Title: Noncomutative Effects in Primordial Density Fluctuations

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Abstract: Near the Planckian scales, quantum gravity is expected to drastically change the structure of spacetime, one feature of which may be noncomutativity of the coordinates. Based on the recent advances in quantum field theories on such noncommutative spaces, I will consider the fluctuations of inflaton and look for possible noncommutative corrections in the CMB. Anisotropy and non-gaussianity are the result. The resultant distribution is then compared with ACBAR, CBI and WMAP data to constrain the scale of noncommutativity parameter.

Plan

$$\langle 0 | \phi(x_1)\, \phi(x_2) | 0 \rangle$$

Plan

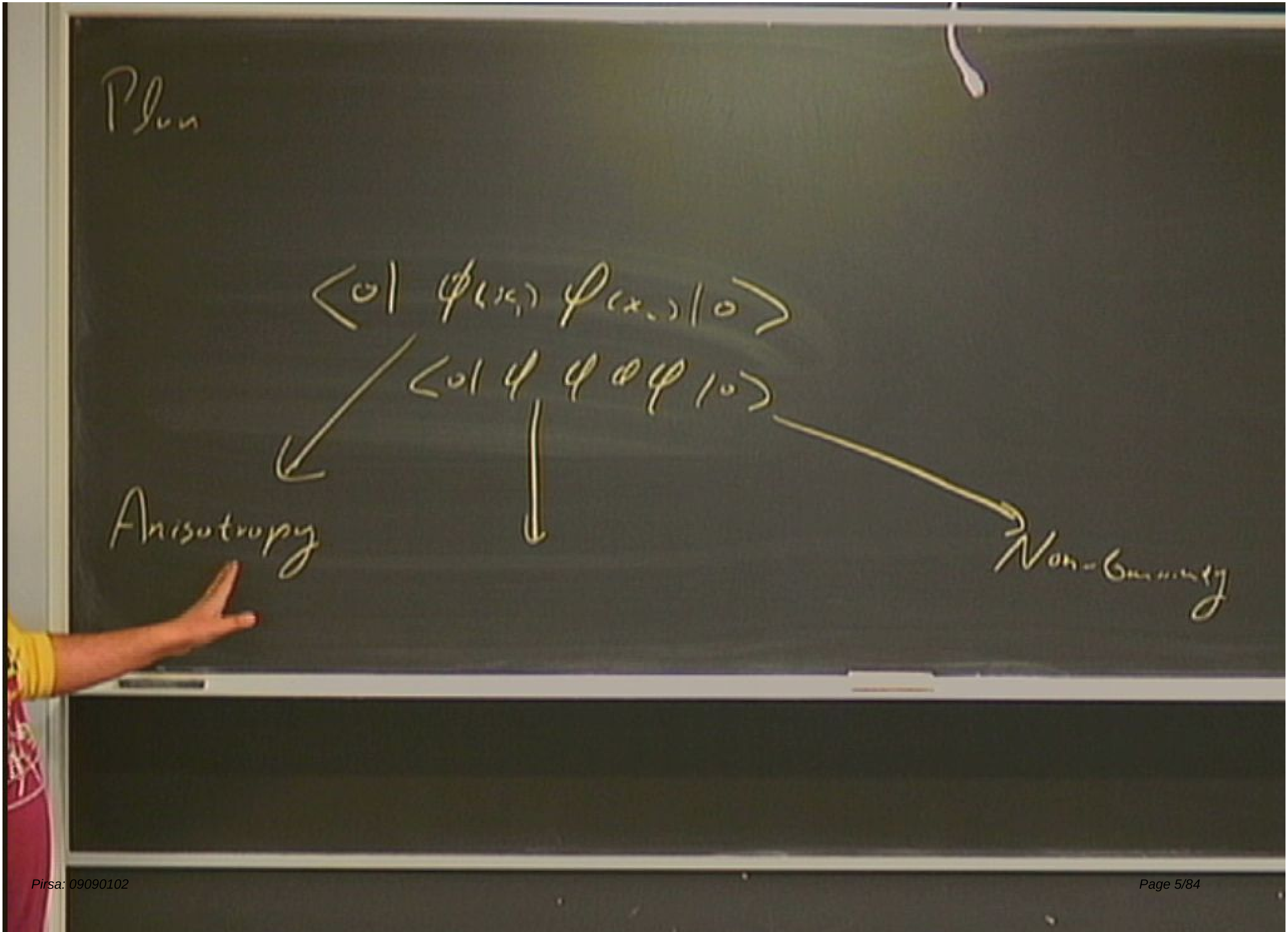
$$\langle 0|\, \phi(x_1)\, \phi(x_2)\, |0\rangle$$

Plan

$$\langle 0 | \phi(x_1) \phi(x_2) | 0 \rangle$$

$$\langle 0 | \phi \phi \phi \phi | 0 \rangle$$

Anisotropy


Plan
$\langle 0| \phi(x_1) \phi(x_2) |0\rangle$
$\langle 0| \phi \phi \phi \phi |0\rangle$
Anisotropy
Non-Gaussianity

Plan

$$\langle 0 | \phi(x_1) \, \phi(x_2) | 0 \rangle$$

$$\langle 0 | \phi \, \phi \, \phi \, \phi | 0 \rangle$$

Anisotropy

Angular Power Spectrum

C_ℓ

Non-Gaussianity

Plan

$$\langle 0|\,\phi(x_1)\,\phi(x_2)\,|0\rangle$$

$$\langle 0|\,\phi\,\phi\,\phi\,\phi\,|0\rangle$$

Anisotropy

Angular Power Spectrum

C_ℓ

Non-Gaussianity

Estimate Θ

Plan

$$\langle 0|\,\phi(x_1)\,\phi(x_2)\,|0\rangle$$

$$\langle 0|\,\phi\,\phi\,\phi\,\phi\,|0\rangle$$

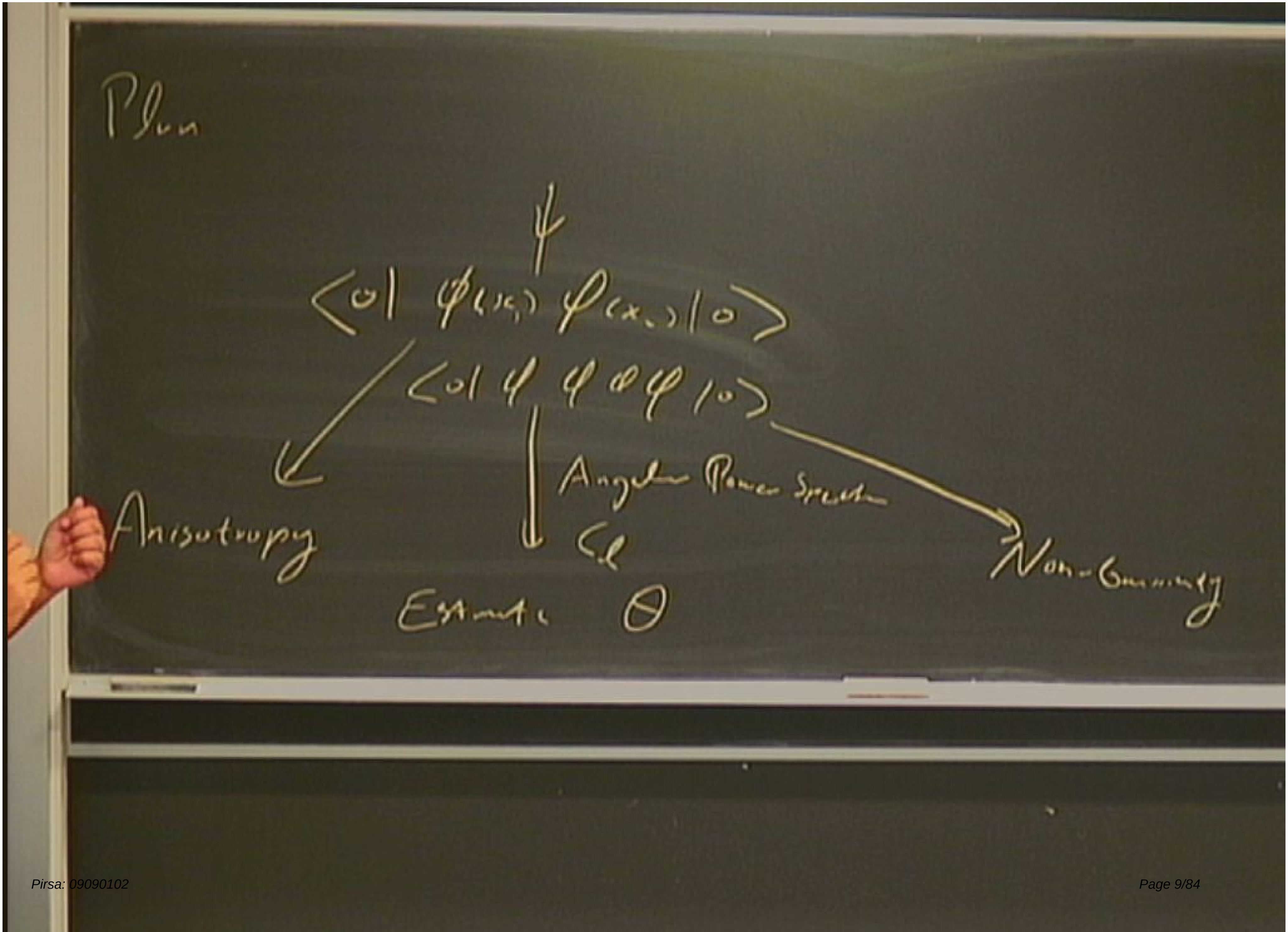
Anisotropy

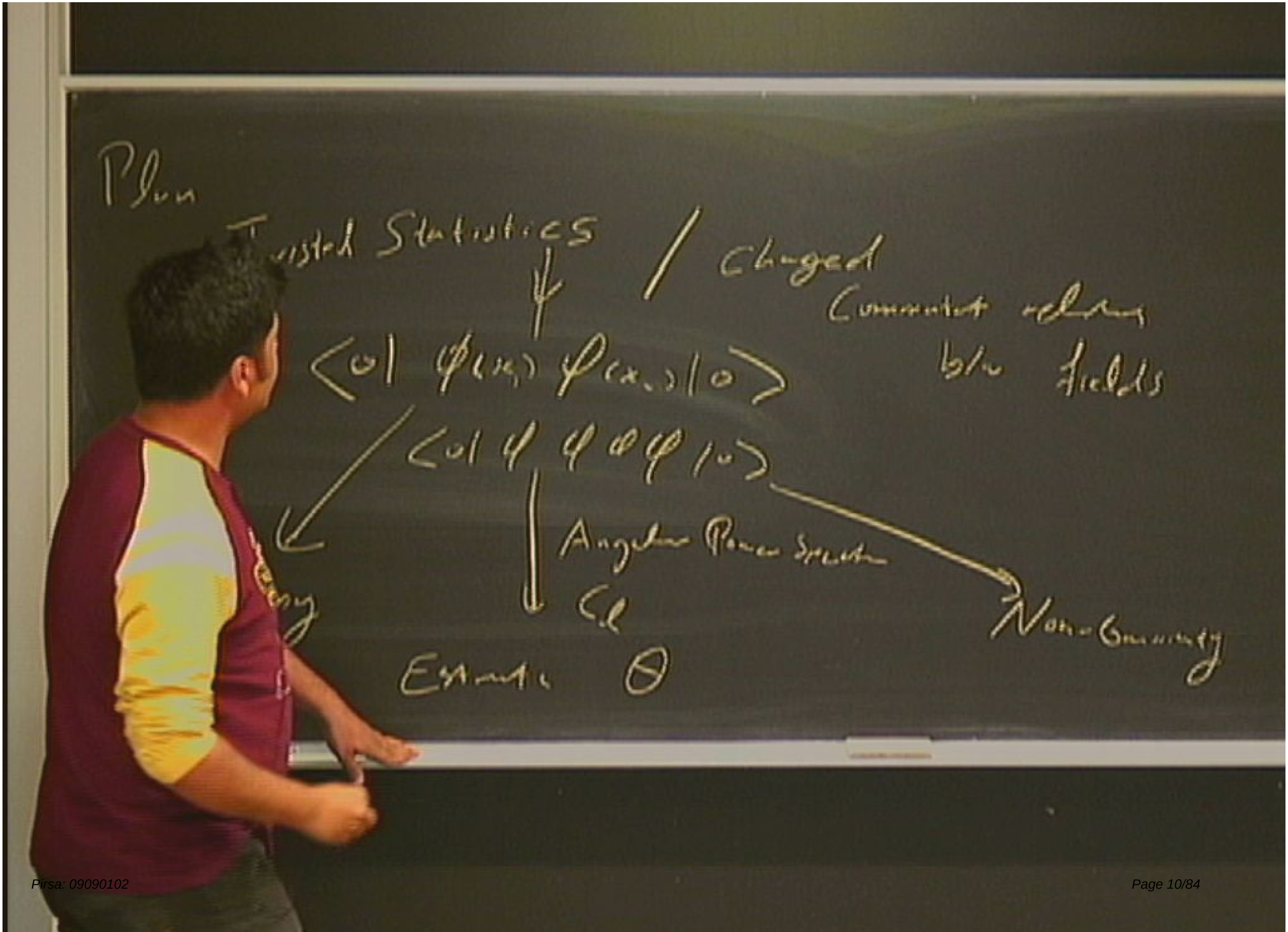
Angular Power Spectrum

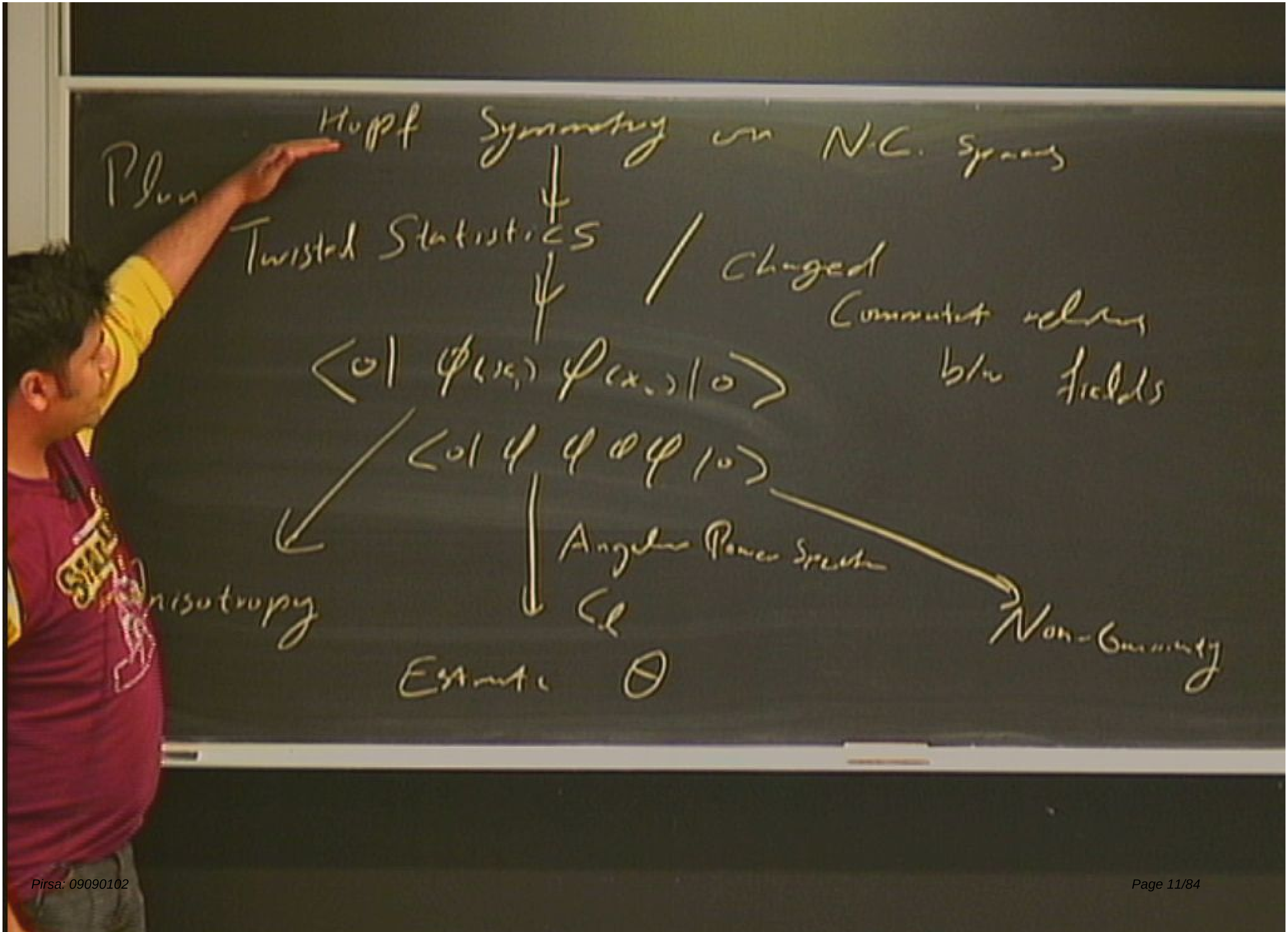
C_l

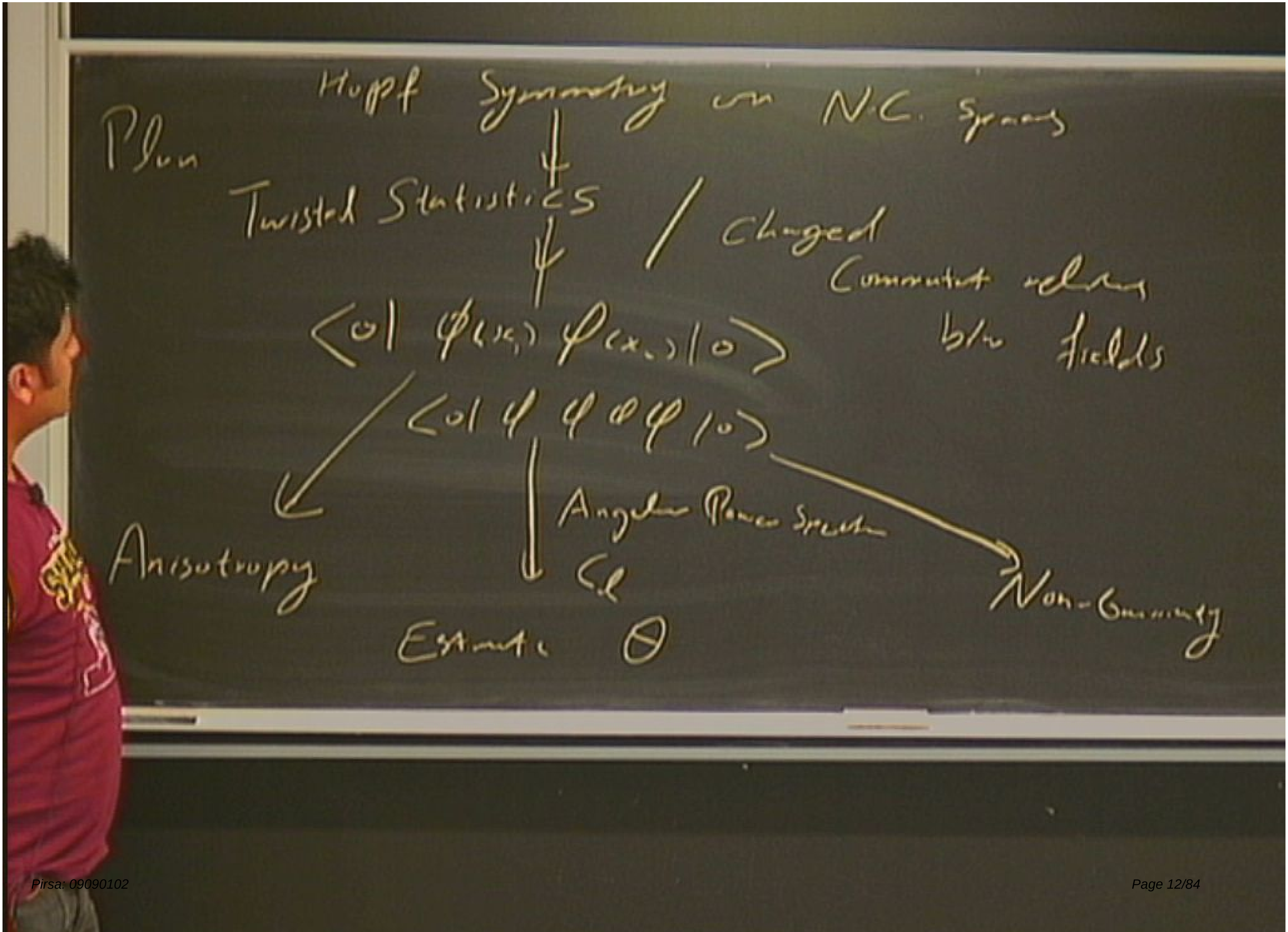
Non-Gaussianity

Estimate Θ


Plan
$\langle 0|\, \varphi(x_1)\, \varphi(x_2)\,|0\rangle$
$\langle 0|\, \varphi\, \varphi\, \varphi\, \varphi\,|0\rangle$
Angular Power Spectrum
C_ℓ
Anisotropy
Non-Gaussianity
Estimate Θ


Plan
Twisted Statistics
Charged
Commutator relations
b/w fields
<0| φ(x1) φ(x2)|0>
<0| φ φ φ φ |0>
Angular Power Spectrum
Cl
Non-Gaussianity
Estimate Θ


Plan
Hopf Symmetry on N.C. Spaces
Twisted Statistics
Changed Commutation relations b/w fields
$\langle 0| \phi(x_1) \phi(x_2) |0\rangle$
$\langle 0| \phi \phi \phi \phi |0\rangle$
Anisotropy
Angular Power Spectrum
C_l
Non-Gaussianity
Estimate θ



Plan
Hopf Symmetry on N.C. Spaces
Twisted Statistics
Changed
Commutation relations
b/w fields
$\langle 0| \phi(x_1)\, \phi(x_2) |0\rangle$
$\langle 0| \phi\, \phi\, \phi\, \phi |0\rangle$
Anisotropy
Angular Power Spectra
C_ℓ
Non-Gaussianity
Estimate Θ

Estimate θ

Non-Gaussianity

$$\Delta 1 \Delta x \geq C^2$$

$$\Delta x \Delta x \geq C^2$$

Estimate Θ Non-Geometry

$$\Delta t \Delta x \geq L^2$$

$$\Delta x \Delta x \geq L^2$$

$$[\hat{x}_\mu , \hat{x}_\nu] = i\Theta_{\mu\nu}$$

Estimate Θ

Non-Geometry

$$\Delta t \, \Delta x \geq l^2$$

$$\Delta x \, \Delta x \geq l^2$$

$$[\hat{x}_\mu, \hat{x}_\nu] = i\Theta_{\mu\nu}$$

$f(x)$

Estimate Θ

Non-Gaussianity

$$\Delta t \Delta x \geq \ell^2$$

$$\Delta x \Delta x \geq \ell^2$$

$$[\hat{x}_\mu, \hat{x}_\nu] = i\Theta_{\mu\nu}$$

$$f(x) * g(x) =$$

Estimate Θ

Non-Commutativity

$$\Delta t\,\Delta x \geq L^2$$

$$\Delta x\,\Delta x \geq L^2$$

$$[\hat{x}_\mu, \hat{x}_\nu] = i\Theta_{\mu\nu}$$

$$f(x) * g(x) = f(x)\, e^{-\frac{i}{2}\overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu}\, g(x)$$

Estimate Θ

Non-Commutativity

$$\Delta t \, \Delta x \geq \ell^2$$

$$\Delta x \, \Delta x \geq \ell^2$$

$$[\hat{x}_\mu, \hat{x}_\nu] = i\Theta_{\mu\nu}$$

$$f(x) * g(x) = f(x) \, e^{-\frac{i}{2} \overleftarrow{\partial}_\mu \Theta^{\mu\nu} \overrightarrow{\partial}_\nu} g(x)$$

$$x_\mu * x_\nu - x_\nu * x_\mu = i\Theta_{\mu\nu}$$

Estimate Θ Non-Commutativity

$$O(O\alpha \geq L^2$$
$$\Delta x \, \Delta x \geq L^2$$

$$[\hat{x}_\mu, \hat{x}_\nu] = i\Theta_{\mu\nu}$$

$$f(x) * g(x) = f(x)\, e^{-\frac{i}{2}\overleftarrow{\partial}_\mu \Theta^{\mu\nu} \overrightarrow{\partial}_\nu}\, g(x)$$

$$x_\mu * x_\nu - x_\nu * x_\mu = i\Theta_{\mu\nu}$$

$$S[\phi] = \int d^4x \; \partial_\mu \varphi * \partial_\mu \varphi$$
$$+ m\, \varphi * \varphi$$
$$+ \varphi * \varphi * \varphi \cdots$$

$S[\phi]$ not invariant under Lorentz symmetry

$S[\phi]$ not invariant under Lorentz symmetry

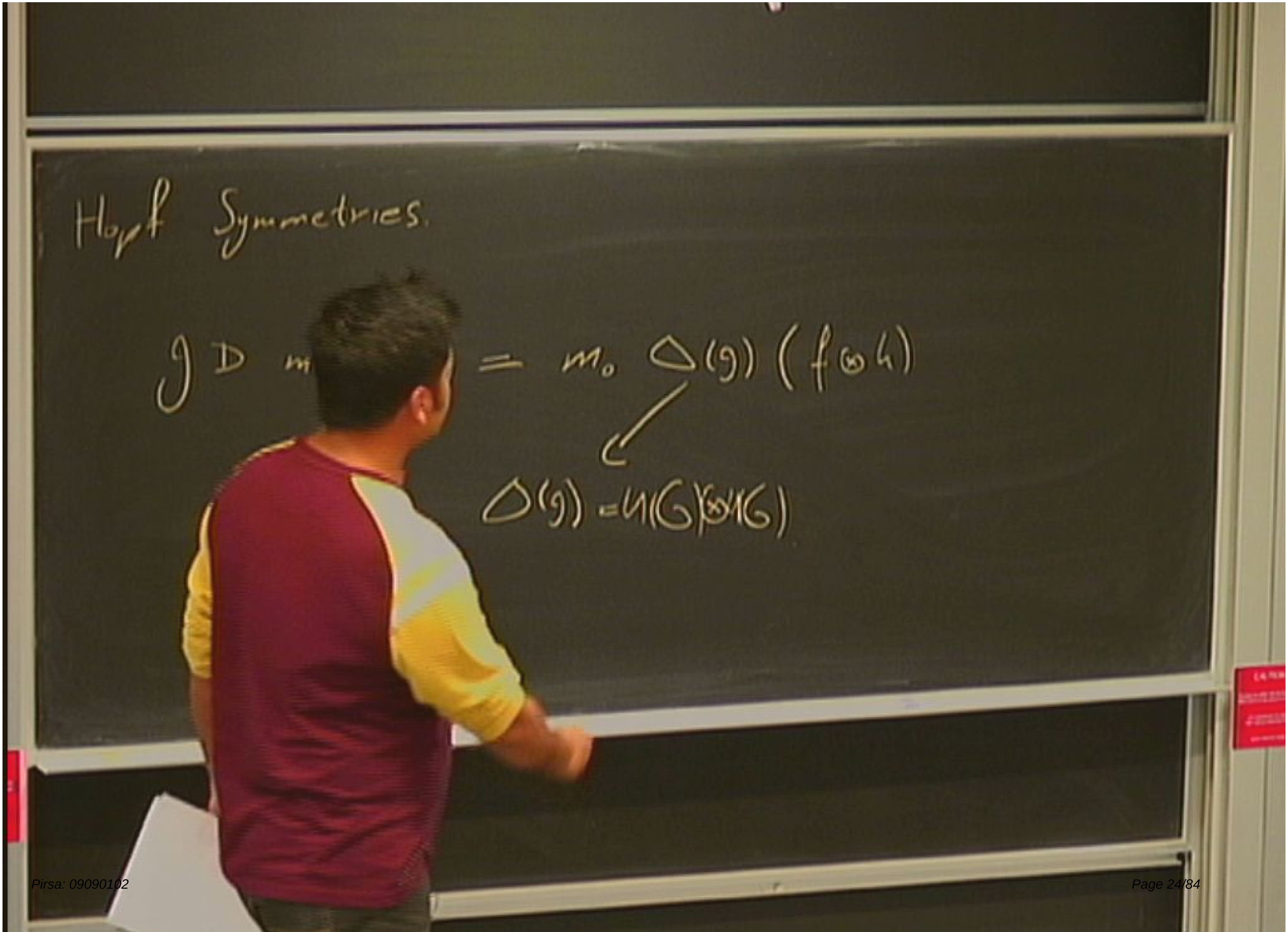
$$g \triangleright \phi(x) = \phi(g^{-1}x)$$

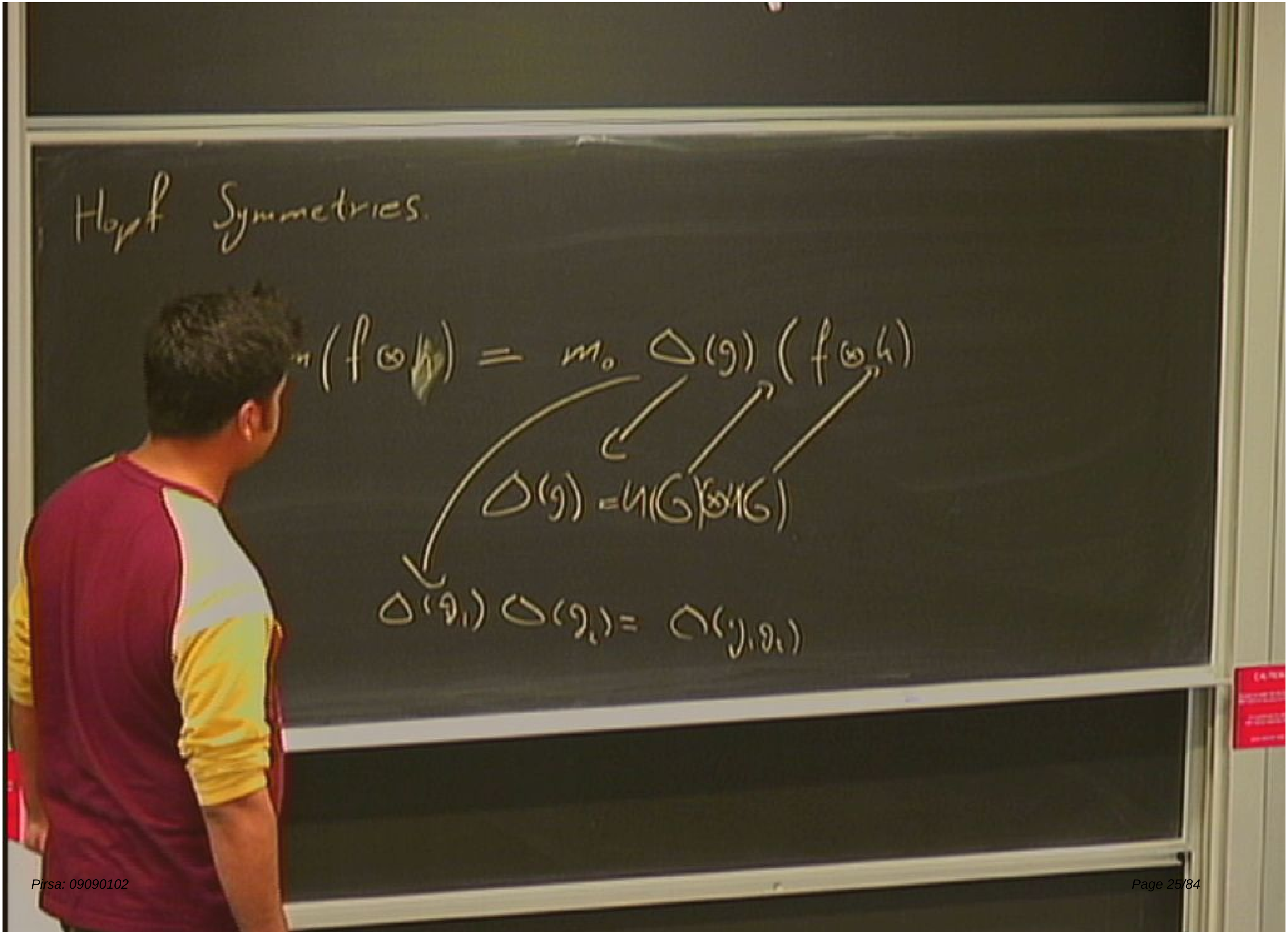
$S[\phi]$ not Invariant under Lorentz symmetry

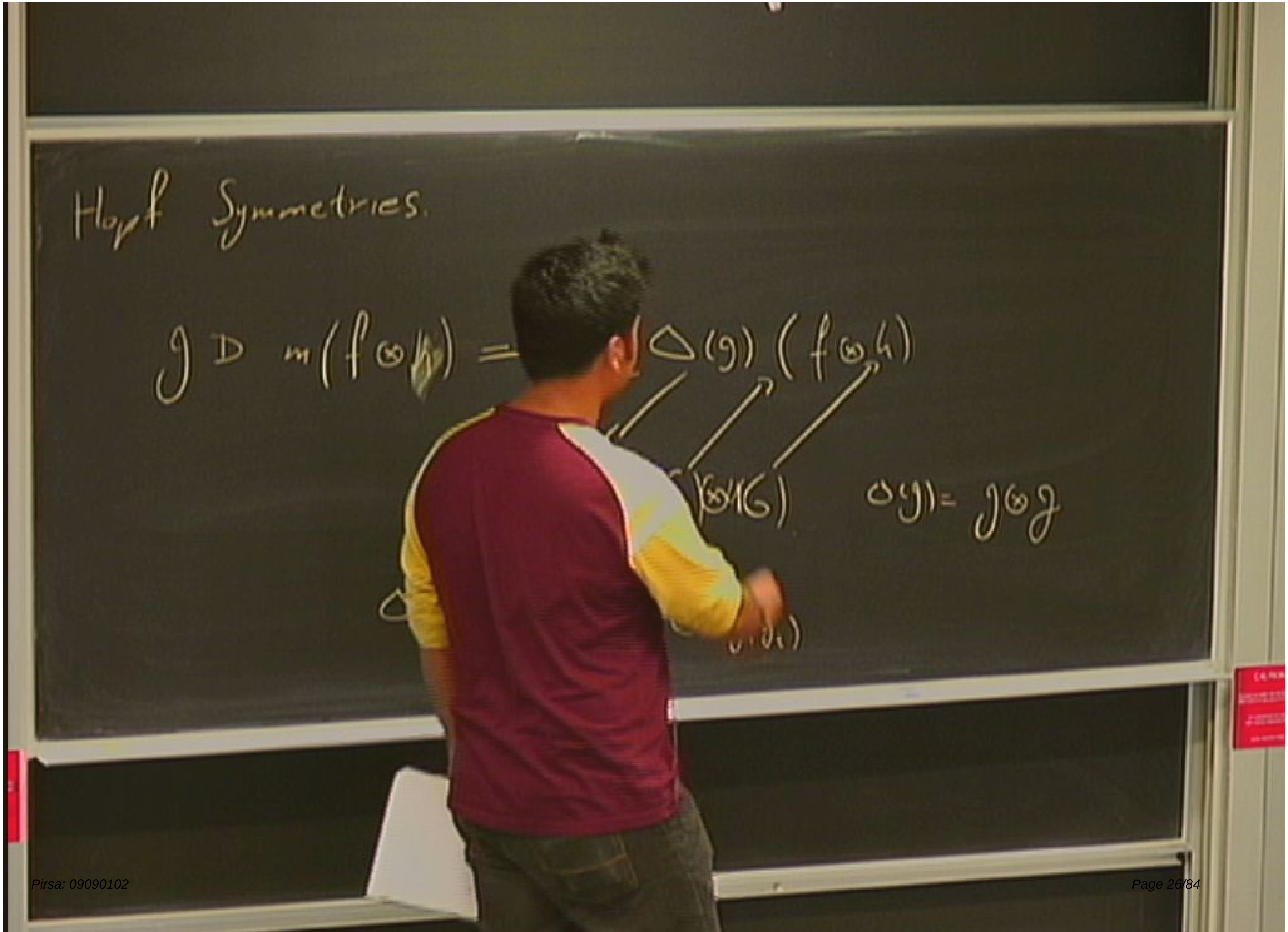
$$g \triangleright \phi(x) = \phi(g^{-1}x)$$

$$g \triangleright \phi(x) * g \triangleright h(x) \neq g \triangleright \phi * h(x)$$

Hopf Symmetries.


Hopf Symmetries.
$g \triangleright m$
$= m \circ \Delta(g)(f \otimes h)$
$\Delta(g) = u(G) \otimes u(G)$


Hopf Symmetries.
$m(f \otimes h) = m_\circ \Delta(g)(f \otimes h)$
$\Delta(g_1)\Delta(g_2) = \Delta(g_1 g_2)$


Hopf Symmetries.
$\Delta(g) = g \otimes g$

Hopf Symmetries.

$$g \triangleright m(f\otimes h) = m\,\Delta(g)(f\otimes h)$$

$$\Delta(g) = U(G)\otimes U(G)$$

$$\Delta(g) = g\otimes g$$

$$\Delta(L) = 1\otimes L + L\otimes 1$$

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries. $m(f \otimes g) = f \cdot g$

$$g \triangleright m(f \otimes h) = m\, \Delta(g)\, (f \otimes h)$$

$$\Delta(g) = \sum g_{(1)} \otimes g_{(2)}$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1)\, \Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries. $\qquad m(f \otimes g) = f \cdot g$

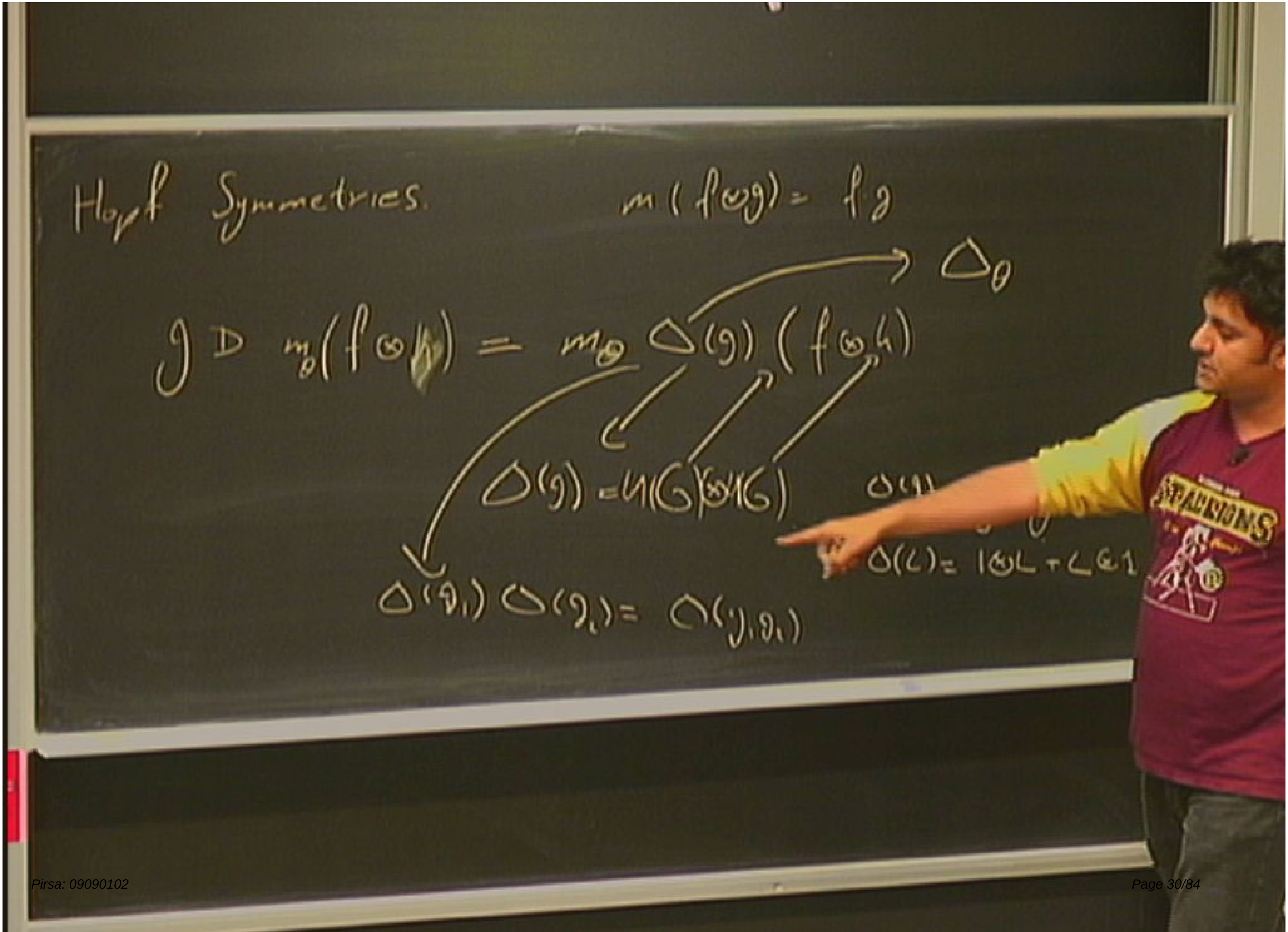
$$g \triangleright m_{\otimes}(f \otimes h) = m_{\otimes}\, \Delta(g)\, (f \otimes h)$$

$$\Delta(g) = \sum g_{(1)} \otimes g_{(2)}$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1)\, \Delta(g_2) = \Delta(g_1 g_2)$$


Hopf Symmetries.
$m(f \otimes g) = f \cdot g$
Δ_θ
$g \triangleright m_\theta(f \otimes h) = m_\theta \Delta(g)(f \otimes h)$
$\Delta(g) = U(G) \otimes U(G)$
$\Delta(L) = 1 \otimes L + L \otimes 1$
$\Delta(g_1)\Delta(g_2) = \Delta(g_1 g_2)$

Hopf Symmetries.

$$m(f \otimes g) = f g$$

$$g \triangleright m_\theta(f \otimes h) = m_\theta \, \Delta(g)\,(f \otimes h)$$

$$\Delta_\theta$$

$$\Delta(g) = U(G) \otimes U(G)$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = \ldots L \otimes 1$$

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = fg$$

$$\Delta_\theta$$

$$g \triangleright m_\theta(f \otimes h) = m_\theta \, \Delta(g)\,(f \otimes h)$$

$$\Delta(g) = u(G) \otimes u(G)$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = fg$$

$$\Delta_\theta$$

$$g \triangleright m_\theta(f \otimes h) = m_\theta \Delta(g)(f \otimes h)$$

$$\Delta(g) = U(G) \otimes U(G) \qquad \Delta(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1)\Delta(g_2) = \Delta(g_1 g_2)$$

$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$

$\Delta(L) = 1 \otimes L + L \otimes 1$

Automorphism

$$f' \cdot g' = f \cdot g$$

$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$

$\Delta(L) = 1 \otimes L + L \otimes 1$

Automorphism

$$f' \cdot g' = f \cdot g$$

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2) \qquad \Delta(L) = 1\otimes L + L\otimes 1$$

Automorphisms

$$f'\cdot g' = f\cdot g$$

$$V_1 \otimes V_2$$

$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$ $\qquad$ $\Delta(L) = 1\otimes L + L\otimes 1$

Automorphism

$$f'\cdot g' = f\cdot g$$

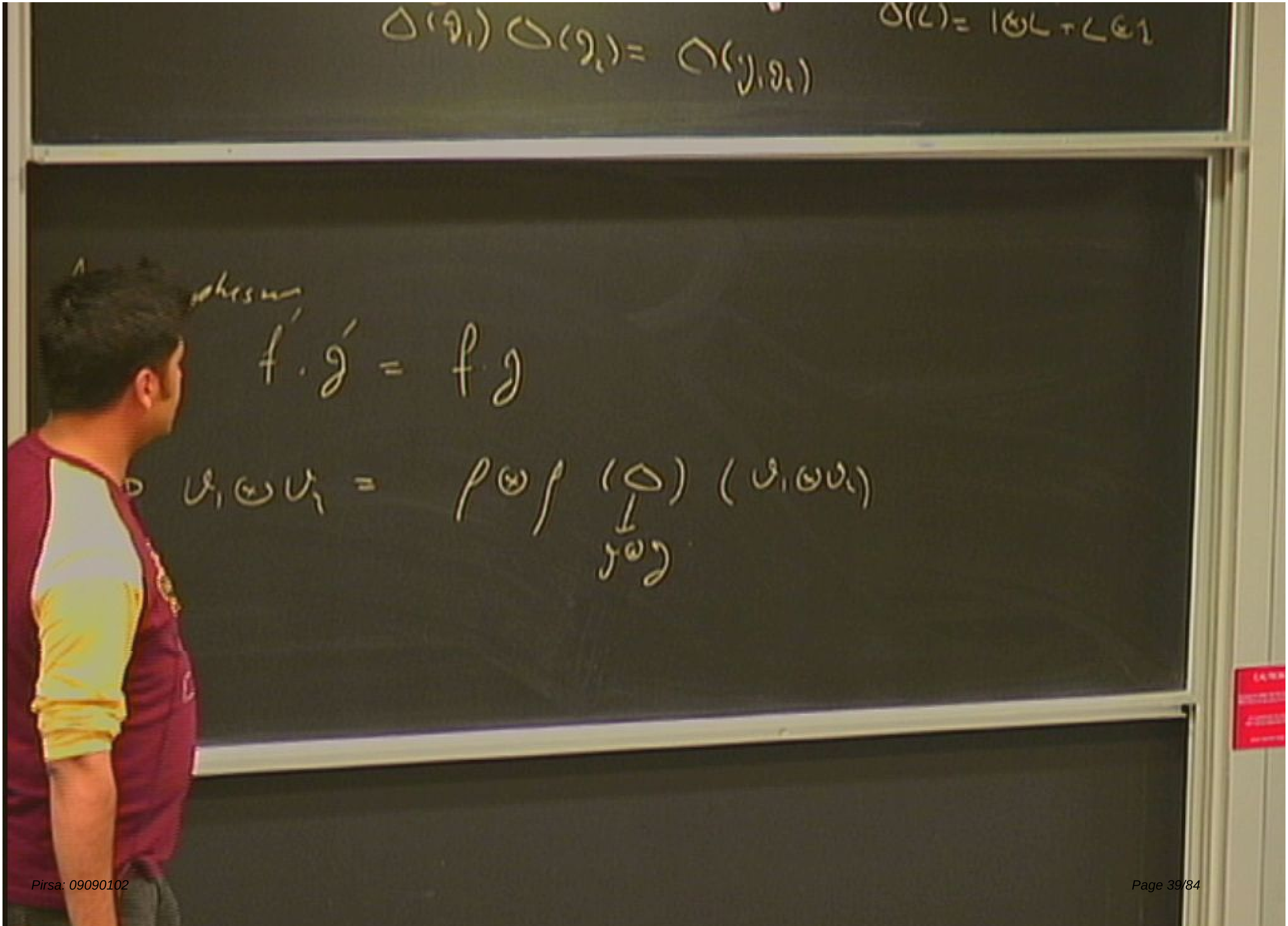
$$g \triangleright v_1 \otimes v_2 = \rho\otimes\rho\,(\Delta)\,(v_1\otimes v_2)$$

$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$ $\Delta(L) = 1 \otimes L + L \otimes 1$

Automorphism

$$f' \cdot g' = f \cdot g$$

$$g \triangleright v_1 \otimes v_2 = \rho \otimes \rho\,(\Delta)\,(v_1 \otimes v_2)$$


Δ(g₁) Δ(g₂) = Δ(g₁g₂)
Δ(L) = 1⊗L + L⊗1
f′ · g′ = f · g
v₁⊗v₂ = ρ⊗ρ (Δ) (v₁⊗v₂)
g⊗g

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2) \qquad \Delta(L) = 1\otimes L + L\otimes 1$$

Automorphism

$$f' \cdot g' = f \cdot g$$

$$g \triangleright v_1 \otimes v_2 = \rho\otimes\rho\,(\Delta)\,(v_1\otimes v_2)$$

$\Delta \to g\otimes g$

Clebsch-Gordan

$\Delta(g_1)\Delta(g_2) = \Delta(g_1 g_2)$ $\Delta(L) = 1\otimes L + L\otimes 1$

$\Delta(g) = g\otimes g$

Automorphism

$f'\cdot g' = f\cdot g$

$g \triangleright v_1\otimes v_2 = \rho\otimes\rho\,(\Delta)\,(v_1\otimes v_2)$

$\Delta \to g\otimes g$

Clebsch-Gordan

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2) \qquad \Delta(L) = 1\otimes L + L\otimes 1$$

$$\Delta(g) = g\otimes g$$

Automorphism

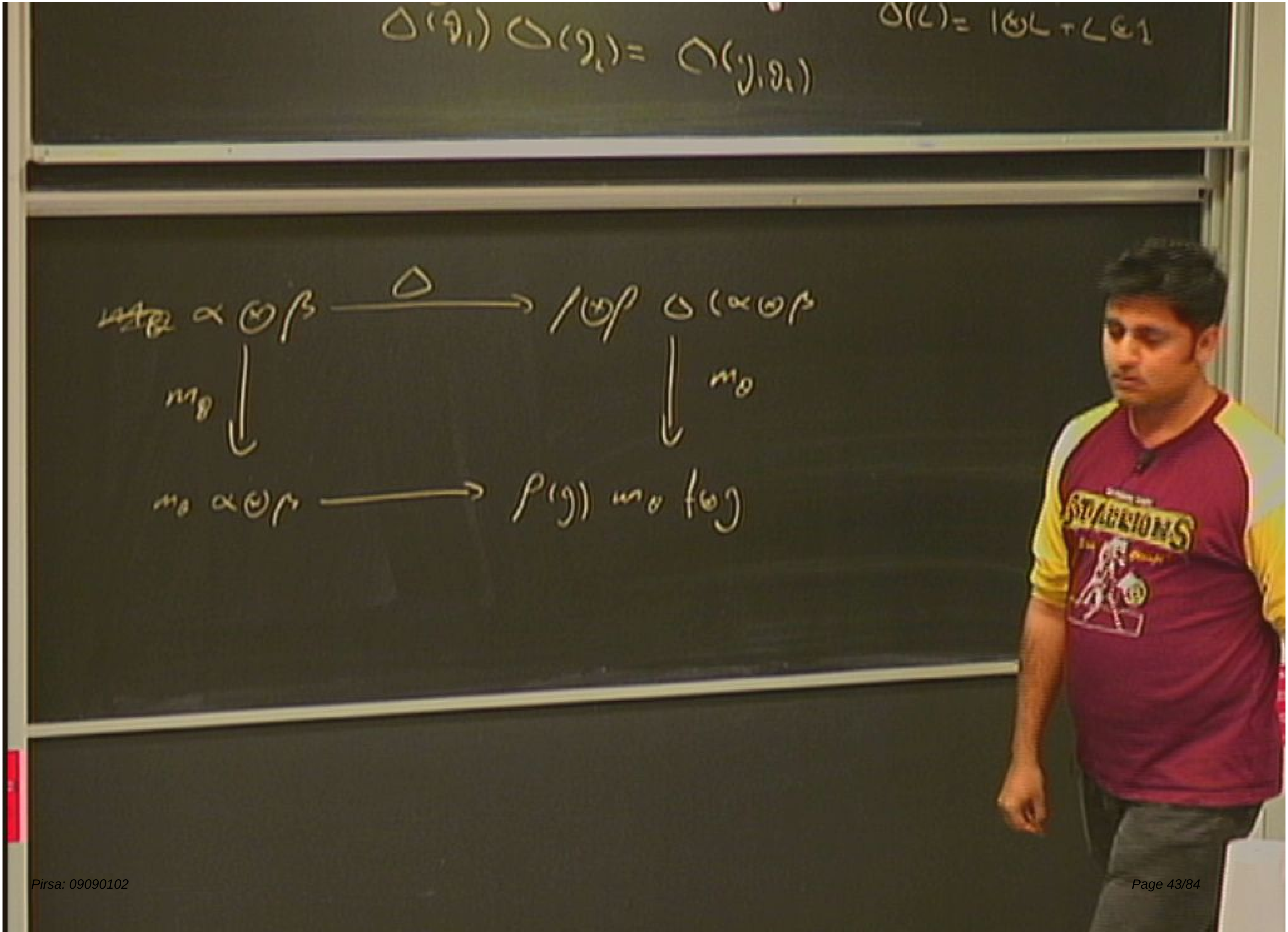
$$f'\cdot g' = f\cdot g$$

$$g \triangleright v_1\otimes v_2 = \rho\otimes\rho\,(\Delta)\,(v_1\otimes v_2)$$

Clebsch–Gordan $\qquad g\otimes g$

Hopf Algebra

$$H = (G, \Delta)$$


Δ(g₁) Δ(g₂) = Δ(g₁g₂)
Δ(L) = 1⊗L + L⊗1
α⊗β
Δ
1⊗1 Δ(α⊗β)
m_g
m_g
m_g α⊗β
ρ(g) m_g f(g)

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2) \qquad \Delta(L) = 1\otimes L + L\otimes 1$$

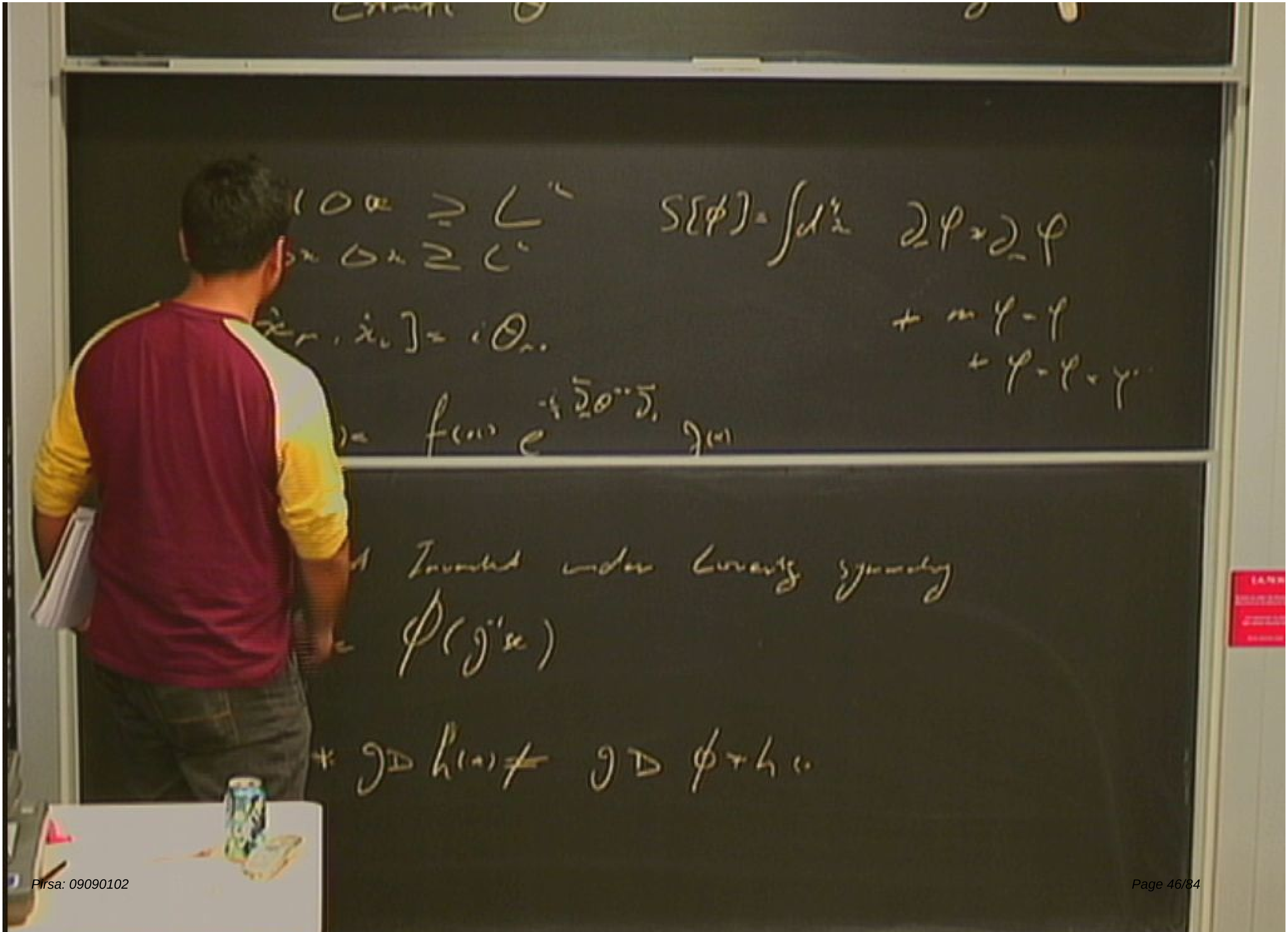
$$\begin{tikzcd}
\alpha\otimes\beta \arrow[r, "\Delta"] \arrow[d, "F"] & f\otimes f \quad \Delta(\alpha\otimes\beta) \arrow[d, "m_\theta"] \\
m_\theta\, \alpha\otimes\beta \arrow[r] & \rho(g)\, m_\theta\, f\otimes g
\end{tikzcd}$$

$$= m_0\, F = e^{i\theta_{\cdot} P_\mu \otimes P_\nu}$$

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2) \qquad \Delta(L) = 1\otimes L + L\otimes 1$$

$$\begin{tikzcd}
\alpha\otimes\beta \arrow[r, "\Delta"] \arrow[d, "m_\theta"'] & f\otimes f\ \Delta(\alpha\otimes\beta) \arrow[d, "m_\theta"] \\
m_\theta\ \alpha\otimes\beta \arrow[r] & \rho(g)\ m_\theta\ f\otimes g
\end{tikzcd}$$

$$m_\theta = m_0 F = m_0\, e^{i\theta_{\mu\nu} P_\mu\otimes P_\nu}$$



$$O(O x \geq C^{n}$$
$$O x \, O x \geq C^{n}$$

$$[\hat{x}_\mu , \hat{x}_\nu] = i\Theta_{\mu\nu}$$

$$f(x) * g(x) = f(x)\, e^{-\frac{i}{2}\overleftarrow{\partial}_\mu \Theta^{\mu\nu} \overrightarrow{\partial}_\nu}\, g(x)$$

$$S[\phi] = \int d^4x \;\; \partial_\mu \varphi * \partial_\mu \varphi + m \varphi * \varphi + \varphi * \varphi * \varphi \cdots$$

$S[\phi]$ not Invariant under Lorentz symmetry

$$g \triangleright \phi(x) = \phi(g^{-1}x)$$

$$g \triangleright \phi(x) * g \triangleright h(x) \neq g \triangleright \phi * h(x)$$

$$\ldots) = \alpha(g_1, g_2)$$

$$\begin{array}{ccc} \alpha \otimes \beta & \xrightarrow{\Delta} & f \otimes f \quad \Delta(\alpha \otimes \beta) \\ m_\theta \big\downarrow & & \big\downarrow m_\theta \\ m_\theta\, \alpha \otimes \beta & \longrightarrow & \rho(g)\; m_\theta\, f(g) \end{array}$$

$$\Delta_\theta = F^{-1} \Delta_0 F$$

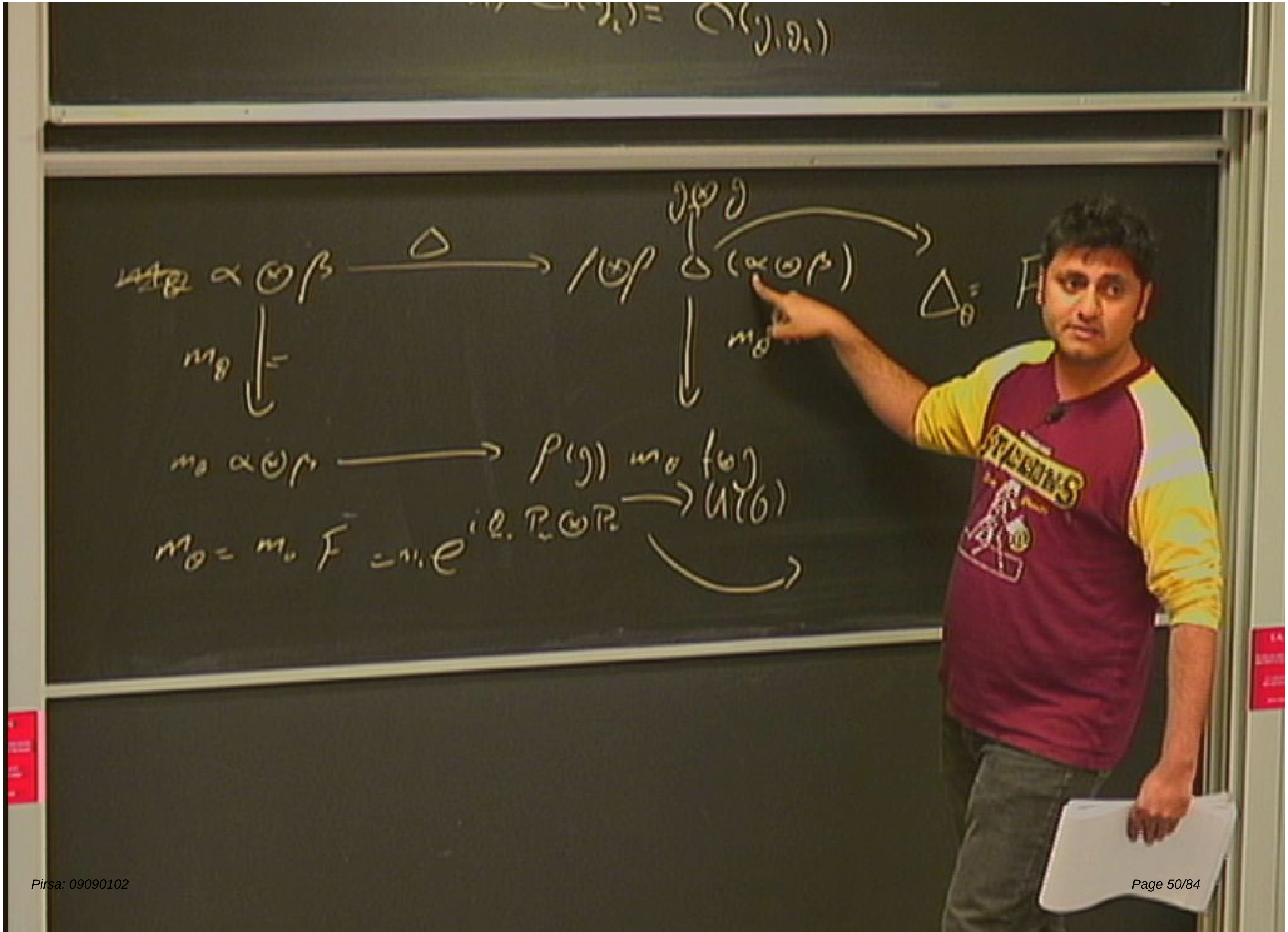
$$\longrightarrow \alpha(g)$$

$$m_\theta = m_0 F = m_0\, e^{i\theta\, P_\mu \otimes P_\nu}$$

$$\ldots(g_1) = \alpha(\cdot, g_1)$$

$$\begin{array}{ccc} \alpha \otimes \beta & \xrightarrow{\Delta} & f \otimes f \quad \Delta(\alpha \otimes \beta) \\ \Big\downarrow m_\theta & & \Big\downarrow m_\theta \\ m_\theta\, \alpha \otimes \beta & \longrightarrow & \rho(g)\, m_\theta\, f(g) \end{array} \qquad \Delta_\theta = F^{-1} \Delta_0 F$$

$$m_\theta = m_0 F = m_0 e^{i\theta\, P \otimes P}$$


α ⊗ β → ρ ⊗ ρ
Δ(α ⊗ β)
Δ_θ = F
m_θ
m_0 α ⊗ β → ρ(g) m_θ f(g)
U(θ)
m_θ = m_0 F = m_0 e^{iθ P ⊗ P}

$$\ldots(g_1) = \Omega(g, g_1)$$

$$\begin{array}{ccc}
\alpha \otimes \beta & \xrightarrow{\ \Delta\ } & \Delta(\alpha \otimes \beta) = \sum_\alpha g^{(1)}_\alpha \otimes g^{(1)}_\beta \\
m_\theta \Big\downarrow & & \Big\downarrow m_\theta \\
m_\theta\, \alpha \otimes \beta & \longrightarrow & \rho(g)\, m_\theta\, f \otimes g
\end{array}$$

$$\Delta_\theta = F^{-1} \Delta_0 F$$

$$\longrightarrow U(\theta)$$

$$m_\theta = m_0 F = m_0 e^{i\theta_{\mu\nu} P_\mu \otimes P_\nu}$$

$$|k_1, k_2\rangle = \alpha \, |k_2, k_1\rangle$$

$$\sigma |k_1, k_2\rangle = F^{-2} \, |k_1, k_2\rangle$$

$$\begin{array}{ccc} V & \sim & W \\ \Big\downarrow g & & \Big\downarrow g \\ V' & \sim & W' \end{array}$$

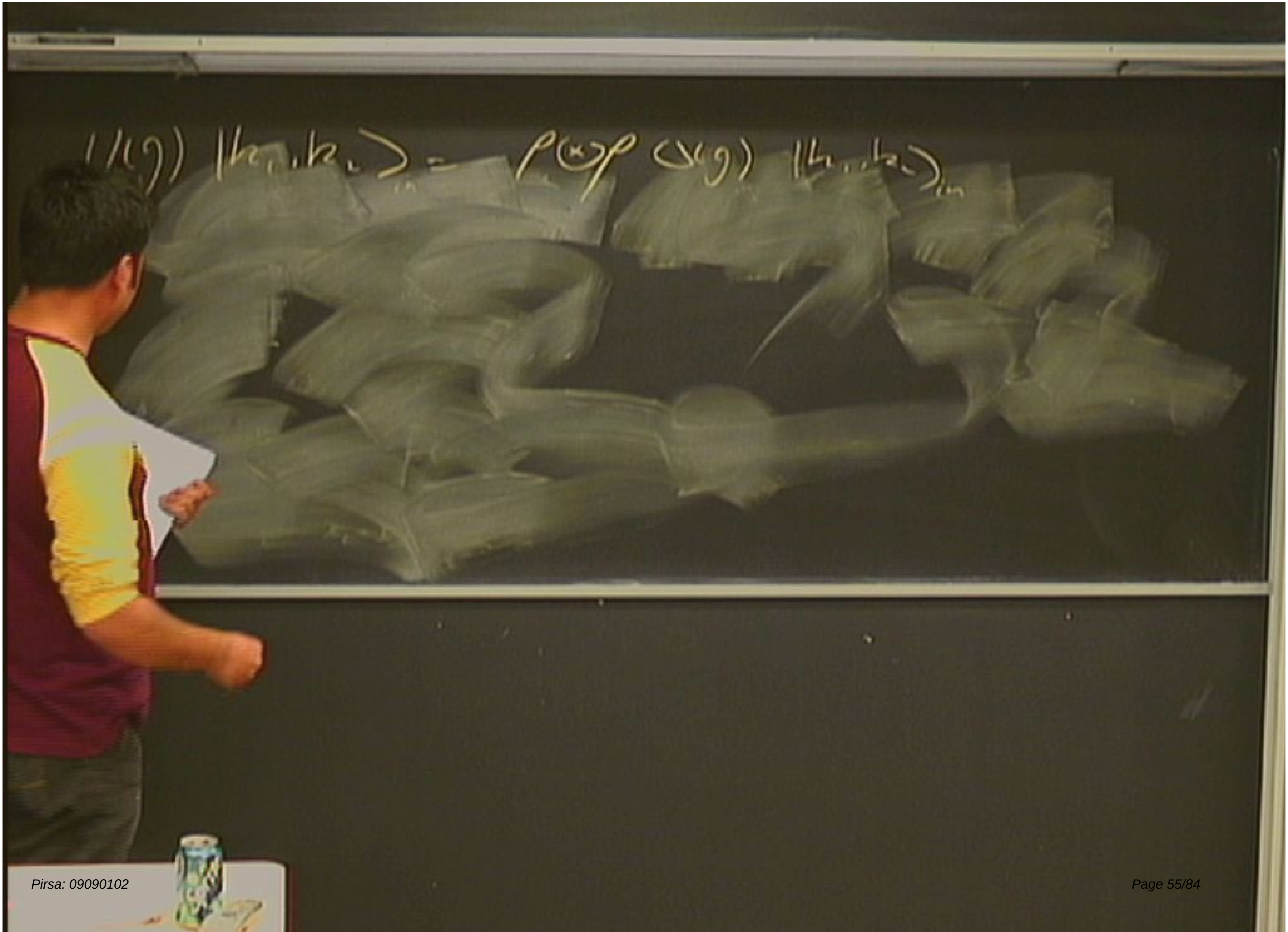
$$|k_1, k_2\rangle = \alpha \, |k_2, k_1\rangle$$

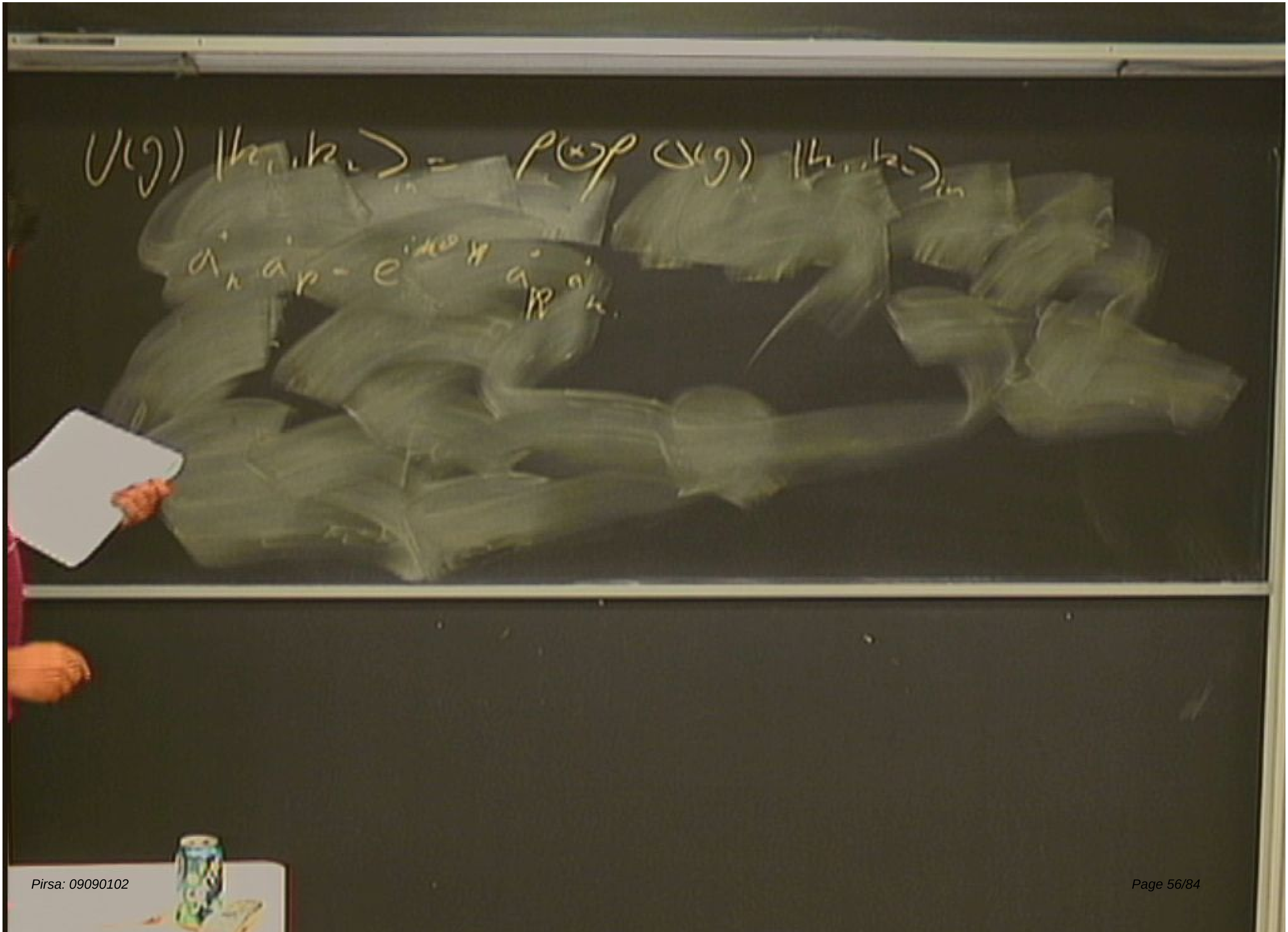
$$\sigma |k_1, k_2\rangle = F^{-2} \, |k_1, k_2\rangle$$

$$e^{i\theta_{\mu\nu} k_1^\mu k_2^\nu} |k_1, k_2\rangle$$

$$\begin{tikzcd} V \arrow[r, phantom, "\sim"] \arrow[d, "g"] & W \arrow[d, "g"] \\ V' \arrow[r, phantom, "\sim"] & W' \end{tikzcd}$$

$$U(g)\,|k_1, k_2\rangle_{in} = \rho_1 \otimes \rho_2\,(U(g))\,|k_1, k_2\rangle_{in}$$

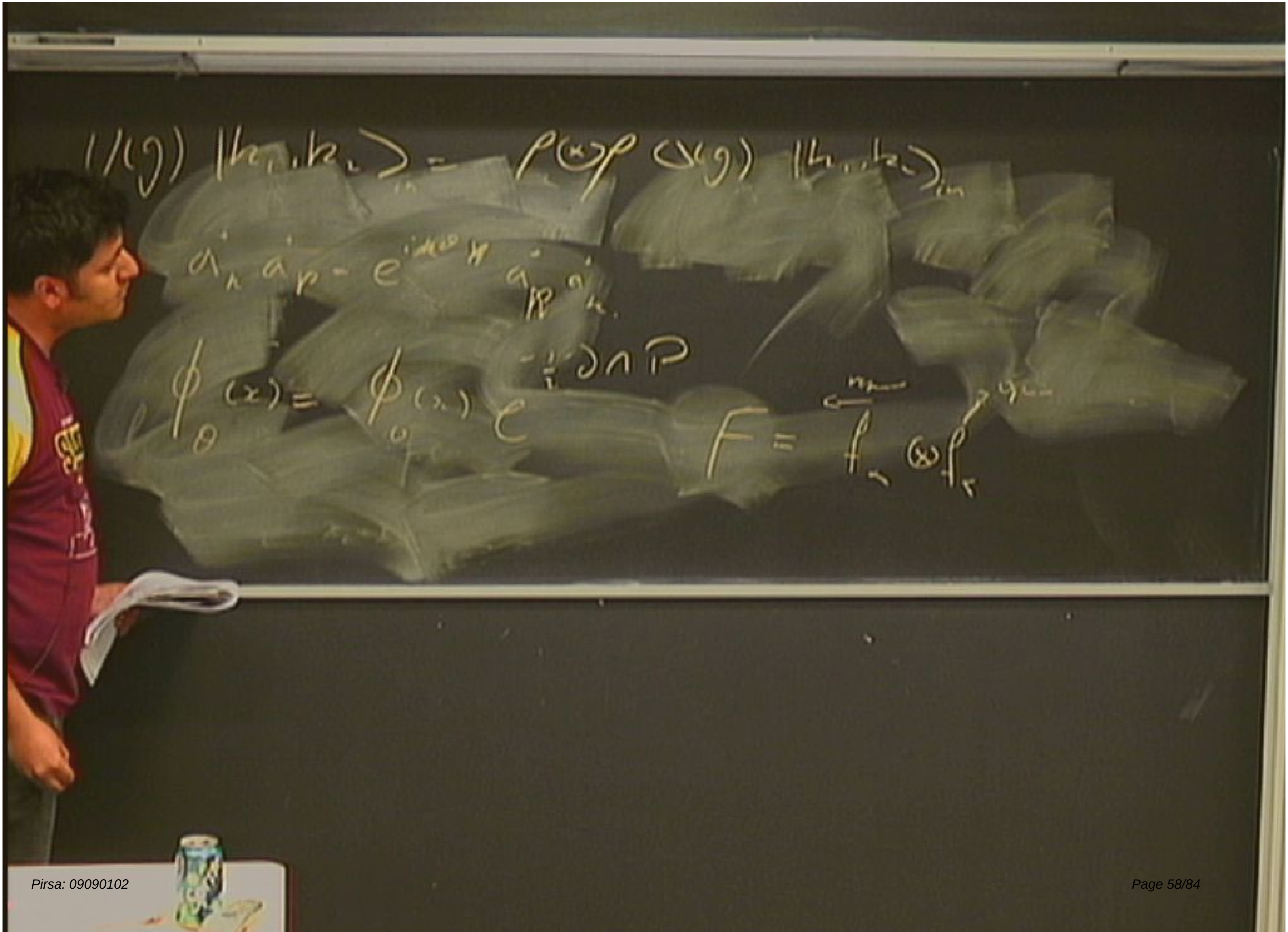

$U(g)\,|k_1,k_2\rangle_{in} = \rho \otimes \rho\,(\Lambda(g))\,|k_1,k_2\rangle_{in}$

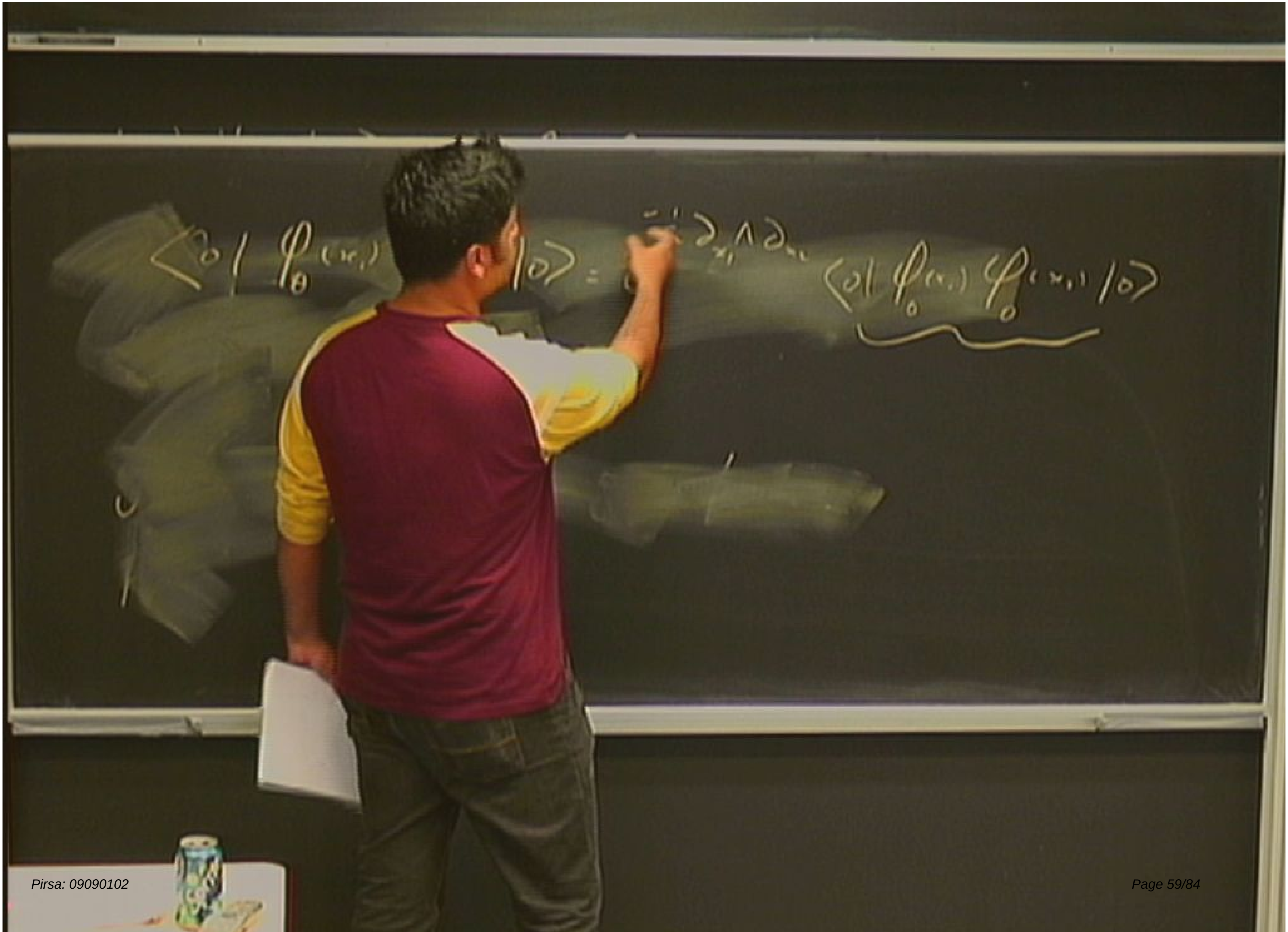


$$U(g)\,|k_1, k_2\rangle_{in} = \rho(x)\rho\,(\Delta(g))\,|k_1, k_2\rangle_{in}$$

$$a^{\dagger}_{k}\, a^{\dagger}_{p} = e^{i k \theta p}\, a^{\dagger}_{p}\, a^{\dagger}_{k}$$

$$\phi_{\theta}(x) = \phi_{0}(x)\, e^{-\frac{i}{2}\partial \wedge P}$$

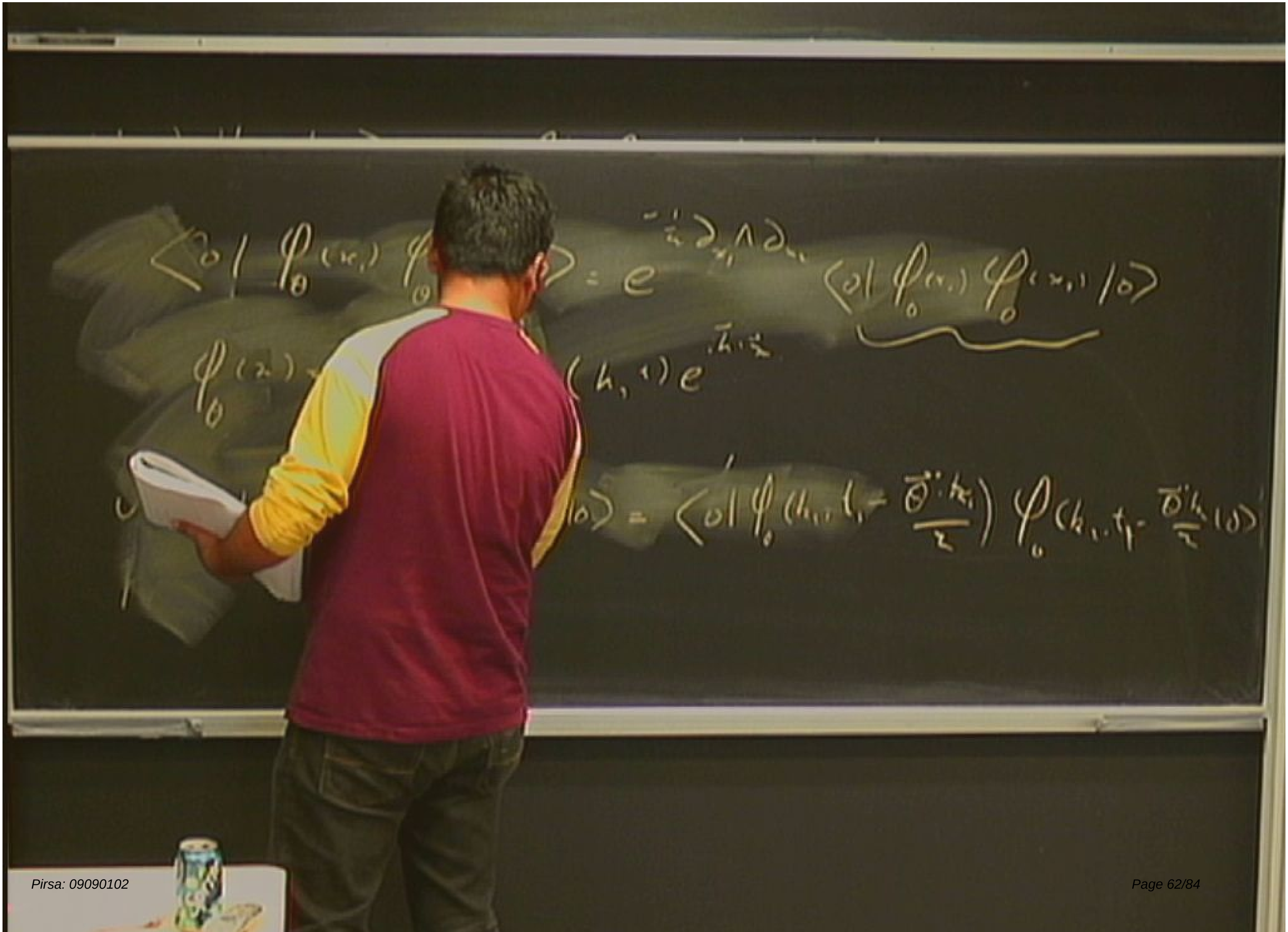

$U(g)\,|k_1,k_2\rangle_{in} = \rho\otimes\rho\,(\Delta(g))\,|k_1,k_2\rangle_{in}$
$a_k\, a_p = e^{i\ldots}\, a_p\, a_k$
$\phi_\theta(x) = \phi_0(x)\, e^{-\frac{i}{2}\partial\wedge P}$
$F = f_n \otimes f_r$


$\langle 0| \varphi_0(x_1)\, \varphi_0(x_2) |0\rangle$

$$\langle 0|\, \phi_\theta(x_1)\, \phi_\theta(x_2)\,|0\rangle = e^{-\frac{i}{2}\partial_{x_1}\wedge\partial_{x_2}}\, \langle 0|\, \phi_0(x_1)\, \phi_0(x_2)\,|0\rangle$$

$$\langle 0 | \varphi_\theta(x_1)\, \varphi_\theta(x_2) | 0 \rangle = e^{-\frac{i}{2} \partial_{x_1} \wedge \partial_{x_2}} \underline{\langle 0 | \varphi_0(x_1)\, \varphi_0(x_2) | 0 \rangle}$$

$$\varphi_\theta(x) = \int \frac{d^4k}{(2\pi)^4}\, \tilde{\varphi}_0(k, t)\, e^{i k \cdot x}$$



$$\langle 0|\, \phi_\theta(x_1)\, \phi_\theta(x_2)\,|0\rangle = e^{-\frac{i}{2}\partial_{x_1}\wedge\partial_{x_2}} \underbrace{\langle 0|\, \phi_0(x_1)\, \phi_0(x_2)\,|0\rangle}$$

$$\phi_\theta(x) = \int \frac{d^4k}{(2\pi)^4}\, \tilde{\Phi}_0(k,t)\, e^{i k\cdot x}$$

$$\langle 0|\, \phi_\theta(\vec{k}_1,t_1)\, \phi_\theta(\vec{k}_2,t_2)\,|0\rangle = \langle 0|\, \phi_0\!\left(k_1, t_1 - \frac{\vec{\theta}\cdot k_2}{2}\right) \phi_0\!\left(k_2, t_2 - \frac{\vec{\theta}\cdot k_1}{2}\right)|0\rangle$$

$$\langle 0|\, \varphi_\theta(x_1)\, \varphi_\theta(x_2)|0\rangle = e^{-\frac{i}{2}\partial_{x_1}\wedge\partial_{x_2}} \langle 0|\, \varphi_0(x_1)\, \varphi_0(x_2)\,|0\rangle$$

$$\varphi_\theta(x) = \int \frac{d^3k}{(2\pi)^3}\, \hat{\varphi}_\theta(k,t)\, e^{ik\cdot x}$$

$$\langle 0|\, \varphi_\theta(\vec{k}_1,t_1)\, \varphi_\theta(\vec{k}_2,t_2)|0\rangle = \langle 0|\, \varphi_0\left(k_1, t_1 - \frac{\vec{\theta}\cdot k_1}{2}\right) \varphi_0\left(k_2, t_2 - \frac{\vec{\theta}\cdot k_2}{2}\right)|0\rangle$$

$$\langle 0|\,\varphi_\theta(x_1)\,\varphi_\theta(x_2)|0\rangle = e^{-\frac{i}{2}\partial_{x_1}\wedge\partial_{x_2}}\,\langle 0|\,\varphi_0(x_1)\,\varphi_0(x_2)\,|0\rangle$$

$$\varphi_\theta(x) = \int \frac{d^4k}{(2\pi)^4}\,\hat{\Phi}_0(k,t)\,e^{ik\cdot x}$$

$$\langle 0|\,\varphi_\theta(\vec{k}_1,t)\,\varphi_\theta(\vec{k}_2,t_2)|0\rangle = \langle 0|\,\varphi_0\!\left(k_1,\,t_1-\frac{\vec{\theta}\cdot k_1}{2}\right)\varphi_0\!\left(k_2,\,t_2-\frac{\vec{\theta}\cdot k_2}{2}\right)|0\rangle$$

$$\langle 0|\,\varphi_\theta(x_1)\,\varphi_\theta(x_2)|0\rangle = e^{-\frac{i}{2}\partial_{x_1}\wedge\partial_{x_2}}\,\underbrace{\langle 0|\,\varphi_0(x_1)\,\varphi_0(x_2)\,|0\rangle}$$

$$\varphi_0(x) = \int \frac{d^4k}{(2\pi)^4}\,\tilde{\varphi}_0(k,t)\,e^{ik\cdot x}$$

$$\langle 0|\,\varphi_\theta(\vec{k}_1,t)\,\varphi_\theta(\vec{k}_2,t_2)|0\rangle = \langle 0|\,\varphi_0\!\left(k_1, t_1 - \frac{\vec{\theta}\cdot k_1}{2}\right)\varphi_0\!\left(k_2, t_2 - \frac{\vec{\theta}\cdot k_2}{2}\right)|0\rangle$$

$x_0,\ x_3 \qquad \varphi(x)$

$$\alpha \otimes \beta \xrightarrow{\Delta} \sum \Delta(\alpha \otimes \beta) \qquad g \otimes g$$

$$\Delta_\theta = F^{-1} \Delta_0 F$$

$$\sum_\alpha g_{m_\theta}^{(1)} \otimes g^{(1)}$$

$$m_\theta \alpha \otimes \beta \longrightarrow \rho(g)\, m_\theta f(g) \to U(\theta)$$

$$m_\theta = m_0 F^{-1} = m_0 e^{i\theta_* P_\mu \otimes P_\nu}$$

$$P_F = P_0(h) \cosh(H\vec{\theta} \cdot \vec{h})$$

$$\alpha \otimes \beta \xrightarrow{\Delta} \Delta(\alpha\otimes\beta)$$

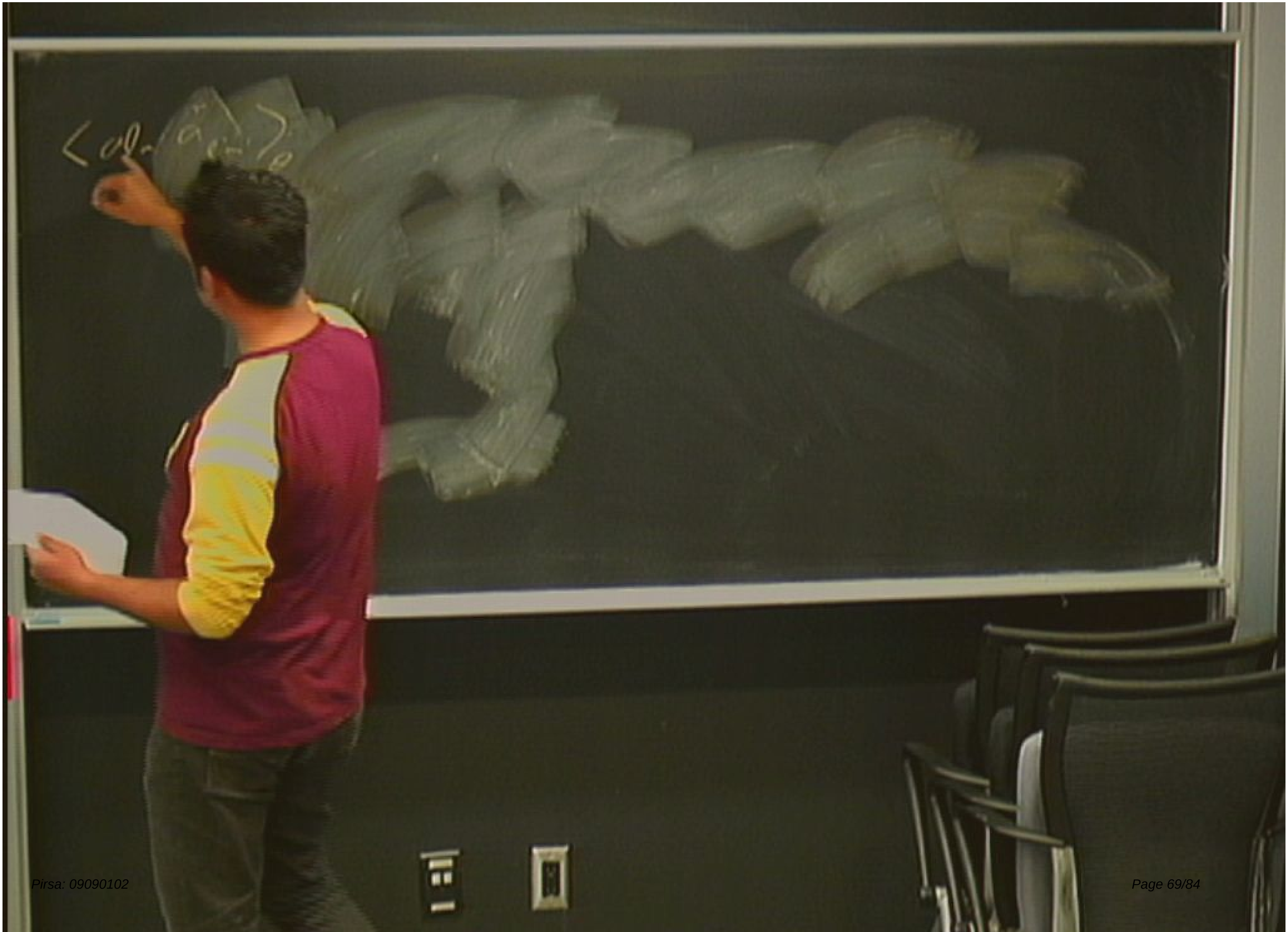
$$\Delta_\theta = F^{-1} \Delta_0 F$$

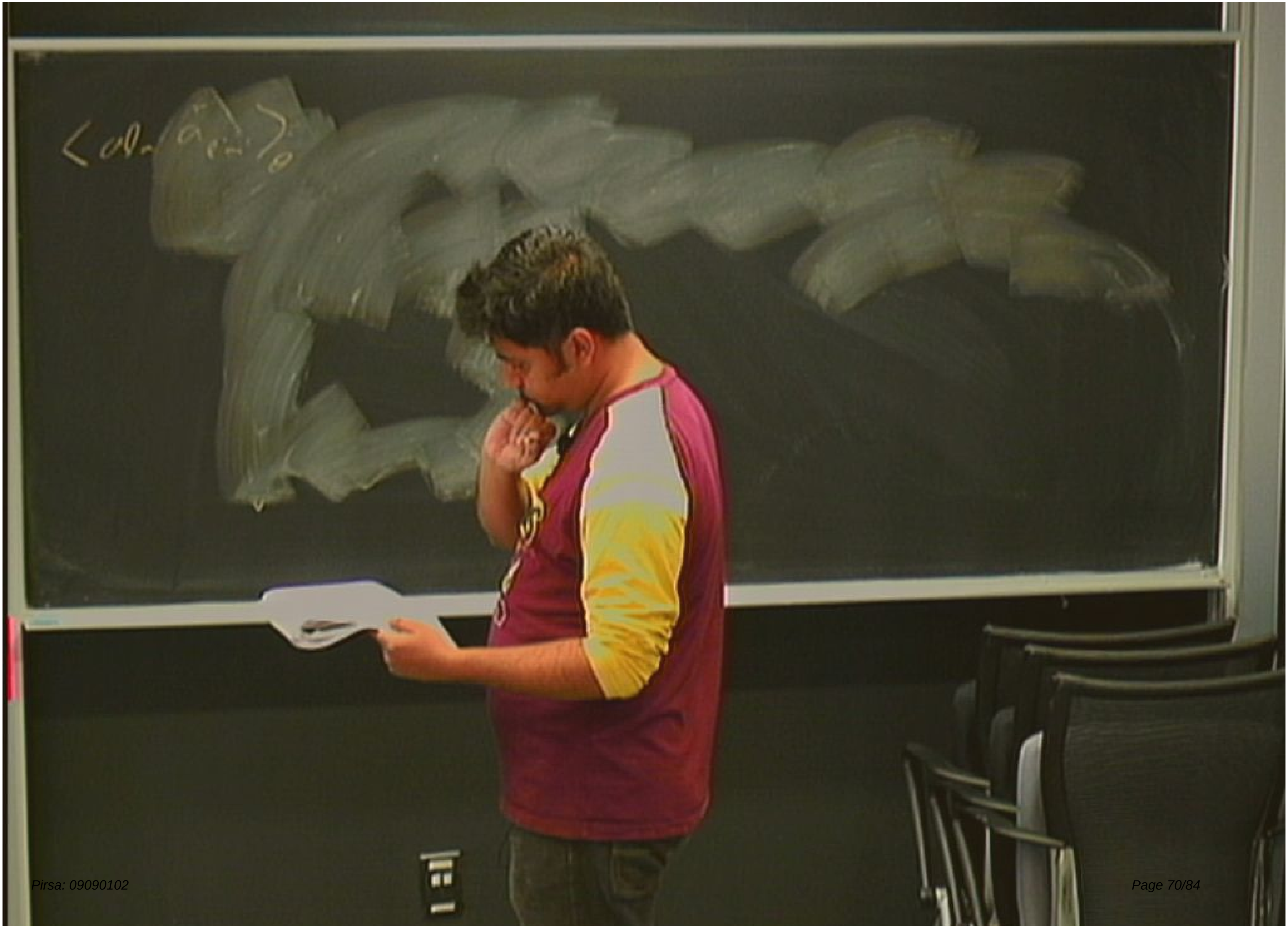
$$m_\theta \downarrow = \qquad \sum_\alpha \big\downarrow\, g^{(1)}_{m_\theta} \otimes g^{(1)}_\mu$$

$$P_{\mathcal{F}} = P_0(h)\cosh(H\vec{\theta}\cdot\vec{h})$$

$$m_\theta\, \alpha\otimes\beta \longrightarrow \rho(g))\, m_\theta\, f(g) \rightarrow (U(g))$$

$$m_\theta = m_0 F^{-1} = m_0\, e^{i\theta_0 P_\mu \otimes P_\nu}$$





$$\Delta(\mathfrak{g}_1 \ldots) = \ldots(\cdot \mathfrak{g}_1, \mathfrak{g}_2)$$

$$\begin{array}{ccc} \alpha \otimes \beta & \xrightarrow{\Delta} & 1 \otimes \beta \quad \Delta(\alpha \otimes \beta) \\ m_\theta \downarrow & & \sum_\alpha \mathfrak{g}^{(1)}_{\alpha} \otimes \mathfrak{g}^{(2)}_{\alpha} \downarrow m_\theta \\ m_\theta\, \alpha \otimes \beta & \longrightarrow & \rho(g)\; m_\theta\; f(g) \end{array}$$

$$\Delta_\theta = F^{-1} \Delta_0 F$$

$$m_\theta = m_0 F^{-1} = m_0 e^{i\theta \cdot P_\mu \otimes P_\nu} \longrightarrow (U\gamma(\theta))$$

$$P_\xi = P_0(h) \cosh(H \vec{\theta} \cdot \vec{h})$$

$$\Delta(g_1 g_2) = \Delta(g_1)\Delta(g_2)$$

$$\alpha \otimes \beta \xrightarrow{\Delta} 1 \otimes \rho \quad \Delta(\alpha \otimes \beta) \quad g \otimes g$$

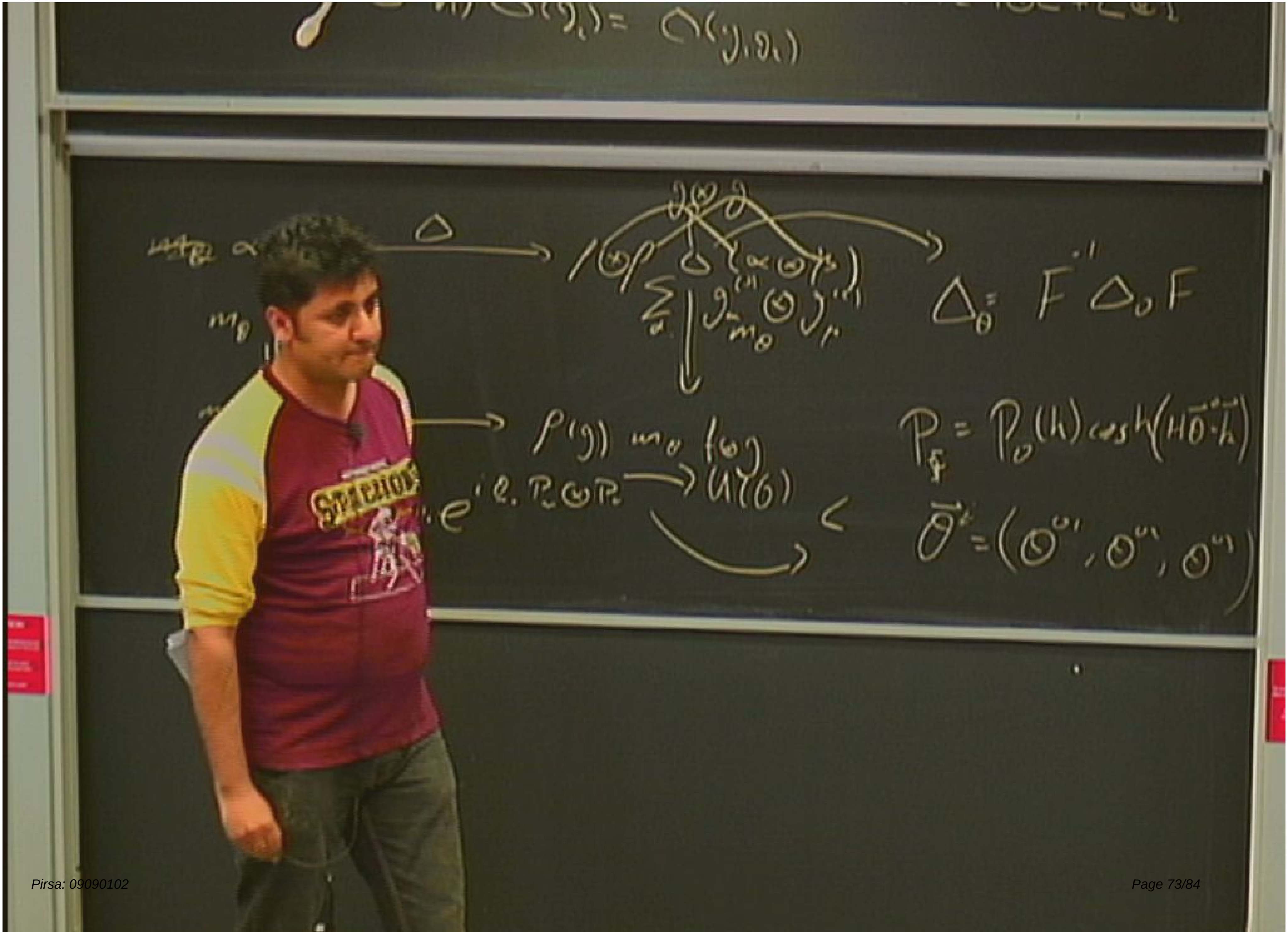
$$m_\theta \downarrow \qquad \sum_\alpha g^{(1)}_\alpha \otimes g^{(2)}_\beta \quad m_\theta \downarrow$$

$$\Delta_\theta = F^{-1} \Delta_0 F$$

$$m_0\, \alpha \otimes \beta \longrightarrow \rho(g)\, m_\theta\, f(g)$$

$$\mathcal{P}_\theta = \mathcal{P}_0(h) \cosh(H \vec{\theta} \cdot \vec{h})$$

$$m_\theta = m_0 F^{-1} = m_0 e^{i\theta \cdot P_\mu \otimes P_\nu} \longrightarrow (h)\gamma(\theta)$$



$$\alpha \otimes \beta \xrightarrow{\Delta} \sum_{(\alpha)} \mathfrak{g}^{(1)}_{\alpha} \otimes \mathfrak{g}^{(2)}_{\beta}$$

$$m_\theta \alpha \otimes \beta \longrightarrow$$

$$\Delta_\theta = F^{-1} \Delta_0 F$$

$$m_\theta = m_0 F^{-1} = m_0 e^{i\theta \cdot P_\mu \otimes P_\nu}$$

$$P_\theta = P_0(h) \cosh(H \vec{\theta} \cdot \vec{h})$$

$$\vec{\theta} = (\theta^{01}, \theta^{02}, \theta^{03})$$

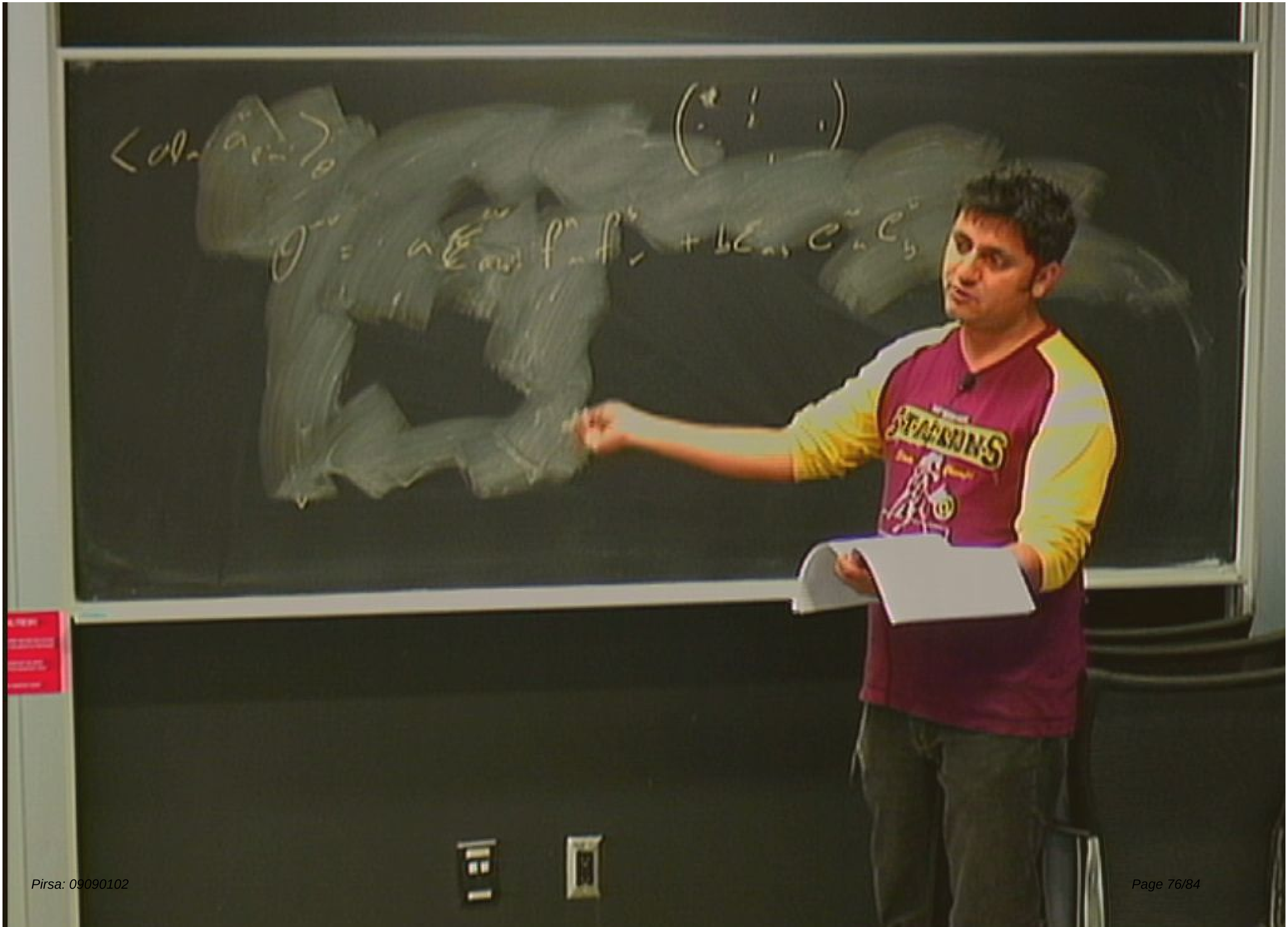
$$\begin{array}{ccc} \alpha\otimes\beta & \xrightarrow{\Delta} & \Delta(\alpha\otimes\beta) = \sum_{(i)} g^{(i)}_{m_\theta}\otimes g^{(i)}_{\mu} \\ \downarrow m_\theta & & \downarrow \\ m_0\, \alpha\otimes\beta & \longrightarrow & \rho(g)\; m_\theta\, f(g) \end{array}$$

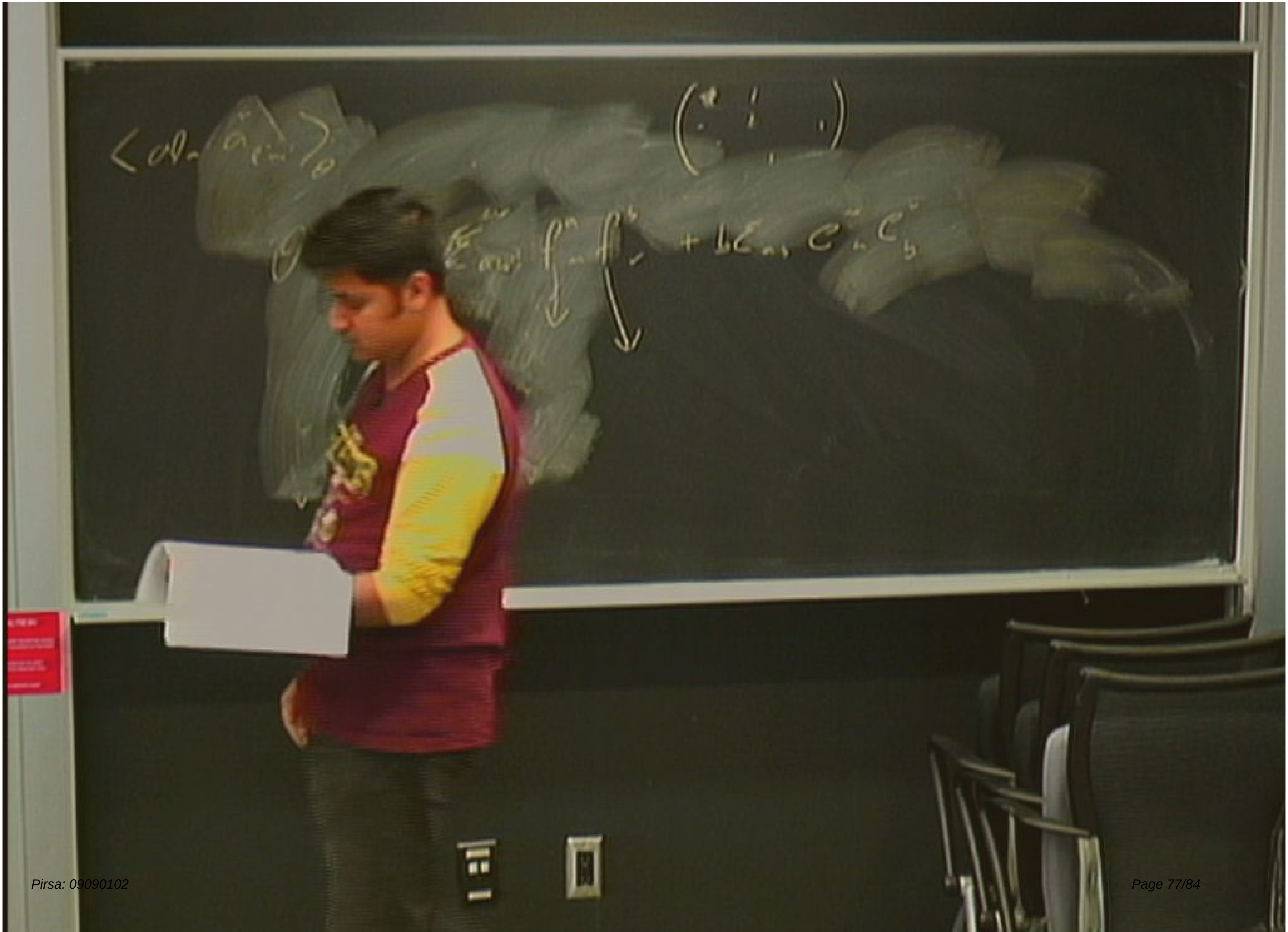
$$\Delta_\theta = F^{-1}\Delta_0 F$$

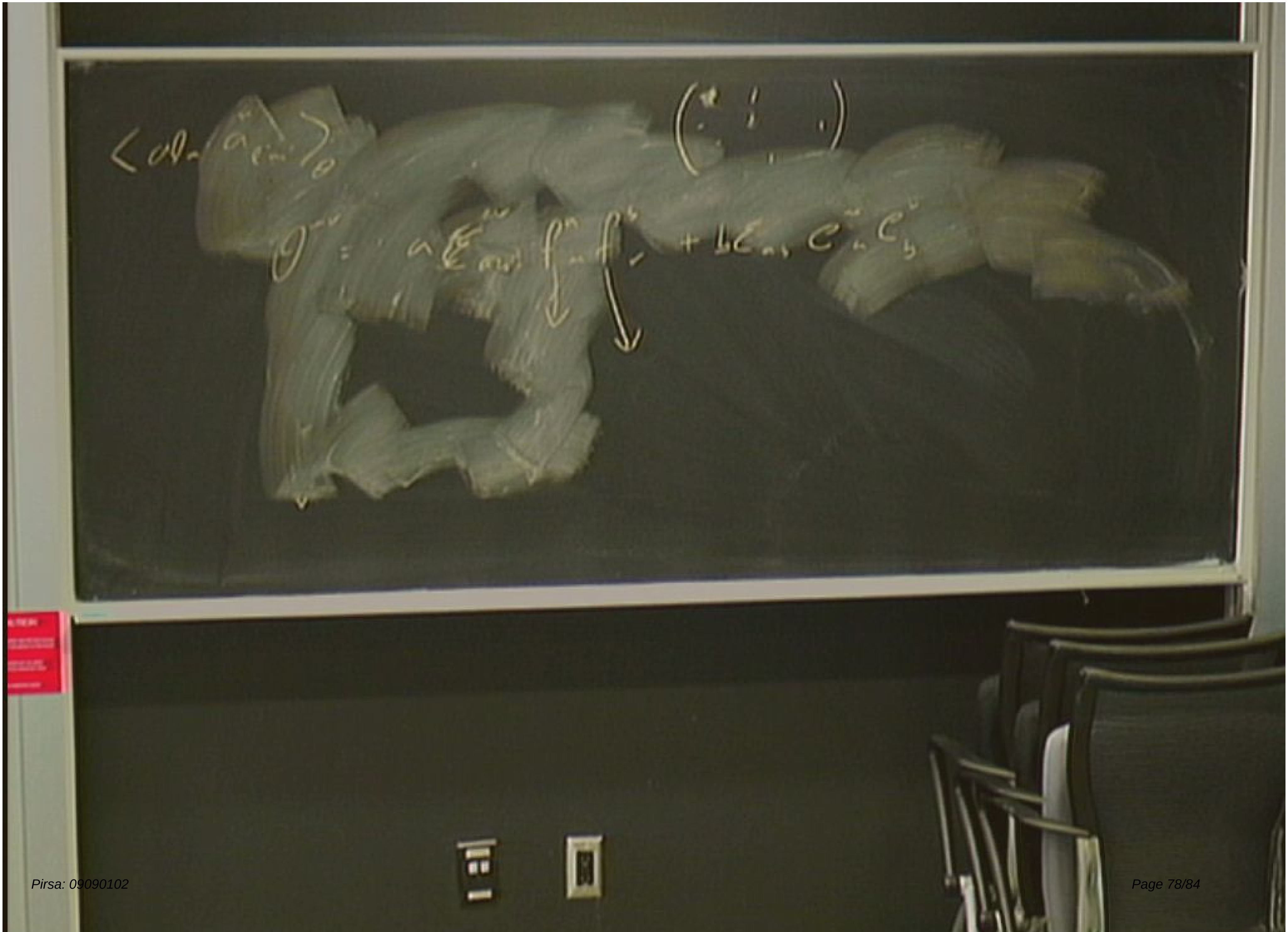
$$m_\theta = m_0 F^{-1} = m_0 e^{i\theta\cdot P_\mu\otimes P_\nu} \longrightarrow (h)\tau(\theta)$$

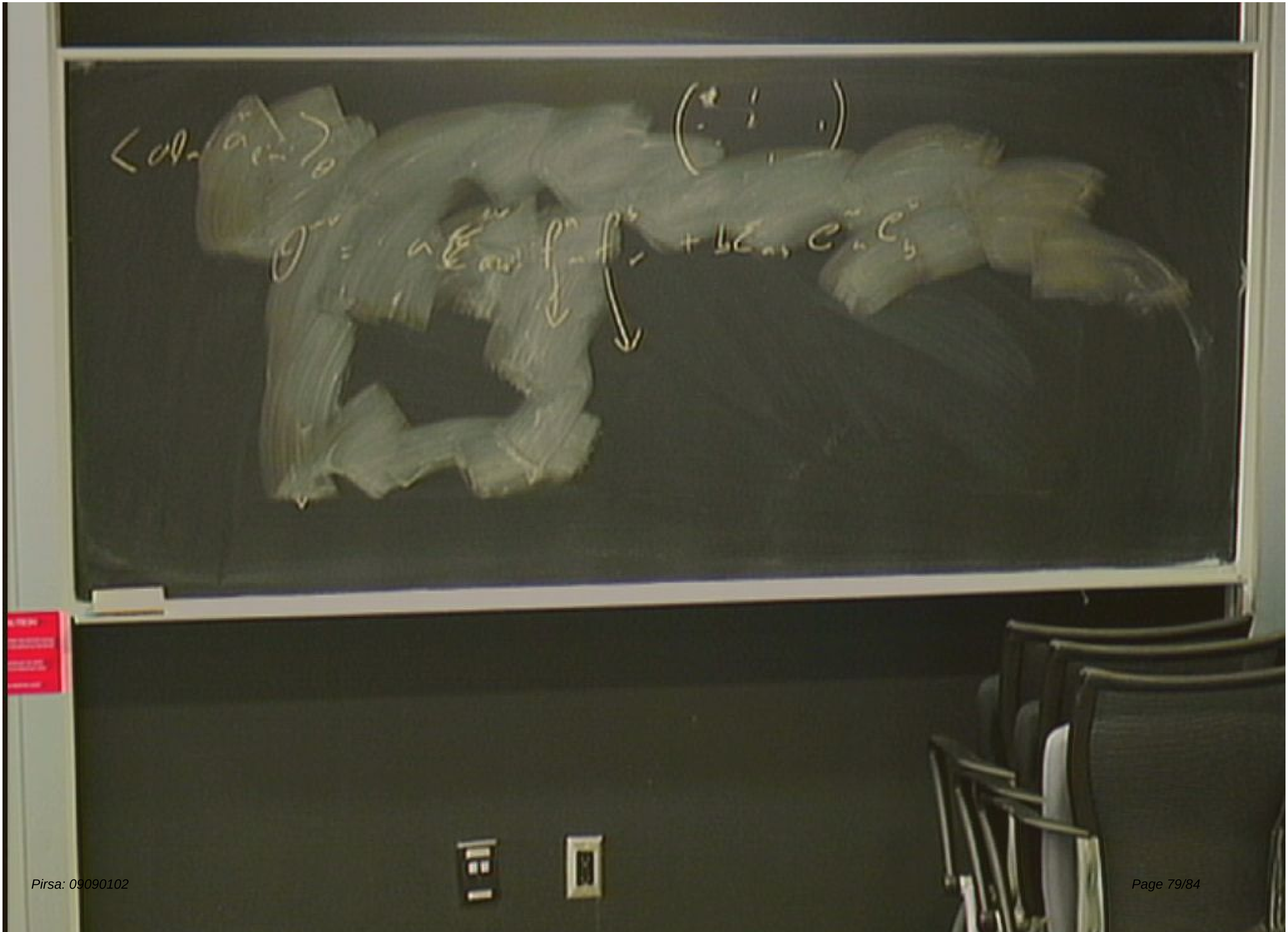
$$P_\xi = P_0(h)\cosh(H\vec{\theta}\cdot\vec{h})$$

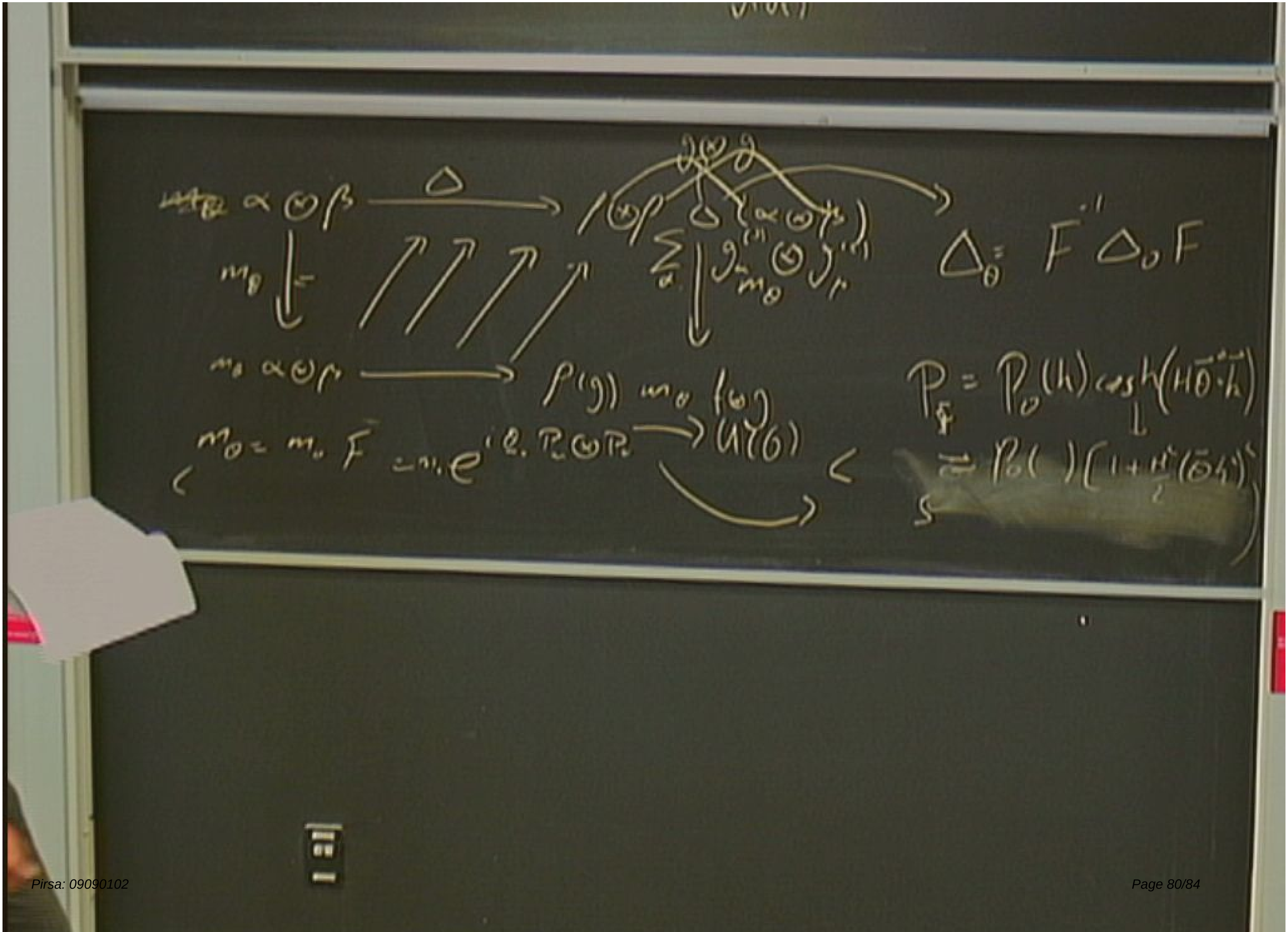
$$\vec{\theta} = (\theta^{01}, \theta^{02}, \theta^{03})$$

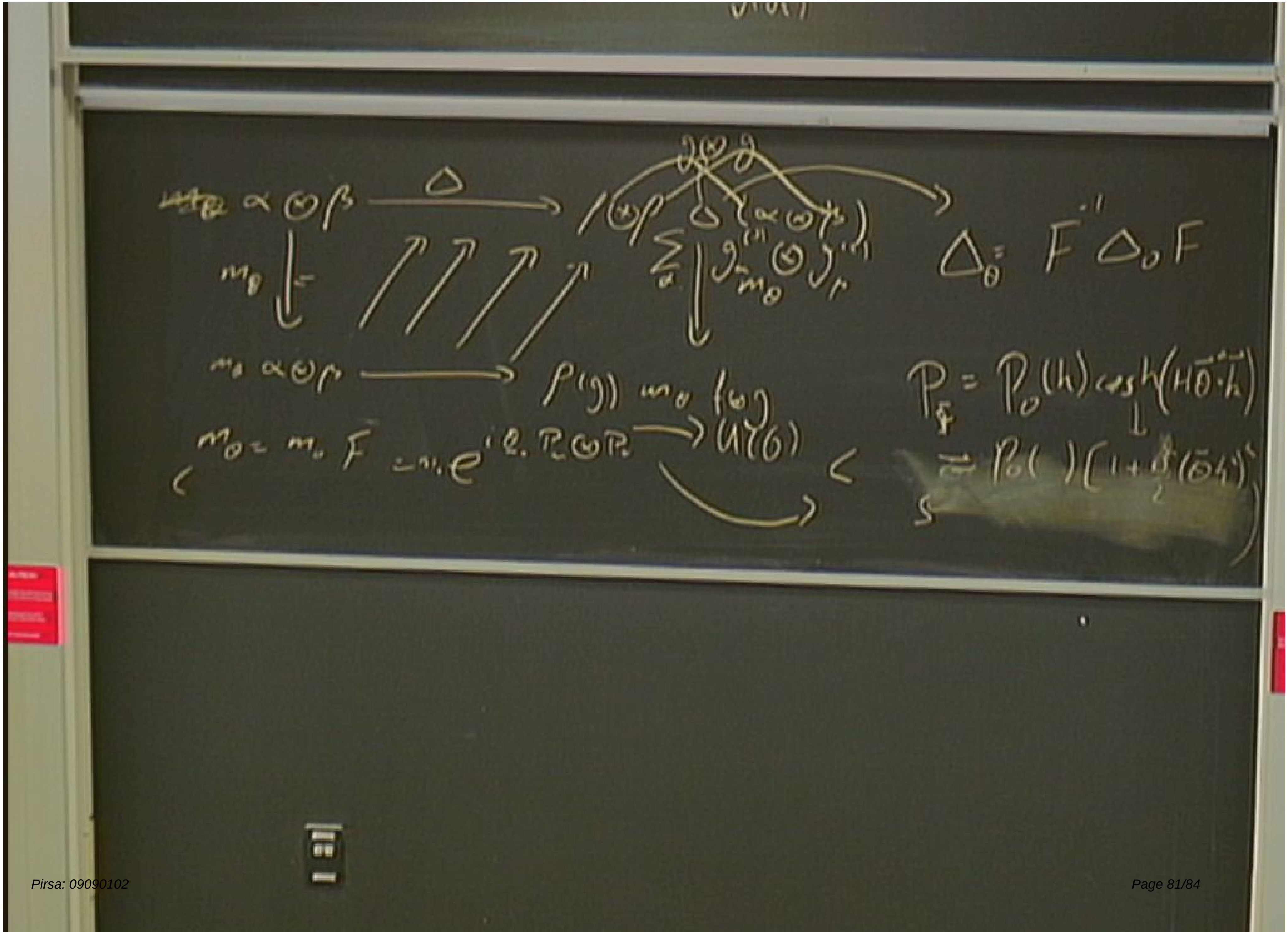




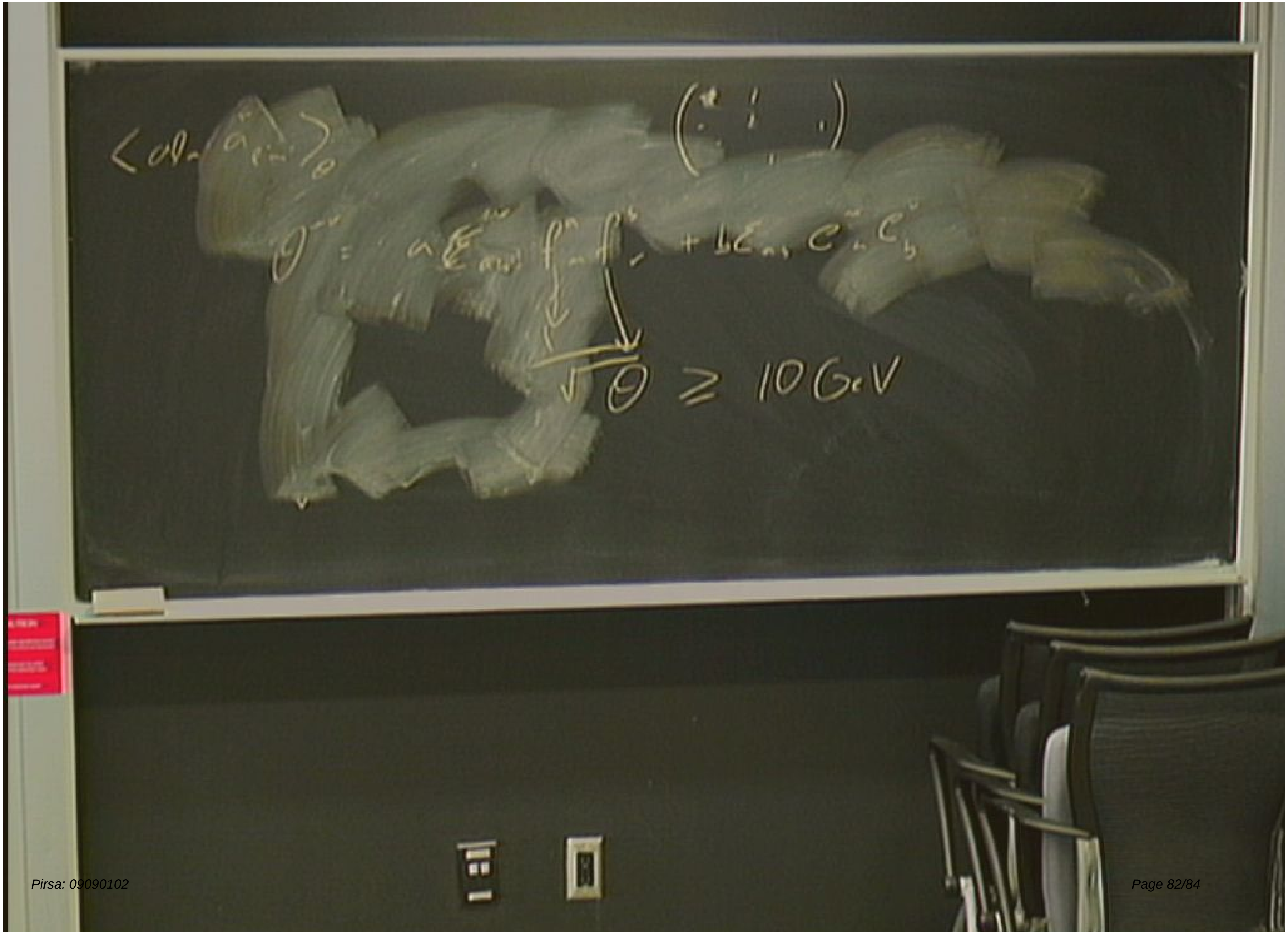


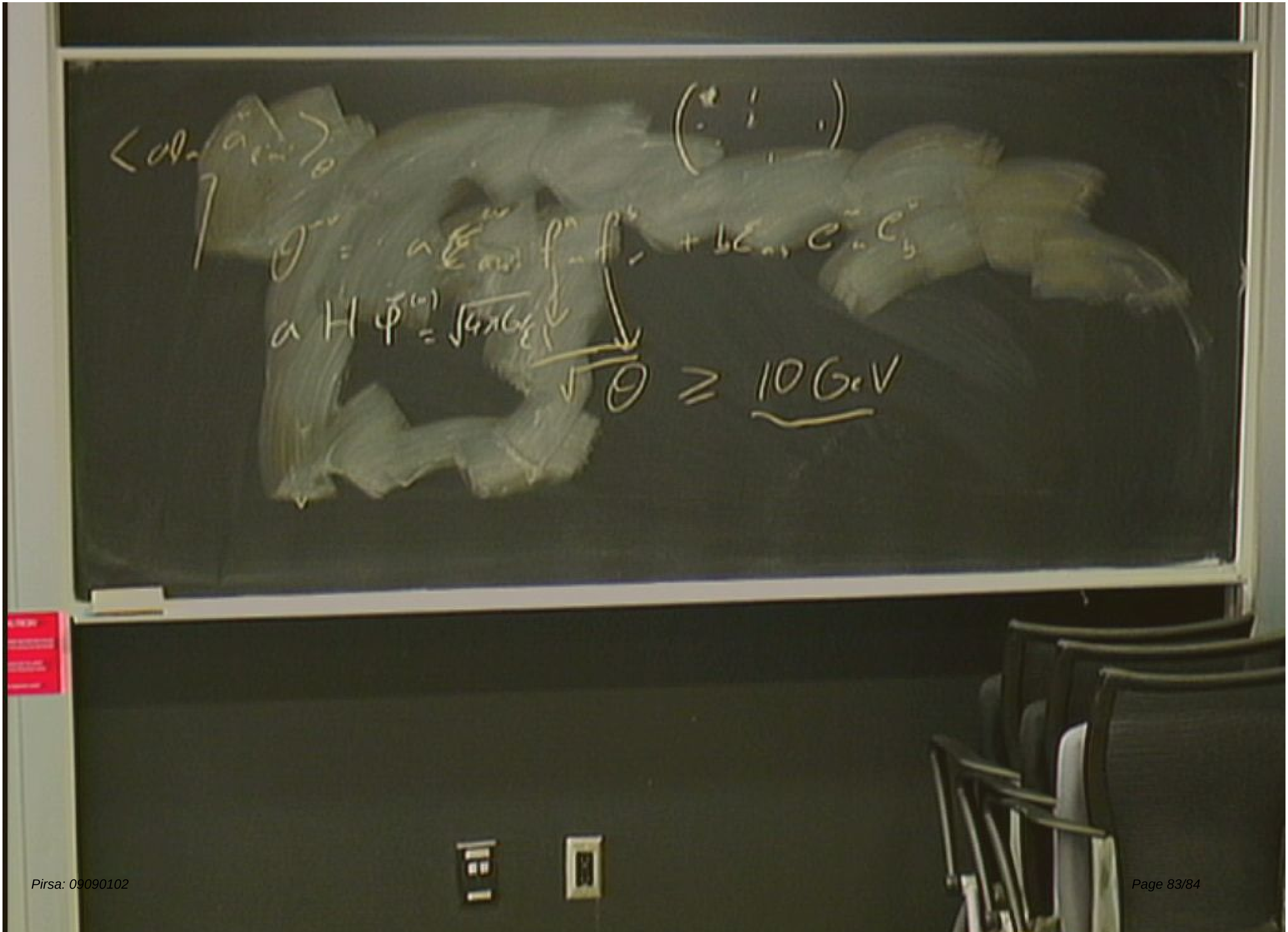


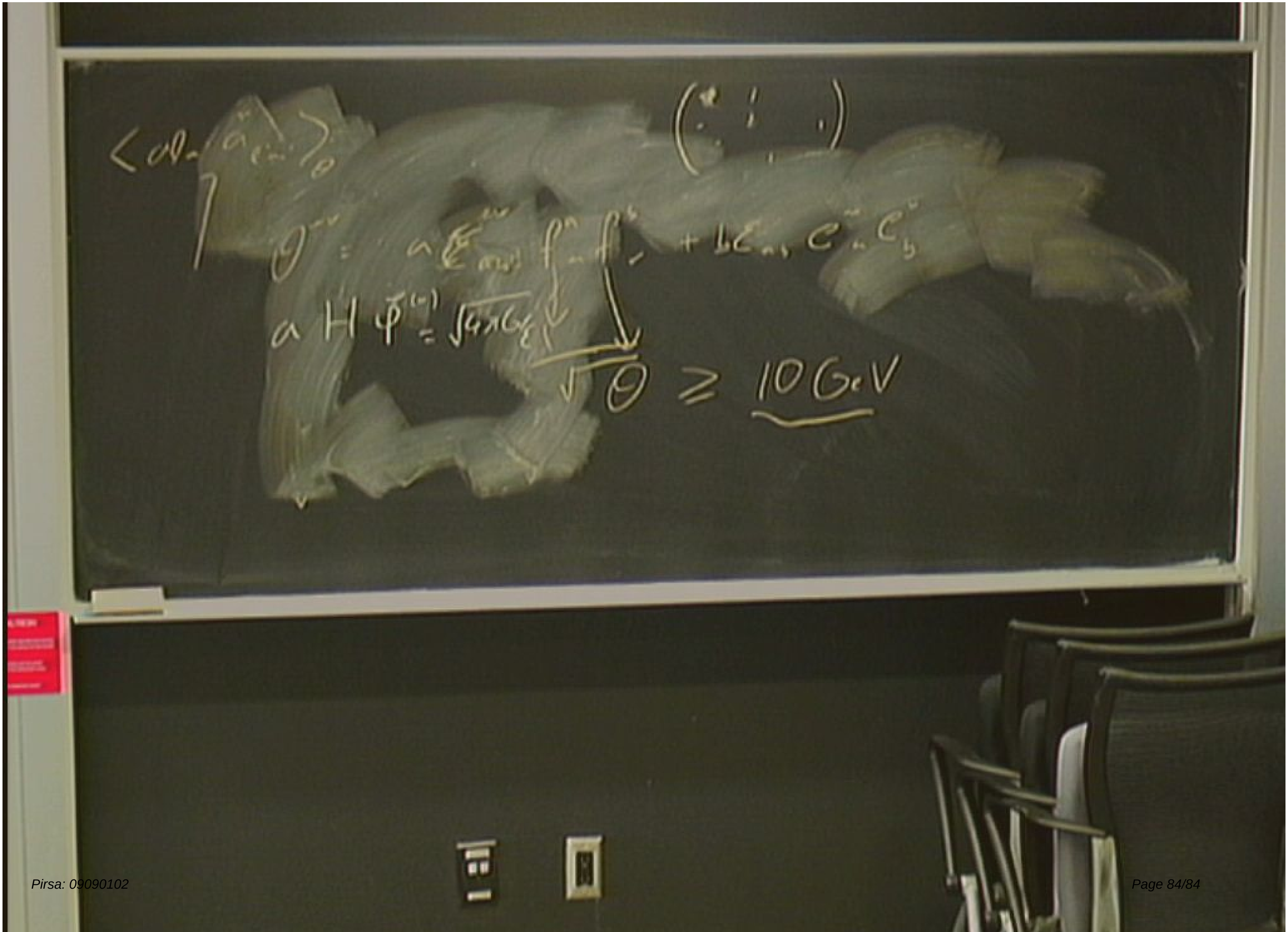

$\Delta_\theta = F^{-1} \Delta_0 F$
$m_\theta = m_0 F^{-1}$
$P_\Phi = P_0(h) \cosh(H\vec{\theta}\cdot\vec{h})$



Δθ = F⁻¹ Δ₀ F
m_θ = m₀ F


√Θ ≥ 10 GeV


√Θ ≥ 10 GeV


√Θ ≥ 10 GeV