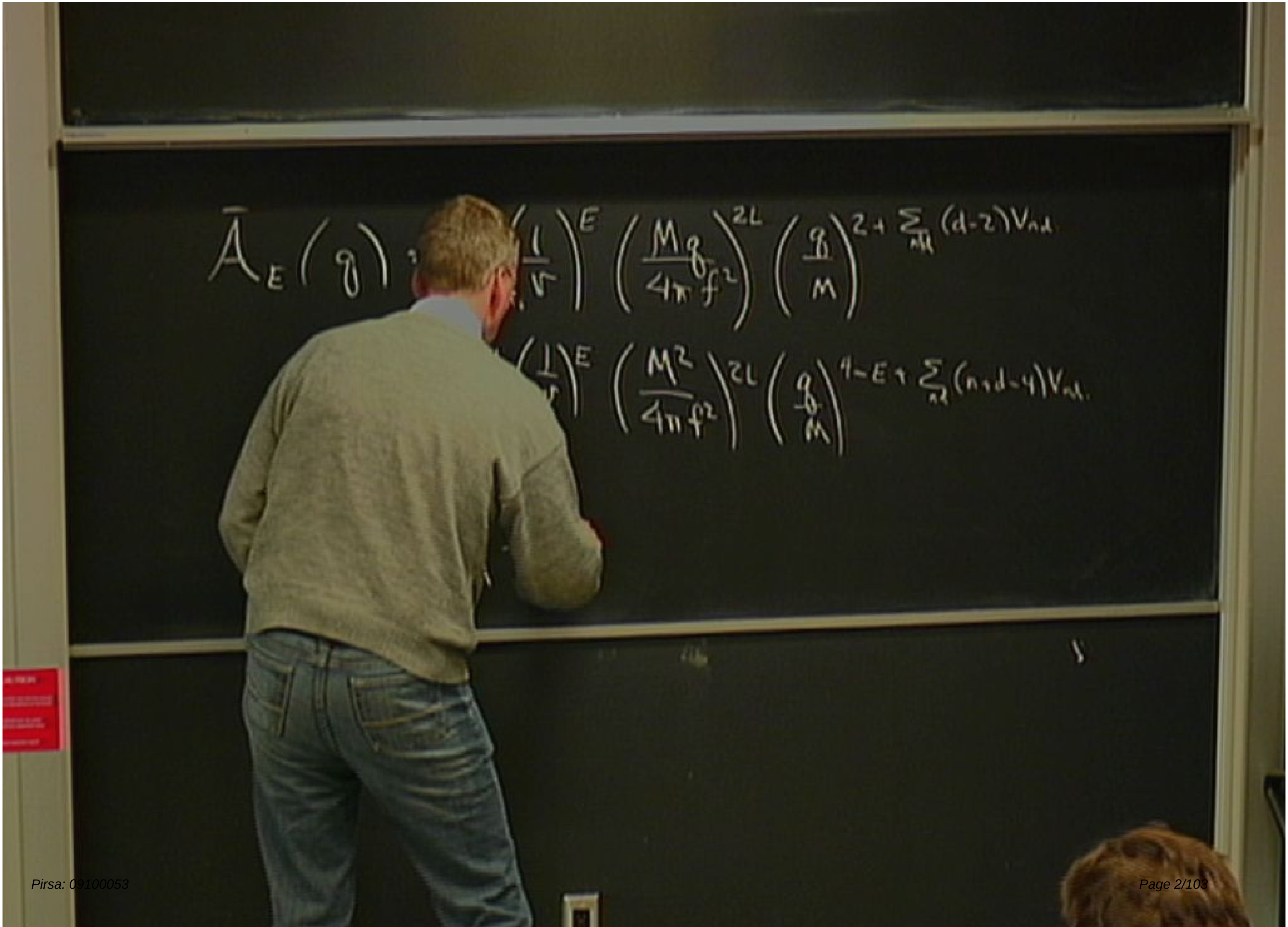
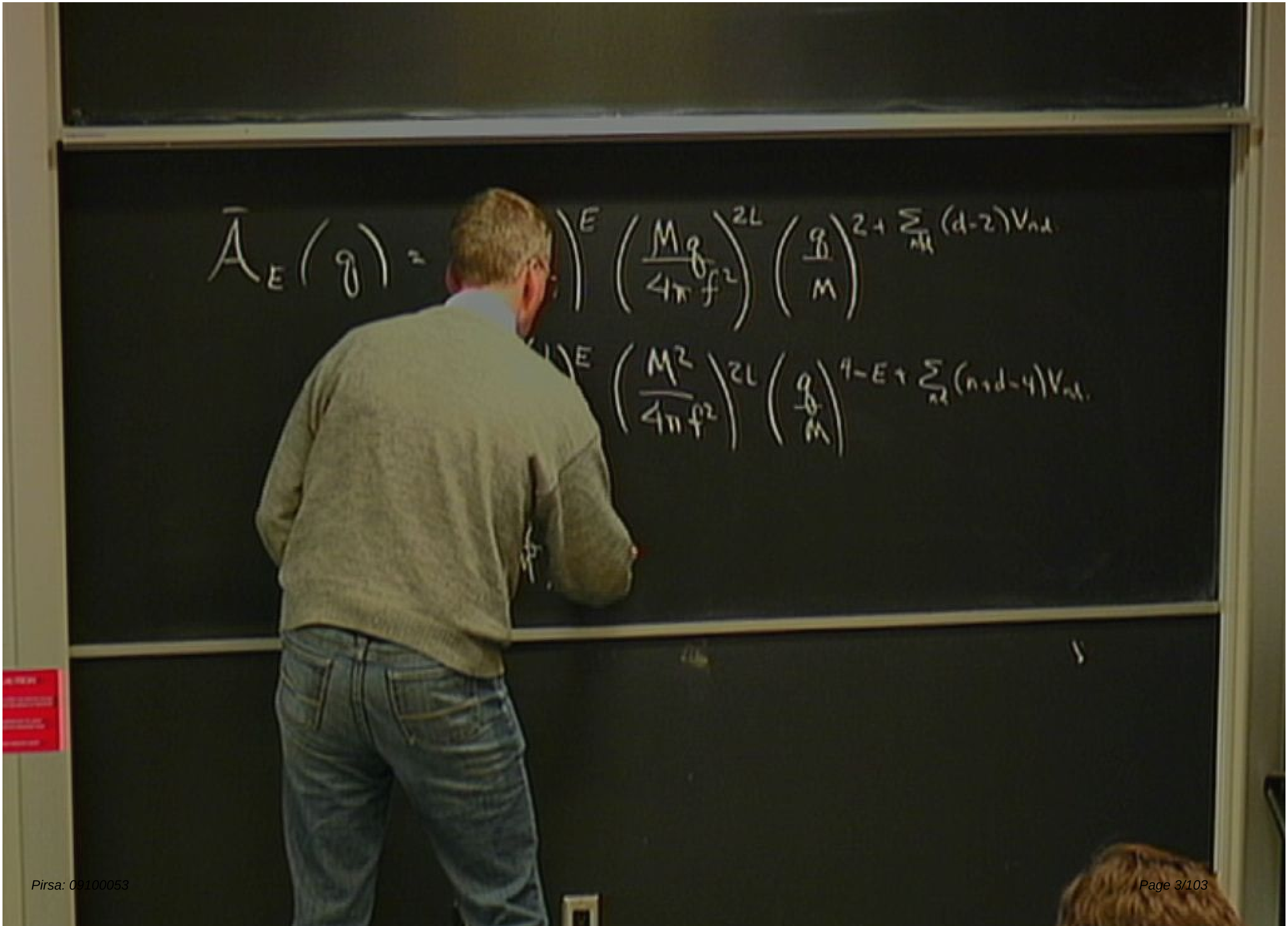
Title: Introduction to Effective Field Theory - Lecture 6A

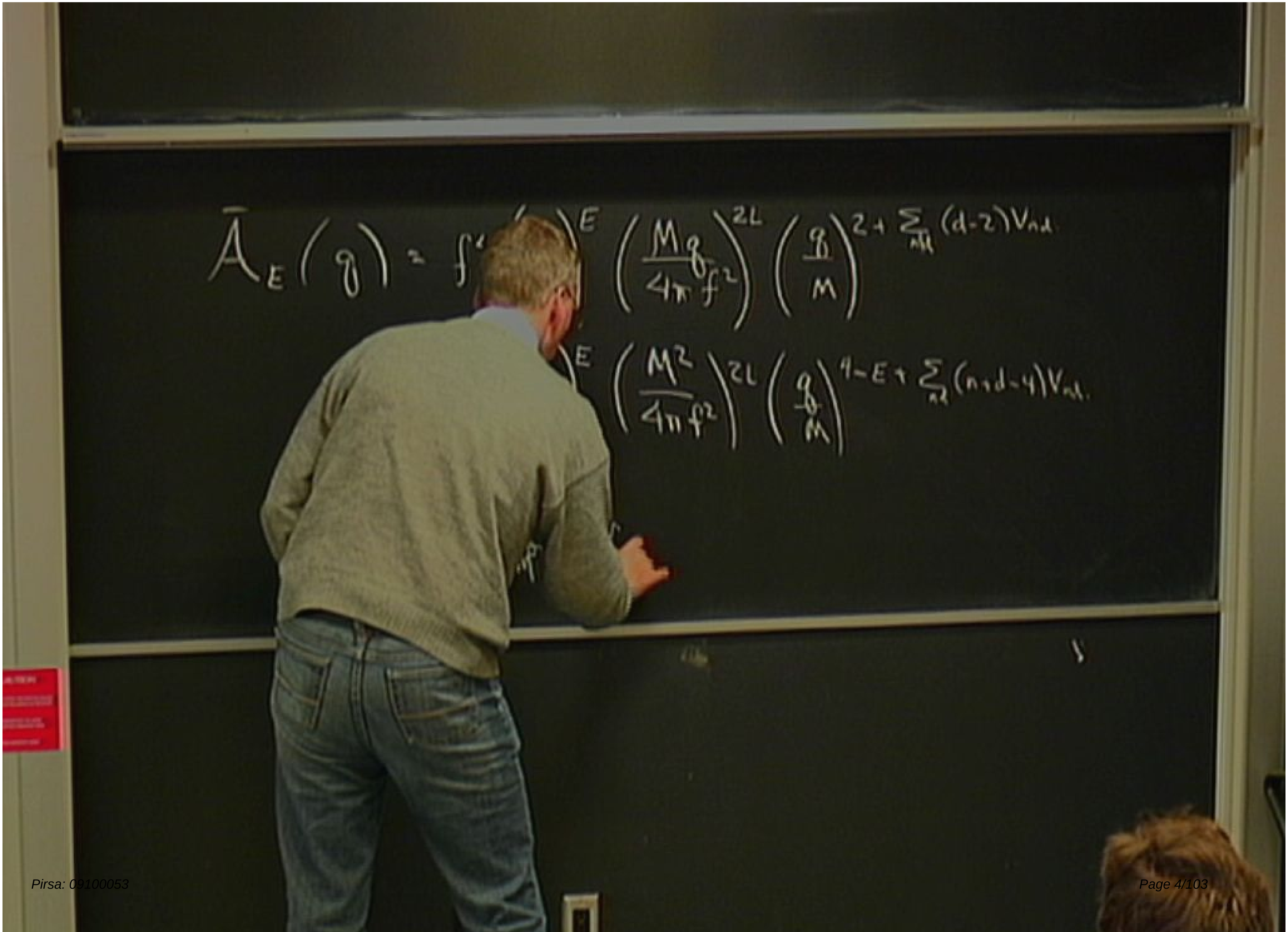
Date: Oct 28, 2009 10:00 AM

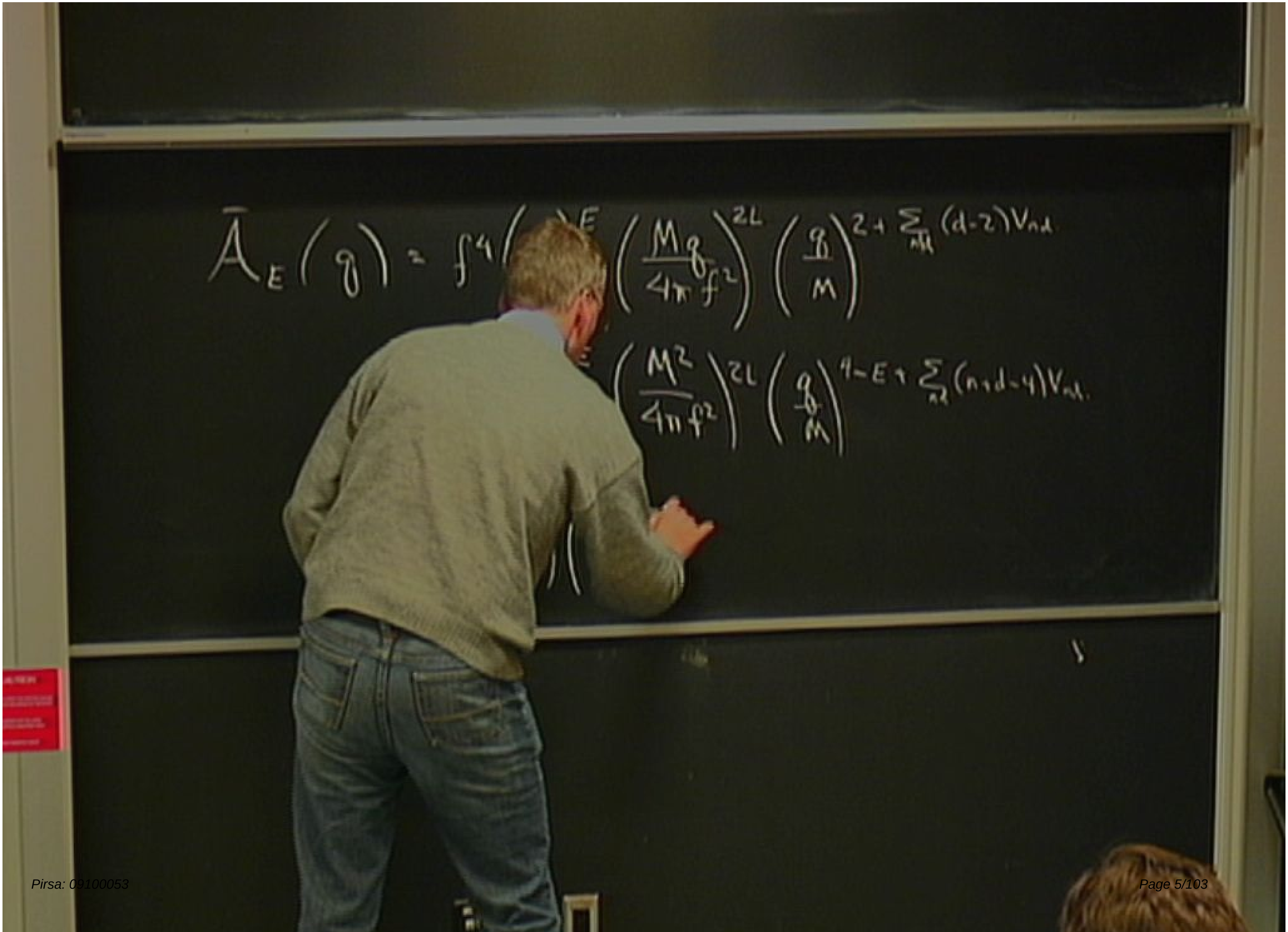
URL: http://pirsa.org/09100053

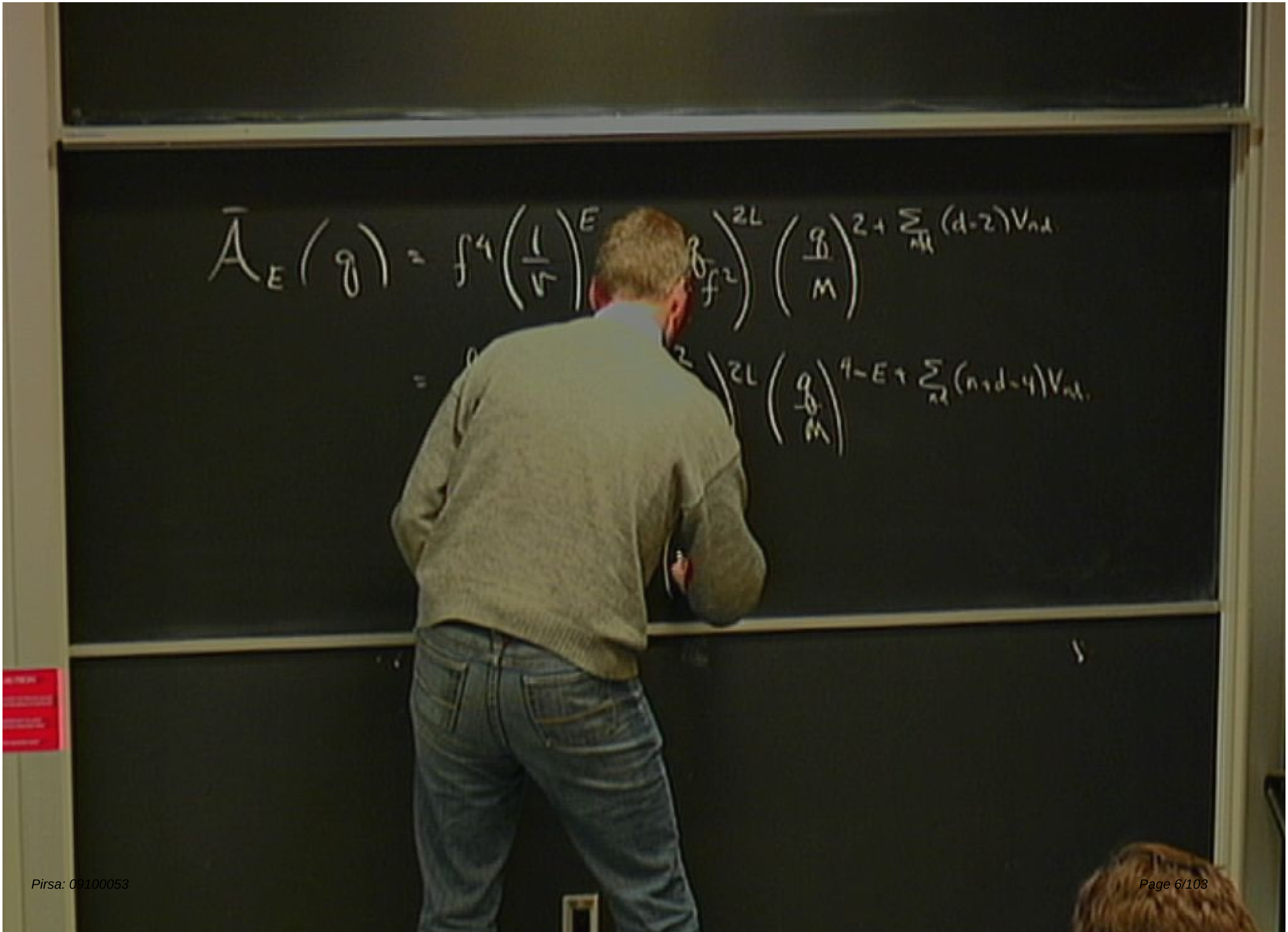
Abstract:

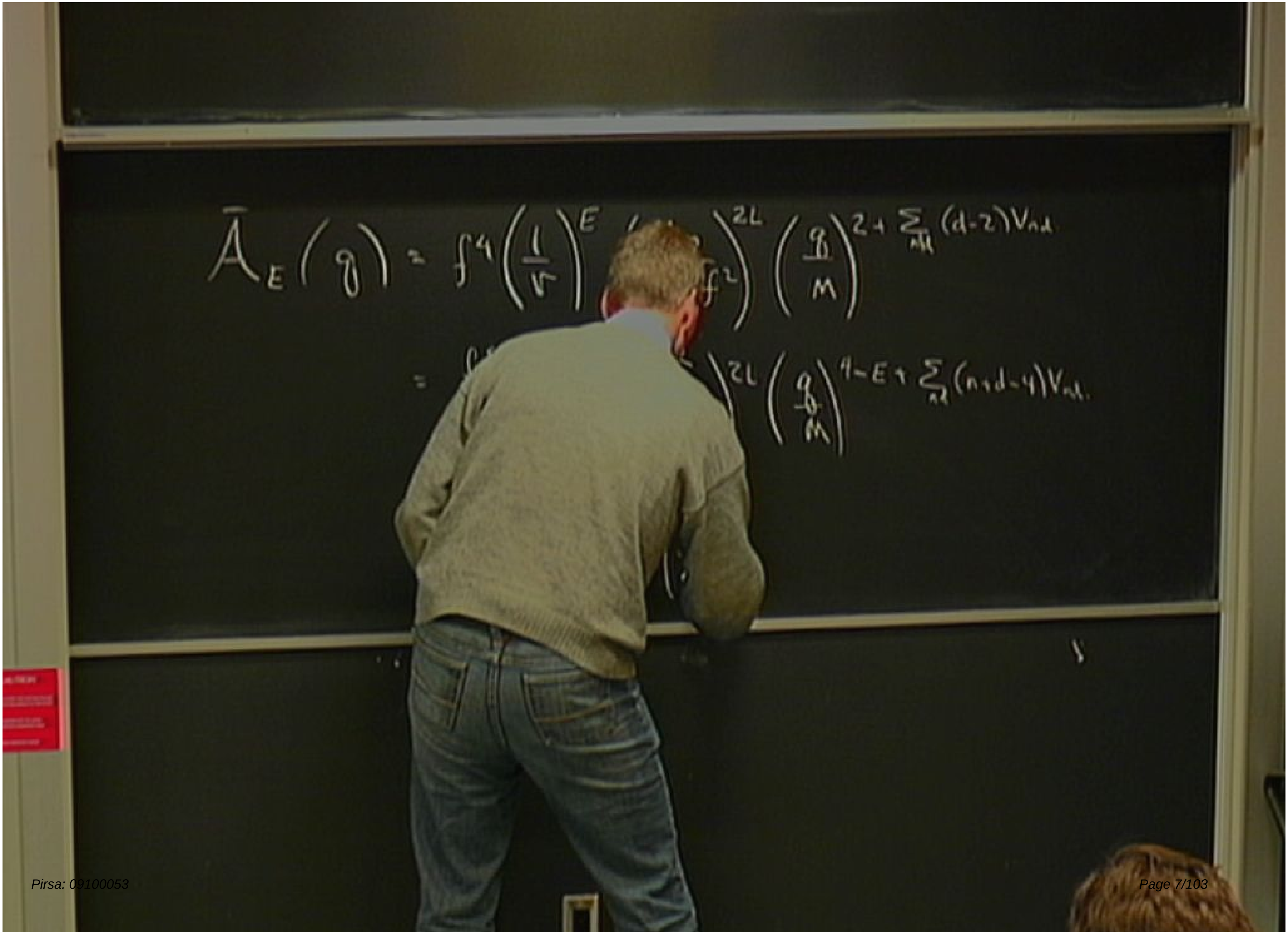


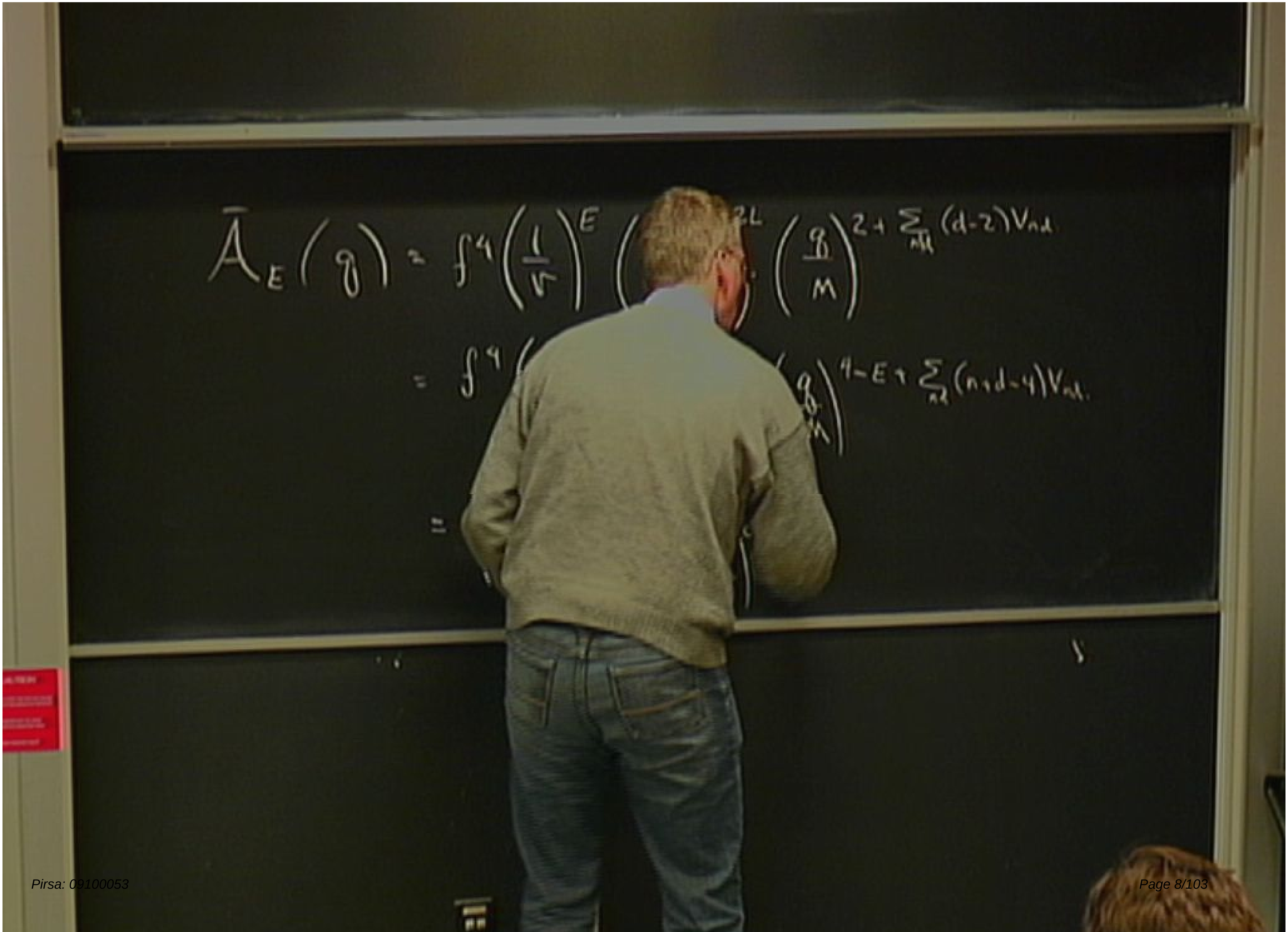

$\bar{A}_E(q) =$
$)^E \left(\frac{Mq}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{2+\sum_{nd}(d-2)V_{nd}}$
$)^E \left(\frac{M^2}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{4-E+\sum_{nd}(n+d-4)V_{nd}}$

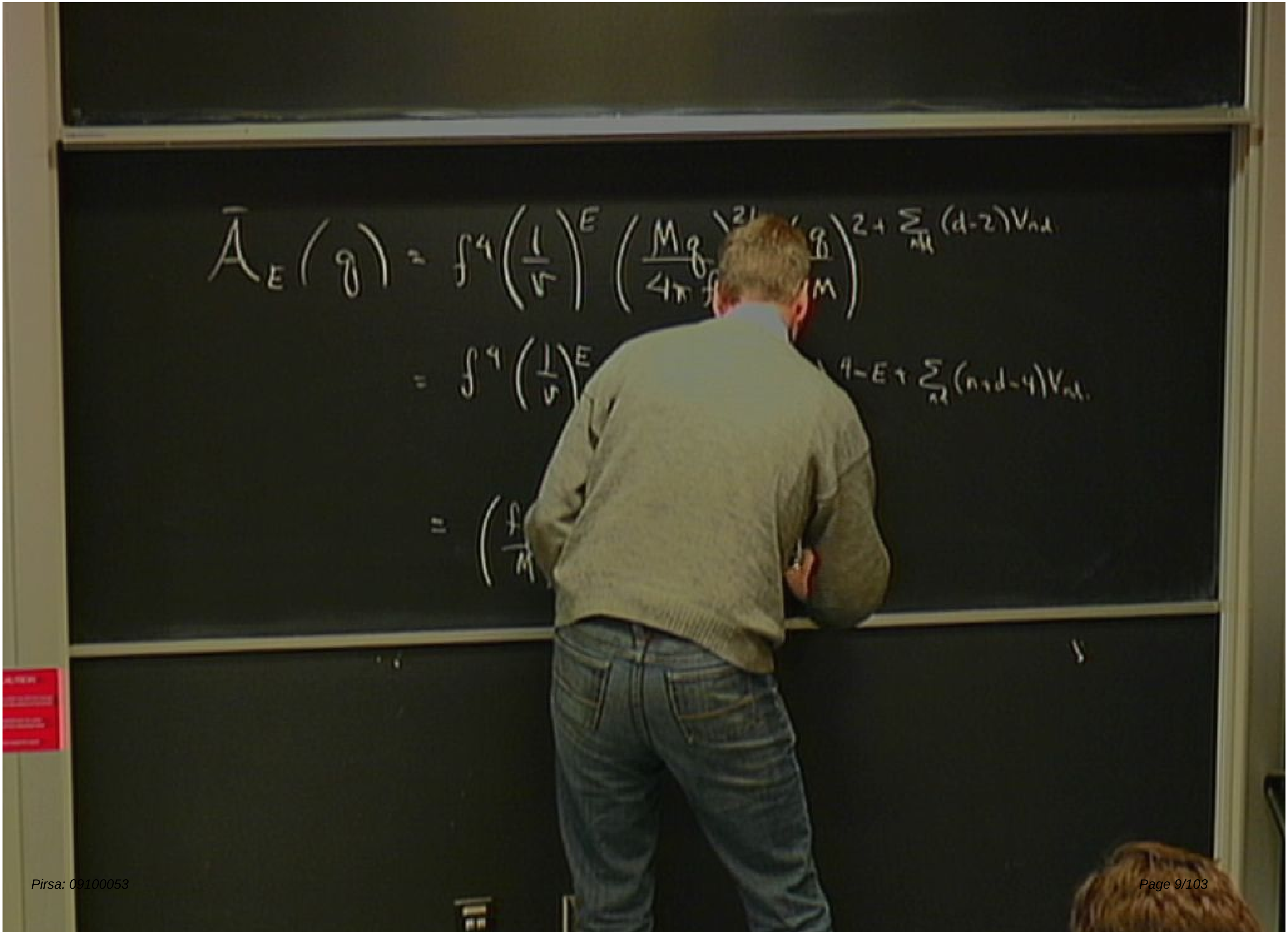

$\bar{A}_E(q) = \int$ $)^E \left(\frac{Mq}{4\pi f^2}\right)^{2L}\left(\frac{q}{M}\right)^{2+\sum_{nd}(d-2)V_{nd}}$
$)^E \left(\frac{M^2}{4\pi f^2}\right)^{2L}\left(\frac{q}{M}\right)^{4-E+\sum_{nd}(n+d-4)V_{nd}}$

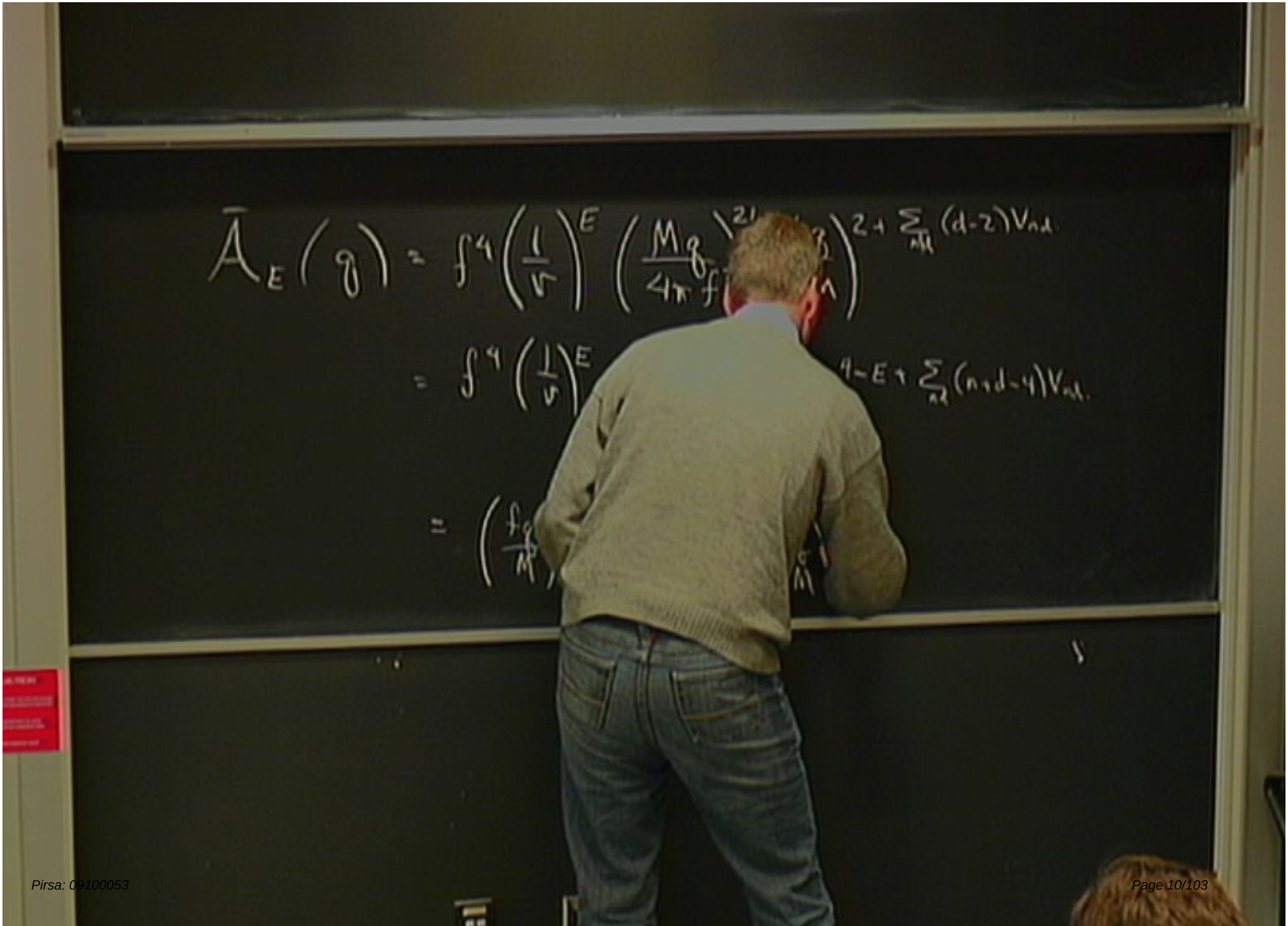

$\bar{A}_E(q) = f^4 \left(\frac{Mq}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{2+\sum_{nd}(d-2)V_{nd}}$
$\left(\frac{M^2}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{4-E+\sum_{nd}(n+d-4)V_{nd}}$




$\bar{A}_E(q) = f^4\left(\frac{1}{v}\right)^E \quad f^2\Big)^{2L}\left(\frac{q}{M}\right)^{2+\sum_{nd}(d-2)V_{nd}}$
$= \quad \Big)^{2L}\left(\frac{q}{M}\right)^{4-E+\sum_{nd}(n+d-4)V_{nd}}$


$\bar{A}_E(q) = \int^4 \left(\frac{1}{v}\right)^E$
$\left(\frac{q}{M}\right)^{2+\sum_{nd}(d-2)V_{nd}}$
$= \int^4$
$\left(\frac{q}{M}\right)^{4-E+\sum_{nd}(n+d-4)V_{nd}}$


$\bar{A}_E(q) = \int^4 \left(\frac{1}{v}\right)^E \left(\frac{Mq}{4\pi f}\right)$
$2 + \sum_{nd} (d-2) V_{nd}$
$= \int^4 \left(\frac{1}{v}\right)^E$
$4 - E + \sum_{nd} (n+d-4) V_{nd}$


$\bar{A}_E(q) = \int^4 \left(\frac{1}{v}\right)^E \left(\frac{Mq}{4\pi f}\right)^{2L}$
$2 + \sum_{nd} (d-2) V_{nd}$
$= \int^4 \left(\frac{1}{v}\right)^E$
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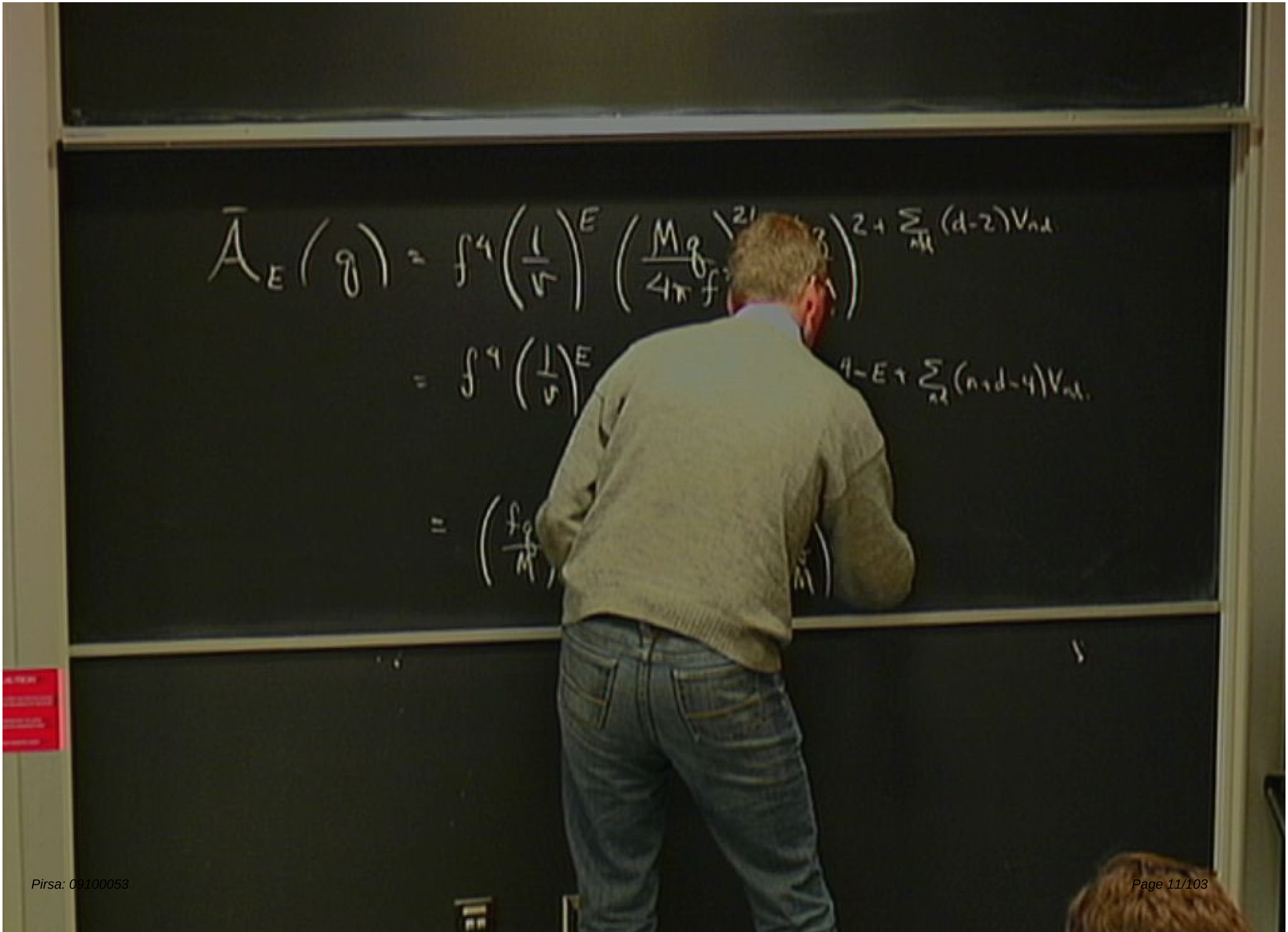
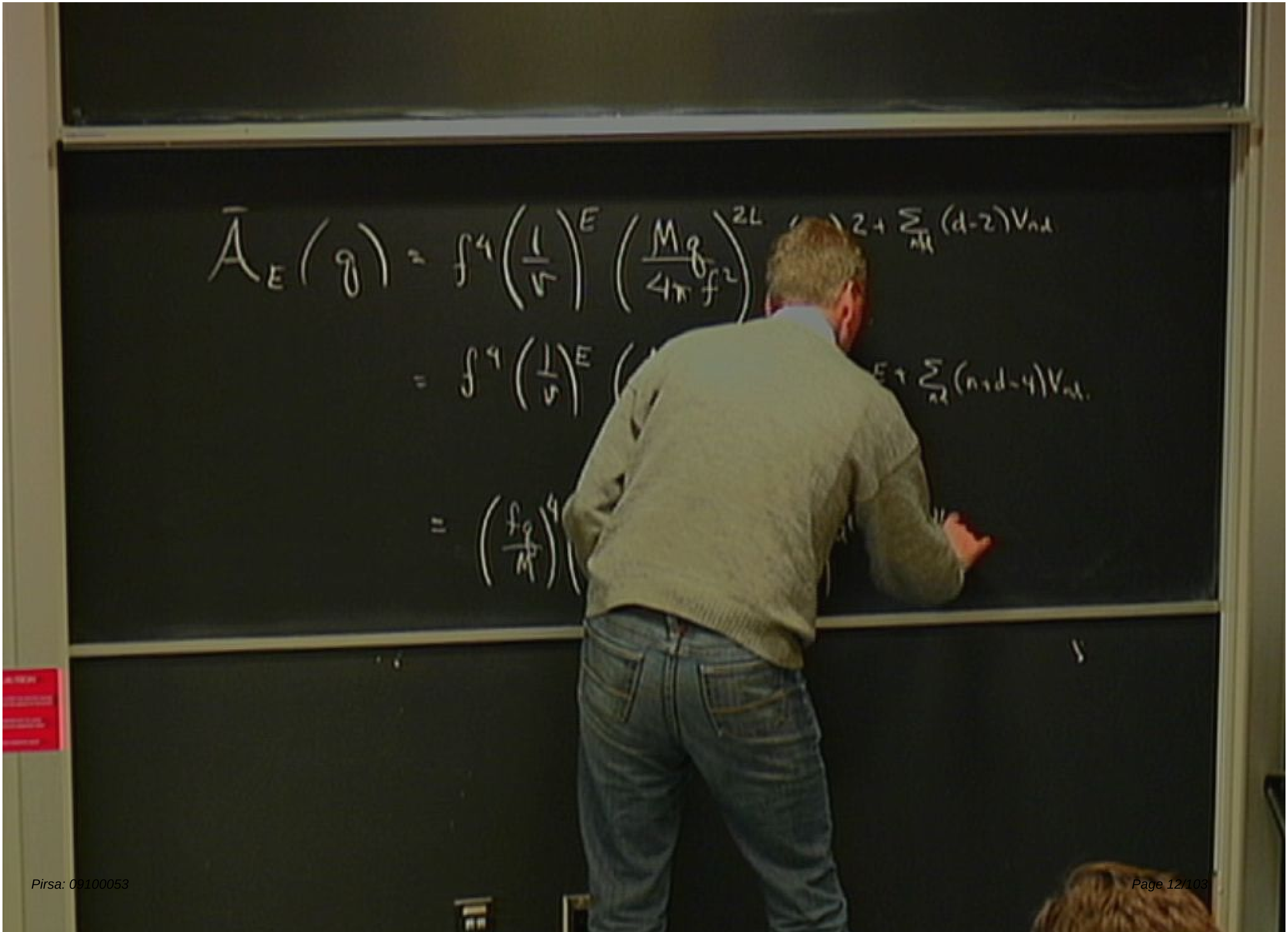
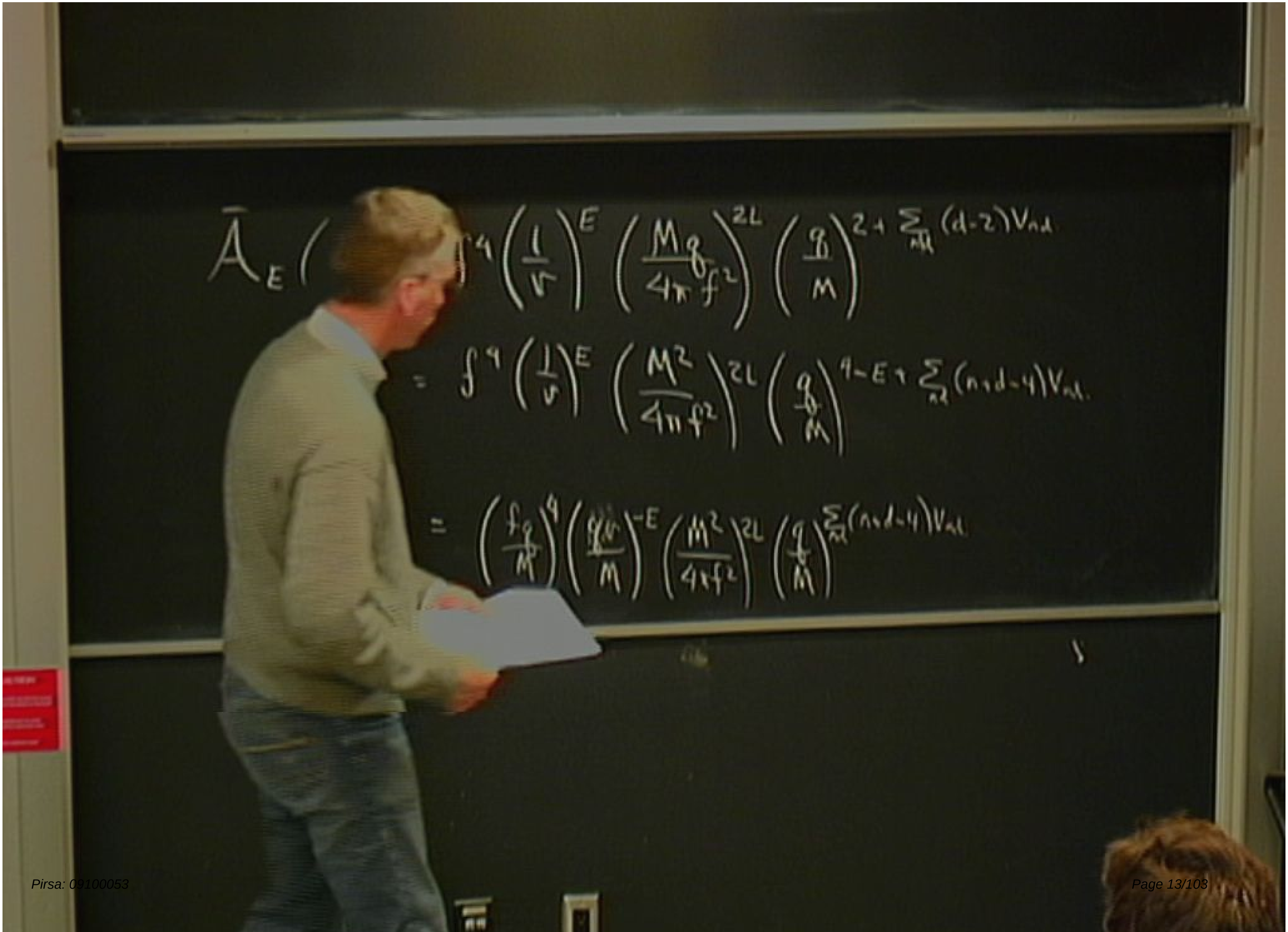





$\bar{A}_E\big($ $f^4\left(\frac{1}{v}\right)^E\left(\frac{Mg}{4\pi f^2}\right)^{2L}\left(\frac{g}{M}\right)^{2+\sum_{nd}(d-2)V_{nd}}$
$= f^4\left(\frac{1}{v}\right)^E\left(\frac{M^2}{4\pi f^2}\right)^{2L}\left(\frac{g}{M}\right)^{4-E+\sum_{nd}(n+d-4)V_{nd}}$
$= \left(\frac{fg}{M}\right)^4\left(\frac{gv}{M}\right)^{-E}\left(\frac{M^2}{4\pi f^2}\right)^{2L}\left(\frac{g}{M}\right)^{\sum_{nd}(n+d-4)V_{nd}}$

$$\bar{A}_E(q) = f^4 \left(\frac{1}{v}\right)^E \left(\frac{Mq}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{2+\sum_{n,d}(d-2)V_{n,d}}$$

$$= f^4 \left(\frac{1}{v}\right)^E \left(\frac{M^2}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{4-E+\sum_{n,d}(n+d-4)V_{n,d}}$$

$$= \left(\frac{fq}{M}\right)^4 \left(\frac{qv}{M}\right)^{-E} \left(\frac{M^2}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{\sum_{n,d}(n+d-4)V_{n,d}}$$

$$\bar{A}_E(q) = f^4 \left(\frac{1}{v}\right)^E \left(\frac{Mq}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{2+\sum_{nd}(d-2)V_{nd}}$$

$$= f^4 \left(\frac{1}{v}\right)^E \left(\frac{M^2}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{4-E+\sum_{nd}(n+d-4)V_{nd}}$$

$$= \left(\frac{fq}{M}\right)^4 \left(\frac{qv}{M}\right)^{-E} \left(\frac{M^2}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{\sum_{nd}(n+d-4)V_{nd}}$$

Effective Lagrangian Logic:

1) Decide on the accuracy required. (eg λ to 1% accuracy)

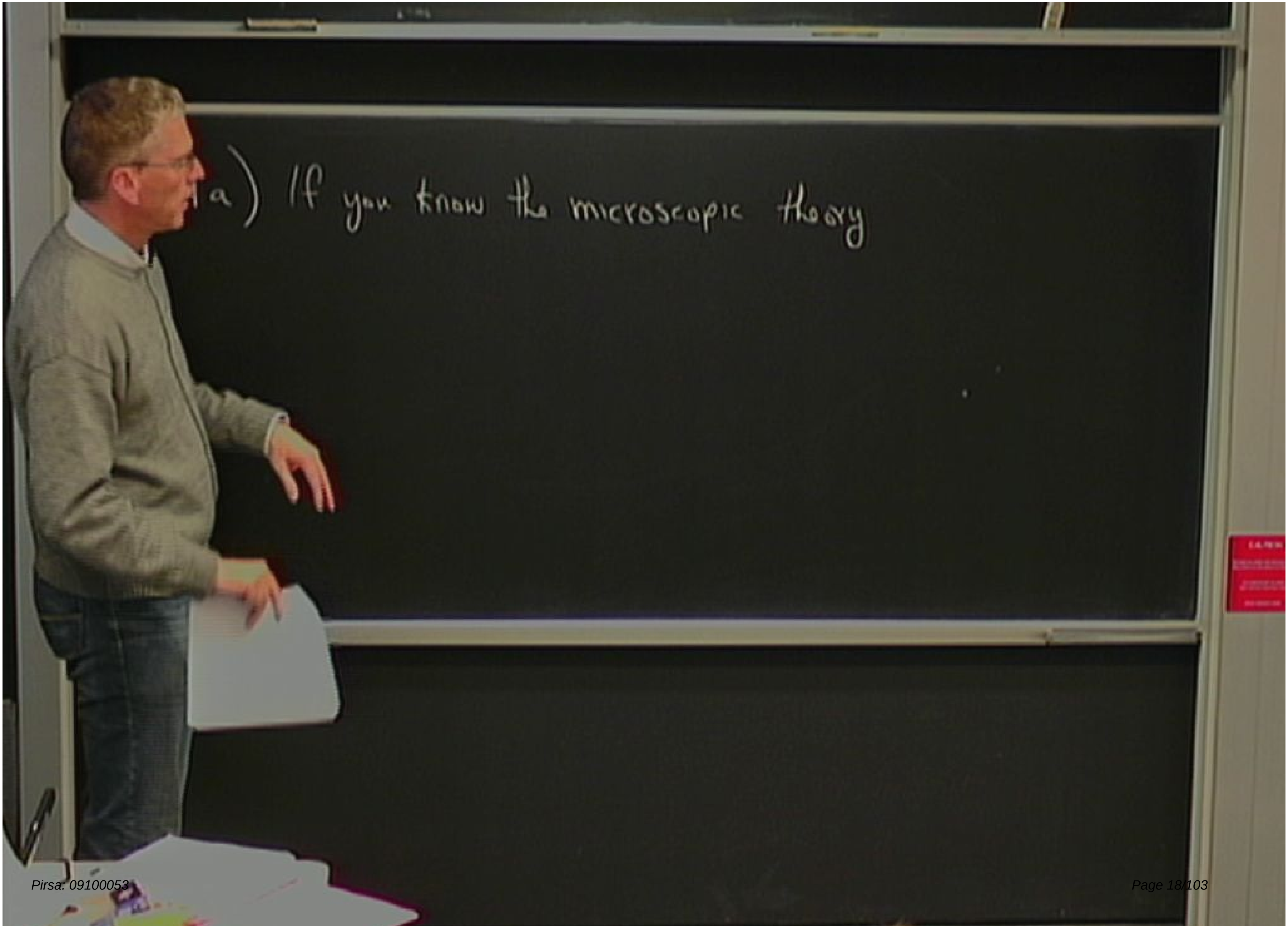
2)

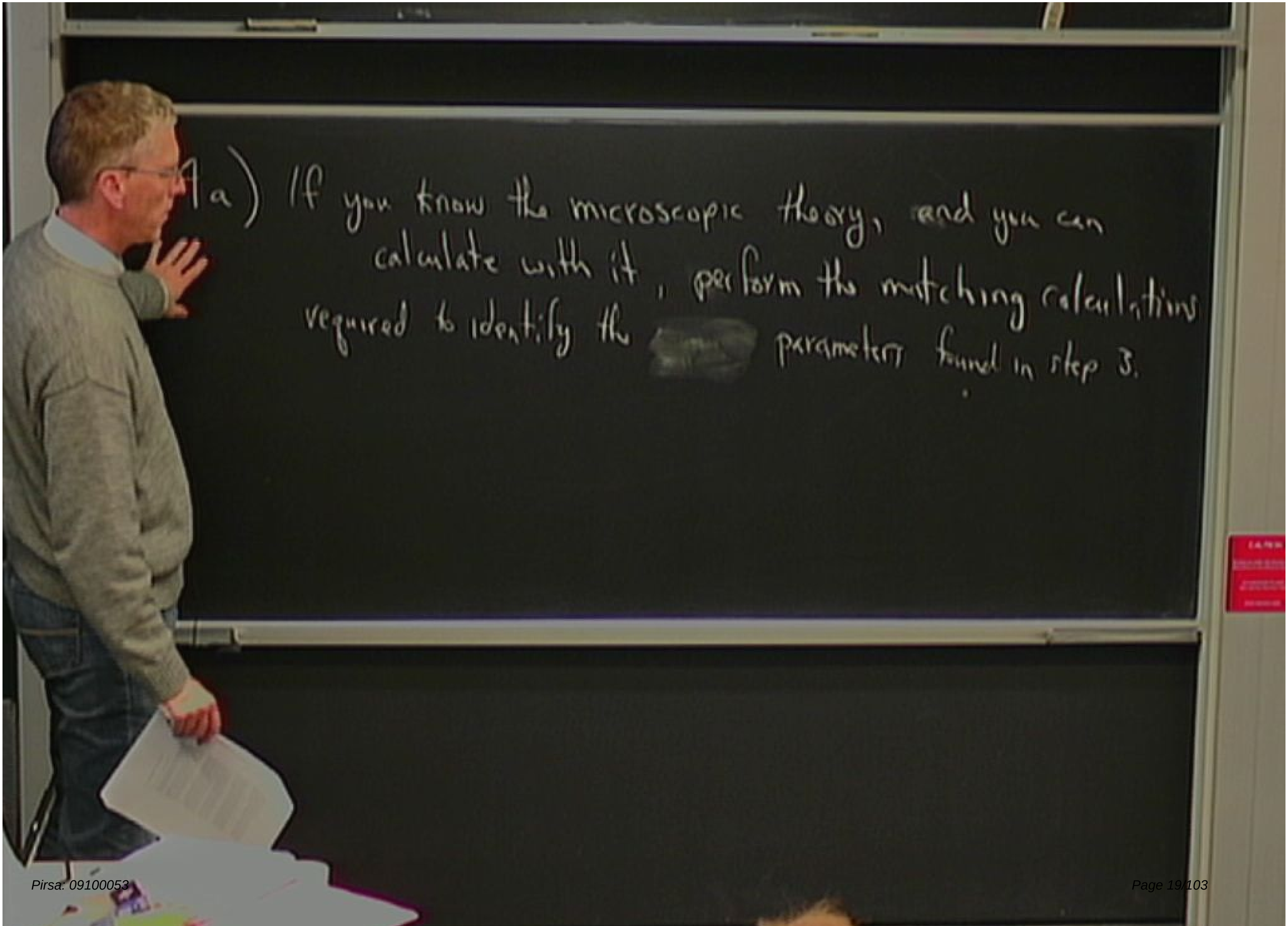
Effective Lagrangian Logic:

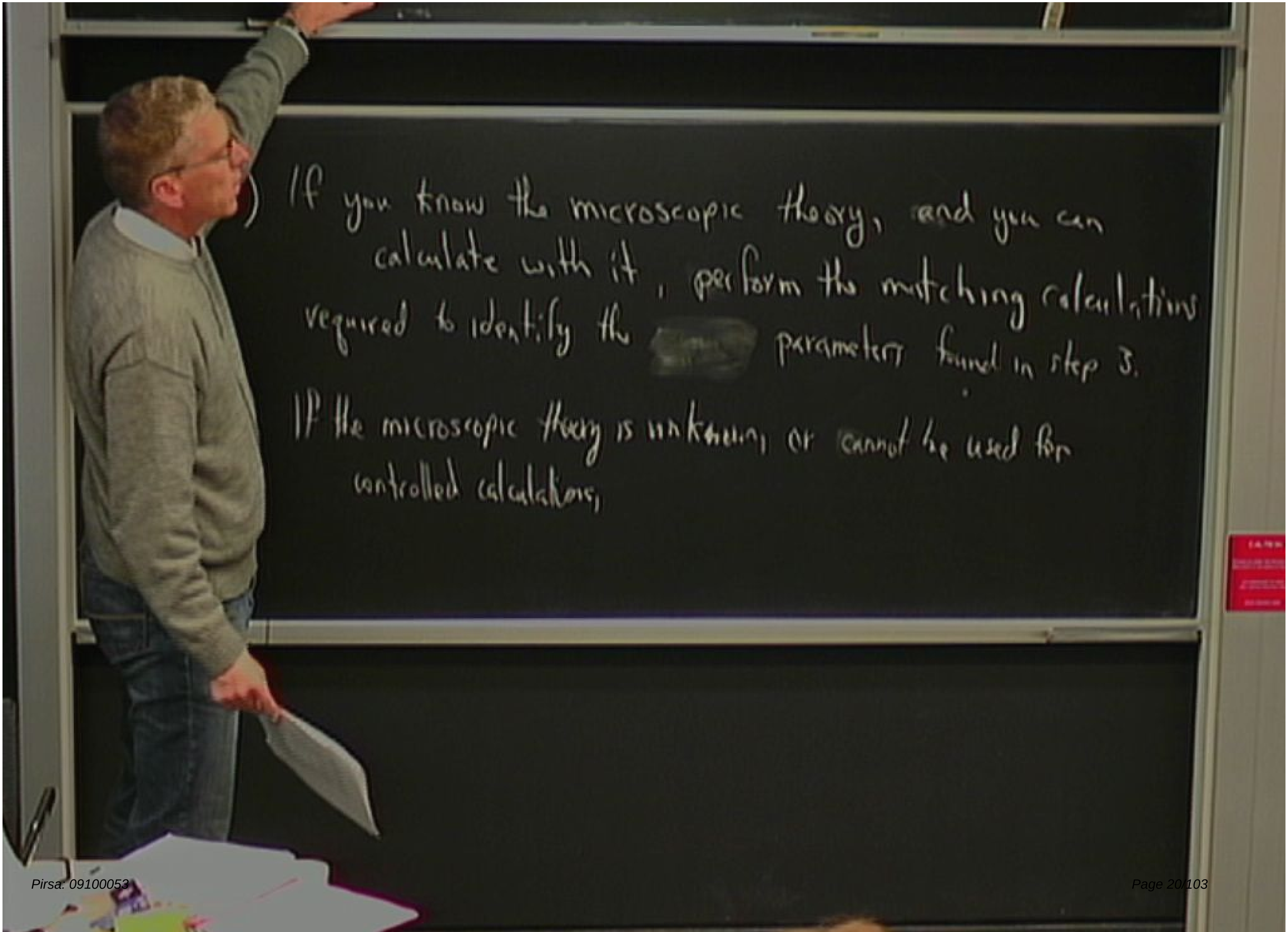
1) Decide on the accuracy required. (eg λ to 1% accuracy)

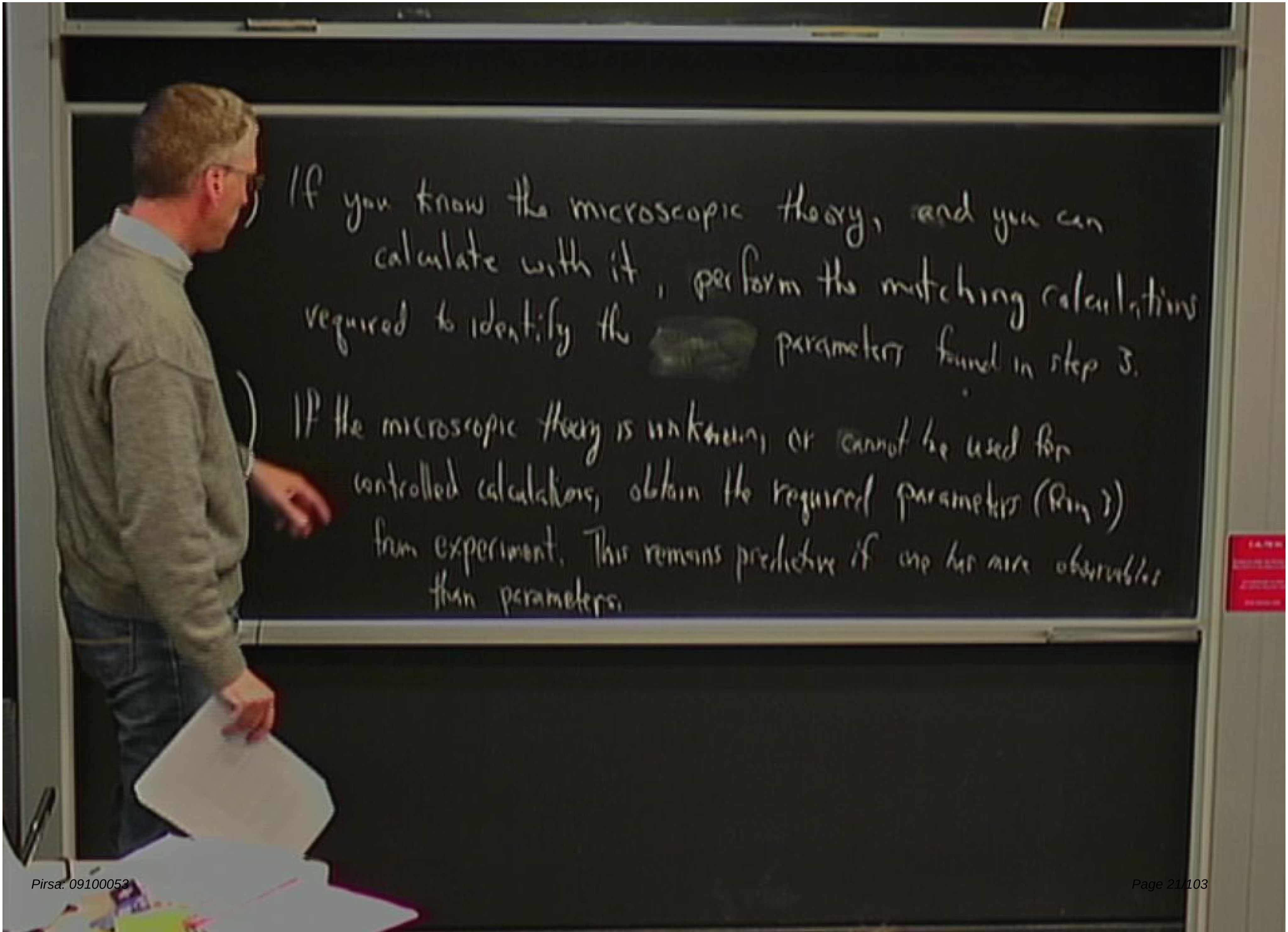
2) Identify how many powers of (E/M) are required to achieve this accuracy.

3) Use power counting formula, to identify which effective interactions can contribute to this order.

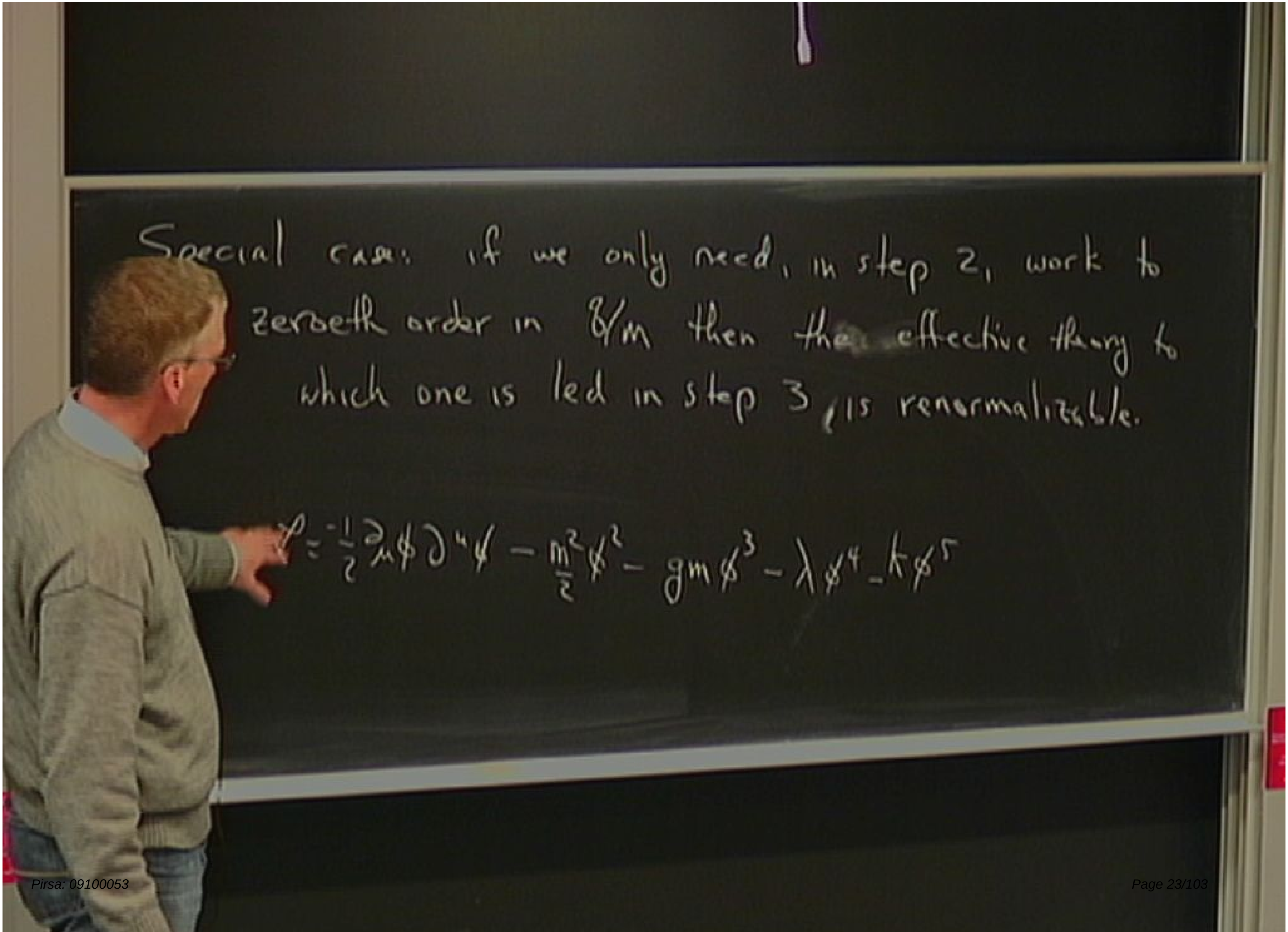

a) If you know the microscopic theory


7a) If you know the microscopic theory, and you can
calculate with it, perform the matching calculations
required to identify the parameters found in step 3.


) If you know the microscopic theory, and you can
calculate with it, perform the matching calculations
required to identify the parameters found in step 3.
If the microscopic theory is unknown, or cannot be used for
controlled calculations,


If you know the microscopic theory, and you can calculate with it, perform the matching calculations required to identify the parameters found in step 3.
If the microscopic theory is unknown, or cannot be used for controlled calculations, obtain the required parameters (from 3) from experiment. This remains predictive if one has more observables than parameters.

Special case: if we only need, in step 2, work to zeroth order in E/m then the effective theory to which one is led in step 3, is renormalizable.


Special case: if we only need, in step 2, work to zeroeth order in E/m then the effective theory to which one is led in step 3, is renormalizable.
$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{m^2}{2}\phi^2 - gm\phi^3 - \lambda\phi^4 - k\phi^5$

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$$\bar{A}_E(q) = f^4 \left(\frac{1}{v}\right)^E \left(\frac{Mq}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{2+\sum_{nd}(d-2)V_{nd}}$$

$f, v, M \gg q$

$$= f^4 \left(\frac{1}{v}\right)^E \left(\frac{M^2}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{4-E+\sum_{nd}(n+d-4)V_{nd}}$$

$$= \left(\frac{fq}{M}\right)^4 \left(\frac{qv}{M}\right)^{-E} \left(\frac{M^2}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{\sum_{nd}(n+d-4)V_{nd}}$$

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Special case: if we only need, in step 2, work to zeroeth order in g/m then the effective theory to which one is led in step 3, is renormalizable.

$$\begin{array}{ccccc} n+d=4 & 2 & 3 & 4 & 5 \end{array}$$

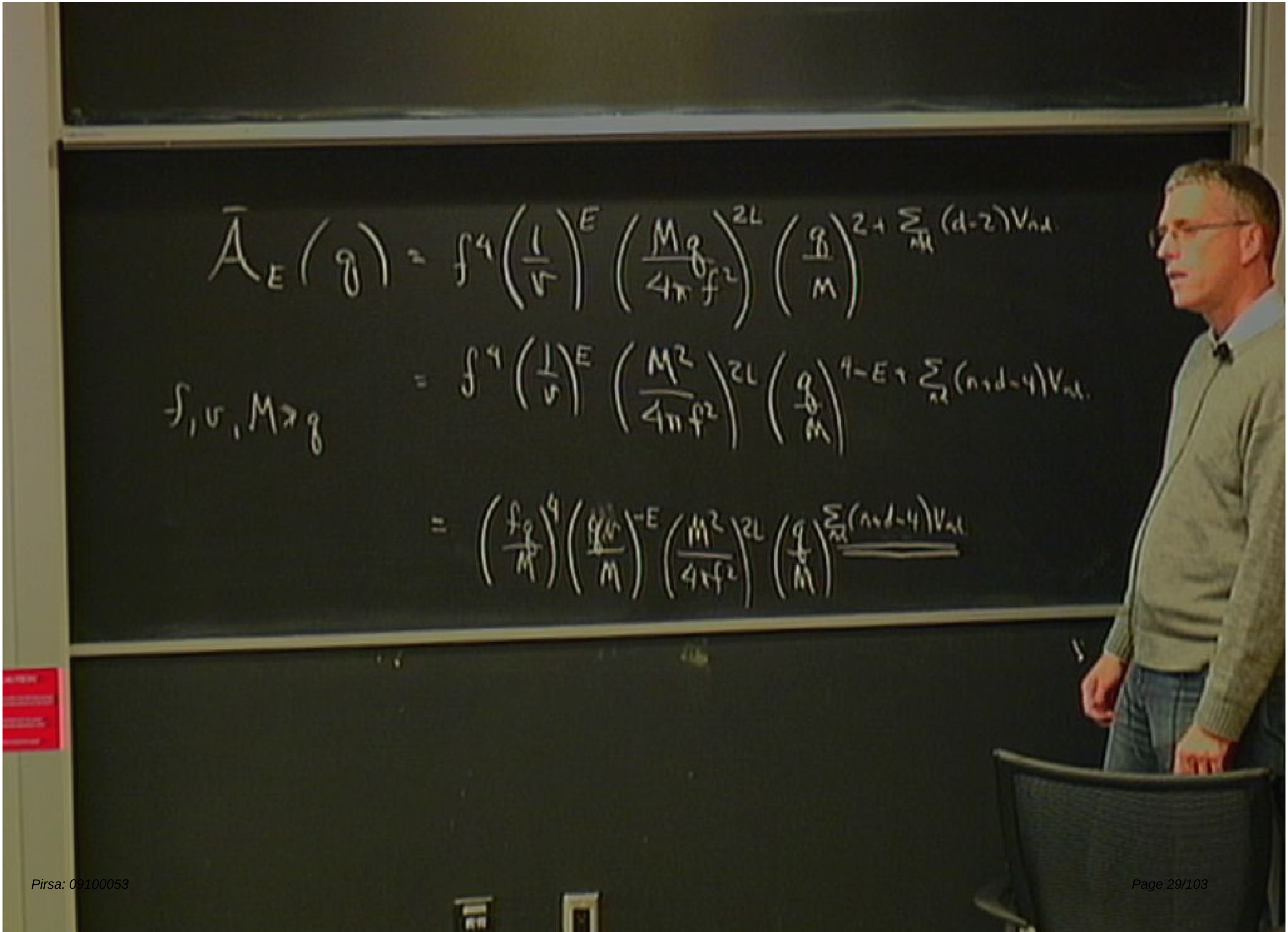
$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{m^2}{2}\phi^2 - gm\phi^3 - \lambda\phi^4 - k\phi^5$$

Special case: if we only need, in step 2, work to zeroeth order in $\mathcal{E}/m$ then the effective theory to which one is led in step 3, is renormalizable.

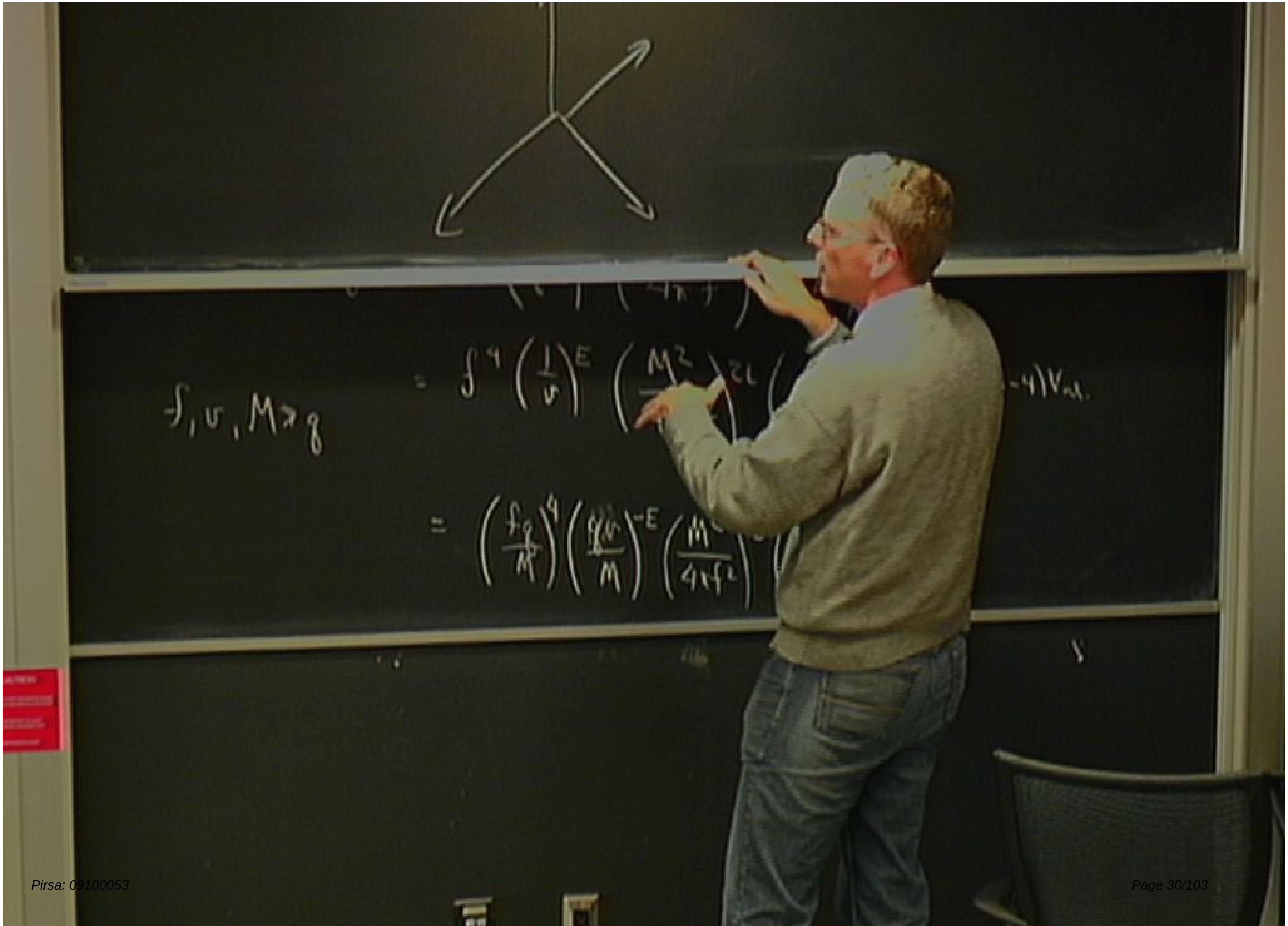
$n+d=4$ 2 3 4 5

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{m^2}{2}\phi^2 - gm\phi^3 - \lambda\phi^4 - k\phi^5$$

Using this in 4b is the usual way one calculate with a renormalizable theory.



$\bar{A}_E(q) = f^4 \left(\frac{1}{v}\right)^E \left(\frac{Mq}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{2+\sum_{nd}(d-2)V_{nd}}$
$= f^4 \left(\frac{1}{v}\right)^E \left(\frac{M^2}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{4-E+\sum_{nd}(n+d-4)V_{nd}}$
$f, v, M \gg q$
$= \left(\frac{fq}{M}\right)^4 \left(\frac{qv}{M}\right)^{-E} \left(\frac{M^2}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{\sum_{nd}(n+d-4)V_{nd}}$


f, v, M ≫ g

$$\bar{A}_E(q) = f^4 \left(\frac{1}{v}\right)^E \left(\frac{Mq}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{2+\sum_{nd}(d-2)V_{nd}}$$

$f, v, M \gg q$

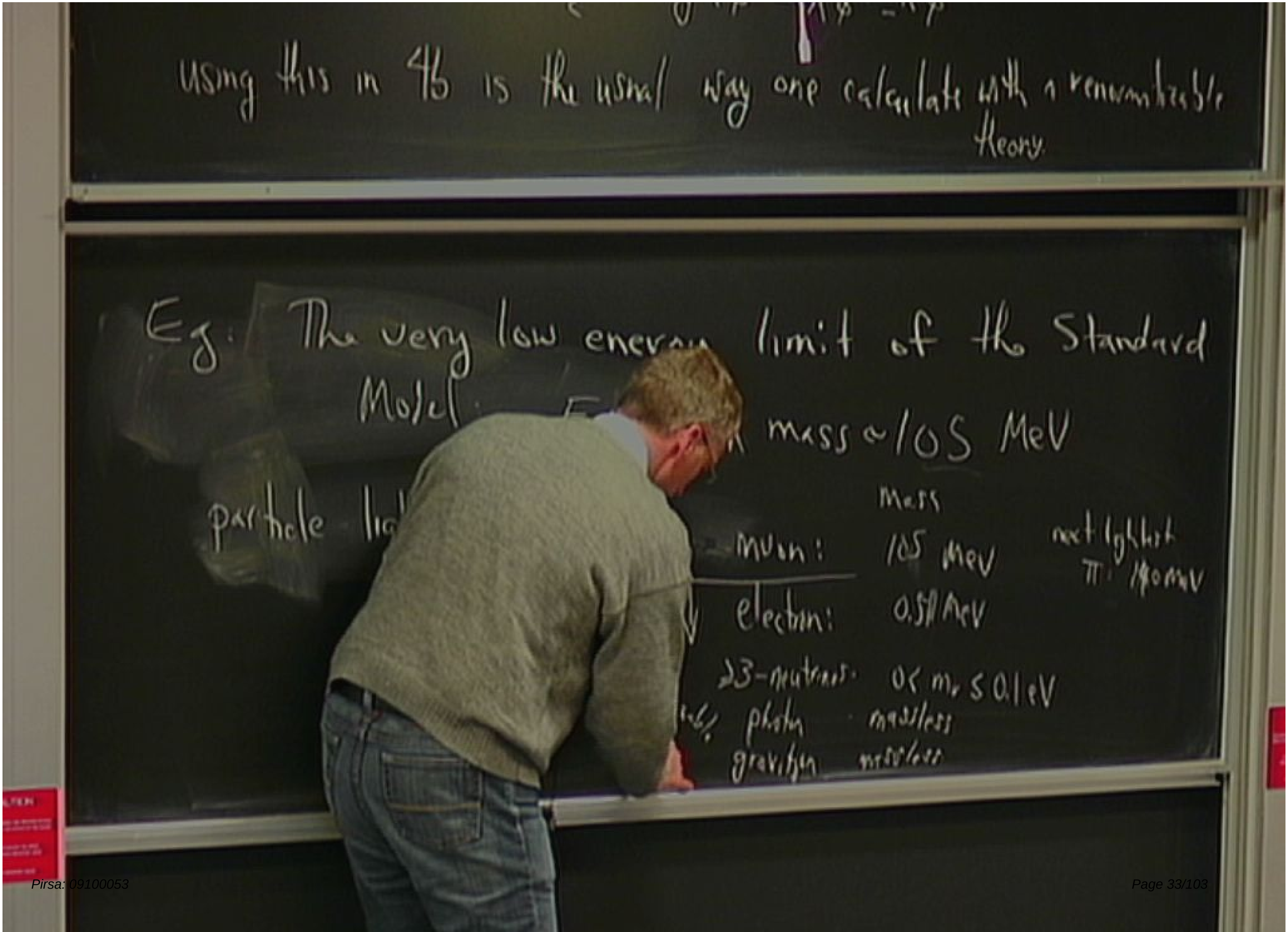
$$= f^4 \left(\frac{1}{v}\right)^E \left(\frac{M^2}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{4-E+\sum_{nd}(n+d-4)V_{nd}}$$

$$= \left(\frac{fq}{M}\right)^4 \left(\frac{qv}{M}\right)^{-E} \left(\frac{M^2}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{\underline{\underline{\sum_{nd}(n+d-4)V_{nd}}}}$$

Using this in 4b is the usual way one calculates with a renormalizable theory.

$$\mathcal{L} = -\frac{1}{2} f(\phi^2)\,\partial_\mu \phi\, \partial^\mu \phi$$

$$f = 1 + c_2 \phi^2 + c_4 \phi^4 + \dots$$


using this in 4b is the usual way one calculate with a renormalizable theory.
Eg: The very low
limit of the Standard
Model:
mass ~ 105 MeV
particle
mass
muon: 105 MeV
next lightest
π: 140 MeV
electron: 0.511 MeV
3-neutrinos: 0 < m_ν ≤ 0.1 eV
photon massless
graviton massless

using this in 4b is the usual way one calculate with a renormalizable theory.

Eg: The very low energy limit of the Standard Model: $E \leq$ muon mass ~ 105 MeV

particle lighter than this:

	mass	
muon:	105 MeV	next lightest π: 140 MeV
electron:	0.511 MeV	
3-neutrinos:	$0 < m_\nu \leq 0.1$ eV	
photon	massless	
graviton	massless	

renormalizable theory should be okay

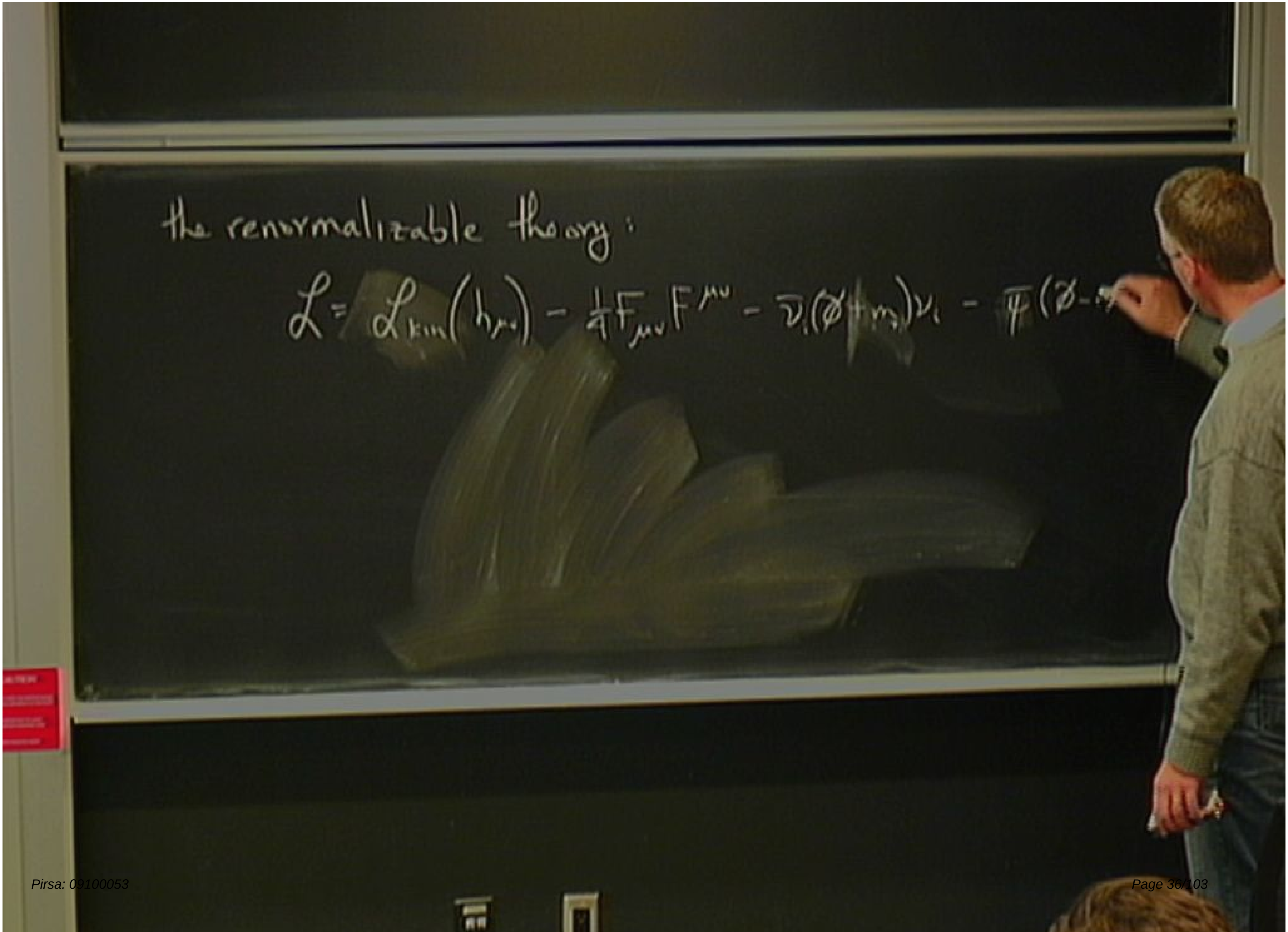
Using this in 4b is the usual way one calculate with a renormalizable theory.

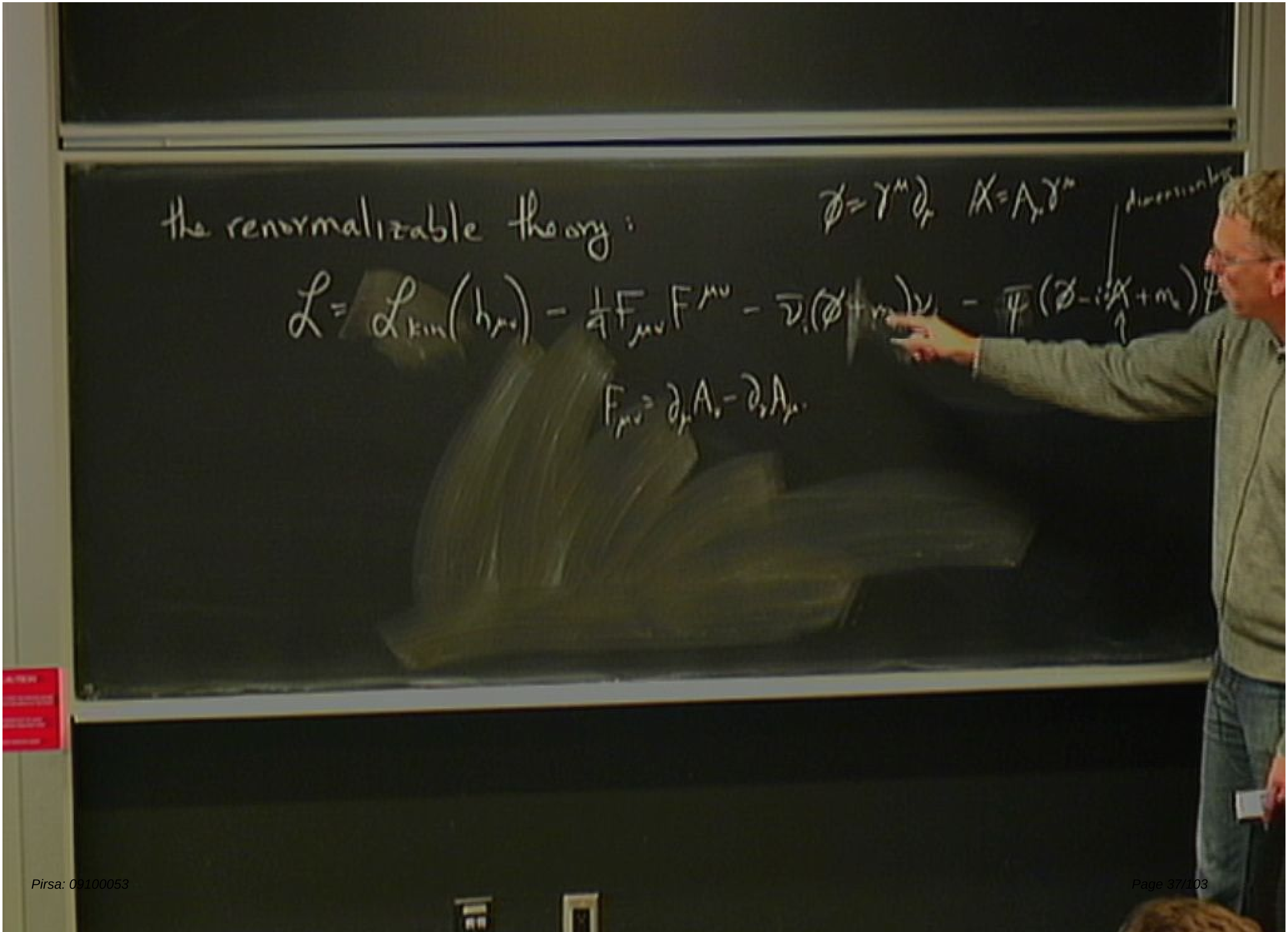
Eg: The very low energy limit of the Standard Model: $E \lesssim$ muon mass $\simeq 105$ MeV

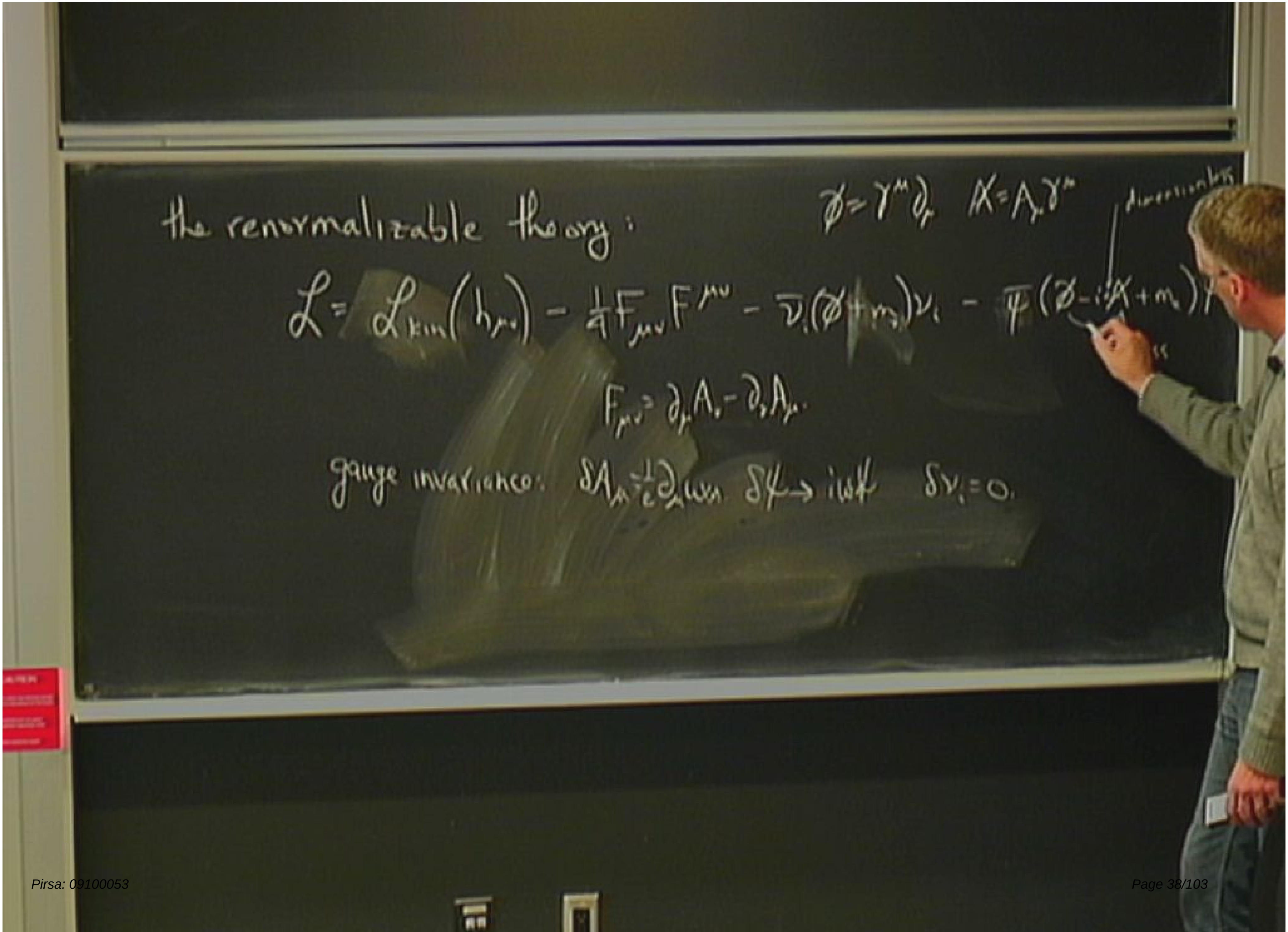
particle lighter than this:

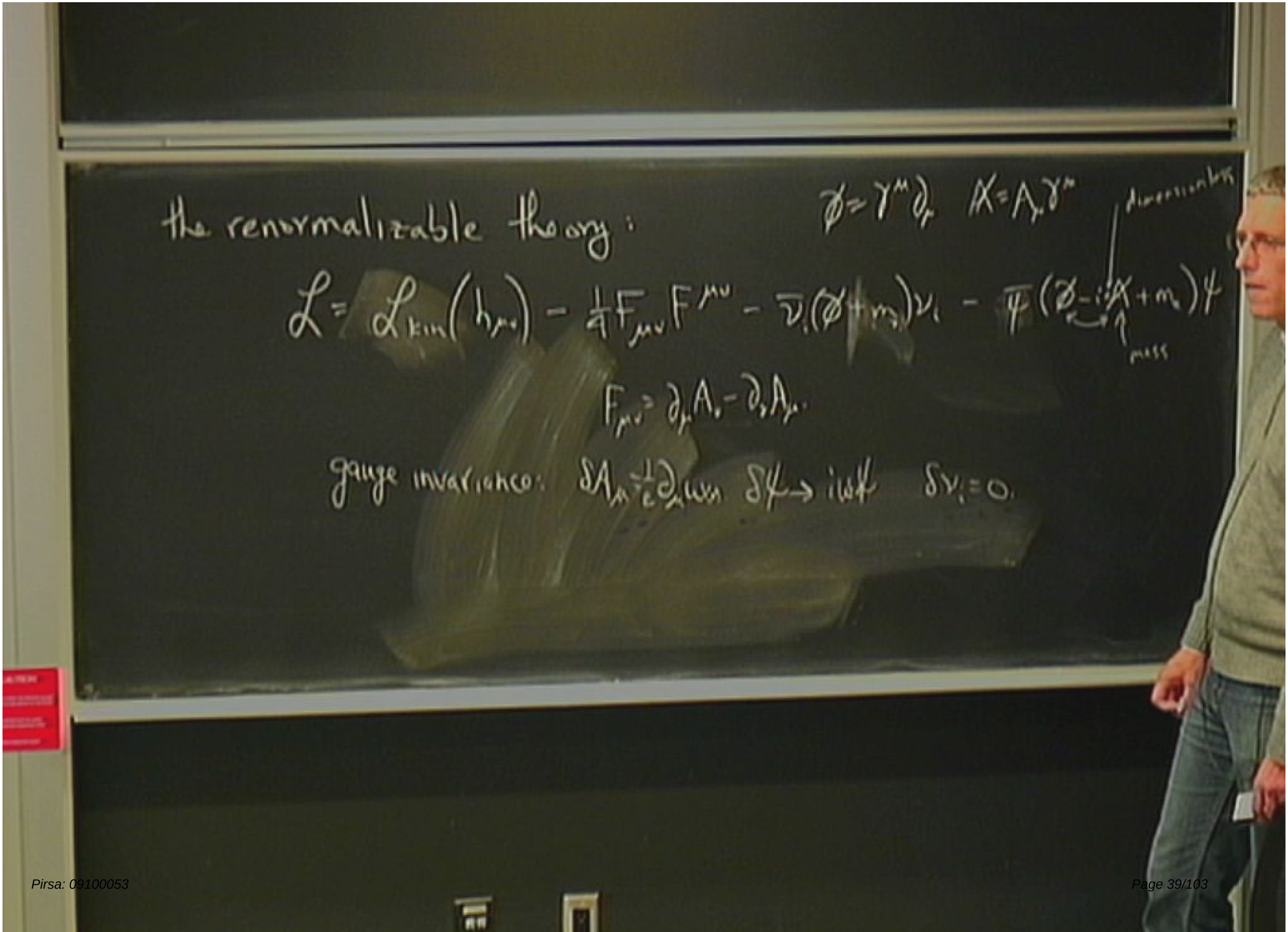
	mass	
muon:	105 MeV	next lightest π: 140 MeV
electron:	0.511 MeV	e
3-neutrinos:	$0 < m_\nu \lesssim 0.1$ eV	ν_i
photon	massless	A_μ
graviton	massless	$g_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$

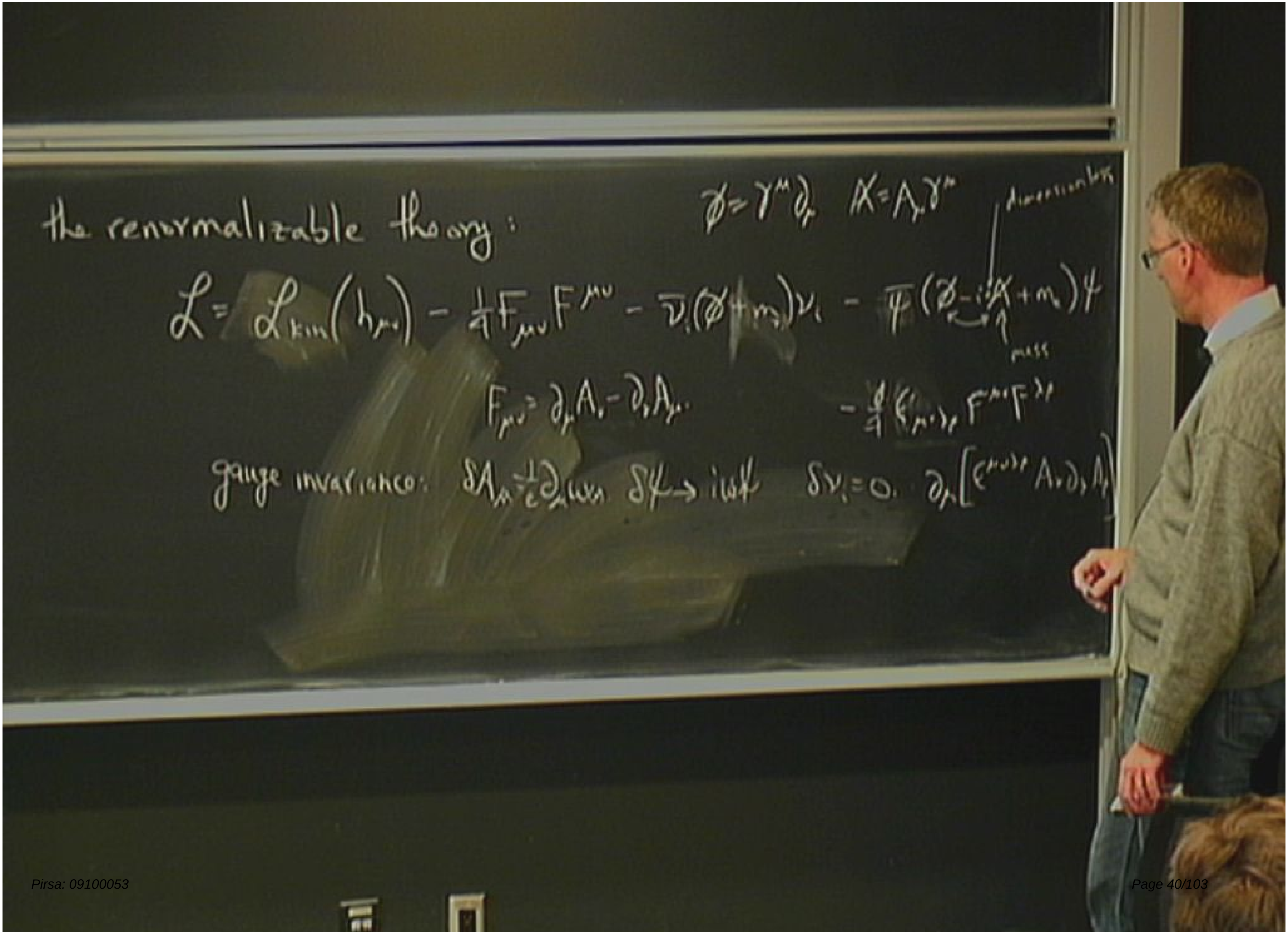
renormalizable theory should [illegible]


the renormalizable theory:
$\mathcal{L} = \mathcal{L}_{kin}(h_{\mu\nu}) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\nu}_i(\not{\partial} + m_i)\nu_i - \bar{\psi}(\not{\partial} -$


the renormalizable theory:
$\not{\partial} = \gamma^\mu \partial_\mu \quad \not{A} = A_\mu \gamma^\mu$
$\mathcal{L} = \mathcal{L}_{kin}(h_{\mu\nu}) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}_i(\not{\partial} + m_i)\psi_i - \bar{\psi}(\not{\partial} - i\not{A} + m)\psi$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$


the renormalizable theory:
$\not{\partial} = \gamma^\mu \partial_\mu \quad \not{A} = A_\mu \gamma^\mu$
dimensionless
$\mathcal{L} = \mathcal{L}_{kin}(h_{\mu\nu}) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\nu}_i (\not{\partial} + m_i) \nu_i - \bar{\psi} (\not{\partial} - i\not{A} + m_e) \psi$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
gauge invariance: $\delta A_\mu = \frac{1}{e} \partial_\mu \omega \quad \delta\psi \to i\omega\psi \quad \delta\nu_i = 0$


the renormalizable theory:
$\not\partial = \gamma^\mu \partial_\mu$ $\not A = A_\mu \gamma^\mu$
dimensionless
$\mathcal{L} = \mathcal{L}_{kin}(h_{\mu\nu}) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\nu}_i(\not\partial + m_i)\nu_i - \bar{\psi}(\not\partial - i\not A + m_e)\psi$
mass
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
gauge invariance: $\delta A_\mu = \frac{1}{e}\partial_\mu \omega$ $\delta\psi \to i\omega\psi$ $\delta\nu_i = 0$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mass
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$- \frac{\theta}{4} \epsilon_{\mu\nu\lambda\rho} F^{\mu\nu} F^{\lambda\rho}$
gauge invariance: $\delta A_\mu = \frac{1}{e}\partial_\mu \omega$, $\delta\psi \to i\omega\psi$ $\delta\nu_i = 0$. $\partial_\mu[\epsilon^{\mu\nu\lambda\rho} A_\nu \partial_\lambda A_\rho]$

the renormalizable theory:

$\not\partial = \gamma^\mu \partial_\mu \quad \not A = A_\mu \gamma^\mu$

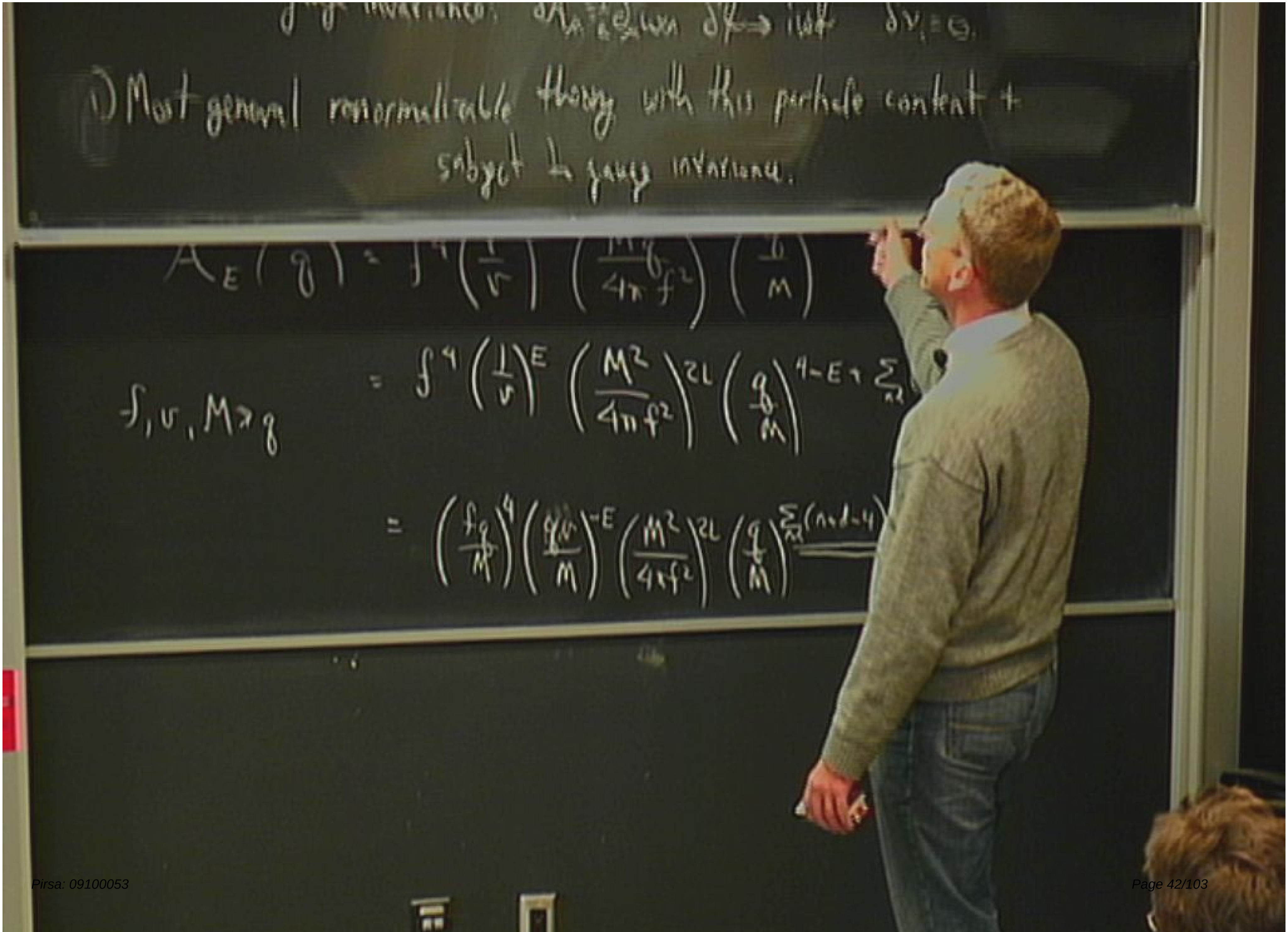
dimensionless

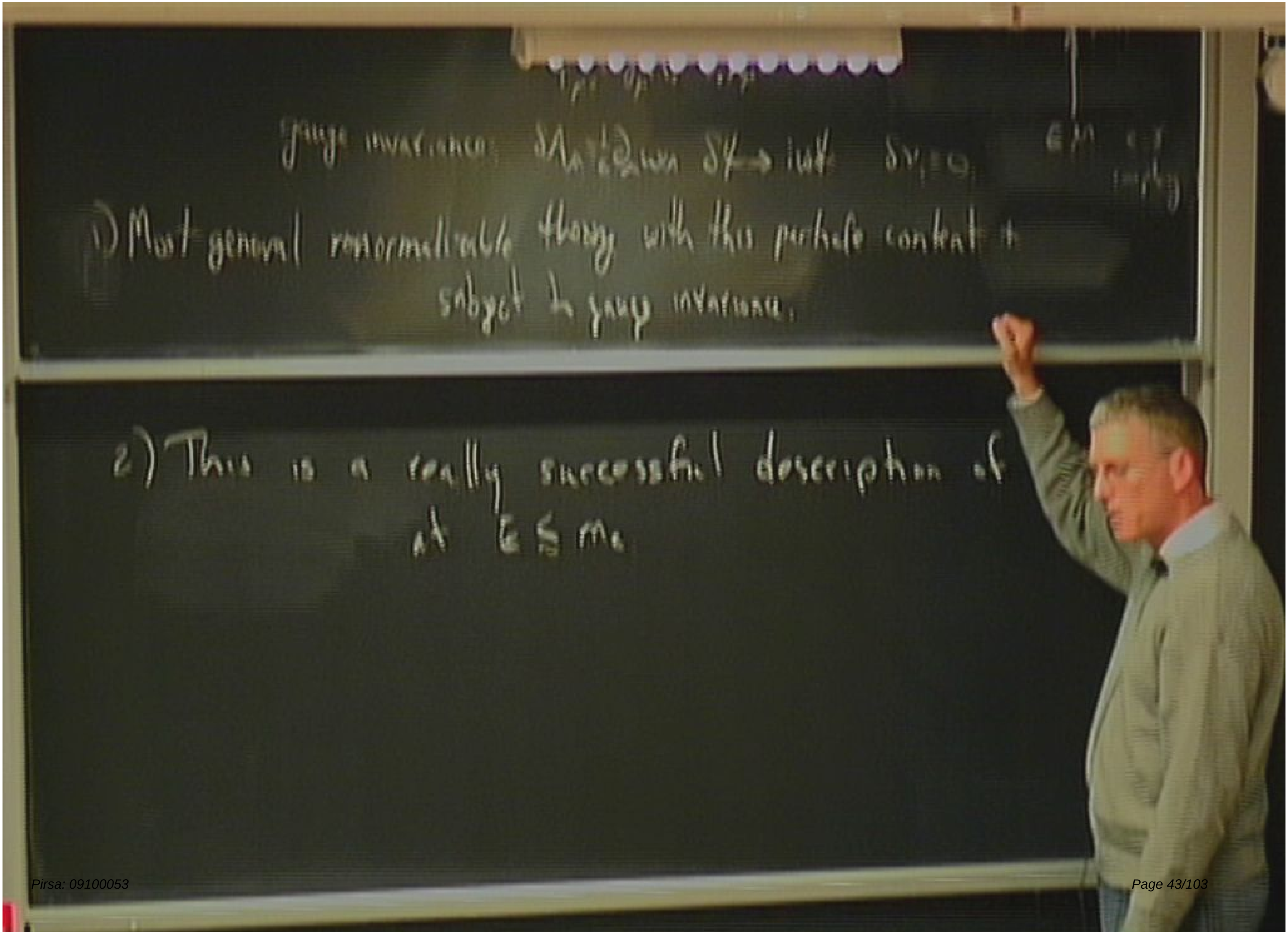
$$\mathcal{L} = \mathcal{L}_{kin}(h_{\mu\nu}) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\nu}_i (\not\partial + m_i) \nu_i - \bar{\psi} (\not\partial - ie\not A + m_e) \psi$$

mass

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

gauge invariance: $\delta A_\mu = \frac{1}{e} \partial_\mu \omega \quad \delta\psi \to i\omega\psi \quad \delta\nu_i = 0$


1) Most general renormalizable theory with this particle content + subject to gauge invariance.
A_E(g) = f^4 (1/v) (M^2 g / 4π f^2) (g/M)
= f^4 (1/v)^E (M^2 / 4π f^2)^{2L} (g/M)^{4-E+Σ}
f, v, M ≫ g
= (fg/M)^4 (gv/M)^{-E} (M^2 / 4π f^2)^{2L} (g/M)^{Σ(n+d-4)}

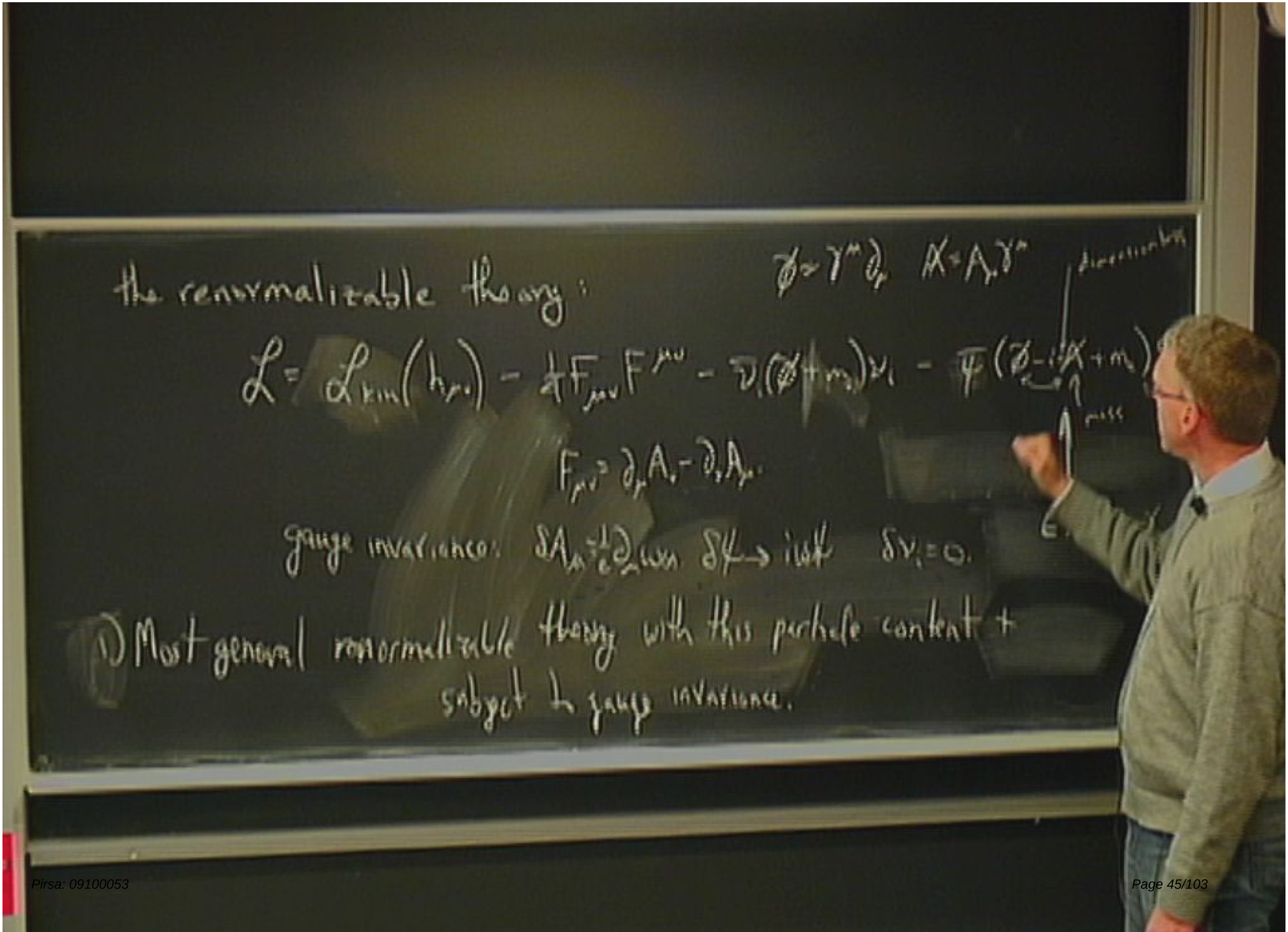

gauge invariance:
1) Most general renormalizable theory with this particle content is
subject to gauge invariance.
2) This is a really successful description of
at E ≲ mₑ

gauge invariance: $\delta A_\mu = \frac{1}{e}\partial_\mu \omega$, $\delta\psi \to i\omega\psi$, $\delta\nu_i = 0$.

EM e-γ coupling

1) Most general renormalizable theory with this particle content + subject to gauge invariance.

2) This is a really successful description of expts at $E \lesssim m_e$.


the renormalizable theory:
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dimensionless
$\mathcal{L} = \mathcal{L}_{kin}(h_{\mu\nu}) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\nu}_i(\not\partial + m_i)\nu_i - \bar{\psi}(\not\partial - i\not A + m)\psi$
mass
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
gauge invariance: $\delta A_\mu = \frac{1}{e}\partial_\mu \omega$ $\delta\psi \to i\omega\psi$ $\delta\nu_i = 0$.
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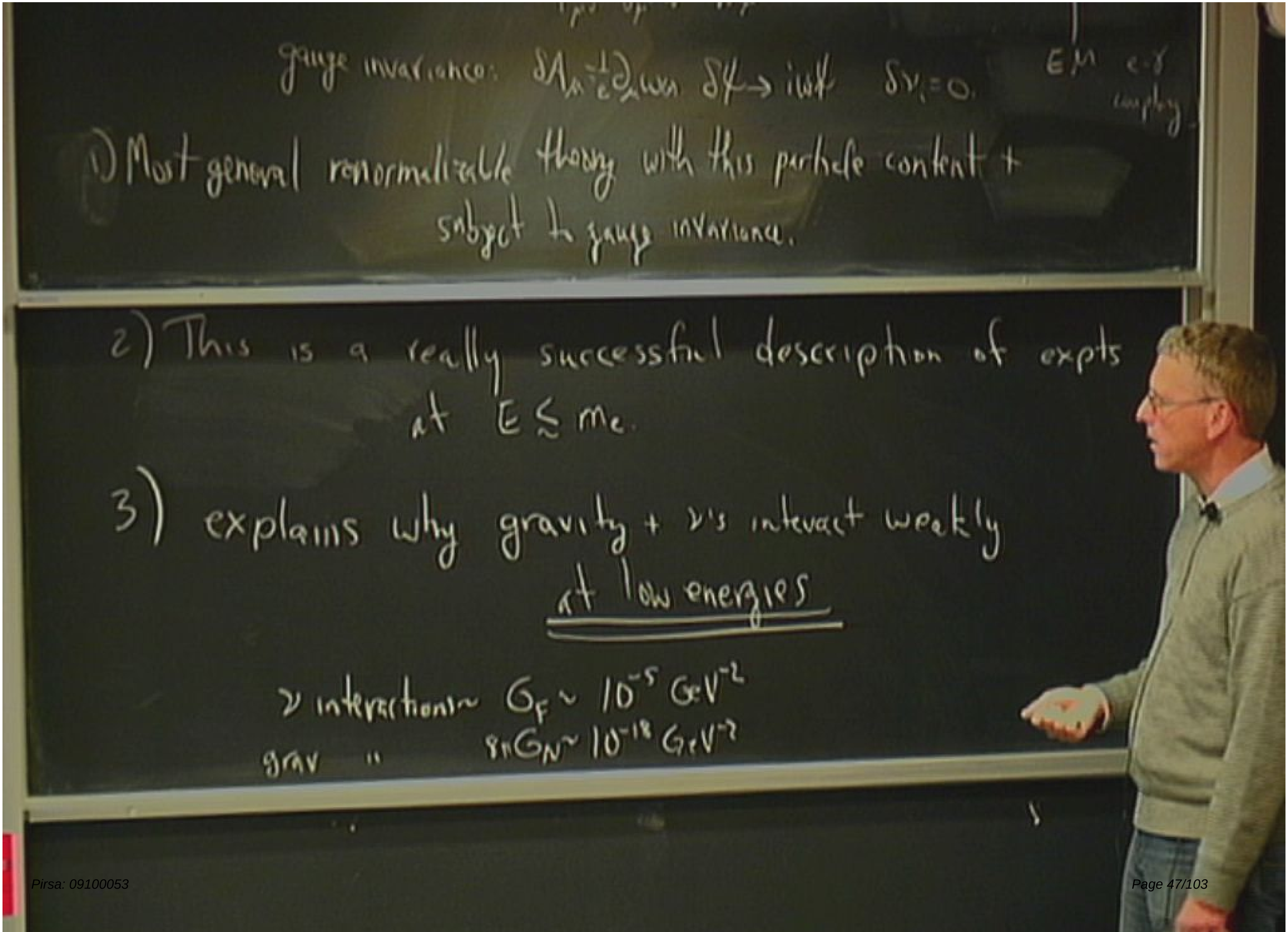
gauge invariance: $\delta A_\mu = \frac{1}{e}\partial_\mu \omega$ $\delta\psi \to i\omega\psi$ $\delta\nu_i = 0$.

EM e-γ coupling

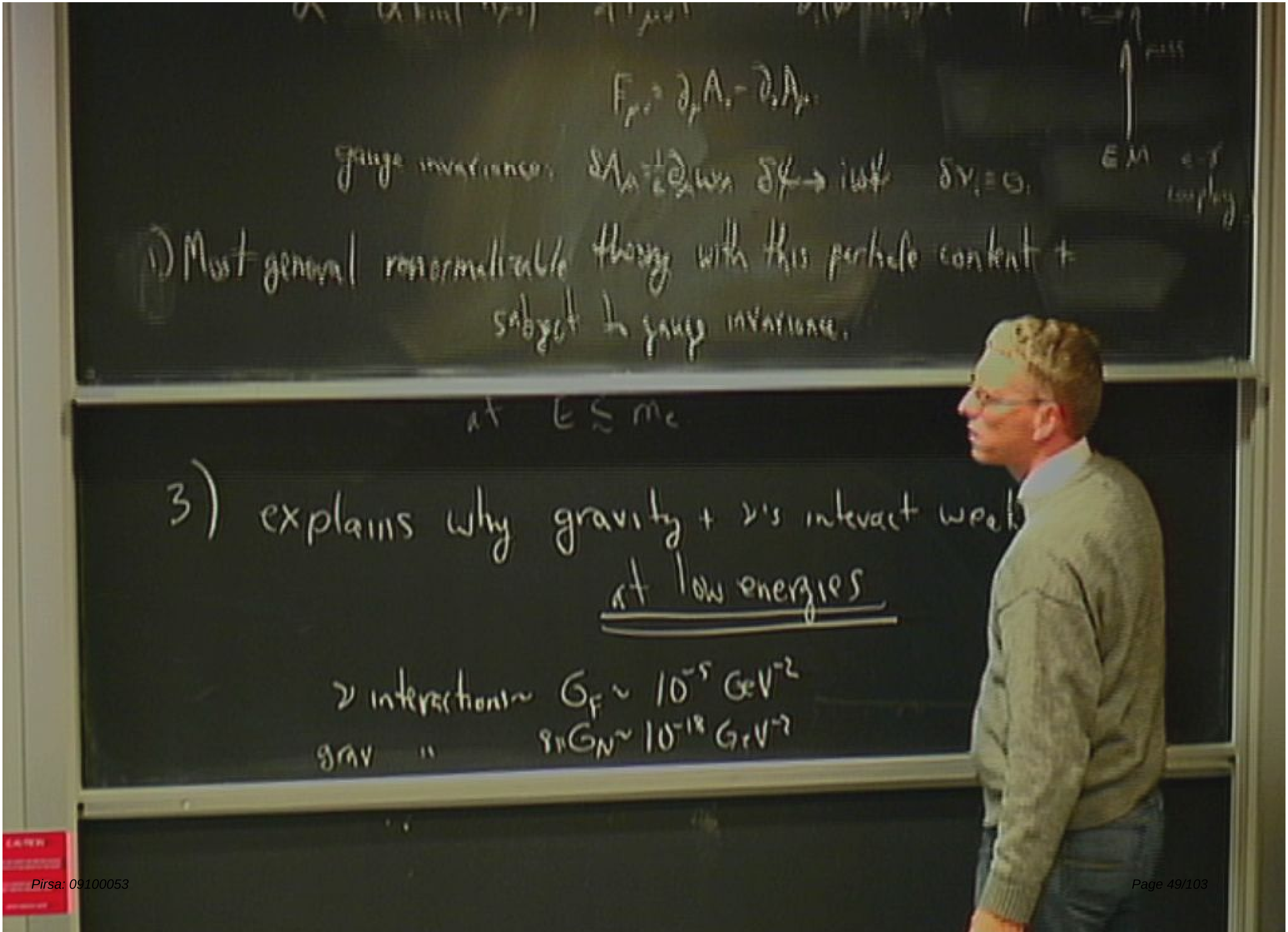
1) Most general renormalizable theory with this particle content + subject to gauge invariance.

2) This is a really successful description of expts at $E \lesssim m_e$.


gauge invariance: δA_μ = -1/e ∂_μ ω(x) δψ → iωψ δν_i = 0.
EM e-γ coupling
1) Most general renormalizable theory with this particle content + subject to gauge invariance.
2) This is a really successful description of expts at E ≲ m_e.
3) explains why gravity + ν's interact weakly at low energies
ν interactions ~ G_F ~ 10^-5 GeV^-2
grav " 8πG_N ~ 10^-18 GeV^-2

Microscopic theory:

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\Psi}(\not{D} + m_e)\Psi \qquad D_\mu = \partial_\mu + ieA_\mu$$



F_μν = ∂_μ A_ν − ∂_ν A_μ
1) Most general renormalizable theory with this particle content +
subject to gauge invariance.
at E ≲ m_e
3) explains why gravity + ν's interact weak
at low energies
ν interactions ~ G_F ~ 10^-5 GeV^-2
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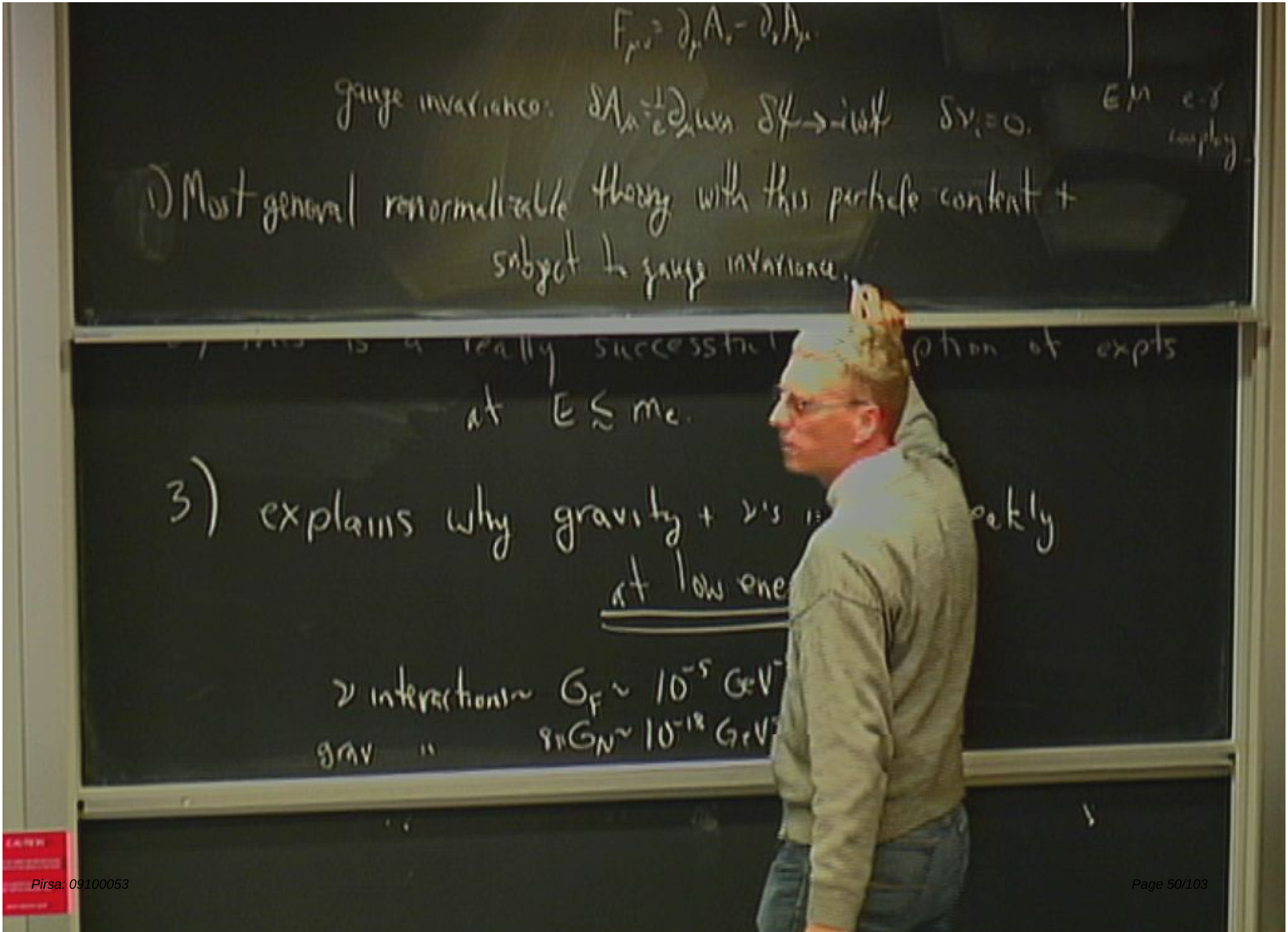
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
gauge invariance: $\delta A_\mu = \frac{1}{e}\partial_\mu \omega(x)$ $\delta\psi \to i\omega\psi$ $\delta\nu_i = 0$.
E^M e.g. coupling
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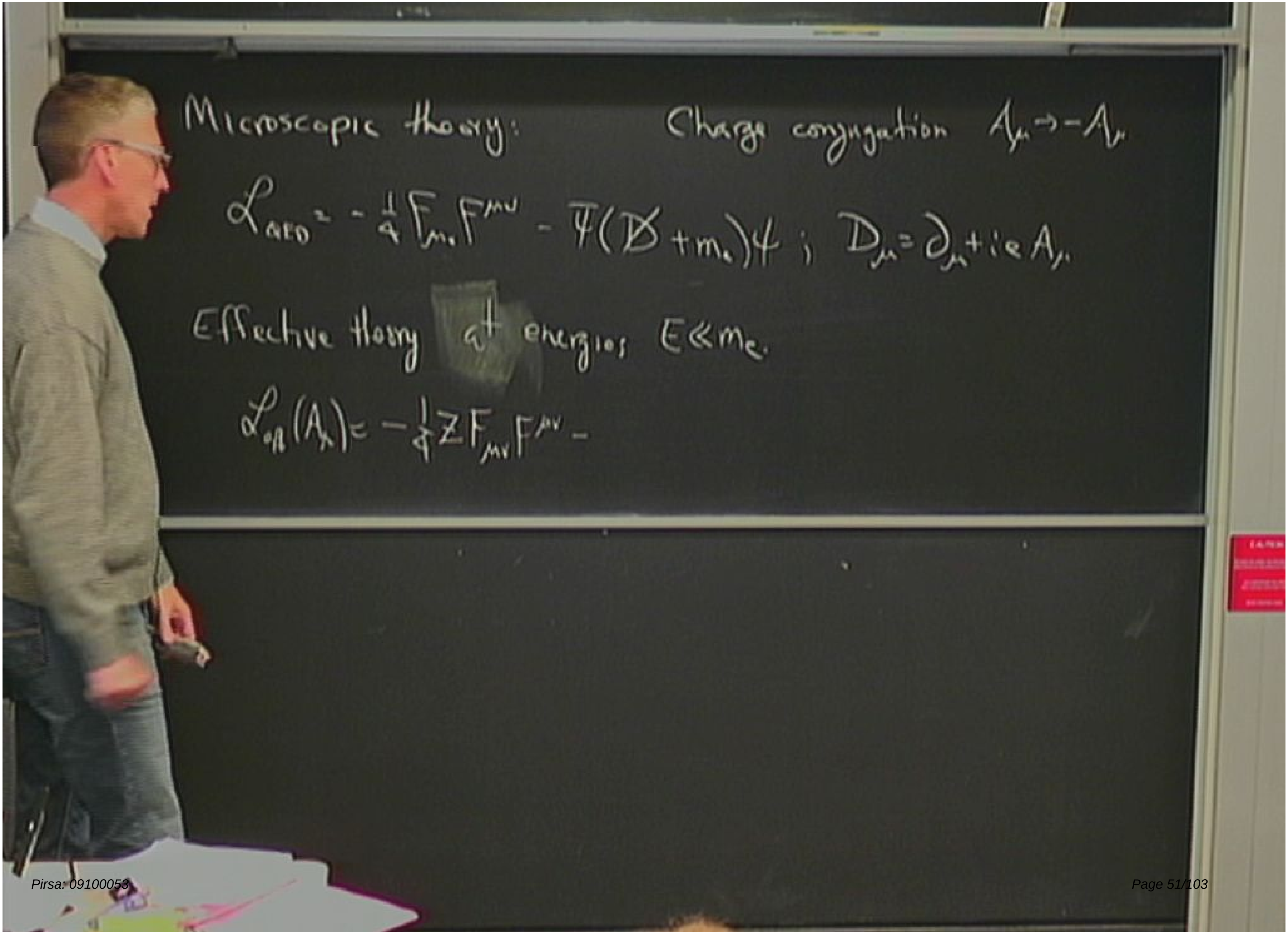
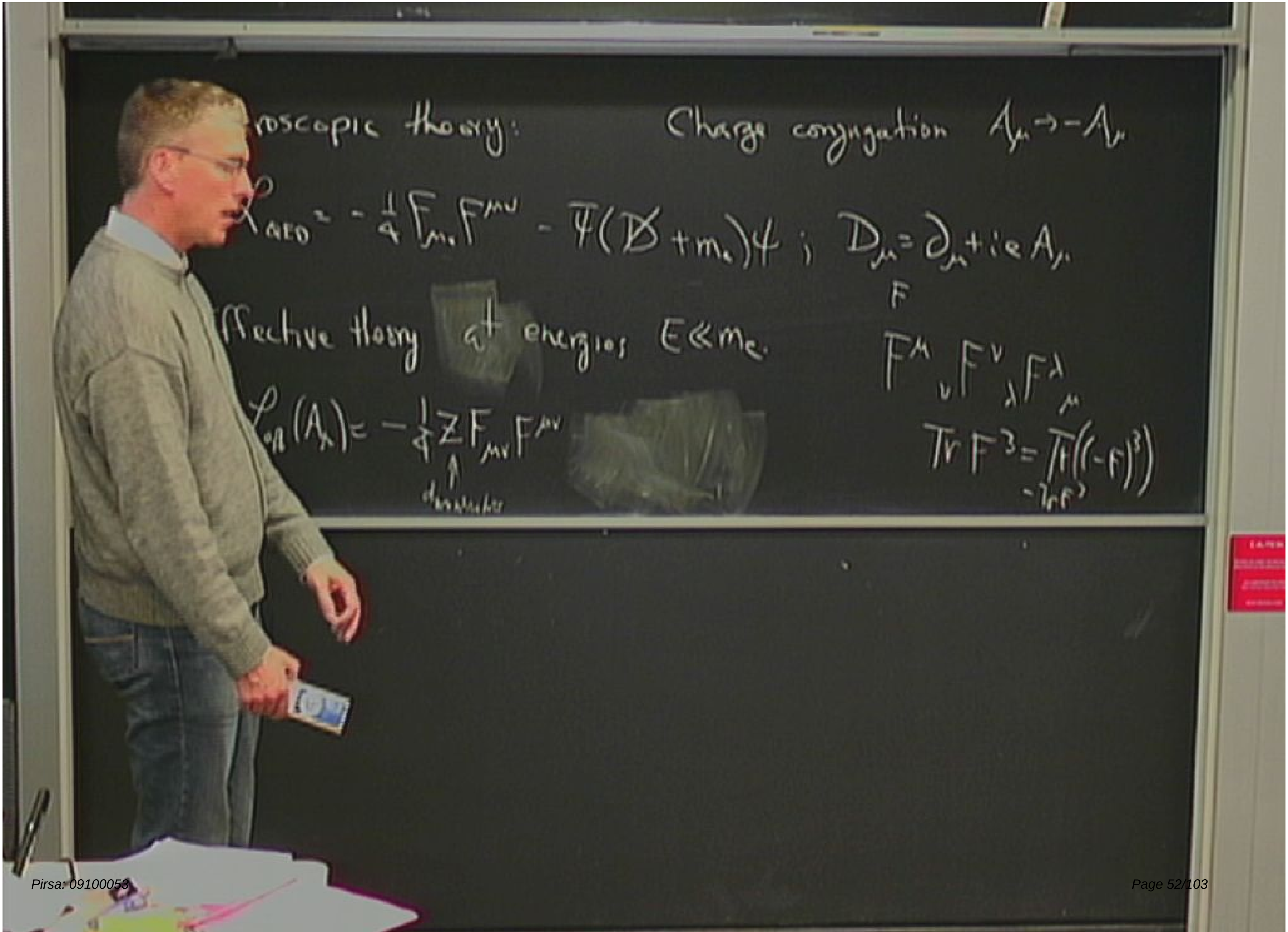

Microscopic theory:
Charge conjugation $A_\mu \to -A_\mu$
$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}(\not{D} + m_e)\psi$; $D_\mu = \partial_\mu + ieA_\mu$
Effective theory at energies $E \ll m_e$.
$\mathcal{L}_{eff}(A_\mu) = -\frac{1}{4} Z F_{\mu\nu} F^{\mu\nu} -$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F
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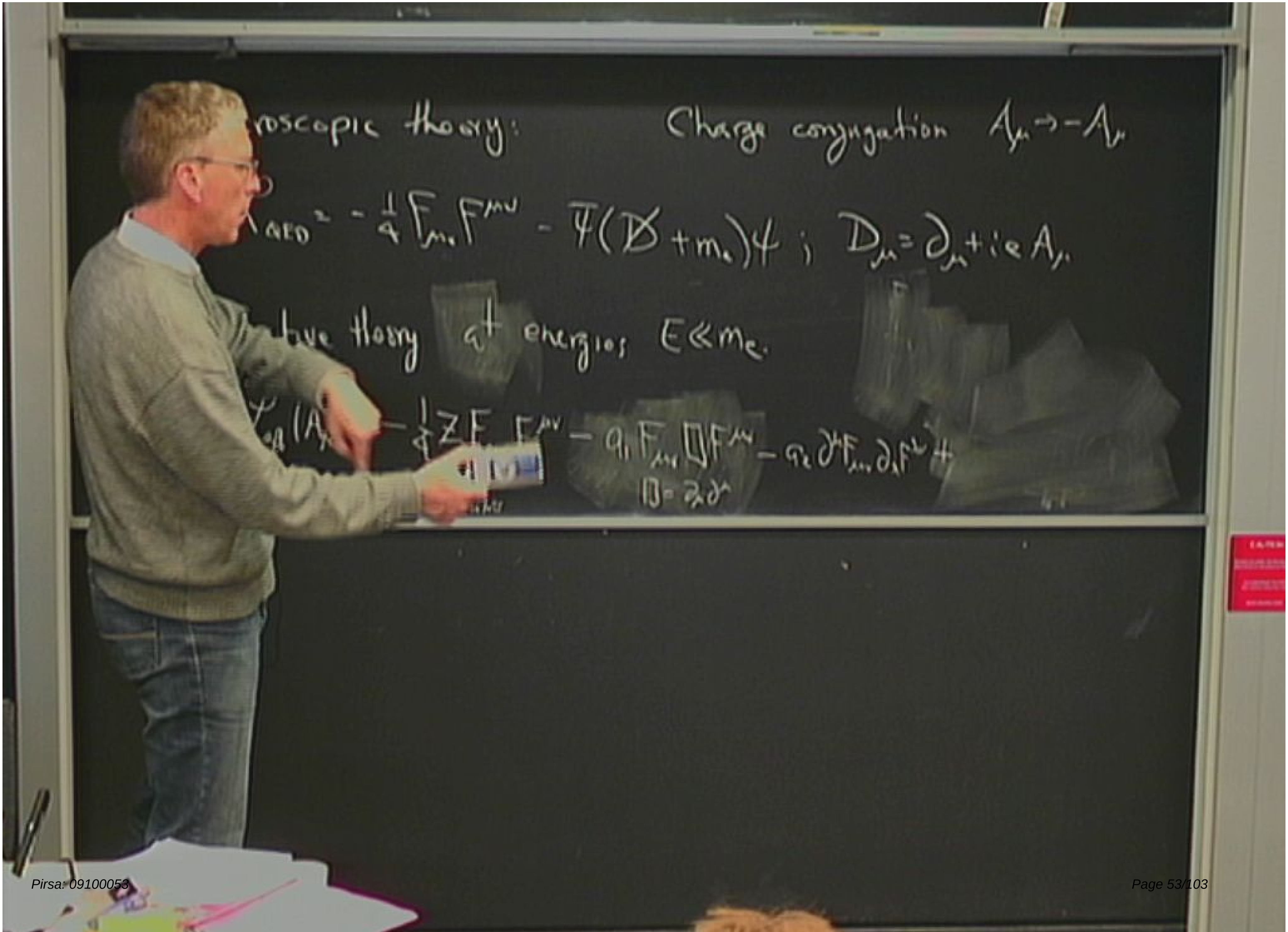
$F^\mu{}_\nu F^\nu{}_\lambda F^\lambda{}_\mu$
$\mathcal{L}_{eff}(A_\mu) = -\frac{1}{4} Z F_{\mu\nu} F^{\mu\nu}$
$\mathrm{Tr}\, F^3 = \mathrm{Tr}((-F)^3)$
$-\mathrm{Tr}\, F^3$



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$\mathcal{L}_{eff}(A_\mu) - \frac{1}{4}Z F_{\mu\nu}F^{\mu\nu} - a_1 F_{\mu\nu}\Box F^{\mu\nu} - a_2 \partial^\mu F_{\mu\nu}\partial_\lambda F^{\lambda\nu} +$
$\Box = \partial_\mu\partial^\mu$

Microscopic theory: Charge conjugation $A_\mu \to -A_\mu$

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}(\not{D} + m_e)\psi \; ; \; D_\mu = \partial_\mu + ie A_\mu$$

Effective theory at energies $E \ll m_e$.

$$\mathcal{L}_{eff}(A_\mu) = -\frac{1}{4} \underset{\substack{\uparrow \\ \text{dimensionless}}}{Z} F_{\mu\nu} F^{\mu\nu} - a_1 F_{\mu\nu} \Box F^{\mu\nu} - a_2 \partial^\mu F_{\mu\nu} \partial_\lambda F^{\lambda\nu} + \ldots$$

$$\Box = \partial_\lambda \partial^\lambda$$

Using this in 4b is the usual way one calculate with a renormalizable theory.

Aside: $\mathcal{L}_0 = -\frac{1}{4} Z F_{\mu\nu} F^{\mu\nu}$

$$\frac{1}{Z} \frac{\delta S_0}{\delta A_\mu} = \partial_\nu F^{\nu\mu} = 0$$

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$$0 = \partial^\mu F_{\mu\nu} = \Box A_\nu - \partial_\nu \partial^\mu A_\mu$$

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$$S = S_0 + \epsilon S_1$$

$$\phi \to \phi + \epsilon \int d^4y \, \frac{\delta S_1}{\delta \phi}$$

Using this in 4b is the usual way one calculate with a renormalizable theory.

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$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

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Caveat: if there are macroscopic sources J^μ, with $\partial^\mu J_\mu = 0$, then:

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Aside: $\mathcal{L}_0 = -\frac{1}{4} Z F_{\mu\nu} F^{\mu\nu}$

$$\frac{1}{Z}\frac{\delta S_0}{\delta A_\mu} = \underline{\underline{\partial_\nu F^{\nu\mu}}} = e J^\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$0 = \partial^\mu F_{\mu\nu} = \Box A_\nu - \partial_\nu \partial^\mu A_\mu$$

$$e(\partial_\mu J_\nu - \partial_\nu J_\mu)\; 0 = \Box \partial_\lambda A_\nu - \cancel{\partial_\lambda \partial_\nu \partial^\mu A_\mu} - \Box \partial_\nu A_\lambda - \cancel{\partial_\nu \partial_\lambda \partial^\mu A_\mu} = \Box F_{\lambda\nu}$$

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EM e-γ coupling

1) Most general renormalizable theory with this particle content + subject to gauge invariance.

$$a_1 F_{\mu\nu}\Box F^{\mu\nu} + a_2\,\partial_\mu F^{\mu\nu}\partial^\lambda F_{\lambda\nu} = e^2(a_1 - 2a_2)\,J^\mu J_\mu$$

is independent of radiation field in A_μ.

gauge invariance: $\delta A_\mu = \frac{1}{e}\partial_\mu \omega$ $\delta\psi \to -i\omega\psi$ $\delta \nu_i = 0$.

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$$+ b\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + c\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2 + \dots \qquad a_1, a_2 \sim \frac{1}{mass^2}$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho} \qquad b, c \sim \frac{1}{mass^4}$$

gauge invariance: $\delta A_\mu = \frac{1}{e}\partial_\mu \omega$, $\delta\psi \to -i\omega\psi$ $\delta \nu_i = 0$.

ϵ^μ e-γ coupling

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is independent of radiation field in A_μ.

$$\mathcal{L}_{eff} = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu} - e A_\mu J^\mu - e^2 (a_1 - 2a_2) J^\mu J_\mu$$

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$a_1 a_2 \sim \frac{1}{\text{mass}^2}$

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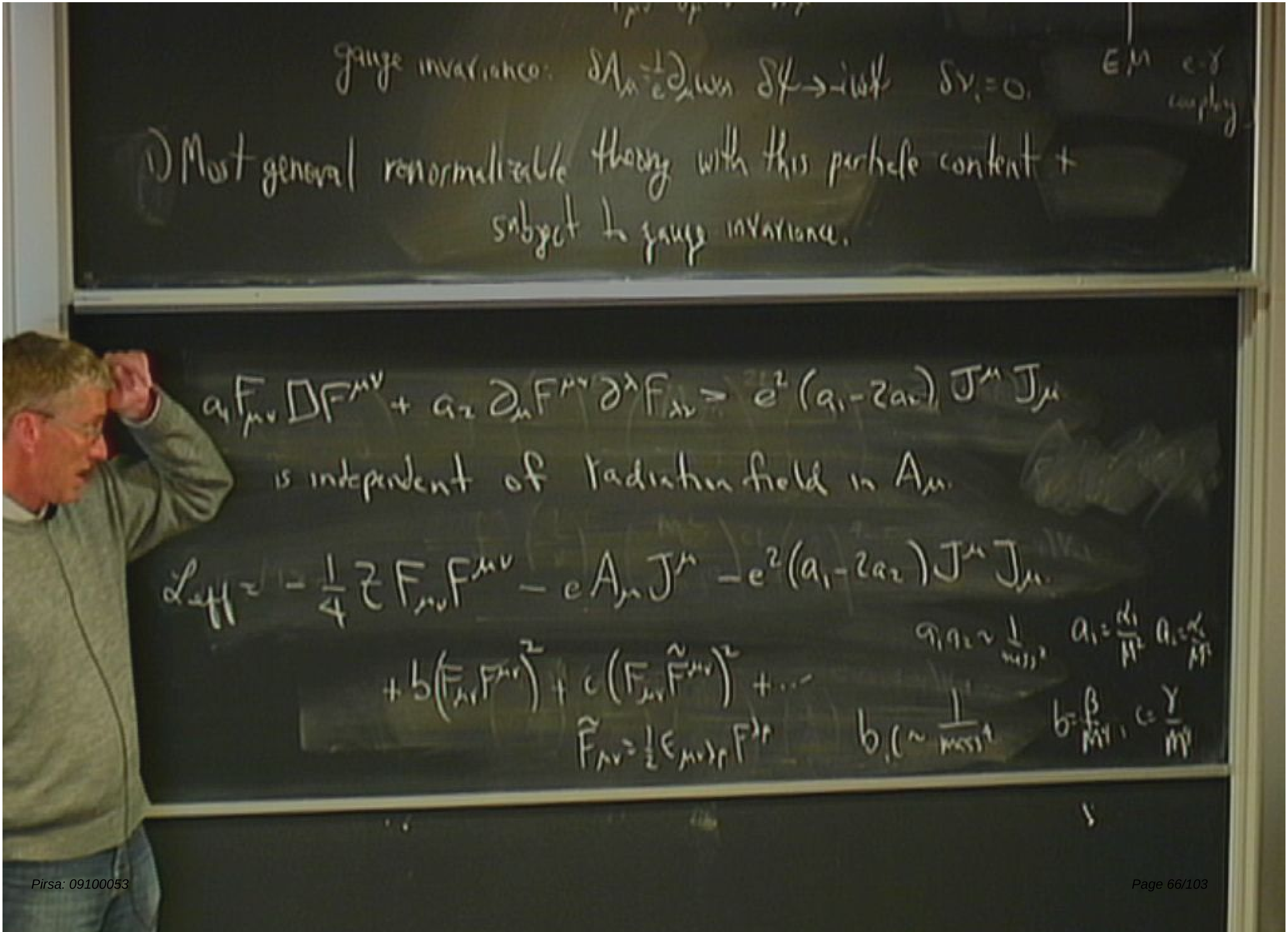
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$\mathcal{L}_{eff} = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu} - e A_\mu J^\mu - e^2(a_1 - 2a_2) J^\mu J_\mu$
$+ b(F_{\mu\nu}F^{\mu\nu})^2 + c(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \cdots$
$a_1 a_2 \sim \frac{1}{(mass)^2}$, $a_1 = \frac{\alpha_1}{M^2}$ $a_2 = \frac{\alpha_2}{M^2}$
$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$
$b, c \sim \frac{1}{(mass)^4}$
$b = \frac{\beta}{M^4}$, $c = \frac{\gamma}{M^4}$

gauge invariance: $\delta A_\mu = \frac{1}{e}\partial_\mu \omega \quad \delta\psi \to -i\omega\psi \quad \delta\nu_i = 0$.

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$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} F^{\lambda\rho}$

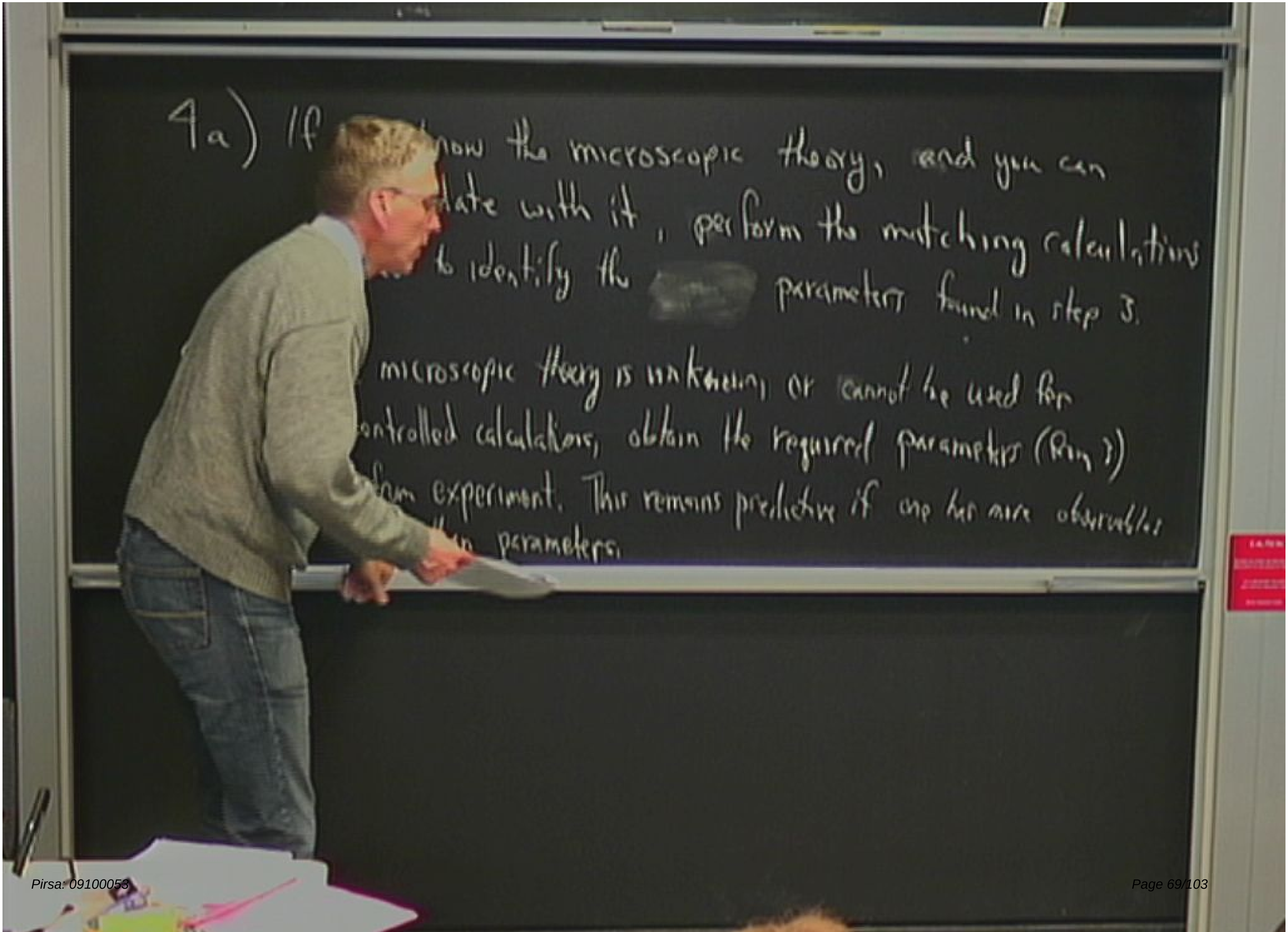
$a_1 a_2 \sim \frac{1}{(mass)^2}$, $a_1 = \frac{\alpha_1}{M^2}$ $a_2 = \frac{\alpha_2}{M^2}$

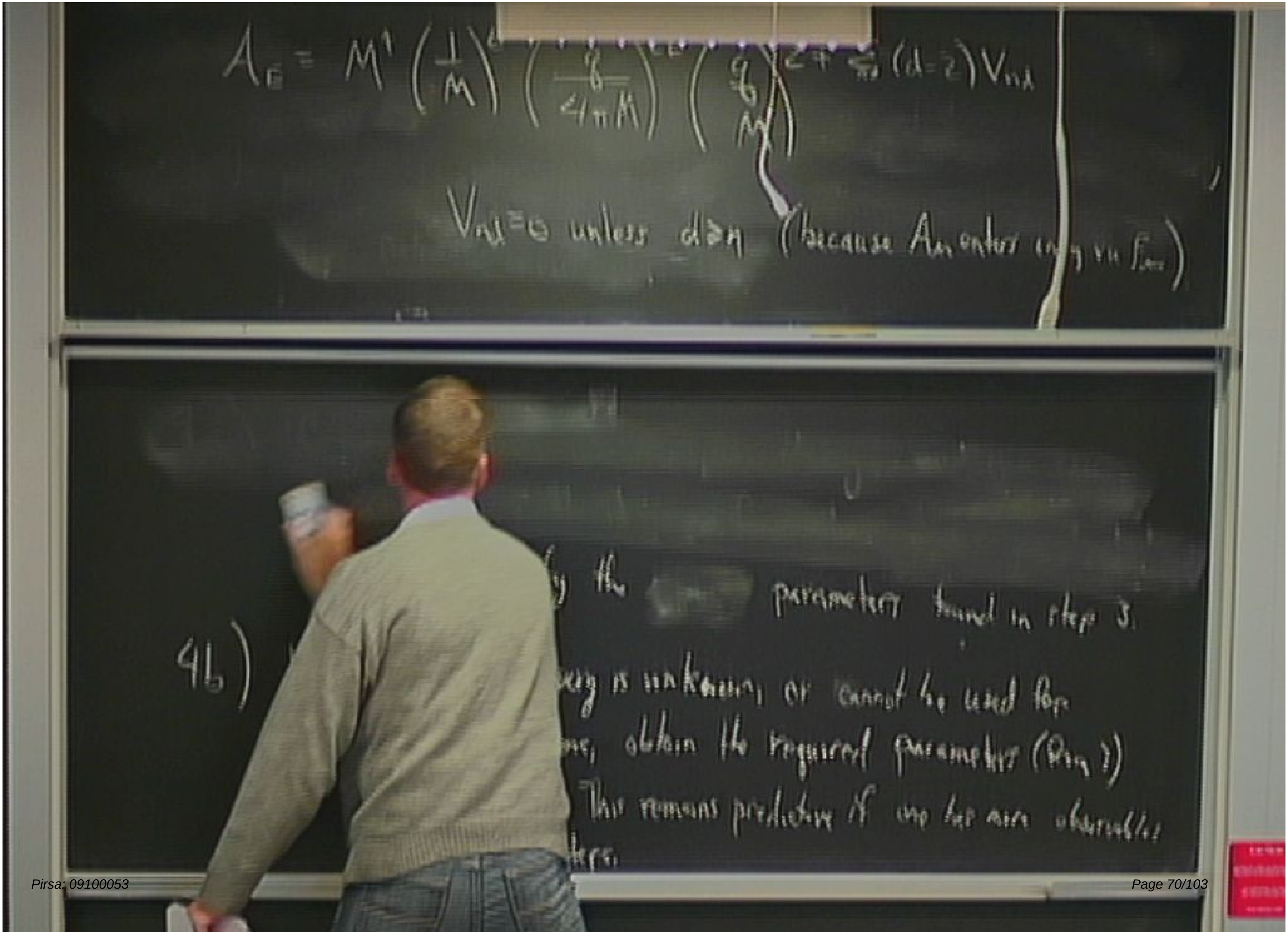
$b, c \sim \frac{1}{(mass)^4}$ $b = \frac{\beta}{M^4}$, $c = \frac{\gamma}{M^4}$

Power counting using $\mathcal{L}_{eff}$ Can use previous

result with $f \sim v \sim M$

$$A_E = M^4 \left(\frac{1}{M}\right)^E \left(\frac{g}{4\pi M}\right)^{2L} \left(\frac{g}{M}\right)^{2+\sum_{n,d}(d-2)V_{nd}}$$


4a) If
now the microscopic theory, and you can
late with it, perform the matching calculations
to identify the parameters found in step 3.
microscopic theory is unknown or cannot be used for
ontrolled calculations, obtain the required parameters (from 3))
from experiment. This remains predictive if one has more observables
parameters.


$V_{nd} = 0$ unless $d \geq 4$ (because A_n enters only via
parameters found in step 3.
4b)
is unknown, or cannot be used for
obtain the required parameters
This remains predictive if one has more observables

$V_{nd}=0$ unless $d \geq n$ (because A_μ enters only via $F_{\mu\nu}$)

For $\gamma\gamma \to \gamma\gamma$ $(E=4)$

$$A_4 \sim \left(\frac{g}{4\pi M}\right)^{2L} \left(\frac{g}{M}\right)^{2+\Sigma(d-2)V_{nd}}$$

dominant piece: $L=0$, $V_{nd}=0$ unless $d=2$ (but $d=2 \Rightarrow n=2$

$V_{nd} = 0$ unless $d \geq n$ (because A_μ enters only via $F_{\mu\nu}$)

For $\gamma\gamma \to \gamma\gamma$ $(E=4)$

$$A_4 \sim \left(\frac{g}{4\pi M}\right)^{2L} \left(\frac{g}{M}\right)^{2 + \Sigma (d-2) V_{nd}}$$

dominant piece: $L=0$, $V_{nd} = 0$ unless $d=2$ (but $d=2 \Rightarrow n=2$ $\rightarrow$ only the kinetic term)

so all vertices have

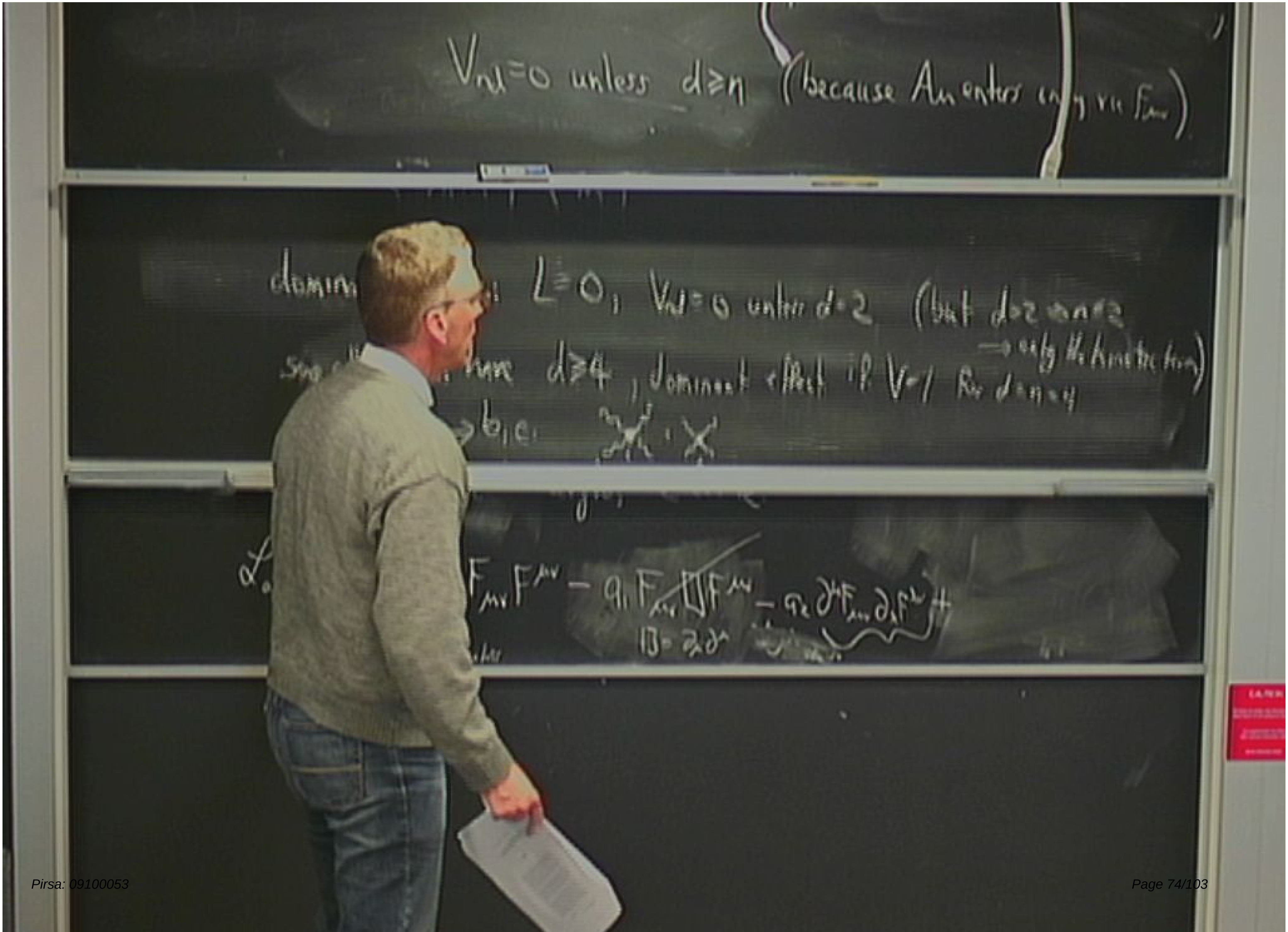
$V_{nd} = 0$ unless $d \geq n$ (because A_μ enters only via $F_{\mu\nu}$)

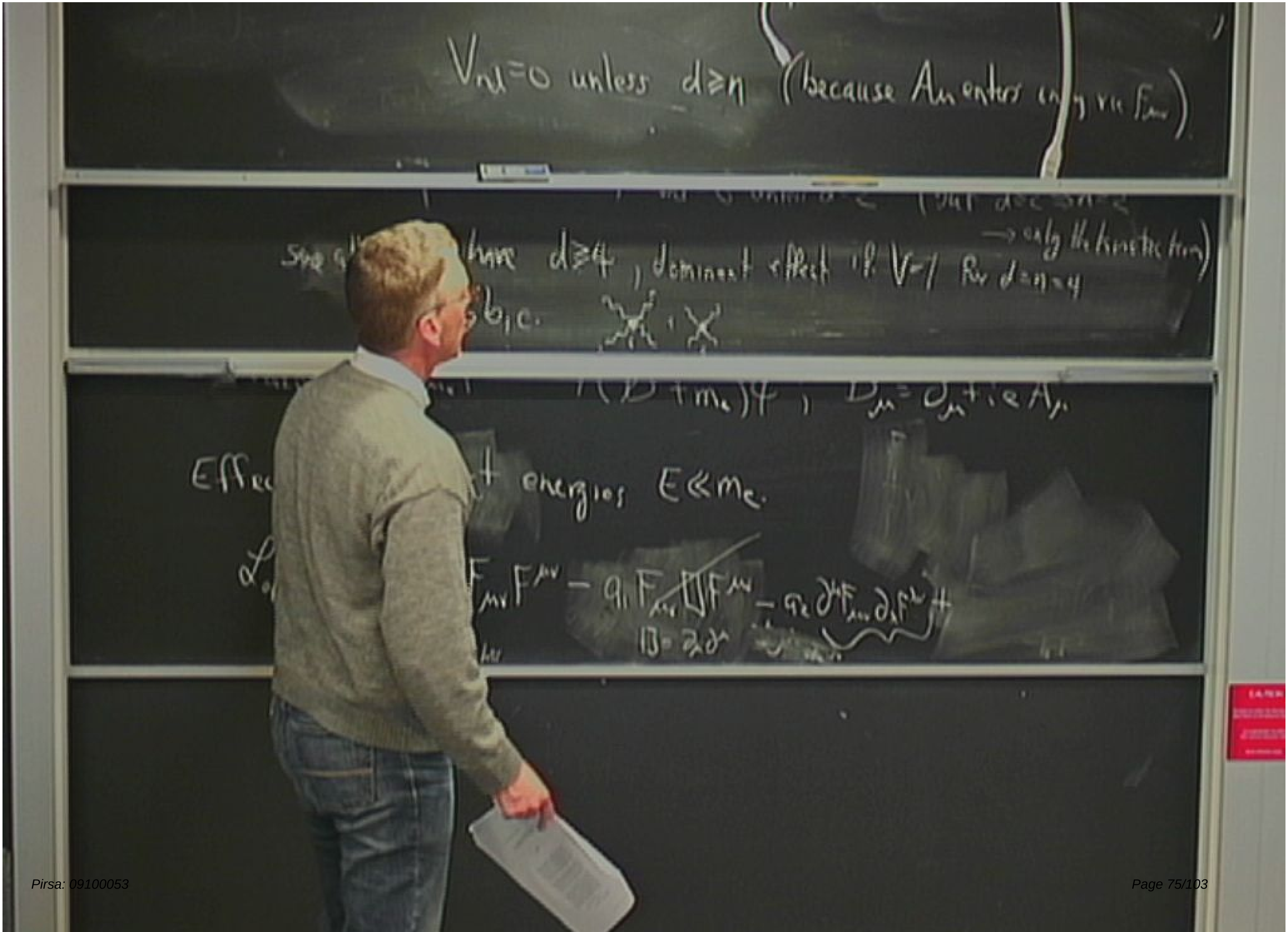
For $\gamma\gamma \to \gamma\gamma$ $(E=4)$

$$A_4 \sim \left(\frac{g}{4\pi M}\right)^{2L} \left(\frac{g}{M}\right)^{2+\Sigma(d-2)V_{nd}}$$

dominant piece: $L=0$, $V_{nd}=0$ unless $d=2$ (but $d=2 \Rightarrow n=2$ $\rightarrow$ only the kinetic term)

Since all vertices have $d \geq 4$, dominant effect is $V=1$ for $d=n=4$


V_nd = 0 unless d ≥ n (because A_n enters only via F_μν)
L = 0, V_nd = 0 unless d = 2 (but d = 2 n = 2
→ only the kinetic term)
d ≥ 4, dominant effect if V = 1 for d = n = 4
F_μν F^μν − g_1 F_μν □ F^μν − g_2 ∂^μ F_μν ∂_λ F^λν +
□ = ∂_λ ∂^λ


Vnd=0 unless d≥n (because Aμ enters only via Fμν)
→ only the kinetic term)
have d≥4, dominant effect if V=1 for d=n=4
b.c.
Dμ=∂μ+ieAμ
t energies E≪me
Fμν Fμν − q1 Fμν □ Fμν − q2 ∂λFμν ∂λFμν +
□=∂λ∂λ

neutrinos: $0 \lesssim m_\nu \lesssim 0.1\,\text{eV}$ ν_i

photon massless A_μ

graviton massless $g_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$

$$\frac{d\sigma_{\gamma\gamma}}{d\Omega} = \frac{278}{65\pi^2}\left[2(b^2+c^2)\right] E_{cm}^6 \left(3+\cos^2\theta\right)^2 \left[1 + \right.$$

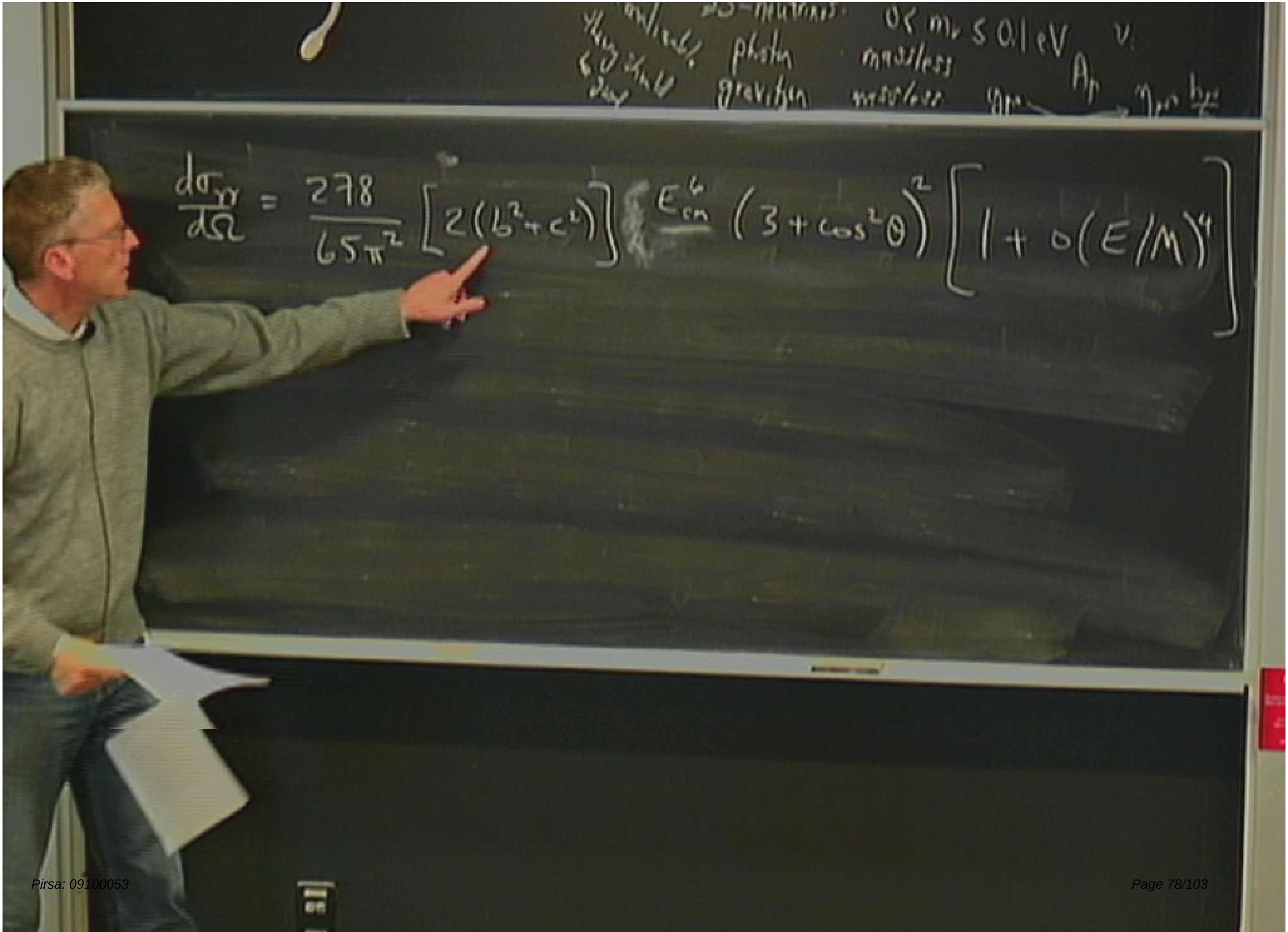
For $\gamma\gamma \to \gamma\gamma$ $\quad (E=4)$

$$A_4 \sim \left(\frac{g}{4\pi M}\right)^{2L} \left(\frac{g}{M}\right)^{2+\Sigma(d-2)V_{nd}}$$

dominant piece: $L=0$, $V_{nd}=0$ unless $d=2$ (but $d=2 \Rightarrow n\leq 2$
$\to$ only the kinetic term)

since all vertices have $d\geq 4$, dominant effect if $V=1$ for $d=n=4$

$\longrightarrow b, c.$ 


$0 < m_\nu \lesssim 0.1\,\mathrm{eV}$
photon massless
graviton massless
$\frac{d\sigma_{\gamma\gamma}}{d\Omega} = \frac{278}{65\pi^2}\left[2(b^2+c^2)\right] E_{cm}^6 \left(3+\cos^2\theta\right)^2 \left[1 + O(E/M)^4\right]$

3-neutrinos: $0 < m_\nu \lesssim 0.1\,\mathrm{eV}$

photon massless

graviton massless

$$\frac{d\sigma_{\gamma\gamma}}{d\Omega} = \frac{278}{65\pi^2}\left[2(b^2+c^2)\right] E_{cm}^6 \left(3+\cos^2\theta\right)^2 \left[1 + O(E/M)^4\right]$$

$$\frac{\beta^2+\gamma^2}{M^8}$$

neutrinos: $0 < m_\nu \lesssim 0.1\,\mathrm{eV}$ ν

photon massless A_μ

graviton massless $g_{\mu\nu}$

$$\frac{d\sigma_{\gamma\gamma}}{d\Omega} = \frac{278}{65\pi^2}\left[2(b^2+c^2)\right] E_{cm}^6 \left(3+\cos^2\theta\right)^2 \left[1 + O\left(E/M\right)^4\right]$$

(unpolarized)

$$\frac{\beta^2+\gamma^2}{M^8}$$

$$|b+ic|^2 + |b-ic|^2$$

neutrinos: $0 < m_\nu \lesssim 0.1\,\text{eV}$ $\quad \nu_i$

photon $\quad$ massless $\quad A_\mu$

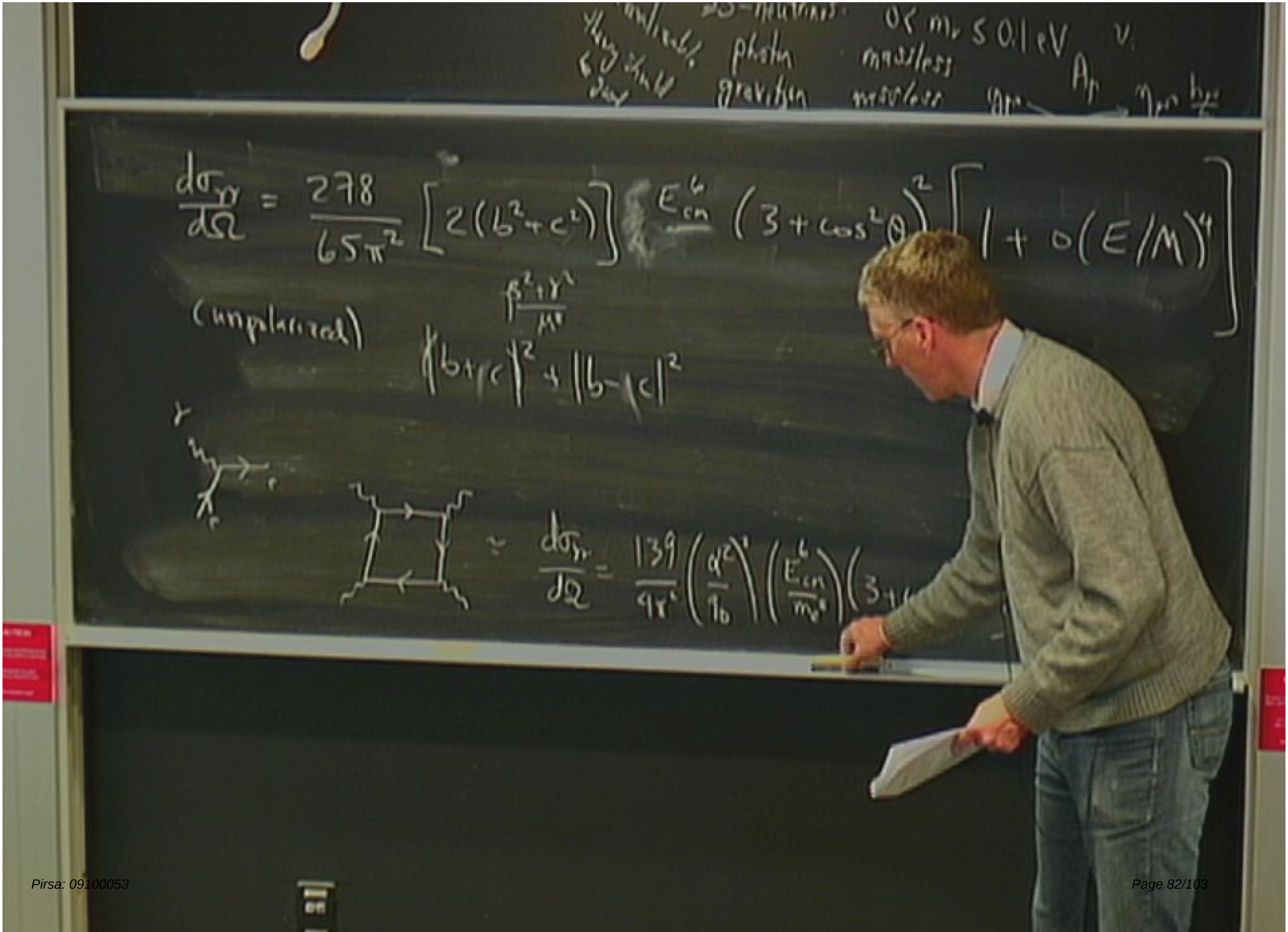
graviton $\quad$ massless $\quad g_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$

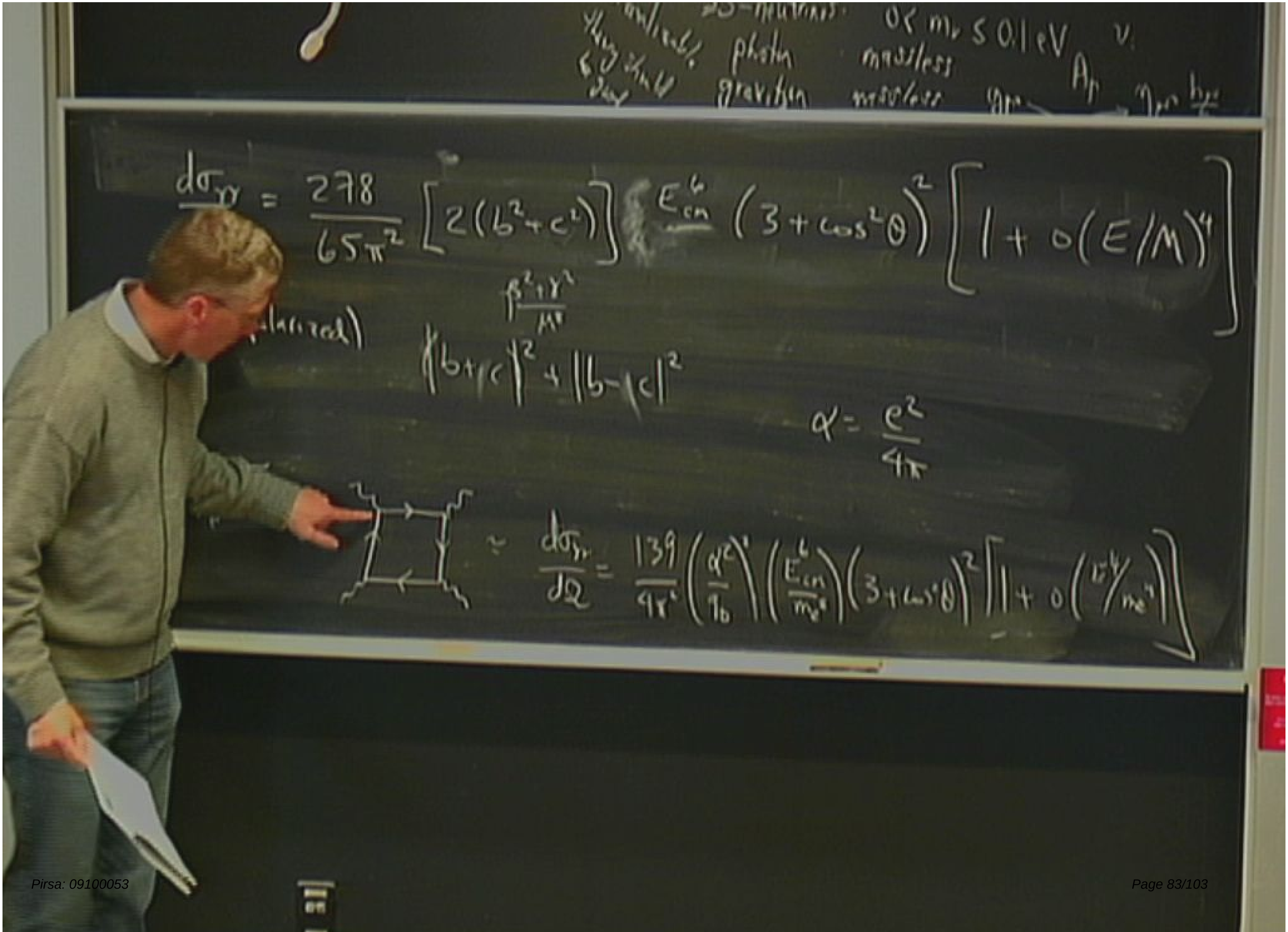
$$\frac{d\sigma_{\gamma\gamma}}{d\Omega} = \frac{278}{65\pi^2}\left[2(b^2+c^2)\right] E_{cm}^6 \left(3+\cos^2\theta\right)^2 \left[1 + O(E/M)^4\right]$$

$$\frac{\beta^2+\gamma^2}{M^8}$$

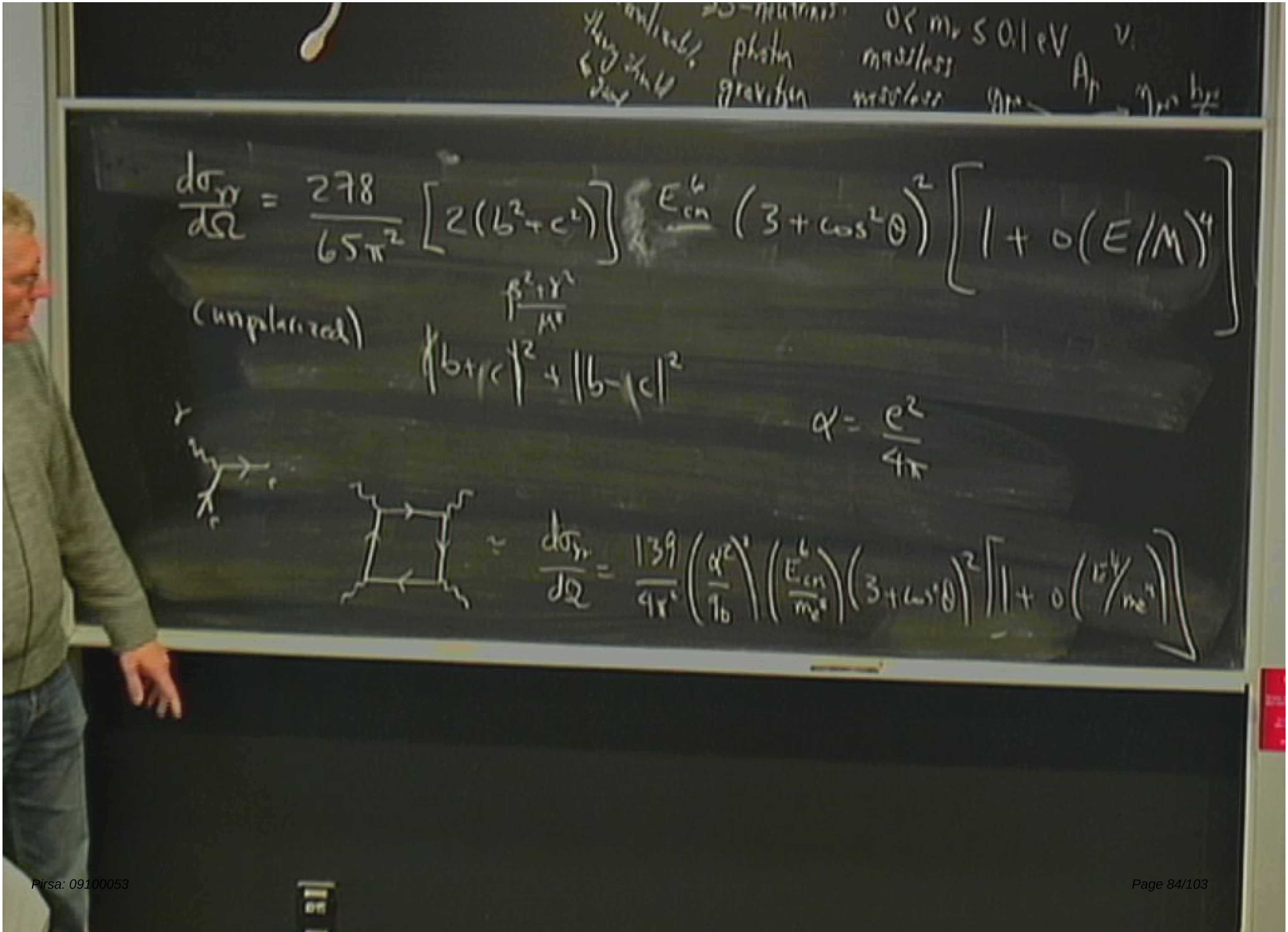
(unpolarized)

$$|b+ic|^2 + |b-ic|^2$$


photon massless
graviton massless
$0 \leq m_\nu \lesssim 0.1\,\mathrm{eV}$
$\frac{d\sigma_{\gamma\gamma}}{d\Omega} = \frac{278}{65\pi^2}\left[2(b^2+c^2)\right]\frac{E_{cm}^6}{}(3+\cos^2\theta)^2\left[1+o(E/M)^4\right]$
(unpolarized)
$|b+ic|^2+|b-ic|^2$
$\frac{d\sigma_{\gamma\gamma}}{d\Omega} = \frac{139}{4\pi^2}\left(\frac{\alpha^2}{a_0}\right)^4\left(\frac{E_{cm}^6}{m_e^8}\right)(3+$



photon massless
graviton massless
0 < mν ≲ 0.1 eV
dσγγ = 278/65π² [2(b²+c²)] E⁶cm (3+cos²θ)² [1+O(E/M)⁴]
α = e²/4π
dσγγ/dΩ = 139/4π² (α²/90)² (E⁶cm/me⁸) (3+cos²θ)² [1+O(E⁴/me⁴)]


photon massless
graviton massless
0.1 eV
(unpolarized)
α = e²/4π

$$\frac{d\sigma_{\gamma\gamma}}{d\Omega} = \frac{139}{4\pi^2}\left(\frac{\alpha^2}{90}\right)\left(\frac{E_{cm}^6}{m_e^8}\right)\left(3+\cos^2\theta\right)^2\left[1+O\left(E^4/m_e^4\right)\right]$$

Eg: The "very low energy" limit of the Standard Model.

$E \leq$ muon mass ≈ 105 MeV

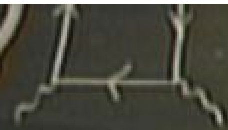
particle lighter than this:

	mass	
muon:	105 MeV	next lightest π: 140 MeV
electron:	0.511 MeV	e
3-neutrinos:	$0 < m_\nu \lesssim 0.1$ eV	ν_i
photon	massless	A_μ
graviton	massless	$g_{\mu\nu} \to \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_p}$

renormalizable theory (all of 'em) today

$$\frac{d\sigma}{d\Omega} = \frac{139}{4\pi^2}\left(\frac{\alpha^2}{90}\right)\left(\frac{E_{cm}^6}{m_e^8}\right)\left(3+\cos^2\theta\right)^2\left[1 + O\left(E^2/m_e^2\right)\right]$$

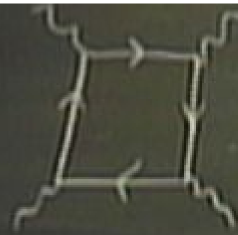
matching: $b = \frac{4}{7}\, c = \frac{\alpha^2}{90}$



$$\frac{d\sigma_{\gamma\gamma}}{d\Omega} = \frac{139}{4\pi^2}\left(\frac{\alpha^2}{90}\right)\left(\frac{E_{cm}^6}{m_e^8}\right)\left(3+\cos^2\theta\right)^2\left[1+O\left(E^4/m_e^4\right)\right]$$

matching: $b = \frac{4}{7}c = \frac{\alpha^2}{90}$

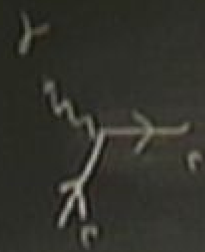
$$\mathcal{L}_{int} = \frac{\alpha^2}{180 m_e^4}\left[5\left(F_{\mu\nu}F^{\mu\nu}\right)^2 - 14 F_{\mu\nu}F^{\nu\lambda}F_{\lambda\rho}F^{\rho\mu}\right]$$

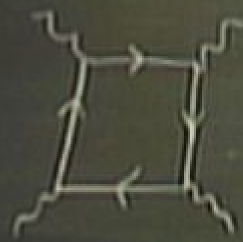


$$\simeq \quad \frac{d\sigma_{\gamma\gamma}}{d\Omega} = \frac{139}{4\pi^2}\left(\frac{\alpha^2}{90}\right)^2\left(\frac{E_{cm}^6}{m_e^8}\right)\left(3+\cos^2\theta\right)^2\left[1+O\left(E^2/m_e^2\right)\right]$$

matching: $b = \frac{4}{7}c = \frac{\alpha^2}{90 m_e^4}$

$$\mathcal{L}_{int} = \frac{\alpha^2}{180 m_e^4}\left[5\left(F_{\mu\nu}F^{\mu\nu}\right)^2 - 14\, F_{\mu\nu}F^{\nu\lambda}F_{\lambda\rho}F^{\rho\mu}\right]$$



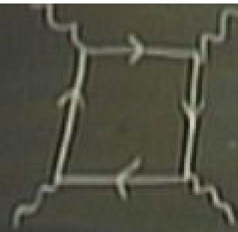


$$\alpha = \frac{e^2}{4\pi}$$

$$\simeq \frac{d\sigma_{\gamma\gamma}}{d\Omega} = \frac{139}{4\pi^2}\left(\frac{\alpha^2}{90}\right)^2\left(\frac{E_{cm}^6}{m_e^8}\right)\left(3+\cos^2\theta\right)^2\left[1+O\left(E^2/m_e^2\right)\right]$$

matching: $c_1 = \frac{4}{7} c_2 = \frac{\alpha^2}{90 m_e^4}$

$$\mathcal{L}_{int} = \frac{\alpha^2}{180 m_e^4}\left[5\left(F_{\mu\nu}F^{\mu\nu}\right)^2 - 14 F_{\mu\nu}F^{\nu\lambda}F_{\lambda\rho}F^{\rho\mu}\right]$$



$$\simeq \frac{d\sigma_{\gamma\gamma}}{d\Omega} = \frac{139}{4\pi^2}\left(\frac{\alpha^2}{90}\right)^2\left(\frac{E_{cm}^6}{m_e^8}\right)\left(3+\cos^2\theta\right)^2\left[1+O\left(E^2/m_e^2\right)\right]$$

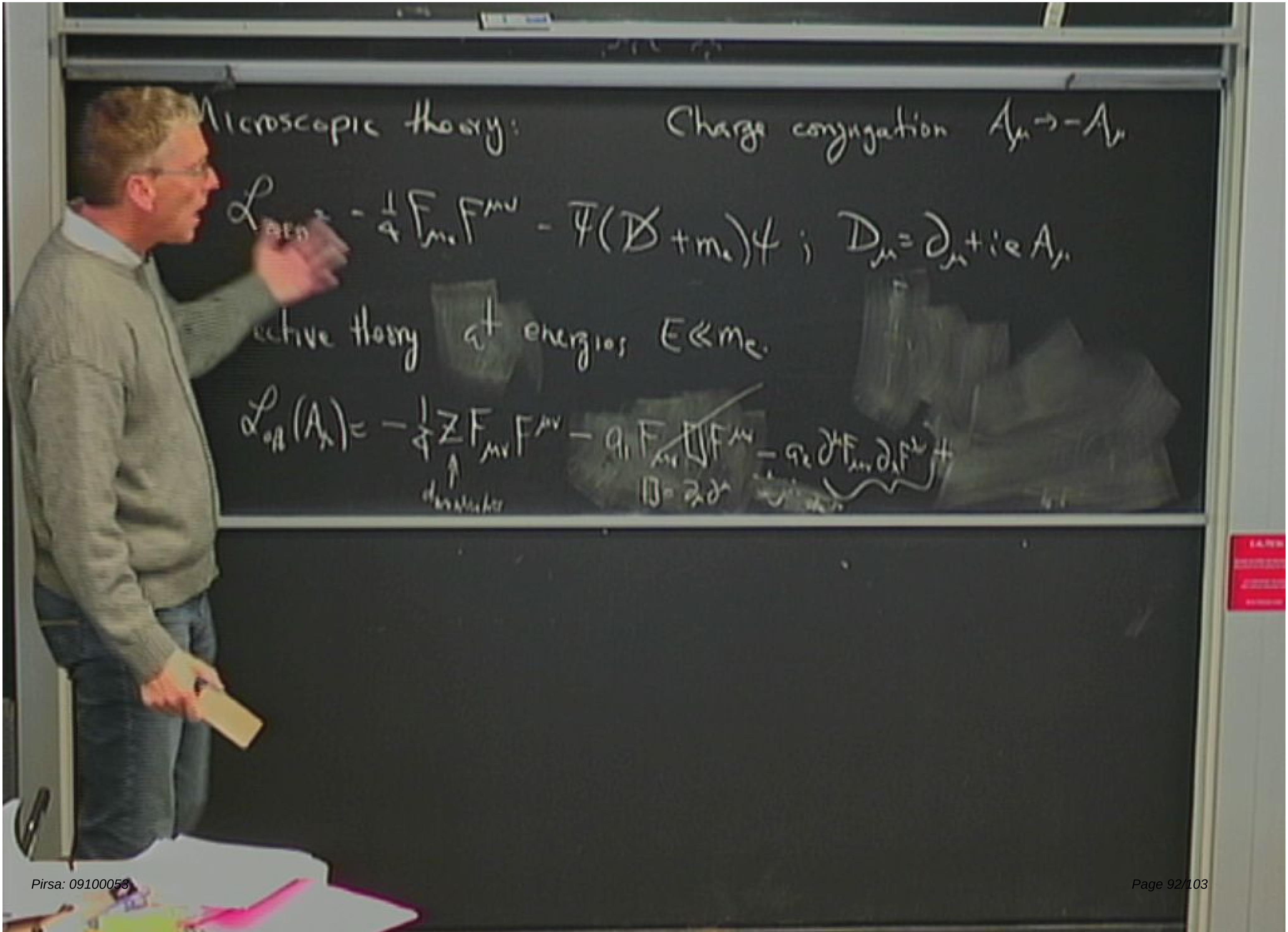
matching: [illegible] $= \frac{4}{7}$ [illegible] $= \dfrac{\alpha^2}{90\,m_e^4}$

$$\mathcal{L}_{int} \simeq \frac{\alpha^2}{180\,m_e^4}\left[5\left(F_{\mu\nu}F^{\mu\nu}\right)^2 - 14\,F_{\mu\nu}F^{\nu\lambda}F_{\lambda\rho}F^{\rho\mu}\right]$$

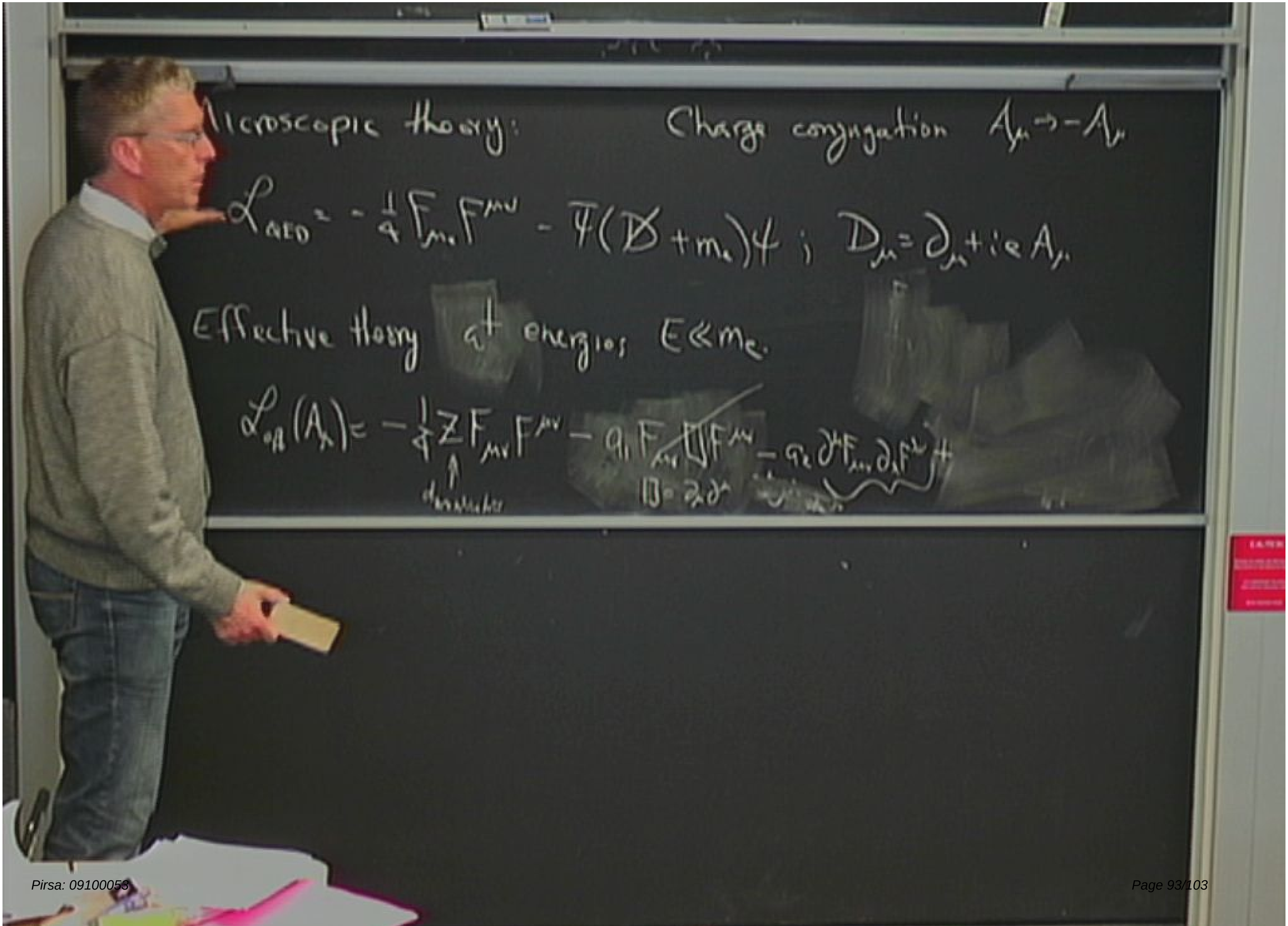
matching: [illegible] $= \frac{4}{7}$ [illegible] $= \frac{\alpha^2}{90 m_e^4}$

$$\mathcal{L}_{int} = \frac{\alpha^2}{180 m_e^4}\left[5\left(F_{\mu\nu}F^{\mu\nu}\right)^2 - 14\, F_{\mu\nu}F^{\nu\lambda}F_{\lambda\rho}F^{\rho\mu}\right]$$

for muons: $m_e \to m_\mu$



Microscopic theory:
Charge conjugation $A_\mu \to -A_\mu$
$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \bar{\Psi}(\not{D} + m_e)\Psi$; $D_\mu = \partial_\mu + ieA_\mu$
ective theory at energies $E \ll m_e$.
$\mathcal{L}_{eff}(A_\mu) = -\frac{1}{4} Z F_{\mu\nu}F^{\mu\nu} - a_1 F_{\mu\nu}\Box F^{\mu\nu} - a_2 \partial^\mu F_{\mu\nu}\partial_\lambda F^{\lambda\nu} +$
$\Box = \partial_\lambda\partial^\lambda$


Microscopic theory:
Charge conjugation $A_\mu \to -A_\mu$
$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}(\not{D} + m_e)\psi$; $D_\mu = \partial_\mu + ieA_\mu$
Effective theory at energies $E \ll m_e$.
$\mathcal{L}_{eff}(A_\mu) = -\frac{1}{4} Z F_{\mu\nu} F^{\mu\nu} - a_1 F_{\mu\nu} \Box F^{\mu\nu} - a_2 \partial^\mu F_{\mu\nu} \partial_\lambda F^{\lambda\nu} +$
$\Box = \partial_\lambda \partial^\lambda$

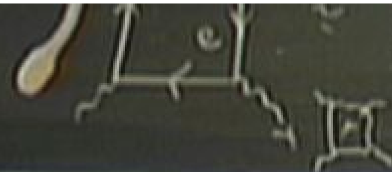
Microscopic theory: Charge conjugation $A_\mu \to -A_\mu$

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}(\not{D} + m_e)\psi + \frac{\alpha^2}{180 m_e^4}\left[5 (F_{\mu\nu}F^{\mu\nu})^2 - 14 \ldots\right]$$

Effective theory at energies $E \ll m_e$.

$$\mathcal{L}_{eff}(A_\lambda) = -\frac{1}{4} \underset{\substack{\uparrow \\ \text{dimensionless}}}{Z} F_{\mu\nu} F^{\mu\nu} - \cancel{a_1 F_{\mu\nu} \Box F^{\mu\nu}} - a_2 \partial^\mu F_{\mu\nu} \partial_\lambda F^{\lambda\nu} + \ldots$$

$\Box = \partial_\lambda \partial^\lambda$



$$\frac{d\sigma}{d\Omega} = \frac{139}{4\pi^2}\left(\frac{\alpha^2}{90}\right)^2\left(\frac{E_{cm}^6}{m_e^8}\right)\left(3+\cos^2\theta\right)^2\left[1 + O\left(E^4/m_e^4\right)\right]$$

matching: $= \frac{4}{7} = \frac{\alpha^2}{90 m_e^4}$

$$\mathcal{L}_{int} = \frac{\alpha^2}{180 m_e^4}\left[5\left(F_{\mu\nu}F^{\mu\nu}\right)^2 - 14 F_{\mu\nu}F^{\nu\lambda}F_{\lambda\rho}F^{\rho\mu}\right] + \frac{\alpha^2}{180 m_\mu^4}\left[5\left(F_{\mu\nu}F^{\mu\nu}\right)^2 - 14 \cdots\right]$$

for muons: $m_e \to m_\mu$

$$\frac{d\sigma}{d\Omega} = \frac{139}{4\pi^2}\left(\frac{\alpha^2}{90}\right)\left(\frac{E_{cm}^6}{m_e^8}\right)\left(3+\cos^2\theta\right)^2\left[1+O\left(E^2/m_e^2\right)\right]$$

matching: $= \frac{4}{7} = \frac{\alpha^2}{90 m_e^4}$

10^{-8} of the e^- piece.

$$\mathcal{L}_{int} = \frac{\alpha^2}{180 m_e^4}\left[5\left(F_{\mu\nu}F^{\mu\nu}\right)^2 - 14\, F_{\mu\nu}F^{\nu\lambda}F_{\lambda\rho}F^{\rho\mu}\right] + \frac{\alpha^2}{180 m_\mu^4}\left[5\left(F_{\mu\nu}F^{\mu\nu}\right)^2 - 14 \ldots\right]$$

for muons: $m_e \to m_\mu$

because $\frac{m_e}{m_\mu} \sim 10^{-2}$

$$\frac{d\sigma}{d\Omega} = \frac{139}{4\pi^2}\left(\frac{\alpha^2}{90}\right)\left(\frac{E_{cm}^6}{m_e^8}\right)\left(3+\cos^2\theta\right)^2\left[1+O\left(E^2/m_e^2\right)\right]$$

Aside: $\mathcal{L} = -\frac{1}{4} Z F_{\mu\nu} F^{\mu\nu}$

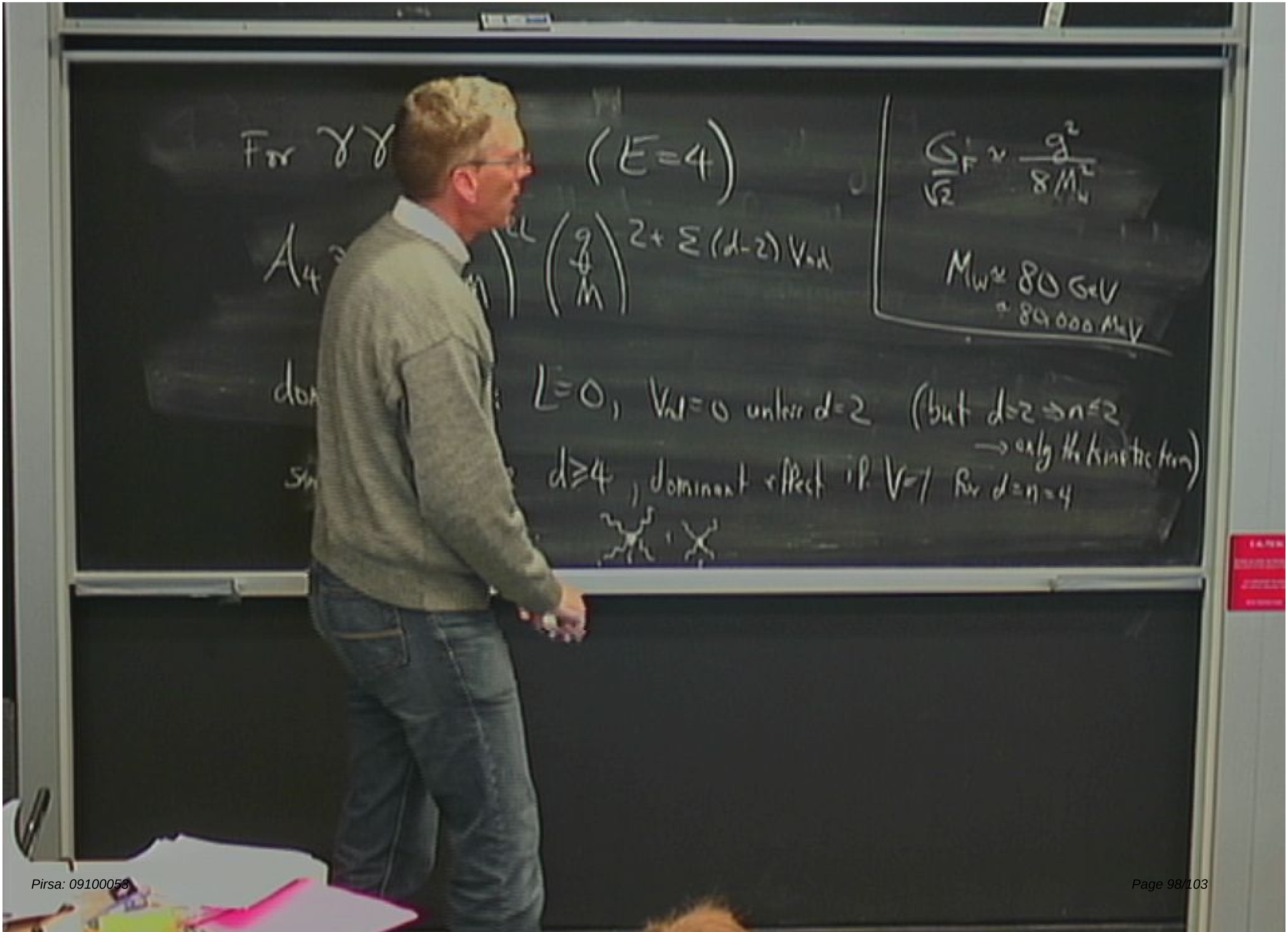
matching: $= \frac{4}{7} = \frac{\alpha^2}{90 m_e^4}$

$$\mathcal{L}_{int} = \frac{\alpha^2}{180 m_e^4}\left[5\left(F_{\mu\nu}F^{\mu\nu}\right)^2 - 14 F_{\mu\nu}F^{\nu\lambda}F_{\lambda\rho}F^{\rho\mu}\right] + \frac{\alpha^2}{180 m_\mu^4}\left[5\left(F_{\mu\nu}F^{\mu\nu}\right)^2 - 14 \cdots\right]$$

10^{-8} of the e^- piece.

for muons: $m_e \to m_\mu$

All other things equal, the smallest mass wins, because $\frac{m_e}{m_\mu} \sim 10^{-2}$



For γγ (E=4)
$\frac{G_F}{\sqrt{2}} \approx \frac{g^2}{8M_W^2}$
A_4 $\left(\frac{g}{M}\right)^{2+\Sigma(d-2)V_{n,d}}$
$M_W \approx 80$ GeV
$\approx 80,000$ MeV
L=0, $V_{nd}=0$ unless d=2 (but d=2 ⇒ n≤2
→ only the kinetic term)
d≥4, dominant effect if V=1 for d=n=4

For $\gamma\gamma \to \gamma\gamma$ $(E=4)$

$$A_4 \sim \left(\frac{g}{4\pi M}\right)^{2L} \left(\frac{g}{M}\right)^{2+\Sigma(d-2)V_{nd}}$$

$\frac{G_F}{\sqrt{2}} \approx \frac{g^2}{8 M_W^2}$

$M_W \approx 80$ GeV $\approx 80\,000$ MeV

$A \sim \frac{1}{M_W^2}$

$\nu\nu\ e\ e$

$\nu\nu \to \gamma\gamma$

$\nu\gamma \to \nu\gamma\gamma$

dominant piece: $L=0$, $V_{nd}=0$ unless $d=2$ (but $d=2 \Rightarrow n=2$ $\to$ only the kinetic term)

since all vertices have $d \geq 4$, dominant effect if $V=1$ for $d=n=4$

$\longrightarrow$ b.c.

For $\gamma\gamma \to \gamma\gamma$ $(E=4)$

$A \sim \frac{1}{M_W^4}$

$$A_4 \sim \left(\frac{g}{4\pi M}\right)^{2L} \left(\frac{g}{M}\right)^{2+\Sigma(d-2)V_{nd}}$$

$\frac{G_F}{\sqrt{2}} \approx \frac{g^2}{8 M_W^2}$

$M_W \approx 80$ GeV $= 80{,}000$ MeV

$\nu\nu e e$

$\nu\nu \to \gamma\gamma$

$\gamma\gamma \to \nu\gamma\gamma$

$\sim \frac{1}{M_W^4} \frac{1}{m_e^4}$

dominant piece: $L=0$, $V_{nd}=0$ unless $d=2$ (but $d=2 \Rightarrow n \leq 2$ $\to$ only the kinetic term)

Since all vertices have $d \geq 4$, dominant effect if $V=1$ for $d=n=4$

$\longrightarrow$ b.c.

For $\gamma\gamma \to \gamma\gamma$ $(E=4)$

$A \sim \frac{1}{M_W^2}$

$$A_4 \sim \left(\frac{g}{4\pi M}\right)^{2L} \left(\frac{g}{M}\right)^{2+\Sigma(d-2)V_{nd}}$$

$\frac{G_F}{\sqrt{2}} \approx \frac{g^2}{8 M_W^2}$

$M_W \approx 80\ \text{GeV}$
$= 80{,}000\ \text{MeV}$

$\nu\ \nu\ e\ e$

$\nu\nu \to \gamma\gamma$

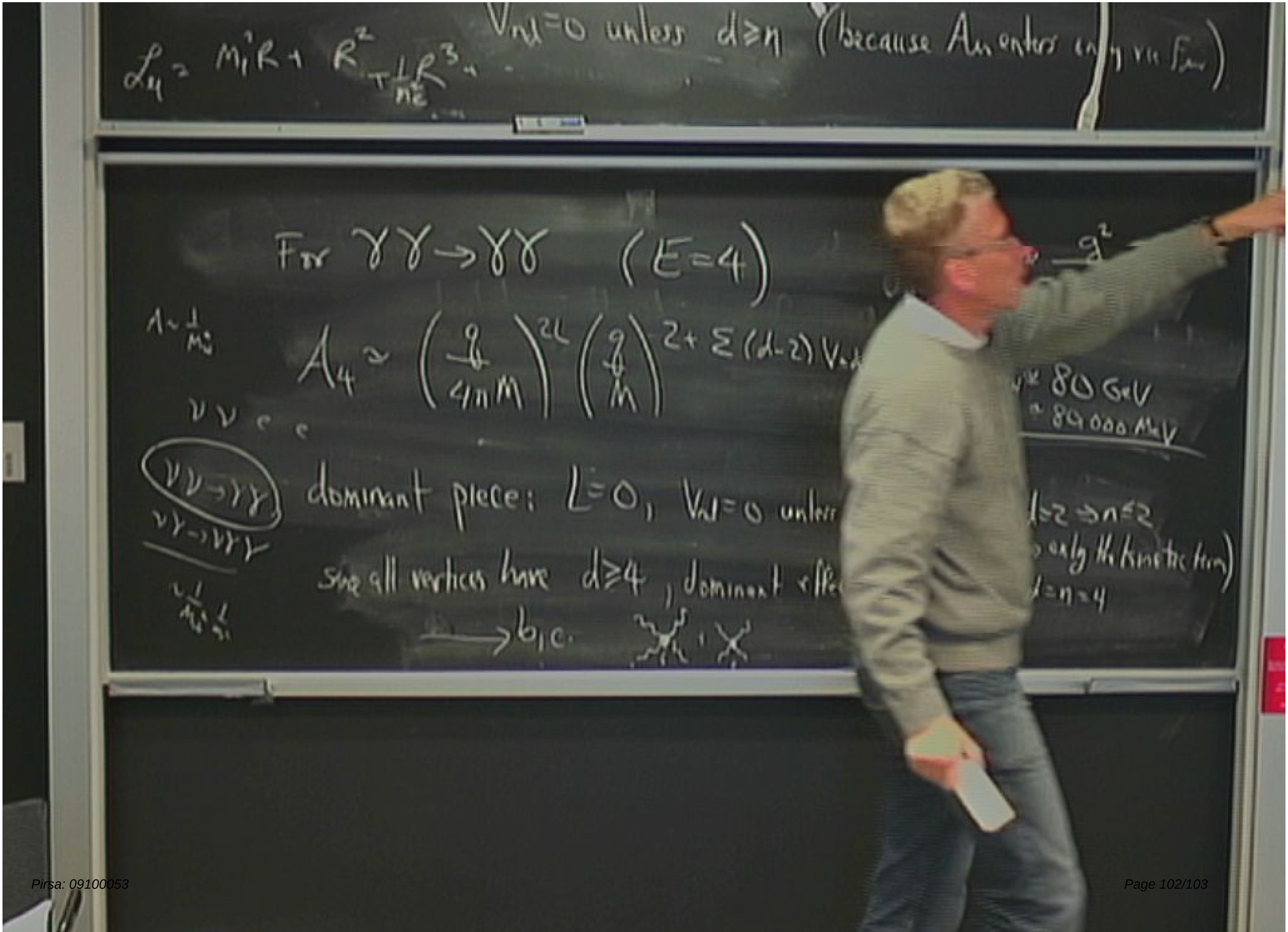
$\nu\gamma \to \nu\gamma\gamma$

$\sim \frac{1}{M_W^4}\frac{1}{M}$

dominant piece: $L=0$, $V_{nd}=0$ unless $d=2$ (but $d=2 \Rightarrow n=2$ $\rightarrow$ only the kinetic term)

Since all vertices have $d \geq 4$, dominant effect if $V=1$ for $d=n=4$

$\longrightarrow$ b.c.



$V_{nd} = 0$ unless $d \geq n$ (because A_μ enters only via $F_{\mu\nu}$)
$\mathcal{L}_4 = M_p^2 R + R^2 + \frac{1}{M_p^2} R^3 + \ldots$
For $\gamma\gamma \to \gamma\gamma$ $(E=4)$
$\Lambda \sim \frac{1}{M_p^2}$
$A_4 \simeq \left(\frac{g}{4\pi M}\right)^{2L} \left(\frac{g}{M}\right)^{2+\Sigma(d-2)V_{nd}}$
80 GeV
= 80 000 MeV
$\nu\nu\ e\ e$
$\nu\nu \to \gamma\gamma$
$\nu\gamma \to \nu\gamma\gamma$
dominant piece: $L=0$, $V_{nd}=0$ unless
$d=2 \Rightarrow n \leq 2$
only the kinetic term)
Since all vertices have $d \geq 4$, dominant effect
$d = n = 4$
$\rightarrow b.c.$

$\mathcal{L}_4 = M_1^2 R + R^2 + \frac{1}{M^2} R^3 + \dots$

$V_{nd} = 0$ unless $d \geq n$ (because A_μ enters only via $F_{\mu\nu}$)

For $\gamma\gamma \to \gamma\gamma$ $(E=4)$

$\Lambda \sim \frac{1}{M_W^2}$

$$A_4 \sim \left(\frac{g}{4\pi M}\right)^{2L} \left(\frac{g}{M}\right)^{2+\sum(d-2)V_{nd}}$$

$\frac{G_F}{\sqrt{2}} \approx \frac{g^2}{8 M_W^2}$

$M_W \approx 80$ GeV
$= 80{,}000$ MeV

$\nu\nu e e$

$\nu\nu \to \gamma\gamma$

$\nu\gamma \to \nu\gamma\gamma$

$\sim \frac{1}{M_W^4}\frac{1}{m_1}$

dominant piece: $L=0$, $V_{nd}=0$ unless $d=2$ (but $d=2 \Rightarrow n \leq 2$
$\to$ only the kinetic term)

Since all vertices have $d \geq 4$, dominant effect if $V=1$ for $d=n=4$

$\longrightarrow$ b.c.