$$\frac{\langle \Psi_2 | e^{i\lambda q \hat{A}} | \Psi_1 \rangle | \Phi \rangle}{}$$

$$\langle \Psi_2 | 1 + i\lambda q \hat{A} | \Psi_1 \rangle$$

$$A_w \equiv \frac{\langle \Psi_2 | \hat{A} | \Psi_1 \rangle}{\langle \Psi_2 | \Psi_1 \rangle}.$$

$$e^{iq\hat{A}} \cdots | \Psi \rangle | \Phi \rangle$$

$$e^{i\lambda q \hat{A}}$$

$$= 1 + i\lambda q \hat{A}$$

$$\frac{\langle \Psi_2 | e^{i\lambda g \hat{A}} | \Psi_1 \rangle}{} \Big/ \langle \Phi \rangle$$

$$\langle \Psi_2 | 1 + i\lambda g \hat{A} | \Psi_1 \rangle$$

$$= \langle \Psi_2 | \Psi_1 \rangle \left(1 + i\lambda g \frac{\langle \Psi_2 \hat{A} \Psi_1 \rangle}{\langle \Psi_2 | \Psi_1 \rangle} \right)$$

$$A_w = \frac{\langle \Psi_2 | \hat{A} |}{\langle \Psi_2 | \Psi_1}$$

$$e^{ig\hat{A}} \ldots | \Psi \rangle / \Phi \rangle$$

$$e^{i\lambda g \hat{A}}$$

$$= 1 + i\lambda g \hat{A}$$

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$$\langle \Psi_2 | e^{i\lambda g \hat{A}} | \Psi_1 \rangle | \Phi \rangle$$

$$\langle \Psi_2 | 1 + i\lambda g \hat{A} | \Psi_1 \rangle$$

$$= \langle \Psi_2 | \Psi_1 \rangle \left(1 + i\lambda g \frac{\langle \Psi_2 \hat{A} \Psi_1 \rangle}{\langle \Psi_2 | \Psi_1 \rangle} \right)$$

$$= e^{i\lambda g A_w}$$

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$$A_w = \frac{\langle \Psi_2 | \hat{A} \ldots}{\langle \Psi_2 | \Psi_1 \ldots}$$

$$e^{ig\hat{A}} | \Psi \rangle | \Phi \rangle$$

$$e^{i\lambda g \hat{A}}$$

$$= 1 + i\lambda g \hat{A} \ldots$$

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$$\frac{\langle \Psi_2 | e^{i\lambda g \hat{A}} | \Psi_1 \rangle}{} |\Phi\rangle$$

$$\langle \Psi_2 | 1 + i\lambda g \hat{A} | \Psi_1 \rangle$$

$$= \langle \Psi_2 | \Psi_1 \rangle \left(1 + i\lambda g \frac{\langle \Psi_2 | \hat{A} | \Psi_1 \rangle}{\langle \Psi_2 | \Psi_1 \rangle} \right)$$

$$= e^{i\lambda g A_w}$$

$$A_w = \frac{\langle \Psi_2 | \hat{A} |}{\langle \Psi_2 | \Psi_1}$$

$$e^{i g \hat{A}} \ldots |\Psi\rangle |\Phi\rangle$$

$$e^{i\lambda g \hat{A}}$$

$$= 1 + i\lambda g \hat{A}$$

$$\langle \Psi_2 | e^{i\lambda g \hat{A}} | \Psi_1 \rangle / \langle \Phi \rangle$$

$$\frac{\langle \Psi_2 | 1 + i\lambda g \hat{A} | \Psi_1 \rangle}{\langle \Psi_2 | \Psi_1 \rangle}$$

$$= \left(1 + i\lambda g \frac{\langle \Psi_2 | \hat{A} | \Psi_1 \rangle}{\langle \Psi_2 | \Psi_1 \rangle} \right)$$

$$= e^{i\lambda g A_w}$$

$$A_w = \frac{\langle \Psi_2 | \hat{A} | \Psi_1 \rangle}{\langle \Psi_2 | \Psi_1 \rangle}$$

$$e^{ig\hat{A}} | \Psi \rangle | \Phi \rangle$$

$$e^{i\lambda g \hat{A}} = 1 + i\lambda g \hat{A}$$

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$$A_w \equiv \frac{\langle \psi_2 | \hat{A} | \psi_1 \rangle}{\langle \psi_2 | \psi_1 \rangle}$$

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$$A_w \equiv \frac{\langle \psi_2 | \hat{A} | \psi_1 \rangle}{\langle \psi_2 | \psi_1 \rangle}$$

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$$A_w \equiv \frac{\langle \psi_2 | \hat{A} | \psi_1 \rangle}{\langle \psi_2 | \psi_1 \rangle}$$

$$x \to x + i x_0$$

$$e^{-\frac{x^2}{\sigma^2}}$$

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$$A_w \equiv \frac{\langle \psi_2 | \hat{A} | \psi \ldots}{\langle \psi_2 | \psi_1 \rangle}$$

$$x \to x + i x_0$$

$$e^{-\frac{x^2}{\sigma^2}} \to e^{-\frac{(x + i x_0)^2}{\sigma^2}}$$

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$$e^{-\frac{x^2}{\Delta^2}} e^{i}$$

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$$A_w \equiv \frac{\langle \psi_2 | \hat{A} | \psi_1 \rangle}{\langle \psi_2 | \psi_1 \rangle}$$

$$x \to x + i x_0$$

$$e^{-\frac{x^2}{\Delta^2}} \to e^{-\frac{(x + i x_0)^2}{\Delta^2}}$$

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$$e^{-\frac{x^2}{2\sigma^2}} e^{ixx_0}$$

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$$A_w \equiv \frac{\langle \psi_2 | \hat{A} | \psi \ldots}{\langle \psi_2 | \psi_1 \rangle \ldots}$$

$$x \to x + ix_0$$

$$e^{-\frac{x^2}{\sigma^2}} \to e^{-\frac{(x+ix_0)^2}{2\sigma^2}}$$

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e

$$A_w \equiv \frac{\langle \psi_2 | \hat{A} | \psi_1 \rangle}{\langle \psi_2 | \psi_1 \rangle}$$

$$x \to x + i x_0$$

$$e^{-\frac{x^2}{\sigma^2}} \to e^{-\frac{(x + i x_0)^2}{\sigma^2}}$$

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$$e^{-\frac{x^2}{2\sigma^2}} e^{i x x_0}$$

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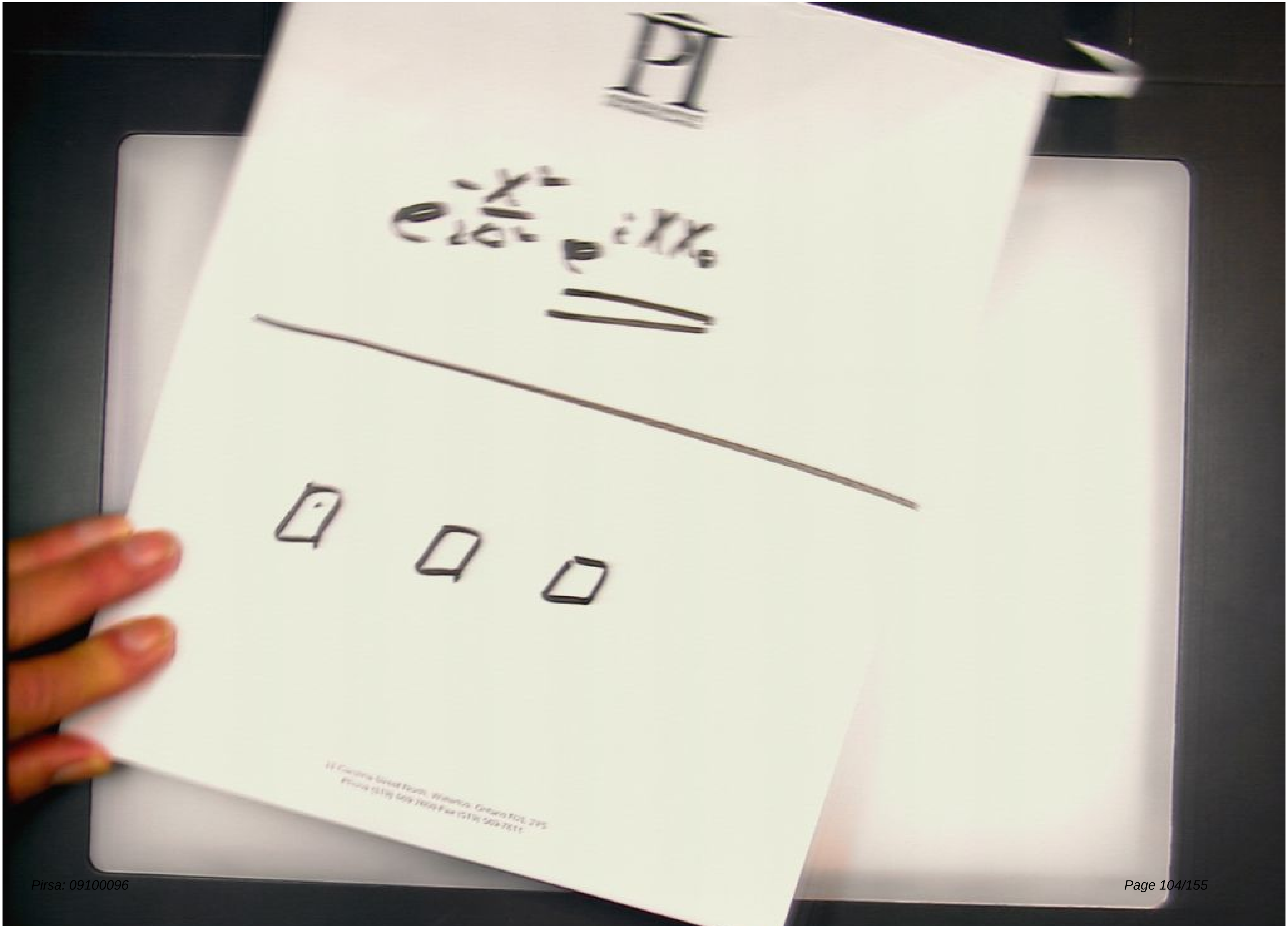

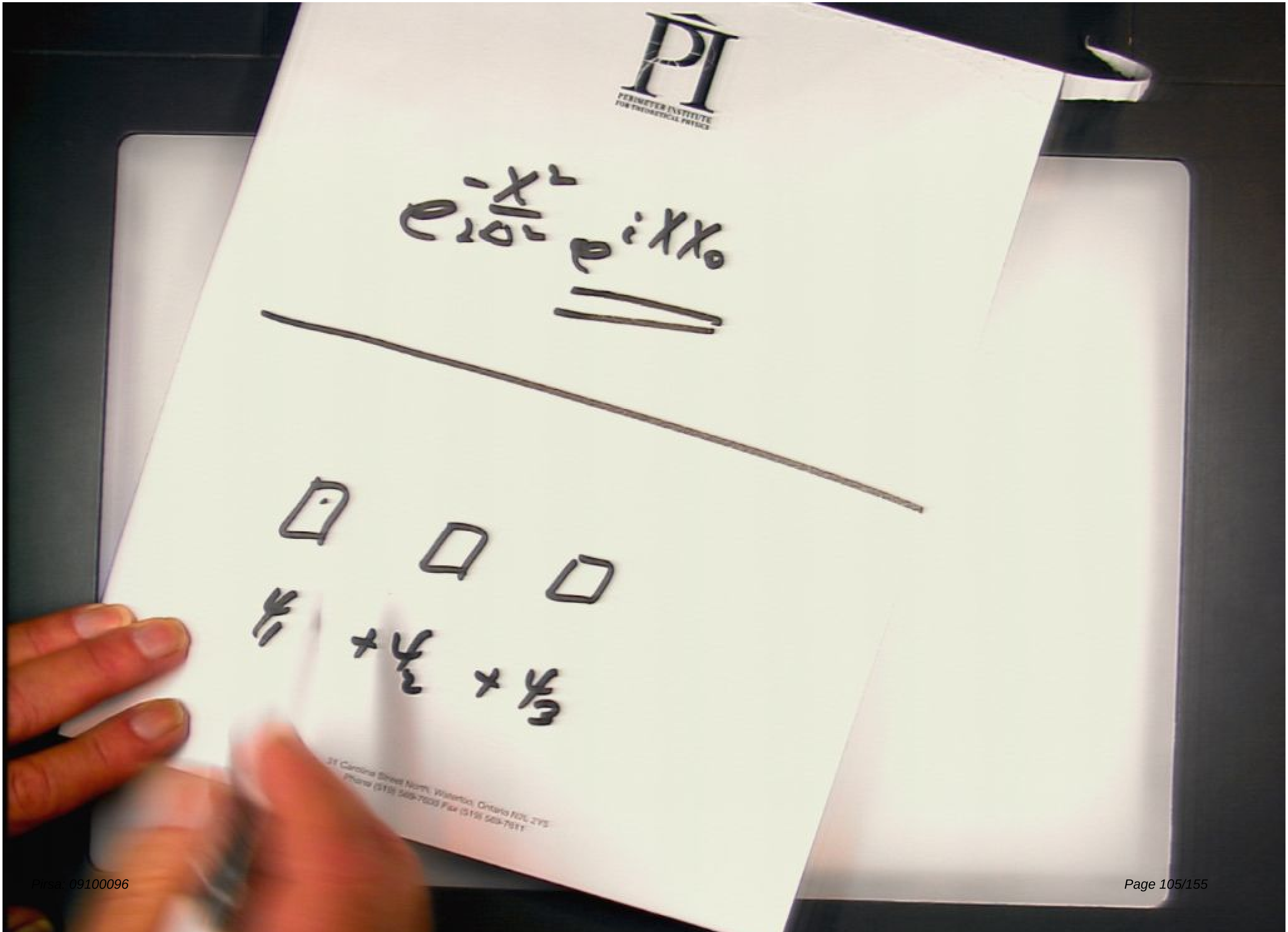
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$e^{-\frac{x^2}{2\sigma^2}} e^{ixx_0}$
$\psi_1 + \psi_2 + \psi_3$

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$$e^{-\frac{x^2}{2\sigma^2}} \, e^{iXX_0}$$

$$\psi_1 + \psi_2 + \psi_3$$

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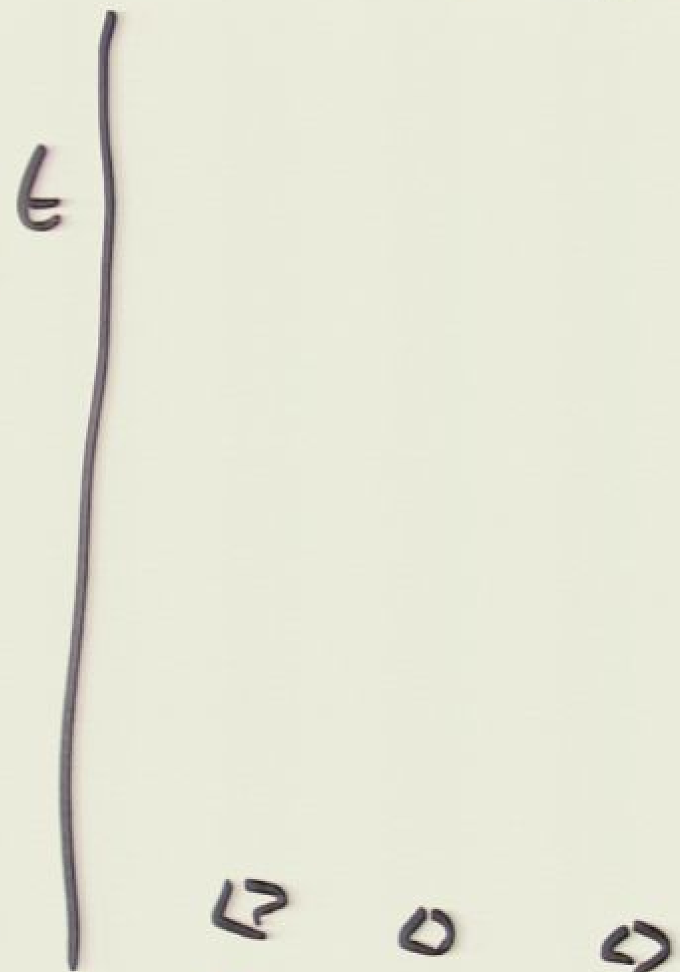

$$\psi_1 + \psi_2 + \psi_3$$

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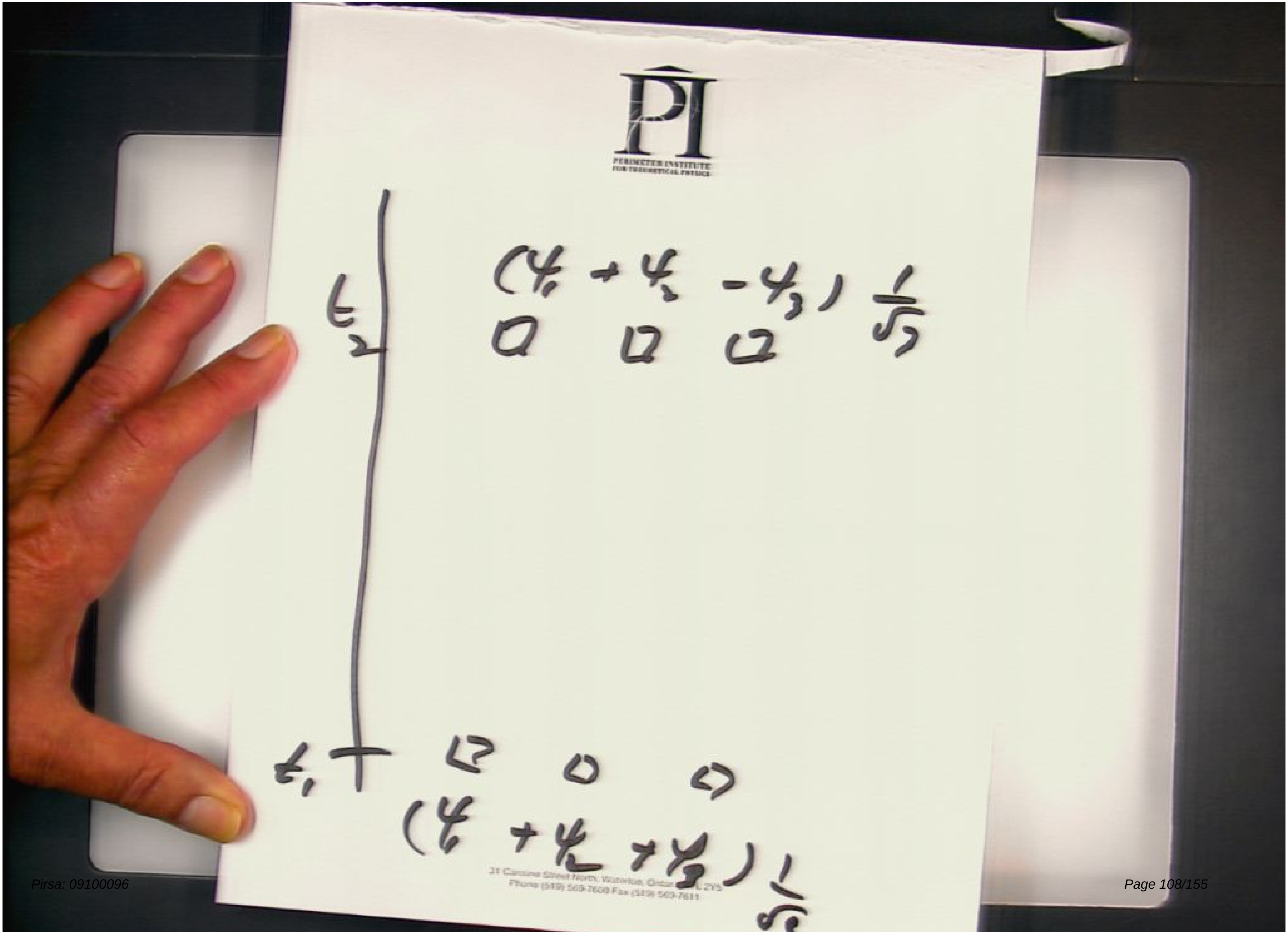
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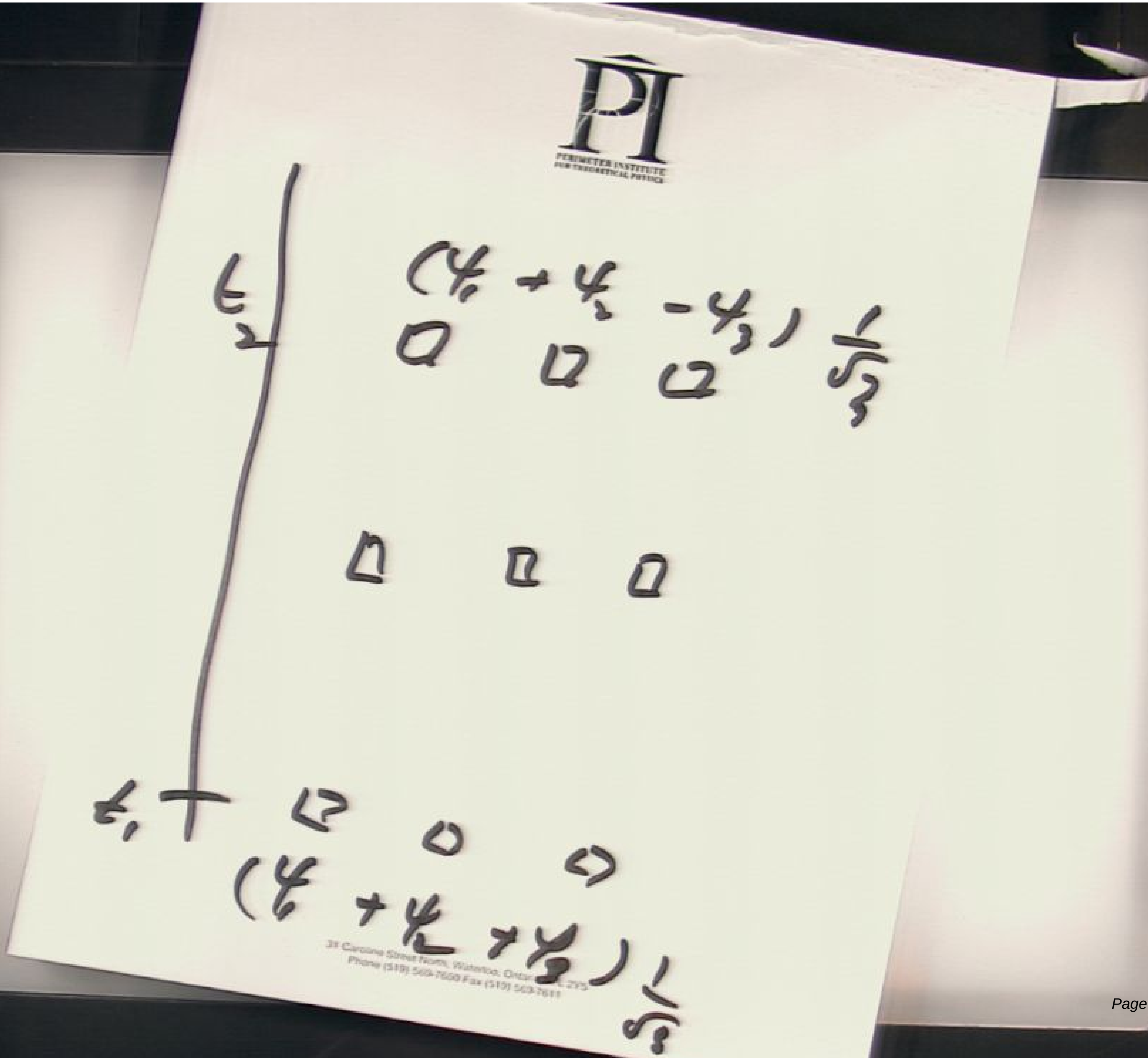
t_2
$(\psi_1 + \psi_2 - \psi_3)\frac{1}{\sqrt{3}}$
t_1
$(\psi_1 + \psi_2 + \psi_3)\frac{1}{\sqrt{3}}$


t_2
$(\psi_1 + \psi_2 - \psi_3)\frac{1}{\sqrt{3}}$
t_1
$(\psi_1 + \psi_2 + \psi_3)\frac{1}{\sqrt{3}}$
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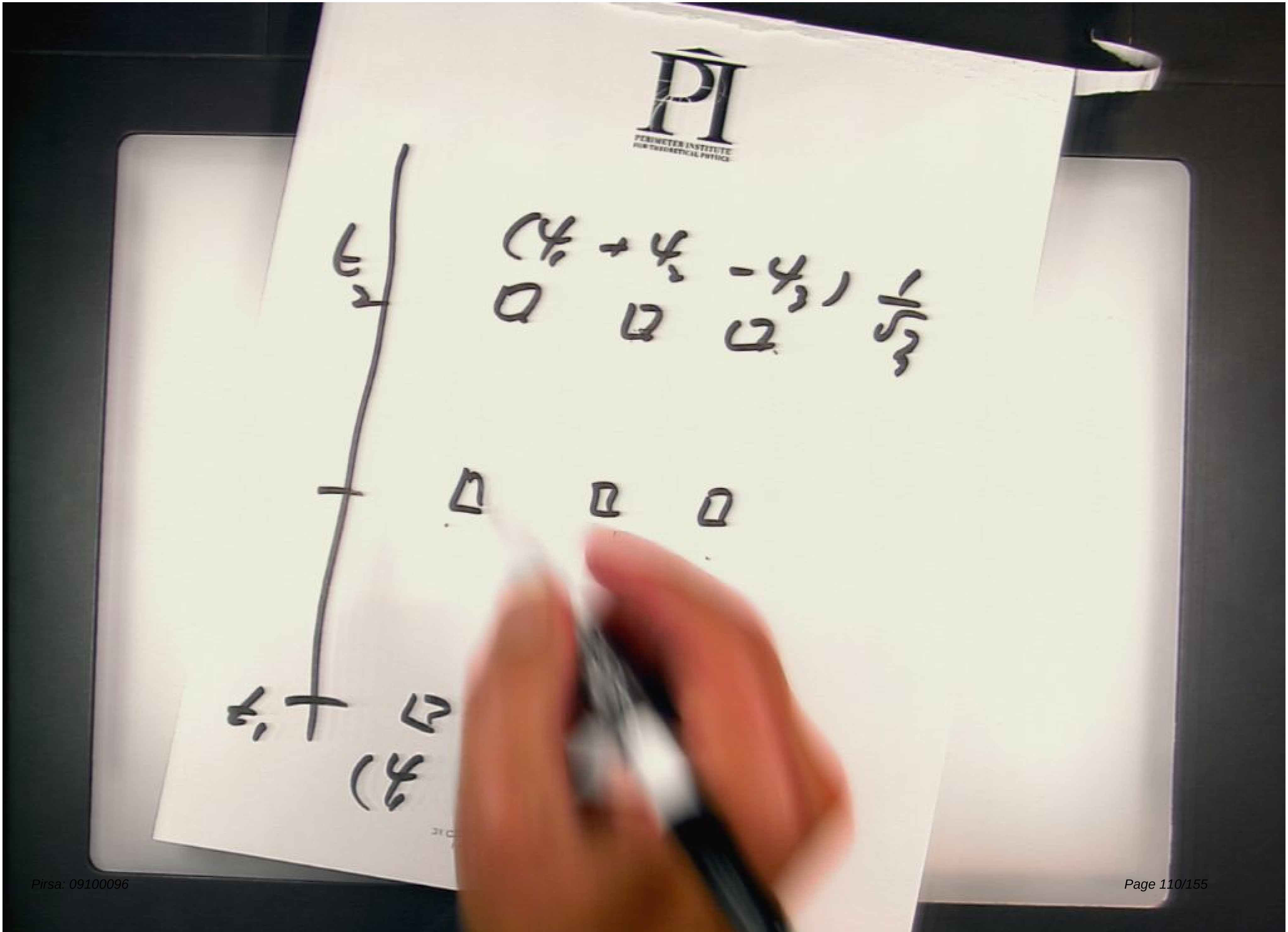
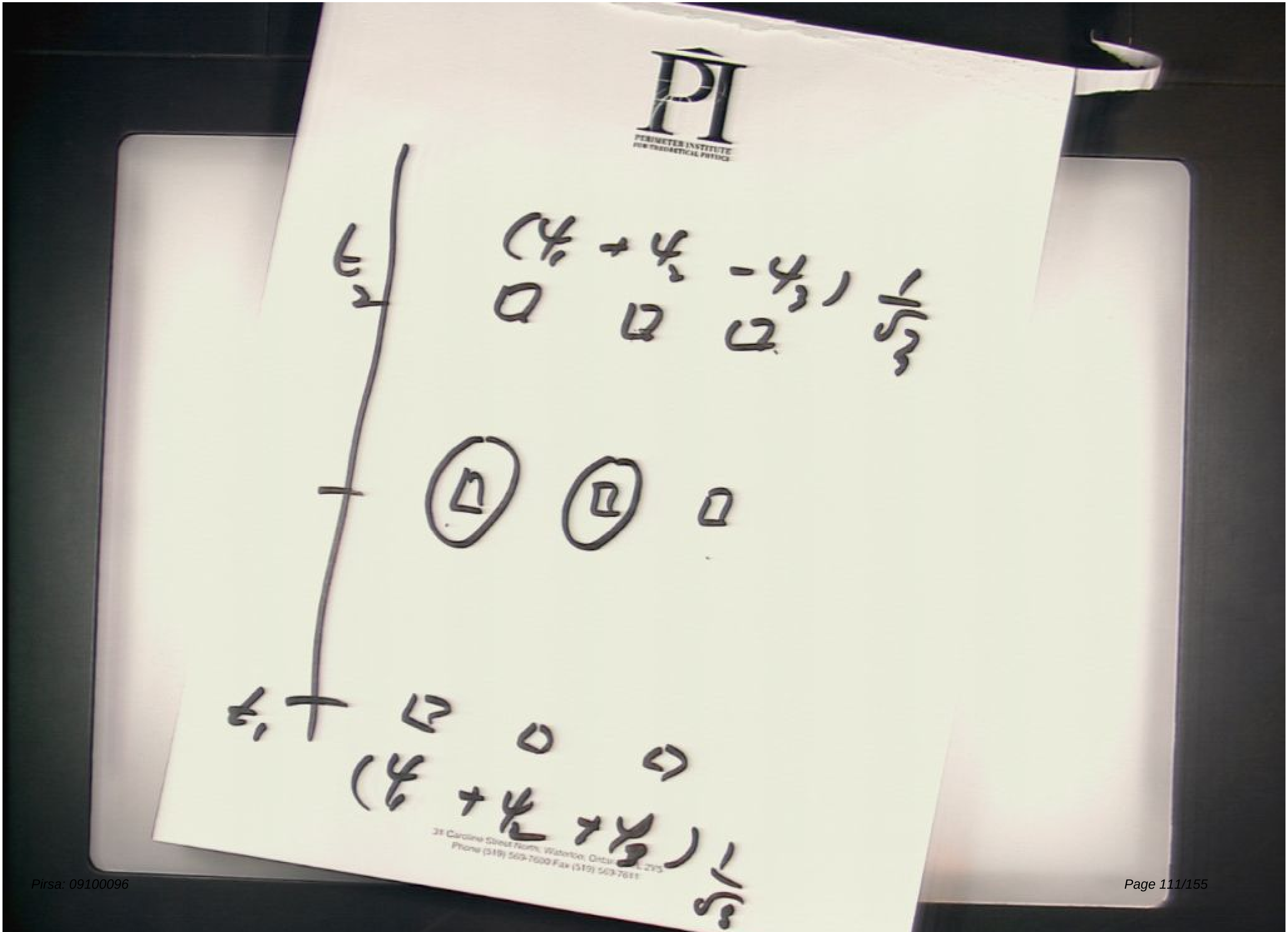

t_2
$(\psi_1 + \psi_2 - \psi_3)\ \frac{1}{\sqrt{3}}$
t_1
$(\psi$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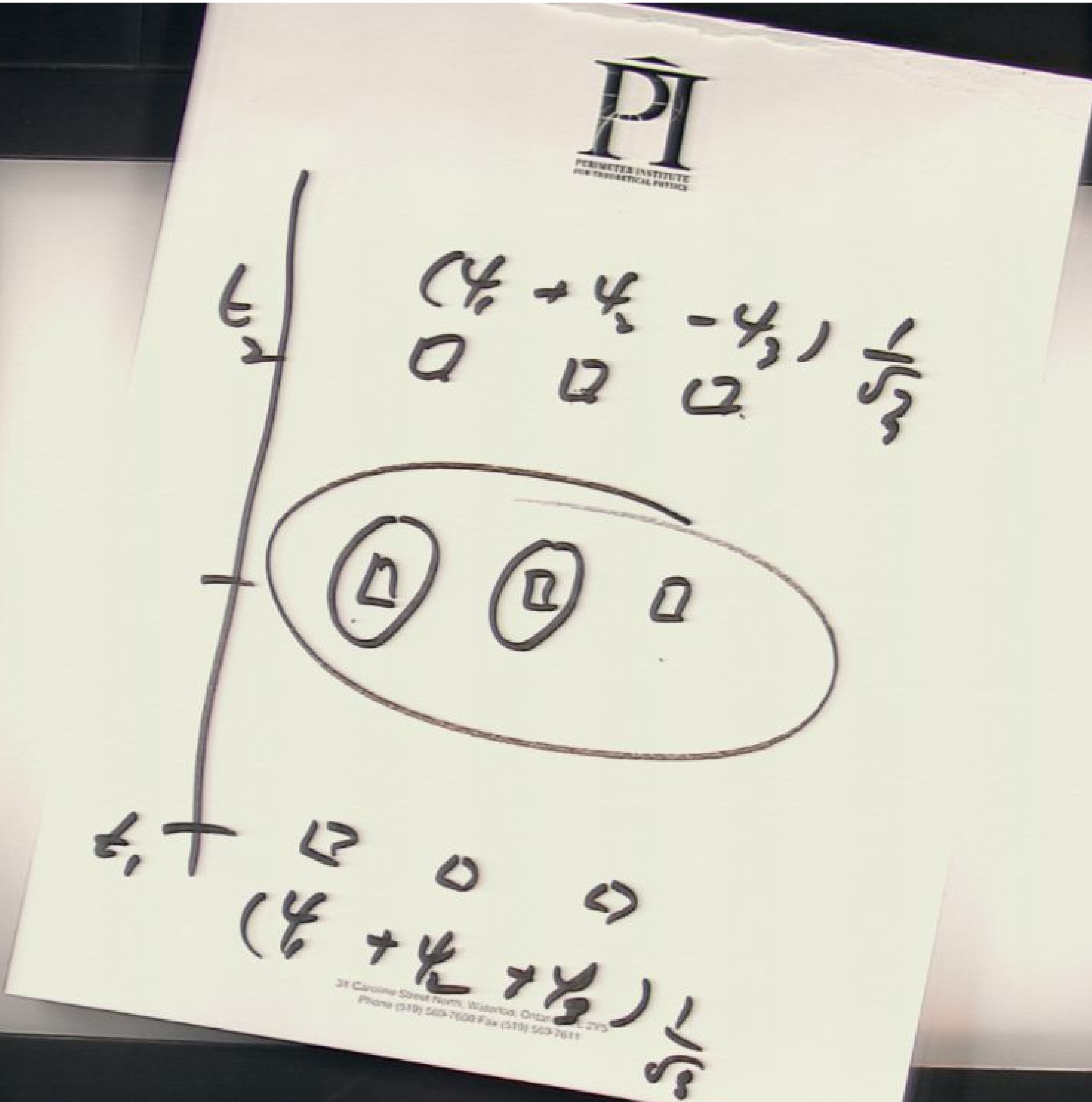
t_2
$(\psi_1 + \psi_2 - \psi_3)\frac{1}{\sqrt{3}}$
t_1
$(\psi_1 + \psi_2 + \psi_3)\frac{1}{\sqrt{3}}$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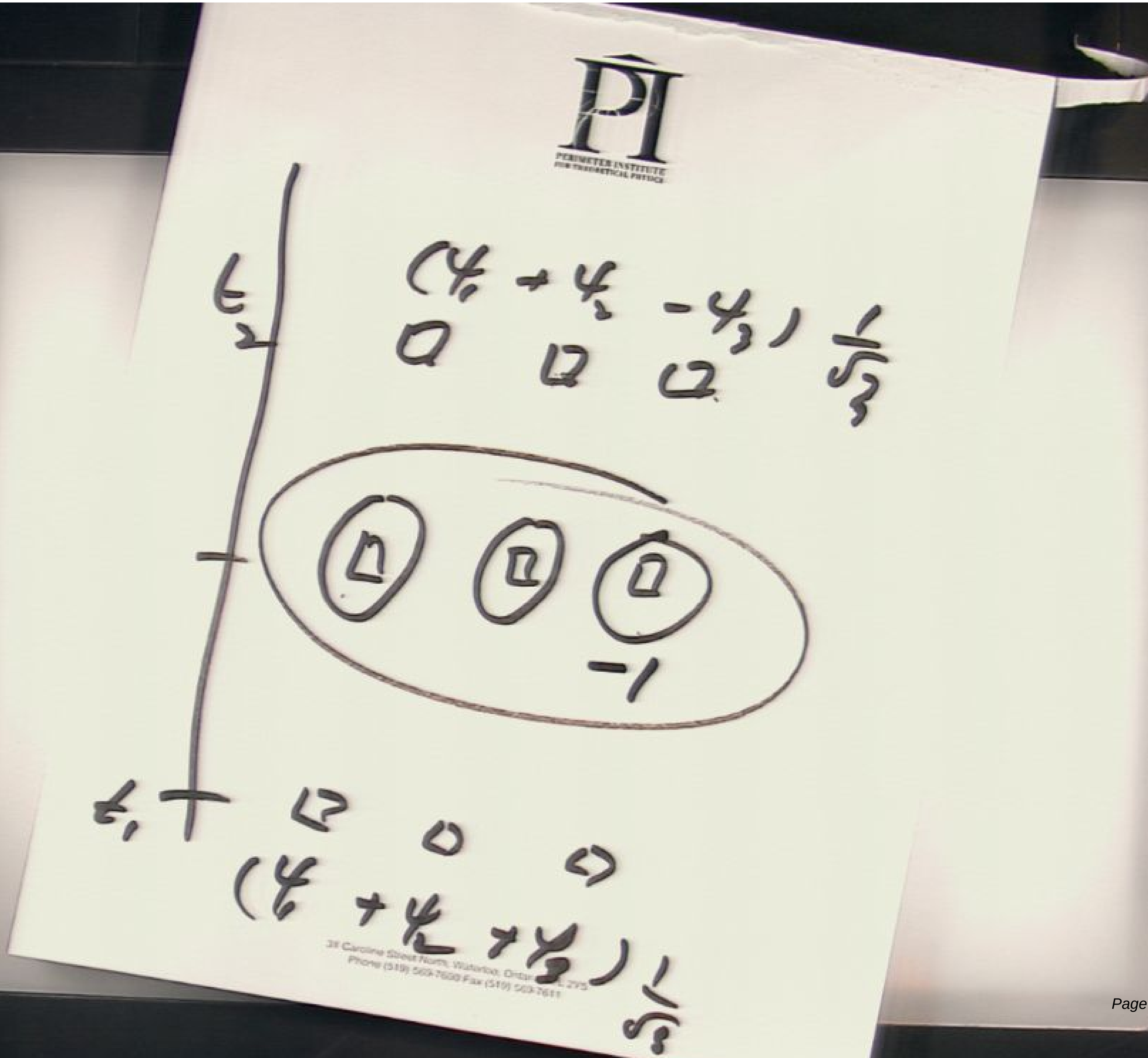
t_2
$(\psi_1 + \psi_2 - \psi_3)\frac{1}{\sqrt{3}}$
t_1
$(\psi_1 + \psi_2 + \psi_3)\frac{1}{\sqrt{3}}$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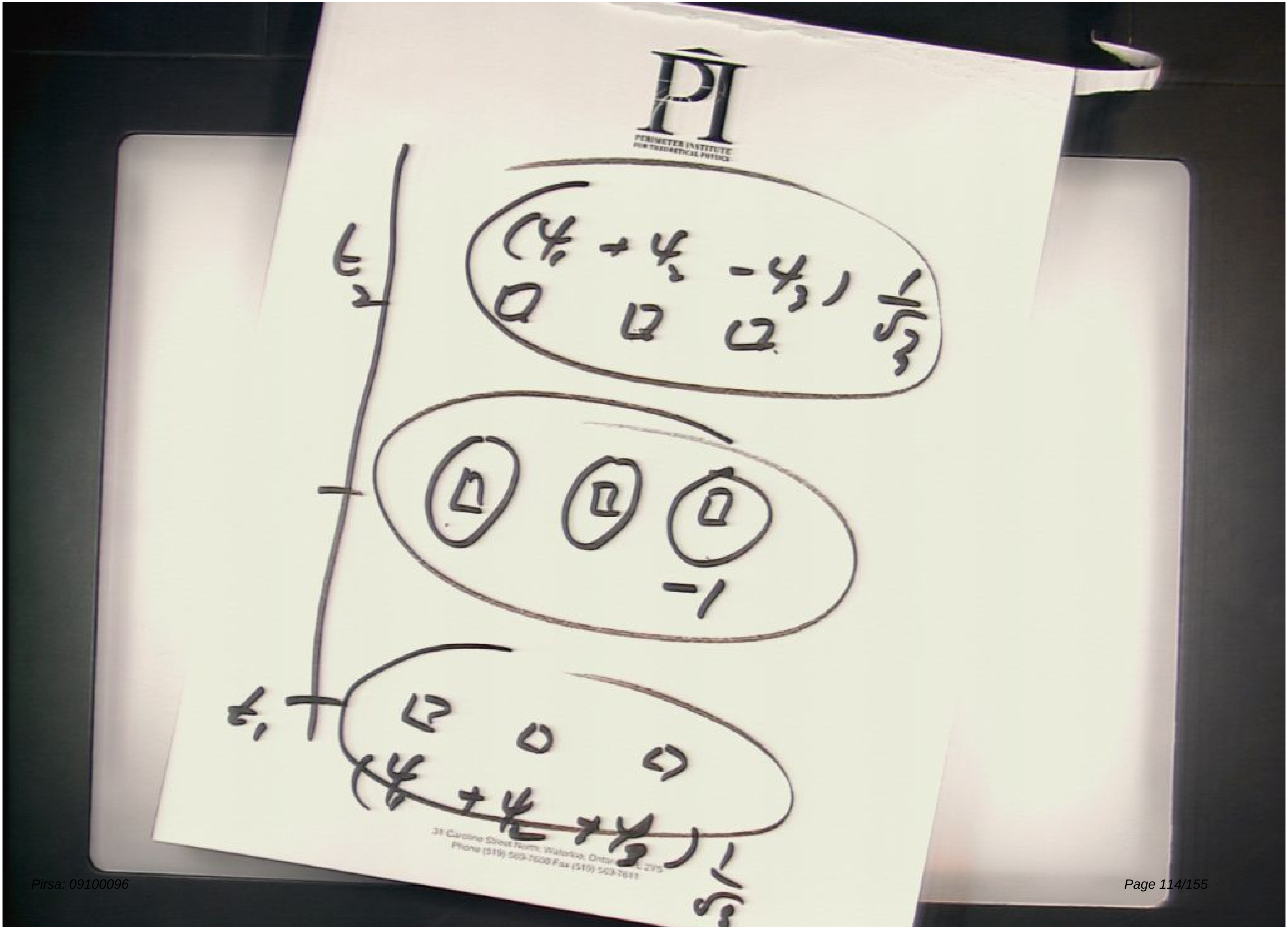
t2
(ψ1 + ψ2 − ψ3) 1/√3
−1
t1
(ψ1 + ψ2 + ψ3) 1/√3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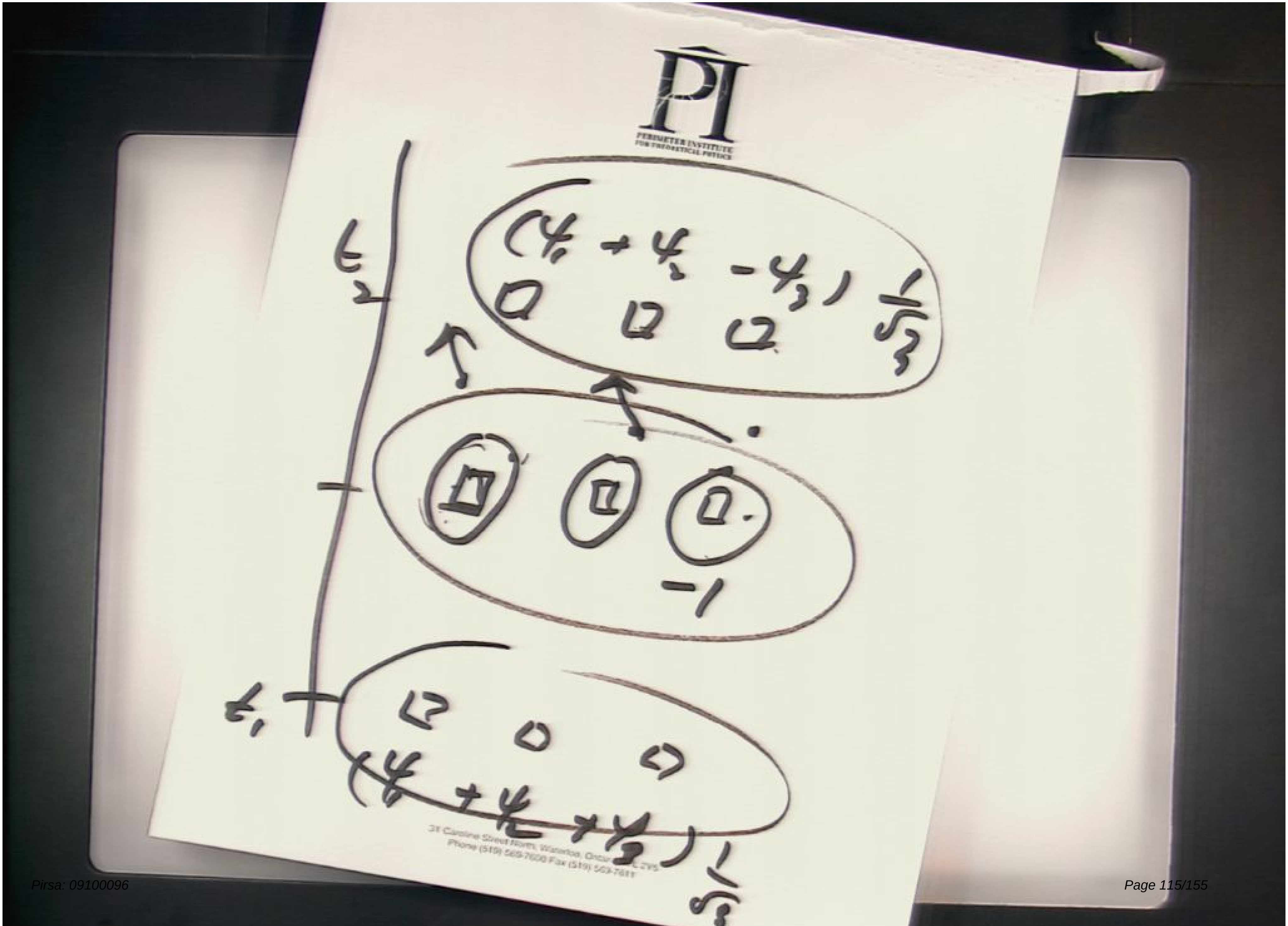
t2
t1
-1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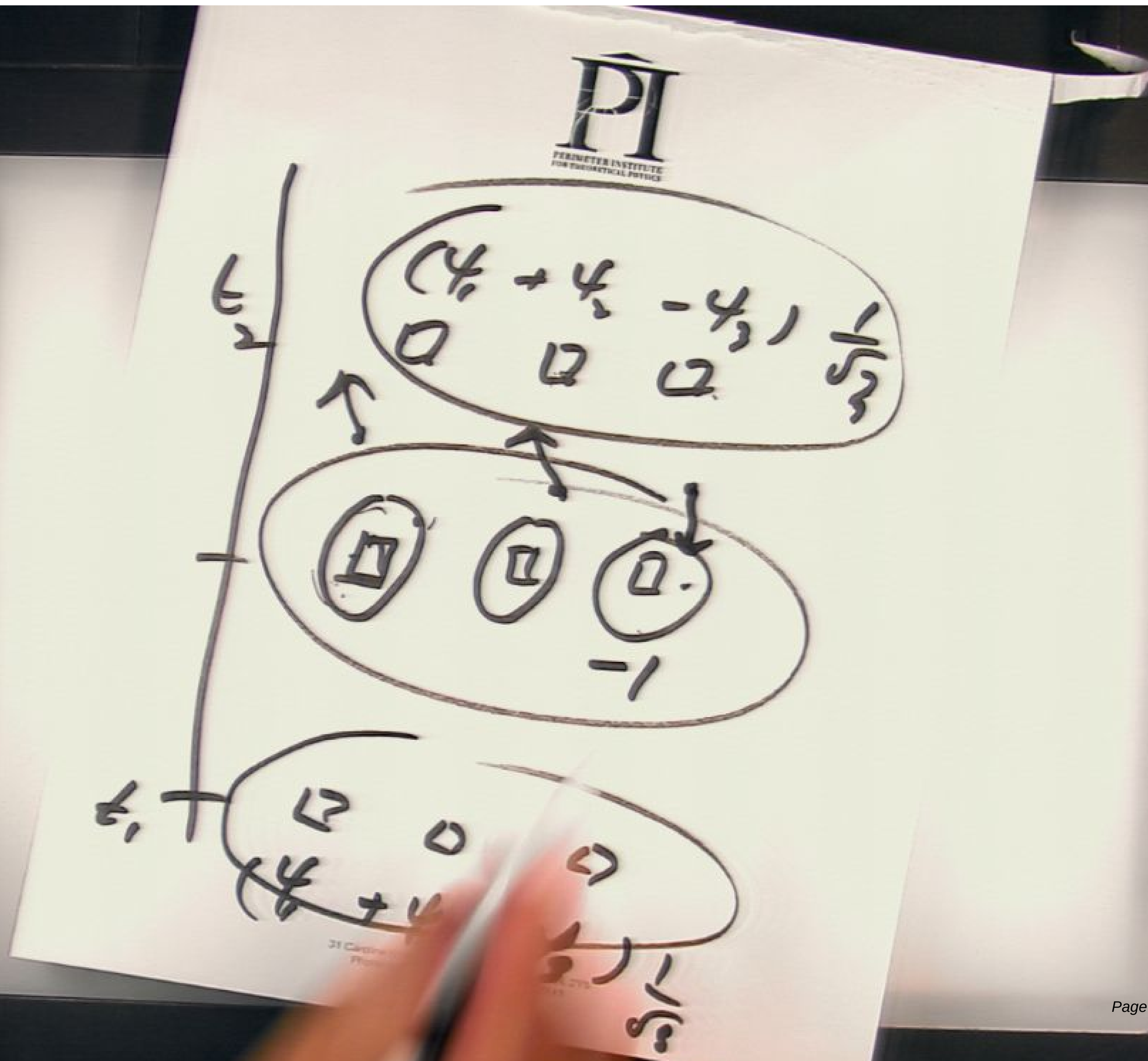
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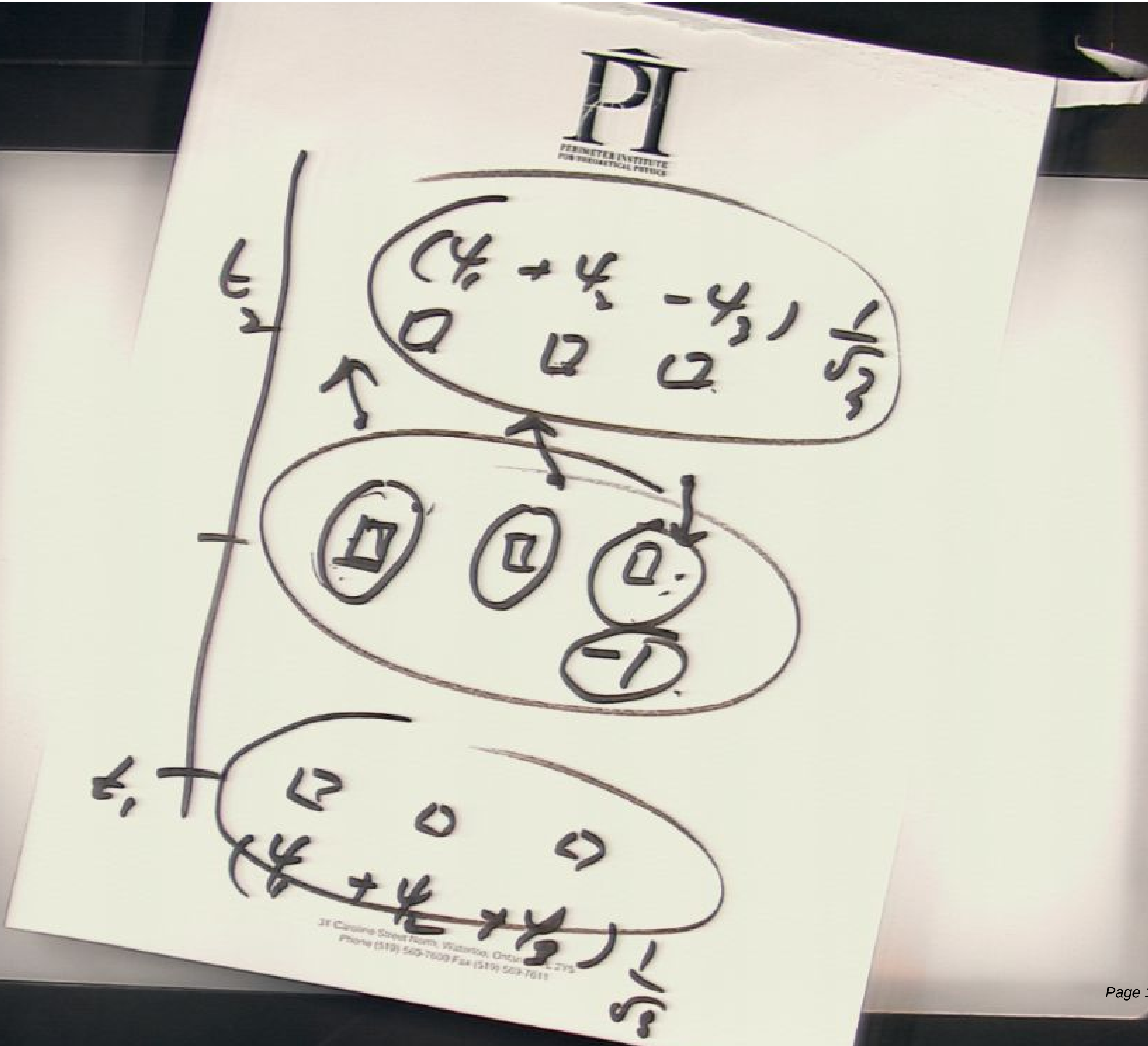
−1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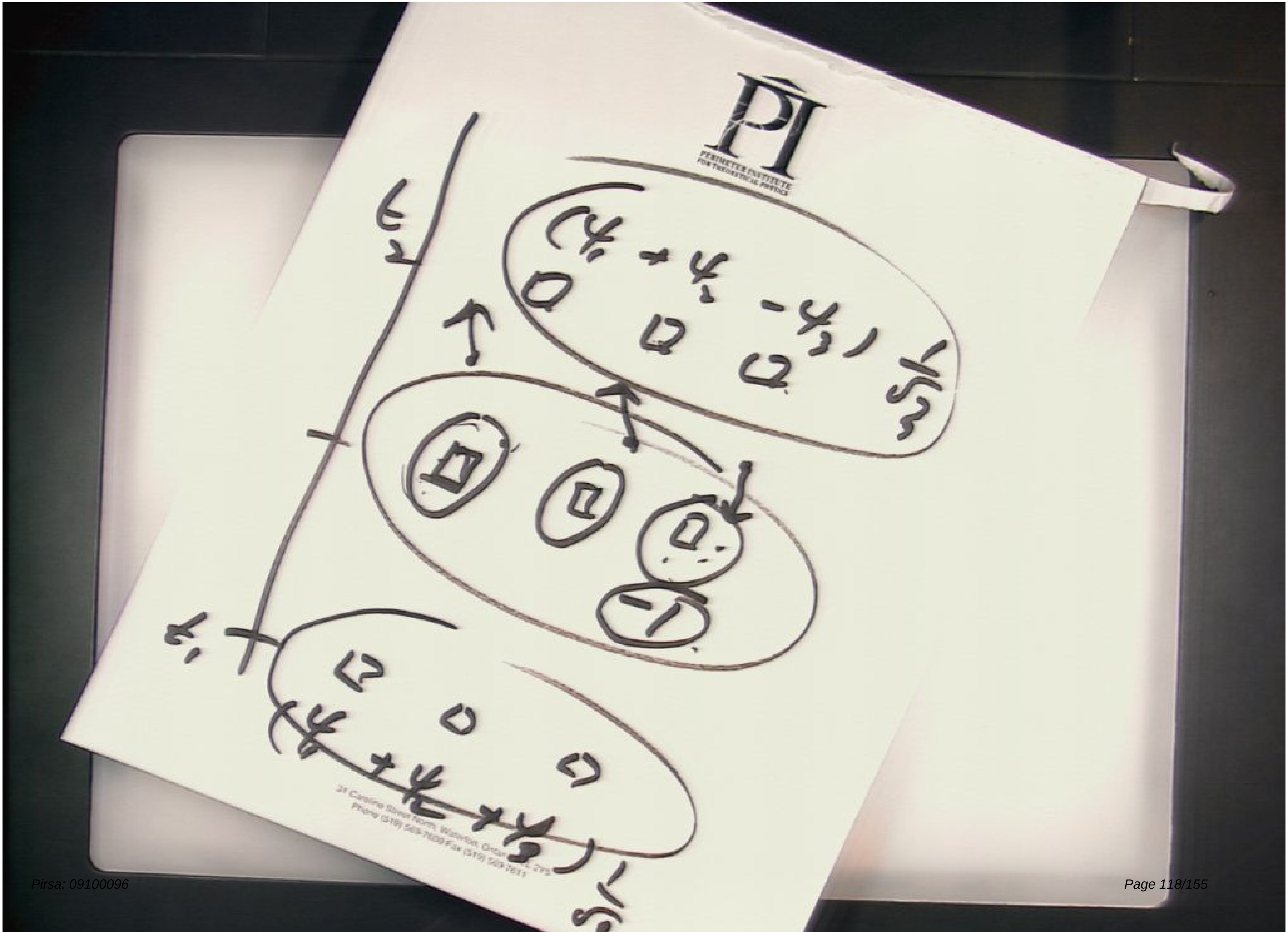
t2
(ψ1 + ψ2 − ψ3) 1/√3
−1
t1
(ψ1 + ψ2 + ψ3) 1/√3


PI
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t_2
t_1

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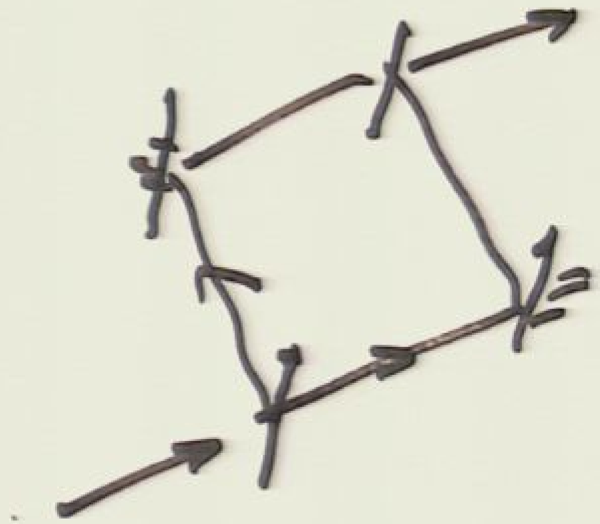

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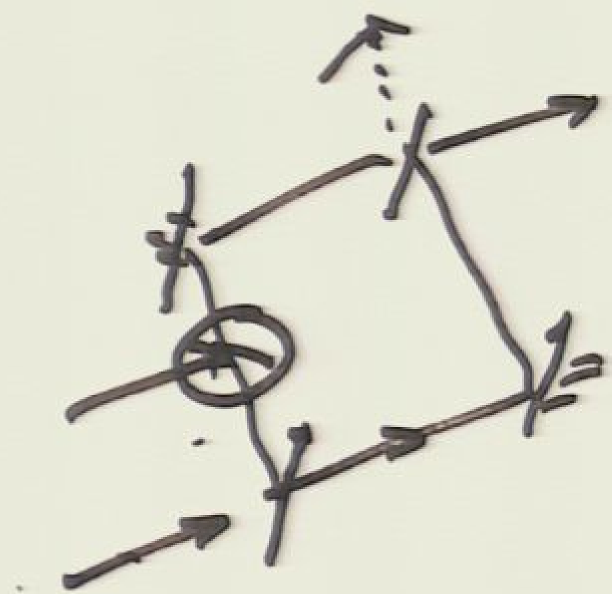

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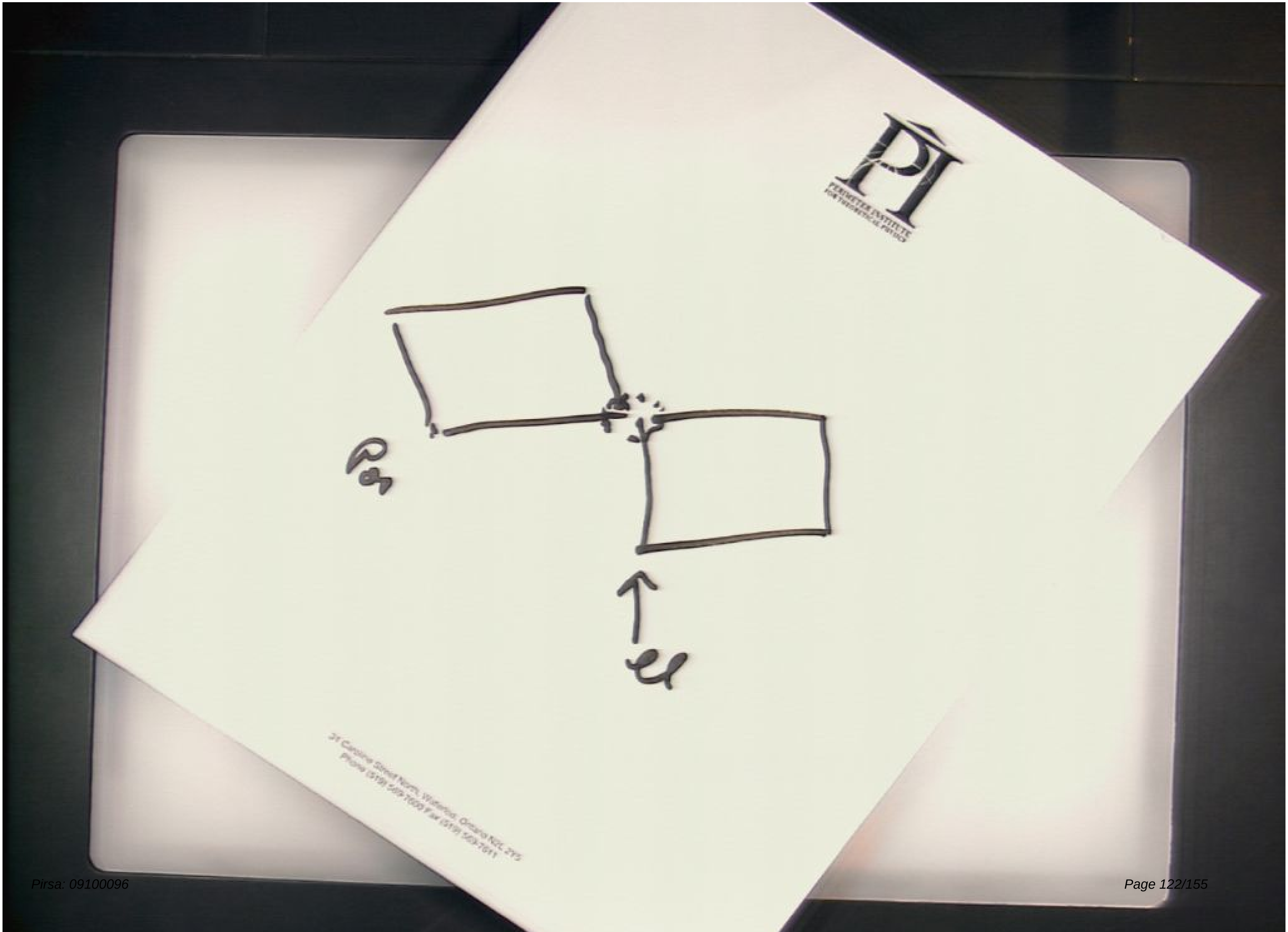
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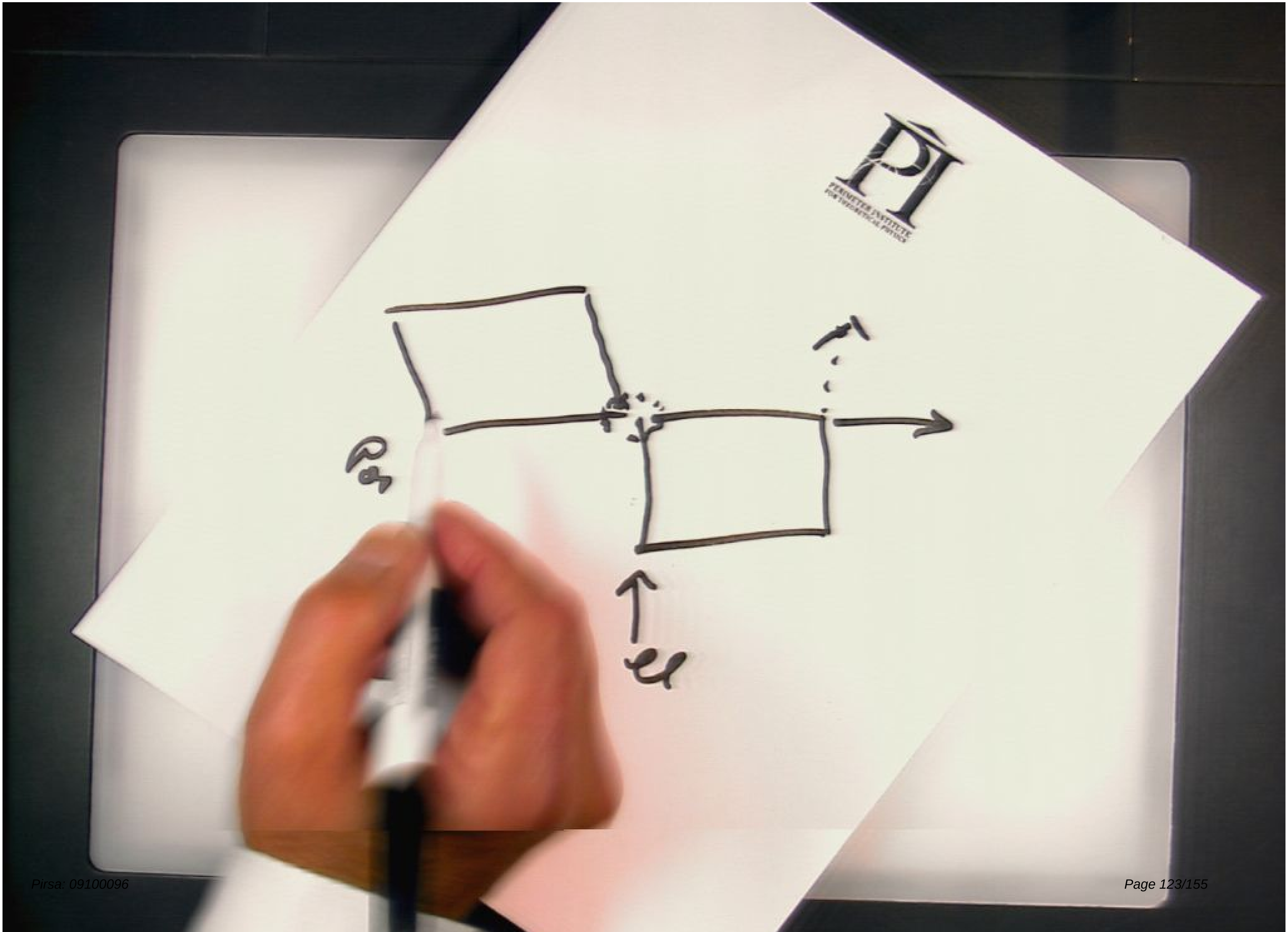

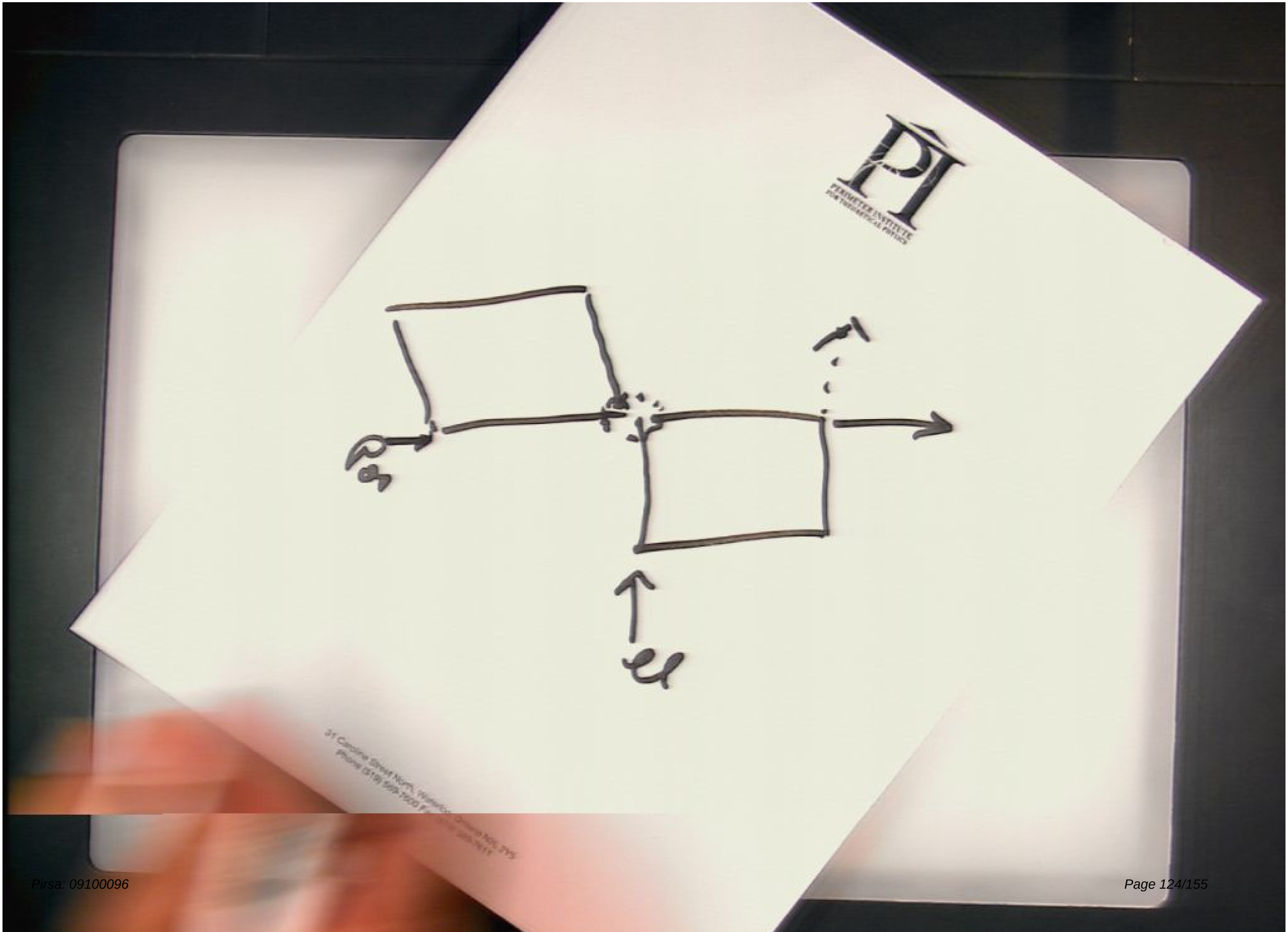

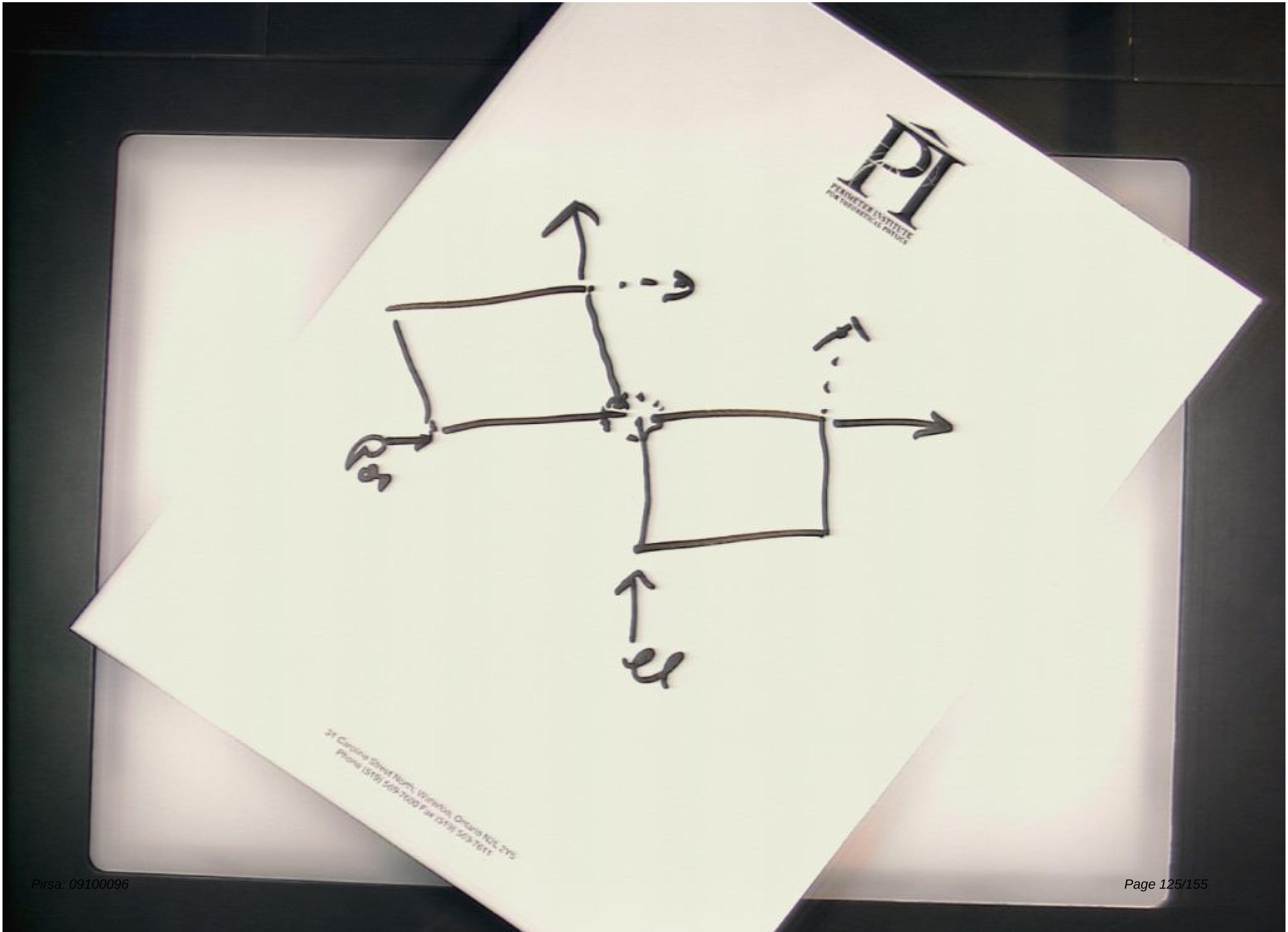

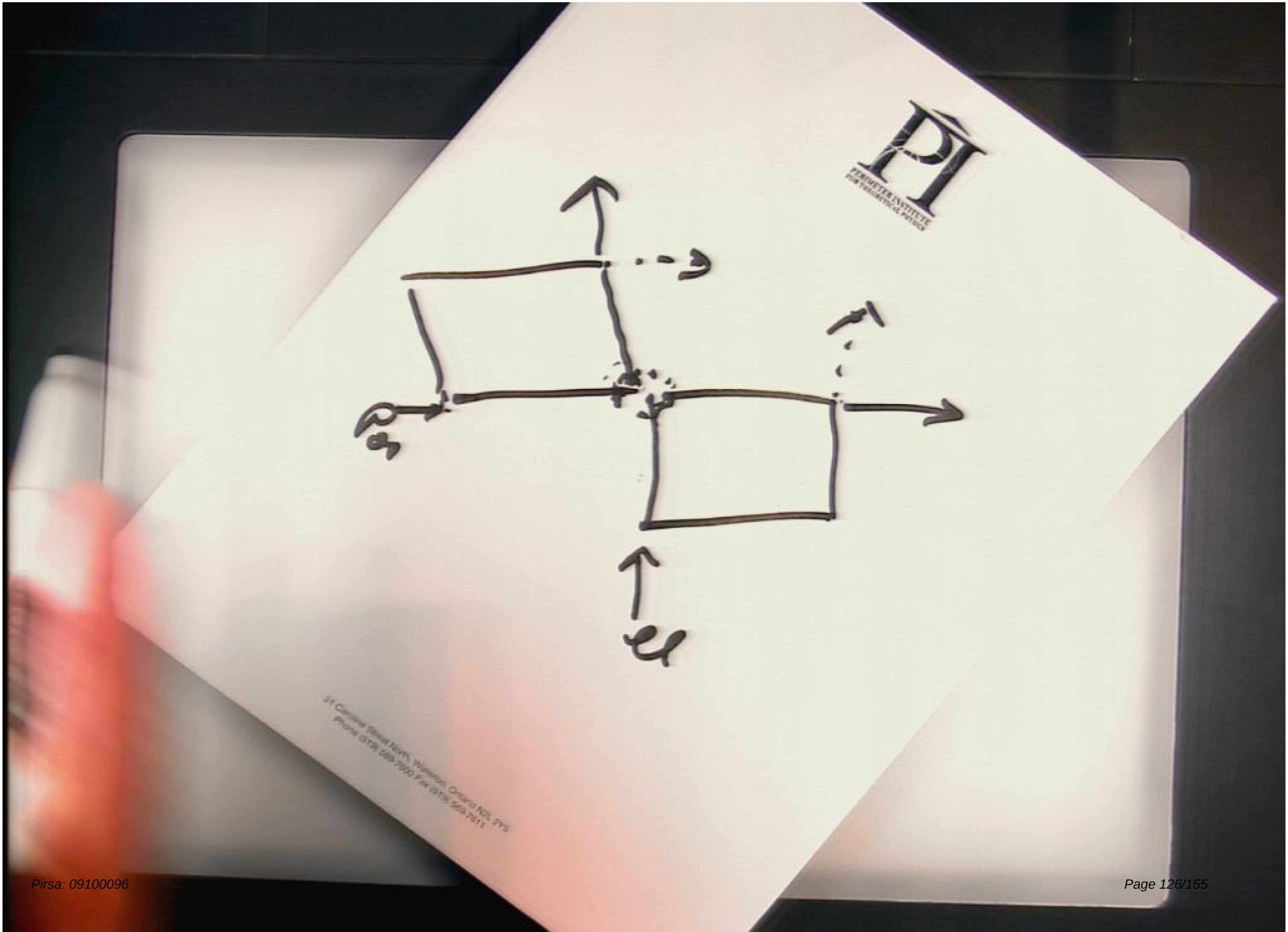
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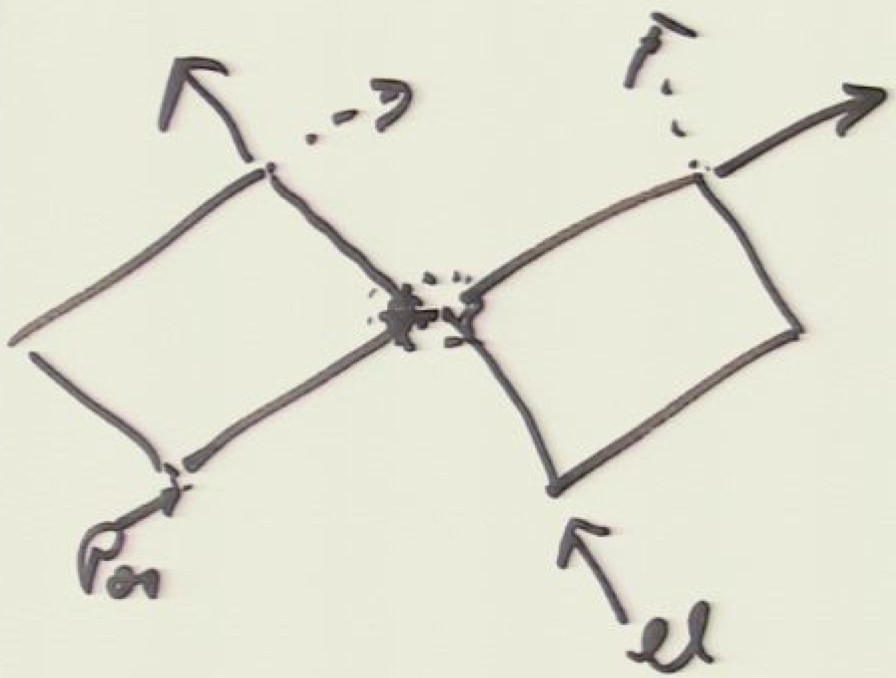

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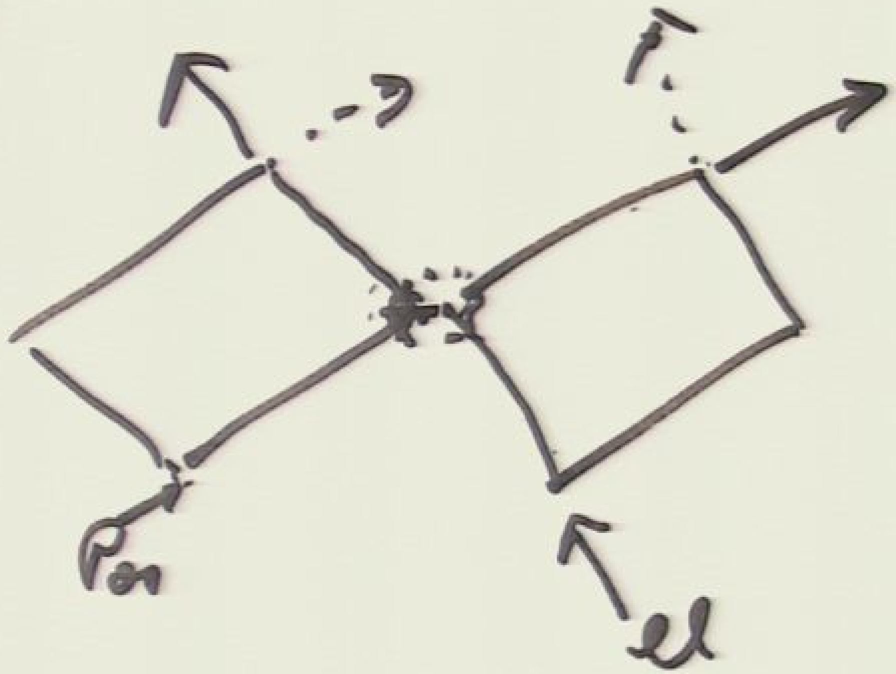

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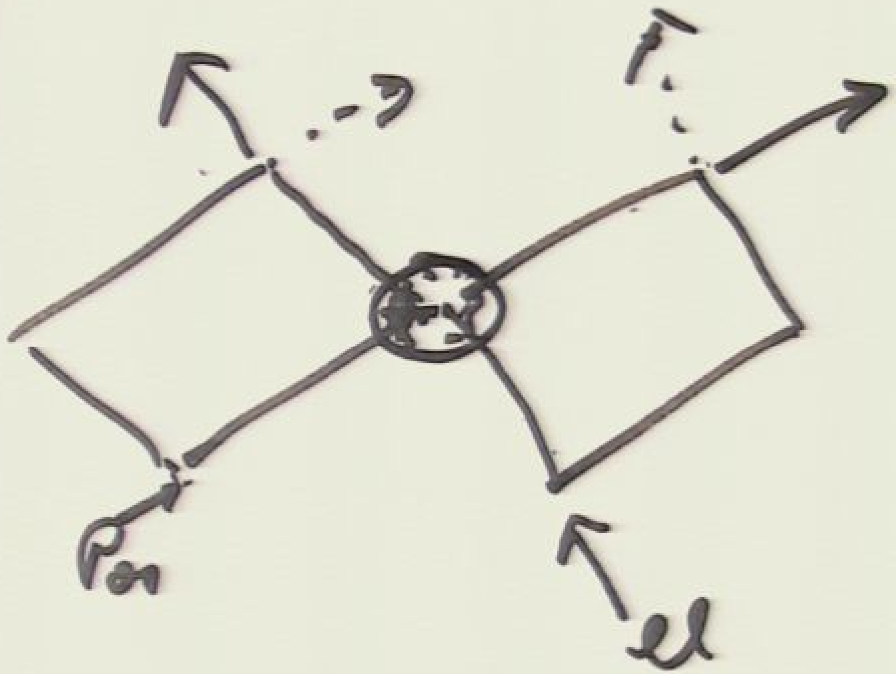

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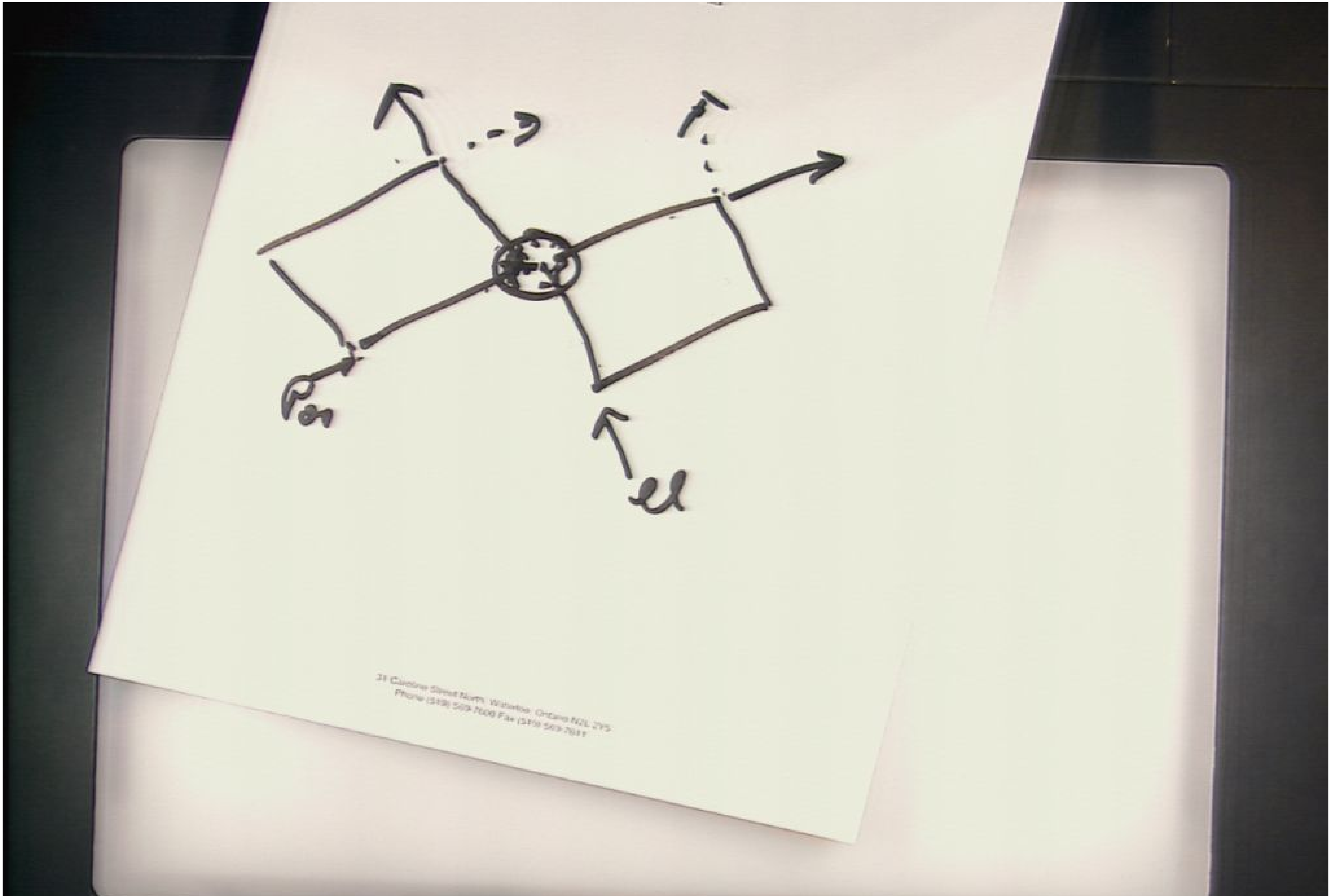
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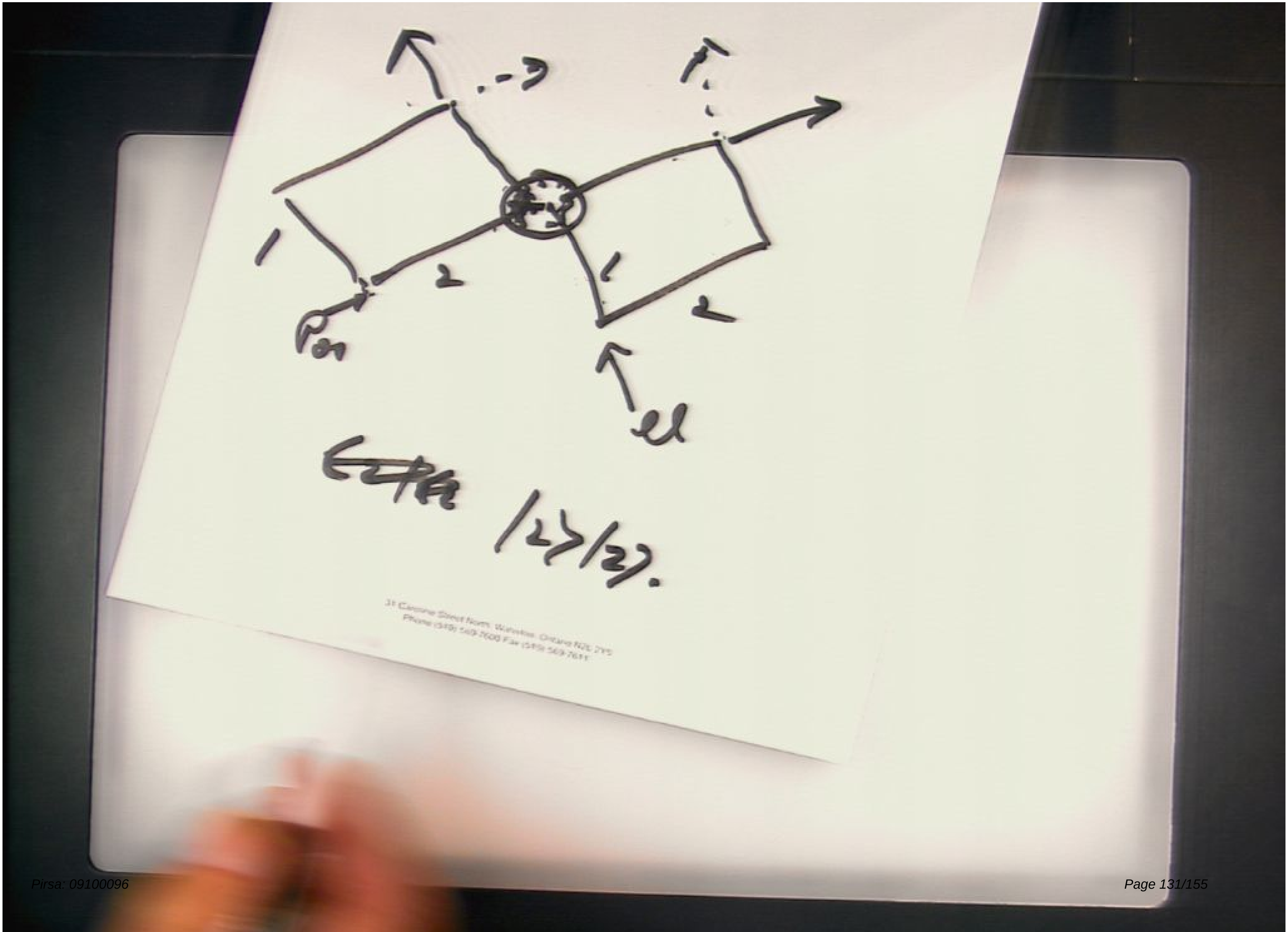
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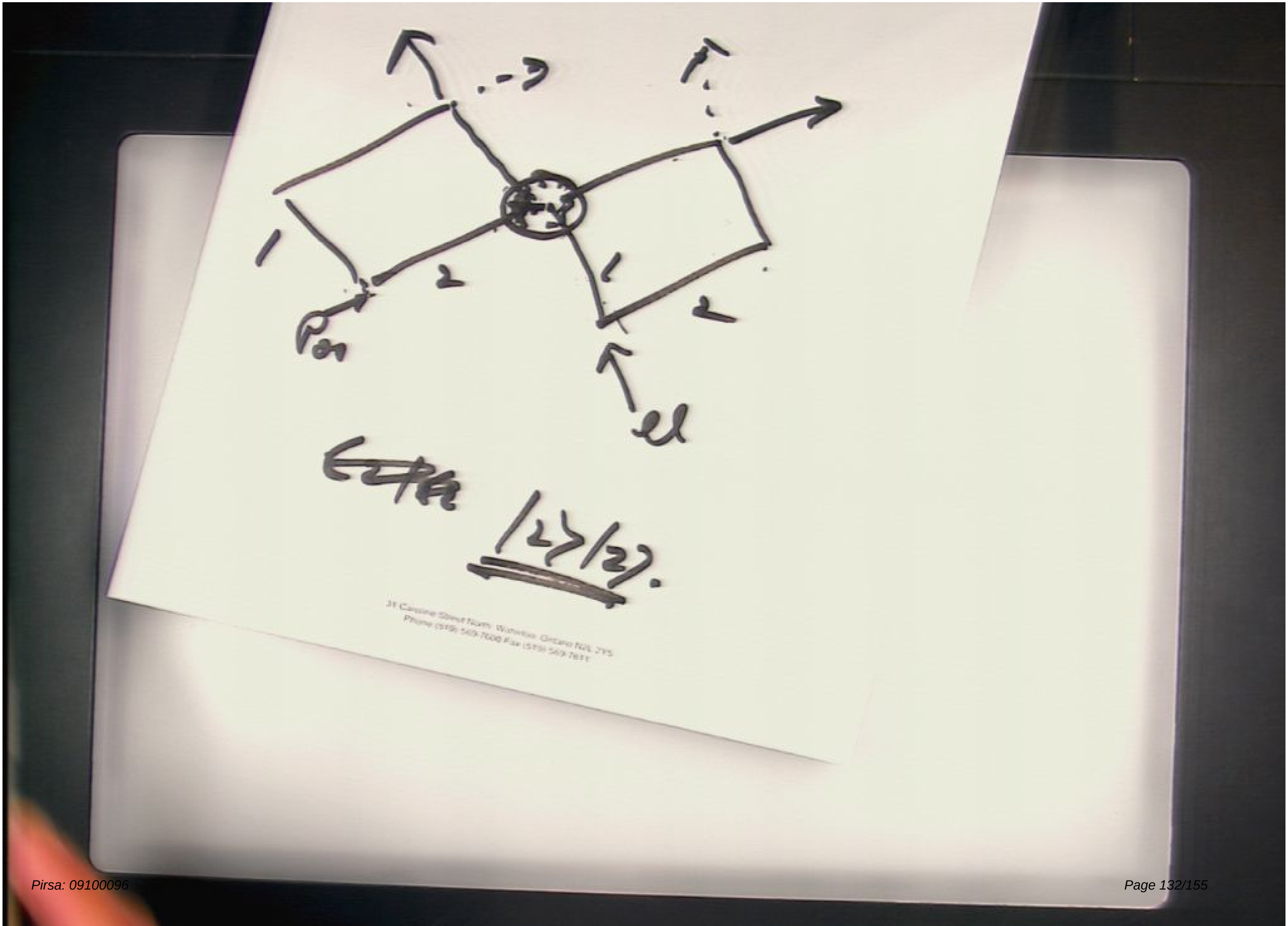
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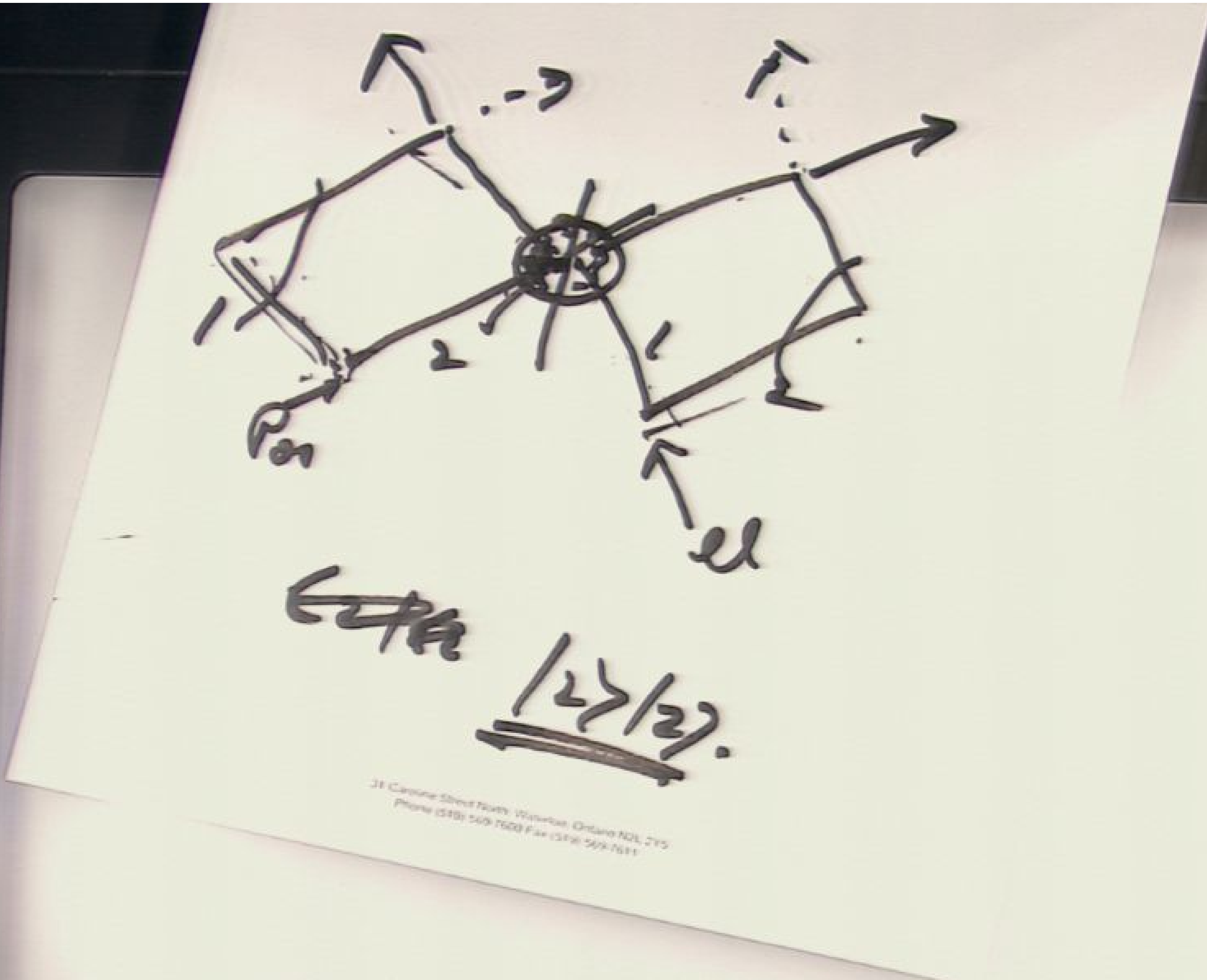

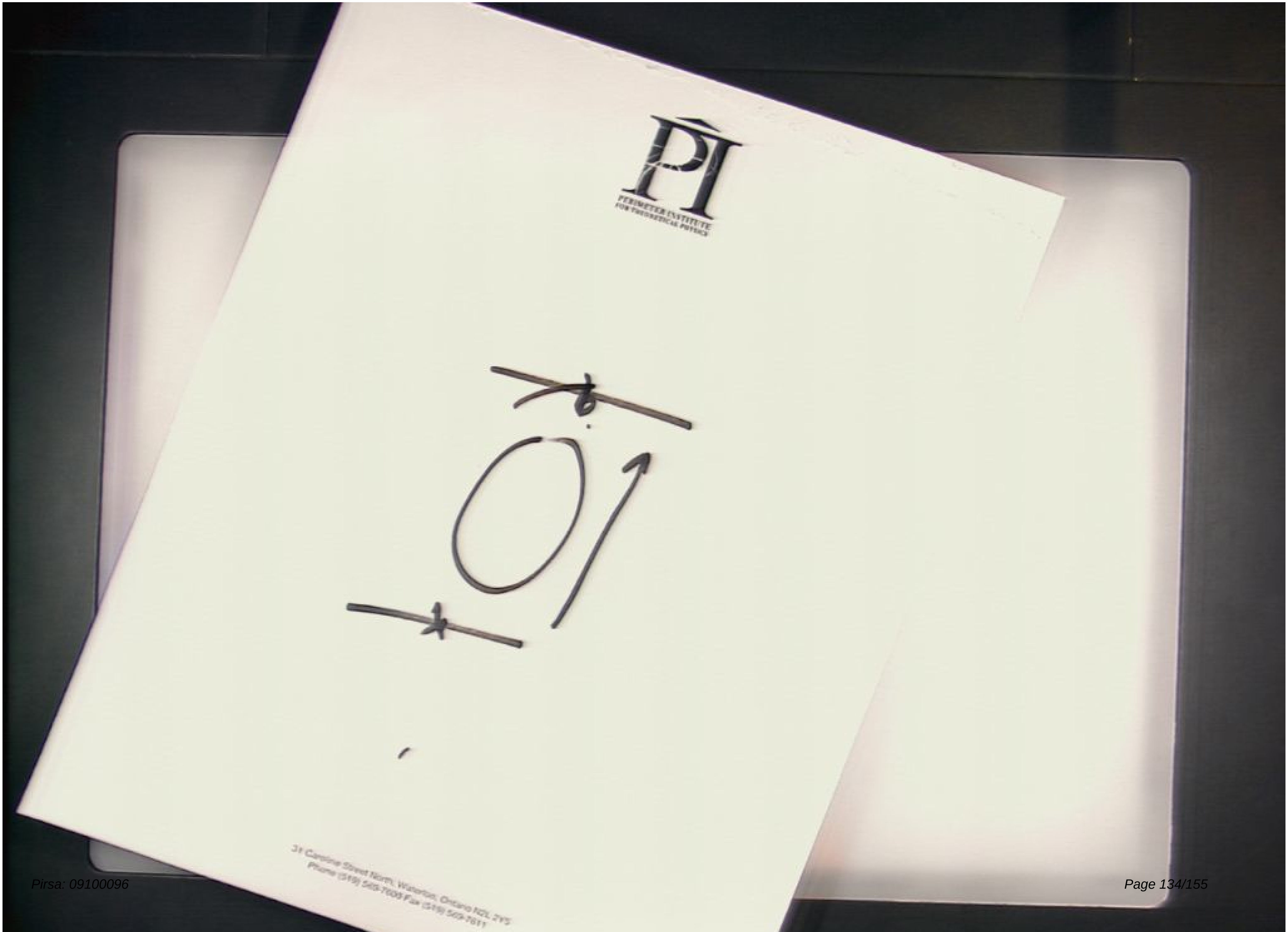
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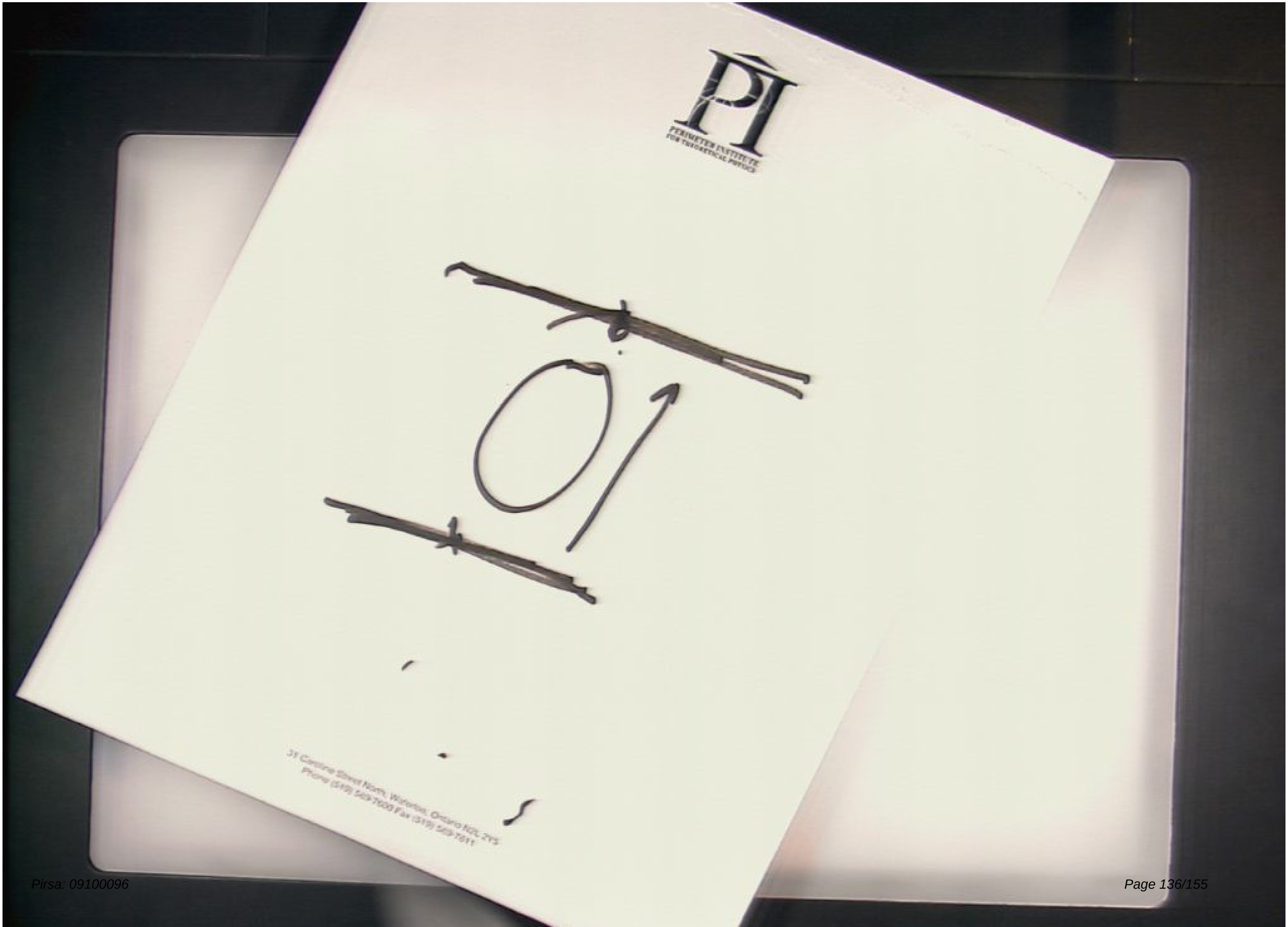
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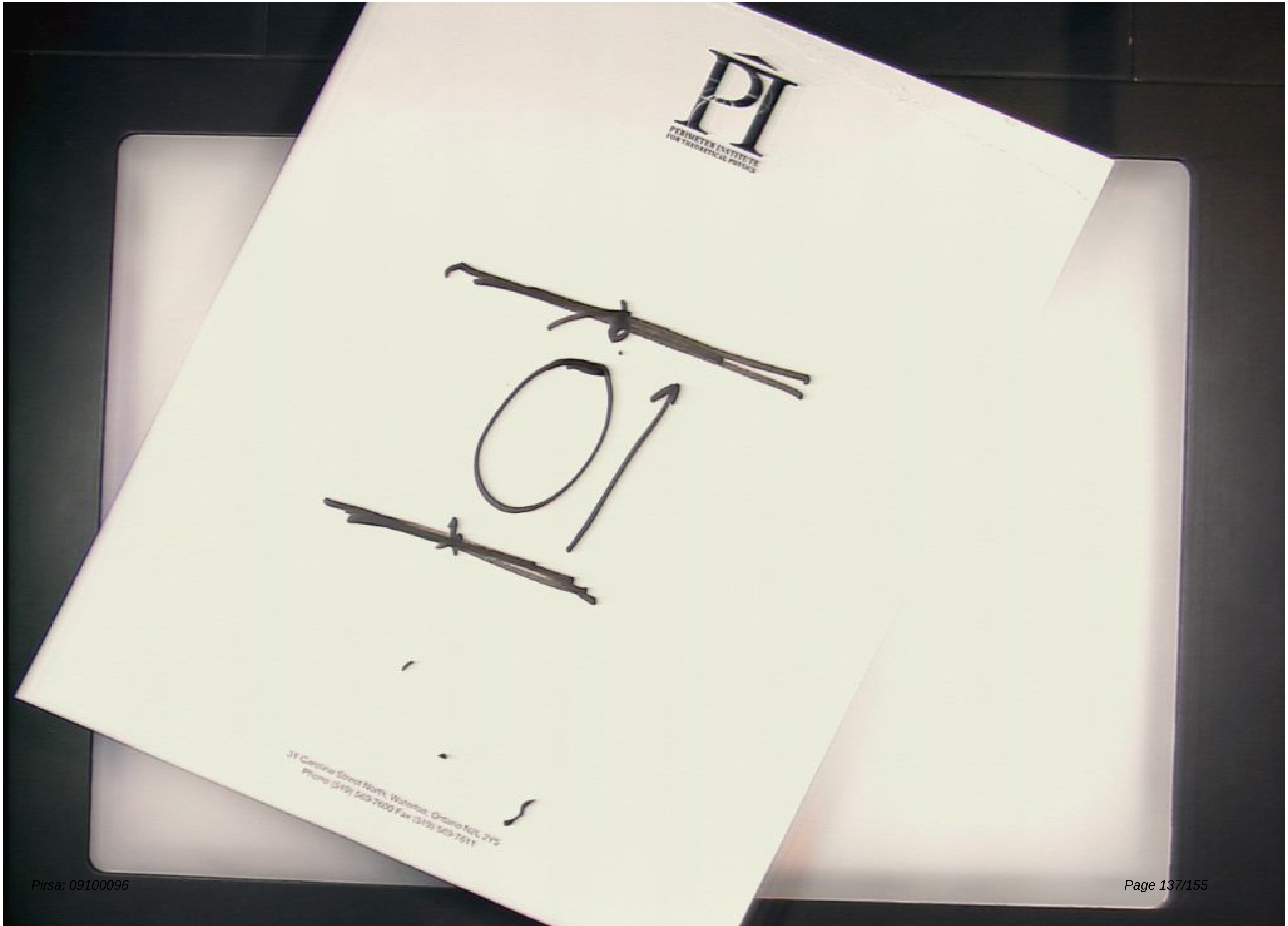
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$$\vec{A}\,|\Psi_1\rangle = a\,|\Psi_1\rangle$$

$$\vec{B}\,|\Psi_2\rangle = b\,|\Psi_2\rangle$$

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$$\vec{A}\,|\Psi_1\rangle = a\,|\Psi_1\rangle$$

$$\vec{B}\,|\Psi_2\rangle = b\,|\Psi_2\rangle$$

$$(\vec{A}+\vec{B})_\omega = a+b.$$

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$$\hat{\vec{A}}\,|\Psi_1\rangle = a\,|\Psi_1\rangle$$

$$\vec{B}\,|\Psi_2\rangle = b\,|\Psi_2\rangle$$

$$(\vec{A} + \vec{B})_\omega = a + b.$$

$$\left[\frac{(x - x_0)^2}{2} + \frac{p^2}{2}\right]|\Psi_1\rangle = \frac{1}{2}\,|\Psi_1\rangle$$

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$$|\Psi_2\rangle = \ell\,|\Psi_2\rangle \qquad |\Psi_1\rangle$$

$$\frac{(\vec{A} + \beta)\,\omega}{} = a + \ell .$$

$$\left[\frac{(x - x_1)^2}{2} + \frac{\rho^2}{2}\right] |\Psi_1\rangle = \frac{1}{2}\,|\Psi_1\rangle$$

$$\frac{(x - x_2)^2}{2} + \frac{\rho^2}{2} = \frac{1}{2}$$

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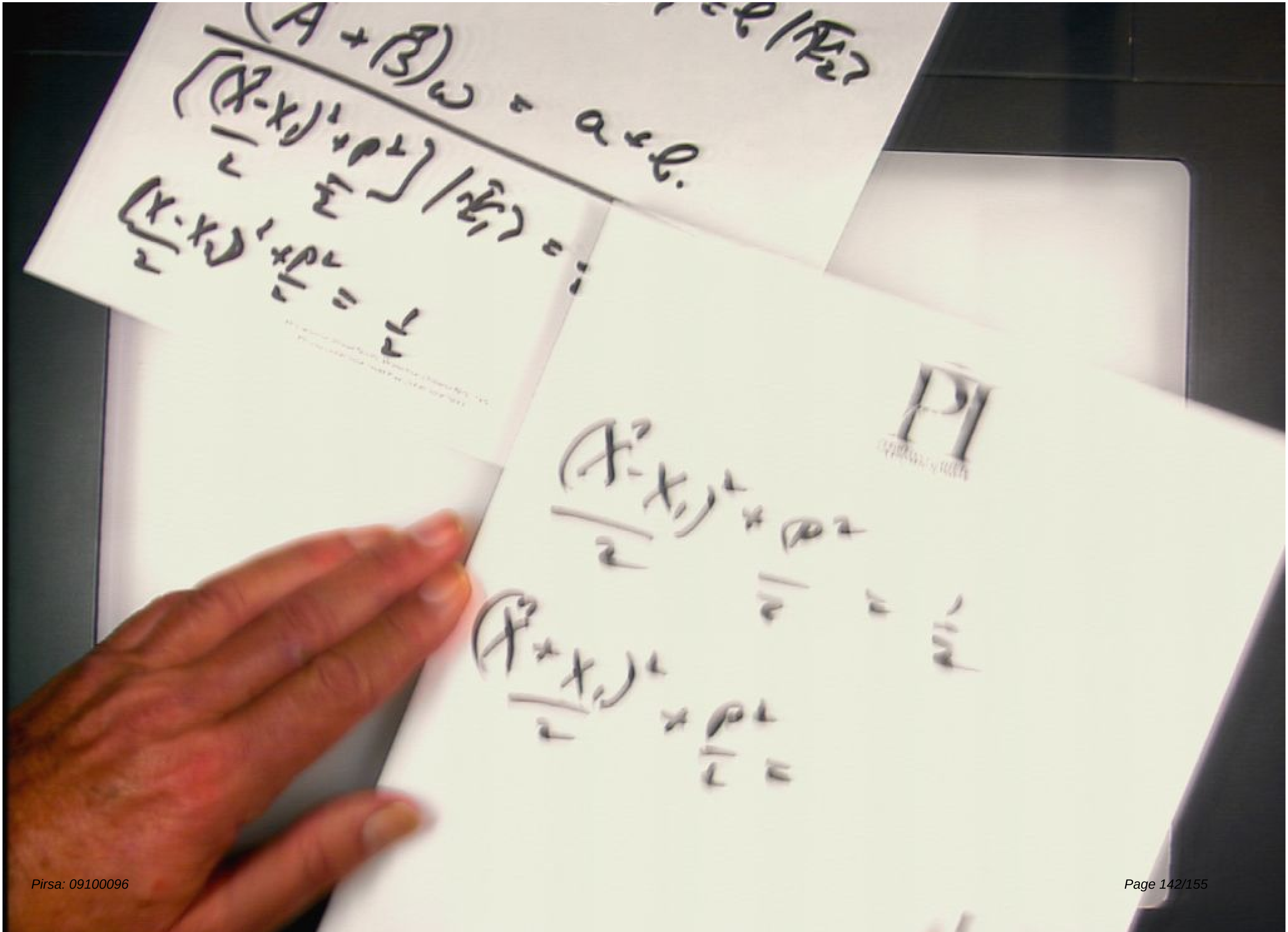

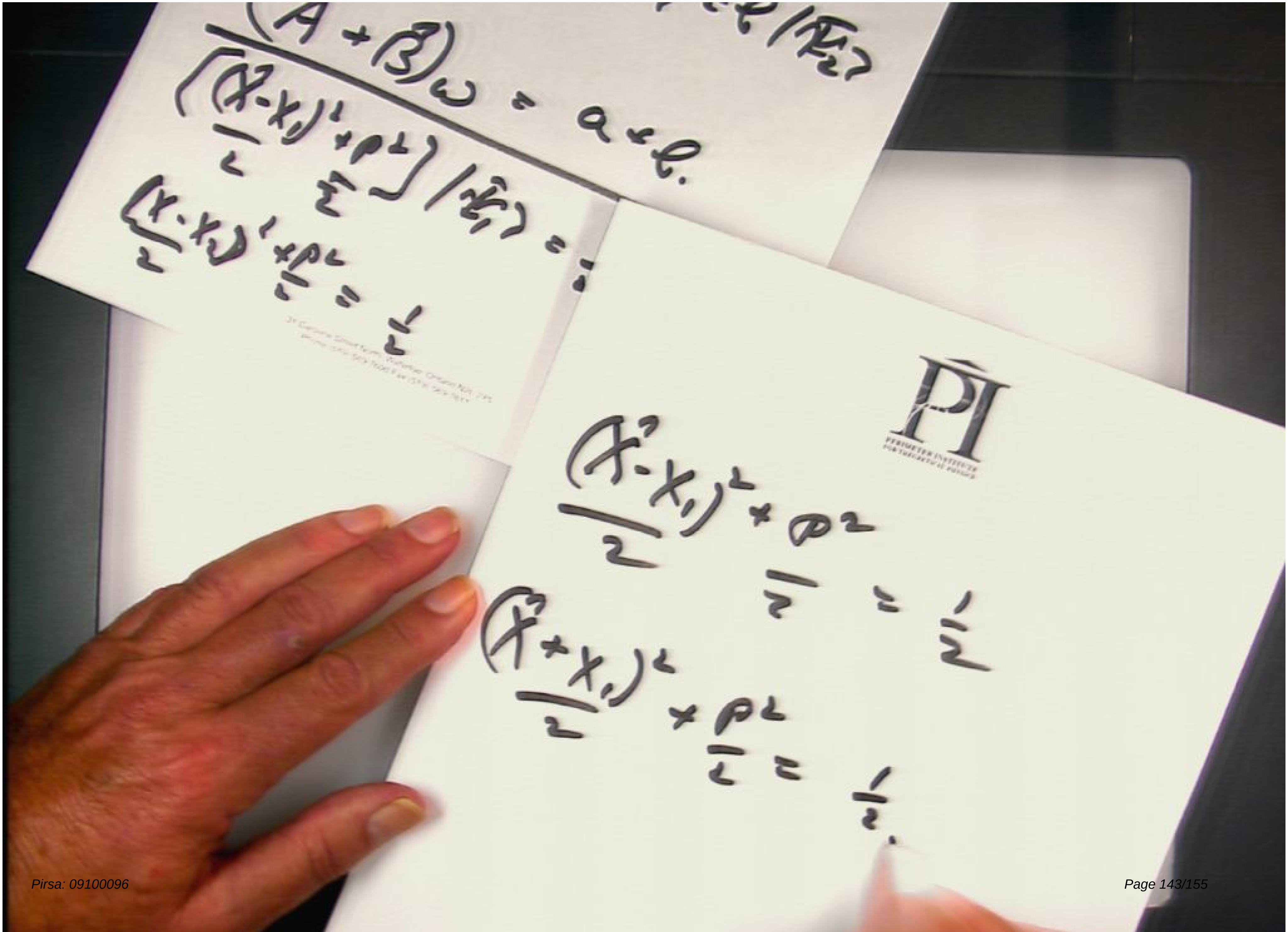

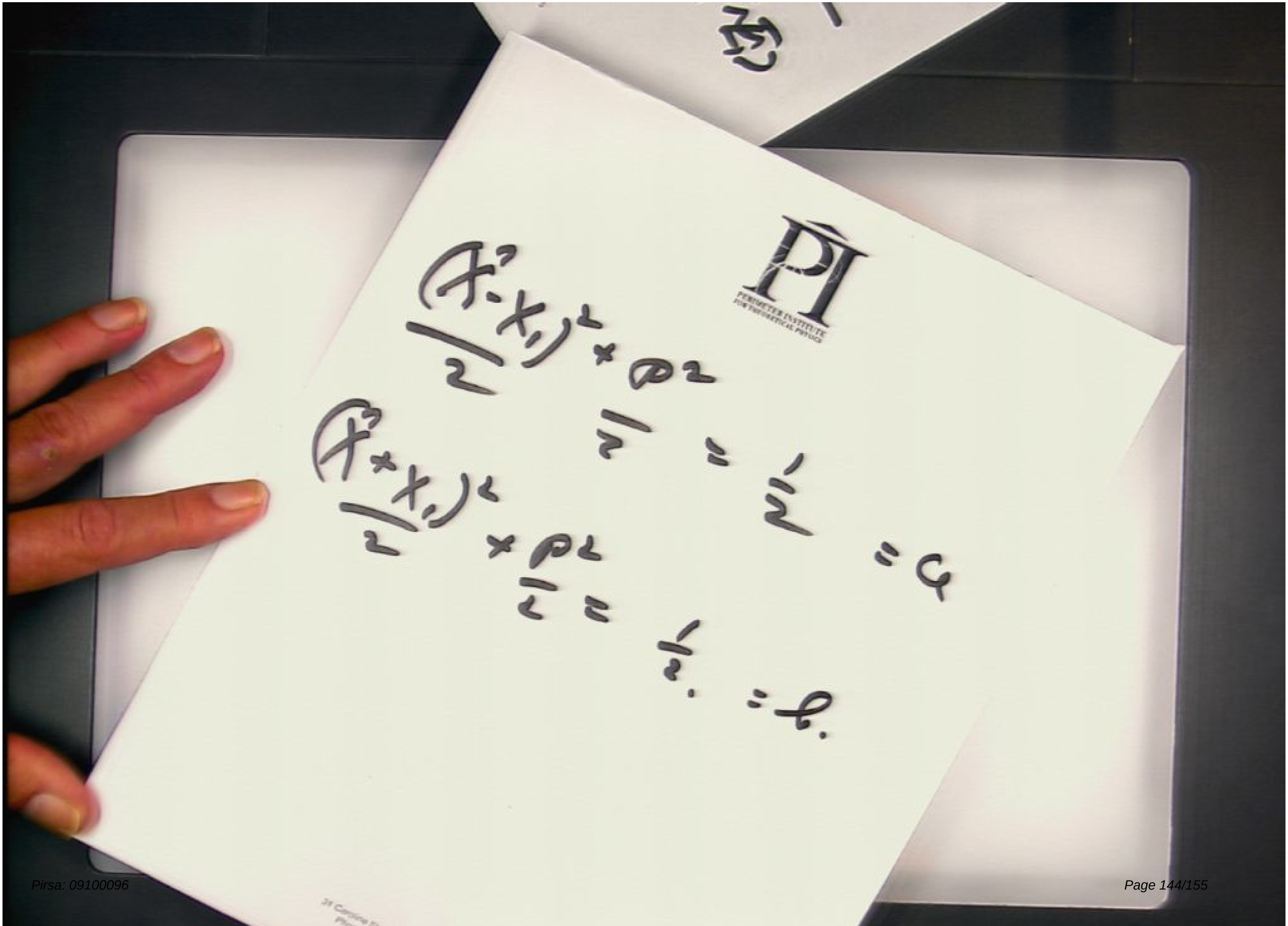
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$\frac{(x_1 - x_2)^2}{2} + \frac{p^2}{2} = \frac{1}{2} = a$
$\frac{(x_1 + x_2)^2}{2} + \frac{p^2}{2} = \frac{1}{2} = b$

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$$\frac{(x - x_1)^2}{2} + \frac{\rho^2}{2} = \frac{1}{2} = a$$

$$\frac{(x + x_1)^2}{2} + \frac{\rho^2}{2} = \frac{1}{2} = b.$$

$$x^2 + \rho^2 + x_1^2 = 1.$$

$$x^2 + \rho^2 = 1 - x_1^2$$

$H_1 + H_2$

$$H_1 + H_2 + \mathcal{H}(1,2).$$

$$H_1 + H_2 + V(1,2).$$

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$$H_1 + H_2 + V(1,2).$$

$|E_1\rangle$

$\overline{|E_1\rangle}$

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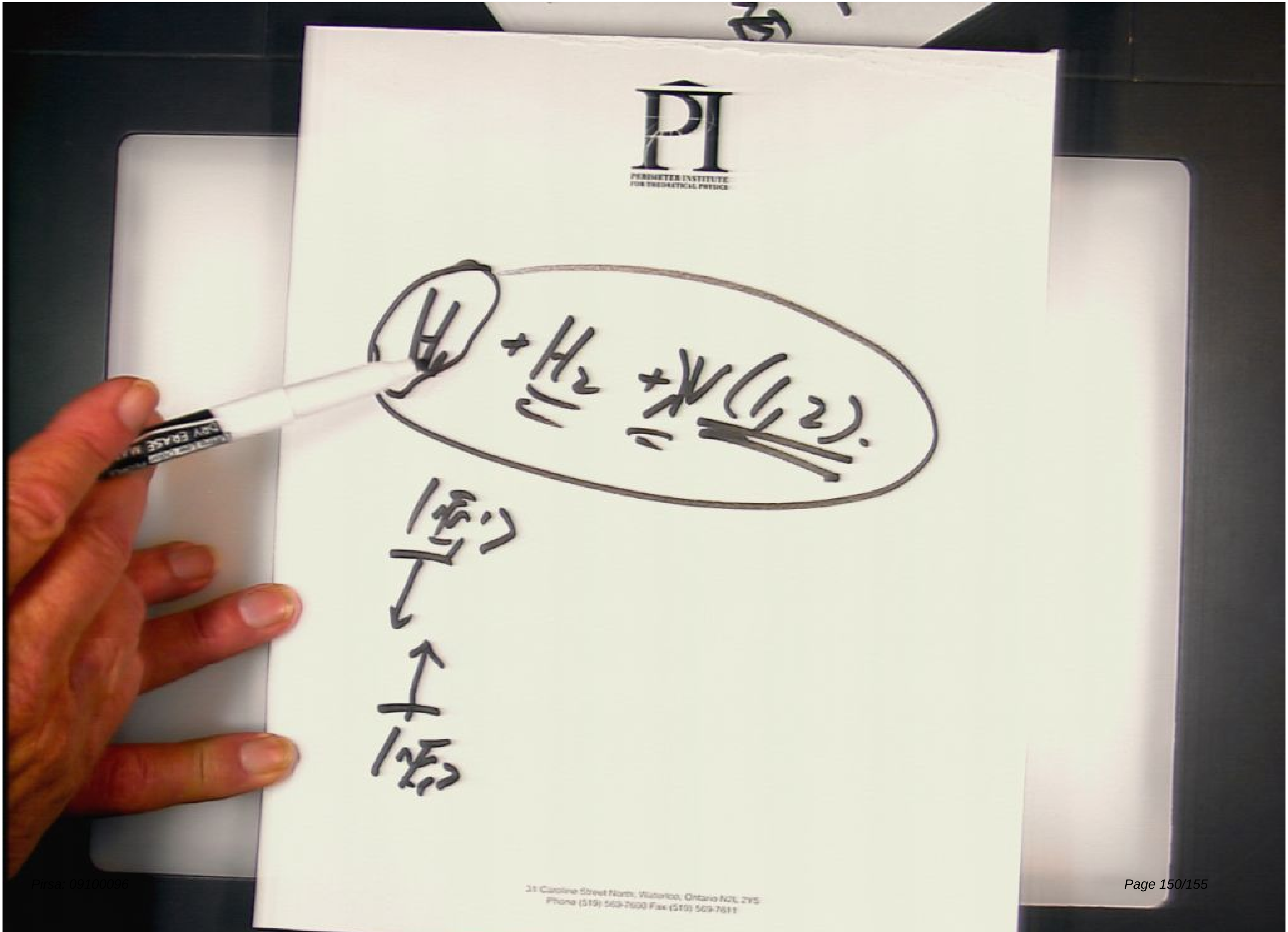
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$H_1 + H_2 + V(1,2)$.
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$|E_2\rangle$
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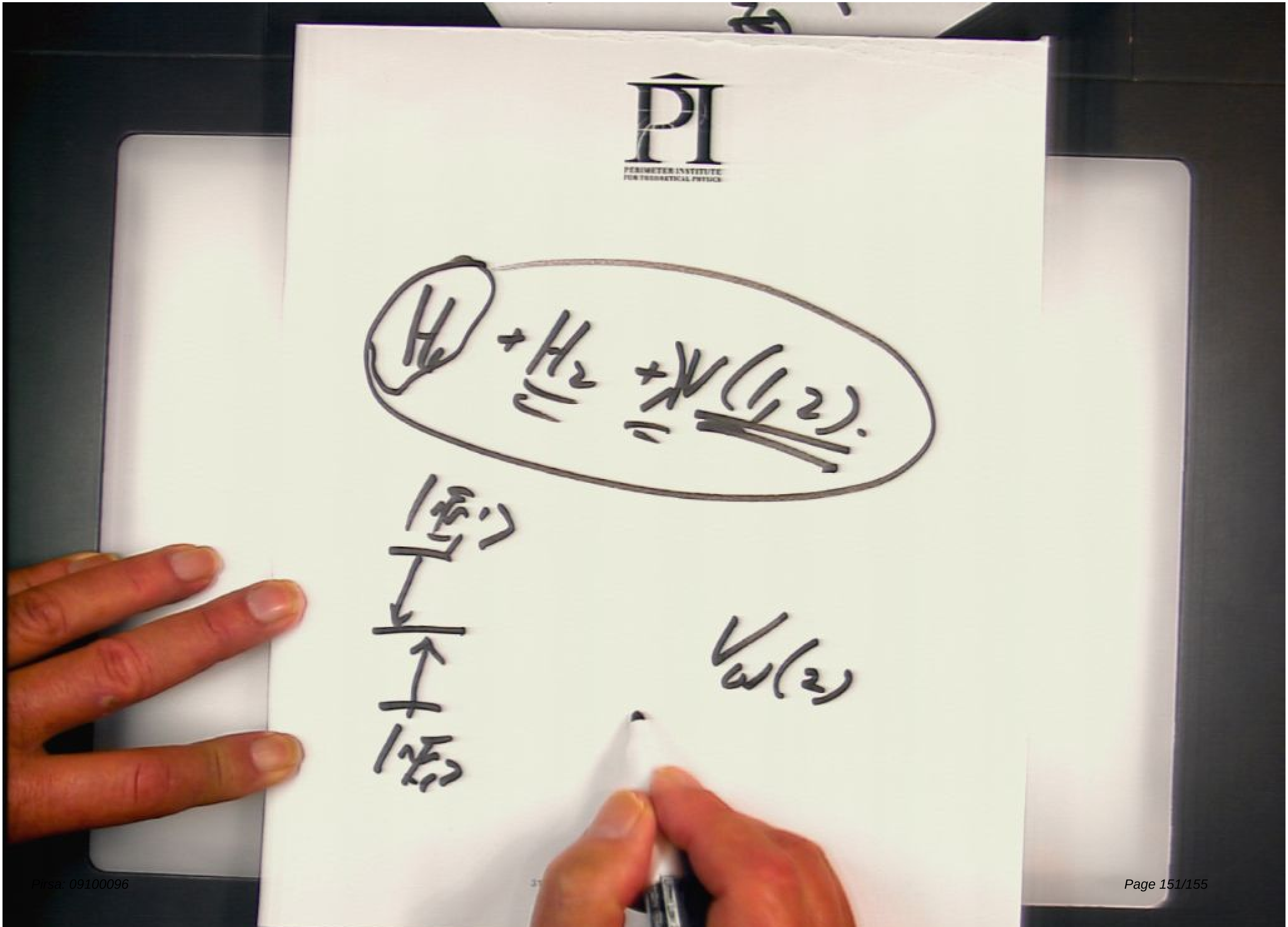
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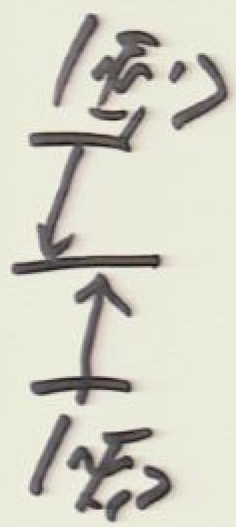
H_0 + H_2 + V(1,2).
V_ω(2)

$$H_1 + H_2 + V(1,2)$$



$$V_{\omega}(2) = \frac{\langle \Psi_1'(1) | V | \Psi_1(1) \rangle}{\langle \Psi_1 | \Psi_1 \rangle}$$

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$$H_1 + H_2 + V(1,2).$$

$|\Psi_1\rangle$

$$V_{eff}(2) = \frac{\langle \Psi_1'(1) | V | \Psi_1(1) \rangle}{\langle \Psi_1' | \Psi_1 \rangle}$$

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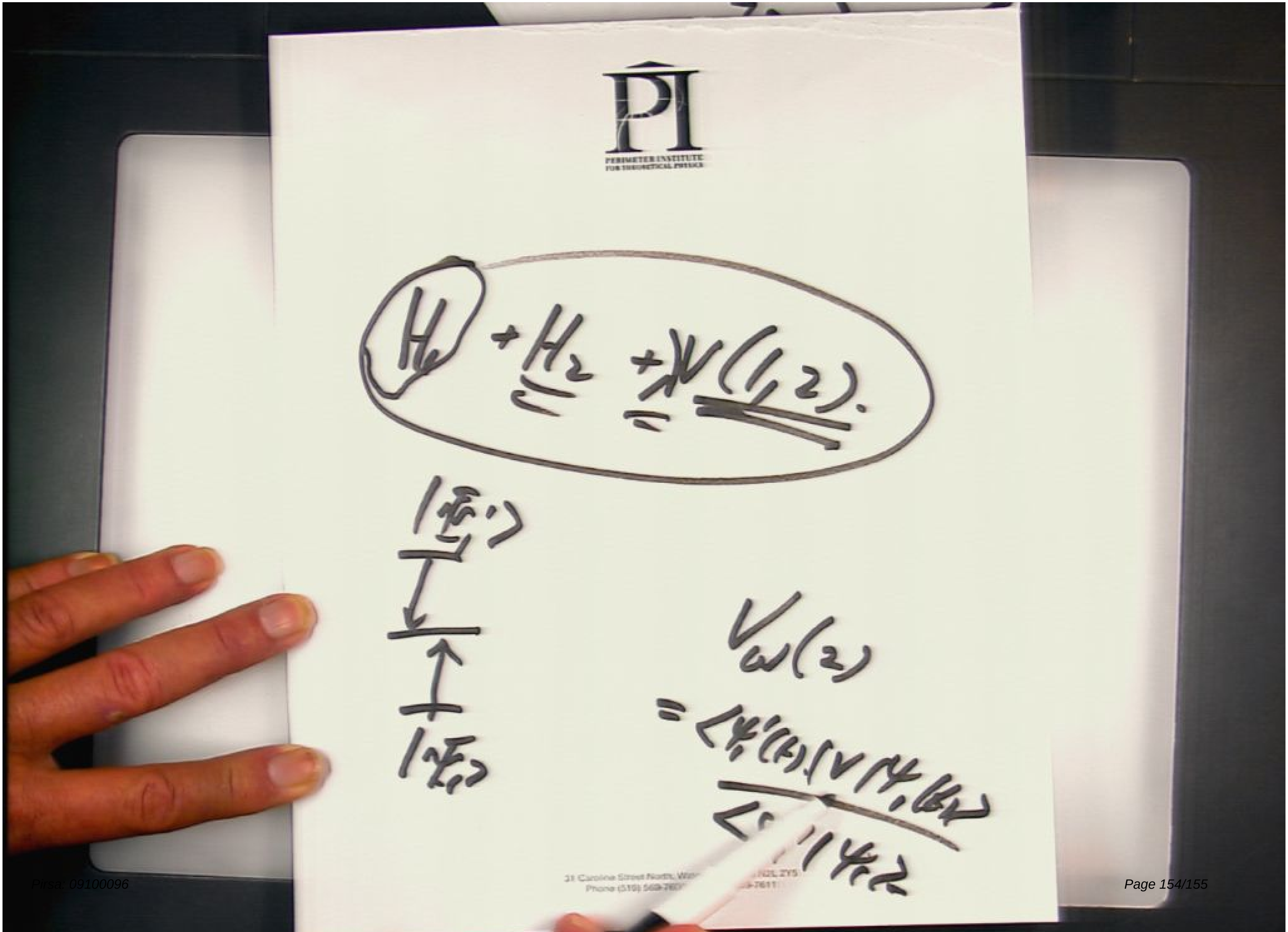
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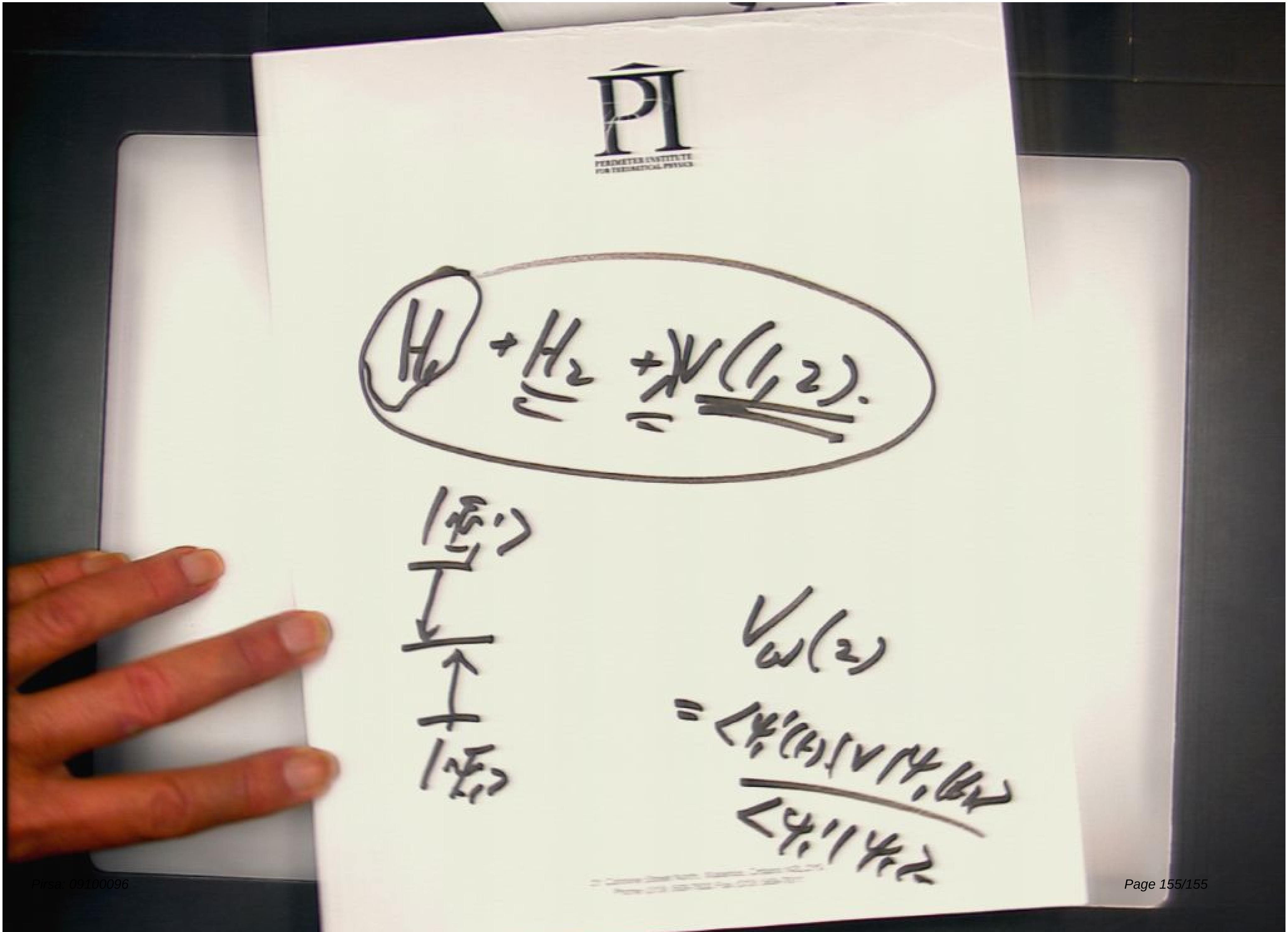
Vω(2)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H1 + H2 + V(1,2).
Vω(2)