

Title: A First-Principles Implementation of Scale Invariance Using Best Matching

Date: Sep 15, 2009 04:00 PM

URL: <http://pirsa.org/09090100>

Abstract: We present a first-principles implementation of $\{em\ spatial\}$ scale invariance as a local gauge symmetry in geometry dynamics using the method of best matching. In addition to the 3-metric, the proposed scale invariant theory also contains a 3-vector potential A_k as a dynamical variable. Although some of the mathematics is similar to Weyl's ingenious, but physically questionable, theory, the equations of motion of this new theory are second order in time-derivatives. It is tempting to try to interpret the vector potential A_k as the electromagnetic field. We exhibit four independent reasons for not giving into this temptation. A more likely possibility is that it can play the role of "dark matter". Indeed, as noted in scale invariance seems to play a role in the MOND phenomenology. Spatial boundary conditions are derived from the free-endpoint variation method and a preliminary analysis of the constraints and their propagation in the Hamiltonian formulation is presented.

$$\oint_C ds = \int \sqrt{g_{ij} dx^i dx^j}$$



$g_{\mu\nu}, A_\rho$

$$\Gamma_{\mu\nu}^\rho = \left\{ \begin{matrix} \rho \\ \mu\nu \end{matrix} \right\} - \frac{1}{2} (\delta_{\mu\nu}^\rho A_\sigma + \delta_{\nu\mu}^\rho A_\sigma - g_{\mu\nu} g^{\sigma\alpha} A_\sigma)$$

$$\nabla_\rho g_{\mu\nu} = A_\rho g_{\mu\nu}$$

$g_{\mu\nu}, A_\rho$

$$\Gamma_{\mu\nu}^\rho = \left\{ \begin{matrix} \rho \\ \mu\nu \end{matrix} \right\} - \frac{1}{2} (\delta_{\mu\nu}^\rho A_\sigma + \delta_{\nu\mu}^\rho A_\sigma - g_{\mu\nu} g^{\sigma\alpha} A_\sigma)$$

$$\nabla_\rho g_{\mu\nu} = A_\rho g_{\mu\nu}$$

$$g_{\mu\nu} \rightarrow e^{2\sigma} g_{\mu\nu}$$

$$A_\rho \rightarrow A_\rho + 2\partial_\rho \sigma$$

A First-Principles Implementation of Scale Invariance Using Best-Matching

$$g_{\mu\nu}, A_p$$

$$\Gamma_{\mu\nu}^{\rho} = \left\{ \begin{matrix} \rho \\ \mu\nu \end{matrix} \right\} - \frac{1}{2} (\delta_{\mu}^{\rho} A_{\nu} + \delta_{\nu}^{\rho} A_{\mu} - g_{\mu\nu} g^{\sigma\rho} A_{\sigma})$$

$$\nabla_p g_{\mu\nu} = A_p g_{\mu\nu}$$

$$g_{\mu\nu} \rightarrow e^{\omega} g_{\mu\nu}$$

$$A_p \rightarrow A_p + 2\partial_p \omega$$

$$R^{\rho}{}_{\sigma\mu\nu} = \partial\Gamma - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$$

$$R_{\sigma\nu}$$

$$R = g^{\mu\nu} R_{\mu\nu} \quad R \rightarrow e^{-\omega} R$$

$$g^{\mu\nu} \rightarrow e^{-\omega} g^{\mu\nu}$$

$$g_{\mu\nu}, A_\rho$$

$$A_\rho = \partial_\rho \phi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \neq 0$$

$$\Gamma_{\mu\nu}^\rho = \left\{ \begin{matrix} \rho \\ \mu\nu \end{matrix} \right\} - \frac{1}{2} (\delta_{\mu\nu}^\rho A_\sigma + \delta_{\nu\mu}^\rho A_\sigma - g_{\mu\nu} g^{\rho\sigma} A_\sigma)$$

$$\nabla_\rho g_{\mu\nu} = A_\rho g_{\mu\nu}$$

$$g_{\mu\nu} \rightarrow e^{\sigma} g_{\mu\nu}$$

$$A_\rho \rightarrow A_\rho + 2\partial_\rho \sigma$$

$$R^\rho{}_{\sigma\mu\nu} = \partial\Gamma - \partial\Gamma + \Gamma\Gamma$$

$$R_{\sigma\nu}$$

$$R = g^{\mu\nu} R_{\mu\nu} \quad R \rightarrow R + 2\partial^2 \sigma$$

$$g^{\mu\nu} \rightarrow e^{-\sigma} g^{\mu\nu}$$

$$\mathcal{L}_W = \sqrt{g} (R^2 + R_{\mu\nu} R^{\mu\nu} + R'_{\mu\nu} R'^{\mu\nu} + F^2 + R_{\mu\nu} F^{\mu\nu})$$

~~$s = ds = \dots$~~

$$\mathcal{L}_W = \sqrt{|g|} (R^2 + R_{\mu\nu} R^{\mu\nu} + R'_{\mu\nu} R'^{\mu\nu} + F^2 + R_{\mu\nu} F^{\mu\nu})$$

Einstein's objections

1) Eq. 4th

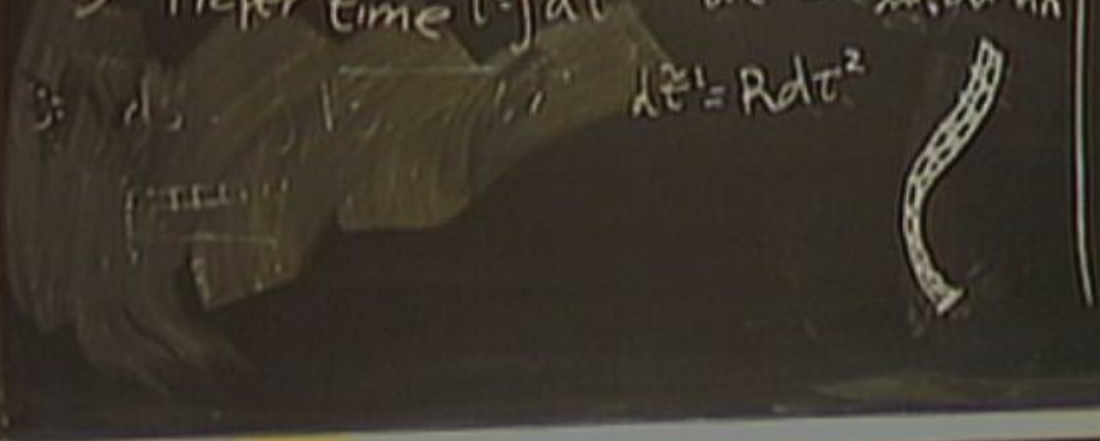
2) Proper time $\tau = \int d\tau$ $d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$

3) ~~...~~

$\Gamma - 2\Gamma + \Gamma - \Gamma$
 $R_{\mu\nu} R^{\mu\nu} = R^2$
 $\Rightarrow g_{\mu\nu}$

Einstein's Equations

- 1) Eq. 4th
- 2) Proper time $\tau = \int d\tau$ $d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$
- 3) ~~...~~ $d\tilde{t}^1 = R dt^2$



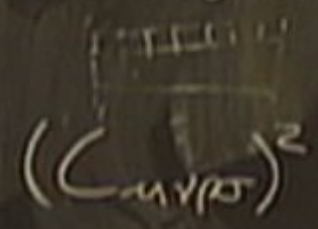
CAUTION
 ...
 ...
 ...



$$\mathcal{L}_W = \sqrt{|g|} (R^2 + R_{\mu\nu} R^{\mu\nu} + R'_{\mu\nu} R'^{\mu\nu} + F^2 + R_{\mu\nu} F^{\mu\nu})$$

Einstein's objections

- 1) Eqs. 4th
- 2) Proper time: $\tau = \int d\tau$ $d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$
- 3) Hydrogen spectra $d\tilde{t}^2 = R dt^2$



$$e^{\theta} \quad e^{\theta+2\pi}$$

$$g_{\mu\nu}, A_{\rho}$$

$$A_{\rho} = \partial_{\rho} \Phi$$

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \neq 0$$

$$\Gamma_{\mu\nu}^{\rho} = \left\{ \begin{matrix} \rho \\ \mu\nu \end{matrix} \right\} - \frac{1}{2} \left(\delta_{\mu}^{\rho} A_{\nu} + \delta_{\nu}^{\rho} A_{\mu} - g_{\mu\nu} g^{\sigma\rho} A_{\sigma} \right)$$

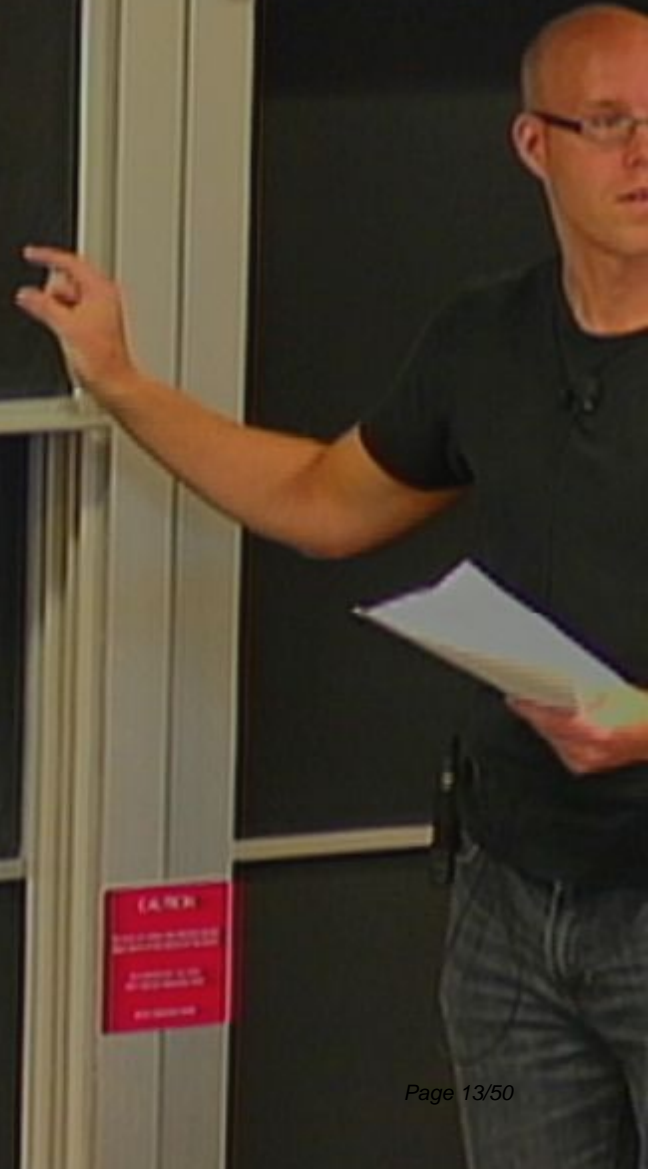
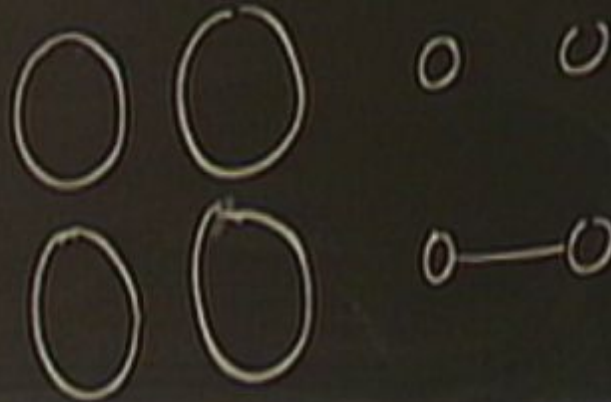
$$\nabla_{\rho} g_{\mu\nu} = A_{\rho} g_{\mu\nu}$$

$$g_{\mu\nu} \rightarrow e^{\alpha} g_{\mu\nu}$$
$$A_{\rho} \rightarrow A_{\rho} + 2\alpha$$

R^{ρ}
 R
 R

$$d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$$

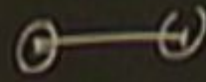
$$d\tilde{\tau}^2 = R d\tau^2$$



$$\tau = \int d\tau \quad d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$$

extra

$$d\tilde{\tau}^2 = R d\tau^2$$

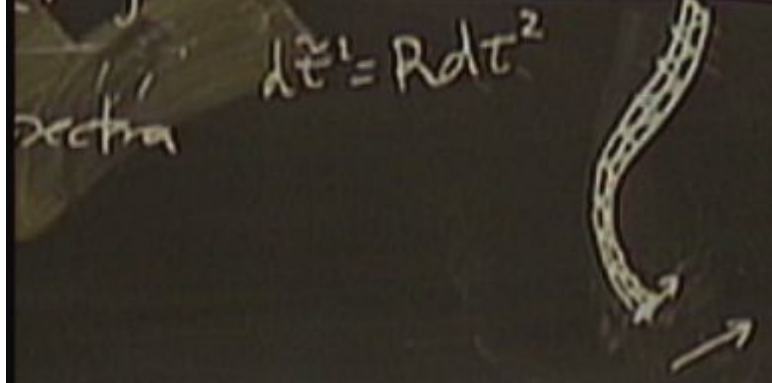


$$R_{\mu\nu}R^{\mu\nu} + R'_{\alpha\beta}R^{\alpha\beta} + F^2 + R_{\mu\nu}F^{\mu\nu}$$

Transport of ruler

ticks

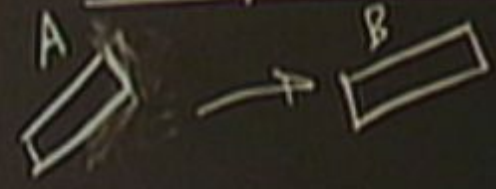
$\tau = \int dt$ $d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$



CAUTION
 DO NOT TOUCH
 THE BOARD

$$R_{\mu\nu}R^{\mu\nu} + R^{\sigma\mu\nu}R_{\sigma\mu\nu} + F^2 + R_{\mu\nu}F^{\mu\nu}$$

Transport of ruler



trans

$\tau = \int d\tau$ $d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$

section $d\tilde{\tau}^2 = R d\tau^2$



CAUTION
 THE UNIVERSITY
 OF TORONTO



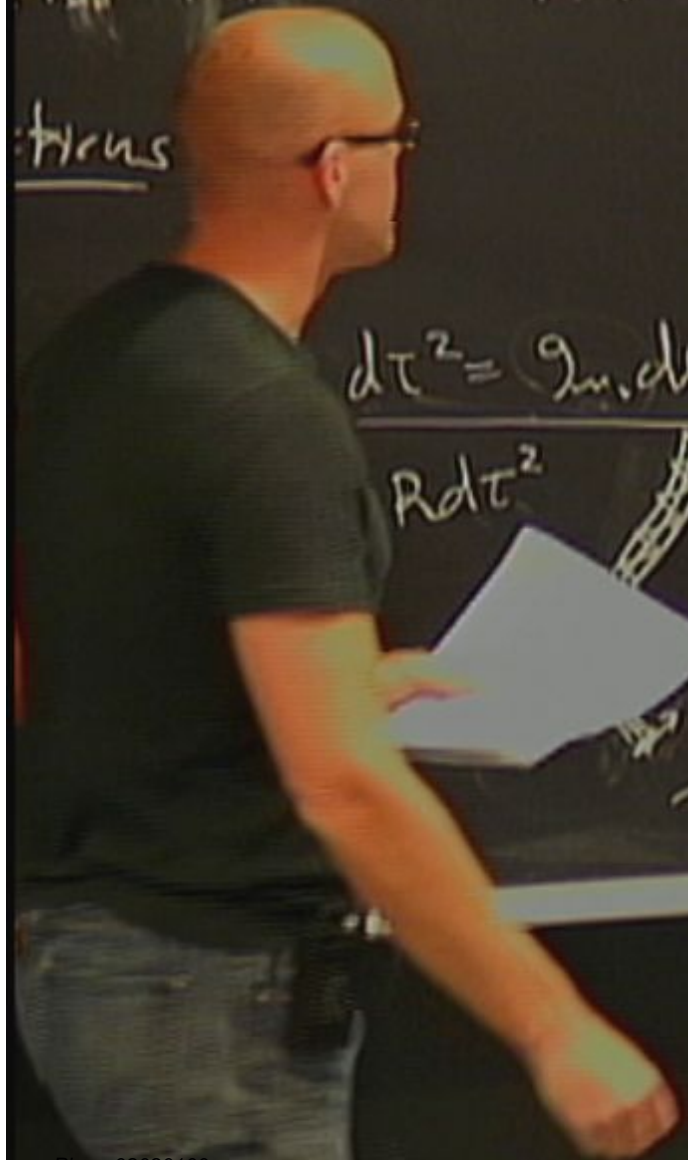
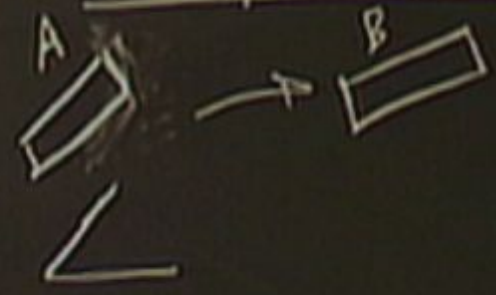
$$R_{\mu\nu}R^{\mu\nu} + R'_{\alpha\beta}R^{\alpha\beta} + F^2 + R_{\alpha\beta}F^{\alpha\beta}$$

terms

$$d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu$$

$$R d\tau^2$$

Transport of ruler



CAUTION
 PLEASE DO NOT TOUCH
 THE BOARD OR THE
 SURROUNDING AREA
 THANK YOU

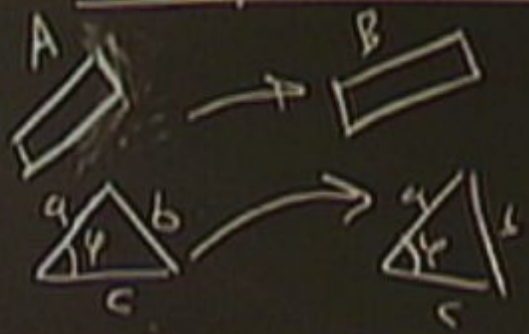
$$R_{\mu\nu}R^{\mu\nu} + R'_{\alpha\beta\gamma\delta}R'^{\alpha\beta\gamma\delta} + F^2 + R_{\mu\nu}F^{\mu\nu}$$

fields

$$\tau^2 = \int ds^2 = \int g_{\mu\nu} dx^\mu dx^\nu$$

$$t' = R dt^2$$

transport of ruler



LATCH
 PULL TO OPEN
 PUSH TO LOCK

$$R_{\mu\nu}R^{\mu\nu} + R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} + F^2 + R_{\mu\nu}F^{\mu\nu}$$

thus

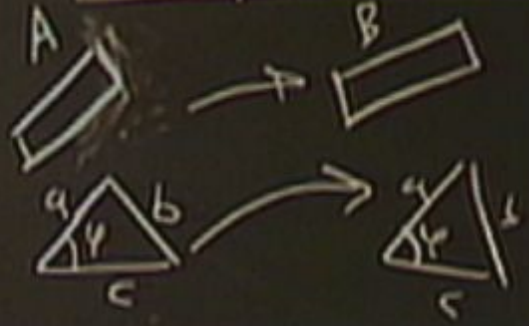
$$\tau = \int d\tau$$

$$d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$d\tau^2 = R dt^2$$



Transport of vectors



$$\frac{D}{DS} (u^i v^j g_{ij}) = 0 \quad \frac{Du^i}{Ds}$$

CAUTION

Notes

$$\tau = \int d\tau \quad d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$$

rectra

$$d\tilde{\tau}^2 = R d\tau^2$$



$$\frac{D}{DS} \left(\frac{u^i v^j g_{ij}}{|u| |v|} \right) = 0 \quad \frac{Du^i}{Ds} = \frac{Dv^i}{Ds}$$

$$\nabla_n g_{ij} \left(\frac{u^i v^j}{|u| |v|} - \frac{1}{2} \frac{u^i u^j}{|u|^2} - \frac{1}{2} \frac{v^i v^j}{|v|^2} \right)$$

$$\nabla_u g_{ij} = A_{ij}$$

$$g_{ij} \rightarrow e^{\phi} g_{ij}$$

$$= 0 \quad \frac{D u_i}{D s} = \frac{D^i}{D s}$$

$$\frac{1}{\Lambda^2} \left(\frac{1}{2} \frac{u_i u^i}{\Lambda^2} \right)$$



$$\nabla_u g_{ij} = A_u g_{ij}$$

$$g_{ij} \rightarrow e^{\sigma} g_{ij} \quad A_k \rightarrow A_k + 2\sigma$$


$$= 0 \quad \frac{D u^i}{D s} = \frac{D^i}{D s}$$

$$\frac{1}{11^L} - \frac{1}{2} \frac{1}{11^L} \rightarrow$$



$$g_{ij} \rightarrow e^{\phi} g_{ij}, \quad A_k \rightarrow A_k + d_k \phi$$

Best-Matching


$$S = \int d\lambda \sqrt{T(E-V)}$$

(x_1, x_2, \dots, x_n)



CAUTION

$$\frac{1}{\sqrt{2}}$$

$$0 \quad \frac{Dv_i}{Dx} = \frac{Dv_i}{D}$$

$$\frac{v_i v_j}{\pi^2} - \frac{1}{2} \frac{v_i v_j}{|v|^2}$$



$$S = \int d\lambda \sqrt{T(E-V)}$$

(x_1, x_2, \dots, x_n)

$$\sum_i m_i (\Delta x_{i1} - T - R x_{i1} - dx_{i1})^2$$



CAUTION
 Do not touch the blackboard
 as it is very hot.

$$\frac{D u_i}{D s} = \frac{D u_i}{D}$$

$$\frac{v_i u_i}{s_i^2} = \frac{1}{2} \frac{v_i}{s_i}$$

$$\nabla_u g_{ij} = A_k g_{ij}$$

$$g_{ij} \rightarrow e^{\sigma} g_{ij} \quad A_k \rightarrow A_k + d_k \sigma$$

$$S_{\text{eff}} = \int d\lambda \sqrt{\frac{1}{2} m_n (\dot{x}_n - T - R \dot{x}_n - d \dot{x}_n)^2 (E - V)}$$

Best-Matching

$$S = \int d\lambda \sqrt{T (E - V)}$$

(x_1, x_2, \dots, x_n)

$$\sum_n m_n (\dot{x}_n - T - R \dot{x}_n - d \dot{x}_n)^2$$

$$\nabla_u g_{ij} = A_{ij} g_{ij}$$

$$g_{ij} \rightarrow e^{\theta} g_{ij}, \quad A_{ij} \rightarrow A_{ij} + \theta$$

$$S_{PI} = \int dx \sqrt{\sum_n m_n (\dot{x}_n - T - R x_n - d x_n)^2 (E - V)}$$

Best-Matching

$$S = \int dt \mathcal{L}(E - V)$$



(x_1, \dots, x_n)

$$\sum_n m_n (\dot{x}_n - T - R x_n - d x_n)^2$$

$$\nabla_u g_{ij} = A_u g_{ij}$$

$$g_{ij} \rightarrow e^{\sigma} g_{ij}, \quad A_u \rightarrow A_u + d_u \sigma$$

$$S_H = \int d^4x \sqrt{-g} \left[\frac{1}{2} \kappa (R - F_{\mu\nu} F^{\mu\nu}) - V \right]$$

Maxwell

Best-Matching

$$N = \int d^3x \left[(A - \nabla \phi)^2 - (\nabla \times A)^2 \right]$$

$$S = \int d^4x \sqrt{-g} (E - V)$$

(x_1, x_2, \dots, x_n)

$$\sum_i m_i (\Delta x_i)^2 = T - R$$

$$g_{ij} \rightarrow e^{\phi} g_{ij}, \quad A_i \rightarrow A_i + d_i \phi$$

Best-Matching

$$S = \int d^4x \sqrt{|T|} (E - V)$$

$$\sum_n m_n (\Delta x_n - d t_n)^2$$

Maxwell

$$L_{MW} = \int d^4x (A_i - \nabla_i \phi)^2 - (\nabla_i A^i)^2$$

$$L_{BMEH} = \sqrt{|g|} (A_i - \nabla_i \phi)^2 (E - \sqrt{|g|} \nabla_i A^i)^2$$

$$g_{ij} \rightarrow e^{\phi} g_{ij}, \quad A_i \rightarrow A_i + d\phi$$

Best-Matching

$$S = \int d\lambda \sqrt{T(E-V)}$$

(x_1, x_2, \dots)

$$\sum_i m_i (\Delta x_i)^2$$

Maxwell

$$L_{MN} = \int d^3x (A_i - \nabla_i \phi)^2 - (\nabla_i A_i)^2$$

$$L_{BMEH} = \int d^3x (A_i - \nabla_i \phi)^2 (E - \nabla_i (\nabla_i \phi)^2)$$

$$\sum_n m_n (\Delta x_{1n} - T - R x_{1n} - dx_{1n})^2$$

$$B_{ADM} = \sqrt{g} (E - \int dx (\nabla_{\mu} A)^{\mu})$$

$$4 ADM = \int d^3x \sqrt{g} (g^{ijkl} (\dot{g}_{ij} - L_{\vec{N}} g_{ij}) (K_{kl}) (R - 2\Lambda))$$

$$T_g = S^{ijkl} (\dot{g}_{ij} - L_{ij}^{\rightarrow} g_{ij} - \phi g_{ij}) (\dot{g}_{kl} - L_{kl}^{\rightarrow} g_{kl} - \phi g_{kl})$$

$$T_g = \int g^{ijkl} (g_{ij} - L_{ij} g_{ij} - \phi g_{ij}) (g_{kl} - L_{kl} g_{kl} - \phi g_{kl})$$

$$T_A =$$



$$T_g = \int g^{ijkl} (\dot{g}_{ij} - L_{\vec{v}} g_{ij} - \phi g_{ij}) (\dot{g}_{kl} - L_{\vec{v}} g_{kl} - \phi g_{kl})$$

$$T_A = g^{ij} (\dot{A}_i - L_{\vec{v}} A_i - \phi A_i) (\dot{A}_j - L_{\vec{v}} A_j - \phi A_j)$$



$\mathcal{L} + \mathcal{L}_\phi$

$$0 \quad T_g = \int_{ij\mu\nu}^{-2} (g_{ij} - L_{\vec{N}} g_{ij} - \phi g_{ij}) (g_{\mu\nu} - L_{\vec{N}} g_{\mu\nu} - \phi g_{\mu\nu})$$



$$-1 \quad T_A = g^{ij} (A_i - L_{\vec{N}} A_i - \partial_i \phi) (A_j - L_{\vec{N}} A_j - \partial_j \phi)$$

$$\phi \rightarrow \phi + \delta\phi - N \partial_\mu \xi \quad g_i \rightarrow g_i + \delta g_i \quad A_\mu \rightarrow A_\mu + \partial_\mu \xi$$

$$L_{3DW} = \sqrt{(T_g R_H a T_A)}$$

$$L_{3DW} = \frac{1}{3} \sqrt{\left(\frac{1}{g} R \hat{a} T_A \right) \left(b F^2 + (R^2 + d R_i R' \dots) \right)}$$

CAUTION
 DO NOT TOUCH THE BOARD
 WHEN IT IS BEING USED
 BY THE INSTRUCTOR

$$L_{3DW} = \frac{1}{2} \sqrt{\left(\frac{1}{g} R_H a T_A \right) \left(b F^2 + (R^2 + d R_i R' \dots) \right)}$$

Hamilton-Formulation

$$\pi^{ij} = \frac{\partial \mathcal{L}}{\partial \dot{g}_{ij}}$$

(14/10)

Hamilton-Formulation

$$\pi^{ij} = \frac{\partial L}{\partial g_{ij}} \quad \pi^i = \frac{\partial L}{\partial \dot{x}^i}$$

CAUTION

Hamiltoni-Formulation

$$\pi^{ij} = \frac{\partial \mathcal{L}}{\partial \dot{g}_{ij}} \quad \pi^i = \frac{\partial \mathcal{L}}{\partial \dot{\lambda}_i}$$

$$\pi_{\dot{\phi}} = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = 0 \quad \pi_{\dot{\lambda}_k} = \frac{\partial \mathcal{L}}{\partial \dot{\lambda}_k} = 0$$

Hamiltoni-Formulation

$$\pi^{ij} = \frac{\partial L}{\partial g_{ij}} \quad \pi^i = \frac{\partial L}{\partial \dot{x}^i}$$

$$\pi_{\dot{x}^i} = \frac{\partial L}{\partial \dot{x}^i} = 0 \quad \pi_{\dot{\alpha}^i} = \frac{\partial L}{\partial \dot{\alpha}^i} = 0$$

CAUTION
DANGER OF ELECTRIC SHOCK
DO NOT TOUCH THE SURFACE
UNLESS YOU ARE QUALIFIED TO DO SO

Hamilton-Formulation

$$\pi^{ij} = \frac{\partial L}{\partial g_{ij}} \quad \pi^i = \frac{\partial L}{\partial \dot{x}^i}$$

$$\pi_{\dot{x}^i} = \frac{\partial L}{\partial \dot{x}^i} = 0 \quad \pi_{\dot{m}^k} = \frac{\partial L}{\partial \dot{m}^k} = 0$$

$$\ddot{\pi}_{\dot{x}^i} = 0 \Rightarrow \pi^{ij} g_{ij} - \nabla \cdot \pi^i = 0$$

Hamiltoni-Formulation

$$\pi^{ij} = \frac{\partial L}{\partial g_{ij}} \quad \pi^i = \frac{\partial L}{\partial \dot{x}^i}$$

$$\pi_{\dot{x}^i} = \frac{\partial L}{\partial \dot{x}^i} = 0 \quad \pi_{D_{\mu}} = \frac{\partial L}{\partial D_{\mu}} = 0$$

$$\ddot{\pi}_{\dot{x}^i} = 0 \Rightarrow \pi^{ij} g_{ij} - \nabla_i \pi^i = 0$$

$$\dot{\pi}_{D_{\mu}} = 0 \Rightarrow \nabla_i \pi^i_j + \frac{1}{2}$$

Hamiltoni-Formulation

$$\pi^{ij} = \frac{\partial L}{\partial g_{ij}} \quad \pi^i = \frac{\partial L}{\partial \dot{x}^i}$$

$$\pi_{\dot{x}^i} = \frac{\partial L}{\partial \dot{x}^i} = 0 \quad \pi_{\dot{A}^i} = \frac{\partial L}{\partial \dot{A}^i} = 0$$

$$\ddot{\pi}_{\dot{x}^i} = 0 \Rightarrow \pi^{ij} g_{ij} - \nabla_i \pi^i = 0$$

$$\ddot{\pi}_{\dot{A}^i} = 0 \Rightarrow \nabla_i \pi^i_j + \frac{1}{2} \pi^i F_{ij} = 0$$



Hamiltoni-Formulation

$$\pi^{ij} = \frac{\partial \mathcal{L}}{\partial g_{ij}} \quad \pi^i = \frac{\partial \mathcal{L}}{\partial \dot{x}^i}$$

$$\pi_{\dot{x}^i} = \frac{\partial \mathcal{L}}{\partial \dot{x}^i} = 0 \quad \pi_{D^k} = \frac{\partial \mathcal{L}}{\partial D^k} = 0$$

$$\ddot{\pi}_{\dot{x}^i} = 0 \Rightarrow \pi^{ij} g_{ij} - \nabla_i \pi^i = 0$$

$$\ddot{\pi}_{D^k} = 0 \Rightarrow \nabla_i \pi^i_j + \frac{1}{2} \pi^i F_{ij} = 0$$

Hamilton-Formulation

$$\pi^{ij} = \frac{\partial L}{\partial g_{ij}} \quad \pi^i = \frac{\partial L}{\partial \dot{x}^i}$$

$$\pi_{\dot{x}^i} = \frac{\partial L}{\partial \dot{x}^i} = 0 \quad \pi_{D^k} = \frac{\partial L}{\partial D^k} = 0$$

$$\ddot{\pi}_{\dot{x}^i} = 0 \Rightarrow \pi^{ij} g_{ij} - \nabla_i \pi^i = 0$$

$$\ddot{\pi}_{D^k} = 0 \Rightarrow \nabla_i \pi^i_j + \frac{1}{2} \pi^i F_{ij} = 0$$

$$a_{ij} \pi^i \pi^j + R g_{ij} \pi^i \pi^j - a g R$$

$$L_{3DW} = \sqrt{g} \sqrt{\left(\frac{1}{g} R + a T_A \right) \left(F^2 + (R^2 + d R_i R^i) \dots \right)}$$

Hamilton-Formulation

$$\pi^{ij} = \frac{\partial L}{\partial g_{ij}} \quad \pi^i = \frac{\partial L}{\partial \dot{\lambda}_i}$$

$$\pi_4 = \frac{\partial L}{\partial \dot{\lambda}_4} = 0 \quad \pi_{D\lambda} = \frac{\partial L}{\partial \dot{\lambda}} = 0$$

$$\ddot{\pi}_4 = 0 \Rightarrow \pi^i g_{ij} - \nabla_i \pi^j = 0$$

$$\dot{\pi}_{D\lambda} = 0 \Rightarrow \nabla_i \pi^j + \frac{1}{2} \pi^i F_{ij} = 0$$

$$a (g_{ij} \pi^{ij} \pi^{kk} + R g_{ij} \pi^i \pi^j) - a g R V$$

$$L_{3DW} = \sqrt{g} \sqrt{\left(T_{ij} R + a T_A \right) \left(F^2 + \left(R^2 + d R_i R^i \right) \dots \right)} \quad R-2.1$$

Hamilton-Formulation

$$\pi^{ij} = \frac{\partial L}{\partial g_{ij}} \quad \pi^i = \frac{\partial L}{\partial \dot{\lambda}_i}$$

$$\pi_4 = \frac{\partial L}{\partial \dot{x}^4} = 0 \quad \pi_{D4} = \frac{\partial L}{\partial \dot{\lambda}^4} = 0$$

$$\dot{\pi}_4 = 0 \Rightarrow \pi^{ij} g_{ij} - \nabla_i \pi^i = 0$$

$$\dot{\pi}_{D4} = 0 \Rightarrow \nabla_i \pi^i_j + \frac{1}{2} \pi^i F_{ij} = 0$$

$$d a g_{ij} \pi^i \pi^j + R g_{ij} \pi^i \pi^j - a g R V = 0$$

$$L_{ADM} = \frac{1}{8\pi} \int_{\Sigma} \sqrt{\gamma} \left(S^{ijkl} (\dot{g}_{ij} - L_{\vec{N}} g_{ij}) (4\epsilon) (R - 2\lambda) \right) d^4x$$



$$\left(\frac{1}{2} \dot{\mathbf{r}}^T \mathbf{A} \dot{\mathbf{r}} + \frac{1}{2} R^2 + d R_i R' \dots \right) \Big|_{g_{\mu\nu} = \begin{pmatrix} \dot{x}_i & N \\ R_i & g_{ij} \end{pmatrix}}$$

$$\ddot{\pi}_4 = 0 \Rightarrow \pi^i g_{ij} - \nabla_i \pi^j = 0 \quad N = \sqrt{\frac{1}{V}}$$

$$\ddot{\pi}_{D_n} = 0 \Rightarrow \nabla_i \pi^j + \frac{1}{2} \pi^i F_{ij} = 0$$

$$\frac{1}{2} a g_{ij} \pi^i \pi^j + R g_{ij} \pi^i \pi^j - a g R V = 0$$

$\frac{\partial L}{\partial \dot{x}_i}$
 $\frac{\partial L}{\partial \dot{x}_i} = 0$

$$g^{ijkl} (g_{ij} - L \dots)$$

