Title: Statistical Mechanics (PHYS 602) - Lecture 1

Date: Sep 28, 2009 10:30 AM

URL: http://pirsa.org/09090111

Abstract:

Outline for Fundamentals of Statistical Physics

Leo P. Kadanoff

text:

Statistical Physics,

Statics, Dynamics, Renormalization

Leo Kadanoff

I also referred often to *Wikipedia* and found it accurate and helpful.

Course Outline

part number	text chapter number	title	length (slides)	number of lectures
		Fundamentals of Statistical Physics		
1	1	Once over Lightly	16	1
2	2 & 3	Basics	24	2
3	4 & 13 &15	Quantum Mechanics and Lattices	28	3
4	5	Diffusion and Hops	24	3
5	6 & 8	Momentum hops	27	3
6	9	Bose and Fermi	16	1
7	10	Phase Transitions & mean fields	40	2
8	11 & 12 & 13 &14 &15	Phase Transitions: Beyond mean fields	42	2
				17

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Part 1: Once over lightly

Concepts which specifically belong to statistical physics
Interesting Physical Science Advances have a Major Statistical Component
Probabilities: One die
Quantum Stat Mech
Classical Stat Mech
Thermodynamics
Random Walk
Brownian Dynamics
From Quantum to Classical: The Ising model
Averages from Derivatives
Degenerate Distributions
Thermodynamic Phases
Phase Transitions
Big Words

Part 2: Statistics

Probabilities
- Simple probabilities
- averages
- Composite probabilities
- Independent events
- simple and complex
- Many dice
- Probability distributions

Statistical Mechanics
- Hamiltonian description
- Statistical Independence
- one and many
- Structural Invariance
- Intensive and Extensive

Gaussian
- Statistical Variables
- Integrals and Probabilities
- Statistical Distributions
- Averages
- Approximate Gaussian integrals

Calculation of Averages and Fluctuations
- The Result
- Going Slowly
- sums and averages in classical mechanics
- more sums and averages
- homework

Part 3: On a Lattice:From Quantum to Ising to RG

- From Classical Stat Mech to Quantum
  - All of quantum mechanics
  - one dimensional lattice
  - from classical to quantum
  - Summary
  - from quantum to classical
  - the path integral for particles
- The Linear Chain
  - Ising model
  - Transfer Matrix
  - Dual Couplings
  - Solution of One-Dimensional Ising Model
- On Quantum Chains
  - Ground state averages
  - Statistical Correlations
  - Average magnetizations
  - Correlations
  - Correlation Length
  - Bloch Wall
- Renormalization for 1D Ising
  - Block Transform
  - New Coupling
- 2D Ising
  - High Temperature Expansion
  - Low Temperature Expansion
  - Duality
  - Specific Heat
  - Block Transform
  - Physical quantities
  - Flows
- Homework

Part 6: Bosons and Fermions

Part 7: More is the Same, continued

- Mean field theory of fluids
  - from van der Waals to Weiss
  - van der Waals' first effect
  - van der Waals' second effect Many Different Phase Transitions
- After van der Waals
- Landau Mean Field Theory
  - order parameter generalized
  - generalized mean field scheme
  - recent example
  - why minimize
  - very M to vary F
  - h=0, no space variation
  - jump like square root of Tc-T
- Summary
  - order parameter and free energy were basic
- Look ahead:
  - a worry

Part 8: Beyond Mean ... continued

After the Revolution
- 2d XY model
- RG point of view is absorbed into particle physics
- Coulomb gas
- Feigenbaum and routes to chaos
- operator product expansion
- field theory
- summary

Conformal Field Theory
- Polyakov, Virasoro algebra
- Friedan, Qiu, Shenker
- correlation functions
- many exact calculations
- quantum gravity

SLE
- Schramm
- critical shapes
- ensemble of critical shapes

Concepts which specifically belong to statistical physics:

Not in few particle quantum mechanics or in Classical Mechanics

- Temperature

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Interesting Physical Science Advances have a Major Statistical Component

Bekenstein-Hawking: entropy of black holes

Fluctuation spectrum of 3 degree kelvin background radiation

Bell's theorem: statistics of quantum measurements

source of complexity in the universe

probabilities of hearing from civilizations elsewhere in universe

Why do markets crash?

Time Reversal Invariance: Nature of Irreversability

Probabilities of major earth-asteroid collision

Probabilistic interpretation of quantum mechanics and of wave functions.

Is our universe likely?

Part 2. Start with Probabilities: Dice

number of times α turns up = N_α; total number of events N

probability of choosing a side with number $\alpha = \rho_\alpha$ $\rho_\alpha = N_\alpha / N$ i.1

total probability =1 --> $$\sum_\alpha \rho_\alpha = 1$$ i.2

r_α = relative probability of … $\Sigma_\alpha r_\alpha$ $\rho = r_\alpha / z$

fair dice --> all probabilities … ll values of α

average number on a throw … $\alpha = 3.5$

general rule: To calculate th… function $f(\alpha)$ that gives the p… will come out will be α, you … $(\alpha)\rho_\alpha$ i.3

Do we understand what th… loaded die? An average from a loaded die?… the other values, and these others were all … hat would we have for the average throw on t…

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probability of choosing a side with number $\alpha = \rho_\alpha$ $\rho_\alpha = N_\alpha / N$ i.1

total probability =1 --> $$\sum_\alpha \rho_\alpha = 1$$ i.2

r_α = relative probability of event α. e.g. for fair dice r_α = const $z = \sum_\alpha r_\alpha$ $\rho = r_\alpha / z$

fair dice --> all probabilities are equal --> $\rho_\alpha = 1/6$ for all values of α

average number on a throw =

$$< \alpha >= \sum_\alpha \rho_\alpha \alpha = 3.5$$

general rule: To calculate the average of any function $f(\alpha)$ that gives the probability that what will come out will be α, you use the formula

$$< f(\alpha) >= \sum_\alpha f(\alpha) \rho_\alpha$$ i.3

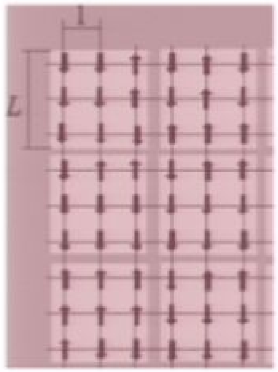
Part 3: Lattices

Renormalization for d-2 Ising model

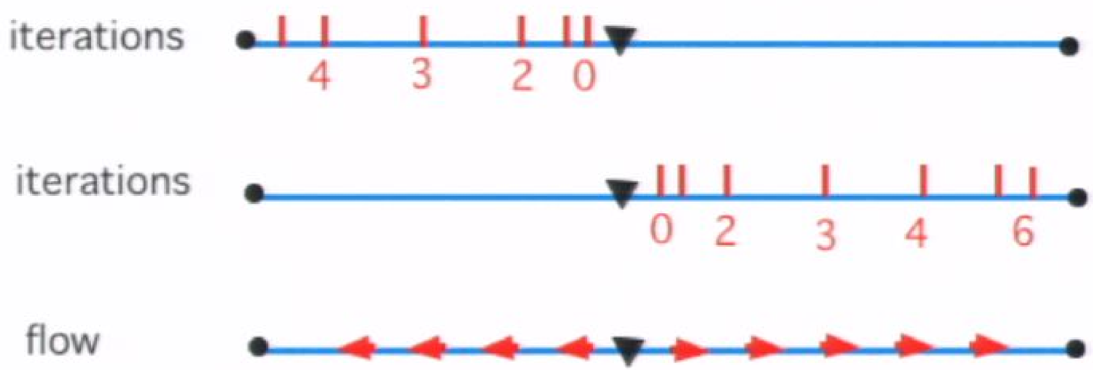
A. Pokrovskii & A. Patashinskii, Ben Widom, myself, Kenneth Wilson.

Z=Trace$_{\{\sigma\}}$ exp($W_K\{\sigma\}$)

nagine that each box in the picture has in it a
iriable called $\mu_{\mathbf{R}}$, where the **R**'s are a set of new
ttice sites with nearest neighbor separation 3*a*. Each
ew variable is tied to an old ones via a
enormalization matrix $G\{\mu, \sigma\} = \prod_{\mathbf{R}} g(\mu_{\mathbf{R}}, \{\sigma\})$
here g couples the $\mu_{\mathbf{R}}$ to the
's in the corresponding box. We take each $\mu_{\mathbf{R}}$ to
e ±1 and define g so that,
Σ_μ g(μ,{s}) =1. For example, μ might be
efined to be an Ising variable with the
ame sign as the sum of σ's in its box



fewer degrees of fi
produces "block renor



• stable fixed point

▼ unstable fixed point

Part 4: Random Walks & Diffusion

http://particlezoo.files.wordpress.com/2008/09/randomwalk.png

Part 5 : Statistics of Motion

Albert Einstein (1905) explained this dancing by many, many collisions with molecules in fluid

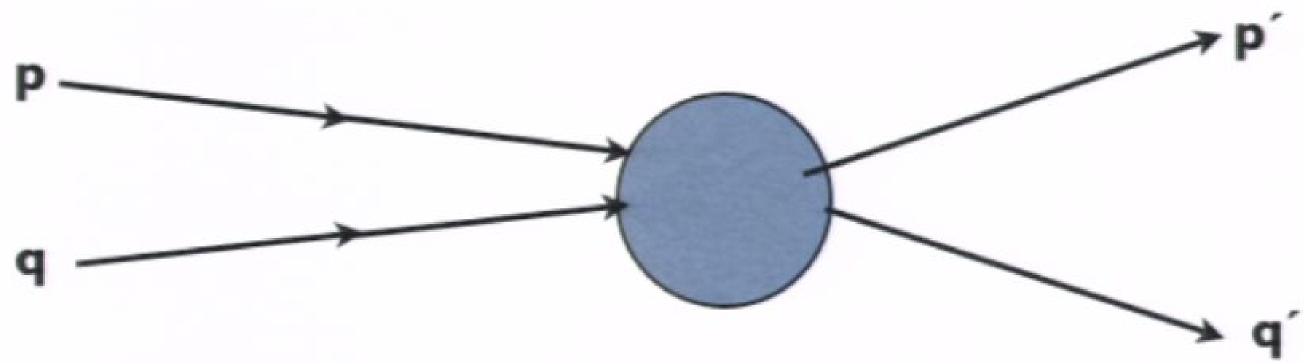
$$dp/dt = ... + \eta(t) - p/\tau$$

$p = (p_x, p_y, p_z)$ $\eta = (\eta_x, \eta_y, \eta_z)$

$\eta(t)$ is a Gaussian random variable resulting from random kicks produced by collisions. Since the kicks have random directions $\langle \eta(t) \rangle = 0$. Different collisions are assumed to be statistically independent

$$\langle \eta_j(t)\, \eta_k(s) \rangle = \Gamma \delta(t-s) \delta_{j,k}$$

$$\partial_t f(\mathbf{p},\mathbf{r},t) + (\mathbf{p}/m) \cdot \nabla_{\mathbf{r}} f(\mathbf{p},\mathbf{r},t) - \nabla_{\mathbf{r}} U(\mathbf{r},t) \cdot \nabla_{\mathbf{p}} f(\mathbf{p},\mathbf{r},t) = \text{ effects of collisions}$$



Part 6: Bose & Fermi

particle statistics, i.e. the symmetry properties of the particles' wave functions, have a major role in determining the behavior of many interesting physical systems. This is especially true when the system is degenerate, i.e. there is a sufficiently high density of identical particles so that there could be a substantial overlap of the wave functions involved. Important degenerate systems include:

for fermions:

- the electrons in atoms
-

for non-conserved bosons

-

for conserved bosons:

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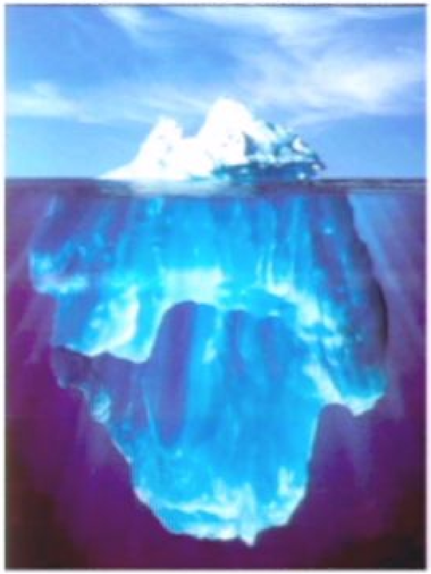
Part 7: Phase Transitions and Mean Fields

phases of matter:

-



which symmetries of nature have been lost in the snowflake?

-
-

are they really lost?

Part 8: After Mean Fields: Big Words

Universality:

In appropriate limits, very different systems can have essentially identical properties

Scale Invariance

Systems look the same at different spatial scales

Renormalization

Take advantage of scale invariance and universality to produce a theory of phase transitions.

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Take advantage of scale invariance and universality to produce a theory of phase transitions.

A start:

Ising system has as its basic variable a spin, σ_z which takes on the values ±1.

We shall use the abbreviation, σ for this spin.

The behavior of a physical system is described by its Hamiltonian. If we put this spin in a magnetic field in the z-direction it has a Hamiltonian $H = -\mu B_z \sigma$.

Statistical Mechanics is defined by a probability. Here the probability is

$\rho(\sigma) = (1/z) \exp[-H/(k_B T] = (1/z) \exp[\mu B_z \sigma/(k_B T)]$

We describe this by using the abbreviation, *h*, for the parameters in the probability

$\rho(\sigma) = (1/z) \exp(h \sigma) \qquad h = \mu B_z /(k_B T)$

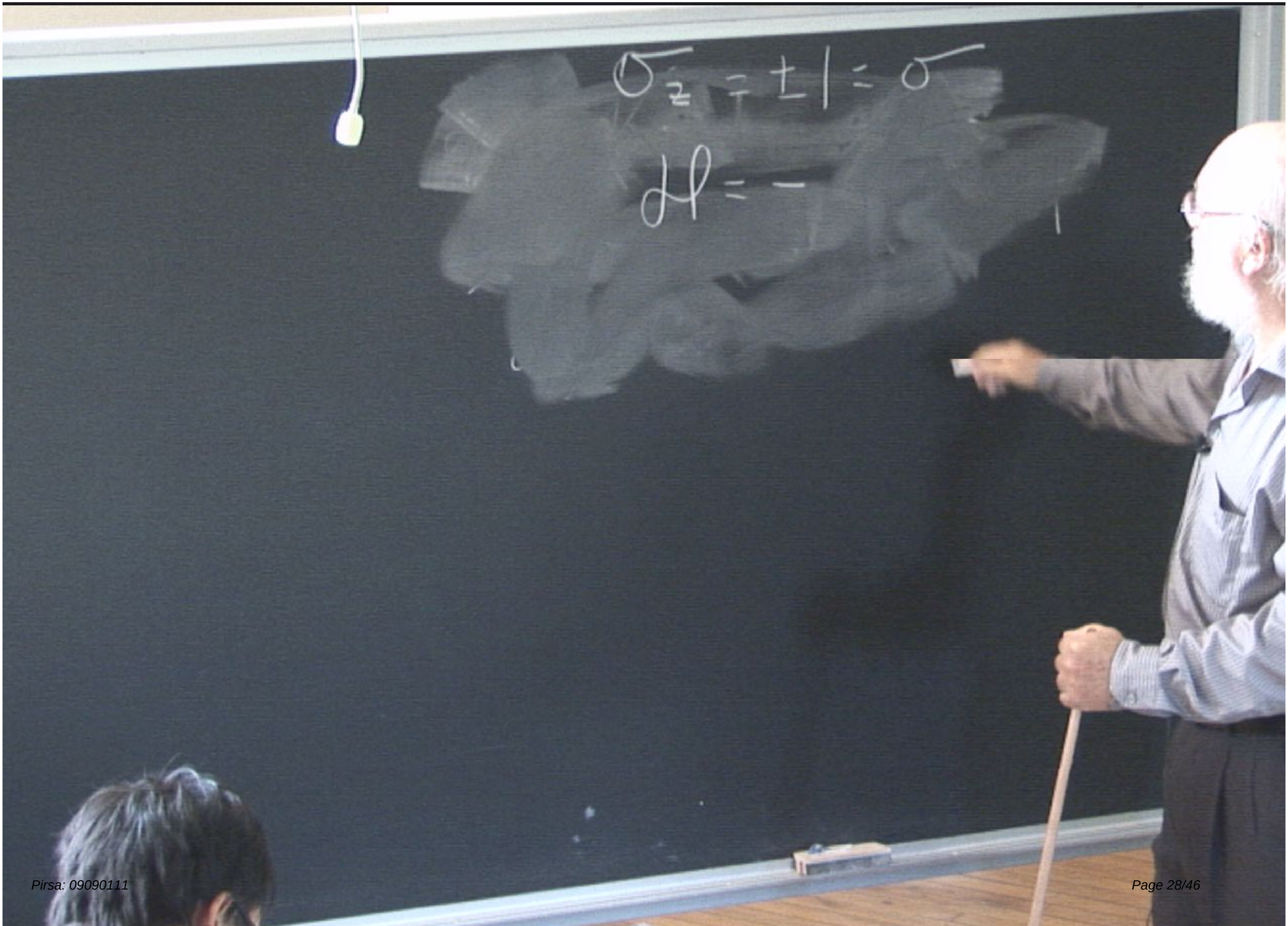
normalization: total probability $=1= \rho(1) + \rho(-1) = (1/z) \exp(h) + (1/z) \exp(-h)$

therefore $z = \exp(h) + \exp(-h) = 2 \cosh h$

average $X = \langle X \rangle = \sum_\alpha \rho(\alpha) X_\alpha$

therefore $\langle \sigma \rangle = \rho(1)1 + \rho(-1)(-1) = 1/(2 \cosh h) \{\exp(h) - \exp(-h)\}$

$= (2 \sinh h) /(2 \cosh h) = \tanh h$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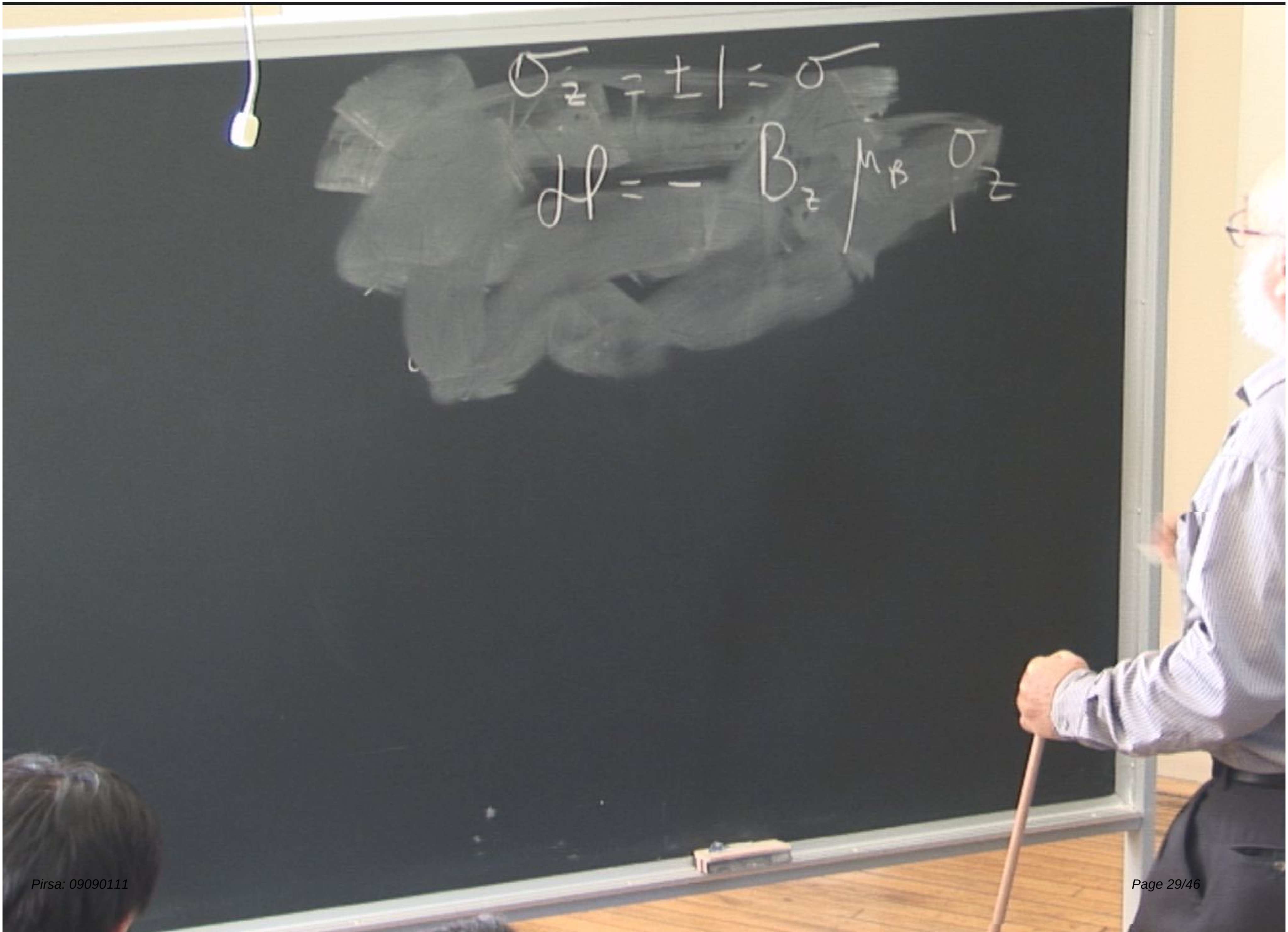
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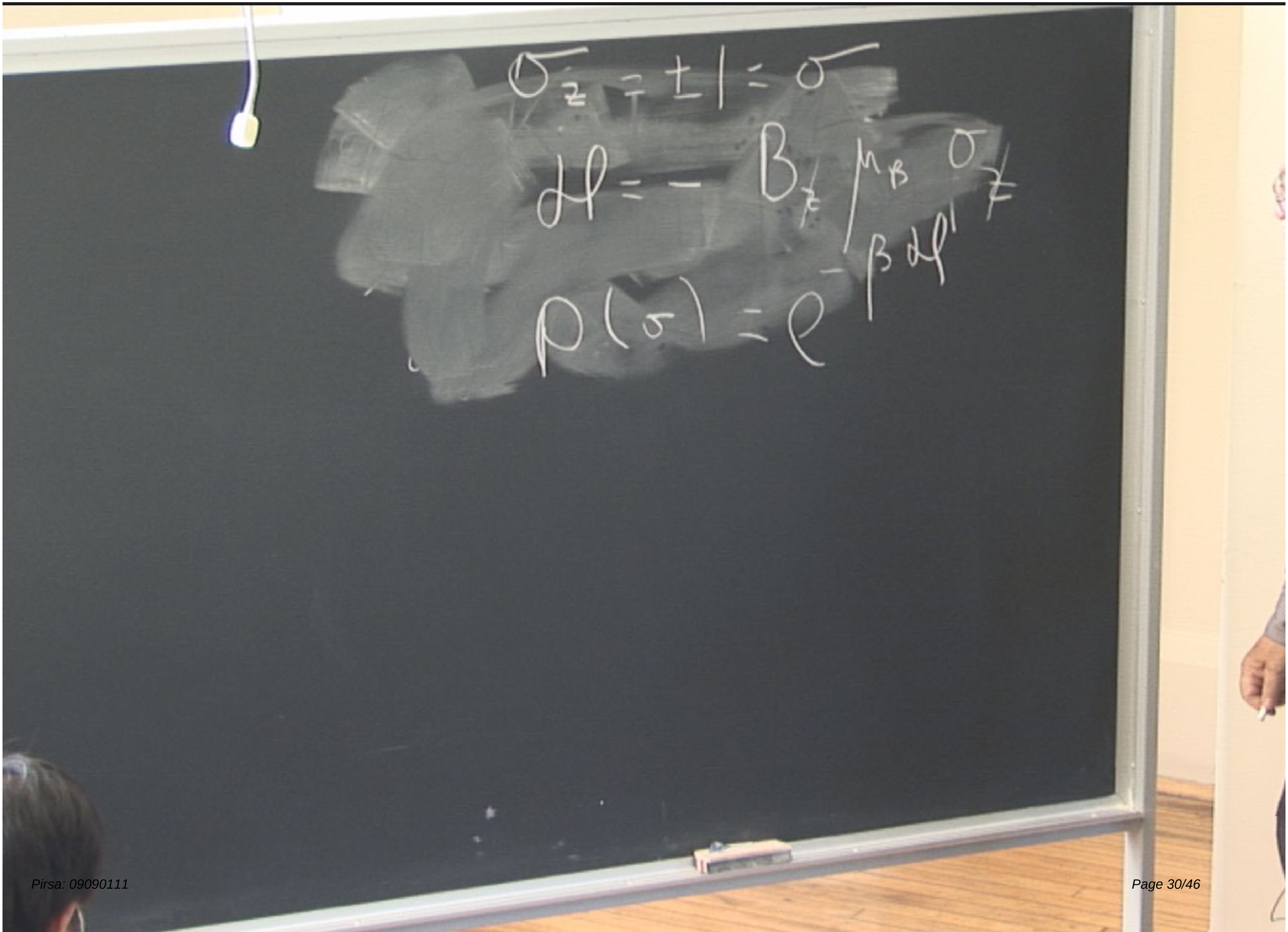
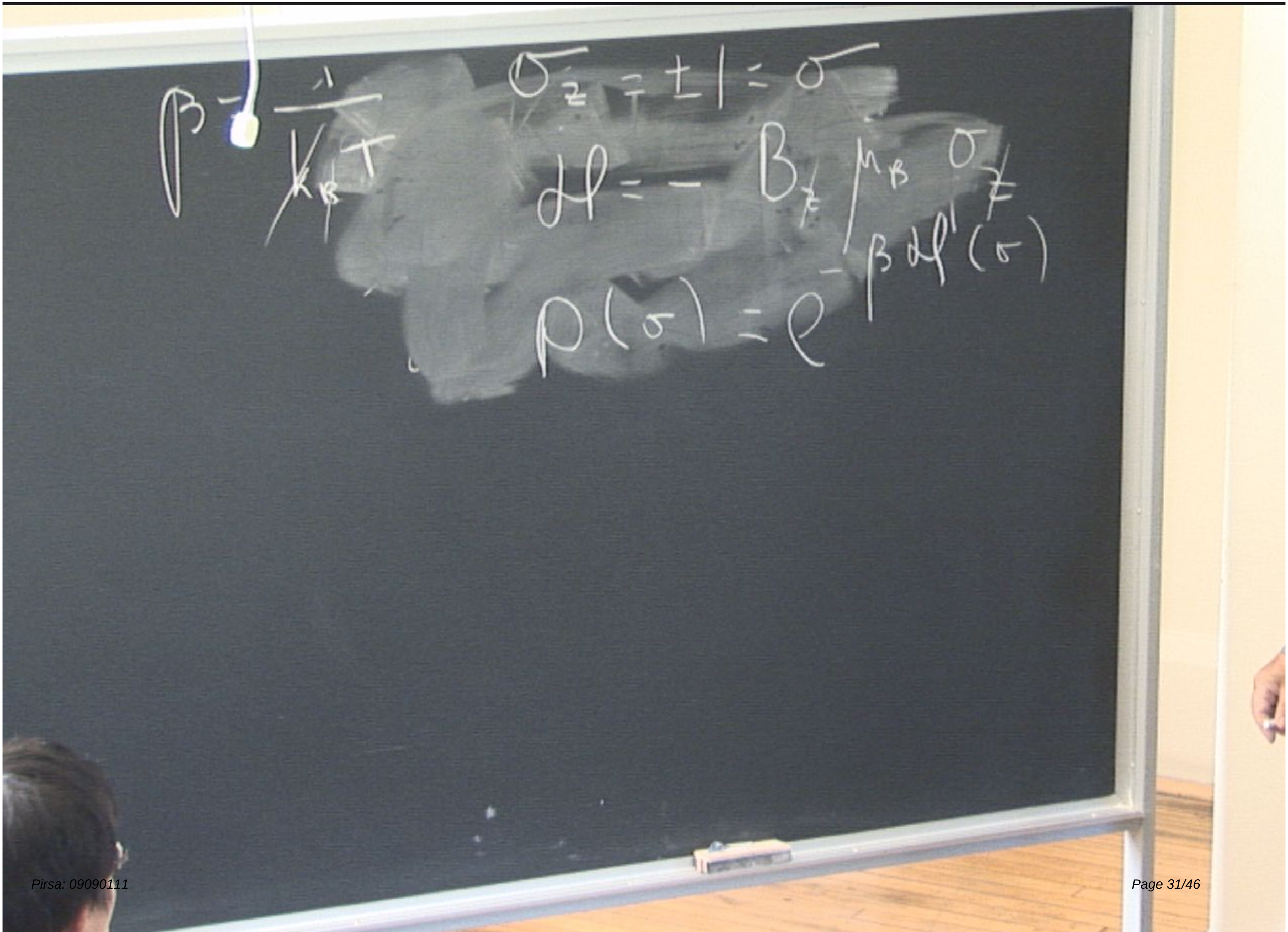

σz = ±1 = σ
H = − Bz μB σz
ρ(σ) = e^{−βH}


$\beta = \frac{1}{k_B T}$
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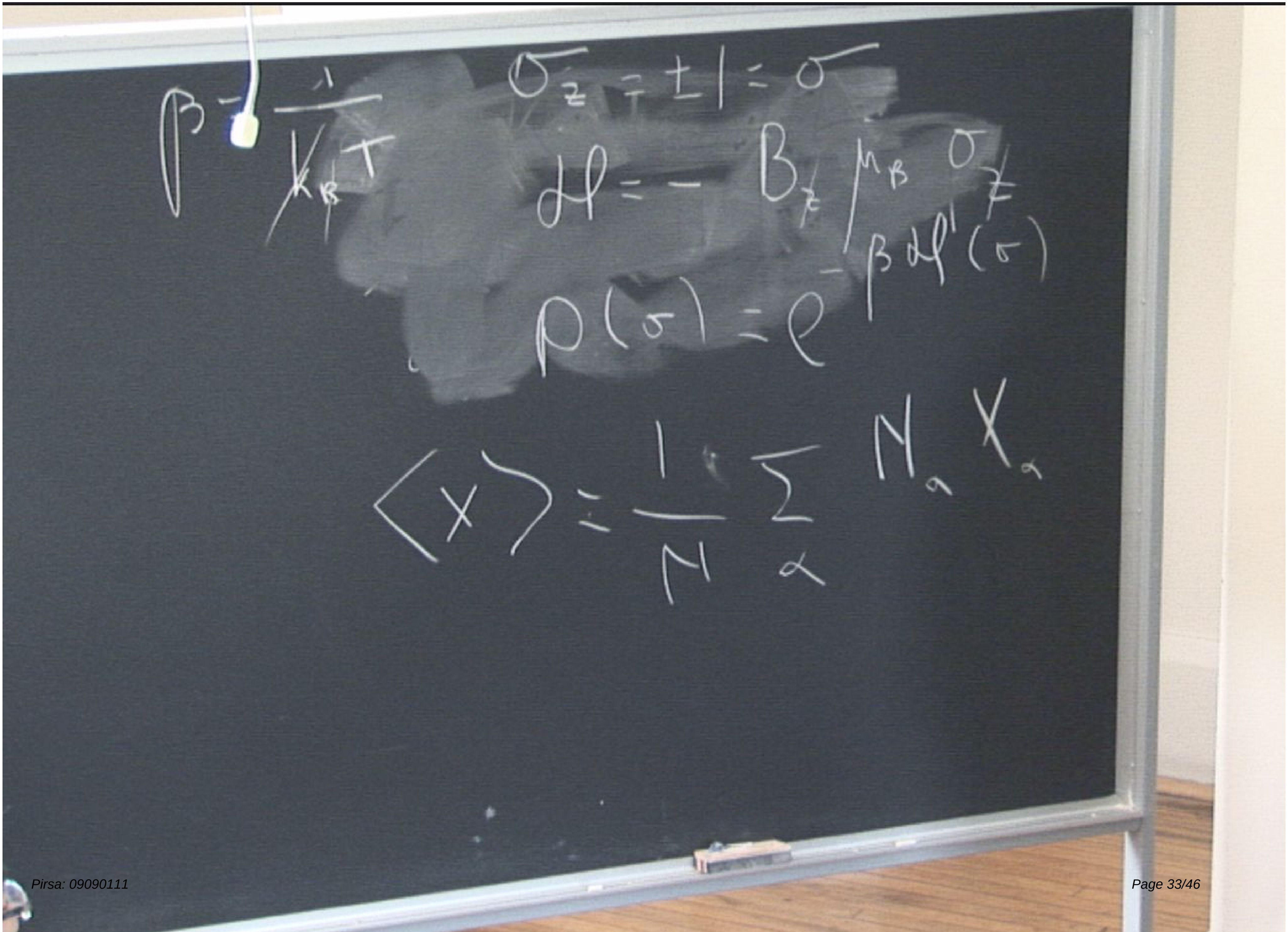
$\beta = \frac{1}{k_B T}$
$\sigma_z = \pm 1 = \sigma$
$\mathcal{H} = - B_z \mu_B \sigma_z$
$p(\sigma) = e^{-\beta \mathcal{H}'(\sigma)}$
$\langle X \rangle = \frac{1}{N} \sum_\alpha N_\alpha X_\alpha$

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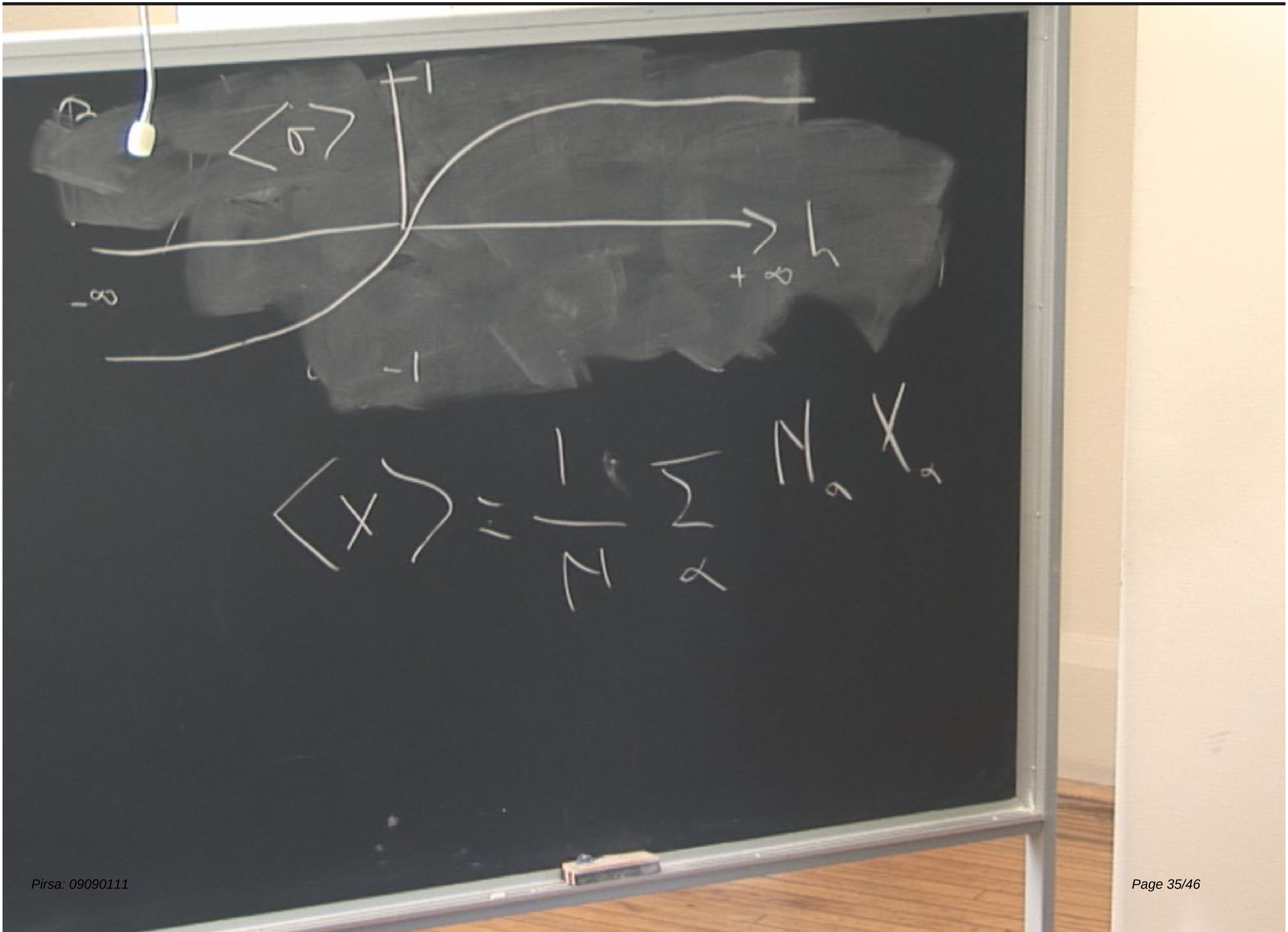


⟨σ⟩
+1
−1
h
+∞
−∞
⟨x⟩ = (1/M) Σ_α M_α X_α

Averages from Derivatives

$$z = \sum_{\sigma} \exp(h\sigma) = 2\cosh h$$

$$d(\ln z) / dh = \sum_{\sigma} \sigma \exp(h\sigma) / z = < \sigma > = \tanh h$$

$$d^2(\ln z) / (dh)^2 = \sum_{\sigma} (\sigma - < \sigma >)^2 \exp(h\sigma) / z = < (\sigma - < \sigma >)^2 >$$

$$= 1 - < \sigma >^2 = 1 - (\tanh h)^2$$

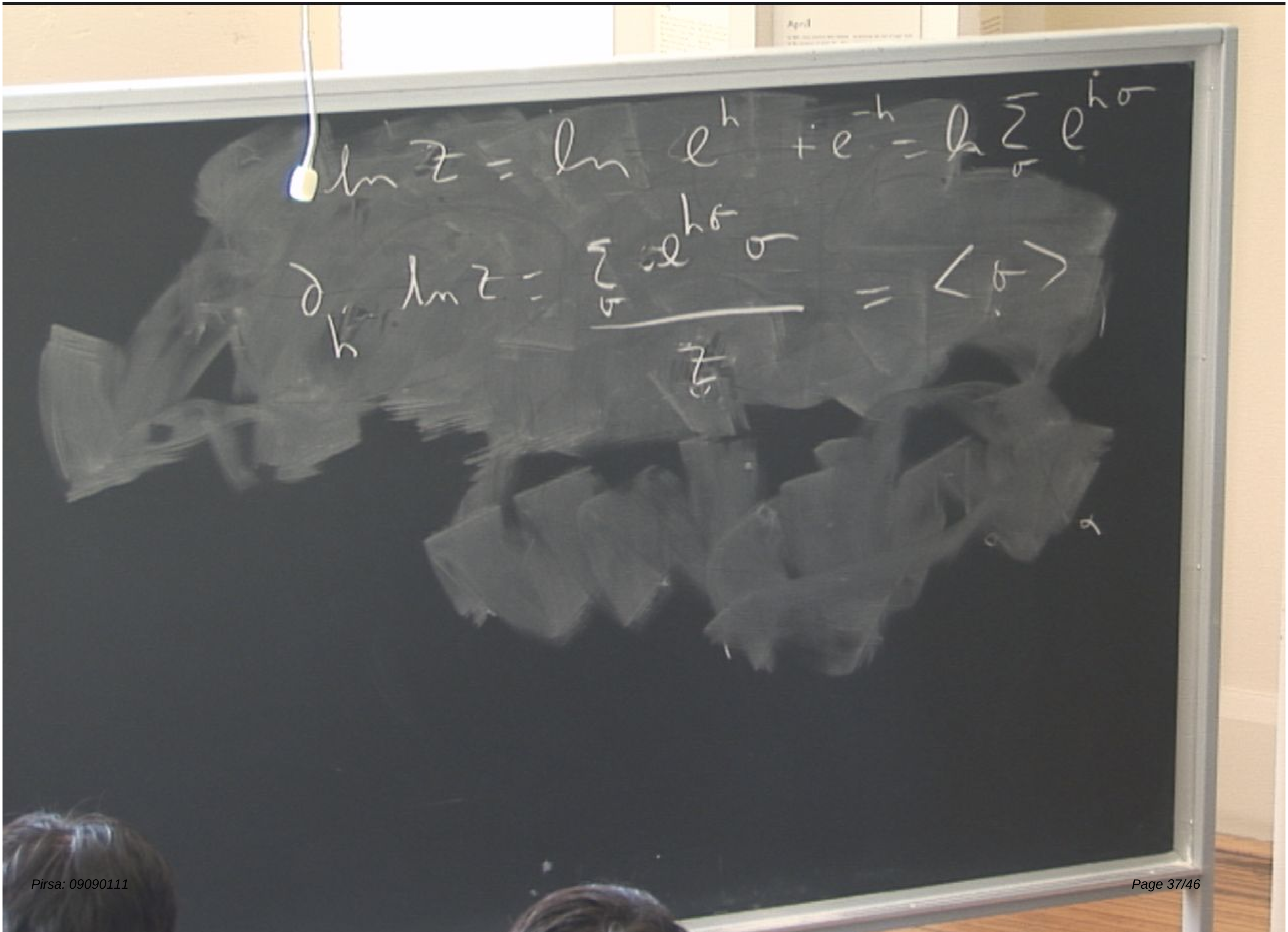
All derivatives of the log of the partition function are thermodynamic functions of some kinds. As I shall say below, we expect simple behavior from the log of Z but not Z itself. The derivatives described above are respectively called the magnetization, $M=\langle\sigma\rangle$ and the magnetic susceptibility, χ, = dM/dH. The analogous first derivative with respect to β is minus the energy. The next derivative with respect to β is proportional to the specific heat, or heat capacity, another traditional thermodynamic quantity. The derivative of partition function with respect to volume is the pressure.

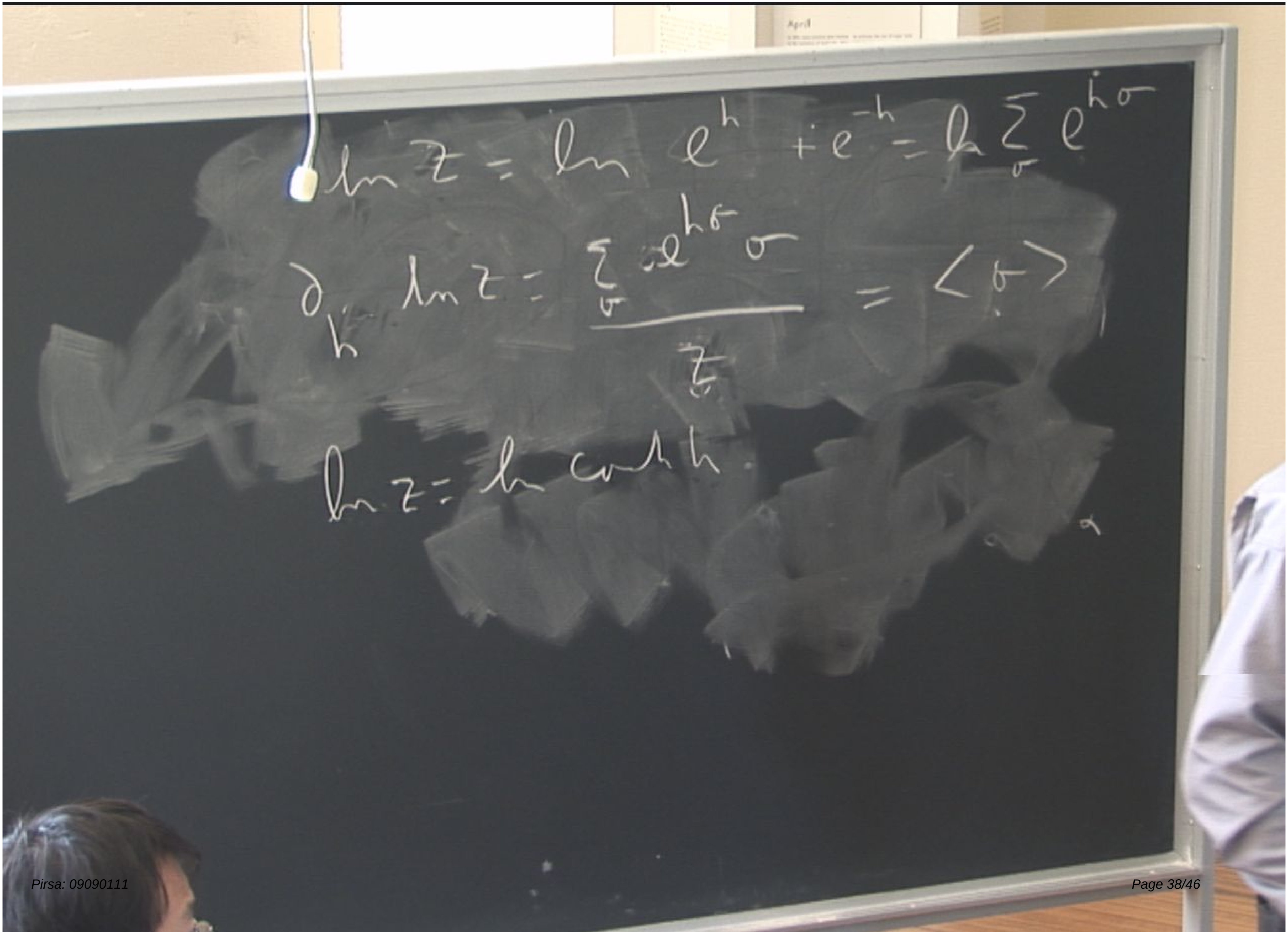
note how the second derivative gives the mean squared fluctuations

homework: Read Chapters 1 and 2 in textbook.

show that $-d(\ln Z)/d\beta = E = \langle H\rangle$ and $d^2(\ln Z)/d\beta^2 = \langle (H-\langle H\rangle)^2\rangle$ and

If $\ln Z = \text{const} + N \ln \Omega + N \gamma \ln T$, with Ω being the volume, find the average pressure and its fluctuations.


$\ln Z = \ln e^{h} + e^{-h} = \ln \sum_{\sigma} e^{h\sigma}$
$\partial_h \ln Z = \frac{\sum_{\sigma} e^{h\sigma} \sigma}{Z} = \langle \sigma \rangle$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$\ln Z = \ln \cosh h$

Averages from Derivatives

$$z = \sum_\sigma \exp(h\sigma) = 2\cosh h$$

$$d(\ln z) / dh = \sum_\sigma \sigma \exp(h\sigma) / z = < \sigma > = \tanh h$$

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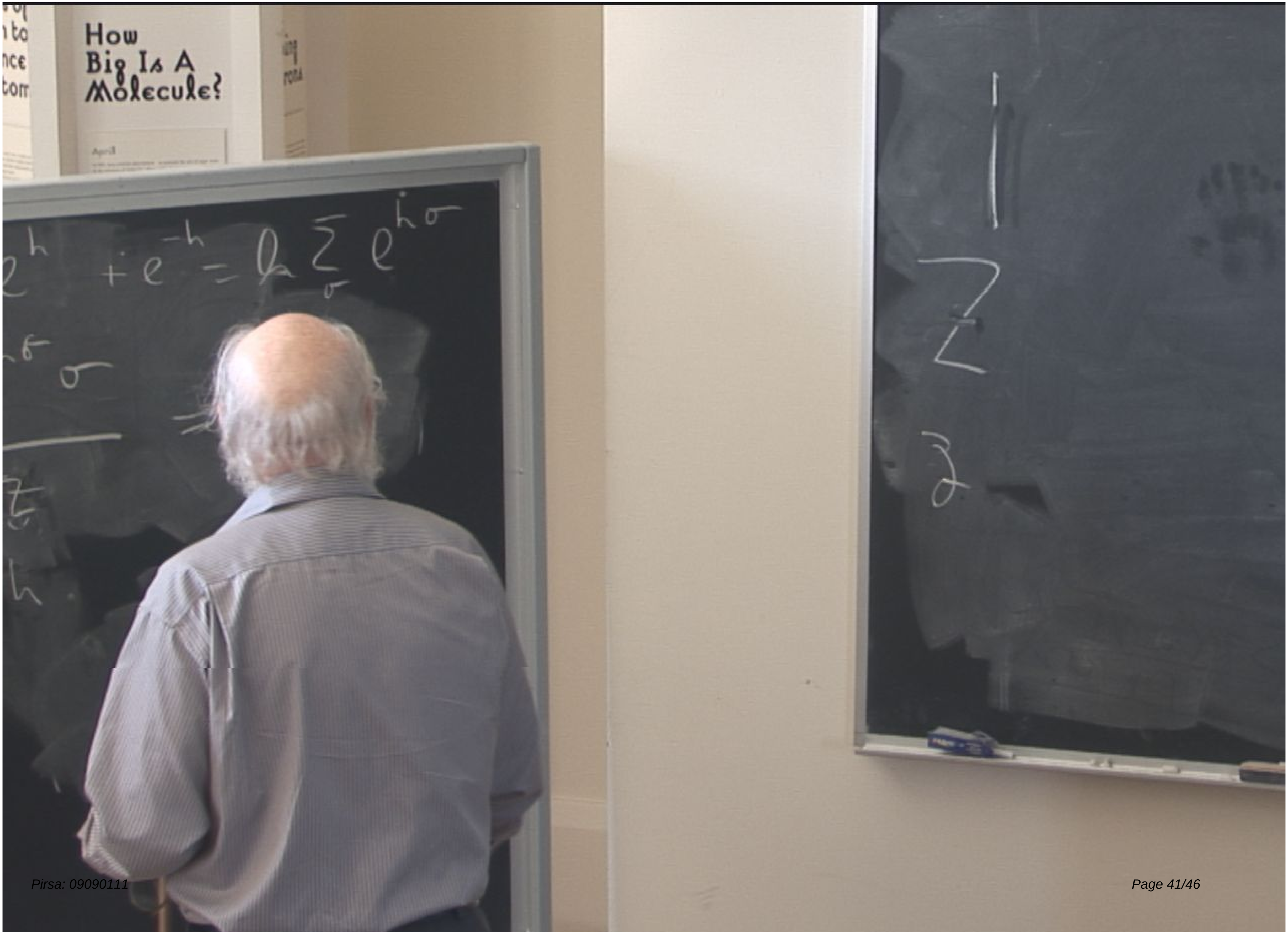
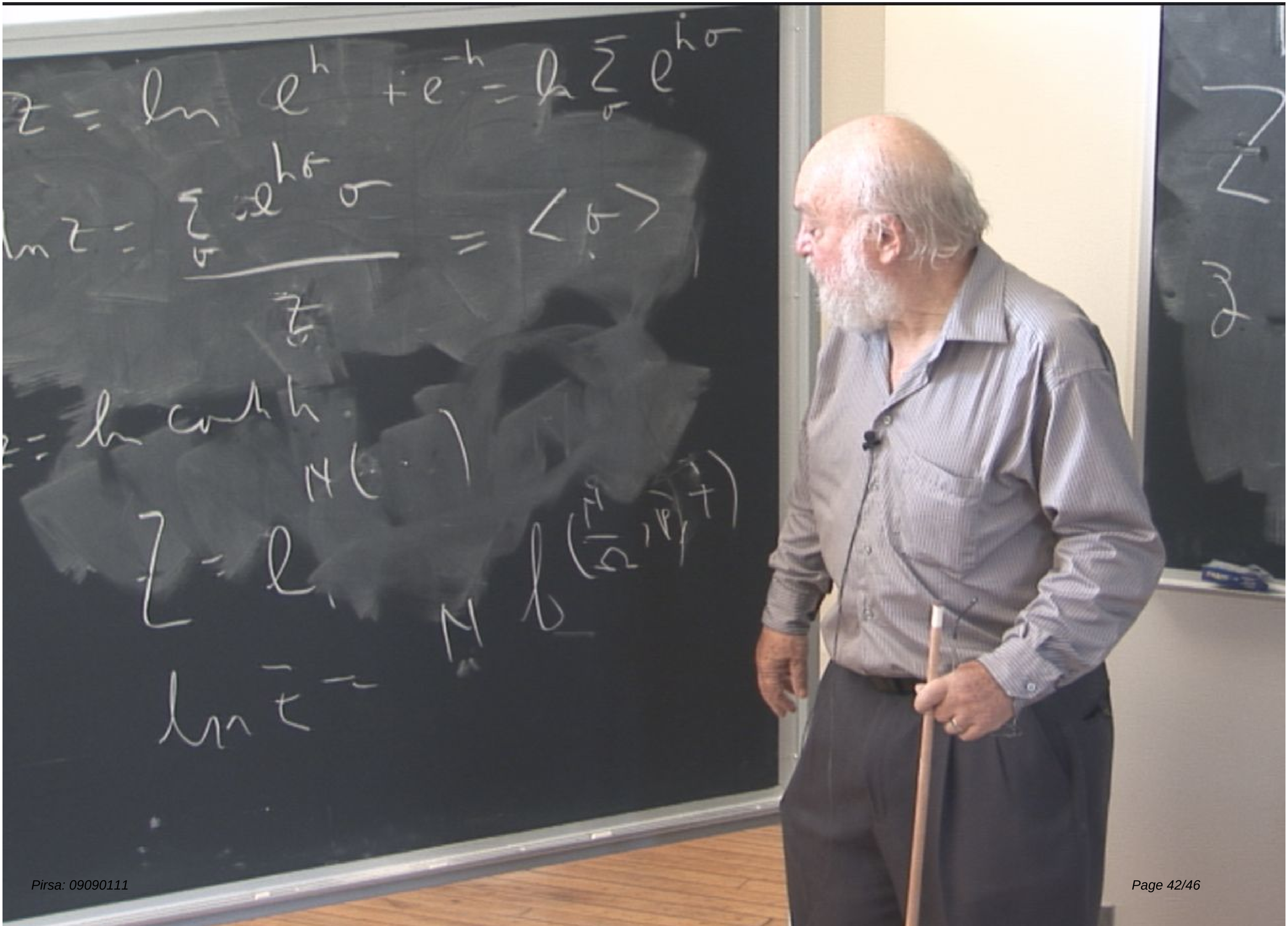
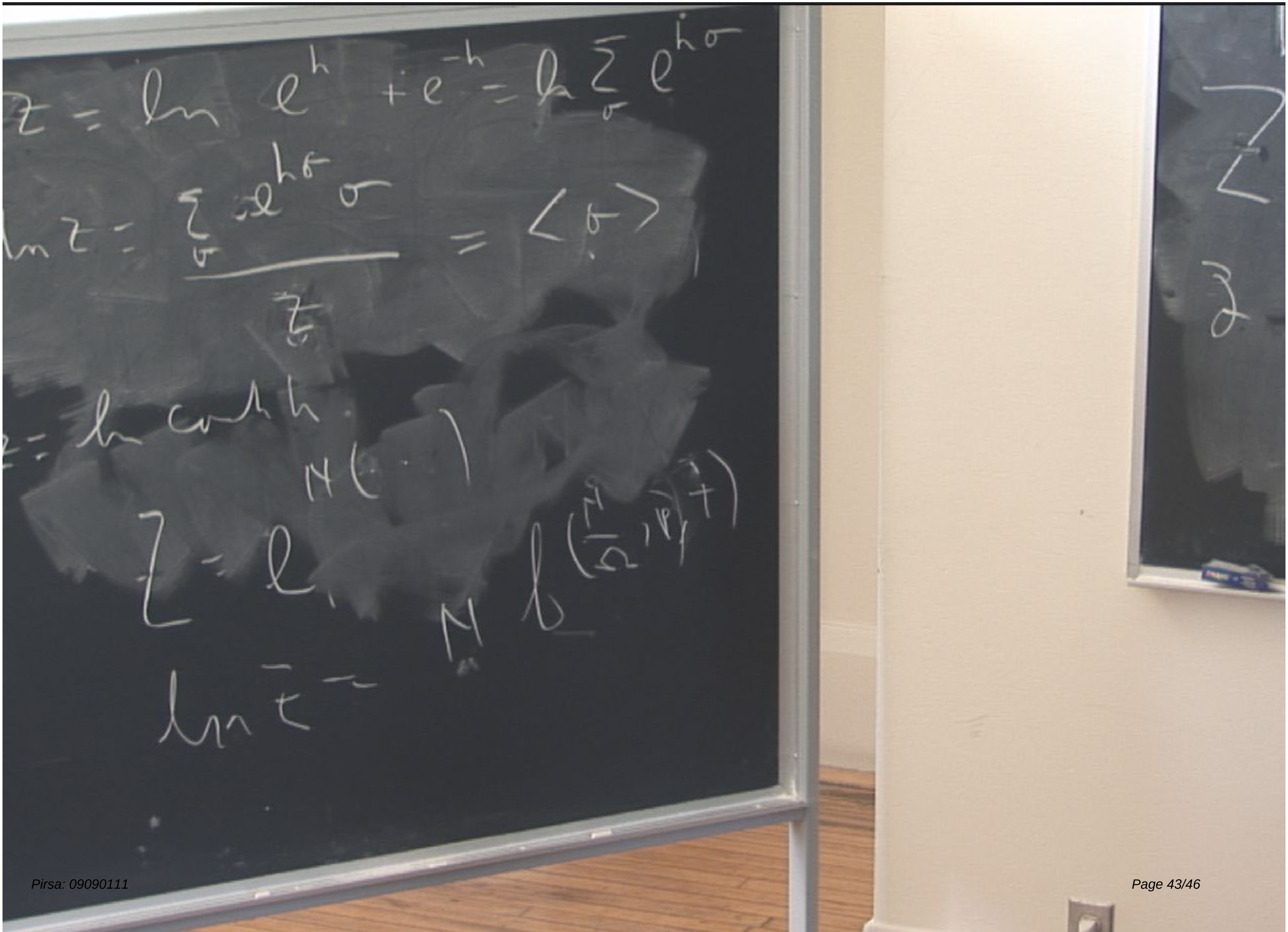
If $\ln Z$ = const + $N \ln \Omega + N \gamma \ln T$, with Ω being the volume, find the average pressure and its fluctuations.

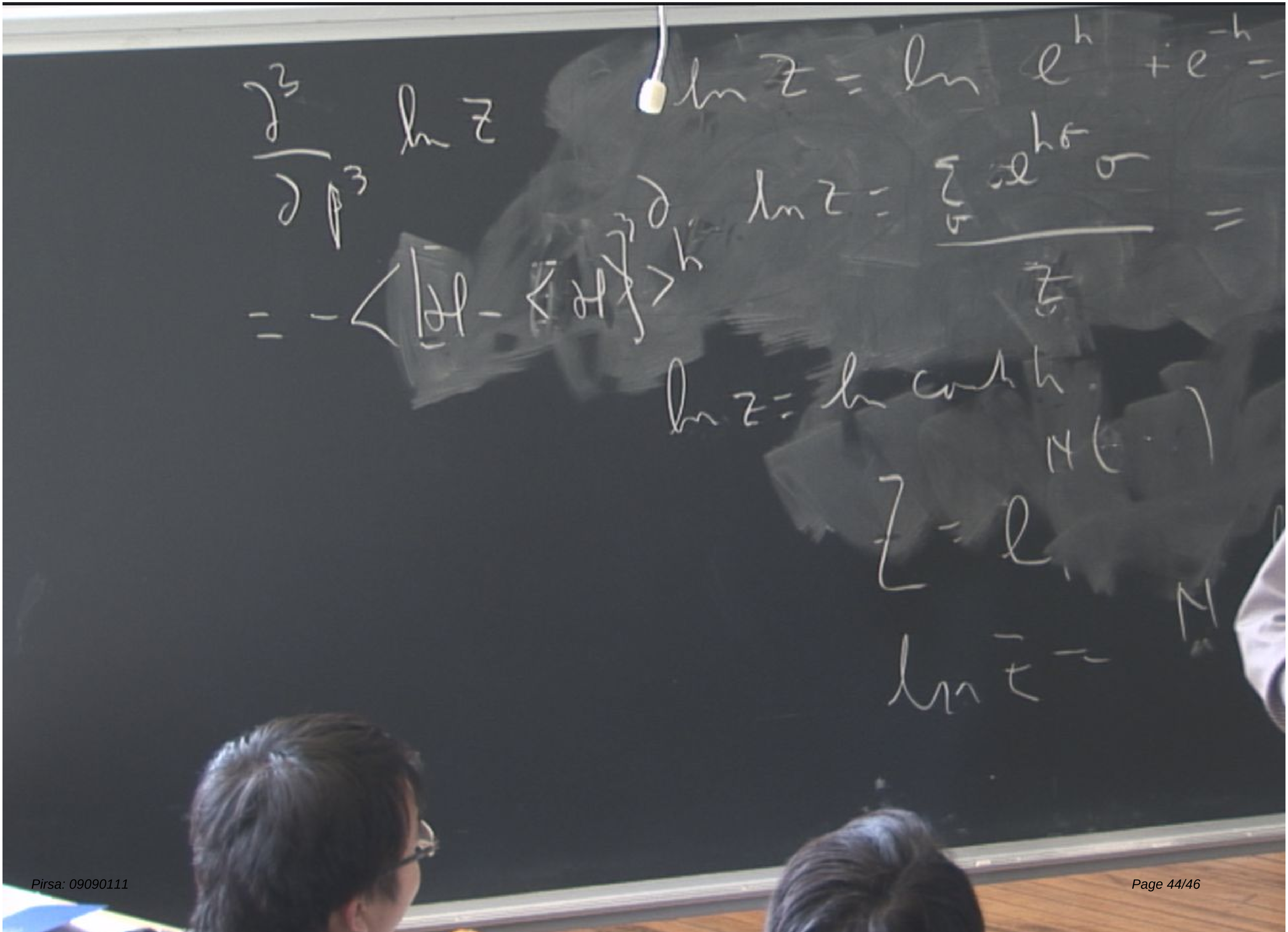
End Survey: Start More Intensive/Extensive Discussions

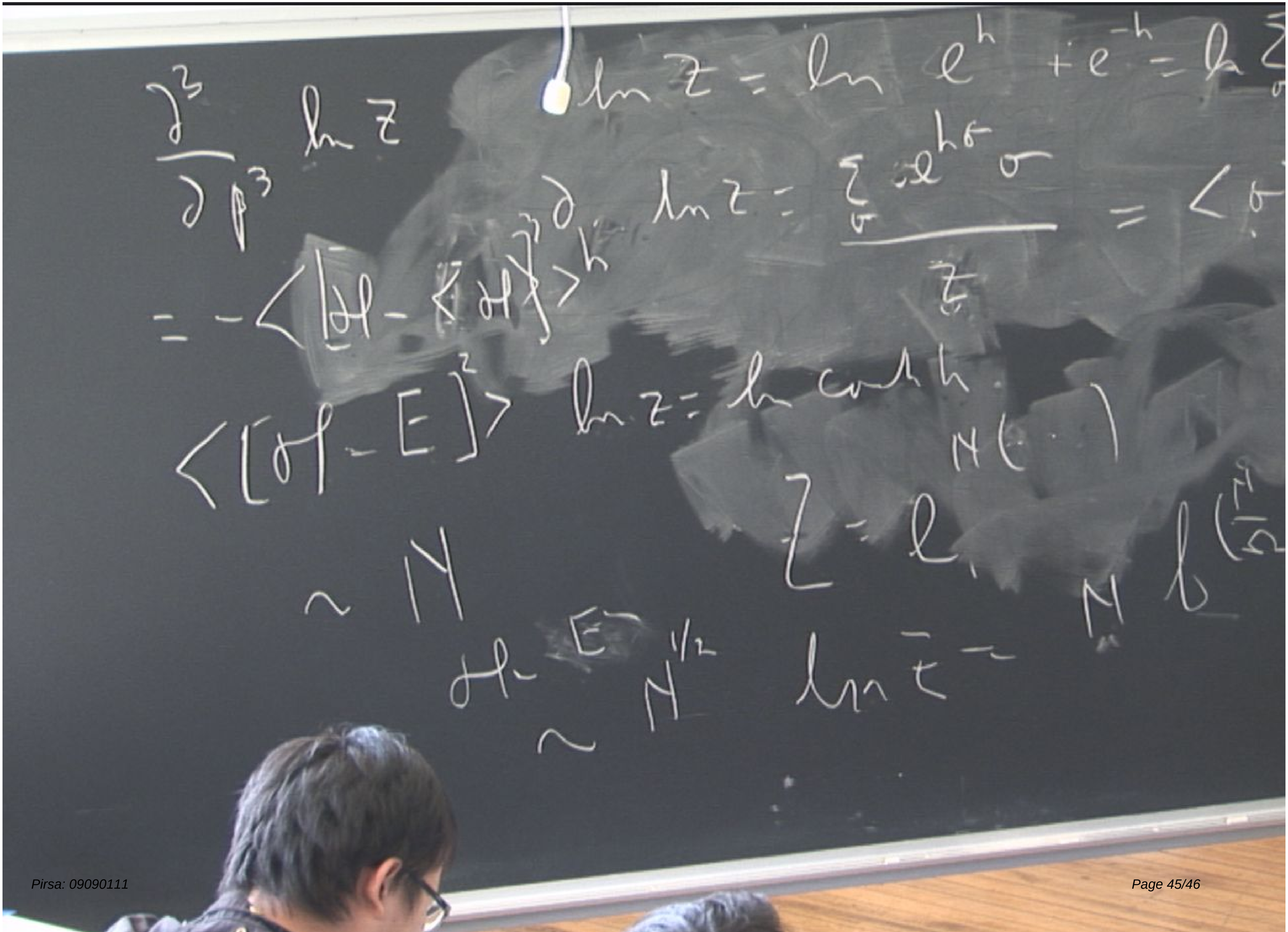
Do you know what intensive and extensive mean in statistical physics?


How
Big Is A
Molecule?






∂³/∂β³ ln Z
= −⟨(∂H − ⟨∂H⟩)³⟩
ln Z = ln cosh h


ln Z = ln e^h + e^-h = ln Σ
∂³/∂β³ ln Z
∂_h ln Z = Σ e^{hσ} σ / Z = <σ
= − <[H − <H>]³>
<[H − E]²>
ln Z = ln cosh h
~ N
N^{1/2}
ln Z