Title: Quantum Field Theory (PHYS 601) - Lecture 1

Date: Sep 28, 2009 09:00 AM

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Abstract:

Quantum Field Theory

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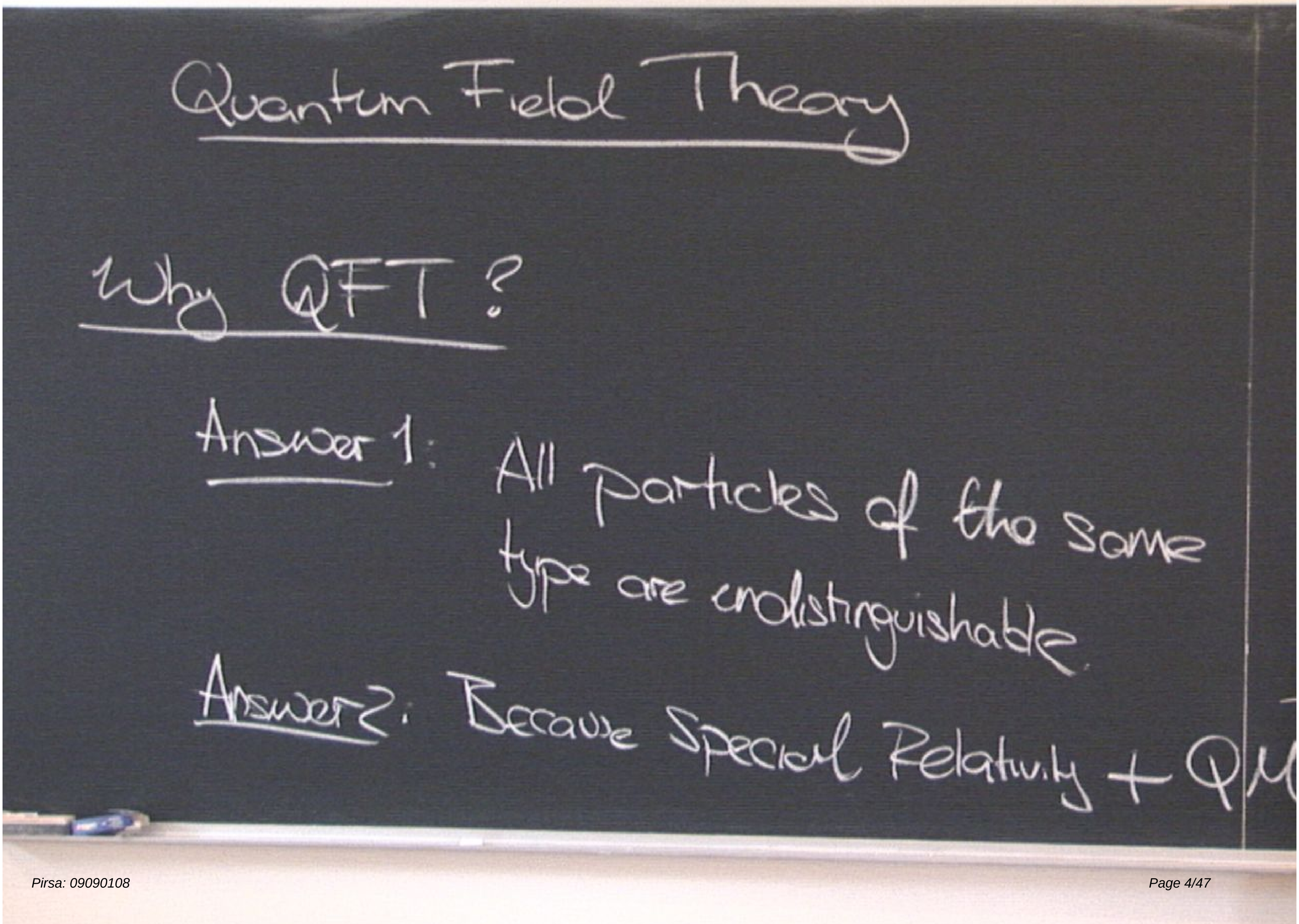
Why QFT?

Answer 1: All particles of the same type are indistinguishable.


Quantum Field Theory
Why QFT?
Answer 1: All particles of the same type are indistinguishable.
Answer 2: Because Special Relativity + Q

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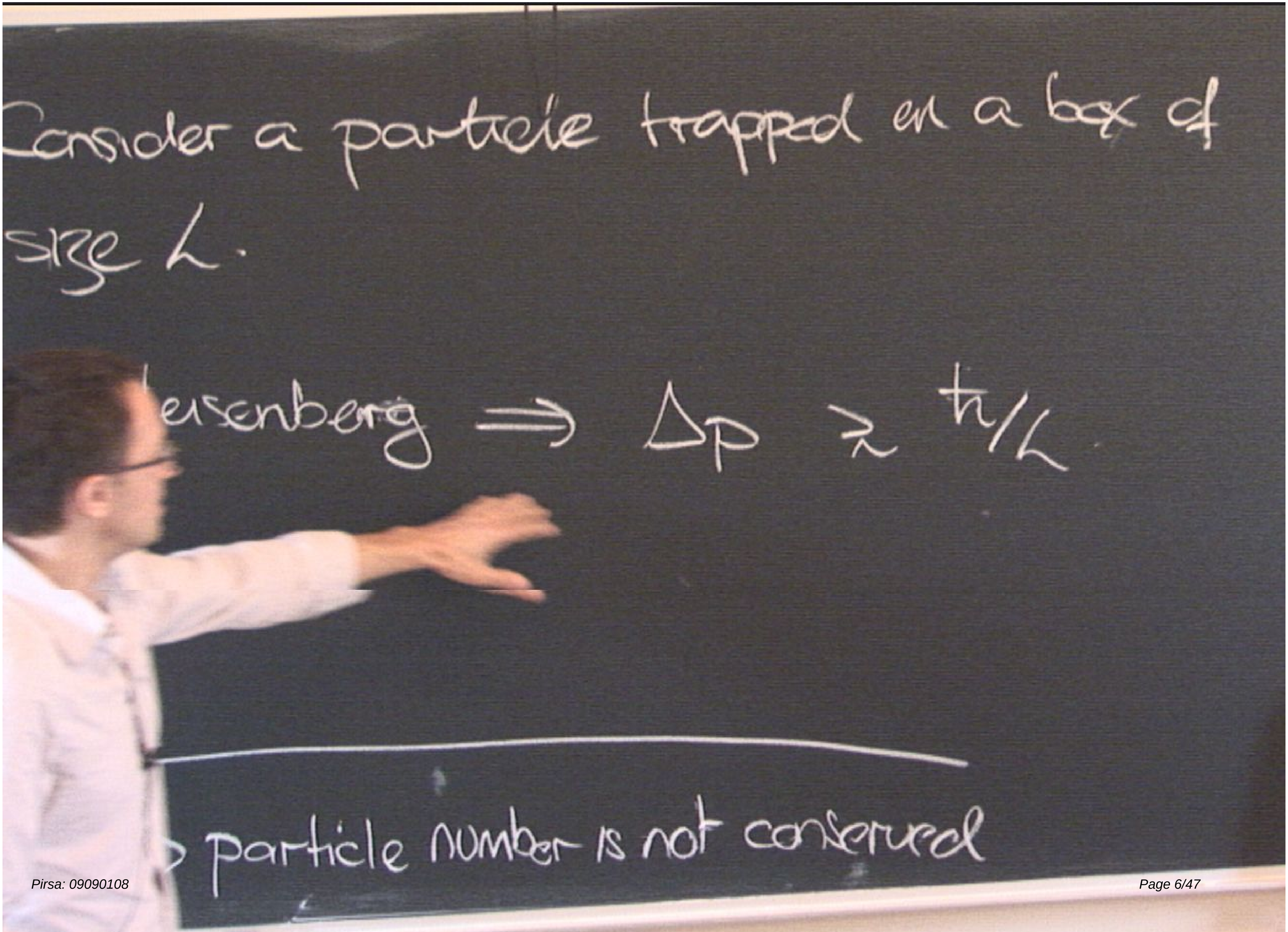
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+ relativity $\Longrightarrow$ $\Delta E \gtrsim \hbar c/L$.

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$\Rightarrow$ if particle is localized in a region $L \lesssim \frac{\hbar}{mc}$.

The length $\lambda = \hbar/mc$ is called the <u>Compton wavelength</u>.

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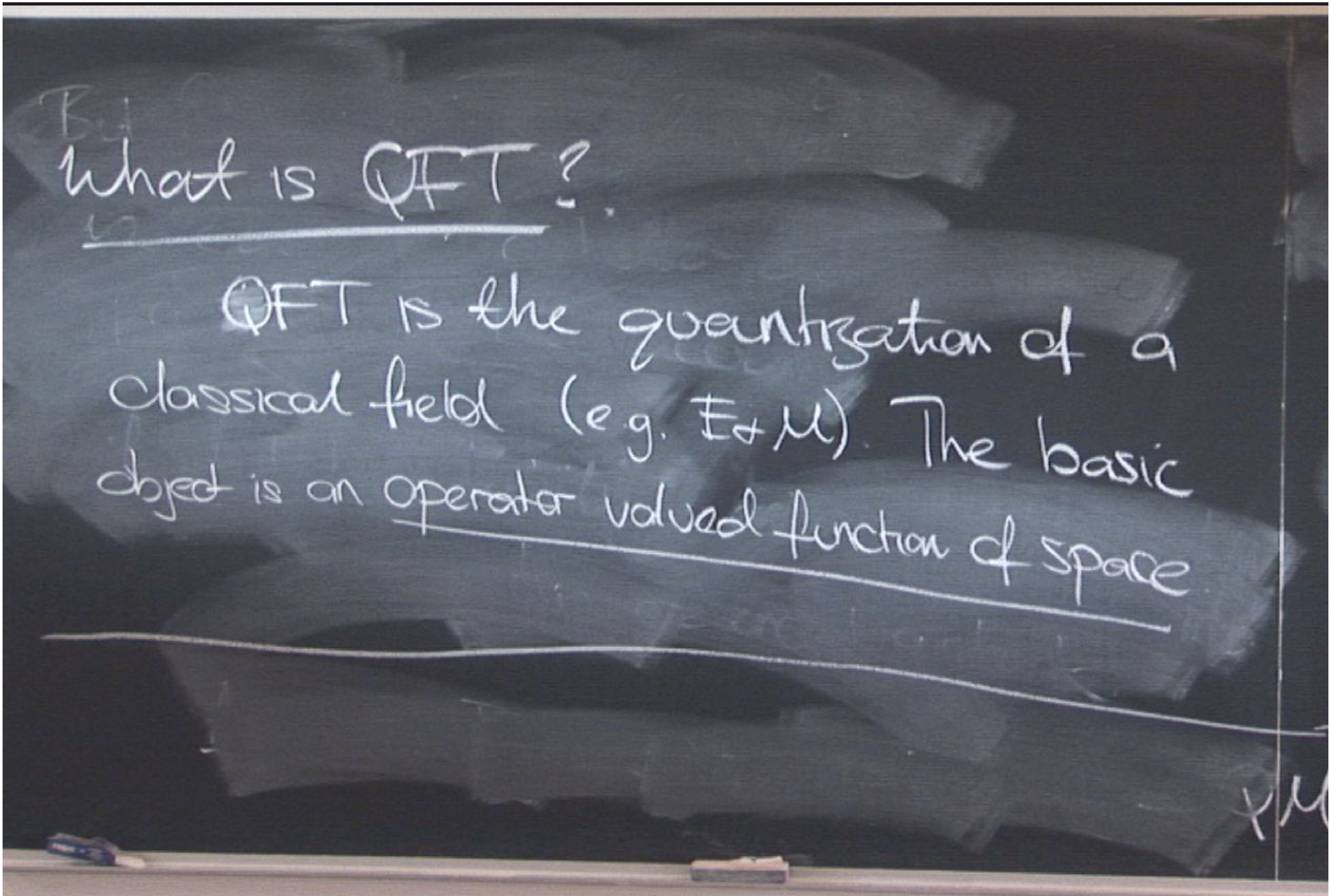
What is QFT?

QFT is the quantization of a classical field (e.g. E&M)


What is QFT ?
QFT is the quantization of a
classical field (e.g. E&M). The basic
object is an operator valued function of space

Units & Scales.

$\Rightarrow$ particle number is not conserved

Units & Scales.

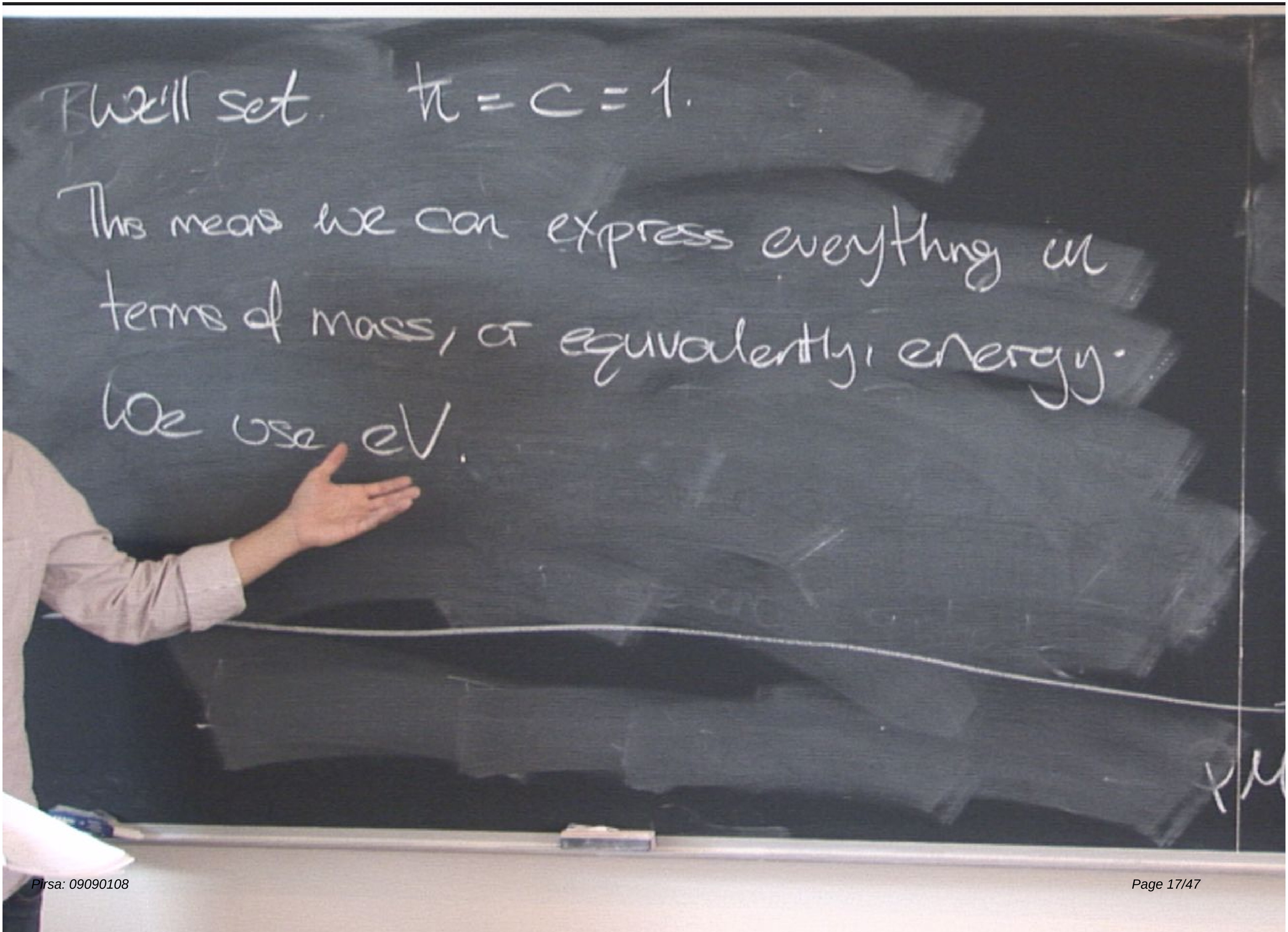
Three fundamental constants

$$[c] = L T^{-1}$$

$$[\hbar] = L^2 M^2 T^{-1}$$

$$[G] = L^3 M^{-1} T^{-2}$$

$\Rightarrow$ particle number is not conserved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To convert back, to m, s, we need to insert c, $\hbar$.

e.g. $\lambda = \hbar / mc$

length

mass

e.g. $m_e = 10^6\ \mathrm{eV} \Longleftrightarrow \lambda_e = 2 \times 10^{-12}\ \mathrm{m}$.

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a quantity has mass dimension $(\text{mass})^\alpha$, we writ

$\Rightarrow$ particle number is not conserved

$$[X] = d.$$

In particular, $[G] = -2$, and we write

$$G = \frac{\hbar c}{M_p^2} \longrightarrow \frac{1}{M_p^2}.$$

where $M_p \approx 10^{19}$ GeV.

§1 Classical Field Theory

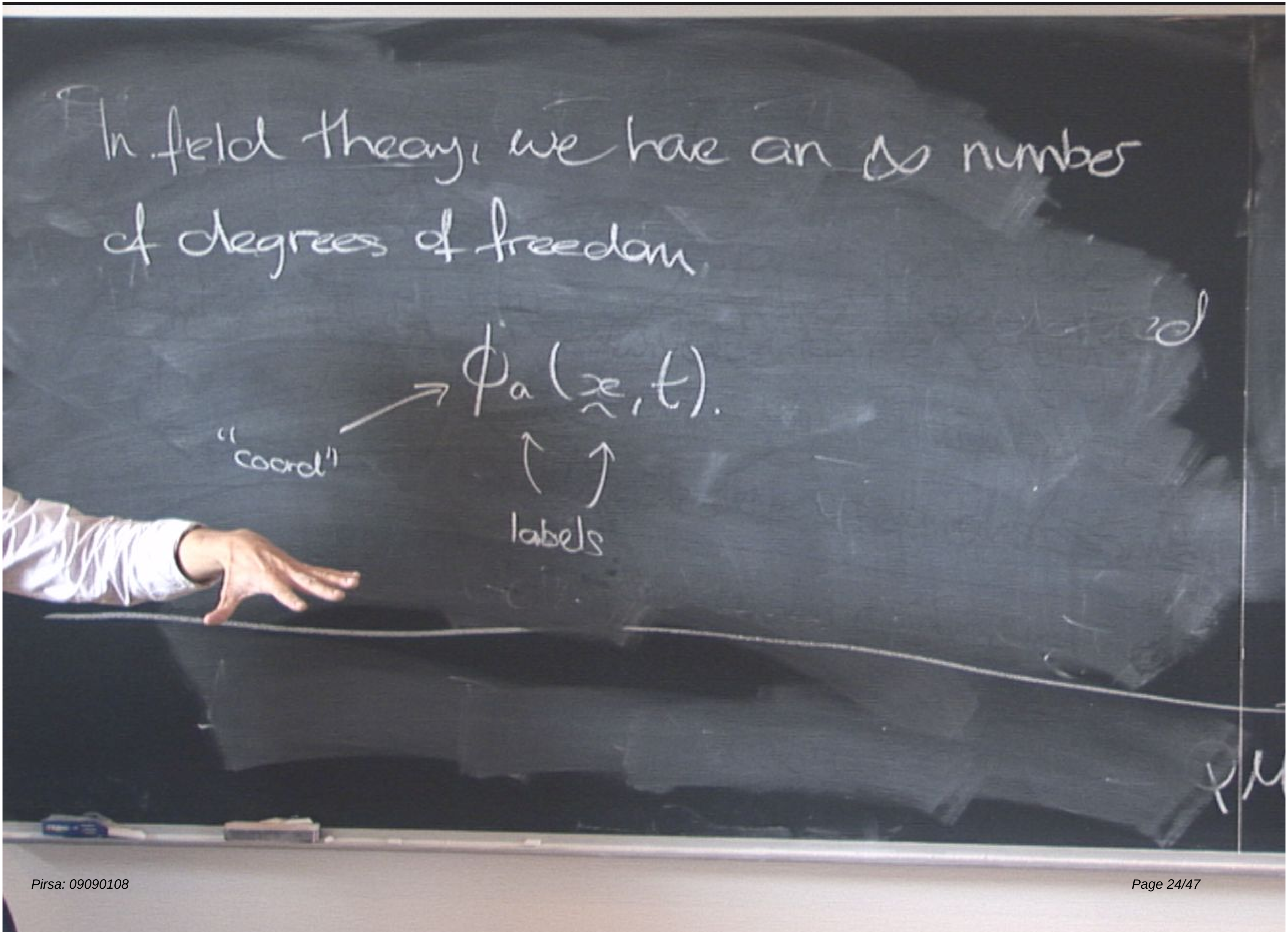
A <u>field</u> is a physical quantity defined at every point of space and time.

In classical particle mechanics, there are a finite number of degrees of freedom,

$$q_a(t).$$

Coordinate

$\Rightarrow$ particle number is not conserved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eg $\underset{\sim}{E}_i(\underset{\sim}{x},t)$ and $\underset{\sim}{B}_i(\underset{\sim}{x},t)$ in E & M

$i=1,2,3$.

Later we learn that these 6 fields are derived from 4: $A_\mu(\underline{x},t)$

$$E_i = \frac{\partial A_i}{\partial t} - \frac{\partial A_0}{\partial x^i}$$

$$B_i = \frac{1}{2}\epsilon_{ijk}\frac{\partial A_k}{\partial A_j}$$

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The dynamics of the fields is governed by a <u>Lagrangian</u>, which is a function of

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$$L(t) = \int d^3x \, \mathcal{L}(\phi_a, \partial_\mu \phi_a) \Longrightarrow \text{action } S = \int dt \, L(t)$$

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<u>Lagrangian density</u>

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Note. In particle mechanics, $\mathcal{L}(q, \dot{q})$, but <u>not</u> a function of $\ddot{q}$. In field theory, $\mathcal{L}$ should not depend on $\ddot{\phi}$, but it could depend on $\nabla\phi$, $\nabla^2\phi$, etc

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$$\delta S = \int d^4x \left\{ \frac{\partial \mathcal{L}}{\partial \phi_a} \delta \phi_a + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_a)} \delta(\partial_\mu \phi_a) \right\}$$

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$$= \int d^4x \left\{ \frac{\partial \mathcal{L}}{\partial \phi_a} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_a)} \right) \right\} \delta\phi_a$$

$$+ \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_a)} \delta\phi_a \right) \Bigg\}$$

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but boundary term = 0

for any variation $\delta\phi_a(\underset{\sim}{x}, t)$ that decays suitably at $\underset{\sim}{x} \to \infty$, and obeys $\delta\phi_a(\underset{\sim}{x}, t_i) = \delta\phi_a(\underset{\sim}{x}, t_f) = 0$.

Then "$\delta S = 0$"

$$\Rightarrow \boxed{\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0}$$

These are Euler-Lagrange equations

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These are Euler-Lagrange equations

Example: Klein-Gordon Eq^{n}:

$$\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}m^2\phi^2$$

$$\eta^{\mu\nu} = \begin{pmatrix} + & & & \\ & - & & \\ & & - & \\ & & & - \end{pmatrix}$$

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$$\eta^{\mu} = \begin{pmatrix} + & & & \\ & - & & \\ & & - & \\ & & & - \end{pmatrix}$$

$$= \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}m^2\phi^2.$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m^2 \phi$$

$\Longrightarrow$ EL equ$^{=}$ is

$$\ddot{\phi} - \nabla^2 \phi + m^2 \phi = 0$$

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This is the Klein-Gordon Equ^{n}.