

Title: Noncommutative Effects in Primordial Density Fluctuations

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Abstract: Near the Planckian scales, quantum gravity is expected to drastically change the structure of spacetime, one feature of which may be noncommutativity of the coordinates. Based on the recent advances in quantum field theories on such noncommutative spaces, I will consider the fluctuations of inflaton and look for possible noncommutative corrections in the CMB. Anisotropy and non-gaussianity are the result. The resultant distribution is then compared with ACBAR, CBI and WMAP data to constrain the scale of noncommutativity parameter.

Plan

$$\langle 0 | \phi(x) \phi(x_0) | 0 \rangle$$

Plan

$$\langle 0 | \phi(x_1) \phi(x_2) | 0 \rangle$$

Plan

$$\langle 0 | \phi(x_1) \phi(x_2) | 0 \rangle$$

$$\langle 0 | \phi \phi \phi \phi | 0 \rangle$$

Anisotropy

Plan

$$\langle 0 | \phi(x_1) \phi(x_2) | 0 \rangle$$

$$\langle 0 | \phi \phi \phi \phi | 0 \rangle$$

Anisotropy

Non-Gaussianity

Plan

$$\langle 0 | \phi(x_1) \phi(x_2) | 0 \rangle$$

$$\langle 0 | \phi \phi \phi \phi | 0 \rangle$$

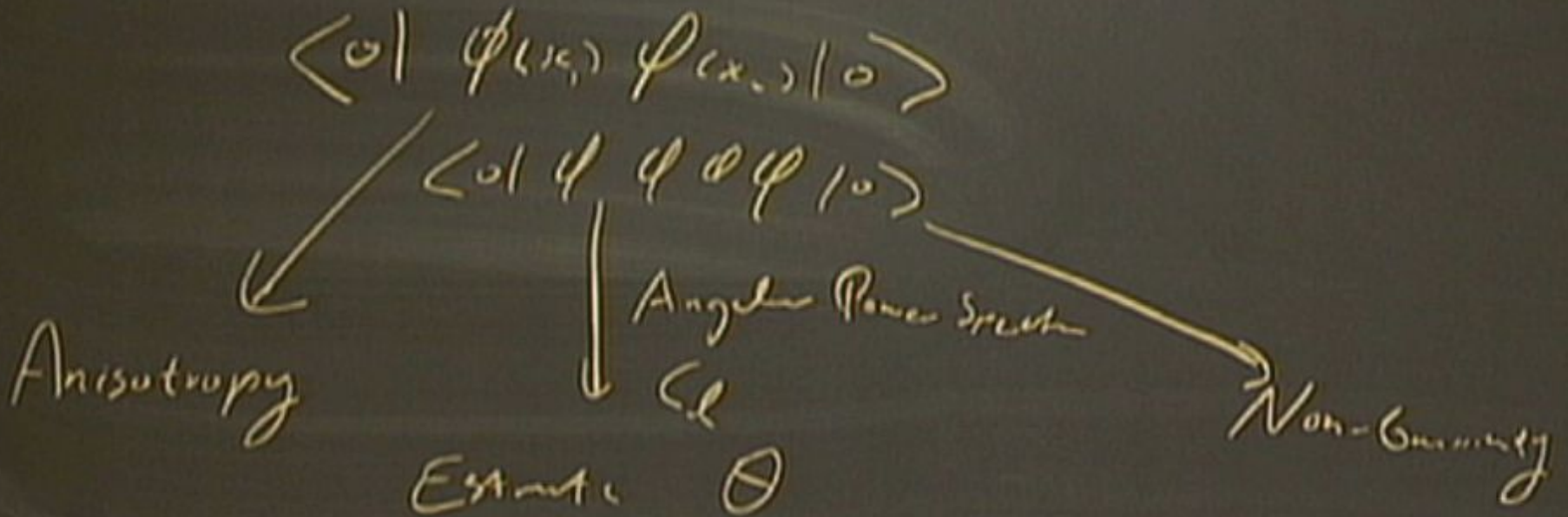
Anisotropy

Angular Power Spectra

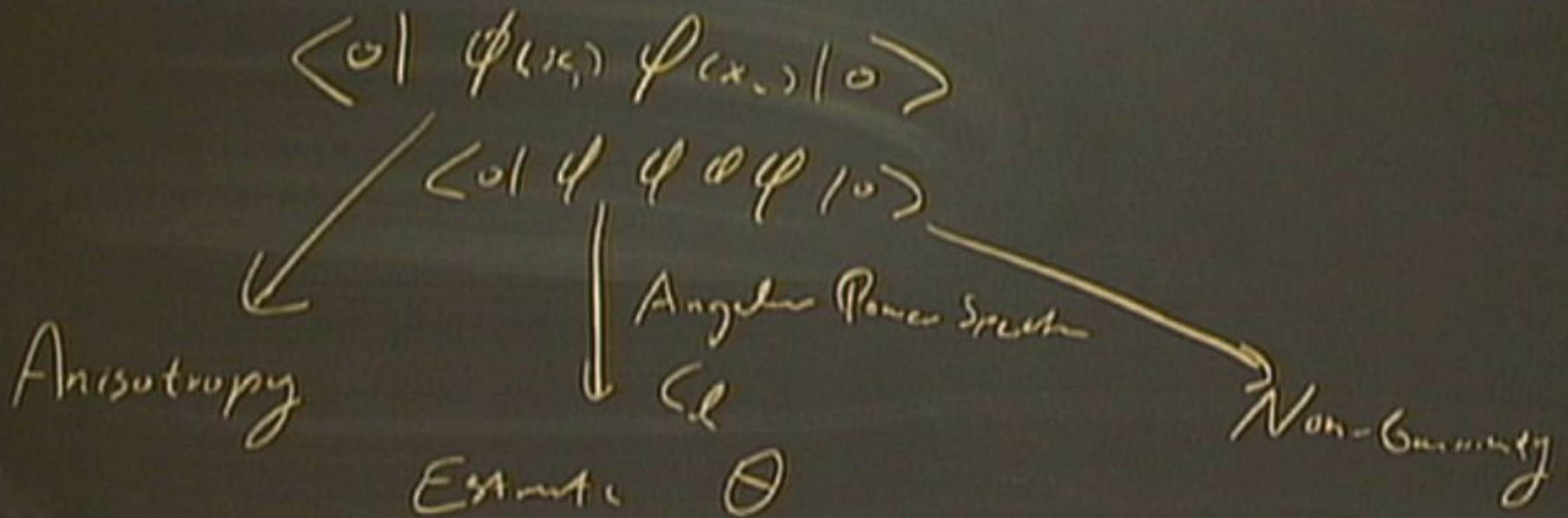
Cl

Non-Gaussianity

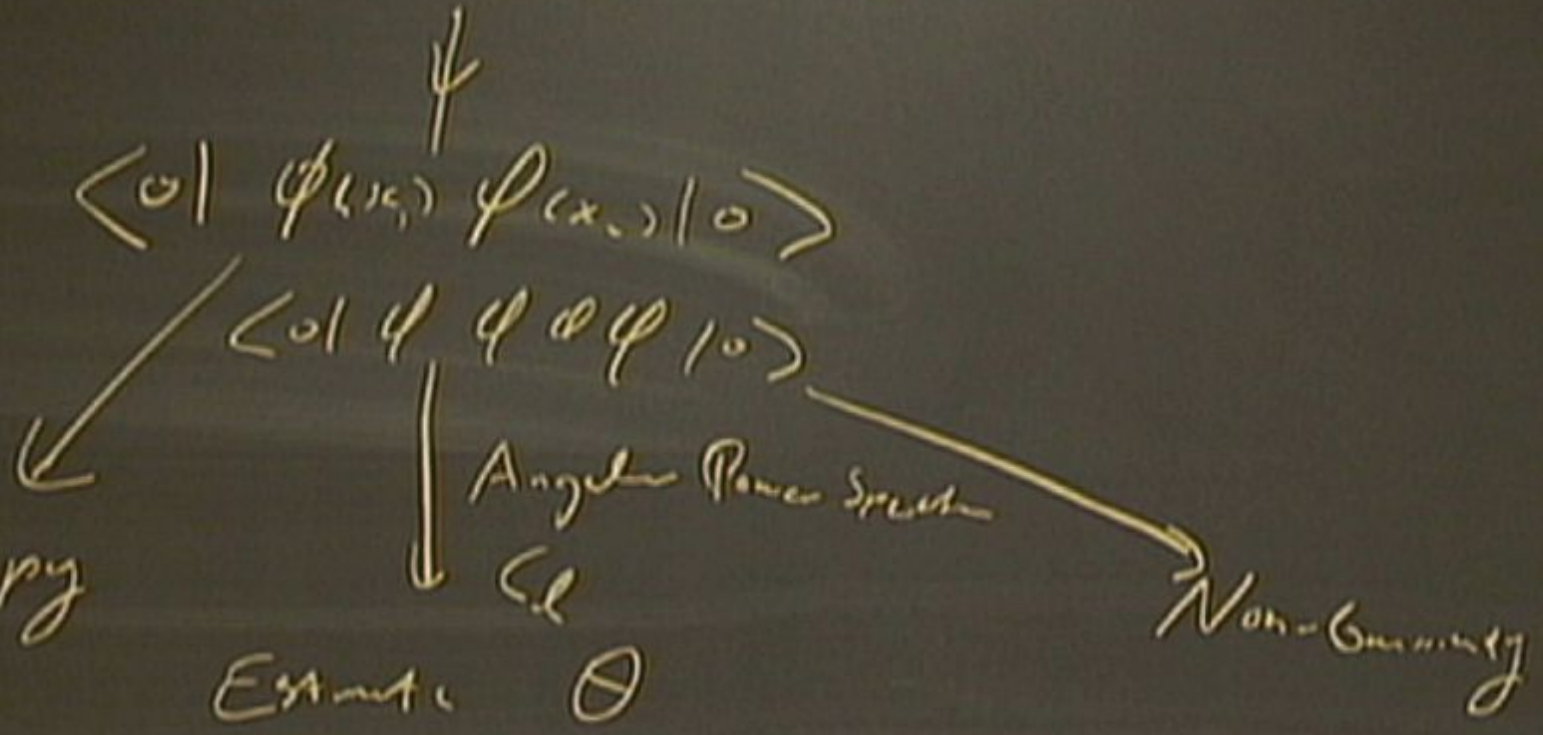
Plan



Plan



Plan



Plan

Trusted Statistics

changed

Commutator relations

between fields

$$\langle \psi | \phi(x_1) \phi(x_2) | 0 \rangle$$

$$\langle \psi | \phi \phi \phi | 0 \rangle$$

Angular Power Spectra

Cl

Non-Commutativity

Example Θ

Plan Hoppf Symmetry on N.C. Spaces

Twisted Statistics

changed

Commutator relations

btw fields

$$\langle 0 | \phi(x_1) \phi(x_2) | 0 \rangle$$

$$\langle 0 | \phi \phi \phi \phi | 0 \rangle$$

Angular Power Spectra

Cl

Non-Commutativity

Anisotropy

Estimate Θ

Hopf Symmetry on N.C. Spaces

Plan

Twisted Statistics

changed

Commutator relations

btw fields

$$\langle \omega | \phi(x_1) \phi(x_2) | \omega \rangle$$

$$\langle \omega | \phi \phi \phi \phi | \omega \rangle$$

Anisotropy

Angular Power Spectra

\mathbb{C}^2

Non-Commutativity

Estimate Θ

Example ①

Von-Neumann

$$0100 \geq L^2$$
$$0x 0x \geq L^2$$

EXAMPLE θ

Von-Neumann

$$O(1) \supseteq \mathcal{L}^n$$

$$O_n \supseteq \mathcal{L}^n$$

$$[\dot{x}_n, \dot{x}_n] = i\theta_n$$

Example θ

Von-Neumann

$$O(0) \geq L^2$$

$$O_n \geq L^2$$

$$[\dot{x}_n, \dot{x}_0] = iO_n$$

$f(x)$

Example θ

Von-Neumann

$$\mathcal{O}(100) \geq \mathcal{L}^n$$

$$\mathcal{O}n \mathcal{O}n \geq \mathcal{L}^n$$

$$[\hat{x}_n, \hat{x}_0] = i\theta_n$$

$$f(x) * g(x) =$$

Example θ

Von-Neumann

$$\mathcal{O}(1) \otimes \mathcal{O}(2) \cong \mathcal{L}^2$$

$$\mathcal{O}(2) \otimes \mathcal{O}(2) \cong \mathcal{L}^2$$

$$[\hat{x}_n, \hat{x}_0] = i\theta_n$$

$$f(x) * g(x) = f(x) e^{-\frac{i}{2}\hat{x}_0^{-1}\hat{x}_n} g(x)$$

EXAMPLE ①

Von-Gurong

$$\mathcal{O}(1) \otimes \mathcal{O} \cong \mathcal{L}^1$$

$$\mathcal{O}_X \otimes \mathcal{O}_X \cong \mathcal{L}^0$$

$$[\tilde{\chi}_n, \tilde{\chi}_0] = i\mathcal{O}_n$$

$$f(x) * g(x) = f(x) e^{-\frac{i}{2} \tilde{\chi}_0^{-1} x} g(x)$$

$$\chi_{-1} * \chi_0 - \chi_0 * \chi_{-1} = i\mathcal{O}_1$$

Example ①

100h - G...ing

$$\partial_1 \partial_2 \geq \mathcal{L}^2$$
$$\partial_x \partial_x \geq \mathcal{L}^2$$

$$S[\phi] = \int d^4x \quad \partial_\mu \phi \partial_\mu \phi$$

$$+ m^2 \phi^2$$
$$+ \phi^4$$

$$[\dot{x}_\mu, \dot{x}_\nu] = i\theta_{\mu\nu}$$

$$f(x) * g(x) = f(x) e^{-\frac{i}{2} \theta^{\mu\nu} \partial_\mu \partial_\nu} g(x)$$

$$x_\mu * x_\nu - x_\nu * x_\mu = i\theta_{\mu\nu}$$

$SL(2)$ not invariant under Lorentz symmetry

$S[\phi]$ not invariant under Lorentz symmetry

$$\mathcal{D}\phi(x) = \mathcal{D}\phi(j^{-1}x)$$

$S[\phi]$ not invariant under Lorentz symmetry

$$\mathcal{G}_D \phi(x) = \phi(j^{-1}x)$$

$$\mathcal{G}_D \phi(x) * \mathcal{G}_D h(x) \neq \mathcal{G}_D (\phi + h)$$

Hopf Symmetries.

Hopf Symmetries.

$$J \otimes m = m_0 \Delta(g) (f \otimes h)$$

↙

$$\Delta(g) = \psi(G) \otimes \psi(G)$$

Hopf Symmetries.

$$\eta(f \otimes h) = m_0 \Delta(g) (f \otimes h)$$

$$\Delta(g) = \eta(G \otimes 1G)$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$g \triangleright m(f \otimes h) = \Delta(g) (f \otimes h)$$

$$(g \triangleright f) \otimes h$$

$$f \otimes (g \triangleright h)$$

$$\sigma(g) = g \otimes g$$

Hopf Symmetries.

$$g \triangleright m(f \otimes h) = m \Delta(g) (f \otimes h)$$

$$\Delta(g) = h(G) \otimes h(G)$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = f \cdot g$$

$$g \triangleright m(f \otimes h) = m_{\Delta(g)}(f \otimes h)$$

$$\Delta(g) = h(G) \otimes h(G)$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = fg$$

$$g \triangleright m_0(f \otimes h) = m_0 \Delta(g)(f \otimes h)$$

$$\Delta(g) = h(G) \otimes h(G)$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = fg$$

$$g \triangleright m_0(f \otimes h) = m_0 \Delta(g)(f \otimes h)$$

$$\Delta(g) = \eta(G) \otimes \eta(G)$$

$$\Delta(1) = 1 \otimes 1$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = fg$$

$$g \triangleright m_0(f \otimes h) = m_0 \Delta(g)(f \otimes h)$$

$$\Delta(g) = \eta(G) \otimes \eta(G)$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = L \otimes 1$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = fg$$

$$g \triangleright m_0(f \otimes h) = m_0 \Delta(g)(f \otimes h)$$

$$\Delta(g) = \eta(G) \otimes \eta(G)$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = fg$$

$$j \triangleright m_0(f \otimes h) = m_0 \Delta(g) (f \otimes h)$$

$$\Delta(g) = \eta(G) \otimes \eta(G)$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

$$\Delta(\varphi_1) \cap \Delta(\varphi_2) = \Delta(\varphi_1, \varphi_2)$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

Automorphism

$$f' \cdot g' = f \cdot g$$

$$\Delta(\varphi_1) \cap \Delta(\varphi_2) = \Delta(\varphi_1, \varphi_2)$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

As a consequence

$$f' \cdot g' = f \cdot g$$

$$\Delta(\varphi_1) \cup \Delta(\varphi_2) = \Delta(\varphi_1, \varphi_2)$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

Anticommutativity

$$f \cdot g = -g \cdot f$$

$$U_1 \otimes U_2$$

$$\Delta(f, g) \cap (g, f) = \Delta(g, f)$$

$$\Delta(L) = L \cup L^c$$

Automorphism

$$f \cdot g' = f \cdot g$$

$$g \supseteq U_1 \cup U_2 = P \cup f(\Delta) \cup (U_1 \cup U_2)$$

$$\Delta(\varphi_1) \cap \Delta(\varphi_2) = \Delta(\varphi_1 \wedge \varphi_2)$$

$$\Delta(L) = L \cup L^c$$

Automorphism

$$f \cdot g = fg$$

$$g \triangleright \varphi_1 \otimes \varphi_2 = \rho \otimes f(\Delta)(\varphi_1 \otimes \varphi_2)$$

$$\Delta(f, g) \cap (f, g) = \Delta(g, f)$$

$$\Delta(L) = L \cup L^c$$

derivation

$$f \cdot g' = f \cdot g$$

$$\Delta(U, U) = \rho \cup \int (\Delta) (U, U)$$

\downarrow
 $\rho \cup \int$

$$\Delta(g_1) \Delta(g_2) = \Delta(g_1 g_2)$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

Automorphisms

$$\Delta(g) = g \otimes g$$

$$f \cdot g = f \otimes g$$

$$g \triangleright u_1 \otimes u_2 = \rho \otimes f \left(\begin{array}{c} \Delta \\ \downarrow \\ g \otimes g \end{array} \right) (u_1 \otimes u_2)$$

Hopf Algebra

Coalgebra

$$H = (G, \alpha)$$

$$\Delta(\varphi_1) \cup (\varphi_2) = \Delta(\varphi_1, \varphi_2)$$

$$\Delta(L) = |L| + L \cup 1$$

$$\begin{array}{ccc}
 \alpha \otimes \beta & \xrightarrow{\Delta} & 1 \otimes \beta \cup (\alpha \otimes 1) \\
 \downarrow m_0 & & \downarrow m_0 \\
 \alpha \otimes \beta & \xrightarrow{\quad} & \rho(\varphi) m_0 \otimes \varphi
 \end{array}$$



$$\Delta(\varphi_1) \cup (\varphi_2) = \Delta(\varphi_1, \varphi_2)$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\alpha \otimes \beta \xrightarrow{\Delta} 1 \otimes \beta \cup (\alpha \otimes 1)$$

$$\downarrow F \qquad \qquad \qquad \downarrow m_0$$

$$m_0 \alpha \otimes \beta \longrightarrow \rho(\varphi) m_0 \tau(\varphi)$$

$$= m_0 F = e^{i \ell \cdot \mathbb{P} \otimes \mathbb{P}}$$

$$\Delta(\varphi_1) \cup (\varphi_2) = \Delta(\varphi_1, \varphi_2)$$

$$\Delta(L) = L \oplus L \oplus 1$$

$$\begin{array}{ccc} \mathbb{R} \oplus \mathbb{R} \xrightarrow{\Delta} \mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R} \\ \downarrow m_0 \quad \quad \quad \downarrow m_0 \\ \mathbb{R} \oplus \mathbb{R} \longrightarrow \mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R} \end{array}$$

$$m_0 = m_0 \cdot F = m_0 \cdot e^{i \cdot \mathbb{R} \oplus \mathbb{R}}$$

Example 0

$$10a \geq L^1$$
$$2x \leq x \geq L^1$$

$$S[\phi] = \int d^4x \quad 2\psi \times 2\psi$$

$$+ m\psi - \psi$$
$$+ \psi - \psi - \psi$$

$$[\dot{x}_r, \dot{x}_l] = iQ_r$$

$$= f(x) e^{i\tilde{\alpha} \cdot \tilde{\alpha}} g(x)$$

Invariant under Lorentz symmetry

$$\phi(g'x)$$

$$+ g \triangleright h(x) \neq g \triangleright \phi + h(x)$$

$$\partial_1 \phi \geq \mathcal{L}^1$$

$$\partial_2 \phi \geq \mathcal{L}^1$$

$$[\dot{x}_r, \dot{x}_s] = i\theta_{rs}$$

$$f^{(n)} * g^{(n)} = f^{(n)} e^{+i\theta_{rs}} g^{(n)}$$

$$S[\phi] = \int d^4x \quad 2\phi * 2\phi$$

$$+ m\phi - \phi$$

$$+ \phi - \phi - \phi$$

$S[\phi]$ not invariant under Lorentz symmetry

$$g \triangleright \phi^{(n)} = \phi(g'x)$$

$$g \triangleright \phi^{(n)} * g \triangleright h^{(n)} \neq g \triangleright \phi + h$$

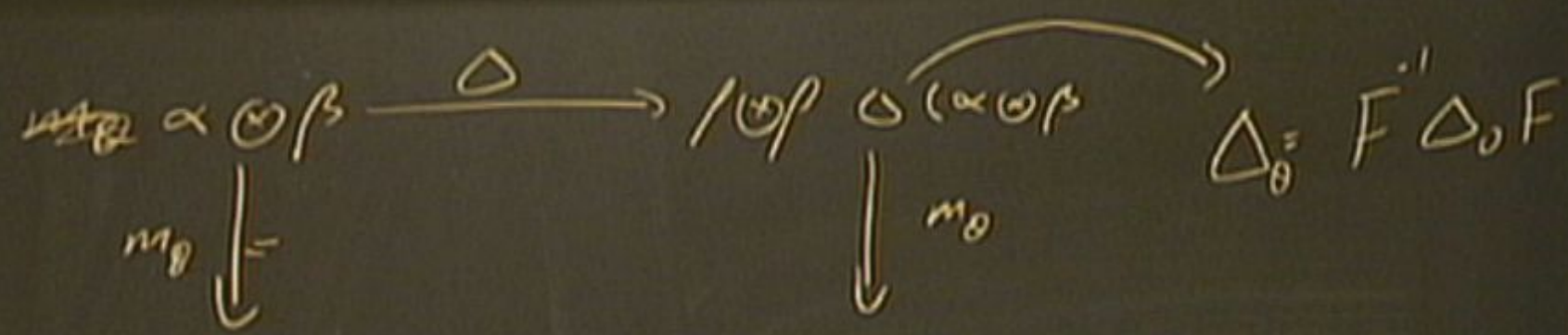
$$(\cdot, \cdot) = \langle \cdot, \cdot \rangle$$

$$\begin{array}{ccc}
 \mathbb{R}^n \times \mathbb{R}^n & \xrightarrow{\Delta} & \mathbb{R}^n \times \mathbb{R}^n \\
 \downarrow m_0 & & \downarrow m_0 \\
 \mathbb{R}^n & & \mathbb{R}^n
 \end{array}
 \quad \Delta = F^{-1} \Delta_0 F$$

$$\begin{array}{ccc}
 m_0 \times \mathbb{R}^n & \longrightarrow & \rho(g) m_0 \times \mathbb{R}^n \\
 & & \downarrow \\
 & & U(1)
 \end{array}$$

$m_0 = m_0 \cdot F = \dots$

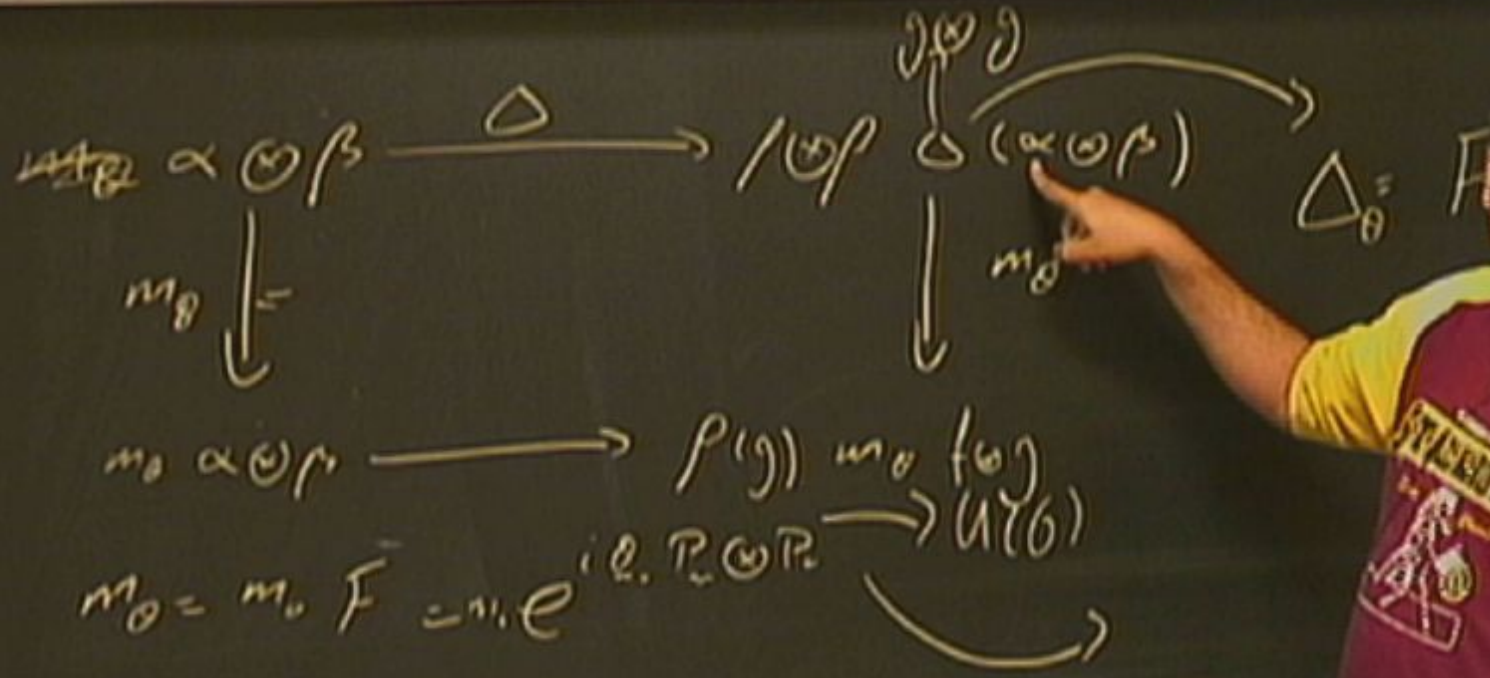
$$C(x, y, z) = C(y, z, x)$$



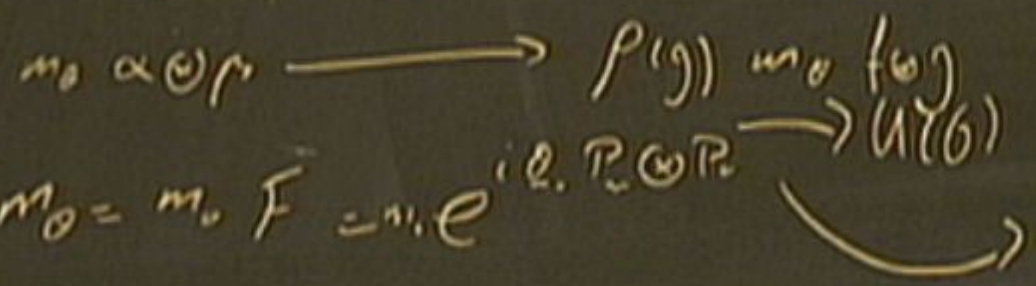
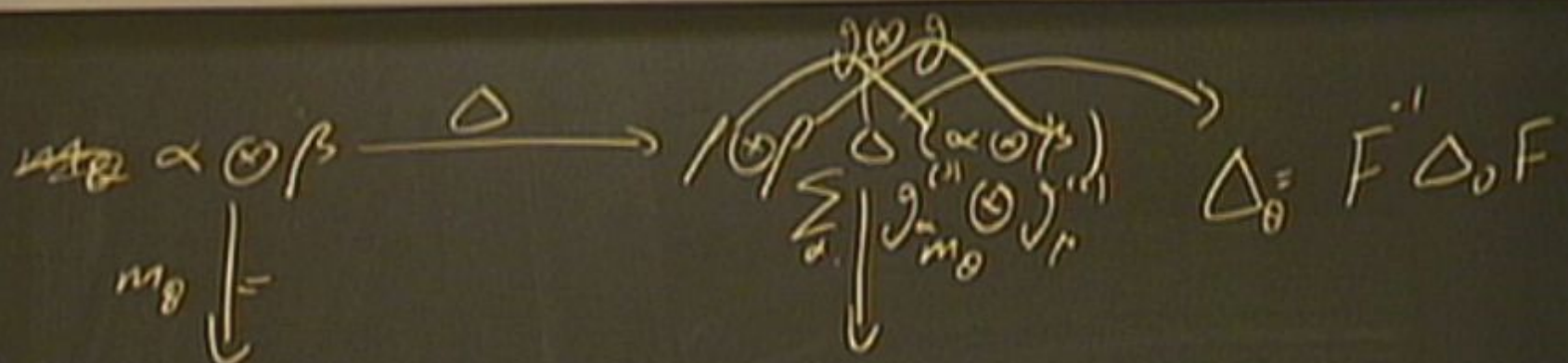
$$m_0 = m_0 \cdot F = \dots e^{i \cdot \rho \cdot \omega} \rightarrow U(0)$$



$$(\cdot, \cdot) = \langle \cdot, \cdot \rangle$$

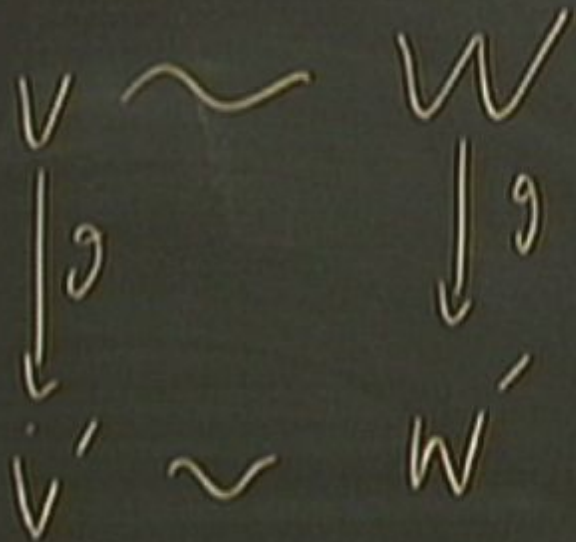


$$C(x, y) = C(y, x)$$



$$|k_1, k_2\rangle = \alpha \langle k_1, k_2 | \dots$$

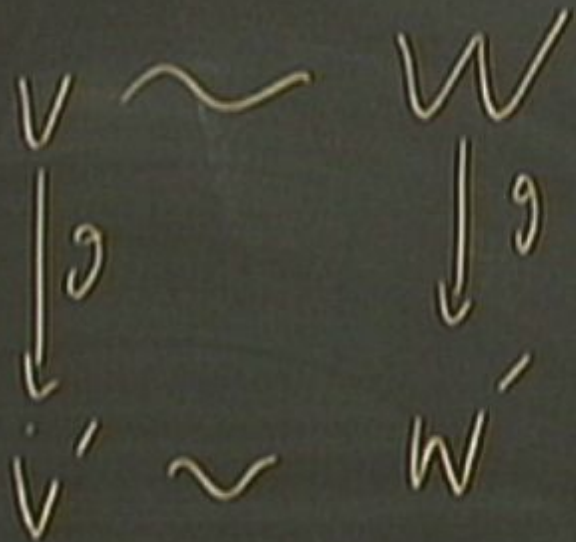
$$\sigma |k_1, k_2\rangle = F |k_1, k_2\rangle$$



$$|k_1, k_2\rangle = \alpha \langle k_1, k_2 | \dots$$

$$\sigma |k_1, k_2\rangle = F |k_1, k_2\rangle$$

$$e^{i2\pi k_1 k_2} |k_1, k_2\rangle$$



$$U(g) |k_1, k_2\rangle = P \otimes P (U(g) |k_1, k_2\rangle)$$

$$1/19) |k_1, k_2\rangle = P \otimes P (g) |k_1, k_2\rangle$$



$$U(g) |k_1, k_2\rangle = P \otimes P (g) |k_1, k_2\rangle$$

$$a_n^\dagger a_p - e^{i\theta} a_p^\dagger a_n$$

$$U(g) |k_1, k_2\rangle = \rho(g) U(g) |k_1, k_2\rangle$$

$$a_{k_1}^{\dagger} a_{k_2} = e^{i k_1 x} a_{k_2}^{\dagger} a_{k_1}$$

$$\phi_0(x) = \phi_0(x) e^{-\frac{i}{\hbar} \partial \wedge P}$$

$$U(g) |k_1, k_2\rangle = \rho(g) U(g) |k_1, k_2\rangle$$

$$a_{\mu} a_{\nu} = e^{i\mu\nu} a_{\nu} a_{\mu}$$

$$\phi_0(x) = \phi_0(x) e^{-\frac{i}{\hbar} \partial \wedge P}$$

$$F = f_1 \otimes f_2$$

$$\langle 0 | \phi(x) | 0 \rangle = \frac{1}{\sqrt{2}} \langle 0 | \phi(x) \phi(x) | 0 \rangle$$

$$\langle 0 | \phi(x) \phi(x) | 0 \rangle = e^{-\frac{i}{2} \partial_x \Delta \partial_x} \langle 0 | \phi(x) \phi(x) | 0 \rangle$$



$$\langle 0 | \phi_0(x) \phi_0(x') | 0 \rangle = e^{-\frac{i}{2} \partial_x \Delta \partial_x} \langle 0 | \phi_0(x) \phi_0(x') | 0 \rangle$$

$$\phi_0(x) = \int \frac{d^4 k}{(2\pi)^4} \tilde{\phi}_0(k, x) e^{i k \cdot x}$$

$$\langle 0 | \psi_0(x) \psi_0(x) \rangle = e^{-\frac{i}{\hbar} \partial_x \wedge \partial_x} \langle 0 | \psi_0(x) \psi_0(x) | 0 \rangle$$

$$\psi_0(x) = (k, t) e^{i k x}$$

$$|0\rangle = \langle 0 | \psi_0(k, t, \frac{\vec{\theta} \cdot \vec{k}_i}{\hbar}) \psi_0(k, t, \frac{\vec{\theta} \cdot \vec{k}_i}{\hbar}) | 0 \rangle$$

$$\langle 0 | \phi_0(x) \phi_0(x) | 0 \rangle = e^{-\frac{i}{2} \partial_x \wedge \partial_x} \langle 0 | \phi_0(x) \phi_0(x) | 0 \rangle$$

$$\phi_0(x) = \int \frac{d^4 k}{(2\pi)^4} \tilde{\phi}_0(k, x) e^{i k \cdot x}$$

$$\langle 0 | \phi_0(\bar{k}_1, t_1) \phi_0(\bar{k}_2, t_2) | 0 \rangle = \langle 0 | \phi_0(\bar{k}_1, t_1 - \frac{\vec{0} \cdot \bar{k}_1}{z}) \phi_0(\bar{k}_2, t_2 - \frac{\vec{0} \cdot \bar{k}_2}{z}) | 0 \rangle$$

$$\langle 0 | \phi_0(x) \phi_0(x) | 0 \rangle = e^{-\frac{i}{2} \partial_x \Delta \partial_x} \langle 0 | \phi_0(x) \phi_0(x) | 0 \rangle$$

$$\phi_0(x) = \int \frac{d^4 k}{(2\pi)^4} \tilde{\phi}_0(k, x) e^{i k \cdot x}$$

$$\langle 0 | \phi_0(\bar{h}, t) \phi_0(\bar{h}, t) | 0 \rangle = \langle 0 | \phi_0(\bar{h}, t, -\frac{\vec{\partial} \cdot \bar{h}}{2}) \phi_0(\bar{h}, t, \frac{\vec{\partial} \cdot \bar{h}}{2}) | 0 \rangle$$

$$\langle 0 | \phi_0(x) \phi_0(x) | 0 \rangle = e^{-\frac{1}{2} \partial_x^2 \Delta \partial_x} \langle 0 | \phi_0(x) \phi_0(x) | 0 \rangle$$

$$\phi_0(x) = \int \frac{d^4 k}{(2\pi)^4} \tilde{\phi}_0(k, x) e^{i k \cdot x}$$

$$\langle 0 | \phi_0(\bar{h}, t) \phi_0(\bar{h}_2, t) | 0 \rangle = \langle 0 | \phi_0(\bar{h}_1, t_1 - \frac{\vec{0} \cdot \bar{h}_1}{z}) \phi_0(\bar{h}_2, t_2 - \frac{\vec{0} \cdot \bar{h}_2}{z}) | 0 \rangle$$

$$\langle 0 | \phi_0(x_1) \phi_0(x_2) | 0 \rangle = e^{-\frac{i}{2} \partial_{x_1} \Delta \partial_{x_2}} \langle 0 | \phi_0(x_1) \phi_0(x_2) | 0 \rangle$$

$$\phi_0(x) = \int \frac{d^4 k}{(2\pi)^4} \tilde{\phi}_0(k, t) e^{i k \cdot x}$$

$$\langle 0 | \phi_0(\bar{x}_1, t_1) \phi_0(\bar{x}_2, t_2) | 0 \rangle = \langle 0 | \phi_0(\bar{x}_1, t_1 - \frac{\vec{\partial} \cdot \vec{k}_1}{2}) \phi_0(\bar{x}_2, t_2 - \frac{\vec{\partial} \cdot \vec{k}_2}{2}) | 0 \rangle$$

$x_0, x_1 \quad f(x)$

$$\begin{array}{ccc}
 \alpha \otimes \beta & \xrightarrow{\Delta} & \mathbb{1} \oplus \rho \oplus (\alpha \otimes \beta) \\
 m_0 \downarrow = & & \downarrow \sum_{\sigma} J_{m_0}^{\sigma} \\
 m_0 \otimes \rho & \longrightarrow & \rho \oplus m_0 \otimes \rho \\
 & & \downarrow \text{i.e. } \mathbb{P}_L \otimes \mathbb{P}_R \\
 m_0 = m_0 \cdot F = m_0 \cdot e & & \downarrow \text{U(6)}
 \end{array}$$

$$\Delta_0 = F^{-1} \Delta_0 F$$

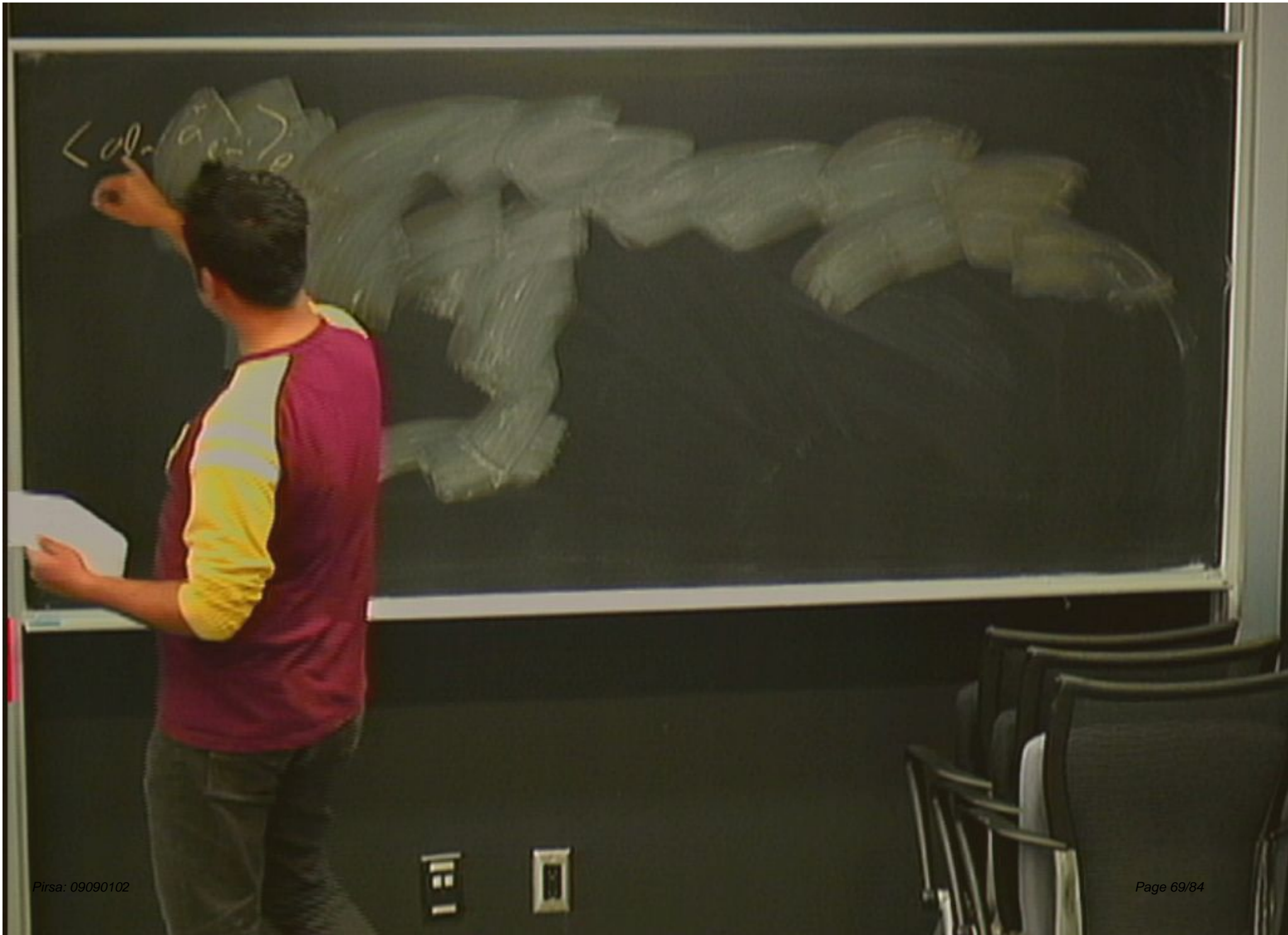
$$P_F = P_0(h) \cosh(H \vec{0} \cdot h)$$

$$\begin{array}{ccc}
 \alpha \otimes \beta & \xrightarrow{\Delta} & 1 \otimes \rho \otimes (\alpha \otimes \beta) \\
 m_0 \downarrow = & & \downarrow \sum_{\sigma} J_{\sigma} m_0 J_{\sigma}^{\dagger} \\
 m_0 \alpha \otimes \beta & \longrightarrow & \rho(g) m_0 f(g)
 \end{array}$$

$$\Delta_0 = F^{-1} \Delta_0 F$$

$$P_F = P_0(h) \cosh(H \vec{0} \cdot h)$$

$$\begin{array}{ccc}
 m_0 = m_0 F = m_0 e^{i \mathcal{L} \cdot \vec{P} \otimes \vec{P}} & \longrightarrow & U(6)
 \end{array}$$



$\langle \text{data} \rangle$

A man with dark hair, wearing a maroon t-shirt with yellow sleeves, stands in profile facing left. He is looking down at a white piece of paper he is holding with both hands. His right hand is near his chin, suggesting he is in deep thought. Behind him is a large black chalkboard. The top left corner of the chalkboard has the handwritten text " $\langle \text{data} \rangle$ ". The rest of the chalkboard is covered in a large, thick, white chalk scribble that obscures any other writing. In the foreground, the backs of several black chairs are visible, indicating a classroom or lecture hall setting. The lighting is somewhat dim, typical of an indoor classroom.

$$\Delta(y, z) = \Delta(y, z)$$

$$\begin{array}{ccc}
 \alpha \otimes \beta & \xrightarrow{\Delta} & \sum_a (\alpha \otimes \beta)_a \\
 \downarrow m_B & & \downarrow \sum_a m_a \\
 m_0 & & m_0
 \end{array}$$

$$\Delta_0 = F^{-1} \Delta_0 F$$

$$\begin{array}{ccc}
 m_0 \otimes \beta & \longrightarrow & P(g) m_0 \otimes \beta \\
 m_0 = m_0 F = \dots & \xrightarrow{i.e. P_1 \otimes P_2} & U(0)
 \end{array}$$

$$P_F = P_0(\hbar) \exp(\hbar \vec{H} \cdot \vec{h})$$

$$\Delta(y_i) = \Delta(y, y_i)$$

$\Delta \rightarrow$ $\left(\sum_a \left| \psi_{m_0}^{(a)} \right\rangle \langle \psi_{m_0}^{(a)} \right)$ $\rightarrow \Delta_0 = F^{-1} \Delta_0 F$

$P(g) \rightarrow U(g)$
 i.e. $P \in \mathcal{P} \rightarrow U(g) \in \mathcal{U}(g)$

$P_F = P_0(h) \cosh(h \vec{H} \cdot \vec{h})$
 $\vec{h} = (h^1, h^2, h^3)$

$$\Delta(j, j_1) = \Delta(j, j_2)$$

$$m_0 \propto \alpha \otimes \beta \xrightarrow{\Delta} \left(\sum_a \left| \begin{matrix} j_1 & j_2 \\ m_1 & m_2 \end{matrix} \right\rangle \left\langle \begin{matrix} j_1 & j_2 \\ m_1 & m_2 \end{matrix} \right| \right) \xrightarrow{\Delta_0} F^{-1} \Delta_0 F$$

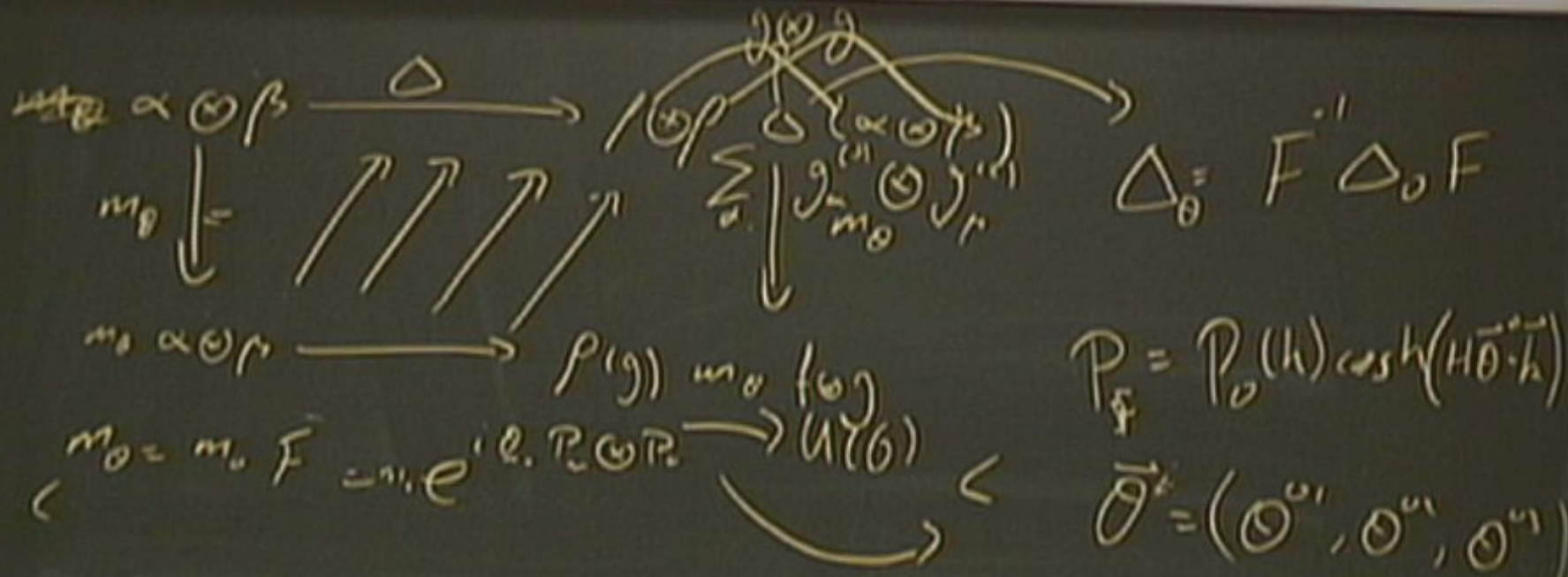
$$m_0 \propto \alpha \otimes \beta \xrightarrow{P(g)} m_0 \text{ for } U(6)$$

$$m_0 = m_0 F = m_0 e^{i \ell \cdot P \otimes P} \xrightarrow{U(6)}$$

$$P_F = P_0(h) \cosh(H \vec{0} \cdot \vec{h})$$

$$\vec{0} = (0^{01}, 0^{02}, 0^{03})$$

$$\Delta(y, z) = \Delta(y, z)$$



$$\langle a_n, a_n \rangle$$

$$\begin{pmatrix} 1 & 1 \\ \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$$

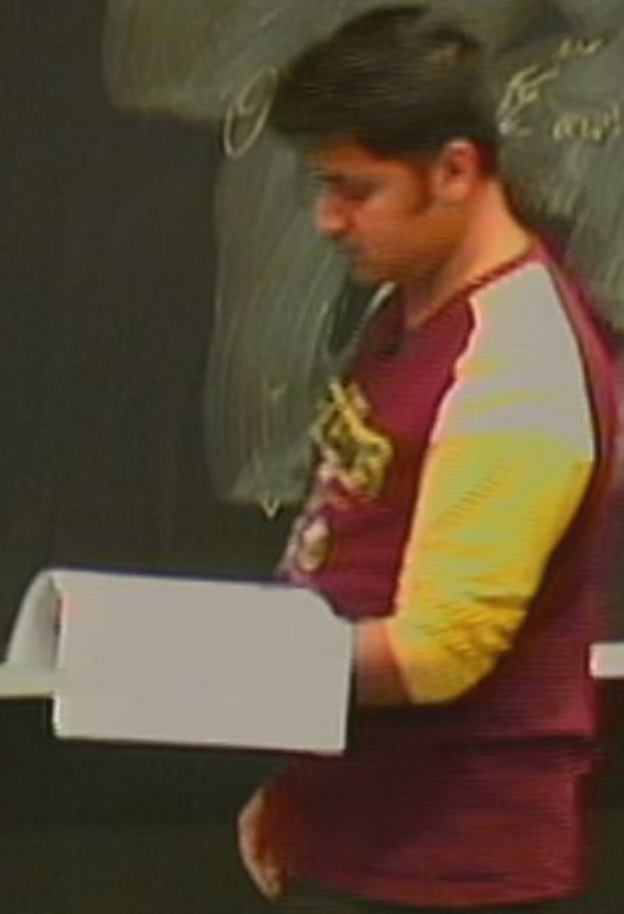
$$D = a \epsilon_{xy} f_{xy} + b \epsilon_{xy} c_{xy} c_{xy}$$



$$\langle a_n | a_m \rangle$$

$$\begin{pmatrix} 1 & & \\ & 1 & \\ & & \ddots \end{pmatrix}$$

$$f_n = f_{n-1} + b e_{n-1} e_n e_{n+1}$$



$\langle a_n | a_m \rangle$

$$\begin{pmatrix} 1 & & \\ & 1 & \\ & & \ddots \\ & & & 1 \end{pmatrix}$$

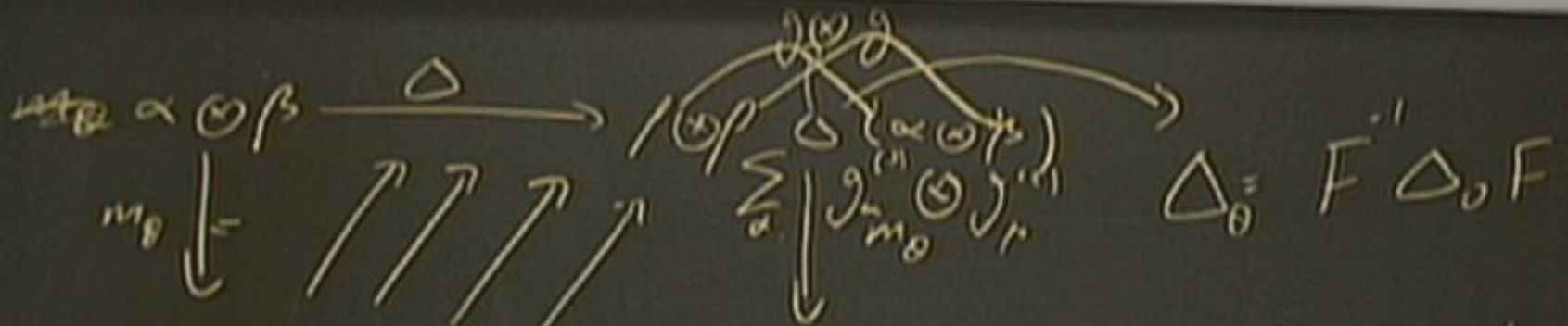
$$D^{-1} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}k^2} e^{ikx} dk + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}k^2} e^{-ikx} dk$$

$$\langle \mathcal{O}_n | \mathcal{O}_n \rangle$$

$$\begin{pmatrix} 1 & & \\ & 1 & \\ & & \ddots \end{pmatrix}$$

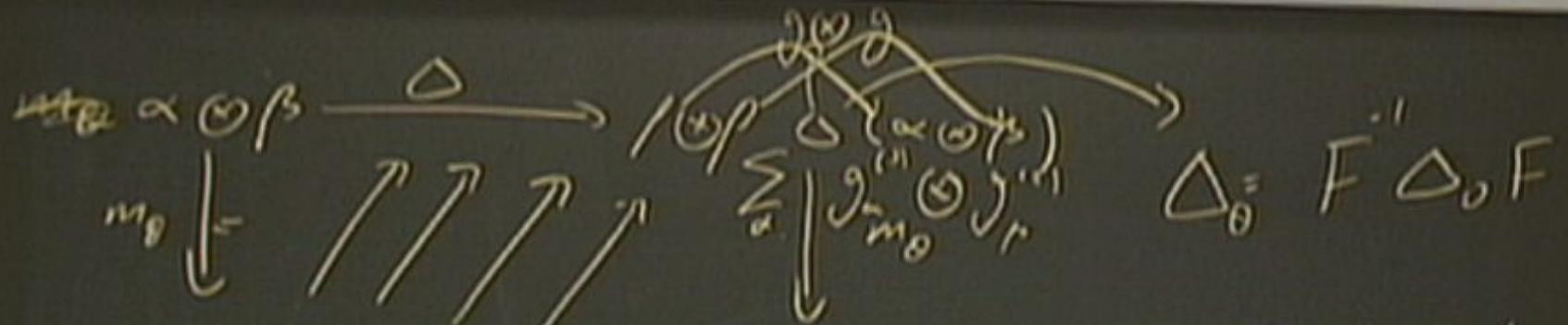
$$\mathcal{O}_n = \sum_{m=0}^n \binom{n}{m} f_{-m} f_+^m + \dots + \binom{n}{m} c_{-m} c_+^m$$





$$\begin{aligned}
 & m_0 \propto \alpha \otimes \beta \longrightarrow P(\gamma) \quad m_0 \text{ (to } \gamma) \\
 & m_0 = m_0 \quad F = m_0 e^{i \mathcal{L} \cdot P \otimes P} \longrightarrow U(6)
 \end{aligned}$$

$$\begin{aligned}
 P &= P_0(h) \cosh(H \vec{0} \cdot \vec{h}) \\
 P_F &= P_0(\dots) \left[1 + \frac{H^2 (\vec{0} \cdot \vec{h})^2}{2} \right]
 \end{aligned}$$



$m_0 \propto \alpha \otimes \beta \longrightarrow \rho(j) \otimes m_0 \otimes \rho(j)$
 $m_0 = m_0 \cdot F = \dots \in \mathbb{R} \otimes \mathbb{R} \longrightarrow \text{UT}(6)$

$$P = P_0(h) \cosh(H \vec{0} \cdot \vec{h})$$

$$= P_0(\dots) \left[1 + \frac{1}{2} (\vec{0} \cdot \vec{h})^2 \right]$$

$\langle \mathcal{O}_n \rangle$

$\begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}$

$$\mathcal{O} = \dots + b \mathcal{E}_{n_1} \mathcal{E}_{n_2} \mathcal{E}_{n_3}$$

↓ ↓ ↓
 $\sqrt{\mathcal{O}} \geq 10 \text{ GeV}$

$\langle a_{\mu} a_{\nu} \rangle_0$

$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$

$$\theta^{-\nu} = a \epsilon_{\mu\nu}^{\alpha\beta} f_{\alpha}^{\mu} f_{\beta}^{\nu} + b \epsilon_{\mu\nu}^{\alpha\beta} c_{\alpha}^{\mu} c_{\beta}^{\nu}$$

$$a H \tilde{\varphi}^{(\mu)} = \sqrt{4\pi G} \xi_{\mu}^{\nu}$$

$$\sqrt{\theta} \geq \underline{10 \text{ GeV}}$$

$\langle a_{\nu} a_{\nu}^{\dagger} \rangle_0$

$\begin{pmatrix} * & & \\ & 1 & \\ & & \cdot \end{pmatrix}$

$\theta^{-1} = a \sum_{\nu} \epsilon_{\nu}^{\mu} f_{\nu}^{\mu} + b \epsilon_{\nu}^{\mu} c_{\nu}^{\mu} c_{\nu}^{\mu}$

$a H \tilde{\varphi}^{(1)} = \sqrt{4\pi G} \xi$

$\sqrt{\theta} \geq \underline{10 \text{ GeV}}$