Title: Lessons from black holes -- Equations of motion, scales & effective coupling of quantum gravity

Date: Sep 30, 2009 12:00 PM

URL: http://pirsa.org/09090106

Abstract: We use black holes to understand some basic properties of theories of quantum gravity. First, we apply ideas from black hole physics to the physics of accelerated observers to show that the equations of motion of generalized theories of gravity are equivalent to the thermodynamic relation $\delta Q = T \delta S$. Our proof relies on extending previous arguments by using a more general definition of the Noether charge entropy. We have thus completed the implementation of Jacobson's proposal to express Einstein's equations as a thermodynamic equation of state. Additionally, we find that the Noether charge entropy obeys the second law of thermodynamics if the energy momentum tensor obeys the null energy condition. Our results support the idea that gravitation on a macroscopic scale is a manifestation of the thermodynamics of the vacuum. Then, we show that the existence of semiclassical black holes of size as small as a minimal length scale l_{UV} implies a bound on a gravitational analogue of 't-Hooft's coupling $\lambda_G(l)\equiv N(l) G_N/l^2$ at all scales $l \ge l_{UV}$. The proof is valid for any metric theory of gravity that consistently extends Einstein's gravity and is based on two assumptions about semiclassical black holes: i) that they emit as black bodies, and ii) that they are perfect quantum emitters. The examples of higher dimensional gravity and of weakly coupled string theory are used to explicitly check our assumptions and to verify that the proposed bound holds. Finally, we discuss some consequences of the bound for theories of quantum gravity in general and for string theory in particular.

Lessons from black holes --
Equations of motion, scales & effective coupling of quantum gravity

Ram Brustein

+ HADAD 0903.0823
+ DVALI, VENEZIANO 0907.5516

אוניברסיטת בן-גוריון

- The Einstein equations for generalized theories of gravity and the thermodynamic relation $\delta Q = T\,\delta S$ are equivalent
- The gravitational analogue of 't Hooft's coupling $\lambda_G(l) = N\, G_N\, l^{-2}$ is bounded $\lambda_G(l) < 1$ for $l > l_{UV}$

The Einstein equations & $\delta Q = T\,\delta S$ are equivalent

Idea: (Einstein-Hilbert, Jacobson '95)

- Equivalence principle → free falling observer can define a local Rindler (acceleration) horizon
- Rindler horizons are associated with thermodynamics δQ, T, δS
- T - Unruh temp., δQ – energy flow across the horizon, δS - entropy (entanglement)

Extension to generalized metric theories of gravity

Idea: (R.B+Hadad)

- Use semiclassical BHs to define $\delta Q, T, \delta S$

for acceleration horizons in generalized theories

$$\mathcal{L} = \mathcal{L}_m(g_{ab}, \phi) + \mathcal{L}_G(R_{abcd}) + \mathcal{L}_{int}(g_{ab}, R_{abcd}, \phi)$$

$\{\phi\}$ - matter

Temperature T

$$\chi_b \nabla^b \chi_a = \kappa \chi_a$$

χ- Rindler horizon killing vector
κ – Surface gravity

Define temperature as for BHs (limiting procedure)

$$\kappa = 2\pi T$$

Heat δQ

$$E = \int_{\mathcal{H}} T_{ab} \tilde{\chi}^a \epsilon^b$$

Energy measured by an observer hoovering outside the horizon

$$\chi^a = \kappa \tilde{\chi}^a$$

$$\epsilon^b = \tilde{\chi}^b \Sigma$$

Σ – volume element

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Energy variation due to causal boundary

$$\delta Q = \int_{\mathscr{H}} T_{ab}\chi^a \epsilon^d$$

Agrees with Jacobson

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Idea: Elizalde+Silva '08 for f (R) differs from Jacobson `95, Eling, Guedens, Jacobson `06

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Another way: find a quantity dS such that the equations δS =1/T δQ are equivalent to Einstein's eqs.

$$\begin{aligned}\delta S &= -\frac{2}{T}\int \chi_m \nabla^m \nabla_c \left(\frac{\partial \mathscr{L}}{\partial R_{abcd}}\hat{\epsilon}_{ab}\right)\epsilon_d \\ &= -4\pi \int \tilde{\chi}_m \nabla^m \nabla_c \left(\frac{\partial \mathscr{L}}{\partial R_{abcd}}\hat{\epsilon}_{ab}\right)\epsilon_d\end{aligned}$$

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For constant ε (fixed horizon)

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Equations of motion for generalized theories of gravity

$$\sqrt{-g}\left(\frac{\partial \mathscr{L}}{\partial g^{ab}}+2\nabla_p\nabla_q\frac{\partial \mathscr{L}}{\partial R_{pabq}}+\frac{\partial \mathscr{L}}{\partial R_{pqr}{}^{a}}R_{pqrb}\right)-\frac{1}{2}g_{ab}\mathscr{L}=0$$

$$\mathscr{L} \;=\; \mathscr{L}_m\left(g_{ab},\phi\right)+\mathscr{L}_G\left(R_{abcd}\right)+\mathscr{L}_{int}\left(g_{ab},R_{abcd},\phi\right)$$

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$$T_m^{ab} = -2/\sqrt{-g}\,\partial\left(\sqrt{-g}\mathscr{L}_m\right)/\partial g_{ab}$$

$$T_{int}^{ab} = -2/\sqrt{-g}\,\partial\left(\sqrt{-g}\mathscr{L}_{int}\right)/\partial g_{ab}$$

$$T^{ab} = T_m^{ab} + T_{int}^{ab}$$

$$T^{ab} = 2\left[2\nabla_p \nabla_q \frac{\partial \mathscr{L}}{\partial R_{pabq}} + \frac{\partial \mathscr{L}}{\partial R_{pqra}} R_{pqr}{}^{b}\right] - g^{ab} \mathscr{L}_G$$

The Einstein equations & $\delta Q = T\,\delta S$ are equivalent

δQ $\qquad\qquad$ $T\,\delta S$

$$\int T_{ab}\tilde{\chi}^a \epsilon^b = -2\int \tilde{\chi}_m \nabla^m \nabla_c \left(\frac{\partial \mathscr{L}}{\partial R_{abcd}} \hat{\epsilon}_{ab} \right) \epsilon_d$$

$$T^{ab} = 2\left[2\nabla_p \nabla_q \frac{\partial \mathscr{L}}{\partial R_{pabq}} + \frac{\partial \mathscr{L}}{\partial R_{pqra}} R_{pqr}{}^{b} \right] + g^{ab} f$$

$$\nabla^m \nabla_c \left(\frac{\partial \mathscr{L}}{\partial R} \hat{\epsilon}_{ab} \right) = \hat{\epsilon}_{nm}\hat{\epsilon}_{ab} \nabla^m \nabla_c \frac{\partial \mathscr{L}}{\partial R} - \frac{\partial \mathscr{L}}{\partial R} R_{abci} \hat{\epsilon}^{mi} \hat{\epsilon}_{nm}$$

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Conservation → $f = -\mathcal{L}_G + \Lambda$

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Einstein equations!

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$\bar{\epsilon}$ is the area element

Idea: Elizalde+Silva '08 for f (R) differs from Jacobson `95, Eling, Guedens, Jacobson `06

For constant ε (fixed horizon)

$$\begin{aligned}\delta S &= -\frac{2}{T}\int \chi_m \nabla^m \nabla_c \left(\frac{\partial \mathscr{L}}{\partial R_{abcd}}\hat{\epsilon}_{ab}\right)\epsilon_d \\ &= -4\pi \int \tilde{\chi}_m \nabla^m \nabla_c \left(\frac{\partial \mathscr{L}}{\partial R_{abcd}}\hat{\epsilon}_{ab}\right)\epsilon_d\end{aligned}$$

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Bonus

The NCE obeys the 2nd law

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$$T_{ab}\tilde{\chi}^a\tilde{\chi}^b \geq 0 \implies \delta S \geq 0$$

Null Energy Condition

The Einstein equations & $\delta Q = T\,\delta S$ are equivalent

- **Assumption**: causal barrier entropy behaves in a similar way to BH entropy.
- **Fact**: causal barrier entropy is associated with entanglement with hidden d.o.f
- **Speculation:** turn the logic around → BH entropy results entanglement with hidden d.o.f

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The Einstein equations & $\delta Q = T\,\delta S$ are equivalent

- **Speculation:** quantum gravity is not fundamental *only* thermodynamic/macroscopic description→ at some scale a microscopic description without gravity should exists (Sakharov's induced gravity ?, gauge-gravity duality?, …?)

A bound on the effective gravitational coupling from semiclassical black holes

- $\lambda_G(l) = N\, G_N\, l^{-2}$ is bounded $\lambda_G(l) < 1$ for $l > l_{UV}$
- N light species $m < \Lambda_{UV}$, $\Gamma < m$, weak coupling
- Metric theories ➔ the equivalence principle
- l_{UV} : scale above which exchanges of metric perturbations processes become strong

* The previous parametrization is not very useful ➔ need another path to prove bound for generalized theories of gravity

Definition: $l_{SCBH} \equiv l_P \sqrt{N}$

$\lambda_G(l) = N\, G_N\, l^{-2}$

Proof of the bound $\lambda_G(l) < 1$ for $l > l_{UV}$

1. $\lambda_G(l) < 1 \text{ for } l > l_{SCBH}$
2. l_{SCBH} is an absolute lower bound on the size of semiclassical BHs in *any* consistent theory of gravity.
3. In any consistent theory of gravity $l_{SCBH} < l_{UV}$

Assumptions about SCBHs

M , R_S , $\beta = 1/T$

$(a) \quad -\frac{dR_S}{dt} < 1$

$(b) \quad -\frac{d\beta}{dt} < 1$

$(c) \quad -\frac{R_S}{M}\frac{dM}{dt} < 1$

$(d) \quad -\frac{\beta}{M}\frac{dM}{dt} < 1$

$(e) \quad \frac{\Gamma}{M} < 1$

Assumptions about SCBH

I $$-\frac{dM}{dt} = N(T)T^4 R_S^2$$

II $$TR_S \leq 1$$

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$$(a) \quad -\frac{dR_S}{dt} < 1$$

$$(b) \quad -\frac{d\beta}{dt} < 1$$

Assumptions about SCBH

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$(a)\quad -\frac{dR_S}{dt} < 1$ $\qquad (b)\quad -\frac{d\beta}{dt} < 1$

Assumptions about SCBH

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$$(d) \quad -\frac{\beta}{M}\frac{dM}{dt} < 1$$

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$$-dM/dt = \Gamma T$$

$$(e)\ \frac{\Gamma}{M} < 1$$

$$(a)\ -\frac{dR_S}{dt} < 1$$

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A bound on the effective gravitational coupling for Einstein gravity

Einstein gravity $M = M_P^2 R_S$

$$R_S M > N$$

$$R_S M > N \frac{d \ln R_S}{d \ln M}$$

$$\Longrightarrow \quad R_S > l_P \sqrt{N} = l_{SCBH}$$

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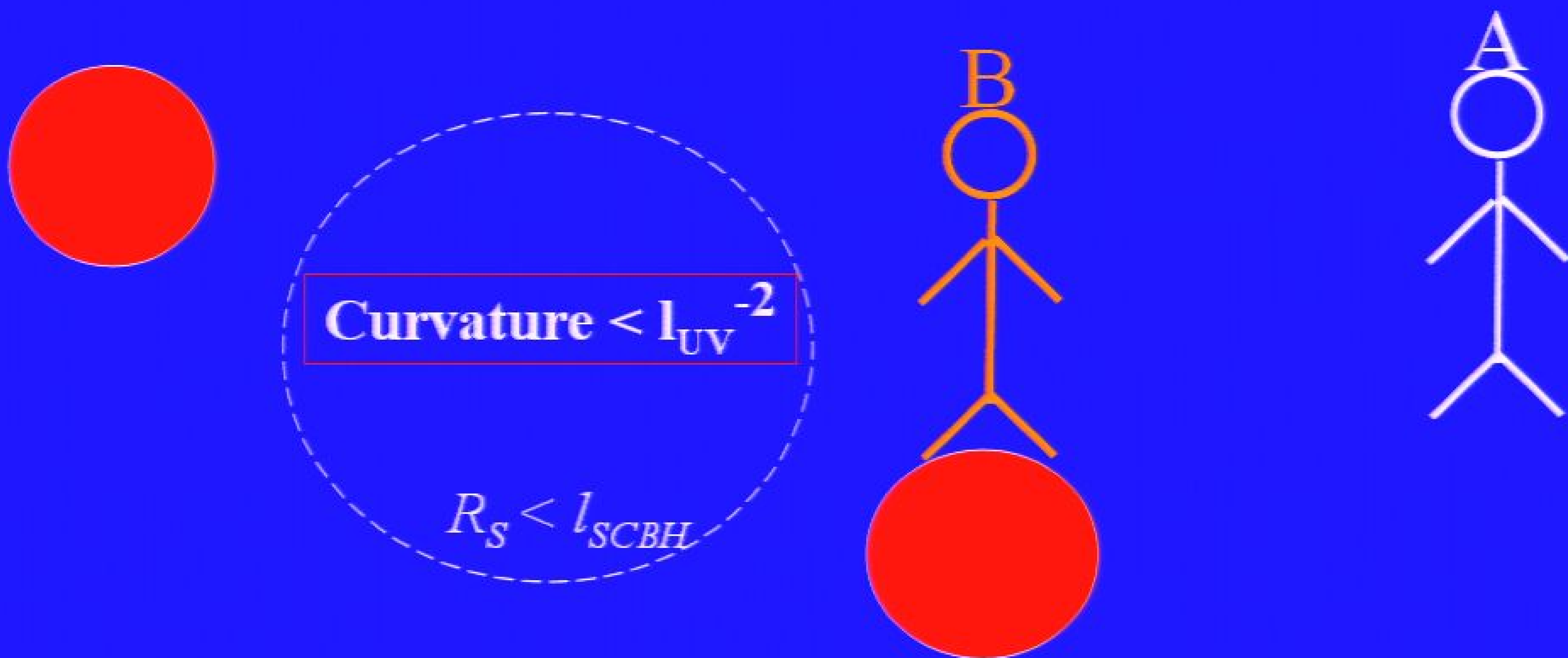
$$\lambda_G(l) = N \frac{l_P^2}{l^2}$$

$$l_{UV} \geq l_{SCBH}^{*}$$

$$\Longrightarrow \quad \lambda_G(l) < 1 \text{ for } l > l_{UV}$$

* Prove next

A though experiment: Assume $l_{UV} < l_{SCBH}$



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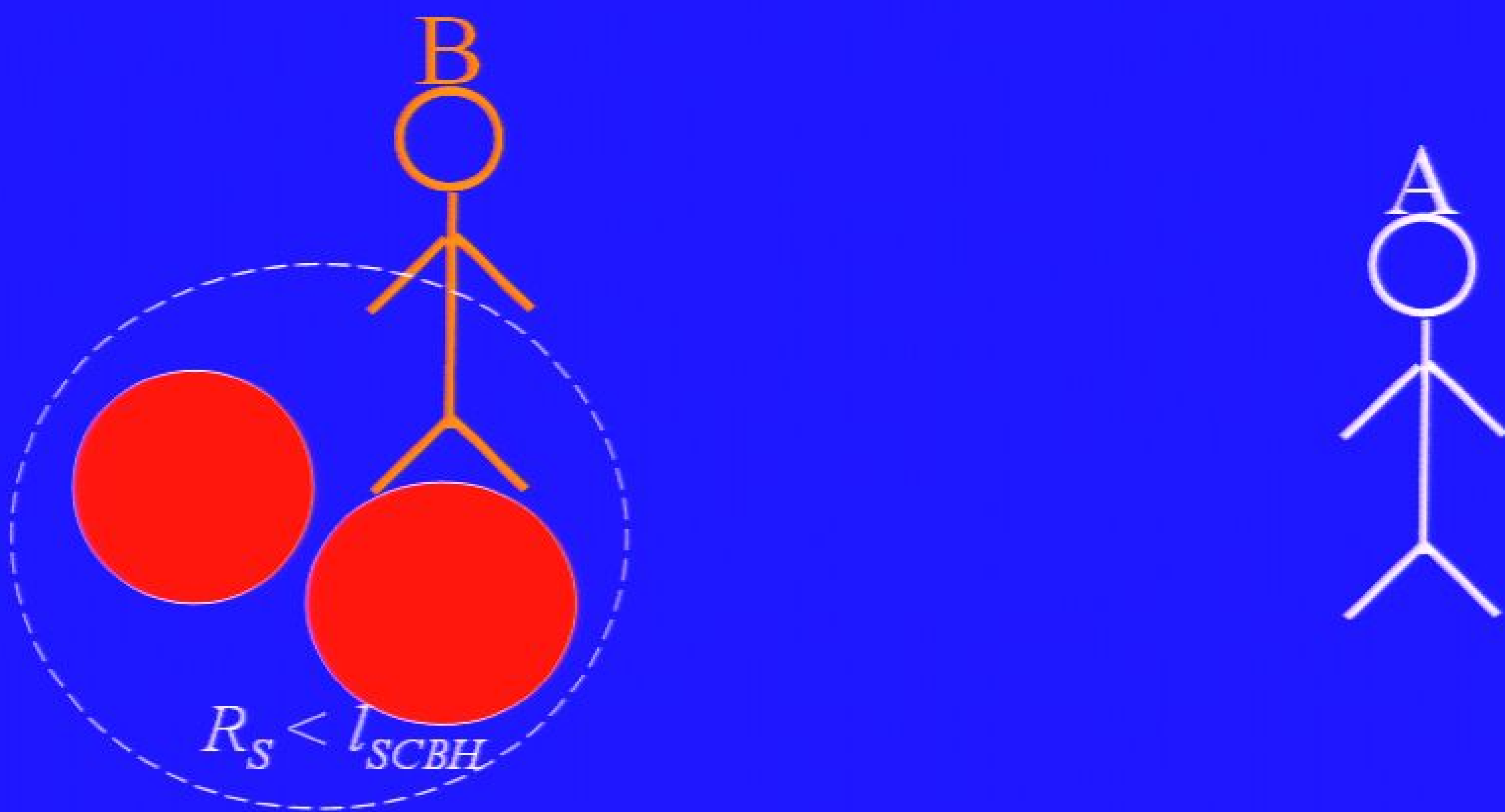

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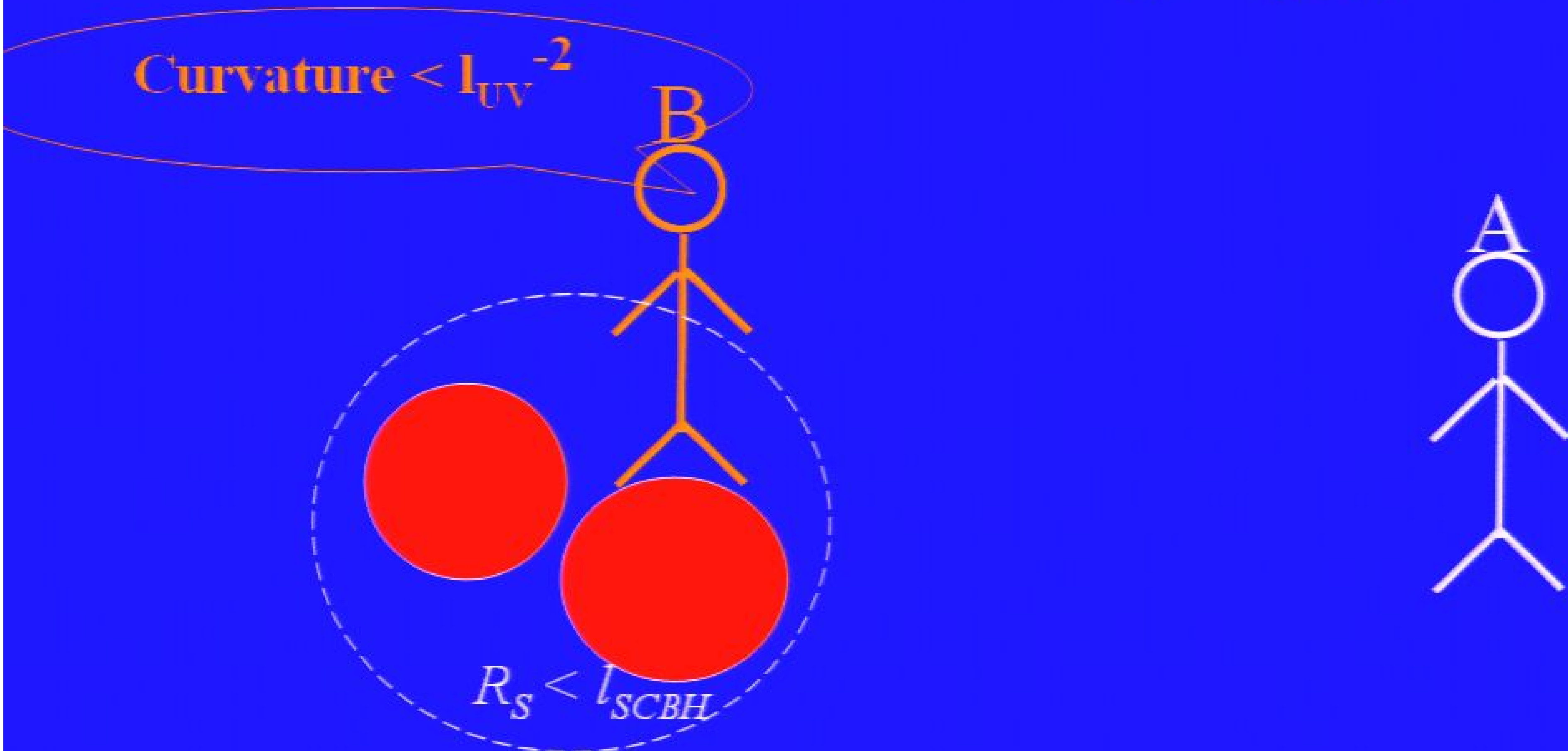

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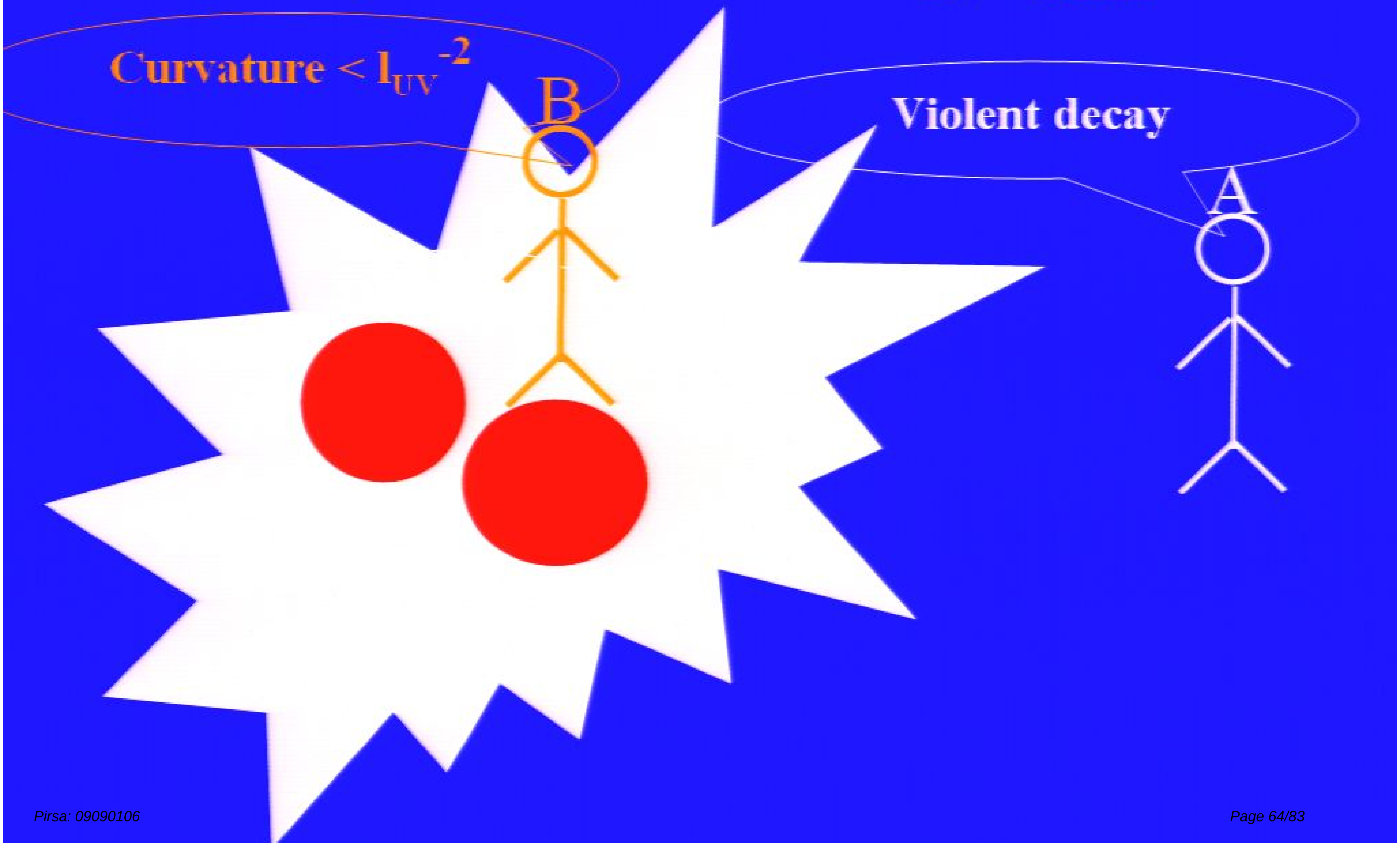

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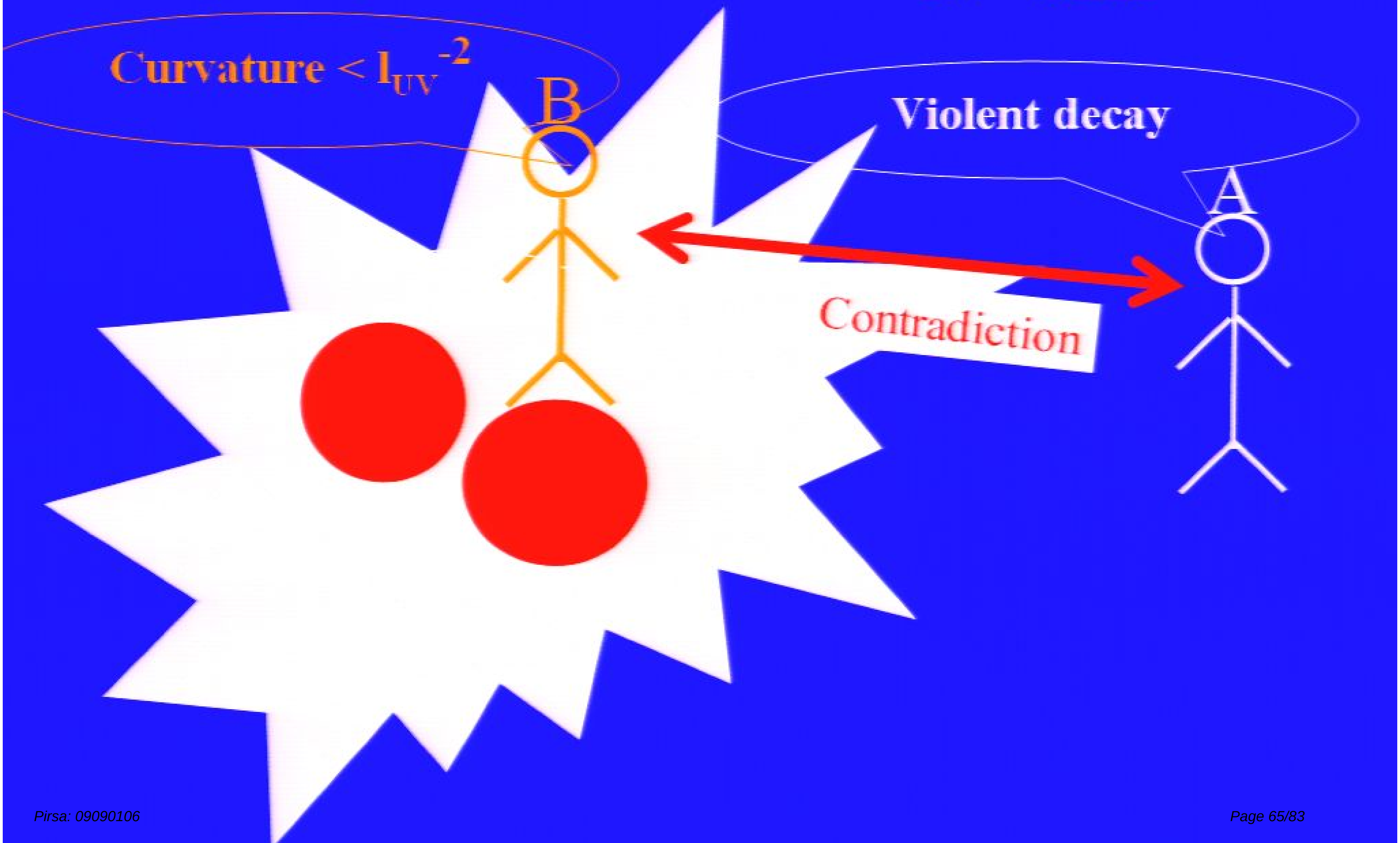

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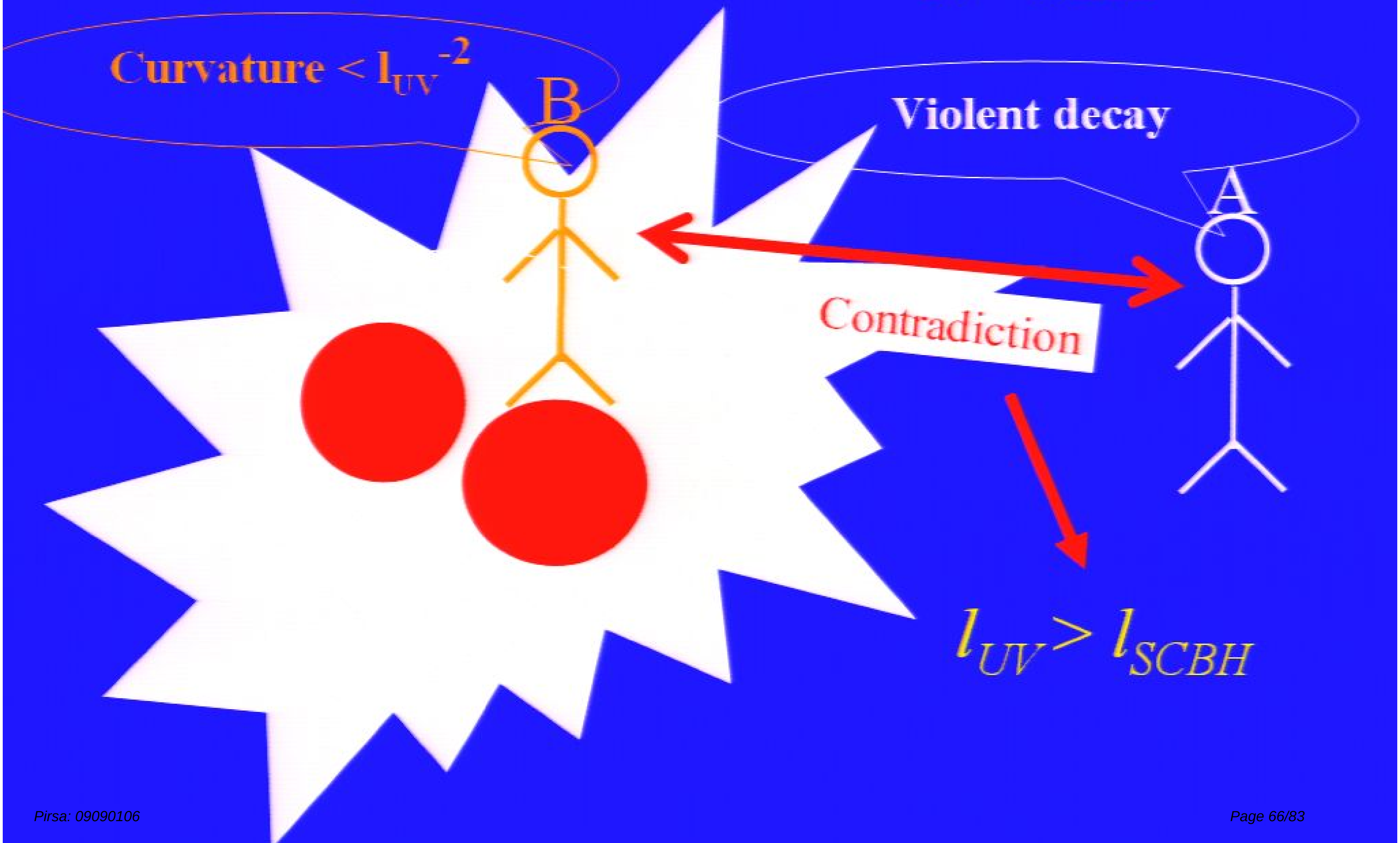

Generalized theories

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

he one particle exchange amplitude $G \equiv t^{\mu\nu}\langle h_{\mu\nu}h_{\alpha\beta}\rangle T^{\alpha\beta}$

$$G = \frac{1}{M_P^2}\frac{t_{\mu\nu}T^{\mu\nu} - \frac{1}{2}t^{\mu}_{\mu}T^{\nu}_{\nu}}{\Box} + \sum_i \frac{1}{M_i^2}\frac{t_{\mu\nu}T^{\mu\nu} - \frac{1}{3}t^{\mu}_{\mu}T^{\nu}_{\nu}}{\Box - m_i^2} + \sum_j \frac{1}{(\overline{M}_j)^2}\frac{t^{\mu}_{\mu}T^{\nu}_{\nu}}{\Box - (\overline{m}_j)^2}$$

All coefficients are +ve for ghost/tachyon free theories, mass screening that reduces the acceleration of the probe not possible!

Vectors irrelevant for conserved sources

Previous parametrization as an expansion in powers of curvature tensor not useful!

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

I(r)>0

$$h_{00}(r) = -\frac{M}{M_P^2}\frac{1}{r}\left(1 + \int_0^\infty dm\rho(m)e^{-mr}\right)$$

Horizon @ $h_{00}(R_S) = -1$

$T = dh_{00}/dr|_{r=R_S}$

$TR_S = 1$ → $\frac{d\ln R_S}{d\ln_M} = 1$

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Examples

- Compactified D=4+n Einstein Gravity for $r < R_C$
- Weakly coupled string theory

$$l_{UV} = l_s \qquad l_P = l_s g_s \qquad \lambda_G|_{l_s} = N l_P^2 / l_s^2 = N g_s^2$$

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number of the energetically-available species in string theory seems to be exponentially large?! Yes, but

$$N \sim \left(\frac{T}{M_s}\right)^2$$

Non-rot.→ non-rot.

Consequences

- Triviality of QG: $G_N \to 0 \text{ for } l_{UV} \to 0 (\Lambda_{UV} \to \infty)$

$$\lambda_G(l_{UV}) = N G_N / l_{UV}^2 < 1 \Longrightarrow G_N < \frac{l_{UV}^2}{N}$$

Not possible to consistently renormalize any theory of QG with a finite fixed number of fields (N=8 SUGRA!)

Consequences

- The Sakharov induced gravity limit for a finite UV cutoff

The Tree-level $G_N \to \infty$ (the Tree-level E-H removed)

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The renormalized G_N remains finite and bounded

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w/. N ~ 100s

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$\lambda_G \sim \lambda_{GUT}$ @ GUT scale

$N \sim \#\, bosons \gg \tilde{N} \sim group\ rank$

Consequences: Entropy bounds

Einstein gravity $$S_{BH}(R_S) = M_P^2 R_S^2$$

$$R_S M \quad > \quad N \Longrightarrow S_{BH} > N$$

more general? proof ?

Saturation is very interesting!

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