Title: The World According to De Finetti

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Abstract: Bruno de Finetti is one of the founding fathers of the subjectivist school of probability, where probabilities are interpreted as rational degrees of belief. His work on the relation between the theorems of the probability calculus and rationality is among the corner stones of modern subjective probability theory. De Finetti maintained that rationality requires that an agent’s degrees of belief be coherent.
I argue that de Finetti held that the coherence conditions of degrees of belief in events depend on their verifiability. On this view, the familiar constraints of coherence only apply to sets of degrees of belief that could in principle be jointly verified. Accordingly, the constraints that coherence imposes on degrees of belief are generally weaker than the familiar ones. I then consider the implications of this interpretation of de Finetti for probabilities in quantum mechanics, focusing on the EPR/Bohm experiment and Bell’s theorem.

The World According to de Finetti

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Bruno de Finetti 1906–1985

The Outline

- 1. The background and motivation
- 2. On the interpretation of Bell's theorem
- 3. De Finetti's subjectivist interpretation
- 4. Verifiability and coherence
- 5. De Finetti's interpretation and QM
- 6. Concluding remarks

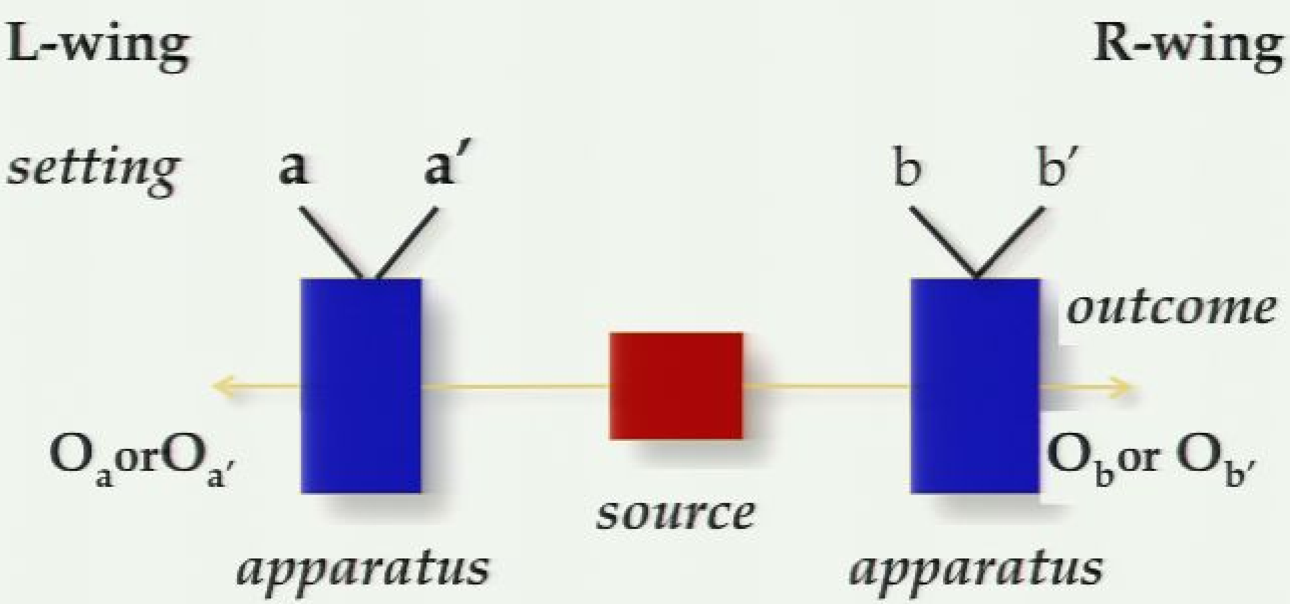
1. The Background and Motivation

- Can the quantum probabilities be interpreted along the lines of de Finetti's subjectivist interpretation?
- This question was raised by Itamar Pitowsky in a course on the Philosophy of Probability in 1990.
- The question was not whether an extreme subjectivist interpretation of probability, like de Finetti's, could be an adequate interpretation of the probabilities in QM.

1. *The background and motivation*

- Rather, it was a question about whether the constraints that this interpretation imposes on probabilities are compatible with QM probabilities.

The EPR/Bohm Experiment



1. The background and motivation

- Bell-type local models of the EPR/Bohm Exp:

(Factorizability)

$$P_{\lambda ab}(O_a \,\&\, O_b) = P_{\lambda a}(O_a)\,P_{\lambda b}(O_b)$$

(λ - independence)

$$\rho_{\psi ab}(\lambda) = \rho_{\psi a}(\lambda) = \rho_{\psi b}(\lambda) = \rho_{\psi}(\lambda)$$

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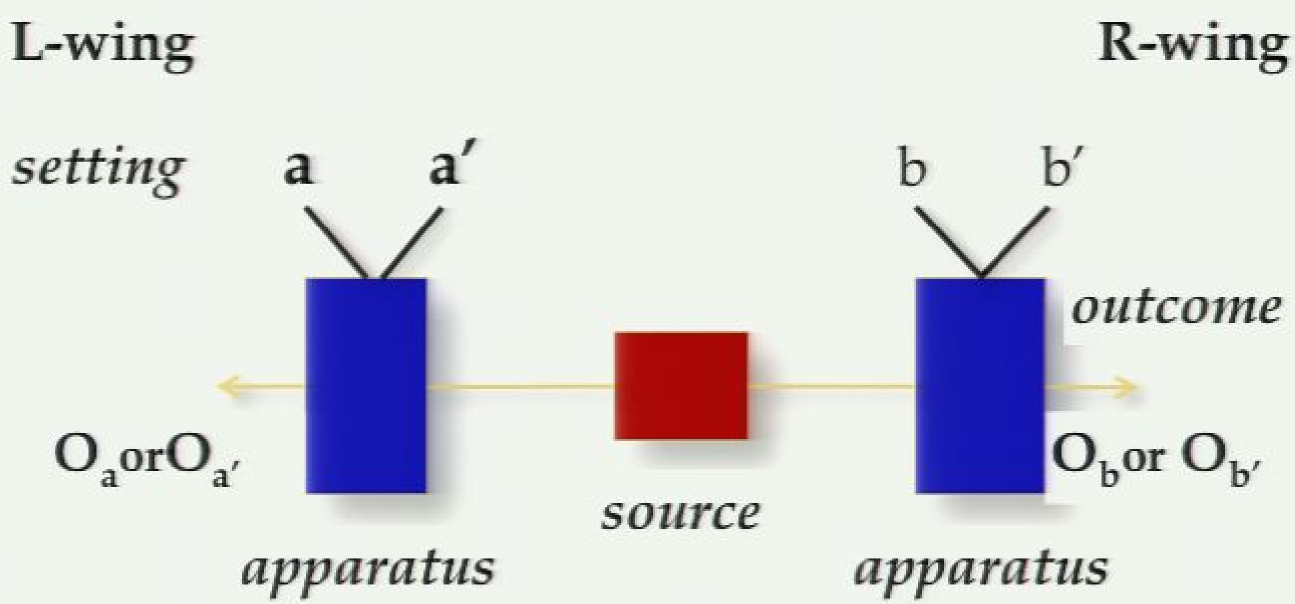

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- Bell's theorem: local Bell-type models predict probabilities of measurement outcomes that satisfy Bell's inequalities.
- The QM probabilities violates these inequalities.
- (λ - independence) seems very plausible.
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- "The first result here is to show that the existence of a deterministic hidden-variables is strictly equivalent to the existence of a joint distribution probability function P(AA'BB') for the four observables of the experiment, one that returns the probabilities of the experiment as marginals." (Fine, Phys. Rev., 1982, p. 291)

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- "Necessary and also sufficient for the existence of deterministic hidden-variables model is that Bell/CH inequalities hold for the probability of the experiment." (ibid., p. 293)

- "There exists a stochastic hidden-variables models for a correlation experiment if and only if there exists a deterministic hidden-variables for the experiment." (ibid., p.293)

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- Fine's claims entail that the existence of a joint distribution over the properties A,A',B,B' is equivalent to the satisfaction of Bell/CH inequalities in the EPR/Bohm experiment.

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- De Finetti maintained that for degrees of belief to be coherent, they have to satisfy the probability theorems.
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- This is tantamount to the assumption of a joint probability distribution over all the propositions the person considers.
- If (i) followers of the subjective interpretation were committed to the assumption of such joint distribution, and
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- Followers of the subjectivist interpretation may agree with Fine's analysis, but reject the assumption that a single probability function represent their degrees of belief.

- The question is whether they have non-ad hoc reasons to do so.

2. On the Interpretation of Bell's Theorem

The term 'Bell inequalities' is ambiguous. It refers to 3 kinds of inequalities.

2. On the interpretation of Bell's theorem

- (Bell/CH - prob)

$$-1 \le P_\lambda(A \,\&\, B) + P_\lambda(A' \,\&\, B) + P_\lambda(A \,\&\, B') - P_\lambda(A' \,\&\, B') - P_\lambda(A) - P_\lambda(B) \le 0$$

- (Bell/CH - phys - λ)

$$-1 \le P_{\lambda ab}(O_a \,\&\, O_b) + P_{\lambda a'b}(O_{a'} \,\&\, O_b) + P_{\lambda ab'}(O_a \,\&\, O_{b'}) - P_{\lambda a'b'}(O_{a'} \,\&\, O_{b'}) - P_{\lambda a}(O_a) - P_{\lambda b}(O_b) \le 0$$

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- (Bell/CH - math) is a theorem of probability theory.
- In this inequality, all the probabilities are of the same probability space (function).
- Any probability space with four events (propositions) will satisfy a similar inequality.

2. On the interpretation of Bell's theorem

- (Bell/CH - phys - λ) is derived from (Bell/CH–phys - ψ) by integrating over all complete pair-states λ, while assuming λ-independence.

$$\rho_{\psi ab}(\lambda) = \rho_{\psi}(\lambda)$$

- These inequalities are not theorems of probability theory.

- Each of the probabilities in these inequalities belongs to a different probability space.

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- Even if rewrite these inequalities so that all the probabilities be in the same probability space, the resulting inequalities are not be theorems of probability.

- $$-1 \le P\left(O_a \,\&\, O_b / \lambda ab\right) + P\left(O_{a'} \,\&\, O_b / \lambda a'b\right) + P\left(O_a \,\&\, O_{b'} / \lambda ab'\right) - P\left(O_{a'} \,\&\, O_{b'} / \lambda a'b'\right) - P\left(O_a / \lambda a\right) - P\left(O_b / \lambda b\right) \le 0$$

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- (Bell/CH - prob) is neither necessary nor sufficient for (Bell/CH - phys - ψ).

- But Bell's theorem is about the incompatibility of (Bell/CH - phys - ψ) with the predictions of QM.

2. On the interpretation of Bell's theorem

- (Bell/CH - prob) is not sufficient for (Bell/CH -phys - λ).
- The derivation of (Bell/CH - phys - λ) from (Bell/CH – prob) requires some assumptions about the physical nature of quantum systems.
- Two natural assumptions, which are implicit in Fine's reasoning, are (λ - independence) and
- (Mirror) $P_{\lambda ab}(O_a \;\&\; O_b) = P_{\lambda}(A \;\&\; B)$

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3. De Finetti's Subjective Theory

- *I. Probability axioms are not formal conventions*

- "Probability theory is not merely a formal, merely arbitrary construction, and its axioms cannot be chosen freely as conventions justified only by mathematical elegance or convenience. They should express what is necessarily inherent in the notion of probability and nothing more." (1972, pp. xiii-xiv)

3. De Finetti's subjectivist probability theory

II. Probabilities are inherently subjective

- Probability and probabilistic reasoning should always be understood as subjective, and "merely stem from our being uncertain about something. It makes no difference whether the uncertainty relates to unforeseeable future, or to unnoticed past, or to past doubtfully reported or forgotten; it may even relate to something more or less knowable (by means of computation, a logical deduction, etc.) but for which we are not willing or able to make the effort; and so on."

3. De Finetti's subjectivist probability theory

- "Moreover, probabilistic reasoning is completely unrelated to general philosophical controversies, such as Determinism versus Indeterminism, Realism versus Solipsism -- including the question of whether the world exists, or simply the scenery of my solipsistic dream ….. The only relevant thing is uncertainty -- the extent of our knowledge and ignorance. The actual fact of whether or not the events considered are in some sense determined, or known by other people, and so on, is of no consequence." (de Finetti 1974, pp. x-xi)

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- "No fact can prove or disprove a belief."
- Yet, facts can have influence on belief revision, though belief revisions "would not be governed by a mechanical rule; it is, in each case, my personal judgment that is responsible for giving a weight to the facts." (de Finetti, 1972, p. 21)

III. Coherence

- Probabilities are coherent degrees of belief in events we are uncertain about.
- The notion of coherent degrees of belief is commonly explicated in terms of Dutch books - contracts in which one lose comes what may.
- The idea is to argue that incoherent degrees of belief are subjected to Dutch books.

IV. Measurement of degrees of belief

- The Dutch book framework provides a way to measure degrees of belief in terms of betting odds that people accept as fair.
- Yet, de Finetti thought that the existence of a bookie could interfere with the measurement of degrees of belief.
- Thus, the latter de Finetti preferred a different framework for measuring degrees of belief (again the Loss function framework).

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3. De Finetti's subjectivist probability theory

- For reasons related to measurement of degrees of belief, the latter de Finetti preferred a different framework for explicating coherence (the so-called "Loss functions" framework).

$$Loss = \frac{\sum_i \left(E_i - DoB\left(E_i\right)\right)^2}{k_i}$$

E_i – event

$E_i = 1$ ($E_i = 0$) if E_i occurs (does not occur)

$DoB(E_i)$ – degree of belief in the occurrence of E_i

k_i – arbitrary constant

3. De Finetti's subjectivist probability theory

- In this framework, a person is coherent if s/he makes decisions -- i.e. choose degrees of belief -- that are not dominated by other corresponding decisions -- i.e. other degrees of belief in the same events.

- $$Loss = \frac{\sum_i \left(E_i - DoB\left(E_i\right)\right)^2}{k_i}$$

3. De Finetti's subjectivist probability theory

V. Operational definition of probability

- Probability is a guide for life.
- Like other notions of great practical importance, probability should have an effective meaning.
- "In order to give an effective meaning to a notion -- and not only an appearance of such in a metaphysical-verbalistic sense -- an operational definition is required."

3. De Finetti's subjectivist probability theory

- By operational definition, de Finetti means "a definition based on a criterion which allows us to measure it." (1970a, p. 76).

VI. Conditional probability

- Every probability is a conditional probability on background information and beliefs.
- Conditional probability is not defined by the ratio formula, as in Kolmogorov's axiomatization. The ratio formula only provides coherent conditions.
- In de Finetti's theory, conditional probability may be interpreted in two different ways.

VI.1. Conditional probability as probability of conditional events

- The conditional probability of E given H is the unconditional probability of the conditional event E|H.

- E|H is true if H and E are true
- is false if H is true and E is false
- is indeterminate or void if H is false

- Probabilities are only assigned to conditional events. that are either true or false

VI.2. Conditional probability as conditional events

- My subjective probability of E given H is equal to p is interpreted as one of the following conditionals:
- (I) If I come to know (believe) H, then my probability of E will be p.
- (II) If I had known (believed) H, then my probability of E would have been p.

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3. *De Finetti's subjectivist probability theory*

- Although these are different ways to model conditional probability, they are not unrelated:

- "We shall write for the probability 'of the event conditional on the event H' (or even the probability 'of the conditional event E|H'), which is the probability that You attribute to if You think that in addition to your present information, i.e. the which we understand implicitly, it will become known to You that is true (and nothing else)." (de Finetti 1974a, p. 134)

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VI.2. Conditional probability as conditional events

- My subjective probability of E given H is equal to p is interpreted as one of the following conditionals:
- (I) If I come to know (believe) H, then my probability of E will be p.
- (II) If I had known (believed) H, then my probability of E would have been p.

3. De Finetti's subjectivist probability theory

- Although these are different ways to model conditional probability, they are not unrelated:
- "We shall write for the probability 'of the event conditional on the event H' (or even the probability 'of the conditional event E|H'), which is the probability that You attribute to if You think that in addition to your present information, i.e. the which we understand implicitly, it will become known to You that is true (and nothing else)." (de Finetti 1974a, p. 134)

4. Verifiability and Coherence

- I. Verifiability
- "In general terms, it will always be a question of examining, if, and in which sense, a statement really constitutes an 'event', permitting in a more or less realistic acceptable form, and in unique way, the 'verification' of whether it is 'true' or 'false'."

4. Verifiability and coherence

- De Finetti acknowledged that verifiability is "a notion that is often vague and illusive" and thought that it is necessary "to recognize that there are various degrees and shades of meaning" attached to it." (1974b, p. 260)
- He took a pragmatic attitude toward the kind and degree of verifiability that is actually required for events to have a definite probability.
- To simplify things, we focus on verifiability in principle.

4. Verifiability and coherence

- "A and B are events (observables), but it is not possible to observe both of them, and, therefore, it is not possible to call the product AB an event (observable)." (1970, p. 34)

- Unlike probabilities, de Finetti was not anti realist about events. Yet, for various reasons, he believed that in the context of probability the domain of events should be restricted to verifiable ones.

II. Motivations

- (1) Events that are not verifiable even in principle may appear to be sensical but in fact be meaningless. (Influence of Positivism)

- (2) Event is a notion of great practical importance, and lack of verifiability would undermine its practicality. (Influence of Positivism; instrumentalism)

4. Verifiability and coherence

- (3) Measurement of degrees of belief in events that are unverifiable are impossible. (Probability as a guide for life.)

- (4) The Dutch book framework for explicating coherent degrees of belief is inapplicable to events that are in principle unverifiable! (Instrumentalism about probs.)

- (5) The Loss Functions framework for explicating coherent degrees of belief is inapplicable to events that are in principle unverifiable. (Instrumentalism about probs.)

4. Verifiability and coherence

- III. Making verifiability explicit

- De Finetti proposes to make his verifiability explicit by conditional probability, where the probability of event E is conditionalized on *either* observation H that enables to verify E *or* a proposition H that E is verifiable.

- There are two different ways to realize this idea, each corresponds to a different interpretation of de Finetti's conception of conditional probability.

4. Verifiability and coherence

- (1) $P(E_i \mid H_i)$

$$E_1 \mid H_1, E_2 \mid H_2, E_{12} \mid H_{12}$$

where $E_i \mid H_i$ is true if both E_i and H_i are true
is false if where H_i is true and E_i is false
is indeterminate or void if E_i is false

$E_{12} \mid H_{12}$ may be indeterminate even if
$E_1 \mid H_1$ and $E_2 \mid H_2$ have determinate truth values.

4. Verifiability and coherence

$P(E_i \mid H_i)$ has probability only if
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5. De Finetti's Theory of Probability and QM

- Consider again (Bell/CH - math):

$$-1 \le P_\lambda(A \,\&\, B) + P_\lambda(A' \,\&\, B) + P_\lambda(A \,\&\, B') - P_\lambda(A' \,\&\, B') - P_\lambda(A) - P_\lambda(B) \le 0$$

- This inequality is derived from the assumption of joint probability over A, A', B and B'.

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5. De Finewtti's theory of Probability and QM

- In de Finetti's theory, such distribution does not exist, as A and A' (B and B') cannot jointly be verified.
- There are 2 ways to represent this idea:
- (i) Events in a big probability space are defined as conditional on the in-principle possibility of their joint verification. The result is a 'big' probability space with 'holes' (where joint probabilities do not exist).

$$P_{big}(A \mid H_A, A' \mid H_{A'}, B \mid H_B, B' \mid H_{B'}, AB \mid H_{AB} \ldots)$$

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5. De Finewtti's theory of Probability and QM

- (ii) Probabilities of events are conditional on their in-principle verifiability.
- As a result, we have 4 smaller spaces, corresponding to the 4 joint probabilities in(Bell/CH – prob).

$$P_{AB}(A, B, A \& B)$$
$$P_{A'B}(A', B, A' \& B)$$
$$P_{AB'}(A, B', A \& B')$$
$$P_{A'B'}(A', B', A' \& B')$$

6. Concluding Remarks

- (1) Even if one rejects de Finetti's motivations for weakening the characteristic coherent conditions in contexts in which propositions cannot jointly be verified, one may still hold that it is rational to do so.

- All that followers of de Finetti need to assume in order to avoid the Bell/CH inequalities is that A and A' (B and B') could not be jointly verified.

6. Concluding remarks

- (2) According to de Finetti, the familiar coherence conditions, expressed in terms of inequalities, are only valid for cases in which the degrees of belief are in propositions that can be jointly verified.

- As Itamar Pitowsky pointed out (BJPS, 1994), George Boole (1862) suggested that the familiar coherence conditions are necessary and sufficient conditions for the existence finite frequencies of events that can be jointly verified (sampled).

- George Boole called them 'conditions of possible experience'.

6. Concluding remarks

- (3) It is very difficult to motivate Bell's argument for non-locality if probabilities are interpreted along de Finetti's theory.
- Since probabilities in this theory only reflect uncertainty, it is difficult to motivate Factorizability as a locality condition.
- It is difficult to formulate (λ- independence). In 'hidden variables' theories, λ is typically impossible to verify, and accordingly the probabilities in (λ- independence) do not exist.

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5. De Finewtti's theory of Probability and QM

- In either case the coherence conditions on degrees of belief in $A, B, A', B', A \& B, A' \& B, A \& B'$ and $A' \& B'$ are weaker than the coherence conditions imposed by (Bell/CH - prob).

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V. Operational definition of probability

- Probability is a guide for life.
- Like other notions of great practical importance, probability should have an effective meaning.
- "In order to give an effective meaning to a notion -- and not only an appearance of such in a metaphysical-verbalistic sense -- an operational definition is required."

3. De Finetti's subjectivist probability theory

- "No fact can prove or disprove a belief."

- Yet, facts can have influence on belief revision, though belief revisions "would not be governed by a mechanical rule; it is, in each case, my personal judgment that is responsible for giving a weight to the facts." (de Finetti, 1972, p. 21)

3. De Finetti's Subjective Theory

2. On the interpretation of Bell's theorem

- De Finetti's notion of coherent degrees of belief embodies a certain kind of verifiability requirement.
- Accordingly, the coherence conditions of degrees of belief are generally weaker than the familiar ones (the ones that are discussed in the literature on subjective probability).

2. On the interpretation of Bell's theorem

- (Bell/CH - prob) is not sufficient for (Bell/CH -phys - λ).
- The derivation of (Bell/CH - phys - λ) from (Bell/CH – prob) requires some assumptions about the physical nature of quantum systems.
- Two natural assumptions, which are implicit in Fine's reasoning, are (λ - independence) and
- (Mirror) $P_{\lambda ab}(O_a \,\&\, O_b) = P_{\lambda}(A \,\&\, B)$

2. On the interpretation of Bell's theorem

- (Bell/CH - math) is not necessary for the derivation of (Bell/CH - phys - ψ).
- The latter inequality follows from (Factorizability) and (λ - independence).
- And neither of these assumptions presupposes a joint distribution over the 'hidden variables'.

2. On the interpretation of Bell's theorem

- Even if rewrite these inequalities so that all the probabilities be in the same probability space, the resulting inequalities are not be theorems of probability.

- $$-1 \le P(O_a \,\&\, O_b / \lambda ab) + P(O_{a'} \,\&\, O_b / \lambda a'b) + P(O_a \,\&\, O_{b'} / \lambda ab') - P(O_{a'} \,\&\, O_{b'} / \lambda a'b') - P(O_a / \lambda a) - P(O_b / \lambda b) \le 0$$

2. On the interpretation of Bell's theorem

- (Bell/CH - prob)

$$-1 \le P_\lambda(A \,\&\, B) + P_\lambda(A' \,\&\, B) + P_\lambda(A \,\&\, B') - P_\lambda(A' \,\&\, B') - P_\lambda(A) - P_\lambda(B) \le 0$$

- (Bell/CH - phys - λ)

$$-1 \le P_{\lambda ab}(O_a \,\&\, O_b) + P_{\lambda a'b}(O_{a'} \,\&\, O_b) + P_{\lambda ab'}(O_a \,\&\, O_{b'}) - P_{\lambda a'b'}(O_{a'} \,\&\, O_{b'}) - P_{\lambda a}(O_a) - P_{\lambda b}(O_b) \le 0$$

- (Bell/CH - phys - ψ)

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- (Mirror) $P_{\lambda ab}\left(O_a \,\&\, O_b\right) = P_{\lambda}\left(A \,\&\, B\right)$

2. On the interpretation of Bell's theorem

- (Bell/CH - math) is a theorem of probability theory.
- In this inequality, all the probabilities are of the same probability space (function).
- Any probability space with four events (propositions) will satisfy a similar inequality.

2. On the interpretation of Bell's theorem

- (Bell/CH - prob)

$$-1 \leq P_\lambda(A \,\&\, B) + P_\lambda(A' \,\&\, B) + P_\lambda(A \,\&\, B') - P_\lambda(A' \,\&\, B') - P_\lambda(A) - P_\lambda(B) \leq 0$$

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