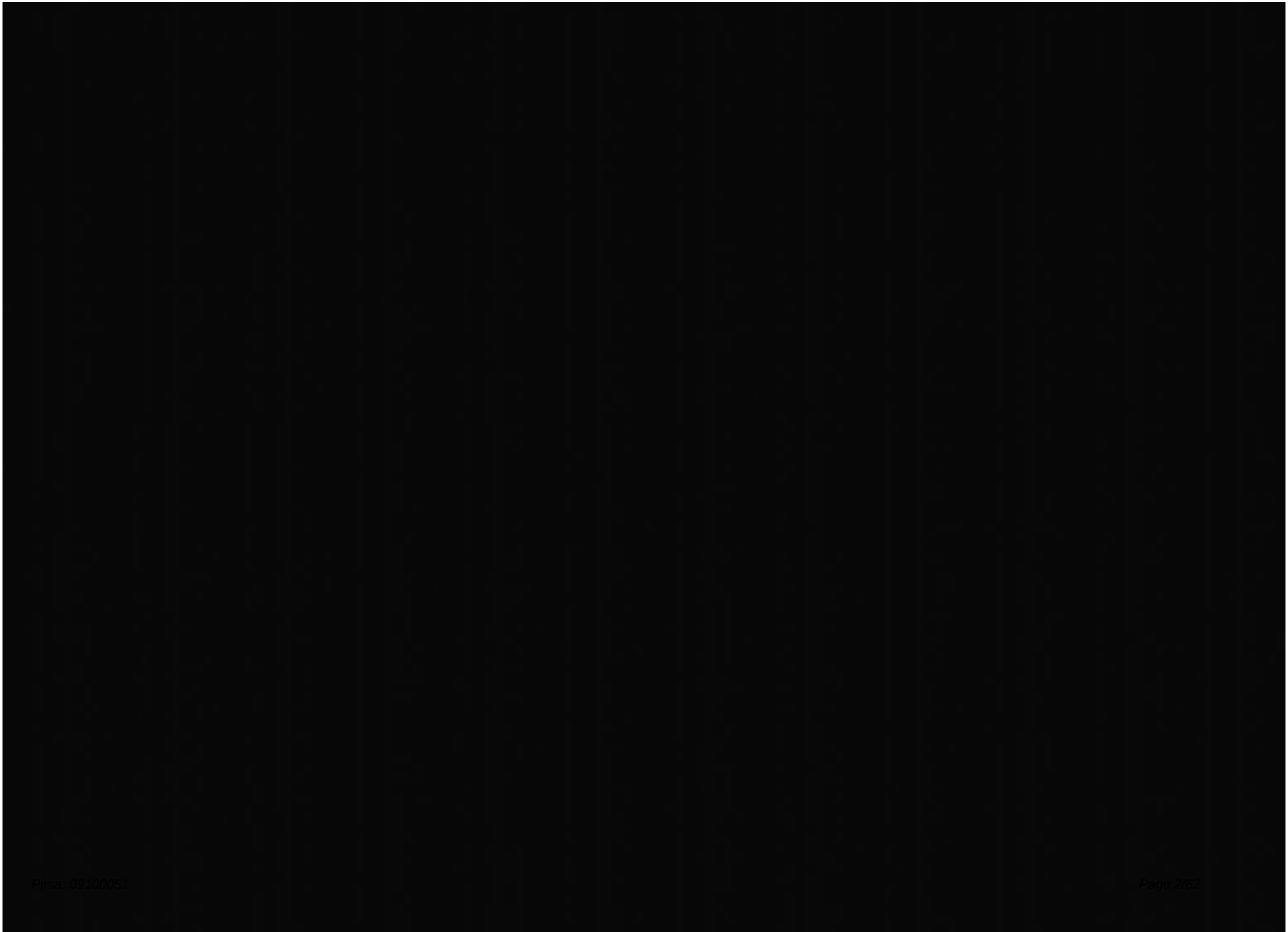
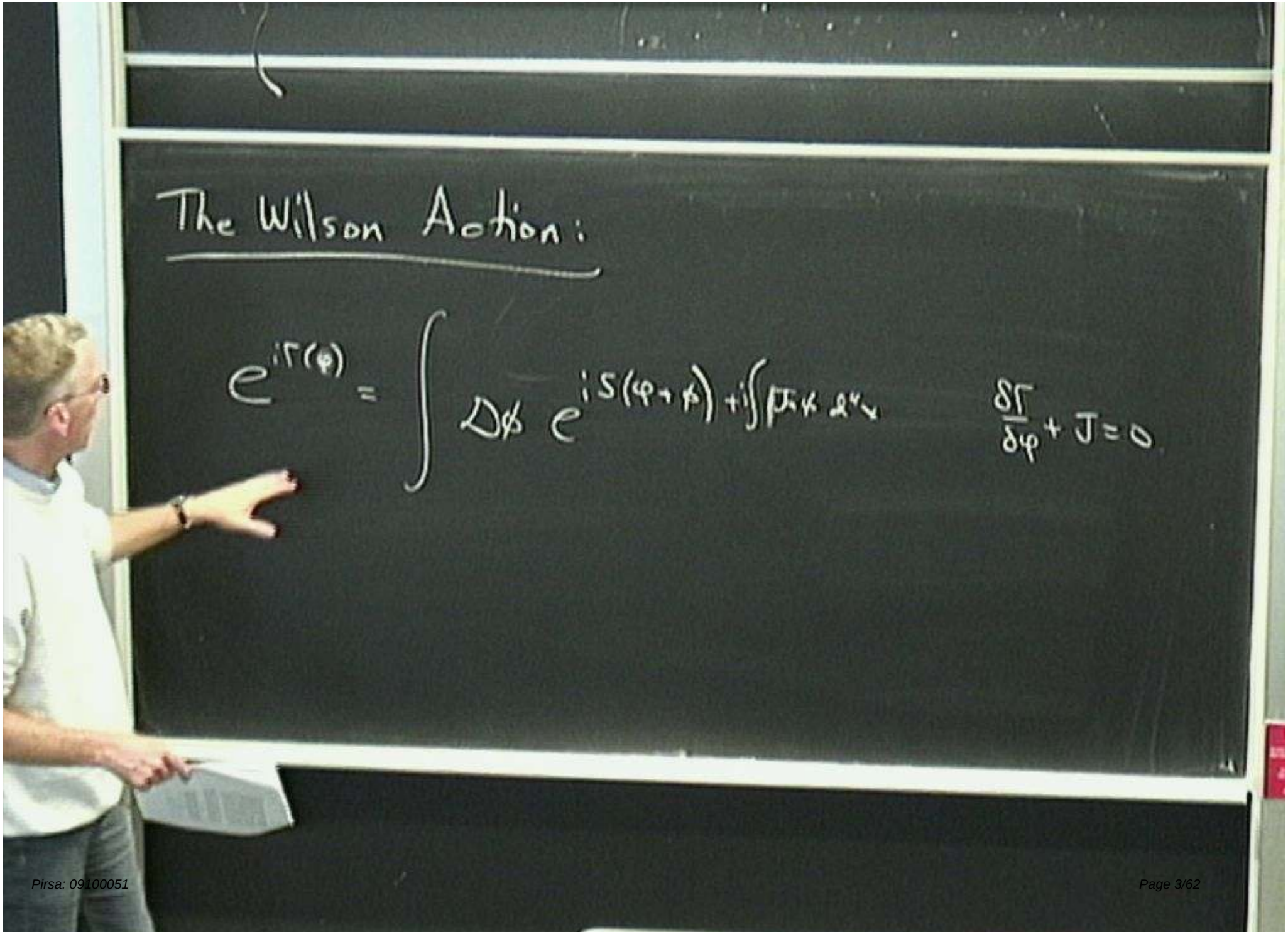
Title: Introduction to Effective Field Theory - Lecture 4A

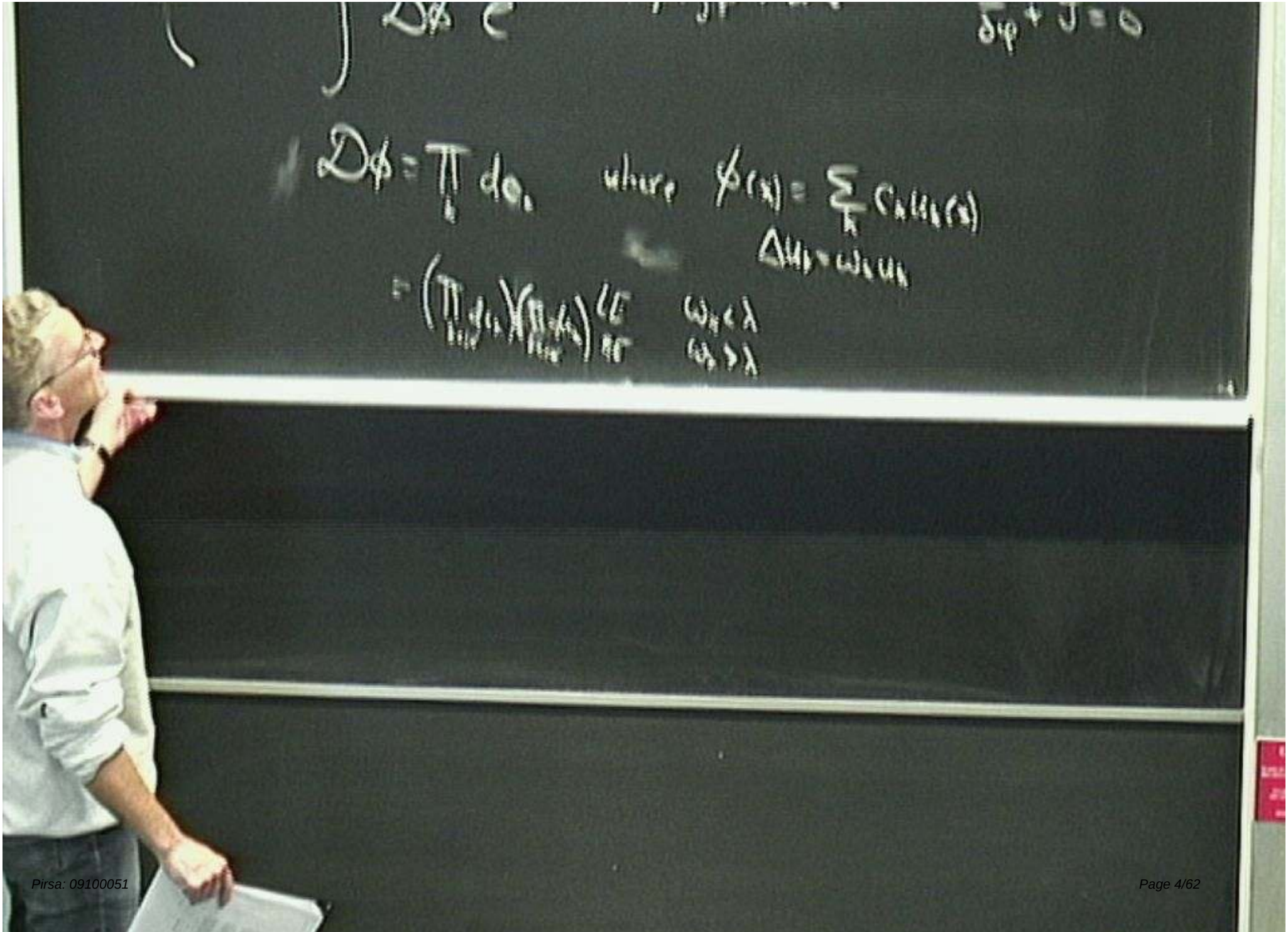
Date: Oct 14, 2009 09:00 AM

URL: http://www.pirsa.org/09100051

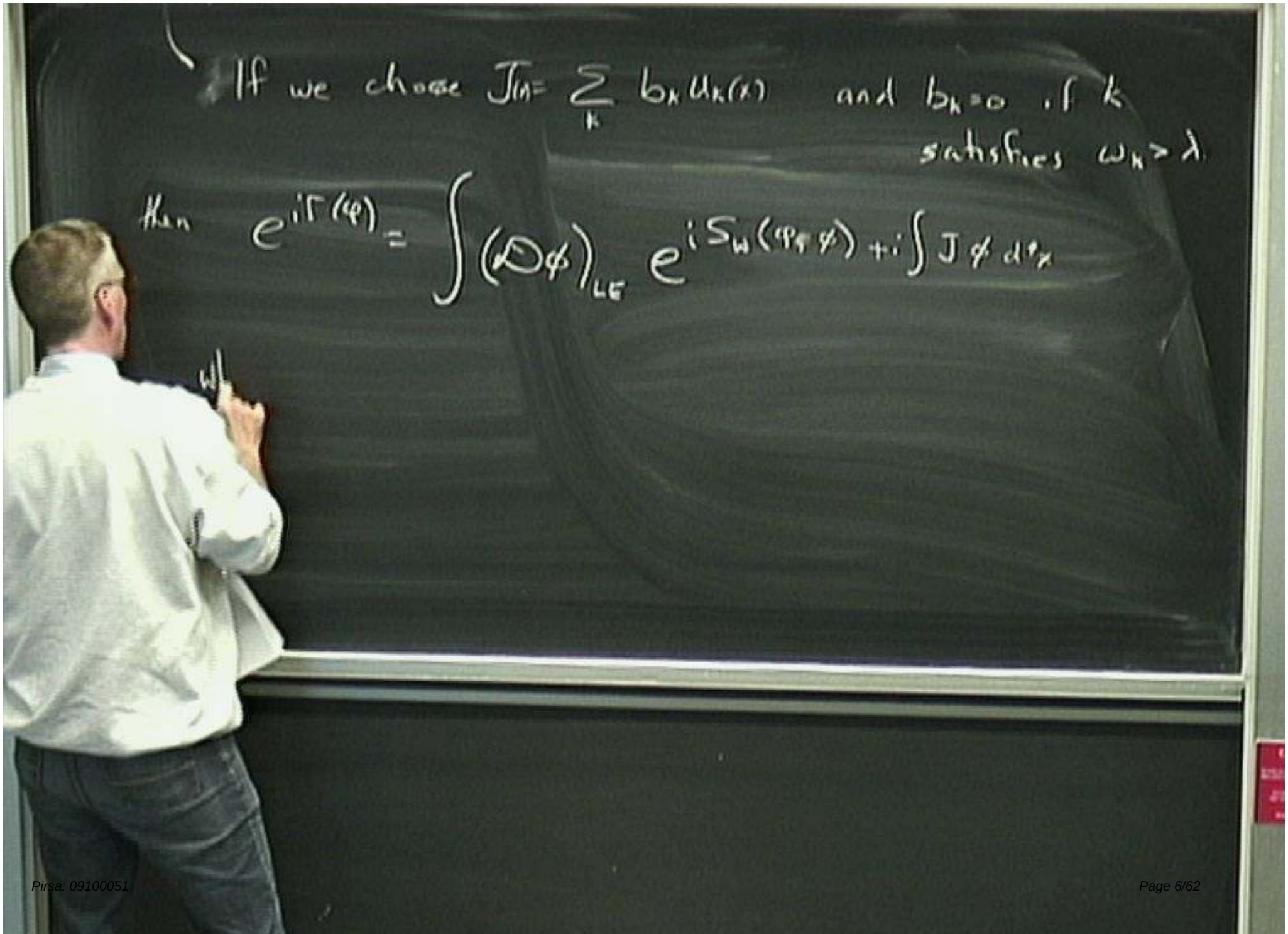
Abstract:

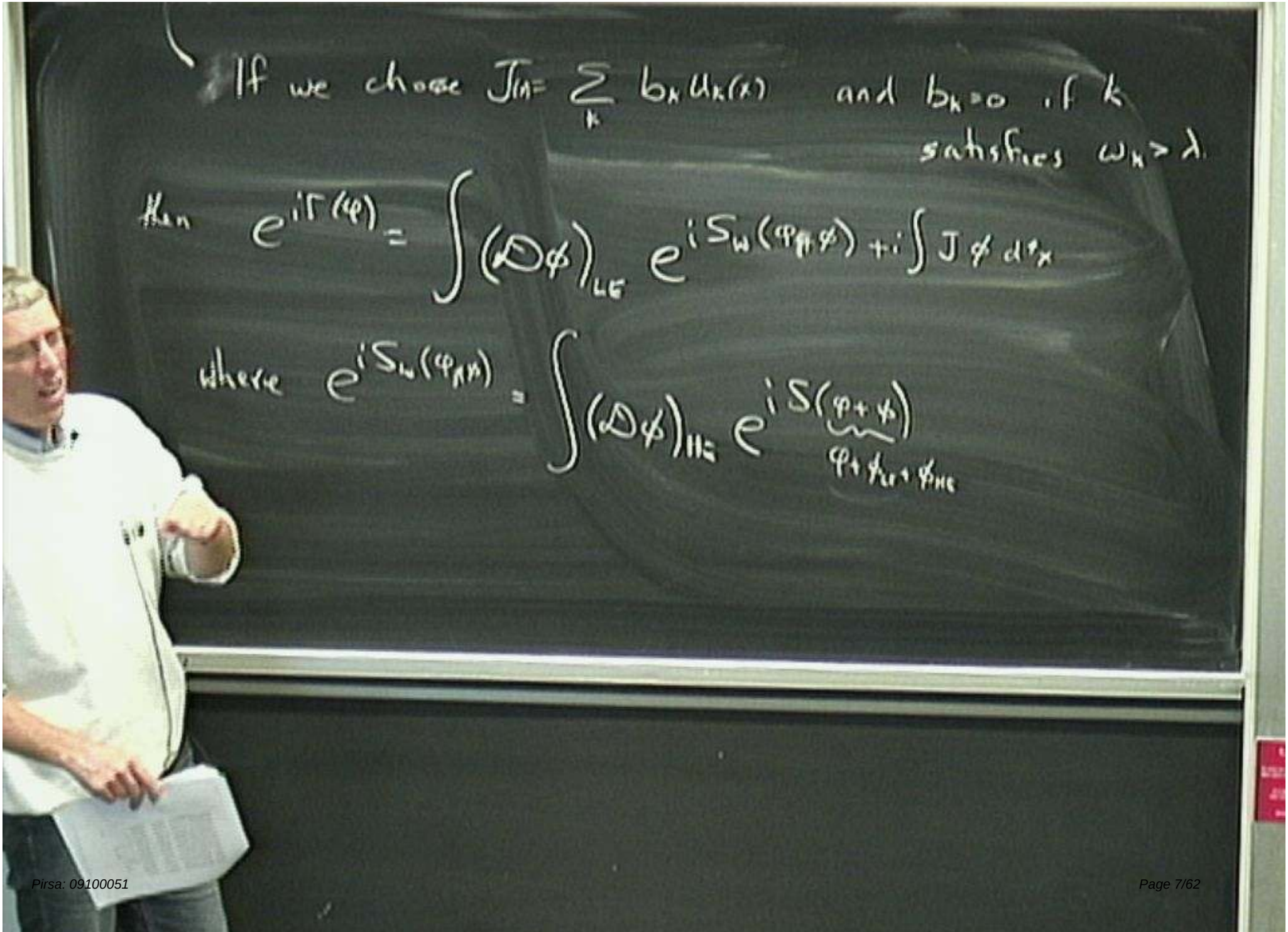


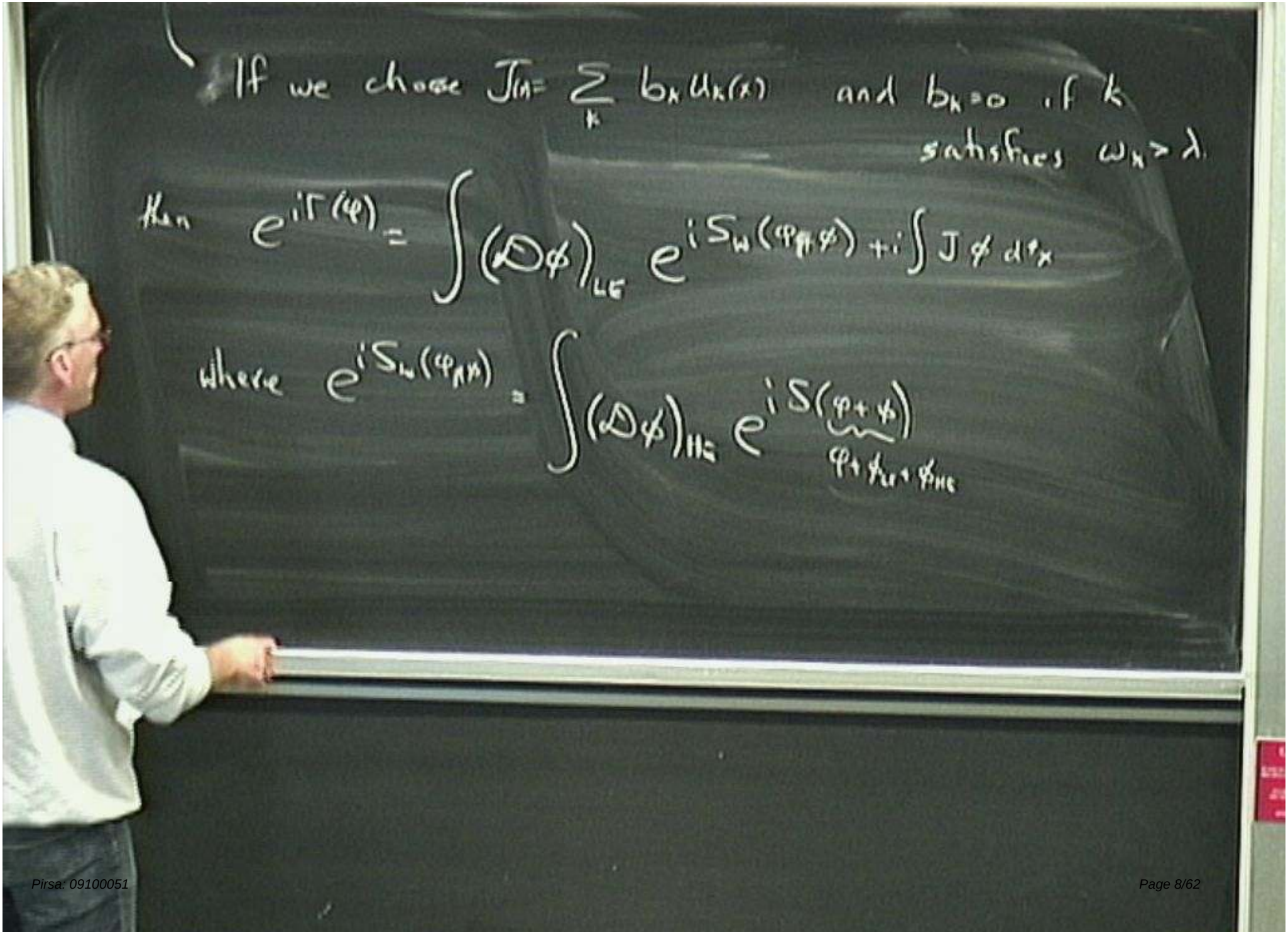

The Wilson Action:
$e^{i\Gamma(\varphi)} = \int D\phi\, e^{iS(\varphi+\phi)+i\int J_\mu \phi^\mu}$
$\frac{\delta\Gamma}{\delta\varphi} + J = 0$


$\delta\varphi + J = 0$
$D\phi = \prod_k d\alpha_k$ where $\phi(x) = \sum_k c_k u_k(x)$
$\Delta u_k = \omega_k u_k$
$\omega_k < \lambda$
$\omega_k > \lambda$

If we choose $J_{in} = \sum_k b_k u_k(x)$ and $b_k = 0$ if k satisfies $\omega_k > \lambda$.


If we choose $J_n = \sum_k b_k u_k(x)$ and $b_k = 0$ if k satisfies $\omega_k > \lambda$
$e^{iW(J)} = \int (\mathcal{D}\phi)_{LE}\, e^{iS_M(\varphi,\phi) + i\int J\phi\, d^4x}$


If we choose $J_n = \sum_k b_k u_k(x)$ and $b_k = 0$ if k satisfies $\omega_k > \lambda$.
Then $e^{iW(J)} = \int (\mathcal{D}\phi)_{LE}\, e^{iS_w(\varphi_0+\phi) + i\int J\phi\, d^4x}$
where $e^{iS_w(\varphi_0)} = \int (\mathcal{D}\phi)_{HE}\, e^{iS(\underbrace{\varphi_0+\phi}_{\varphi_0+\phi_{LE}+\phi_{HE}})}$


If we choose $J_n = \sum_k b_k u_k(x)$ and $b_k = 0$ if k satisfies $\omega_k > \lambda$.
Then $e^{iT(\varphi)} = \int (\mathcal{D}\phi)_{LE}\, e^{iS_w(\varphi_{fl}+\phi) + i\int J\phi\, d^4x}$
where $e^{iS_w(\varphi_{fl})} = \int (\mathcal{D}\phi)_{HE}\, e^{iS(\varphi_{fl}+\phi)}$
$\varphi_{fl} + \phi_{LE} + \phi_{HE}$

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where $e^{iS_W(\varphi_{fl})} := \int (\mathcal{D}\phi)_{HE}\, e^{iS(\underbrace{\varphi_{fl}+\phi}_{\varphi_{fl}+\phi_{UV}+\phi_{HE}})}$

defines the Wilson action.

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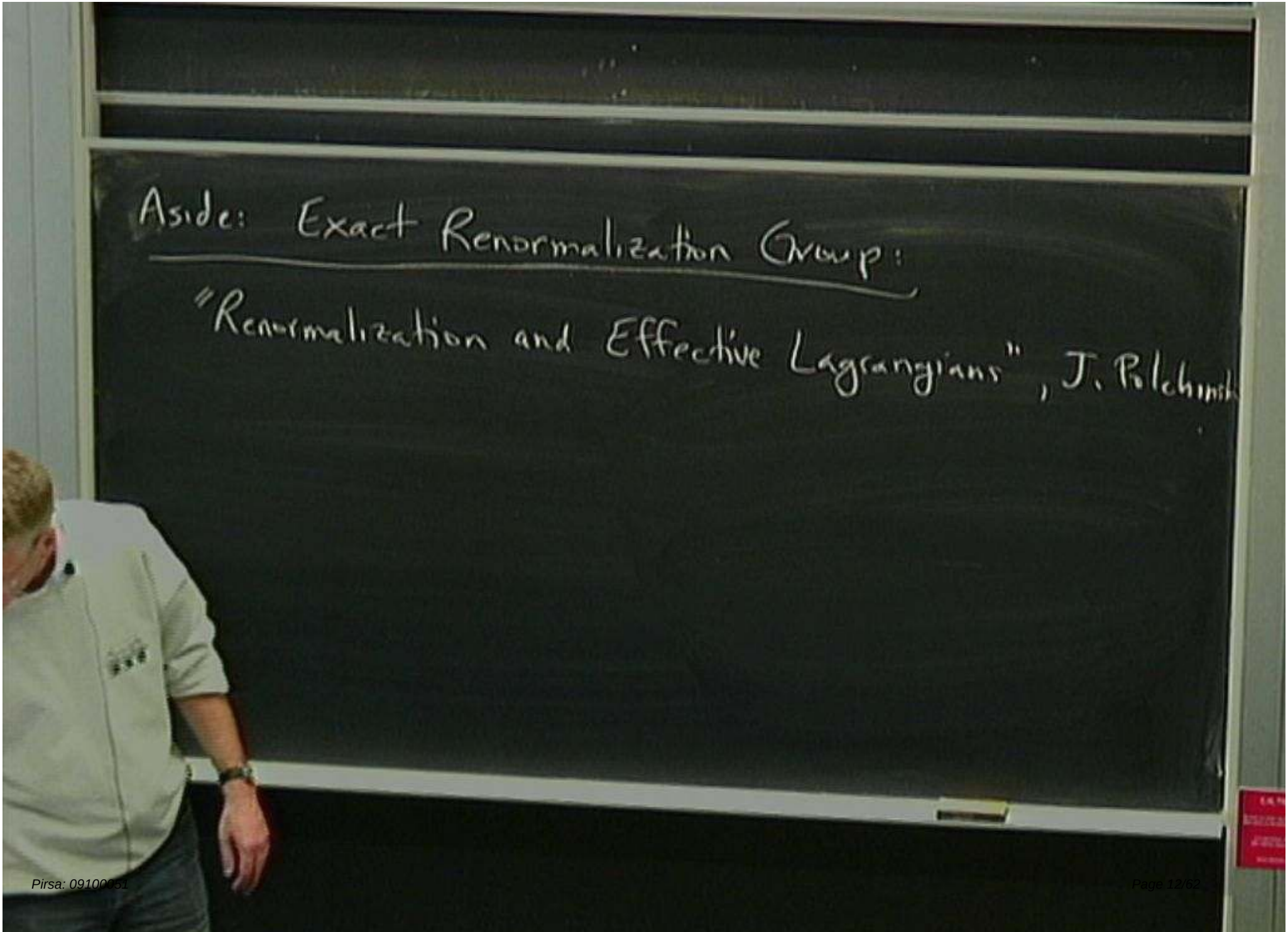
Aside: Exact Renormalization Group:
"Renormalization and Effective Lagrangians", J. Polchinski

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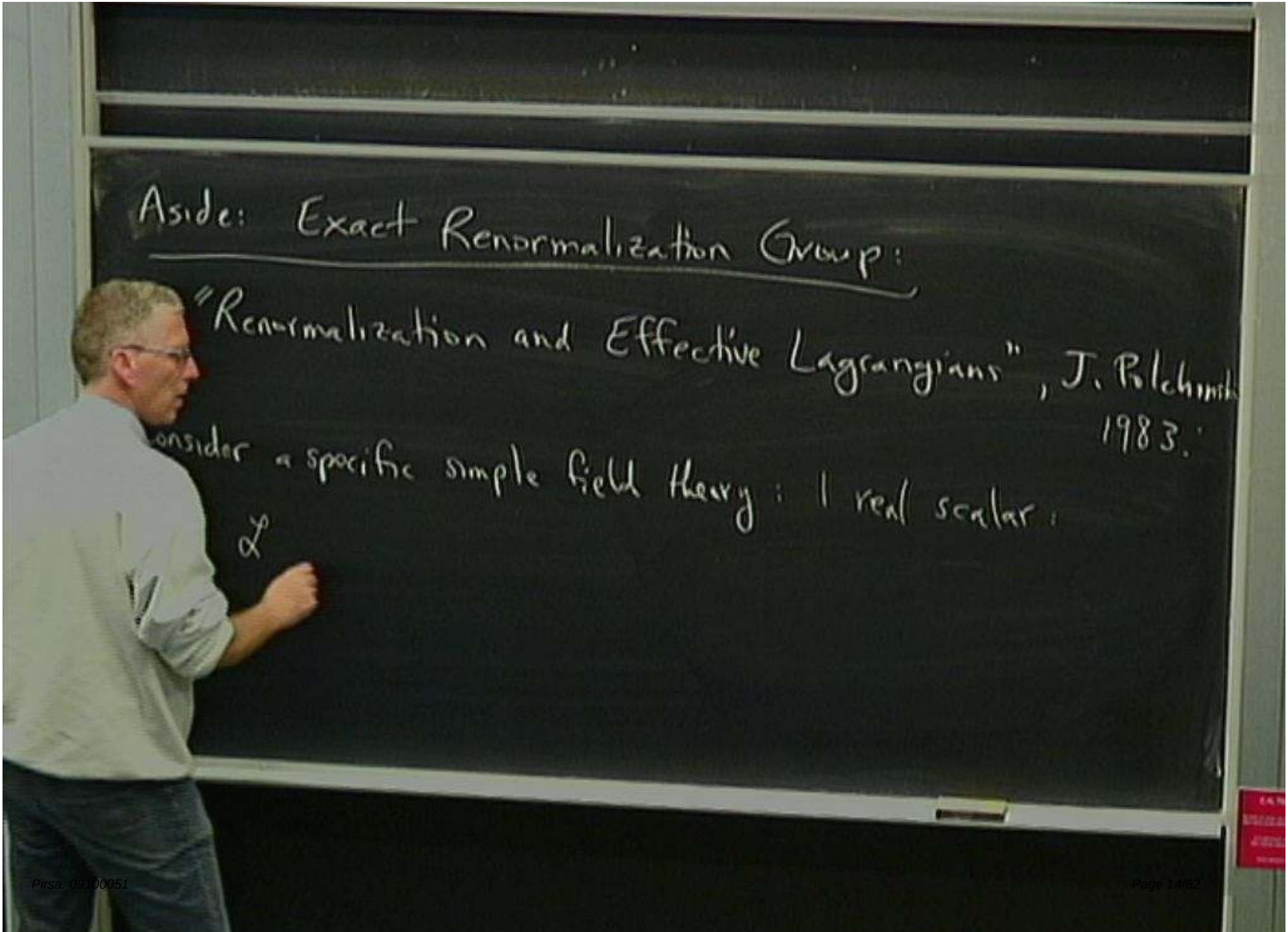
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$\mathcal{L}$

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Consider a specific simple field theory: 1 real scalar: ϕ

$\mathcal{L} =$

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Consider a specific simple field theory: 1 real scalar: ϕ

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m_0^2\phi^2 - \frac{1}{4!}\lambda_0\phi^4$$

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$$\phi(x) = \int \frac{d^4p}{(2\pi)^4}\, e^{ipx}\,\phi(p)$$

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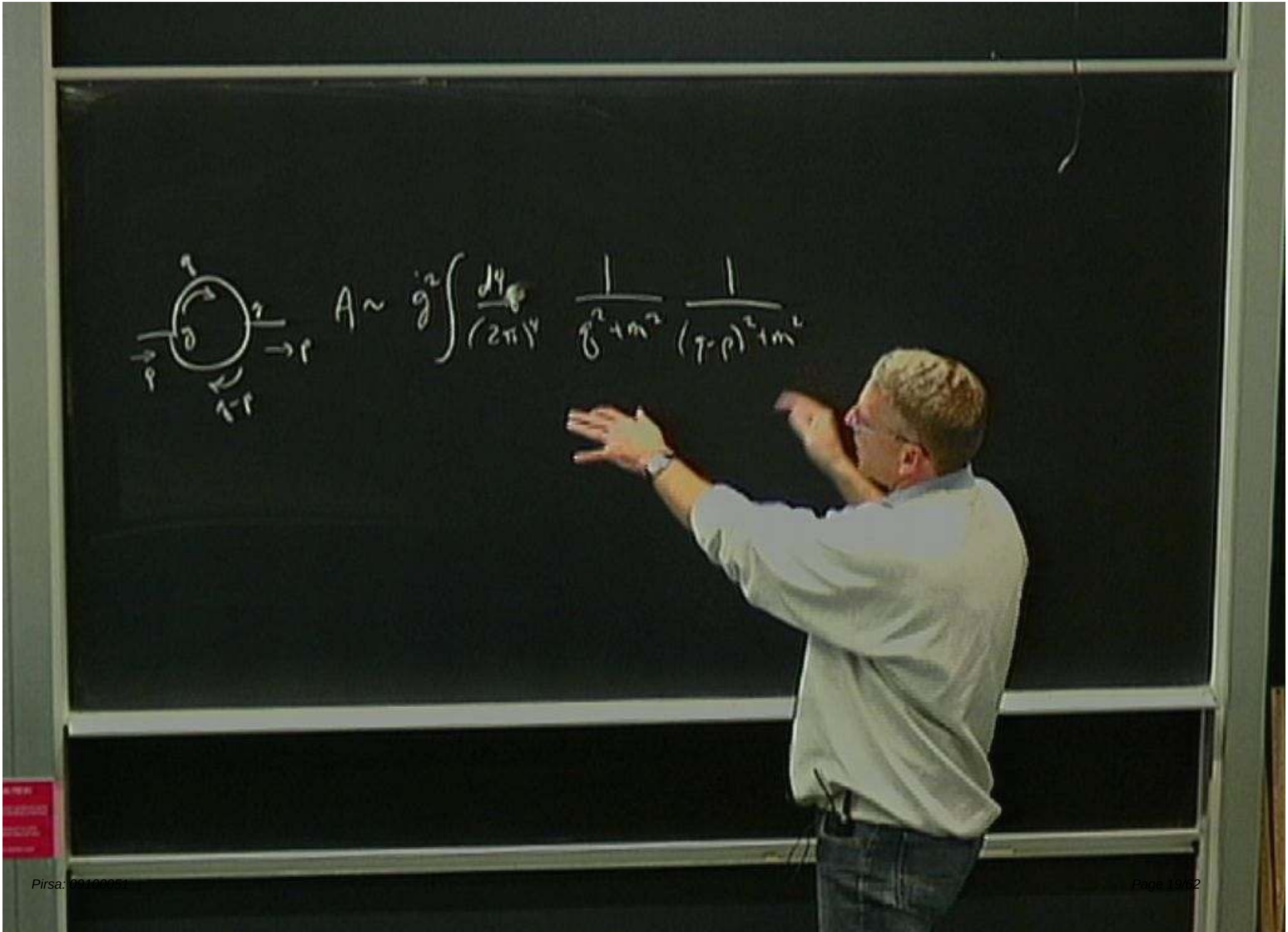
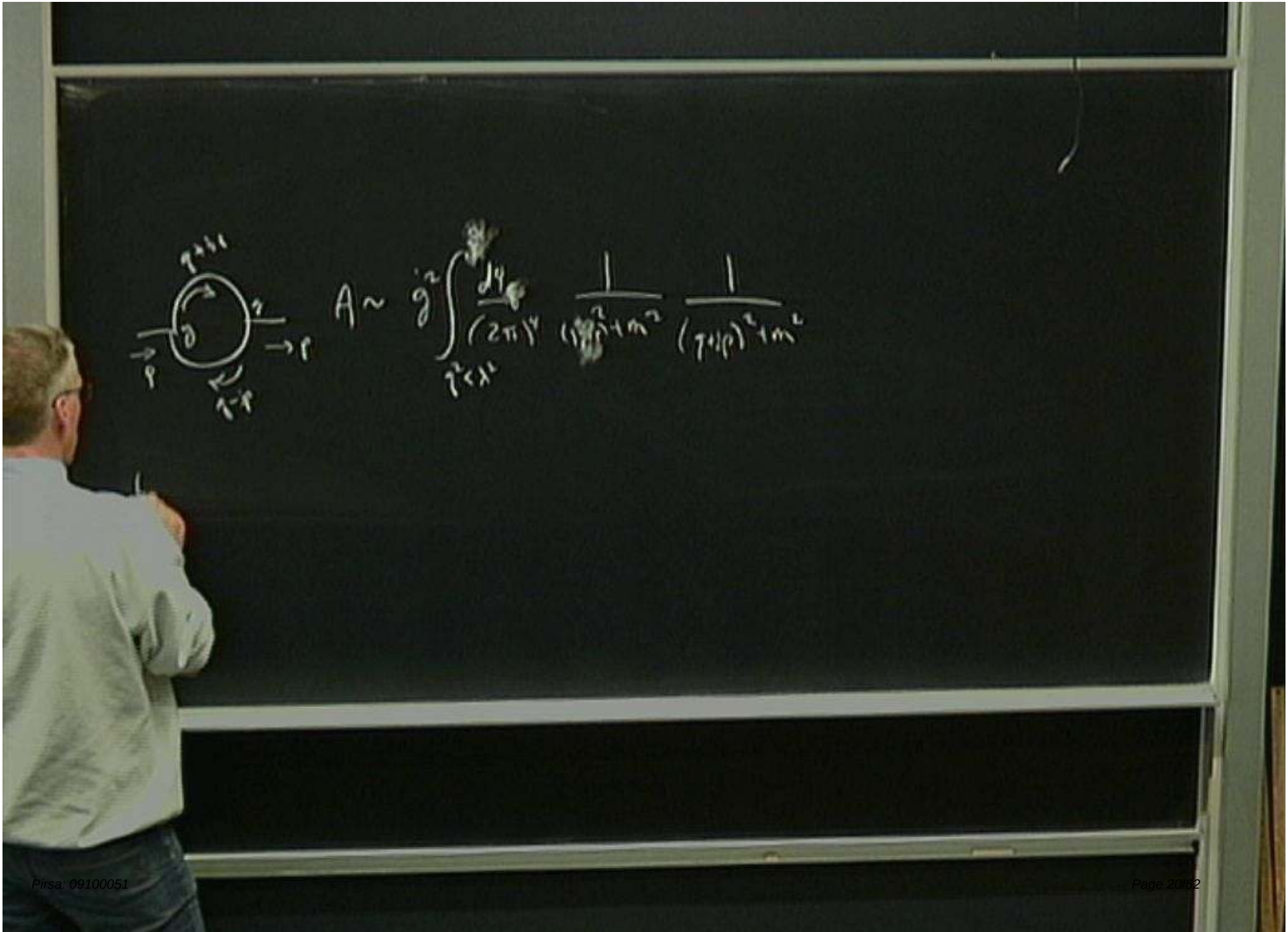
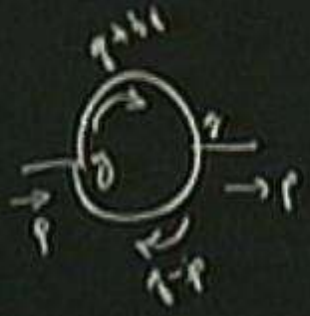

q
p
p-r
A ~ g² ∫ d⁴q / (2π)⁴ 1/(q²+m²) 1/((p-q)²+m²)


A ~ g² ∫ d⁴q / (2π)⁴ · 1/(q² + m²) · 1/((q+p)² + m²)
q²<Λ²



$$A \sim g^2 \int_{q^2<\Lambda^2} \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2+m^2)} \frac{1}{((q+p)^2+m^2)}$$

Warning: answers will depend on how you route momentum through the graph "routing dependence"

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$$S = \int d^4x\,\mathcal{L} = \int \frac{d^4p}{(2\pi)^4}\left[\frac{1}{2}\phi(p)\phi(-p)\frac{(p^2+m_i^2)}{K(p^2/\Lambda^2)}\right] + S_{int}$$

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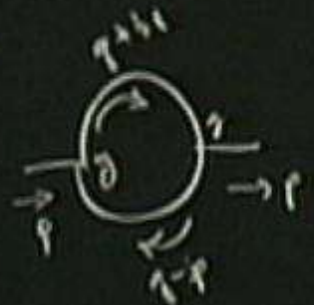
$K=0$ $p^2 \gg \Lambda^2$

$K=1$ $p^2 \ll \Lambda^2$



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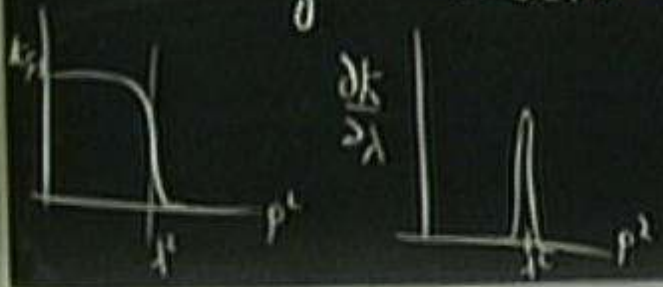

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$$S_M = \int d^4x\,\mathcal{L}_M = \int \frac{d^4p}{(2\pi)^4}\left[\frac{1}{2}\phi(p)\phi(-p)\underbrace{(p^2+m_0^2)}_{K(p^2/\Lambda^2)}\right] + S_{int}$$

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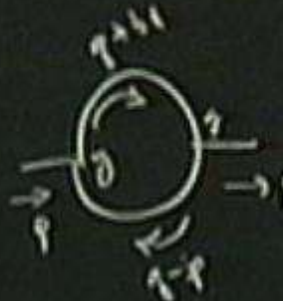

$$A \sim g^2 \int \frac{d^4q}{(2\pi)^4} \frac{K(q+p_1)}{(q+p_1)^2+m^2} \frac{K(q+p_2)}{(q+p)^2+m^2}$$

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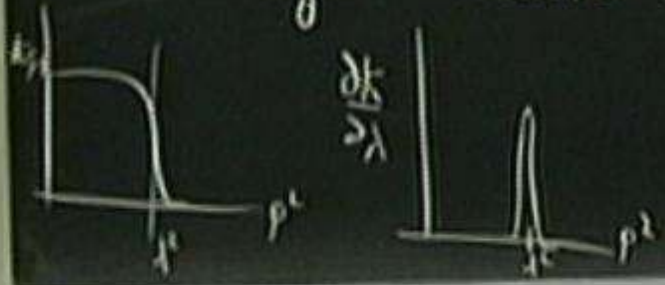
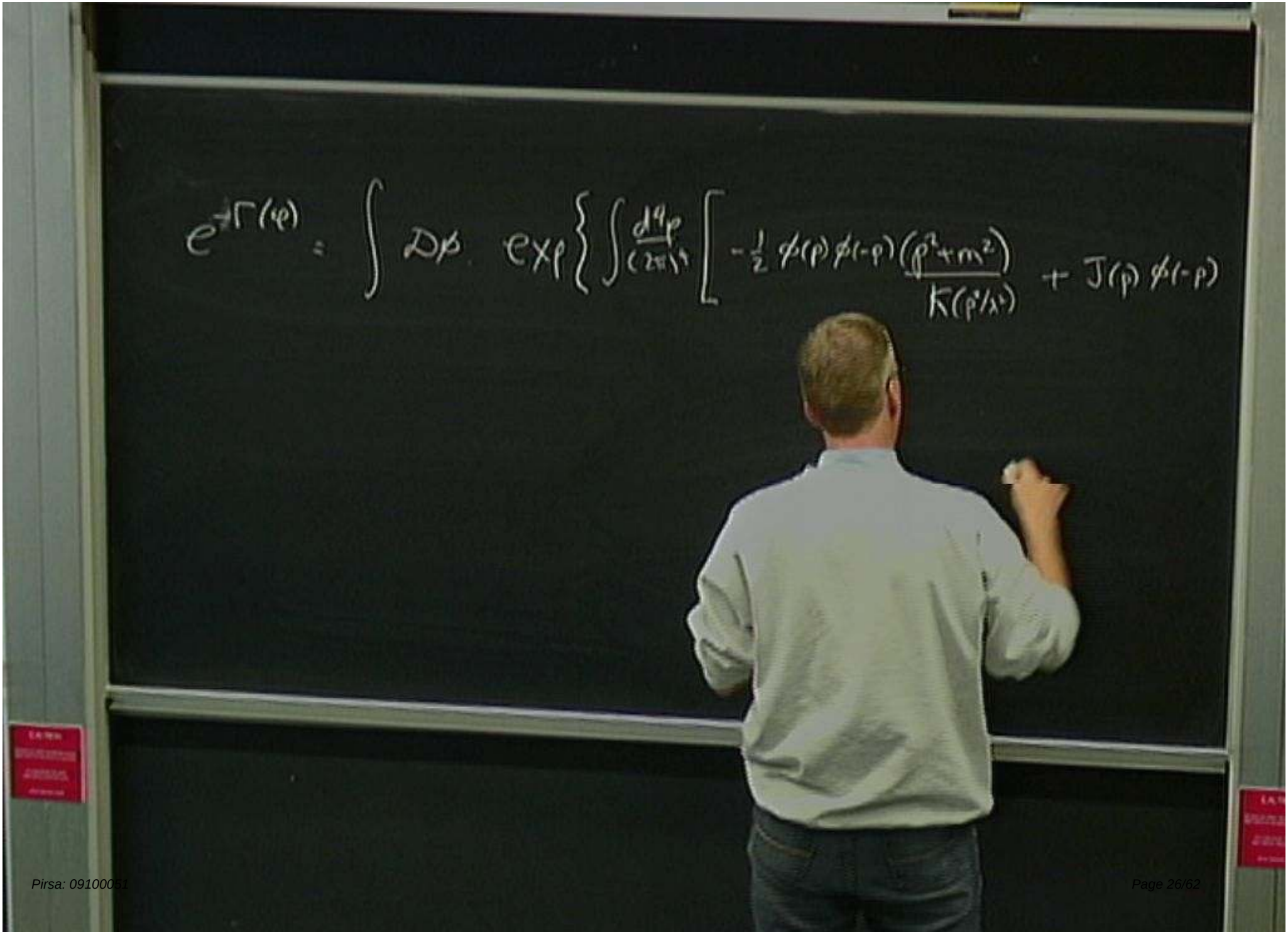
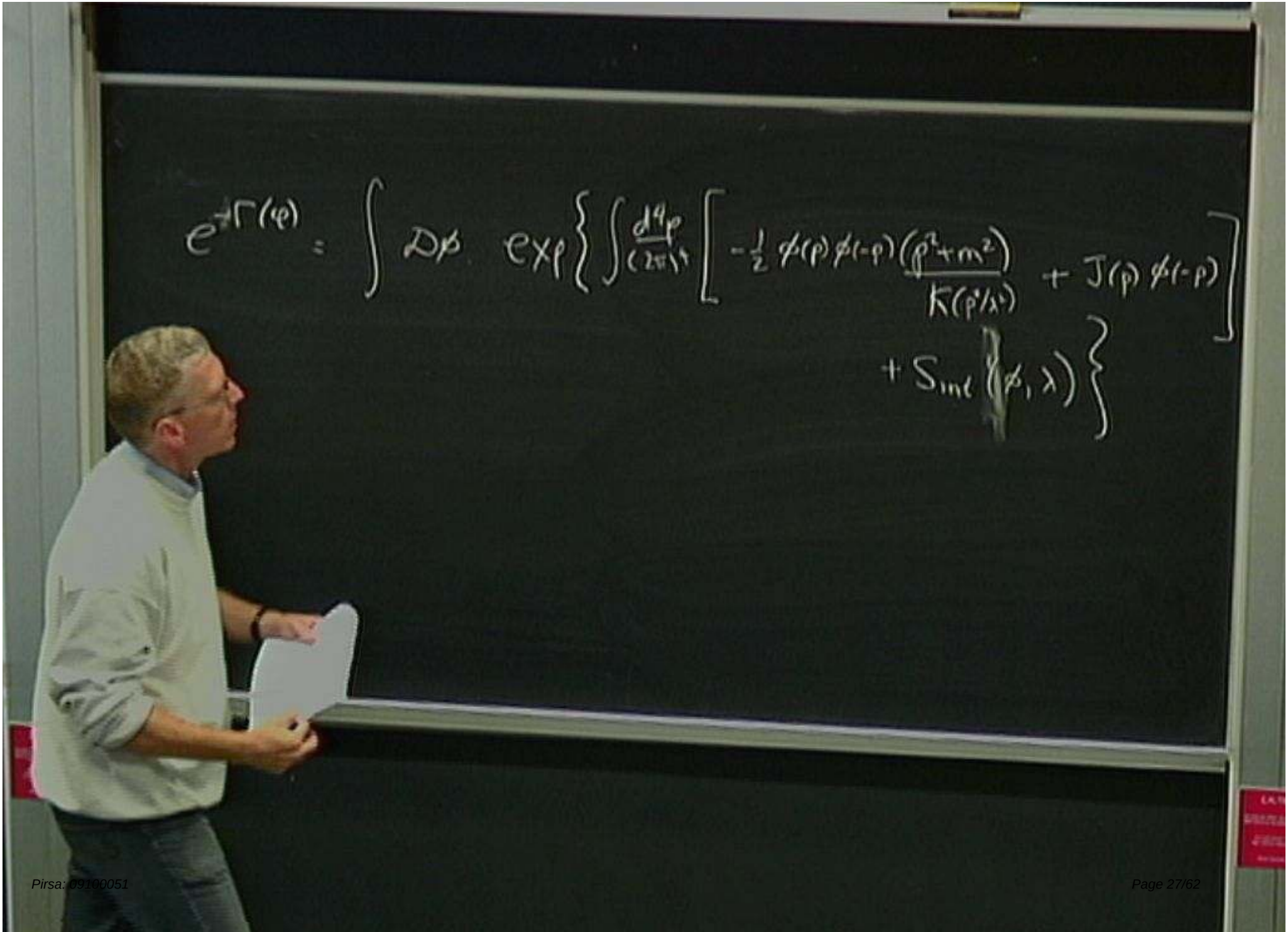



$e^{-\Gamma(\varphi)} = \int \mathcal{D}\phi \; \exp\left\{ \int \frac{d^4p}{(2\pi)^4} \left[-\frac{1}{2}\phi(p)\phi(-p)\frac{(p^2+m^2)}{K(p^2/\Lambda^2)} + J(p)\phi(-p) \right. \right.$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$$Z = e^{-\Gamma(\phi)} = \int \mathcal{D}\phi \; \overbrace{\exp\left\{ \int \frac{d^4p}{(2\pi)^4} \left[-\frac{1}{2}\phi(p)\phi(-p)\frac{(p^2+m^2)}{K(p^2/\Lambda^2)} + J(p)\phi(-p) \right] + S_{int}(\phi,\lambda) \right\}}^{e^S}$$

$$\lambda \frac{\partial Z}{\partial \lambda} = \int \mathcal{D}\phi \left\{ \int \frac{d^4p}{(2\pi)^4} \left[-\frac{1}{2}\phi(p)\phi(-p)(p^2+m^2)\lambda \frac{\partial K^{-1}}{\partial \lambda} \right] + \lambda \frac{\partial S_{int}}{\partial \lambda} \right\} e^{+S}$$

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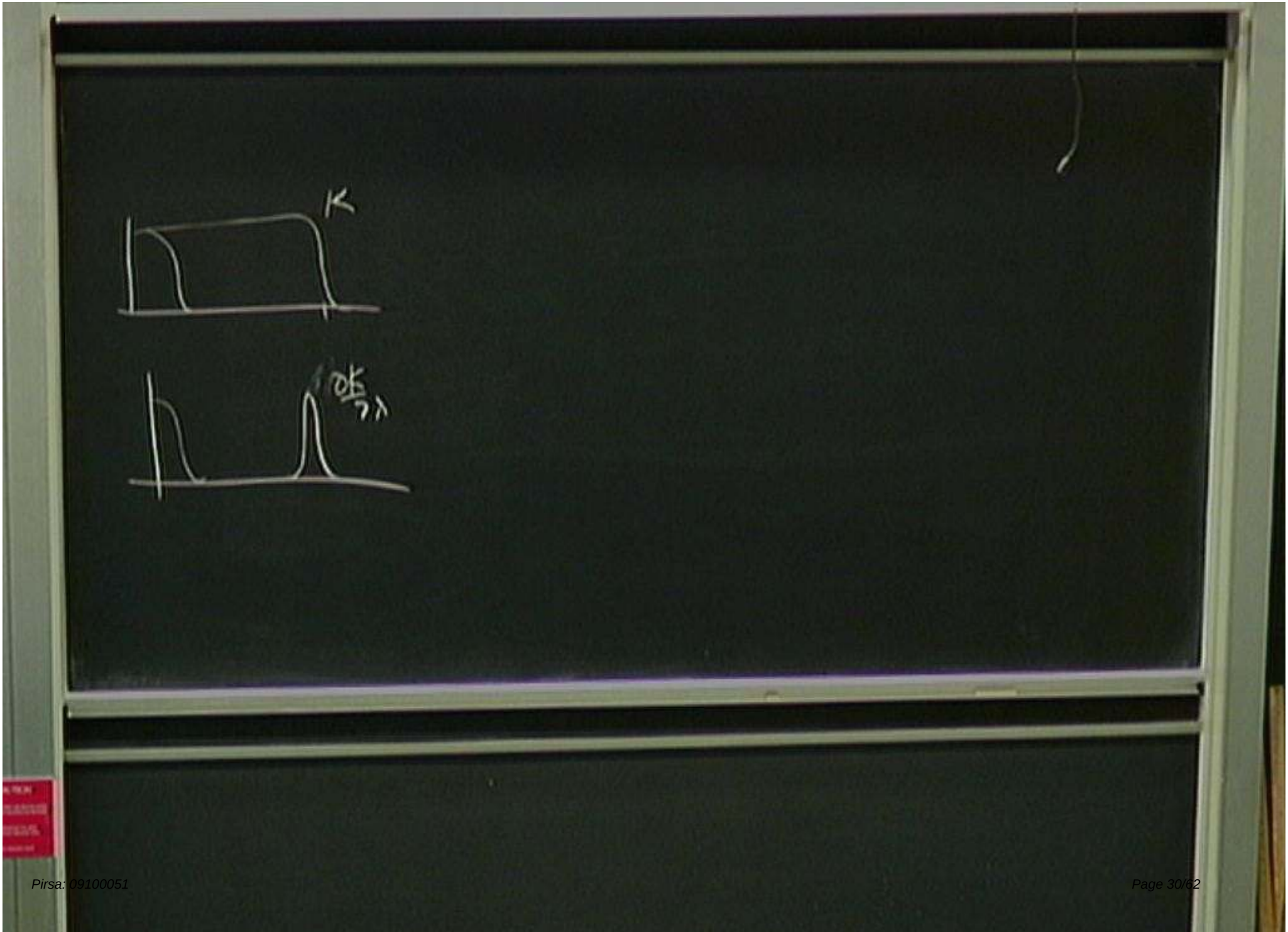
K
OE
??

$$Z = e^{+W(J)} = \int \mathcal{D}\phi \; \overbrace{\exp\left\{ \int \frac{d^4p}{(2\pi)^4} \left[-\frac{1}{2}\phi(p)\phi(-p)\frac{(p^2+m^2)}{K(p^2/\lambda^2)} + J(p)\phi(-p) \right] + S_{int}(\phi,\lambda) \right\}}^{= e^S}$$

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To show $\lambda \frac{\partial Z}{\partial \lambda} = 0$ use the identity $\int \mathcal{D}\phi \frac{\delta}{\delta \phi} F(\phi) = 0$

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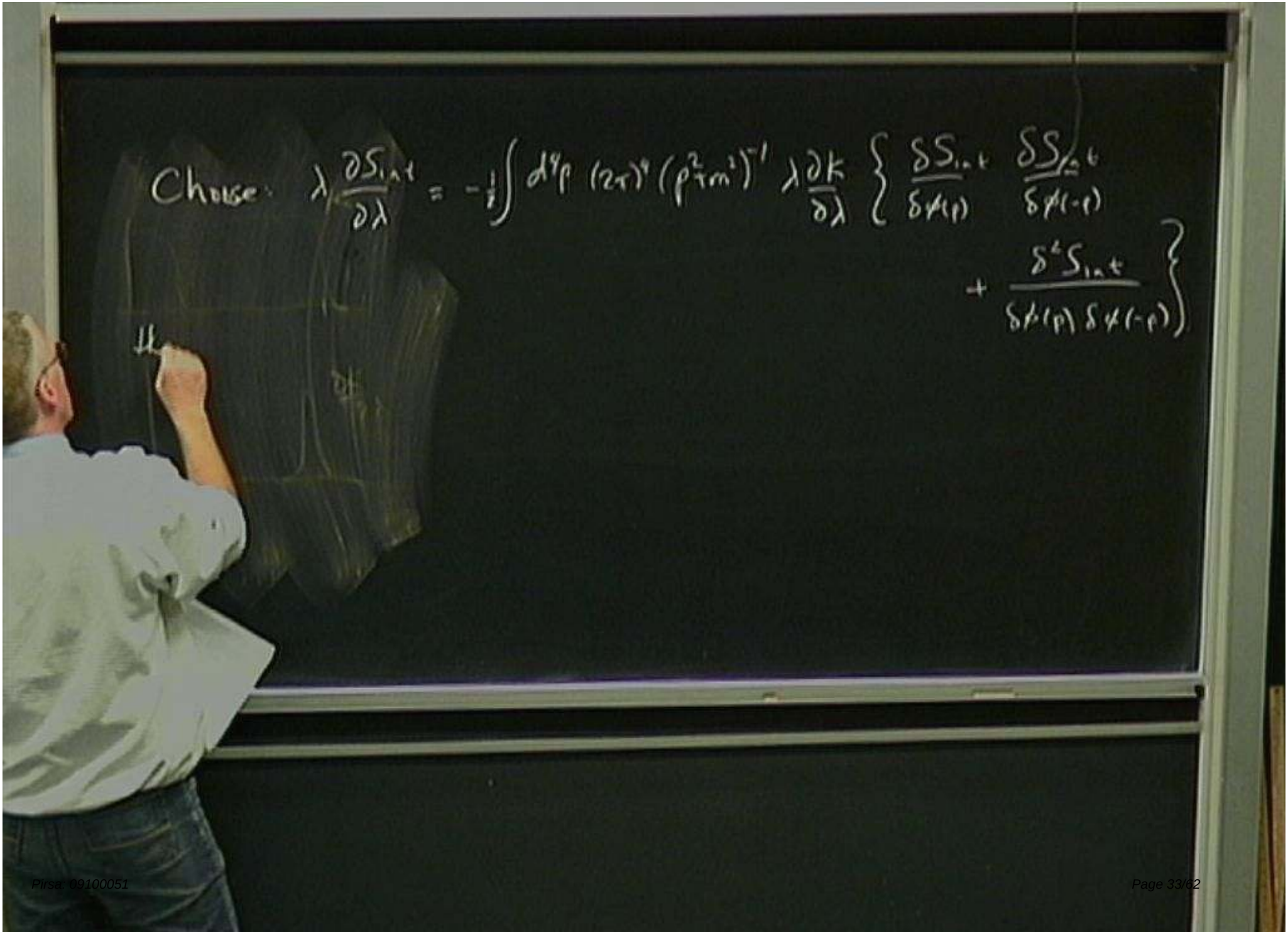
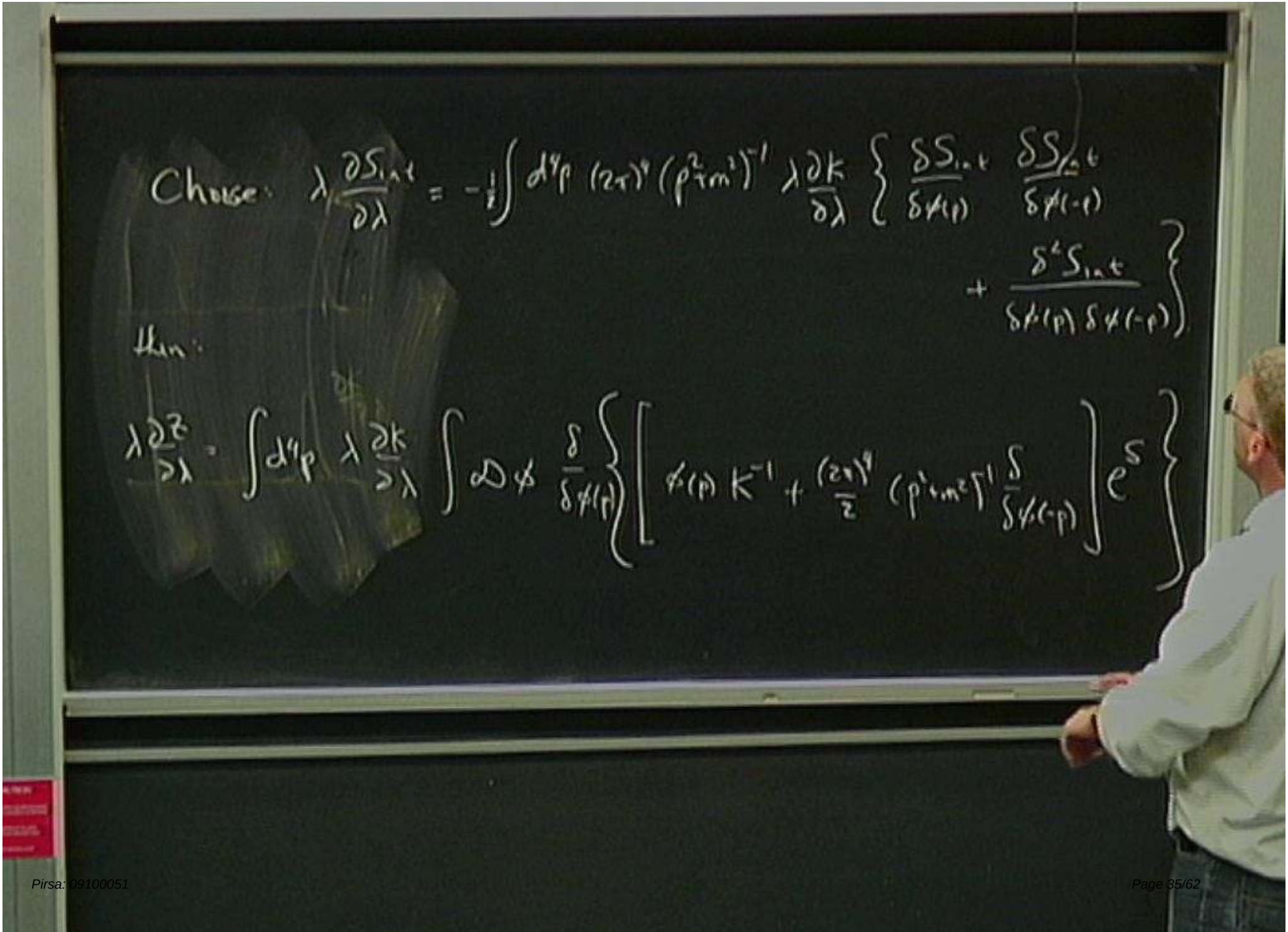
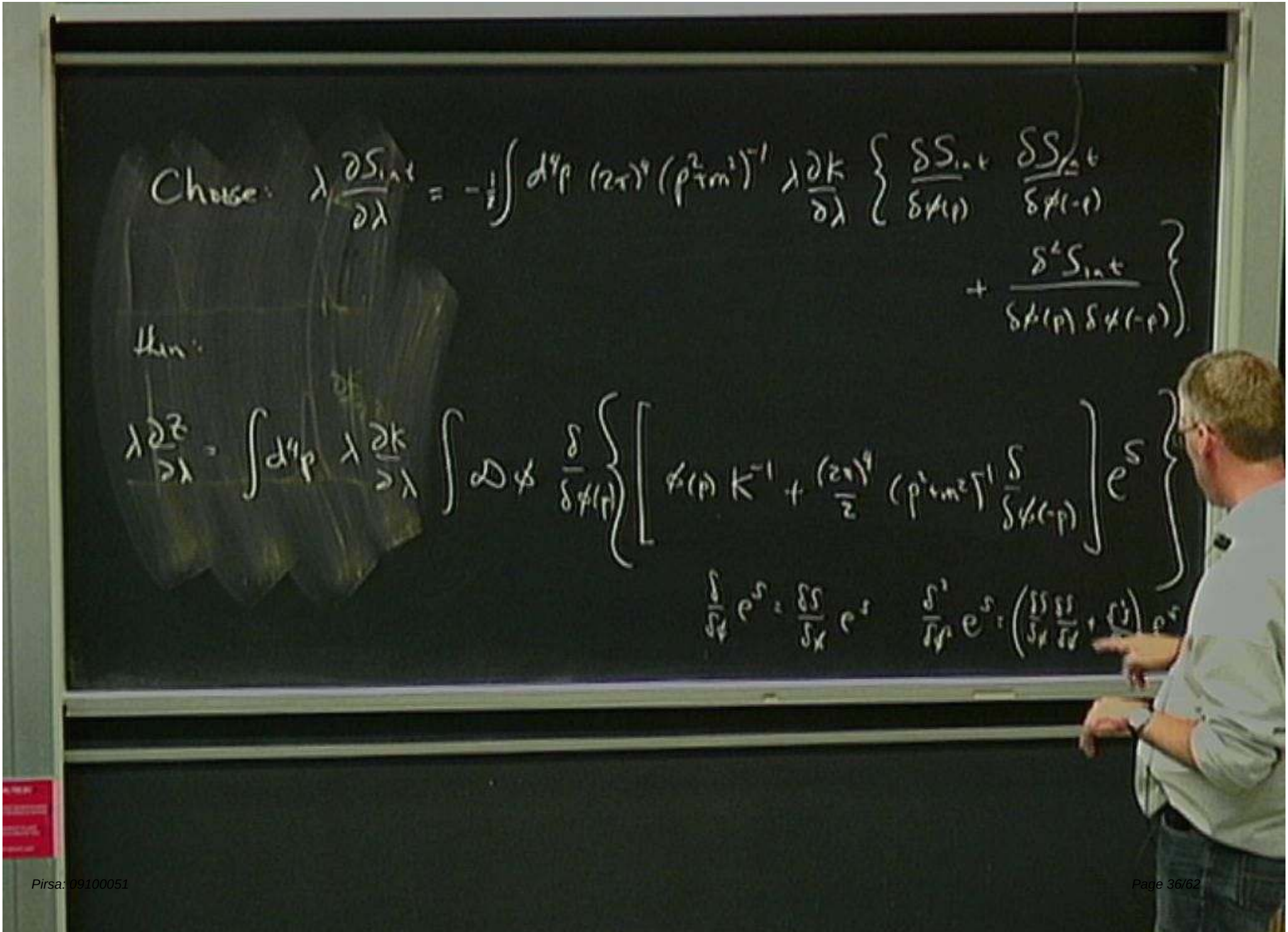

Choose: $\lambda \frac{\partial S_{int}}{\partial \lambda} = -\frac{i}{\hbar} \int d^4p\, (2\pi)^4 (p^2+m^2)^{-1} \lambda \frac{\partial k}{\partial \lambda} \left\{ \frac{\delta S_{int}}{\delta \psi(p)} \frac{\delta S_{int}}{\delta \psi(-p)} + \frac{\delta^2 S_{int}}{\delta \psi(p) \delta \psi(-p)} \right\}$

Change: $\lambda \frac{\partial S_{1.t}}{\partial \lambda} = -\frac{i}{2}\int d^4p\,(2\pi)^4 (p^2+m^2)^{-1}\,\lambda\frac{\partial k}{\partial \lambda}\left\{\frac{\delta S_{1.t}}{\delta \psi(p)}\,\frac{\delta S_{1.t}}{\delta \psi(-p)} + \frac{\delta^2 S_{1.t}}{\delta\psi(p)\,\delta\psi(-p)}\right\}$

then:


Change:
Then:



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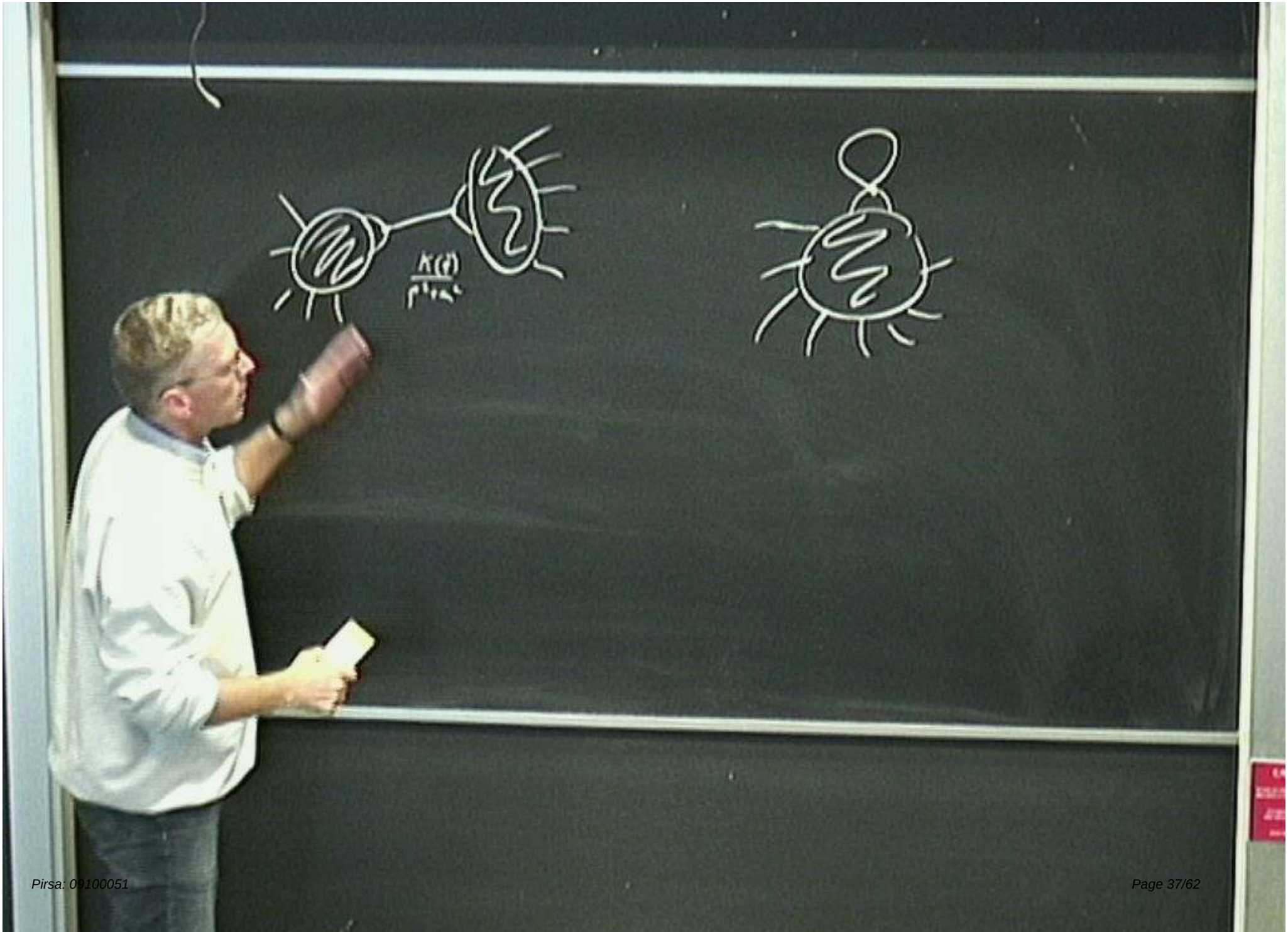
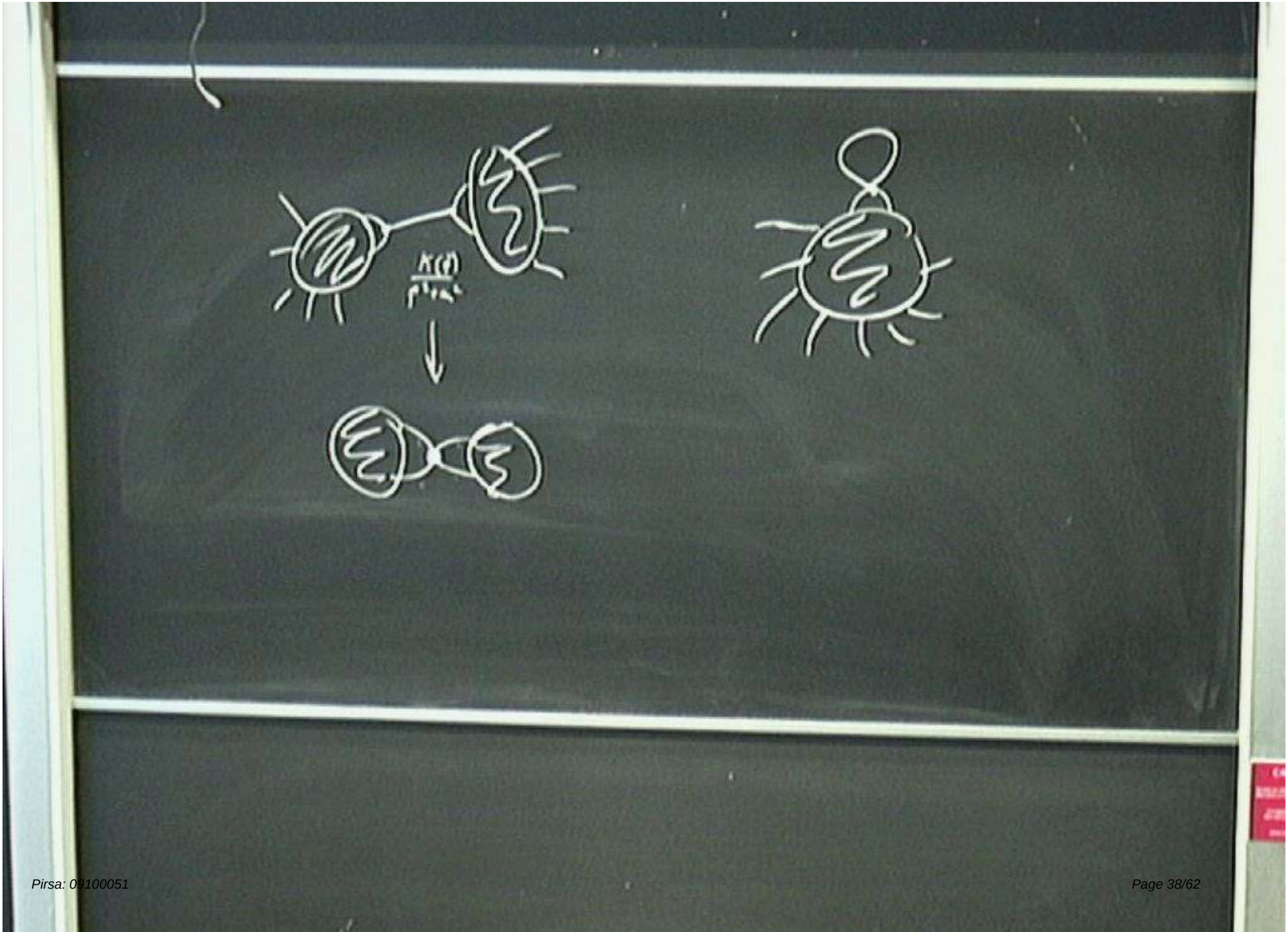
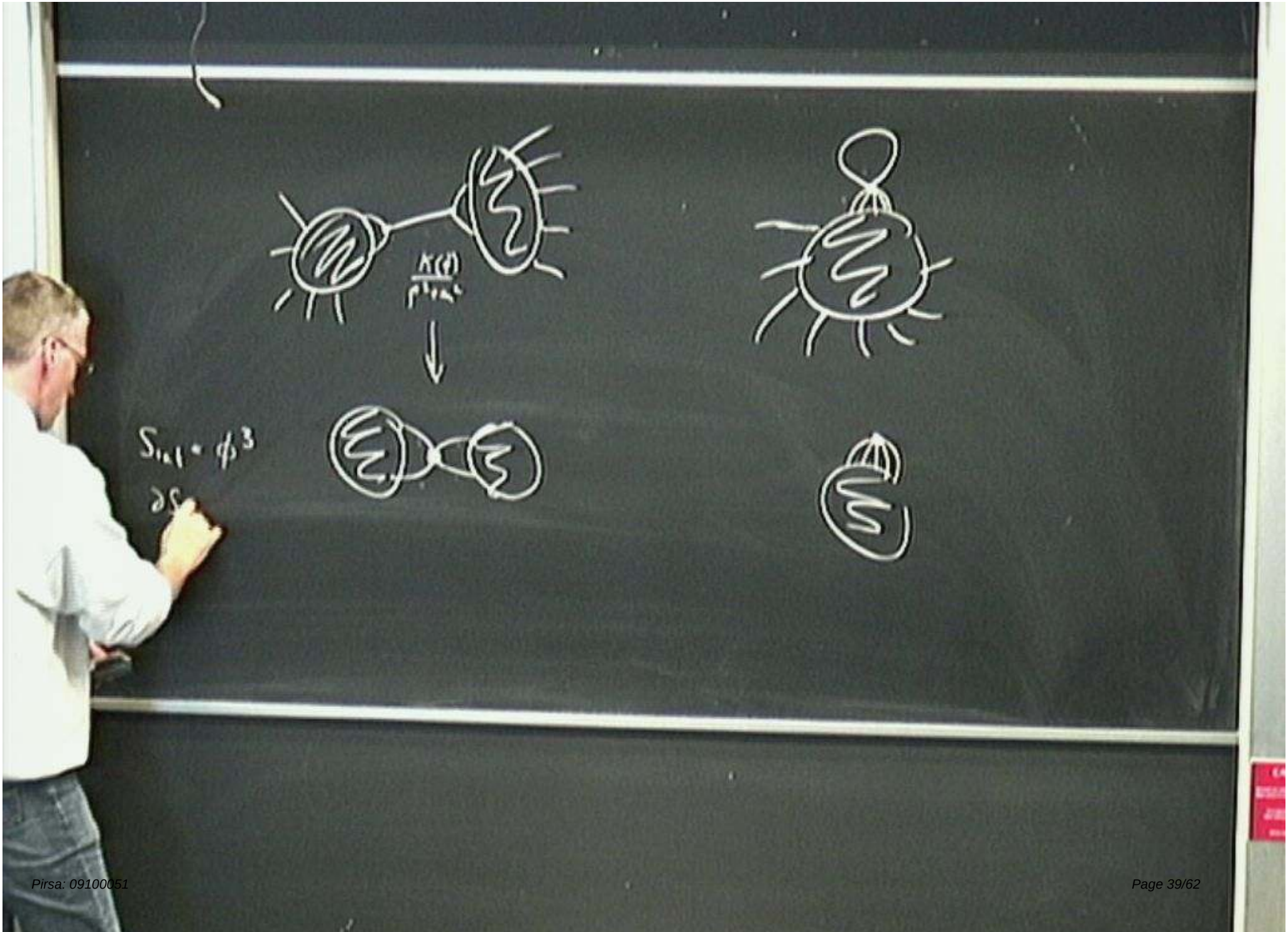
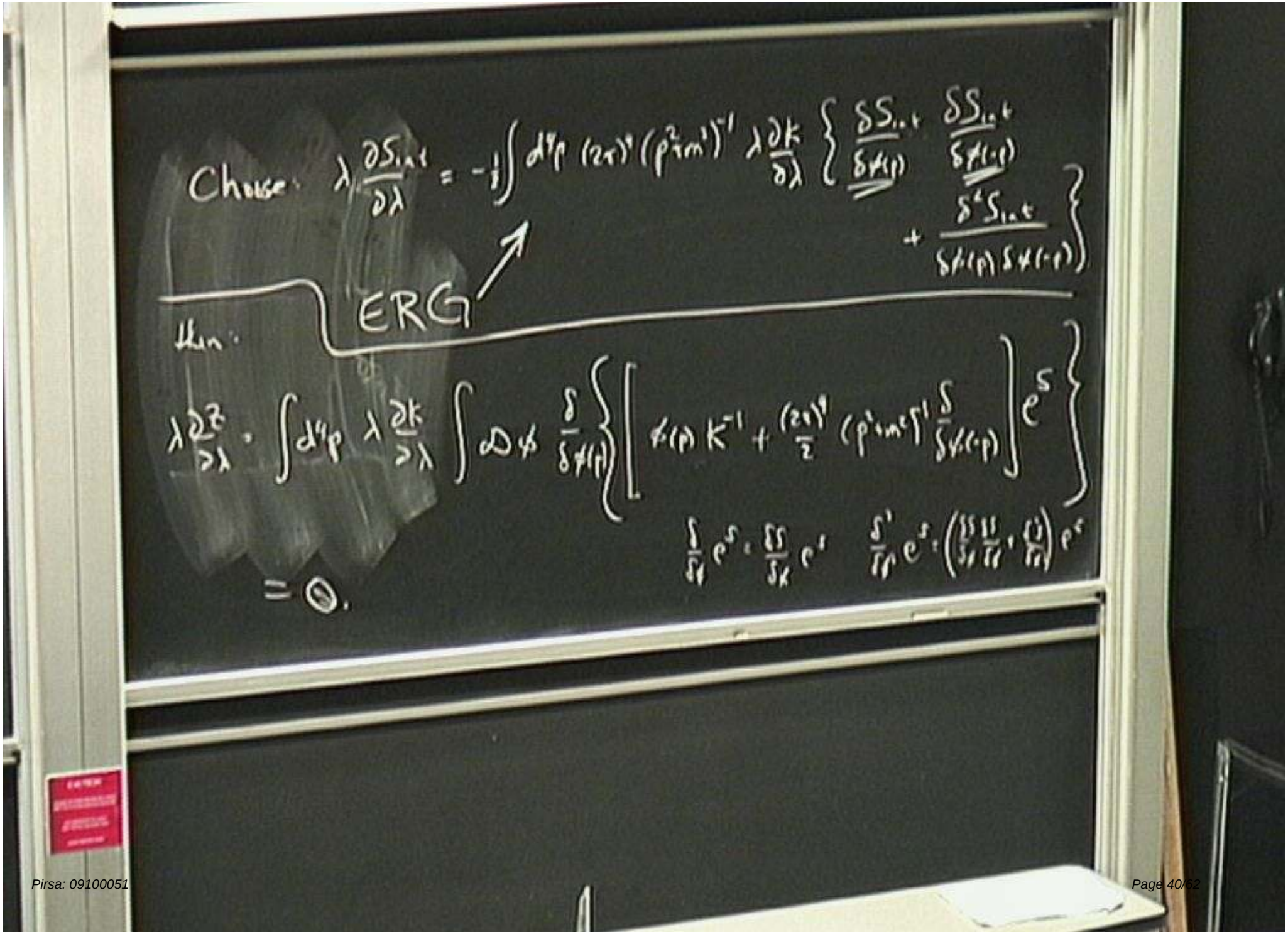





$S_{int} \sim \phi^3$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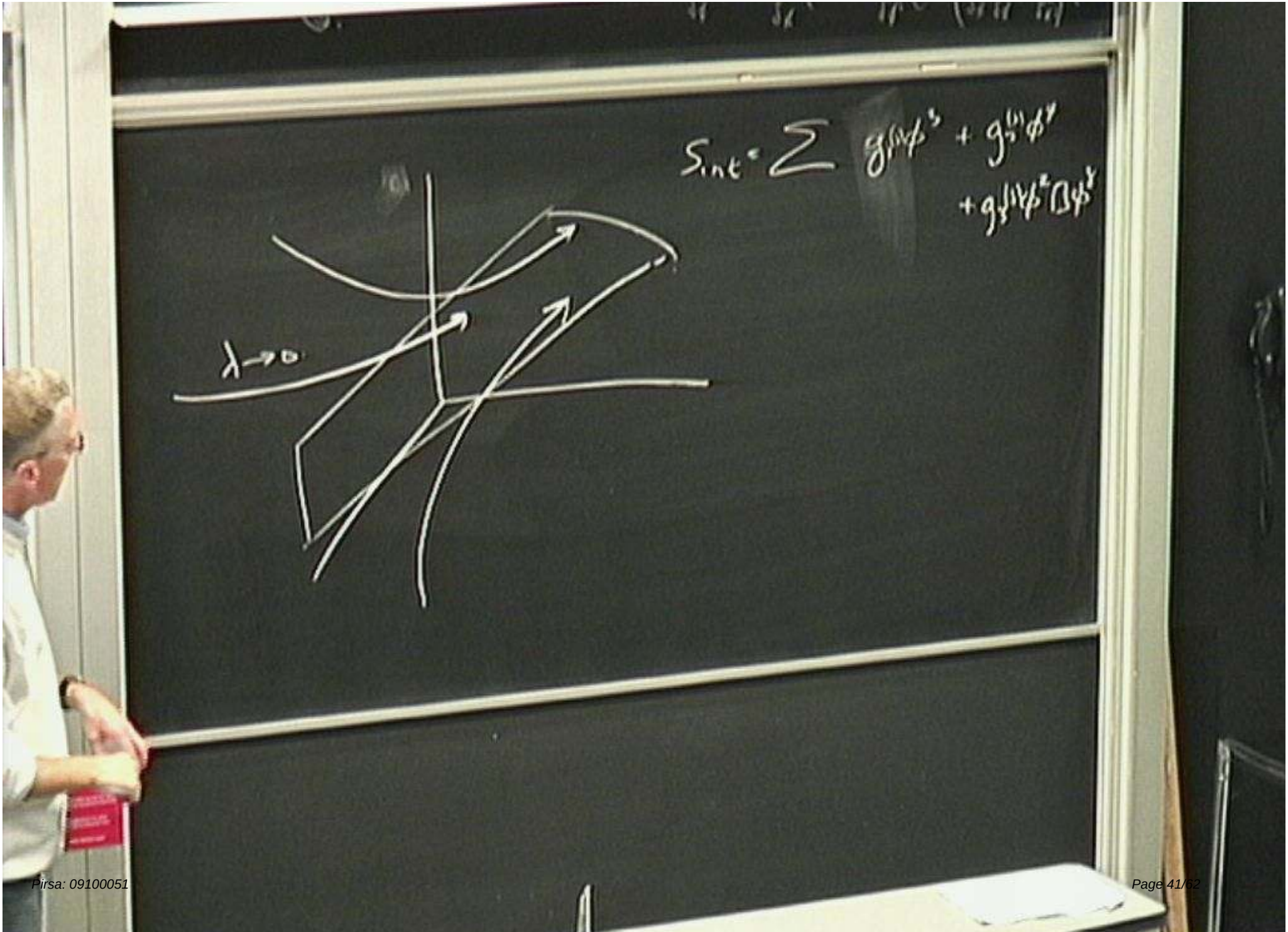
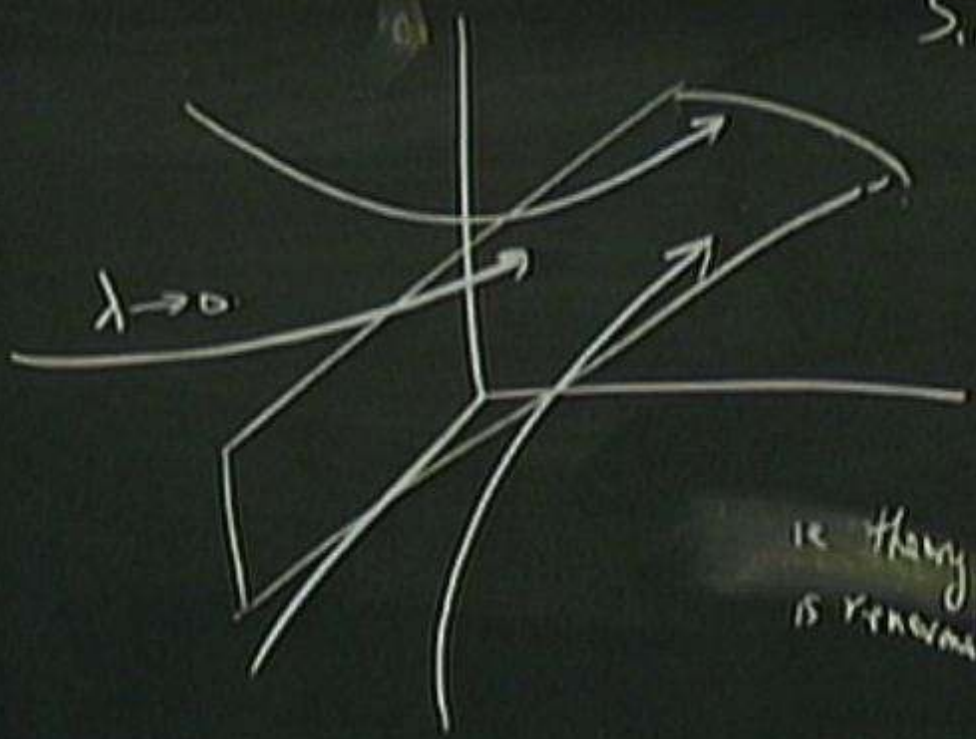

$S_{int} = \sum g_{\mu\nu}\phi^3 + g_{\mu\nu}\phi^4$
$+ g_{\mu\nu}\phi^2 \Box \phi^2$
$\lambda \to 0$

$S_{int} = \sum g_{ijk}\phi^3 + g_{ijkl}\phi^4 + g_{ijkl}\phi^2\Box\phi^2$



Polchinski's claim:

as $\lambda \to 0$ couplings flow to a finite-dimensional subsurface. need only specify a few parameters to know all of S_{int}.

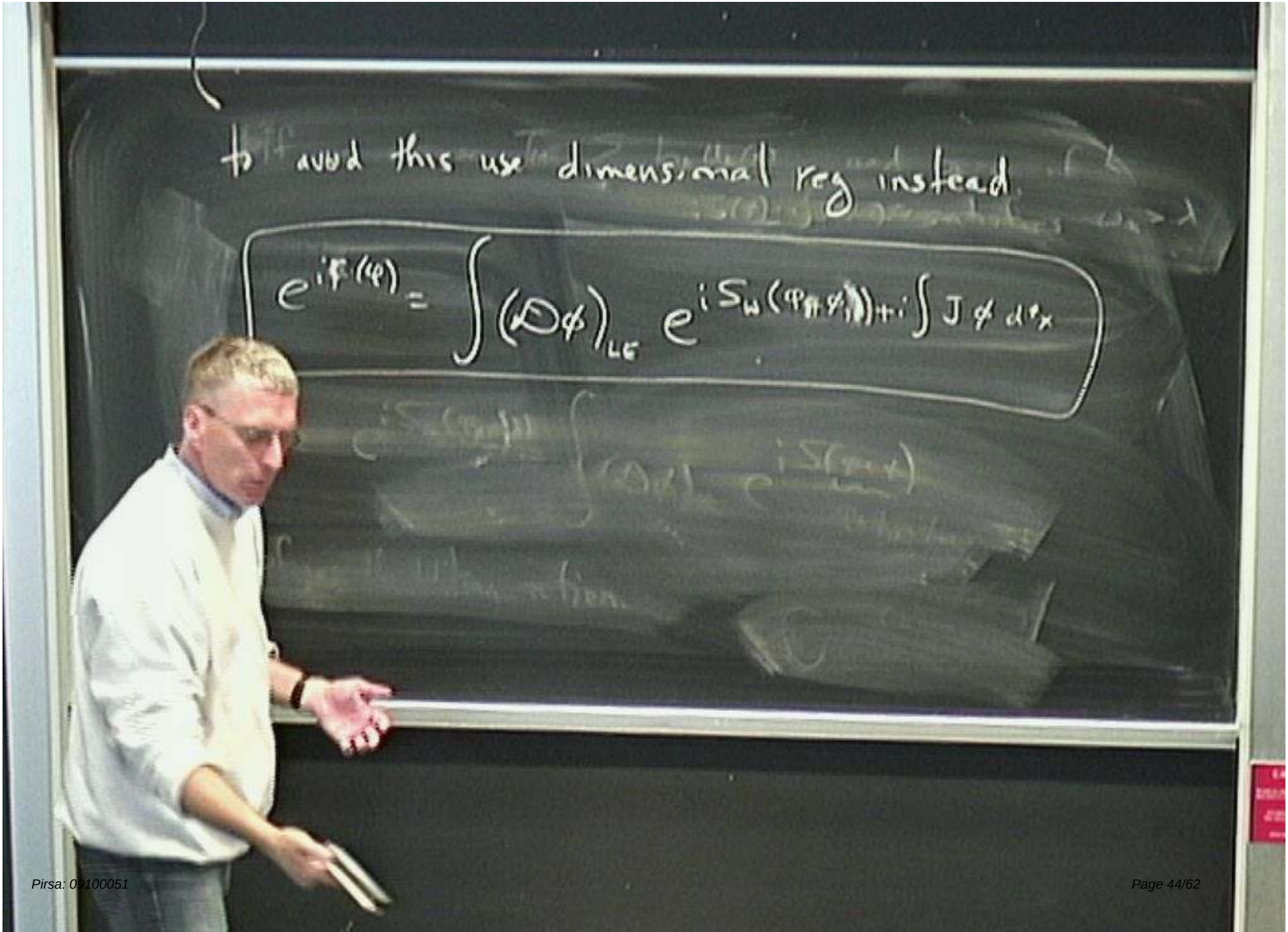
ie theory is renormalizable

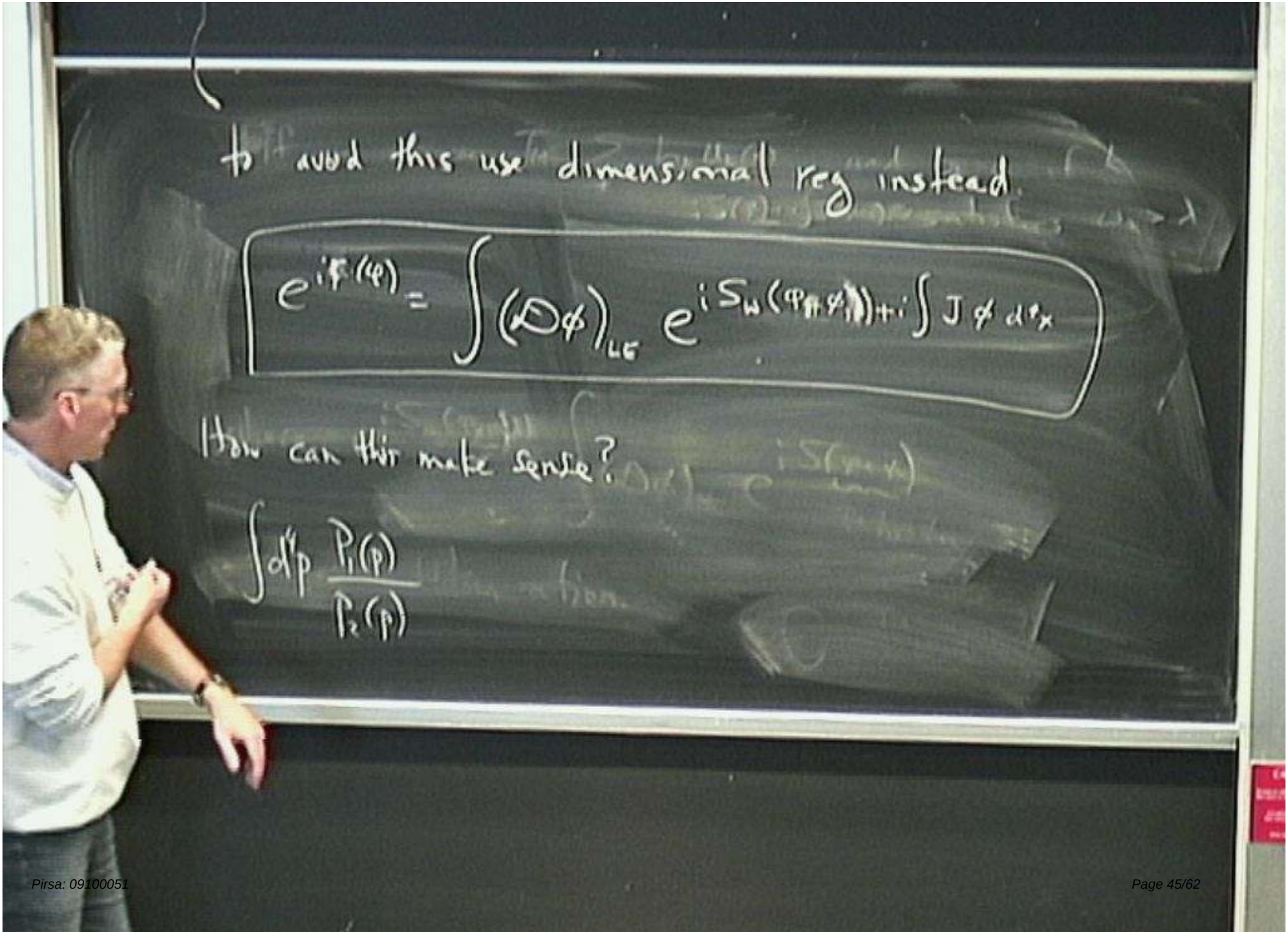
Dimensional Regularization + the Wilson action:

Cutoffs are inconvenient because:

1) introduce a large scale (Λ) for intermediate steps, that are not in physical quantities (thus complicating dimensional analysis)

2) they break gauge invariance + other symmetries.


to avoid this use dimensional reg instead
$e^{iW(J)} = \int (\mathcal{D}\phi)_{LG}\, e^{iS_W(\phi_{\parallel},\phi_{\perp}) + i\int J\phi\, d^4x}$


to avoid this use dimensional reg instead.
$e^{iW(J)} = \int (\mathcal{D}\phi)_{UG}\, e^{iS_W(\varphi+\phi)+i\int J\phi\, d^Dx}$
How can this make sense?
$\int d^Dp \frac{P_1(p)}{P_2(p)}$

to avoid this use dimensional reg instead.

$$e^{iW(\varphi)} = \int (\mathcal{D}\phi)_{UE}\, e^{iS_W(\varphi+\phi_1)+i\int J\phi\, d^4x}$$

How can this make sense?

$I_n(q) \int d^4p \, \frac{P_1(p,q)}{P_2(p)}$ where polynomials P_1, P_2 are such that P_1/P_2 does not fall faster than $1/p^4$ at large p.

to avoid this use dimensional reg instead.

$$e^{iF(\varphi)} = \int (\mathcal{D}\phi)_{LE}\, e^{iS_W(\varphi_{\#}\phi_{\#}) + i\int J\phi\, d^4x}$$

How can this make sense?

$I_n(q) \int d^4p \frac{P_1(p,q)}{P_2(p)}$ where polynomials P_1, P_2 are such that

$n \to 4$

P_1/P_2 does not fall faster than $\frac{1}{p^4}$ at large p.

to avoid this use dimensional reg instead.

$$e^{iF(\varphi)} = \int (\mathcal{D}\phi)_{LE}\, e^{iS_W(\varphi+\phi_i)+i\int J\phi\, d^4x}$$

How can this make sense?

$$I(q) \underset{n\to 4}{\int} d^n p\, \frac{P_1(p,k)}{P_2(p_i)}$$

where polynomials P_1, P_2 are such that

P_1/P_2 does not fall faster than $1/p^4$ at large p.

to avoid this use dimensional reg instead.

$$e^{iW(J)} = \int (\mathcal{D}\phi)_{UE}\, e^{iS_W(\varphi_{ij}\phi_{ij}) + i\int J\phi\, d^4x}$$

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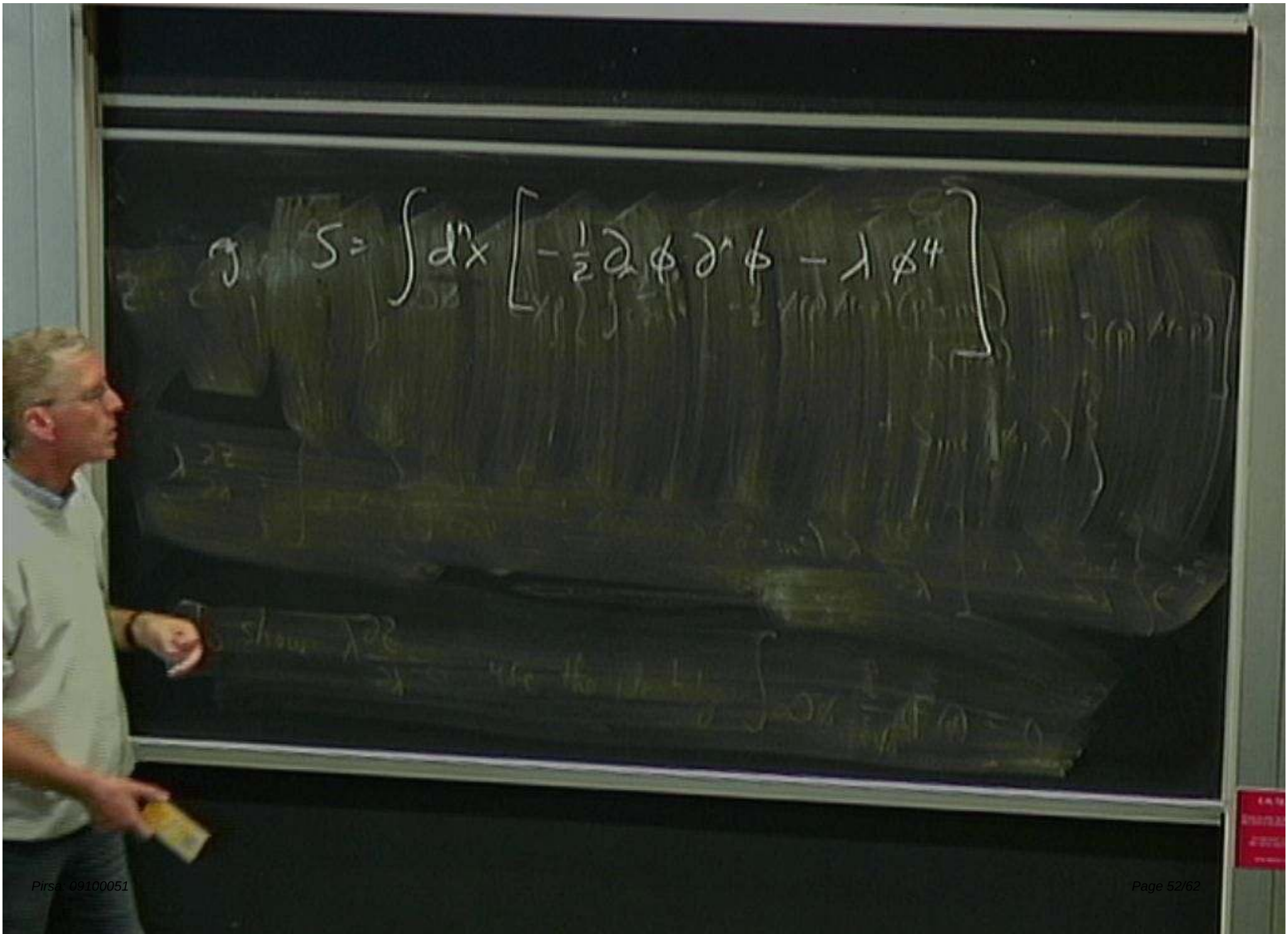
Expect $I_n(q^p)$ to converge for large p if n is sufficiently negative. Define $I_n(q)$ as a function of complex n by analytic continuation from this region.

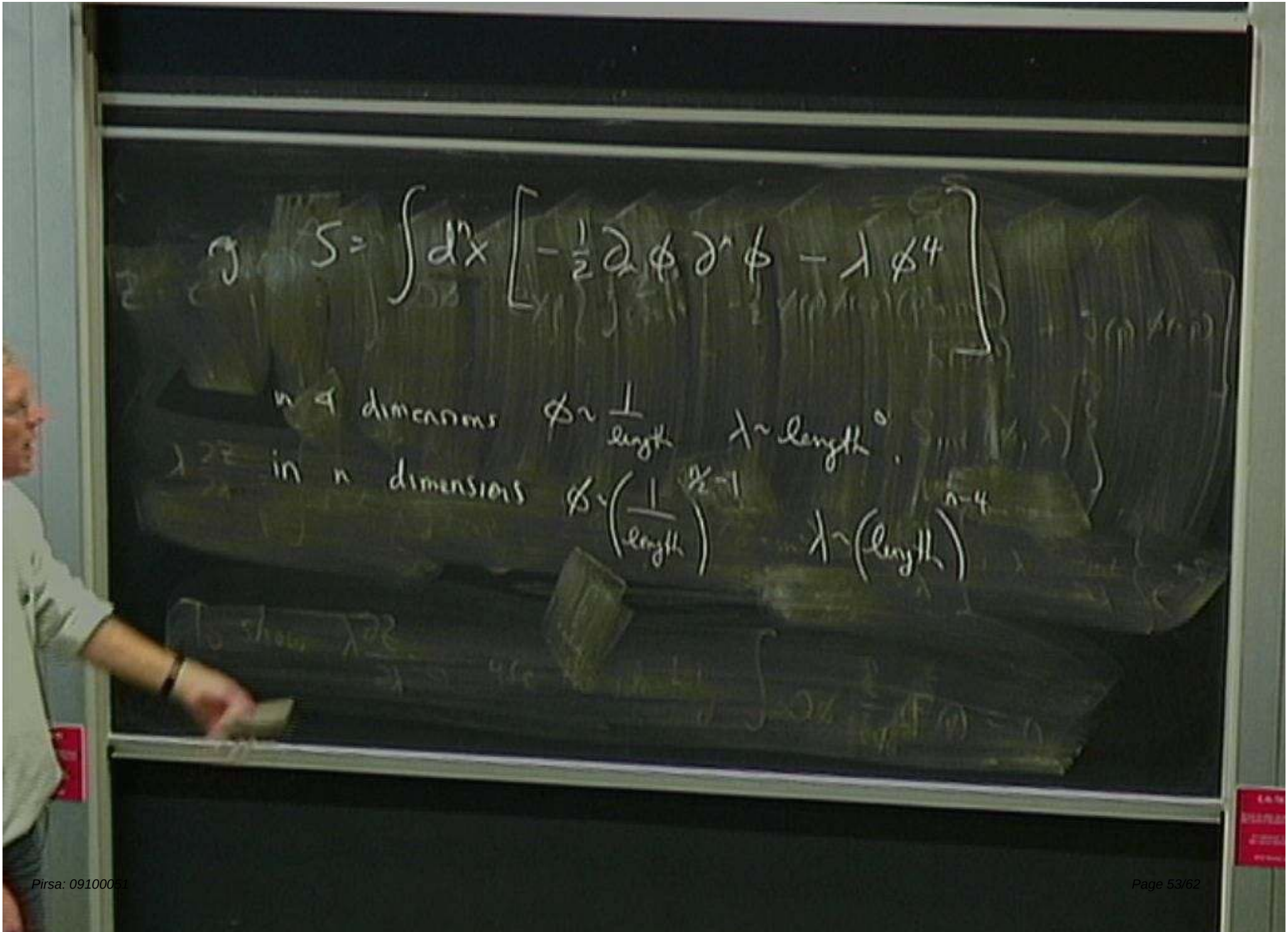
Expect $I_n(q^2)$ to converge for large p if p^2 is sufficiently negative. Define $I_n(q)$ as a function of complex n by analytic continuation from this region.

write $n = 4 - 2\epsilon$ ask for limiting behaviour as $\epsilon \to 0$.

typically get poles in $1/\epsilon$.

Pirsa: 09100051


$S = \int d^n x \left[-\frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - \lambda \phi^4 \right]$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in 4 dimensions $\phi \sim \frac{1}{\text{length}}$ $\lambda \sim \text{length}^0$
in n dimensions $\phi \sim \left(\frac{1}{\text{length}}\right)^{\frac{n}{2}-1}$ $\lambda \sim (\text{length})^{n-4}$

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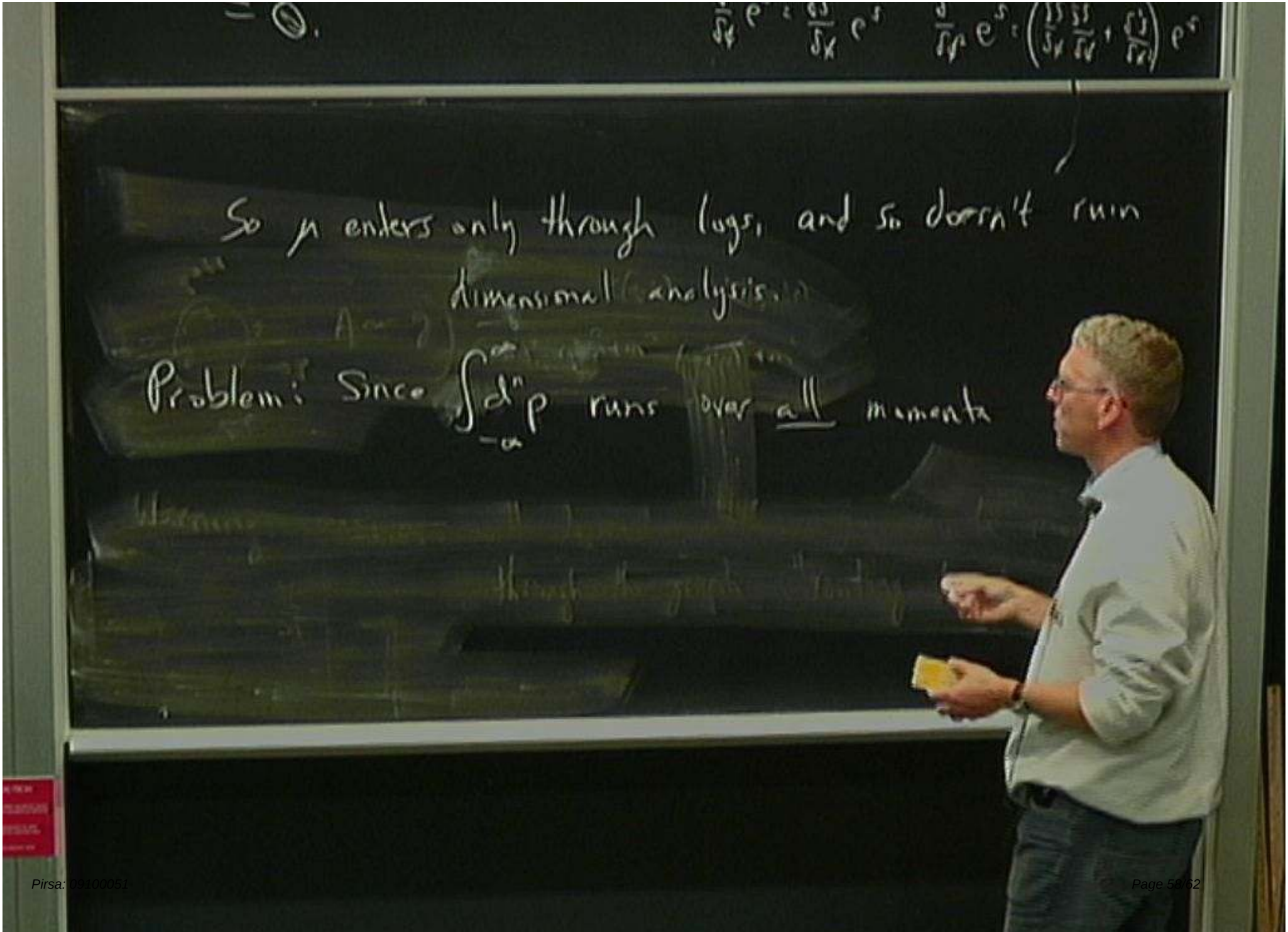
As $n \to 4$ $\mu^{(4-n)\ln}$ $n = 4 - 2\epsilon$ $\mu^{(4-n)\ln} = \mu^{2\epsilon\ln} = e^{2\epsilon \ln \mu} \approx 1 + 2\epsilon \ln \mu$ Physics!

So μ enters only through logs, and so doesn't ruin dimensional analysis.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dimensional analysis.
Problem: Since $\int_{-\infty}^{\infty} d^n p$ runs over all momenta

$= 0.$

$\frac{\partial}{\partial x} e^{s} = \frac{\partial s}{\partial x} e^{s}$ $\quad \frac{\partial}{\partial r} e^{s} = \left(\frac{\partial s}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial s}{\partial r} \right) e^{s}$

So μ enters only through logs, and so doesn't ruin dimensional analysis.

Problem: Since $\int_{-a}^{a} d^n p$ runs over all momenta (including $p > \Lambda$)

how can you remove high frequencies to get $\mathcal{L}_W$?

So μ enters only through logs, and so doesn't ruin dimensional analysis.

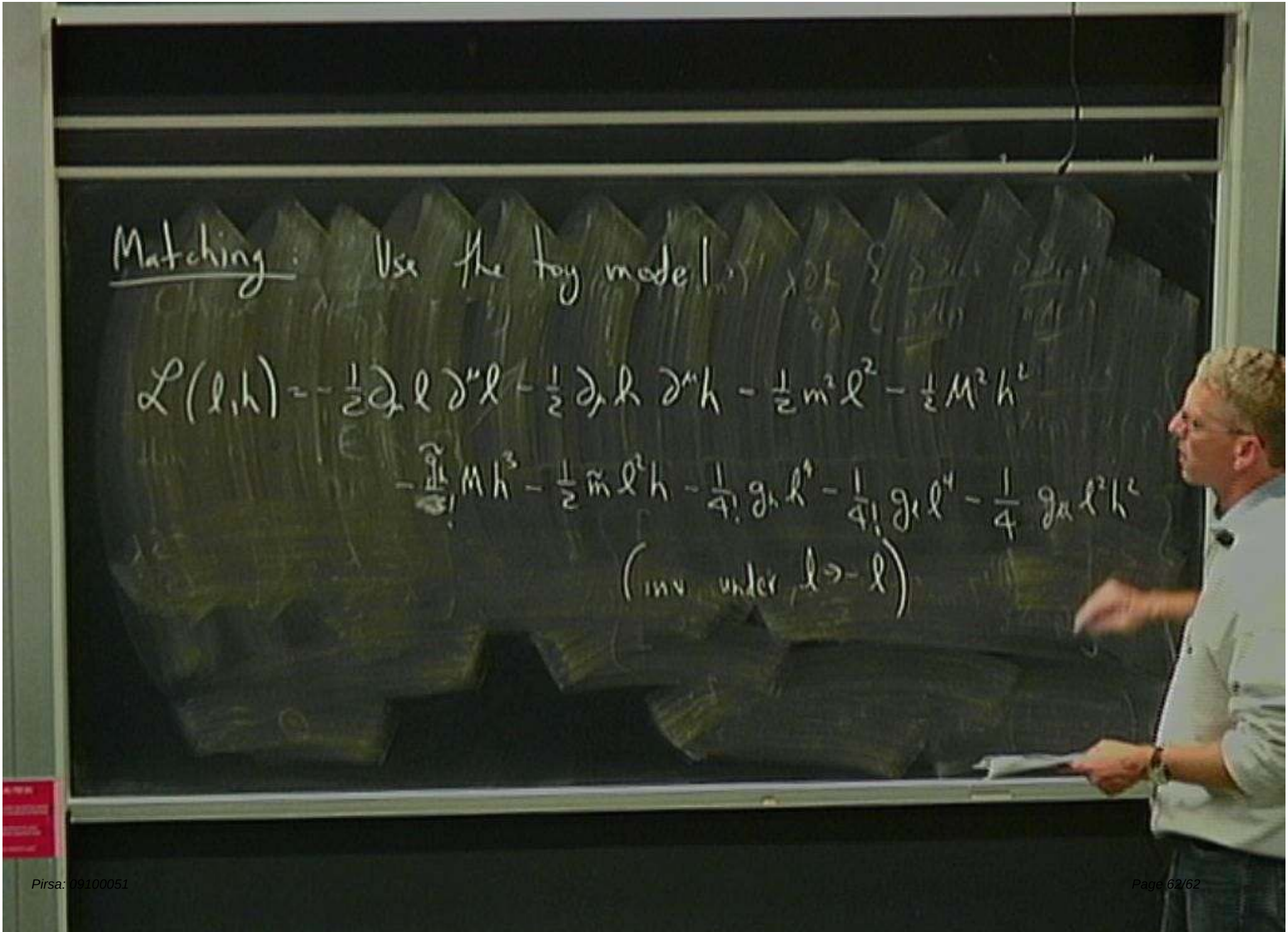
Problem: Since $\int_{-\infty}^{\infty} d^n p$ runs over all momenta (including $p \sim \Lambda$)

How can you remove high frequencies to get $\mathcal{L}_w$?

The error you make by including these high momenta can be absorbed into some local effective interaction.

Never have to calculate it explicitly if you compute α_w by "matching" to full theory.

Matching : Use


Matching: Use the toy model!
$\mathcal{L}(l,h) = -\frac{1}{2}\partial_\mu l\,\partial^\mu l + \frac{1}{2}\partial_\mu h\,\partial^\mu h - \frac{1}{2}m^2 l^2 - \frac{1}{2}M^2 h^2$
$- \frac{\tilde{\lambda}}{3!}M h^3 - \frac{1}{2}\tilde{m}\, l^2 h - \frac{1}{4!}g_h h^4 - \frac{1}{4!}g_l l^4 - \frac{1}{4}g_{hl} l^2 h^2$
(inv. under $l \to -l$)