

Title: Noncommutative Effects in Primordial Density Fluctuations

Date: Sep 29, 2009 02:00 PM

URL: <http://pirsa.org/09090102>

Abstract: Near the Planckian scales, quantum gravity is expected to drastically change the structure of spacetime, one feature of which may be noncommutativity of the coordinates. Based on the recent advances in quantum field theories on such noncommutative spaces, I will consider the fluctuations of inflaton and look for possible noncommutative corrections in the CMB. Anisotropy and non-gaussianity are the result. The resultant distribution is then compared with ACBAR, CBI and WMAP data to constrain the scale of noncommutativity parameter.

Plan

$$\langle \phi | \phi \rangle = \langle \phi | \phi \rangle$$

Plan

$$\langle 0 | \phi(x_1) \phi(x_2) | 0 \rangle$$

Plan

$$\langle 0 | \phi(x_1) \phi(x_2) | 0 \rangle$$

$$\langle 0 | \phi \phi \phi \phi | 0 \rangle$$

Anisotropy

Plan

$$\langle \sigma | \phi(x_1) \phi(x_2) | 0 \rangle$$

$$\langle \sigma | \phi \phi \phi | 0 \rangle$$

Anisotropy

Non-Commuting

Plan

$$\langle \sigma | \phi(x_1) \phi(x_2) | 0 \rangle$$

$$\langle \sigma | \phi \phi \phi | 0 \rangle$$

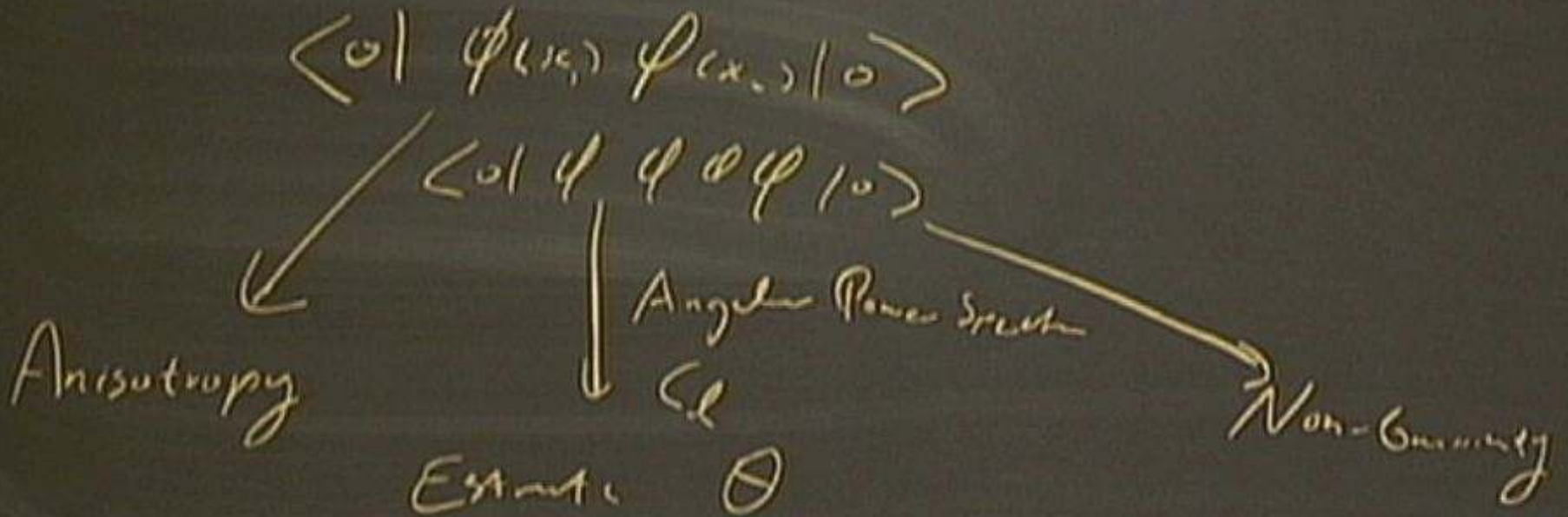
Anisotropy

Angular Power Spectra

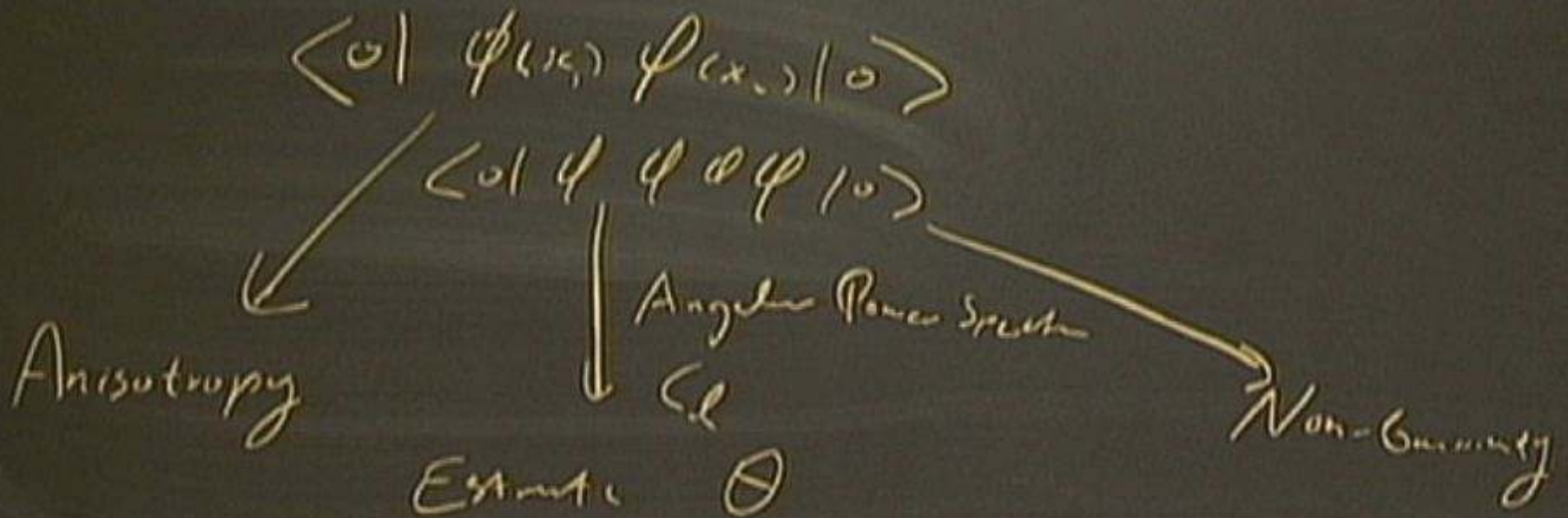
Cl

Non-Gaussianity

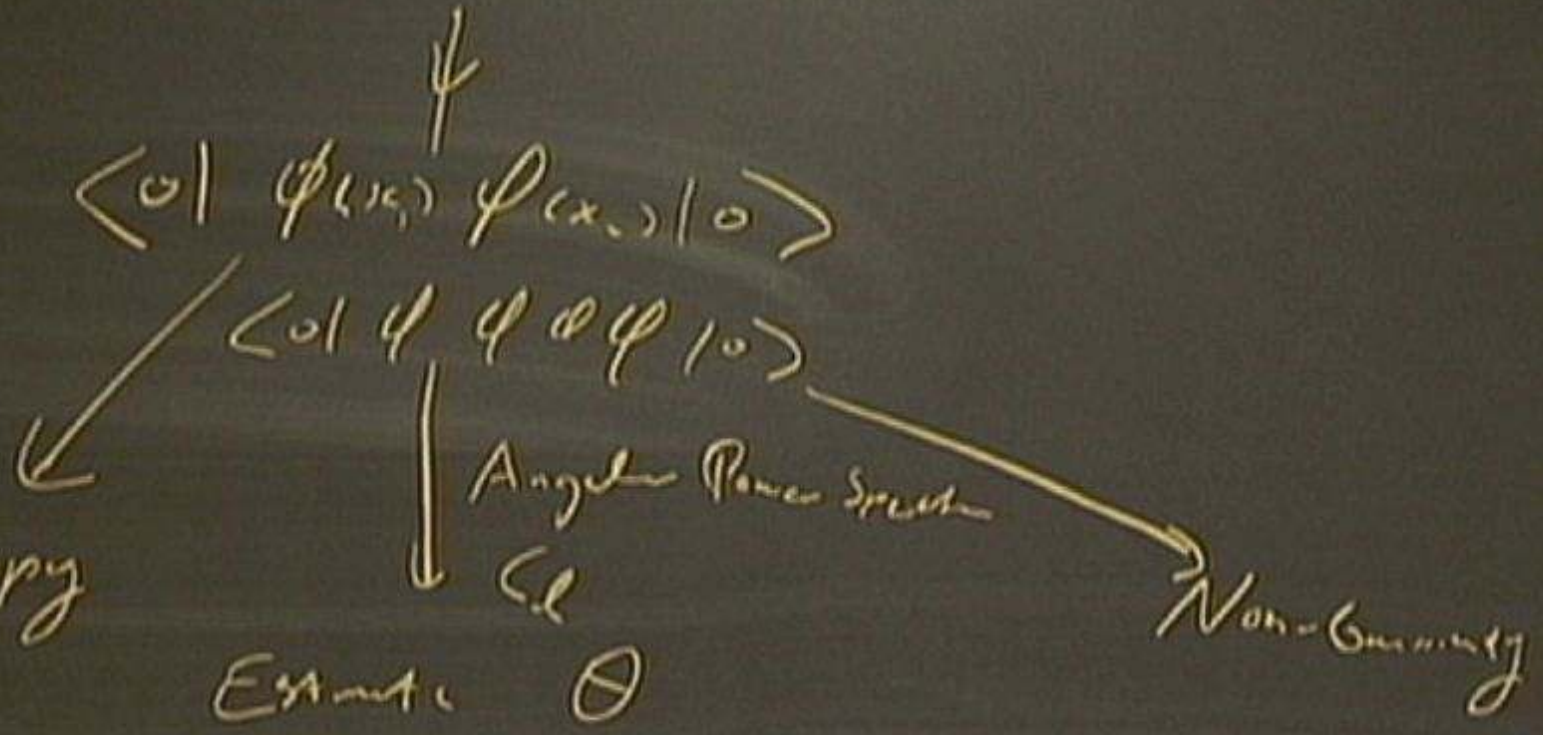
Plan



Plan



Plan



Plan

Trusted Statistics

changed

Commutator relations

between fields

$$\langle \psi | \phi(x_1) \phi(x_2) | 0 \rangle$$

$$\langle \psi | \phi \phi \phi | 0 \rangle$$

Angular Power Spectra

Cl

Non-Commutativity

Example θ

Plan Hoppf Symmetry on N.C. Spaces

Twisted Statistics

changed

Commutator relations

between fields

$$\langle 0 | \phi(x_1) \phi(x_2) | 0 \rangle$$

$$\langle 0 | \psi \psi \psi \psi | 0 \rangle$$

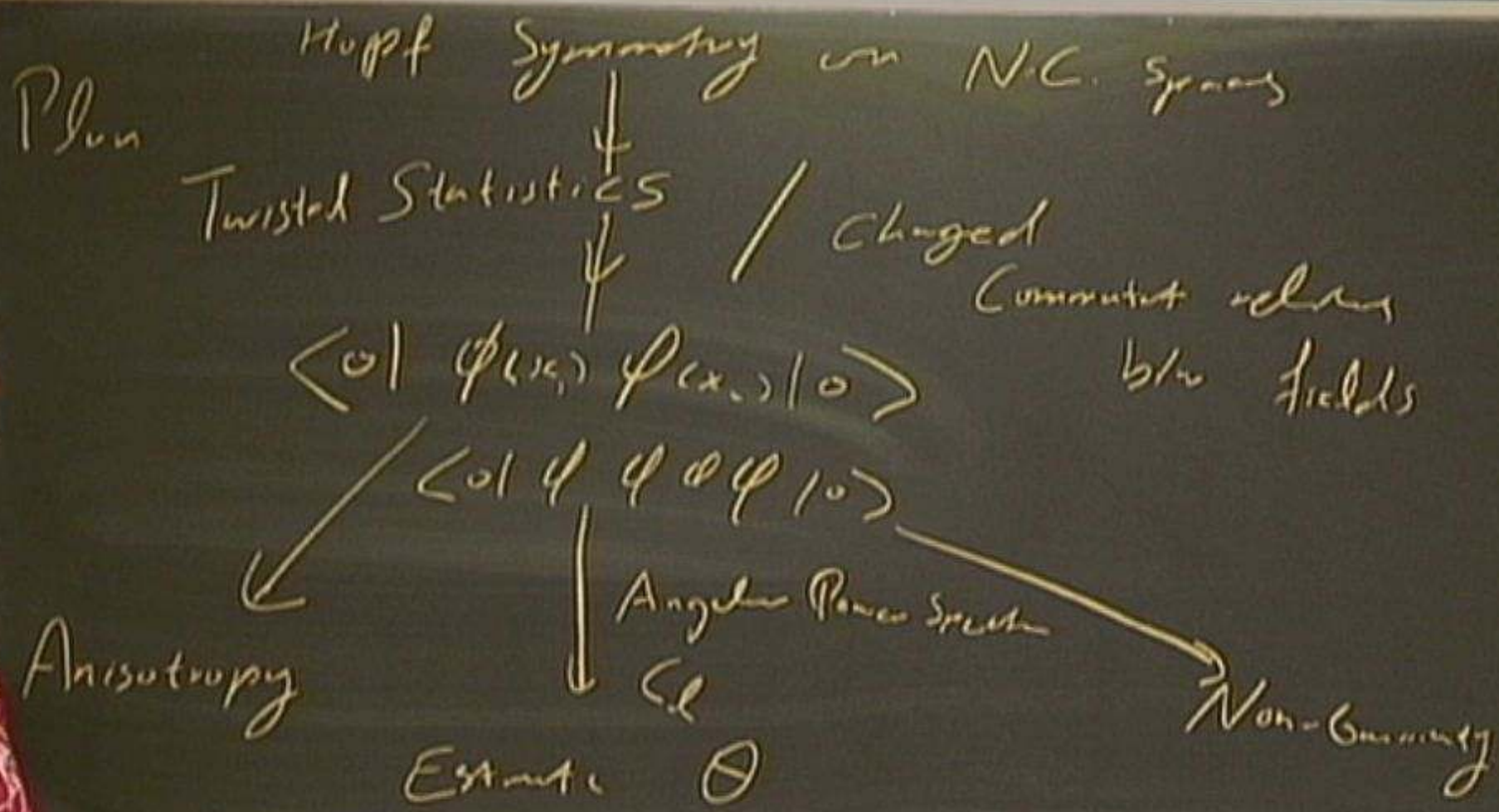
Angular Power Spectra

Anisotropy

Sl

Non-Gaussianity

Estimate Θ



Example ①

Von-Neumann

$$0100 \geq L^2$$
$$0x0x \geq L^2$$

EXAMPLE θ

Von-Neumann

$$O(1) \supseteq \mathcal{L}^2$$

$$O_n \supseteq \mathcal{L}^2$$

$$[\dot{x}_n, \dot{x}_n] = i\theta_n$$

Example θ

Von-Neumann

$$O(1) \supseteq \mathcal{L}^n$$

$$O_n \supseteq \mathcal{L}^n$$

$$[\dot{x}_n, \dot{x}_n] = iO_n$$

$f(x)$

Estimate θ

Von-G...ing

$$O(10^6) \geq L^2$$

$$O_n O_n \geq L^2$$

$$[\hat{x}_n, \hat{x}_n] = i O_n$$

$$f(x) * g(x) =$$

Example θ

Von-Neumann

$$\mathcal{O}(\mathcal{O} \supseteq \mathcal{L}^n \\ \mathcal{O} \supseteq \mathcal{L}^n$$

$$[\hat{x}_n, \hat{x}_0] = i\mathcal{O}_n$$

$$f(x) * g(x) = f(x) e^{-\frac{i}{\hbar} \mathcal{O}_n} g(x)$$

EXAMPLE ①

Von-Gurong

$$O(1) \supseteq \mathbb{C}^n$$

$$O(n) \supseteq \mathbb{C}^n$$

$$[\hat{x}_n, \hat{x}_n] = iQ_n$$

$$f(x) * g(x) = f(x) e^{-\frac{i}{2} \hat{x}_n \hat{x}_n} g(x)$$

$$x_- * x_+ - x_+ * x_- = iQ_n$$

Example ①

1000 - Gummy

$$\begin{aligned} \mathcal{O} \in \mathcal{L} &\Rightarrow \mathcal{L} \\ \mathcal{O} \times \mathcal{O} &\in \mathcal{L} \end{aligned}$$

$$S[\phi] = \int d^4x \quad \partial_\mu \phi \partial^\mu \phi$$

$$\begin{aligned} &+ m^2 \phi^2 \\ &+ \phi^4 \end{aligned}$$

$$[\dot{x}_\mu, \dot{x}_\nu] = iQ_{\mu\nu}$$

$$f(x) * g(x) = f(x) e^{-\frac{i}{2} \dot{x}_\mu \dot{x}_\nu} g(x)$$

$$x_\mu * x_\nu - x_\nu * x_\mu = iQ_{\mu\nu}$$

$SL(4)$ not invariant under Lorentz symmetry

$S[\phi]$ not invariant under Lorentz symmetry

$$g \triangleright \phi(x) = \phi(g^{-1}x)$$

$S(\phi)$ not invariant under Lorentz symmetry

$$\mathcal{L}(\phi(x)) = \mathcal{L}(g^{-1}x)$$

$$\mathcal{L}(\phi(x)) * \mathcal{L}(h(x)) \neq \mathcal{L}(\phi + h)$$

Hopf Symmetries.

Hopf Symmetries.

$$J^D m = m_0 \Delta(g) (f \otimes h)$$

↙

$$\Delta(g) = \psi(G) \otimes \psi(G)$$

Hopf Symmetries.

$$m(f \otimes h) = m_0 \Delta(g) (f \otimes h)$$

$$\Delta(g) = \eta(G) \otimes \eta(G)$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$g \triangleright m(f \otimes h) = \Delta(g) (f \otimes h)$$

$$(g \triangleright f) \otimes h$$

$$f \otimes (g \triangleright h)$$

$$\sigma(g) = g \otimes g$$

Hopf Symmetries

$$g \triangleright m(f \otimes h) = m \Delta(g) (f \otimes h)$$

$$\Delta(g) = h(G) \otimes h(G)$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = fg$$

$$g \triangleright m(f \otimes h) = m_{\Delta(g)}(f \otimes h)$$

$$\Delta(g) = h(G) \otimes h(G)$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = f \cdot g$$

$$g \triangleright m_0(f \otimes h) = m_0 \Delta(g) (f \otimes h)$$

$$\Delta(g) = \eta(G) \psi(G)$$

$$\psi(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = fg$$

$$g \triangleright m_0(f \otimes h) = m_0 \Delta(g)(f \otimes h)$$

$$\Delta(g) = h(G) \otimes h(G)$$

$$\Delta(1) = 1 \otimes 1$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = fg$$

$$g \triangleright m_0(f \otimes h) = m_0 \Delta(g)(f \otimes h)$$

$$\Delta(g) = h(G) \otimes 1(G)$$

$$\Delta(g) = g \otimes 1$$

$$\Delta(L) = L \otimes 1$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1, g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = fg$$

$$j \triangleright m_0(f \otimes h) = m_0 \Delta(g) (f \otimes h)$$

$$\Delta(g) = h(G) \otimes 1(G)$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1, g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = fg$$

$$j \triangleright m_0(f \otimes h) = m_0 \Delta(g) (f \otimes h)$$

$$\Delta(g) = \eta(G) \otimes \eta(G)$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

$$\Delta(\vartheta_1) \cap (\vartheta_2) = \Delta(\vartheta_1, \vartheta_2)$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

Automorphism

$$f' \cdot g' = f \cdot g$$

$$\Delta(\mathfrak{g}_1) \cap \Delta(\mathfrak{g}_2) = \Delta(\mathfrak{g}_1 \cap \mathfrak{g}_2)$$

$$\sigma(L) = 1 \otimes L + L \otimes 1$$

Automorphism

$$f' \cdot g' = f \cdot g$$

$$\Delta(\varphi_1) \cap (\varphi_2) = \Delta(\varphi_1, \varphi_2)$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

Anticommutativity

$$f \cdot g = -g \cdot f$$

$$U_1 \otimes U_2$$

$$\Delta(f, g) \cap (g, h) = \Delta(g, h)$$

$$\Delta(L) = L \cup L^c$$

Automorphism

$$f \cdot g' = f \cdot g$$

$$g \supseteq U_1 \cup U_2 = f \cup f^{-1}(U_1 \cup U_2)$$

$$\Delta(y_1) \cap (y_1) = \Delta(y_1)$$

$$\Delta(L) = L \cup L^c$$

Automorphism

$$f \cdot g' = f \cdot g$$

$$g \triangleright \psi_1 \cup \psi_2 = \rho \cup f(\psi) (\psi_1 \cup \psi_2)$$

$$\Delta(f, g) \cap (f, g) = \Delta(g, f)$$

$$\Delta(L) = L \cup L^c$$

derivation

$$f \cdot g' = f \cdot g$$

$$\Delta(U, V) = \rho \cup \int \underset{g \cup g'}{\Delta} (U, V)$$

$$\Delta(x, y) \cup (y, x) = \Delta(y, x)$$

$$\Delta(L) = L \cup L^R$$

Automorphism

$$f \cdot g' = f \cdot g$$

$$g \triangleright \psi_1 \cup \psi_2 = \rho \cup f \left(\begin{array}{c} \downarrow \\ \psi \cup \gamma \end{array} \right) (\psi_1 \cup \psi_2)$$

\downarrow Check-Union

$$\Delta(g_1) \cap \Delta(g_2) = \Delta(g_1 g_2)$$

$$\Delta(L) = L \cup L^{-1}$$

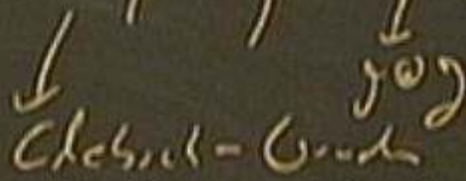
Automorphisms

$$\Delta(g) = g \circ g$$

$$f \cdot g = f \circ g$$

$$g \triangleright U_1 \circ U_2 = \rho \circ f \circ (\sigma) \circ (U_1 \circ U_2)$$

Hopf Algebra



$$H = (G, \sigma)$$

$$\Delta(\varphi_1) \cup (\varphi_2) = \Delta(\varphi_1, \varphi_2)$$

$$\Delta(L) = |L| + L \cup 1$$

$$\begin{array}{ccc}
 \varphi_1 \cup \varphi_2 \xrightarrow{\Delta} \varphi_1 \cup \Delta(\varphi_2) \\
 \downarrow m_0 \qquad \qquad \downarrow m_0 \\
 \varphi_1 \cup \varphi_2 \longrightarrow \varphi_1 \cup m_0(\varphi_2)
 \end{array}$$



$$\Delta(\varphi_1) \cup (\varphi_2) = \Delta(\varphi_1, \varphi_2)$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\alpha \otimes \beta \xrightarrow{\Delta} 1 \otimes \beta \cup (\alpha \otimes 1)$$

$$\downarrow F$$

$$\downarrow m_0$$

$$m_0 \alpha \otimes \beta \longrightarrow \rho(\varphi) m_0 \tau(\varphi)$$

$$= m_0 \bar{F} = e^{i \ell \cdot \mathbb{P} \otimes \mathbb{P}}$$

$$\Delta(\varphi_1) \cup (\varphi_2) = \Delta(\varphi_1, \varphi_2)$$

$$\Delta(L) = L \oplus L \oplus 1$$

$$\mathbb{R} \oplus \mathbb{R} \xrightarrow{\Delta} \mathbb{R} \oplus \mathbb{R} \cup (\mathbb{R} \oplus \mathbb{R})$$

$$\downarrow m_0 \quad \downarrow m_0$$

$$m_0 \oplus m_0 \longrightarrow \rho(\varphi) m_0 \oplus m_0$$

$$m_0 = m_0 \cdot F = m_0 \cdot e^{i \cdot \mathbb{R} \oplus \mathbb{R}}$$

Example 0

$$i\partial_t \psi \geq \mathcal{L} \psi$$

$$S[\phi] = \int d^4x \quad 2\phi \times 2\phi$$

$$[\hat{x}_r, \hat{x}_s] = i\delta_{rs}$$

$$+ m\phi - \phi$$
$$+ \phi - \phi - \phi$$

$$\psi = f(x) e^{i\vec{p}\cdot\vec{x} - iEt}$$

Invariant under Lorentz symmetry

$$\phi(g'x)$$

$$+ g \mathcal{D} h(x) \neq g \mathcal{D} \phi + h(x)$$

$$\mathcal{O}(10^a) \geq \mathcal{L}^n$$

$$\mathcal{O} \times \mathcal{O} \geq \mathcal{L}^n$$

$$[\dot{x}_r, \dot{x}_s] = i\theta_{rs}$$

$$f^{(n)} * g^{(n)} = f^{(n)} e^{i\theta_{rs}} g^{(n)}$$

$$S[\phi] = \int d^4x \quad 2\phi * 2\phi$$

$$+ m\phi - \phi$$

$$+ \phi - \phi - \phi$$

$S[\phi]$ not invariant under Lorentz symmetry

$$g \triangleright \phi^{(n)} = \phi(g'^{rs})$$

$$g \triangleright \phi^{(n)} * g \triangleright h^{(n)} \neq g \triangleright \phi + h^{(n)}$$

$$(\cdot, \cdot) = \langle \cdot, \cdot \rangle$$

$$\begin{array}{ccc}
 \alpha \otimes \beta & \xrightarrow{\Delta} & 1 \otimes \beta \oplus \alpha \otimes 1 \\
 \downarrow m_0 & & \downarrow m_0 \\
 \alpha \otimes \beta & \xrightarrow{\quad} & \rho(g) m_0 f(g) \\
 & & \downarrow \\
 & & U(1)
 \end{array}$$

$$\Delta = F^{-1} \Delta_0 F$$

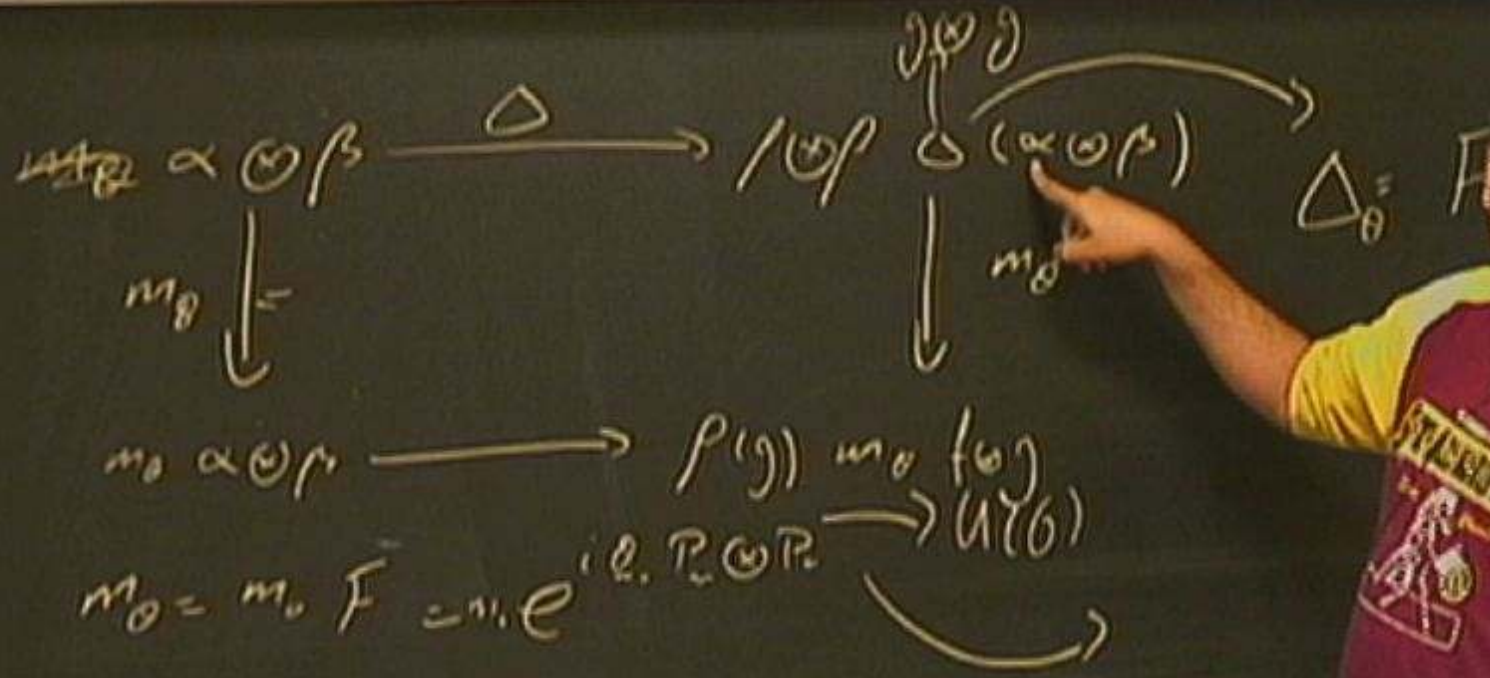
$$\begin{array}{ccc}
 m_0 = m_0 \cdot F = \dots & \xrightarrow{\text{i.e. } \mathbb{R} \otimes \mathbb{R}} & U(1)
 \end{array}$$

$$Q(x, y, z) = Q(y, z, x)$$

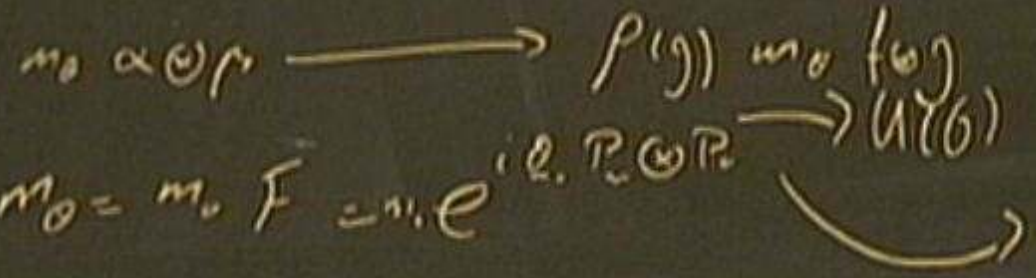
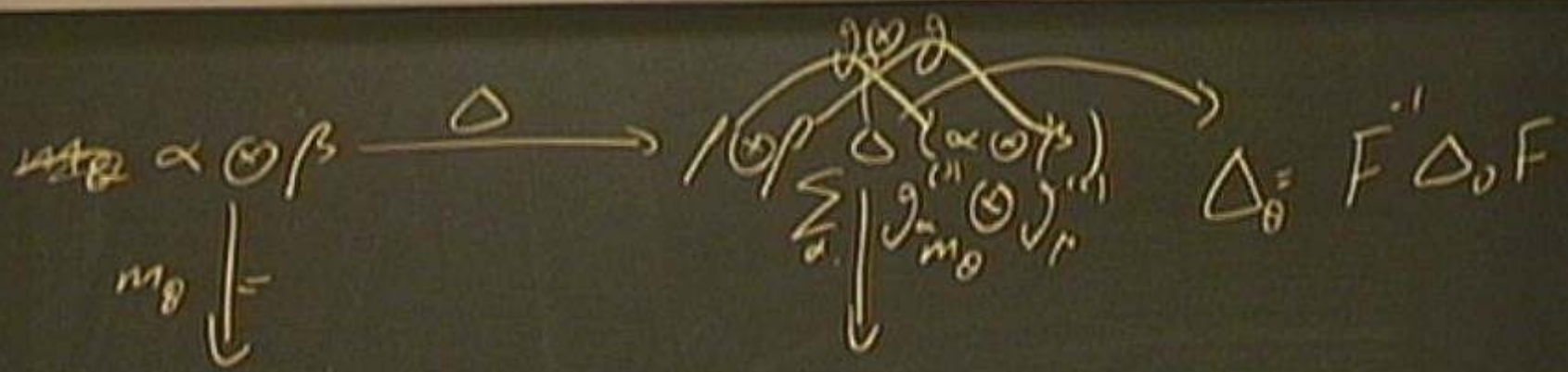
$$\begin{array}{ccc}
 \alpha \otimes \beta & \xrightarrow{\Delta} & 1 \otimes \beta \oplus \alpha \otimes 1 \\
 \downarrow m_0 & & \downarrow m_0 \\
 \alpha \otimes \beta & & \alpha \otimes \beta
 \end{array}
 \quad \Delta_0 = F^{-1} \Delta_0 F$$

$$\begin{array}{ccc}
 m_0 \alpha \otimes \beta & \longrightarrow & \rho(g) m_0 \otimes g \\
 & & \downarrow \\
 & & U(0)
 \end{array}
 \quad \text{i.e. } \mathbb{P} \otimes \mathbb{P} \longrightarrow U(0)$$

$$(\cdot, \cdot) = \langle \cdot, \cdot \rangle$$

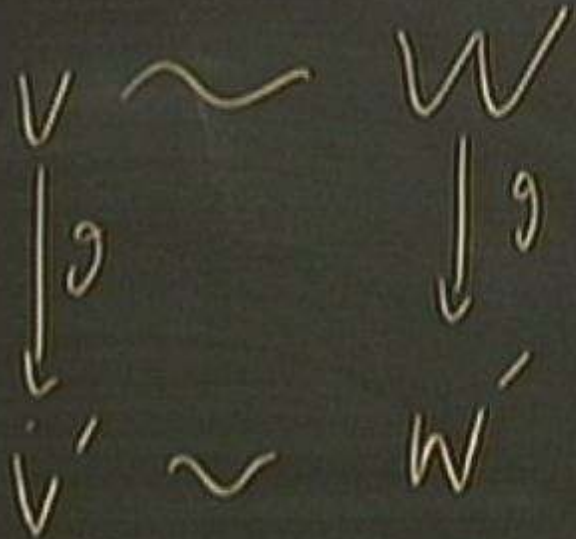


$$C(x, y) = C(y, x)$$



$$|k_1, k_2\rangle = \alpha \langle \dots | k_1, k_2 \rangle$$

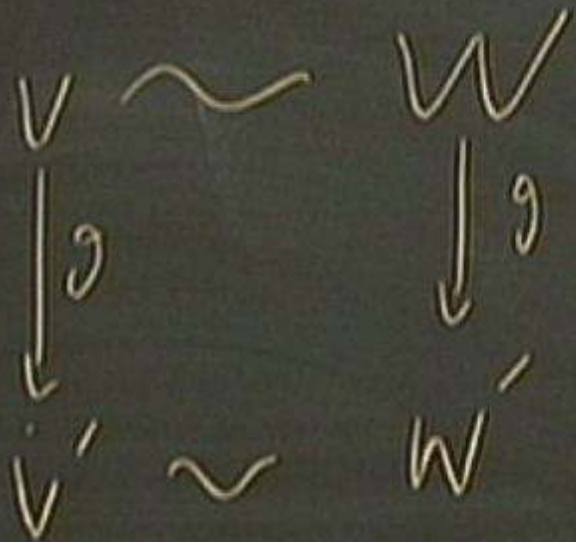
$$\sigma |k_1, k_2\rangle = \dots F |k_1, k_2\rangle$$



$$|k_1, k_2\rangle = \alpha \langle k_1, k_2 | \dots$$

$$\sigma |k_1, k_2\rangle = F |k_1, k_2\rangle$$

$$e^{i2\pi k_1 k_2} |k_1, k_2\rangle$$



$$U(g) |k_1, k_2\rangle = P \otimes P (U(g) |k_1, k_2\rangle)$$

$$1/19) |k_1, k_2\rangle = \rho(x) \rho(y) |k_1, k_2\rangle$$



$$U(g) |k_1, k_2\rangle = P \otimes P (g) |k_1, k_2\rangle$$

$$a_p^\dagger a_p - e^{i\theta} a_p^\dagger a_p$$

$$U(g) |k_1, k_2\rangle = \rho(g) U(g) |k_1, k_2\rangle_{in}$$

$$a_{k_1}^{\dagger} a_{k_2}^{\dagger} = e^{i\alpha} a_{k_2}^{\dagger} a_{k_1}^{\dagger}$$

$$\phi_0(x) = \phi_0(x) e^{-\frac{i}{\hbar} \partial \wedge P}$$

$$U(g) |k_1, k_2\rangle = \rho(g) U(g) |k_1, k_2\rangle$$

$$a_n a_p = e^{i\alpha_{np}} a_p a_n$$

$$\phi_0(x) = \phi_0(x) e^{-\frac{i}{\hbar} \partial \wedge P}$$

$$F = f_1 \otimes f_2$$

$$\langle 0 | \phi(x) | 0 \rangle = \langle 0 | \phi(x) \phi(x) | 0 \rangle$$

$$\langle 0 | \phi_0(x) \phi_0(x) | 0 \rangle = e^{-\frac{1}{2} \int dx \Lambda \partial_x} \langle 0 | \phi_0(x) \phi_0(x) | 0 \rangle$$



$$\langle 0 | \phi_0(x) \phi_0(x) | 0 \rangle = e^{-\frac{1}{2} \int dx \Lambda \partial_x} \langle 0 | \phi_0(x) \phi_0(x) | 0 \rangle$$

$$\phi_0(x) = \int \frac{d^4 k}{(2\pi)^4} \bar{\phi}_0(k, x) e^{ikx}$$

$$\langle 0 | \psi_0(x) \psi_0(x) \rangle = e^{-\frac{i}{\hbar} \Delta \partial_x} \langle 0 | \psi_0(x) \psi_0(x) | 0 \rangle$$

$$\psi_0(x) = (k, t) e^{i k x}$$

$$|0\rangle = \langle 0 | \psi_0(k_1, t_1 - \frac{\vec{0} \cdot k_1}{\hbar}) \psi_0(k_2, t_2 - \frac{\vec{0} \cdot k_2}{\hbar}) | 0 \rangle$$

$$\langle 0 | \phi_0(x) \phi_0(x) | 0 \rangle = e^{-\frac{1}{2} \partial_x^2 \Delta x} \langle 0 | \phi_0(x) \phi_0(x) | 0 \rangle$$

$$\phi_0(x) = \int \frac{d^4 k}{(2\pi)^4} \tilde{\phi}_0(k, x) e^{i k \cdot x}$$

$$\langle 0 | \phi_0(\bar{k}_1, t) \phi_0(\bar{k}_2, t) | 0 \rangle = \langle 0 | \phi_0(\bar{k}_1, t, \frac{\vec{\theta} \cdot \bar{k}_1}{2}) \phi_0(\bar{k}_2, t, \frac{\vec{\theta} \cdot \bar{k}_2}{2}) | 0 \rangle$$

$$\langle 0 | \phi_0(x) \phi_0(x) | 0 \rangle = e^{-\frac{1}{2} \partial_x \Delta \partial_x} \langle 0 | \phi_0(x) \phi_0(x) | 0 \rangle$$

$$\phi_0(x) = \int \frac{d^4 k}{(2\pi)^4} \tilde{\phi}_0(k, x) e^{i k \cdot x}$$

$$\langle 0 | \phi_0(\bar{h}, t) \phi_0(\bar{h}, t) | 0 \rangle = \langle 0 | \phi_0(\bar{h}, t, \frac{\vec{\partial} \cdot \bar{h}}{2}) \phi_0(\bar{h}, t, \frac{\vec{\partial} \cdot \bar{h}}{2}) | 0 \rangle$$

$$\langle 0 | \phi_0(\vec{r}_1) \phi_0(\vec{r}_2) | 0 \rangle = e^{-\frac{1}{2} \vec{r}_1 \cdot \vec{r}_2} \langle 0 | \phi_0(\vec{r}_1) \phi_0(\vec{r}_2) | 0 \rangle$$

$$\phi_0(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} \tilde{\phi}_0(\vec{k}, t) e^{i\vec{k} \cdot \vec{r}}$$

$$\langle 0 | \phi_0(\vec{r}_1, t) \phi_0(\vec{r}_2, t) | 0 \rangle = \langle 0 | \phi_0(\vec{k}_1, t_1 = \frac{\vec{0} \cdot \vec{k}_1}{\omega}) \phi_0(\vec{k}_2, t_2 = \frac{\vec{0} \cdot \vec{k}_2}{\omega}) | 0 \rangle$$

$$\langle 0 | \phi_0(x_1) \phi_0(x_2) | 0 \rangle = e^{-\frac{1}{2} \partial_{x_1} \Delta \partial_{x_2}} \langle 0 | \phi_0(x_1) \phi_0(x_2) | 0 \rangle$$

$$\phi_0(x) = \int \frac{d^4 k}{(2\pi)^4} \tilde{\phi}_0(k, x) e^{i k \cdot x}$$

$$\langle 0 | \phi_0(\bar{x}_1, t_1) \phi_0(\bar{x}_2, t_2) | 0 \rangle = \langle 0 | \phi_0(\bar{x}_1, t_1 - \frac{\vec{0} \cdot \vec{k}_1}{z}) \phi_0(\bar{x}_2, t_2 - \frac{\vec{0} \cdot \vec{k}_2}{z}) | 0 \rangle$$

$x_0, x_1 \quad f(x)$

$$\begin{array}{ccc}
 \alpha \otimes \beta & \xrightarrow{\Delta} & \mathbb{1} \oplus \rho \oplus (\alpha \otimes \beta) \\
 m_0 \downarrow = & & \downarrow \sum_{\sigma} J_{\sigma}^{m_0} \\
 m_0 \otimes \rho & \longrightarrow & \rho \oplus m_0 \otimes \rho \\
 m_0 = m_0 \cdot F = m_0 \cdot e^{i \mathcal{L} \cdot \mathbb{P} \otimes \mathbb{P}} & \longrightarrow & U(6)
 \end{array}$$

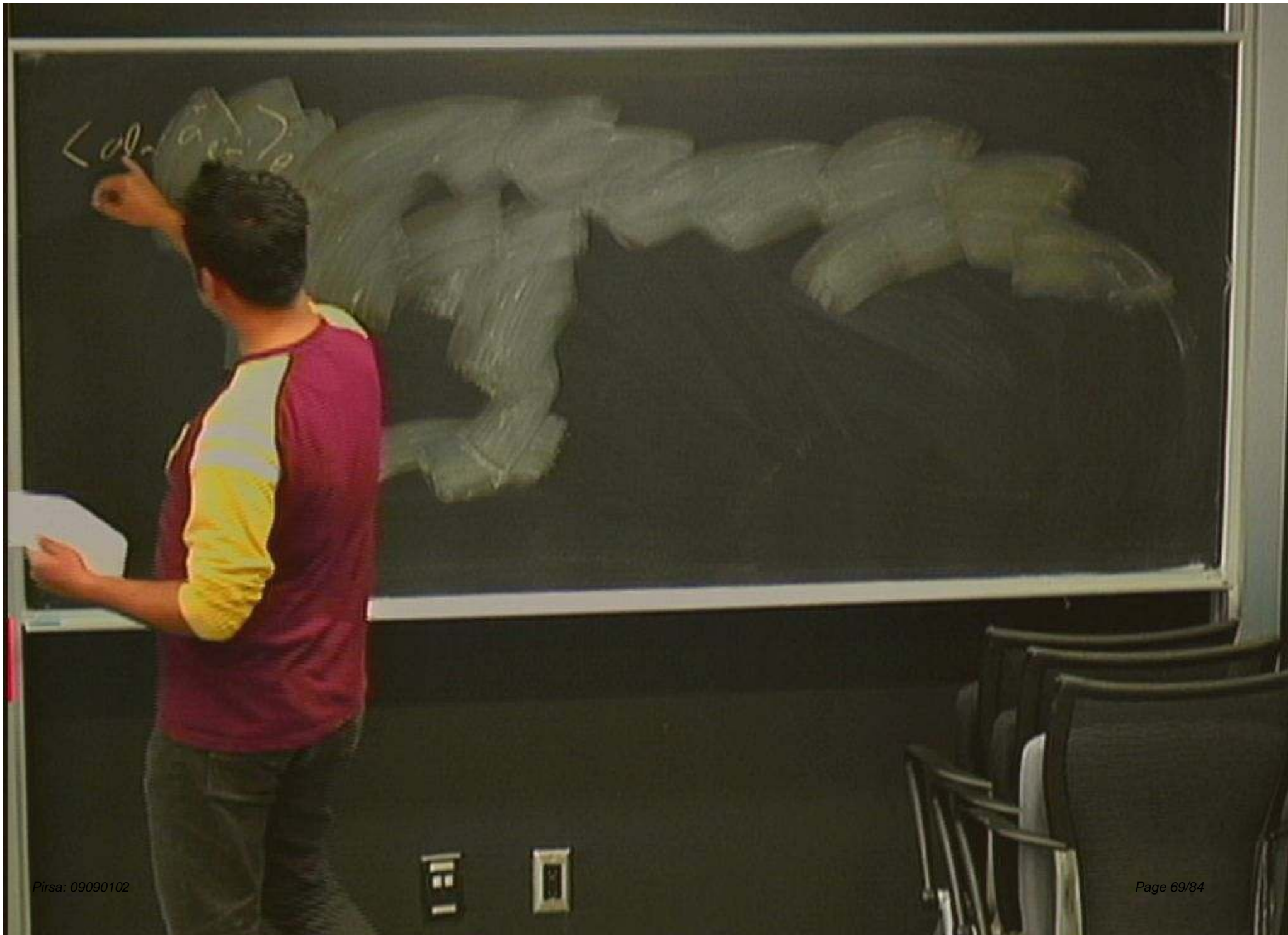
$$\Delta_0 = F^{-1} \Delta_0 F$$

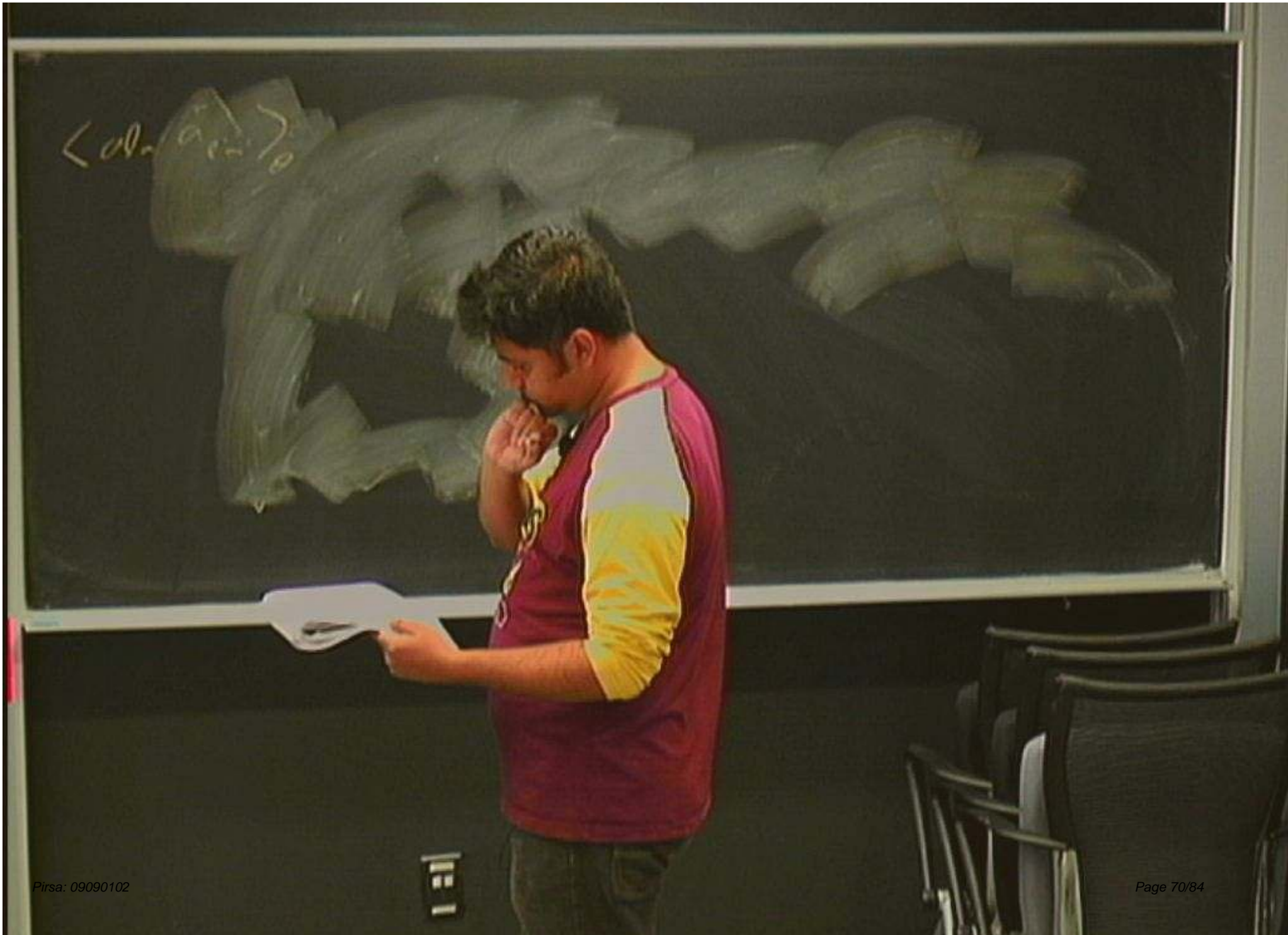
$$P_F = P_0(k) \cosh(H \vec{0} \cdot \vec{k})$$

$$\begin{array}{ccc}
 \alpha \otimes \beta & \xrightarrow{\Delta} & 1 \oplus \rho \oplus (\alpha \otimes \beta) \\
 m_0 \downarrow = & & \downarrow \sum_{\sigma} \rho_{\sigma} \otimes \rho_{\sigma} \\
 m_0 \otimes \rho & \longrightarrow & \rho(g) m_0 \otimes \rho(g) \\
 & & \xrightarrow{i.e. \rho \otimes \rho} U(6)
 \end{array}$$

$$\Delta_0 = F^{-1} \Delta_0 F$$

$$P_F = P_0(h) \cosh(H \vec{0} \cdot \vec{h})$$





$$\Delta(y, z) = \Delta(y, z)$$

$$\begin{array}{ccc}
 \alpha \otimes \beta & \xrightarrow{\Delta} & \sum_a (\alpha \otimes \beta)_a \\
 \downarrow m_0 & & \downarrow m_0 \otimes 1 \\
 \alpha \otimes \beta & & \alpha \otimes \beta
 \end{array}$$

$$\Delta_0 = F^{-1} \Delta_0 F$$

$$\begin{array}{ccc}
 \alpha \otimes \beta & \xrightarrow{P(g)} & m_0 \otimes g \\
 m_0 = m_0 F = \dots & \xrightarrow{P_0 \otimes P_0} & U(0)
 \end{array}$$

$$P_F = P_0(\hbar) \cosh(\hbar \vec{0} \cdot \vec{h})$$

$$\Delta(j, j_1) = \Delta(j, j_2)$$

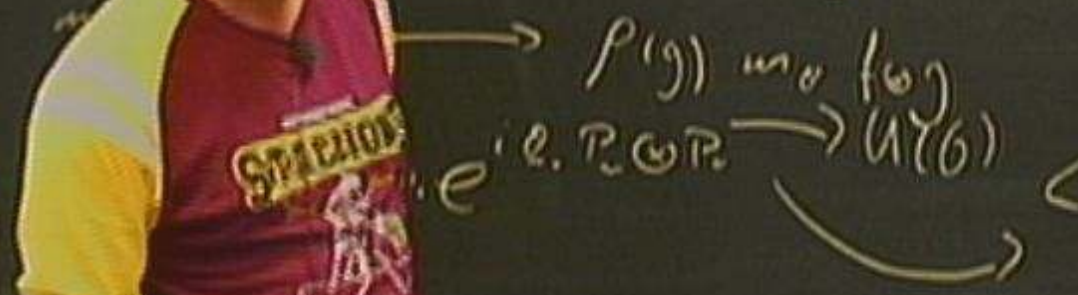
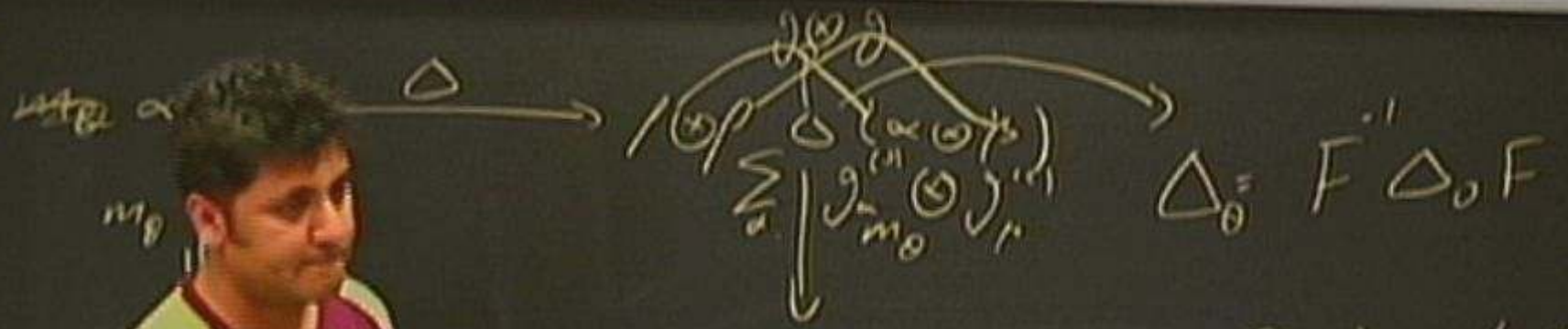
$$\begin{array}{ccc}
 \psi \propto \alpha \otimes \beta & \xrightarrow{\Delta} & \sum_a \psi_a \otimes \psi_a \\
 m_B \downarrow & & \downarrow \\
 & & \psi_a \otimes \psi_a
 \end{array}$$

$$\Delta_0 = F^{-1} \Delta_0 F$$

$$\begin{array}{ccc}
 m_0 \propto \alpha \otimes \beta & \longrightarrow & P(j) m_0 \otimes m_0 \\
 m_0 = m_0 F = m_0 e^{i\ell \cdot P_0 \otimes P_0} & \longrightarrow & U(0)
 \end{array}$$

$$P_F = P_0(h) \cosh(H\vec{0} \cdot h)$$

$$\Delta(j, j_1) = \Delta(j, j_2)$$



$$P = P_0(h) \exp(i\vec{h} \cdot \vec{J})$$

$$\vec{\theta} = (\theta^1, \theta^2, \theta^3)$$

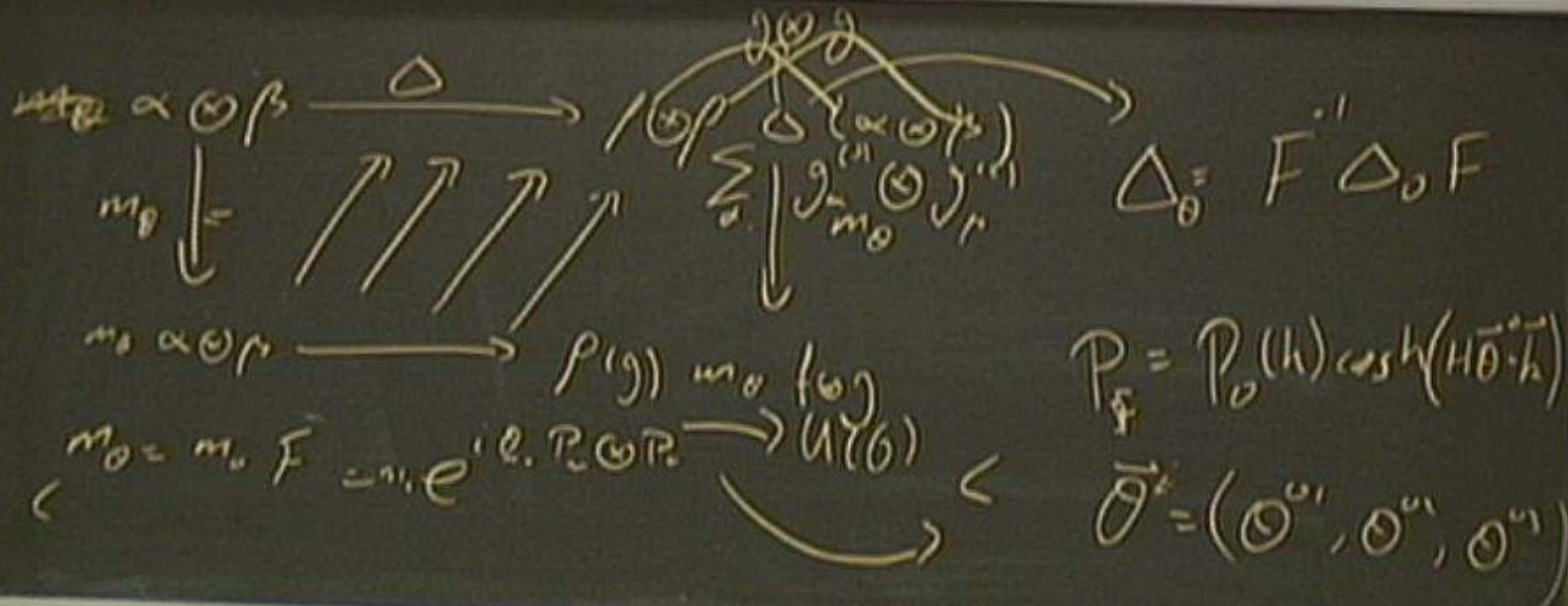
$$\Delta(j, j_1) = \Delta(j, j_2)$$

$$\begin{array}{ccc}
 \alpha \otimes \beta & \xrightarrow{\Delta} & \sum_a (j_a \otimes j_a) \\
 \downarrow m_B & & \downarrow \sum_a j_a \\
 m_0 & & m_0
 \end{array}
 \qquad
 \Delta_0 = F^{-1} \Delta_0 F$$

$$\begin{array}{ccc}
 m_0 \otimes \beta & \longrightarrow & P(j) \otimes m_0 \otimes j \\
 m_0 = m_0 F = m_0 e^{i \ell \cdot P \otimes P} & \longrightarrow & U(6)
 \end{array}$$

$$\begin{aligned}
 P_F &= P_0(h) \cosh(H \vec{0} \cdot \vec{h}) \\
 \vec{0} &= (0^{01}, 0^{01}, 0^{01})
 \end{aligned}$$

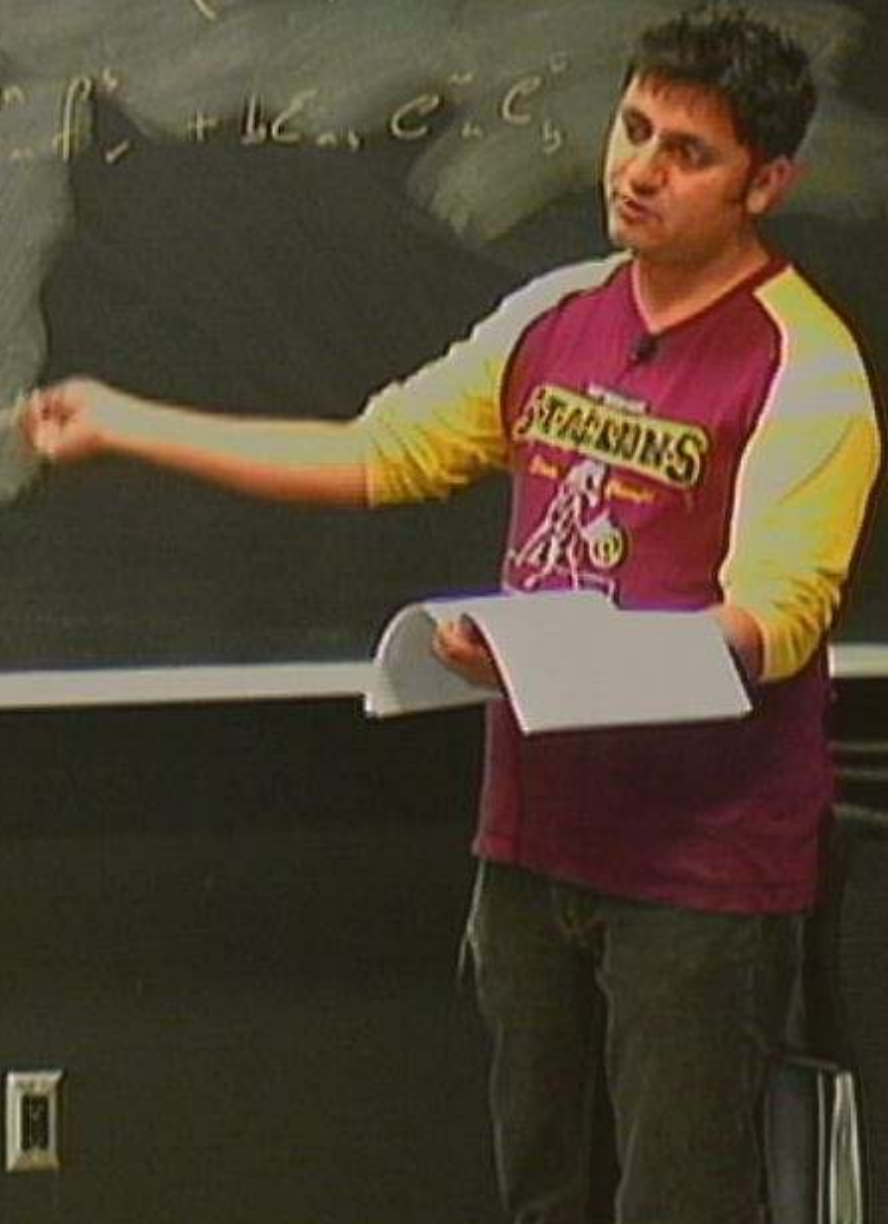
$$\Delta(\mathbf{r}, t) = \Delta(\mathbf{r}', t')$$



$$\langle a_n, a_n \rangle$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

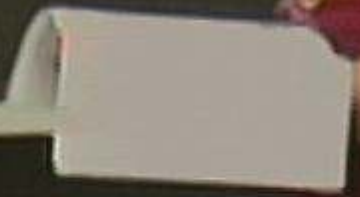
$$D_{n+1} = a_n E_{n+1} f_{n+1} + b_n E_{n+1} c_n c_n$$



$$\langle a_n | a_m \rangle$$

$$\begin{pmatrix} 1 & & \\ & 1 & \\ & & \dots \end{pmatrix}$$

$$\mathcal{E}_{a_n} f_{a_n} + b \mathcal{E}_{a_n} c_{a_n} c_{a_n}$$

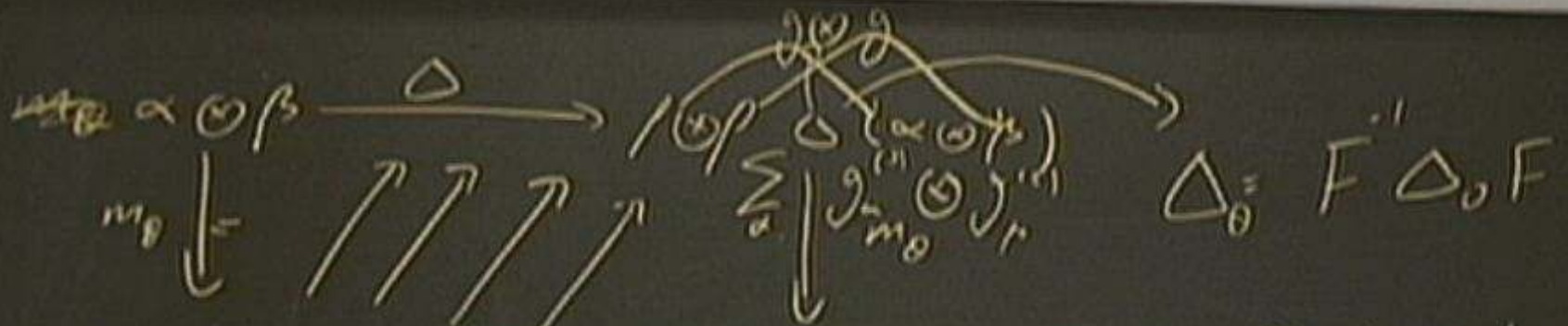


$\langle \alpha_n | \alpha_m \rangle$ $\begin{pmatrix} 1 & & \\ & 1 & \\ & & \ddots \\ & & & 1 \end{pmatrix}$

$$Q_{nm} = \frac{1}{\sqrt{2}} (\alpha_n + \alpha_m) f_{nm} + \frac{1}{\sqrt{2}} (\alpha_n - \alpha_m) g_{nm}$$

$\langle \mathcal{O}_n | \mathcal{O}_m \rangle$ $\begin{pmatrix} 1 & & \\ & 1 & \\ & & \ddots \end{pmatrix}$

$$\mathcal{O}_n = a \mathcal{E}_{nn} + b \mathcal{E}_{nn} \mathcal{E}_{nn} \mathcal{E}_{nn}$$



$$m_0 \propto \alpha \otimes \beta \xrightarrow{\Delta} 1 \otimes p$$

$$m_0 \propto \alpha \otimes \beta \xrightarrow{m_0} \sum_a g_{m_0}^{(a)}$$

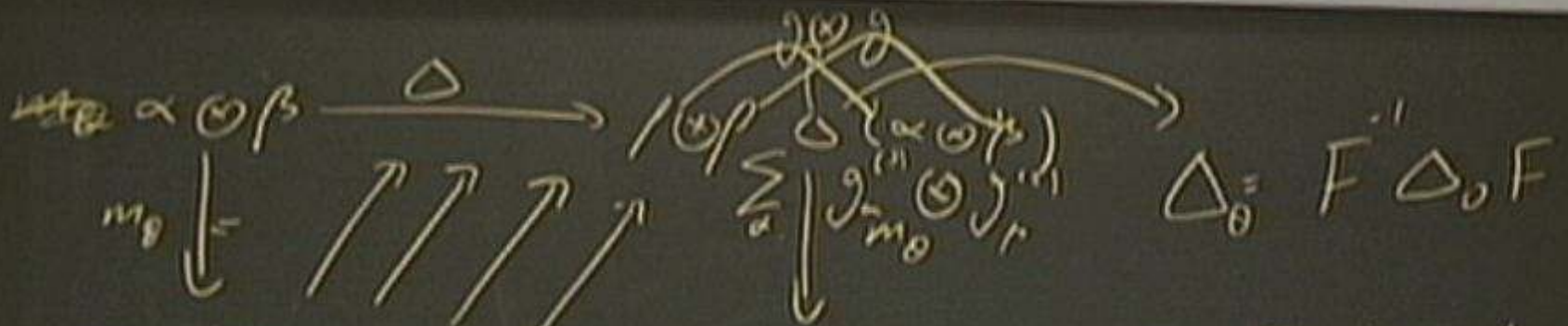
$$m_0 \propto \alpha \otimes \beta \xrightarrow{m_0} P(g) m_0 f(g)$$

$$m_0 = m_0 \quad F = m_0 \cdot e^{i \cdot \mathcal{P} \otimes \mathcal{P}} \rightarrow U(6)$$

$$\Delta_0 = F^{-1} \Delta_0 F$$

$$P_F = P_0(h) \cosh(H \vec{0} \cdot \vec{h})$$

$$= P_0(\dots) \left[1 + \frac{H^2 (\vec{0} \cdot \vec{h})^2}{2} \right]$$



$$\begin{aligned}
 & m_0 \propto \alpha \otimes \beta \xrightarrow{\Delta} 1 \otimes p \otimes \left(\sum_a |g_{m_0}^{(a)}\rangle \langle g_{m_0}^{(a)}| \right) \\
 & m_0 \propto \alpha \otimes \beta \xrightarrow{\quad} P^{(g)} |m_0\rangle \langle g| \\
 & m_0 = m_0 \cdot F = m_0 \cdot e^{i \cdot \mathcal{P} \otimes \mathcal{P}} \rightarrow U(\mathcal{G})
 \end{aligned}$$

$$\begin{aligned}
 P &= P_0(h) \cosh(H \vec{0} \cdot \vec{h}) \\
 P_F &= P_0(\cdot) \left[1 + \frac{1}{2} (\vec{0} \cdot \vec{h})^2 \right]
 \end{aligned}$$

$\langle \psi | \psi \rangle$ $\begin{pmatrix} * & & \\ & 1 & \\ & & \dots \end{pmatrix}$

$$\mathcal{O} = a \epsilon_{\mu\nu\rho\sigma} f_{\mu\nu} f_{\rho\sigma} + b \epsilon_{\mu\nu\rho\sigma} e_{\mu\nu} e_{\rho\sigma}$$

$\mathcal{O} \geq 10 \text{ GeV}$

$\langle \psi, \psi \rangle$

$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$

$$\theta = a \epsilon_{\mu\nu} f_{\mu\nu} + b \epsilon_{\mu\nu} c_{\mu} c_{\nu}$$

$$a H \tilde{\phi} = \sqrt{4\pi} \epsilon_{\mu\nu}$$

$$\sqrt{\theta} \geq \underline{10 \text{ GeV}}$$

$$\langle a_{\nu} a_{\nu}^{\dagger} \rangle_0$$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$\mathcal{L} = \frac{1}{2} \bar{\psi} \not{\partial} \psi + \bar{\psi} \not{A} \psi + \bar{\psi} \not{B} \psi + \dots$$

$$a H \bar{\psi} = \sqrt{4\pi\alpha} \bar{\psi} \not{A} \psi$$

$$\sqrt{\theta} \geq \underline{10 \text{ GeV}}$$