Title: Noncomutative Effects in Primordial Density Fluctuations

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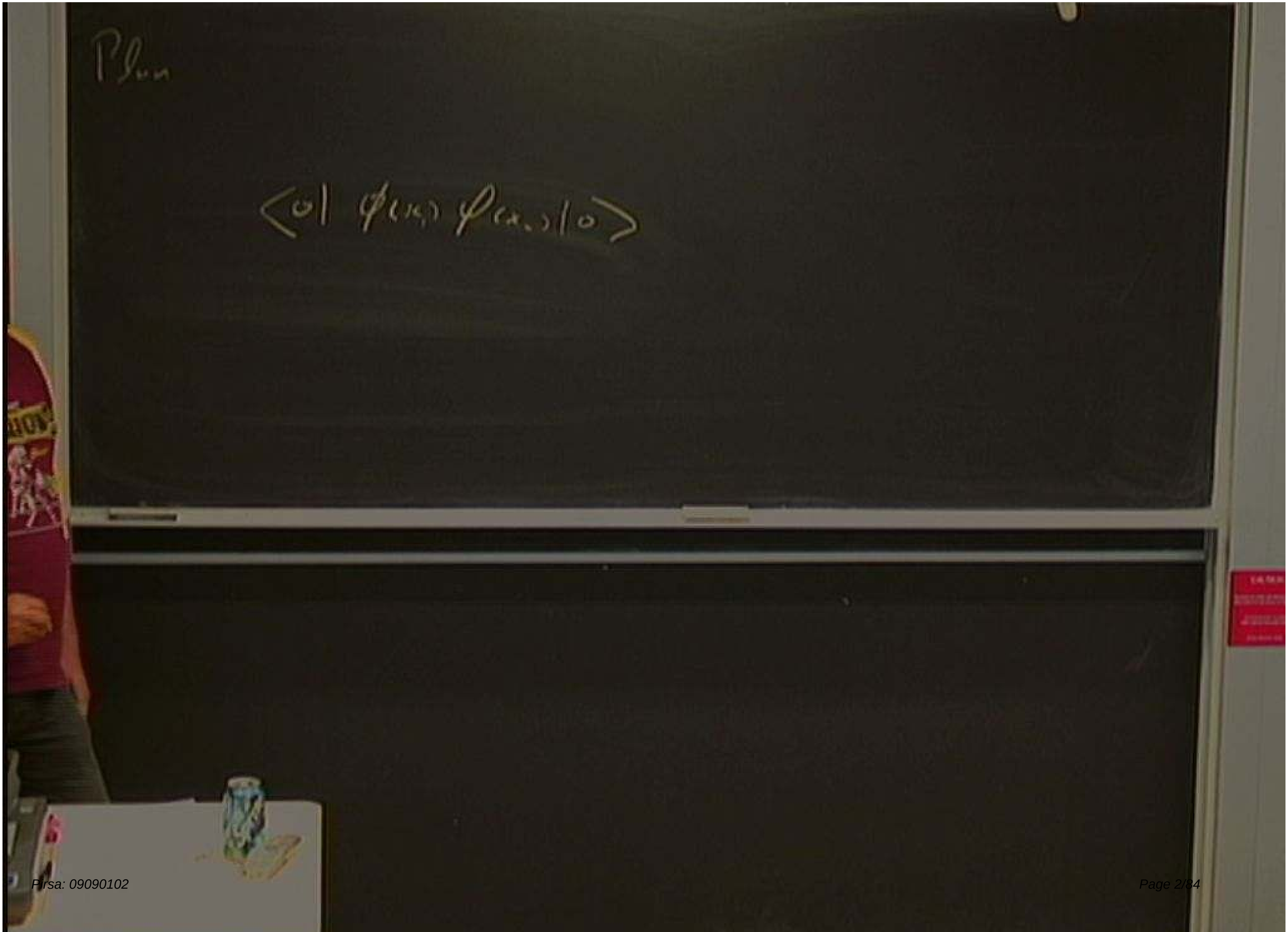
Abstract: Near the Planckian scales, quantum gravity is expected to drastically change the structure of spacetime, one feature of which may be noncomutativity of the coordinates. Based on the recent advances in quantum field theories on such noncommutative spaces, I will consider the fluctuations of inflaton and look for possible noncommutative corrections in the CMB. Anisotropy and non-gaussianity are the result. The resultant distribution is then compared with ACBAR, CBI and WMAP data to constrain the scale of noncommutativity parameter.


Plan
$\langle 0|\, \phi(x_1)\, \phi(x_2)\,|0\rangle$

Plan

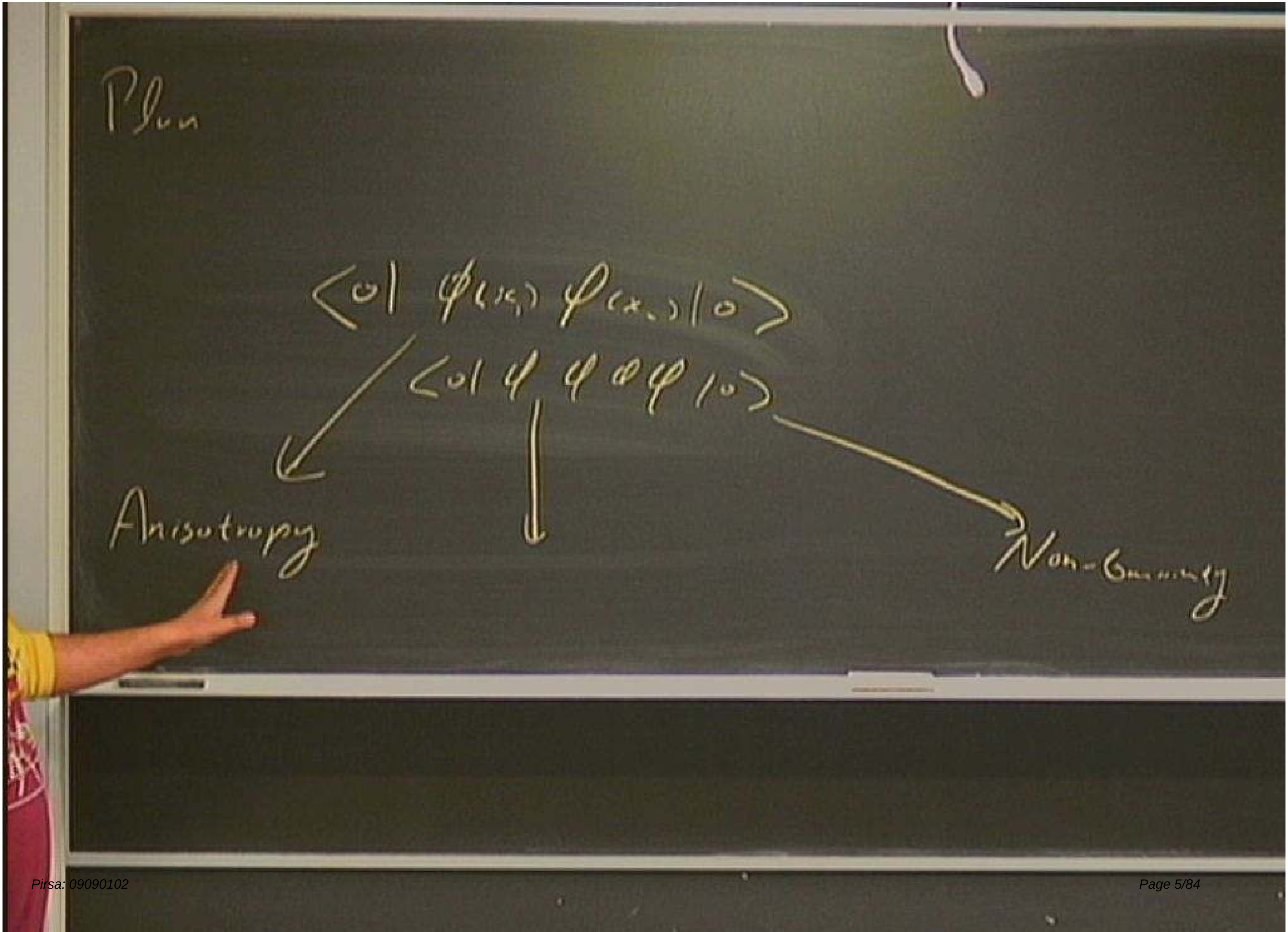
$$\langle 0 | \phi(x_1) \phi(x_2) | 0 \rangle$$

Plan

$$\langle 0 | \phi(x_1)\, \phi(x_2) | 0 \rangle$$

$$\langle 0 | \phi\ \phi\ \phi\ \phi | 0 \rangle$$

Anisotropy


Plan
$\langle 0| \phi(x_1) \phi(x_2) |0\rangle$
$\langle 0| \phi \phi \phi \phi |0\rangle$
Anisotropy
Non-Gaussianity

Plan

$$\langle 0 | \phi(x_1)\, \phi(x_2) | 0 \rangle$$

$$\langle 0 | \phi\, \phi\, \phi\, \phi | 0 \rangle$$

Anisotropy

Angular Power Spectrum

C_ℓ

Non-Gaussianity

Plan

$$\langle 0|\, \phi(x_1)\, \phi(x_2) \,|0\rangle$$

$$\langle 0|\, \phi\, \phi\, \phi\, \phi \,|0\rangle$$

Anisotropy

Angular Power Spectrum

C_ℓ

Non-Gaussianity

Estimate Θ

Plan

$$\langle 0| \, \phi(x_1)\, \phi(x_2) \,|0\rangle$$

$$\langle 0| \, \phi \, \phi \, \phi \, \phi \,|0\rangle$$

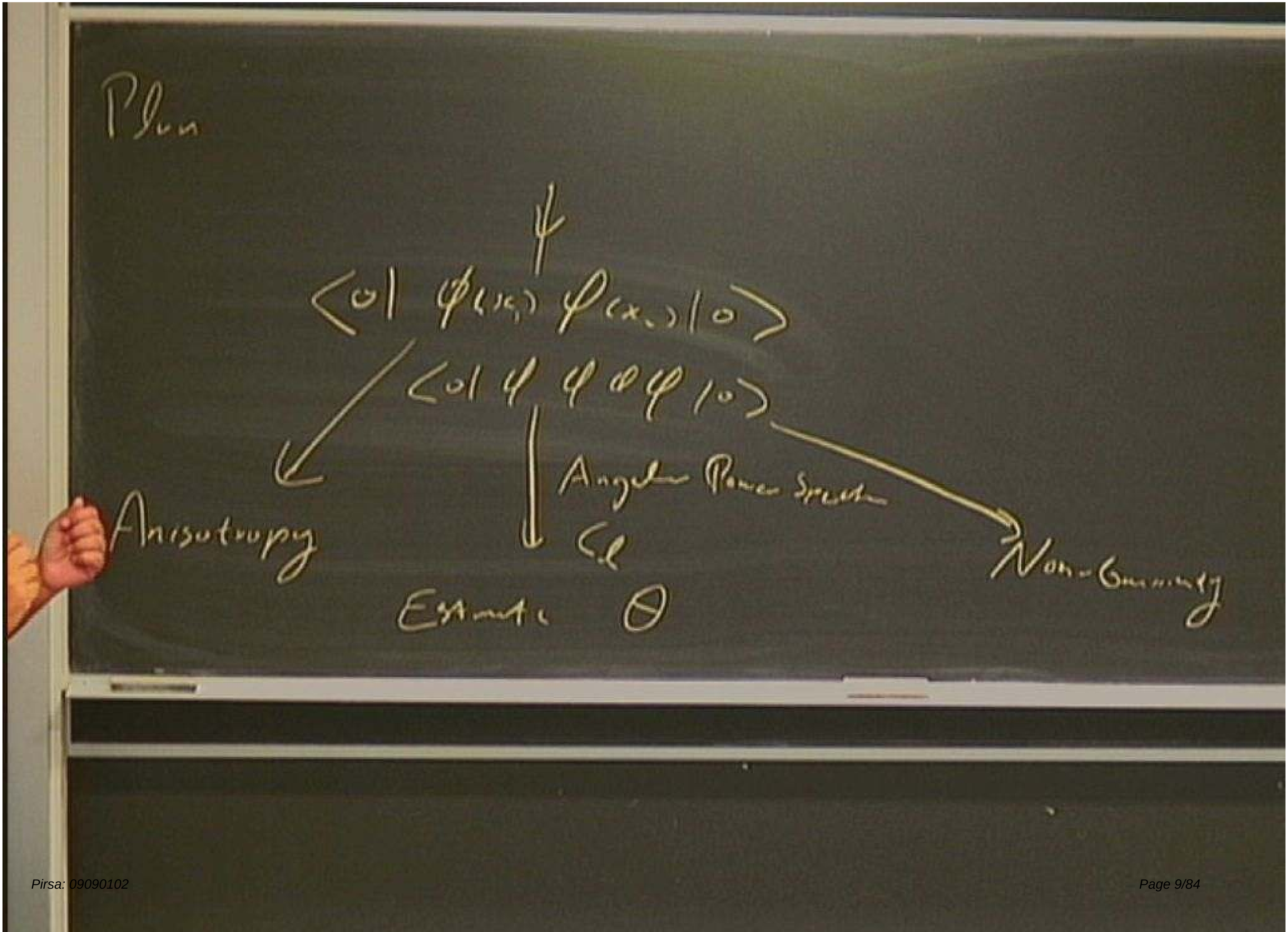
Anisotropy

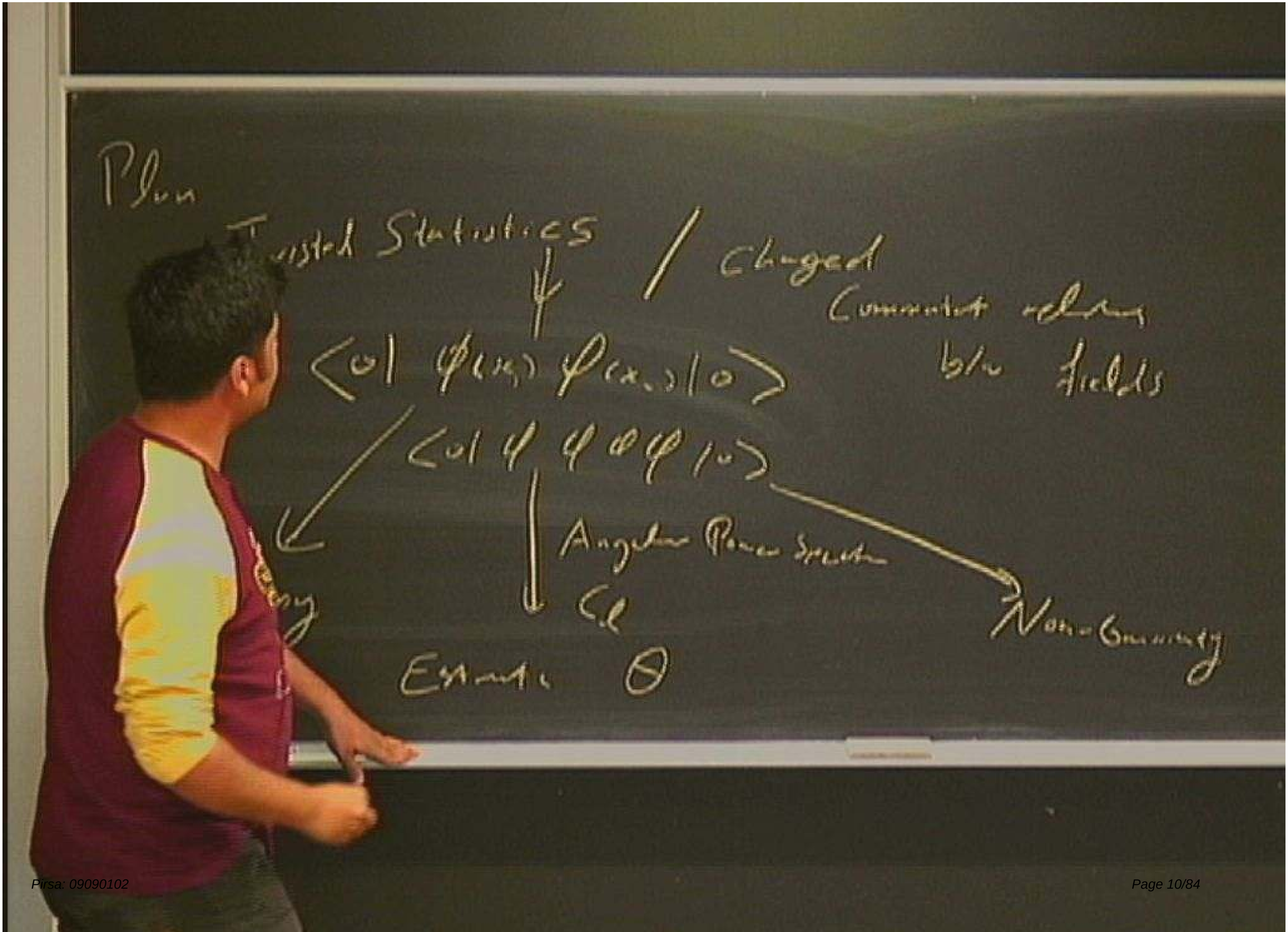
Angular Power Spectrum

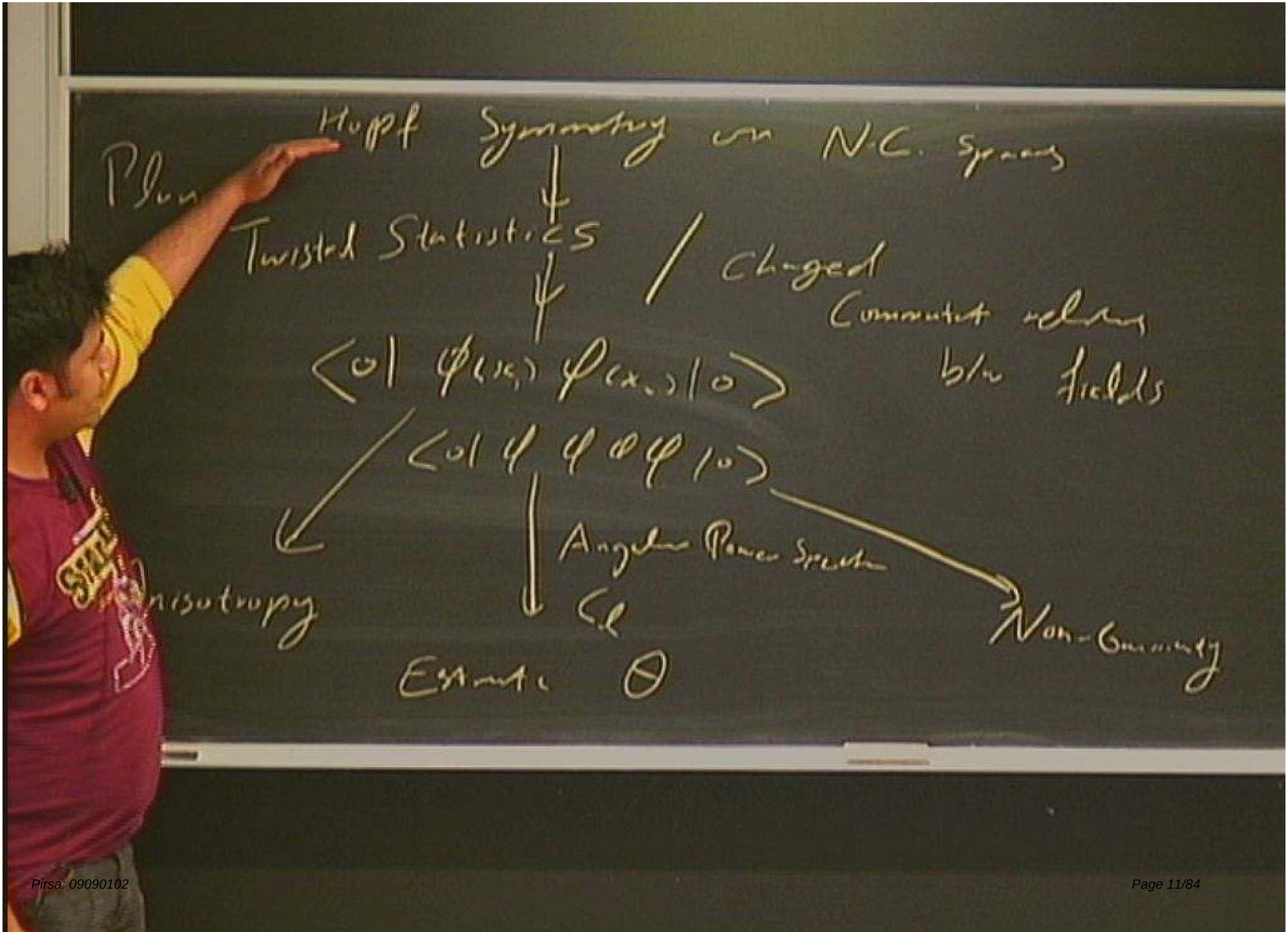
C_ℓ

Estimate Θ

Non-Gaussianity


Plan
ψ
$\langle 0|\,\phi(x_1)\,\phi(x_2)\,|0\rangle$
$\langle 0|\,\phi\,\phi\,\phi\,\phi\,|0\rangle$
Anisotropy
Angular Power Spectrum
C_ℓ
Non-Gaussianity
Estimate Θ


Plan
Twisted Statistics / Charged
Commutation relations b/w fields
$\langle 0| \phi(x_1) \phi(x_2) |0\rangle$
$\langle 0| \phi \phi \phi \phi |0\rangle$
Angular Power Spectrum
C_l
Non-Gaussianity
Estimate Θ


Plan
Hopf Symmetry on N.C. Spaces
Twisted Statistics
Changed Commutation relations b/w fields
$\langle 0| \phi(x_1) \phi(x_2) |0\rangle$
$\langle 0| \phi \phi \phi \phi |0\rangle$
Angular Power Spectrum
C_ℓ
Non-Gaussianity
Estimate Θ

Plan

Hopf Symmetry on N.C. Spaces

Twisted Statistics / Changed Commutation relations b/w fields

$\langle 0|\, \phi(x_1)\, \phi(x_2)\, |0\rangle$

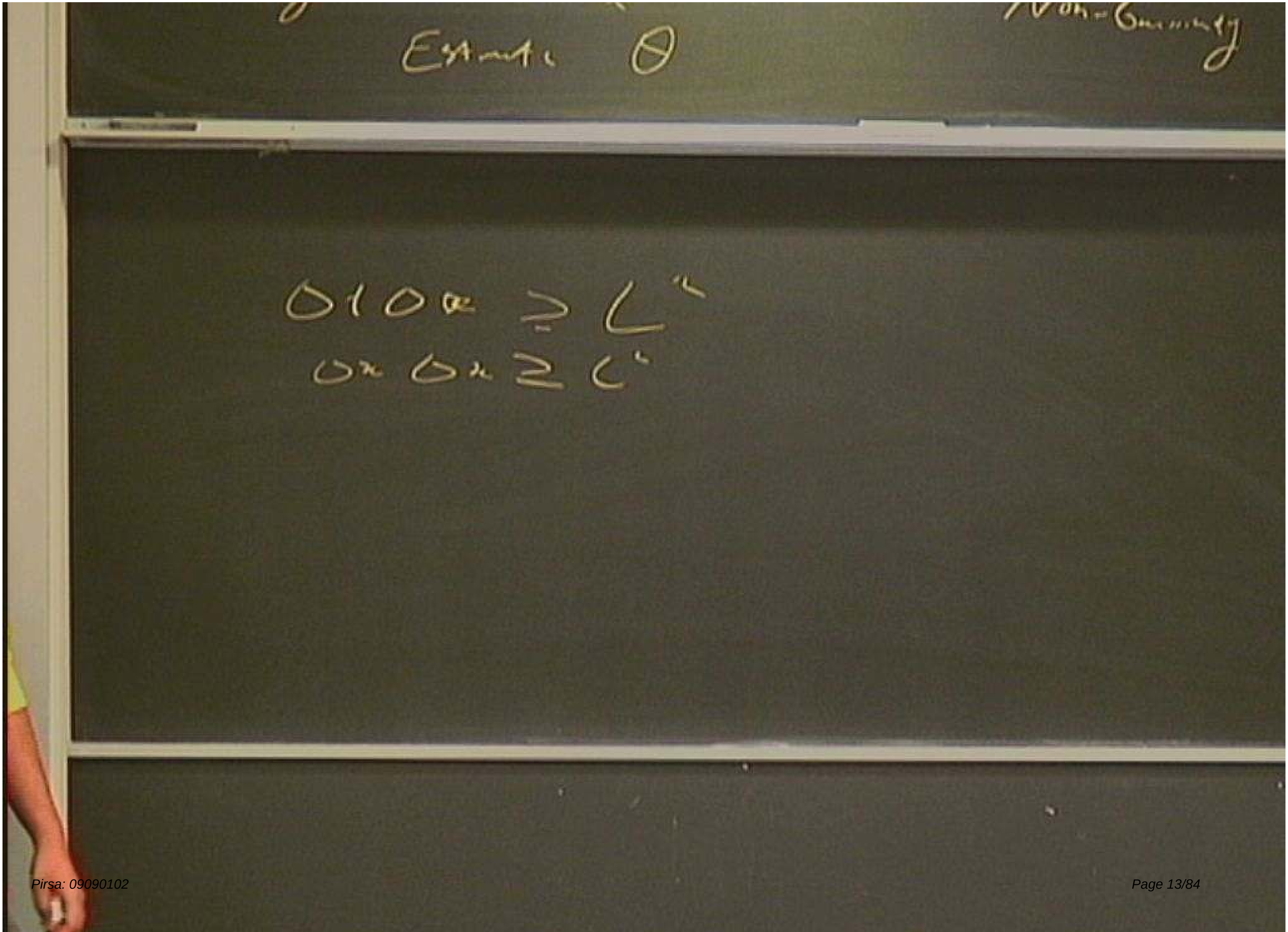
$\langle 0|\, \phi\, \phi\, \phi\, \phi\, |0\rangle$

Anisotropy

Angular Power Spectra C_ℓ

Estimate Θ

Non-Gaussianity


Estimate θ

Estimate Θ

Non-Geometry

$$\Delta t \, \Delta x \geq L^2$$

$$\Delta x \, \Delta x \geq L^2$$

$$[\hat{x}_\mu, \hat{x}_\nu] = i\Theta_{\mu\nu}$$

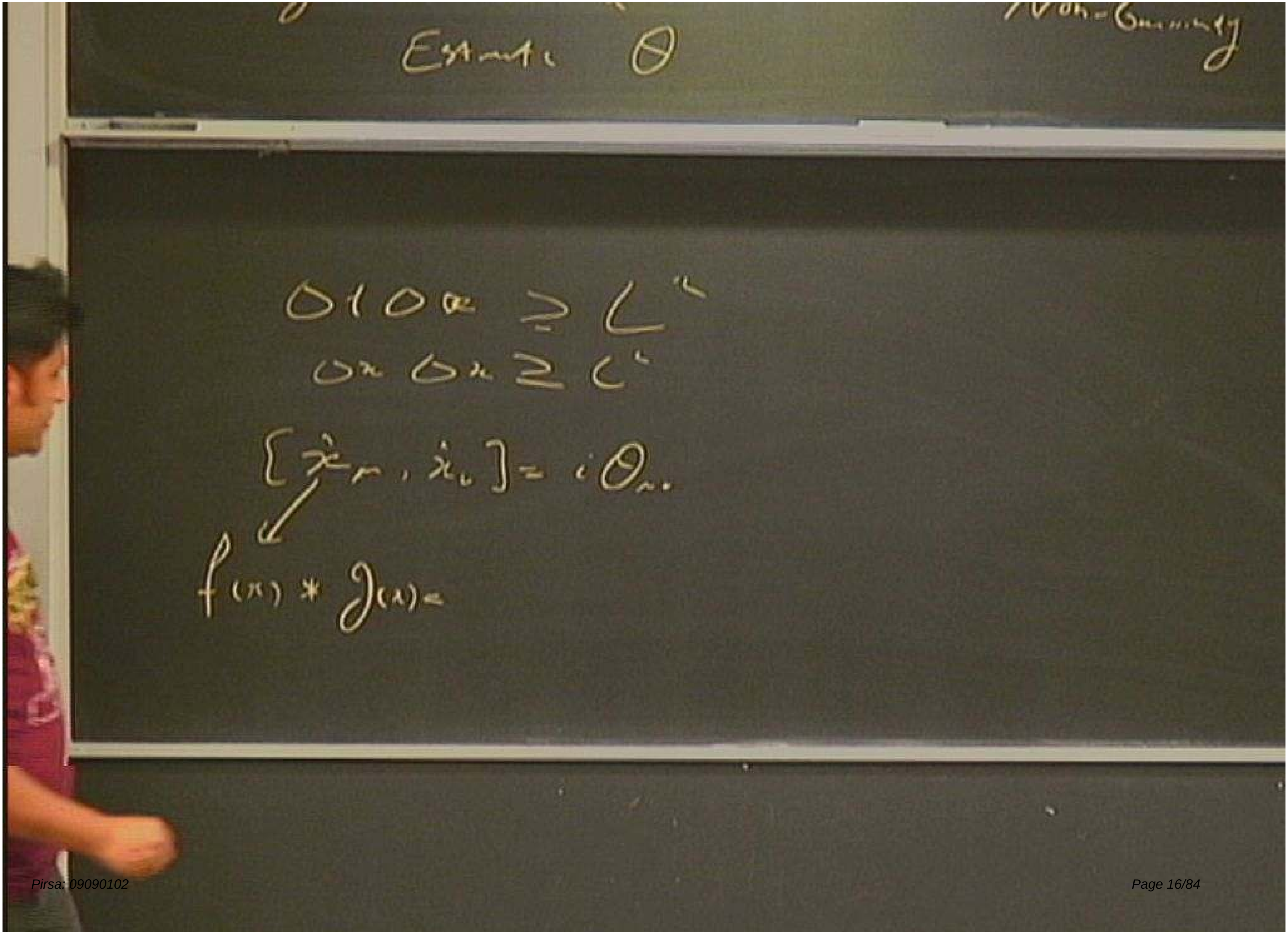
Estimate θ Non-Geometry

$$\Delta t \, \Delta x \geq L^2$$

$$\Delta x \, \Delta x \geq L^2$$

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}$$

$f(x)$


Estimate θ
[x̂_μ, x̂_ν] = iθ_μν
f(x) * g(x) =

Estimate Θ

Non-Commuting

$$\Delta t \, \Delta x \geq L^2$$

$$\Delta x \, \Delta x \geq L^2$$

$$[\hat{x}_\mu, \hat{x}_\nu] = i\Theta_{\mu\nu}$$

$$f(x) * g(x) = f(x) \, e^{-\frac{i}{2} \overleftarrow{\partial_\mu} \theta^{\mu\nu} \overrightarrow{\partial_\nu}} \, g(x)$$

Estimate Θ

Non-Geometry

$$\Delta t \, \Delta x \geq L^2$$

$$\Delta x \, \Delta x \geq L^2$$

$$[\hat{x}_\mu, \hat{x}_\nu] = i\Theta_{\mu\nu}$$

$$f(x) * g(x) = f(x)\, e^{-\frac{i}{2}\overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu}\, g(x)$$

$$x_\mu * x_\nu - x_\nu * x_\mu = i\Theta_{\mu\nu}$$

Estimate Θ

Non-Commutativity

$\Delta t \, \Delta x \geq L^2$

$\Delta x \, \Delta x \geq L^2$

$[\hat{x}_\mu, \hat{x}_\nu] = i\Theta_{\mu\nu}$

$f(x) * g(x) = f(x) \, e^{-\frac{i}{2} \overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu} \, g(x)$

$x_\mu * x_\nu - x_\nu * x_\mu = i\Theta_{\mu\nu}$

$S[\phi] = \int d^4x \;\; \partial_\mu \varphi * \partial_\mu \varphi$

$+ m \, \varphi * \varphi$

$+ \varphi * \varphi * \varphi \cdots$

$S[\phi]$ not Invariant under Lorentz symmetry

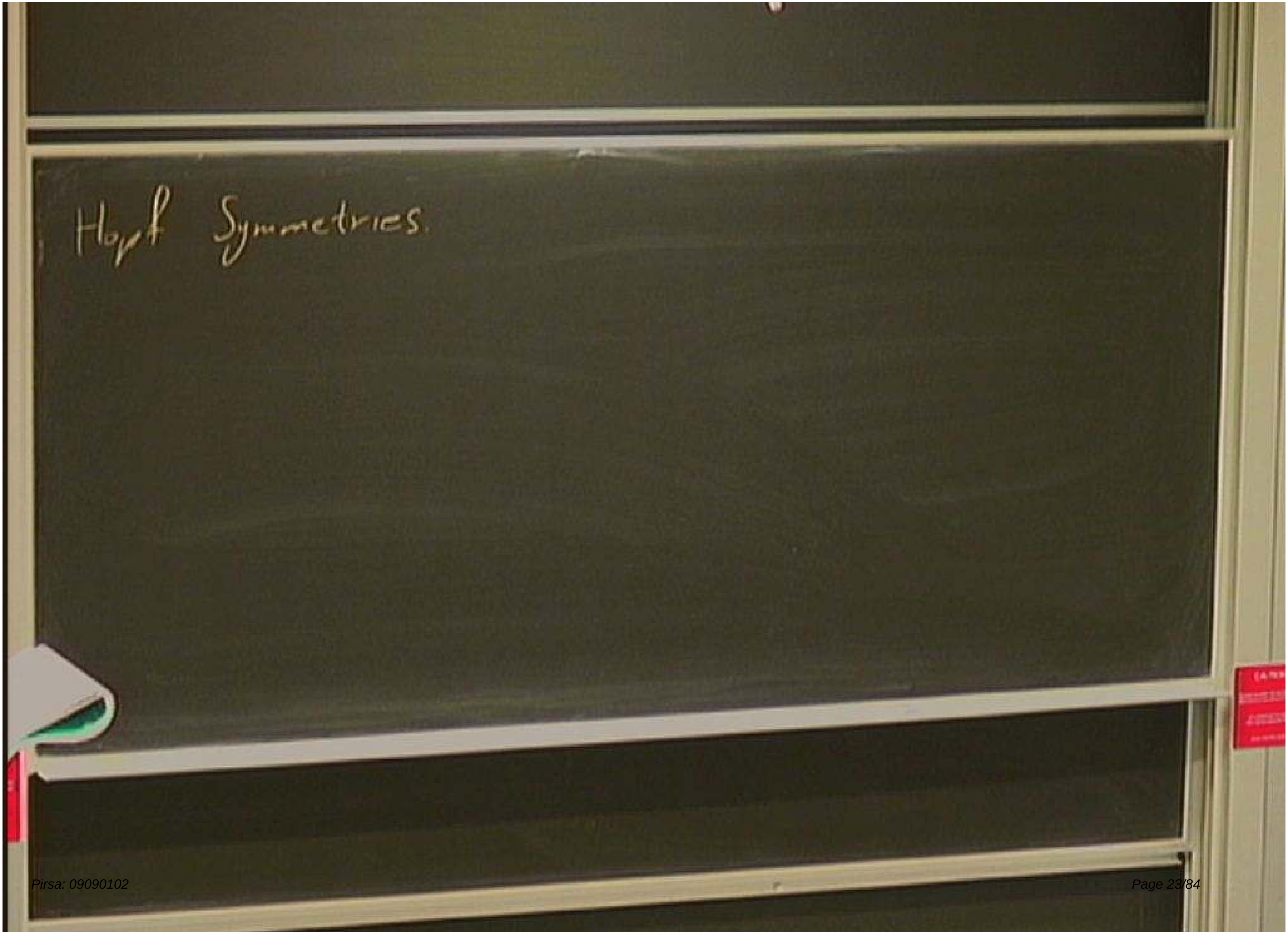
$S[\phi]$ not invariant under Lorentz symmetry

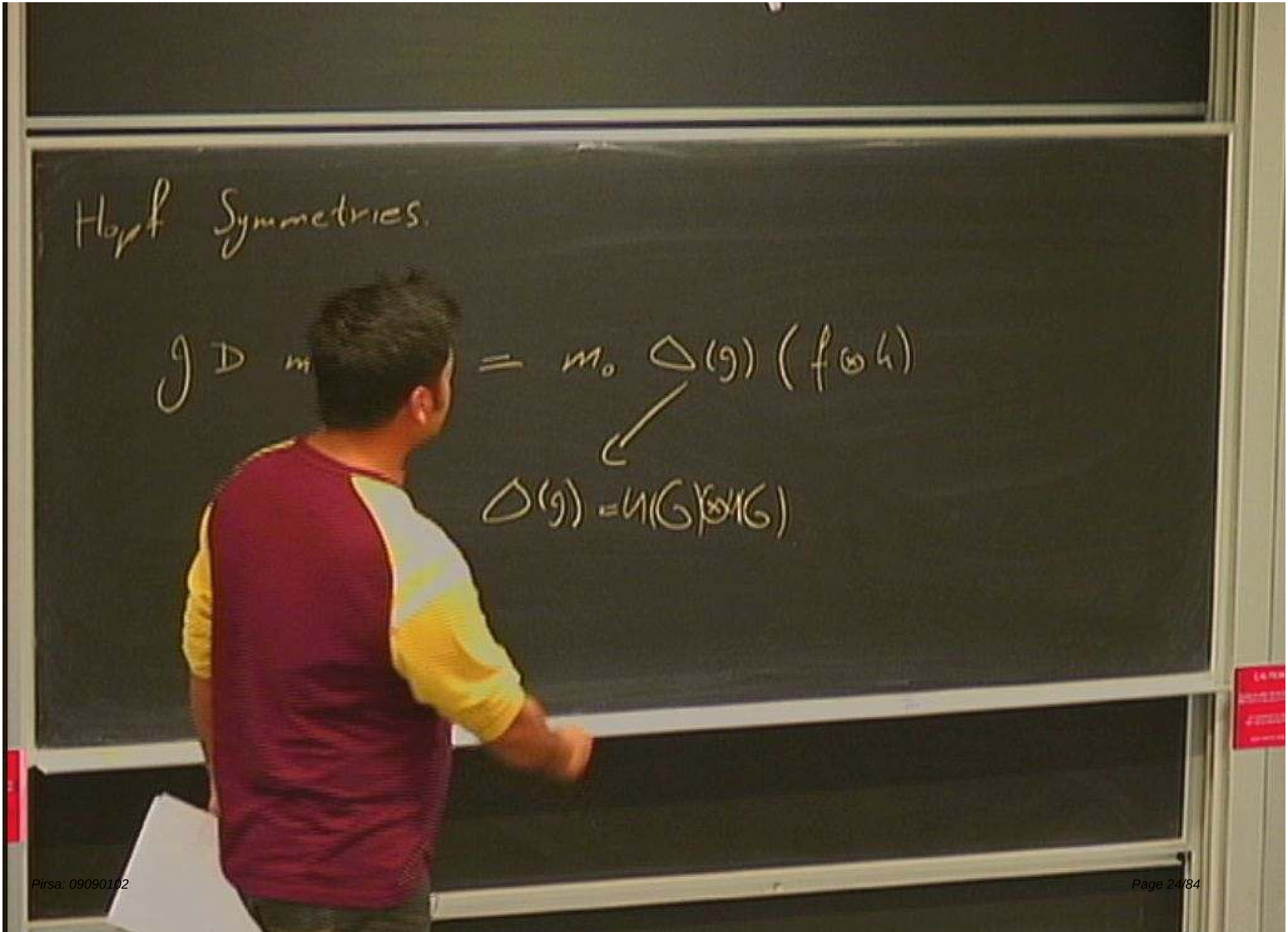
$$g \triangleright \phi(x) = \phi(g^{-1}x)$$

$S[\phi]$ not Invariant under Lorentz symmetry

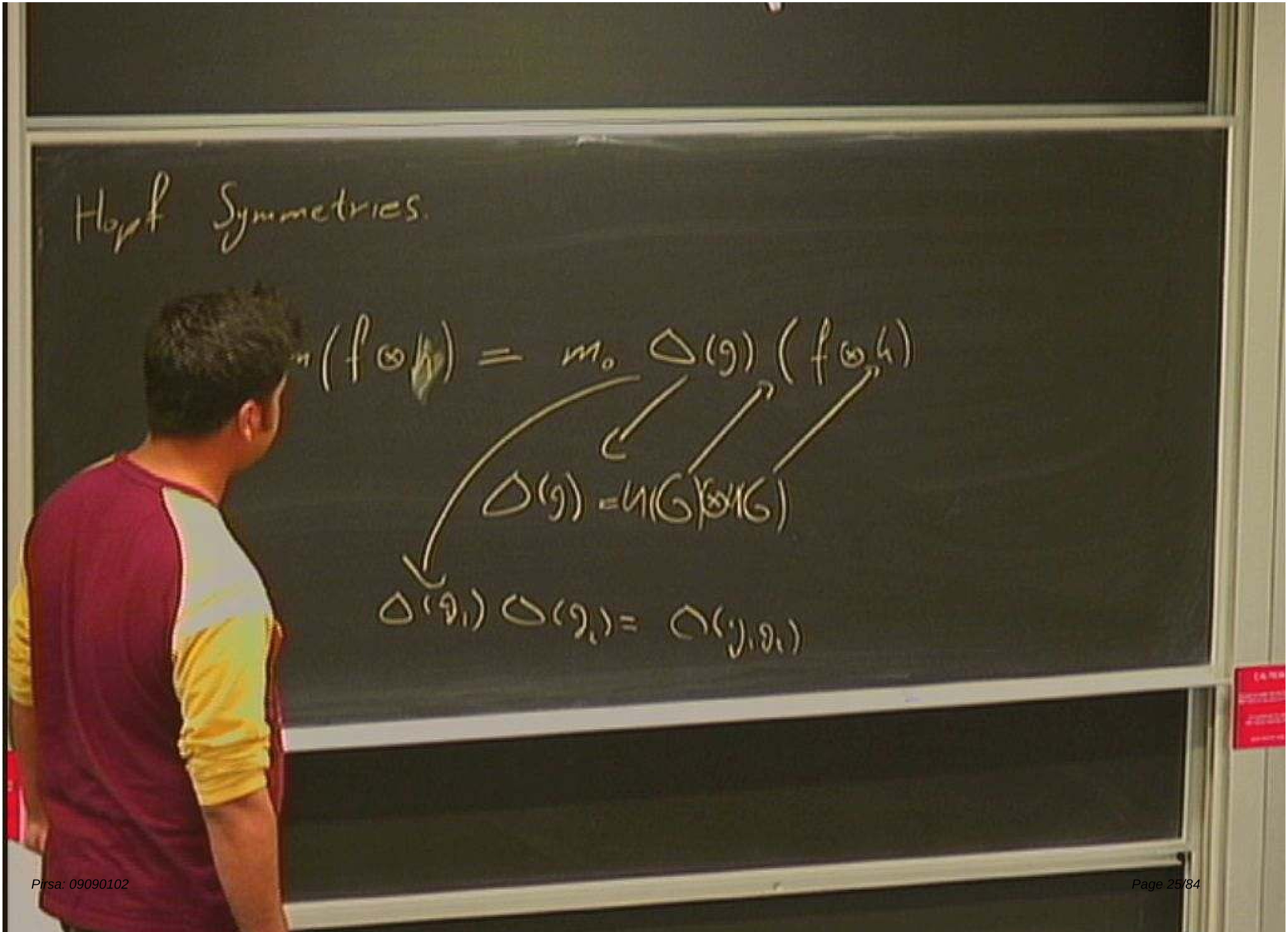
$$g \triangleright \phi(x) = \phi(g^{-1}x)$$

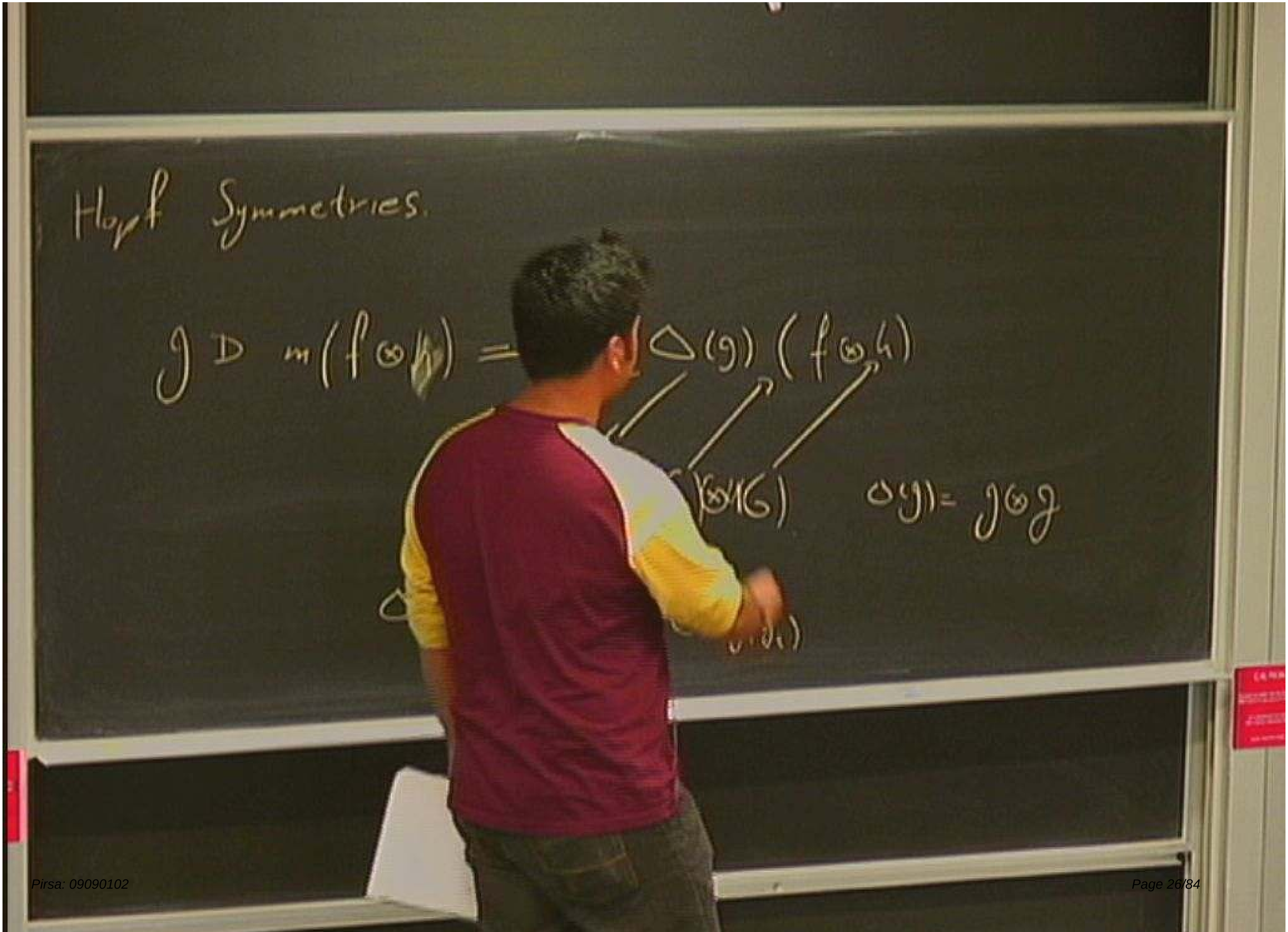
$$g \triangleright \phi(x) * g \triangleright h(x) \neq g \triangleright \phi * h(x)$$


Hopf Symmetries.



Hopf Symmetries.
g ▷ m
= m∘ Δ(g) (f ⊗ h)
Δ(g) = U(G)⊗U(G)


Hopf Symmetries.
m(f⊗h) = m∘Δ(g)(f⊗h)
Δ(g) = Π(G)⊗Π(G)
Δ(g₁)Δ(g₂) = Δ(g₁g₂)


Hopf Symmetries.
$g \triangleright m(f \otimes h) =$
$\Delta(g)\,(f \otimes h)$
$\Delta(g) = g \otimes g$

Hopf Symmetries.

$$g \triangleright m(f\otimes h) = m\,\Delta(g)\,(f\otimes h)$$

$$\Delta(g) = g_{(1)}\otimes g_{(2)} \qquad \Delta(g) = g\otimes g$$

$$\Delta(L) = 1\otimes L + L\otimes 1$$

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries. $\quad m(f\otimes g) = f\,g$

$$g \triangleright m(f\otimes h) = m\,\Delta(g)\,(f\otimes h)$$

$$\Delta(g) = U(G)\otimes U(G) \qquad \Delta(g) = g\otimes g$$

$$\Delta(L) = 1\otimes L + L\otimes 1$$

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$$m(f \otimes g) = f \cdot g$$

$$g \triangleright m_\otimes(f \otimes h) = m_\otimes \Delta(g)(f \otimes h)$$

$$\Delta(g) = \sum g_{(1)} \otimes g_{(2)}$$

$$\Delta(g) = g \otimes g$$

$$\Delta(L) = 1 \otimes L + L \otimes 1$$

$$\Delta(g_1)\Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries. $\quad m(f\otimes g) = f\,g$

$$g \triangleright m_\theta(f\otimes h) = m_\theta\, \Delta(g)\,(f\otimes h) \qquad \Delta_\theta$$

$$\Delta(g) = U(G)\otimes U(G) \qquad \Delta(g)$$

$$\Delta(L) = 1\otimes L + L\otimes 1$$

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$m(f \otimes g) = f \cdot g$

$$g \triangleright m_\theta(f \otimes h) = m_\theta \, \Delta(g)\,(f \otimes h) \longrightarrow \Delta_\theta$$

$\Delta(g) = U(G) \otimes U(G)$

$\Delta(g) = g \otimes g$

$\Delta(L) = \ldots L \otimes 1$

$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$

Hopf Symmetries.

$$m(f\otimes g) = fg$$

$$\Delta_\theta$$

$$g \triangleright m_\theta(f\otimes h) = m_\theta\, \Delta(g)\,(f\otimes h)$$

$$\Delta(g) = \mathcal{U}(G)\otimes\mathcal{U}(G) \qquad \Delta(g) = g\otimes g$$

$$\Delta(L) = 1\otimes L + L\otimes 1$$

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$$

Hopf Symmetries.

$m(f \otimes g) = fg$

Δ_θ

$$g \triangleright m_\theta(f \otimes h) = m_\theta \, \Delta(g)(f \otimes h)$$

$\Delta(g) = \mathcal{U}(G) \otimes \mathcal{U}(G)$

$\Delta(g) = g \otimes g$

$\Delta(L) = 1 \otimes L + L \otimes 1$

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$$

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2) \qquad \Delta(L) = 1 \otimes L + L \otimes 1$$

Automorphism

$$f' \cdot g' = f \cdot g$$

$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$ $\quad$ $\Delta(L) = 1\otimes L + L \otimes 1$

Automorphism

$$f' \cdot g' = f \cdot g$$

$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$ $\Delta(L) = 1 \otimes L + L \otimes 1$

Automorphism

$$f' \cdot g' = f \cdot g$$

$V_1 \otimes V_2$

$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$ $\quad$ $\Delta(L) = 1\otimes L + L\otimes 1$

Automorphism

$$f' \cdot g' = f \cdot g$$

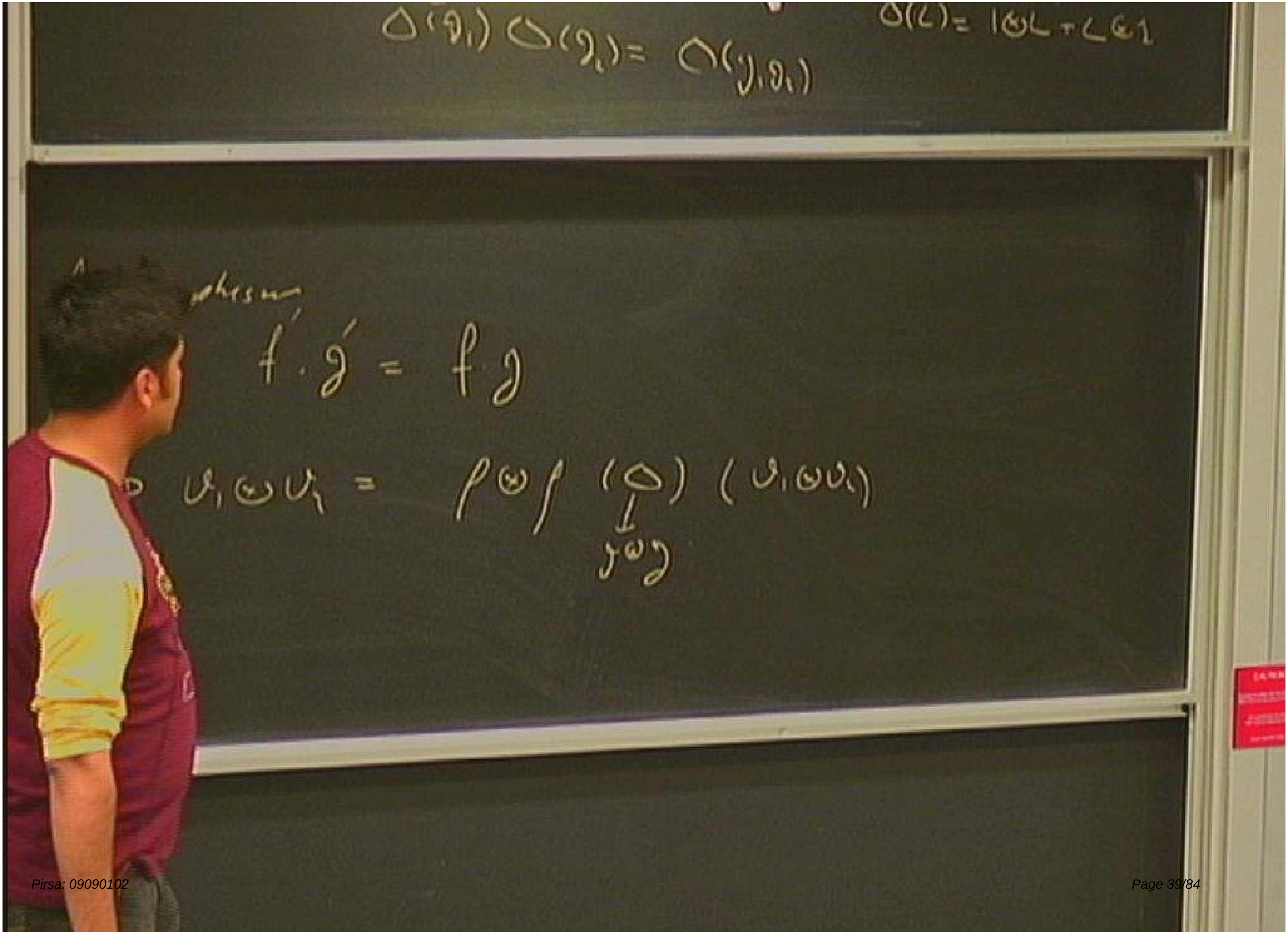
$$g \triangleright v_1 \otimes v_2 = \rho \otimes \rho\,(\Delta)\,(v_1 \otimes v_2)$$

$$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2) \qquad \Delta(L) = 1\otimes L + L\otimes 1$$

Automorphisms

$$f' \cdot g' = f \cdot g$$

$$g \triangleright v_1 \otimes v_2 = \rho\otimes\rho\,(\Delta)\,(v_1\otimes v_2)$$


Δ(g₁) Δ(g₂) = Δ(g₁g₂)
Δ(L) = 1⊗L + L⊗1
f'. g' = f. g
v₁ ⊗ v₂ = ρ⊗ρ (Δ) (v₁⊗v₂)
g⊗g

$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$ $\quad \Delta(L) = 1\otimes L + L\otimes 1$

Automorphism

$$f' \cdot g' = f \cdot g$$

$$g \triangleright v_1 \otimes v_2 = \rho\otimes\rho\,(\Delta)\,(v_1\otimes v_2)$$

$\Delta \to g\otimes g$

Clebsch–Gordan

$$\Delta(g_1)\Delta(g_2) = \Delta(g_1 g_2) \qquad \Delta(L) = 1\otimes L + L\otimes 1$$

$$\Delta(g) = g\otimes g$$

Automorphism

$$f' \cdot g' = f \cdot g$$

$$g \triangleright v_1 \otimes v_2 = \rho\otimes\rho\,(\Delta)\,(v_1\otimes v_2)$$

Clebsch–Gordan $\qquad g\otimes g$

$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$ $\qquad \Delta(L) = 1 \otimes L + L \otimes 1$

$\Delta(g) = g \otimes g$

Automorphism

$$f' \cdot g' = f \cdot g$$

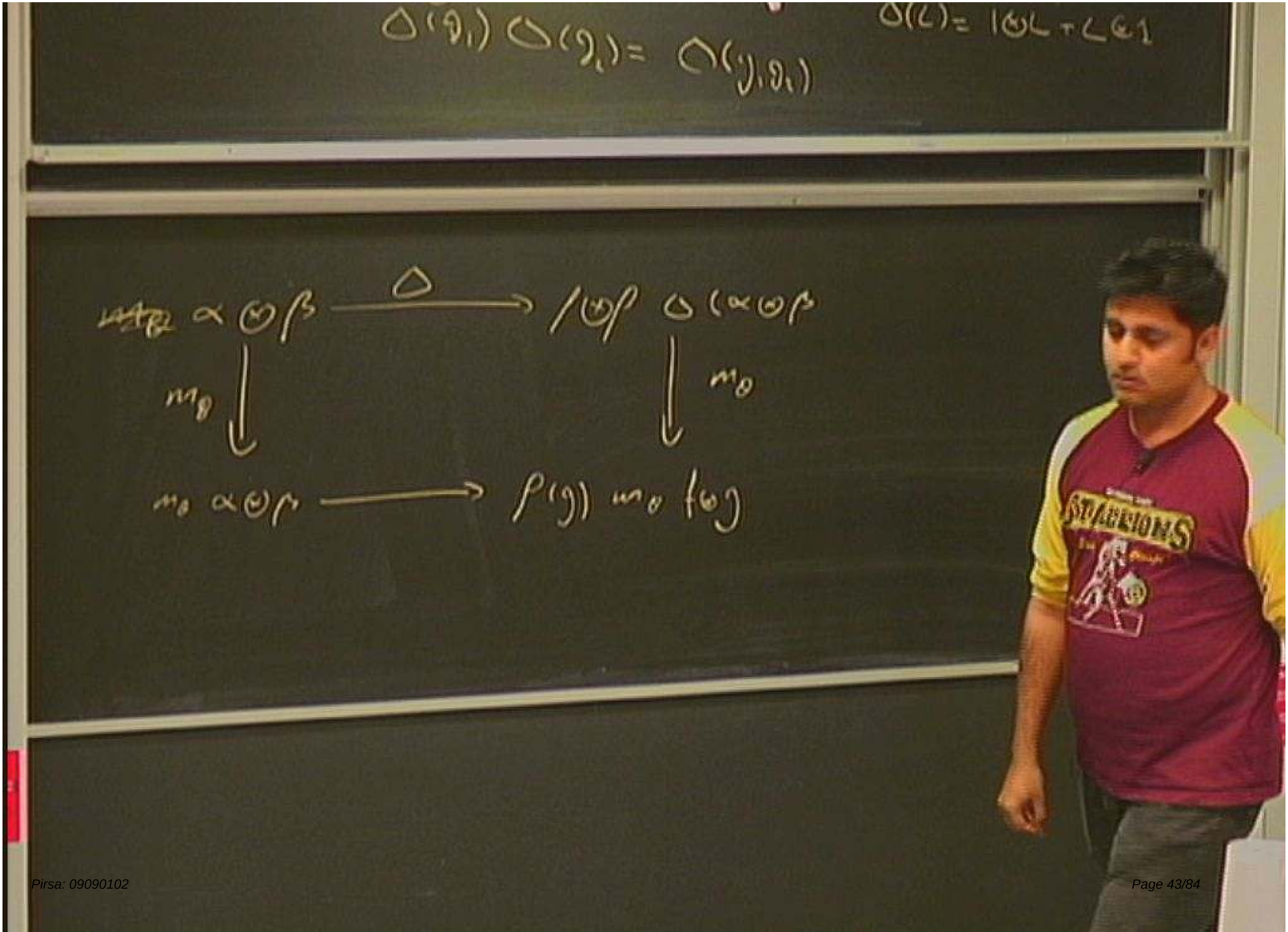
$$g \triangleright v_1 \otimes v_2 = \rho \otimes \rho\,(\Delta)\,(v_1 \otimes v_2)$$

$\Delta \to g \otimes g$

Clebsch-Gordan

Hopf Algebra

$$H = (G, \Delta)$$


$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$
$\Delta(L) = 1 \otimes L + L \otimes 1$
$\alpha \otimes \beta \xrightarrow{\Delta} \rho \otimes \rho\ \Delta(\alpha \otimes \beta)$
m_g
m_g
$m_g\ \alpha \otimes \beta \longrightarrow \rho(g)\ m_g\ f(g)$

$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$ $\qquad \Delta(L) = 1 \otimes L + L \otimes 1$

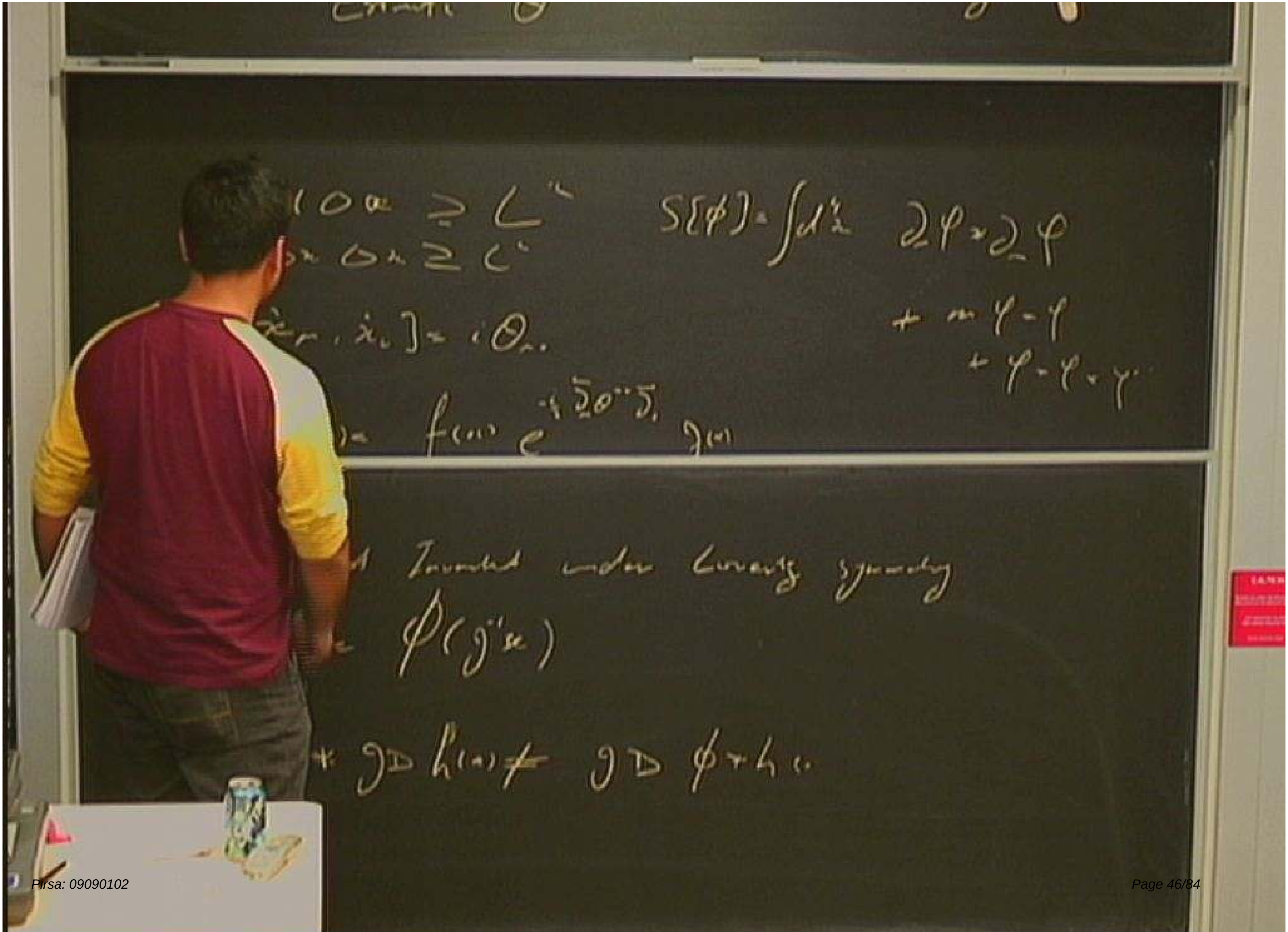
$$\begin{array}{ccc} \alpha \otimes \beta & \xrightarrow{\ \Delta\ } & f \otimes g \quad \Delta(\alpha \otimes \beta) \\ \downarrow F & & \downarrow m_\theta \\ m_\theta\, \alpha \otimes \beta & \longrightarrow & \rho(g)\, m_\theta\, f \otimes g \end{array}$$

$= m_0 F = e^{i\theta\, P_\mu \otimes P_\nu}$

$\Delta(g_1)\,\Delta(g_2) = \Delta(g_1 g_2)$ $\qquad$ $\Delta(L) = 1 \otimes L + L \otimes 1$

$$
\begin{array}{ccc}
\alpha \otimes \beta & \xrightarrow{\ \Delta\ } & f \otimes f \quad \Delta(\alpha \otimes \beta) \\
\Big\downarrow m_\theta & & \Big\downarrow m_\theta \\
m_\theta\, \alpha \otimes \beta & \longrightarrow & \rho(g)\, m_\theta\, f \otimes g
\end{array}
$$

$m_\theta = m_0\, F = m_0\, e^{i\theta \cdot P \otimes P}$


S[φ] = ∫d²x ∂φ * ∂φ
+ φ * φ * φ
Invariant under Lorentz symmetry
φ(g⁻¹x)

$$\Delta t \, \Delta x \geq L^2$$
$$\Delta x \, \Delta x \geq L^2$$

$$S[\phi] = \int d^4x \; \partial_\mu \phi * \partial^\mu \phi + m\,\phi * \phi + \phi * \phi * \phi$$

$$[\hat{x}_\mu , \hat{x}_\nu] = i\,\Theta_{\mu\nu}$$

$$f(x) * g(x) = f(x)\, e^{\frac{i}{2}\overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu}\, g(x)$$

$S[\phi]$ not Invariant under Lorentz symmetry

$$g \triangleright \phi(x) = \phi(g^{-1}x)$$

$$g \triangleright \phi(x) * g \triangleright h(x) \neq g \triangleright (\phi * h)$$

$$\alpha(\cdot, g_1, g_2)$$

$$
\begin{array}{ccc}
\alpha \otimes \beta & \xrightarrow{\Delta} & \Delta(\alpha \otimes \beta) \\
\downarrow m_\theta & & \downarrow m_\theta \\
m_\theta\, \alpha \otimes \beta & \longrightarrow & \rho(g)\, m_\theta\, f(g)
\end{array}
$$

$$\Delta_\theta = F^{-1} \Delta_0 F$$

$$m_\theta = m_0 F^{-1} = m_0\, e^{i\theta\, P_\mu \otimes P_\nu}$$

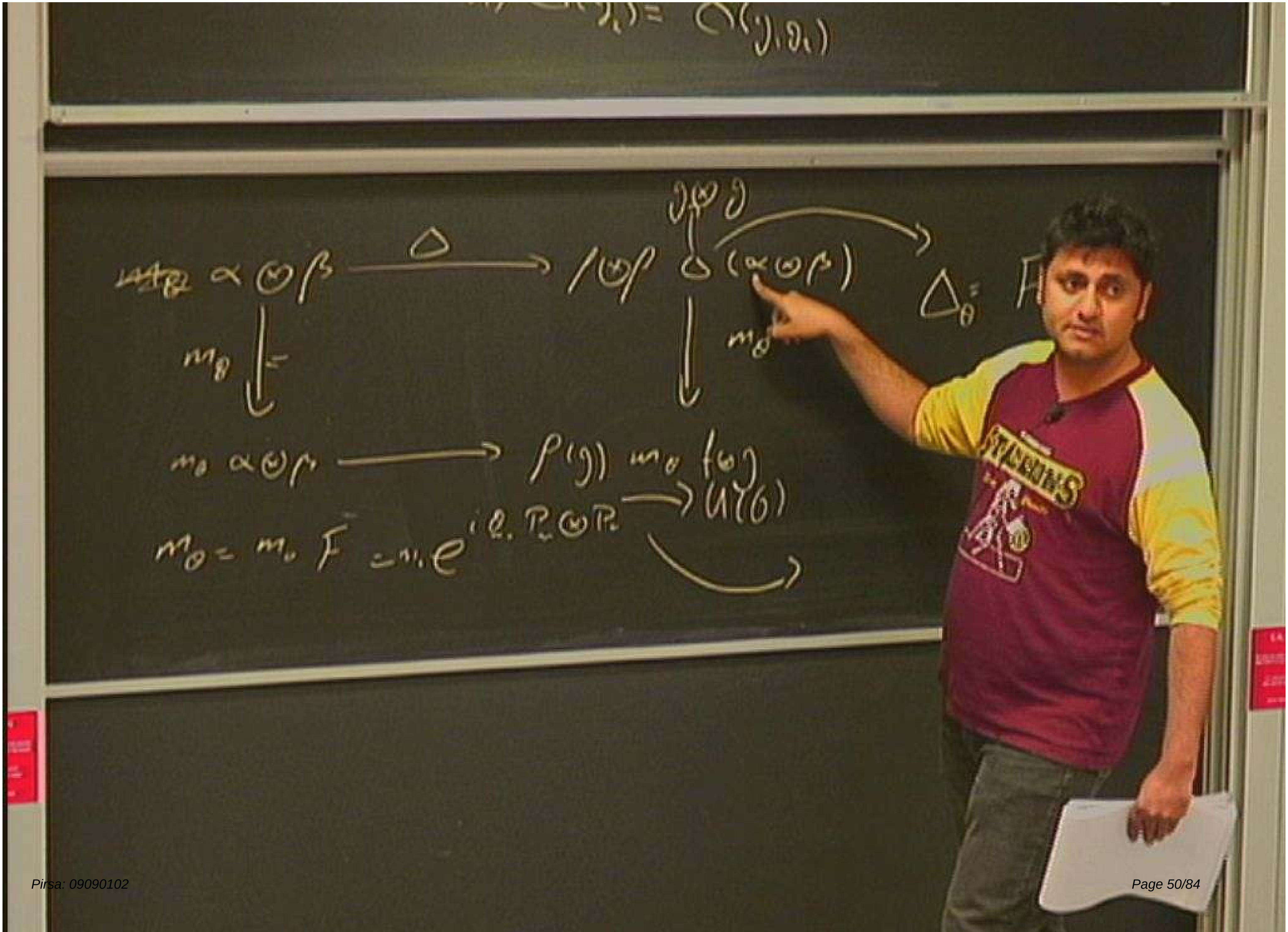
$$\alpha \otimes \beta \xrightarrow{\Delta} f \otimes f \quad \Delta(\alpha \otimes \beta) \qquad \Delta_\theta = F^{-1} \Delta_0 F$$

$$m_\theta \downarrow = \qquad\qquad \downarrow m_\theta$$

$$m_0\, \alpha \otimes \beta \longrightarrow f(g)\; m_\theta\, f \otimes g$$

$$\longrightarrow (\alpha \gamma(\theta))$$

$$m_\theta = m_0 F = m_0 e^{i\theta_\cdot P_0 \otimes P_1}$$


α ⊗ β
Δ
m_θ
m_θ α ⊗ β
m_θ = m_0 F = m_0 e
Δ_θ =

$$\ldots) = \Delta(\cdot, g, g_\cdot)$$

$$\alpha \otimes \beta \xrightarrow{\Delta} f(\otimes)\beta \quad \Delta(\alpha\otimes\beta) \quad \sum_\alpha g^{(1)}_{m_\theta} \otimes g^{(1)}_{\beta}$$

$$\Delta_\theta = F^{-1}\Delta_0 F$$

$$m_\theta \Big\downarrow$$

$$m_0\, \alpha\otimes\beta \longrightarrow \rho(g)\, m_0\, f(\otimes g)$$

$$\longrightarrow (u\gamma(\theta))$$

$$m_\theta = m_0 F = m_0 e^{i\theta_\cdot P_\mu \otimes P_\nu}$$

$$|k_1, k_2\rangle = \alpha \, |k_2, k_1\rangle$$

$$\sigma |k_1, k_2\rangle = F^{-2} \, |k_1, k_2\rangle$$

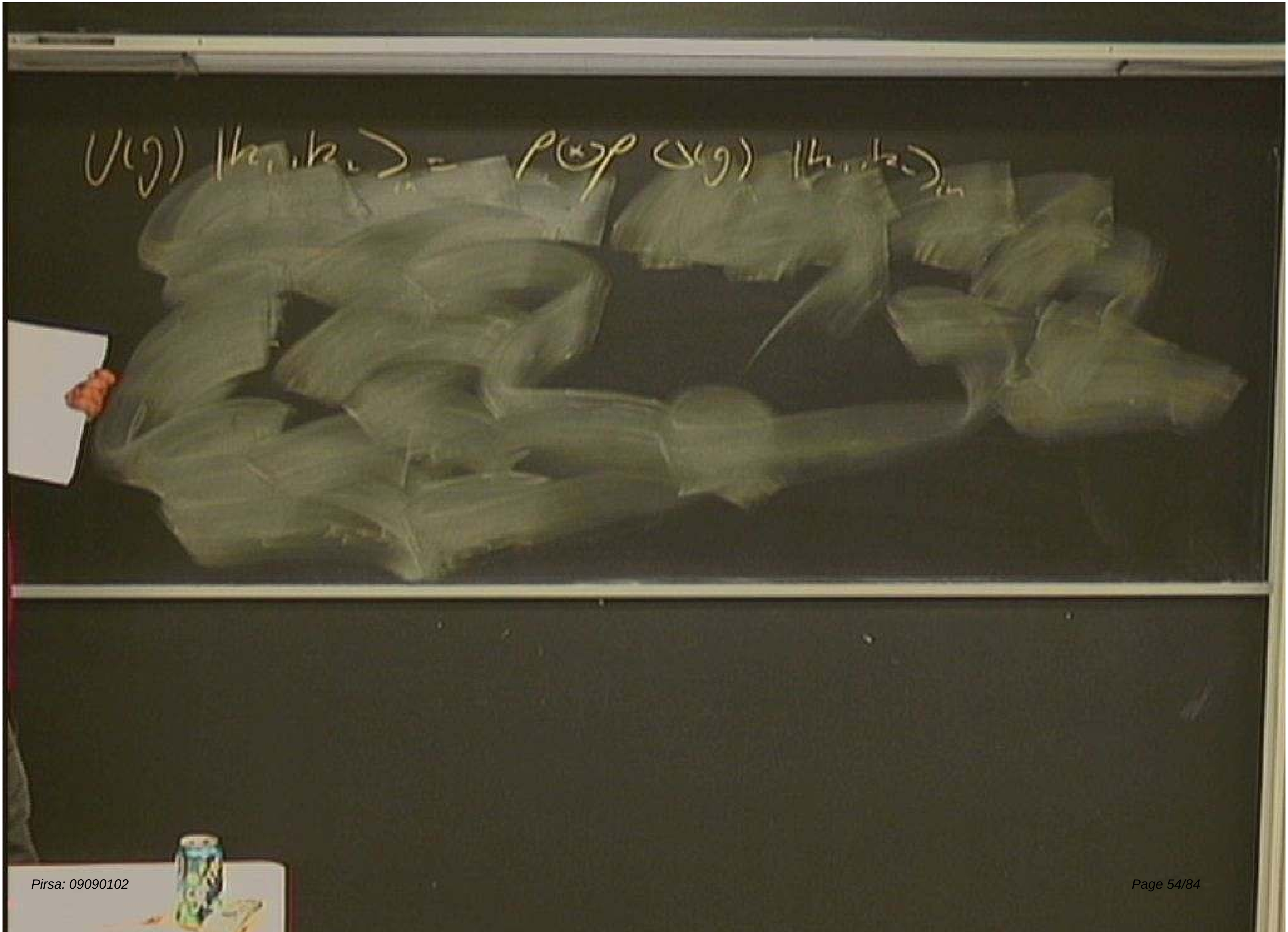
$$\begin{array}{ccc} V & \sim & W \\ \downarrow g & & \downarrow g \\ V' & \sim & W' \end{array}$$

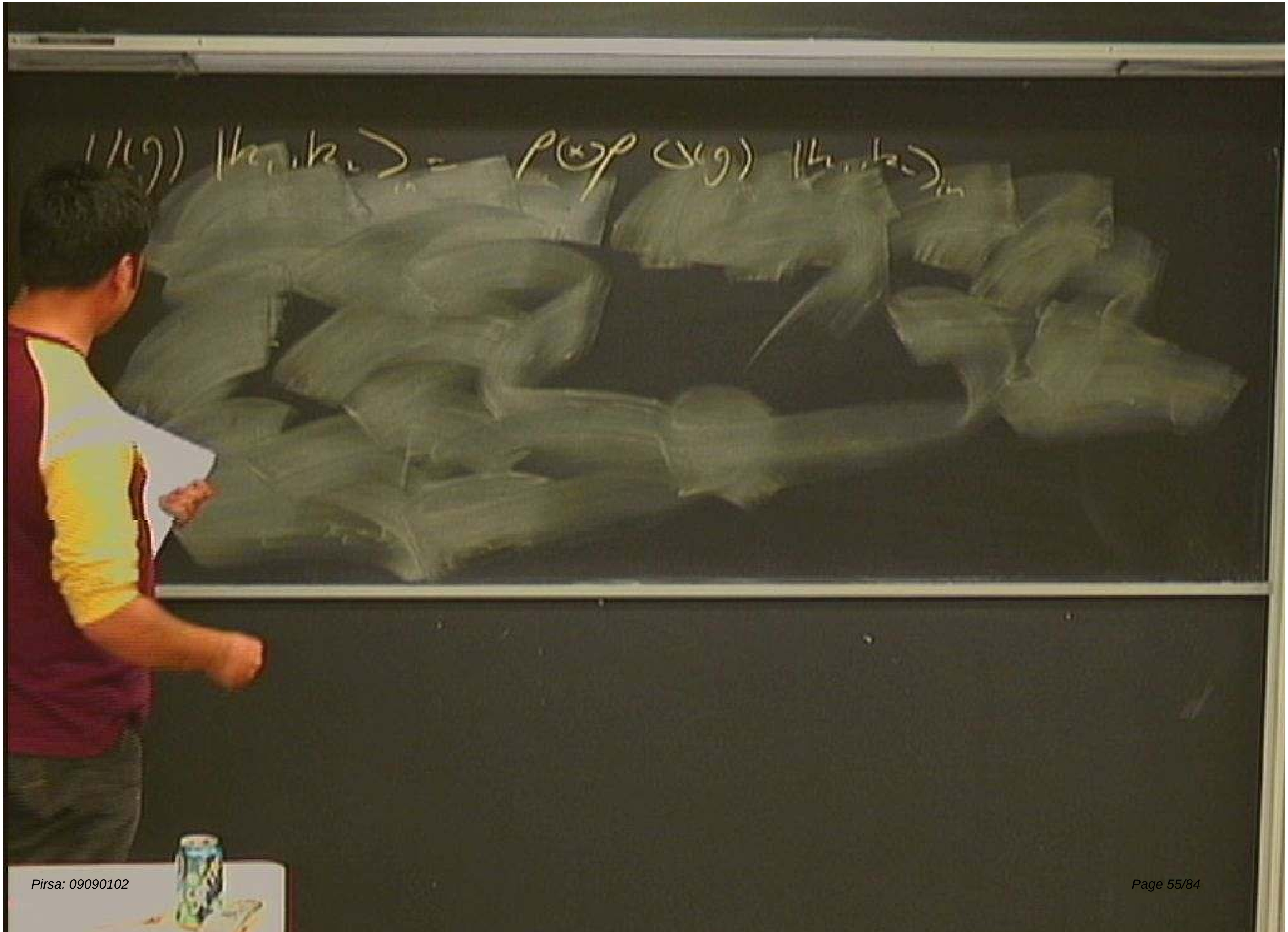
$$|k_1, k_2\rangle = \alpha \, |k_2, k_1\rangle$$

$$\sigma |k_1, k_2\rangle = \hat{F}^{-2} \, |k_1, k_2\rangle$$

$$e^{i\theta_{\mu\nu} k_1^{\mu} k_2^{\nu}} |k_1, k_2\rangle$$

$$\begin{array}{ccc} V & \sim & W \\ \downarrow g & & \downarrow g \\ V' & \sim & W' \end{array}$$


$U(g)\,|k_1,k_2\rangle_{in} = \rho \otimes \rho\, U(g)\,|k_1,k_2\rangle_{in}$


$U(g)\,|k_1,k_2\rangle_{in} = \rho\otimes\rho\,(\Delta(g))\,|k_1,k_2\rangle_{in}$

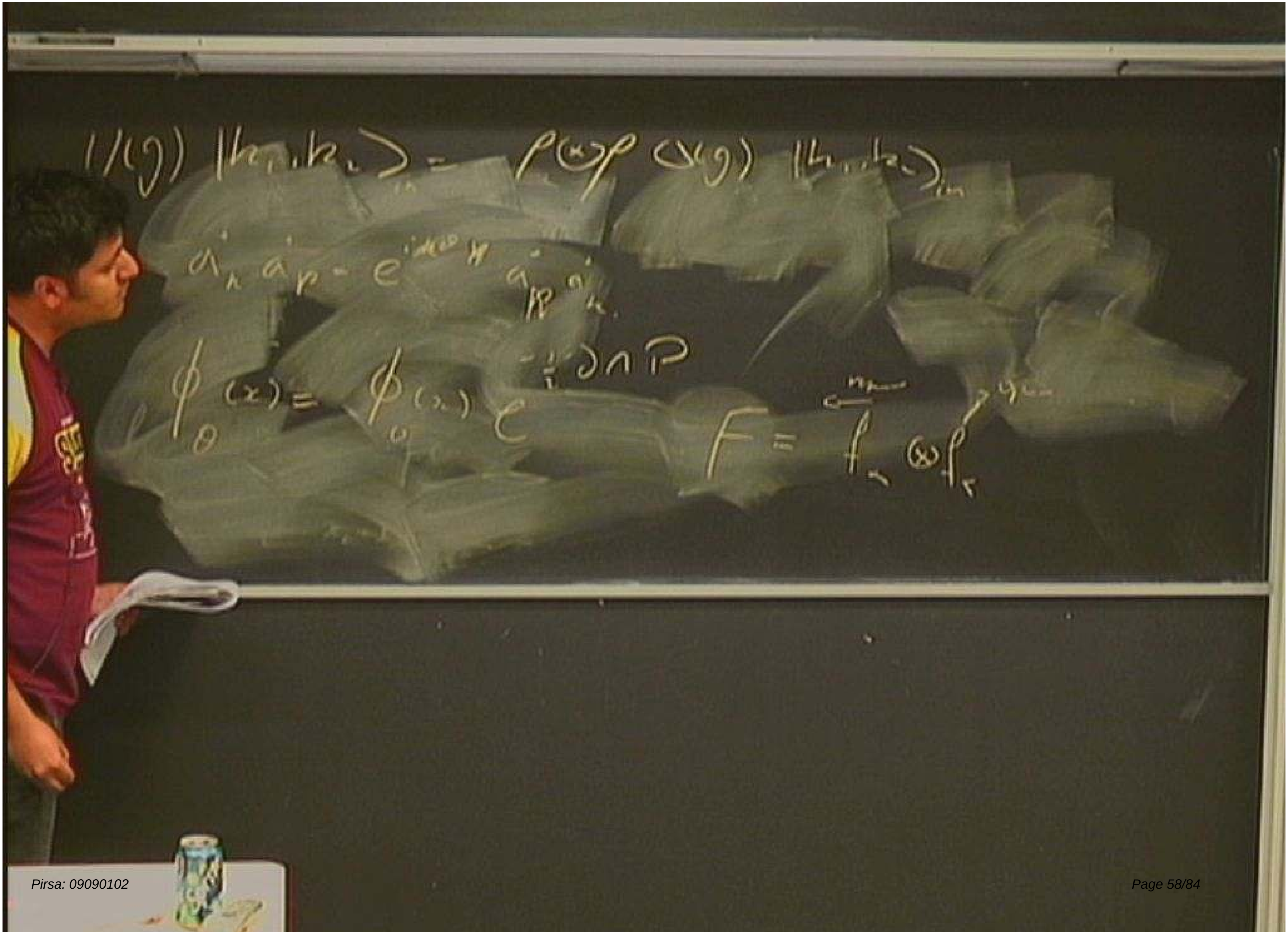
$$U(g)\,|k_1,k_2\rangle_{in} = \rho \otimes \rho\,(\Delta(g))\,|k_1,k_2\rangle_{in}$$

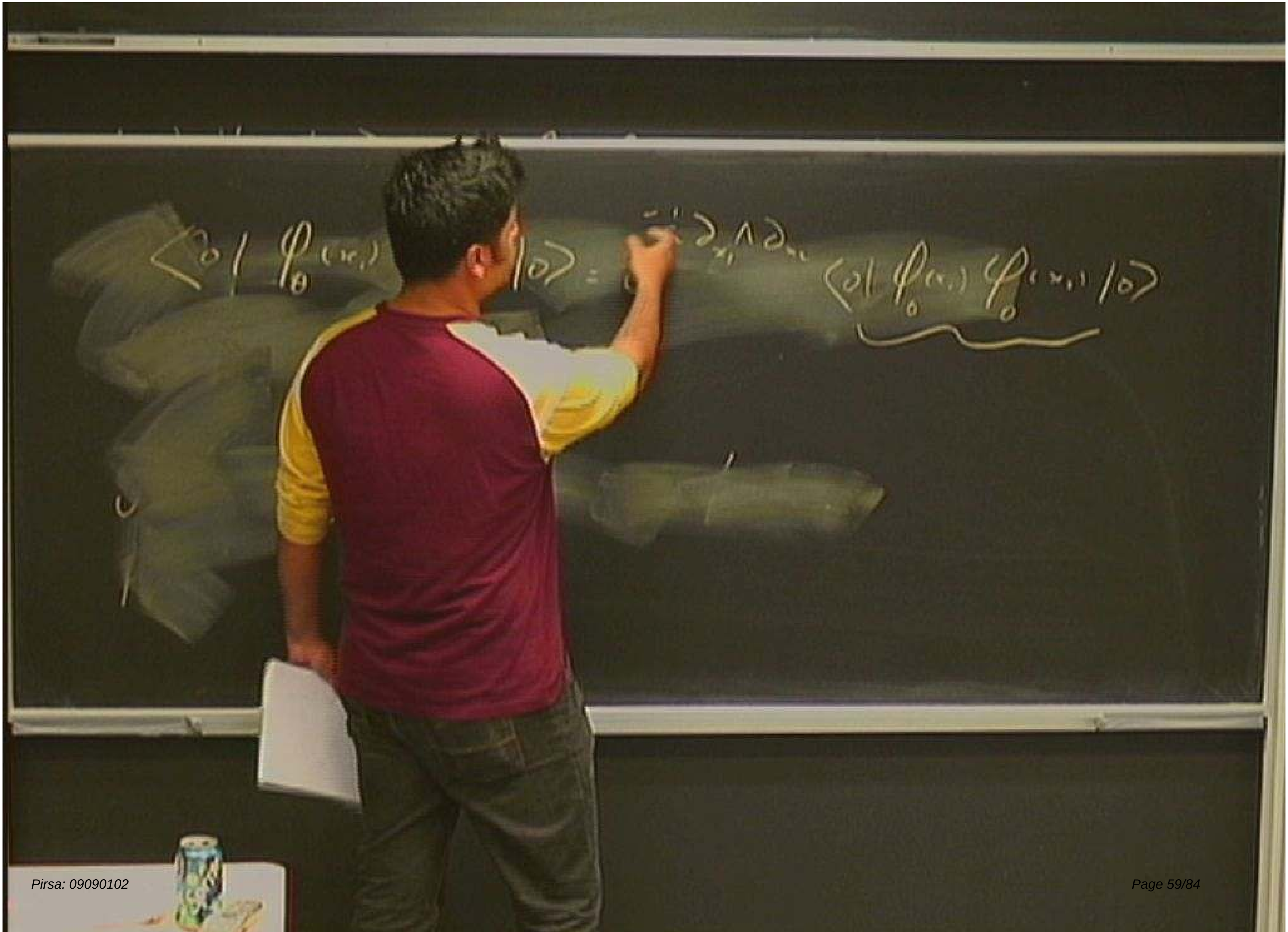
$$a^i_k\, a^j_p = e^{ik\theta p}\, a^j_p\, a^i_k$$

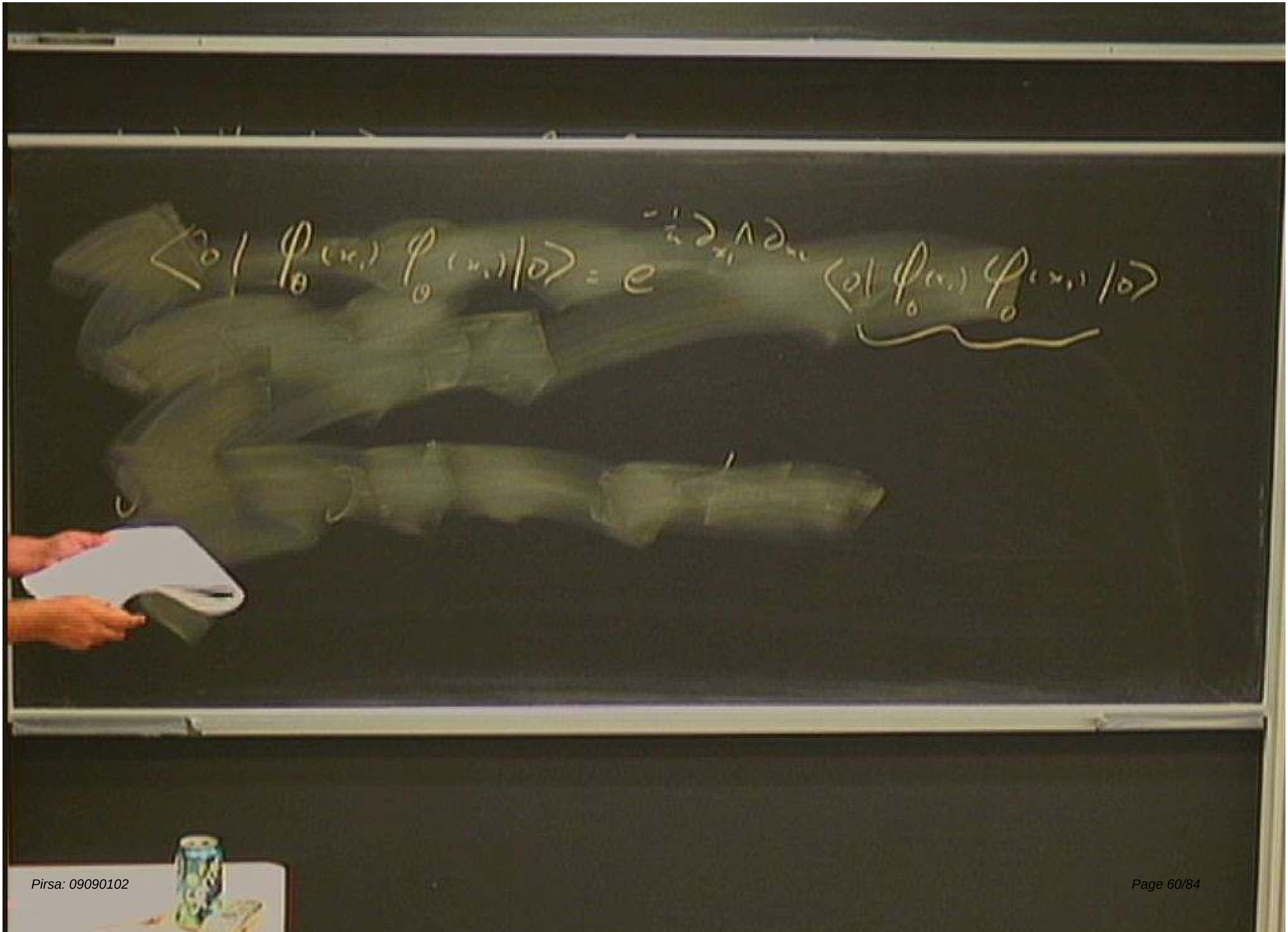
$$U(g)\,|k_1,k_2\rangle_{in} = \rho(\star)\rho\,U(g)\,|k_1,k_2\rangle_{in}$$

$$a_k^{\dagger} a_p^{\dagger} = e^{i k \theta p}\, a_p^{\dagger} a_k^{\dagger}$$

$$\phi_\theta(x) = \phi_0(x)\, e^{-\frac{1}{2}\partial \wedge P}$$

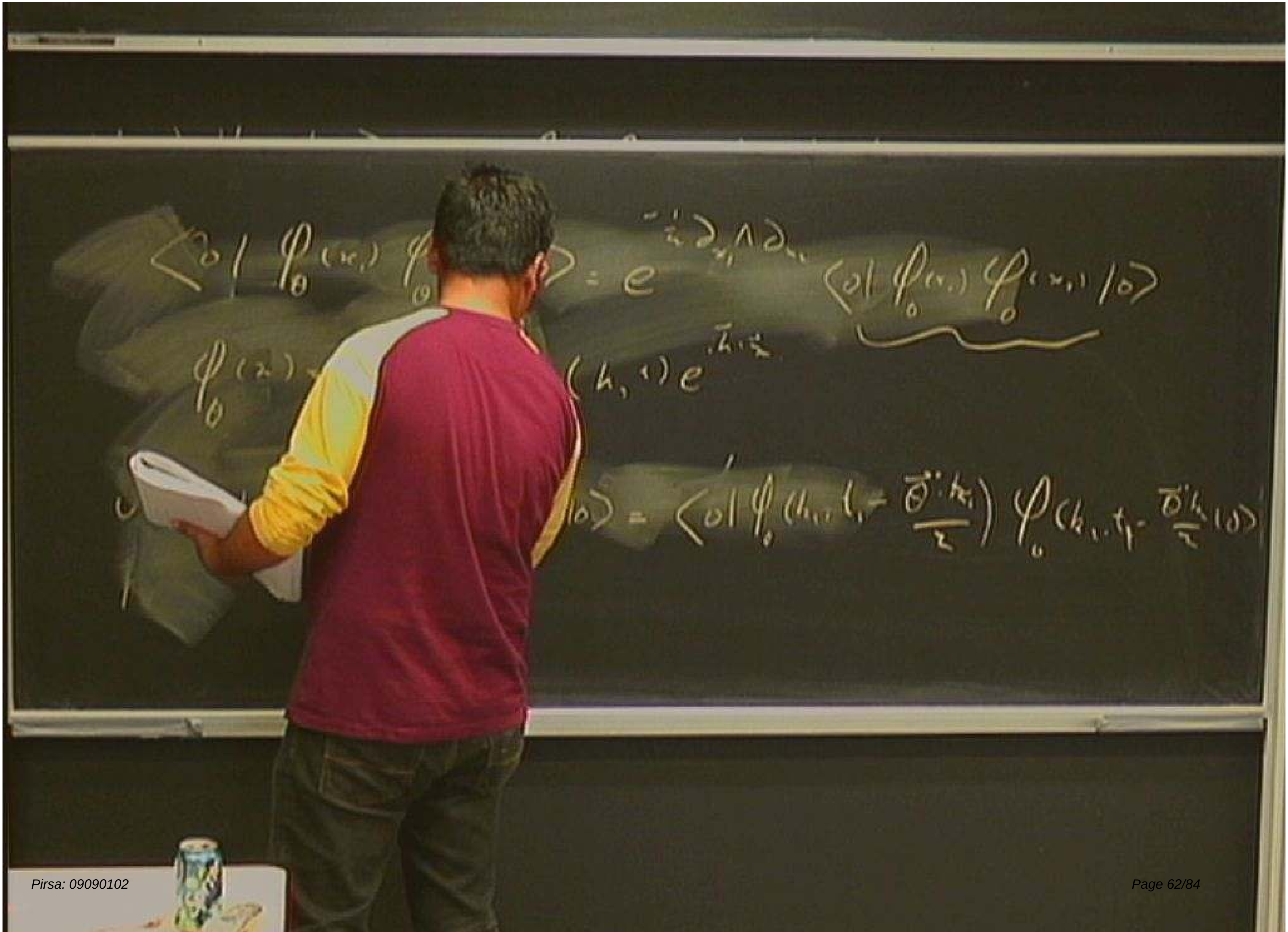


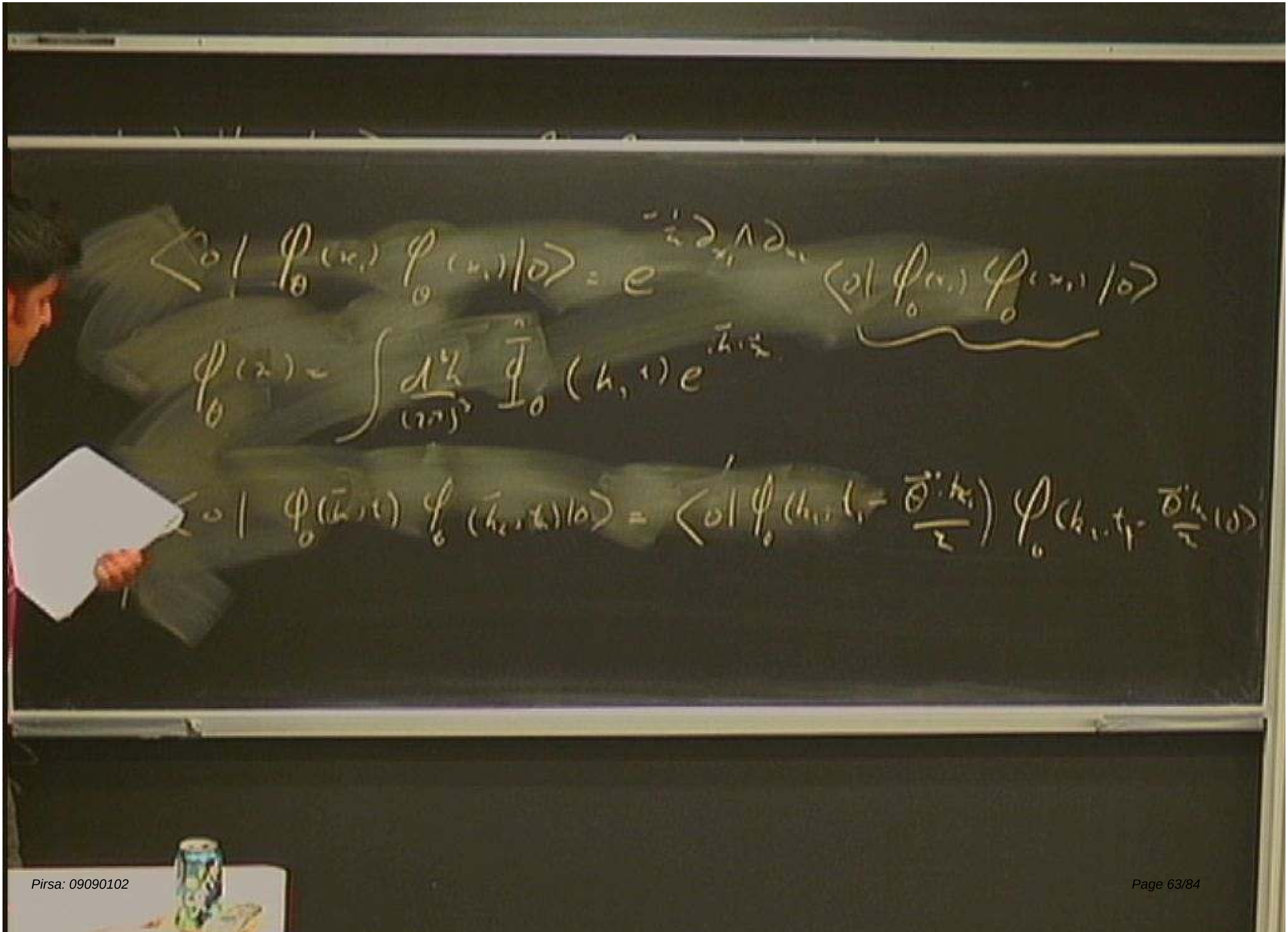



$\langle 0| \varphi_\theta(x_1)\, \varphi_\theta(x_2) |0\rangle = e^{-\frac{i}{2}\partial_{x_1}\wedge\partial_{x_2}} \langle 0| \varphi_0(x_1)\, \varphi_0(x_2) |0\rangle$

$$\langle 0 | \phi_\theta(x_1)\, \phi_\theta(x_2) | 0 \rangle = e^{-\frac{i}{2}\partial_{x_1} \wedge \partial_{x_2}} \langle 0 | \phi_0(x_1)\, \phi_0(x_2) | 0 \rangle$$

$$\phi_\theta(x) = \int \frac{d^4k}{(2\pi)^4}\, \tilde{\phi}_0(k, t)\, e^{ik \cdot x}$$





$\langle 0 | \varphi_\theta(x_1)\, \varphi_\theta(x_2) | 0 \rangle = e^{-\frac{i}{2} \partial_{x_1} \wedge \partial_{x_2}} \langle 0 | \varphi_0(x_1)\, \varphi_0(x_2) | 0 \rangle$
$\varphi_\theta(x) = \int \frac{d^4 k}{(2\pi)^4}\, \tilde{\varphi}_0(k, t)\, e^{i k \cdot x}$
$\langle 0 | \varphi_\theta(\vec{k}_1, t)\, \varphi_\theta(\vec{k}_2, t_2) | 0 \rangle = \langle 0 | \varphi_0(k_1, t_1 - \frac{\vec{\theta} \cdot k_1}{2})\, \varphi_0(k_2, t_2 - \frac{\vec{\theta} \cdot k_2}{2}) | 0 \rangle$

$$\langle 0 | \phi_\theta(x_1)\, \phi_\theta(x_2) | 0 \rangle = e^{-\frac{i}{2}\partial_{x_1}\wedge\partial_{x_2}} \underbrace{\langle 0 | \phi_0(x_1)\, \phi_0(x_2) | 0 \rangle}$$

$$\phi_\theta(x) = \int \frac{d^3k}{(2\pi)^3}\, \hat{\Phi}_\theta(k,t)\, e^{i\vec{k}\cdot\vec{x}}$$

$$\langle 0 | \phi_\theta(\vec{k}_1, t_1)\, \phi_\theta(\vec{k}_2, t_2) | 0 \rangle = \langle 0 | \phi_0\left(k_1, t_1 - \frac{\vec{\theta}\cdot k_2}{2}\right) \phi_0\left(k_2, t_2 - \frac{\vec{\theta}\cdot k_1}{2}\right) | 0 \rangle$$

$$\langle 0|\, \varphi_\theta(x_1)\, \varphi_\theta(x_2)\,|0\rangle = e^{-\frac{i}{2}\partial_{x_1}\wedge\partial_{x_2}} \langle 0|\, \varphi_0(x_1)\, \varphi_0(x_2)\,|0\rangle$$

$$\varphi_\theta(x) = \int \frac{d^4k}{(2\pi)^4}\, \hat{\Phi}_0(k,t)\, e^{ik\cdot x}$$

$$\langle 0|\, \varphi_\theta(\vec{k}_1,t_1)\, \varphi_\theta(\vec{k}_2,t_2)\,|0\rangle = \langle 0|\, \varphi_0\left(k_1, t_1 - \frac{\vec{\theta}\cdot k_1}{2}\right) \varphi_0\left(k_2, t_2 - \frac{\vec{\theta}\cdot k_2}{2}\right)|0\rangle$$

$$\langle 0 | \varphi_\theta(x_1)\, \varphi_\theta(x_2) | 0 \rangle = e^{-\frac{i}{2}\partial_{x_1}\wedge\partial_{x_2}} \underbrace{\langle 0 | \varphi_0(x_1)\, \varphi_0(x_2) | 0 \rangle}$$

$$\varphi_\theta(x) = \int \frac{d^3k}{(2\pi)^3}\, \hat{\Phi}_\theta(k, t)\, e^{i\vec{k}\cdot\vec{x}}$$

$$\langle 0 | \varphi_\theta(\vec{k}_1, t_1)\, \varphi_\theta(\vec{k}_2, t_2) | 0 \rangle = \langle 0 | \varphi_0\left(k_1, t_1 - \frac{\vec{\theta}\cdot k_1}{2}\right) \varphi_0\left(k_2, t_2 - \frac{\vec{\theta}\cdot k_2}{2}\right) | 0 \rangle$$

$x_0,\ x_3 \qquad \varphi(x)$

$$\alpha \otimes \beta \xrightarrow{\ \Delta\ } 1 \otimes \beta \quad \Delta(\alpha \otimes \beta)$$

$$g \otimes g$$

$$\sum_{\alpha} g^{(1)}_{m_\theta} \otimes g^{(1)}_{r}$$

$$\Delta_\theta = F^{-1} \Delta_0 F$$

$$m_\theta \downarrow$$

$$P_\theta = P_0(h)\cosh(H\vec{\theta}\cdot\vec{h})$$

$$m_\theta\ \alpha \otimes \beta \longrightarrow \rho(g)\ m_\theta\ f(\omega) \longrightarrow U(\theta)$$

$$m_\theta = m_0 F^{-1} = m_0\, e^{i\theta_{\mu\nu} P_\mu \otimes P_\nu}$$

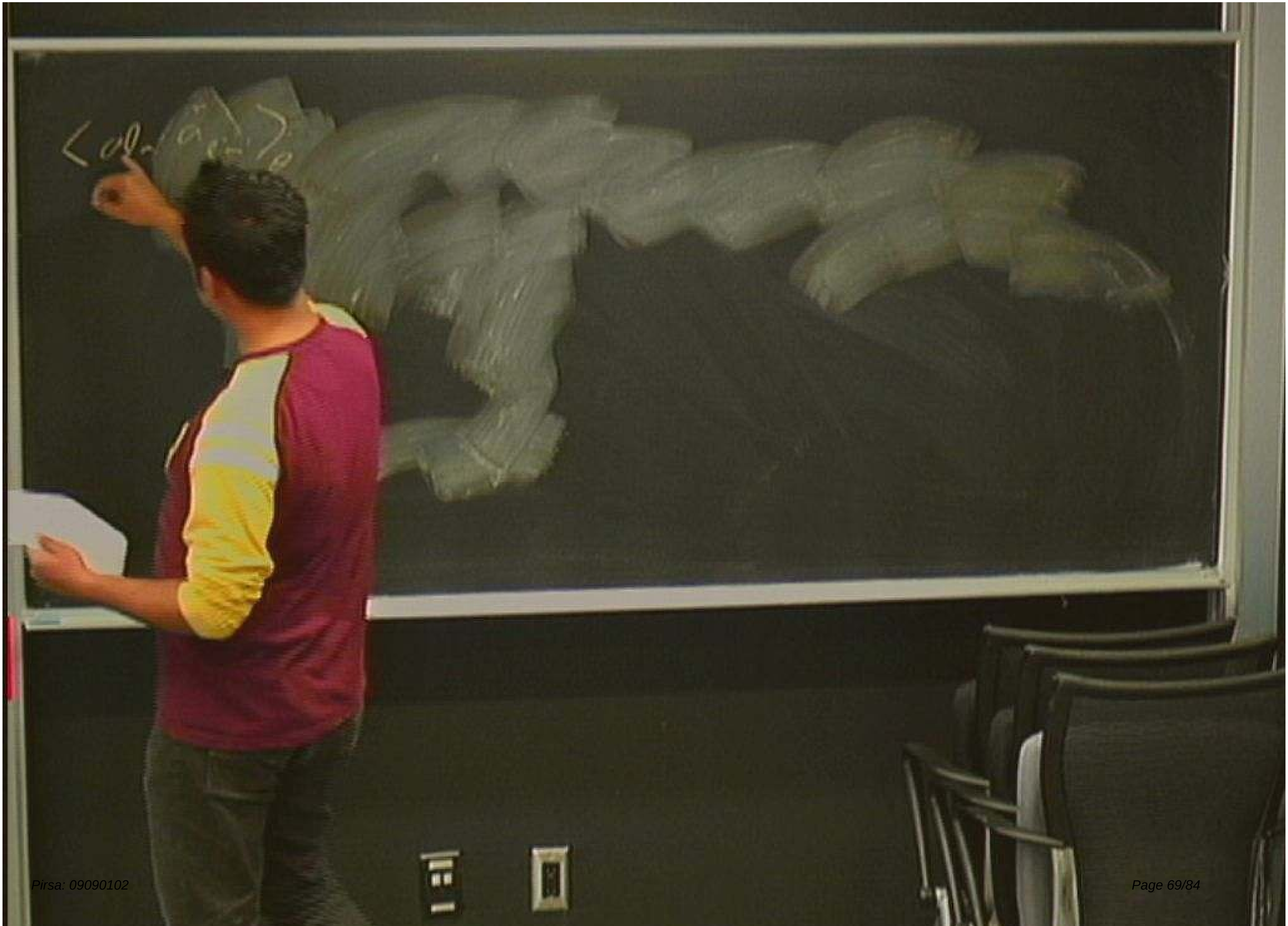
$$\alpha \otimes \beta \xrightarrow{\Delta} \sum_\alpha g^{(1)}_{m_\theta} \otimes g^{(1)}_r \quad \Delta_\theta = F^{-1} \Delta_0 F$$

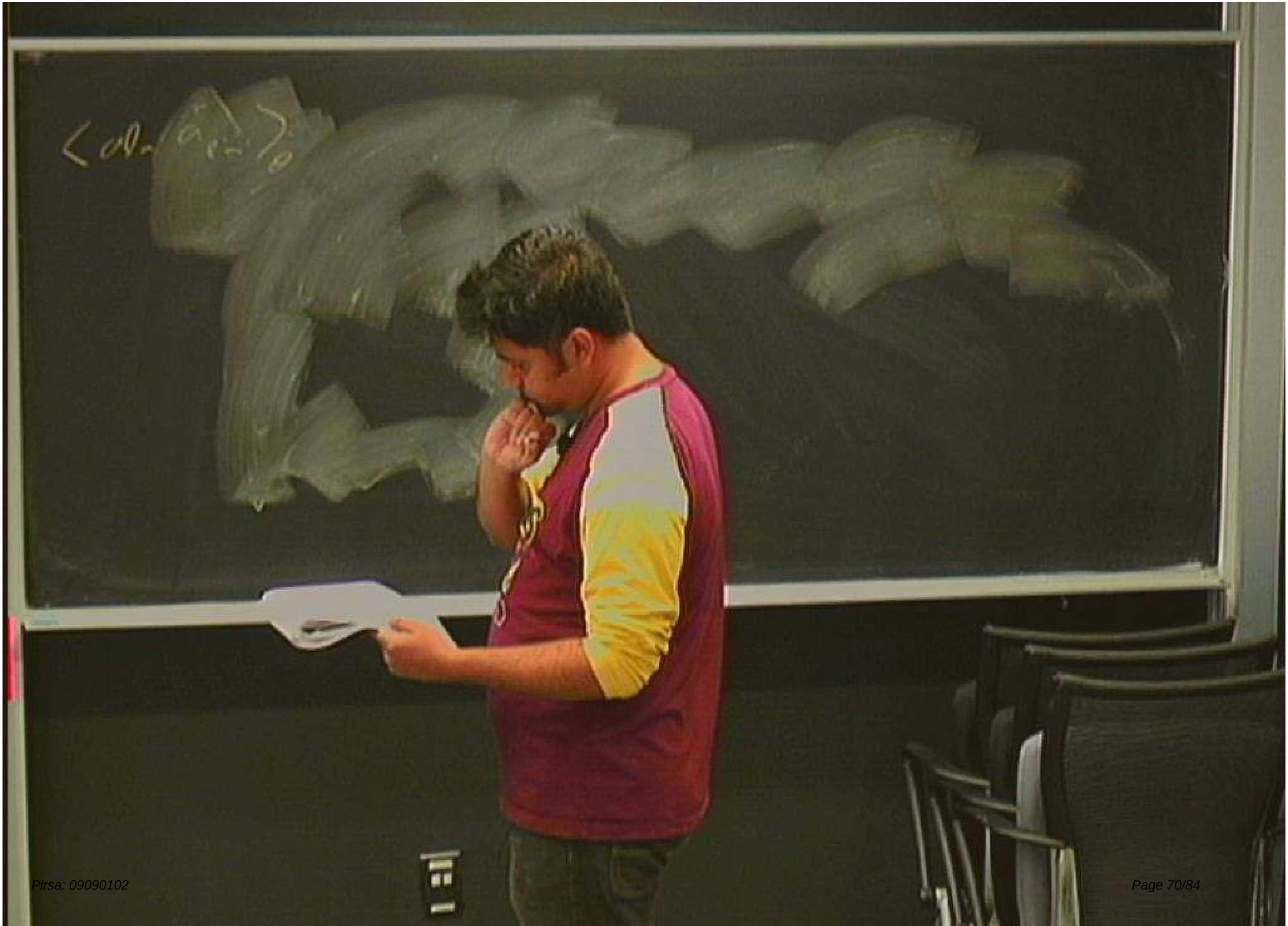
$$m_\theta \downarrow \qquad \downarrow$$

$$m_\theta\, \alpha \otimes \beta \longrightarrow \rho(g)\, m_\theta\, f \otimes g$$

$$P_\Phi = P_0(h) \cosh(H\vec{\theta} \cdot \vec{h})$$

$$m_\theta = m_0 F^{-1} = m_0 e^{i\theta_\cdot P_\mu \otimes P_\nu} \to (U(\theta))$$





$$\Delta(g_1) = \Delta(\cdot\, g_1 g_2)$$

$$\begin{array}{ccc}
\alpha \otimes \beta & \xrightarrow{\Delta} & \int \otimes \beta \quad \Delta(\alpha \otimes \beta) \\
m_\theta \Big\downarrow & & \Big\downarrow \; \sum_\alpha g^{(1)}_{m_\theta} \otimes g^{(1)}_{(1)} \\
m_\theta\, \alpha \otimes \beta & \longrightarrow & \rho(g)\; m_\theta\, f(\omega)
\end{array}$$

$$\Delta_\theta = F^{-1} \Delta_0 F$$

$$m_\theta = m_0 F = m_0 e^{i\theta \cdot P_\mu \otimes P_\nu} \longrightarrow (\mathcal{U}Y(\theta))$$

$$P_\theta = P_0(h) \cosh\left(H \vec{\theta} \cdot \vec{h}\right)$$

$\ldots) = \Delta(g_1 g_2)$

$$\alpha \otimes \beta \xrightarrow{\Delta} 1 \otimes \beta \quad \Delta(\alpha \otimes \beta)$$

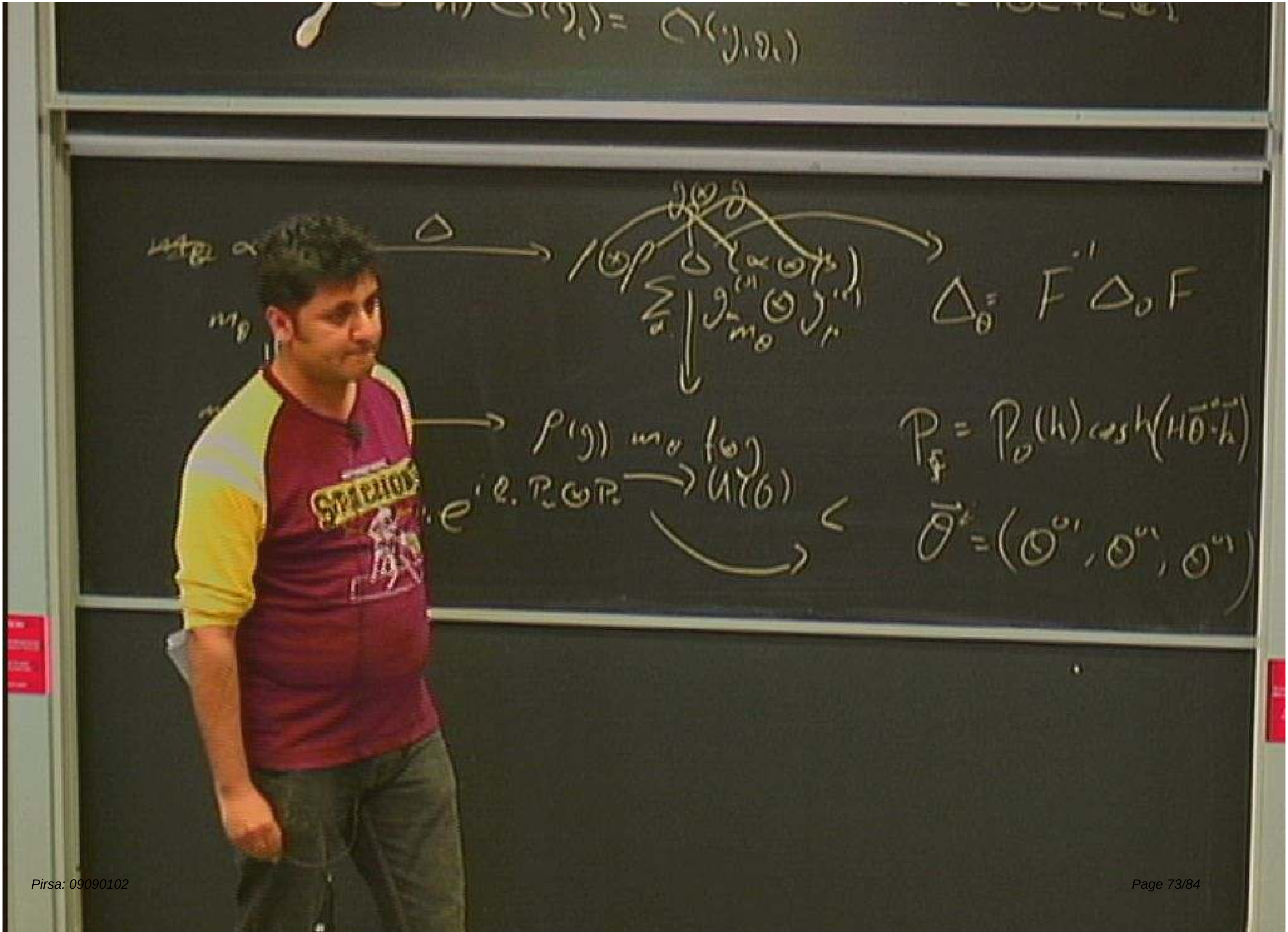
$$\sum_{g} g^{(1)} \otimes g^{(2)}$$

$$\Delta_\theta = F^{-1} \Delta_0 F$$

$$m_\theta \; \alpha \otimes \beta \longrightarrow \rho(g)) \; m_\theta \, f(\omega)$$

$$P_\theta = P_0(h) \cosh(H \vec{\theta} \cdot \vec{h})$$

$$m_\theta = m_0 F^{-1} = m_0 e^{i\theta \cdot P_\mu \otimes P_\nu} \longrightarrow (h\gamma(\theta))$$



$$\cdots (g_2) = \cap(g_1, g_2)$$

$$\alpha \otimes \beta \xrightarrow{\Delta} f \otimes f \quad \Delta(\alpha\otimes\beta)$$

$$m_\theta \downarrow \qquad \sum_\alpha \mathcal{g}^{(1)}_\alpha \otimes \mathcal{g}^{(1)}_{\alpha} \quad m_\theta$$

$$\Delta_\theta = F^{-1} \Delta_0 F$$

$$m_0\, \alpha\otimes\beta \longrightarrow f(g)\; m_0\; f(g)$$

$$P_\theta = P_0(h) \cosh(H \vec\theta \cdot \vec h)$$

$$m_\theta = m_0 F^{-1} = m_0 e^{i\theta P_\alpha \otimes P_\alpha} \longrightarrow (U\gamma(G)) <$$

$$\vec\theta = (\theta^{01}, \theta^{02}, \theta^{03})$$

$$\Delta(g_1) = \Delta(g_1, g_2)$$

$$\alpha \otimes \beta \xrightarrow{\Delta} \gamma \otimes \rho \quad \Delta(\alpha \otimes \beta) \quad \sum_{(g)} g_{(1)} \otimes g_{(2)}$$

$$m_\theta \downarrow \qquad \downarrow m_\theta$$

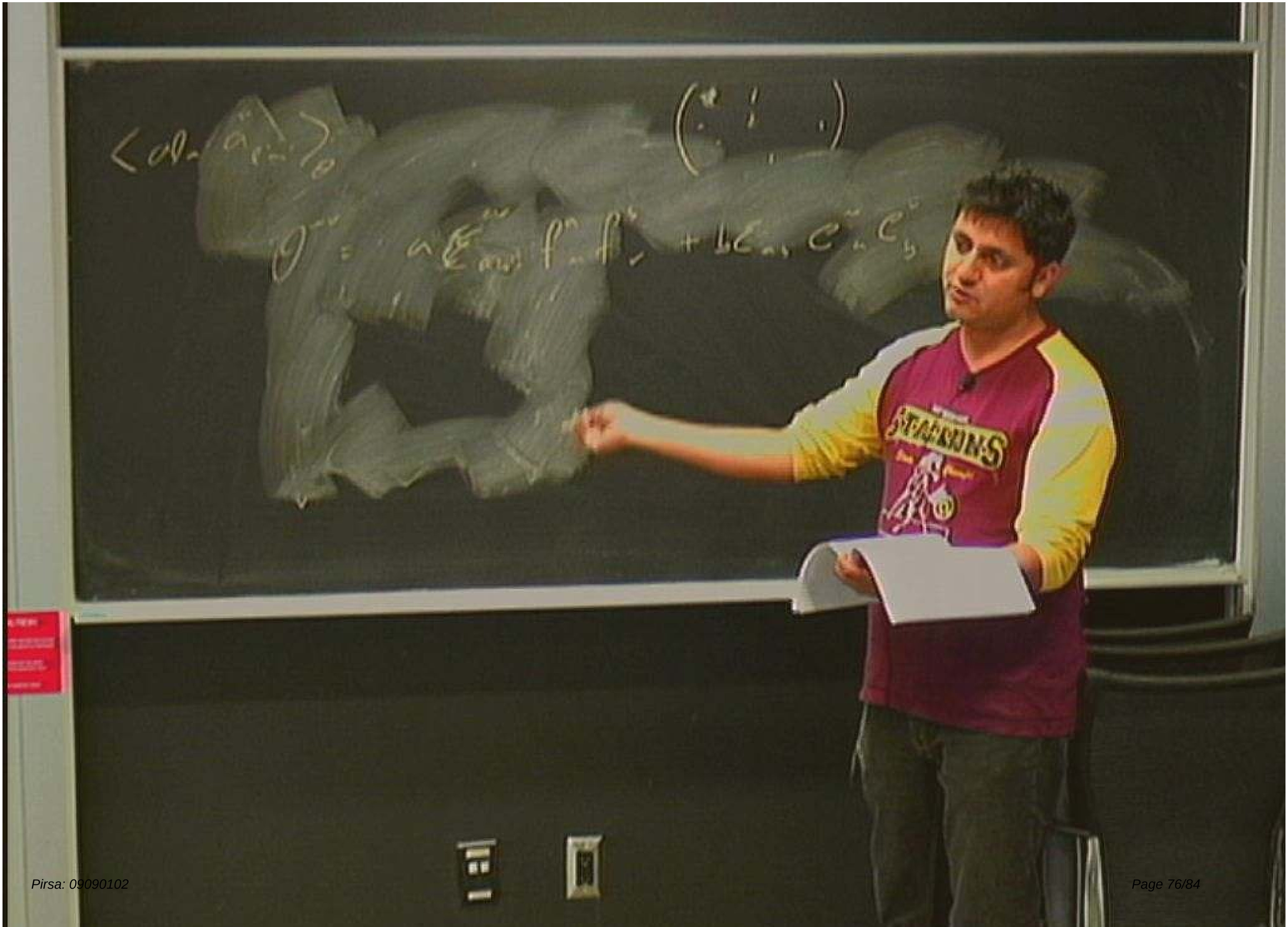
$$m_0\, \alpha \otimes \beta \longrightarrow \rho(g) \quad m_\theta\, f(g)$$

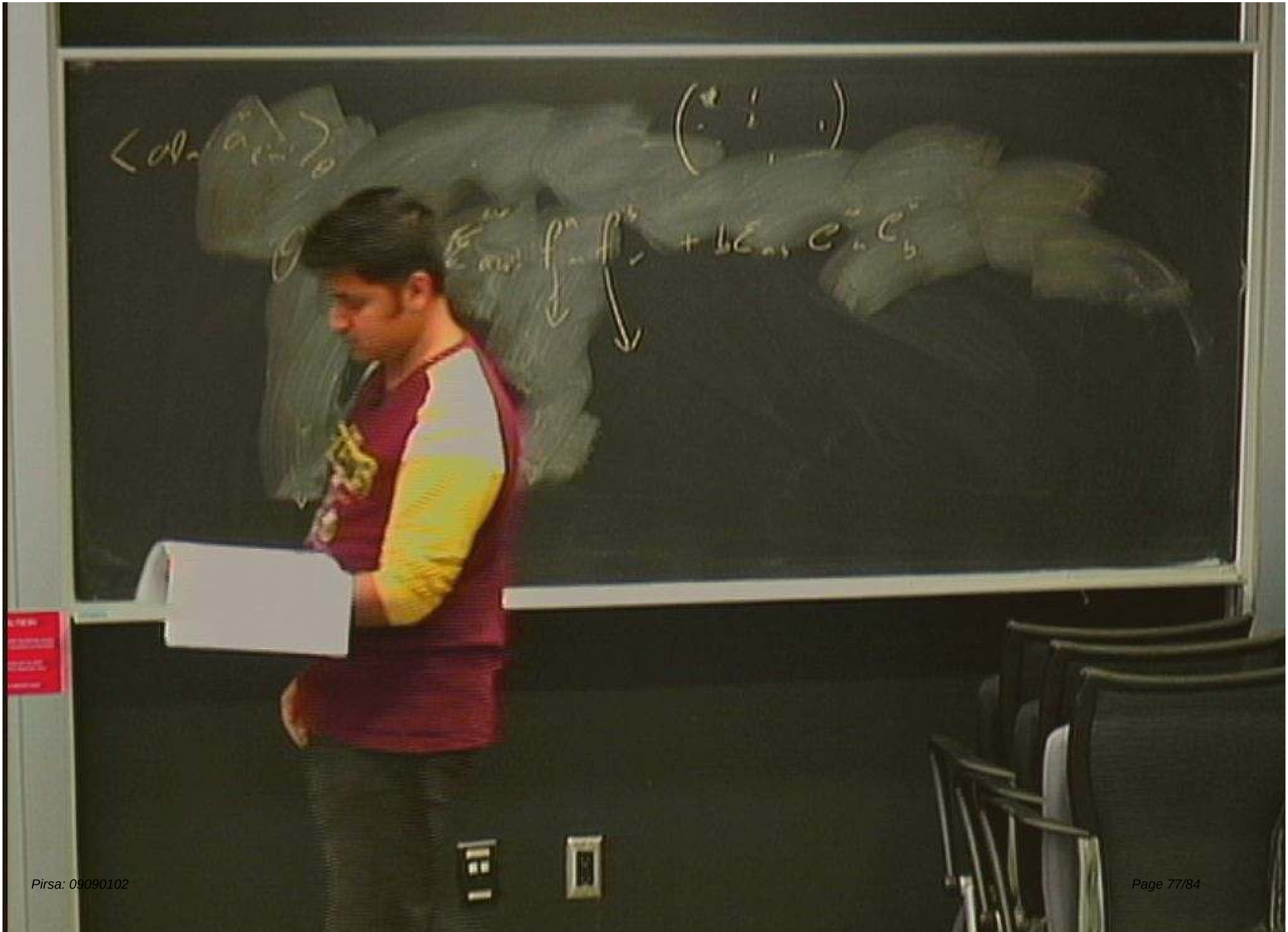
$$m_\theta = m_0 F^{-1} = m_0\, e^{i\theta \cdot P_0 \otimes P_0} \longrightarrow (\alpha\gamma(\beta))$$

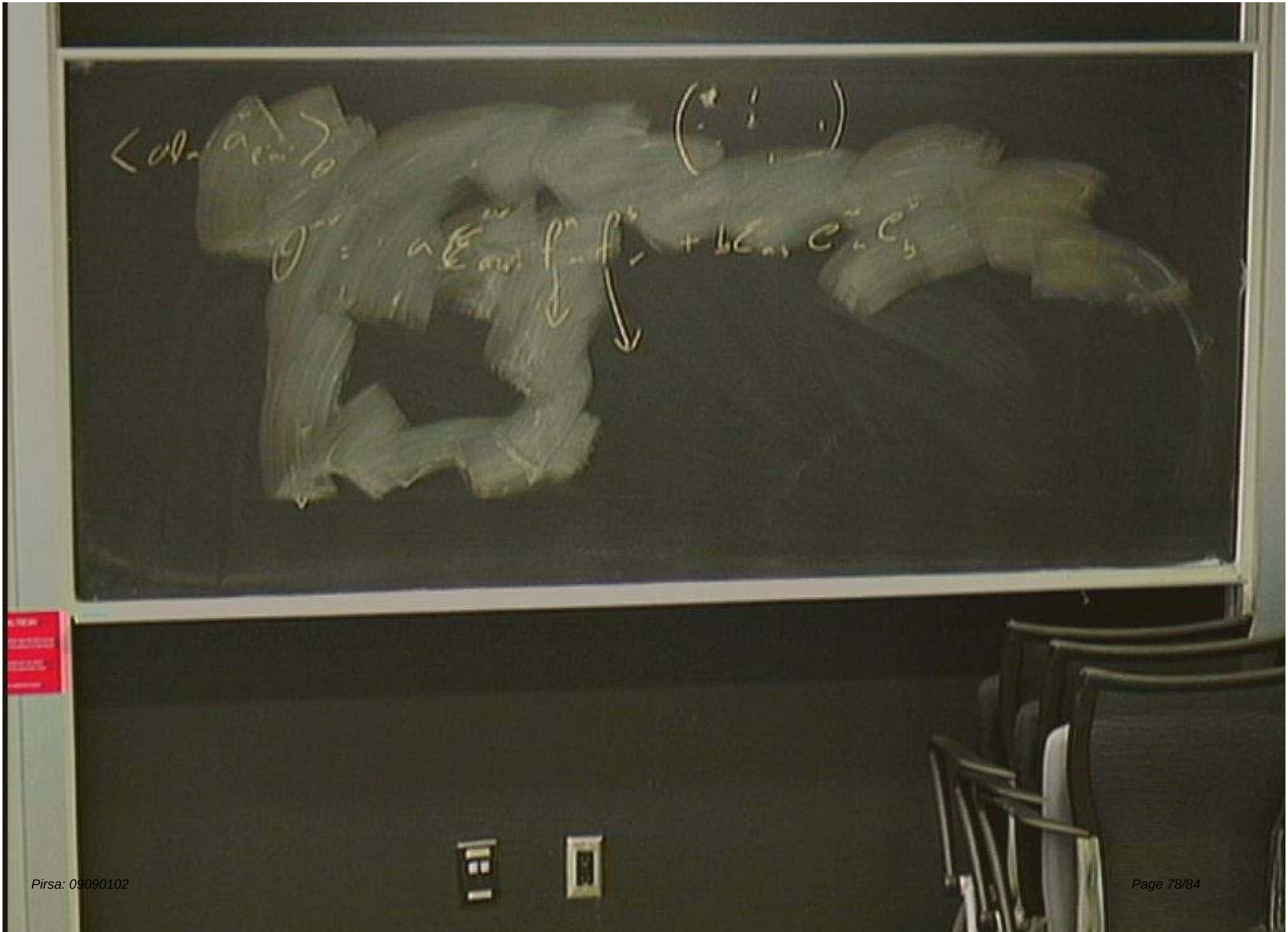
$$\Delta_\theta = F^{-1} \Delta_0 F$$

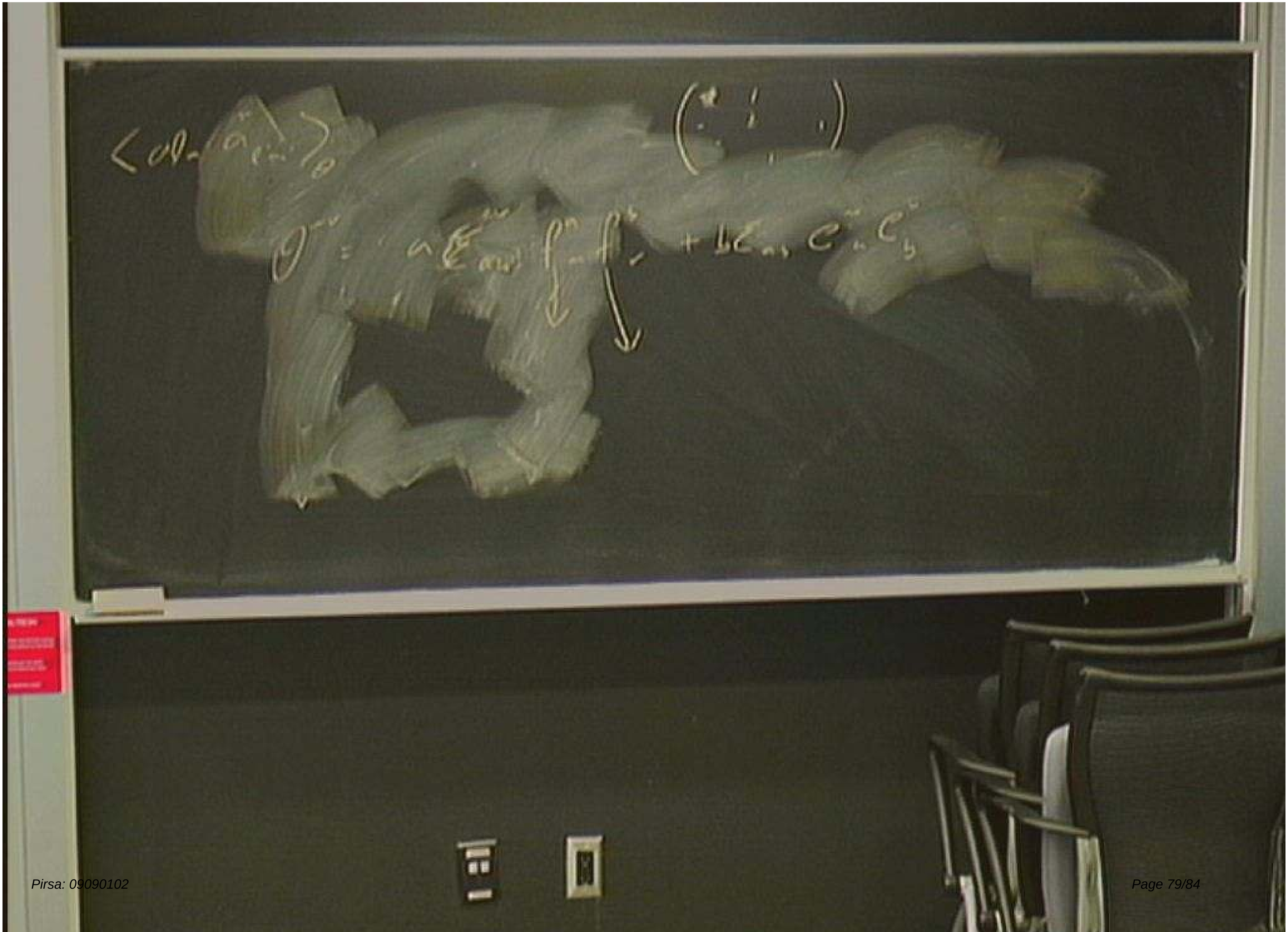
$$P_\theta = P_0(h) \cosh(H \vec{\theta} \cdot \vec{h})$$

$$\vec{\theta} = (\theta^{01}, \theta^{02}, \theta^{03})$$









$$\alpha \otimes \beta \xrightarrow{\Delta} (\ \otimes\ )\,\Delta(\alpha \otimes \beta) \longrightarrow \Delta_\theta = F^{-1}\Delta_0 F$$

$$\sum_\alpha g^{(1)}_\alpha \otimes g^{(2)}_\alpha$$

$$m_\theta\, \alpha \otimes \beta \longrightarrow \rho(g)\, m_\theta\, \ell(g)$$

$$m_\theta = m_0 F^{-1} = m_0\, e^{i\theta_{\mu\nu} P_\mu \otimes P_\nu} \longrightarrow (U(\theta))$$

$$P_\theta = P_0(k)\cosh(H\vec{\theta}\cdot\vec{k})$$

$$\Rightarrow P_0(\ )\left(1 + \frac{H^2}{2}(\vec{\theta}\cdot\vec{k})^2\right)$$

$$\alpha \otimes \beta \xrightarrow{\Delta} \Delta(\alpha \otimes \beta)$$

$$m_\theta\, \alpha \otimes \beta \longrightarrow \rho(g)\; m_\theta\, (\omega)$$

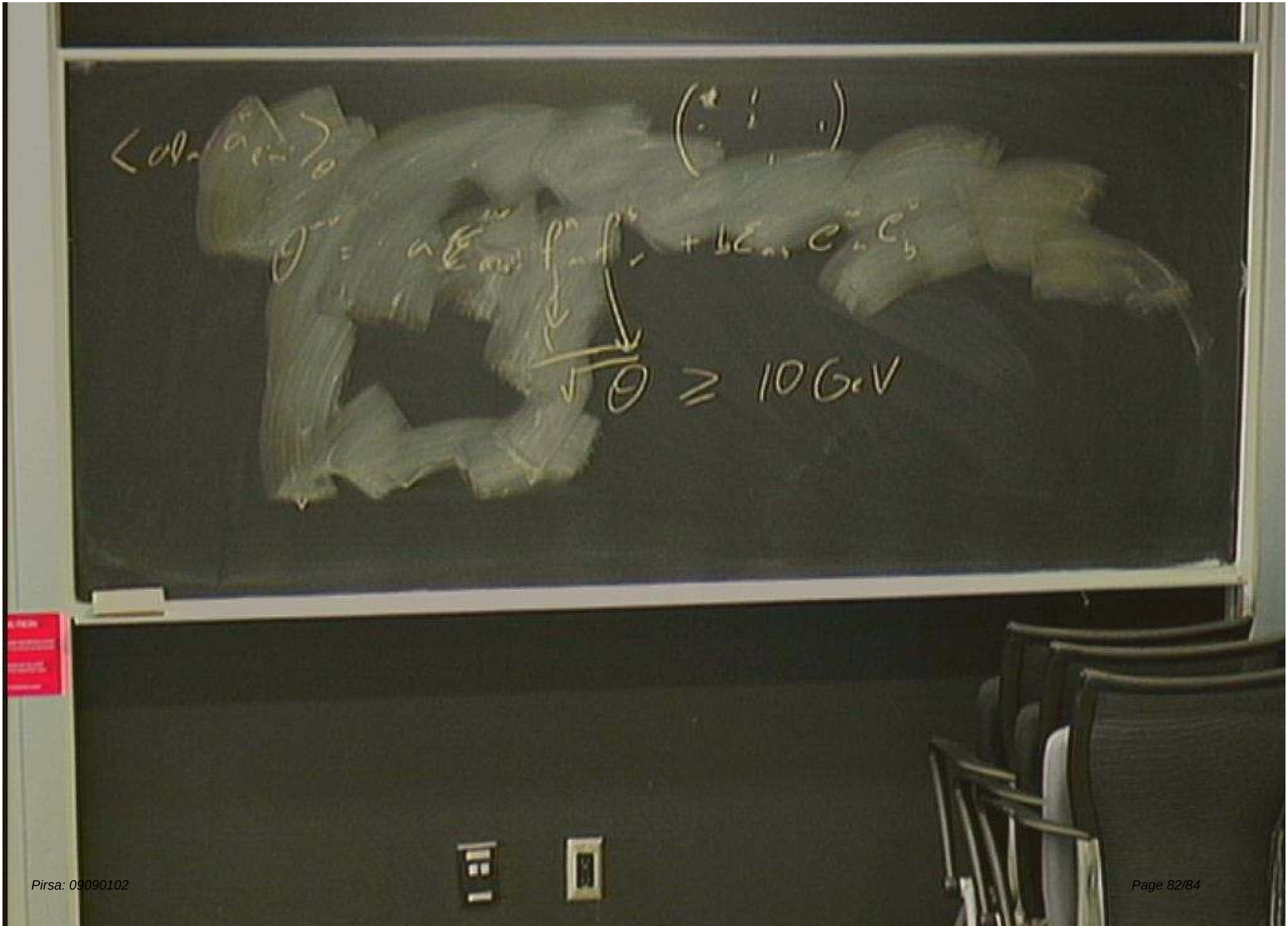
$$\sum_\alpha g^{(1)}_\alpha \otimes g^{(2)}_\alpha$$

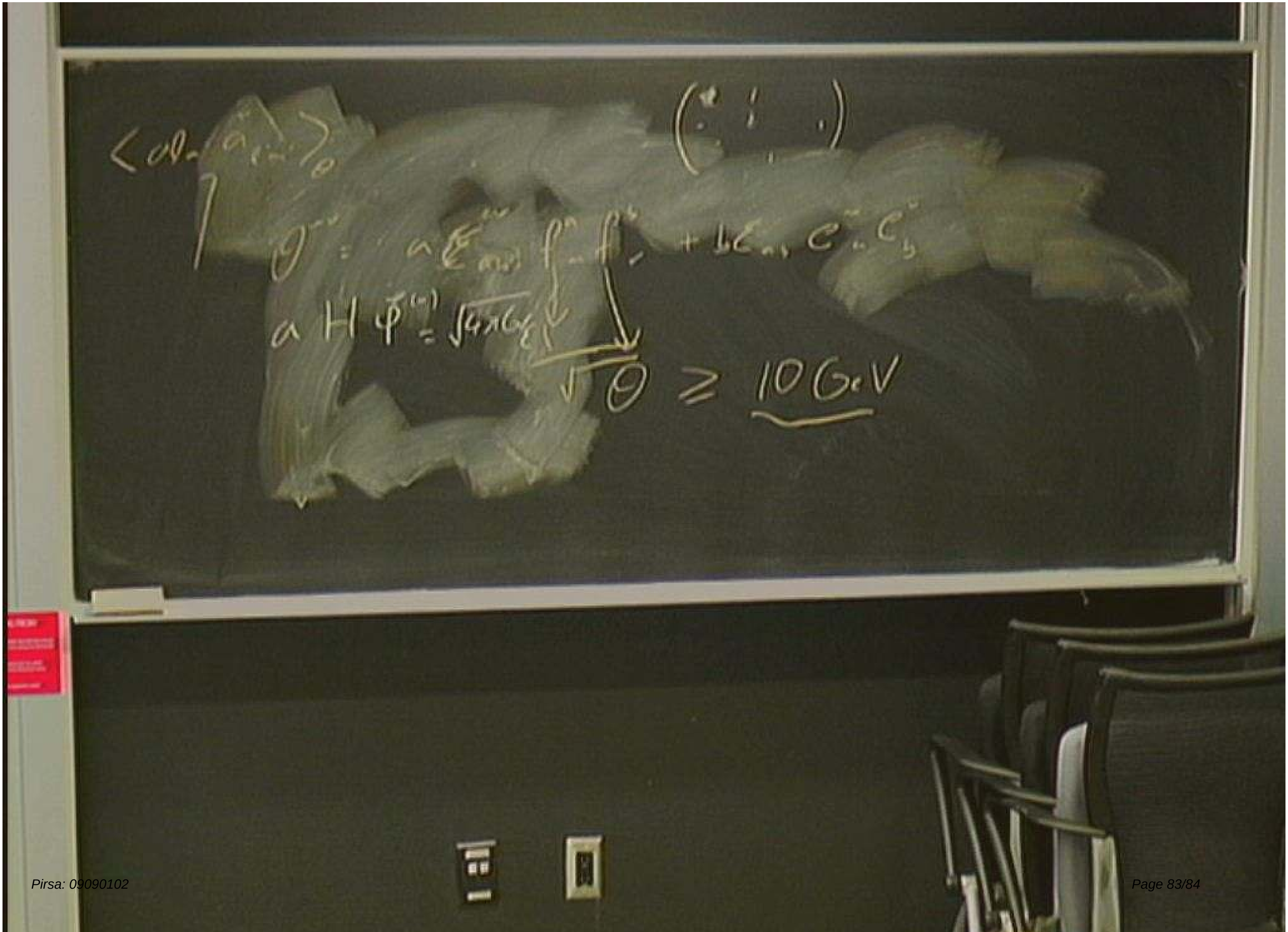
$$\Delta_\theta = F^{-1} \Delta_0 F$$

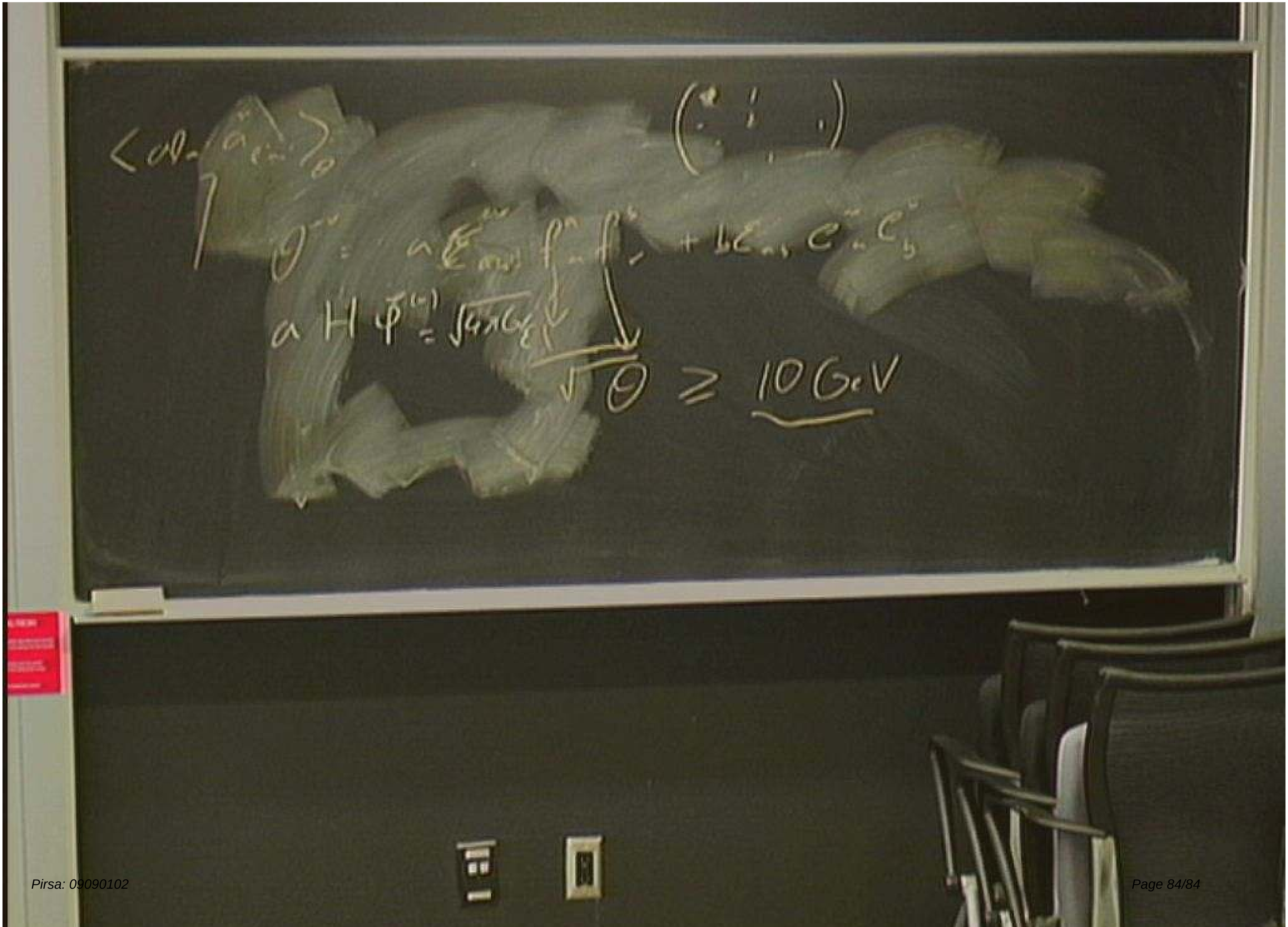
$$m_\theta = m_0 F = m_0\, e^{i\theta \cdot P \otimes P}$$

$$P_\theta = P_0(h) \cosh(H \vec{\theta} \cdot \vec{h})$$

$$= P_0(\;)\left(1 + \frac{\theta^2}{2}(\vec{\theta} \cdot \vec{h})^2\right)$$


√Θ ≥ 10 GeV


√Θ ≥ 10 GeV


√Θ ≥ 10 GeV