Title: Screening effects in plasma with charged Bose condensate

Date: Sep 28, 2009 02:00 PM

URL: http://pirsa.org/09090105

Abstract: The screening of electric charge in plasma with Bose condensate of a charged scalar field is calculated. In all previous calculations before 2009 the effects of Bose condensation have not been considered. Due to the condensate the time-time component of the photon polarization tensor in addition to the usual terms k-squared and Debye mass squared, contains infrared singular terms inversely proportional to k and k-squared. Such terms lead to power law oscillation behaviour of the screened potential, which is different form Friedel oscillations known for fermions. An analogue of Friedel oscillations in bosonic case is also considered.

Similar results, at $T = 0$, obtained by G. Gabadadze, R.Rosen:
Phys. Lett. B666 (2008) 277;
JCAP 0810 (2008) 030;
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Why it may be interesting?

1. He-condenstation in dense stars.
2. Cosmological plasma with large lepton asymmetry.
3. Normal solid state physics?

Chemical potential is introduced to describe asymmetry between particles and antiparticles in thermal equilibrium:

$$f = \left[e^{(E-\mu)/T} \pm 1\right]^{-1}$$

with $\mu = -\bar{\mu}$.

Maximum value of bosonic chemical potential is m_B. If charge asymmetry is so large that $\mu = m_B$ cannot ensure it, bosons would condense:

$$f_B = C\delta^{(3)}(p) + \left[e^{(E-m_B)/T} - 1\right]^{-1},$$

equilibrium solution of kinetic equation, if and only if $\mu = m_B$.

Well known that electric charge, Q, in plasma is screened according to the Debye law:

$$U(r) = \frac{Q}{4\pi r} \to \frac{Q\,\exp(-m_D r)}{4\pi r},$$

because the time-time component of the photon polarization tensor, due to interaction with medium, acquires the form:

$$\Pi_{00}(k, \omega = 0) = k^2 + m_D^2\,.$$

Effects of bosonic condensate lead to infrared singular terms:

$$\Pi_{00} = k^2 + e^2 \left(m_0^2 + \frac{m_1^3}{k} + \frac{m_2^4}{k^2} \right) ,$$

and creates oscillating screening:

$$U \sim \frac{\exp[-\sqrt{e/2}\, m_2 r] \cos[\sqrt{e/2}\, m_2 r]}{r} .$$

Moreover, $1/k$ term, present only at $T \neq 0$, leads to power law screening.

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Surprised by such outrageous behaviour we did not publish the paper for some time but then found out that similar, but not exactly the same properties of screening, are known in purely fermionic plasma: Friedel oscillations discovered half a century ago and observed in experiment.

Theory: electrodynamics with charged fermions ψ and bosons ϕ:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |(\partial_\mu + i\,eA_\mu)\phi|^2 - -m_B^2|\phi^2 + \bar{\psi}(i\not{\partial} - e\not{A} - m_F)\psi\,.$$

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$$(i\partial\!\!\!/ - m)\psi(x) = eA\!\!\!/\psi(x)\,,$$

$$(\partial_\mu\partial^\mu + m^2)\phi(x) = eJ_\phi(x)\,,$$

$$\partial_\nu F^{\mu\nu}(x) = eJ^\mu(x)\,.$$

where the currents J are defined as

$$J_\phi = -i\left[\partial_\mu A^\mu + 2A_\mu\partial^\mu\right]\phi + eA^\mu A_\mu\phi\,,$$

$$J^\mu = -i\left[(\phi^\dagger\partial^\mu\phi) - (\partial^\mu\phi^\dagger)\phi\right] + 2eA^\mu|\phi|^2 - \bar{\psi}\gamma^\mu\psi\,.$$

Solve these operator equations perturbatively:

$$\phi = \phi_0 + eG_B * \mathcal{J}_\phi ,$$

$$\psi = \psi_0 + e\,G_F * \not{A}\psi ,$$

substitute the expressions for the currents into Maxwell equations and average over medium.
The usual imaginary time (applicable only for equilibrium case) or real-time thermal field theories may be used but with Bose condensate we found it simpler to start from the first principles.

The zeroth order fields satisfies the free equations of motion:

$$(\partial_\mu \partial^\mu + m_B^2)\phi_0(x) = 0,$$
$$(i\partial\!\!\!/ - m_F)\psi_0(x) = 0$$

and are quantized in the usual way

$$\phi_0(x) = \int \frac{d^3q}{\sqrt{(2\pi)^3 2E}}[a(q)e^{-iqx} + b^\dagger(q$$

$$\psi_0(x) = \int \frac{d^3q}{\sqrt{(2\pi)^3}}\sqrt{\frac{m_F}{E}}[c(q)u(\mathrm{q})e^{-iqx} +$$

Calculate electromagnetic current:

$$J_\mu = J_\mu[A_\alpha, \phi_0, \psi_0]\,,$$

substitute this quantum current into Maxwell equation for classical electromagnatic field, take average over medium, separating vacuum and particle states:

$$\langle a^\dagger(\mathrm{q})a(\mathrm{q}')\rangle = f_B(E_q)\delta^{(3)}(\mathrm{q}-\mathrm{q}'),$$
$$\langle a(\mathrm{q})a^\dagger(\mathrm{q}')\rangle = [1+f_B(E_p)]\delta^{(3)}(\mathrm{q}-\mathrm{q}'),$$
$$\langle c^\dagger(\mathrm{q})c(\mathrm{q}')\rangle = f_F(E_p)\delta^{(3)}(\mathrm{q}-\mathrm{q}'),$$
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Obtain the well known result, but with arbitrary occupation numbers:

$$[k^2 g^{\mu\nu} - k^\mu k^\nu + \Pi^{\mu\nu}(k)] A_\nu(k) = e\, J^\mu(k)$$

where the bosonic part of the photon polarization tensor has the form:

$$\Pi^B_{\mu\nu}(k) = e^2 \int \frac{d^3q}{(2\pi)^3 E} \left[f_B(E) + \bar{f}_B(E) \right]$$

$$\left[\frac{(2q-k)_\mu (2q-k)_\nu}{2((q-k)^2 - m_B^2)} + (k \to -k) - g_{\mu\nu} \right].$$

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$$\left. + \frac{q_\nu (q-k)_\mu + q^\rho k_\rho g_{\mu\nu} + q_\mu (q-k)_\nu}{(k-q)^2 - m_F^2} \right] .$$

Integrate over angles in istropic plasma:

$$\Pi_{00}^{B}(0,k) = \frac{e^2}{2\pi^2}\int_0^\infty \frac{dq\, q^2}{E_B}\left[f_B + \bar{f}_B\right]$$
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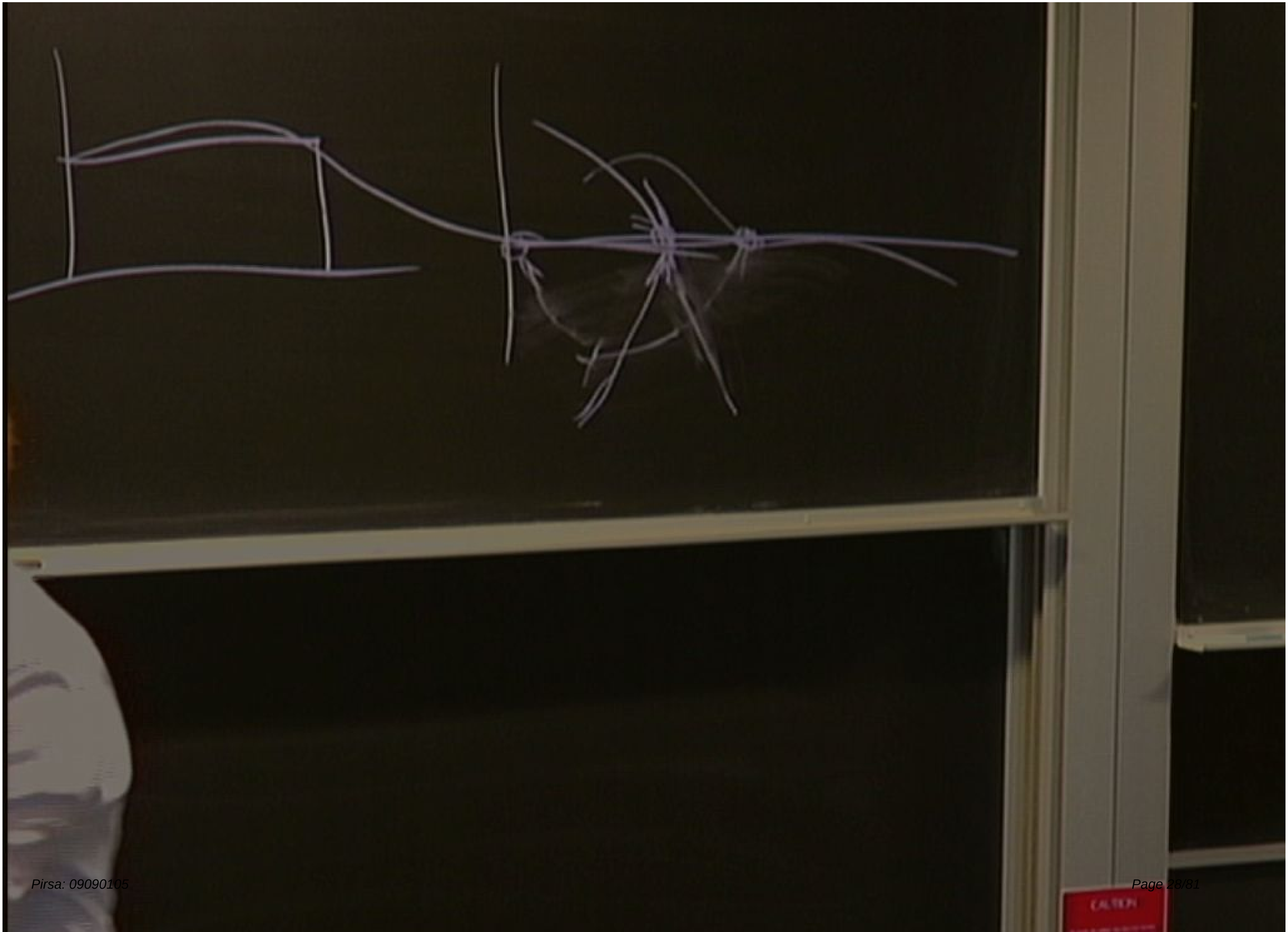

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where function $h(T)$ is independent of k and has the limiting values:

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More generally, they are induced by the singularities of Π_{00} due to pinching of the integration contour over dq by the poles of $f_F(E)$ at:

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NB: k^{-1} appears only if $\mu_B = m_B$.

Integrate over angles in istropic plasma:

$$\Pi_{00}^{B}(0,k) = \frac{e^2}{2\pi^2}\int_0^\infty \frac{dq\, q^2}{E_B}\left[f_B + \bar{f}_B\right]$$

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Friedel oscillations in fermionic plasma: usually precribed to an abrupt cut-off of the integral over dq for strongly degenerate plasma at $T = 0$.

More generally, they are induced by the singularities of Π_{00} due to pinching of the integration contour over dq by the poles of $f_F(E)$ at:

$$q_n^2 = [\mu \pm i\pi T(2n+1)]^2 - m_F^2,$$

where n runs from 0 to infinity, and log branch points at $q_b = \pm k/2$.

Π_{00} has singularities at $k = 2q_n$.

Contribution of n-th singularity to $U(r)$ at large r:

$$U_n(r) = \frac{Q}{2\pi^2 r} \mathcal{I}m \int_0^\infty idy\, ke^{-yr+2iq_n r}$$

$$\frac{(-\Delta\Pi_{00})}{\left[k^2 + \Pi_{00}^{(n)+}\right]\left[k^2 + \Pi_{00}^{(n)-}\right]}.$$

Here $k = 2q_n + iy$.

Discontinuity $\Delta\Pi_{00} = ie^2 Ty$ – can be found by moving the integration contour in q-plane over n-th pole.

After simple integration:

$$U_n(r) = \frac{Qe^2T}{16\pi^2 q_n^3 r^3} \sin(2\mu r) e^{-2\pi(2n+1)Tr}.$$

If $Tr \gg 1$ the lowest term with $n = 1$ dominates.

For $T \to 0$ all terms give comparble contribution and summation over n in relativistic limit gives:

$$U = \frac{e^2 Q T}{16\pi^2 r^3 \mu^3} \frac{\sin(2\mu r)\, e^{-2\pi r T}}{1 - e^{-4\pi r T}}$$

$$\approx \frac{e^2 Q}{64\pi^3} \frac{\sin(2\mu r)}{r^4 \mu^3}, \quad \text{for } Tr \ll 1\,.$$

In non-relativistic case:

$$U(r) = \frac{Qe^2 m_F \cos(2q_F r)}{64\pi^3 r^3 q_F^3},$$

where $q_F = \sqrt{2\tilde{\mu} m_F}$.

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$$\approx \frac{e^2 Q}{64\pi^3} \frac{\sin(2\mu r)}{r^4 \mu^3}, \;\; \text{for} \;\; Tr \ll 1\,.$$

In the limit of small k the integral can be taken analytically:

$$\Pi_{00}^{B} = e^2 \left[h + \frac{m_B^2 T}{2k} + \frac{C(1 + 4m_B^2/k^2)}{(2\pi)^3 m_B} \right],$$

where function $h(T)$ is independent of k and has the limiting values:

$$h(T) = T^2/3, \quad \text{for highT},$$

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Screening in bosonic plasma.

I. Contributions of the poles:

$$k^2 + e^2\left(m_0^2 + \frac{m_1^3}{k} + \frac{m_2^4}{k^2}\right) = 0,$$

where

$$m_0^2 = \frac{C}{(2\pi^3)m_B} + h(T) + m_D^{(F)2}(T,\mu_F),$$

$$m_1^3 = m_B^2 T/2,$$

$$m_2^4 = 4m_B C/(2\pi)^3.$$

Many different limiting cases. E.g. for $m_2^2 \ll e m_0^2$:

$$U(r) \approx \frac{Q}{4\pi r}\left[e^{-e m_0 r} - \frac{m_2^4}{e^2 m_0^4} e^{-\frac{m_2^2 r}{m_0}}\right].$$

In the opposite limit, $e^4 m_0^4 < 4 e^2 m_2^4$ and especially large m_2:

$$U(r) = \frac{Q}{4\pi r} e^{-\sqrt{\frac{e}{2}} m_2 r} \cos\left(\sqrt{\frac{e}{2}} m_2 r\right).$$

Since Π_{00} is an odd function of k, the integral over imaginary axis of k does not vanish and gives power law asymptotics. If $m_2 \neq 0$ it behaves as

$$U(r) = -\frac{12Qm_1^3}{\pi^2 e^2 r^6 m_2^8}.$$

If $T \neq 0$, $\mu = m_B$, but $C = 0$:

$$U(r) = -\frac{2Q}{\pi^2 e^2 r^4 m_B^2 T}.$$

So the signal of the condensate formation is a strong decrease of screening.

Bosonic analogue of Friedel oscillations.

The same type of singularties due to pinching of the q-contour by poles of $f_B(E)$ and the branch points of the logarithm leads to singularities of Π_{00}^B in the complex k-plane.

The difference is that the poles move to zero if $T \to 0$:

$$q_n = (4i\pi n T m_B)^{1/2} (1 + i\pi n T/m_B)^{1/2} .$$

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At large n and $T \neq 0$ the screened potential, in relativistic limit, is dominated by $n = 1$:

$$U_1(r) = -\frac{Q\pi^2}{2e^2}\frac{Tm_B^2}{r^2\mu_F^4}e^{-Z}\cos Z\,,$$

where $Z = 2r\sqrt{2\pi m_B T} > 1$.

For a small Z the effective number of the singular terms is $n_{eff} \sim 1/Z^2 \gg 1$ and summation shold be taken over them. On the other hand, the singular terms dominate over k^2 term in Π_{00} if $n < 10^{-3}(m_B/T)^{1/3}$. In this interval of the parameter values the potential behaves as

$$U(r) \approx -\frac{3Q}{2e^2T^2m_B^3r^6\ln^3\left(\sqrt{8m_BT}r\right)}.$$

THE END

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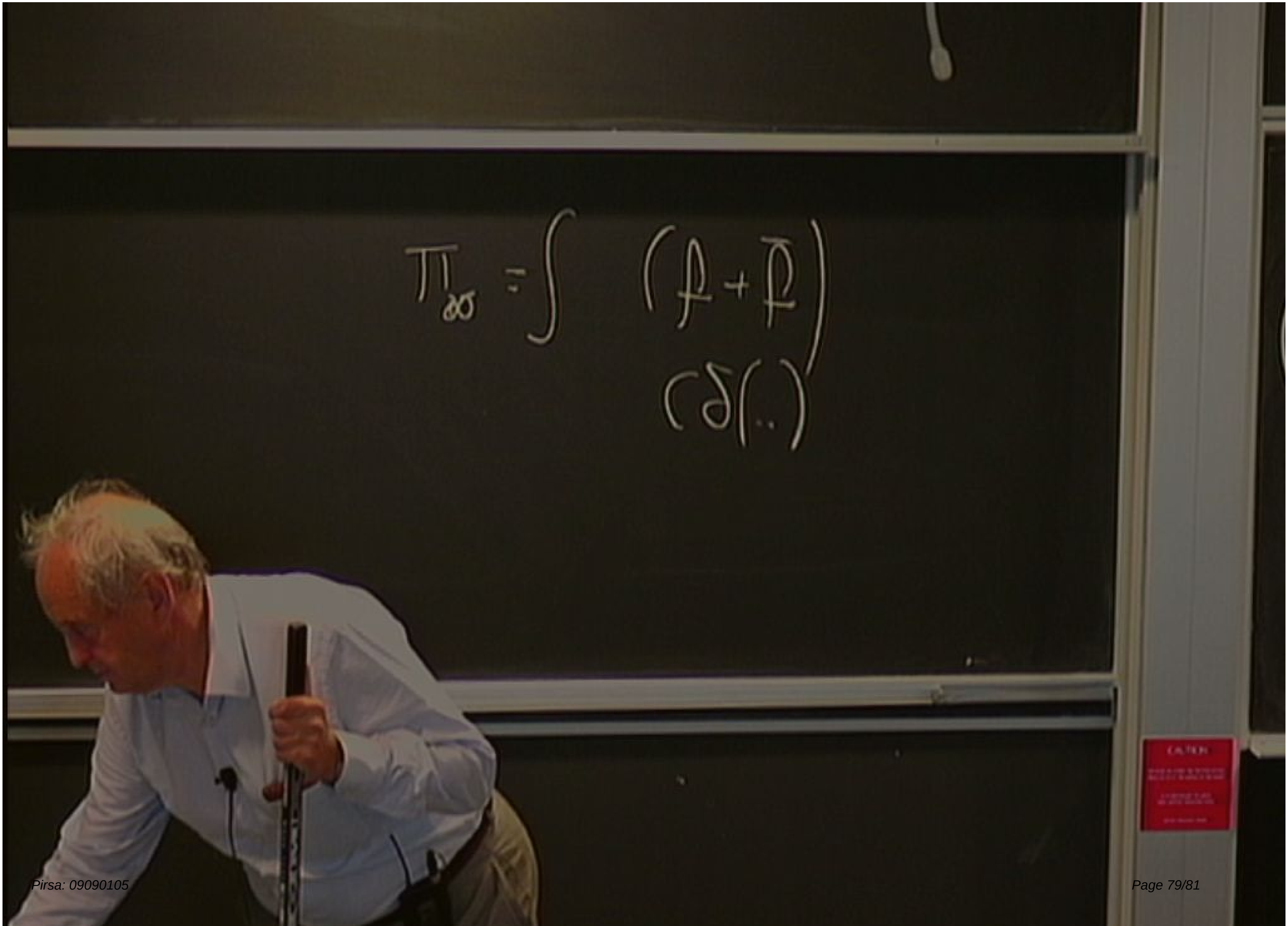

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