Title: A First-Principles Implementation of Scale Invariance Using Best Matching

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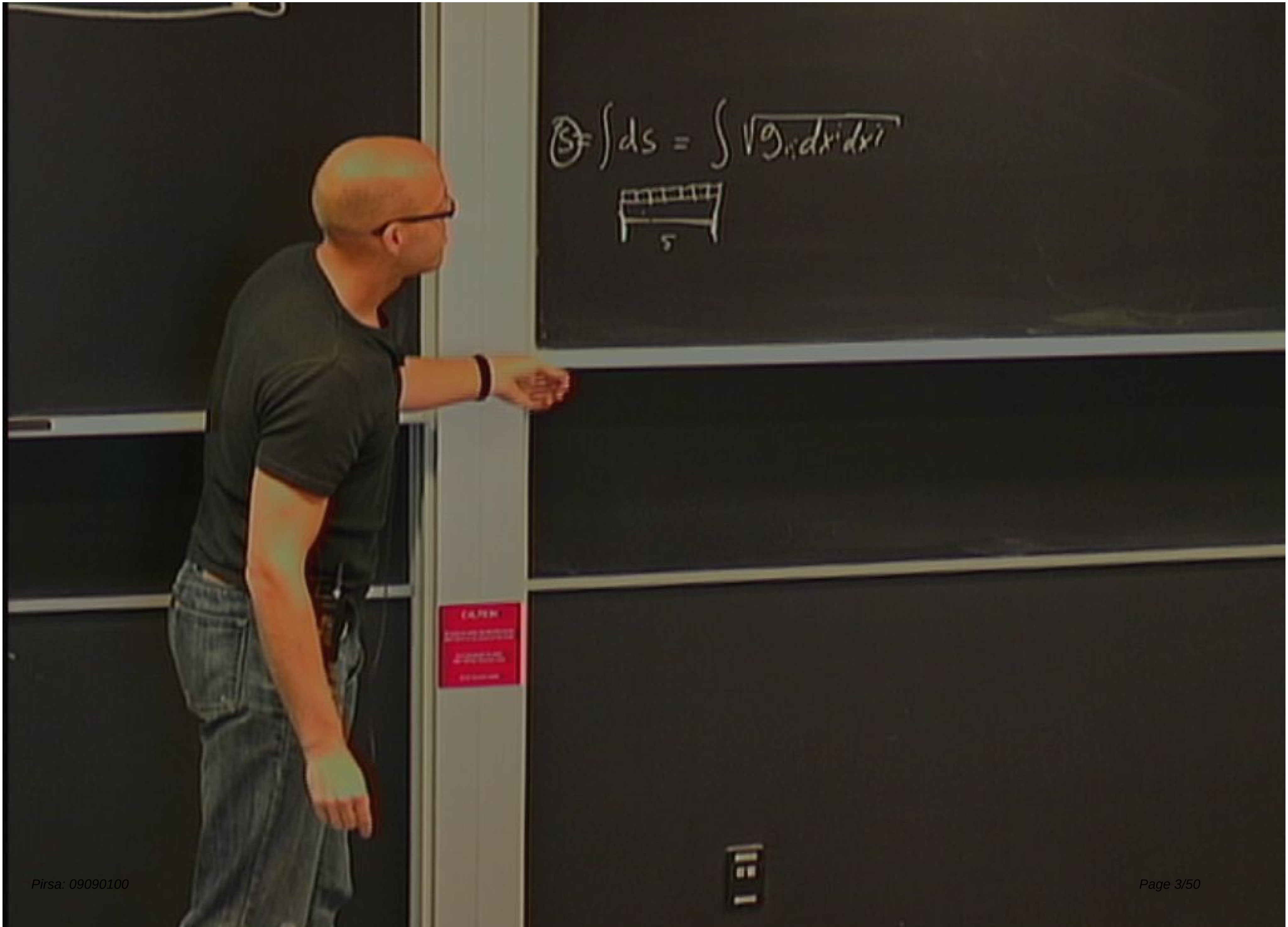
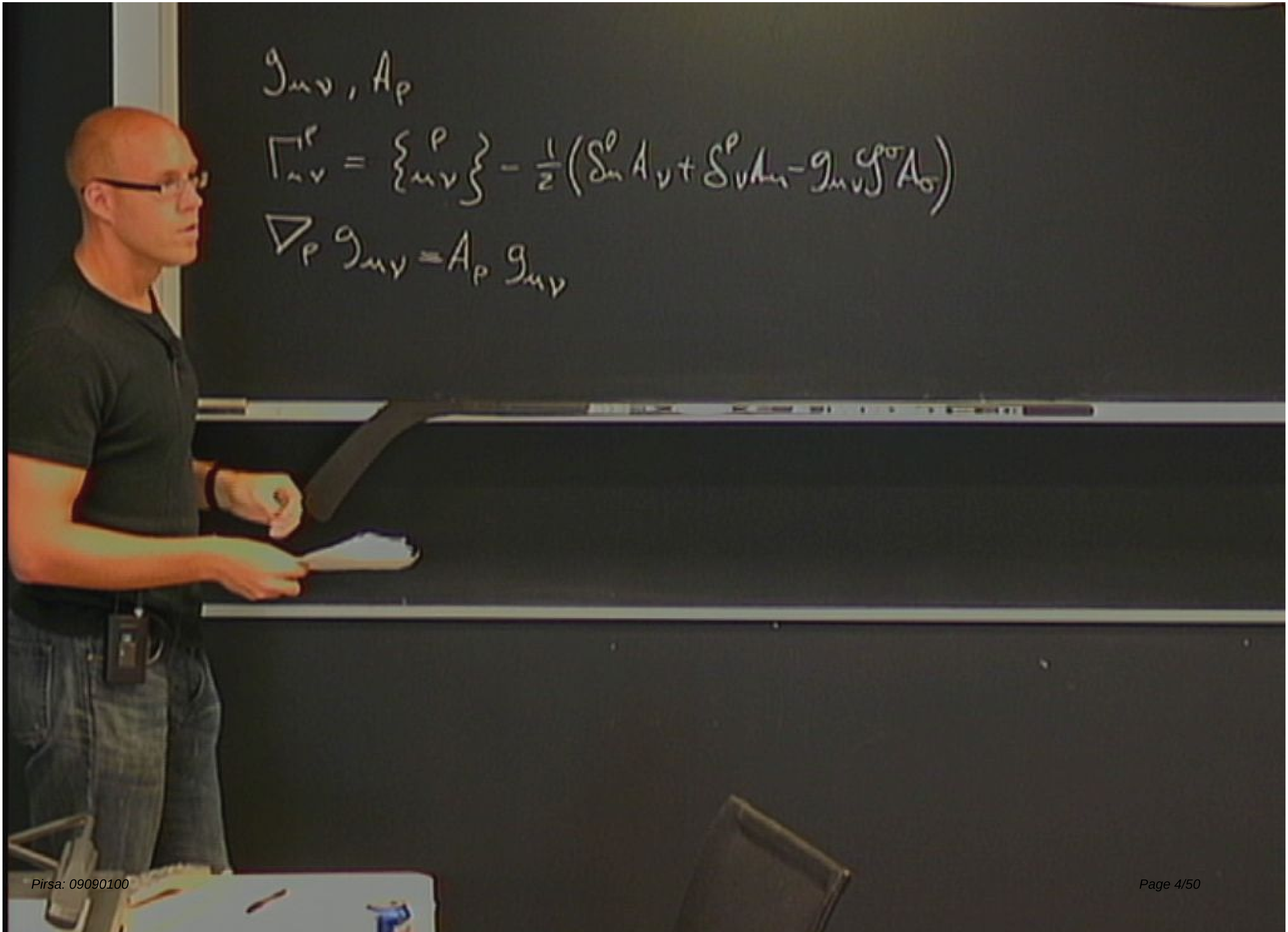
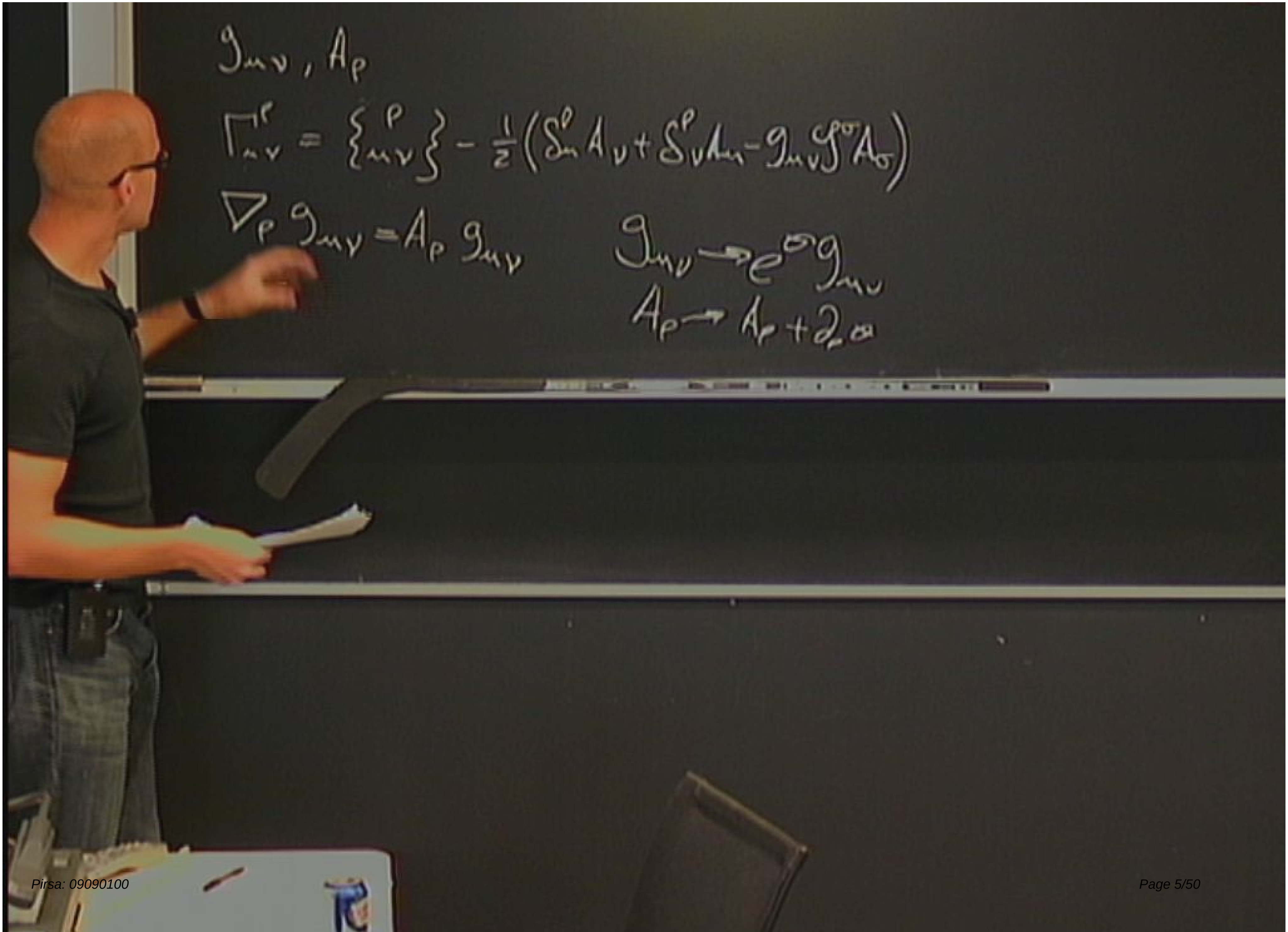
Abstract: We present a first-principles implementation of {\em spatial} scale invariance as a local gauge symmetry in geometry dynamics using the method of best matching. In addition to the 3-metric, the proposed scale invariant theory also contains a 3-vector potential A_k as a dynamical variable. Although some of the mathematics is similar to Weyl's ingenious, but physically questionable, theory, the equations of motion of this new theory are second order in time-derivatives. It is tempting to try to interpret the vector potential A_k as the electromagnetic field. We exhibit four independent reasons for not giving into this temptation. A more likely possibility is that it can play the role of ``dark matter''. Indeed, as noted in scale invariance seems to play a role in the MOND phenomenology. Spatial boundary conditions are derived from the free-endpoint variation method and a preliminary analysis of the constraints and their propagation in the Hamiltonian formulation is presented.


$S = \int ds = \int \sqrt{g_{ij} dx^i dx^j}$
S


$g_{\mu\nu}\,,\ A_\rho$
$\Gamma^\rho_{\mu\nu} = \left\{ {\rho \atop \mu\nu} \right\} - \frac{1}{2}\left(\delta^\rho_\mu A_\nu + \delta^\rho_\nu A_\mu - g_{\mu\nu} g^{\rho\sigma} A_\sigma\right)$
$\nabla_\rho g_{\mu\nu} = A_\rho g_{\mu\nu}$


$g_{\mu\nu}\,,\ A_\rho$
$\Gamma^\rho_{\mu\nu} = \left\{ {\rho \atop \mu\nu} \right\} - \frac{1}{2}\left(\delta^\rho_\mu A_\nu + \delta^\rho_\nu A_\mu - g_{\mu\nu} g^{\rho\sigma} A_\sigma\right)$
$\nabla_\rho g_{\mu\nu} = A_\rho g_{\mu\nu}$
$g_{\mu\nu} \to e^\sigma g_{\mu\nu}$
$A_\rho \to A_\rho + \partial_\rho \sigma$

A First-Principles Implementation of Scale Invariance Using Best-Matching

$g_{\mu\nu}, A_\rho$

$\Gamma^\rho_{\mu\nu} = \left\{ {\rho \atop \mu\nu} \right\} - \frac{1}{2}\left(\delta^\rho_\mu A_\nu + \delta^\rho_\nu A_\mu - g_{\mu\nu} g^{\rho\sigma} A_\sigma\right)$

$\nabla_\rho g_{\mu\nu} = A_\rho g_{\mu\nu}$

$g_{\mu\nu} \to e^{\omega} g_{\mu\nu}$

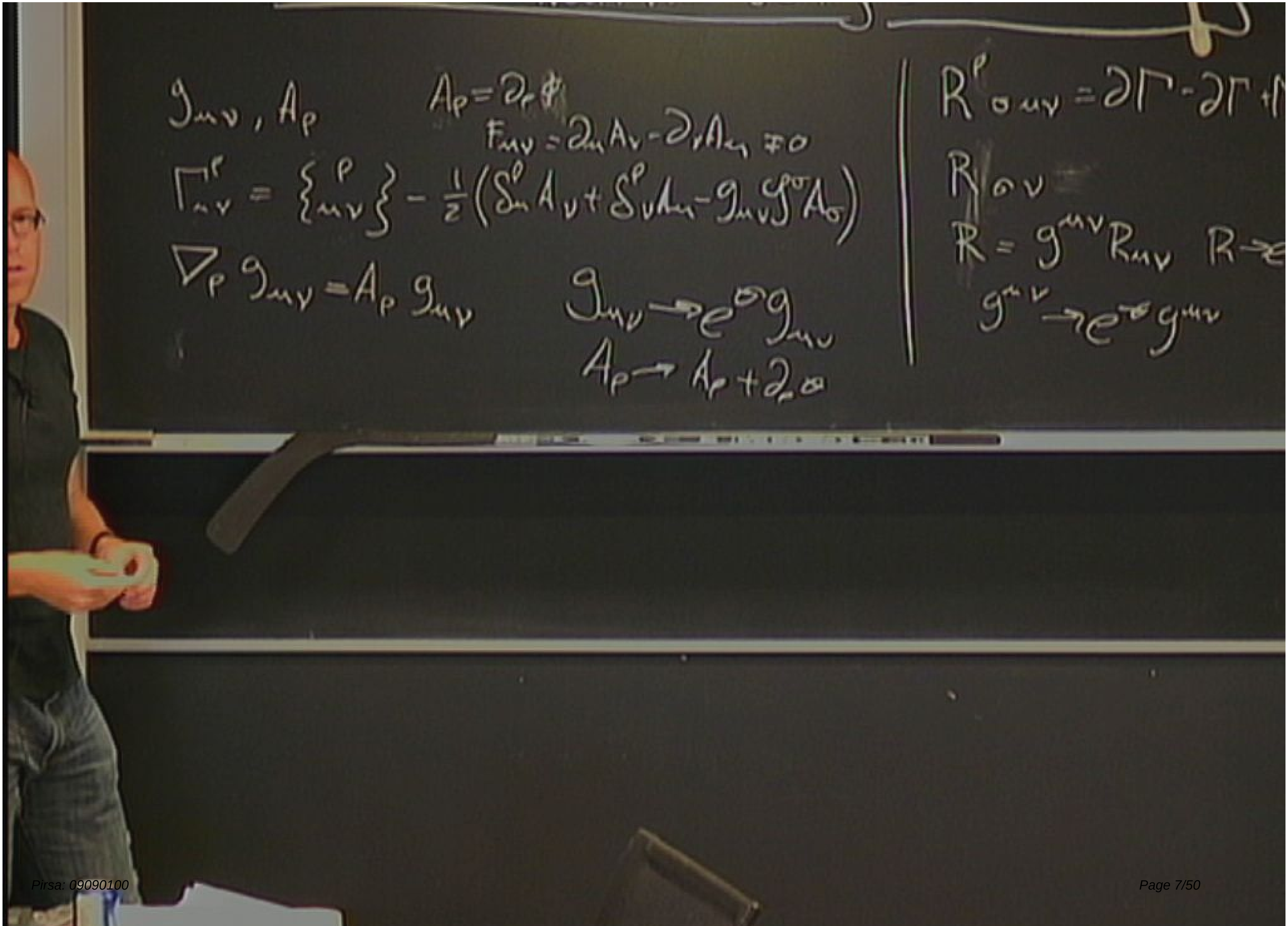
$A_\rho \to A_\rho + \partial_\rho \omega$

$R^\rho{}_{\sigma\mu\nu} = \partial\Gamma - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$

$R_{\sigma\nu}$

$R = g^{\mu\nu} R_{\mu\nu}$ $\quad R \to e^{-\omega} R$

$g^{\mu\nu} \to e^{-\omega} g^{\mu\nu}$



$g_{\mu\nu}\,,\ A_\rho$
$A_\rho = \partial_\rho \phi$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \neq 0$
$\Gamma^\rho_{\mu\nu} = \left\{{\rho \atop \mu\nu}\right\} - \frac{1}{2}\left(\delta^\rho_\mu A_\nu + \delta^\rho_\nu A_\mu - g_{\mu\nu} g^{\rho\sigma} A_\sigma\right)$
$\nabla_\rho g_{\mu\nu} = A_\rho g_{\mu\nu}$
$g_{\mu\nu} \to e^{\sigma} g_{\mu\nu}$
$A_\rho \to A_\rho + \partial_\rho \sigma$
$R^\rho{}_{\sigma\mu\nu} = \partial\Gamma - \partial\Gamma +$
$R_{\sigma\nu}$
$R = g^{\mu\nu} R_{\mu\nu}$
$R \to e$
$g^{\mu\nu} \to e^{-\sigma} g^{\mu\nu}$

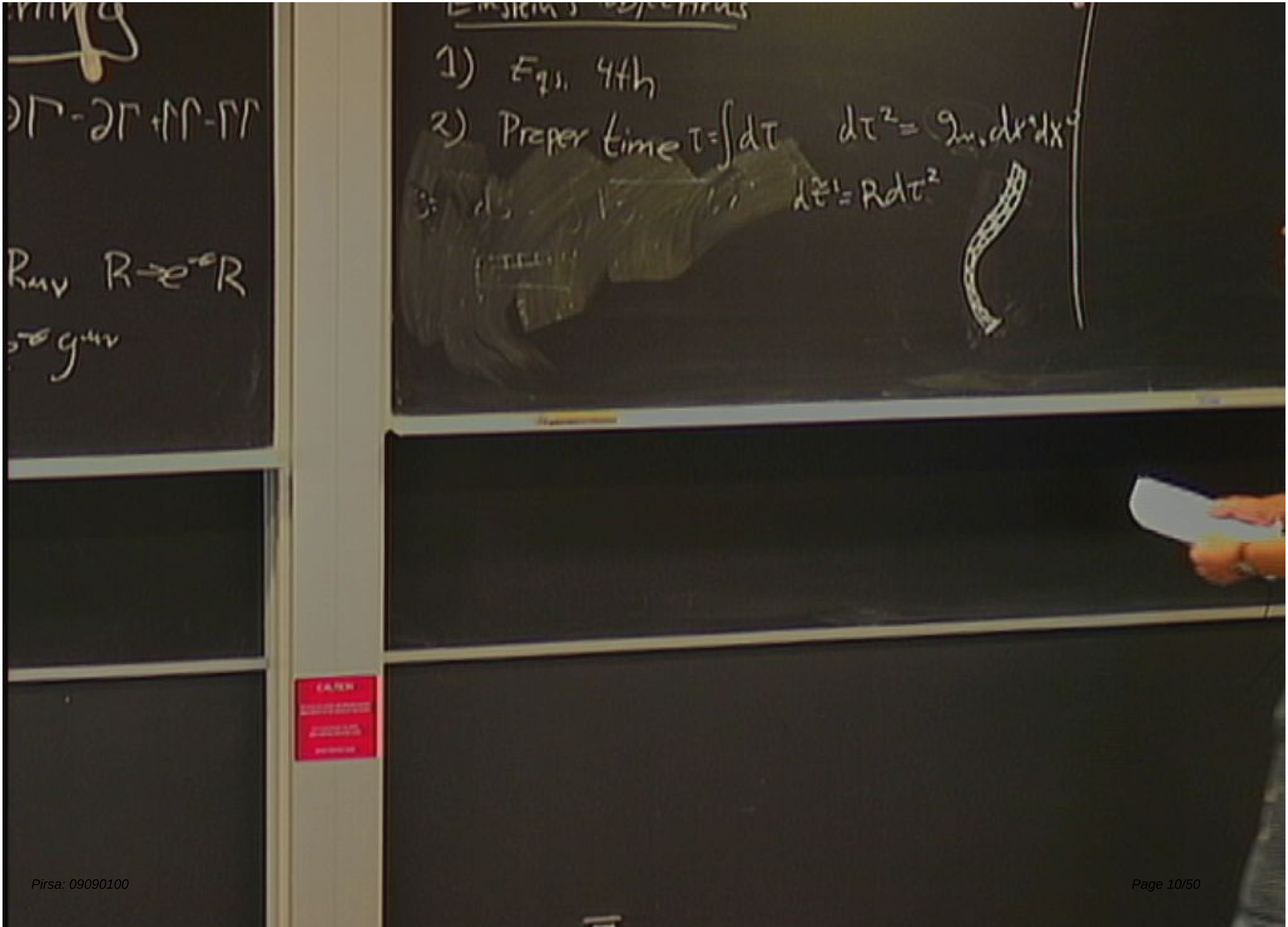
$$\mathcal{L}_W = \sqrt{g}^{\,+2}\left(R^2 + R_{\mu\nu}R^{\mu\nu} + R'_{\sigma\mu\nu}R^{\sigma\mu\nu} + F^2 + R_{\mu\nu}F^{\mu\nu}\right)$$

$$\mathcal{L}_W = \sqrt{g}\,\left(R^2 + R_{\mu\nu}R^{\mu\nu} + R_{\sigma\mu\nu\rho}R^{\sigma\mu\nu\rho} + F^2 + R_{\mu\nu}F^{\mu\nu}\right)$$

Einstein's objections

1) Eqs. 4th

2) Proper time $\tau = \int d\tau \quad d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu$


1) Eq. 4th
2) Proper time τ=∫dτ dτ² = g dx dx
dτ' = R dτ²

$$\mathcal{L}_W = \sqrt{g}\,\left(R^2 + R_{\mu\nu}R^{\mu\nu} + R'_{\sigma\mu\nu\rho}R^{\sigma\mu\nu\rho} + F^2 + R_{\mu\nu}F^{\mu\nu}\right)$$

Einstein's objections

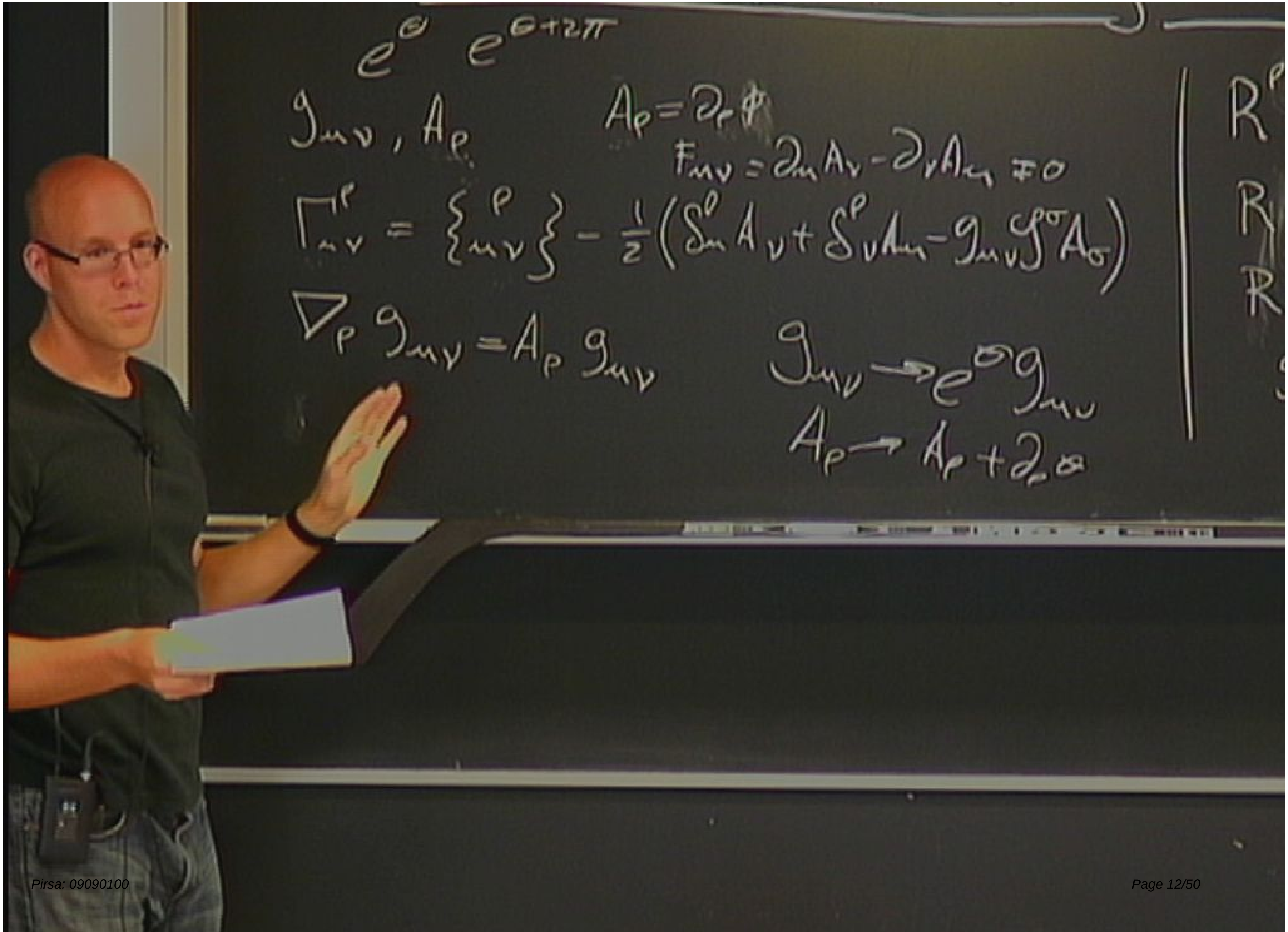
1) Eqs. 4th

2) Proper time: $\tau = \int d\tau$ $\quad d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu$

$d\tilde{\tau}^2 = R\,d\tau^2$

3) Hydrogen spectra

$(C_{\mu\nu\rho\sigma})^2$


e^{Θ} $e^{\Theta+2\pi}$
$g_{\mu\nu}$, A_ρ
$A_\rho = \partial_\rho \phi$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \neq 0$
$\Gamma^\rho_{\mu\nu} = \left\{ {\rho \atop \mu\nu} \right\} - \frac{1}{2}\left(\delta^\rho_\mu A_\nu + \delta^\rho_\nu A_\mu - g_{\mu\nu} g^{\rho\sigma} A_\sigma\right)$
$\nabla_\rho g_{\mu\nu} = A_\rho g_{\mu\nu}$
$g_{\mu\nu} \to e^{\sigma} g_{\mu\nu}$
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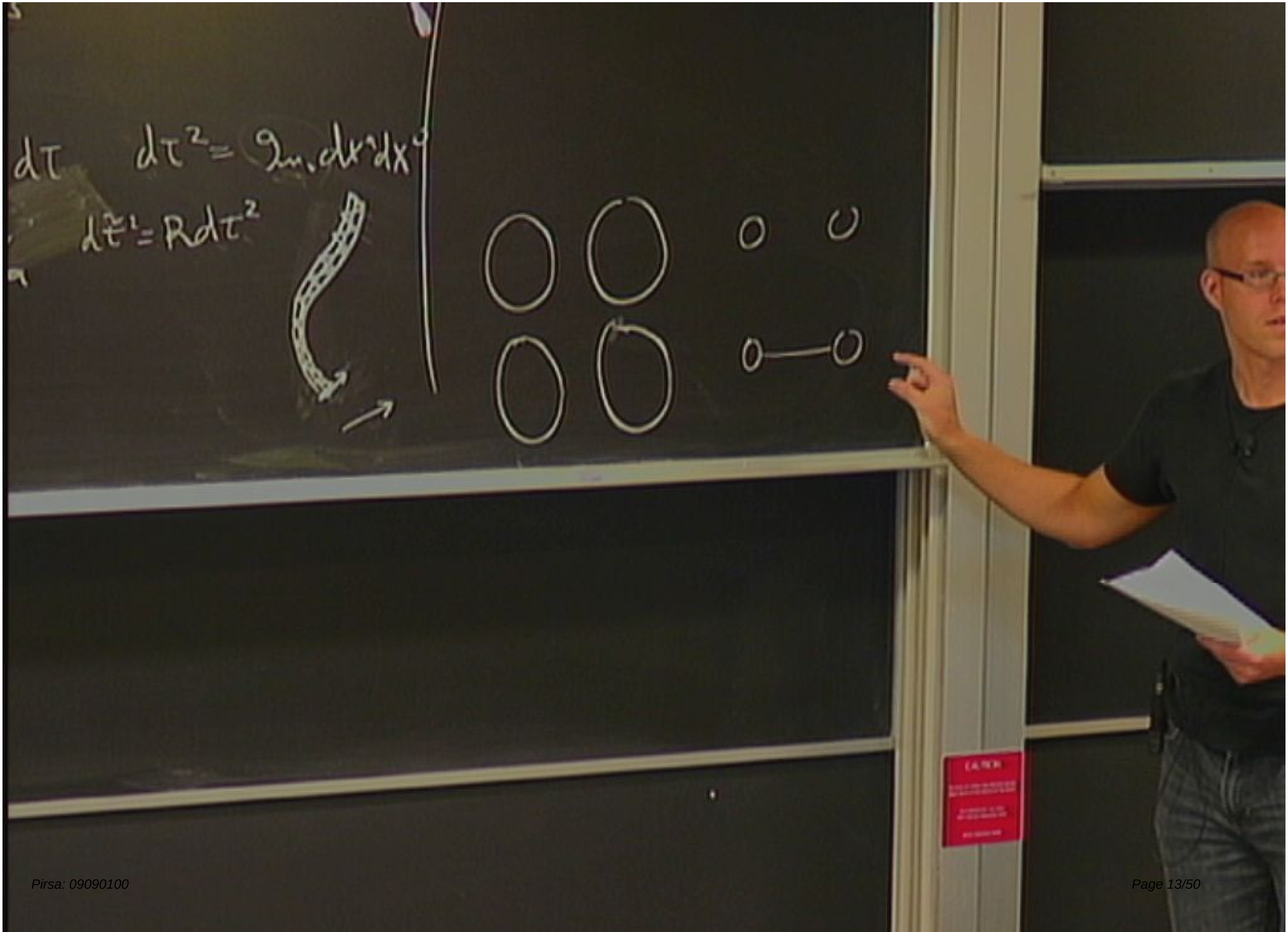
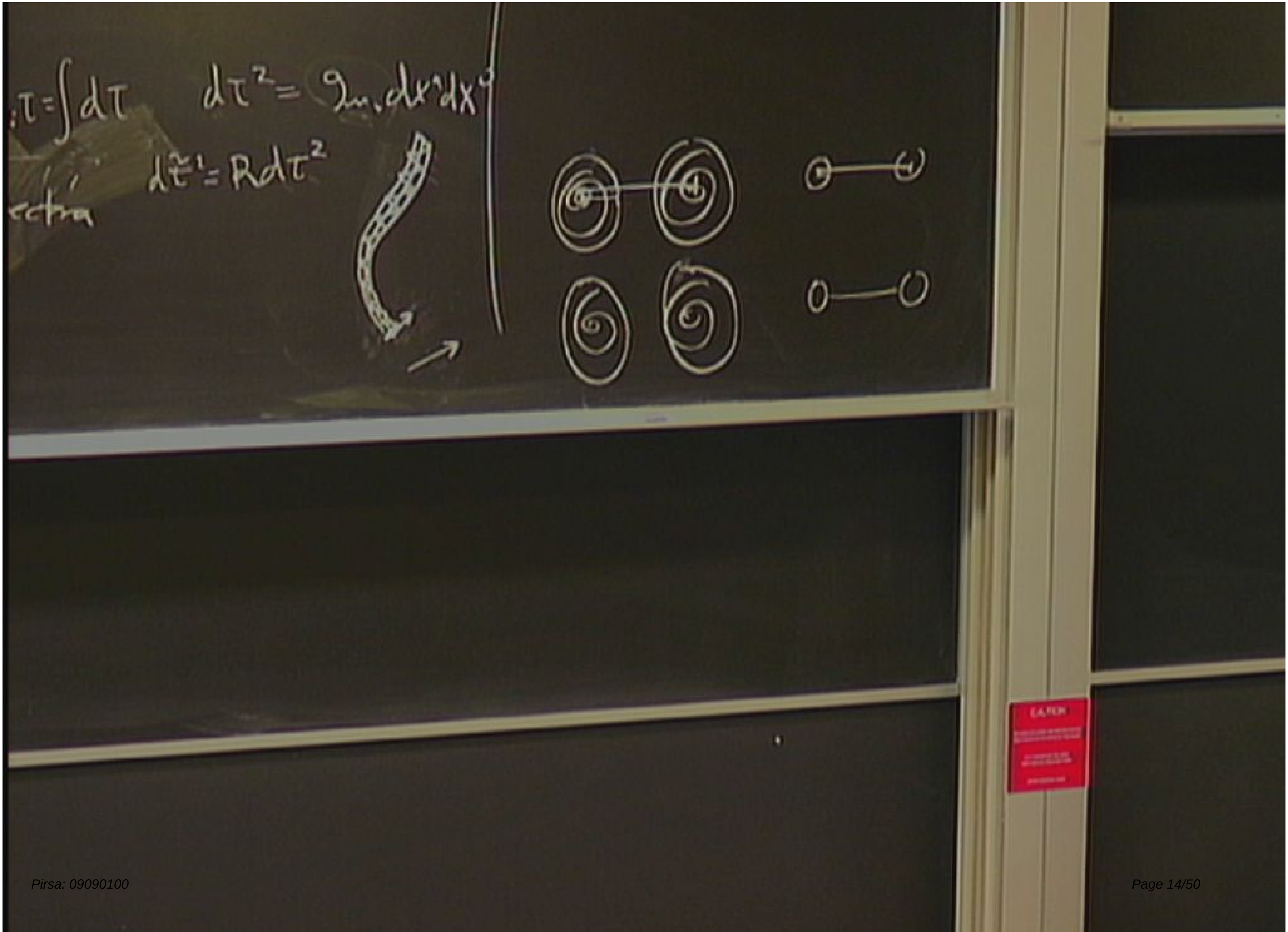
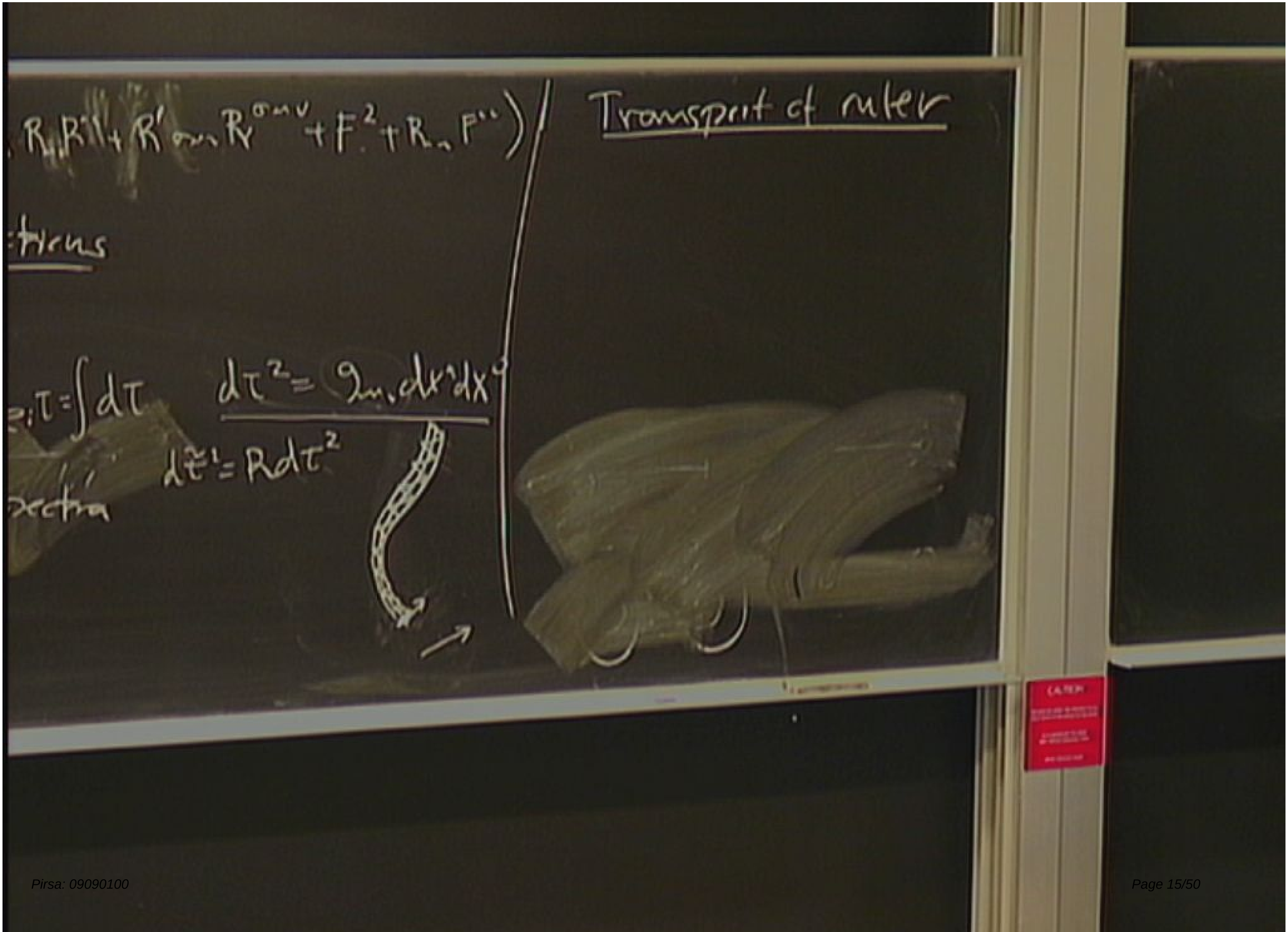
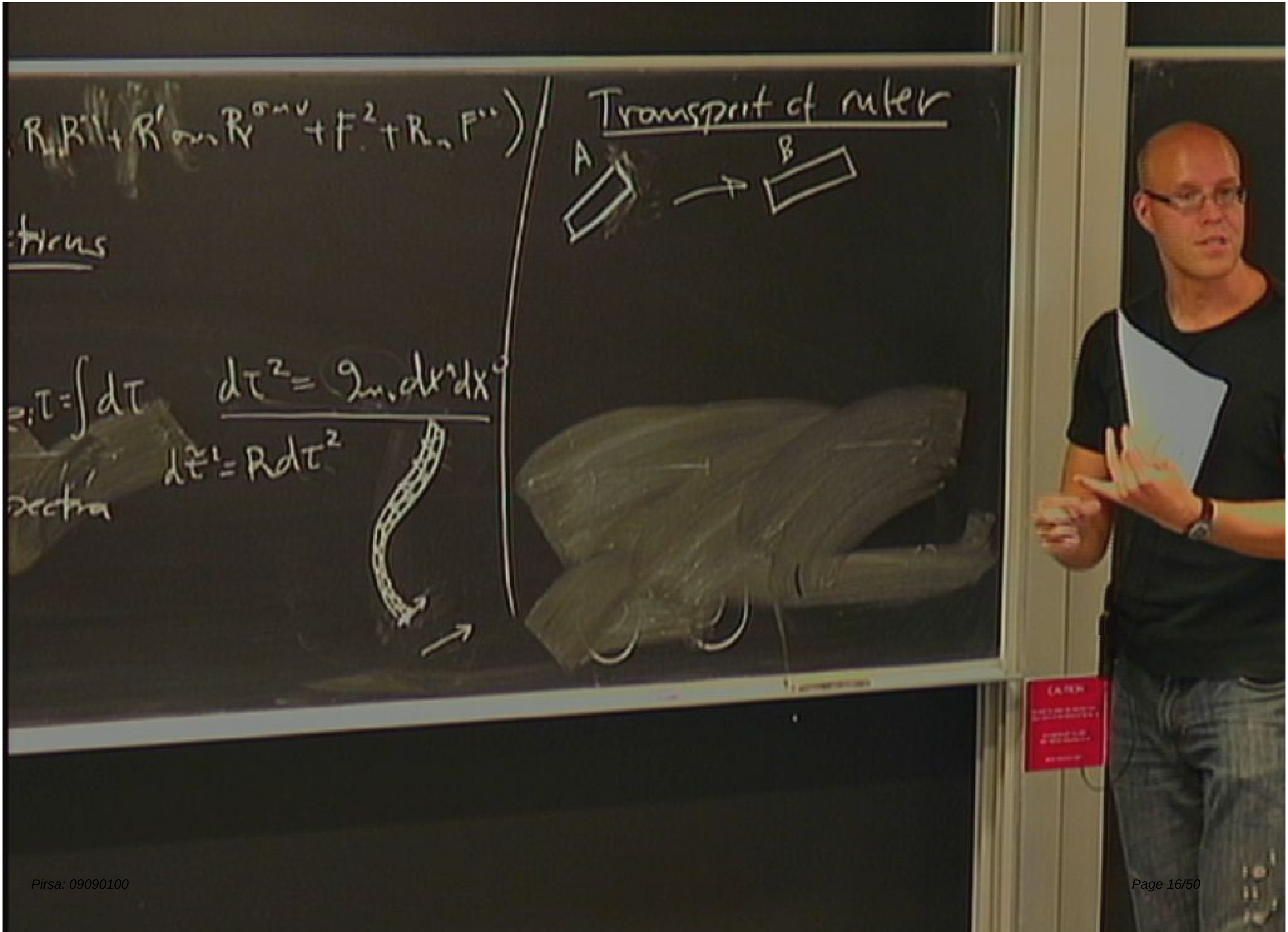
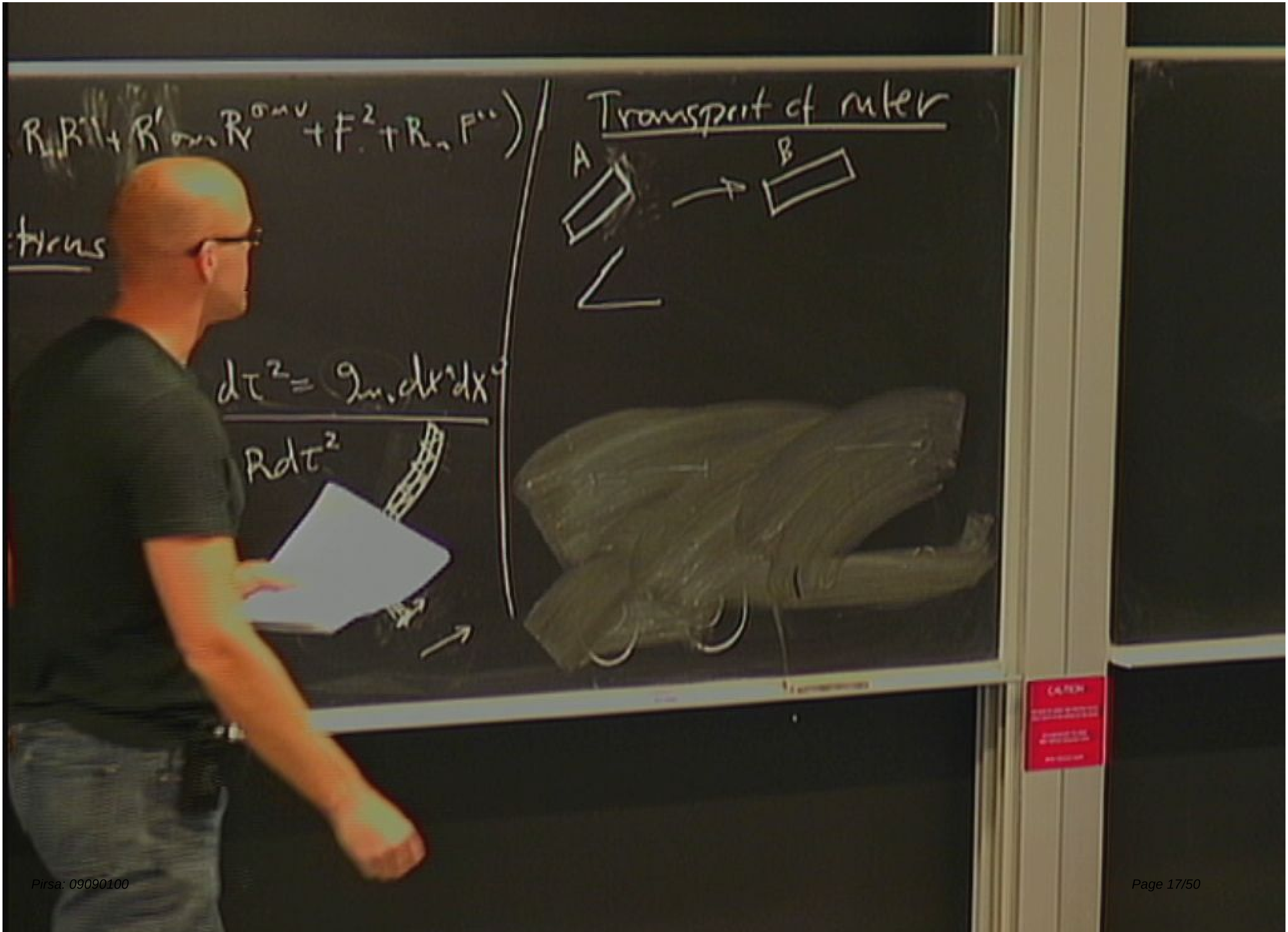

dτ dτ² = g_{μν} dx^μ dx^ν
dτ̃² = R dτ²

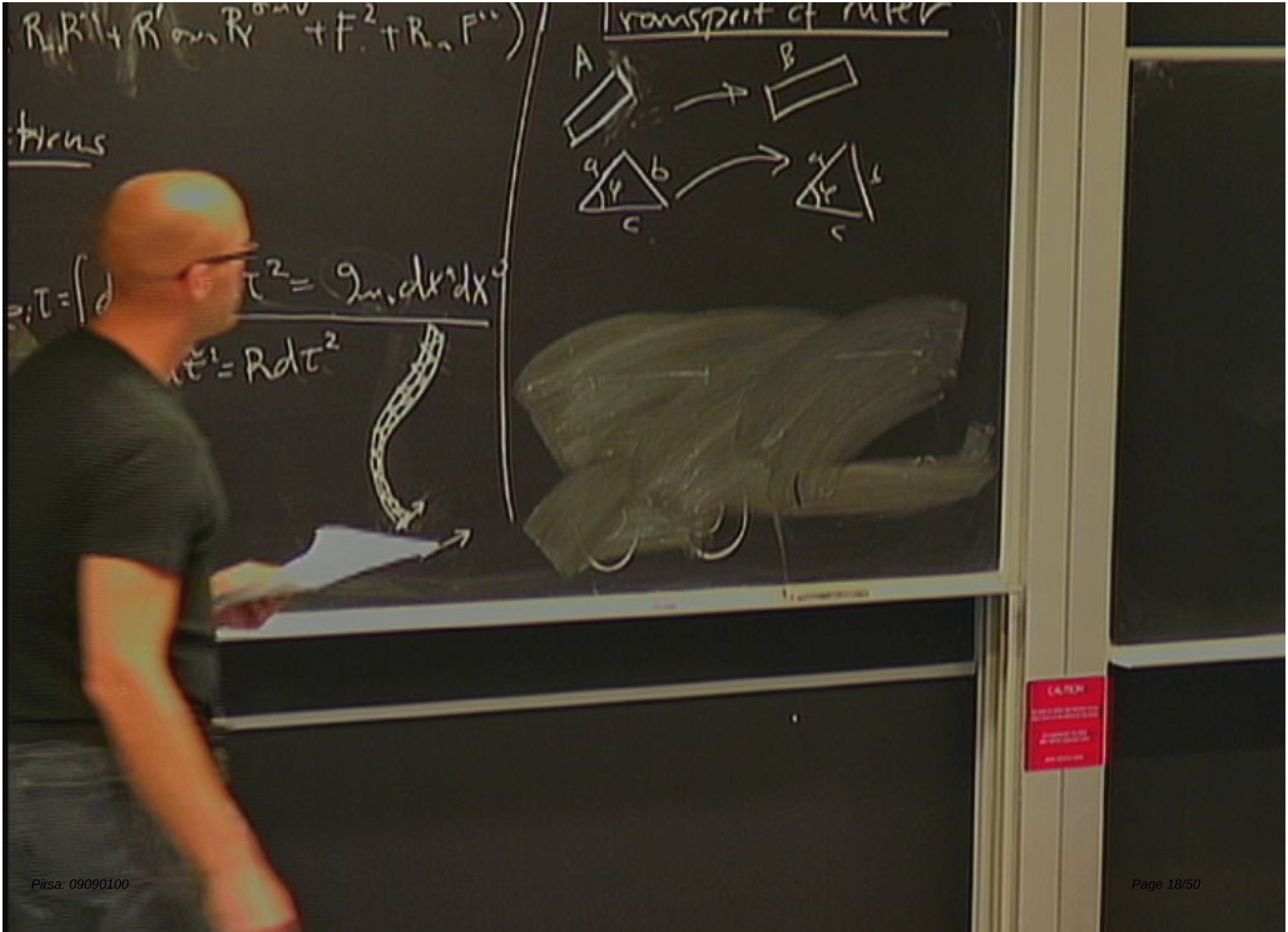

τ = ∫dτ
dτ² = g dx dx
ectra

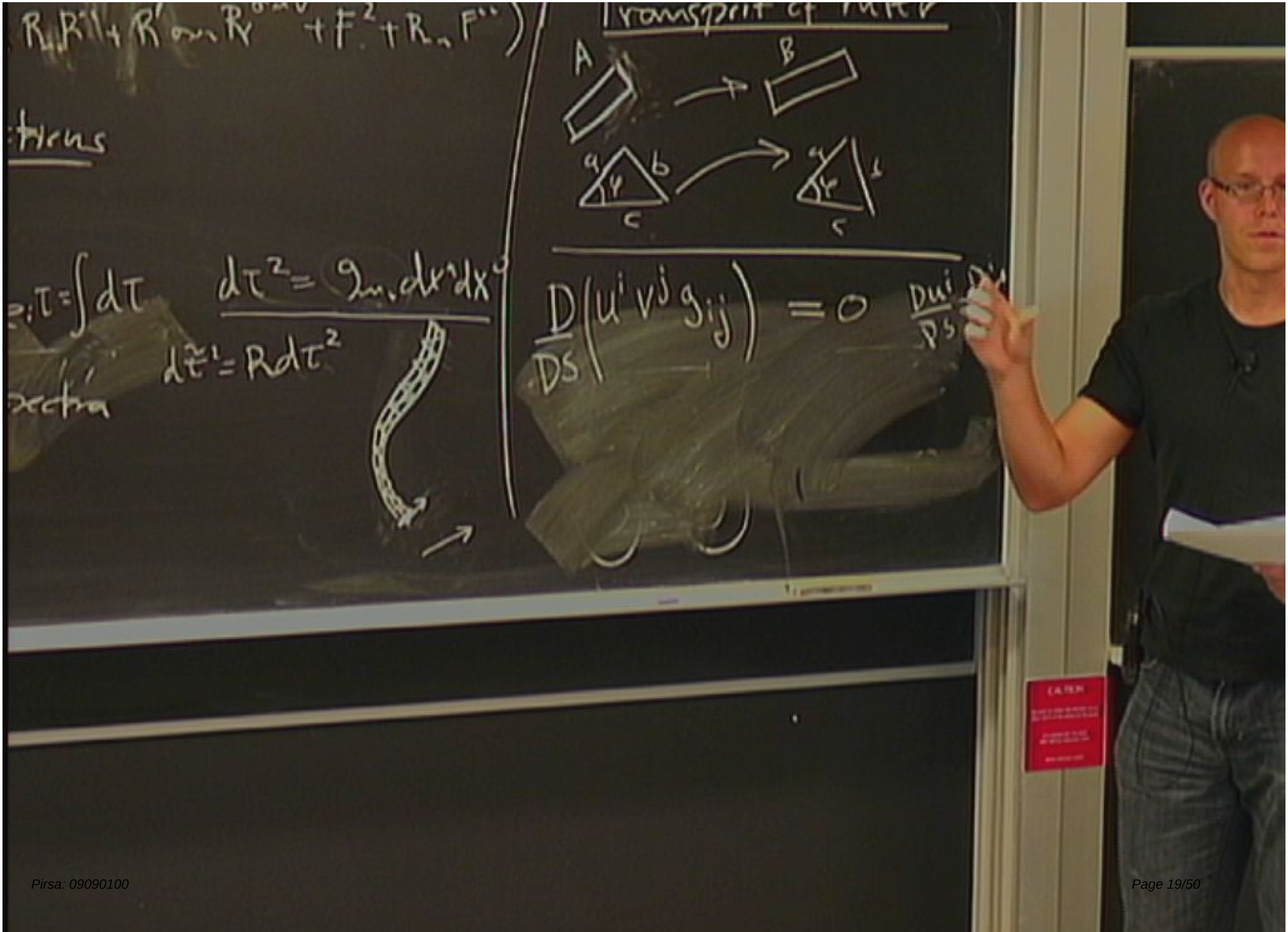


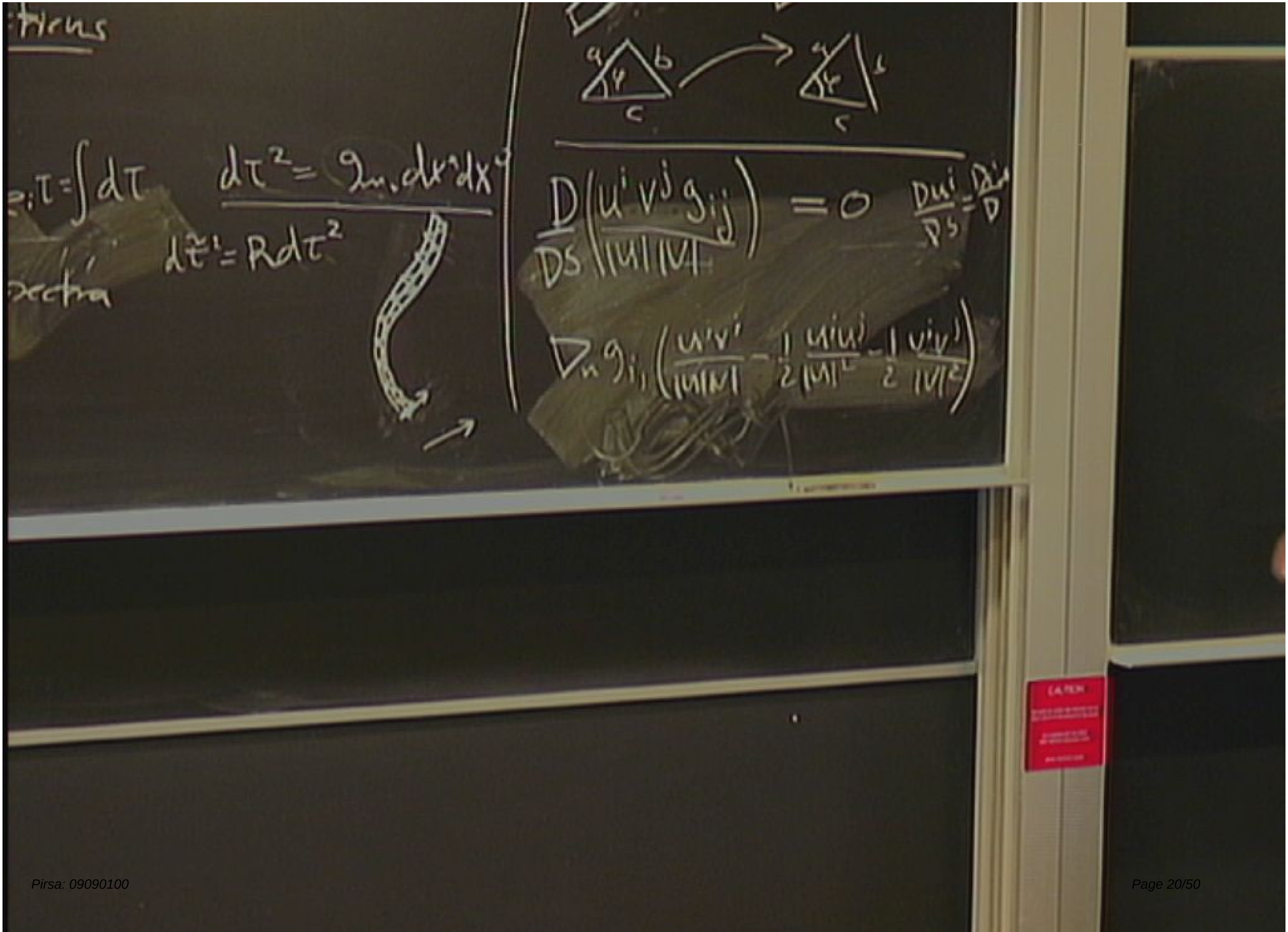
Transport of matter
$d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu$
$d\tilde{\tau}^2 = R\,d\tau^2$
$\tau = \int d\tau$

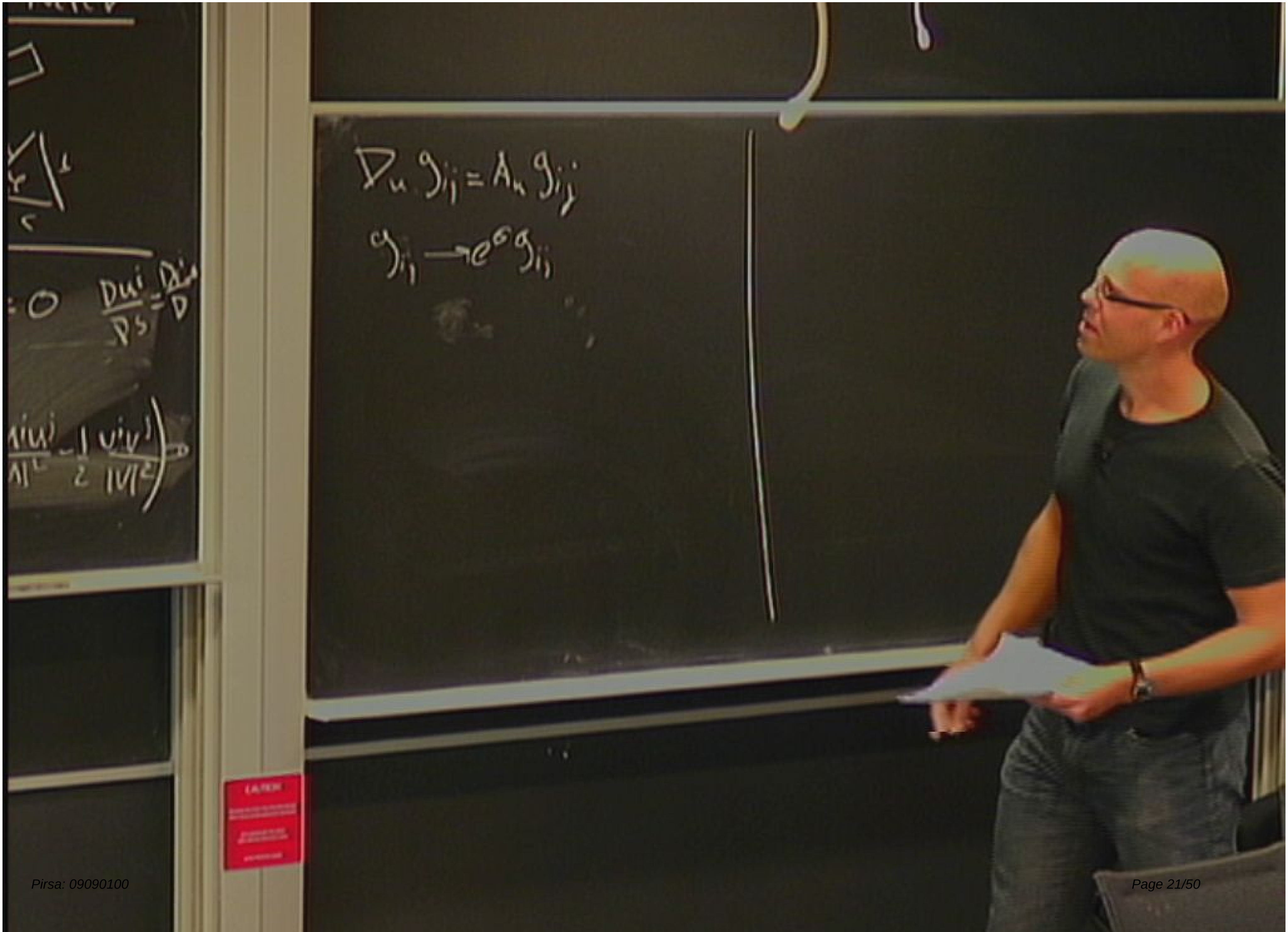

Transport of matter
A
B
$d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu$
$d\tilde{\tau}^2 = R\,d\tau^2$
$\tau = \int d\tau$


Transport of ruler
A
B
dτ² = g_μν dx^μ dx^ν
Rdτ²


A
B
a
b
c

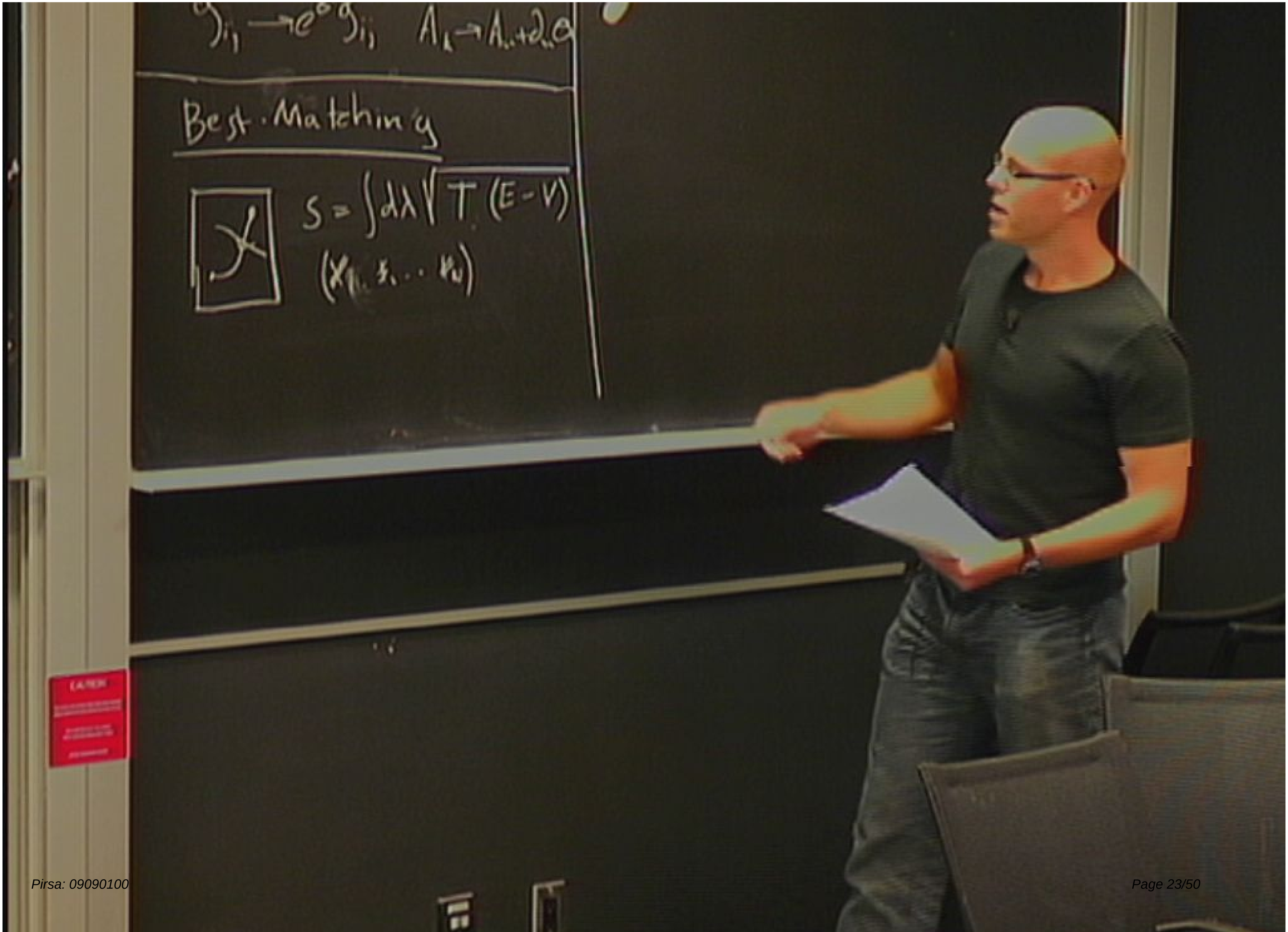

Transport of vector
A
B
a
b
c
$\frac{D}{DS}\left(u^i v^j g_{ij}\right) = 0$
$d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu$
$d\tilde{\tau}^2 = R\,d\tau^2$

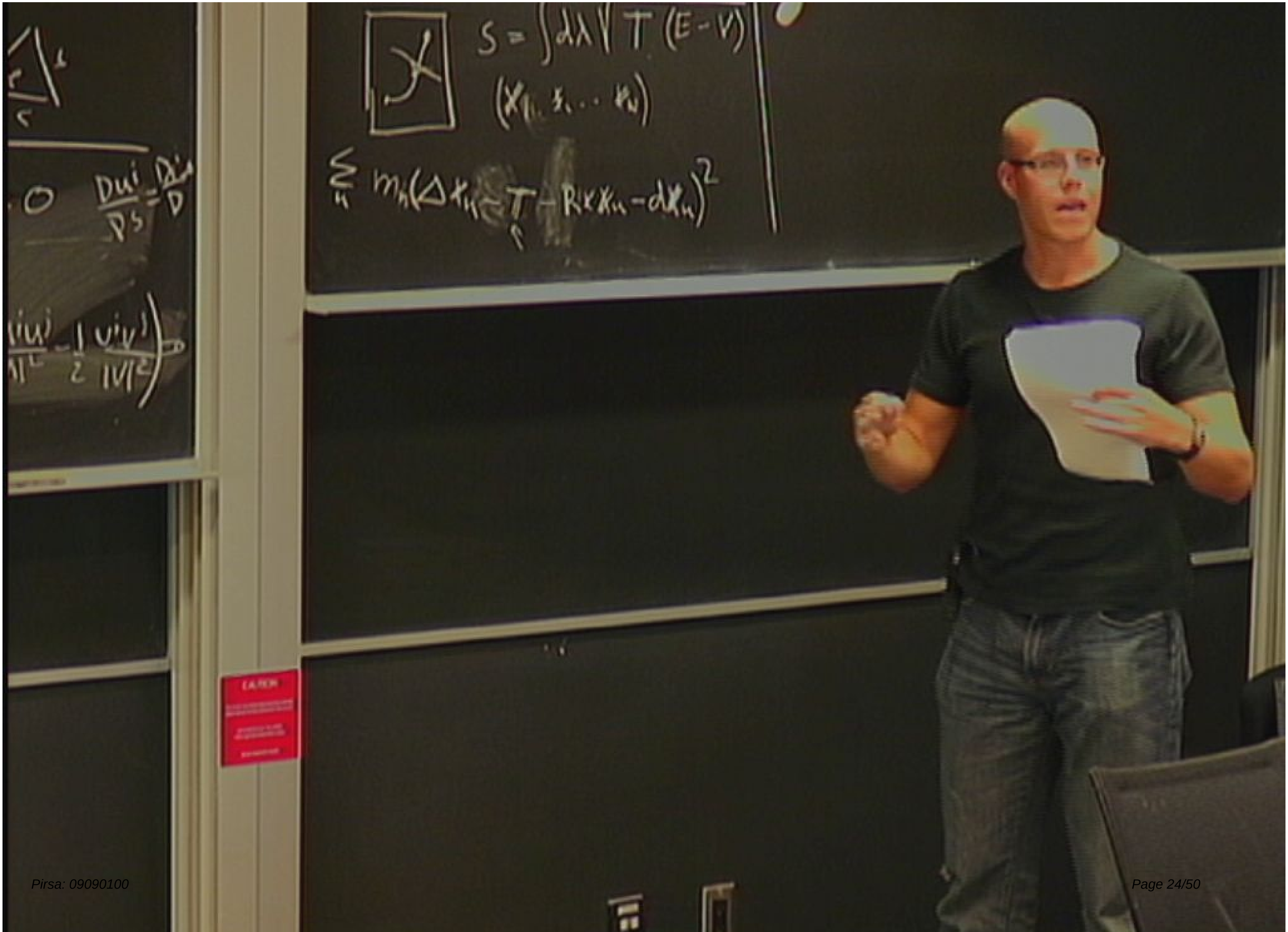

τ = ∫dτ
dτ² = g_{μν} dx^μ dx^ν
dτ̃² = R dτ²
a b c
D/DS (u^i v^j g_ij / |u||v|) = 0
Du^i/DS
∇_n g_ij (u^i v^j/|u||v| − 1/2 u^i u^j/|u|² − 1/2 v^i v^j/|v|²)
CAUTION

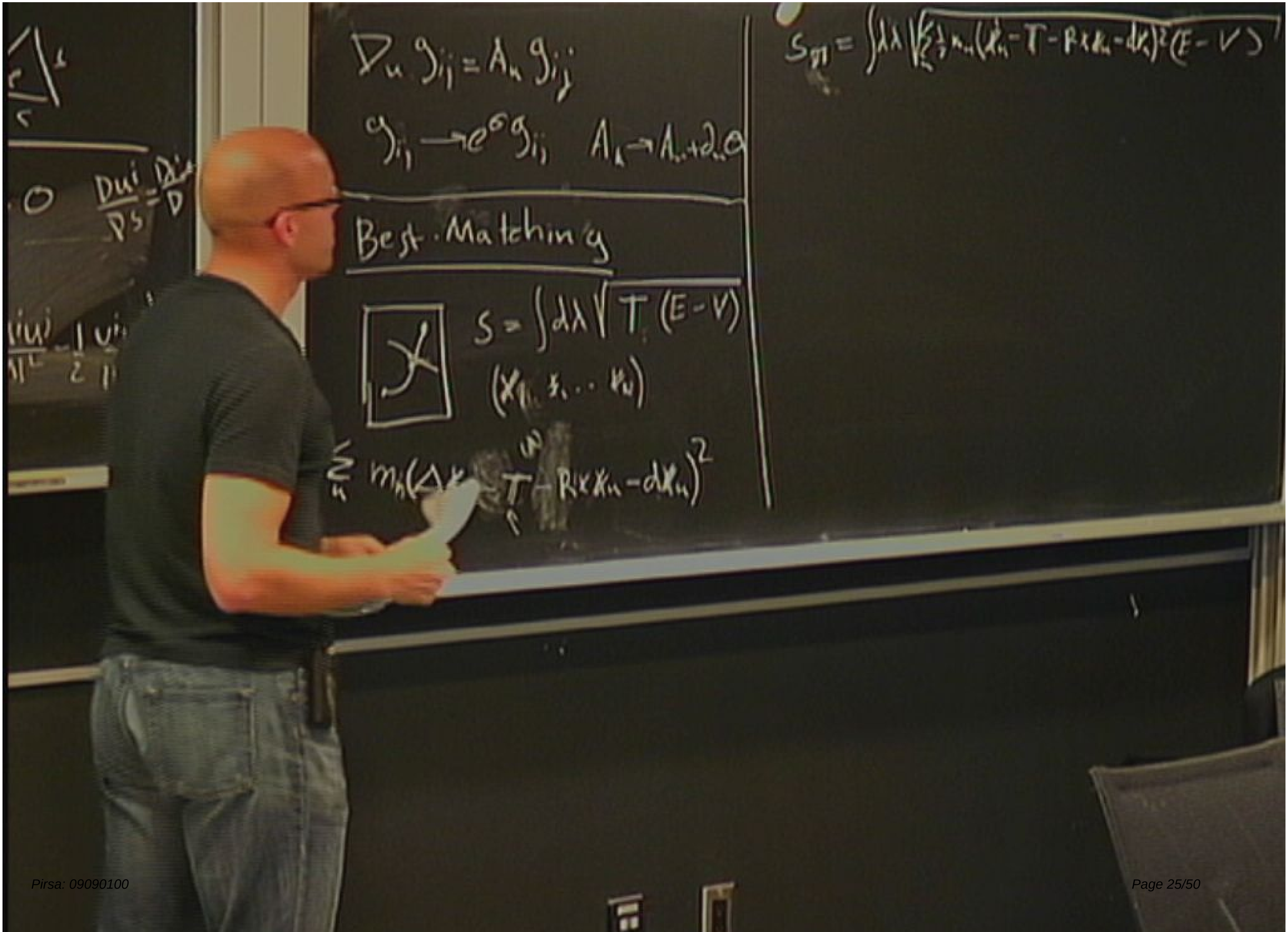

$D_u g_{ij} = A_u g_{ij}$
$g_{ij} \to e^{\sigma} g_{ij}$

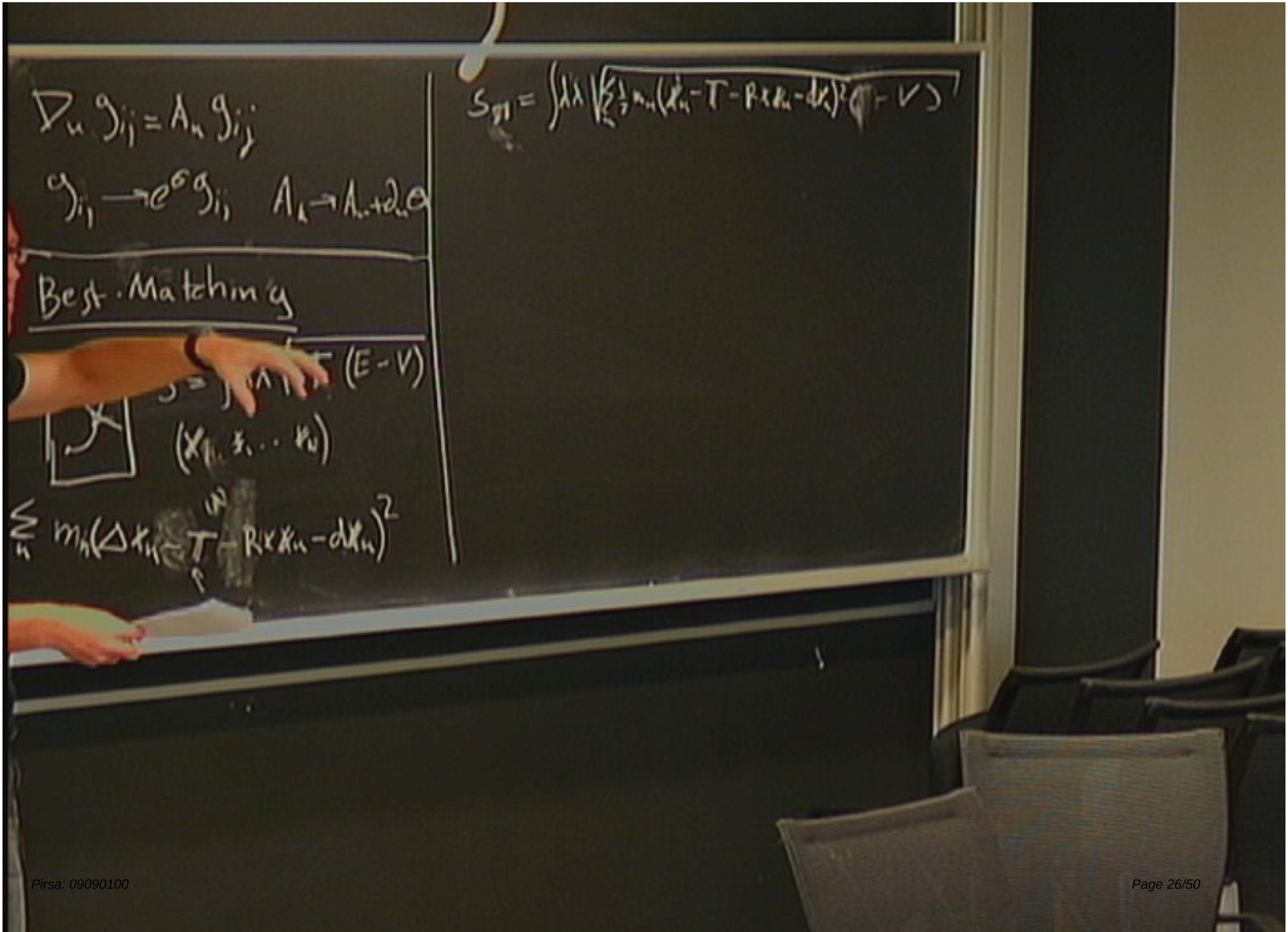
$$D_\mu g_{ij} = A_\mu g_{ij}$$

$$g_{ij} \to e^{\sigma} g_{ij} \qquad A_\mu \to A_\mu + \partial_\mu \sigma$$

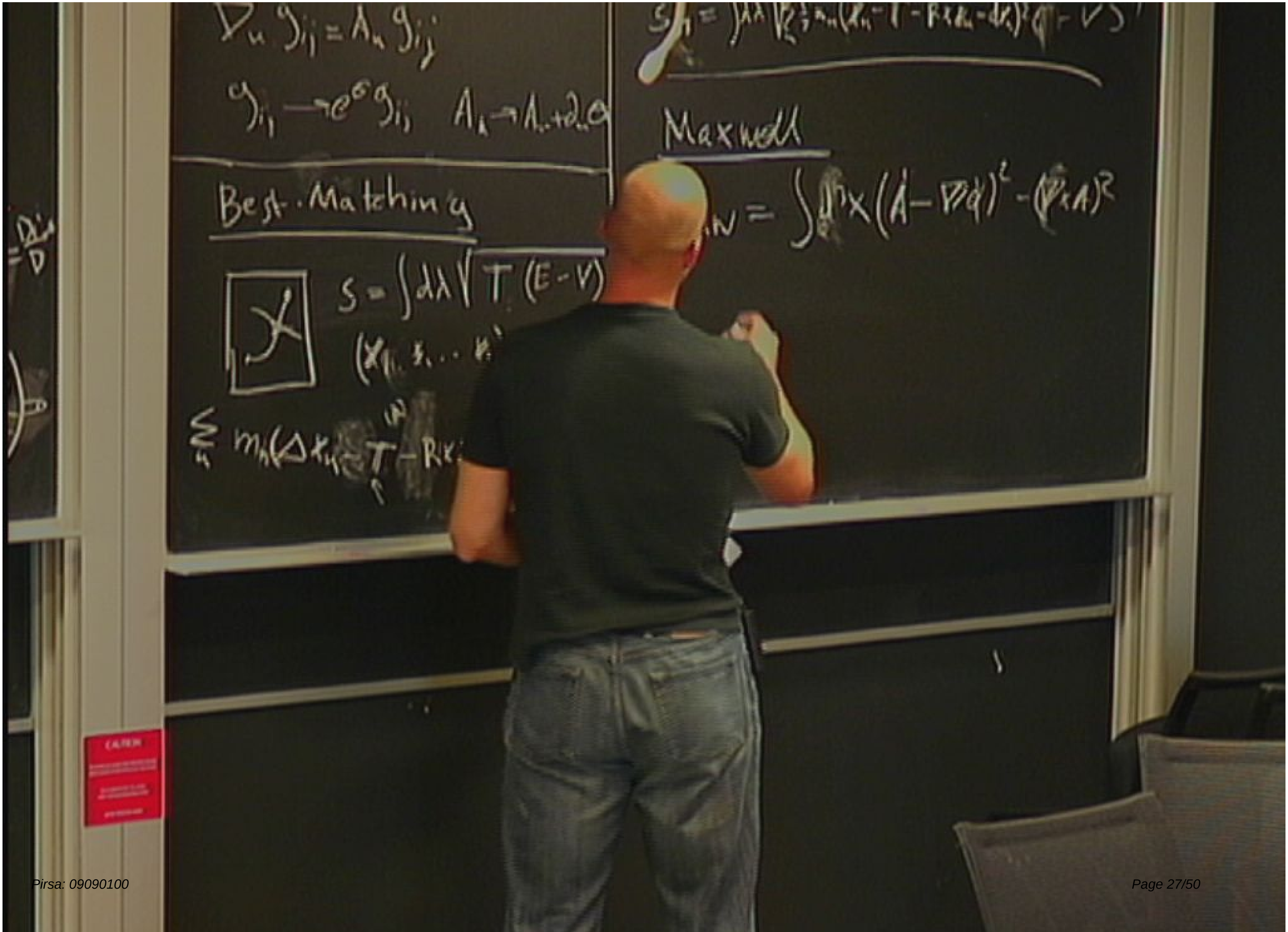

Best Matching
S = ∫dλ √T (E - V)

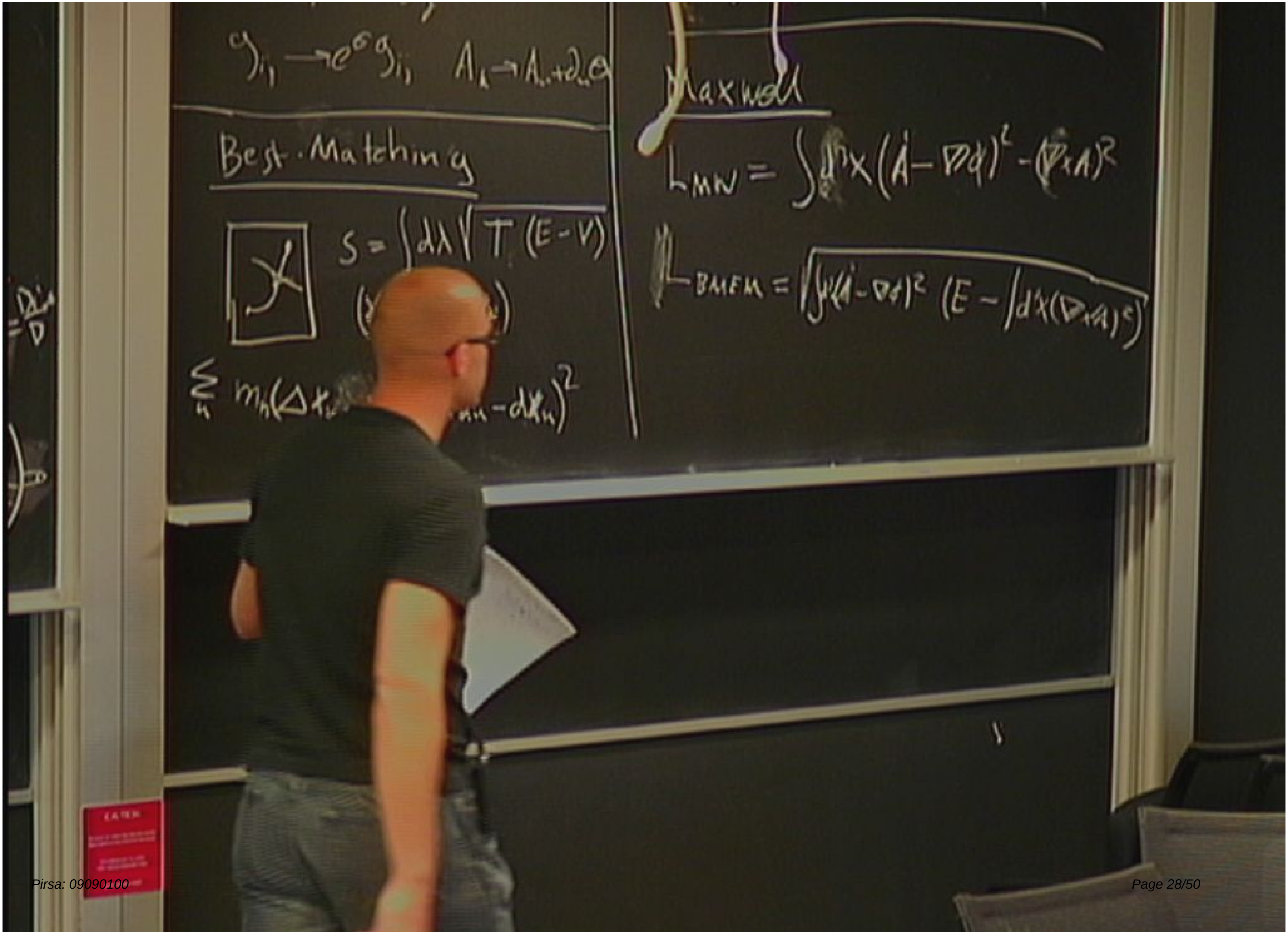

S = ∫dλ √T (E − V)
(X_1, X_2, … X_N)

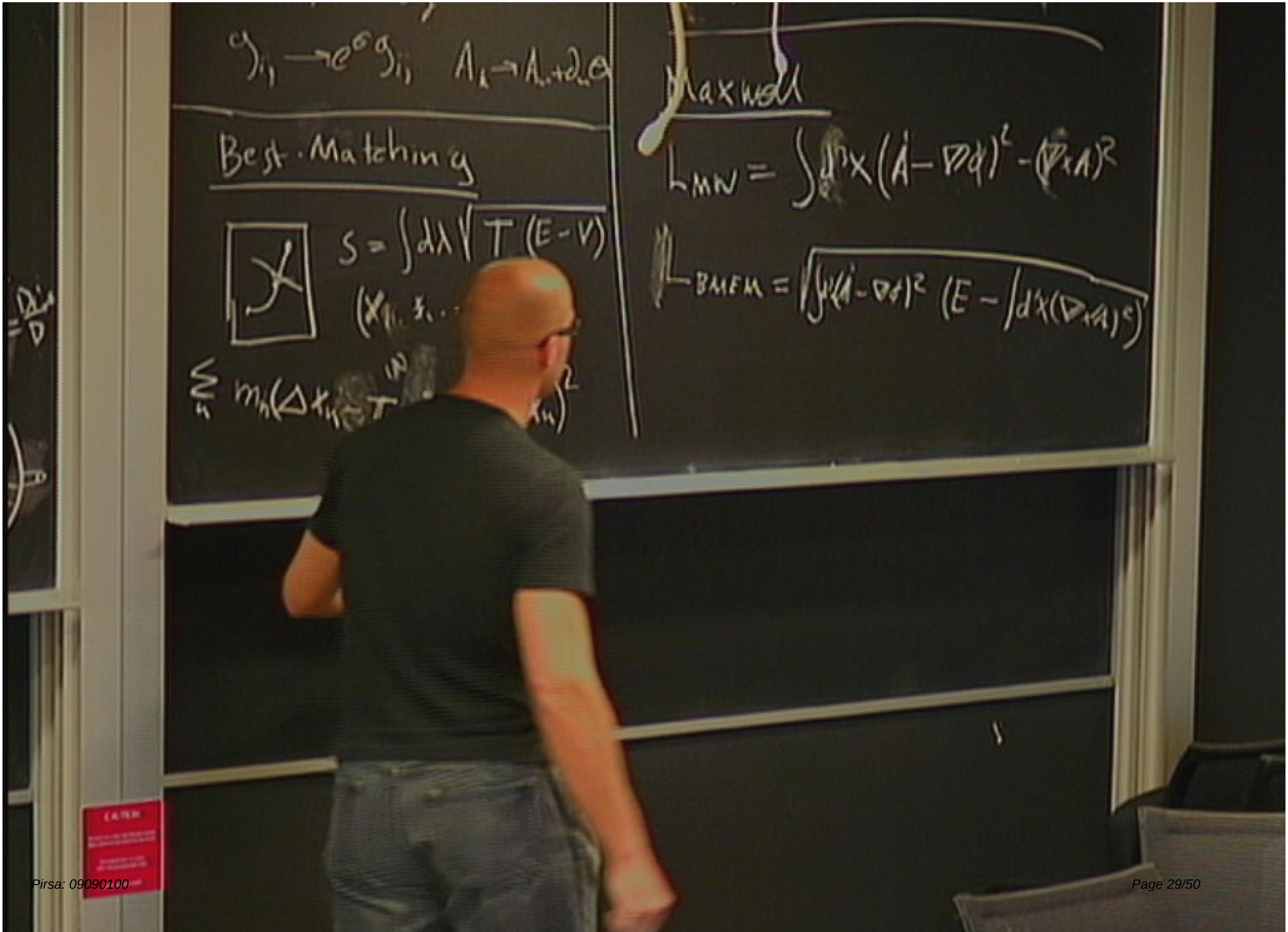

D_μ g_ij = A_μ g_ij
g_ij → e^φ g_ij   A_μ → A_μ + ∂_μ φ
Best Matching
S = ∫dλ √T (E − V)

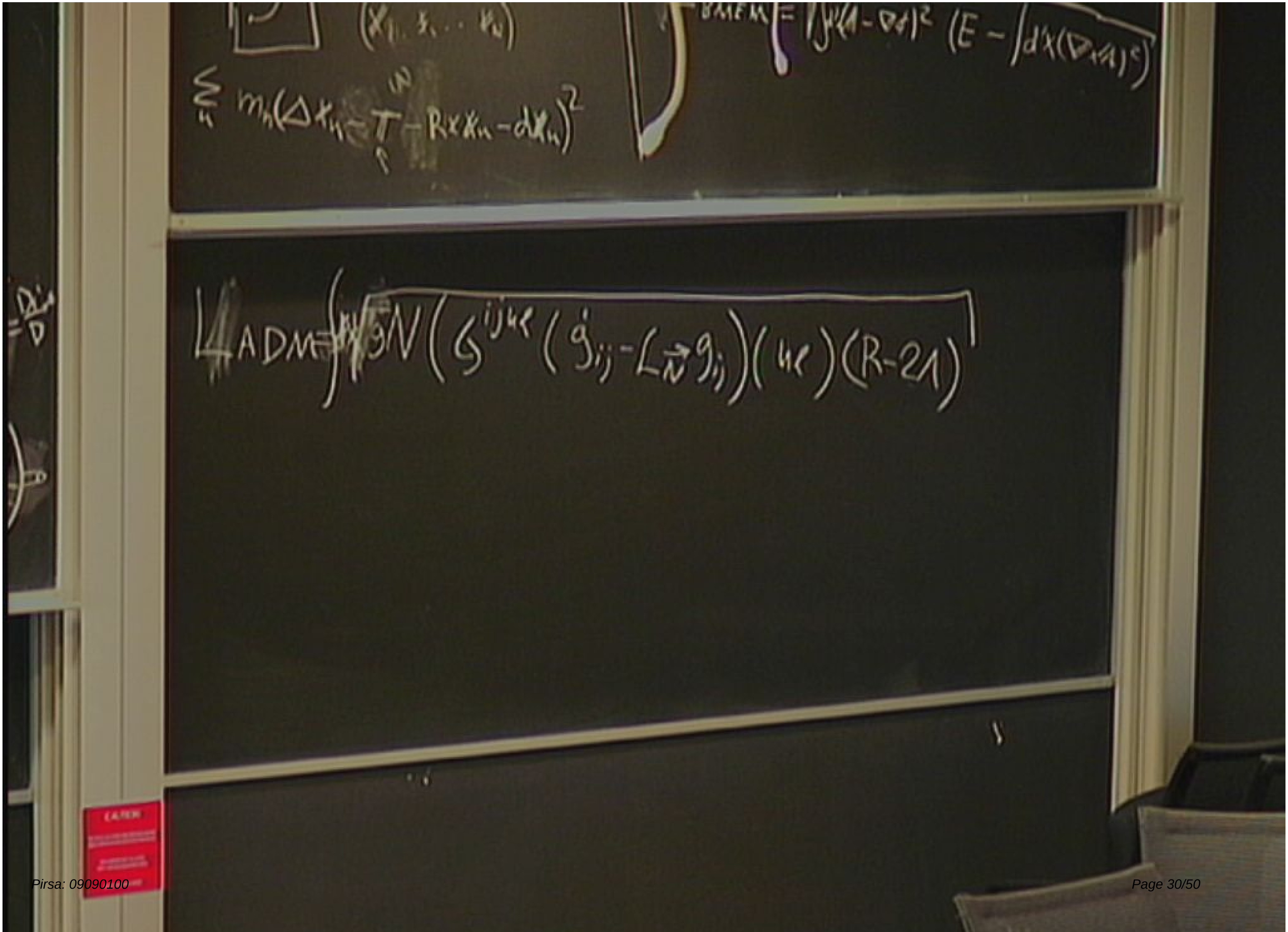


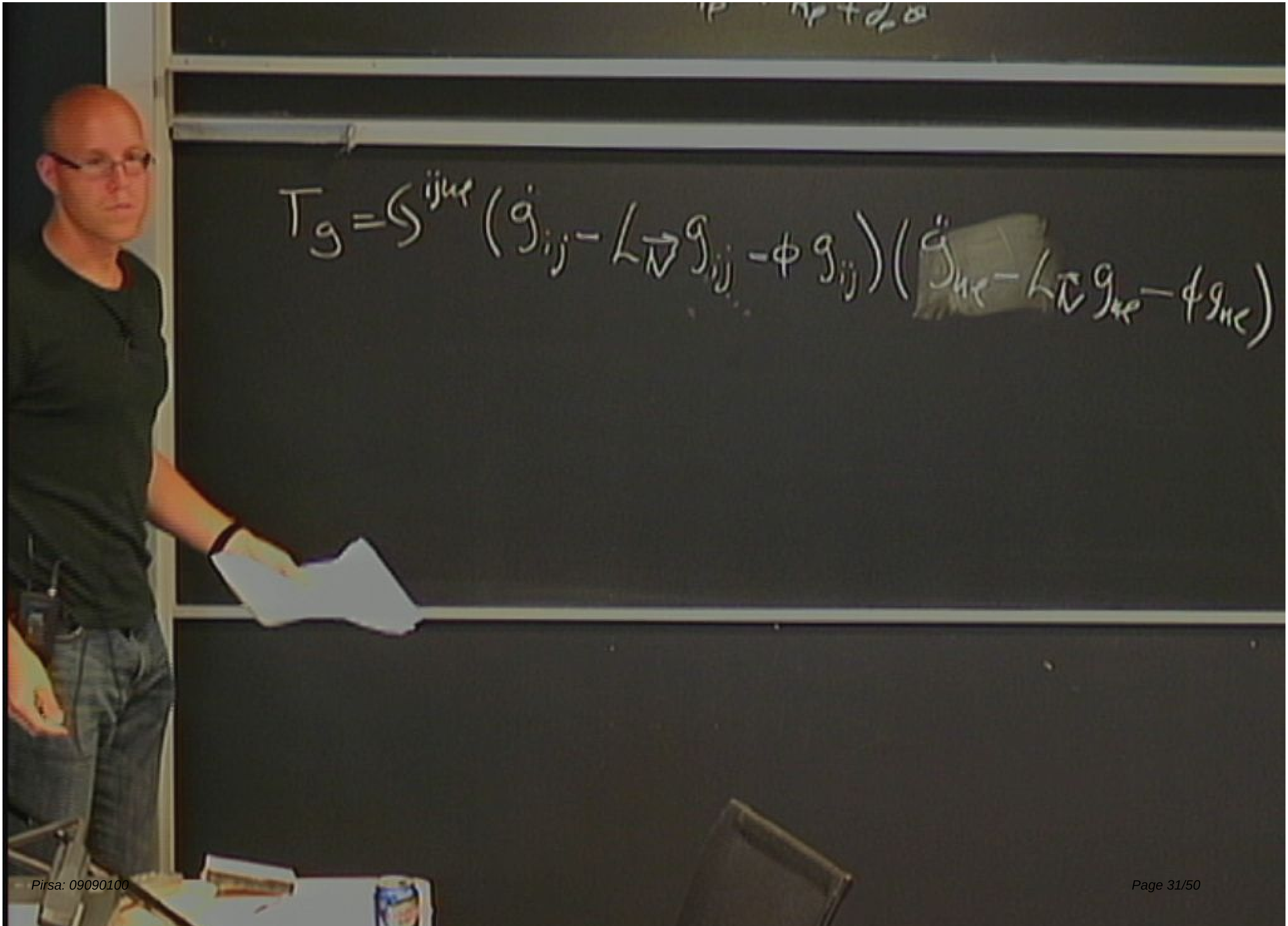
D_μ g_ij = A_μ g_ij
g_ij → e^σ g_ij   A_μ → A_μ + ∂_μ σ
Best Matching
(E − V)


Maxwell
Best Matching


Best Matching
Maxwell

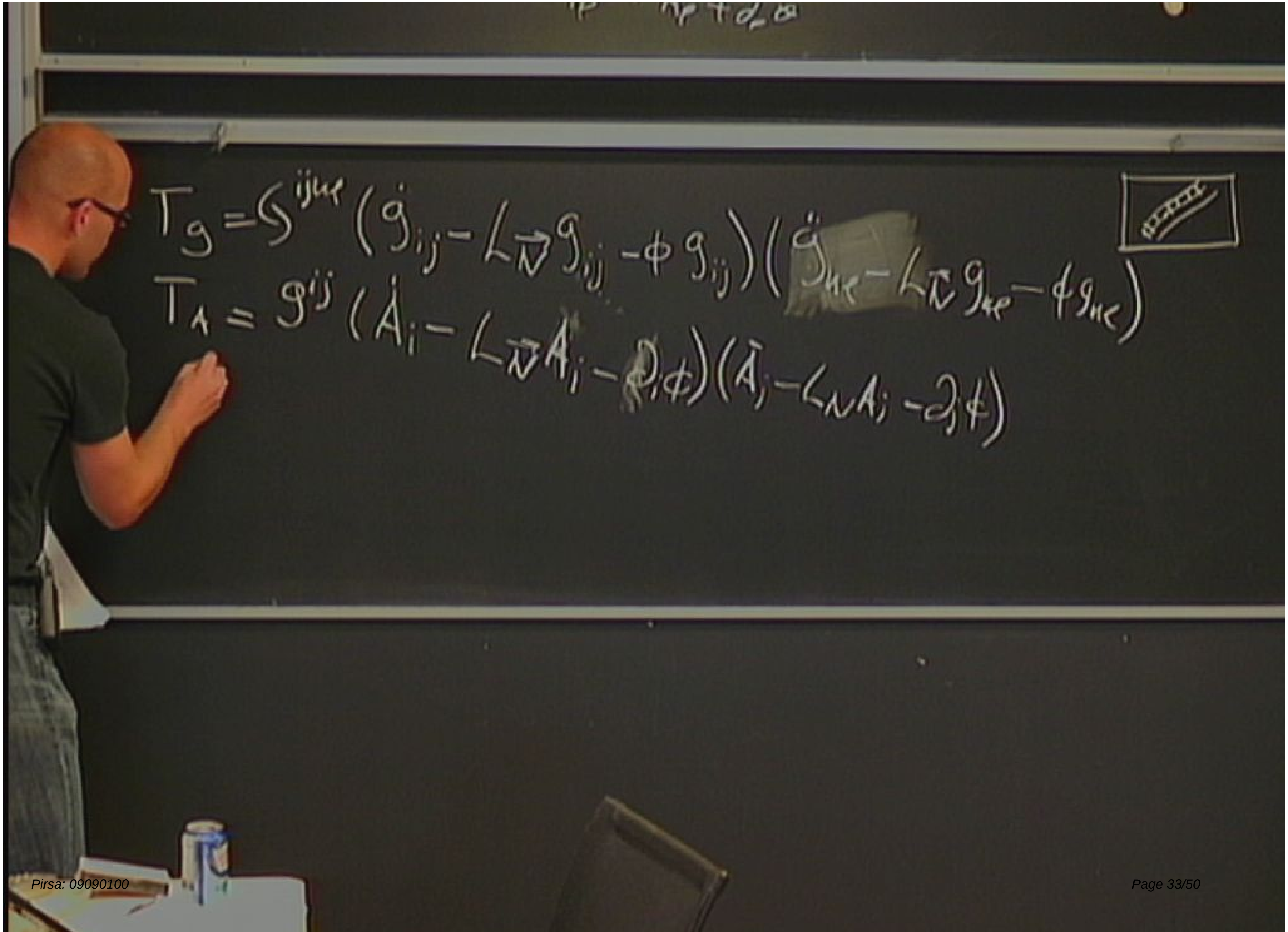

Best Matching
Maxwell


$L_{ADM} = \int \sqrt{g}\, N \left(G^{ijkl} (\dot{g}_{ij} - \mathcal{L}_{\vec{N}} g_{ij})(kl)(R-2\Lambda) \right)$


$T_g = G^{ijkl}(\dot{g}_{ij} - \mathcal{L}_{\vec{\xi}}\, g_{ij} - \phi\, g_{ij})(\dot{g}_{kl} - \mathcal{L}_{\vec{\xi}}\, g_{kl} - \phi\, g_{kl})$

$$T_g = G^{ijkl}\left(\dot g_{ij} - \mathcal{L}_{\vec N} g_{ij} - \phi\, g_{ij}\right)\left(\dot g_{kl} - \mathcal{L}_{\vec N} g_{kl} - \phi\, g_{kl}\right)$$

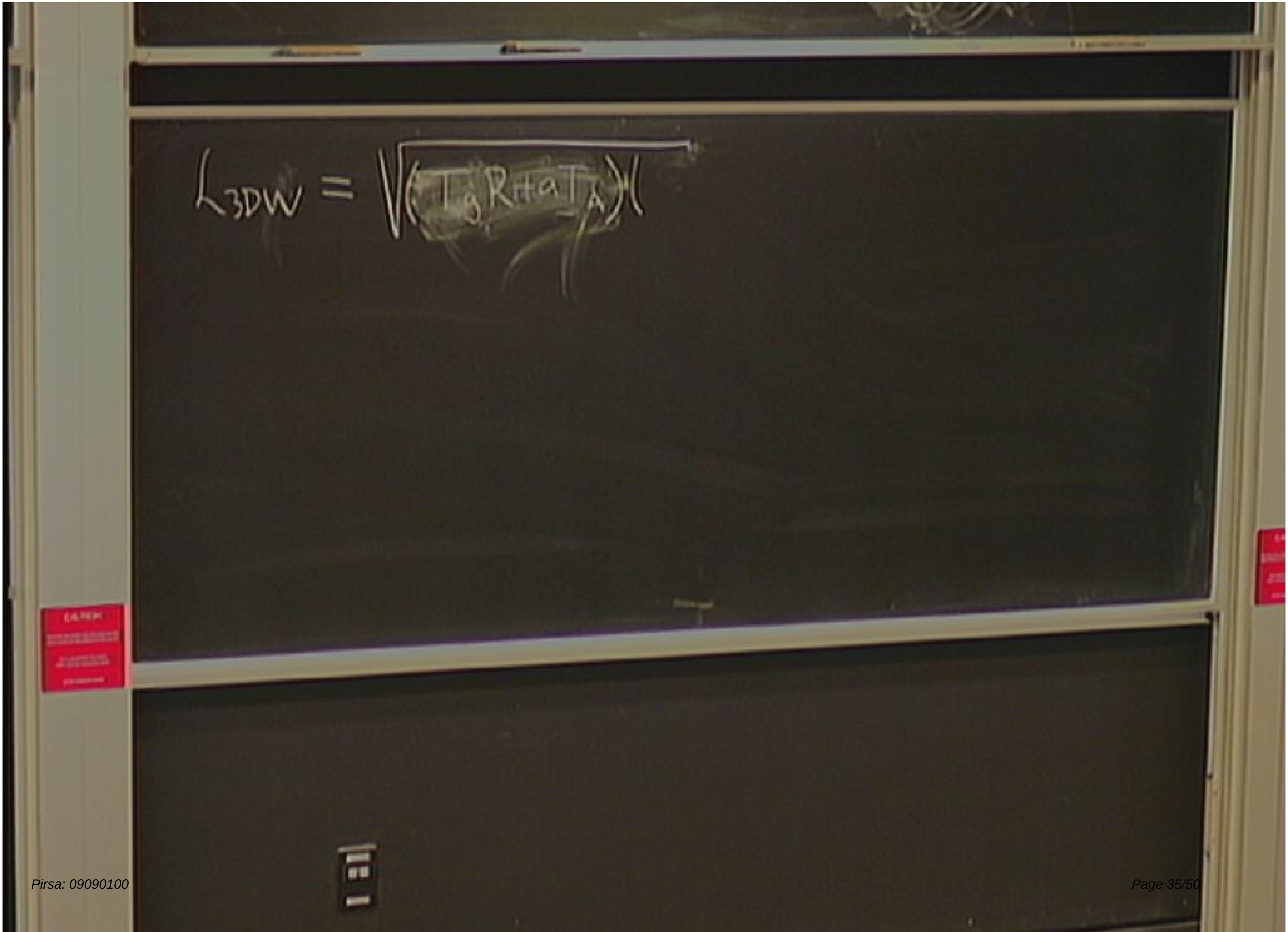
$$T_\Lambda =$$


$T_g = G^{ijkl}(\dot{g}_{ij} - \mathcal{L}_{\vec{N}} g_{ij} - \phi g_{ij})(\dot{g}_{kl} - \mathcal{L}_{\vec{N}} g_{kl} - \phi g_{kl})$
$T_A = g^{ij}(\dot{A}_i - \mathcal{L}_{\vec{N}} A_i - \partial_i \phi)(\dot{A}_j - \mathcal{L}_{N} A_j - \partial_j \phi)$

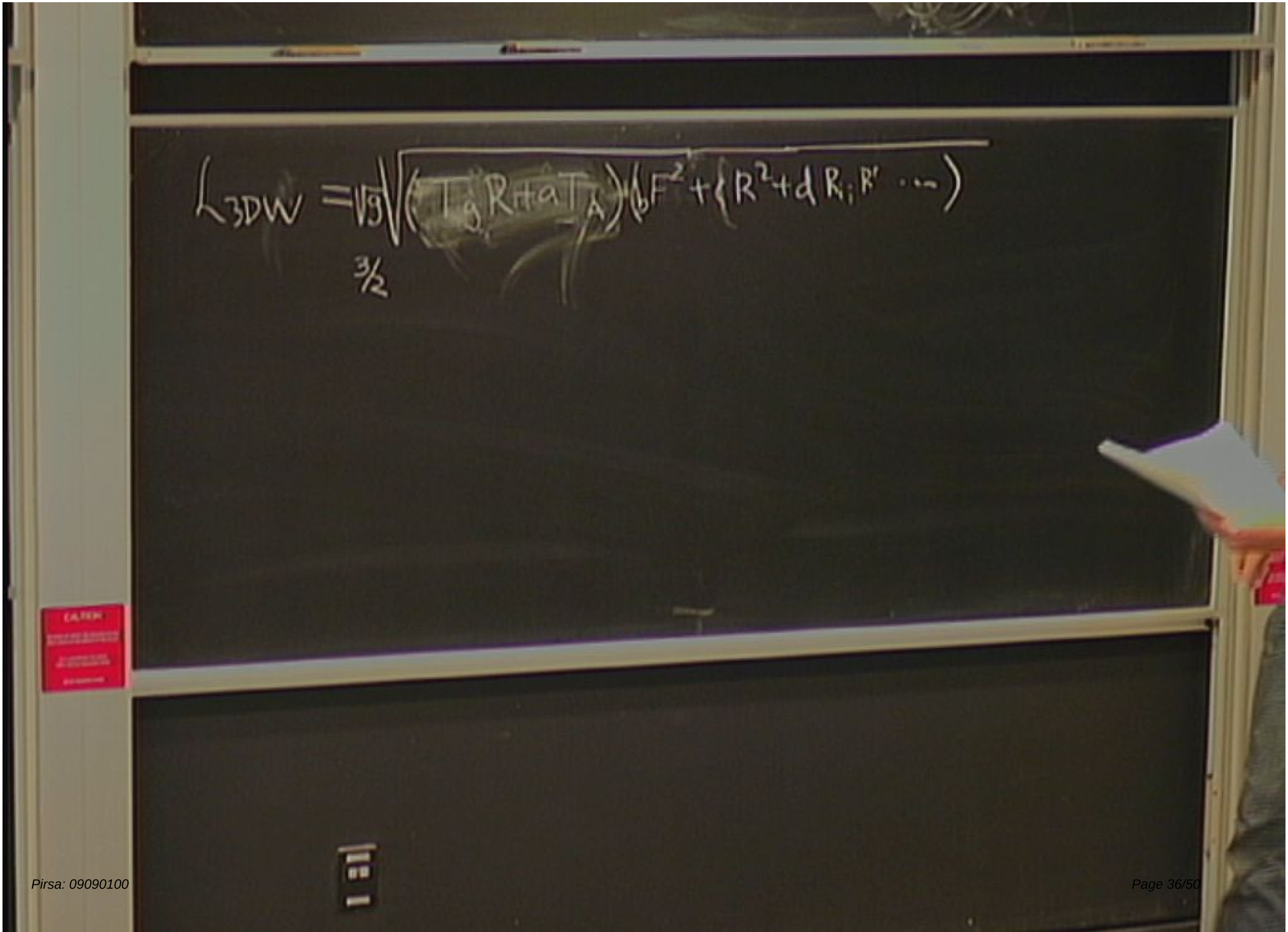
$$0 \quad T_g = G^{ijkl}\,(\dot g_{ij} - \mathcal{L}_{\vec N} g_{ij} - \phi\, g_{ij})(\dot g_{kl} - \mathcal{L}_{\vec N} g_{kl} - \phi\, g_{kl})$$

$$-1 \quad T_A = g^{ij}\,(\dot A_i - \mathcal{L}_{\vec N} A_i - \partial_i \phi)(\dot A_j - \mathcal{L}_N A_j - \partial_j \phi)$$

$$\phi \to \phi + \dot\epsilon - N^i \partial_i \epsilon \qquad g_{ij} \to e^{\epsilon} g_{ij} \qquad A_i \to A_i + \partial_i \epsilon$$

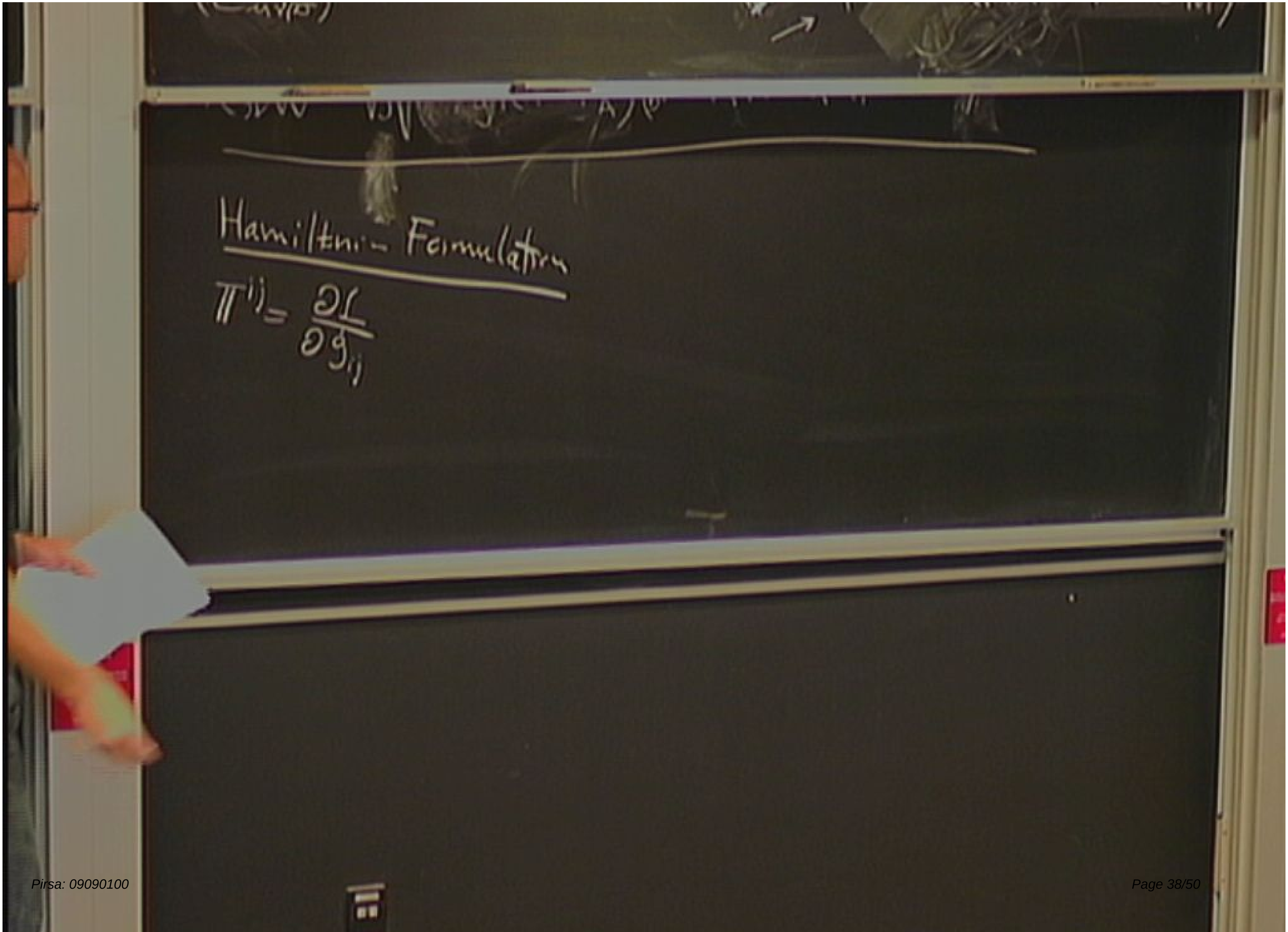


L3DW = √((Tg R+aTA)(

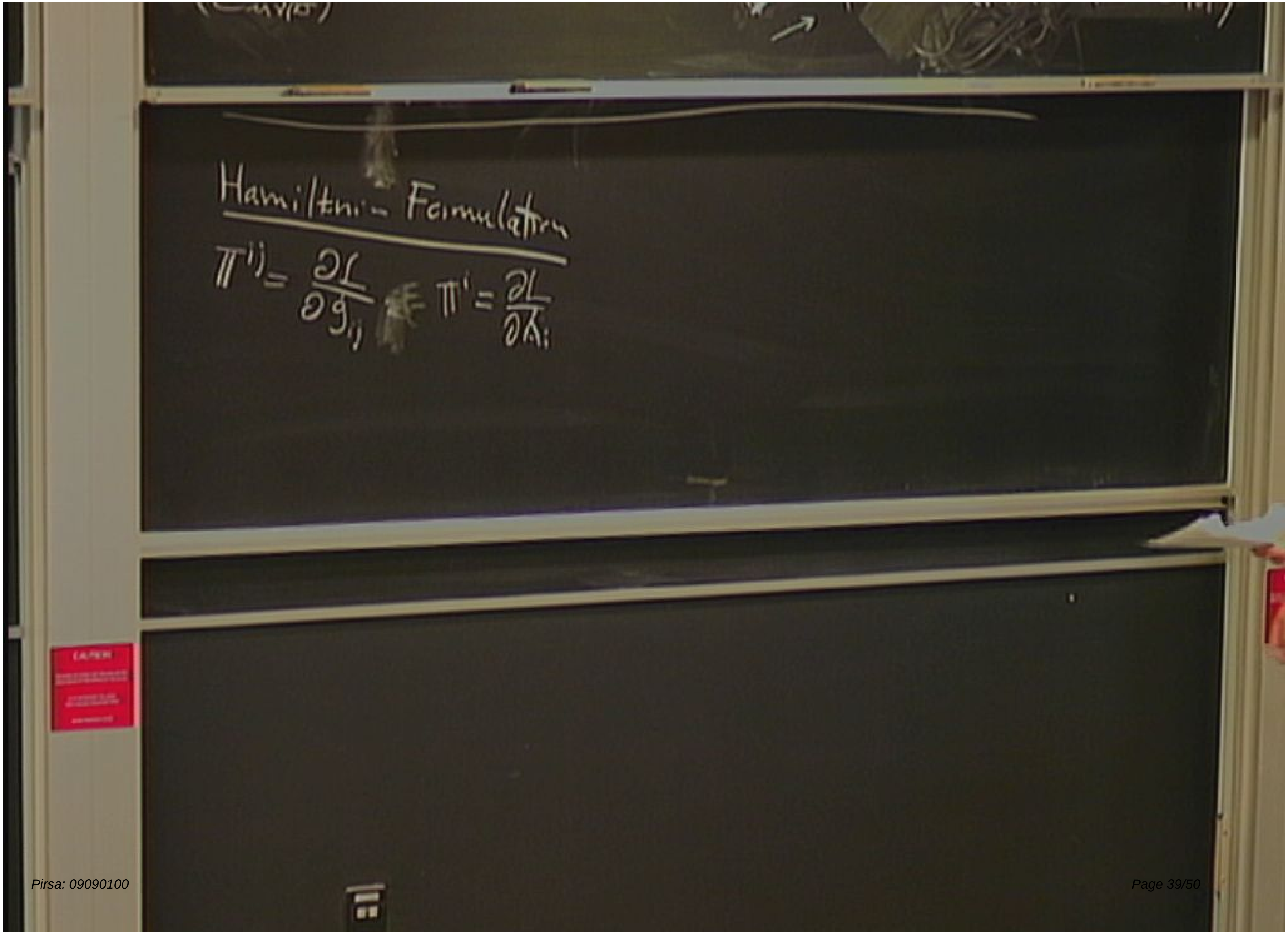

$L_{3DW} = \sqrt{g}\sqrt{(T_g R + a T_A)(b F^2 + c R^2 + d R_{ij} R^{ij} \cdots)}$
3/2

$$\mathcal{L}_{3DW} = \sqrt{g}\sqrt{(T_g R + a T_A)(b F^2 + c R^2 + d R_{ij} R^{ij} \cdots)}$$

$$3/2$$



Hamiltonian Formulation
$\pi^{ij} = \frac{\partial L}{\partial g_{ij}}$


Hamiltonian Formulation
$\pi^{ij} = \frac{\partial L}{\partial g_{ij}}$
$\pi^{i} = \frac{\partial L}{\partial \lambda_i}$

Hamiltonian Formulation

$$\pi^{ij} = \frac{\partial L}{\partial \dot{g}_{ij}} \qquad \pi^{i} = \frac{\partial L}{\partial \dot{A}_i}$$

$$\pi_{\phi} = \frac{\partial L}{\partial \dot{\phi}} \equiv 0 \qquad \pi_{A_0} = \frac{\partial L}{\partial \dot{A}_0} \equiv 0$$

Hamiltonian Formulation

$$\pi^{ij} = \frac{\partial L}{\partial \dot{g}_{ij}} \qquad \pi^{i} = \frac{\partial L}{\partial \dot{\lambda}_i}$$

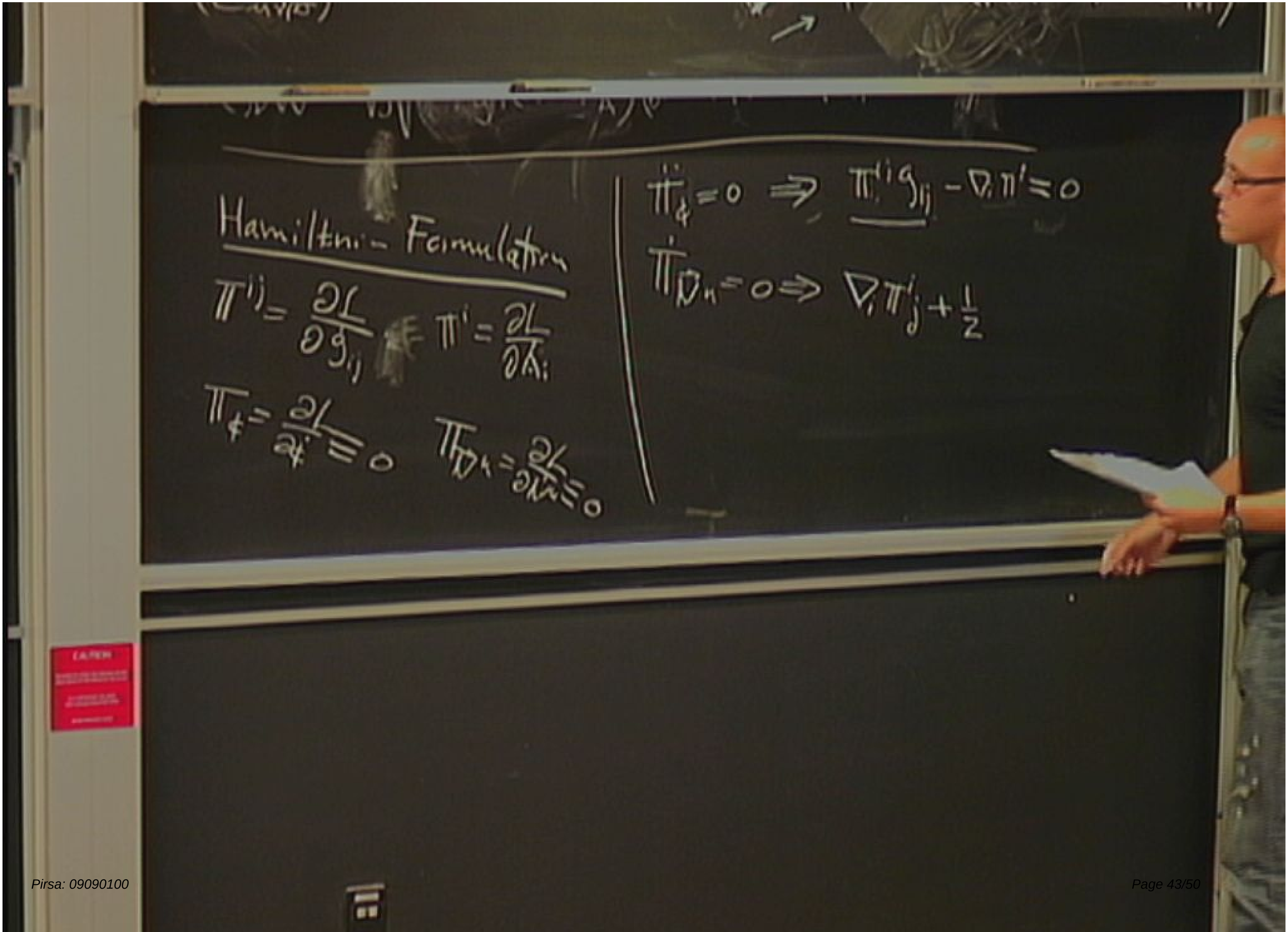
$$\pi_{\phi} = \frac{\partial L}{\partial \dot{\phi}} \equiv 0 \qquad \pi_{\nabla\lambda} = \frac{\partial L}{\partial \dot{\nabla\lambda}} \equiv 0$$

Hamiltonian Formulation

$$\pi^{ij} = \frac{\partial L}{\partial \dot g_{ij}} \qquad \pi^{i} = \frac{\partial L}{\partial \dot A_i}$$

$$\pi_{\phi} = \frac{\partial L}{\partial \dot\phi} \equiv 0 \qquad \pi_{\nabla A} = \frac{\partial L}{\partial \dot{\nabla A}} \equiv 0$$

$$\dot\pi_{\phi} = 0 \Rightarrow \pi^{ij}_{;i}\, g_{ij} - \nabla_i \pi^{i} = 0$$


Hamiltonian Formulation
$\pi^{ij} = \frac{\partial L}{\partial \dot g_{ij}}$
$\pi^{i} = \frac{\partial L}{\partial \dot\lambda_i}$
$\pi_{\phi} = \frac{\partial L}{\partial \dot\phi} \equiv 0$
$\pi_{\nabla\lambda} = \frac{\partial L}{\partial \nabla\lambda} \equiv 0$
$\dot\pi_{\phi} = 0 \Rightarrow \pi^{ij} g_{ij} - \nabla_i \pi^i = 0$
$\dot\pi_{\nabla\lambda} = 0 \Rightarrow \nabla_i \pi^i_j + \frac{1}{2}$

Hamiltonian Formulation

$$\pi^{ij} = \frac{\partial L}{\partial \dot{g}_{ij}} \qquad \pi^{i} = \frac{\partial L}{\partial \dot{A}_{i}}$$

$$\pi_{\phi} = \frac{\partial L}{\partial \dot{\phi}} \equiv 0 \qquad \pi_{N_k} = \frac{\partial L}{\partial \dot{N}_k} \equiv 0$$

$$\dot{\pi}_{\phi} = 0 \Rightarrow \underline{\pi^{ij} g_{ij}} - \nabla_i \pi^i = 0$$

$$\dot{\pi}_{N_k} = 0 \Rightarrow \nabla_i \pi^i_j + \frac{1}{2} \pi^i F_{ij} = 0$$

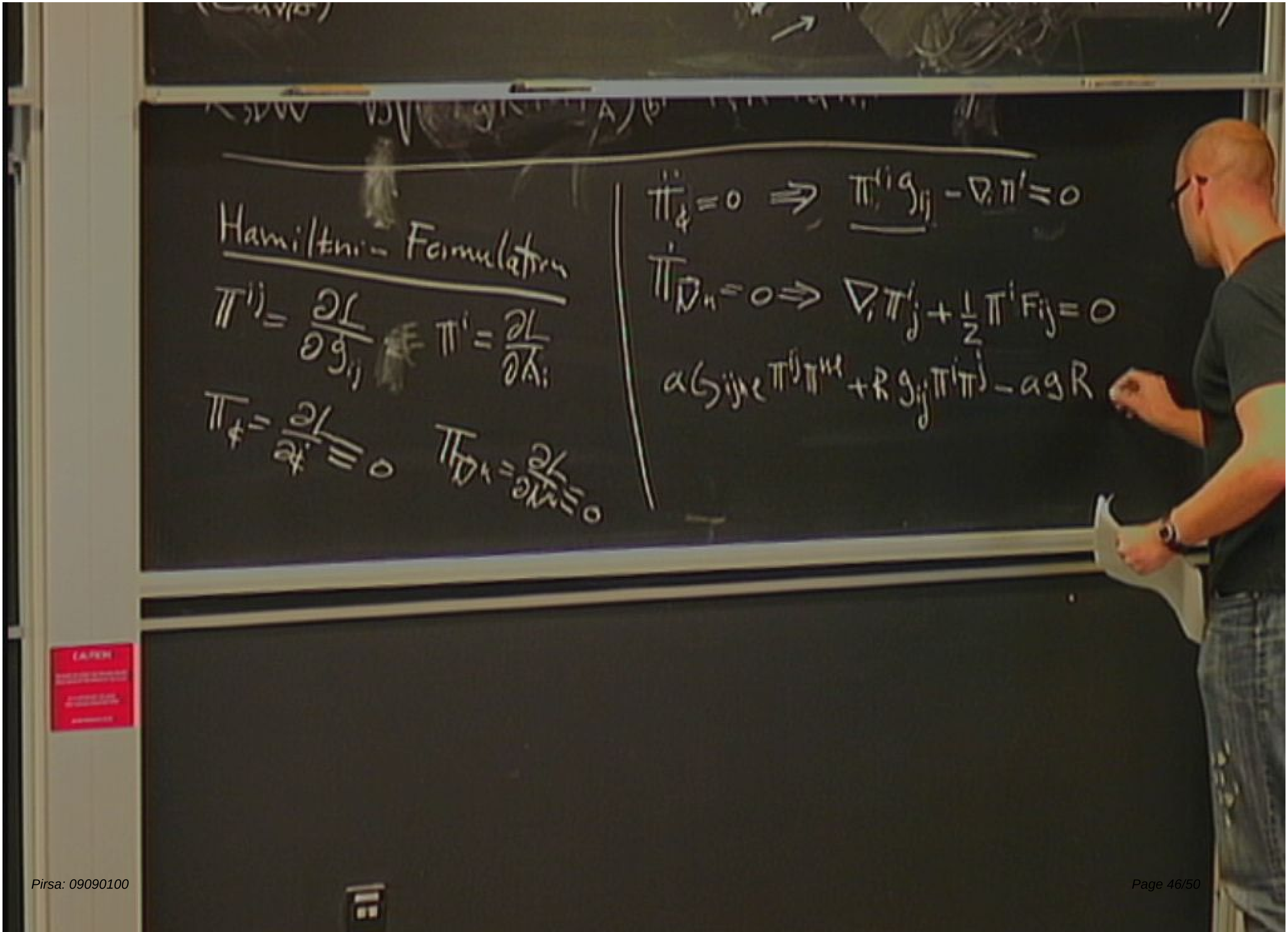
Hamiltonian Formulation

$$\pi^{ij} = \frac{\partial L}{\partial \dot g_{ij}} \qquad \pi^i = \frac{\partial L}{\partial \dot A_i}$$

$$\pi_\phi = \frac{\partial L}{\partial \dot\phi} \equiv 0 \qquad \pi_{\mathcal{D}_n} = \frac{\partial L}{\partial \dot{\mathcal{D}}_n} \equiv 0$$

$$\dot\pi_\phi = 0 \Rightarrow \underline{\pi^i g_{ij}} - \nabla_i \pi^i = 0$$

$$\dot\pi_{\mathcal{D}_n} = 0 \Rightarrow \nabla_i \pi^i_j + \frac{1}{2}\pi^i F_{ij} = 0$$


Hamiltonian Formulation
π^ij = ∂L/∂g_ij   π^i = ∂L/∂A_i
π_φ = ∂L/∂φ̇ ≡ 0   π_{DN} = ∂L/∂Ṅ ≡ 0
π̇_φ = 0 ⇒ π^ij g_ij − ∇_i π^i = 0
π̇_{DN} = 0 ⇒ ∇_i π^i_j + ½ π^i F_ij = 0
a G_ijkl π^ij π^kl + R g_ij π^i π^j − a g R

$$L_{3DW} = \sqrt{g}\sqrt{(\ldots R + a T_A)(bF^2 + c R^2 + d R_{ij}R^{ij} \ldots)}$$

Hamiltonian Formulation

$$\pi^{ij} = \frac{\partial L}{\partial \dot{g}_{ij}} \qquad \pi^i = \frac{\partial L}{\partial \dot{A}_i}$$

$$\pi_\phi = \frac{\partial L}{\partial \dot{\phi}} = 0 \qquad \pi_{D_n} = \frac{\partial L}{\partial \dot{N}} = 0$$

$$\dot{\pi}_\phi = 0 \Rightarrow \pi^{ij} g_{ij} - \nabla_i \pi^i = 0$$

$$\dot{\pi}_{D_n} = 0 \Rightarrow \nabla_i \pi^i_j + \frac{1}{2}\pi^i F_{ij} = 0$$

$$a G_{ijkl}\pi^{ij}\pi^{kl} + R g_{ij}\pi^i\pi^j - a g R V$$

$$\mathcal{L}_{3DW} = \sqrt{g}\sqrt{(T_g R + a T_\lambda)(bF^2 + cR^2 + dR_{ij}R^{ij} \cdots)}$$

R-2.1

Hamiltonian Formulation

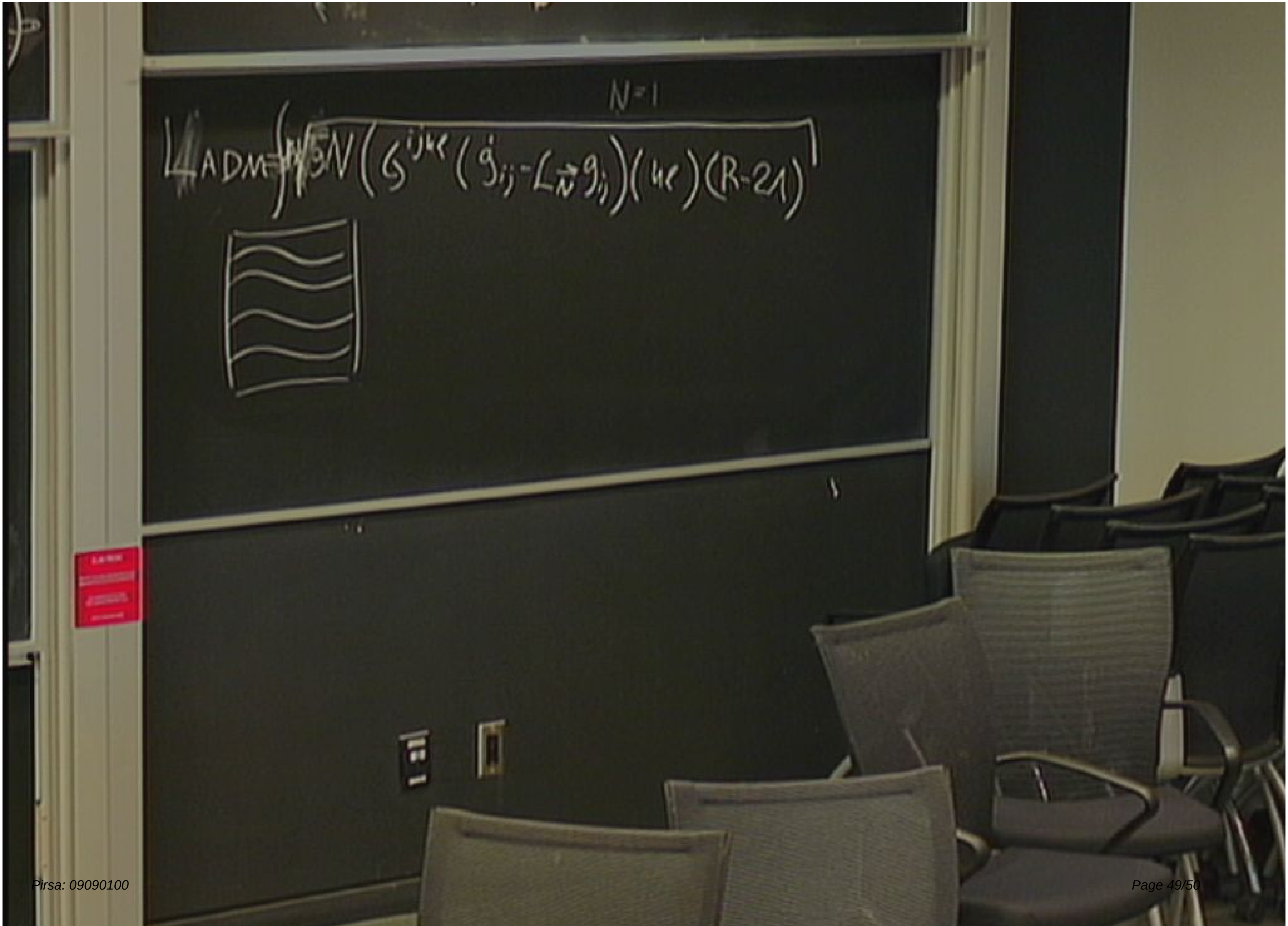
$$\pi^{ij} = \frac{\partial L}{\partial \dot{g}_{ij}} \qquad \pi^i = \frac{\partial L}{\partial \dot{\lambda}_i}$$

$$\pi_\phi = \frac{\partial L}{\partial \dot{\phi}} = 0 \qquad \pi_{\mathcal{D}_n} = \frac{\partial L}{\partial \dot{\lambda}_n} = 0$$

$$\dot{\pi}_\phi = 0 \Rightarrow \pi^{ij} g_{ij} - \nabla_i \pi^i = 0$$

$$\dot{\pi}_{\mathcal{D}_n} = 0 \Rightarrow \nabla_i \pi^i_j + \frac{1}{2}\pi^i F_{ij} = 0$$

$$a G_{ijkl}\pi^{ij}\pi^{kl} + R g_{ij}\pi^i\pi^j - a g R V = 0$$


N=1