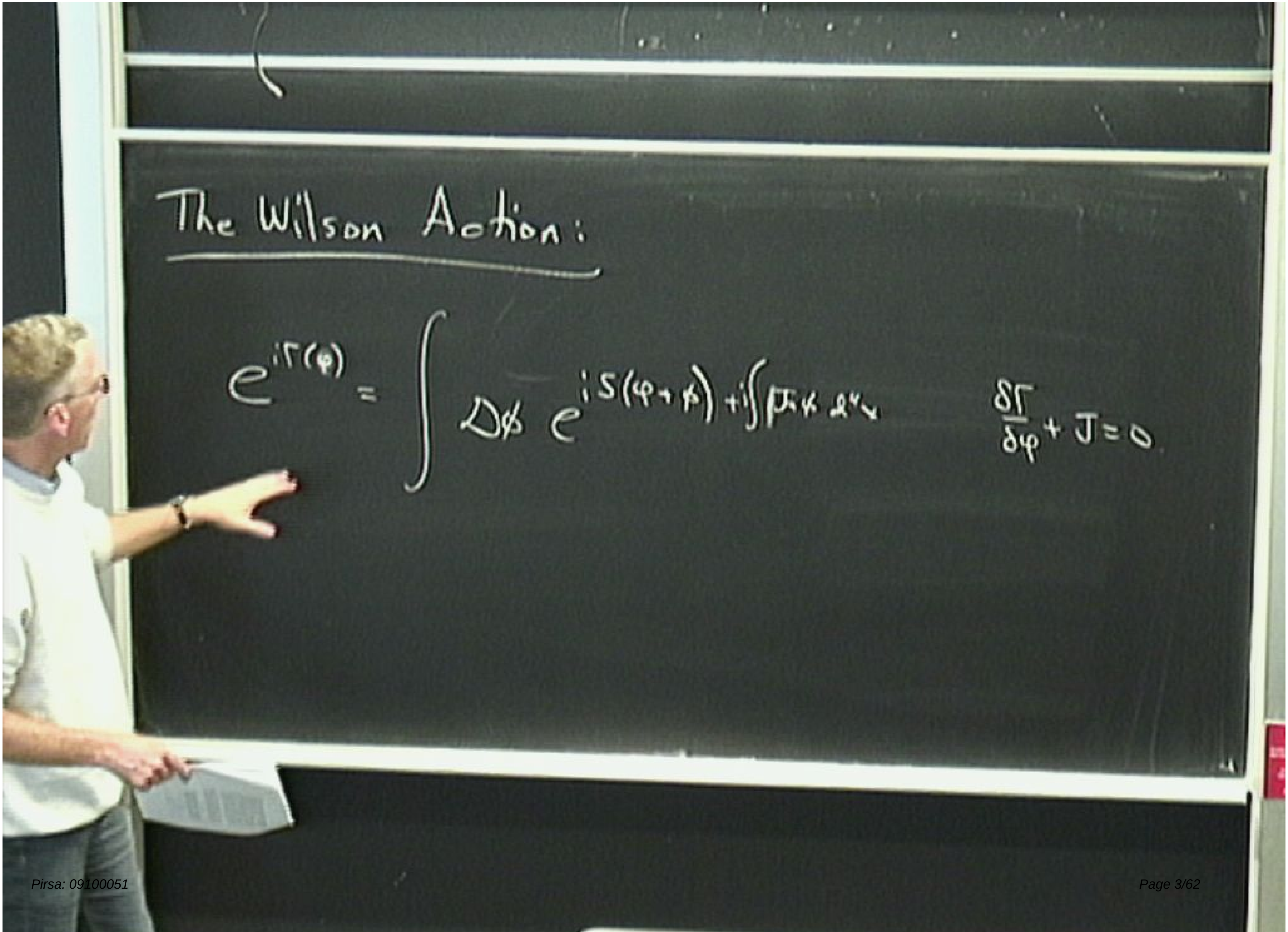
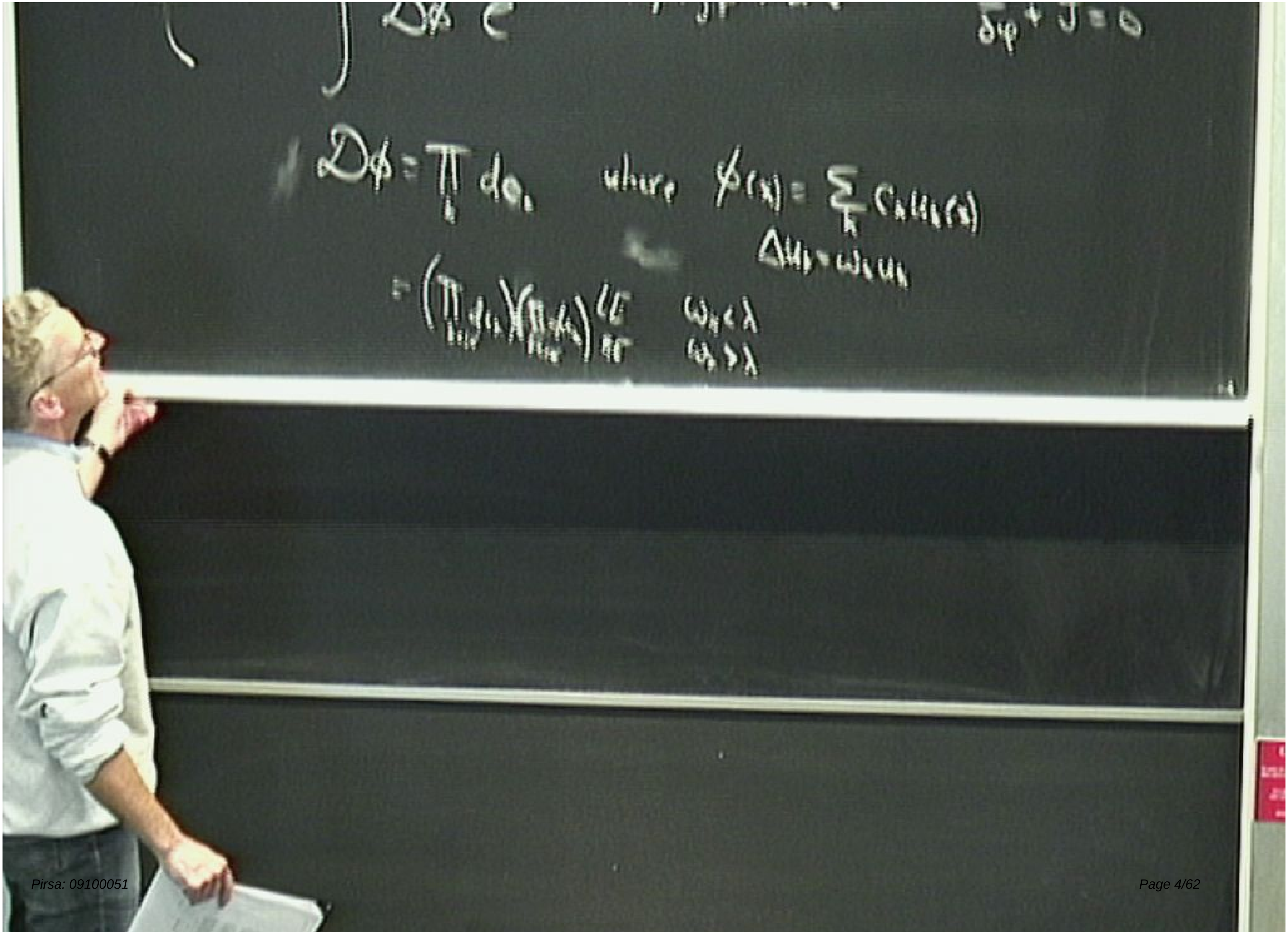
Title: Introduction to Effective Field Theory - Lecture 4A

Date: Oct 14, 2009 09:00 AM

URL: http://www.pirsa.org/09100051

Abstract:


The Wilson Action:
$e^{i\Gamma(\varphi)} = \int \mathcal{D}\phi \, e^{iS(\varphi+\phi) + i\int J\phi \, d^4x}$
$\frac{\delta\Gamma}{\delta\varphi} + J = 0$


Dφ = ∏_k dc_k   where   φ(x) = Σ_k c_k u_k(x)
Δu_k = ω_k u_k
= (∏ dc_k)(∏ dc_k)
ω_k < λ
ω_k > λ

If we choose $J(x) = \sum_k b_k u_k(x)$ and $b_k = 0$ if k satisfies $\omega_k > \lambda$

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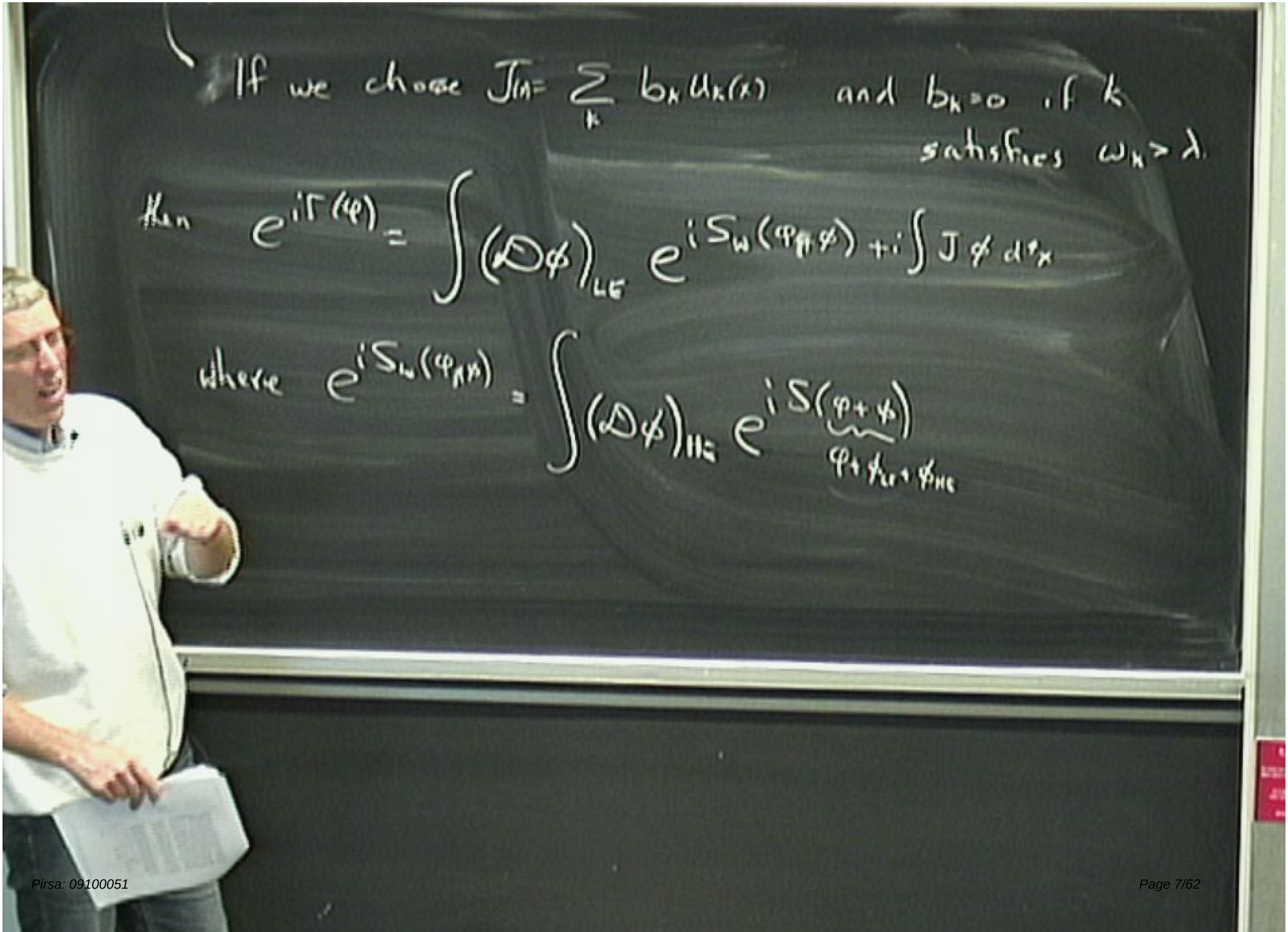
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defines the Wilson action.

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Aside: Exact Renormalization Group:

"Renormalization and Effective Lagrangians", J. Polchinski

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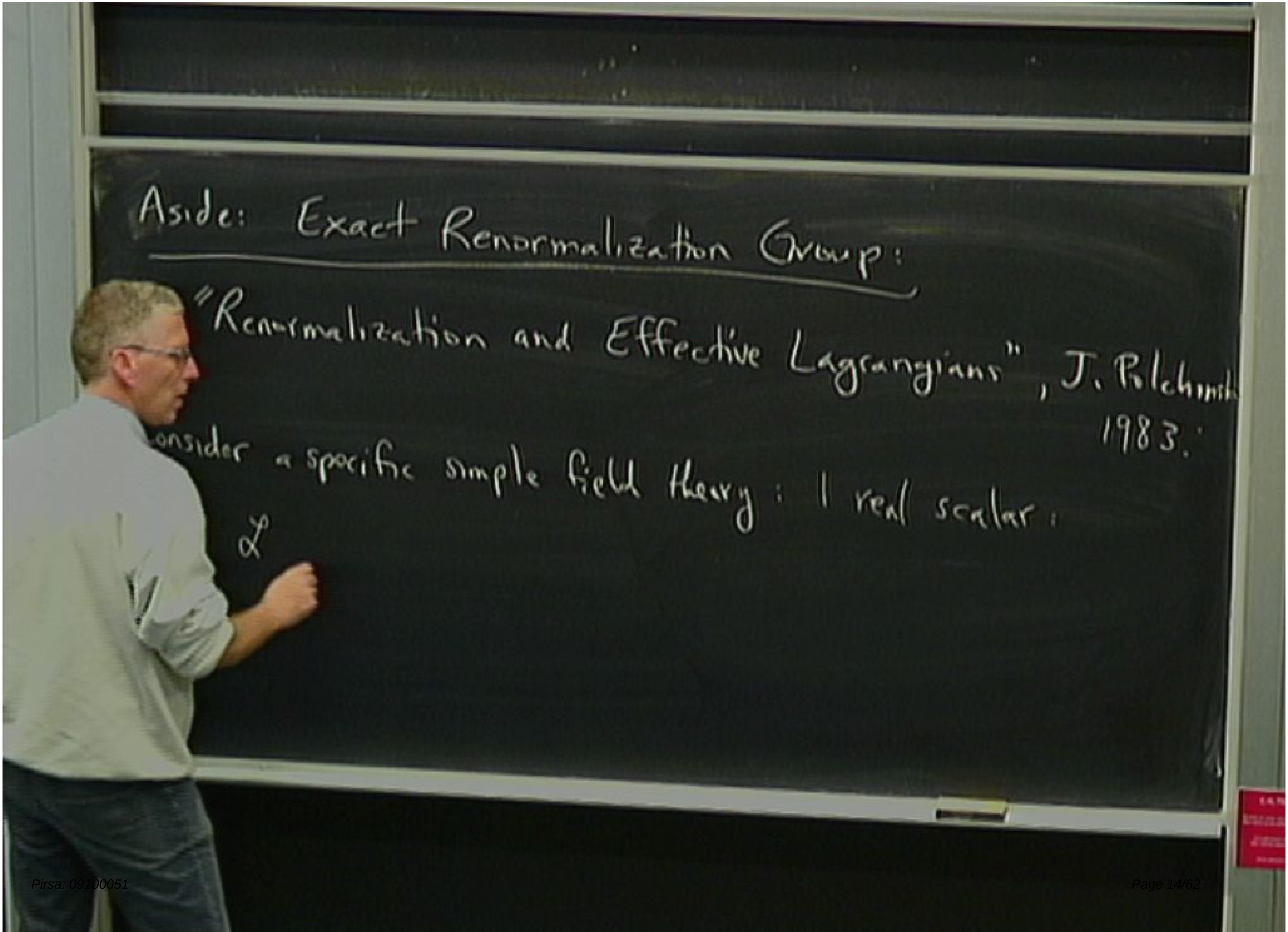
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$\mathcal{L} =$

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$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m_0^2\phi^2 - \frac{1}{4!}\lambda_0\phi^4$$

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$$S = \int d^4x\,\mathcal{L} = \int d$$

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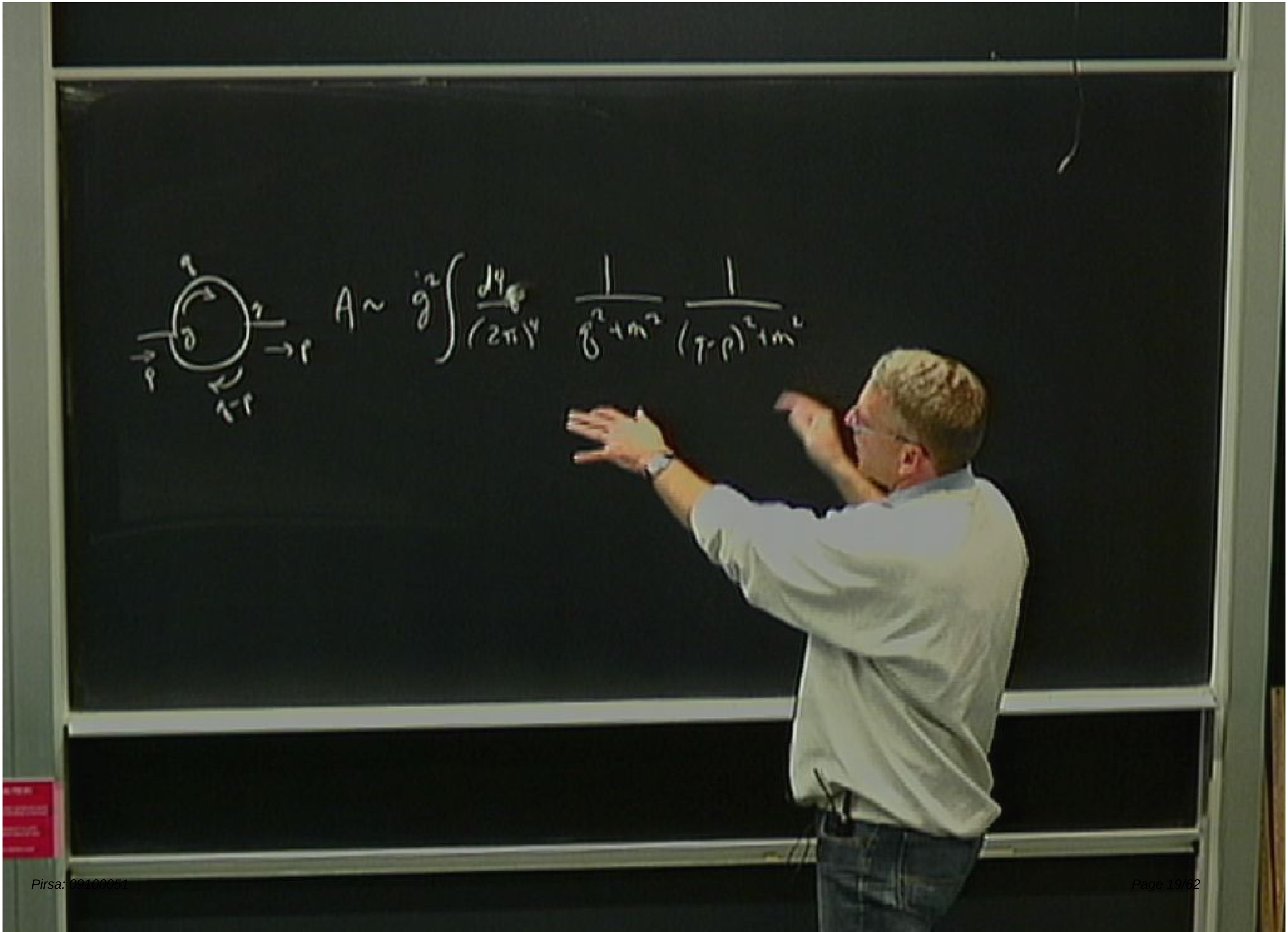
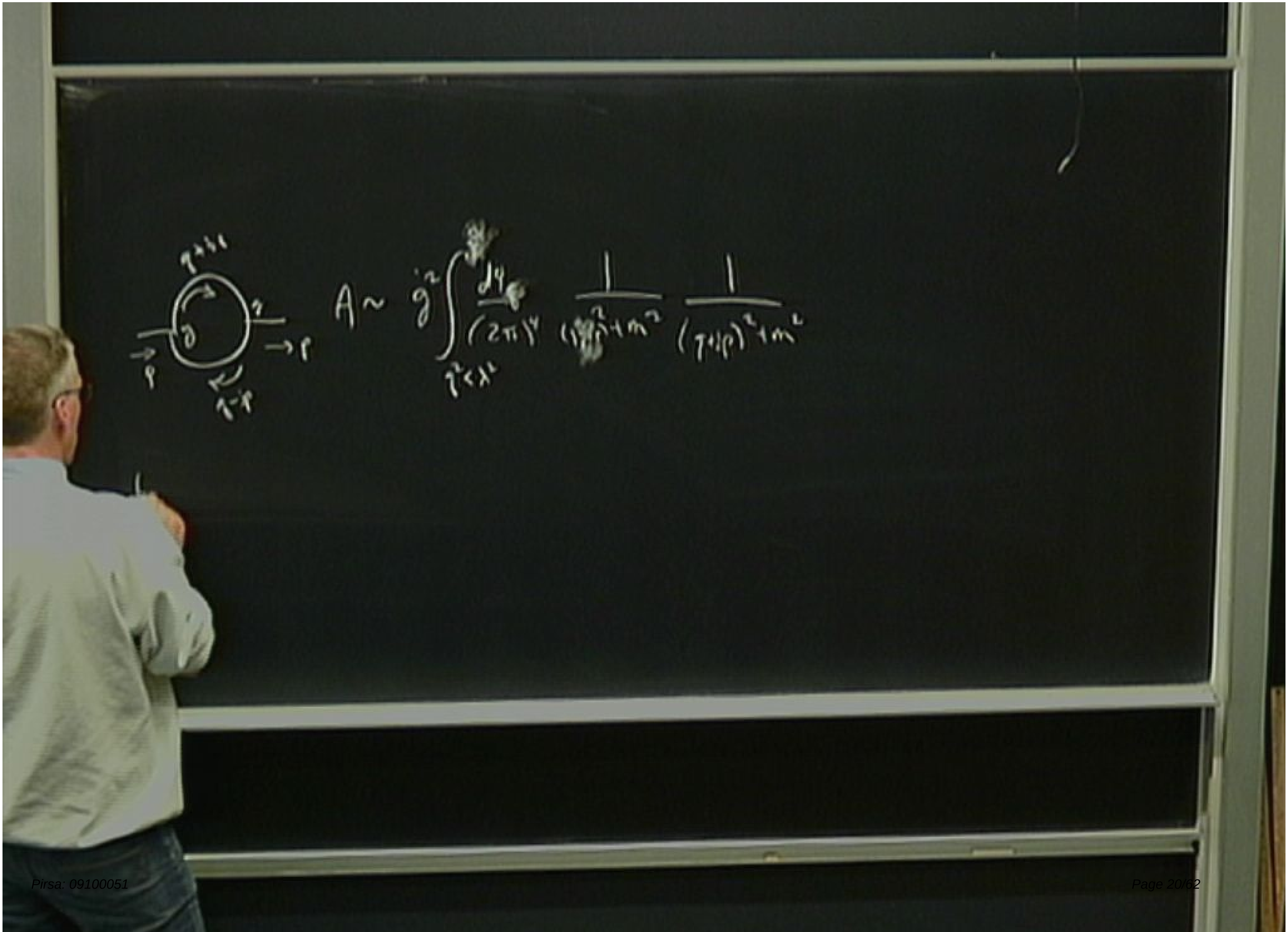
$$S = \int d^4x\,\mathcal{L} = \int \frac{d^4p}{(2\pi)^4}\left[\frac{1}{2}\phi(p)\phi(-p)(p^2+m_0^2)\right] + S_{int}$$

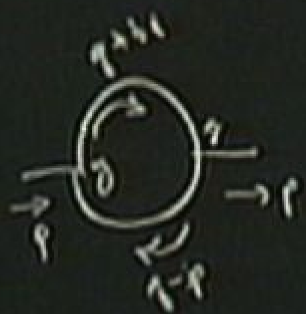
$$\phi(x) = \int \frac{d^4p}{(2\pi)^4}e^{ipx}\phi(p)$$



$A \sim g^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2+m^2} \frac{1}{(q-p)^2+m^2}$



A ~ g² ∫ d⁴q/(2π)⁴ 1/(q²+m²) 1/((q+p)²+m²)
q²<Λ²



$$A \sim g^2 \int_{q^2<\Lambda^2} \frac{d^4q}{(2\pi)^4} \frac{1}{(q)^2+m^2} \frac{1}{(q+p)^2+m^2}$$

Warning: answers will depend on how you route momentum through the graph "routing dependence"

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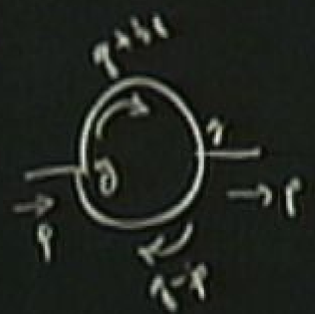
$$S = \int d^4x\,\mathcal{L} = \int \frac{d^4p}{(2\pi)^4}\Big[\frac{1}{2}\phi(p)\phi(-p)\underbrace{(p^2+m_0^2)}_{K(p^2/\lambda^2)}\Big] + S_{int}$$

$$\phi(x) = \int \frac{d^4p}{(2\pi)^4} e^{ipx}\ldots$$

$K = 0 \quad p^2 \gg \lambda^2$

$K = 1 \quad p^2 \ll \lambda^2$



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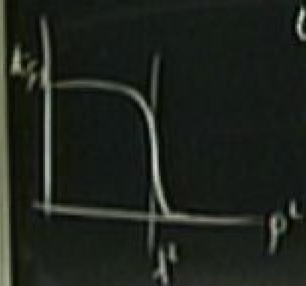

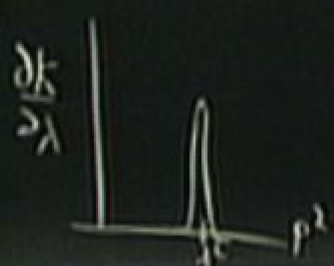

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$$S_H = \int d^4x\,\mathcal{L}_H = \int \frac{d^4p}{(2\pi)^4}\left[\frac{1}{2}\phi(p)\,\phi(-p)\,\frac{(p^2+m_0^2)}{K(p^2/\lambda^2)}\right] + S_{int}$$

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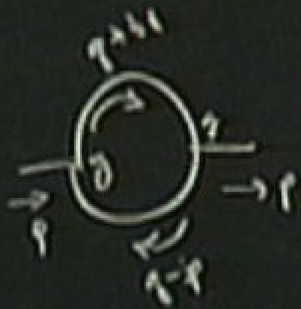

$$A \sim g^2 \int \frac{d^4q}{(2\pi)^4} \frac{K(q-\frac{1}{2}p)}{(q-\frac{1}{2}p)^2+m^2} \frac{K(q+\frac{1}{2}p)}{(q+\frac{1}{2}p)^2+m^2}$$

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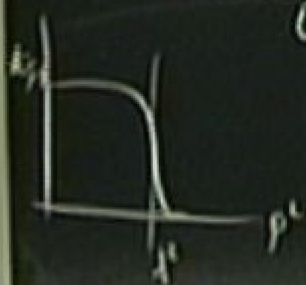
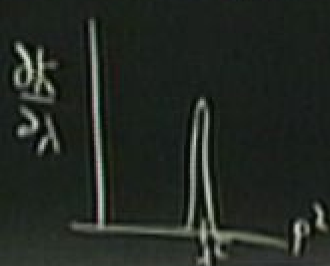
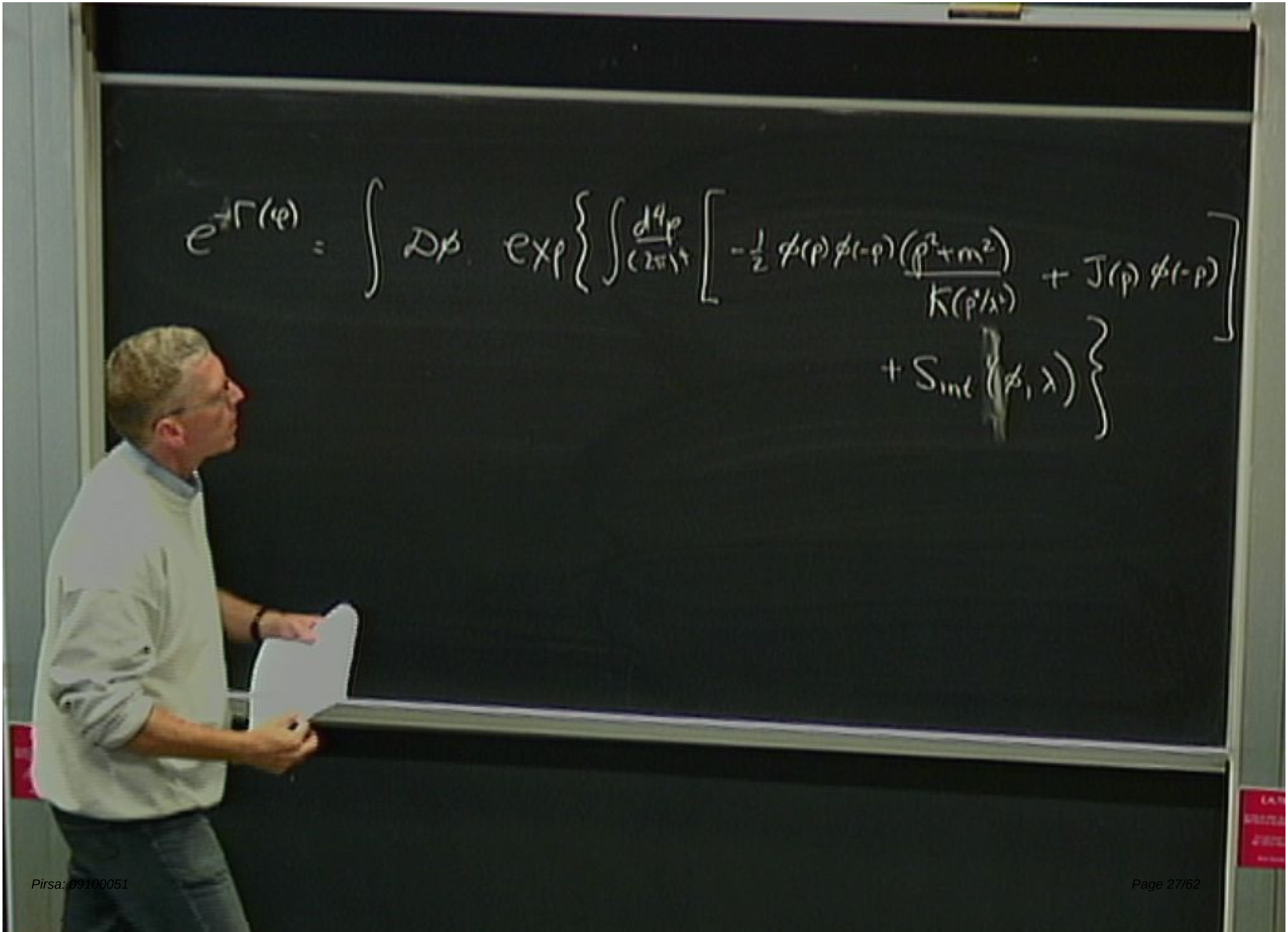




$$e^{-\Gamma(\varphi)} = \int \mathcal{D}\phi \; \exp\left\{ \int \frac{d^4p}{(2\pi)^4} \left[-\frac{1}{2}\phi(p)\phi(-p)\frac{(p^2+m^2)}{K(p^2/\Lambda^2)} + J(p)\phi(-p) \right.\right.$$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$$Z = e^{-\Gamma(\varphi)} = \int \mathcal{D}\phi \; \overbrace{\exp\left\{ \int \frac{d^4p}{(2\pi)^4} \left[-\frac{1}{2}\phi(p)\phi(-p)\frac{(p^2+m^2)}{K(p^2/\lambda^2)} + J(p)\phi(-p) \right] + S_{int}(\phi,\lambda) \right\}}^{e^{S}}$$

$$\lambda \frac{\partial Z}{\partial \lambda} = \int \mathcal{D}\phi \left\{ \int \frac{d^4p}{(2\pi)^4} \left[-\frac{1}{2}\phi(p)\phi(-p)(p^2+m^2)\,\lambda \frac{\partial K^{-1}}{\partial \lambda} \right] + \lambda \frac{\partial S_{int}}{\partial \lambda} \right\} e^{+S}$$

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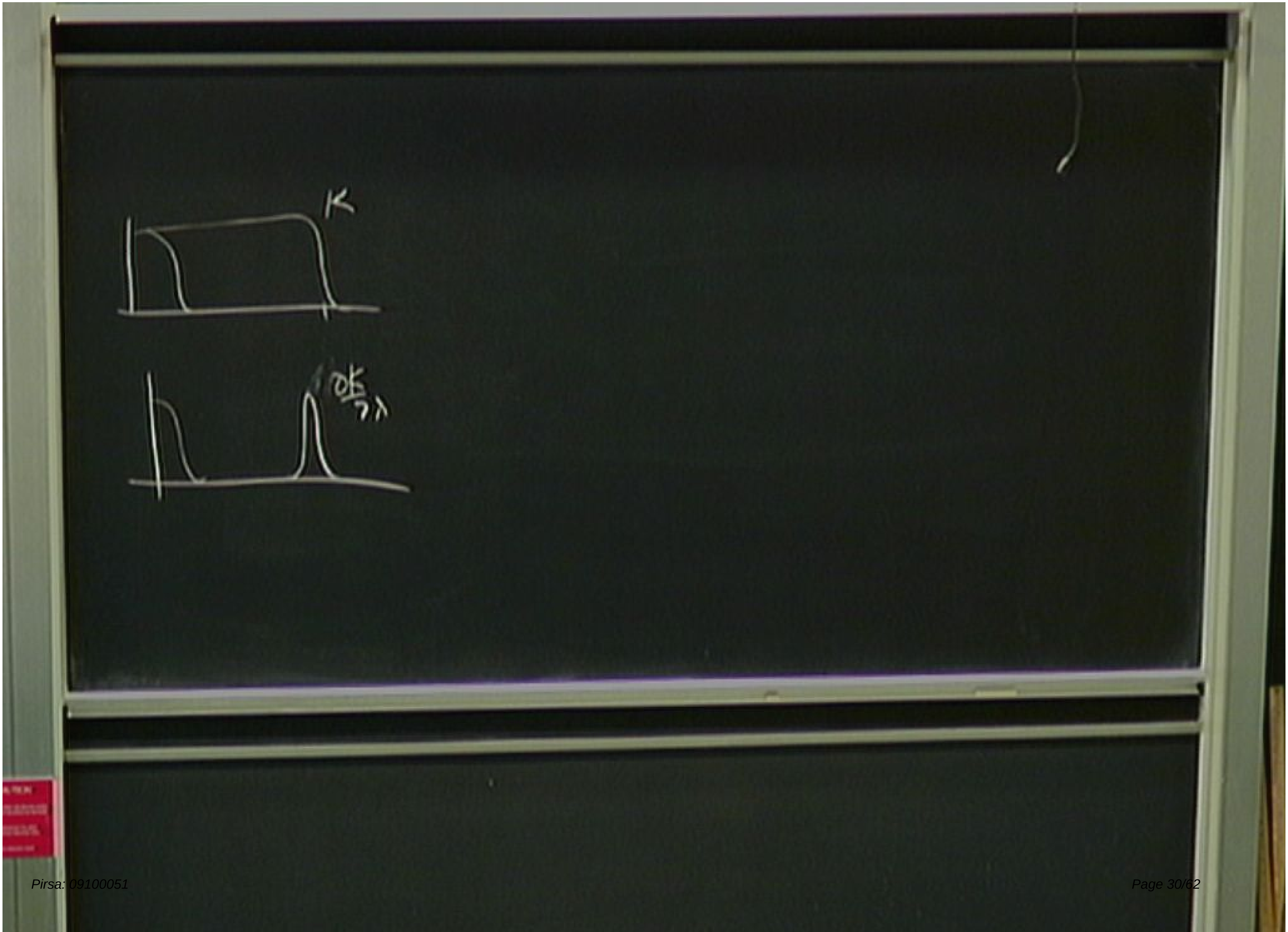
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$$Z = e^{-W(J)} = \int \mathcal{D}\phi \, \exp\Big\{ \overbrace{\int \frac{d^4p}{(2\pi)^4} \Big[-\tfrac{1}{2}\phi(p)\phi(-p)\frac{(p^2+m^2)}{K(p^2/\lambda^2)} + J(p)\phi(-p) \Big] + S_{int}(\phi, \lambda)}^{:= e^S} \Big\}$$

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To show $\lambda \frac{\partial Z}{\partial \lambda} = 0$ use the identity $\int \mathcal{D}\phi \frac{\delta}{\delta\phi} F(\phi) = 0$

$$:= e^{S}$$

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Choose: $$\lambda \frac{\partial S_{int}}{\partial \lambda} = -\frac{1}{2}\int d^4p \,(2\pi)^4 \left(p^2+m^2\right)^{-1} \lambda \frac{\partial K}{\partial \lambda} \left\{ \frac{\delta S_{int}}{\delta \phi(p)} \frac{\delta S_{int}}{\delta \phi(-p)} + \frac{\delta^2 S_{int}}{\delta \phi(p)\, \delta \phi(-p)} \right\}$$

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then:

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then:

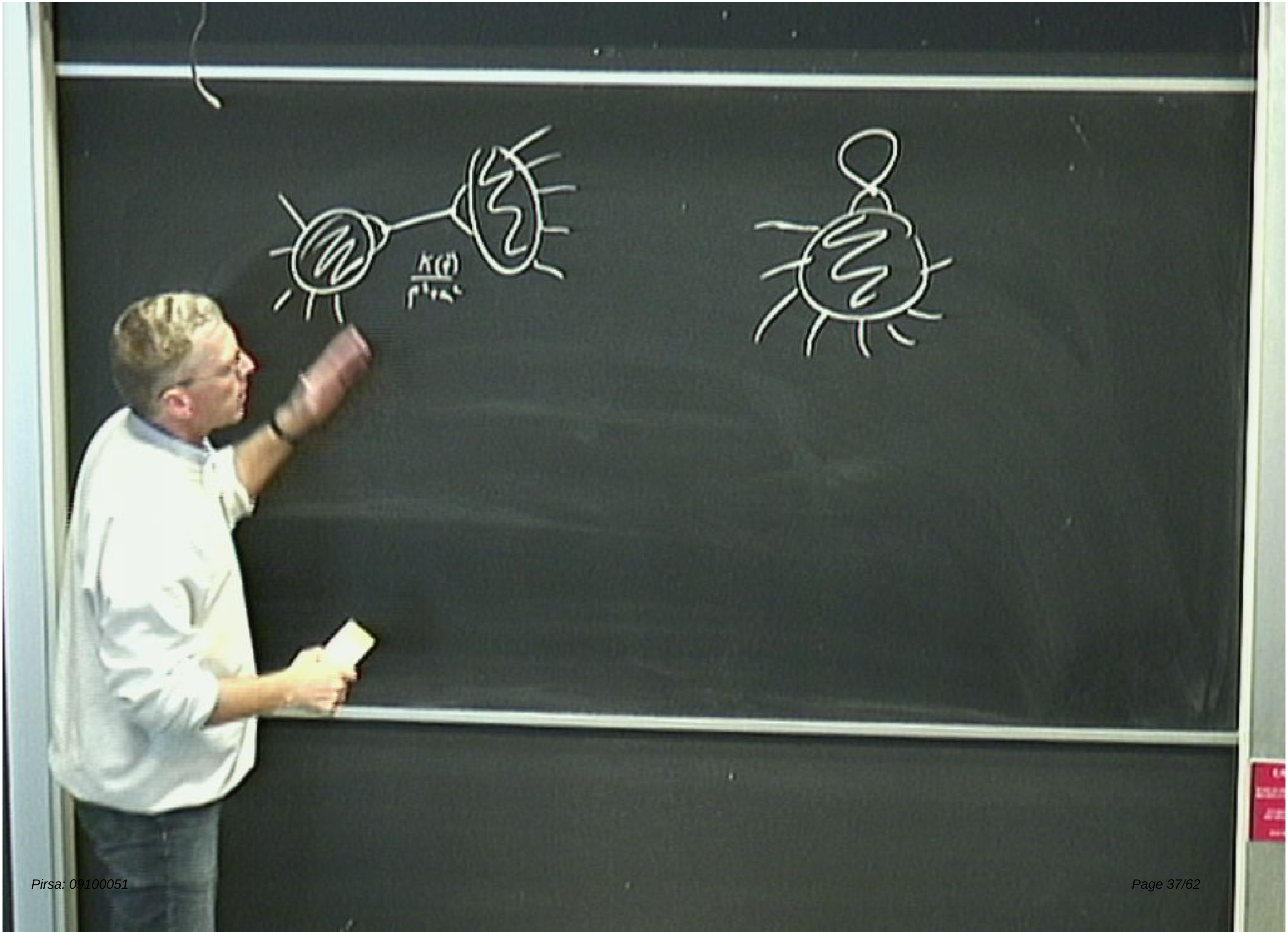
$$\lambda \frac{\partial Z}{\partial \lambda} = \int d^4p\, \lambda \frac{\partial K}{\partial \lambda} \int \mathcal{D}\phi\, \frac{\delta}{\delta\phi(p)} \left\{ \left[\phi(p)\, K^{-1} + \frac{(2\pi)^4}{2} \left(p^2+m^2\right)^{-1} \frac{\delta}{\delta\phi(-p)} \right] e^{S} \right\}$$

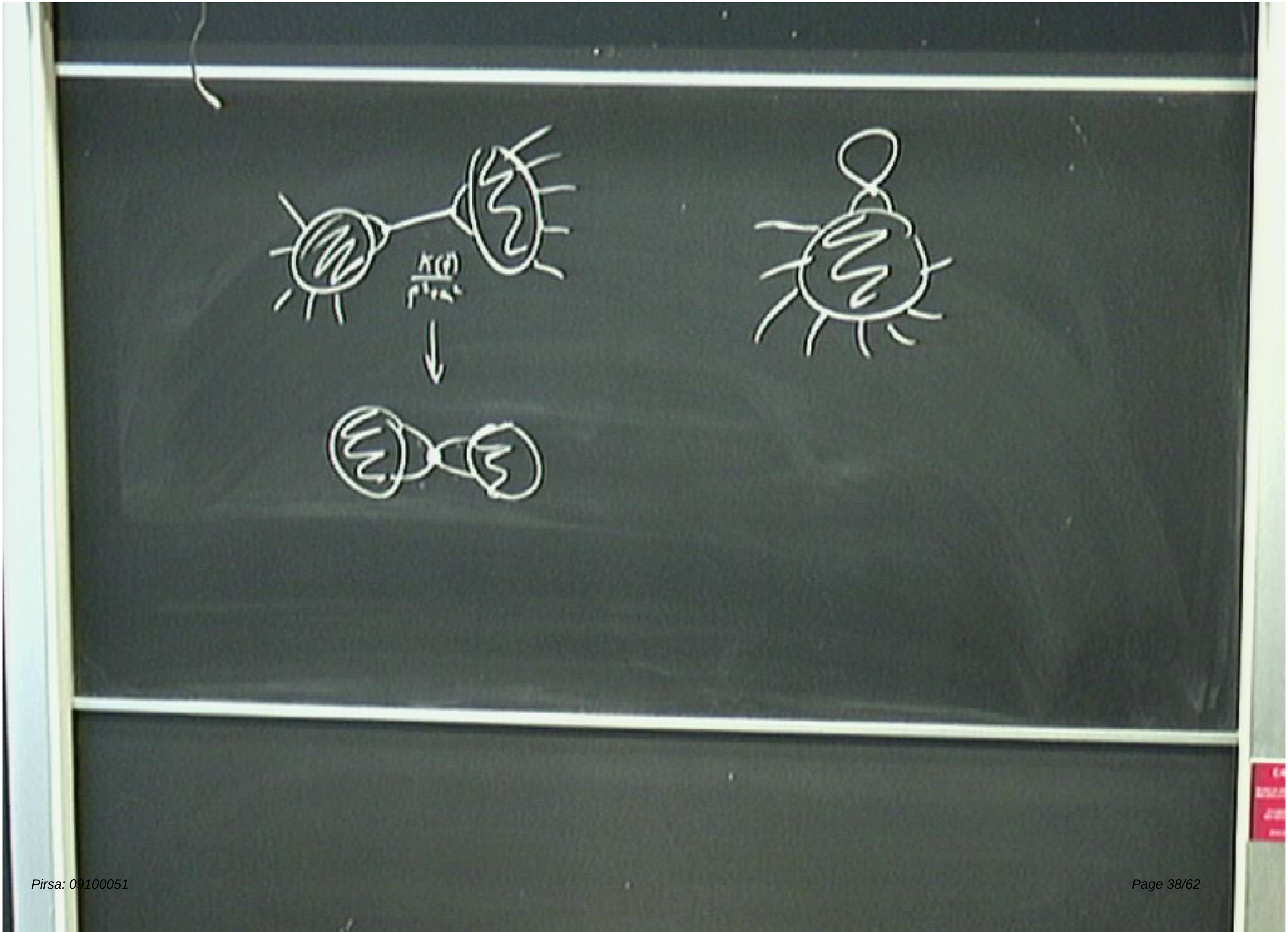
Choose: $\lambda \dfrac{\partial S_{int}}{\partial \lambda} = -\frac{1}{2}\int d^4p \, (2\pi)^4 \left(p^2+m^2\right)^{-1} \lambda \dfrac{\partial K}{\partial \lambda} \left\{ \dfrac{\delta S_{int}}{\delta \phi(p)} \dfrac{\delta S_{int}}{\delta \phi(-p)} + \dfrac{\delta^2 S_{int}}{\delta \phi(p)\, \delta \phi(-p)} \right\}$

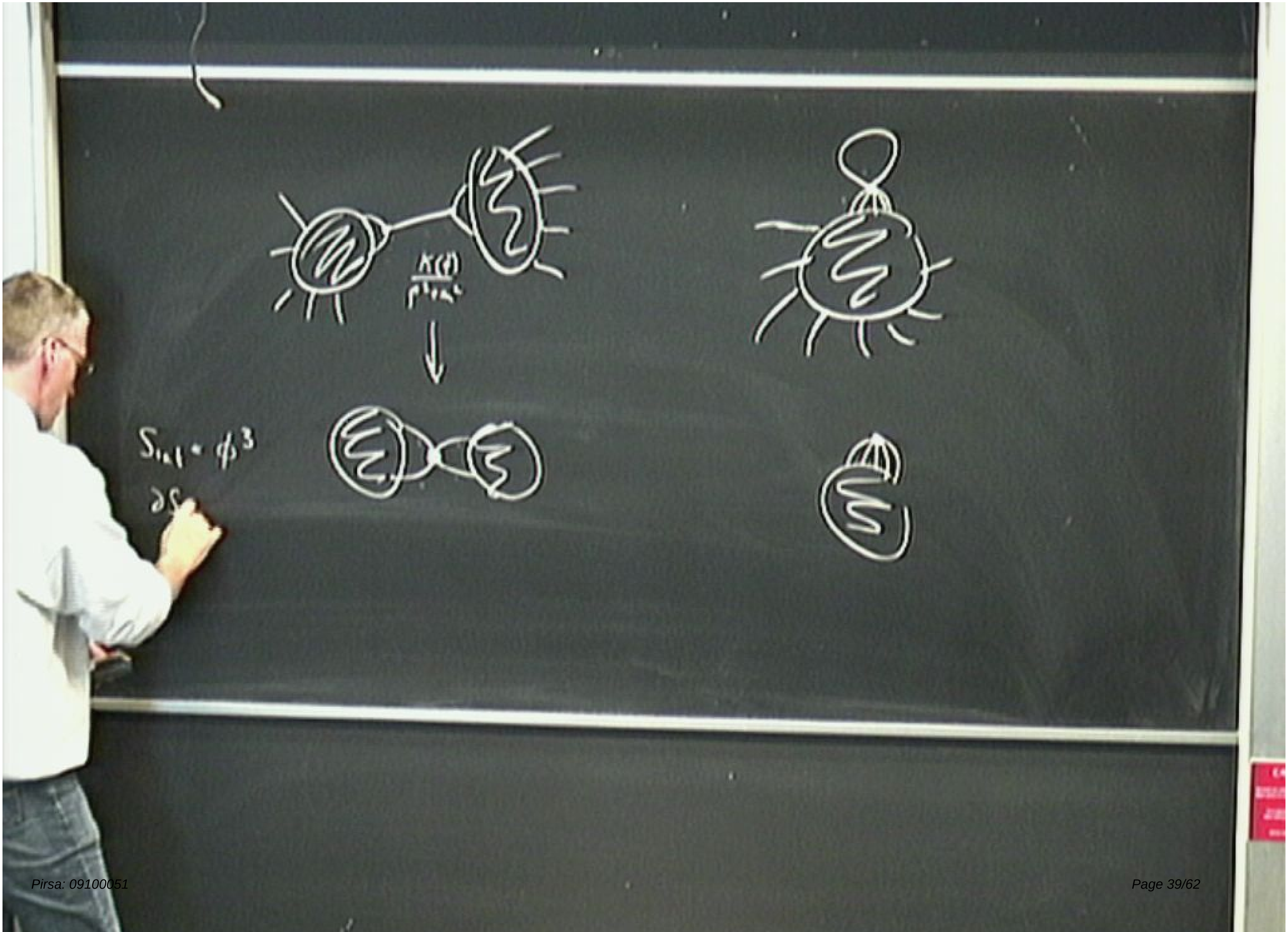
then:

$$\lambda \frac{\partial Z}{\partial \lambda} = \int d^4p \, \lambda \frac{\partial K}{\partial \lambda} \int \mathcal{D}\phi \, \frac{\delta}{\delta \phi(p)} \left\{ \left[\phi(p) K^{-1} + \frac{(2\pi)^4}{2} \left(p^2+m^2\right)^{-1} \frac{\delta}{\delta \phi(-p)} \right] e^S \right\}$$

$$\frac{\delta}{\delta \phi} e^S = \frac{\delta S}{\delta \phi} e^S \qquad \frac{\delta^2}{\delta \phi^2} e^S = \left(\frac{\delta S}{\delta \phi} \frac{\delta S}{\delta \phi} + \frac{\delta^2 S}{\delta \phi^2} \right) e^S$$







Choose: $\lambda \frac{\partial S_{int}}{\partial \lambda} = -\frac{1}{2}\int d^4p\,(2\pi)^4\,(p^2+m^2)^{-1}\,\lambda\frac{\partial K}{\partial\lambda}\left\{\frac{\delta S_{int}}{\delta\phi(p)}\frac{\delta S_{int}}{\delta\phi(-p)} + \frac{\delta^2 S_{int}}{\delta\phi(p)\,\delta\phi(-p)}\right\}$

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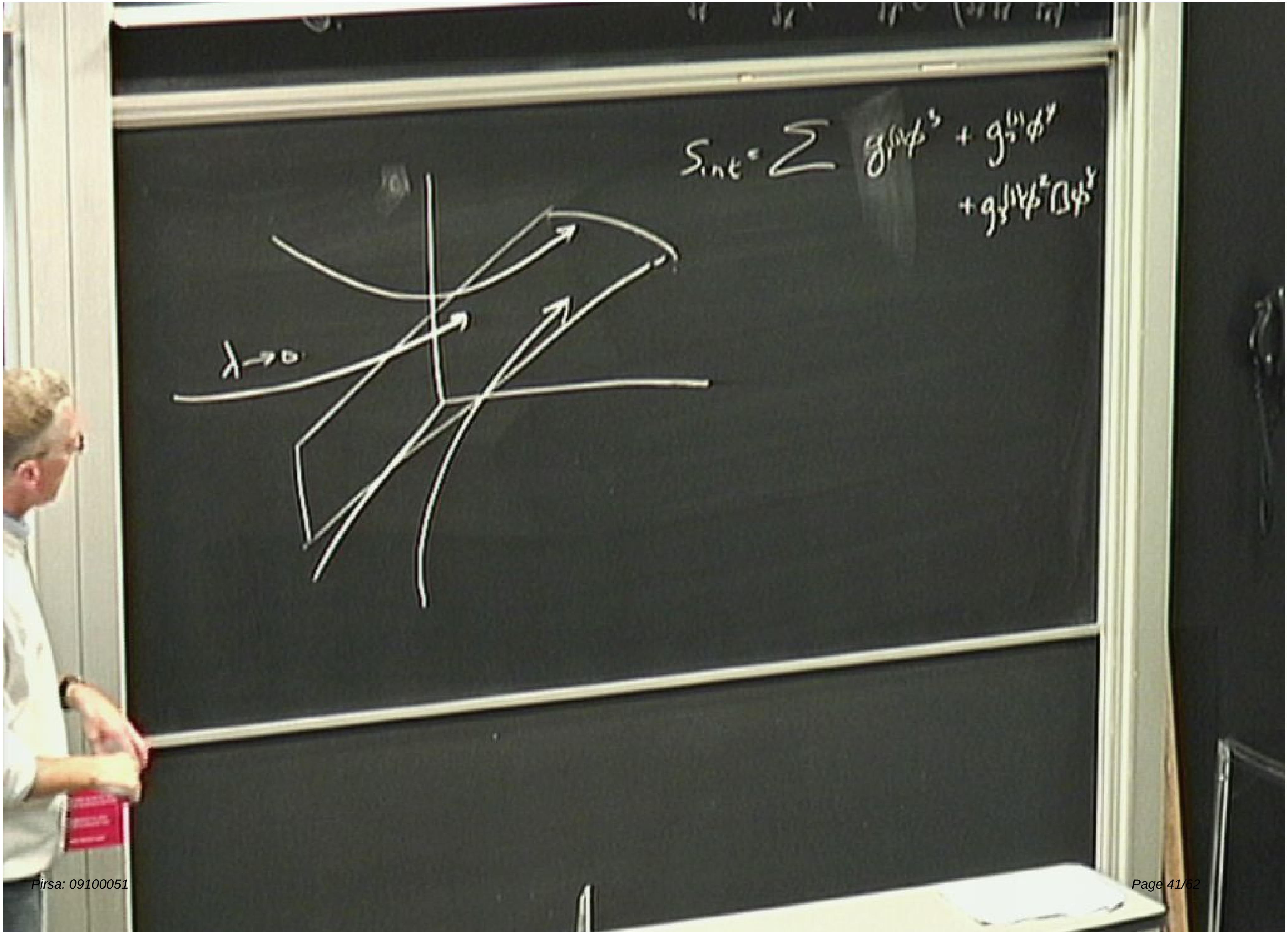
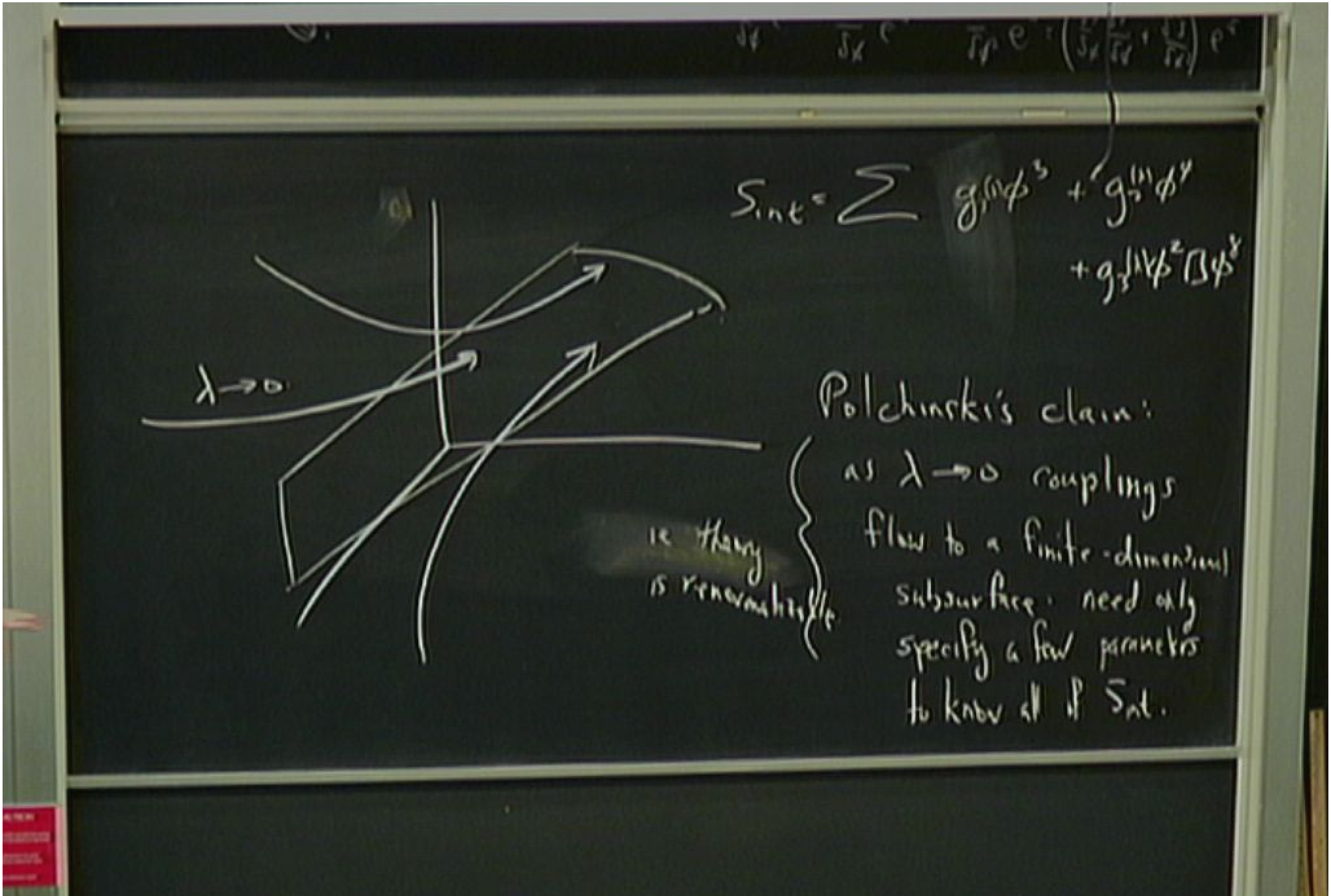
then:

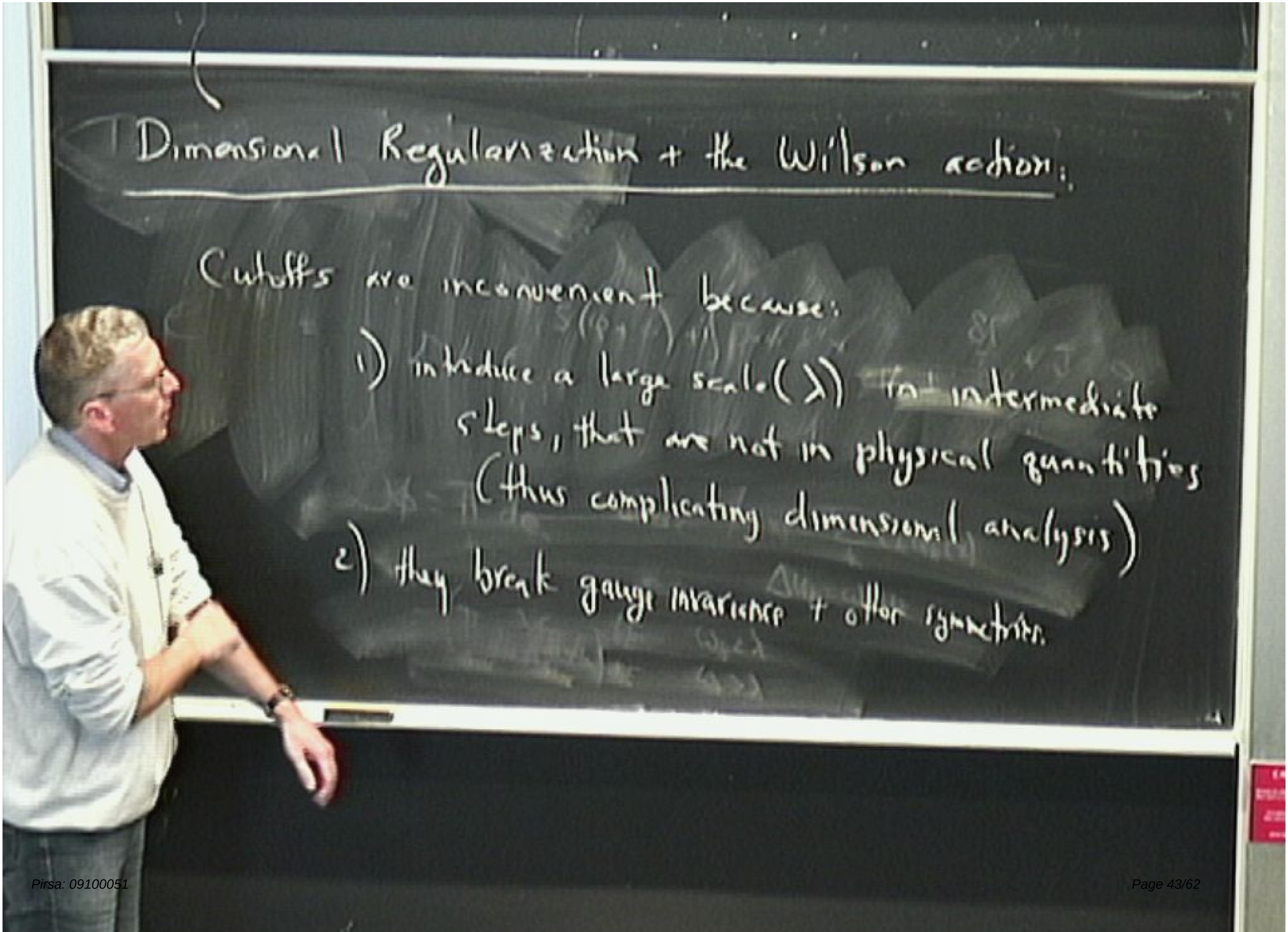
$$\lambda\frac{\partial Z}{\partial\lambda} = \int d^4p\;\lambda\frac{\partial K}{\partial\lambda}\int \mathcal{D}\phi\;\frac{\delta}{\delta\phi(p)}\left\{\left[\phi(p)K^{-1} + \frac{(2\pi)^4}{2}(p^2+m^2)^{-1}\frac{\delta}{\delta\phi(-p)}\right]e^{S}\right\}$$

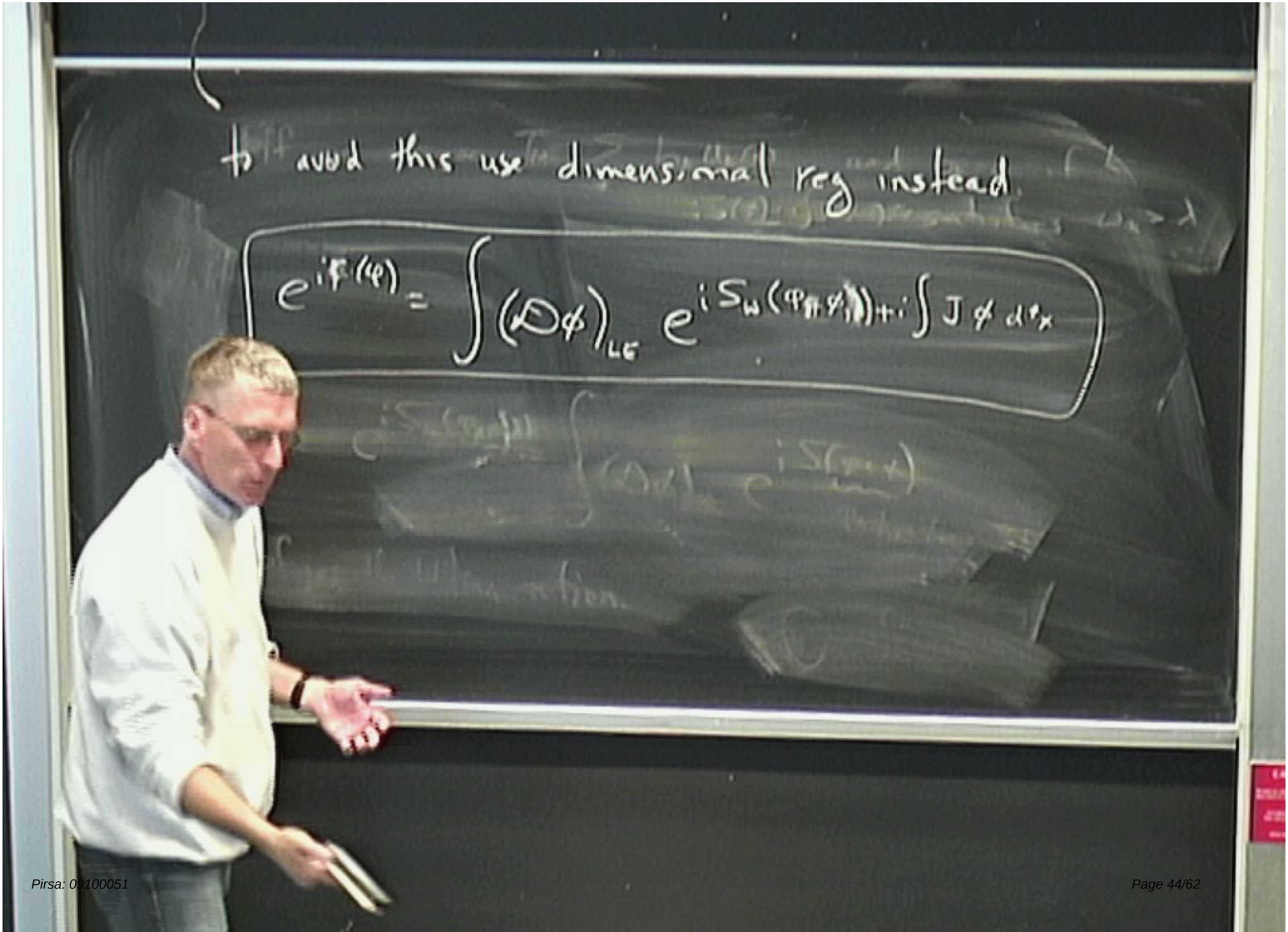
$$\frac{\delta}{\delta\phi}e^S = \frac{\delta S}{\delta\phi}e^S \qquad \frac{\delta^2}{\delta\phi^2}e^S = \left(\frac{\delta S}{\delta\phi}\frac{\delta S}{\delta\phi} + \frac{\delta^2 S}{\delta\phi\delta\phi}\right)e^S$$

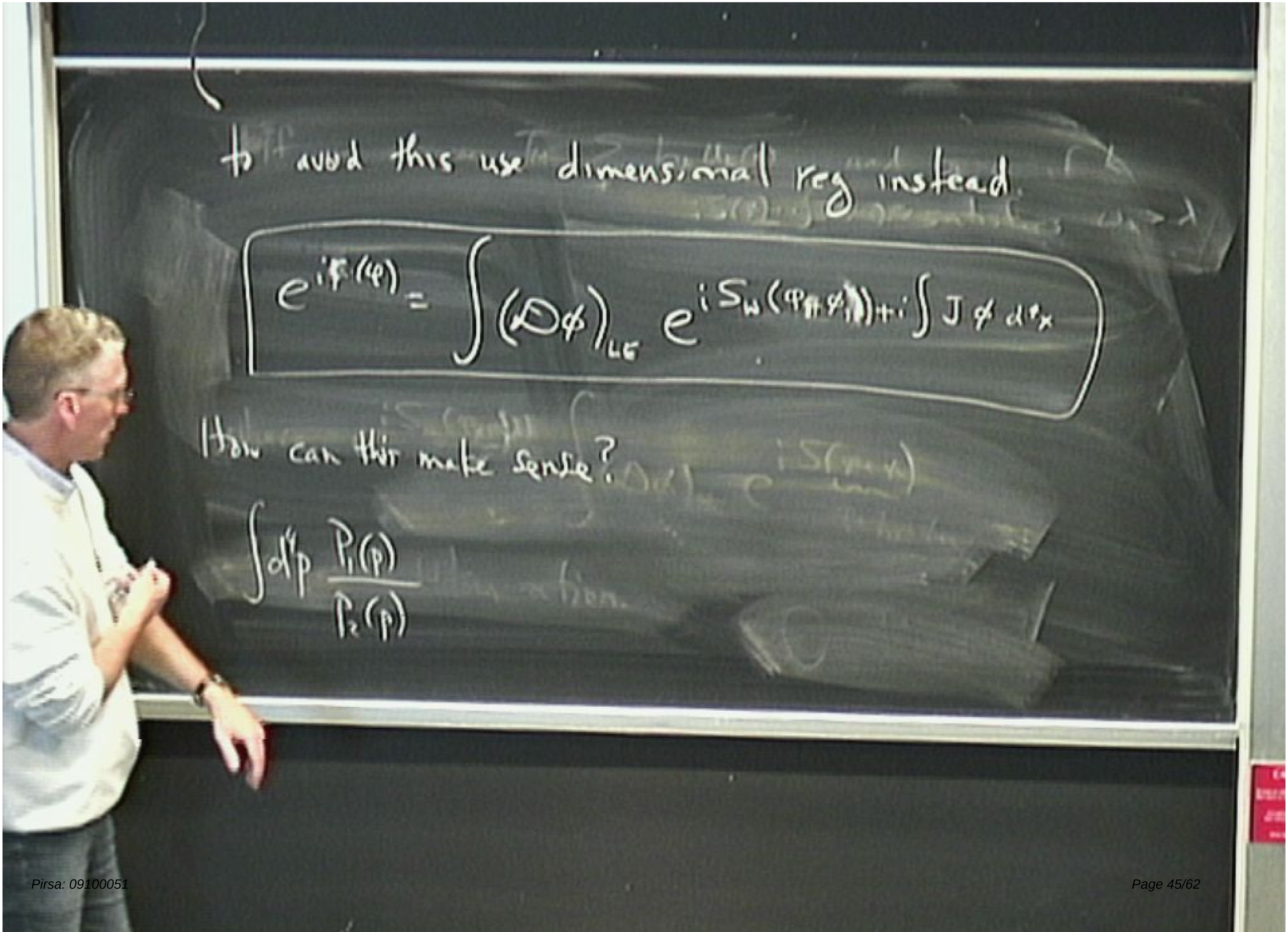
$= 0.$


$S_{int} = \sum g_1^{(i)}\phi^3 + g_2^{(i)}\phi^4$
$+ g_3^{(i)}\phi^2\Box\phi^4$
$\lambda \to 0$


$S_{int} = \sum g_1^{(i)}\phi^3 + g_2^{(i)}\phi^4 + g_3^{(i)}\phi^2\Box\phi^8$
$\lambda \to 0$
Polchinski's claim:
as $\lambda \to 0$ couplings flow to a finite-dimensional subsurface. need only specify a few parameters to know all of S_{int}.
i.e. theory is renormalizable


Dimensional Regularization + the Wilson action:
Cutoffs are inconvenient because:
1) introduce a large scale (λ) in intermediate steps, that are not in physical quantities (thus complicating dimensional analysis)
2) they break gauge invariance + other symmetries.


to avoid this use dimensional reg instead
$e^{i\Gamma(\varphi)} = \int (\mathcal{D}\phi)_{LG}\, e^{iS_W(\varphi+\phi)+i\int J\phi\, d^4x}$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$e^{i\Gamma(\varphi)} = \int (\mathcal{D}\phi)_{LG}\, e^{iS_W(\varphi+\phi)+i\int J\phi\, d^4x}$
How can this make sense?
$\int d^4p\, \frac{P_1(p)}{P_2(p)}$

to avoid this use dimensional reg instead

$$\boxed{e^{i\Gamma(\varphi)} = \int (\mathcal{D}\phi)_{LE}\, e^{iS_W(\varphi_{||}+\phi_{||}) + i\int J\phi\, d^4x}}$$

How can this make sense?

$I_n(q) \int d^np \dfrac{P_1(p,q)}{P_2(p)}$ where polynomials P_1, P_2 are such that

P_1/P_2 does not fall faster than $\frac{1}{p^4}$ at large p

to avoid this use dimensional reg instead

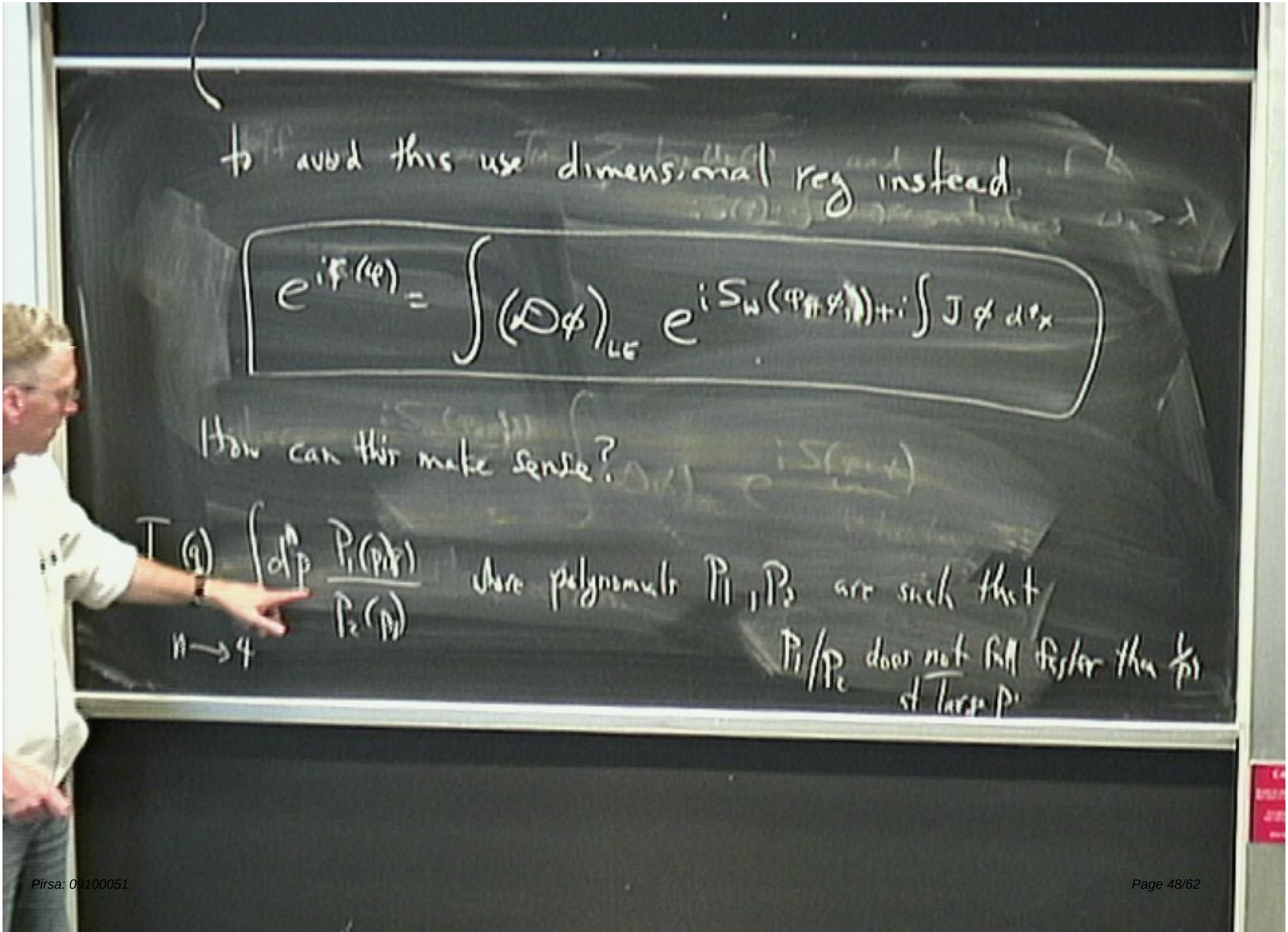
$$\boxed{e^{i\Gamma(\varphi)} = \int (\mathcal{D}\phi)_{LE}\, e^{iS_W(\varphi_B + \phi_{fl}) + i\int J\phi\, d^4x}}$$

How can this make sense?

$I_n(q) \int d^n p \, \frac{P_1(p,q)}{P_2(p)}$ where polynomials P_1, P_2 are such that

$n \to 4$

P_1/P_2 does not fall faster than $1/p^4$ at large p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e^{iF(φ)} = ∫(Dφ)_{LE} e^{iS_W(φ, φ)) + i∫ J φ d^4x}
How can this make sense?
I(q) ∫ d^n p P_1(p,q) / P_2(p)
where polynomials P_1, P_2 are such that
n → 4
P_1/P_2 does not fall faster than 1/p^4
at large p

to avoid this use dimensional reg instead

$$\boxed{e^{i\Gamma(\varphi)} = \int (\mathcal{D}\phi)_{LE}\, e^{iS_W(\varphi_{\Pi}+\phi_{\Pi}) + i\int J\phi\, d^4x}}$$

How can this make sense?

$$I_n(q) = \int d^n p\, \frac{P_1(p,q)}{P_2(p)}$$

where polynomials P_1, P_2 are such that

$n \to 4$

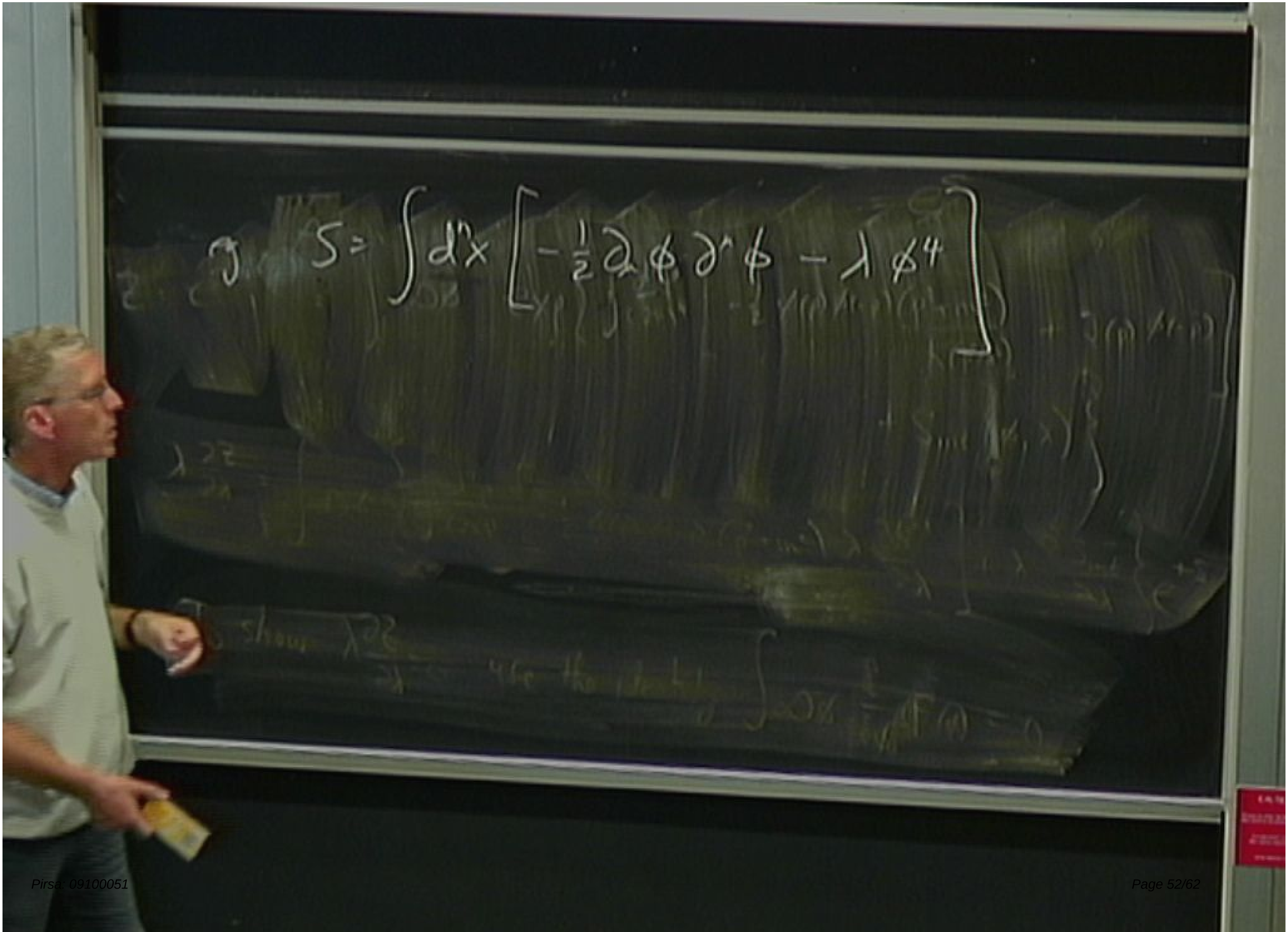
P_1/P_2 does not fall faster than $1/p^4$ at large p

Expect $I_n(q^n)$ to converge for large ρ if $\mathrm{Re}\, n$ is sufficiently negative. Define $I_n(q)$ as a function of complex n by analytic continuation from this region.

Expect $I_n(q^2)$ to converge for large p if $\mathrm{Re}\, n$ is sufficiently negative. Define $I_n(q)$ as a function of complex n by analytic continuation from this region.

write $n = 4 - 2\epsilon$ ask for limiting behaviour as $\epsilon \to 0$.

typically get poles in $1/\epsilon$.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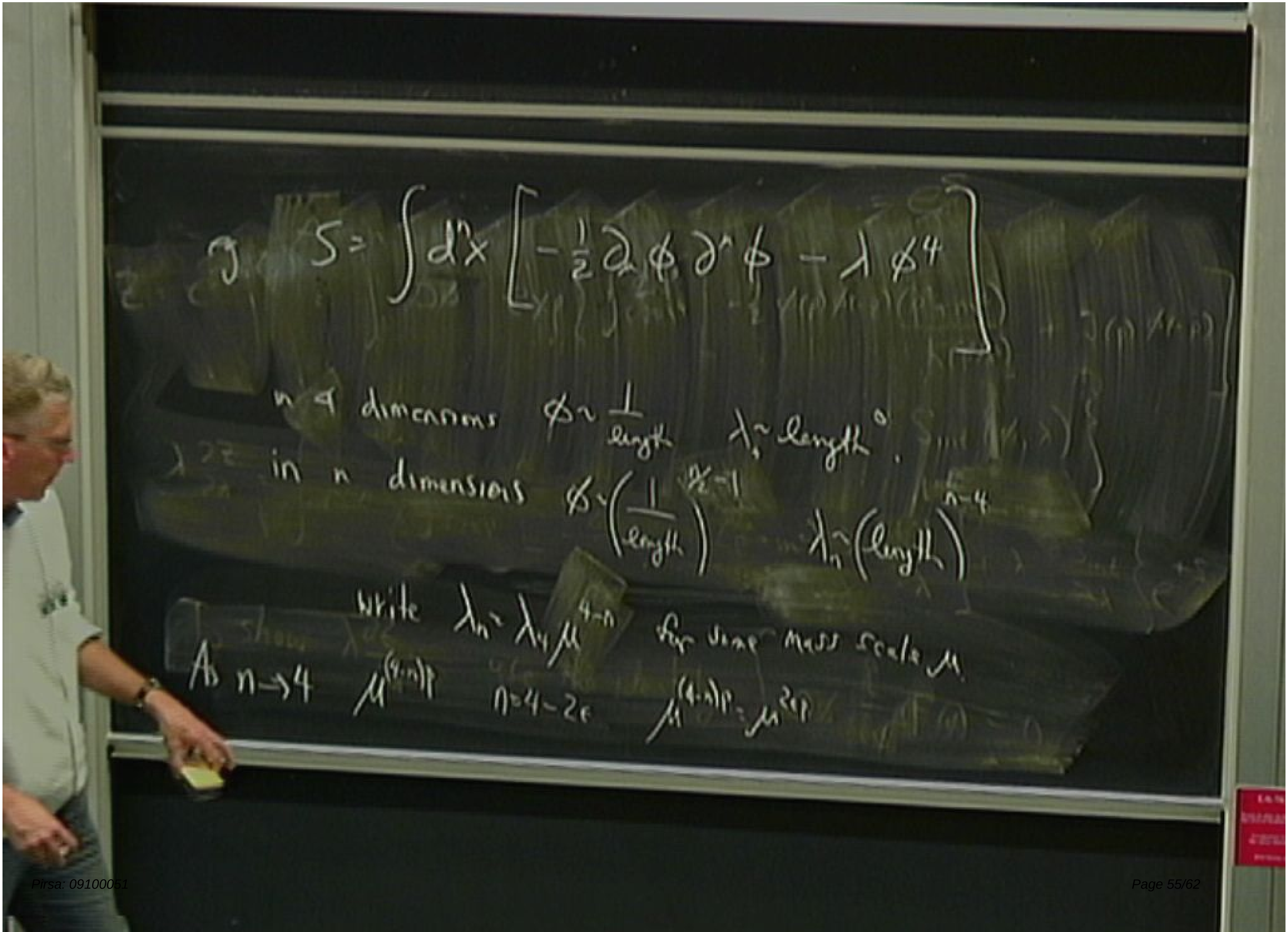

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$= 0.$

$\frac{\delta}{\delta\phi} e^{S} = \frac{\delta S}{\delta\phi} e^{S}$ $\quad$ $\frac{\delta}{\delta\phi} e^{S} = \left(\frac{\delta S}{\delta\phi}\frac{\delta S}{\delta\phi} + \frac{\delta^2 S}{\delta\phi^2}\right) e^{S}$

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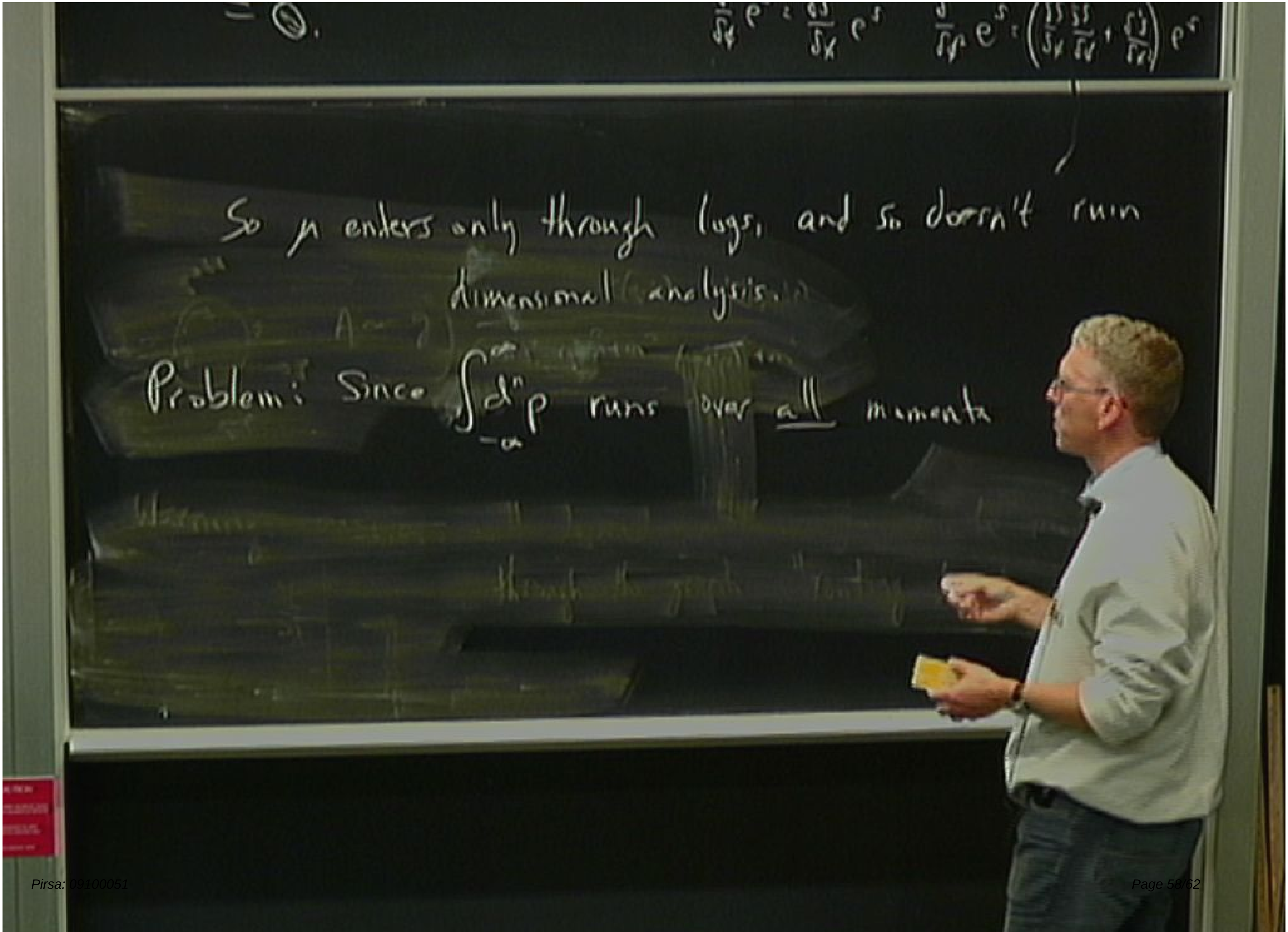
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Problem: Since $\int_{-\infty}^{\infty} d^n p$ runs over all momenta (including $p \to \infty$)

how can you remove high frequencies to get $\mathcal{L}_W$?

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Problem: Since $\int_{-\infty}^{\infty} d^n p$ runs over all momenta (including $p > \Lambda$)

how can you remove high frequencies to get $\mathcal{L}_w$?

The error you make by including these high momenta can be absorbed into some local effective interaction.

Never have to calculate it separately if you compute $\mathcal{L}_w$ by "matching" to full theory.

Matching: Use

Matching: Use the toy model:

$$\mathcal{L}(\ell,h) = -\frac{1}{2}\partial_\mu \ell\, \partial^\mu \ell - \frac{1}{2}\partial_\mu h\, \partial^\mu h - \frac{1}{2}m^2 \ell^2 - \frac{1}{2}M^2 h^2$$

$$- \frac{\tilde{g}_h}{3!} M h^3 - \frac{1}{2}\tilde{m}\, \ell^2 h - \frac{1}{4!} g_h h^4 - \frac{1}{4!} g_\ell \ell^4 - \frac{1}{4} g_{\ell h} \ell^2 h^2$$

(inv under $\ell \to -\ell$)