Title: Dynamical origin of quantum probabilities revisited

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Abstract: The de Broglie-Bohm theory is about non-relativistic point-particles that move deterministically along trajectories. The theory reproduces the predictions of standard quantum theory given that the distribution of particle positions over an ensemble of systems, all described by the same wavefunction psi, equals the quantum equilibrium distribution |psi| squared. Numerical simulations by Valentini and Westman have illustrated that non-equilibrium particle distributions may relax to quantum equilibrium after some time. Here we consider non-equilibrium distributions and their relaxation properties for a particular class of trajectory theories, first studied in detail by Deotto and Ghirardi, that are empirically equivalent to the de Broglie-Bohm theory in quantum equilibrium. Joint work with Ward Struyve (KUL, Belgium).

Overview

The de Broglie-Bohm pilot-wave theory is about non-relativistic point-particles that move deterministically along trajectories. The theory reproduces the predictions of standard quantum theory given that the distribution of particle positions over an ensemble of systems, all described by the same wavefunction ψ equals the quantum equilibrium distribution ψ^2. Numerical simulations by Valentini and Westman (Dynamical origin of quantum probabilities) have illustrated that non-equilibrium particle distributions may relax to quantum equilibrium after some time. Here we consider non-equilibrium distributions and their relaxation properties for a particular class of trajectory theories, first studied in detail by Deotto and Ghirardi (Bohmian mechanics revisited), that are empirically equivalent to the de Broglie-Bohm theory in quantum equilibrium.

Overview

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Plan

- The de Broglie-Bohm pilot-wave theory
- Quantum non-equilibrium and relaxation
- Non-uniqueness of the guidance equation
- Simulations

Non-relativistic pilot-wave theory

- Complete description for one particle: $(\psi(t,\vec{x}), \vec{X}(t))$
- Schrödinger equation:

$$i\hbar\frac{\partial\psi(t,\vec{x})}{\partial t} = \left(-\hbar^2\frac{\Delta}{2m} + V(\vec{x})\right)\psi(t,\vec{x}) \tag{1}$$

- Guidance equation:

$$\vec{v}(t) = \left.\frac{\vec{j}(t,\vec{x})}{|\psi(t,\vec{x})|^2}\right|_{\vec{x}=\vec{X}(t)} = \frac{\hbar}{m}\mathfrak{Im}\left.\left(\frac{\vec{\nabla}\psi(t,\vec{x})}{\psi(t,\vec{x})}\right)\right|_{\vec{x}=\vec{X}(t)} \tag{2}$$

where $\partial_t|\psi(t,\vec{x})|^2 + \vec{\nabla}\cdot\vec{j}(t,\vec{x}) = 0$.

Equivariance

- $\rho(t, \vec{x}) = |\psi(t, \vec{x})|^2$ if $\rho(t_0, \vec{x}) = |\psi(t_0, \vec{x})|^2$.
- Consequence of

$$\vec{v} = \frac{\vec{j}}{|\psi|^2} \quad \text{where} \quad \partial_t |\psi|^2 + \vec{\nabla} \cdot \vec{j} = 0 \qquad (3)$$

- $\rho(t, \vec{x}) = |\psi(t, \vec{x})|^2$: quantum equilibrium distribution

Quantum non-equilibrium and relaxation

- What about systems for which $\rho(t_0, \vec{x}) \neq |\psi(t_0, \vec{x})|^2$?
- Do they relax to quantum equilibrium?

$$\rho(\tau, \vec{x}) \simeq |\psi(\tau, \vec{x})|^2 \tag{4}$$

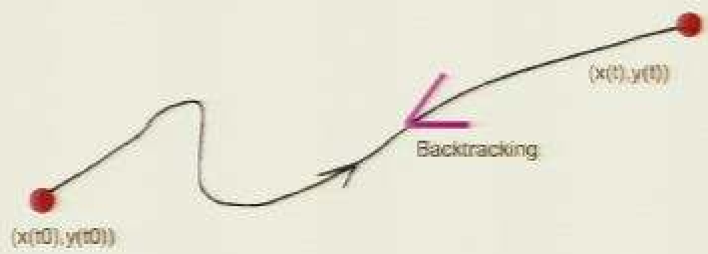
Evolution of the non-equilibrium density

- Liouville-like equation:

$$\frac{d}{dt}\left(\frac{\rho(t,\vec{x})}{|\psi(t,\vec{x})|^2}\right) = 0 \qquad (5)$$

- $\rho(t,\vec{x})$? $\vec{X}(t) = \vec{x}$

$$\rho(t,\vec{X}(t)) = \frac{\rho(t_0,\vec{X}(t_0))}{|\psi(t_0,\vec{X}(t_0))|^2}|\psi(t,\vec{X}(t))|^2 \qquad (6)$$



- Relaxation?

Valentini's conjecture

- Coarse-grained quantities:

$$\bar{\rho}(t,\vec{x}) = \int_{cube\ at\ \vec{x}} \rho(t,\vec{x}')d^3x' \ . \tag{7}$$

- Relaxation to quantum equilibrium:

$$\bar{\rho}(t,\vec{x}) \to \overline{|\psi(t,\vec{x})|^2} \ . \tag{8}$$

$\tau \simeq \frac{\hbar^2}{m^{1/2}\lambda(\Delta E)^{3/2}}$ (one particle).

- Quantum non-equilibrium in the early universe:
  Test pilot-wave theory against standard quantum theory[1].

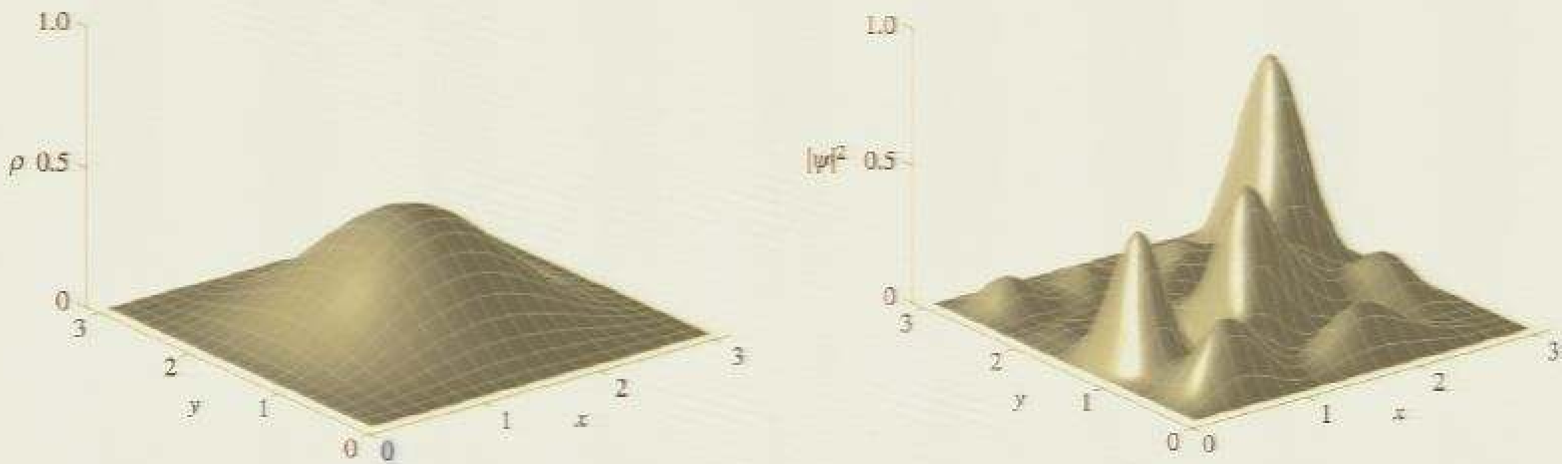
[1] A. Valentini, Inflationary Cosmology as a Probe of Primordial Quantum Mechanics, 0805.0163

Relaxation simulations (A.Valentini - H.Westman)[2]

- Particle in a 2-D Box $[0,\pi]x[0,\pi]$
- Wave-function:

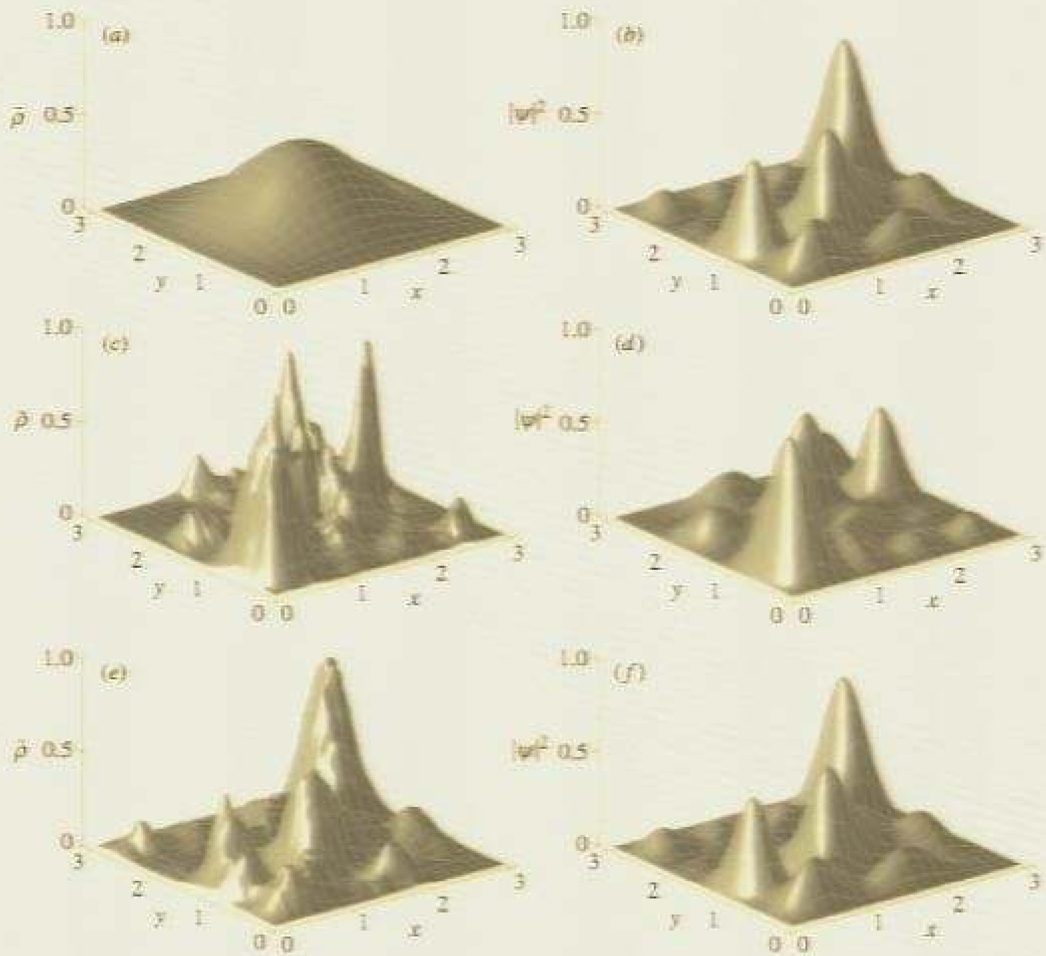
$$\psi(t,x,y) = \frac{1}{2\pi}\sum_{m=1}^{4}\sum_{n=1}^{4}\sin(mx)\sin(ny)e^{i(\theta_{mn}-\frac{m^2+n^2}{2}t)} \qquad (9)$$

- Initial non equilibrium density and $|\psi|^2$:



[2]A. Valentini and H. Westman, Proc. R. Soc. A **461**, 253-272 (2005)

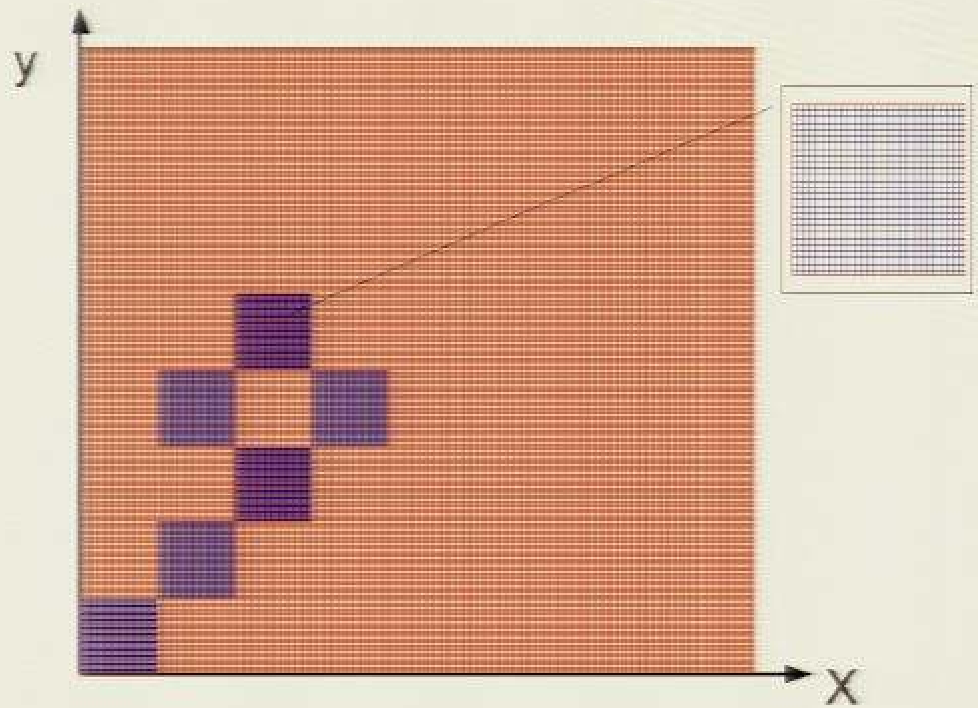
Evolution of $\bar{\rho}(t, x, y)$



$$a)b) : t = 0 \quad c)d) : t = 2\pi \quad e)f) : t = 4\pi.$$

Method

- Divide box in coarse-graining cells (CG cells):



- Sample of lattice points inside CG cell.
- For each of these lattice points, compute the density at final time t using the "Liouville equation", then average.

Evolution of the non-equilibrium density

- Liouville-like equation:

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- $\rho(t,\vec{x})$? $\vec{X}(t) = \vec{x}$

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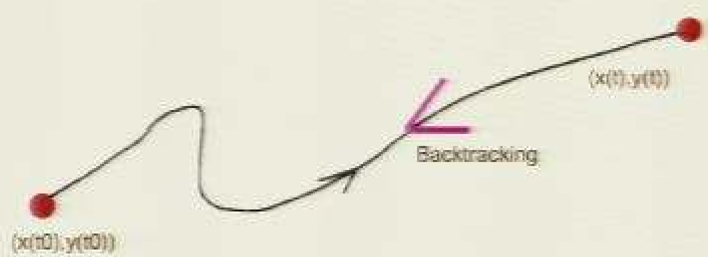

- Relaxation?

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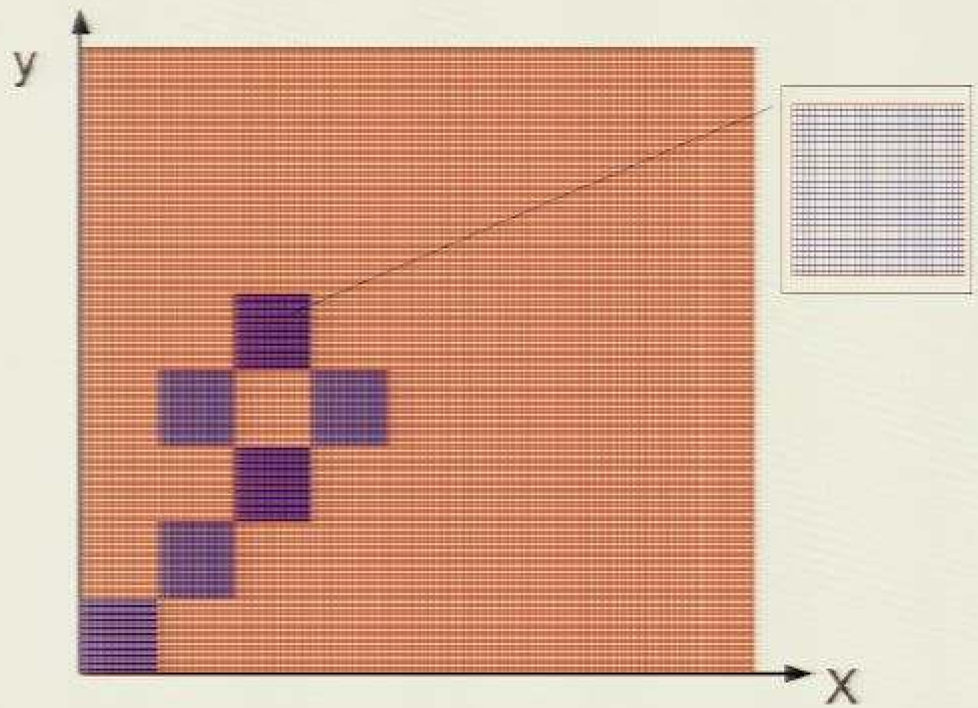

- Sample of lattice points inside CG cell.
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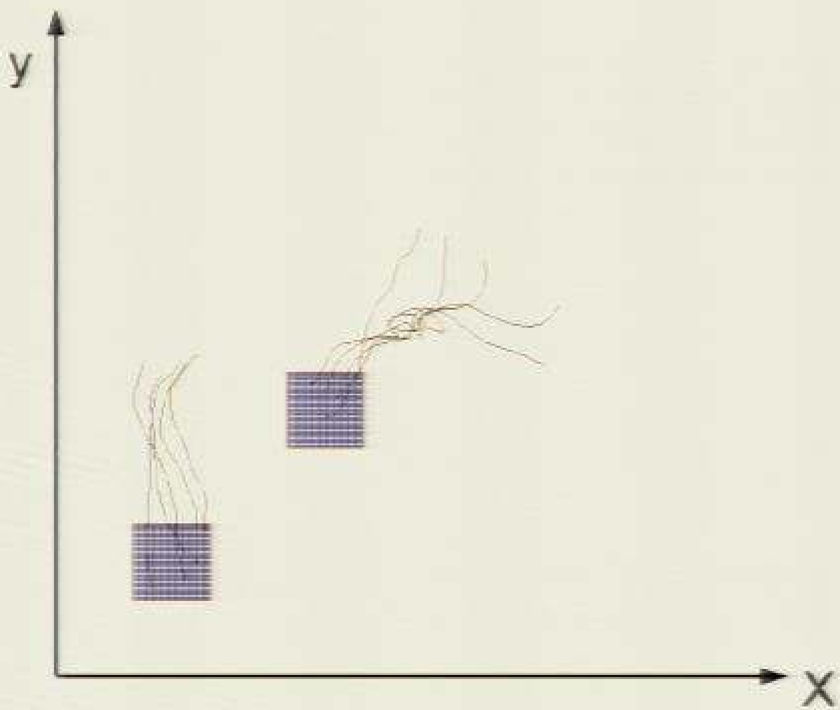
Chaos and nodes of the wave-function



Nodes are important for chaotic trajectories[3]. Usually vorticity around nodes.

[3] H.Frisk-P. Falsaperla and G. Fonte - D.A. Wisniacki, E.R. Pujals and F.

Back to equivariance

- $\rho(t, \vec{x}) = |\psi(t, \vec{x})|^2$ if $\rho(t_0, \vec{x}) = |\psi(t_0, \vec{x})|^2$.
- Consequence of

$$\vec{v} = \frac{\vec{j}}{|\psi|^2} \quad \text{where} \quad \partial_t |\psi|^2 + \vec{\nabla} \cdot \vec{j} = 0 \tag{10}$$

- $\rho(t, \vec{x}) = |\psi(t, \vec{x})|^2$: quantum equilibrium distribution

Equivariance again

- $\rho(t,\vec{x}) = |\psi(t,\vec{x})|^2$ if $\rho(t_0,\vec{x}) = |\psi(t_0,\vec{x})|^2$.
- Consequence of

$$\vec{v} = \frac{\vec{j} + \vec{\nabla} \times \vec{a}}{|\psi|^2} \quad \text{where} \quad \partial_t |\psi|^2 + \vec{\nabla} \cdot (\vec{j} + \vec{\nabla} \times \vec{a}) = 0 \qquad (11)$$

- $\rho(t,\vec{x}) = |\psi(t,\vec{x})|^2$: quantum equilibrium distribution

Non-uniqueness of the guidance equation

- ∞ number of de Broglie-Bohm-like models for which $|\psi(t,\vec{x})|^2$ is equivariant (E. Deotto and G.C. Ghirardi[4]):

$$\vec{v}' = \vec{v} + \frac{\vec{\nabla} \times \vec{a}}{|\psi(t,\vec{x})|^2} \, . \tag{12}$$

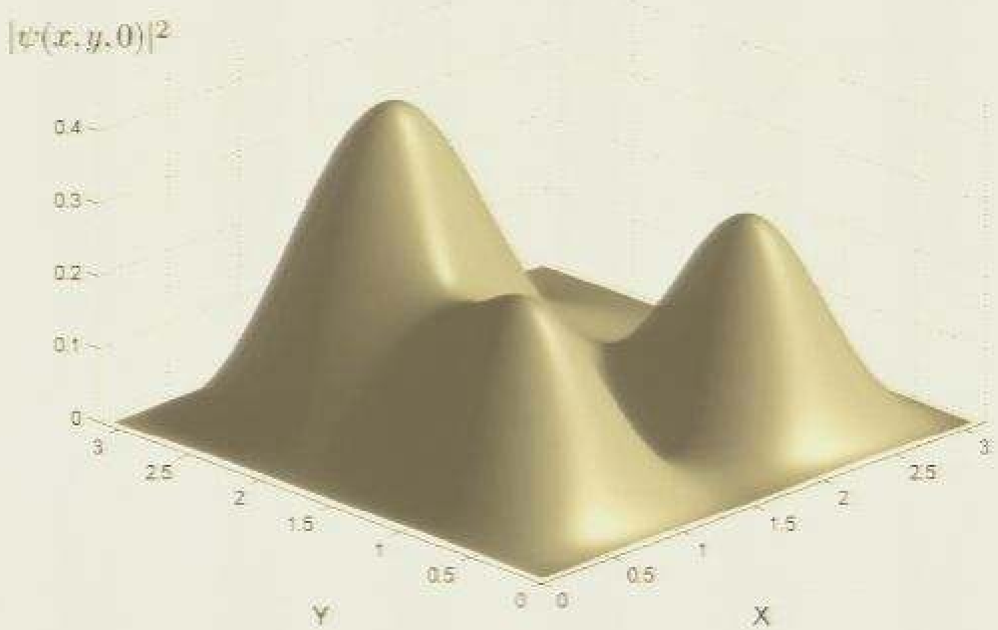
- The Pauli equation as the NR limit of the Dirac equation (D. Bohm and B. Hiley). New term: spin term.
- Arguments for the uniqueness of the guidance equation (Dürr, Goldstein & Zanghì, P. Holland, H. Wiseman (weak meas.))

[4] E. Deotto and G.C. Ghirardi, Found. Phys. **28**, 1 (1998).

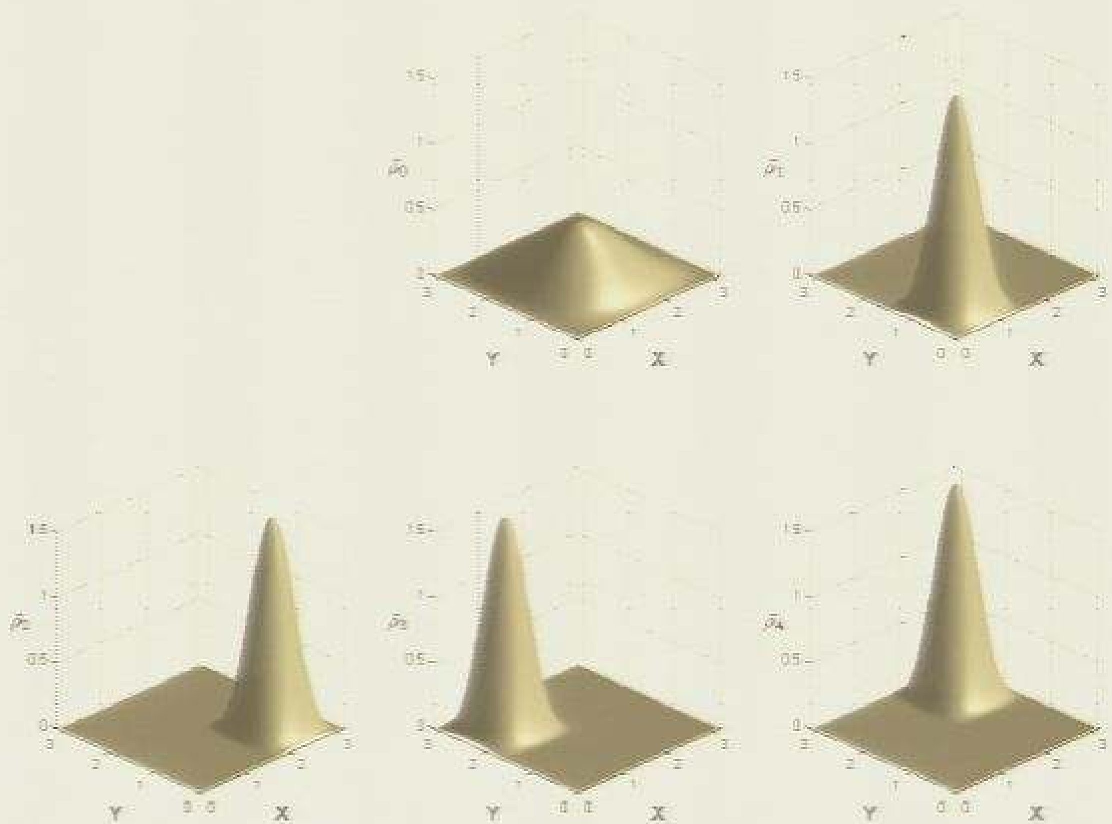
Simulations

Particle in a 2-D Box $[0,\pi]x[0,\pi]$

$$\psi(t,x,y)=\frac{1}{\pi}\sum_{m=1}^{2}\sum_{n=1}^{2}\sin(mx)\sin(ny)e^{i(\theta_{mn}-\frac{m^2+n^2}{2}t)} \tag{13}$$



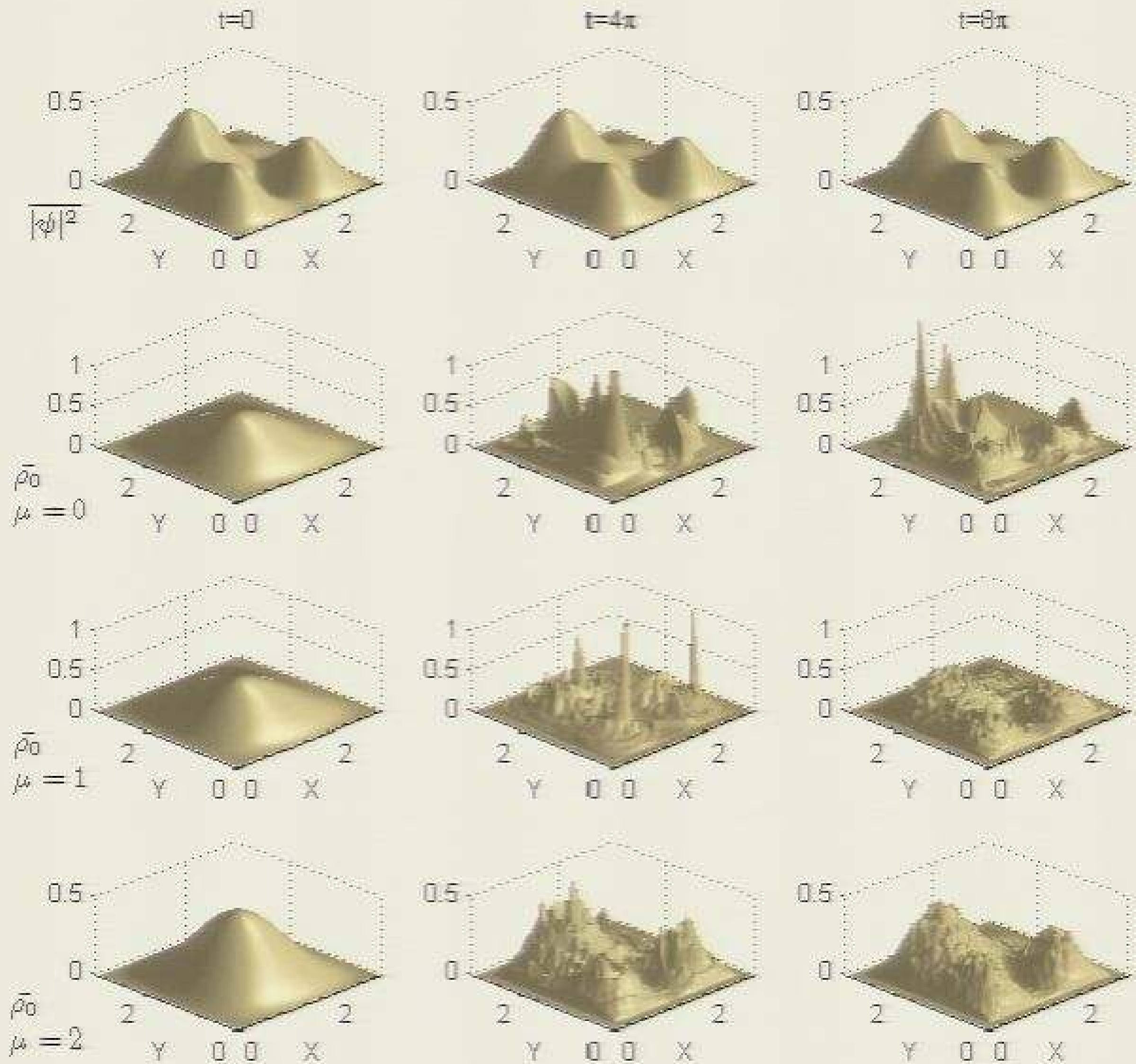
- Different initial non-equilibrium distributions:



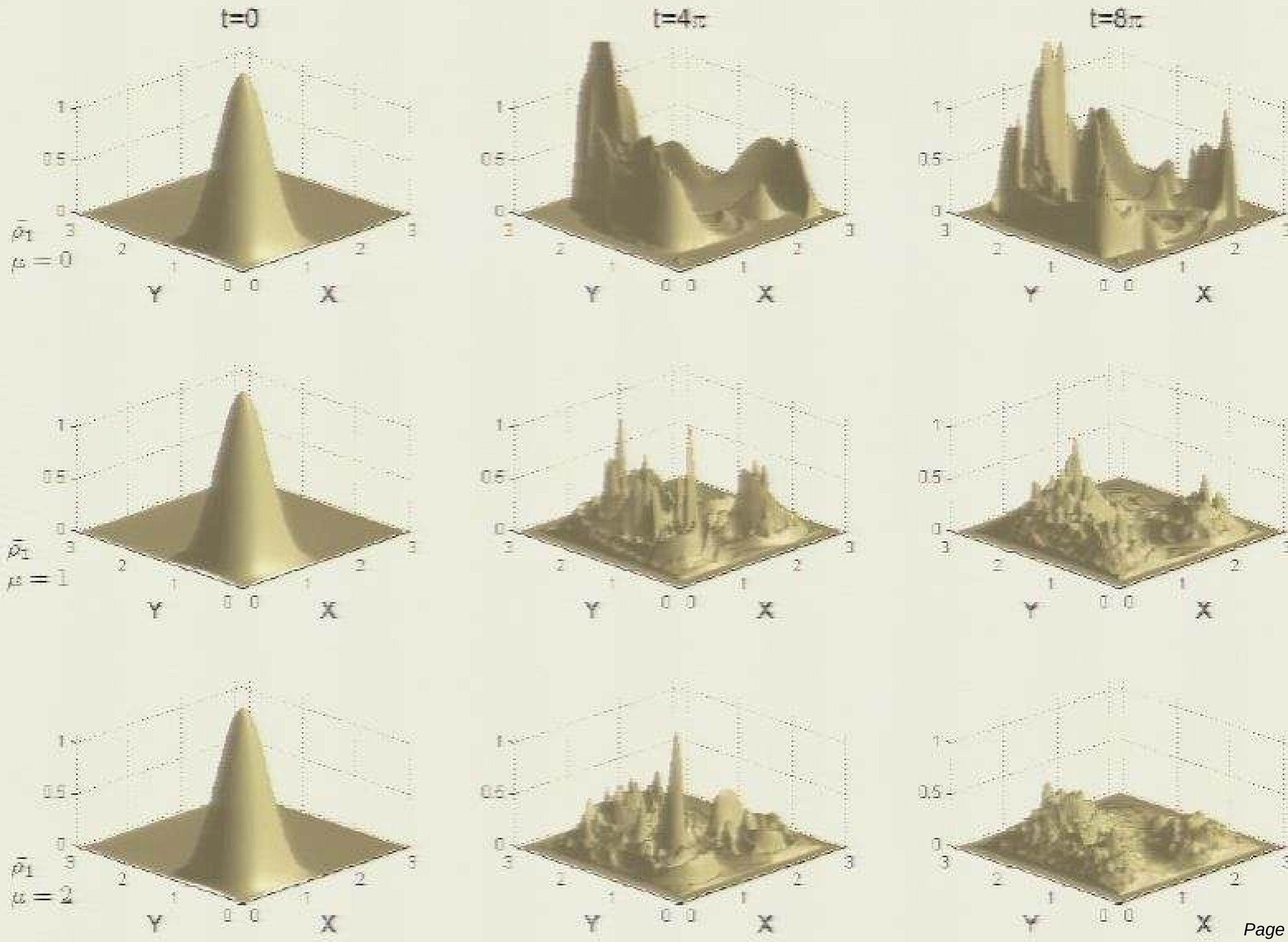
- Different guidance equations:

$$v_k' = v_k + \mu \frac{\epsilon_{kl}\partial_l f^{\psi}}{|\psi|^2} \qquad (14)$$

Amplitude of the curl term



Different initial non-equilibrium distributions



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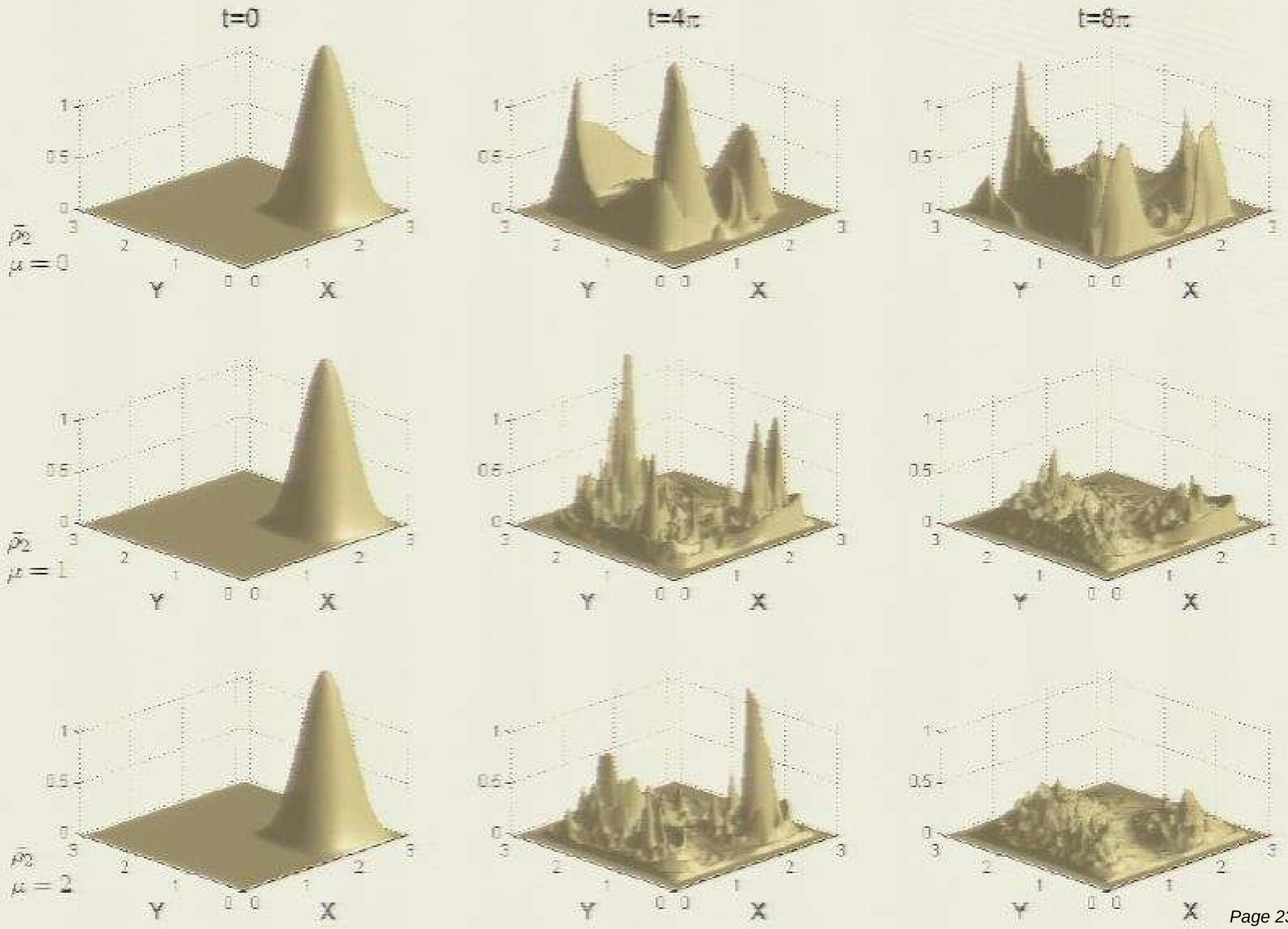

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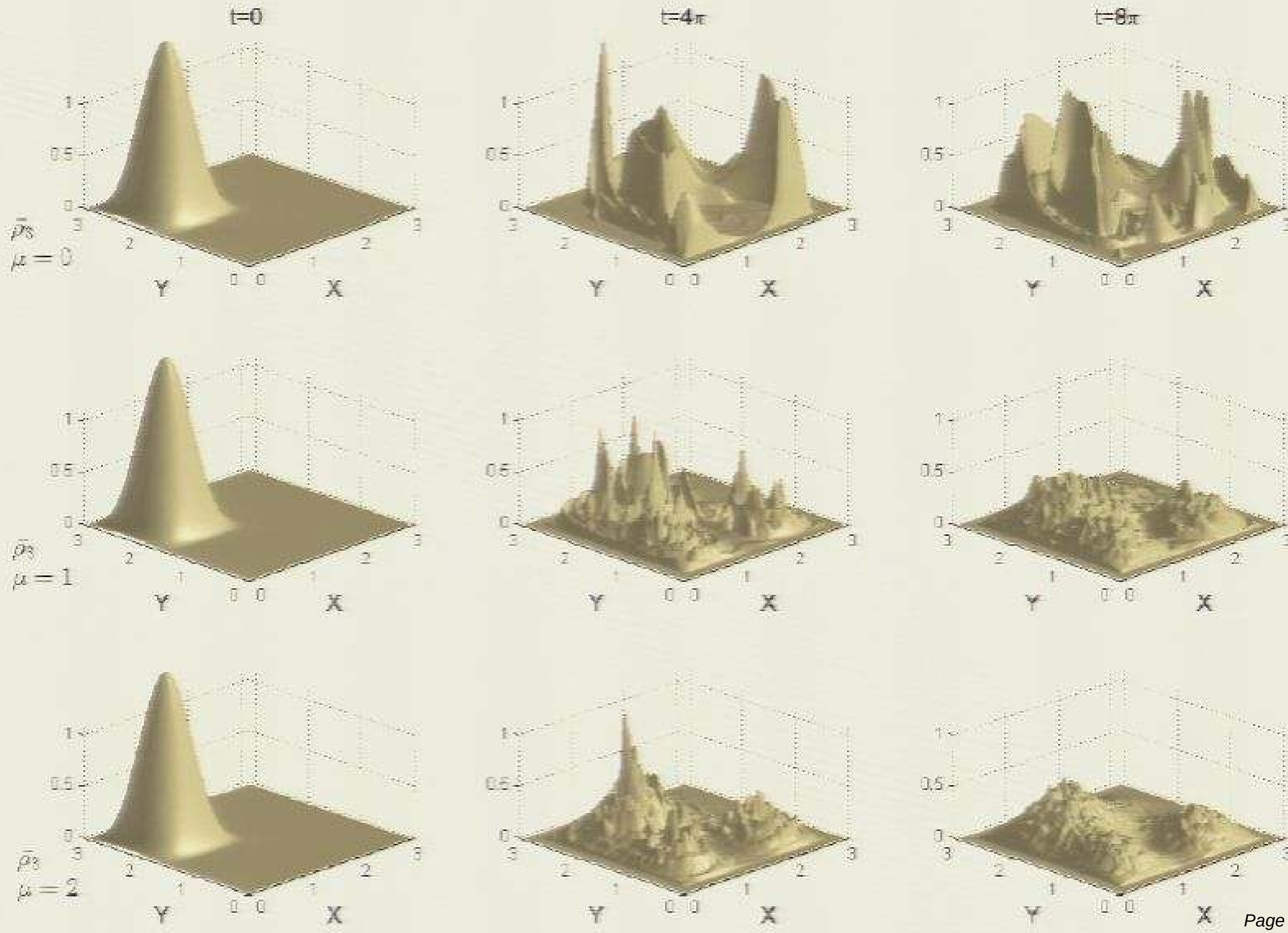
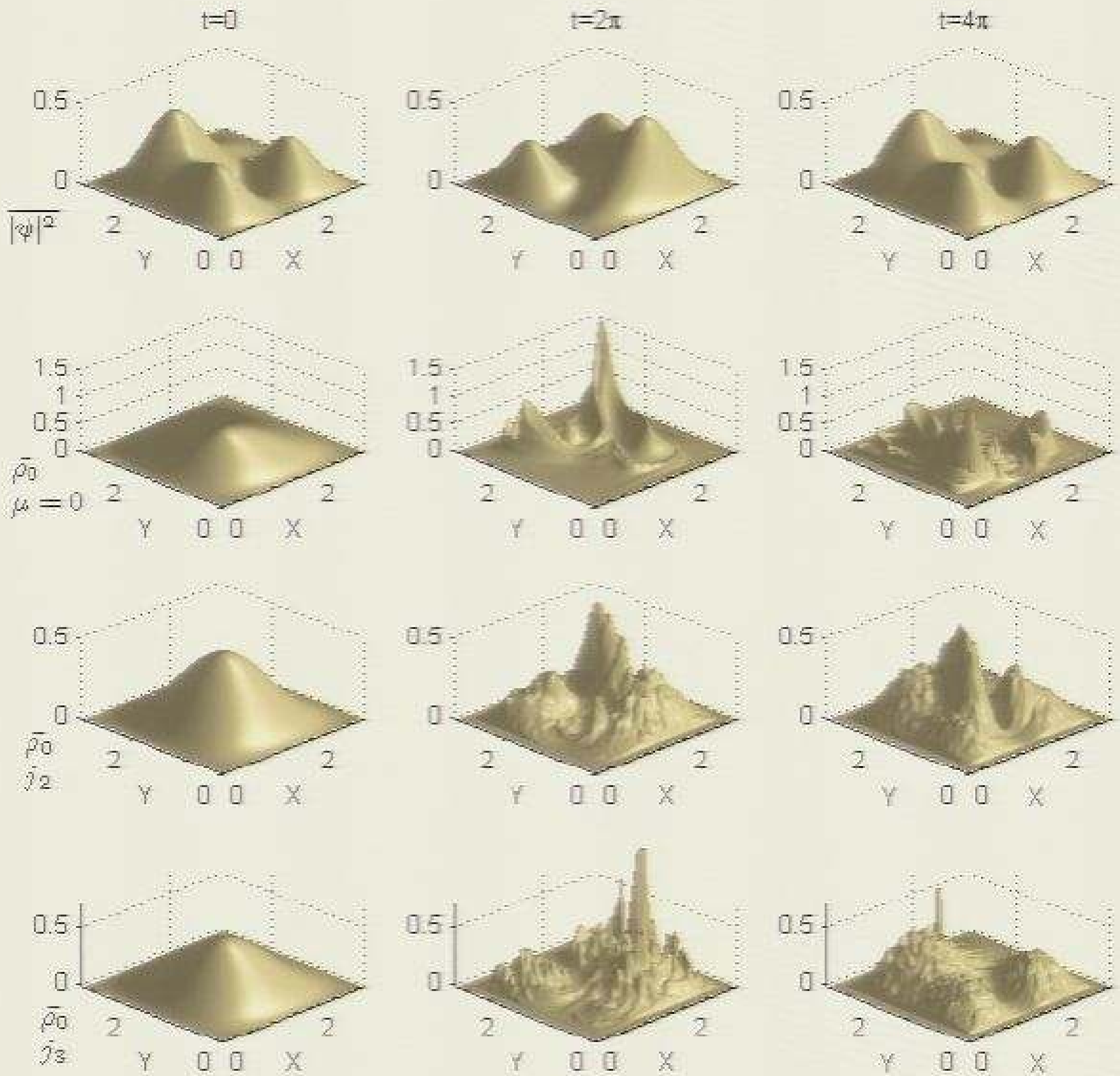
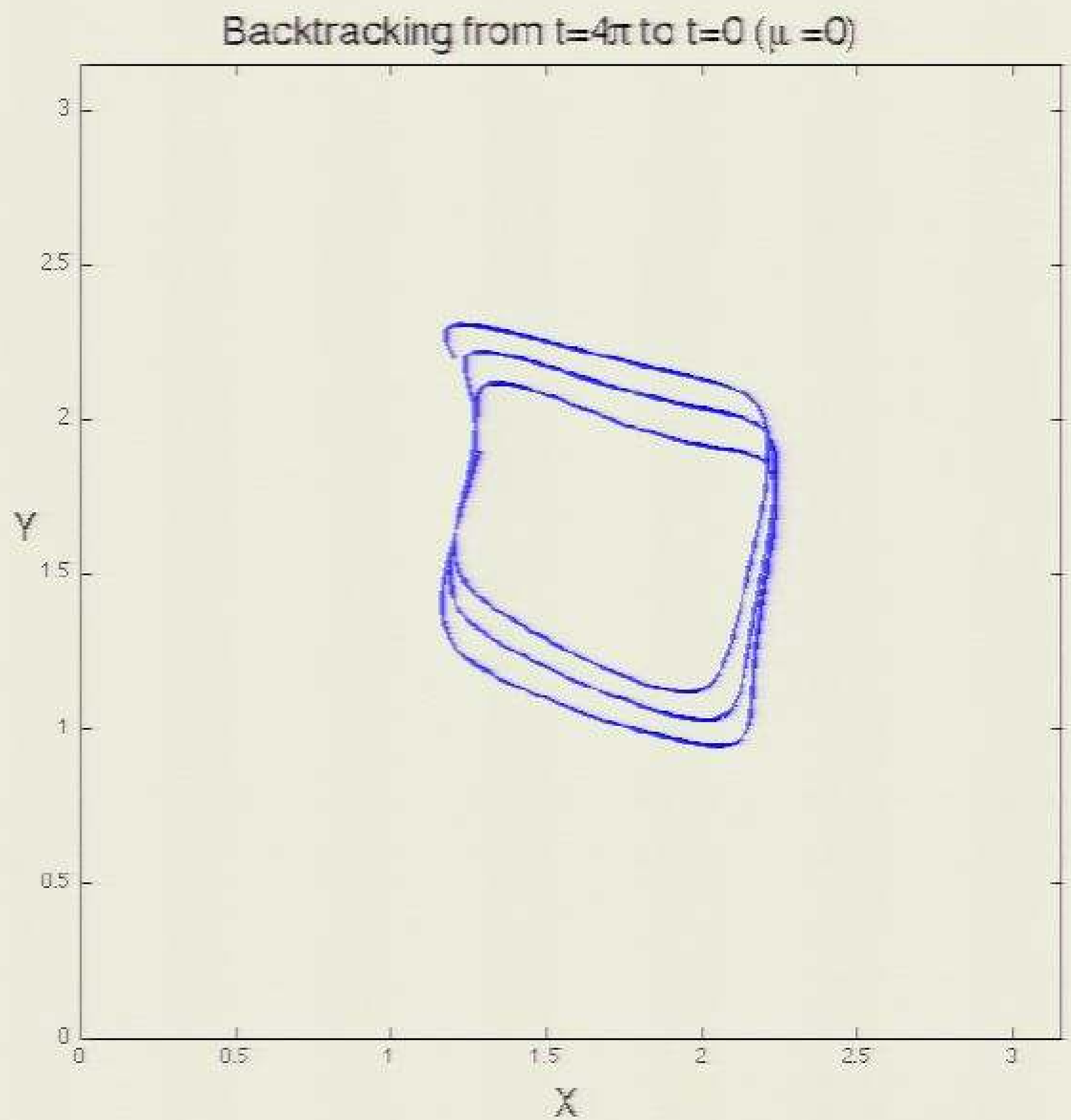


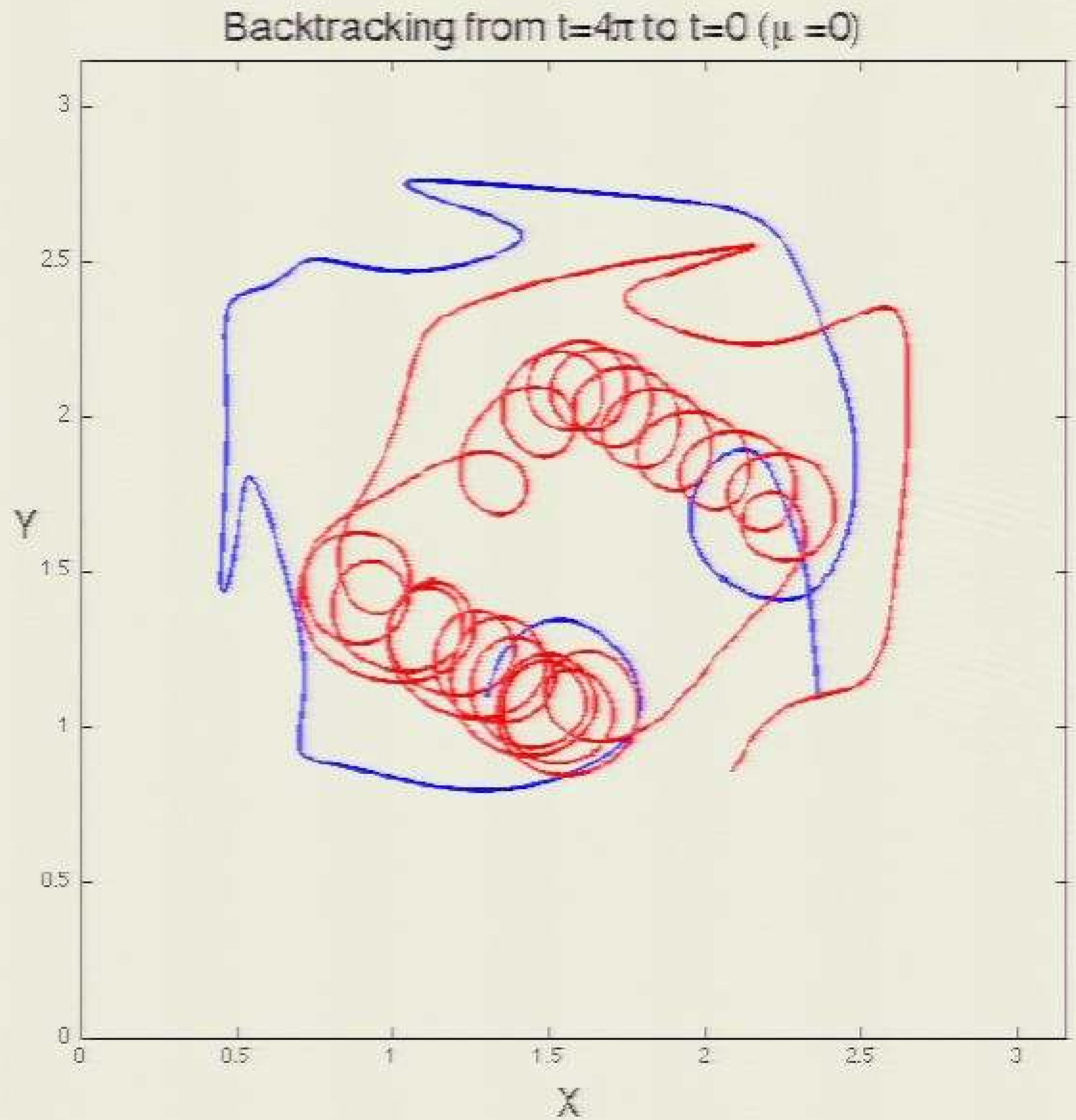
Other curl terms



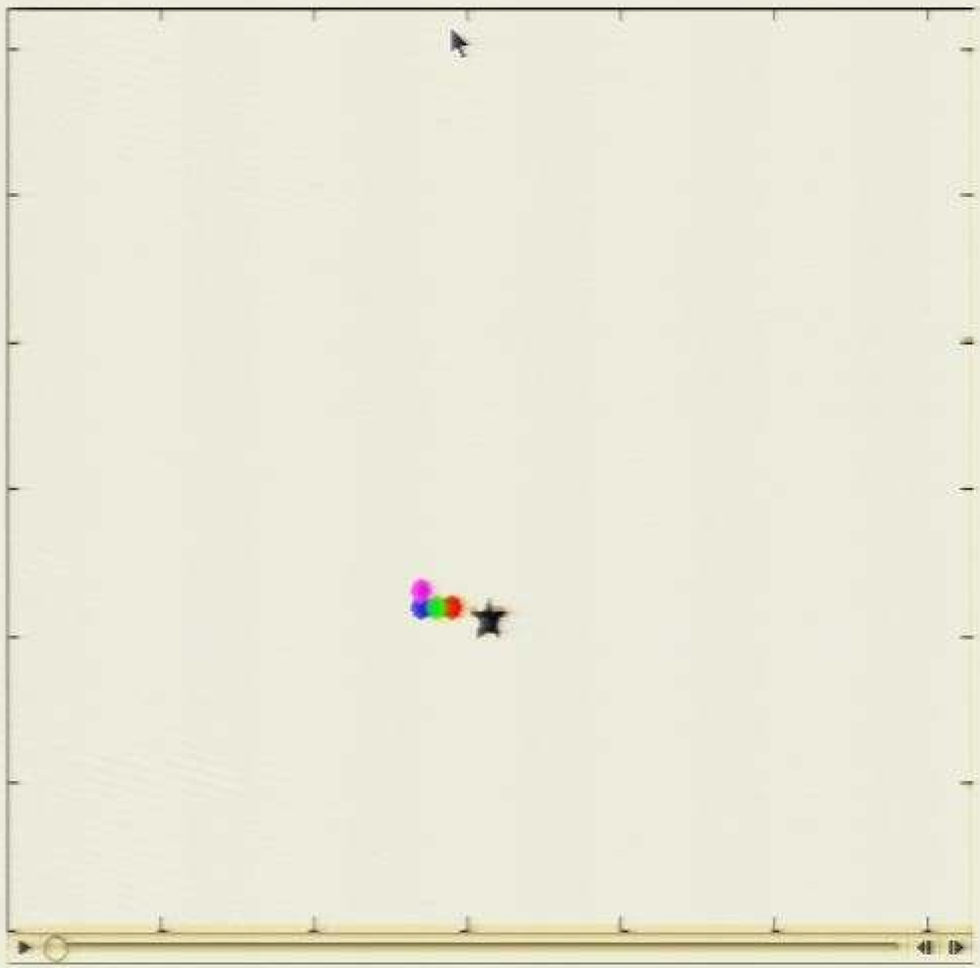
No curl term. Away from the node.



No curl term. Going closer to a node.



No curl term. 4 particles close to the node.



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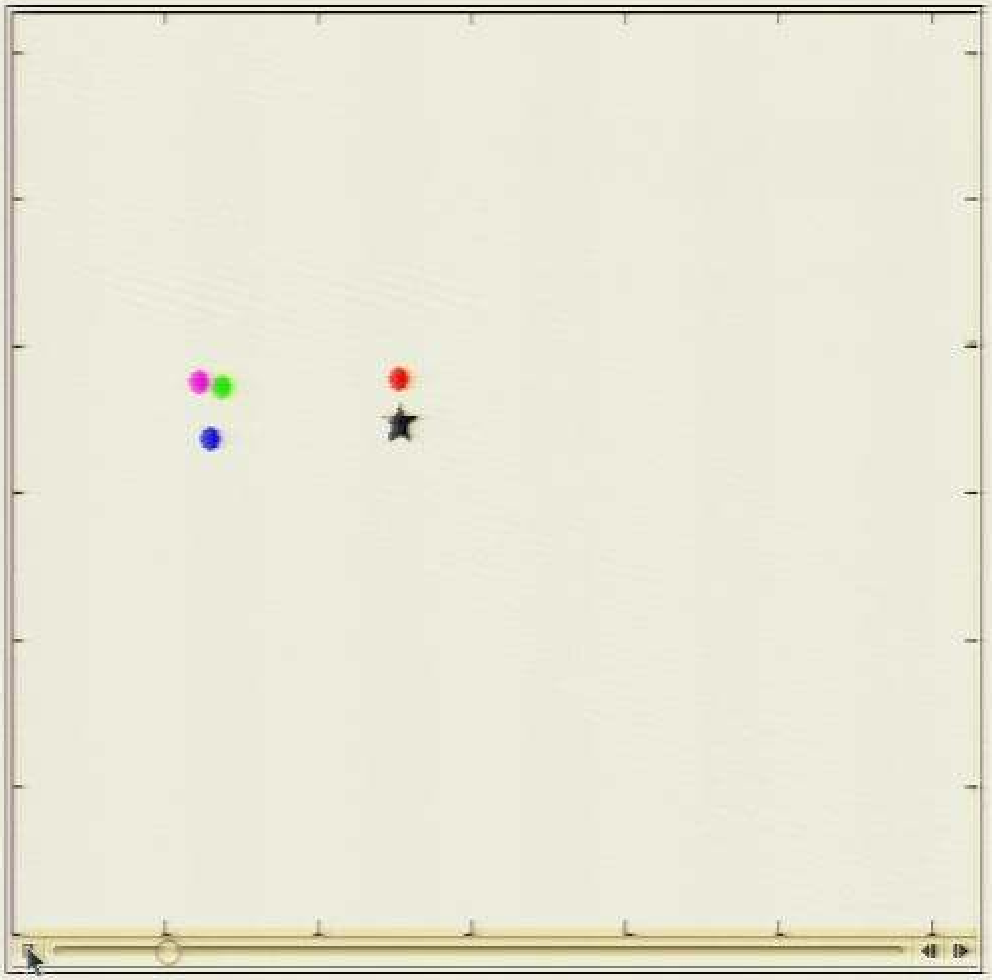

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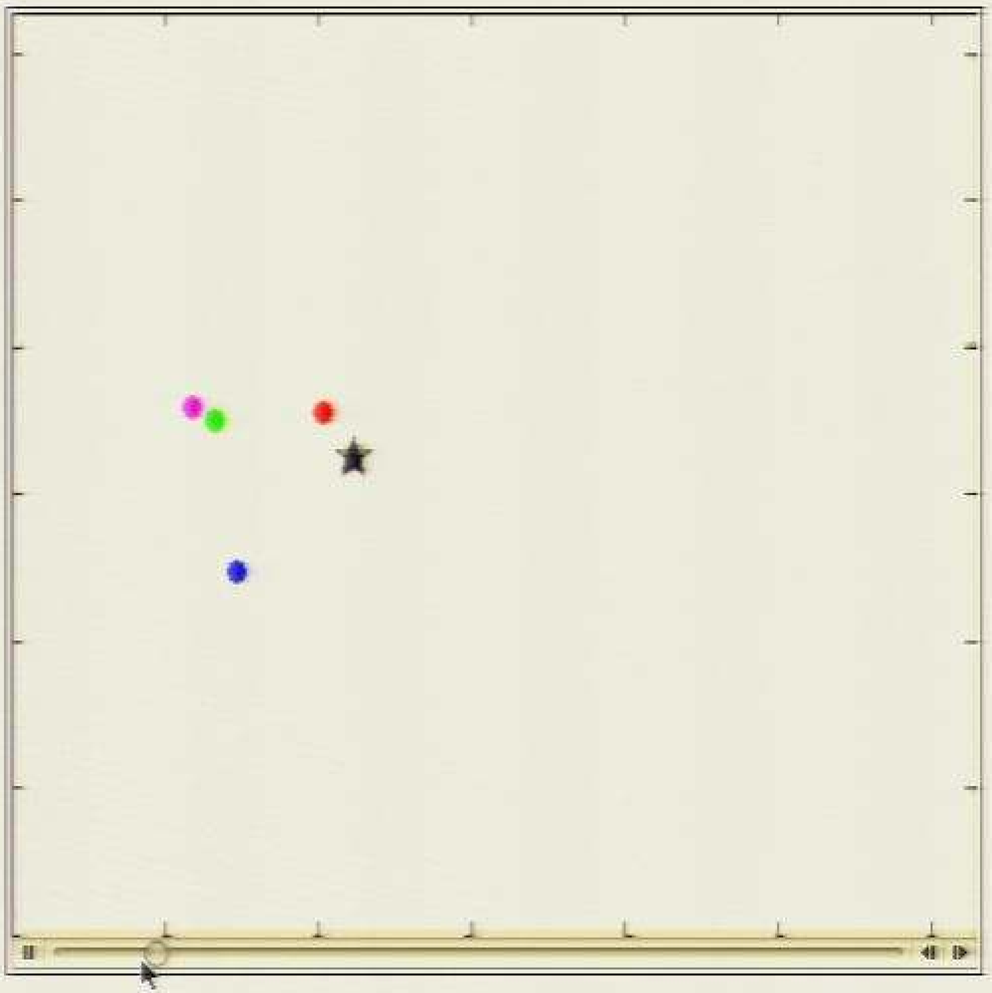

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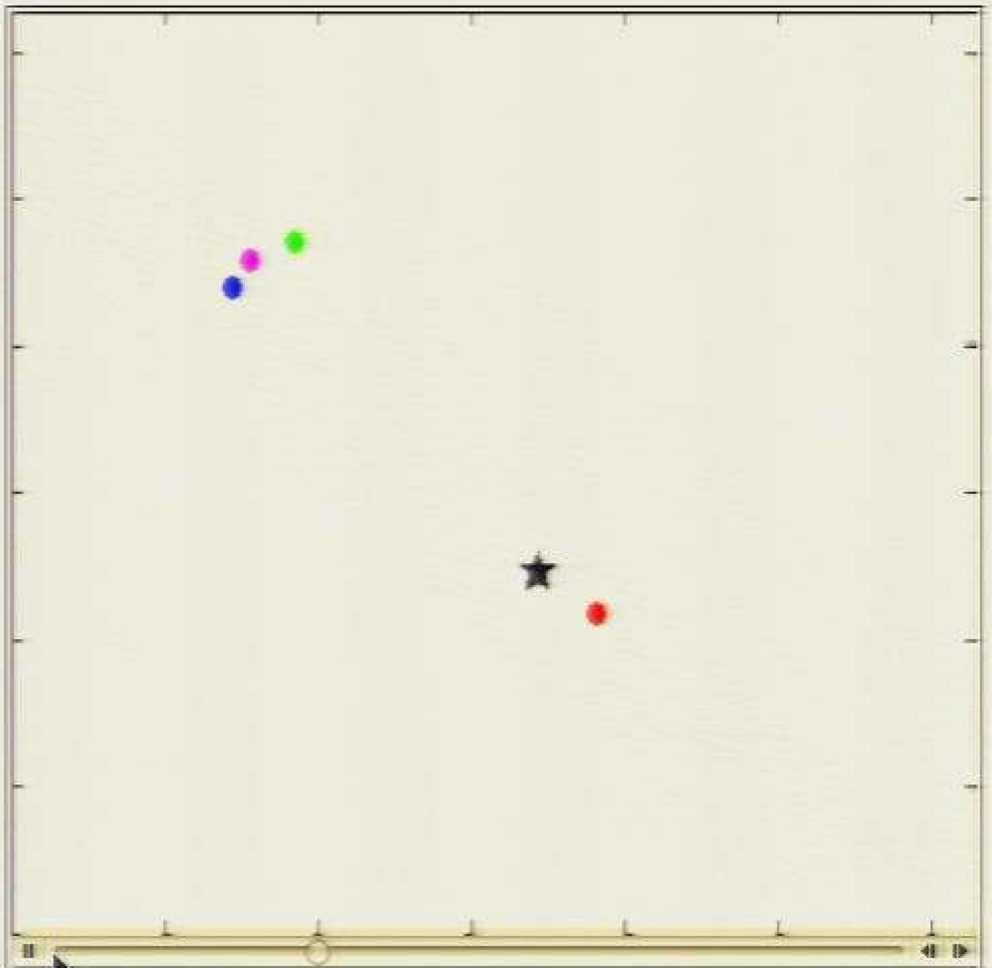
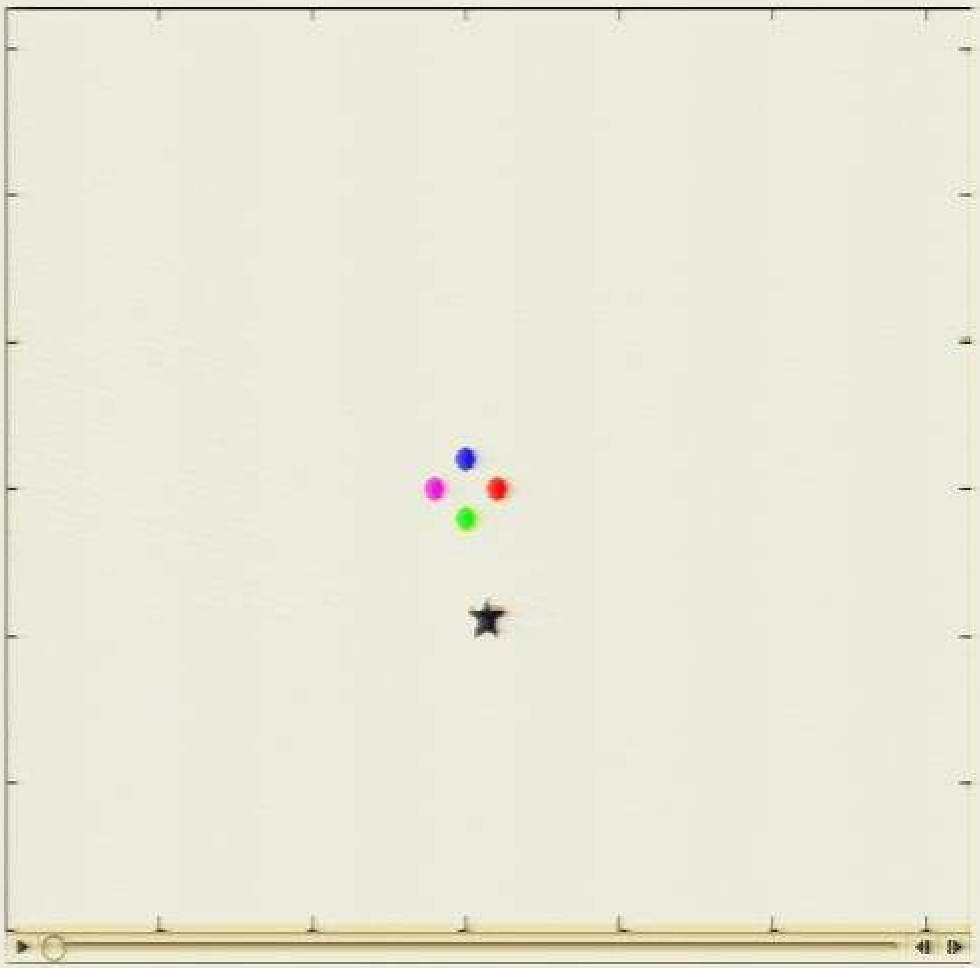


No curl term. 4 particles at the centre.



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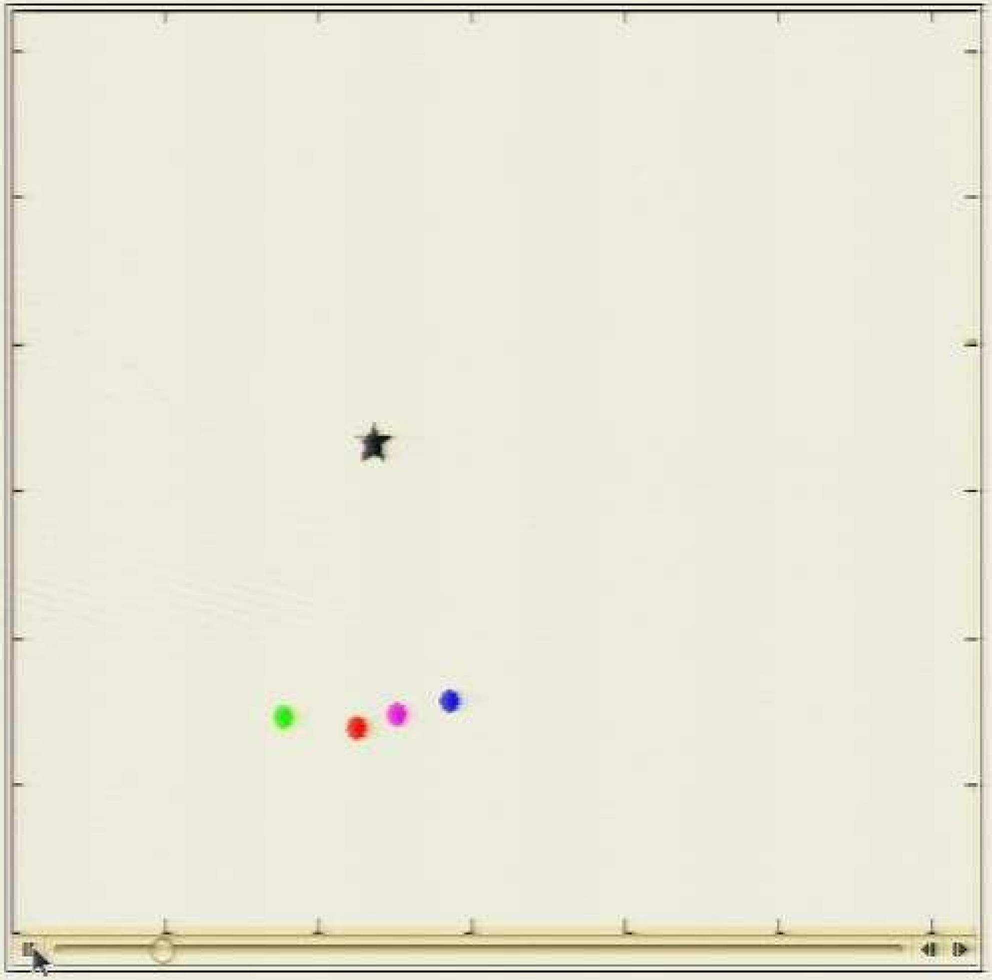

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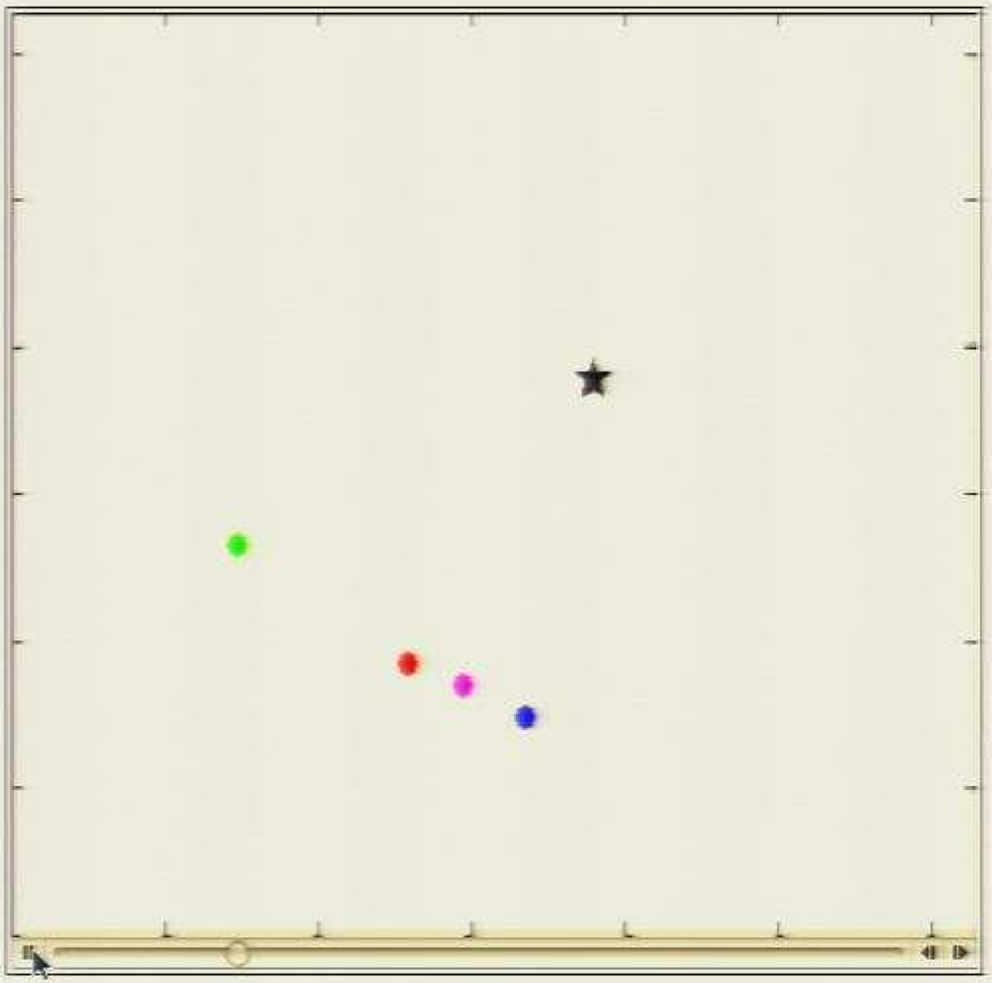

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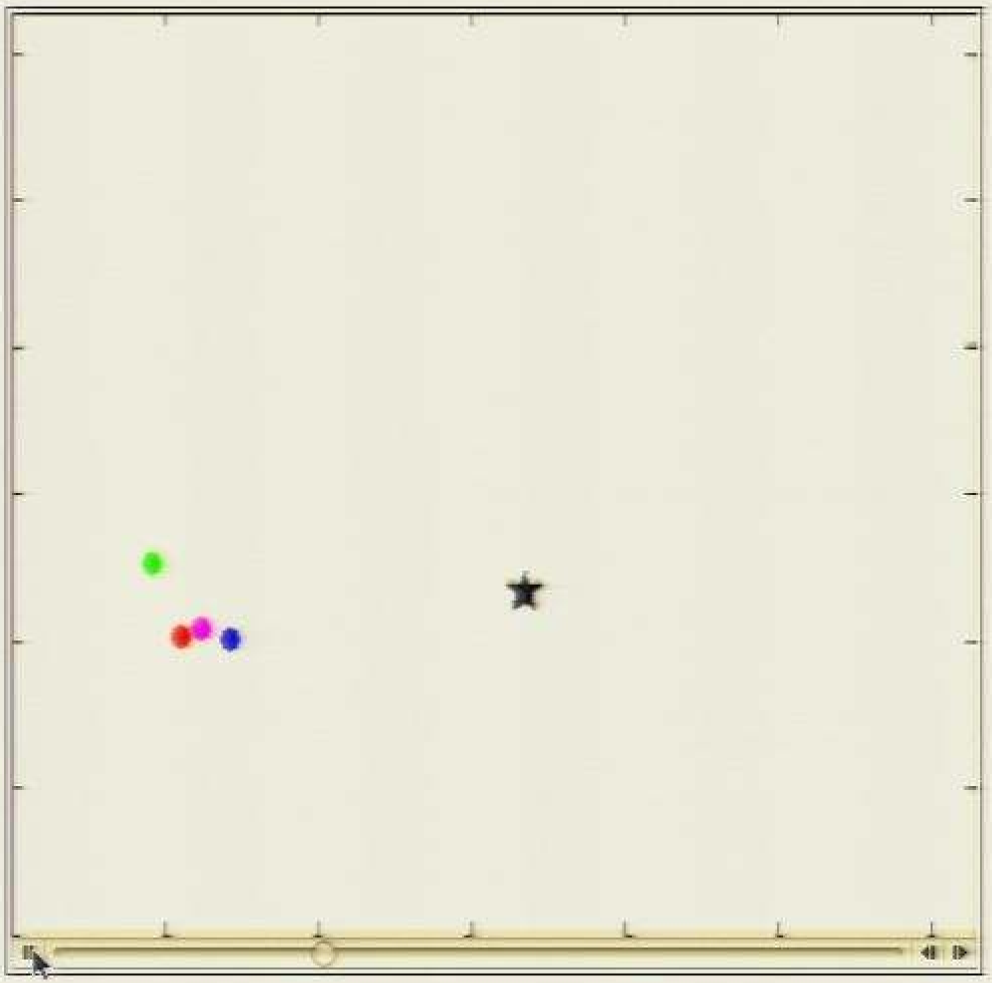

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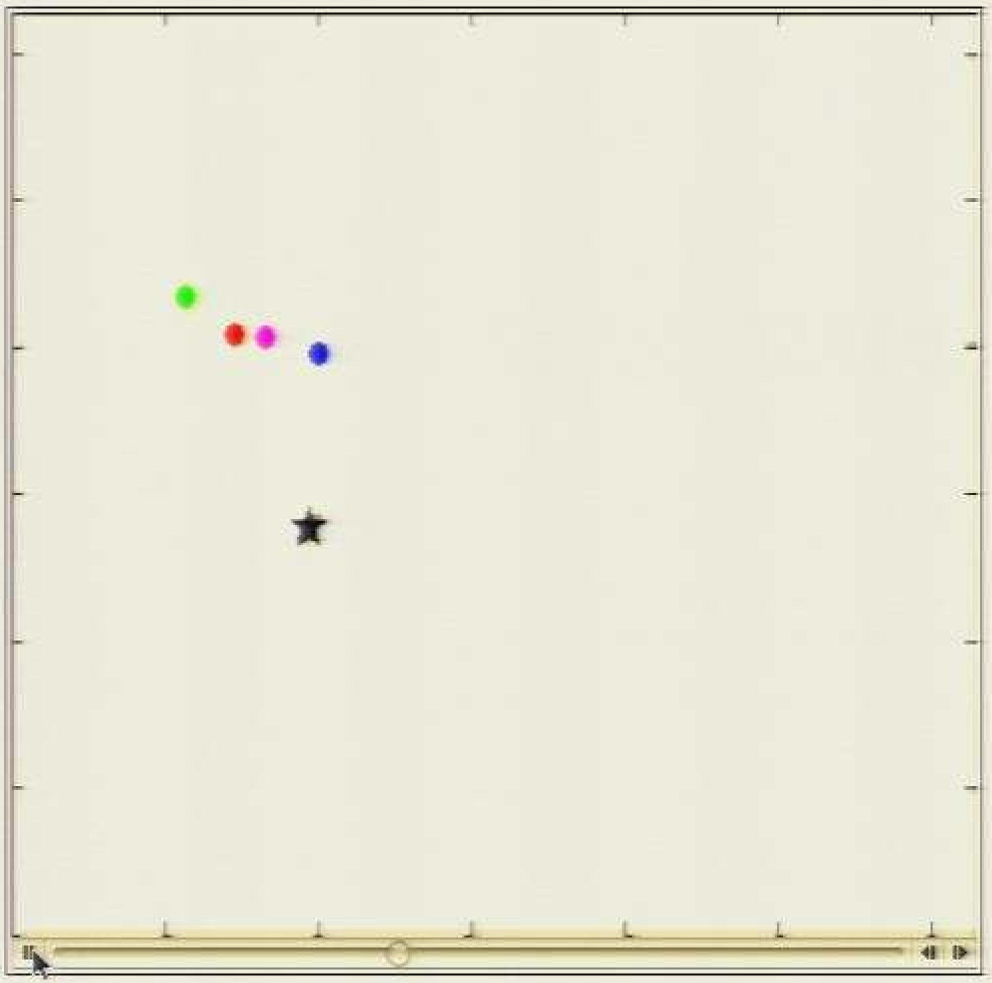

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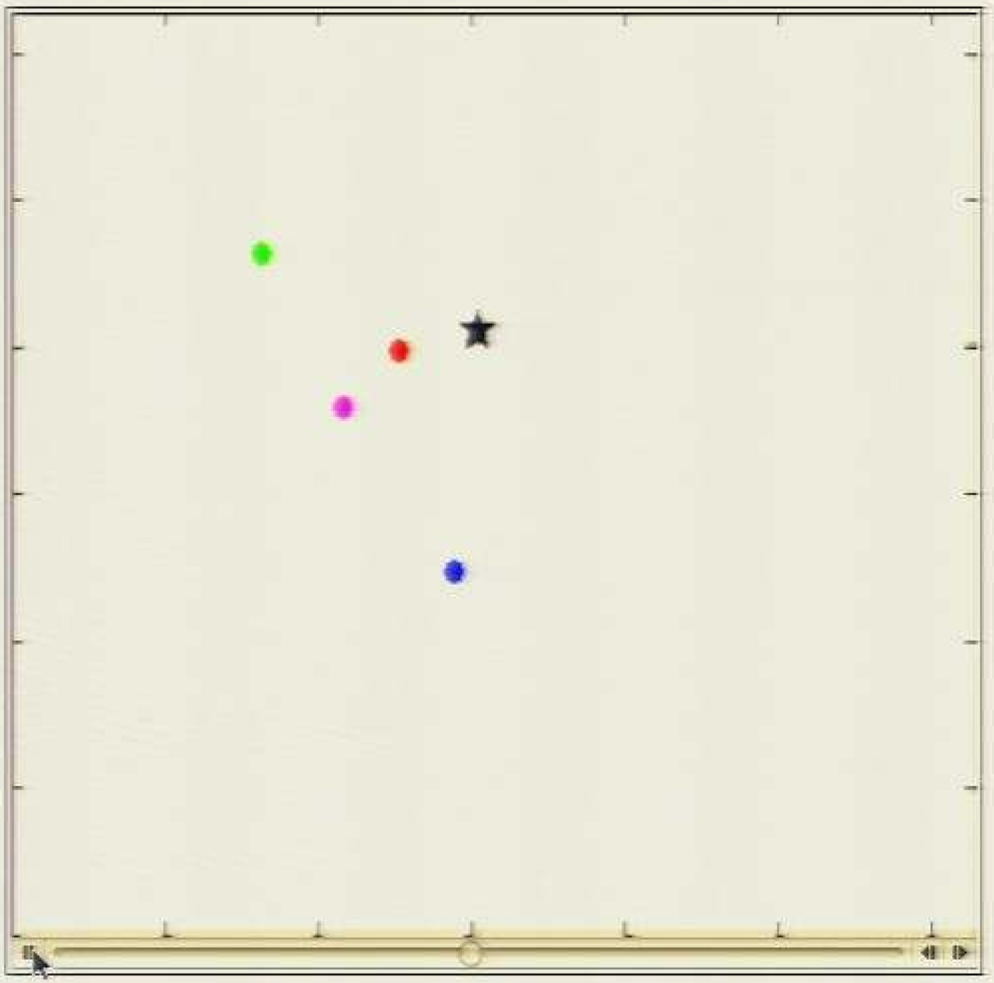

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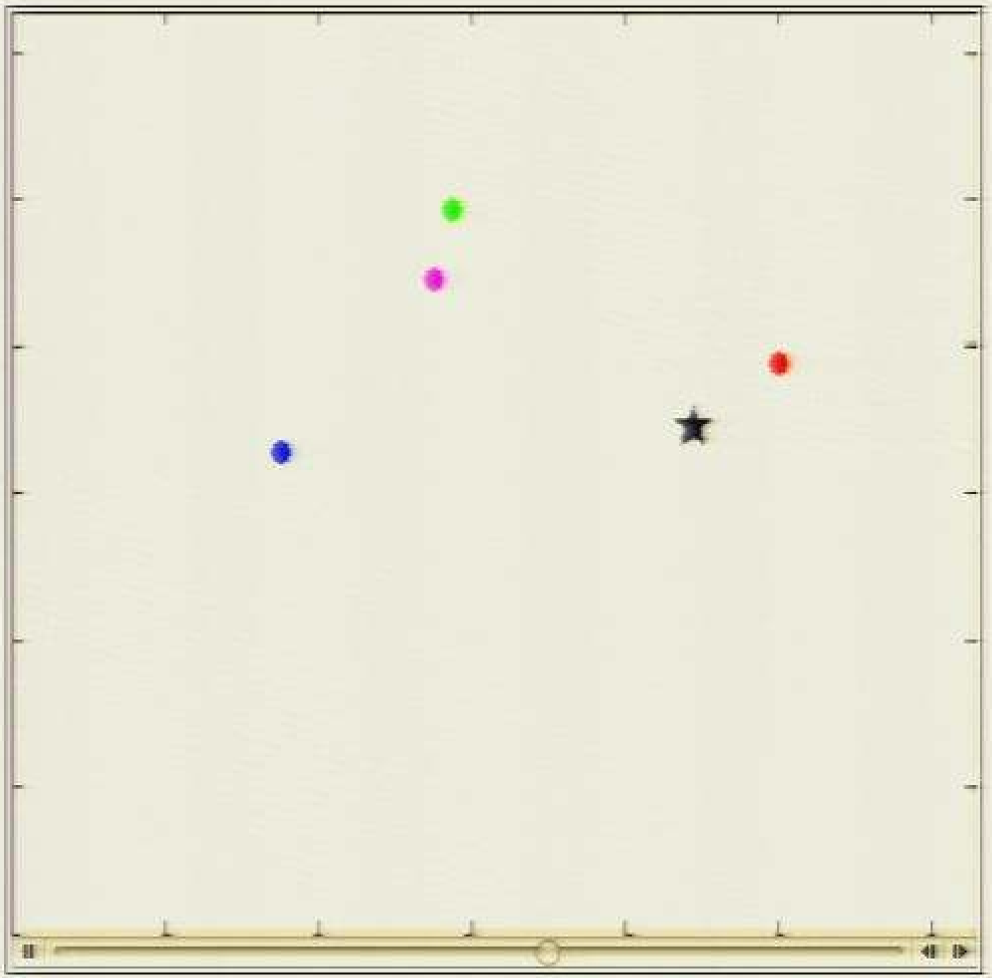
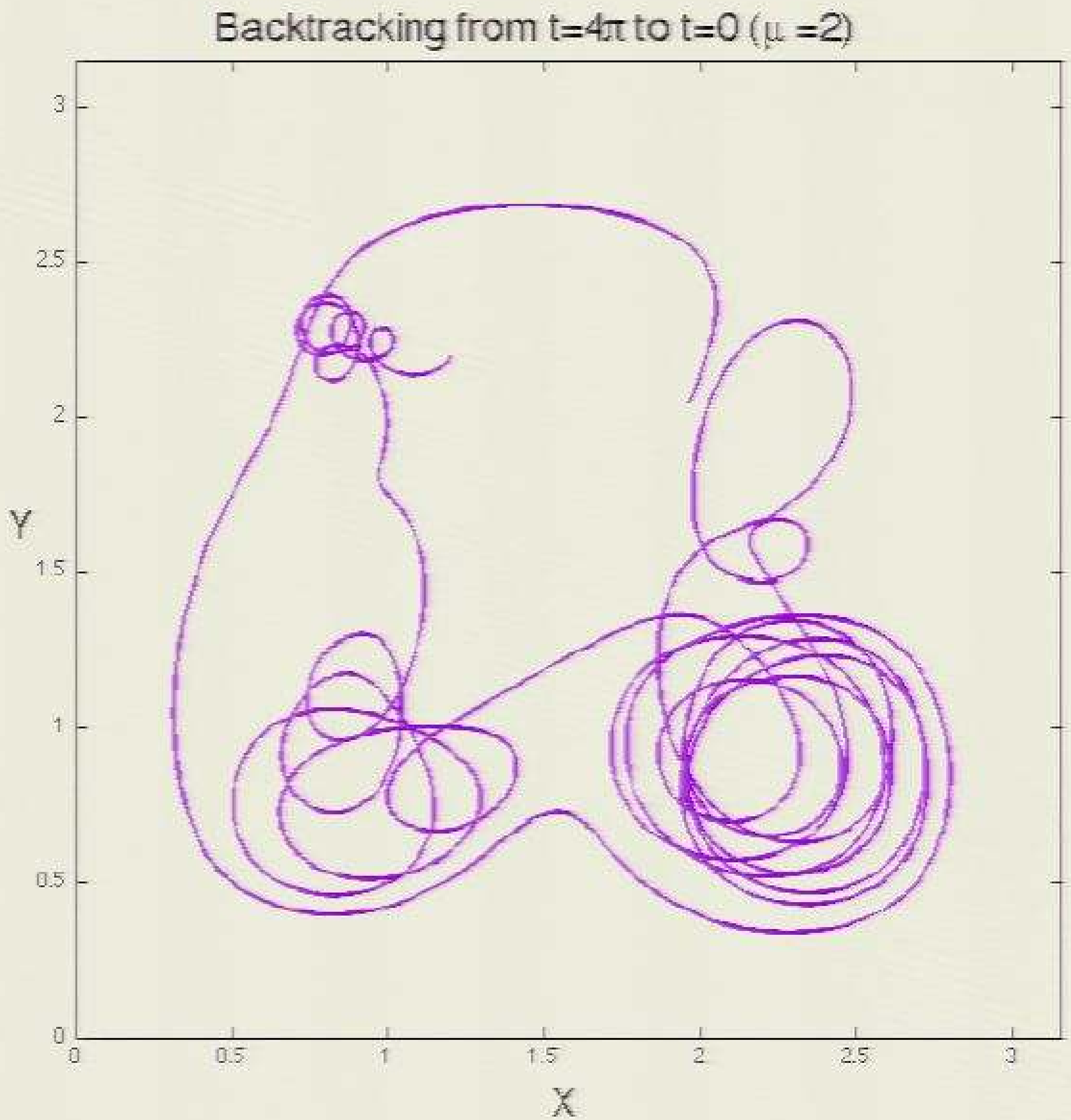


Curl term. Away from the node.



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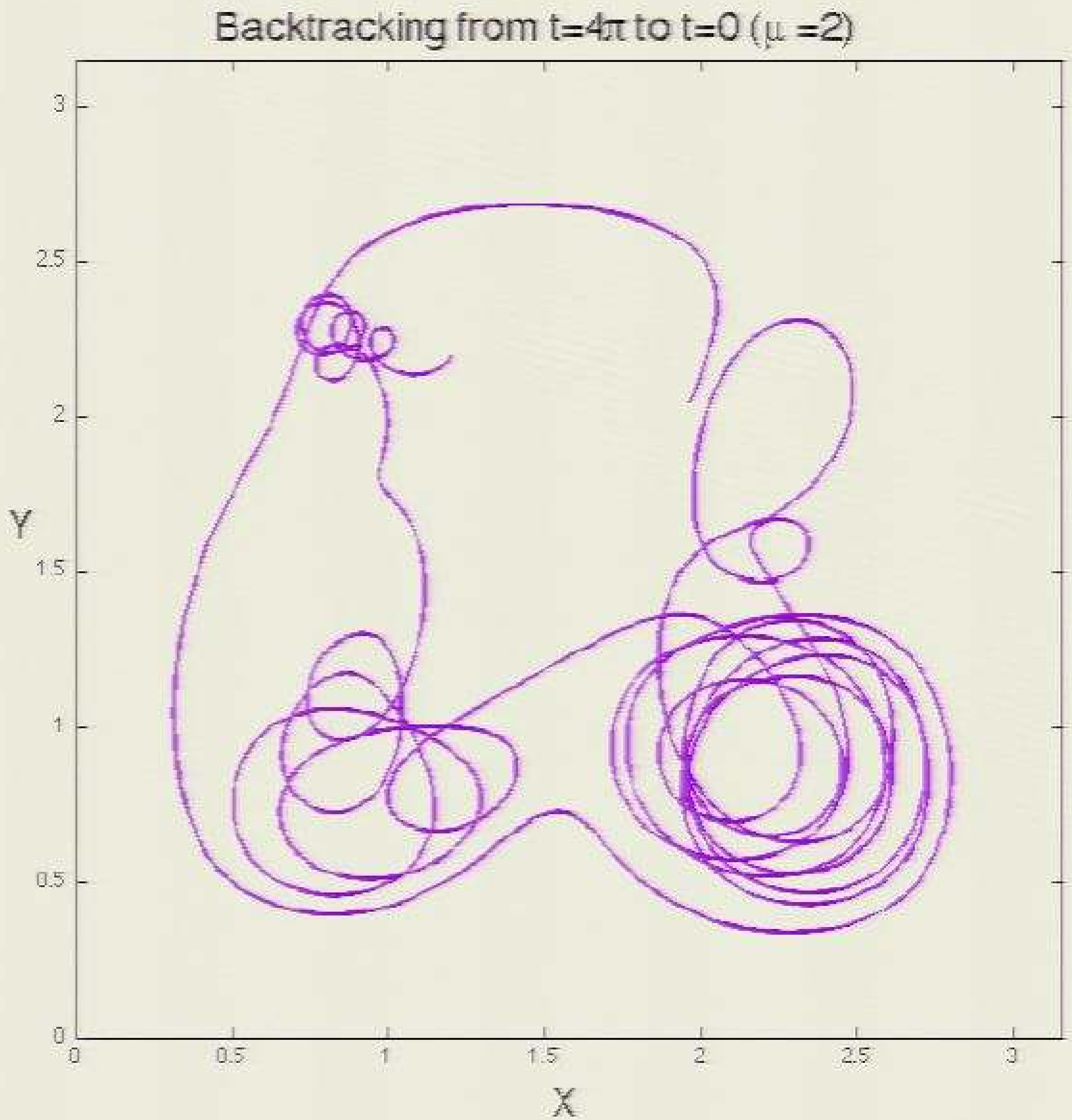
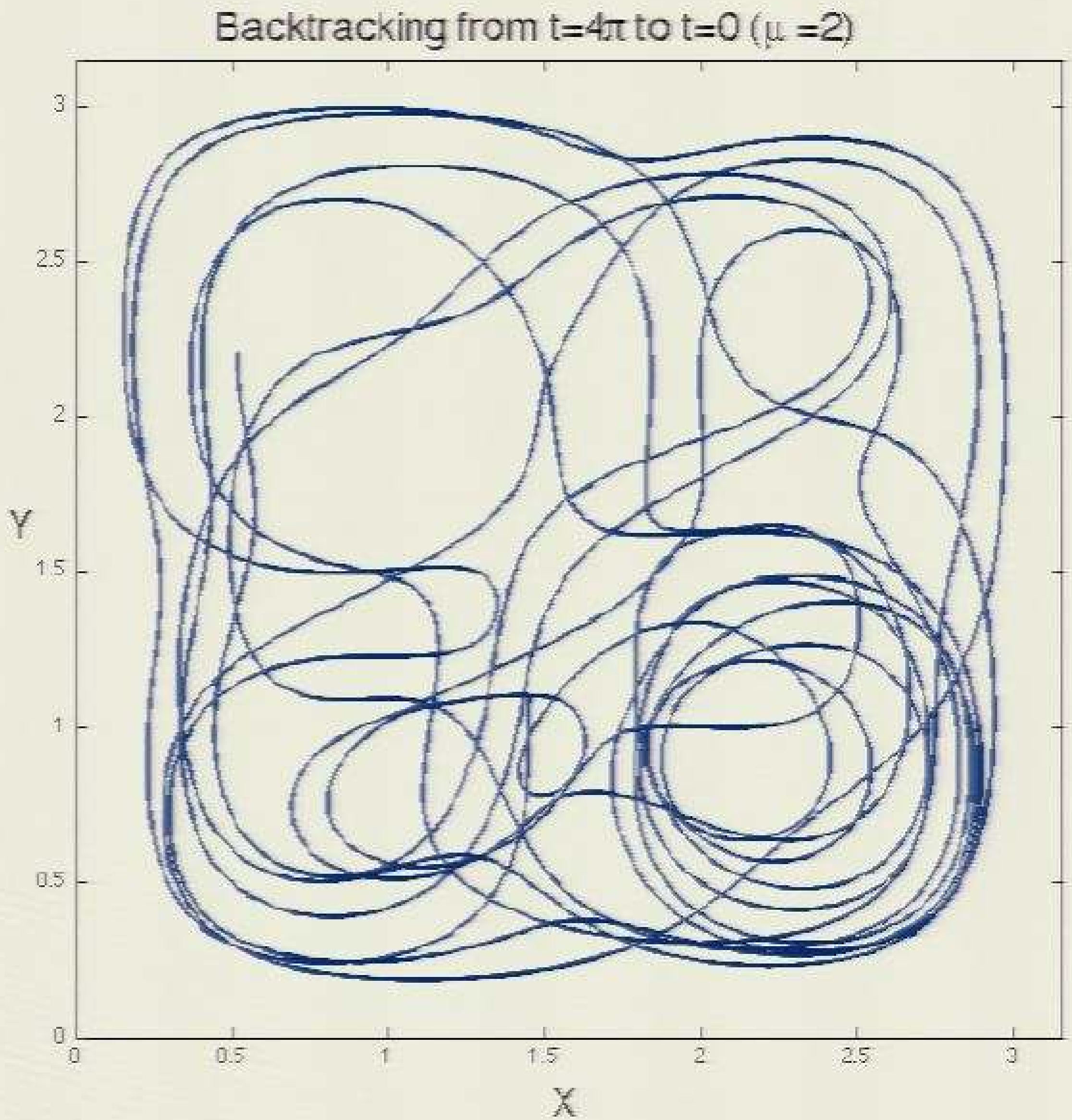


Curl term. Close to the node.



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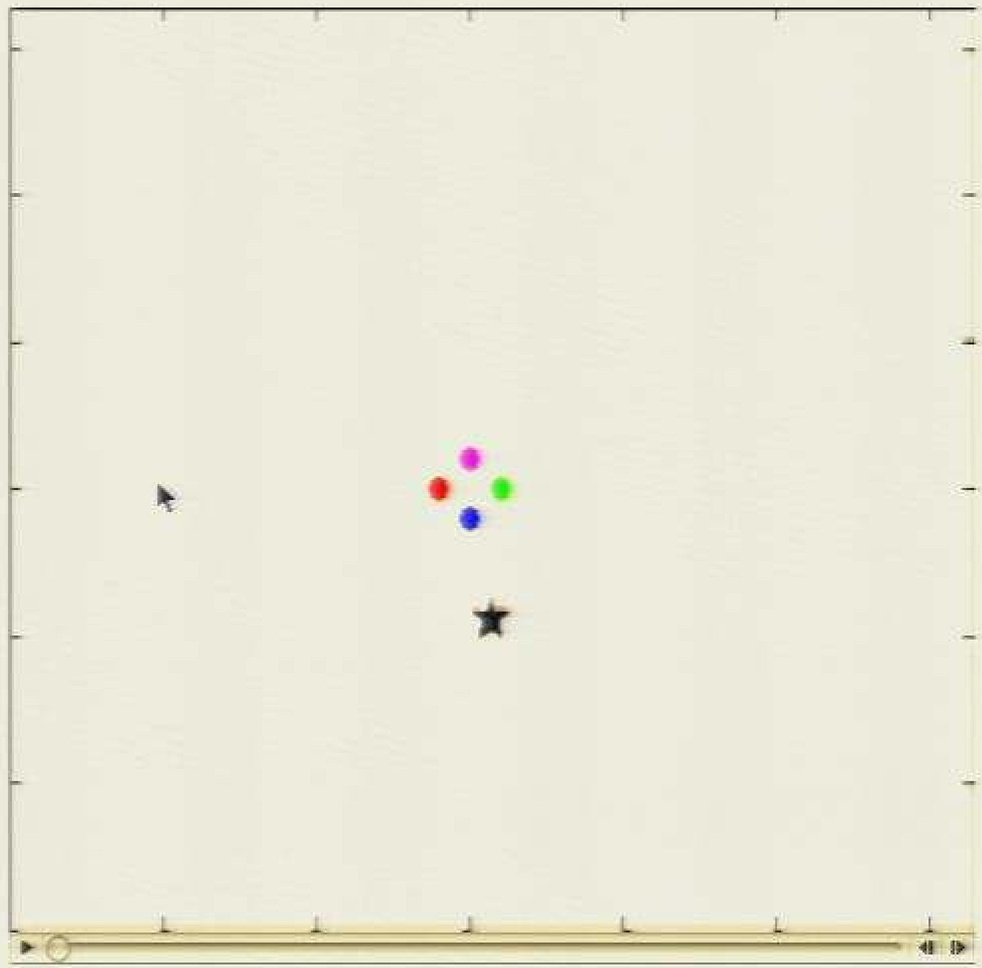

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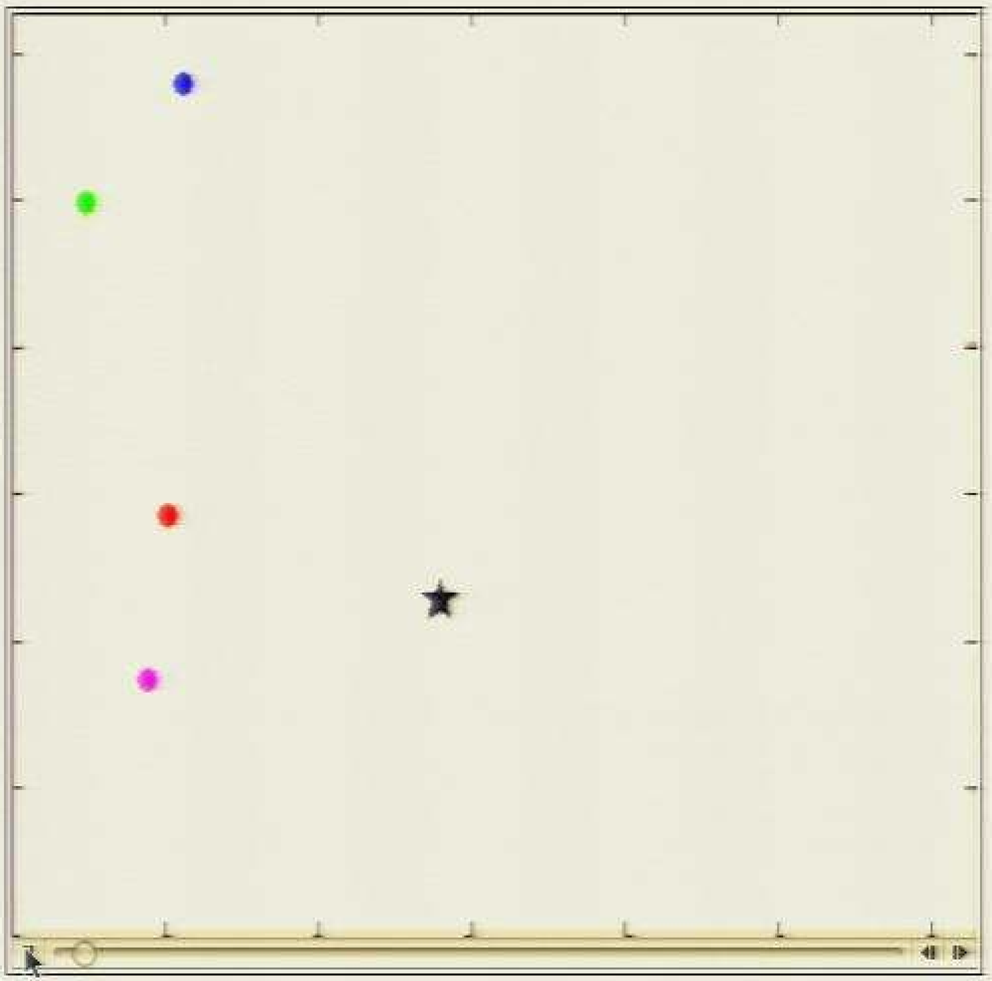

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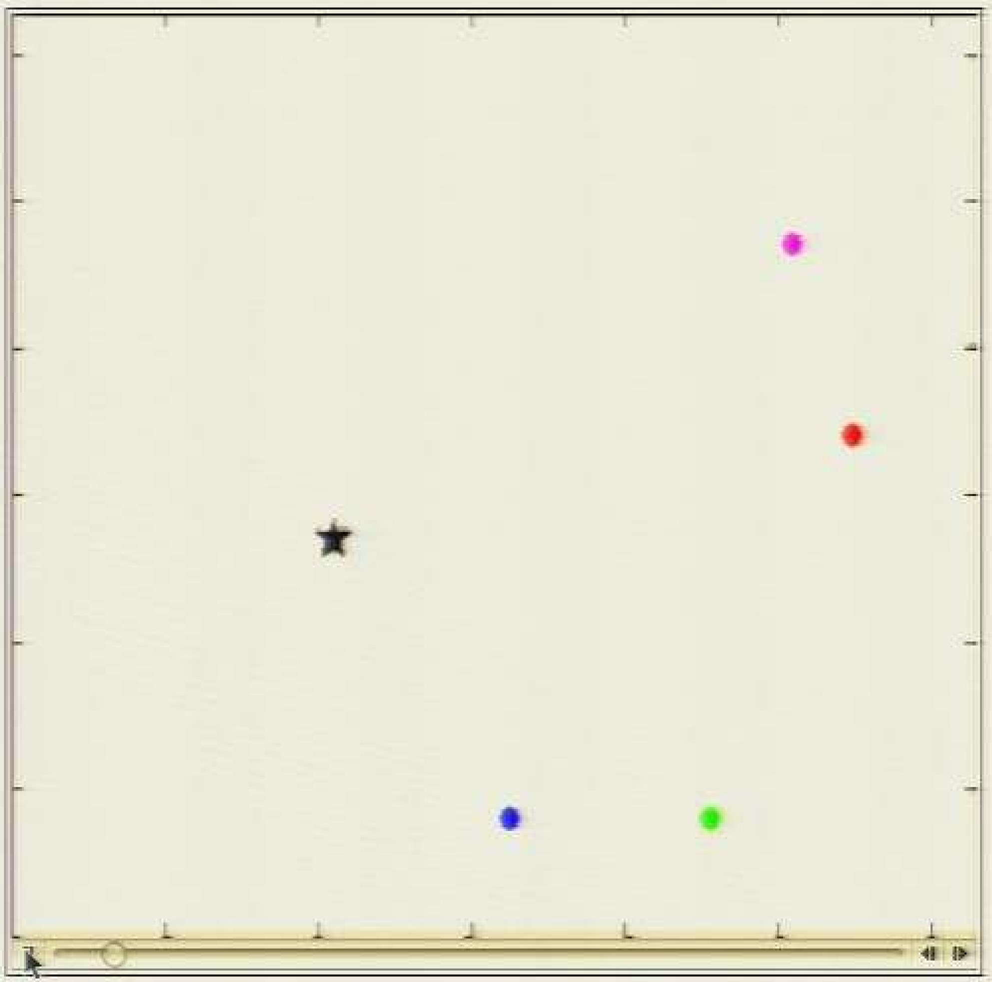

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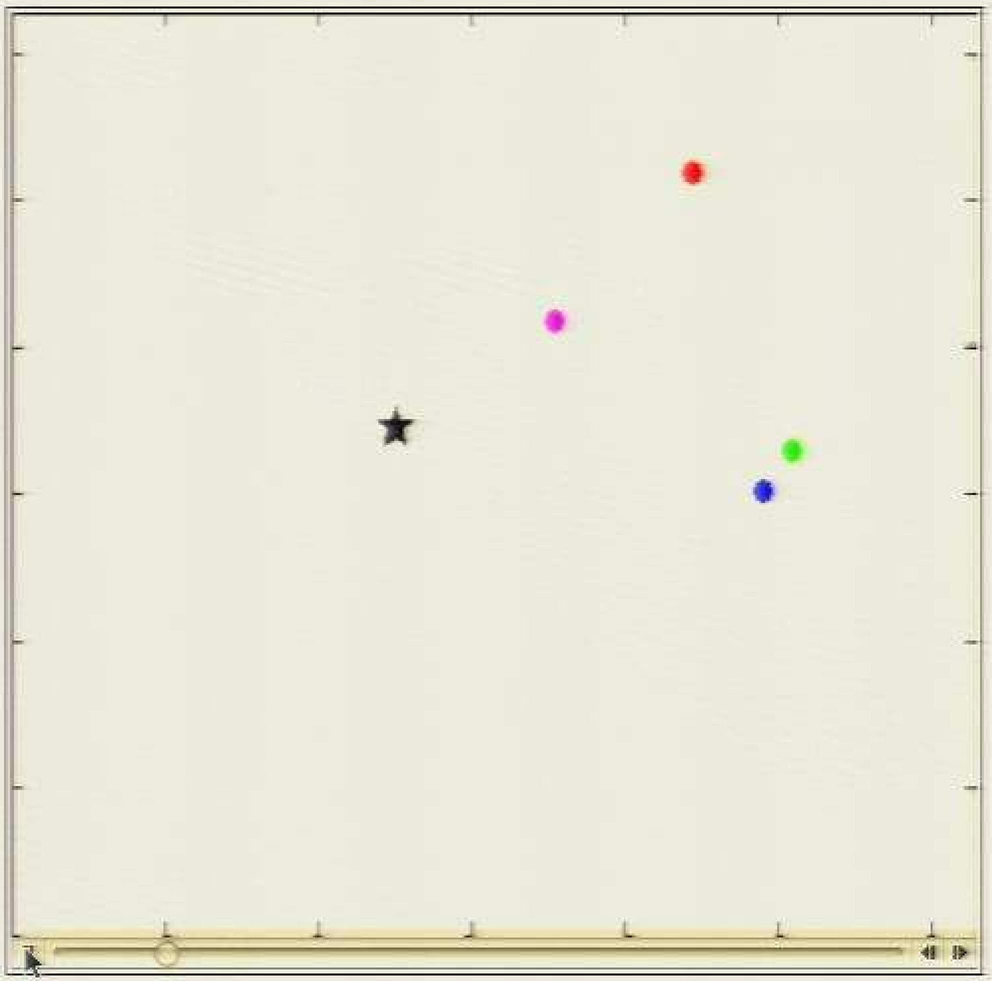

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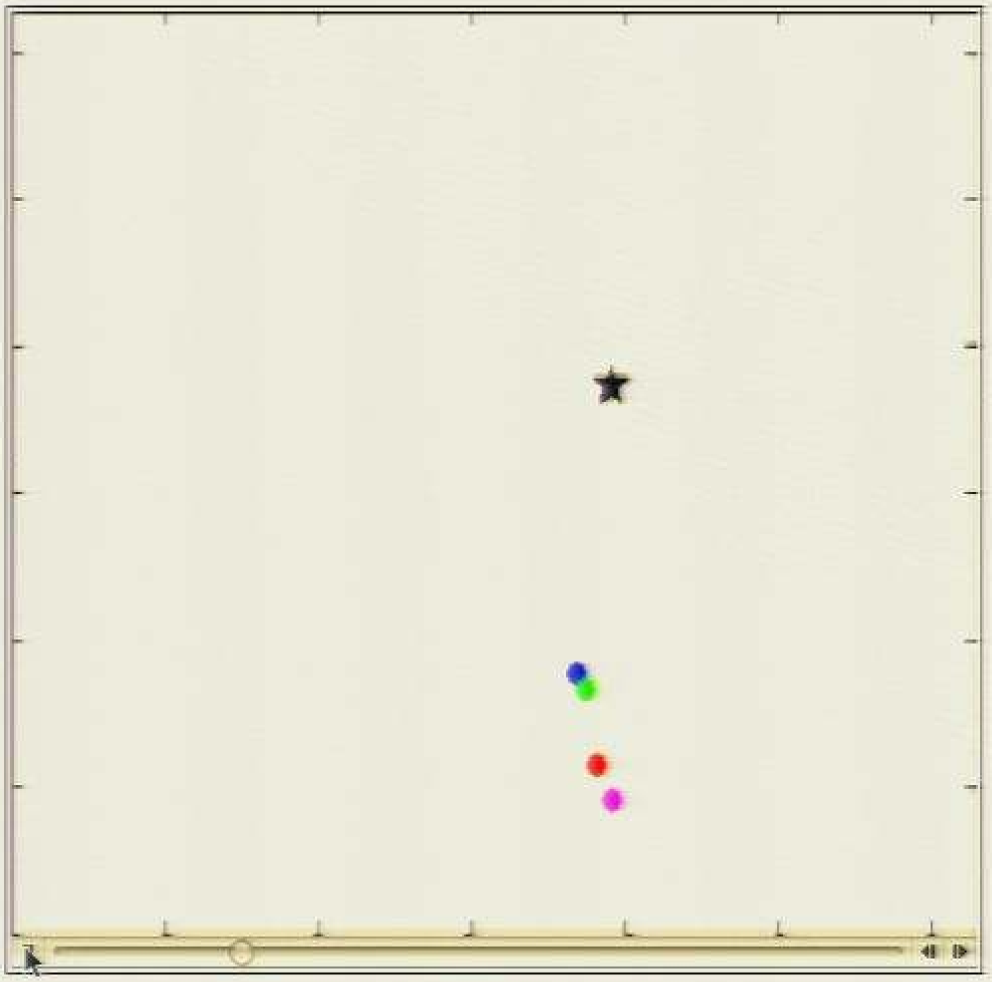

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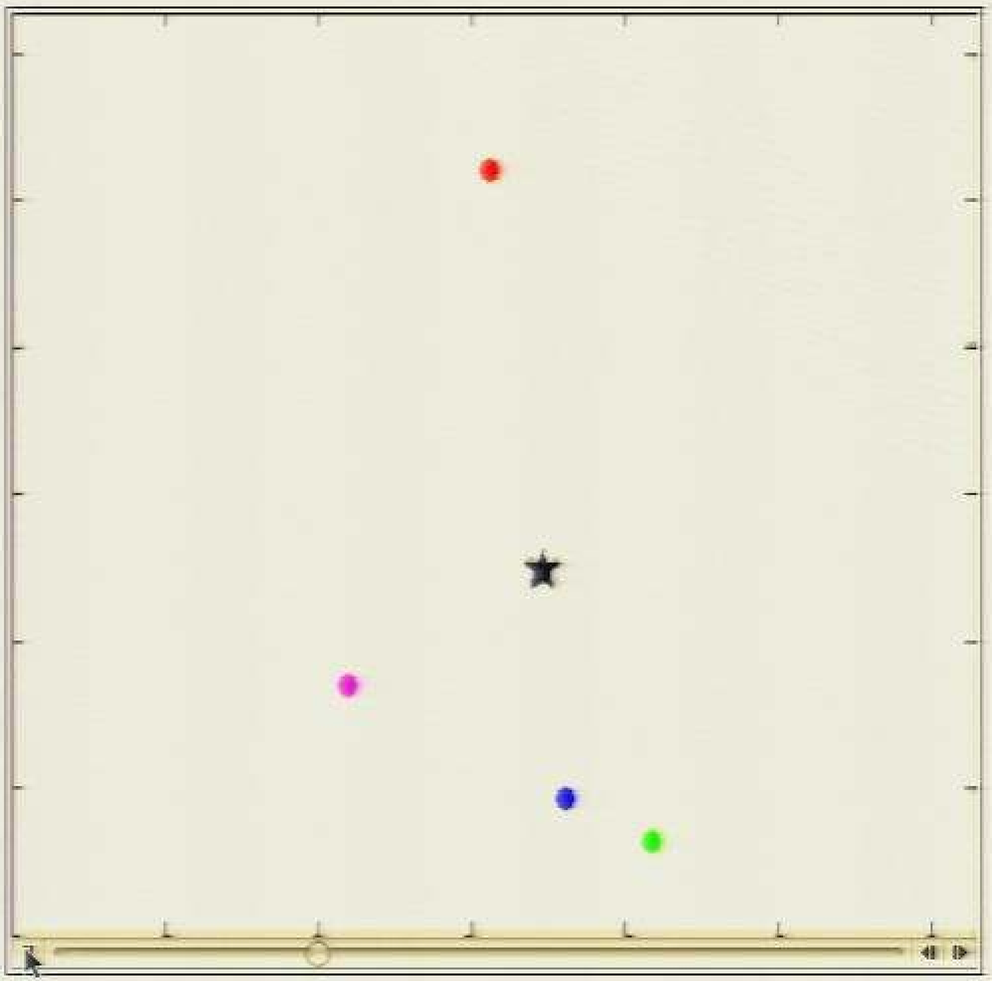

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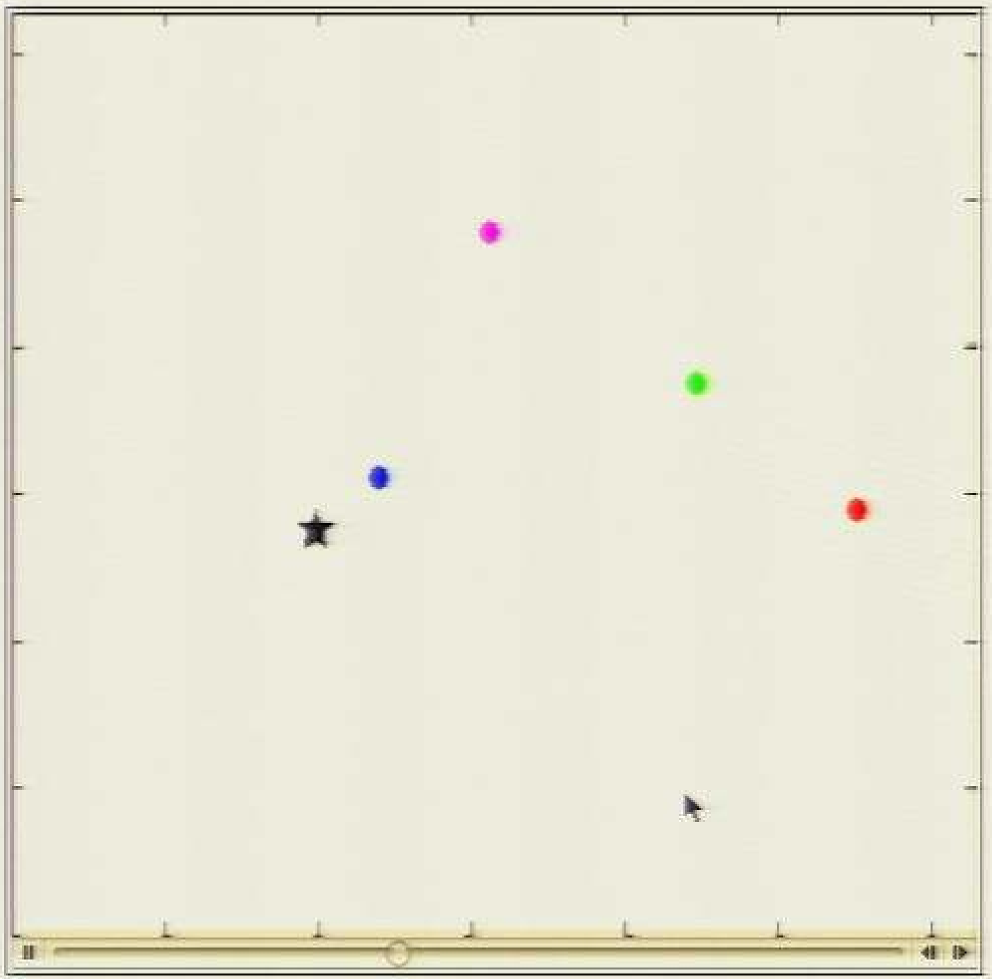
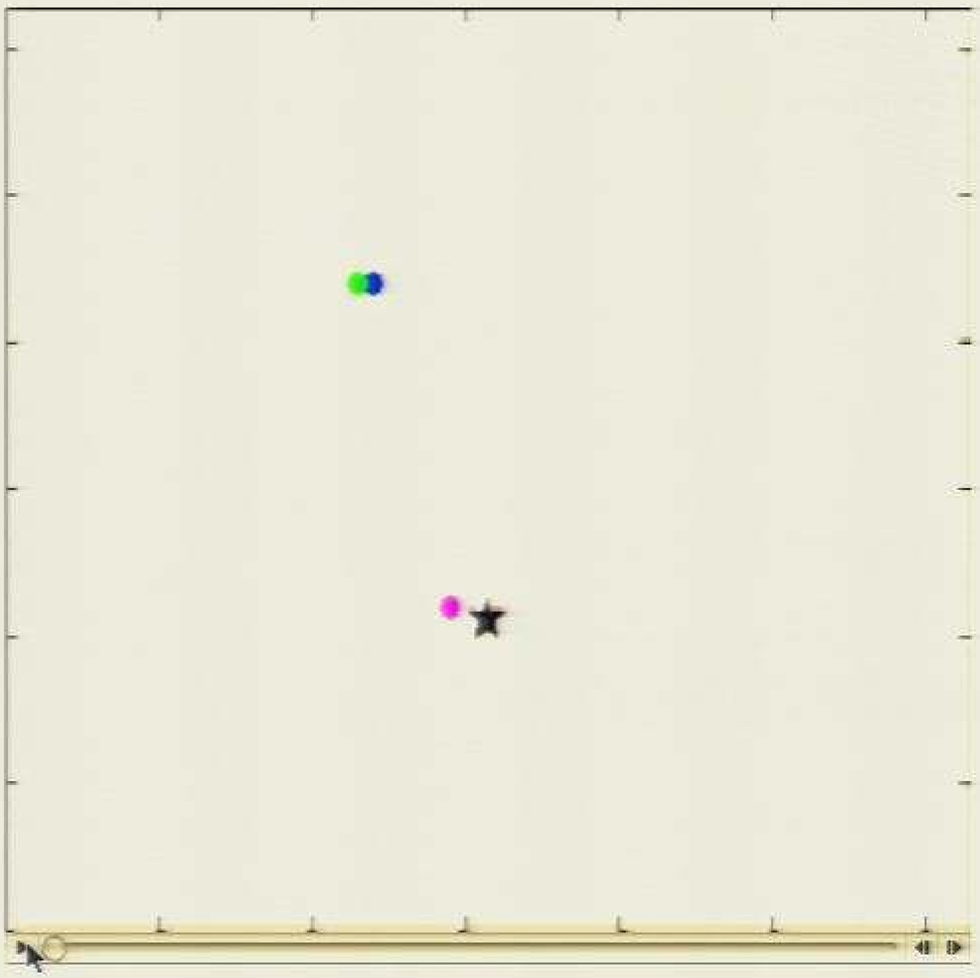


Curl term. 2 particles close. Another one near the node.



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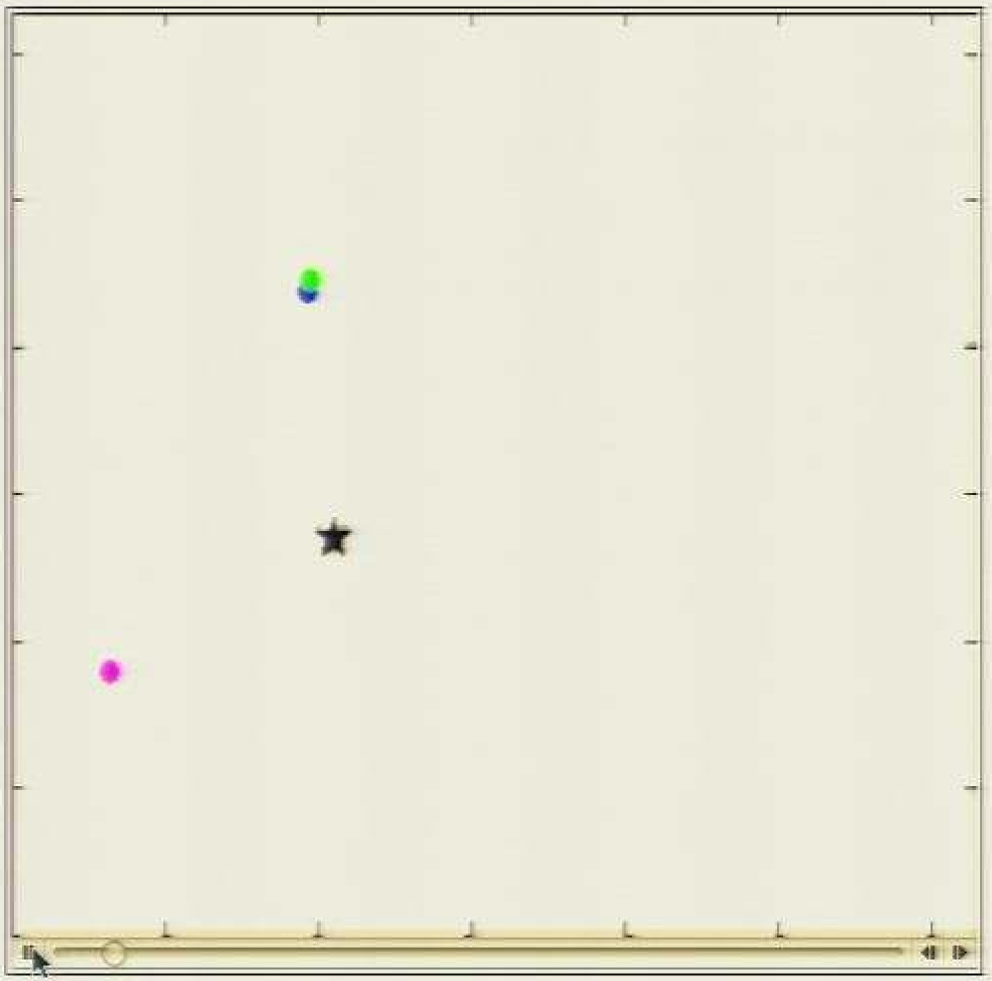

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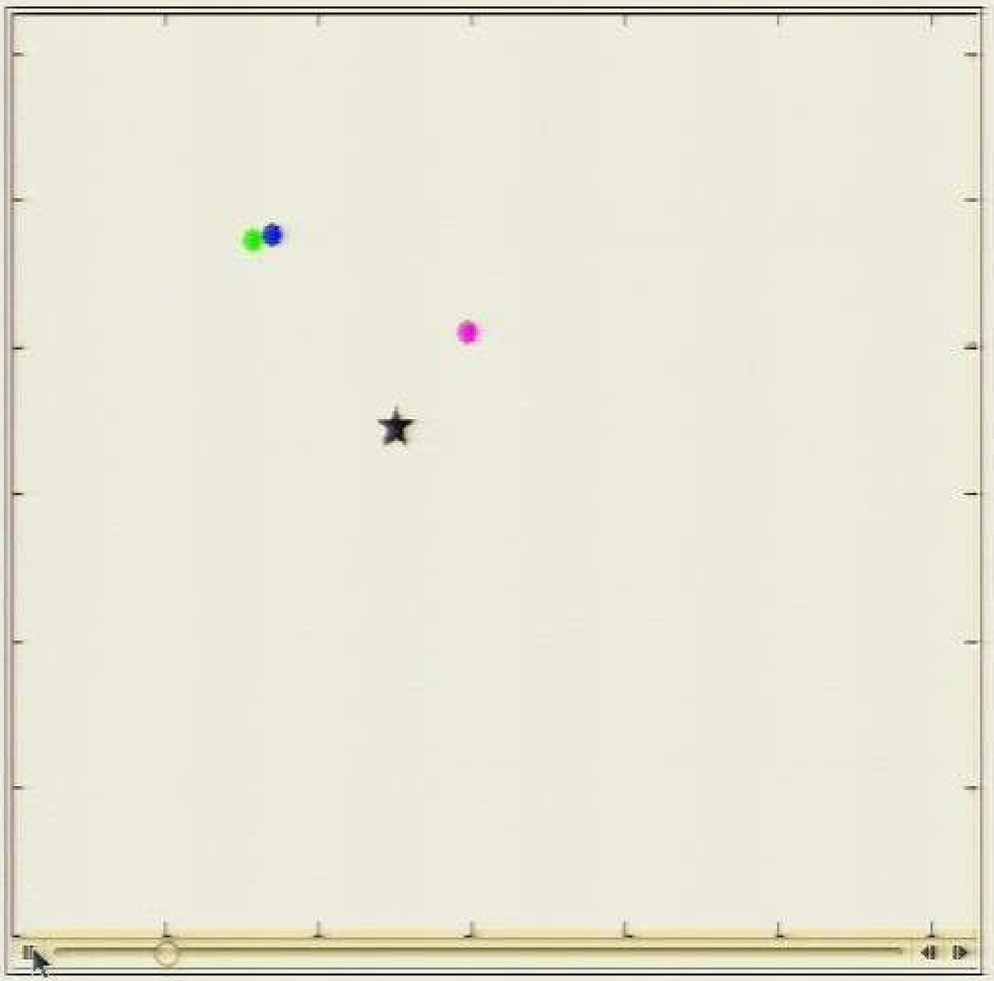

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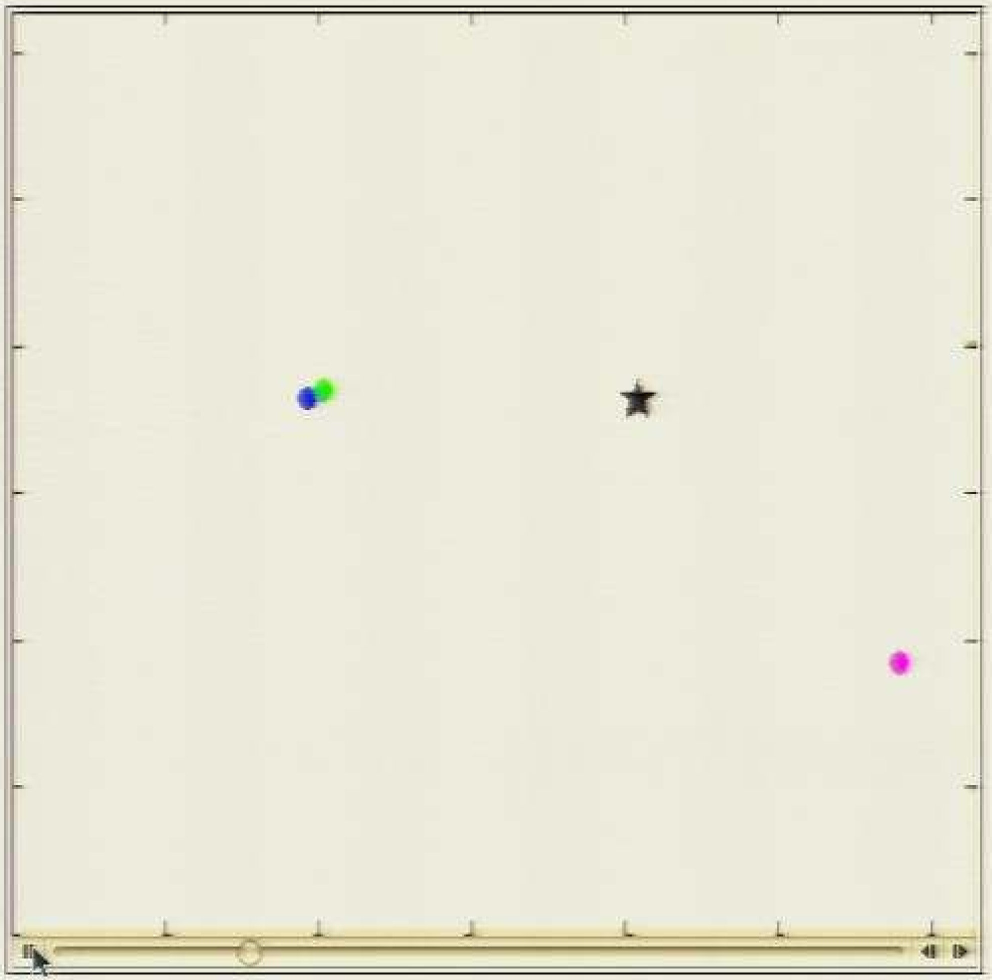

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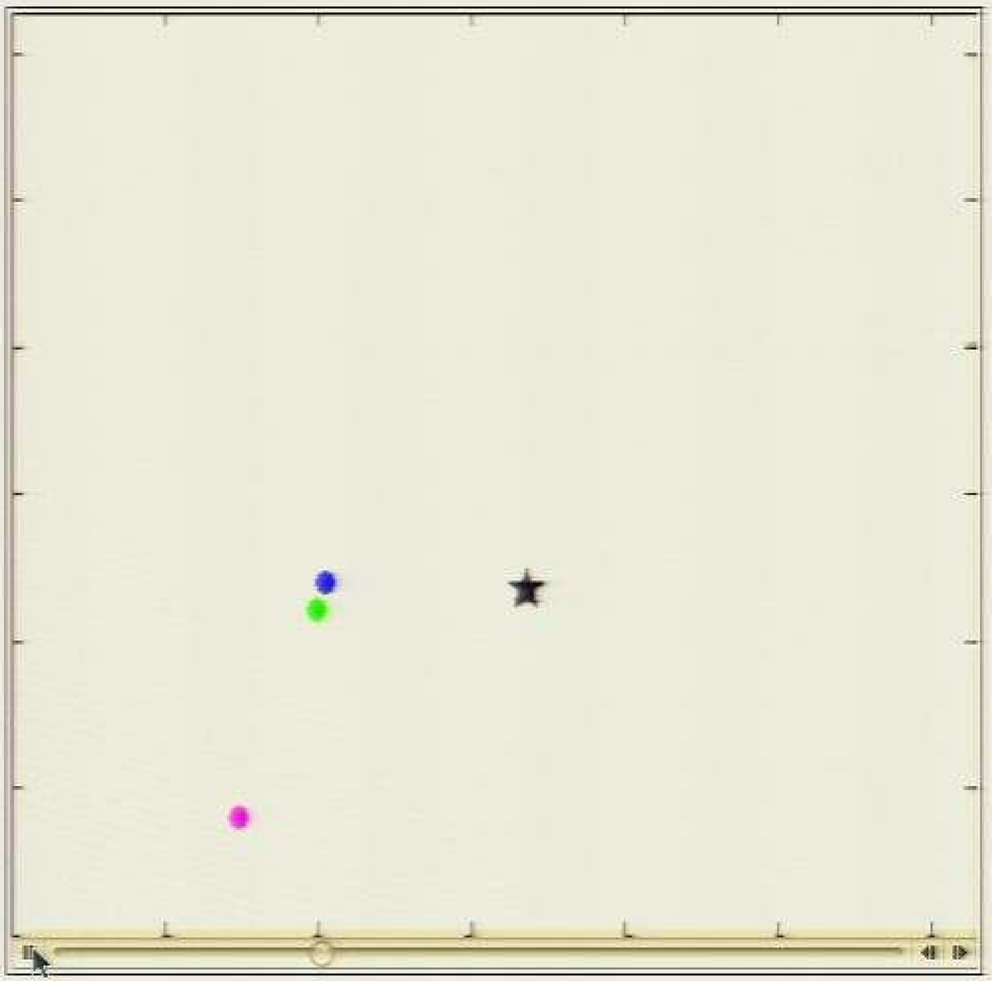

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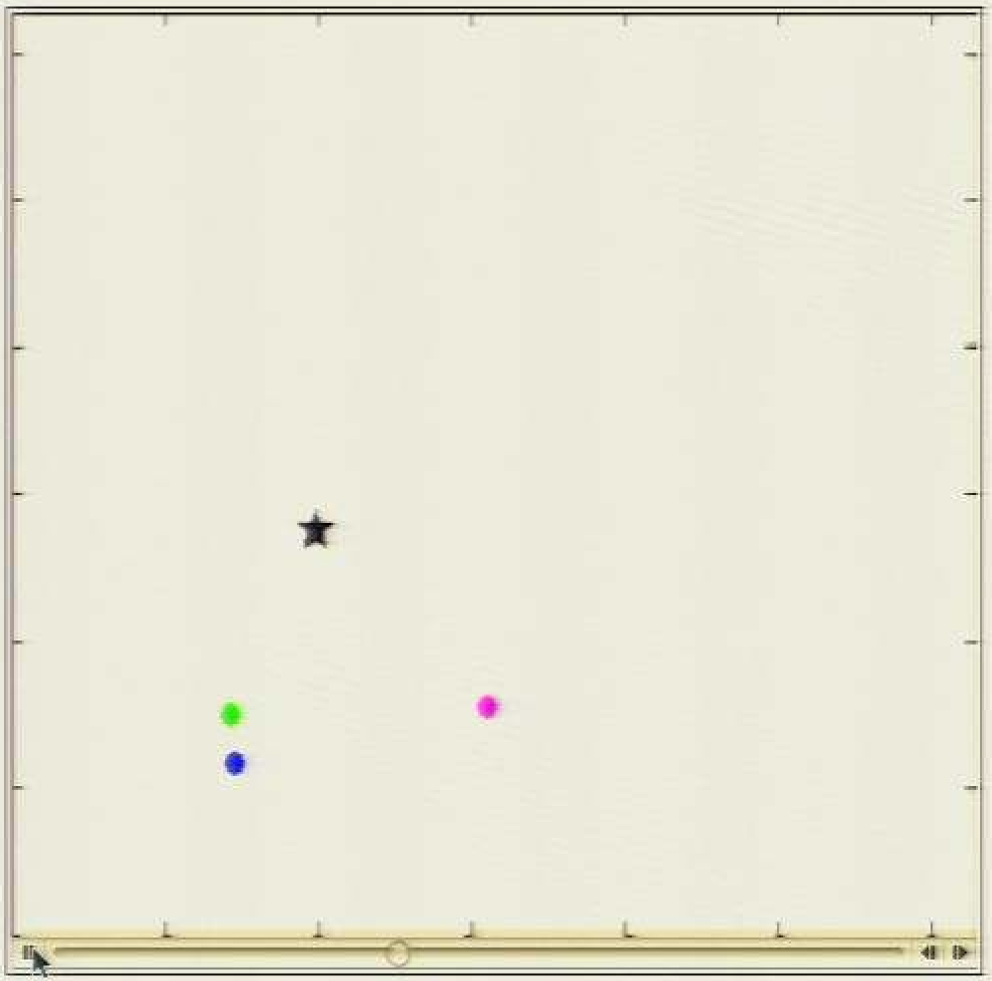

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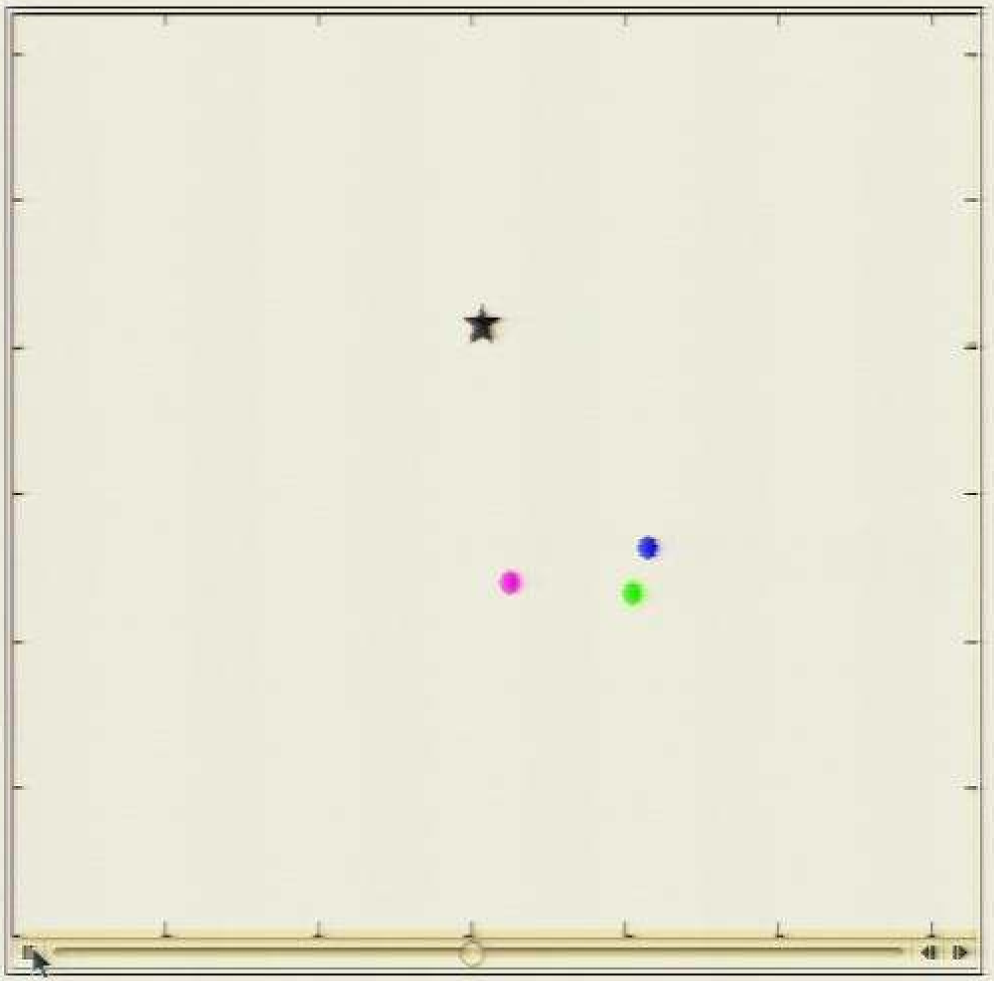

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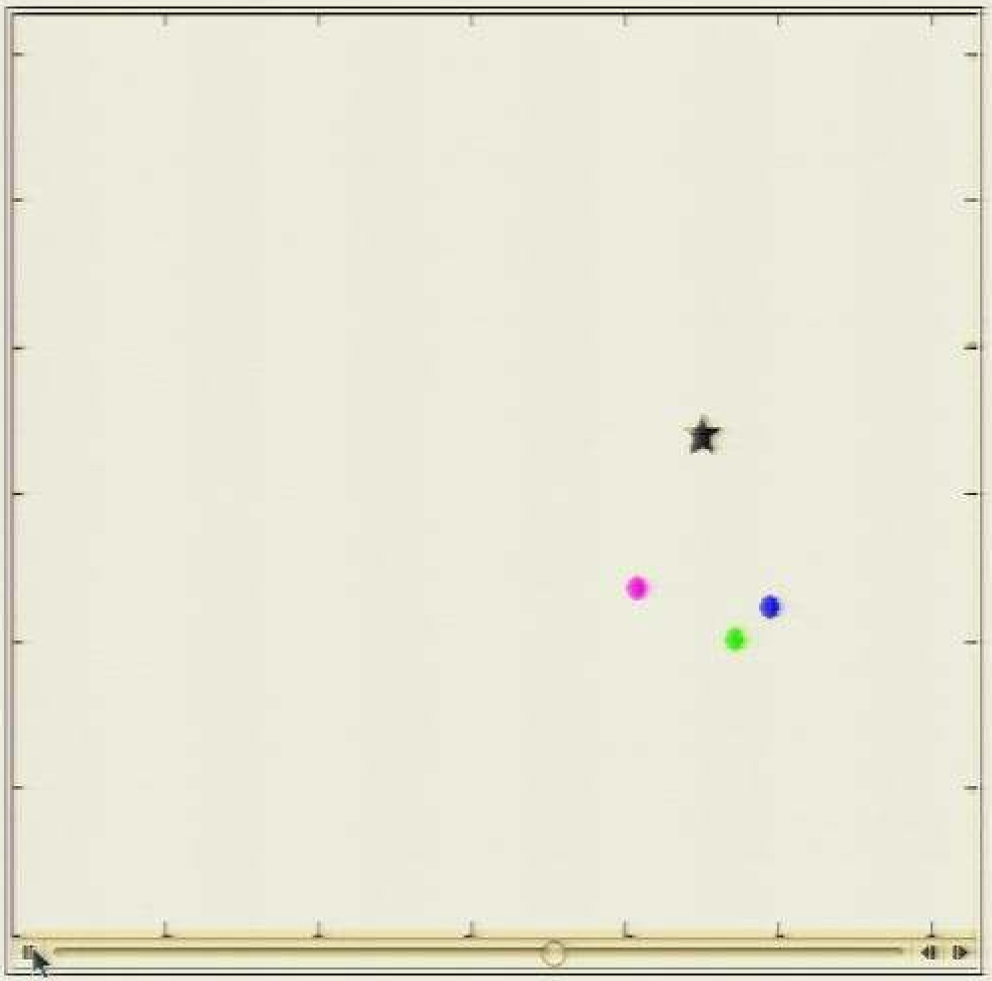

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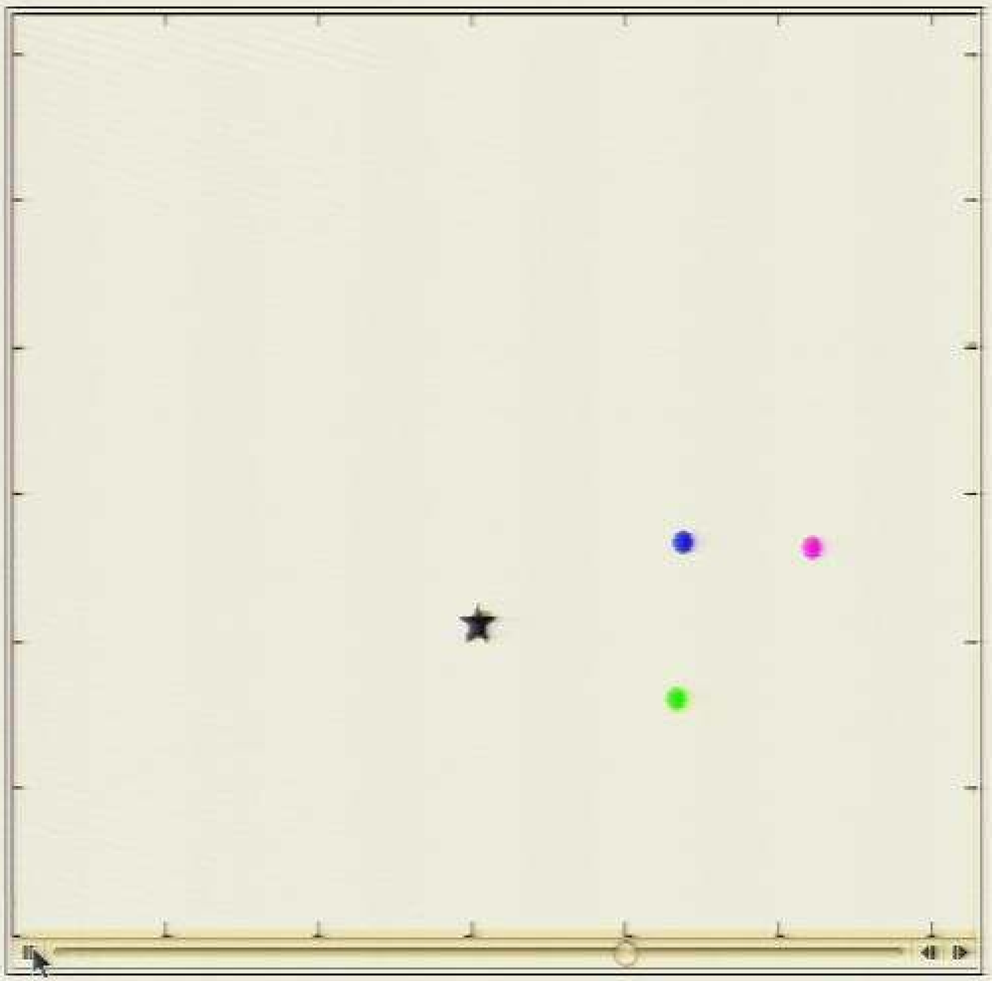

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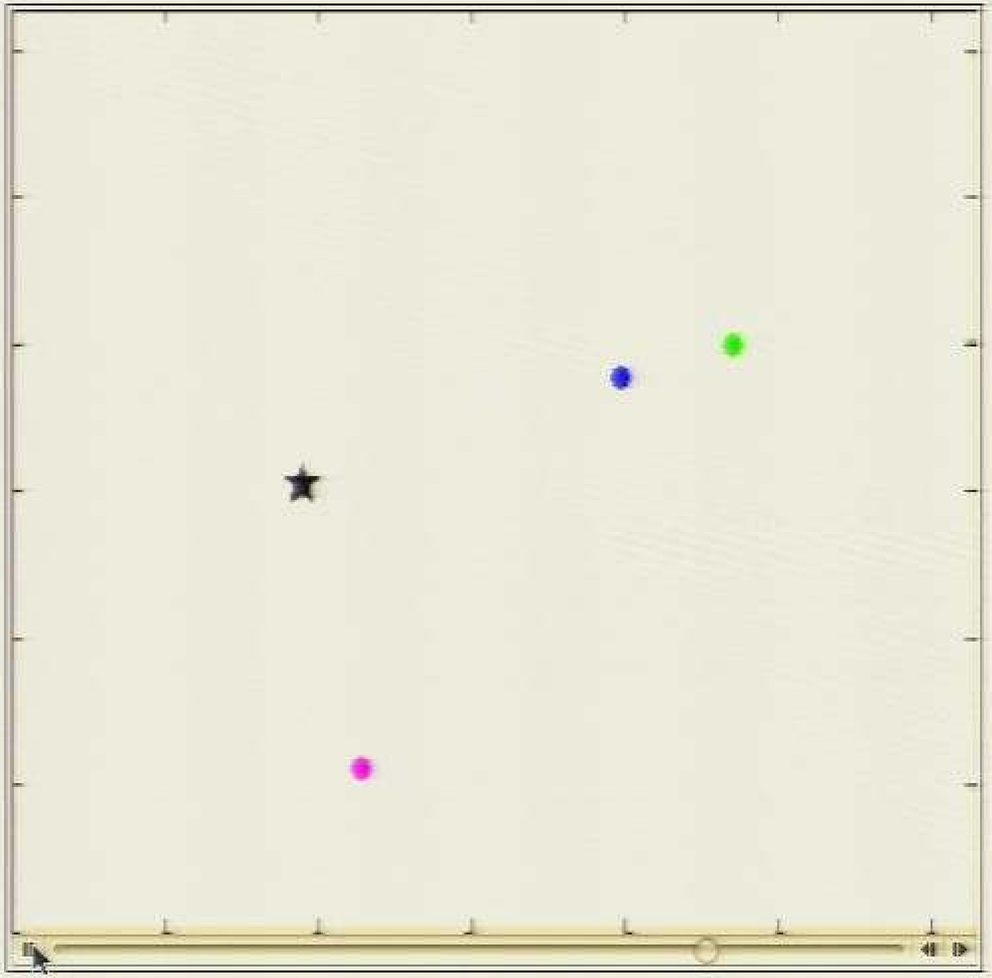

Conclusion

- Relaxation speed-up with the curl-term
- Curl-term implies vorticity but nodes remain important for chaotic behaviour.
- Any prediction will also depend crucially on the preferred beable.

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