Title: Adding Flavor to Ads4/CFT3 correspondence

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Abstract: TBA

Adding Flavor to AdS_4/CFT_3 correspondence

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October 13th, 2009

Based on:

MA, J. Erdmenger, R. Meyer, A. O'Bannon, T. Wrase, arXiv: 0909.3845

(see also [Hohenegger, Kirsch], [Hikida, Li, Takayanagi], [Gaiotto, Jafferis], [Myers, Thomson], [Fujita, Li, Ryu, Takayanagi],...)

Outline

1. Motivation
2. Adding Flavor to AdS_5/CFT_4
3. The AdS_4/CFT_3 Duality
   - Dictionary
   - Deriving AdS_4/CFT_3 from type IIB setup
   - The Chern-Simons Matter Theory
4. Adding Flavor to AdS_4/CFT_3
   - General Recipe to determine Field theory / Gravity theory
   - $\mathcal{N} = 3$, codimension zero Flavor
   - $\mathcal{N} = (0, 6)$, codimension one Flavor
5. Conclusion and Future Directions

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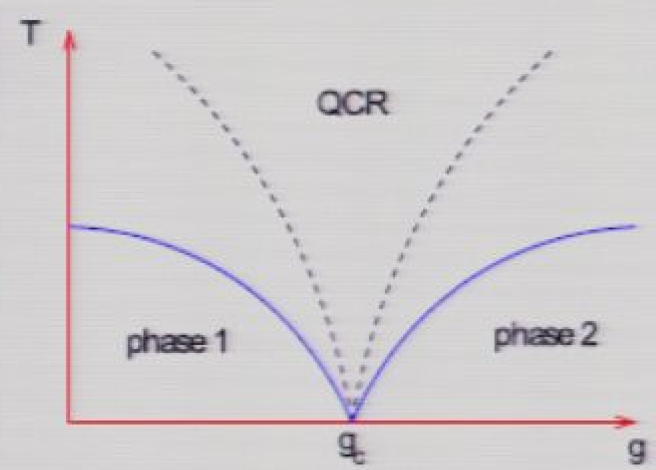
Many real–world systems are strongly–coupled

Quark-gluon plasma (QGP)

- observed at RHIC
- behaves as a liquid → Hydrodynamics
- inputs for hydrodynamic description
  - thermodynamics (equation of state)
  - transport coefficients (shear and bulk viscosity, charge diffusion constant, conductivities)

Condensed matter systems

- consider condensed matter systems near quantum critical points
- quantum critical theory is scale invariant and often strongly coupled



Many real–world systems are strongly–coupled

AdS/CFT dualities

- map strongly coupled gauge theories to weakly coupled gravity theories
- For some strongly–coupled system Gauge/gravity dualities are the only way to compute certain observables

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

[Kovtun, Son, Starinets, '03]

In most AdS/CFT systems, the CFT is

- supersymmetric,
- a non–Abelian gauge theory.
- Fields only in adjoint representation of the gauge group

Adding Flavor

Flavor fields are important

- QGP / RHIC: quarks
- Condensed matter systems: "electrons"

For AdS_5/CFT_4

Procedure to add flavors highly developed

many successes: Meson Spectra, Thermodynamics, Phase diagram, Chiral Symmetry Breaking, Hydrodynamics, Condensed matter physics (Quantum Hall Effect, Superfluids), ...

For AdS_4/CFT_3

- CFT was discovered only recently
- Adding flavors poorly developed

Adding Flavor

Goal

Determine general "recipe" for adding supersymmetric Flavor to AdS_4/CFT_3 in *gravity and field theory* description

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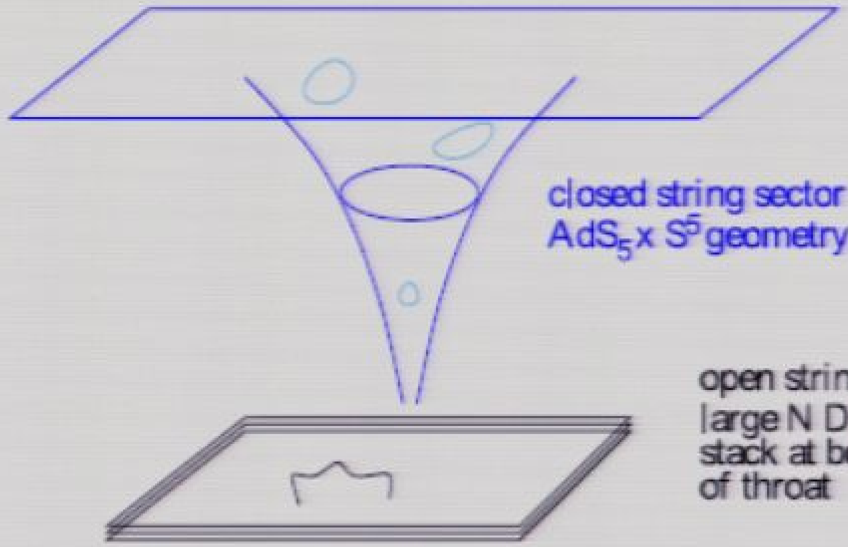
AdS5/CFT4



Duality

$\mathcal{N} = 4$ $SU(N_c)$ SYM in 4d $\longleftrightarrow$ IIB String Theory in asympt. $AdS_5 \times S^5$ [Maldacena, '97]

$N_c \gg 1$, $\lambda = g^2_{YM} N_c \gg 1$ $\longleftrightarrow$ SUGRA reliable

source and vev of op. $\longleftrightarrow$ boundary asym. of gravity field

partition function Z $=$ $e^{-S_{\text{sugra,on-shell}}}$ [Witten '98]

Adding Flavor

Idea

Add another Dp-brane stack. Fundamental strings between Dp- and D3-branes are interpreted as quarks q_i^a

Possible Dp - brane stacks (1/2 BPS intersections)

	0	1	2	3	4	5	6	7	8	9
N_c $D3$	•	•	•	•	-	-	-	-	-	-
N_f $D7$	•	•	•	•	•	•	•	•	-	-
N_f $D7$	•	•	-	-	•	•	•	•	•	•
N_f $D5$	•	•	•	-	•	•	•	-	-	-
N_f $D5$	•	-	-	-	•	•	•	•	•	-
N_f $D3$	•	•	-	-	•	•	-	-	-	-

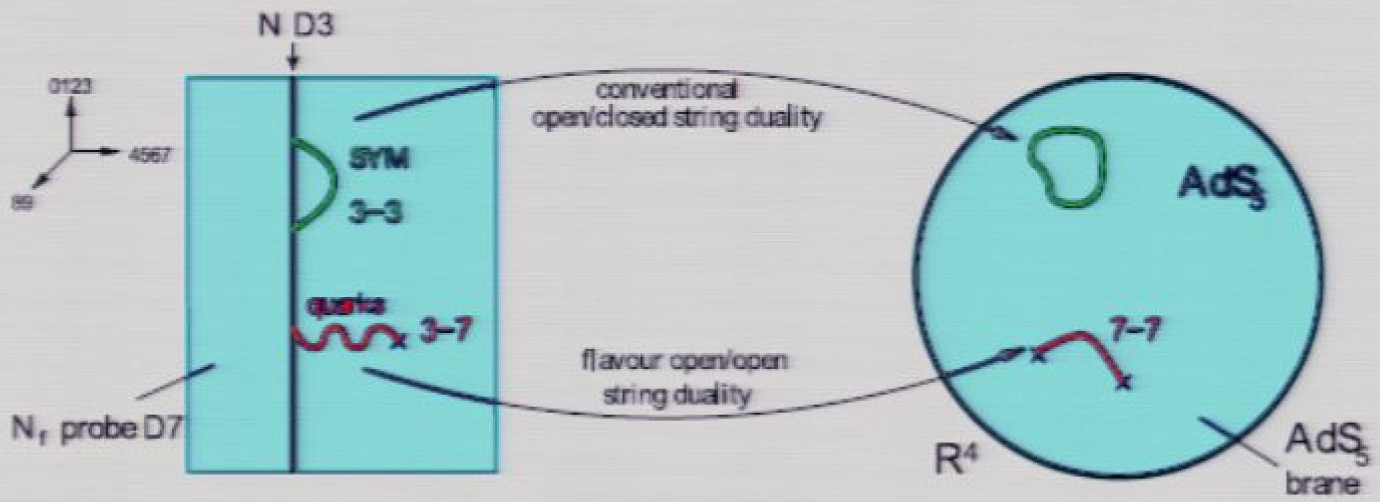
For all these configurations the field theory is known explicitly.

New ideas (<6 month) [Karch, Kulaxizi, Parnachev][Karch, Kachru, Yaida][Hung, Sinha][Wapler][Alanen, Keski-Vakkuri, Kraus,Suur-Uski][Chen, Hashimoto, Matsuura][Hikida, Li, Takayanagi][Hoyos-Badajoz, Karch][...]

Adding Flavor: Gravity side

	0	1	2	3	4	5	6	7	8	9
N_c $D3$	•	•	•	•	-	-	-	-	-	-
N_f $D7$	•	•	•	•	•	•	•	•	-	-

[Karch, Katz, '02]



Probe Limit: N_f fixed, $N_c \to \infty$

- *Gravity side:* Ignore back–reaction of $D7$–branes, study probe $D7$–branes wrapping asymp. $AdS_5 \times S^3$ in $AdS_5 \times S^5$.
- *Gauge theory side:* Ignore quantum (loop) effects of flavor degrees $\Rightarrow \beta \sim +\mathcal{O}(N_f/N_c) \approx 0$

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N_f $D7$	•	•	-	-	•	•	•	•	•	•
N_f $D5$	•	•	•	-	•	•	•	-	-	-
N_f $D5$	•	-	-	-	•	•	•	•	•	-
N_f $D3$	•	•	-	-	•	•	-	-	-	-

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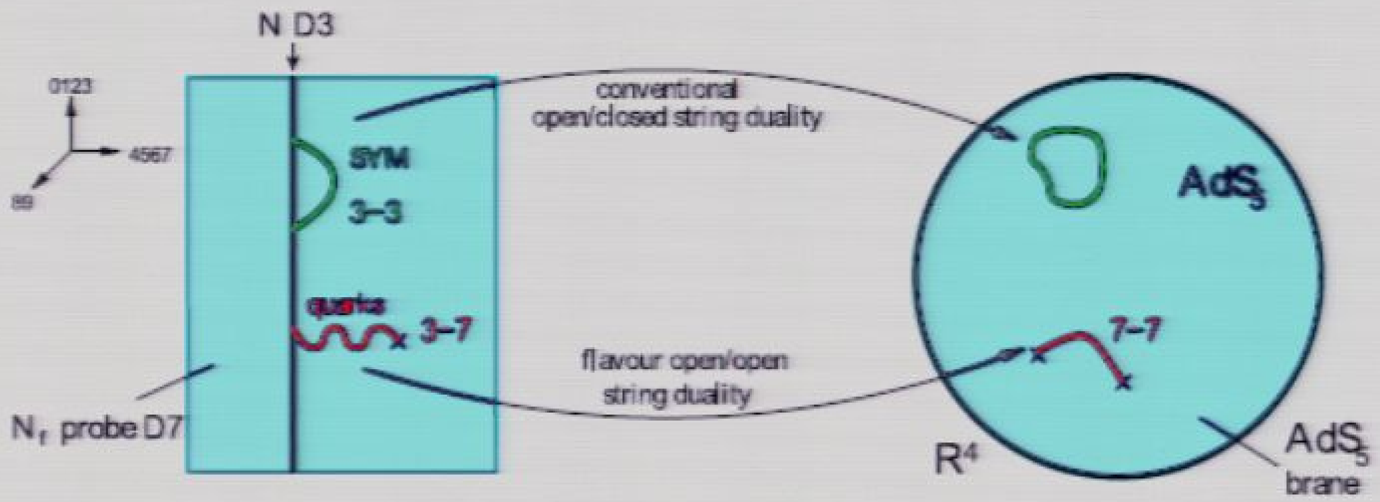

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Adding Flavor: Field theory side

field content of D3/D7 setup

$\mathcal{N}=2$	components	spin	$SU(2)_\Phi \times SU(2)_\mathcal{R}$	$U(1)_\mathcal{R}$	Δ	$U(N_f)$	$U(1)_B$
(Φ_1,Φ_2)	X^4,X^5,X^6,X^7	0	$(\frac{1}{2},\frac{1}{2})$	0	1	1	0
hyper	λ_1,λ_2	$\frac{1}{2}$	$(\frac{1}{2},0)$	-1	$\frac{3}{2}$	1	0
(Φ_3, W_α)	$X_V^A=(X^8,X^9)$	0	$(0,0)$	$+2$	1	1	0
vector	λ_3,λ_4	$\frac{1}{2}$	$(0,\frac{1}{2})$	$+1$	$\frac{3}{2}$	1	0
	v_μ	1	$(0,0)$	0	1	1	0
$(Q,\tilde{Q})$	$q^m=(q,\bar{\tilde{q}})$	0	$(0,\frac{1}{2})$	0	1	N_f	+1
fund. hyper	$\psi_i=(\psi,\tilde{\psi}^\dagger)$	$\frac{1}{2}$	$(0,0)$	∓ 1	$\frac{3}{2}$	N_f	+1

Lagrangian

$$\mathcal{L} = \mathrm{Im}\left[\tau \int d^4\theta \left(\mathrm{tr}\,(\bar{\Phi}_I e^V \Phi_I e^{-V}) + Q_r^\dagger e^V Q^r + \tilde{Q}_r^\dagger e^{-V}\tilde{Q}^r\right)\right.$$
$$\left. + \tau \int d^2\theta (\mathrm{tr}\,(W^\alpha W_\alpha) + W) + \mathrm{c.c.}\right],$$

where the superpotential W is $W = \mathrm{tr}\,(\varepsilon_{IJK}\Phi_I\Phi_J\Phi_K) + \tilde{Q}_r(m_q+\Phi_3)Q^r$.

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Low–Energy descriptions of M2–Branes

Consider N_c M2–Branes

	0	1	2	3	4	5	6	7	8	9	10
N_c *M2*	•	•	•	-	-	-	-	-	-	-	-

N_c M2–Branes on C^4/Z_k have two different low–energy descriptions for $N_c \to \infty$ and $N_c \gg k$: [Aharony, Bergman, Jafferis, Maldacena, '08]

Gravity side

- For $N_c \gg k^5$: 11D Supergravity on asymptotically $AdS_4 \times S^7/Z_k$

Gauge theory side

- $U(N_c)_k \times U(N_c)_{-k}$ Chern-Simons Matter Theory (CSM)
- $\mathcal{N} = 6$ supersymmetric for general k
- conformal, invariant under parity, $SU(4)_\mathcal{R} \simeq SO(6)_\mathcal{R}$ symmetry

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- $\mathcal{N} = 8$ supersymmetric for $k = 1, 2$
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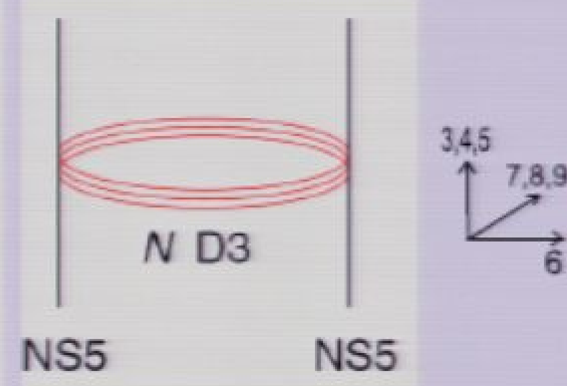
M2–Brane Theory: IIB Brane construction (Step 1)

Brane Setup

	0	1	2	3	4	5	6	7	8	9
$N\,D3$	•	•	•	-	-	-	•	-	-	-
$2\,NS5$	•	•	•	•	•	•	-	-	-	-

IIB Picture



Low–Energy Field Theory

$\mathcal{N} = 4$, $3+1$ dim. $U(N_c) \times U(N_c)$ gauge theory + bifundamental fields, Vector Multiplet (A_μ^{0126}, 345789).

If we dim. reduce along 6 direction: Vector Multiplet split into

$\mathcal{N} = 4$, $2+1$ dim. Vector (A_μ^{012}, 345) and

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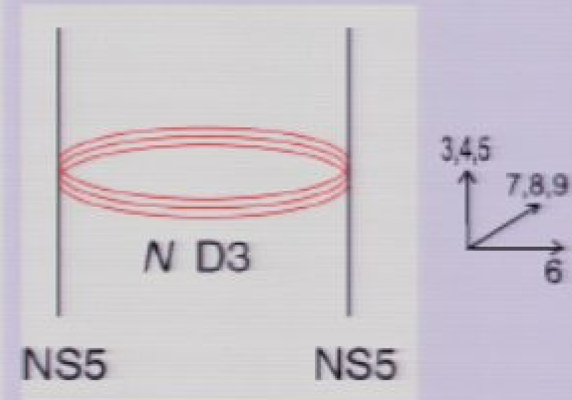

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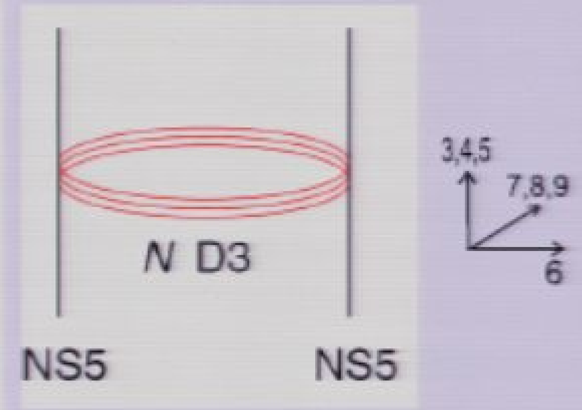

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Hyper killed by NS5 – Branes

M2–Brane Theory: IIB Brane construction (Step 2)

How to obtain Chern – Simons terms?

[Bergman, Hanany, Karch,Kol '99]

Brane Setup

	0	1	2	3	4	5	6	7	8	9
N $D3$	•	•	•	-	-	-	•	-	-	-
2 $NS5$	•	•	•	•	•	•	-	-	-	-
k $D5$	•	•	•	•	•	-	-	-	-	•

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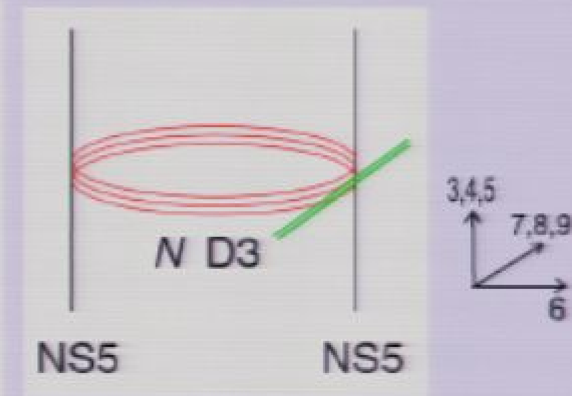

Low – Energy Field Theory

- 3–5 strings ⇒ "Flavor" Fields
- Supersymmetry broken down to $\mathcal{N} = 2$.
- Give mass to "flavor" fields and integrate them out
- Via parity anomaly generate Chern–Simons terms.

M2–Brane Theory: IIB Brane construction (Step 2)

Maximally supersymmetric mass deformation: Bind k D5–Branes to NS5 forming $(1,k)$5–Brane and rotate in (37), (48), (59) plane.

	0	1	2	3	4	5	6	7	8	9
N $D3$	•	•	•	-	-	-	•	-	-	-
1 $NS5$	•	•	•	•	•	•	-	-	-	-
$(1,k)5$	•	•	•	$[3,7]_\theta$	$[4,8]_\theta$	$[5,9]_\theta$	-	-	-	-

Low – Energy Field Theory

- Chern-Simons term generated
- $\mathcal{N} = 3$ $U(N_c)_k \times U(N_c)_{-k}$ Yang-Mills theory with a Chern-Simons term
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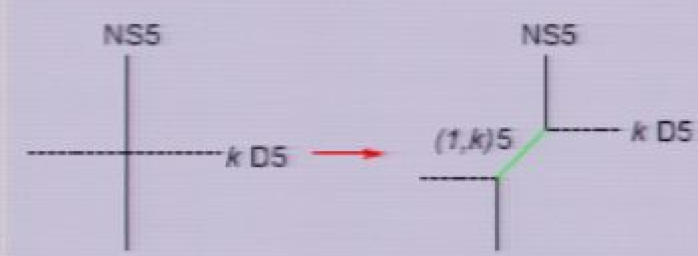
Web deformation



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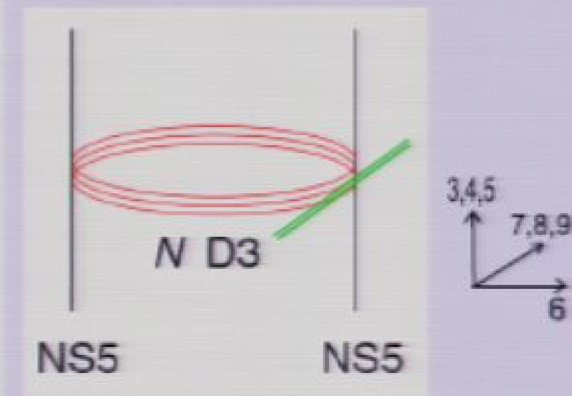

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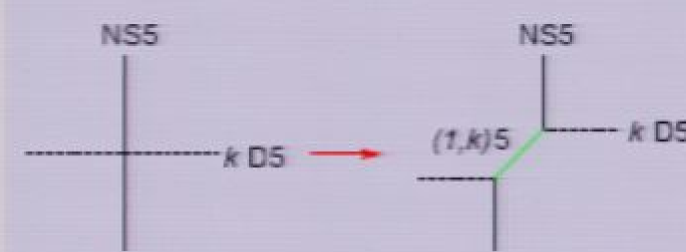

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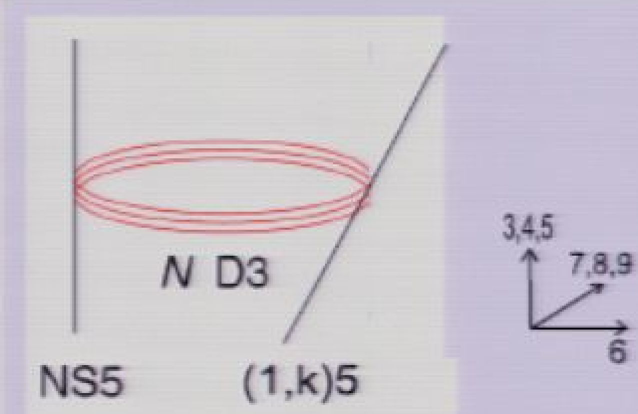

M2–Brane Theory: Uplift to M-theory (Step 3)

T–Dualize in x^6–direction

	0	1	2	3	4	5	6	7	8	9
N $D2$	•	•	•	-	-	-	-	-	-	-
KK	•	•	•	•	•	•	-	-	-	-
$KK + D6$ flux	•	•	•	$[3,7]_\theta$	$[4,8]_\theta$	$[5,9]_\theta$	-	-	-	-

Lift to M-theory

	0	1	2	3	4	5	6	7	8	9	10
N $M2$	•	•	•	-	-	-	-	-	-	-	-
X_8				•	•	•	•	•	•	•	•

where X_8 is the intersection of two KK monopoles.

Field Theory

Still $\mathcal{N} = 3$ SYM + CS + matter

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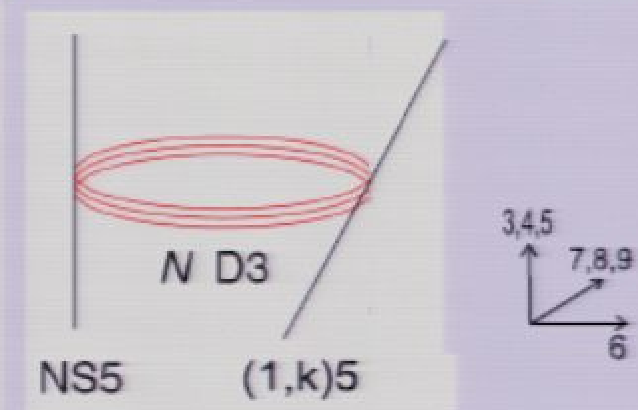

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T–Dualize in x^6–direction

	0	1	2	3	4	5	6	7	8	9
N $D2$	•	•	•	-	-	-	-	-	-	-
KK	•	•	•	•	•	•	-	-	-	-
$KK + D6$ flux	•	•	•	$[3,7]_\theta$	$[4,8]_\theta$	$[5,9]_\theta$	-	-	-	-

Lift to M-theory

	0	1	2	3	4	5	6	7	8	9	10
N $M2$	•	•	•	-	-	-	-	-	-	-	-
X_8				•	•	•	•	•	•	•	•

where X_8 is the intersection of two KK monopoles.

Field Theory

Still $\mathcal{N} = 3$ SYM + CS + matter

M2–Brane Theory: IIB Brane construction (Step 2)

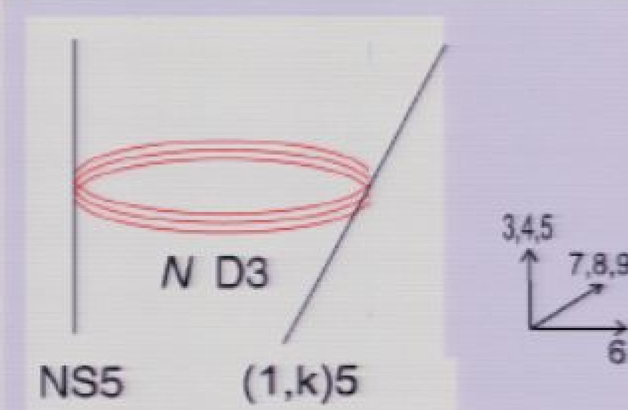
Maximally supersymmetric mass deformation: Bind k D5–Branes to NS5 forming $(1,k)$5–Brane and rotate in (37), (48), (59) plane.

	0	1	2	3	4	5	6	7	8	9
N $D3$	•	•	•	-	-	-	•	-	-	-
1 $NS5$	•	•	•	•	•	•	-	-	-	-
$(1,k)5$	•	•	•	$[3,7]_\theta$	$[4,8]_\theta$	$[5,9]_\theta$	-	-	-	-

Low – Energy Field Theory

- Chern-Simons term generated
- $\mathcal{N}=3$ $U(N_c)_k \times U(N_c)_{-k}$ Yang-Mills theory with a Chern-Simons term
- 4 massless bifundamental matter fields (A_a, B_a)

IIB Picture



M2–Brane Theory: Uplift to M-theory (Step 3)

T–Dualize in x^6–direction

	0	1	2	3	4	5	6	7	8	9
N $D2$	•	•	•	-	-	-	-	-	-	-
KK	•	•	•	•	•	•	-	-	-	-
$KK + D6$ flux	•	•	•	$[3,7]_\theta$	$[4,8]_\theta$	$[5,9]_\theta$	-	-	-	-

Lift to M-theory

	0	1	2	3	4	5	6	7	8	9	10
N $M2$	•	•	•	-	-	-	-	-	-	-	-
X_8				•	•	•	•	•	•	•	•

where X_8 is the intersection of two KK monopoles.

Field Theory

Still $\mathcal{N} = 3$ SYM + CS + matter

M2–Brane Theory: Near–Horizon Limit (Step 4)

Enhancement to $\mathcal{N} = 6$ supersymmetry.

Gravity side

- X_8 has sinularity; near singularity spacetime locally C^4/Z_k.
- take "near-horizon" limit: discard all geometry except the singularity

Field Theory side

- low–energy limit
- write effectiv theory at scales below $\sim g_{YM}^2 k$
  $\Rightarrow$ discard YM terms, only CS terms survive
- $\mathcal{N} = 6$ supersymmetric.

The gauge theory (in $\mathcal{N}=2$ language)

Field content

- Two $\mathcal{N}=2$ vector superfields V_i, one for each gauge group,
- Two $\mathcal{N}=2$ chiral superfields Φ_i in the adjoint representation,
- Four $\mathcal{N}=2$ chiral superfields, A_1, A_2, B_1 and B_2, where A_k in $(N_c, \overline{N_c})$ and B_k in $(\overline{N_c}, N_c)$ representation.

Action

$$S_{\text{ABJM}} = S_{\text{CS}} + S_{\text{bifund}} + S_{\text{pot}}$$

with

- $S_{\text{CS}} = kS_{\text{CS},1} - kS_{\text{CS},2}$,
- $S_{\text{bifund}} = \int d^3x d^4\theta \left[\bar{A}_a e^{-V_1} A_a e^{V_2} + \bar{B}_a e^{-V_2} B_a e^{V_1}\right]$,
- $S_{\text{pot}} = \int d^3x\, d^2\theta\, W + c.c.$,

and superpotential $W = -\frac{k}{8\pi}\text{Tr}\left(\Phi_1^2 - \Phi_2^2\right) + \text{Tr}\left(B_a\Phi_1 A_a\right) + \text{Tr}\left(A_a\Phi_2 B_a\right)$

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Action

$$S_{\text{ABJM}} = S_{\text{CS}} + S_{\text{bifund}} + S_{\text{pot}}$$

with

- $S_{\text{CS}} = kS_{\text{CS},1} - kS_{\text{CS},2}$, $\qquad S_{\text{CS,k}} = -\frac{i}{4\pi}\int d^3x\, d^4\theta \int_0^1 dt\, \text{Tr}\, V_k \bar{D}^\alpha \left(e^{tV_k} D_\alpha e^{-tV_k}\right)$
- $S_{\text{bifund}} = \int d^3x d^4\theta \left[\bar{A}_a e^{-V_1} A_a e^{V_2} + \bar{B}_a e^{-V_2} B_a e^{V_1}\right]$,
- $S_{\text{pot}} = \int d^3x\, d^2\theta\, W + c.c.$,

and superpotential $W = -\frac{k}{8\pi}\text{Tr}\left(\Phi_1^2 - \Phi_2^2\right) + \text{Tr}\left(B_a\Phi_1 A_a\right) + \text{Tr}\left(A_a\Phi_2 B_a\right)$

The gauge theory (in $\mathcal{N} = 2$ language)

Field content

- Two $\mathcal{N} = 2$ vector superfields V_i, one for each gauge group,
- Two $\mathcal{N} = 2$ chiral superfields Φ_i in the adjoint representation,
- Four $\mathcal{N} = 2$ chiral superfields, A_1, A_2, B_1 and B_2, where A_k in $(N_c, \overline{N_c})$ and B_k in $(\overline{N_c}, N_c)$ representation.

Action

$$S_{\text{ABJM}} = S_{\text{CS}} + S_{\text{bifund}} + S_{\text{pot}}$$

with

- $S_{\text{CS}} = kS_{\text{CS},1} - kS_{\text{CS},2}$, $\quad S_{\text{CS,k}} = -\frac{i}{4\pi}\int d^3x\, d^4\theta \int_0^1 dt\, \text{Tr}\, V_k \bar{D}^{\alpha}\left(e^{tV_k} D_{\alpha} e^{-tV_k}\right)$
- $S_{\text{bifund}} = \int d^3x d^4\theta \left[\bar{A}_a e^{-V_1} A_a e^{V_2} + \bar{B}_a e^{-V_2} B_a e^{V_1}\right]$,
- $S_{\text{pot}} = \int d^3x\, d^2\theta\, W + c.c.$,

and superpotential $W = \frac{2\pi}{k}\varepsilon^{ab}\,\varepsilon^{cd}\, \text{Tr}\left(A_a B_c A_b B_d\right)$ after integrating out Φ_1 and Φ_2

Outline

Add in type IIB flavor branes and follow the four steps

Supersymmetric flavor branes in type IIB

Type IIB	Type IIA	M theory	codim	wrapping	SUSY	SUSY (anti)
D1	D2	M2	2	0(7)	2	2
D3	D2	M2	0	0126	6	0
D3	D4	M5	1	01(37)	3	3
D3	D4	M5	1	01(38)	2	2
D3	D2	M2	2	0(34)6	2	2
D3	D2	M2	2	06(78)	2	2
D5	D6	KK	0	012(347)	2	2
D5	D6	KK	0	012(349)	4	2
D5	D6	KK	0	012789	6	0
D5	D4	M5	1	013456	3	3
D5	D4	M5	1	01(378)6	2	2
D5	D4	M5	1	01(389)6	3	3
D5	D6	KK	2	0(34)789	2	2
D7	D6	KK	0	0126(3478)	2	4
D7	D6	KK	0	0126(3479)	2	2
D7	D8	M9	1	01345789	3	3

Codimension zero Flavor, Step 1

Consider for example D5–Brane in 012789 direction. [Hohenegger, Kirsch], [Hikida, Li, Takayanagi], [Gaiotto, Jafferis]

N_c D3–Branes and N_f D5–Branes

	0	1	2	3	4	5	6	7	8	9
N_c $D3$	•	•	•	-	-	-	•	-	-	-
N_f $D5$	•	•	•	-	-	-	-	•	•	•

- 2+1 dimensional $\mathcal{N}=4$ supersymmetry
- Action of Flavor degrees in $\mathcal{N}=2$ superspace

$$S_{fl} = \int d^3x\, d^4\theta \left(\bar{Q}e^V Q + \tilde{Q}e^{-V}\bar{\tilde{Q}}\right) + \int d^3x\, d^2\theta \tilde{Q}\Phi Q\,,$$

where (V,Φ) is the $\mathcal{N}=4$ Vector Multiplet

Codimension zero Flavor, Step 1

Add NS5–Branes

	0	1	2	3	4	5	6	7	8	9
$N_c\,D3$	•	•	•	-	-	-	•	-	-	-
$N_f\,D5$	•	•	•	-	-	-	-	•	•	•
$1\,NS5$	•	•	•	•	•	•	-	-	-	-

Does S_{fl} change?

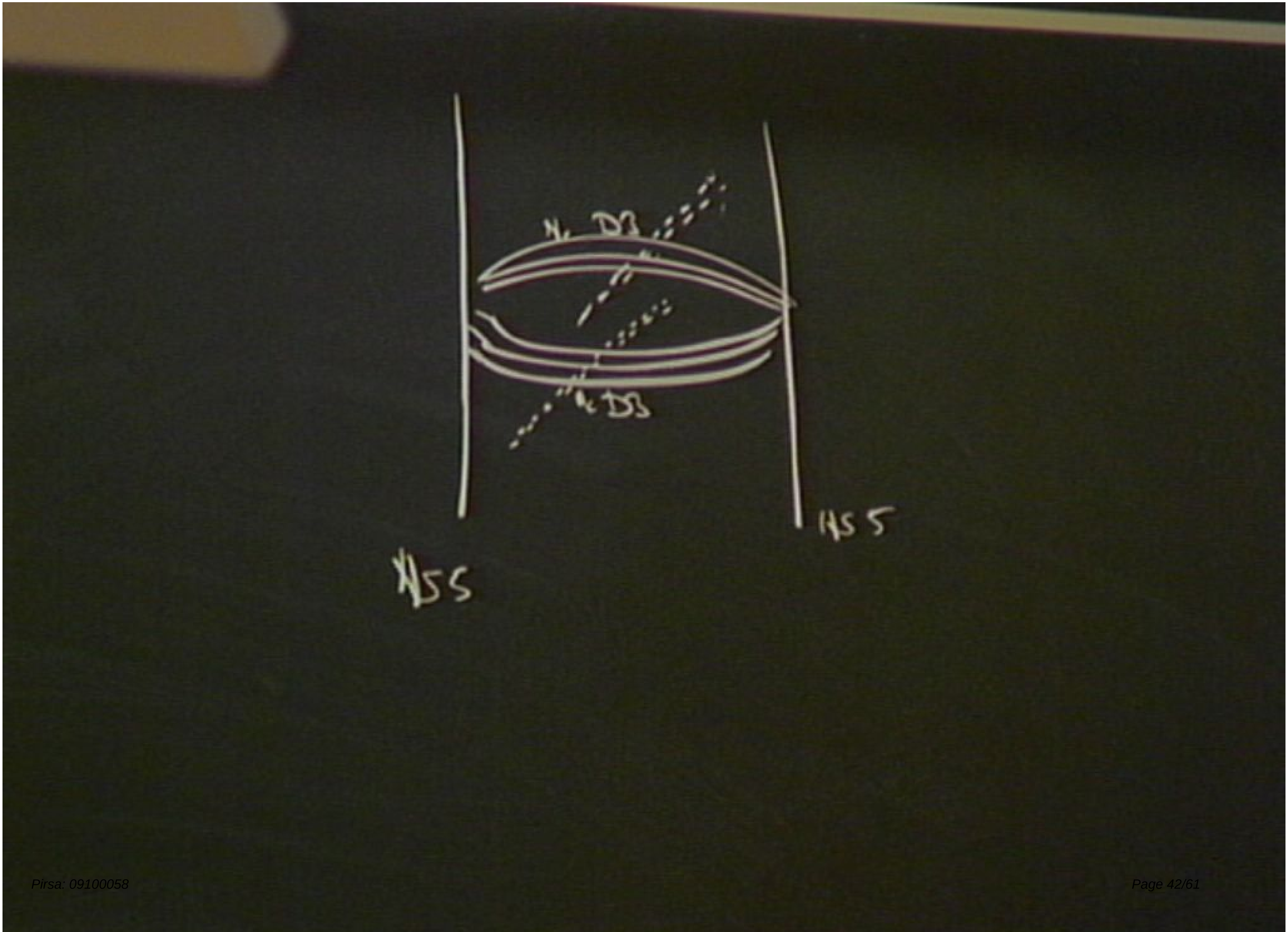
- dimensional reduce on $x_6 \rightarrow$ No!
- Set 2+1 dim. $\mathcal{N}=4$ Hyper Multiplet to zero $\rightarrow$ No!

Compactify x_6

Add N_f D5–Branes intersecting <u>each</u> stack of N_c $D3$–Branes

$\Rightarrow$ massless flavor in each gauge group.

$$S_{fl} = \int d^3x\, d^4\theta \left(\bar{Q}_k e^{V_k} Q_k + \tilde{Q}_k e^{-V_k} \bar{\tilde{Q}}_k\right) + \int d^3x\, d^2\theta\, \tilde{Q}_k (-1)^k \Phi_k Q_k\,,$$



D3
D3

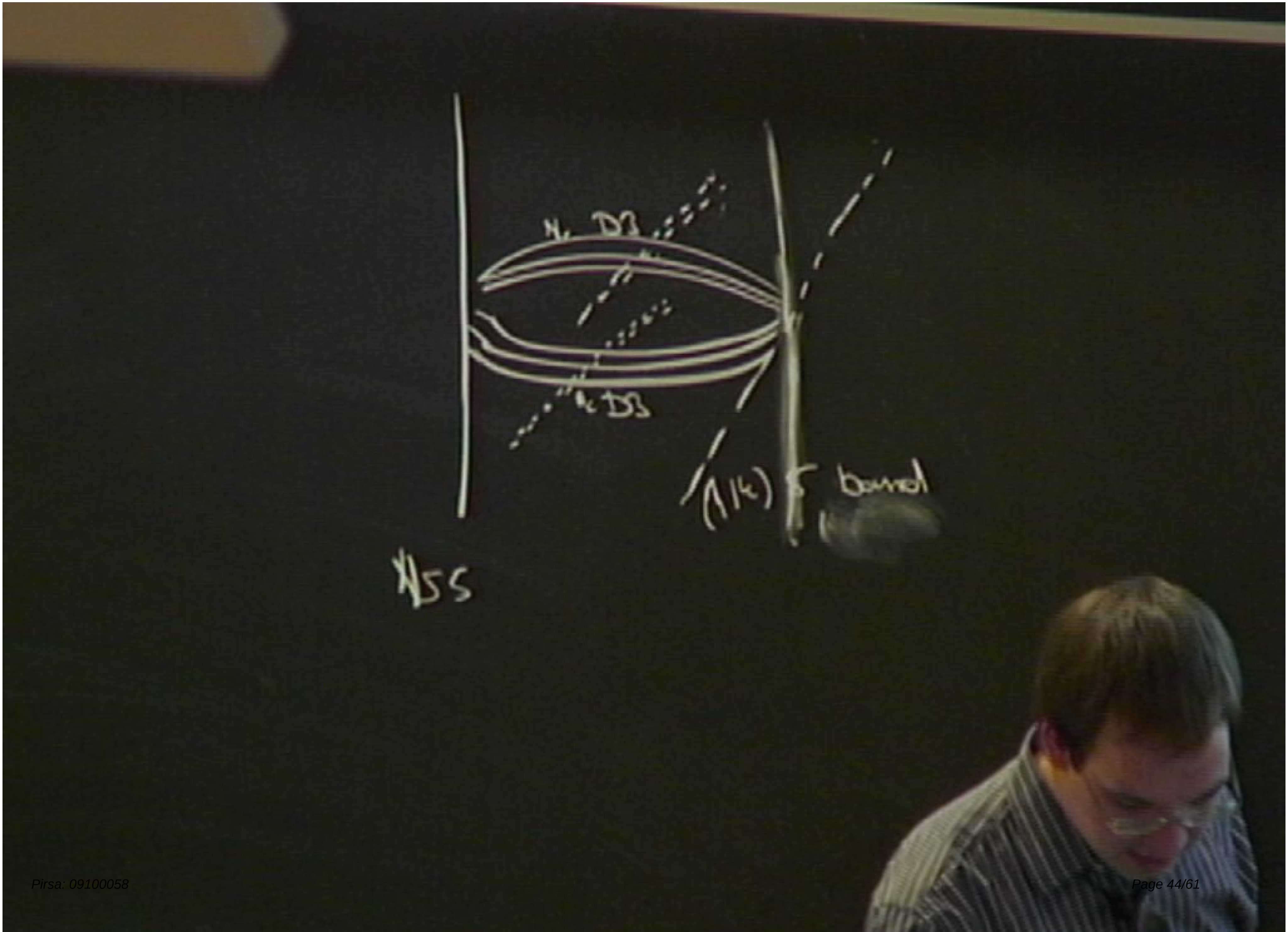
Codimension zero Flavor, Step 2+3

$(1, k)$5–Brane

- Supersymmetry broken to $\mathcal{N} = 3$.
- Flavor action unchanged

T-duality along x_6 and Lift to M-theory

- type IIA configuration:
  N_f $D5 \rightarrow N_f$ $D6$–Branes.
- M-theory configuration:
  N_f $D6 \rightarrow KK$–Monopole associated with M–theory circle.
- action S_{fl} unchanged.


D3
D3
NS5
(1,k)

Codimension zero Flavor, Step 4

Gravity side

- zoom in on C^4/Z_k.
- For $N_c \gg k^5$: KK–Monopole wrapping a three cycle on S^7/Z_k.
- For $N_c \ll k^5$: $D6$–Brane wrapping $AdS_4 \times RP^3$.
- preserves 12 supercharges, i.e. $\mathcal{N}=3$ in 2+1 dimensions, as well as $U(1)_b$ and $SU(2)_R \times SU(2)_D \simeq SO(4) \subset SO(6)_{\mathcal{R}}$

Field theory side

- Determine effective theory valid on scales $\ll g^2_{YM} k$.
- In S_{fl}, write down all terms consistent with 2+1 dimensional $\mathcal{N}=3$ supersymmetry and $SO(3)_R$.

  $\Rightarrow$ No such terms! $\Rightarrow$ S_{fl} unchanged. [Gaiotto, Yin '07]
- Integrate out Φ_1 and Φ_2.

Codimension zero Flavor, Action

Action

$$S = S_{\text{fl}} + S_{\text{ABJM}} = S_{\text{fl}} + S_{\text{CS}} + S_{\text{bifund}} + S_{\text{pot}}\,,$$

where

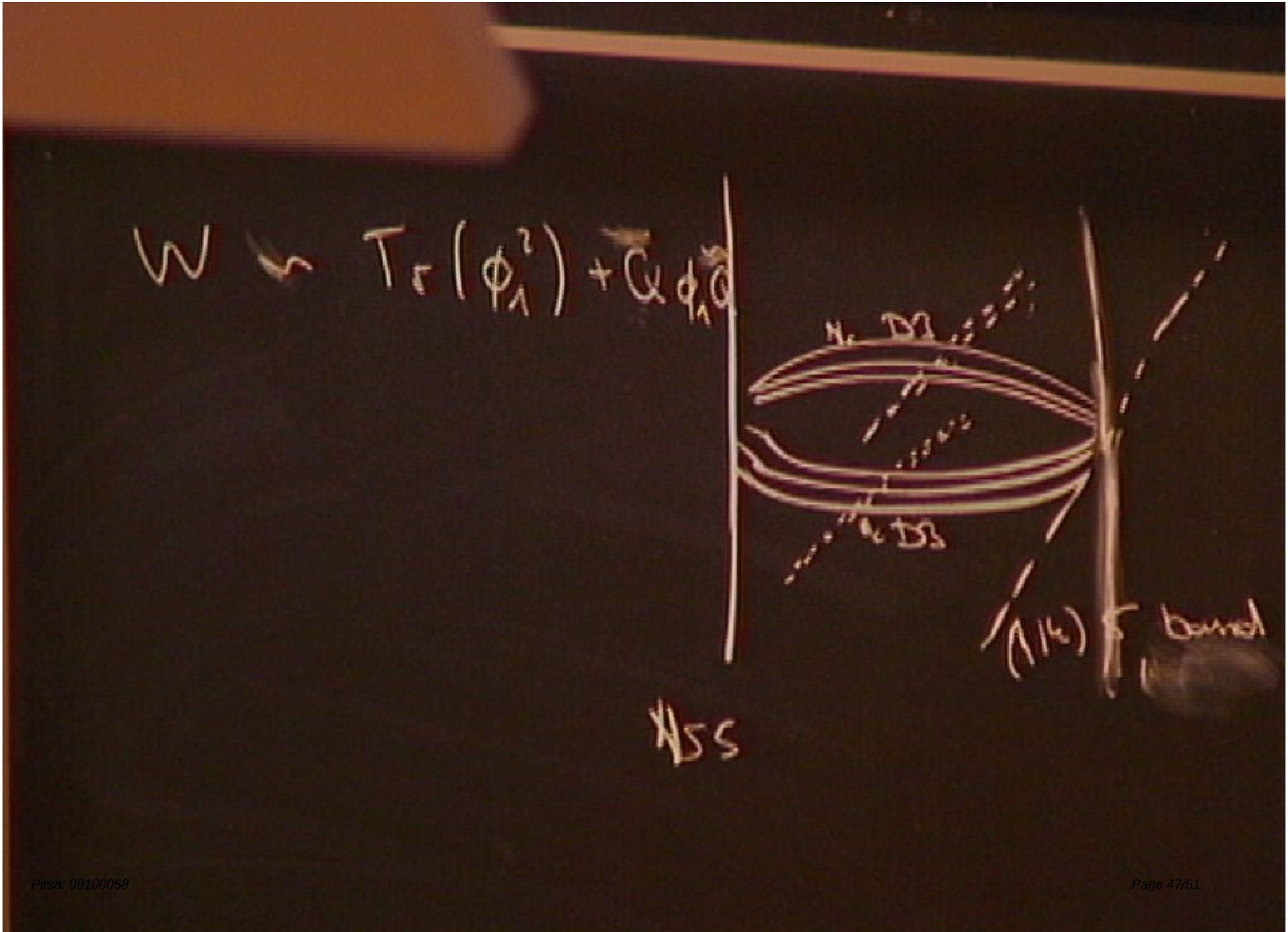
- S_{CS} and S_{bifund} unchanged, $S_{\text{pot}} = \int d^3x\, d^2\theta\, W + c.c.$, with

$$W = \frac{2\pi}{k}\,\text{Tr}\left[(A_a B_a + Q_1\tilde{Q}_1)^2 - (B_a A_a - Q_2\tilde{Q}_2)^2\right].$$

- $S_{\text{fl}} = \int d^3x\, d^4\theta \left(\bar{Q}_k e^{V_k} Q_k + \tilde{Q}_k e^{-V_k}\bar{\tilde{Q}}_k\right)$,

Symmetries of the action

- preserves 12 supersymmetry charges, i.e. $\mathcal{N}=3$ in 2+1 D
- $U(1)_b$ Symmetry as well as $SU(2)_D \times SU(2)_R = SO(4)_{\mathcal{R}}$
- Symmetries on gravity and field theory side match


W ~ Tr(φ₁²) + Q φ₁ Q̃
N_c D3
N_c D3
(1,k) 5-brane
NS5

Codimension zero Flavor, Action

Action

$$S = S_{\text{fl}} + S_{\text{ABJM}} = S_{\text{fl}} + S_{\text{CS}} + S_{\text{bifund}} + S_{\text{pot}}\,,$$

where

- S_{CS} and S_{bifund} unchanged, $S_{\text{pot}} = \int d^3x\, d^2\theta\, W + c.c.$, with

$$W = \frac{2\pi}{k}\,\text{Tr}\left[(A_a B_a + Q_1\tilde{Q}_1)^2 - (B_a A_a - Q_2\tilde{Q}_2)^2\right].$$

- $S_{fl} = \int d^3x\, d^4\theta \left(\bar{Q}_k e^{V_k} Q_k + \tilde{Q}_k e^{-V_k} \bar{\tilde{Q}}_k\right),$

Symmetries of the action

- preserves 12 supersymmetry charges, i.e. $\mathcal{N}=3$ in 2+1 D
- $U(1)_b$ Symmetry as well as $SU(2)_D \times SU(2)_R = SO(4)_{\mathcal{R}}$
- Symmetries on gravity and field theory side match

Codimension one Flavor, Step 1

Consider D7–Brane in 01345789 direction. [Fujita, Li, Ryu, Takayanagi]

N_c D3–Branes and N_f D7–Branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	•	•	•	-	-	-	•	-	-	-
N_f D7	•	•	-	•	•	•	-	•	•	•

8 supercharges, Flavor fields confined to 1+1 dimensional defect.

What are the flavor fields?

- study spectrum of 3–7 strings → single 1+1 dim. Weyl fermion ψ.
- But how can fermions alone be supersymmetric?

  Fermions are *left–handed*, preserved supercharges *right–handed*, i.e. $\mathcal{N} = (0, 8)$.
- $S_{def} = \int dx_+ dx_- \psi^\dagger (i\partial_- - A_-)\psi$. [Harvey, Royston '08]

Codimension one Flavor, Step 1

Add NS5–Branes

	0	1	2	3	4	5	6	7	8	9
$N_c\, D3$	•	•	•	-	-	-	•	-	-	-
$N_f\, D7$	•	•	-	•	•	•	-	•	•	•
$1\, NS5$	•	•	•	•	•	•	-	-	-	-

Does S_{def} change?

- dimensional reduce on $x_6 \rightarrow$ No!
- Set $\mathcal{N} = 4$ Hyper Multiplet to zero $\rightarrow$ No!
  Supersymmetry broken down to $\mathcal{N} = (0,4)$.

Compactify x_6

Add N_f D7–Branes intersecting <u>each</u> stack of N_c $D3$–Branes

$$S_{def} = \int dx_+ dx_- \psi^\dagger_{(k)}(i\partial_- - A_{(k)\,-})\psi_{(k)}.$$

Codimension one Flavor, Step 1

Consider D7–Brane in 01345789 direction. [Fujita, Li, Ryu, Takayanagi]

N_c D3–Branes and N_f D7–Branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	•	•	•	-	-	-	•	-	-	-
N_f D7	•	•	-	•	•	•	-	•	•	•

8 supercharges, Flavor fields confined to 1+1 dimensional defect.

What are the flavor fields?

- study spectrum of 3–7 strings $\to$ single 1+1 dim. Weyl fermion ψ.
- But how can fermions alone be supersymmetric?

  Fermions are *left–handed*, preserved supercharges *right–handed*, i.e. $\mathcal{N}=(0,8)$.
- $S_{def}=\int dx_+dx_-\psi^\dagger(i\partial_- - A_-)\psi$. [Harvey, Royston '08]

Codimension one Flavor, Step 1

Add NS5–Branes

	0	1	2	3	4	5	6	7	8	9
$N_c\ D3$	•	•	•	-	-	-	•	-	-	-
$N_f\ D7$	•	•	-	•	•	•	-	•	•	•
$1\ NS5$	•	•	•	•	•	•	-	-	-	-

Does S_{def} change?

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Compactify x_6

Add N_f D7–Branes intersecting each stack of N_c $D3$–Branes

$$S_{def} = \int dx_+ dx_- \psi^\dagger_{(k)}(i\partial_- - A_{(k)\,-})\psi_{(k)}.$$

Codimension one Flavor, Step 2+3

$(1,k)$5–Brane

- Bind k D5 and NS5 into $(1,k)$5–Brane

	0	1	2	3	4	5	6	7	8	9
N_c $D3$	•	•	•	-	-	-	•	-	-	-
N_f $D7$	•	•	-	•	•	•	-	•	•	•
1 $NS5$	•	•	•	•	•	•	-	-	-	-
$(1,k)5$	•	•	•	$[3,7]_\theta$	$[4,8]_\theta$	$[5,9]_\theta$	-	-	-	-

- Supersymmetry broken to $\mathcal{N} = (0,3)$.
- Flavor action S_{def} unchanged

T-duality along x_6 and Lift to M-theory

- type IIA configuration: N_f $D7 \to N_f$ $D8$–Branes.
- M-theory configuration: N_f $D8 \to$ "M9"–Branes.
- action S_{def} unchanged.

Codimension one Flavor, Step 4

Gravity side

- zoom in on C^4/Z_k.
- For $N_c \gg k^5$: "M9"–Branes wrapping $AdS_3 \times S^7/Z_k$.
- For $N_c \ll k^5$: D8–Branes wrapping $AdS_3 \times CP^3$.
- preserves 6 real supercharges, as well as $U(1)_b$ and $SU(4)_\mathcal{R}$.

Field theory side

- Determine effective theory valid on scales $\ll g_{YM}^2 k$.
- For S_{def}, write down all terms consistent with 1+1 dimensional $\mathcal{N} = (0,3)$ supersymmetry, $SO(3)_\mathcal{R}$, 1+1 D Lorentz- and gauge invarance $\Rightarrow$ No such terms! $\Rightarrow$ S_{def} unchanged.
- Integrate out Φ_1 and Φ_2 (trivial) $\Rightarrow$ action $S = S_{\text{ABJM}} + S_{def}$.

Codimension one Flavor, Step 4

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- Integrate out Φ_1 and Φ_2 (trivial) $\Rightarrow$ action $S = S_{ABJM} + S_{def}$.
- $\mathcal{N} = (0, 6)$ susy, $SU(4)_{\mathcal{R}} \times U(1)_b$

Codimension one Flavor, Step 4

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- Integrate out Φ_1 and Φ_2 (trivial) $\Rightarrow$ action $S = S_{\text{ABJM}} + S_{def}$.
- $\mathcal{N} = (0,6)$ susy, $SU(4)_{\mathcal{R}} \times U(1)_b \Rightarrow$ Symmetries match!

Generalizations

Other interesting field theories

The flavor branes in type IIB give rise to other field theories, which we studied in arXiv: 0909.3845.

- A D3-brane in type IIB (D4-brane in type IIA, M5-brane in M-theory) $\rightarrow$ codimension one, non-chiral, $\mathcal{N} = (3,3)$ flavor fields.
- A D3-brane in type IIB (D2-brane in type IIA, M2-brane in M-theory) $\rightarrow$ codimension two, $\mathcal{N} = 4$ flavor fields.

$SU(4)_{\mathcal{R}}$ equivalence

Different Dp–Branes in IIB become the same object in M–Theory (e.g. $D5$–Brane in 012789 and $\overline{D7}$–Brane in 01263478)

$\Rightarrow$ The same dual field theory

$\rightarrow$ Better understanding of IR–flow needed.

Outline

1. Motivation
2. Adding Flavor to AdS_5/CFT_4
3. The AdS_4/CFT_3 Duality
   - Dictionary
   - Deriving AdS_4/CFT_3 from type IIB setup
   - The Chern-Simons Matter Theory
4. Adding Flavor to AdS_4/CFT_3
   - General Recipe to determine Field theory / Gravity theory
   - $\mathcal{N} = 3$, codimension zero Flavor
   - $\mathcal{N} = (0, 6)$, codimension one Flavor
5. **Conclusion and Future Directions**

Conclusion

Summary

General *recipe* for adding flavors to AdS_4/CFT_3, in particular

- $\mathcal{N} = 3$, codimension zero flavor
  dual gravity description for $N_c \to \infty$, $N_c \ll k$
  - $N_c \gg k^5$: KK–Monopole in $AdS_4 \times S^7/Z_k$,
  - $N_c \ll k^5$: D6–brane wrapping $AdS_4 \times RP^3$ in $AdS_4 \times CP^3$.
- $\mathcal{N} = (0, 6)$, codimension one flavor
  - $N_c \gg k^5$: "M9–Brane" wrapping $AdS_3 \times S^7/Z_k$,
  - $N_c \ll k^5$: D8–brane wrapping $AdS_3 \times CP^3$.

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Future Directions

- More examples, complete classification!
- Introduce mass, compute meson spectra.
- Study thermodynamics and hydrodynamics.

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Thank you!