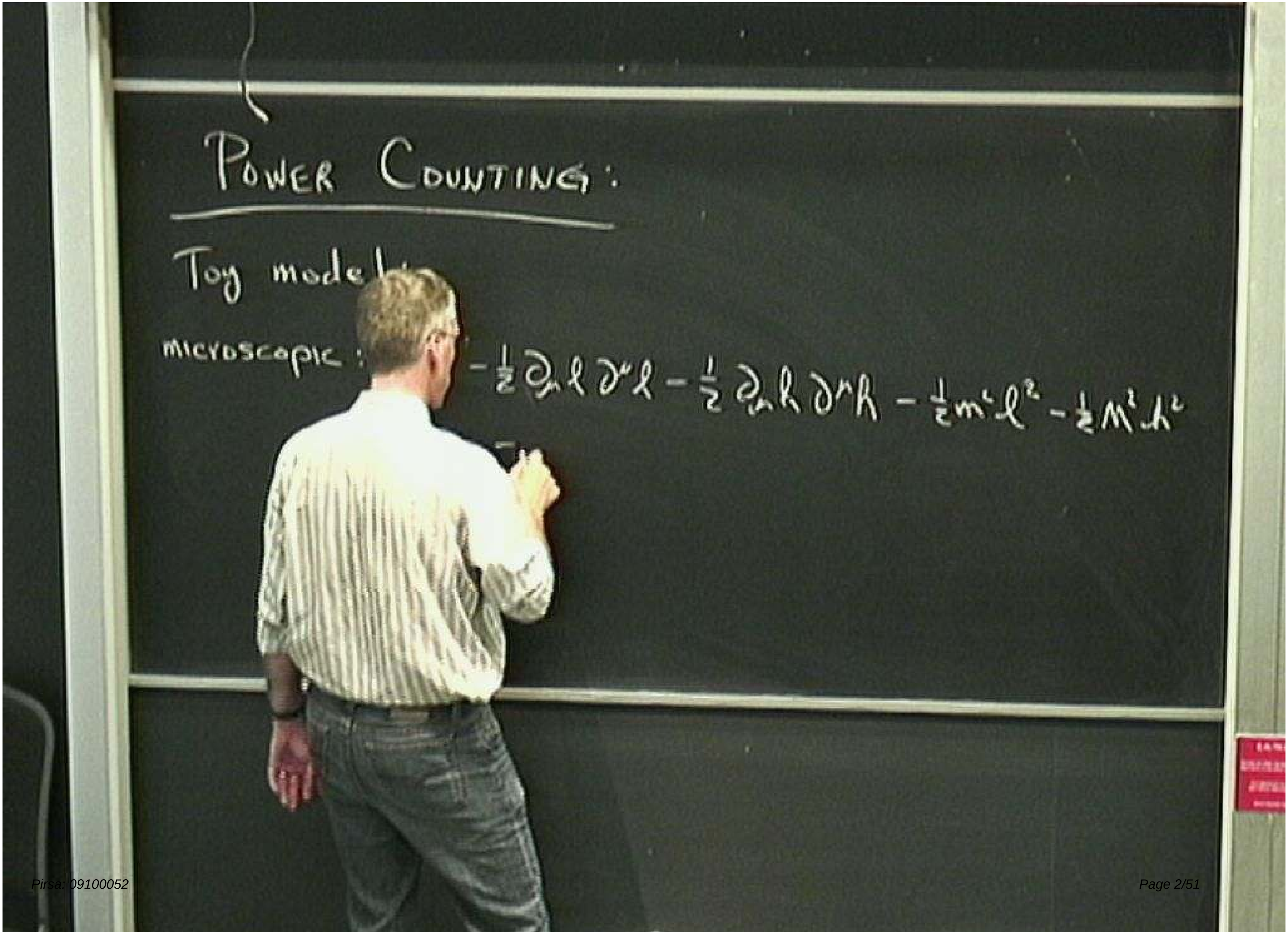
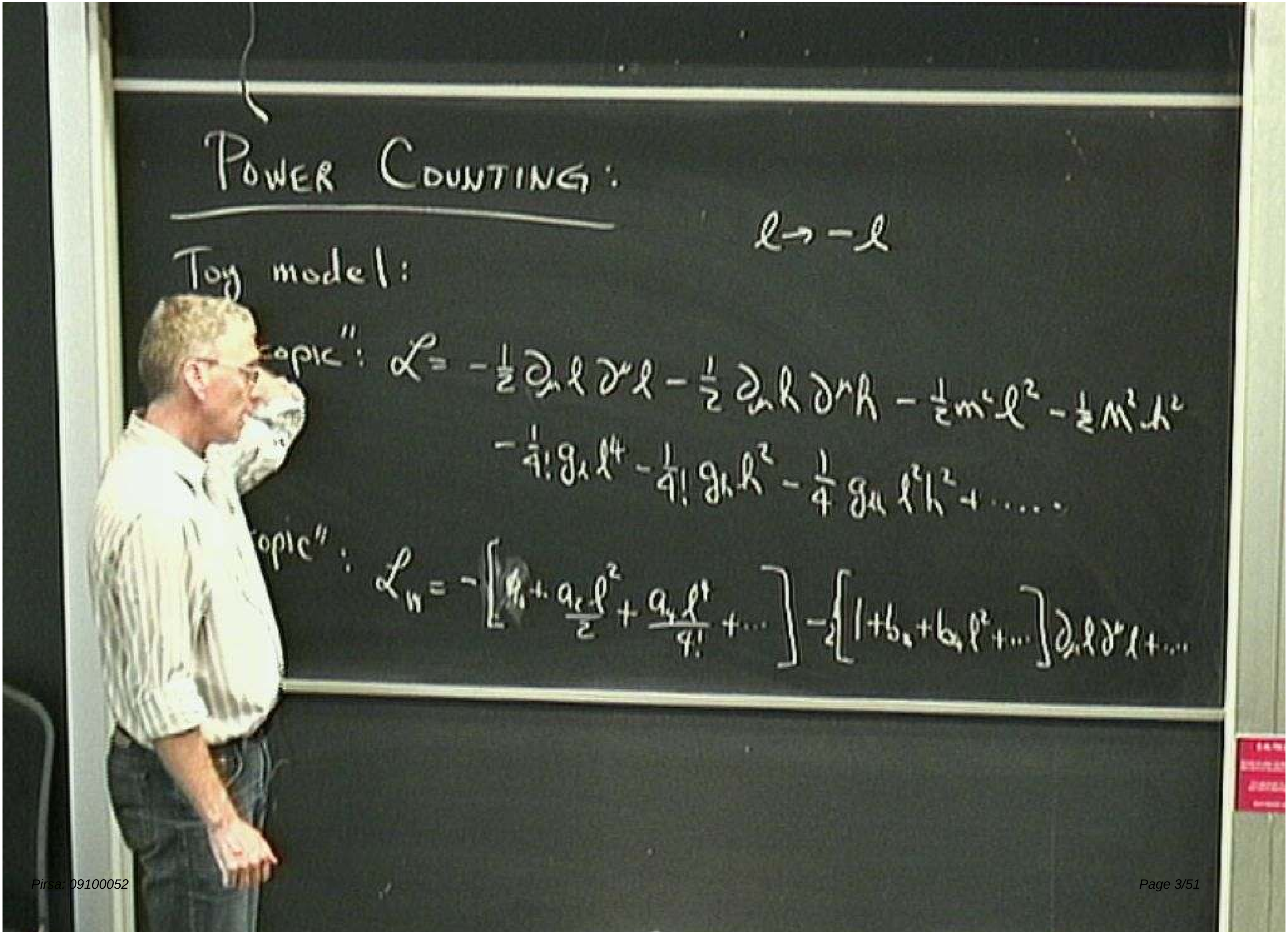
Title: Introduction to Effective Field Theory - Lecture 5A

Date: Oct 21, 2009 09:00 AM

URL: http://pirsa.org/09100052

Abstract:


POWER COUNTING:
Toy model
microscopic: $-\frac{1}{2}\partial_\mu \ell \, \partial^\mu \ell - \frac{1}{2}\partial_\mu h \, \partial^\mu h - \frac{1}{2}m^2 \ell^2 - \frac{1}{2}M^2 h^2$


POWER COUNTING:
Toy model:
$\ell \to -\ell$
opic": $\mathcal{L} = -\frac{1}{2}\partial_\mu \ell\, \partial^\mu \ell - \frac{1}{2}\partial_\mu h\, \partial^\mu h - \frac{1}{2}m^2\ell^2 - \frac{1}{2}M^2 h^2$
$- \frac{1}{4!} g_\ell \ell^4 - \frac{1}{4!} g_h h^2 - \frac{1}{4} g_{\ell h} \ell^2 h^2 + \dots$
opic": $\mathcal{L}_n = -\left[a_0 + \frac{a_2 \ell^2}{2} + \frac{a_4 \ell^4}{4!} + \dots \right] - \frac{1}{2}\left[1 + b_0 + b_2 \ell^2 + \dots \right] \partial_\mu \ell\, \partial^\mu \ell + \dots$

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$\ell \to -\ell$

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"microscopic": $\mathcal{L} = -\frac{1}{2}\partial_\mu \ell\, \partial^\mu \ell - \frac{1}{2}\partial_\mu h\, \partial^\mu h - \frac{1}{2}m^2\ell^2 - \frac{1}{2}M^2 h^2$
$\qquad - \frac{1}{4!}g_\ell \ell^4 - \frac{1}{4!}g_h h^2 - \frac{1}{4}g_{\ell h}\ell^2 h^2 + \dots$

"macroscopic": $\mathcal{L}_h = -\left[a_0 + \frac{a_2 \ell^2}{2} + \frac{a_4 \ell^4}{4!} + \dots\right] - \frac{1}{2}\left[1 + b_0 + b_2 \ell^2 + \dots\right]\partial_\mu \ell\, \partial^\mu \ell + \dots$

Power Counting:

Toy model:

$\ell \to -\ell$ $\qquad$ $m \ll M$

"microscopic": $\mathcal{L} = -\frac{1}{2}\partial_\mu \ell\, \partial^\mu \ell - \frac{1}{2}\partial_\mu h\, \partial^\mu h - \frac{1}{2}m^2 \ell^2 - \frac{1}{2}M^2 h^2$
$- \frac{1}{4!} g_\ell \ell^4 - \frac{1}{4!} g_h h^4 - \frac{1}{4} g_{\ell h} \ell^2 h^2 + \dots\dots$

"macroscopic": $\mathcal{L}_{\text{eff}} = -\left[a_0 + \frac{a_2 \ell^2}{2} + \frac{a_4 \ell^4}{4!} + \dots \right] - \frac{1}{2}\left[1 + b_0 + b_2 \ell^2 + \dots \right] \partial_\mu \ell\, \partial^\mu \ell + \dots$

Effective couplings: $a_0, a_2, \ldots$

fixed by "matching": require $a_0, \ldots$ to be whatever is required to reproduce full result for physical observables to some fixed order in $1/M$, and possibly $g_e, \ldots$

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Q: since the insertion of an eff. interaction $\sim 1/M^2$ into a complicated Feynman graph can produce effects enhanced by powers of M^2 how do you decide which interactions are relevant to fixed order in $1/M^2$?

A: Suppose

$$\mathcal{L}_w = f^4 \sum_{k} \mathcal{O}_k\left(\frac{\phi}{v}\right)$$

$\partial^\mu h - \frac{1}{2} m^2 \ell^2 - \frac{1}{2} M^2 h^2$

$- \frac{1}{4} g_{4} \ell^2 h^2 + \cdots$

$\ell = \frac{\ell}{v} v$

$\ell^2 \partial_\lambda \partial \ell = v^4 \left(\frac{\ell}{v}\right)^2 \partial\left(\frac{\ell}{v}\right) \partial$

$-\frac{1}{2}\left[1 + b_2 + b_4 \ell^2 + \cdots\right] \partial_\mu \ell \, \partial^\mu \ell + \cdots$

A: Suppose

$$\mathcal{L}_W = f^4 \sum_k \frac{c_k}{M^{d_k}} \mathcal{O}_k\left(\frac{\phi}{v}\right)$$

f, v, M are arbitrary mass scales.

c_k are dimensionless

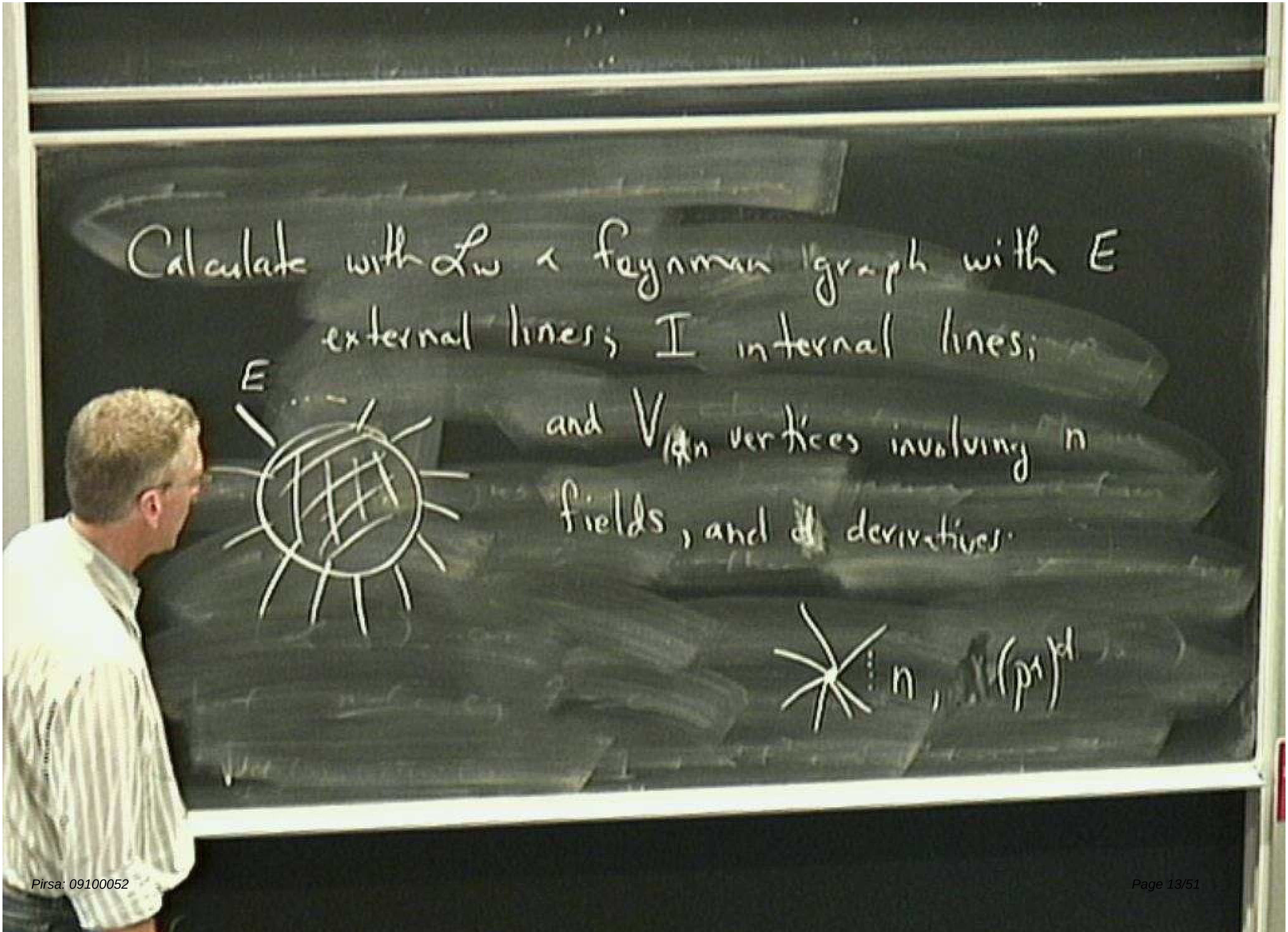
d_k = # derivatives in $\mathcal{O}_k$ = dimension of $\mathcal{O}_k$.

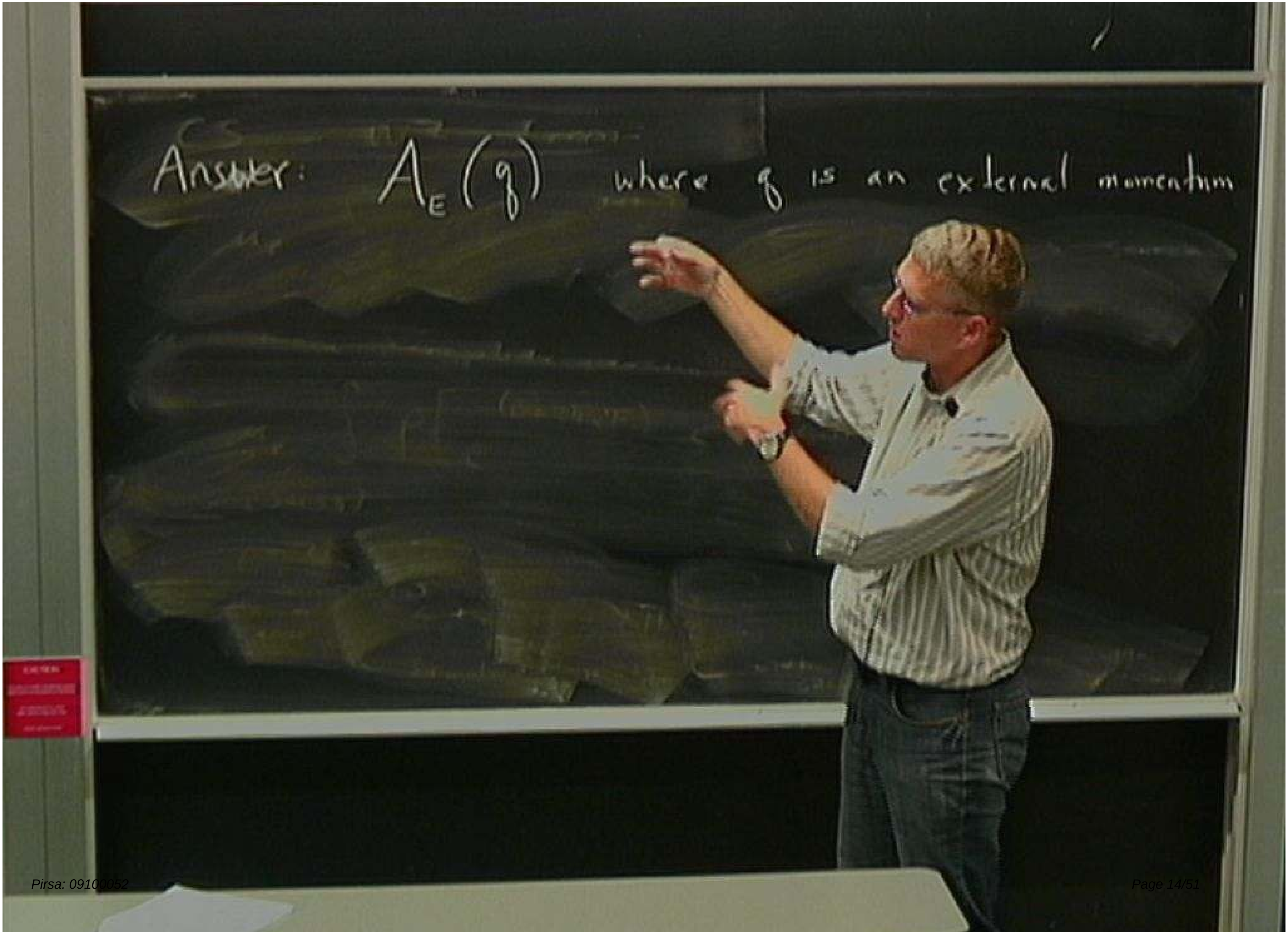
$\mathcal{O}_k$ = power of $\frac{\phi}{v}$, and its derivatives, all at a single point x.

We put this into a path integral
and separate out the quadratic piece in ϕ
as the unperturbed action, perturbing in rest.

$\mathcal{L}_{w_0}$ = terms with two powers of $\left(\phi/v\right)$
and two (or zero) powers of ∂ $\quad (d_k = 2, 0)$

Calculate with $\mathcal{L}_W$ a feynman graph with E
external lines; I internal lines;


Calculate with $\mathcal{L}_W$ a Feynman graph with E
external lines; I internal lines;
and $V_{i,n}$ vertices involving n
fields, and d_i derivatives.
E
n, $(p)^d$


Answer: A_E(q) where q is an external momentum

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Schematically:

$$A_E(q) \approx \Bigg[-i \int \frac{d^4 p}{(2\pi)^4} \, \frac{1}{p^2 + m^2}$$

assumed $\mathcal{L}_0 = \partial\phi\partial\phi + m^2\phi^2$

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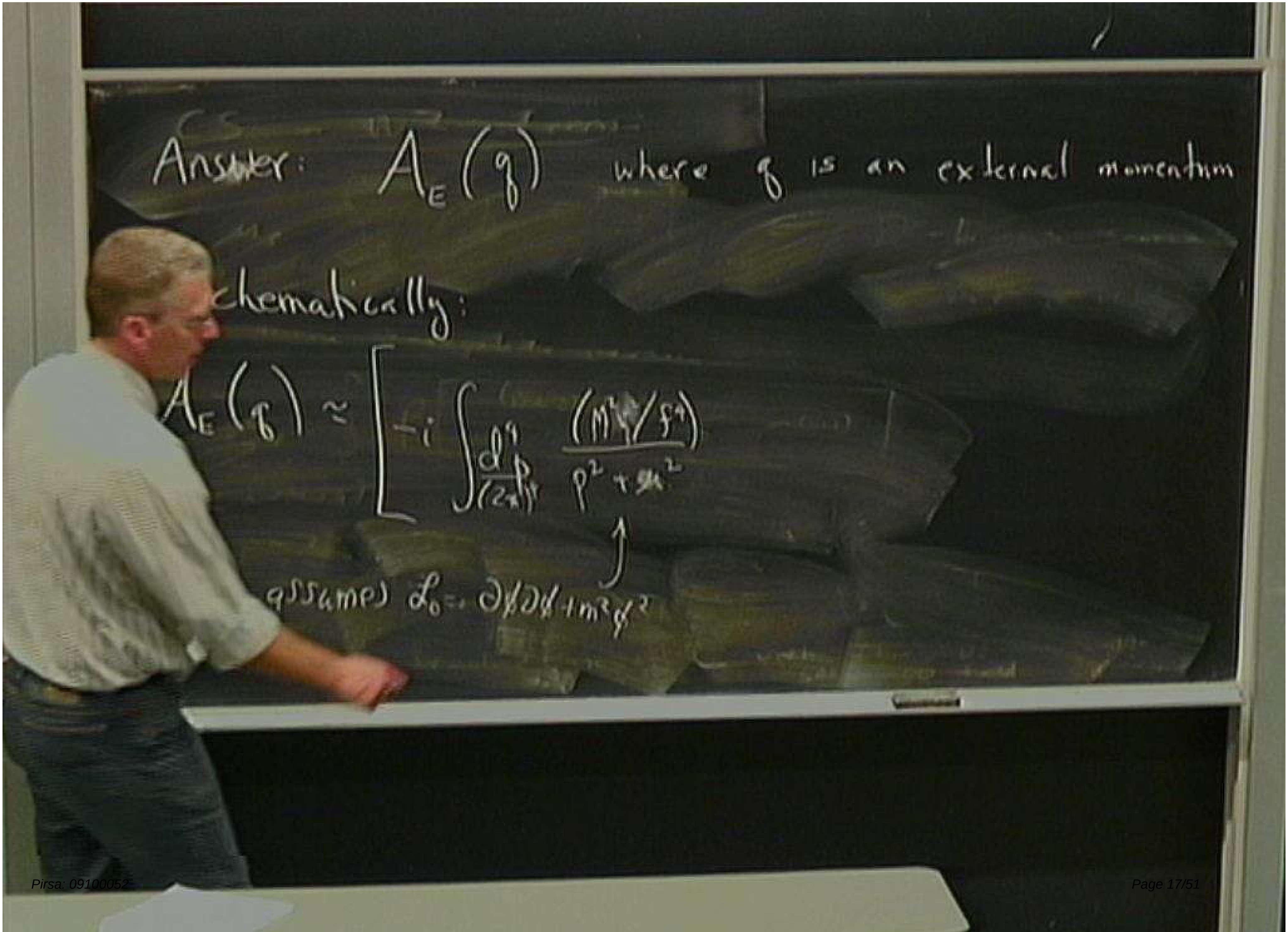
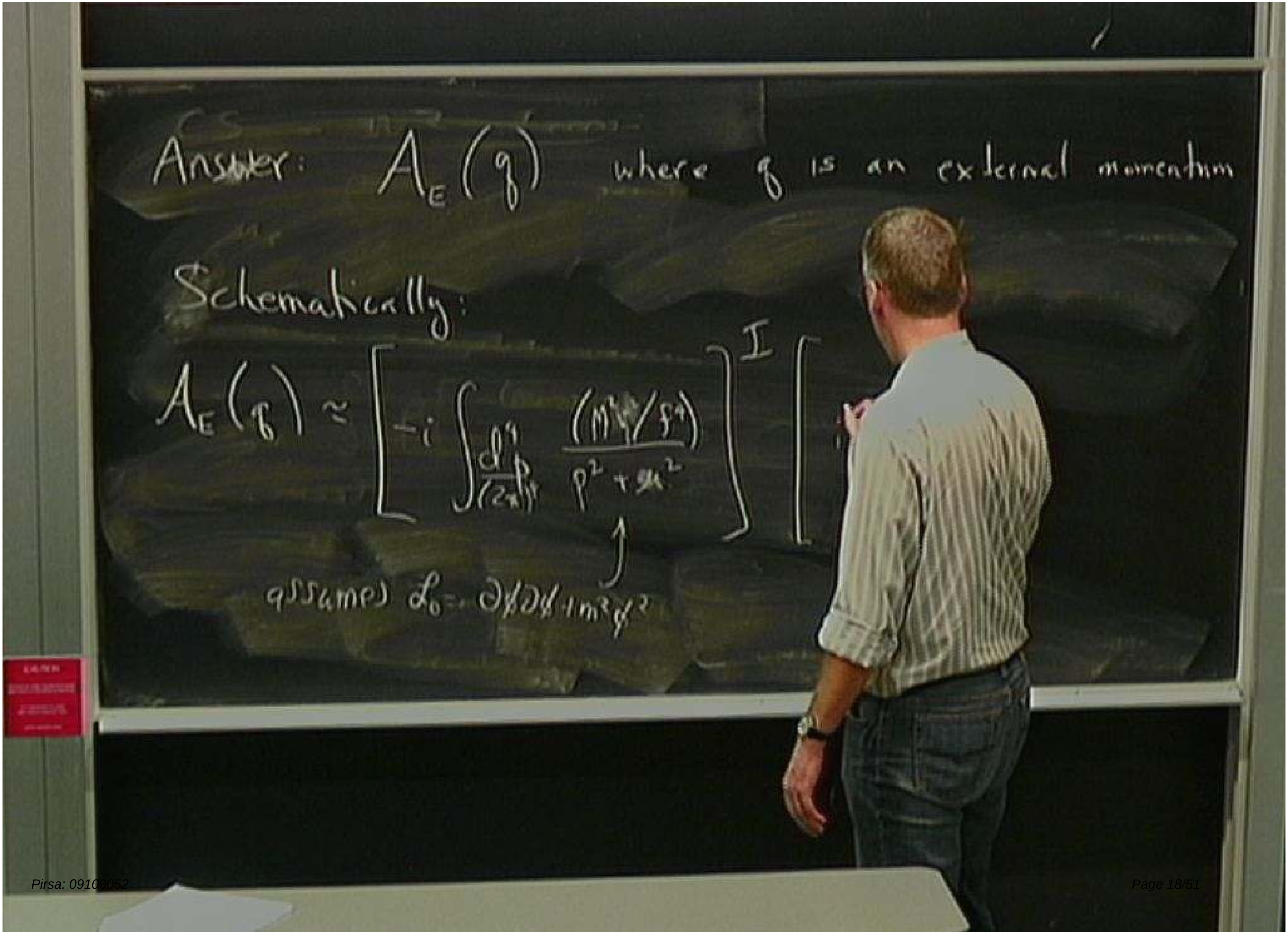
eg $d=2$, with 2 fields: $\mathcal{L}_{kin} = f^4 \frac{c_{22}}{M^2} \partial\left(\frac{\phi}{v}\right)\partial\left(\frac{\phi}{v}\right) = \frac{f^4 c_{22}}{M^2 v^2} \partial\phi\,\partial\phi$


Answer: A_E(q) where q is an external momentum
chematically:
A_E(q) ≈ [-i ∫ d^4p/(2π)^4 (m^4/f^4)/(p^2 + m^2)
assumes L_0 = ∂φ∂φ + m^2 φ^2

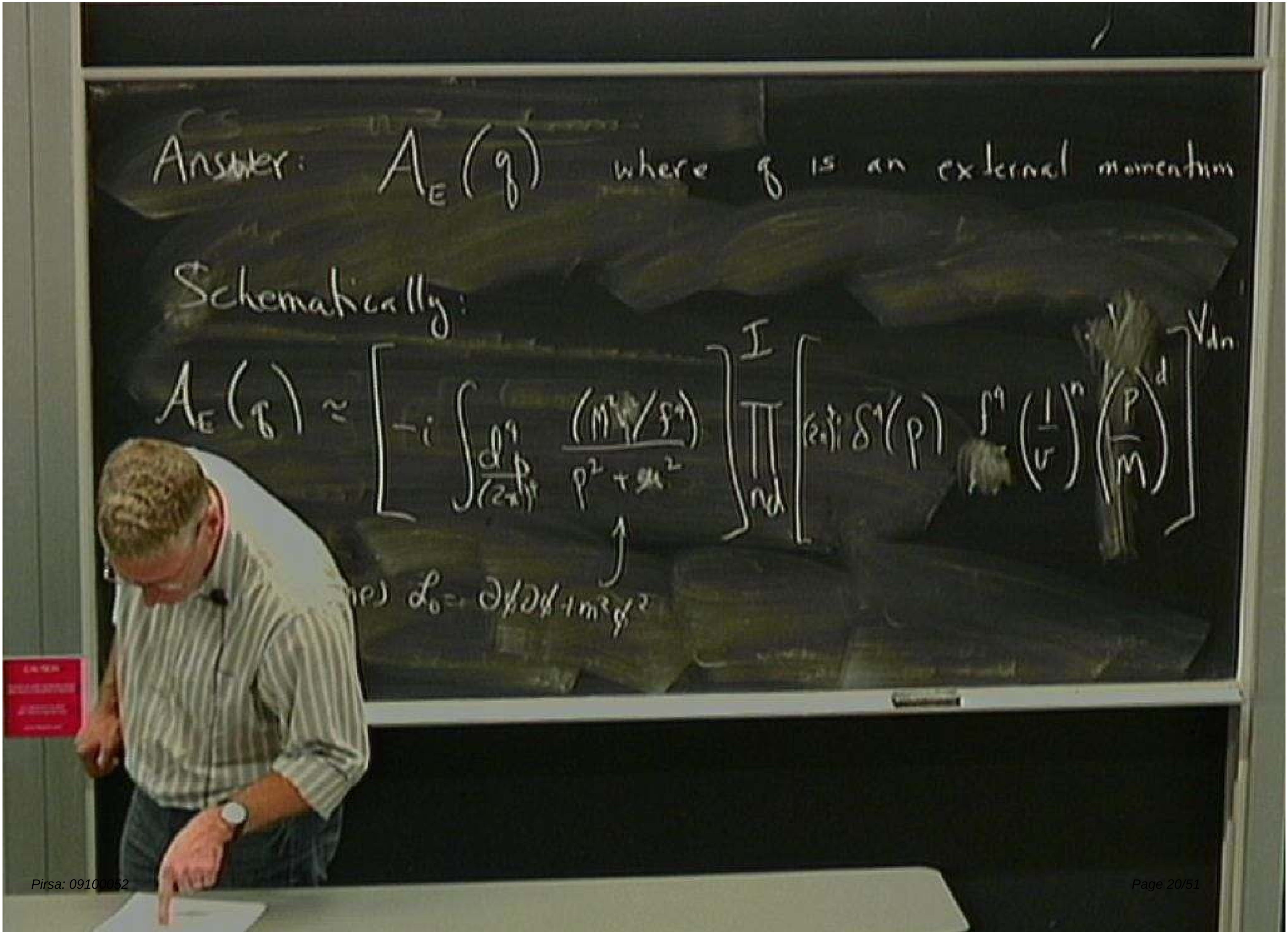

Answer: A_E(q) where q is an external momentum
Schematically:
A_E(q) ≃ [-i ∫ d^4p/(2π)^4 (m^4/f^4)/(p^2 + m^2)]^I [
assumes L_0 = ∂φ∂φ + m^2 φ^2

Answer: $A_E(q)$ where q is an external momentum

Schematically:

$$A_E(q) \simeq \left[-i \int \frac{d^4p}{(2\pi)^4} \, \frac{(M^4/f^4)}{p^2+m^2} \right]^{I} \prod_{nd} \left[i\delta^4(p) \, \frac{f^n}{M^4} \left(\frac{1}{v}\right)^n \right]^{V_{dn}}$$

assumes $\mathcal{L}_0 = \partial\phi\partial\phi + m^2\phi^2$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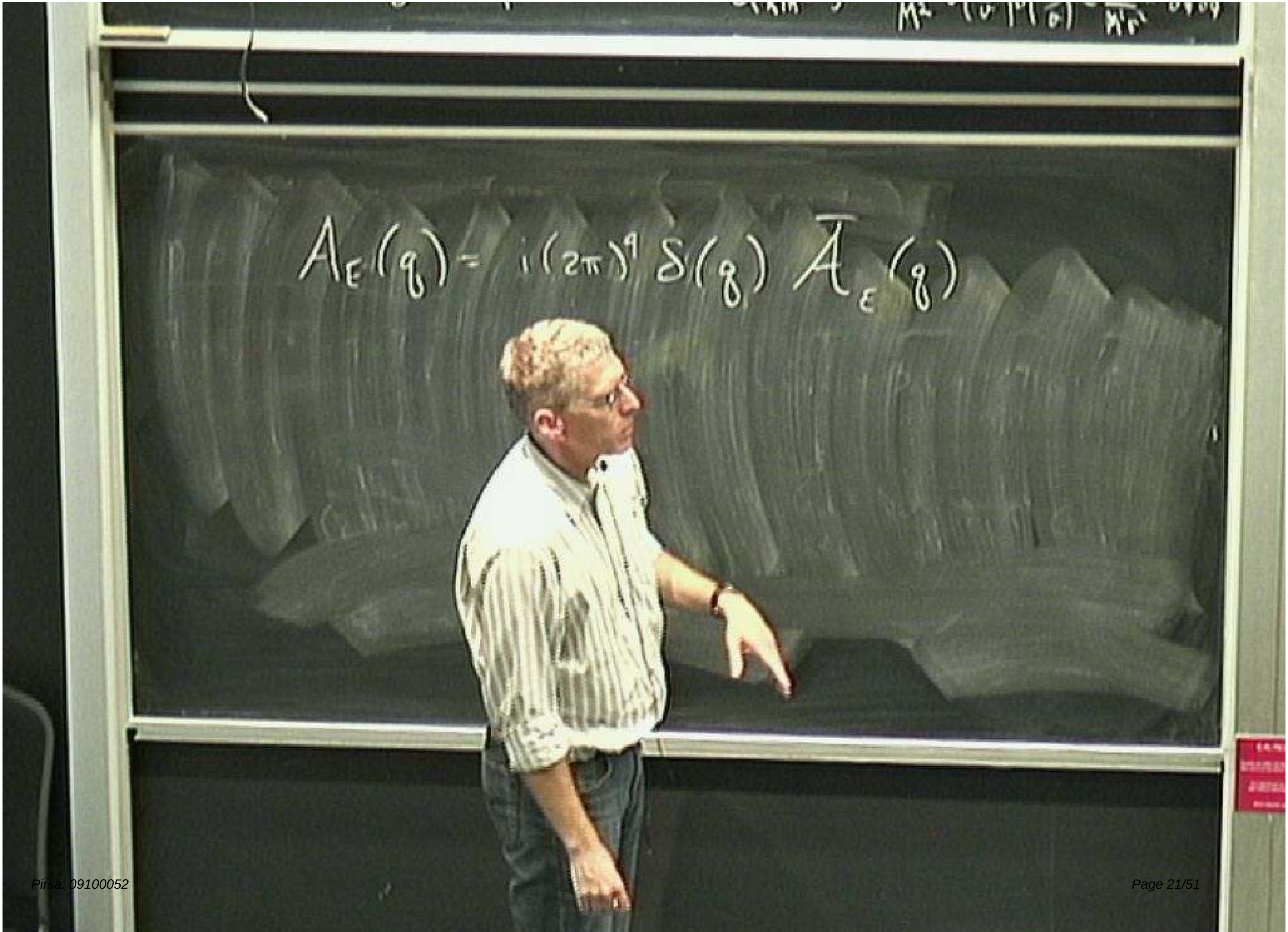
$A_E(q) = i(2\pi)^q \delta(q)\, \bar{A}_E(q)$

$$A_E(q) = i(2\pi)^4\,\delta^4(q)\,\bar{A}_E(q)$$

$$\bar{A}_E(q) \approx$$

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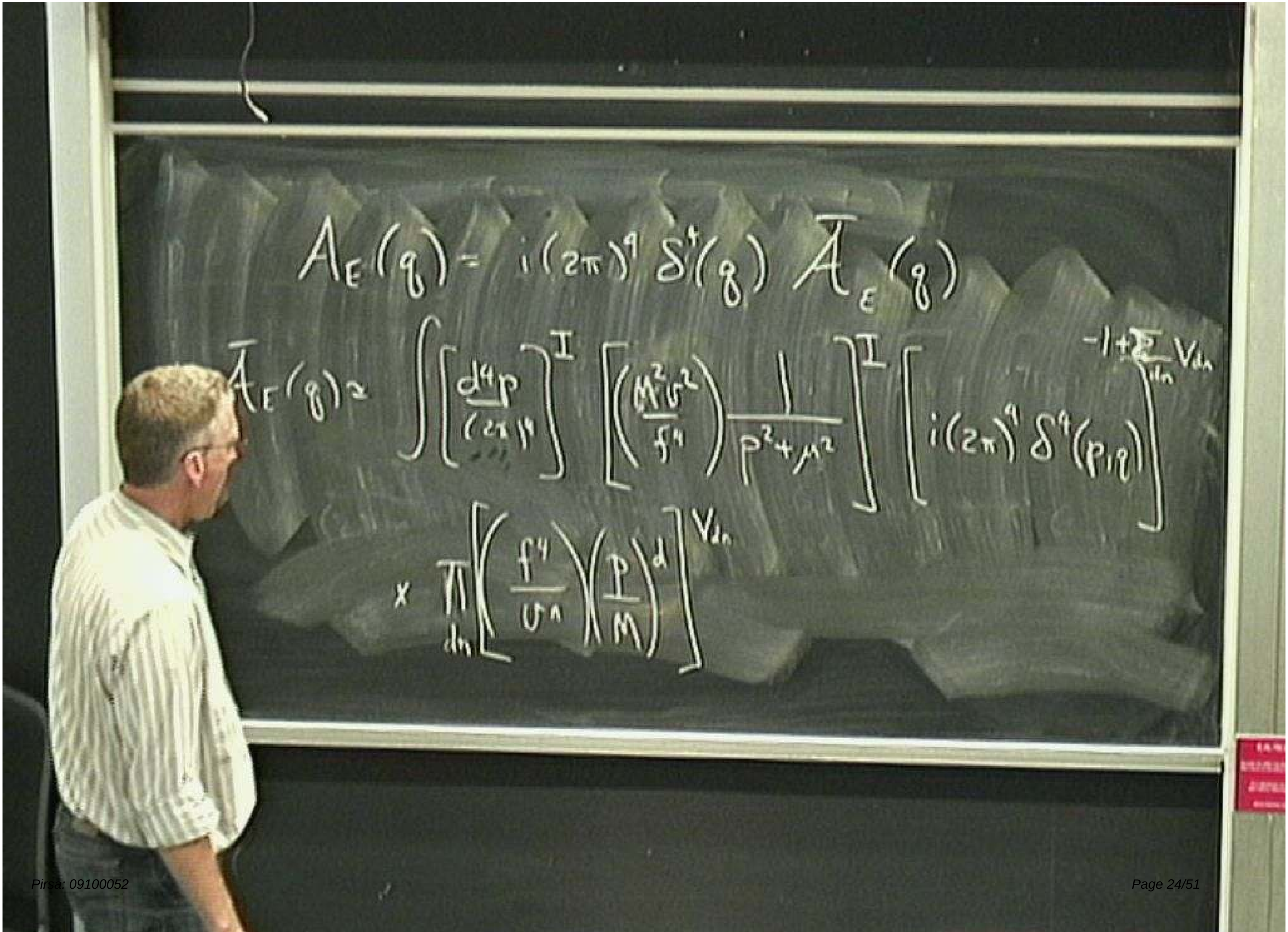
$$\bar{A}_E(q) \approx \int \left[\frac{d^4p}{(2\pi)^4}\right]^I \left[\left(\frac{M^2 v^2}{f^4}\right)\frac{1}{p^2+\mu^2}\right]^I \left[i(2\pi)^4\,\delta^4(p,q)\right]^{-1+\sum_{v,n} V_{v,n}}$$


$A_E(q) = i(2\pi)^4\,\delta^4(q)\,\bar{A}_E(q)$
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$\times \prod_{dn}\left[\left(\frac{f^4}{v^n}\right)\left(\frac{p}{M}\right)^d\right]^{V_{dn}}$

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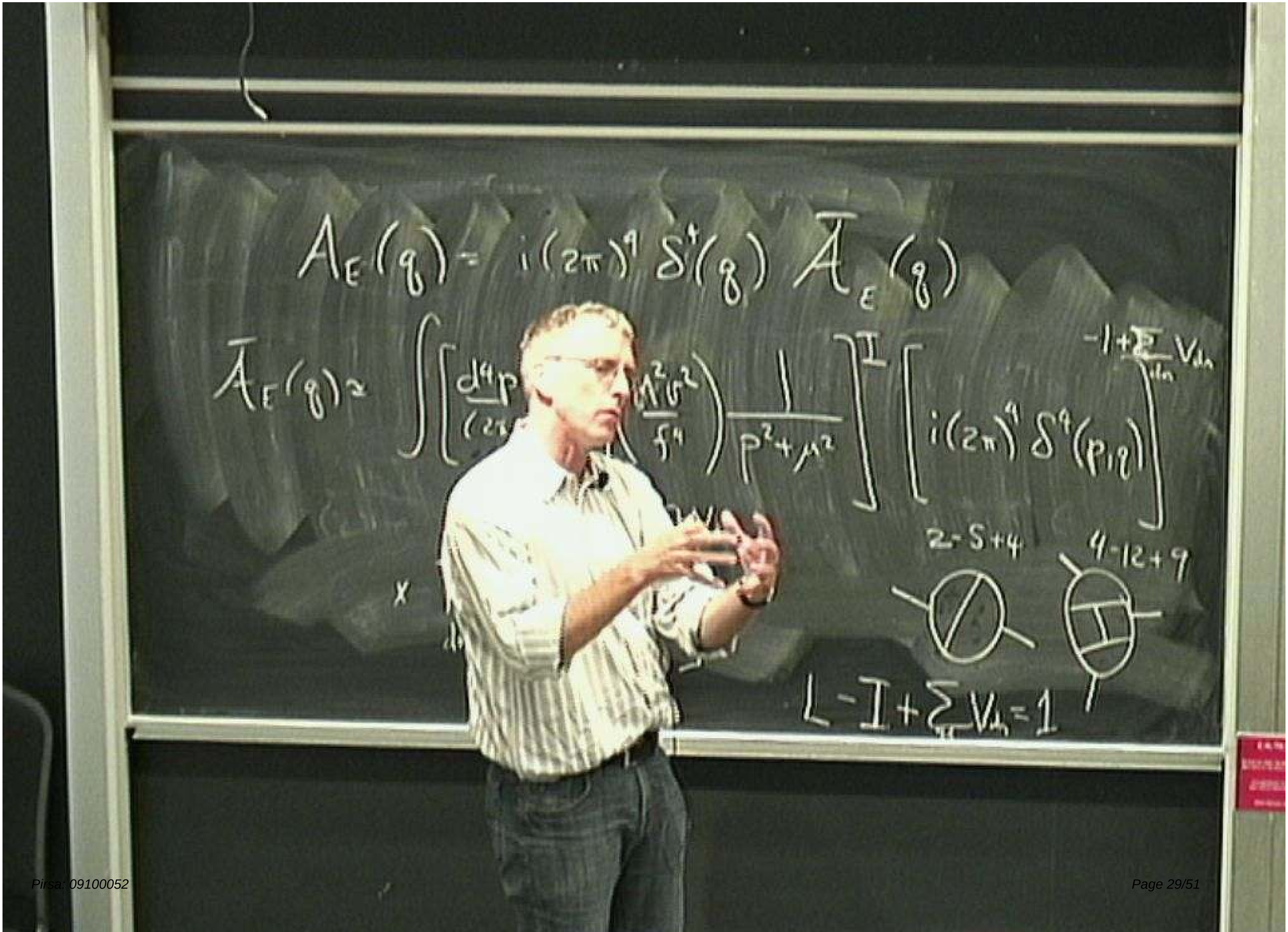
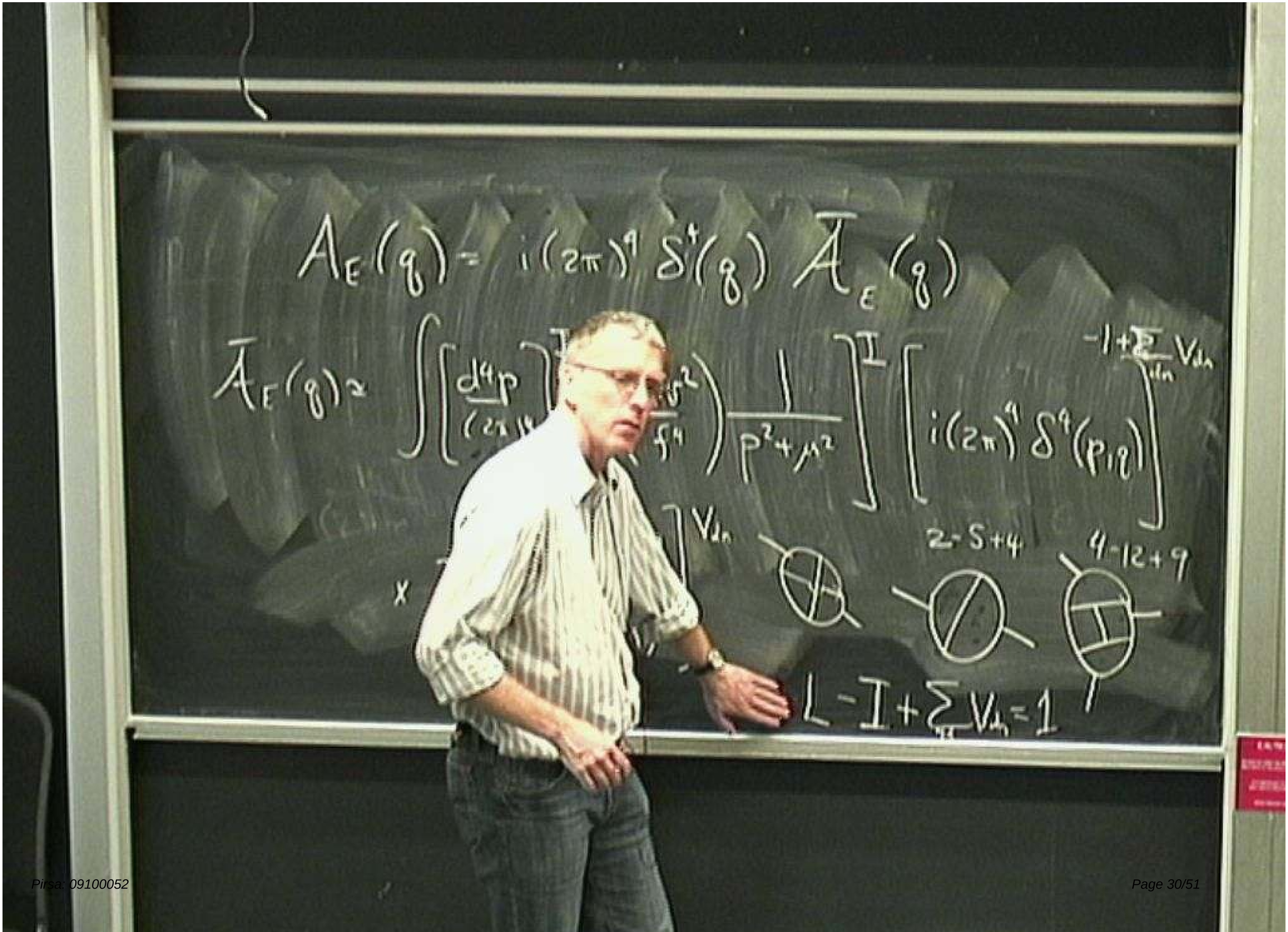
2) Definition of the number of loops:



$A_E(q) = i(2\pi)^4 \delta^4(q) \bar{A}_E(q)$
$\bar{A}_E(q) =$
$i(2\pi)^4 \delta^4(p, q)$
2-5+4
4-12+9
$L - I + \sum V_n = 1$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Two identities relate E, I, V_{dn} in any graph:

1) "Conservation of ends":

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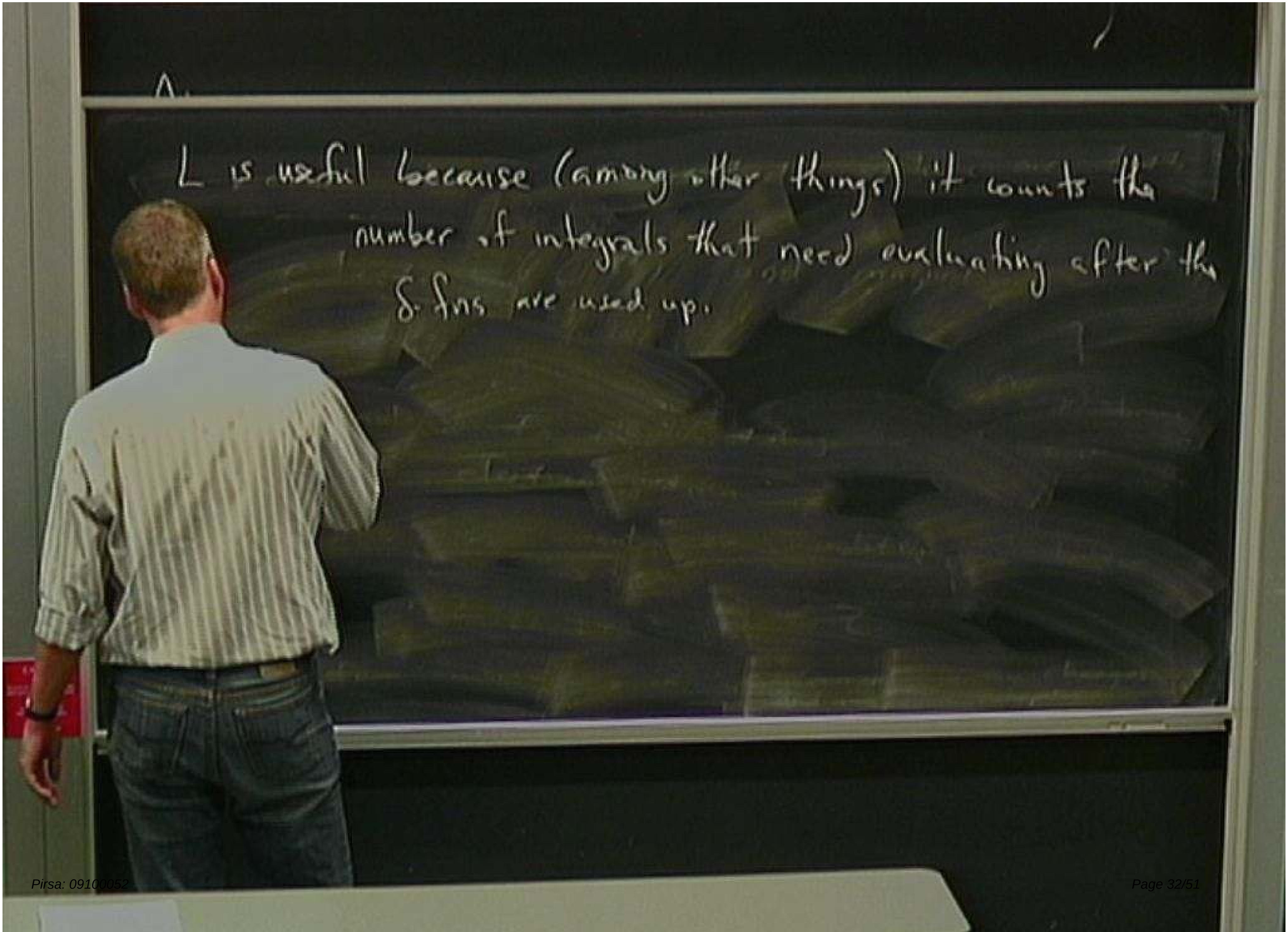
$$L := 1 + I - \sum_{nd} V_{nd}$$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$$\bar{A}_E(q) = \left[\int \frac{d^4p}{(2\pi)^4}\right]^L$$

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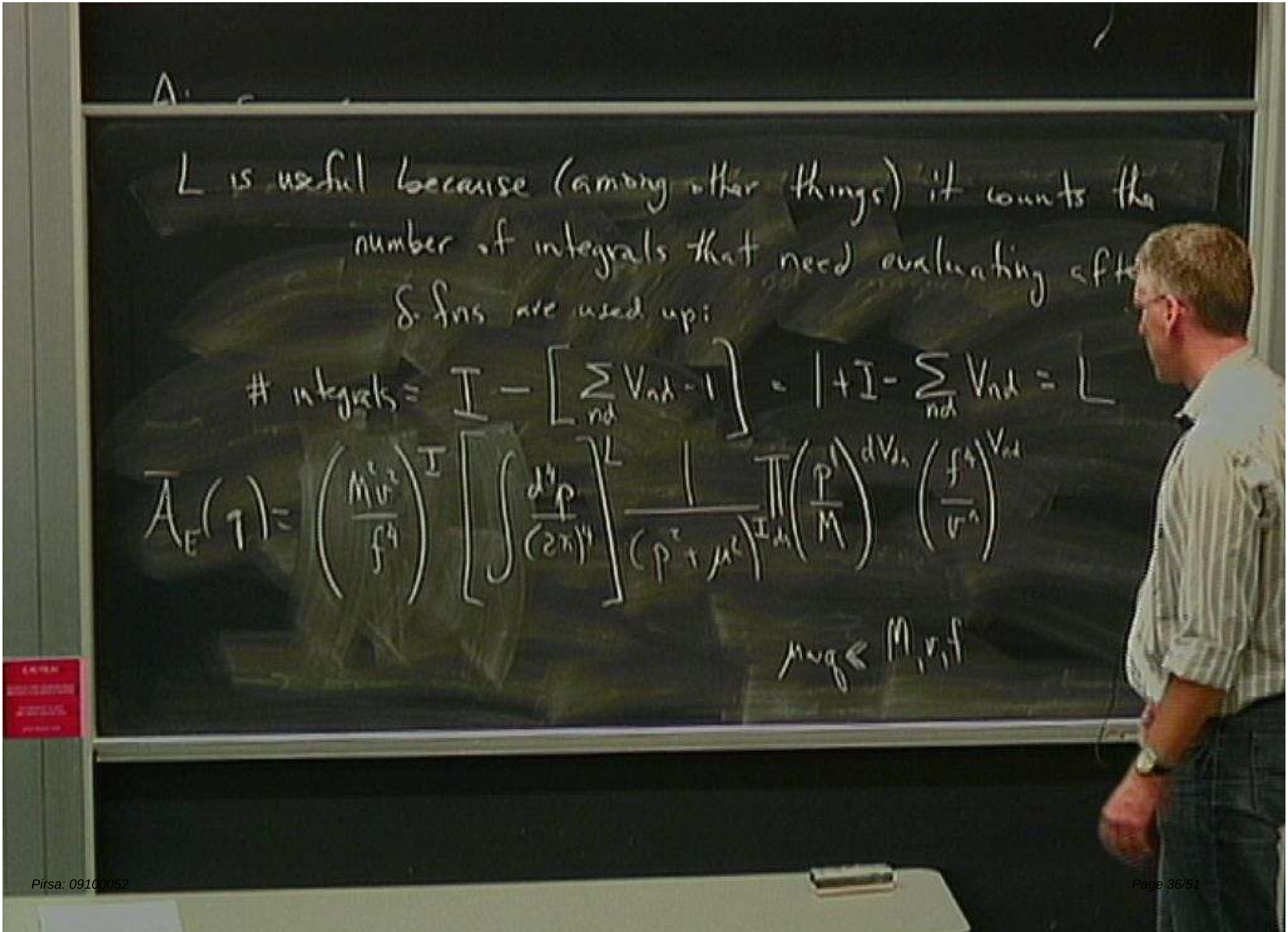
$$\bar{A}_E(q) = \left(\frac{M^2 v^2}{f^4}\right)^{I} \left[\int \frac{d^4p}{(2\pi)^4}\right]^{L} \frac{1}{(p^2+\mu^2)^{I_{nd}}} \prod \left(\frac{p}{M}\right)^{dV_{dn}} \left(\frac{f^4}{v^n}\right)^{V_{nd}}$$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$\# \text{ integrals} = I - \left[\sum_{nd} V_{nd} - 1\right] = 1 + I - \sum_{nd} V_{nd} = L$
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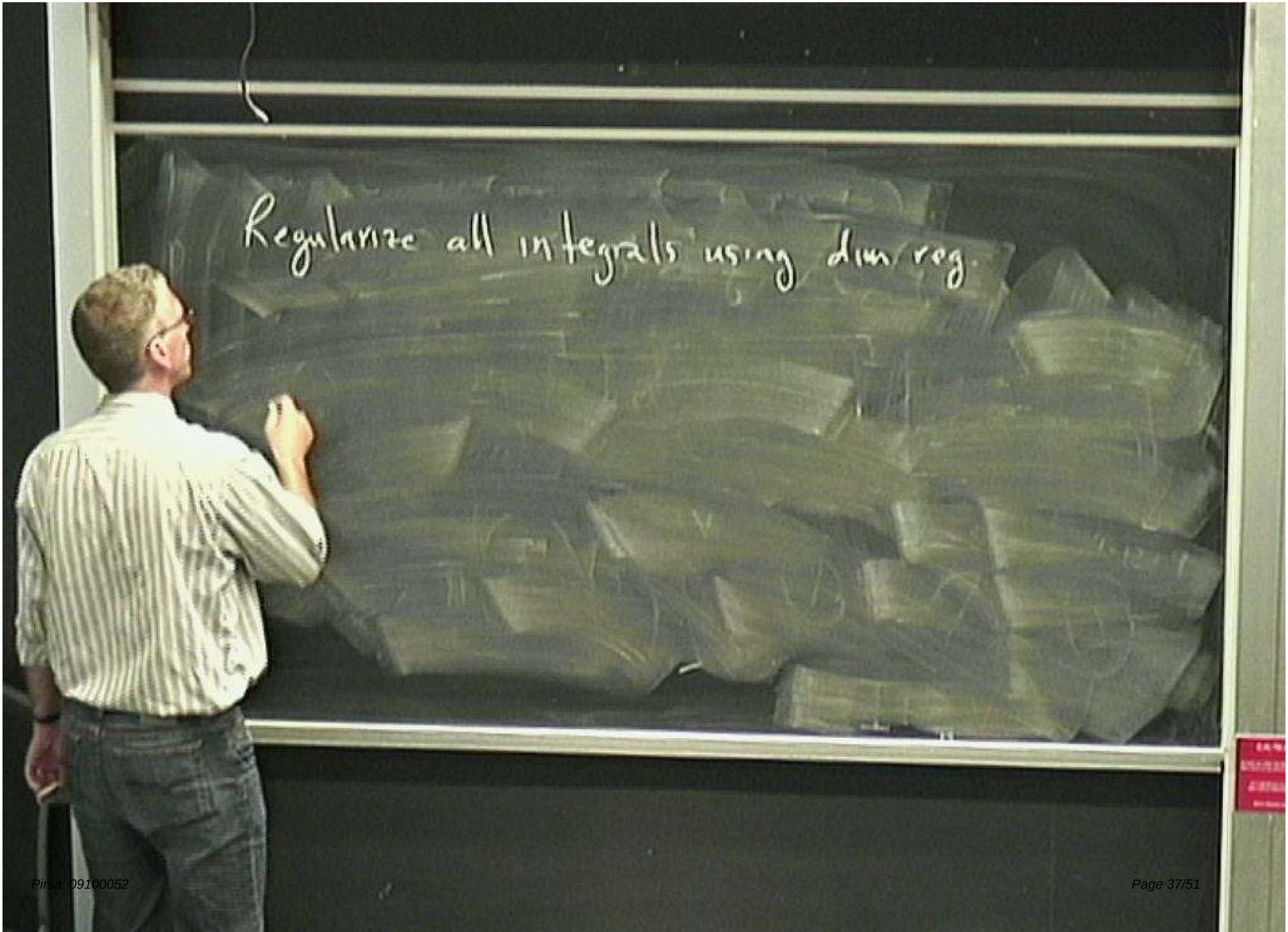
$\mu, q \ll M, v, f$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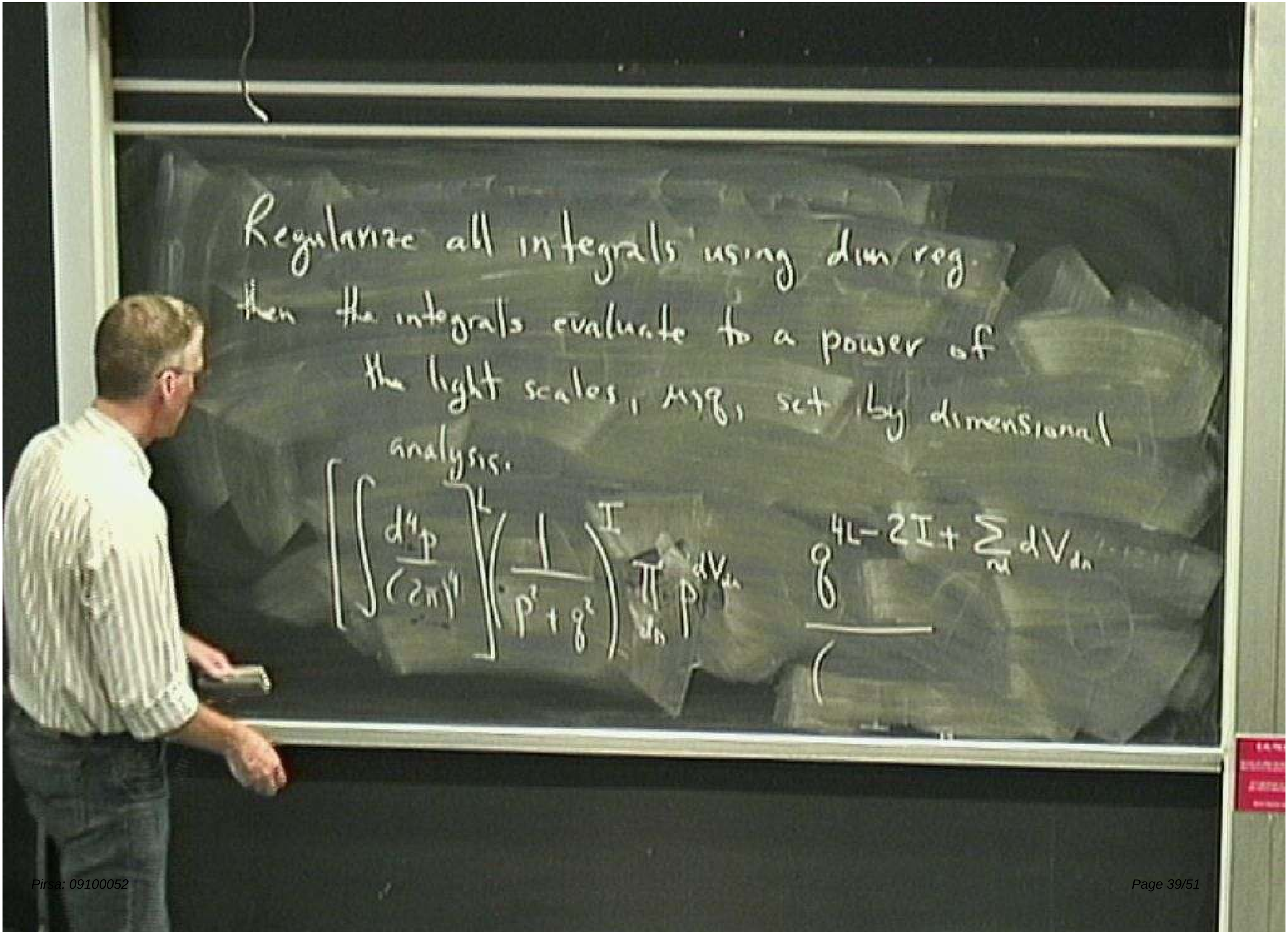
then the integrals evaluate to a power of the light scales, μ, q, set by dimensional analysis.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analysis.
$\left[\int \frac{d^4p}{(2\pi)^4}\right]^L \left(\frac{1}{p^2+q^2}\right)^I \prod_{dn} p^{dV_{dn}}$
$\frac{q^{4L-2I+\sum_n dV_{dn}}}{(}$

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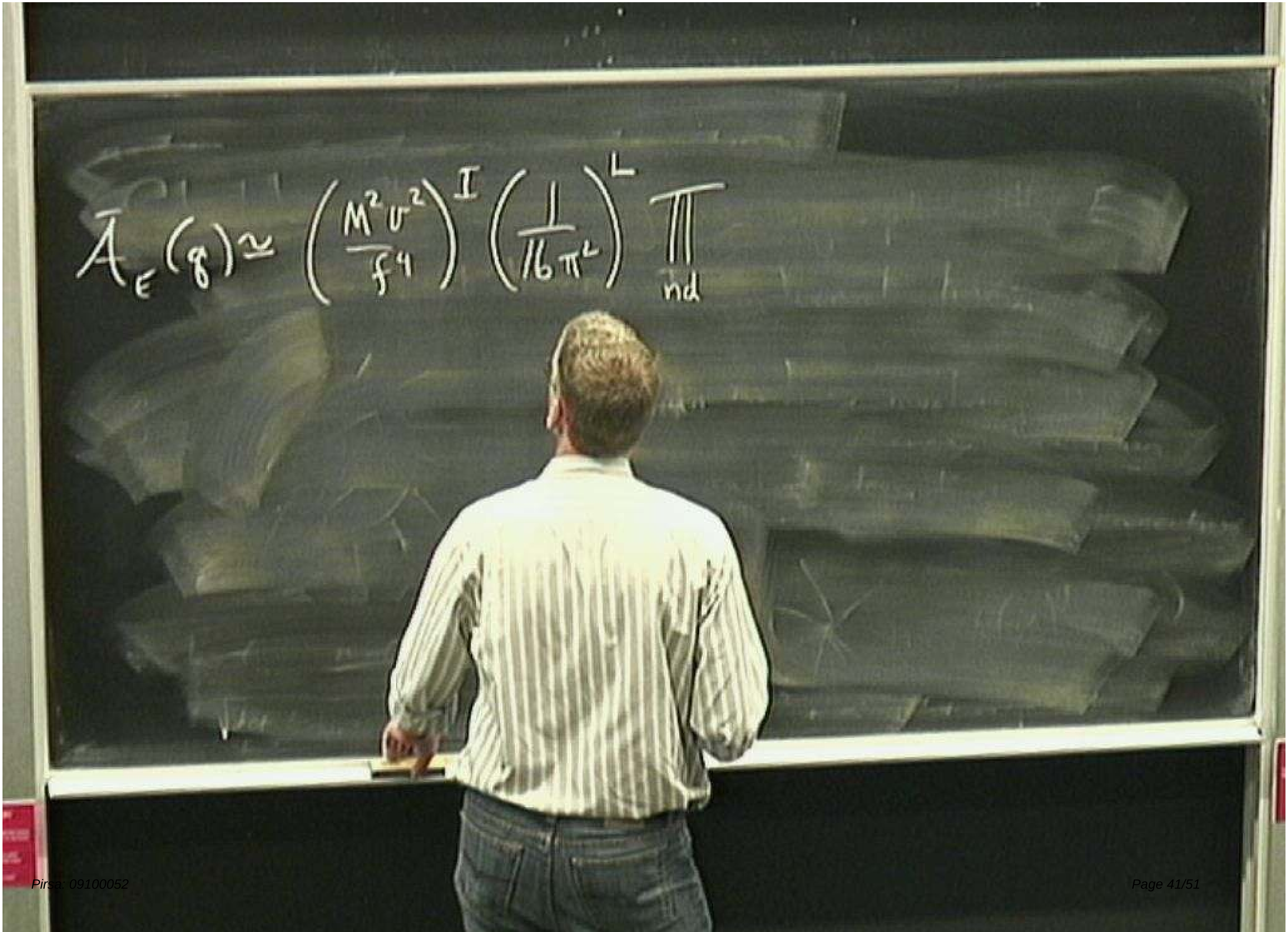
$$\int d^4p\, f(p^2)$$

$$\int p^3 dp\, d\Omega_3\, f(p^2)$$

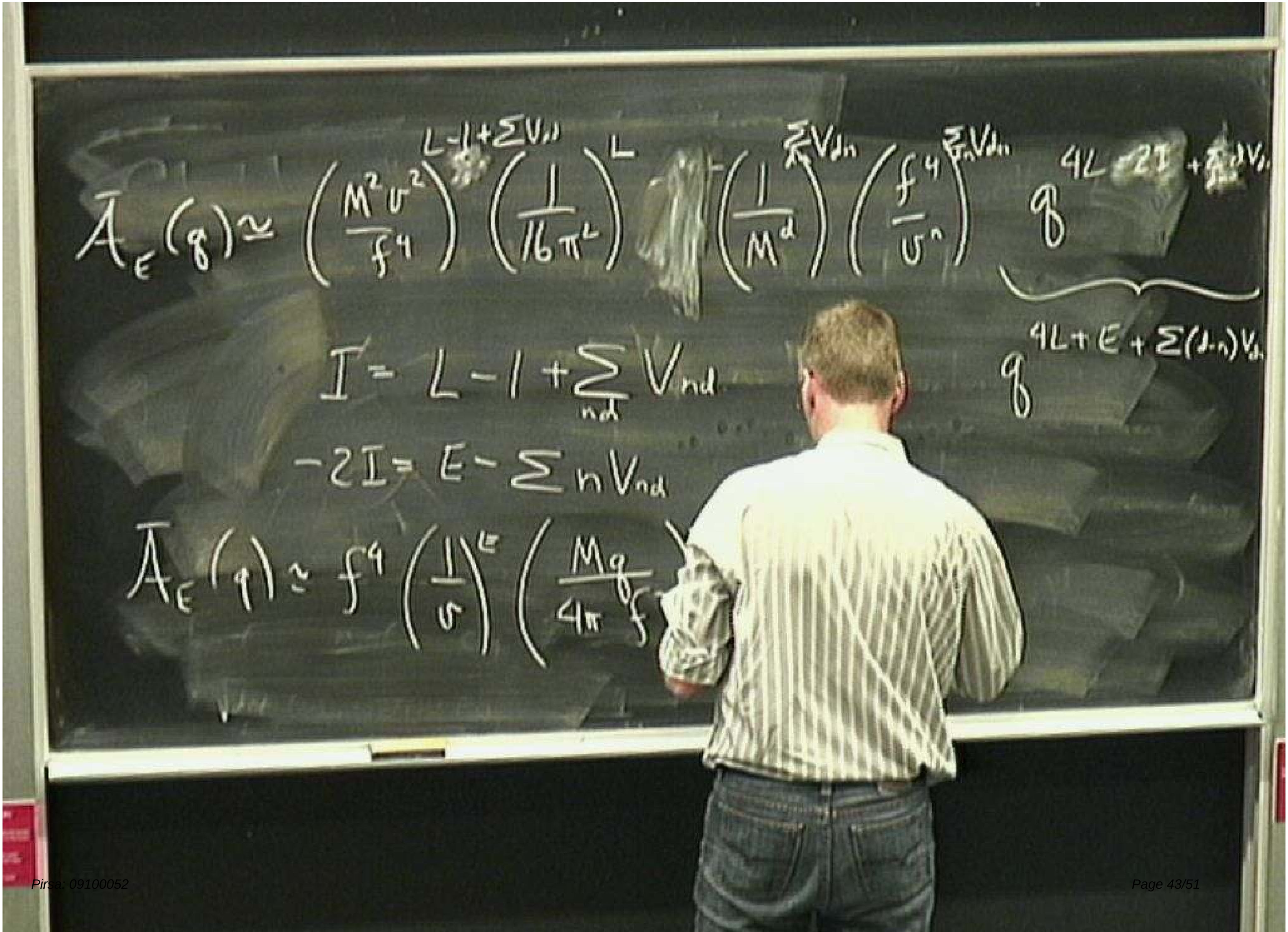
$$2\pi^2$$

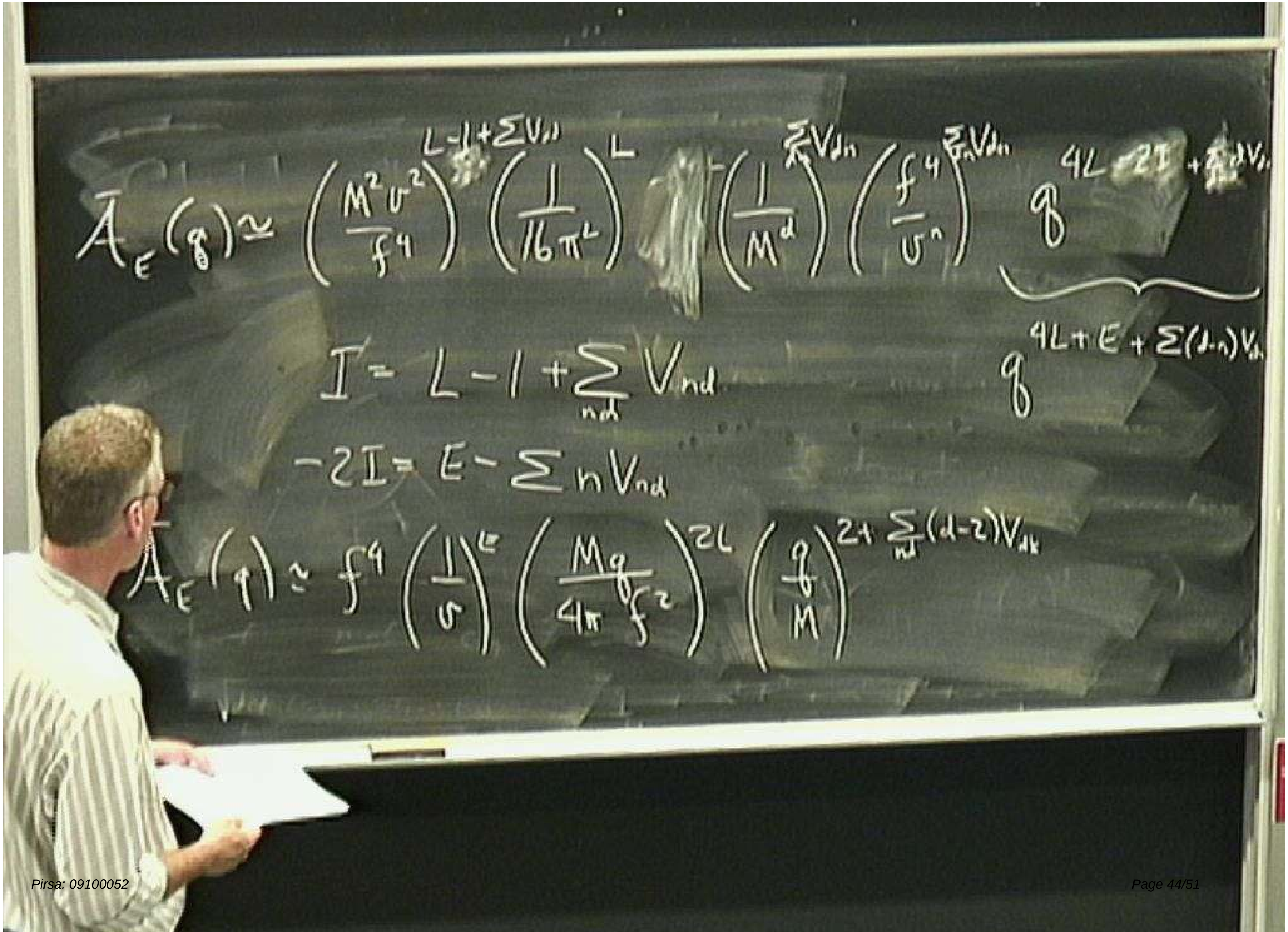
$$\left[\int \frac{d^4p}{(2\pi)^4}\right]^L \left(\frac{1}{p^2+q^2}\right)^I \prod_{dn} p^{dV_{dn}} \left(\frac{\pi^2}{(2\pi)^4}\right)^L q^{4L-2I+\sum_{n} dV_{dn}}$$

$$\left(\frac{\pi^2}{(2\pi)^4}\right) \longrightarrow \left(\frac{1}{16\pi^2}\right)^L$$


$\bar{A}_E(g) \sim \left(\frac{M^2 v^2}{f^4}\right)^I \left(\frac{1}{16\pi^2}\right)^L \prod_{nd}$

$$\bar{A}_{E}(g) \simeq \left(\frac{M^2 v^2}{f^4}\right)^{I} \left(\frac{1}{16\pi^2}\right)^{L} \left(\frac{1}{M^d}\right)^{\sum_{n} V_{dn}} \left(\frac{f^4}{v^n}\right)^{\sum_n V_{dn}} g^{4L-2I+\sum V_i}$$


$\bar{A}_E(q) \sim \left(\frac{M^2 v^2}{f^4}\right)^{L-1+\sum V_{..}} \left(\frac{1}{16\pi^2}\right)^{L} \left(\frac{1}{M^d}\right)^{\sum V_{dn}} \left(\frac{f^4}{v^n}\right)^{\sum_n V_{dn}} q^{4L-2I+\sum d V_{..}}$
$q^{4L+E+\sum(d-n)V_{dn}}$
$I = L-1+\sum_{nd} V_{nd}$
$-2I = E - \sum n V_{nd}$
$\bar{A}_E(q) \simeq f^4 \left(\frac{1}{v}\right)^E \left(\frac{Mq}{4\pi f}\right.$



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$\bar{A}_E(q) \simeq f^4 \left(\frac{1}{v}\right)^{E} \left(\frac{Mq}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{2+\sum_{nd}(d-2)V_{nd}}$

$$\bar{A}_E(g) \simeq \left(\frac{M^2 v^2}{f^4}\right)^{L-1+\sum V_{n}} \left(\frac{1}{16\pi^2}\right)^{L} \left(\frac{1}{M^d}\right)^{\sum_n V_{dn}} \left(\frac{f^4}{v^n}\right)^{\sum_n V_{dn}} \underbrace{g^{4L+2I+\sum_n V}}_{g^{4L+E+\sum(d-n)V_{dn}}}$$

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$$g, \left(\frac{1}{a}\right)^{-L+1-\sum V+\sum_i V}$$

$$I = L - 1 + \sum V$$

$$E = \sum nV - 2I$$

$$= \sum nV - 2L + 2 - 2\sum V$$

$$= 2 - 2L + \sum (n-2)V$$

$$\bar{A}_E(q) \simeq \left(\frac{M^2 v^2}{f^4}\right)^{L-1+\Sigma V_{nd}} \left(\frac{1}{16\pi^2}\right)^L \left(\frac{1}{M^d}\right)^{\Sigma V_{dn}} \left(\frac{f^4}{v^n}\right)^{\Sigma V_{dn}} q^{4L-2I+\Sigma d V_{dn}}$$

$$I = L - 1 + \sum_{nd} V_{nd} \qquad q^{4L+E+\Sigma(d-n)V_{dn}}$$

$$-2I = E - \sum n V_{nd}$$

$$\bar{A}_E(q) \simeq f^4 \left(\frac{1}{v}\right)^E \left(\frac{Mq}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{2+\sum_{nd}(d-2)V_{dn}}$$

$$g_i \left(\frac{1}{u}\right)^{-2+2L-2\Sigma V+\Sigma i V}$$

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Remember $q \ll f, M, v$:

1) Loops are suppressed by $\left(\frac{Mq}{4\pi f^2}\right)$
which is small only if $\frac{Mq}{4\pi f^2} \ll 1$.

2) $\left(\frac{q}{M}\right)^{2+\Sigma(d-2)V}$ $\ll 1$ if $2+\Sigma(d-2)V \geq 0$
because $q \ll M$

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Since $d = 0, 2, 4, \ldots$ by Lorentz invariance, only $d=0$ can produce $P<0$.

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(scalar potential)