Title: Bohmian Quantum Cosmology

Date: Sep 29, 2009 11:00 AM

URL: http://pirsa.org/09090090

Abstract: Quantum cosmology is the arena where the interpretations of quantum mechanics are pushed to their limits. For instance, the Copenhaguen interpretation cannot even be applied to this framework. With this in mind, I will describe the main results which emerge from the application of the Bohm-de Broglie interpretation to quantum cosmology, not only for an investigation of the later, but also to get a better understanding of the former in comparison with other interpretations. At first, without imposing any spacetime symmetry from the beginning, we show explicitly the breakdown of spacetime into space and time due to quantum effects, and an investigation of these latter structures within the Bohm-de Broglie picture. Afterwards, in the case of minisuperspace quantum cosmology, I will present how the notions of an evolution time parameter, cosmological singularities, and classical limit can be unambiguously defined. Cosmological non singular quantum bouncing solutions emerge, which are naturally led to th e standard cosmological model evolution before nucleosynthesis: large classical universes can be obtained without any traditional primordial inflationary expansion. A theory of quantum cosmological perturbations on these backgrounds is constructed, and almost scale invariant spectra are obtained. I argue about the possibility of testing these models against inflation. Use of the Bohm interpretation is crucial to obtain these results, which are otherwise unclear within other interpretations. Finally, I show potential discrepant results about the avoidance of cosmological singularities when different interpretations of quantum mechanics are used, and I speculate about constructing analog models where such differences could be tested.

NEW PERSPECTIVES ON THE QUANTUM STATE
Perimeter Institute, Waterloo, September 2009

BOHMIAN QUANTUM COSMOLOGY

Nelson Pinto Neto

CBPF
ICRA

If quantum mechanics is a universal theory, it should be aplied to the Universe!

Problems with the Copenhaguen interpretation: where is the classical domain, the observer?

Need of alternative interpretations:

Many Worlds
Consistent Histories
Bohm-de Broglie
....

OUTLINE

1) General case: superspace:
how is quantum geometry?

2) Homogeneous case: minisuperspace:
what about singularities and time?

3) Homogeneous background+perturbations
can we test these hypothesis?

1) General case

The classical Hamiltonian of GR with a scalar field is given by

$$H=\int d^3x(N\mathcal{H}+N^j\mathcal{H}_j) \tag{3.1}$$

where

$$\mathcal{H}=\kappa G_{ijkl}\Pi^{ij}\Pi^{kl}+\frac{1}{2}h^{-1/2}\Pi_\phi^2+h^{1/2}$$
$$\times\left[-\kappa^{-1}(R^{(3)}-2\Lambda)+\frac{1}{2}h^{ij}\partial_i\phi\partial_j\phi+U(\phi)\right] \tag{3.2}$$

$$\mathcal{H}_j=-2D_i\Pi_j^i+\Pi_\phi\partial_j\phi. \tag{3.3}$$

OUTLINE

1) General case: superspace:
how is quantum geometry?

2) Homogeneous case: minisuperspace:
what about singularities and time?

3) Homogeneous background+perturbations
can we test these hypothesis?

1) General case

The classical Hamiltonian of GR with a scalar field is given by

$$H = \int d^3x (N\mathcal{H} + N^j \mathcal{H}_j) \tag{3.1}$$

where

$$\mathcal{H} = \kappa G_{ijkl} \Pi^{ij} \Pi^{kl} + \frac{1}{2} h^{-1/2} \Pi_\phi^2 + h^{1/2} \times \left[-\kappa^{-1} (R^{(3)} - 2\Lambda) + \frac{1}{2} h^{ij} \partial_i \phi \partial_j \phi + U(\phi) \right] \tag{3.2}$$

$$\mathcal{H}_j = -2 D_i \Pi^i_j + \Pi_\phi \partial_j \phi. \tag{3.3}$$

Dynamics of GR in Hamilton-Jacobi form

$$\kappa G_{ijkl}\frac{\delta S}{\delta h_{ij}}\frac{\delta S}{\delta h_{kl}}+\frac{1}{2}h^{-1/2}\left(\frac{\delta S}{\delta \phi}\right)^2+V\cdot \;=0$$

$$-2h_{li}D_j\frac{\delta S(h_{ij},\phi)}{\delta h_{lj}}+\frac{\delta S(h_{ij},\phi)}{\delta \phi}\partial_i\phi=0,$$

$$\dot{h}_{ij}=2NG_{ijkl}\frac{\delta S}{\delta h_{kl}}+D_iN_j+D_jN_i$$

and

$$\dot{\phi}=Nh^{-1/2}\frac{\delta S}{\delta \phi}+N^i\partial_i\phi.$$

DIRAC ALGEBRA

$$\{\mathcal{H}(x),\mathcal{H}(x')\}=\mathcal{H}^i(x)\partial_i\delta^3(x,x')-\mathcal{H}^i(x')\partial_i\delta^3(x',x)$$

$$\{\mathcal{H}_i(x),\mathcal{H}(x')\}=\mathcal{H}(x)\partial_i\delta^3(x,x')$$

$$\{\mathcal{H}_i(x),\mathcal{H}_j(x')\}=\mathcal{H}_i(x)\partial_j\delta^3(x,x')+\mathcal{H}_j(x')\partial_i\delta^3(x,x'). \tag{3.16}$$

Dynamics of GR in Hamilton-Jacobi form

$$\kappa G_{ijkl}\frac{\delta S}{\delta h_{ij}}\frac{\delta S}{\delta h_{kl}}+\frac{1}{2}h^{-1/2}\left(\frac{\delta S}{\delta \phi}\right)^2+V\cdot \;=0$$

$$-2h_{li}D_j\frac{\delta S(h_{ij},\phi)}{\delta h_{lj}}+\frac{\delta S(h_{ij},\phi)}{\delta \phi}\partial_i\phi=0,$$

$$\dot{h}_{ij}=2NG_{ijkl}\frac{\delta S}{\delta h_{kl}}+D_iN_j+D_jN_i$$

and

$$\dot{\phi}=Nh^{-1/2}\frac{\delta S}{\delta \phi}+N^i\partial_i\phi.$$

DIRAC ALGEBRA

$$\{\mathcal{H}(x),\mathcal{H}(x')\}=\mathcal{H}^i(x)\partial_i\delta^3(x,x')-\mathcal{H}^i(x')\partial_i\delta^3(x',x)$$

$$\{\mathcal{H}_i(x),\mathcal{H}(x')\}=\mathcal{H}(x)\partial_i\delta^3(x,x')$$

$$\{\mathcal{H}_i(x),\mathcal{H}_j(x')\}=\mathcal{H}_i(x)\partial_j\delta^3(x,x')+\mathcal{H}_j(x')\partial_i\delta^3(x,x'). \quad (3.16)$$

Dynamics of GR in Hamilton-Jacobi form

$$\kappa G_{ijkl} \frac{\delta S}{\delta h_{ij}} \frac{\delta S}{\delta h_{kl}} + \frac{1}{2} h^{-1/2} \left(\frac{\delta S}{\delta \phi} \right)^2 + V \cdot \quad = 0$$

$$-2 h_{li} D_j \frac{\delta S(h_{ij}, \phi)}{\delta h_{lj}} + \frac{\delta S(h_{ij}, \phi)}{\delta \phi} \partial_i \phi = 0,$$

$$\dot{h}_{ij} = 2 N G_{ijkl} \frac{\delta S}{\delta h_{kl}} + D_i N_j + D_j N_i$$

and

$$\dot{\phi} = N h^{-1/2} \frac{\delta S}{\delta \phi} + N^i \partial_i \phi.$$

DIRAC ALGEBRA

$$\{\mathcal{H}(x), \mathcal{H}(x')\} = \mathcal{H}^i(x)\partial_i\delta^3(x,x') - \mathcal{H}^i(x')\partial_i\delta^3(x',x)$$

$$\{\mathcal{H}_i(x), \mathcal{H}(x')\} = \mathcal{H}(x)\partial_i\delta^3(x,x')$$

$$\{\mathcal{H}_i(x), \mathcal{H}_j(x')\} = \mathcal{H}_i(x)\partial_j\delta^3(x,x') + \mathcal{H}_j(x')\partial_i\delta^3(x,x'). \tag{3.16}$$

Dirac quantization

$$\hat{\mathcal{H}}_i|\Psi\rangle=0 \tag{3.17}$$

$$\hat{\mathcal{H}}|\Psi\rangle=0. \tag{3.18}$$

In the metric and field representation, the first equation is

$$-2h_{li}D_j\frac{\delta\Psi(h_{ij},\phi)}{\delta h_{lj}}+\frac{\delta\Psi(h_{ij},\phi)}{\delta\phi}\partial_i\phi=0, \tag{3.19}$$

which implies that the wave functional Ψ is an invariant under space coordinate transformations.

The second equation is the Wheeler-DeWitt equation [34,35]. Writing it unregulated in the coordinate representation we get

$$\left\{-\hbar^2\left[\kappa G_{ijkl}\frac{\delta}{\delta h_{ij}}\frac{\delta}{\delta h_{kl}}+\frac{1}{2}h^{-1/2}\frac{\delta^2}{\delta\phi^2}\right]+V\right\}\Psi(h_{ij},\phi)=0, \tag{3.20}$$

where V is the classical potential given by

$$V=h^{1/2}\left[-\kappa^{-1}(R^{(3)}-2\Lambda)+\frac{1}{2}h^{ij}\partial_i\phi\partial_j\phi+U(\phi)\right].$$

Writing in polar form

$$\Psi = A\exp(iS/\hbar)$$

the equation $\hat{\mathcal{H}}_i|\Psi\rangle=0$ **yields**

$$-2h_{li}D_j\frac{\delta S(h_{ij},\phi)}{\delta h_{lj}}+\frac{\delta S(h_{ij},\phi)}{\delta\phi}\partial_i\phi=0, \qquad (3.22)$$

$$-2h_{li}D_j\frac{\delta A(h_{ij},\phi)}{\delta h_{lj}}+\frac{\delta A(h_{ij},\phi)}{\delta\phi}\partial_i\phi=0. \qquad (3.23)$$

The equation $\hat{\mathcal{H}}|\Psi\rangle = 0.$ yields

$$\kappa G_{ijkl}\frac{\delta S}{\delta h_{ij}}\frac{\delta S}{\delta h_{kl}} + \frac{1}{2}h^{-1/2}\left(\frac{\delta S}{\delta \phi}\right)^2 + V + Q = 0,$$

$$\kappa G_{ijkl}\frac{\delta}{\delta h_{ij}}\left(A^2\frac{\delta S}{\delta h_{kl}}\right) + \frac{1}{2}h^{-1/2}\frac{\delta}{\delta \phi}\left(A^2\frac{\delta S}{\delta \phi}\right) = 0.$$

where

$$Q = -\hbar^2\frac{1}{A}\left(\kappa G_{ijkl}\frac{\delta^2 A}{\delta h_{ij}\delta h_{kl}} + \frac{h^{-1/2}}{2}\frac{\delta^2 A}{\delta \phi^2}\right).$$

The equation $\hat{\mathcal{H}}|\Psi\rangle = 0.$ yields

$$\kappa G_{ijkl} \frac{\delta S}{\delta h_{ij}} \frac{\delta S}{\delta h_{kl}} + \frac{1}{2} h^{-1/2} \left(\frac{\delta S}{\delta \phi} \right)^2 + V + Q = 0,$$

$$\kappa G_{ijkl} \frac{\delta}{\delta h_{ij}} \left(A^2 \frac{\delta S}{\delta h_{kl}} \right) + \frac{1}{2} h^{-1/2} \frac{\delta}{\delta \phi} \left(A^2 \frac{\delta S}{\delta \phi} \right) = 0.$$

where

$$Q = -\hbar^2 \frac{1}{A} \left(\kappa G_{ijkl} \frac{\delta^2 A}{\delta h_{ij} \delta h_{kl}} + \frac{h^{-1/2}}{2} \frac{\delta^2 A}{\delta \phi^2} \right).$$

Guidance relations

$$\dot{h}_{ij} = 2NG_{ijkl}\frac{\delta S}{\delta h_{kl}} + D_i N_j + D_j N_i$$

and

$$\dot{\phi} = Nh^{-1/2}\frac{\delta S}{\delta \phi} + N^i \partial_i \phi.$$

The Bohmian dynamics can be generated by the Hamiltonian

$$H_Q = \int d^3x [N(\mathcal{H} + Q) + N^i \mathcal{H}_i]$$

where we define

$$\mathcal{H}_Q \equiv \mathcal{H} + Q.$$

and constraints

$$\Phi^{ij} \equiv \Pi^{ij} - \frac{\delta S(h_{ab}, \phi)}{\delta h_{ij}} \approx 0$$

and

$$\Phi_\phi \equiv \Pi_\phi - \frac{\delta S(h_{ij}, \phi)}{\delta \phi} \approx 0.$$

The constraints close.

Teitelboim result:

$$\overline{H} = \int d^3x (N\overline{\mathcal{H}} + N^i \overline{\mathcal{H}}_i),$$

algebra to form a 4-geometry is

$$\{\overline{\mathcal{H}}(x), \overline{\mathcal{H}}(x')\} = -\epsilon[\overline{\mathcal{H}}^i(x)\partial_i\delta^3(x',x)] - \overline{\mathcal{H}}^i(x')\partial_i\delta^3(x,x') \tag{4.15}$$

$$\{\overline{\mathcal{H}}_i(x), \overline{\mathcal{H}}(x')\} = \overline{\mathcal{H}}(x)\partial_i\delta^3(x,x') \tag{4.16}$$

$$\{\overline{\mathcal{H}}_i(x), \overline{\mathcal{H}}_j(x')\} = \overline{\mathcal{H}}_i(x)\partial_j\delta^3(x,x') - \overline{\mathcal{H}}_j(x')\partial_i\delta^3(x,x'). \tag{4.17}$$

Hojman, Kuchar and Teitelboim: together with other conditions the Hamiltonian must have the form

$$\bar{\mathcal{H}} = \kappa G_{ijkl}\Pi^{ij}\Pi^{kl} + \frac{1}{2}h^{-1/2}\pi_{\phi}^{2} + V_G, \qquad (4.18)$$

where

$$V_G \equiv -\epsilon h^{1/2}\left[-\kappa^{-1}(R^{(3)} - 2\bar{\Lambda}) + \frac{1}{2}h^{ij}\partial_i\phi\partial_j\phi + \bar{U}(\phi)\right]. \qquad (4.19)$$

1. Spacetime is hyperbolic ($\epsilon=-1$)

In this case Q is

$$Q=-h^{1/2}\left[\frac{2}{\kappa}(-\overline{\Lambda}+\Lambda)-\overline{U}(\phi)+U(\phi)\right]. \qquad (4.21)$$

2. Spacetime is Euclidean ($\epsilon=1$)

In this case Q is

$$Q=-h^{1/2}\left[2\left(-\kappa^{-1}R^{(3)}+\frac{1}{2}h^{ij}\partial_i\phi\partial_j\phi\right)+\frac{2}{\kappa}(\overline{\Lambda}+\Lambda)\right.$$

$$\left.+\overline{U}(\phi)+U(\phi)\right]. \qquad (4.22)$$

3. There is no spacetime: privileged foliation.

Trivial example: real solutions of the Wheeler-DeWitt equation:
S = 0, Q = -V

$$V = h^{1/2}\left[-\kappa^{-1}(R^{(3)} - 2\Lambda) + \frac{1}{2}h^{ij}\partial_i\phi\partial_j\phi + U(\phi)\right]. \qquad (3.21)$$

$$\{\mathcal{H}_Q(x), \mathcal{H}_Q(x')\} = 0.$$

E. S. Santini and NPN, Phys. Rev. D 59, 123517, (1999).

2) MINISUPERSPACE MODELS

Degrees of freedom: $a(t)$, $\Phi(t)$ or $\varepsilon(t)$ (homogeneity and isotropy)

Simplification implemented to answer qualitative questions concerning quantum cosmology:

1) How can we decide whether a quantum spacetime has a singularity?

2) Does quantum effects eliminate them?

Bohmian answers:

1) As we have quantum trajectories, quantum singularities are situations where it goes to zero.

2) Yes, in many cases, but not always.

ACTION FOR HOMOGENEOUS COSMOLOGY WITH RADIATION

$$S = \int d\eta (pa' - p^2/4).$$

Like a free particle when written in conformal time ($d\eta = dt/a(t)$): $a = \eta \Longrightarrow a = t^{1/2}$ (classical solution)

Schrödinger equation:

$$i\frac{\partial\Psi}{\partial\eta} = -\frac{\partial^2\Psi}{4\partial a^2}.$$

Initial condition: gaussian

$$\Psi_0(a) = \left(\frac{8b}{\pi}\right)^{1/4} \exp(-ba^2),$$

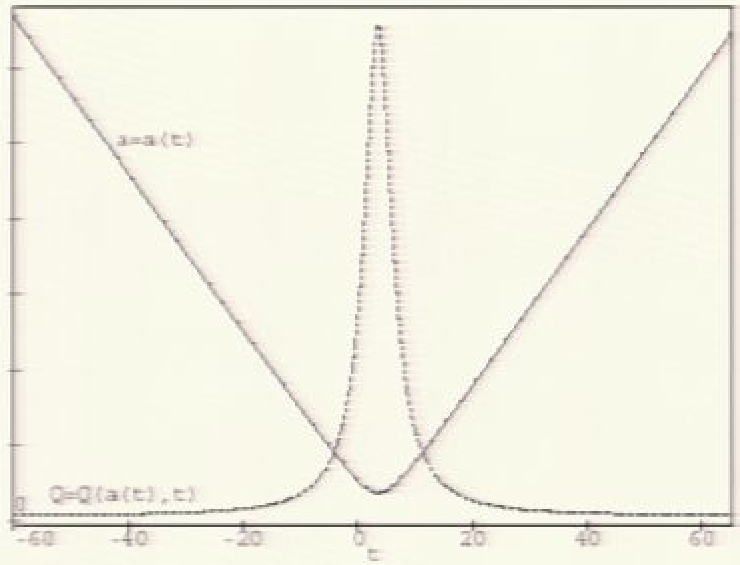
gerando

$$\Psi(a,\eta) = \left(\frac{8b}{\pi}\right)^{1/4} \left(\frac{1}{b\eta - i}\right)^{1/2} \exp\left\{\frac{i}{\eta}\left[1 + \frac{i}{(b\eta - i)}\right]a^2\right\}.$$

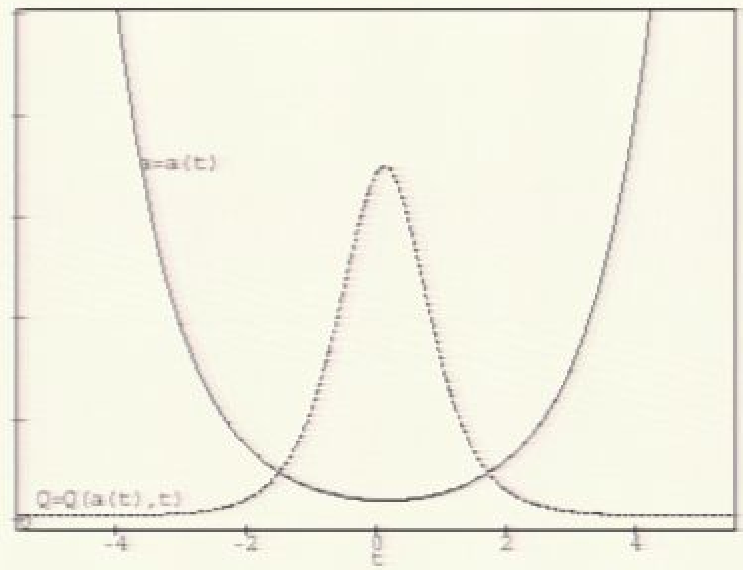
Bohmian quantum trajectory: $da/d\eta = \partial S/\partial a$

$$a(\eta) = a_0 (b^2 \eta^2 + 1)^{1/2}$$

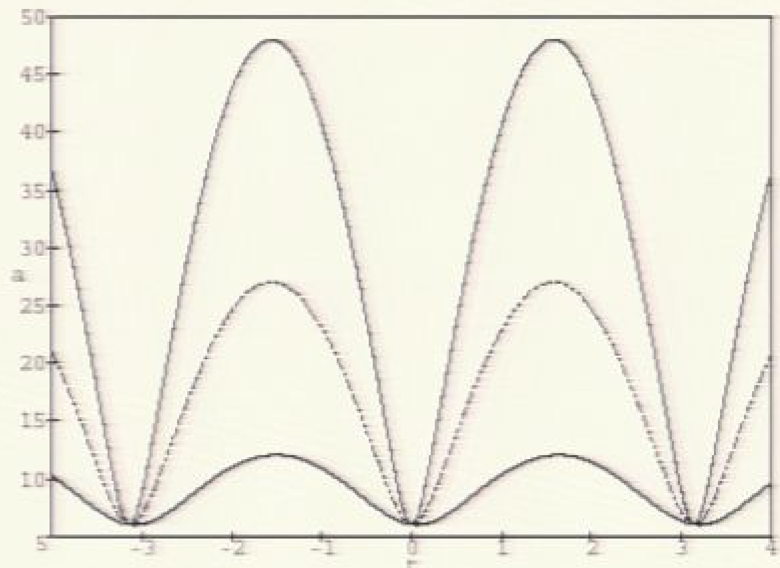
J. A.de Barros, M. A. Saggioro-Leal, NPN, Phys. Lett. A 241, 229 (1998)


a=a(t)
Q=Q(a(t),t)
-60
-40
-20
0
20
40
60
t

$k = 0$


a=a(t)
Q=Q(a(t),t)
-4
-2
0
2
4
t

$k = -1$


50
45
40
35
30
25
20
15
10
5
a
-3
-2
-1
0
1
2
3
4
t

$k = 1$

Initial condition: gaussian

$$\Psi_0(a) = \left(\frac{8b}{\pi}\right)^{1/4} \exp(-ba^2),$$

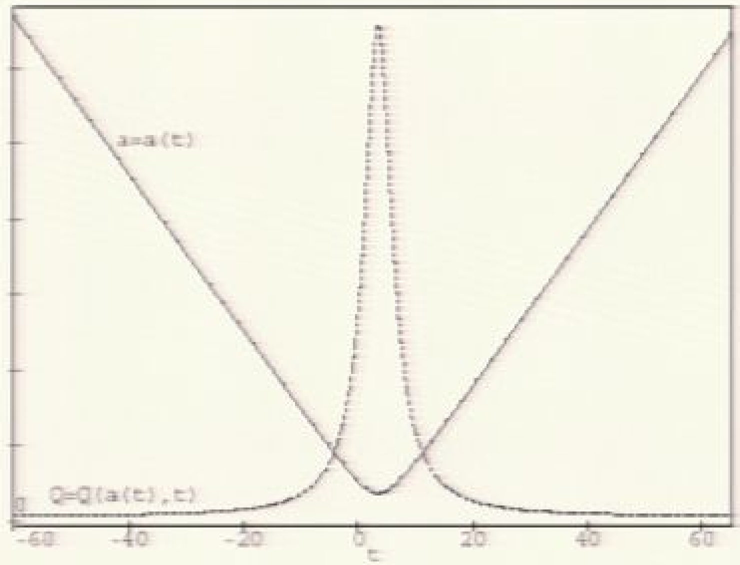
gerando

$$\Psi(a,\eta) = \left(\frac{8b}{\pi}\right)^{1/4} \left(\frac{1}{b\eta - i}\right)^{1/2} \exp\left\{\frac{i}{\eta}\left[1 + \frac{i}{(b\eta - i)}\right] a^2\right\}.$$

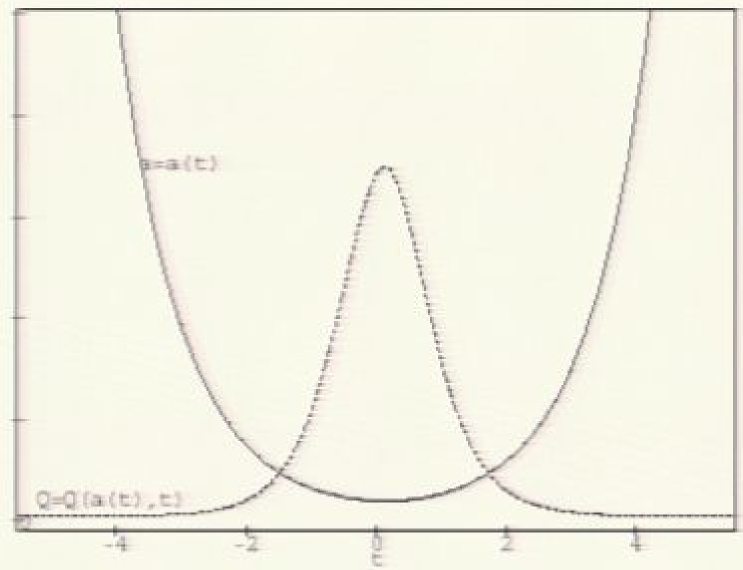
Bohmian quantum trajectory: $da/d\eta = \partial S/\partial a$

$$a(\eta) = a_0 \, (b^2 \eta^2 + 1)^{1/2}$$

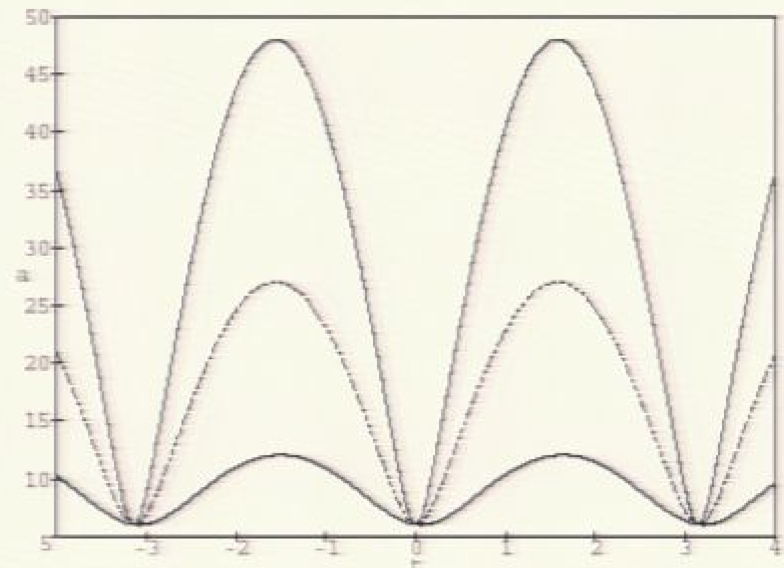
J. A.de Barros, M. A. Saggioro-Leal, NPN, Phys. Lett. A 241, 229 (1998)


a=a(t)
Q=Q(a(t),t)
-60
-40
-20
0
20
40
60
t

$k = 0$


a=a(t)
Q=Q(a(t),t)
-4
-2
0
2
4
t

$k = -1$


50
45
40
35
30
25
20
15
10
5
a
-3
-2
-1
0
1
2
3
4
t

$k = 1$

General perfect fluids: p=λρ

$$\imath\frac{\partial}{\partial T}\Psi_{(0)}(a,T)=\frac{1}{4}\left\{a^{(3\lambda-1)/2}\frac{\partial}{\partial a}\left[a^{(3\lambda-1)/2}\frac{\partial}{\partial a}\right]\right\}\Psi_{(0)}(a,T),$$

$$\Psi_{(0)}^{(\mathrm{init})}(\chi)=\left(\frac{8}{T_0\pi}\right)^{1/4}\exp\left(-\frac{\chi^2}{T_0}\right),$$

$$\chi=\frac{2}{3}(1-\lambda)^{-1}a^{3(1-\lambda)/2}$$

$$\Psi_{(0)}(a,T),=\left[\frac{8T_0}{\pi(T^2+T_0^2)}\right]^{1/4}\exp\left[\frac{-4T_0a^{3(1-\lambda)}}{9(T^2+T_0^2)(1-\lambda)^2}\right]\exp\left\{-i\left[\frac{4Ta^{3(1-\lambda)}}{9(T^2+T_0^2)(1-\lambda)^2}+\frac{1}{2}\arctan\left(\frac{T_0}{T}\right)-\frac{\pi}{4}\right]\right\}$$

$$\frac{\partial\rho}{\partial T}-\frac{\partial}{\partial a}\left[\frac{a^{(3\lambda-1)}}{2}\frac{\partial S}{\partial a}\rho\right]=0,$$

$$\dot{a}=-\frac{a^{(3\lambda-1)}}{2}\frac{\partial S}{\partial a},$$

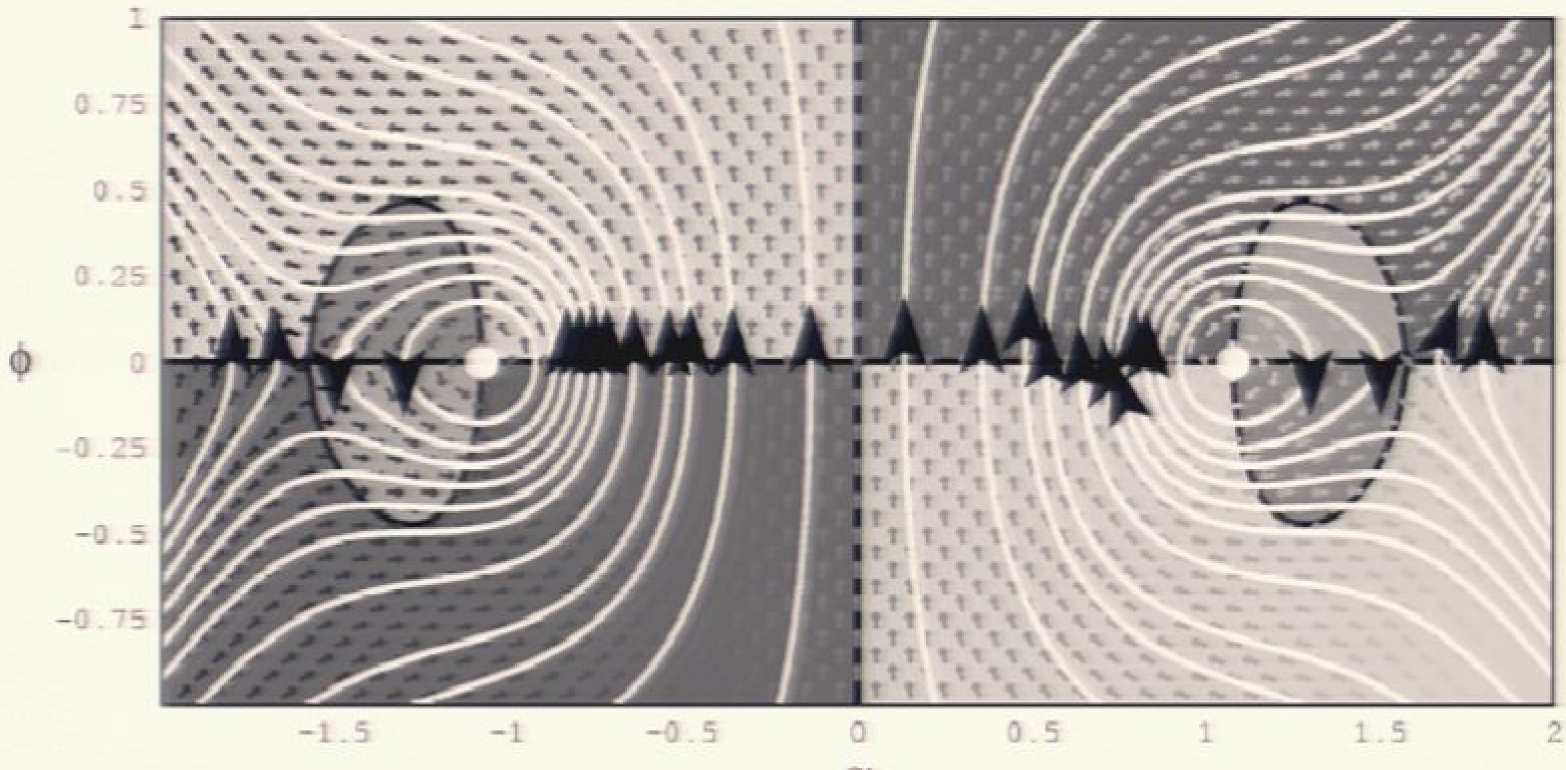
$$a(T)=a_0\left[1+\left(\frac{T}{T_0}\right)^2\right]^{\frac{1}{3(1-\lambda)}}.$$

$$\mathrm{d}\eta=[a(T)]^{3\lambda-1}\,\mathrm{d}T.$$

Gaussian Superpositions in Scalar Tensor Quantum Cosmological Models

Phys. Rev. D62, 83507 (2000)

com R. Colistete Jr. e J. C. Fabris



3) LINEAR COSMOLOGICAL PERTURBATIONS

The action:

$$\mathcal{S} = \mathcal{S}_{\rm GR} + \mathcal{S}_{\rm fluid} = -\frac{1}{6\ell_{\rm Pl}^2}\int \sqrt{-g}R\mathrm{d}^4x - \int \sqrt{-g}\epsilon\mathrm{d}^4x,$$

The metric:

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu},$$

$$ds^2 = g_{\mu\nu}^{(0)} dx^\mu dx^\nu = N^2(t)dt^2 - a^2(t)\delta_{ij}dx^i dx^j,$$

$$
\begin{aligned}
h_{00} &= 2N^2\phi \\
h_{0i} &= -NaA_i \\
h_{ij} &= a^2\epsilon_{ij}
\end{aligned}
$$

Scalar perturbations:

$$
\begin{aligned}
h_{00} &= 2N^2\phi \\
h_{0i} &= -NaB_{,i} \\
h_{ij} &= 2a^2(\psi\gamma_{ij} - E_{,i,j})
\end{aligned}
$$

The total Lagrangian

$$
\begin{aligned}
L \; = \; & -\frac{\dot{a}^2 a V}{l^2 N} - N a^3 \varepsilon_0 V \\
& + \frac{N a}{6 l^2} \int d^3 x \gamma^{\frac{1}{2}} [A^{i|j} A_{[i|j]} - \frac{1}{4} \epsilon^{ij|k} \epsilon_{ij|k} \\
& + \frac{a}{N} \dot{A}_i \epsilon^{ij}{}_{|j} + \frac{1}{2} \epsilon^{ij}{}_{|j} \epsilon_i{}^k{}_{|k} + \phi_{|i} \epsilon^{ij}{}_{|j} \\
& - \frac{1}{2} \epsilon_{|i} \epsilon^{ij}{}_{|j} - \phi_{|i} \epsilon^{|i} + \frac{1}{4} \epsilon_{|i} \epsilon^{|i}] + \frac{a^3}{24 l^2 N} \int d^3 x \gamma^{\frac{1}{2}} \dot{\epsilon}^{ij} \dot{\epsilon}_{ij} \\
& - \frac{a^3}{24 l^2 N} \int d^3 x \gamma^{\frac{1}{2}} \dot{\epsilon}^2 + \frac{a \dot{a}^2}{6 l^2 N} \int d^3 x \gamma^{\frac{1}{2}} (-9 \phi^2 - 3 \epsilon \phi \\
& - \frac{3}{4} \epsilon^2 + 3 A^i A_i + \frac{3}{2} \epsilon^{ij} \epsilon_{ij}) - \frac{2 a \dot{a}}{3 l^2} \int d^3 x \gamma^{\frac{1}{2}} (\phi A^i{}_{|i} \\
& - \frac{1}{2} A_i \epsilon^{ij}{}_{|j}) + \frac{a^2 \dot{a}}{3 l^2 N} \int d^3 x \gamma^{\frac{1}{2}} (\epsilon^{ij} \dot{\epsilon}_{ij} - \frac{1}{2} \epsilon \dot{\epsilon} - \phi \dot{\epsilon}) \\
& - \frac{a^2}{6 l^2} \int d^3 x \gamma^{\frac{1}{2}} \dot{\epsilon} A^i{}_{|i} - N a^3 \varepsilon_0 \int d^3 x \gamma^{\frac{1}{2}} (-\frac{1}{2} \phi^2 + \frac{1}{2} A^i A_i \\
& - \phi \xi^i{}_{|i}) - N a^3 p_0 \int d^3 x \gamma^{\frac{1}{2}} (\frac{1}{2} \epsilon \phi + \frac{1}{4} \epsilon^{ij} \epsilon_{ij} - \frac{1}{8} \epsilon^2 - \phi \xi^i{}_{|i}) \\
& + \frac{1}{2} N a^3 (\varepsilon_0 + p_0) \int d^3 x \gamma^{\frac{1}{2}} (\frac{a^2}{N^2} \dot{\xi}^i \dot{\xi}_i + 2 \frac{a}{N} A_i \dot{\xi}^i + A_i A^i) \\
& - \frac{1}{2} c_s^2 N a^3 (\varepsilon_0 + p_0) \int d^3 x \gamma^{\frac{1}{2}} (\frac{1}{4} \epsilon^2 + \xi^i{}_{|i} \xi^j{}_{|j} - \epsilon \xi^i{}_{|i})
\end{aligned}
$$

Scalar perturbations

Background classical equations of motion and Mukhanov-Sasaki variables: tremendous simplification of the hamiltonian.

$$H_0^{(2)} \equiv \frac{1}{2}\int d^3x\pi^2 + \frac{1}{2}\int d^3x\frac{1}{2}\left(\lambda v^{,i}v_{,i} - \frac{z''}{z}v^2\right).$$

$$\mathbf{p} = \lambda\,\varepsilon$$

Conformal time: dη= dt/a(t)

$$\Phi^{,i}{}_{,i} = -\sqrt{\frac{3}{2}}\frac{\ell_{\rm Pl}\sqrt{\mathcal{H}^2-\mathcal{H}'}}{\lambda}\left(\frac{v}{z}\right)',$$

$$z = \frac{a}{\mathcal{H}\sqrt{\lambda}}\sqrt{\mathcal{H}^2-\mathcal{H}'}$$

QUANTUM EQUATIONS FOR PERTURBATIONS

$$i\frac{\partial\Psi_{(2)}[v,\eta]}{\partial\eta} = \int \mathrm{d}^3x \left(-\frac{1}{2}\frac{\delta^2}{\delta v^2} + \frac{\lambda}{2}v_{,i}v^{,i} - \frac{a''}{2a}v^2\right)\Psi_{(2)}[v,\eta].$$

In the Heisenberg representation

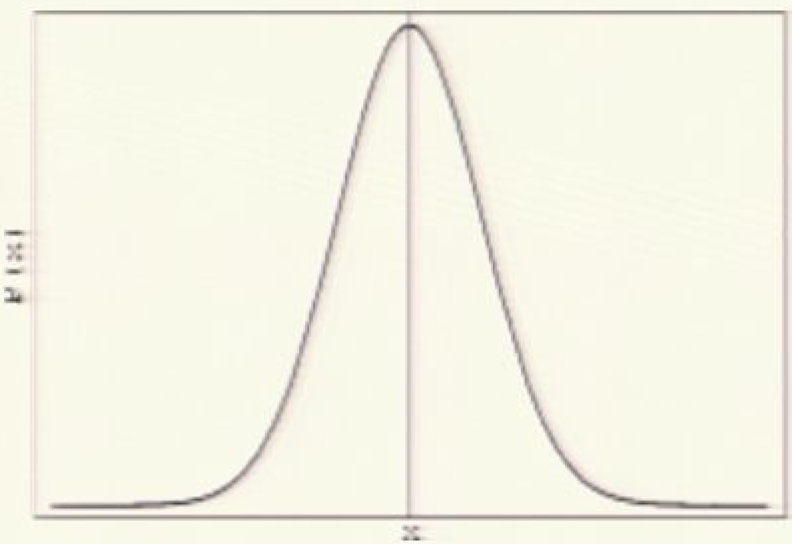
$$\boxed{v''_k + \left(\lambda k^2 - \frac{a''}{a}\right)v_k = 0.}$$

Like a scalar field with time dependent mass.

INFLATION

$$v''_k + \left(\lambda k^2 - \frac{a''}{a}\right) v_k = 0.$$

Expanding accelerated phase



Expanding decelerated phase

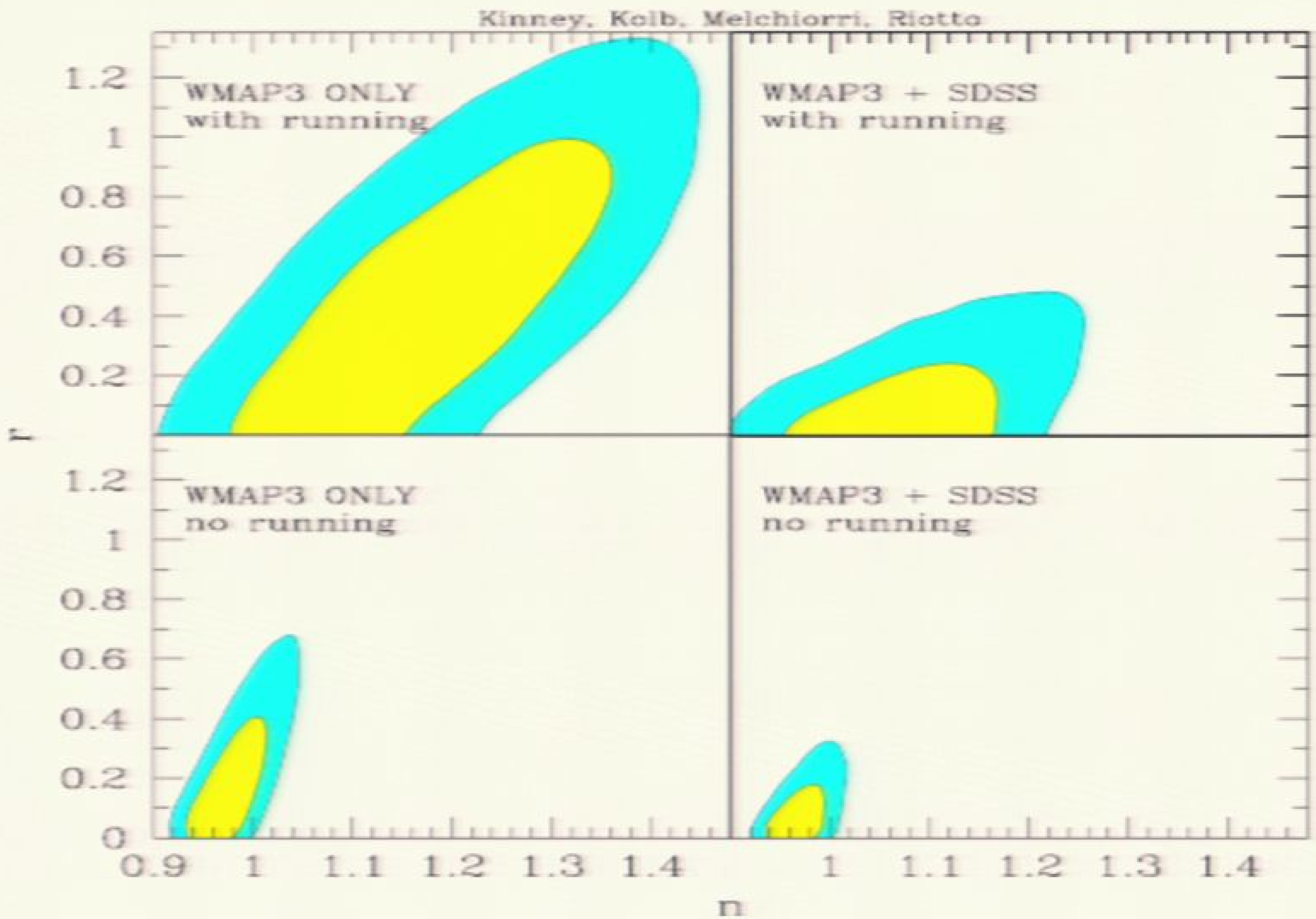
Point of matching: $\lambda k^2 = a''/a \Longrightarrow l_{phys} = \lambda^{1/2}\, l_c$

Primordial universe models: yield initial conditions for the evolution of the perturbations.

Spectral indices for large wavelengths::
(k is the wavenumber)

Scalar perturbations: $k^3\mathcal{P}_s \propto k^{n_s-1},$

Tensor perturbations: $k^3\mathcal{P}_T \propto k^{n_\mathrm{T}},$


Kinney, Kolb, Melchiorri, Riotto
WMAP3 ONLY
with running
WMAP3 + SDSS
with running
WMAP3 ONLY
no running
WMAP3 + SDSS
no running
r
n
0
0.2
0.4
0.6
0.8
1
1.2
0.9
1.1
1.3
1.4

What happens near the singularity?

What about the perturbations?
(They have quantum mechanical origin but evolve in a classical background)

Quantum cosmological models may have bounces: what happens if the Universe did not have a beginning?

Are there observational consequences of a primordial contracting phase in our Universe?

No use of background equations of motion.

The first to try: Halliwell and Hawking.

Hamiltonian → Quantization

$H_0\,\psi = 0$: too complicated

Is it possible to obtain a simplification without using the background classical equations?

YES: through canonical transformations and redefinitions of N.

$$F_1 \;=\; T\tilde{P}_T + a\tilde{P}_a + \int d^3x[\frac{1}{\sqrt{6}l}a^{-\frac{1}{2}(1-3\lambda)}\tilde{\varphi}_{(c)}\pi + \psi\tilde{\pi}_\psi$$
$$+\frac{2\sqrt{V}\sqrt{(\lambda+1)\tilde{P}_T}}{l^2\tilde{P}_a\sqrt{\lambda}}a^{\frac{3}{2}(1-\lambda)}\psi\pi - \frac{\gamma^{\frac{1}{2}}}{2}\alpha\tilde{\varphi}^2_{(c)}]$$

$$\mathcal{F}_2 \;=\; a\tilde{P}_a + \int d^3x\Big\{\psi\tilde{\pi_\psi} + v\tilde{\pi} + \Big[\frac{6\sqrt{V}[(\lambda+1)P_T]^{\frac{3}{2}}}{l^2\tilde{P_a}^2\sqrt{\lambda}}a^{\frac{3}{2}(1-3\lambda)} - \frac{(1+3\lambda)\sqrt{(\lambda+1)P_T}}{2\sqrt{\lambda}\sqrt{V}}a^{\frac{1}{2}(1-3\lambda)}\Big]\gamma^{\frac{1}{2}}v\psi$$
$$+\;\Big[-\frac{6V[(\lambda+1)P_T]^2}{l^4\tilde{P_a}^3\lambda}a^{3(1-2\lambda)} + \frac{(1+3\lambda)(\lambda+1)P_T}{2l^2\tilde{P}_a\lambda}a^{2-3\lambda}\Big]\gamma^{\frac{1}{2}}\psi^2 + \frac{2a^3V}{3l^4\tilde{P}_a}\psi\psi^{i}{}_{;i}\Big\}$$

$$\mathcal{F}_3 = a\tilde{P}_a + \frac{1}{a}\int d^3x v\tilde{\pi} - \frac{l^2\tilde{P}_a}{4aV}\int d^3x\gamma^{\frac{1}{2}}v^2$$

SCALAR FIELD

$$H_0 = \frac{\sqrt{2V}}{2\ell_{Pl} e^{3\alpha}} \left[-P_\alpha^2 + P_\varphi^2 + \int d^3x \left(\frac{\pi^2}{\sqrt{\gamma}} + \sqrt{\gamma} e^{4\alpha} v^{,i} v_{,i} \right) \right].$$

Background:
$\alpha = \ln a$, φ
v: perturbation

Classical Hamilton-Jacobi treatment

The respective Hamilton-Jacobi equation reads

$$-\frac{1}{2}\left(\frac{\partial S_T}{\partial \alpha}\right)^2 + \frac{1}{2}\left(\frac{\partial S_T}{\partial \varphi}\right)^2 + \frac{1}{2}\int d^3x \left[\frac{1}{\sqrt{\gamma}} \left(\frac{\delta S_T}{\delta v}\right)^2 + \sqrt{\gamma} e^{4\alpha} v^{,i} v_{,i} \right], \quad (26)$$

where the classical trajectories can be obtained from a solution S_T of Eq. (26) through

$$\dot{\alpha} = -P_\alpha = -\frac{\partial S_T}{\partial \alpha}, \qquad \dot{\varphi} = P_\varphi = \frac{\partial S_T}{\partial \varphi},$$
$$\dot{v} = \frac{1}{\sqrt{\gamma}} \pi = \frac{1}{\sqrt{\gamma}} \frac{\delta S_T}{\delta v}, \quad (27)$$

v is the unique combination of the canonical variables which has null Poisson brackets with all first order first class constraints, generators of linear gauge transformations

$$S_T(\alpha, \varphi, v) = S_0(\alpha, \varphi) + S_2(\alpha, \varphi, v), \qquad (28)$$

where it is assumed that $S_2(\alpha, \varphi, v)$ cannot be splitted again into a sum involving a function of the background variables alone (which would just impose a redefinition of S_0). Noting that, in order to be a solution of the Hamilton-Jacobi Eq. (26), S_2 must be at least a second order functional of v (see Ref. [21]), then $S_2 \ll S_0$ as well as their partial derivatives with respect to the background variables. Hence one obtains for the background that

$$\dot{\alpha} \approx -\frac{\partial S_0}{\partial \alpha}, \qquad \dot{\varphi} \approx \frac{\partial S_0}{\partial \varphi}. \qquad (29)$$

Inserting the splitting given in Eq. (28) into Eq. (26), one obtains, order by order:

$$-\frac{1}{2}\left(\frac{\partial S_0}{\partial \alpha}\right)^2 + \frac{1}{2}\left(\frac{\partial S_0}{\partial \varphi}\right)^2 = 0, \qquad (30)$$

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right) + \left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right) + \frac{1}{2}\int d^3x\left[\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2 + \sqrt{\gamma}e^{4\alpha}v^{i}v_{,i}\right] = 0, \qquad (31)$$

$$-\frac{1}{2}\left(\frac{\partial S_2}{\partial \alpha}\right)^2 + \frac{1}{2}\left(\frac{\partial S_2}{\partial \varphi}\right)^2 + O(4) = 0. \qquad (32)$$

SCALAR FIELD

$$H_0 = \frac{\sqrt{2V}}{2\ell_{Pl} e^{3\alpha}}\left[-P_\alpha^2 + P_\varphi^2 + \int d^3x\left(\frac{\pi^2}{\sqrt{\gamma}} + \sqrt{\gamma} e^{4\alpha} v^{,i} v_{,i}\right)\right].$$

Background:
$\alpha = \ln a$, φ
v: perturbation

v is the unique combination of the canonical variables which has null Poisson brackets with all first order first class constraints, generators of linear gauge transformations

Classical Hamilton-Jacobi treatment

The respective Hamilton-Jacobi equation reads

$$-\frac{1}{2}\left(\frac{\partial S_T}{\partial \alpha}\right)^2 + \frac{1}{2}\left(\frac{\partial S_T}{\partial \varphi}\right)^2 + \frac{1}{2}\int d^3x\left[\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_T}{\delta v}\right)^2 + \sqrt{\gamma} e^{4\alpha} v^{,i} v_{,i}\right], \quad (26)$$

where the classical trajectories can be obtained from a solution S_T of Eq. (26) through

$$\dot{\alpha} = -P_\alpha = -\frac{\partial S_T}{\partial \alpha}, \qquad \dot{\varphi} = P_\varphi = \frac{\partial S_T}{\partial \varphi},$$
$$\dot{v} = \frac{1}{\sqrt{\gamma}}\pi = \frac{1}{\sqrt{\gamma}}\frac{\delta S_T}{\delta v}, \quad (27)$$

$$S_T(\alpha, \varphi, v) = S_0(\alpha, \varphi) + S_2(\alpha, \varphi, v), \tag{28}$$

where it is assumed that $S_2(\alpha, \varphi, v)$ cannot be splitted again into a sum involving a function of the background variables alone (which would just impose a redefinition of S_0). Noting that, in order to be a solution of the Hamilton-Jacobi Eq. (26), S_2 must be at least a second order functional of v (see Ref. [21]), then $S_2 \ll S_0$ as well as their partial derivatives with respect to the background variables. Hence one obtains for the background that

$$\dot{\alpha} \approx -\frac{\partial S_0}{\partial \alpha}, \qquad \dot{\varphi} \approx \frac{\partial S_0}{\partial \varphi}. \tag{29}$$

Inserting the splitting given in Eq. (28) into Eq. (26), one obtains, order by order:

$$-\frac{1}{2}\left(\frac{\partial S_0}{\partial \alpha}\right)^2 + \frac{1}{2}\left(\frac{\partial S_0}{\partial \varphi}\right)^2 = 0, \tag{30}$$

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right) + \left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right) + \frac{1}{2}\int d^3x\left[\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2 + \sqrt{\gamma}e^{4\alpha}v^{i}v_{,i}\right] = 0, \tag{31}$$

$$-\frac{1}{2}\left(\frac{\partial S_2}{\partial \alpha}\right)^2 + \frac{1}{2}\left(\frac{\partial S_2}{\partial \varphi}\right)^2 + O(4) = 0. \tag{32}$$

one obtains the classical trajectories $\alpha(t)$, $\varphi(t)$, the functional $S_2(\alpha, \varphi, v)$ becomes a functional of v and a function of t, $S_2(\alpha, \varphi, v) \to S_2(\alpha(t), \varphi(t), v) = \bar{S}_2(t, v)$. Hence Eq. (31), using Eqs. (29), can be written as

$$\frac{\partial S_2}{\partial t} + \frac{1}{2}\int d^3x\left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2 + \sqrt{\gamma}e^{4\alpha(t)}v^{i}v_{,i}\right) = 0. \tag{33}$$

Equation (33) can now be understood as the Hamilton-Jacobi equation coming from the Hamiltonian

$$H_2 = \frac{1}{2}\int d^3x\left(\frac{\pi^2}{\sqrt{\gamma}} + \sqrt{\gamma}e^{4\alpha(t)}v^{i}v_{,i}\right), \tag{34}$$

which is the generator of time t translations (and not anymore constrained to be null).

If one wants to quantize the perturbations, the corresponding Schrödinger equation should be

Quantization of the perturbation only:

$$i\frac{\partial \chi}{\partial t} = \hat{H}_2\chi, \tag{35}$$

Let us define this unitary transformation by

$$U = e^{iA} e^{-iB}$$

with,

$$A = \frac{1}{2} \int d^3x \sqrt{\gamma} \frac{\dot{a}}{a^3} \hat{v}^2,$$

$$B = \frac{1}{2} \int d^3x (\hat{\pi}\, \hat{v} + \hat{v}\, \hat{\pi}) \log(a).$$

$$\hat{H}_{2U} = \frac{a^2}{2} \int d^3x \left[\frac{\hat{\pi}^2}{\sqrt{\gamma}} + \sqrt{\gamma} \hat{v}^{,i} \hat{v}_{,i} - \frac{a''}{a} \sqrt{\gamma} \hat{v}^2 \right].$$

Heisenberg representation

$$v_k'' + (k^2 - a''/a)\, v_k = 0$$

one obtains the classical trajectories $\alpha(t)$, $\varphi(t)$, the functional $S_2(\alpha, \varphi, v)$ becomes a functional of v and a function of t, $S_2(\alpha, \varphi, v) \to S_2(\alpha(t), \varphi(t), v) = \bar{S}_2(t, v)$. Hence Eq. (31), using Eqs. (29), can be written as

$$\frac{\partial S_2}{\partial t} + \frac{1}{2}\int d^3x \left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2 + \sqrt{\gamma} e^{4\alpha(t)} v^{,i} v_{,i}\right) = 0. \tag{33}$$

Equation (33) can now be understood as the Hamilton-Jacobi equation coming from the Hamiltonian

$$H_2 = \frac{1}{2}\int d^3x \left(\frac{\pi^2}{\sqrt{\gamma}} + \sqrt{\gamma} e^{4\alpha(t)} v^{,i} v_{,i}\right), \tag{34}$$

which is the generator of time t translations (and not anymore constrained to be null).

If one wants to quantize the perturbations, the corresponding Schrödinger equation should be

Quantization of the perturbation only:

$$i\frac{\partial \chi}{\partial t} = \hat{H}_2 \chi, \tag{35}$$

Let us define this unitary transformation by

$$U = e^{iA} e^{-iB}$$

with,

$$A = \frac{1}{2} \int d^3x \sqrt{\gamma} \frac{\dot{a}}{a^3} \hat{v}^2,$$

$$B = \frac{1}{2} \int d^3x (\hat{\pi}\, \hat{v} + \hat{v}\, \hat{\pi}) \log(a).$$

$$\hat{H}_{2U} = \frac{a^2}{2} \int d^3x \left[\frac{\hat{\pi}^2}{\sqrt{\gamma}} + \sqrt{\gamma} \hat{v}^{,i} \hat{v}_{,i} - \frac{a''}{a} \sqrt{\gamma} \hat{v}^2 \right].$$

Heisenberg representation

$$v_k'' + (k^2 - a''/a)\, v_k = 0$$

one obtains the classical trajectories $\alpha(t)$, $\varphi(t)$, the functional $S_2(\alpha, \varphi, v)$ becomes a functional of v and a function of t, $S_2(\alpha, \varphi, v) \to S_2(\alpha(t), \varphi(t), v) = \bar{S}_2(t, v)$. Hence Eq. (31), using Eqs. (29), can be written as

$$\frac{\partial \bar{S}_2}{\partial t} + \frac{1}{2}\int d^3x \left(\frac{1}{\sqrt{\gamma}} \left(\frac{\delta \bar{S}_2}{\delta v} \right)^2 + \sqrt{\gamma} e^{4\alpha(t)} v^{,i} v_{,i} \right) = 0. \tag{33}$$

Equation (33) can now be understood as the Hamilton-Jacobi equation coming from the Hamiltonian

$$H_2 = \frac{1}{2}\int d^3x \left(\frac{\pi^2}{\sqrt{\gamma}} + \sqrt{\gamma} e^{4\alpha(t)} v^{,i} v_{,i} \right), \tag{34}$$

which is the generator of time t translations (and not anymore constrained to be null).

If one wants to quantize the perturbations, the corresponding Schrödinger equation should be

Quantization of the perturbation only:

$$i\frac{\partial \chi}{\partial t} = \hat{H}_2 \chi, \tag{35}$$

$$S_T(\alpha, \varphi, v) = S_0(\alpha, \varphi) + S_2(\alpha, \varphi, v), \qquad (28)$$

where it is assumed that $S_2(\alpha, \varphi, v)$ cannot be splitted again into a sum involving a function of the background variables alone (which would just impose a redefinition of S_0). Noting that, in order to be a solution of the Hamilton-Jacobi Eq. (26), S_2 must be at least a second order functional of v (see Ref. [21]), then $S_2 \ll S_0$ as well as their partial derivatives with respect to the background variables. Hence one obtains for the background that

$$\dot{\alpha} \approx -\frac{\partial S_0}{\partial \alpha}, \qquad \dot{\varphi} \approx \frac{\partial S_0}{\partial \varphi}. \qquad (29)$$

Inserting the splitting given in Eq. (28) into Eq. (26), one obtains, order by order:

$$-\frac{1}{2}\left(\frac{\partial S_0}{\partial \alpha}\right)^2 + \frac{1}{2}\left(\frac{\partial S_0}{\partial \varphi}\right)^2 = 0, \qquad (30)$$

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right) + \left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right) + \frac{1}{2}\int d^3x \left[\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2 + \sqrt{\gamma} e^{4\alpha} v^{i} v_{,i}\right] = 0, \qquad (31)$$

$$-\frac{1}{2}\left(\frac{\partial S_2}{\partial \alpha}\right)^2 + \frac{1}{2}\left(\frac{\partial S_2}{\partial \varphi}\right)^2 + O(4) = 0. \qquad (32)$$

one obtains the classical trajectories $\alpha(t)$, $\varphi(t)$, the functional $S_2(\alpha, \varphi, v)$ becomes a functional of v and a function of t, $S_2(\alpha, \varphi, v) \rightarrow S_2(\alpha(t), \varphi(t), v) = \bar{S}_2(t, v)$. Hence Eq. (31), using Eqs. (29), can be written as

$$\frac{\partial S_2}{\partial t} + \frac{1}{2}\int d^3x\left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2 + \sqrt{\gamma}e^{4\alpha(t)}v^{,i}v_{,i}\right) = 0. \tag{33}$$

Equation (33) can now be understood as the Hamilton-Jacobi equation coming from the Hamiltonian

$$H_2 = \frac{1}{2}\int d^3x\left(\frac{\pi^2}{\sqrt{\gamma}} + \sqrt{\gamma}e^{4\alpha(t)}v^{,i}v_{,i}\right), \tag{34}$$

which is the generator of time t translations (and not anymore constrained to be null).

If one wants to quantize the perturbations, the corresponding Schrödinger equation should be

Quantization of the perturbation only:

$$i\frac{\partial \chi}{\partial t} = \hat{H}_2\chi, \tag{35}$$

Let us define this unitary transformation by

$$U = e^{iA} e^{-iB}$$

with,

$$A = \frac{1}{2} \int d^3x \sqrt{\gamma} \frac{\dot{a}}{a^3} \hat{v}^2,$$

$$B = \frac{1}{2} \int d^3x (\hat{\pi}\,\hat{v} + \hat{v}\,\hat{\pi}) \log(a).$$

$$\hat{H}_{2U} = \frac{a^2}{2} \int d^3x \left[\frac{\hat{\pi}^2}{\sqrt{\gamma}} + \sqrt{\gamma}\hat{v}^{,i}\hat{v}_{,i} - \frac{a''}{a}\sqrt{\gamma}\hat{v}^2 \right].$$

Heisenberg representation

$$v_k'' + (k^2 - a''/a)\, v_k = 0$$

Quantization of perturbation AND background

$$(\hat{H}_0^{(0)} + \hat{H}_2)\Psi = 0,$$

where

$$\hat{H}_0^{(0)} = -\frac{\hat{P}_\alpha^2}{2} + \frac{\hat{P}_\varphi^2}{2},$$

$$\hat{H}_2 = \frac{1}{2}\int d^3x \left(\frac{\hat{\pi}^2}{\sqrt{\gamma}} + \sqrt{\gamma} e^{4\hat{\alpha}} \hat{v}^{,i} \hat{v}_{,i}\right).$$

Hamilton-Jacobi like equation

1) Background

$$-\frac{1}{2}\left(\frac{\partial S_0}{\partial \alpha}\right)^2 + \frac{1}{2}\left(\frac{\partial S_0}{\partial \varphi}\right)^2 + \frac{1}{2R_0}\left(\frac{\partial^2 R_0}{\partial \alpha^2} - \frac{\partial^2 R_0}{\partial \varphi^2}\right) \approx 0.$$

2) Perturbation

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right) + \left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right) + \frac{1}{2}\int \mathrm{d}^3x \left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2 + \sqrt{\gamma}e^{4\alpha}v^{,i}v_{,i}\right)$$

$$+ \frac{1}{2R_2}\left(\frac{\partial^2 R_2}{\partial \alpha^2} - \frac{\partial^2 R_2}{\partial \varphi^2}\right) - \frac{1}{2}\int \frac{\mathrm{d}^3x}{\sqrt{\gamma}}\frac{1}{R_2}\frac{\delta^2 R_2}{\delta v^2} = 0\,.$$

From the background equations one can obtain the Bohmian trajectories $\alpha(t)$ and $\varphi(t)$ ---- insert into the perturbation equation

Assume wave function which can yield semi-classical gravity.

Quantization of perturbation AND background

$$(\hat{H}_0^{(0)} + \hat{H}_2)\Psi = 0,$$

where

$$\hat{H}_0^{(0)} = -\frac{\hat{P}_\alpha^2}{2} + \frac{\hat{P}_\varphi^2}{2},$$

$$\hat{H}_2 = \frac{1}{2}\int d^3x \left(\frac{\hat{\pi}^2}{\sqrt{\gamma}} + \sqrt{\gamma}e^{4\hat{\alpha}}\hat{v}^{,i}\hat{v}_{,i}\right).$$

Hamilton-Jacobi like equation

1) Background

$$-\frac{1}{2}\left(\frac{\partial S_0}{\partial \alpha}\right)^2+\frac{1}{2}\left(\frac{\partial S_0}{\partial \varphi}\right)^2+\frac{1}{2R_0}\left(\frac{\partial^2 R_0}{\partial \alpha^2}-\frac{\partial^2 R_0}{\partial \varphi^2}\right)\approx 0.$$

2) Perturbation

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right)+\left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right)+\ \frac{1}{2}\int \mathrm{d}^3x\left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2+\sqrt{\gamma}e^{4\alpha}v^{,i}v_{,i}\right)$$

$$+\ \frac{1}{2R_2}\left(\frac{\partial^2 R_2}{\partial \alpha^2}-\frac{\partial^2 R_2}{\partial \varphi^2}\right)-\frac{1}{2}\int\frac{\mathrm{d}^3x}{\sqrt{\gamma}}\frac{1}{R_2}\frac{\delta^2 R_2}{\delta v^2}=0\,.$$

From the background equations one can obtain the Bohmian trajectories α(t) and φ(t) ---- insert into the perturbation equation

Assume wave function which can yield semi-classical gravity.

one obtains the classical trajectories $\alpha(t)$, $\varphi(t)$, the functional $S_2(\alpha, \varphi, v)$ becomes a functional of v and a function of t, $S_2(\alpha, \varphi, v) \rightarrow S_2(\alpha(t), \varphi(t), v) = \bar{S}_2(t, v)$. Hence Eq. (31), using Eqs. (29), can be written as

$$\frac{\partial S_2}{\partial t} + \frac{1}{2}\int d^3x \left(\frac{1}{\sqrt{\gamma}} \left(\frac{\delta S_2}{\delta v} \right)^2 + \sqrt{\gamma} e^{4\alpha(t)} v^{,i} v_{,i} \right) = 0. \tag{33}$$

Equation (33) can now be understood as the Hamilton-Jacobi equation coming from the Hamiltonian

$$H_2 = \frac{1}{2}\int d^3x \left(\frac{\pi^2}{\sqrt{\gamma}} + \sqrt{\gamma} e^{4\alpha(t)} v^{,i} v_{,i} \right), \tag{34}$$

which is the generator of time t translations (and not anymore constrained to be null).

If one wants to quantize the perturbations, the corresponding Schrödinger equation should be

Quantization of the perturbation only:

$$i\frac{\partial \chi}{\partial t} = \hat{H}_2 \chi, \tag{35}$$

$$S_T(\alpha, \varphi, v) = S_0(\alpha, \varphi) + S_2(\alpha, \varphi, v), \tag{28}$$

where it is assumed that $S_2(\alpha, \varphi, v)$ cannot be splitted again into a sum involving a function of the background variables alone (which would just impose a redefinition of S_0). Noting that, in order to be a solution of the Hamilton-Jacobi Eq. (26), S_2 must be at least a second order functional of v (see Ref. [21]), then $S_2 \ll S_0$ as well as their partial derivatives with respect to the background variables. Hence one obtains for the background that

$$\dot{\alpha} \approx -\frac{\partial S_0}{\partial \alpha}, \qquad \dot{\varphi} \approx \frac{\partial S_0}{\partial \varphi}. \tag{29}$$

Inserting the splitting given in Eq. (28) into Eq. (26), one obtains, order by order:

$$-\frac{1}{2}\left(\frac{\partial S_0}{\partial \alpha}\right)^2 + \frac{1}{2}\left(\frac{\partial S_0}{\partial \varphi}\right)^2 = 0, \tag{30}$$

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right) + \left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right) + \frac{1}{2}\int d^3x\left[\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2 + \sqrt{\gamma}e^{4\alpha}v^{i}v_{,i}\right] = 0, \tag{31}$$

$$-\frac{1}{2}\left(\frac{\partial S_2}{\partial \alpha}\right)^2 + \frac{1}{2}\left(\frac{\partial S_2}{\partial \varphi}\right)^2 + O(4) = 0. \tag{32}$$

Quantization of perturbation AND background

$$(\hat{H}_0^{(0)} + \hat{H}_2)\Psi = 0,$$

where

$$\hat{H}_0^{(0)} = -\frac{\hat{P}_\alpha^2}{2} + \frac{\hat{P}_\varphi^2}{2},$$

$$\hat{H}_2 = \frac{1}{2}\int d^3x \left(\frac{\hat{\pi}^2}{\sqrt{\gamma}} + \sqrt{\gamma} e^{4\hat{\alpha}} \hat{v}^{,i} \hat{v}_{,i}\right).$$

Hamilton-Jacobi like equation

1) Background

$$-\frac{1}{2}\left(\frac{\partial S_0}{\partial \alpha}\right)^2+\frac{1}{2}\left(\frac{\partial S_0}{\partial \varphi}\right)^2+\frac{1}{2R_0}\left(\frac{\partial^2 R_0}{\partial \alpha^2}-\frac{\partial^2 R_0}{\partial \varphi^2}\right)\approx 0.$$

2) Perturbation

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right)+\left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right)+\frac{1}{2}\int \mathrm{d}^3x\left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2+\sqrt{\gamma}e^{4\alpha}v^{,i}v_{,i}\right)$$

$$+\frac{1}{2R_2}\left(\frac{\partial^2 R_2}{\partial \alpha^2}-\frac{\partial^2 R_2}{\partial \varphi^2}\right)-\frac{1}{2}\int\frac{\mathrm{d}^3x}{\sqrt{\gamma}}\frac{1}{R_2}\frac{\delta^2 R_2}{\delta v^2}=0\,.$$

From the background equations one can obtain the Bohmian trajectories $\alpha(t)$ and $\varphi(t)$ ---- insert into the perturbation equation

Assume wave function which can yield semi-classical gravity.

one obtains the classical trajectories $\alpha(t)$, $\varphi(t)$, the functional $S_2(\alpha, \varphi, v)$ becomes a functional of v and a function of t, $S_2(\alpha, \varphi, v) \to S_2(\alpha(t), \varphi(t), v) = \bar{S}_2(t, v)$. Hence Eq. (31), using Eqs. (29), can be written as

$$\frac{\partial S_2}{\partial t} + \frac{1}{2}\int d^3x\left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2 + \sqrt{\gamma}e^{4\alpha(t)}v^{,i}v_{,i}\right) = 0. \tag{33}$$

Equation (33) can now be understood as the Hamilton-Jacobi equation coming from the Hamiltonian

$$H_2 = \frac{1}{2}\int d^3x\left(\frac{\pi^2}{\sqrt{\gamma}} + \sqrt{\gamma}e^{4\alpha(t)}v^{,i}v_{,i}\right), \tag{34}$$

which is the generator of time t translations (and not anymore constrained to be null).

If one wants to quantize the perturbations, the corresponding Schrödinger equation should be

Quantization of the perturbation only:

$$i\frac{\partial \chi}{\partial t} = \hat{H}_2\chi, \tag{35}$$

$$S_T(\alpha,\varphi,v) = S_0(\alpha,\varphi) + S_2(\alpha,\varphi,v), \qquad (28)$$

where it is assumed that $S_2(\alpha,\varphi,v)$ cannot be splitted again into a sum involving a function of the background variables alone (which would just impose a redefinition of S_0). Noting that, in order to be a solution of the Hamilton-Jacobi Eq. (26), S_2 must be at least a second order functional of v (see Ref. [21]), then $S_2 \ll S_0$ as well as their partial derivatives with respect to the background variables. Hence one obtains for the background that

$$\dot{\alpha} \approx -\frac{\partial S_0}{\partial \alpha}, \qquad \dot{\varphi} \approx \frac{\partial S_0}{\partial \varphi}. \qquad (29)$$

Inserting the splitting given in Eq. (28) into Eq. (26), one obtains, order by order:

$$-\frac{1}{2}\left(\frac{\partial S_0}{\partial \alpha}\right)^2 + \frac{1}{2}\left(\frac{\partial S_0}{\partial \varphi}\right)^2 = 0, \qquad (30)$$

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right) + \left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right) + \frac{1}{2}\int d^3x\left[\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2 + \sqrt{\gamma}e^{4\alpha}v^{i}v_{,i}\right] = 0, \qquad (31)$$

$$-\frac{1}{2}\left(\frac{\partial S_2}{\partial \alpha}\right)^2 + \frac{1}{2}\left(\frac{\partial S_2}{\partial \varphi}\right)^2 + O(4) = 0. \qquad (32)$$

Let us define this unitary transformation by

$$U = e^{iA} e^{-iB}$$

with,

$$A = \frac{1}{2} \int d^3x \sqrt{\gamma} \frac{\dot{a}}{a^3} \hat{v}^2,$$

$$B = \frac{1}{2} \int d^3x (\hat{\pi} \hat{v} + \hat{v} \hat{\pi}) \log(a).$$

$$\hat{H}_{2U} = \frac{a^2}{2} \int d^3x \left[\frac{\hat{\pi}^2}{\sqrt{\gamma}} + \sqrt{\gamma} \hat{v}^{,i} \hat{v}_{,i} - \frac{a''}{a} \sqrt{\gamma} \hat{v}^2 \right].$$

Heisenberg representation

$$v_k'' + (k^2 - a''/a)\, v_k = 0$$

Hamilton-Jacobi like equation

1) Background

$$-\frac{1}{2}\left(\frac{\partial S_0}{\partial \alpha}\right)^2+\frac{1}{2}\left(\frac{\partial S_0}{\partial \varphi}\right)^2+\frac{1}{2R_0}\left(\frac{\partial^2 R_0}{\partial \alpha^2}-\frac{\partial^2 R_0}{\partial \varphi^2}\right)\approx 0.$$

2) Perturbation

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right)+\left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right)+\ \frac{1}{2}\int d^3x\left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2+\sqrt{\gamma}e^{4\alpha}v^{,i}v_{,i}\right)$$

$$+\ \frac{1}{2R_2}\left(\frac{\partial^2 R_2}{\partial \alpha^2}-\frac{\partial^2 R_2}{\partial \varphi^2}\right)-\frac{1}{2}\int\frac{d^3x}{\sqrt{\gamma}}\frac{1}{R_2}\frac{\delta^2 R_2}{\delta v^2}=0\,.$$

From the background equations one can obtain the Bohmian trajectories α(t) and φ(t) ---- insert into the perturbation equation

Assume wave function which can yield semi-classical gravity.

$$\frac{\partial \bar{R}_2^2}{\partial t} + \int \frac{d^3x}{\sqrt{\gamma}} \frac{\delta}{\delta v}\left(\bar{R}_2^2 \frac{\delta \bar{S}_2}{\delta v} d^3x\right) = 0, \qquad (47)$$

$$\frac{\partial \bar{S}_2}{\partial t} + \frac{1}{2}\int d^3x\left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta \bar{S}_2}{\delta v}\right)^2 + \sqrt{\gamma}e^{4\alpha(t)} v^{,i} v_{,i}\right) - \frac{1}{2}\int \frac{d^3x}{\bar{R}_2\sqrt{\gamma}} \frac{\delta^2 \bar{R}_2}{\delta v^2} = 0, \qquad (48)$$

where $\bar{R}_2(t, v) \equiv \exp(\bar{A}_2(t, v))$. In order to obtain these equations we used that

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right) + \left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right) = \frac{\partial \bar{S}_2}{\partial t}, \qquad (49)$$

and the same for R_2 and $\bar{R}_2$.

These two equations can be grouped into a single Schrödinger equation

$$i\frac{\partial \bar{\chi}}{\partial t} = \hat{H}_2 \bar{\chi}, \qquad (50)$$

Let us define this unitary transformation by

$$U = e^{iA}e^{-iB}$$

with,

$$A = \frac{1}{2}\int d^3x\sqrt{\gamma}\frac{\dot{a}}{a^3}\hat{v}^2,$$

$$B = \frac{1}{2}\int d^3x(\hat{\pi}\,\hat{v} + \hat{v}\,\hat{\pi})\log(a).$$

$$\hat{H}_{2U} = \frac{a^2}{2}\int d^3x\left[\frac{\hat{\pi}^2}{\sqrt{\gamma}} + \sqrt{\gamma}\hat{v}^{,i}\hat{v}_{,i} - \frac{a''}{a}\sqrt{\gamma}\hat{v}^2\right].$$

Heisenberg representation

$$v_k'' + (k^2 - a''/a)\, v_k = 0$$

Has the same form as in the case where the background is classical but now a(t) is a quantum trajectory.

ONLY POSSIBLE IN THE BOHM INTERPRETATION!

F. T. Falciano and NPN, Phys. Rev. D 79, 023507 (2009).

$$\frac{\partial \bar{R}_2^2}{\partial t} + \int \frac{d^3x}{\sqrt{\gamma}} \frac{\delta}{\delta v}\left(\bar{R}_2^2 \frac{\delta \bar{S}_2}{\delta v} d^3x\right) = 0, \tag{47}$$

$$\frac{\partial \bar{S}_2}{\partial t} + \frac{1}{2} \int d^3x \left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta \bar{S}_2}{\delta v}\right)^2 + \sqrt{\gamma} e^{4\alpha(t)} v^{,i} v_{,i}\right) - \frac{1}{2} \int \frac{d^3x}{\bar{R}_2 \sqrt{\gamma}} \frac{\delta^2 \bar{R}_2}{\delta v^2} = 0, \tag{48}$$

where $\bar{R}_2(t, v) \equiv \exp(\bar{A}_2(t, v))$. In order to obtain these equations we used that

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right) + \left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right) = \frac{\partial \bar{S}_2}{\partial t}, \tag{49}$$

and the same for R_2 and $\bar{R}_2$.

These two equations can be grouped into a single Schrödinger equation

$$i \frac{\partial \bar{\chi}}{\partial t} = \hat{H}_2 \bar{\chi}, \tag{50}$$

Let us define this unitary transformation by

$$U = e^{iA}e^{-iB}$$

with,

$$A = \frac{1}{2}\int d^3x\sqrt{\gamma}\frac{\dot{a}}{a^3}\hat{v}^2,$$

$$B = \frac{1}{2}\int d^3x(\hat{\pi}\,\hat{v} + \hat{v}\,\hat{\pi})\log(a).$$

$$\hat{H}_{2U} = \frac{a^2}{2}\int d^3x\left[\frac{\hat{\pi}^2}{\sqrt{\gamma}} + \sqrt{\gamma}\hat{v}^{,i}\hat{v}_{,i} - \frac{a''}{a}\sqrt{\gamma}\hat{v}^2\right].$$

Heisenberg representation

$$v_k'' + (k^2 - a''/a)\, v_k = 0$$

Has the same form as in the case where the background is classical but now a(t) is a quantum trajectory.

ONLY POSSIBLE IN THE BOHM INTERPRETATION!

F. T. Falciano and NPN, Phys. Rev. D 79, 023507 (2009).

$$\frac{\partial \bar{R}_2^2}{\partial t} + \int \frac{d^3x}{\sqrt{\gamma}} \frac{\delta}{\delta v}\left(\bar{R}_2^2 \frac{\delta \bar{S}_2}{\delta v} d^3x\right) = 0, \tag{47}$$

$$\frac{\partial \bar{S}_2}{\partial t} + \frac{1}{2}\int d^3x\left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta \bar{S}_2}{\delta v}\right)^2 + \sqrt{\gamma}e^{4\alpha(t)}v^{,i}v_{,i}\right) - \frac{1}{2}\int \frac{d^3x}{\bar{R}_2\sqrt{\gamma}} \frac{\delta^2 \bar{R}_2}{\delta v^2} = 0, \tag{48}$$

where $\bar{R}_2(t, v) \equiv \exp(\bar{A}_2(t, v))$. In order to obtain these equations we used that

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right) + \left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right) = \frac{\partial \bar{S}_2}{\partial t}, \tag{49}$$

and the same for R_2 and $\bar{R}_2$.

These two equations can be grouped into a single Schrödinger equation

$$i\frac{\partial \bar{\chi}}{\partial t} = \hat{H}_2\bar{\chi}, \tag{50}$$

Hamilton-Jacobi like equation

1) Background

$$-\frac{1}{2}\left(\frac{\partial S_0}{\partial \alpha}\right)^2+\frac{1}{2}\left(\frac{\partial S_0}{\partial \varphi}\right)^2+\frac{1}{2R_0}\left(\frac{\partial^2 R_0}{\partial \alpha^2}-\frac{\partial^2 R_0}{\partial \varphi^2}\right)\approx 0.$$

2) Perturbation

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right)+\left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right)+\ \frac{1}{2}\int \mathrm{d}^3x\left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2+\sqrt{\gamma}e^{4\alpha}v^{,i}v_{,i}\right)$$

$$+\ \frac{1}{2R_2}\left(\frac{\partial^2 R_2}{\partial \alpha^2}-\frac{\partial^2 R_2}{\partial \varphi^2}\right)-\frac{1}{2}\int\frac{\mathrm{d}^3x}{\sqrt{\gamma}}\frac{1}{R_2}\frac{\delta^2 R_2}{\delta v^2}=0\,.$$

From the background equations one can obtain the Bohmian trajectories $\alpha(t)$ and $\varphi(t)$ ---- insert into the perturbation equation

Assume wave function which can yield semi-classical gravity.

$$\frac{\partial \bar{R}_2^2}{\partial t} + \int \frac{d^3x}{\sqrt{\gamma}} \frac{\delta}{\delta v}\left(\bar{R}_2^2 \frac{\delta \bar{S}_2}{\delta v} d^3x\right) = 0, \tag{47}$$

$$\frac{\partial \bar{S}_2}{\partial t} + \frac{1}{2}\int d^3x \left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta \bar{S}_2}{\delta v}\right)^2 + \sqrt{\gamma} e^{4\alpha(t)} v^{,i} v_{,i}\right) - \frac{1}{2}\int \frac{d^3x}{\bar{R}_2\sqrt{\gamma}} \frac{\delta^2 \bar{R}_2}{\delta v^2} = 0, \tag{48}$$

where $\bar{R}_2(t, v) \equiv \exp(\bar{A}_2(t, v))$. In order to obtain these equations we used that

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right) + \left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right) = \frac{\partial \bar{S}_2}{\partial t}, \tag{49}$$

and the same for R_2 and $\bar{R}_2$.

These two equations can be grouped into a single Schrödinger equation

$$i\frac{\partial \bar{\chi}}{\partial t} = \hat{H}_2 \bar{\chi}, \tag{50}$$

Let us define this unitary transformation by

$$U = e^{iA}e^{-iB}$$

with,

$$A = \frac{1}{2}\int d^3x \sqrt{\gamma}\frac{\dot{a}}{a^3}\hat{v}^2,$$

$$B = \frac{1}{2}\int d^3x (\hat{\pi}\,\hat{v} + \hat{v}\,\hat{\pi}) \log(a).$$

$$\hat{H}_{2U} = \frac{a^2}{2}\int d^3x \left[\frac{\hat{\pi}^2}{\sqrt{\gamma}} + \sqrt{\gamma}\hat{v}^{,i}\hat{v}_{,i} - \frac{a''}{a}\sqrt{\gamma}\hat{v}^2\right].$$

Heisenberg representation

$$v_k'' + (k^2 - a''/a)\, v_k = 0$$

Has the same form as in the case where the background is classical but now a(t) is a quantum trajectory.

ONLY POSSIBLE IN THE BOHM INTERPRETATION!

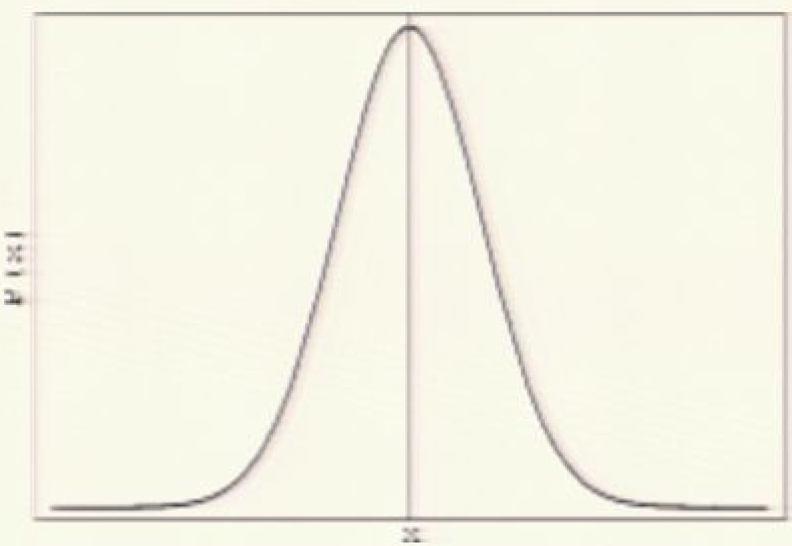
F. T. Falciano and NPN, Phys. Rev. D 79, 023507 (2009).

FOR A PERFECT FLUID: $p = \lambda\,\varepsilon$

$$v''_k + \left(\lambda k^2 - \frac{a''}{a}\right) v_k = 0.$$

$$a(\eta) = a_0\,(b^2\,\eta^2 + 1)^{1/2}$$

Before the bounce, in the contracting phase



After the bounce, In the expanding phase.

Point of matching: $\lambda k^2 = a''/a \Longrightarrow l_{phys} = \lambda^{1/2}\, l_c$

- The two key ingredients:
1) The fluid that dominates when crossing the potential.
2) The mixing of the modes when crossing the bounce.

THE RESULTING SPECTRAL INDICES:

$$n_s = 1 + \frac{12\lambda}{1+3\lambda}$$

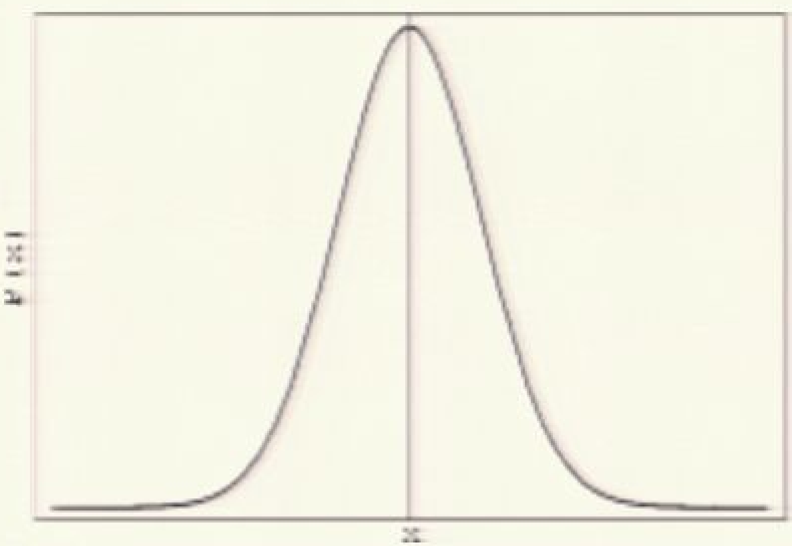
$$n_T = \frac{12\lambda}{1+3\lambda}$$

FOR A PERFECT FLUID: $p = \lambda \varepsilon$

$$v''_k + \left(\lambda k^2 - \frac{a''}{a}\right) v_k = 0.$$

$$a(\eta) = a_0 (b^2 \eta^2 + 1)^{1/2}$$

Before the bounce, in the contracting phase



After the bounce, In the expanding phase.

Point of matching: $\lambda k^2 = a''/a \Longrightarrow l_{phys} = \lambda^{1/2} l_c$

- The two key ingredients:
1) The fluid that dominates when crossing the potential.
2) The mixing of the modes when crossing the bounce.

THE RESULTING SPECTRAL INDICES:

$$n_s = 1 + \frac{12\lambda}{1+3\lambda}$$

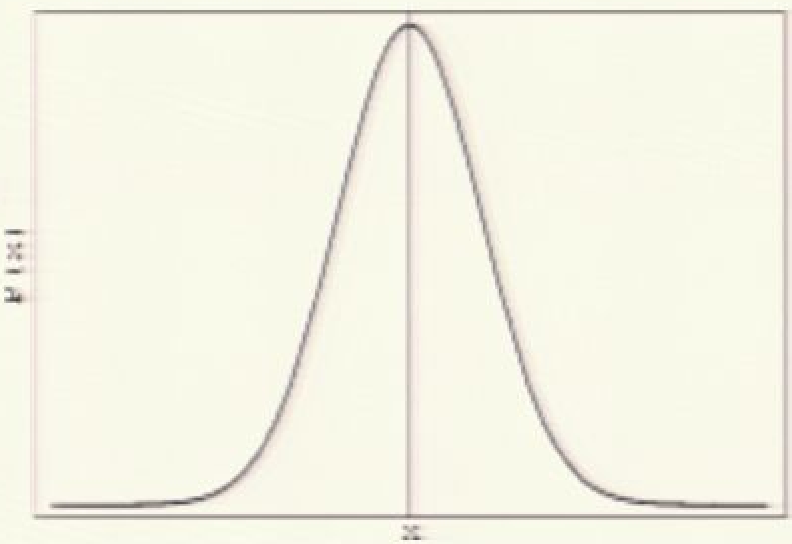
$$n_T = \frac{12\lambda}{1+3\lambda}$$

FOR A PERFECT FLUID: $p = \lambda \varepsilon$

$$v''_k + \left(\lambda k^2 - \frac{a''}{a}\right) v_k = 0.$$

$$a(\eta) = a_0 \, (b^2 \eta^2 + 1)^{1/2}$$

Before the bounce, in the contracting phase



After the bounce, In the expanding phase.

Point of matching: $\lambda k^2 = a''/a \Longrightarrow l_{phys} = \lambda^{1/2} \, l_c$

- The two key ingredients:
1) The fluid that dominates when crossing the potential.
2) The mixing of the modes when crossing the bounce.

THE RESULTING SPECTRAL INDICES:

$$n_s = 1 + \frac{12\lambda}{1+3\lambda}$$

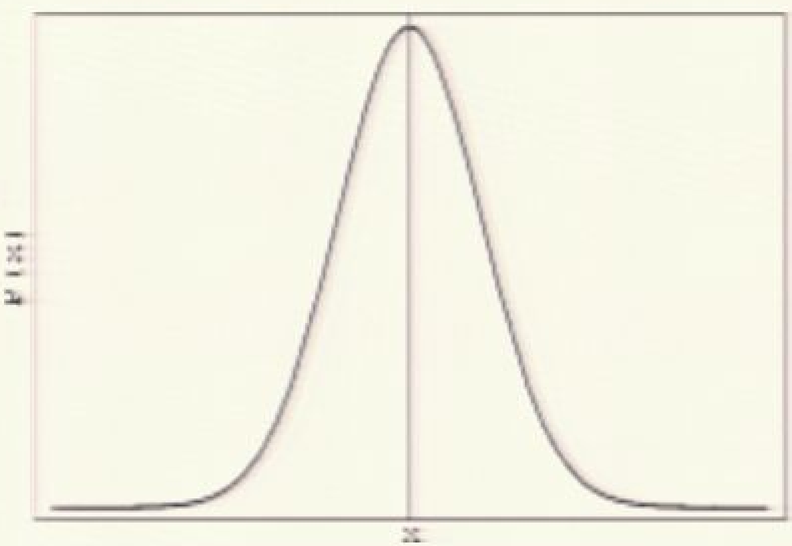
$$n_T = \frac{12\lambda}{1+3\lambda}$$

FOR A PERFECT FLUID: $p = \lambda \varepsilon$

$$v''_k + \left(\lambda k^2 - \frac{a''}{a}\right) v_k = 0.$$

$$a(\eta) = a_0 (b^2 \eta^2 + 1)^{1/2}$$

Before the bounce, in the contracting phase



After the bounce, In the expanding phase.

Point of matching: $\lambda k^2 = a''/a \Longrightarrow l_{phys} = \lambda^{1/2}\, l_c$

Let us define this unitary transformation by

$$U = e^{iA}e^{-iB}$$

with,

$$A = \frac{1}{2}\int d^3x\sqrt{\gamma}\frac{\dot{a}}{a^3}\hat{v}^2,$$

$$B = \frac{1}{2}\int d^3x(\hat{\pi}\,\hat{v} + \hat{v}\,\hat{\pi})\log(a).$$

$$\hat{H}_{2U} = \frac{a^2}{2}\int d^3x\left[\frac{\hat{\pi}^2}{\sqrt{\gamma}} + \sqrt{\gamma}\hat{v}^{,i}\hat{v}_{,i} - \frac{a''}{a}\sqrt{\gamma}\hat{v}^2\right].$$

Heisenberg representation

$$v_k'' + (k^2 - a''/a)\, v_k = 0$$

Has the same form as in the case where the background is classical but now a(t) is a quantum trajectory.

ONLY POSSIBLE IN THE BOHM INTERPRETATION!

F. T. Falciano and NPN, Phys. Rev. D 79, 023507 (2009).

- The two key ingredients:

1) The fluid that dominates when crossing the potential.
2) The mixing of the modes when crossing the bounce.

THE RESULTING SPECTRAL INDICES:

$$n_s = 1 + \frac{12\lambda}{1+3\lambda}$$

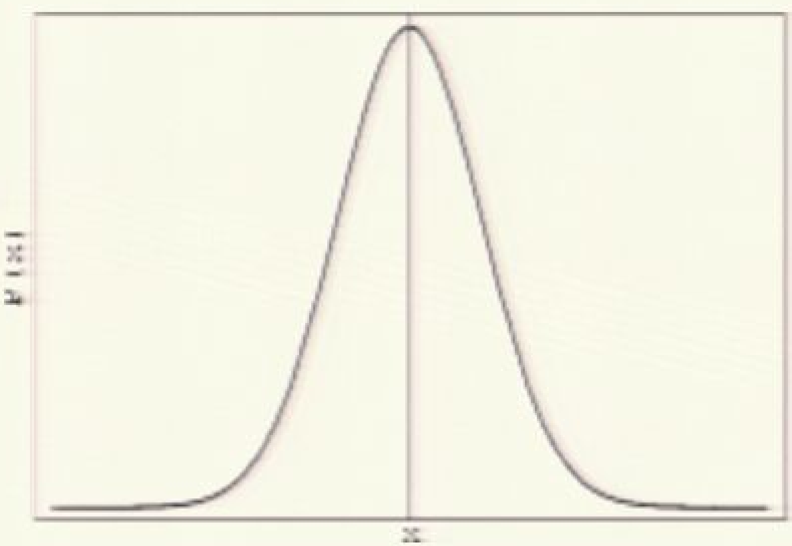
$$n_T = \frac{12\lambda}{1+3\lambda}$$

FOR A PERFECT FLUID: $p = \lambda \varepsilon$

$$v''_k + \left(\lambda k^2 - \frac{a''}{a}\right) v_k = 0.$$

$$a(\eta) = a_0 (b^2 \eta^2 + 1)^{1/2}$$

Before the bounce, in the contracting phase



After the bounce, In the expanding phase.

Point of matching: $\lambda k^2 = a''/a \Longrightarrow l_{phys} = \lambda^{1/2}\, l_c$

- The two key ingredients:
1) The fluid that dominates when crossing the potential.
2) The mixing of the modes when crossing the bounce.

THE RESULTING SPECTRAL INDICES:

$$n_s = 1 + \frac{12\lambda}{1+3\lambda}$$

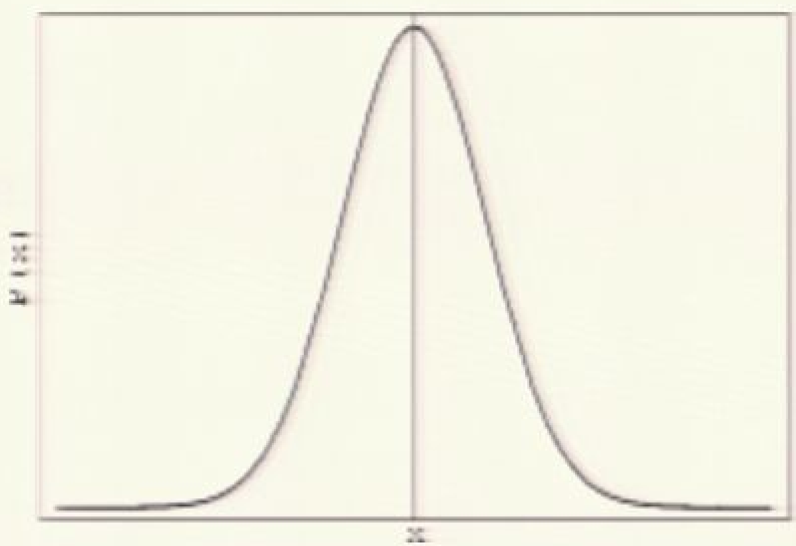
$$n_T = \frac{12\lambda}{1+3\lambda}$$

FOR A PERFECT FLUID: $p = \lambda \varepsilon$

$$v''_k + \left(\lambda k^2 - \frac{a''}{a}\right) v_k = 0.$$

$$a(\eta) = a_0 (b^2 \eta^2 + 1)^{1/2}$$

Before the bounce, in the contracting phase



After the bounce, In the expanding phase.

Point of matching: $\lambda k^2 = a''/a \Longrightarrow l_{phys} = \lambda^{1/2} l_c$

- The two key ingredients:
1) The fluid that dominates when crossing the potential.
2) The mixing of the modes when crossing the bounce.

THE RESULTING SPECTRAL INDICES:

$$n_s = 1 + \frac{12\lambda}{1+3\lambda}$$

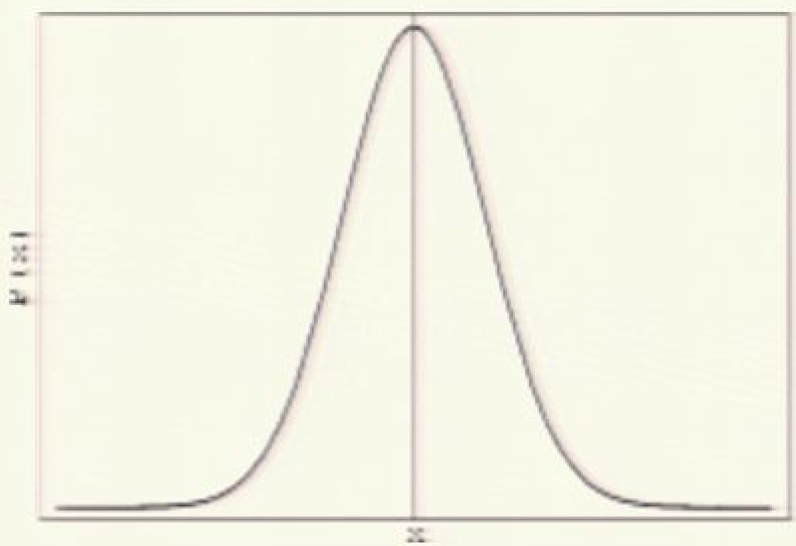
$$n_T = \frac{12\lambda}{1+3\lambda}$$

FOR A PERFECT FLUID: $p = \lambda\,\varepsilon$

$$v''_k + \left(\lambda k^2 - \frac{a''}{a}\right) v_k = 0.$$

$$a(\eta) = a_0\,(b^2\,\eta^2 + 1)^{1/2}$$

Before the bounce, in the contracting phase



After the bounce, In the expanding phase.

Point of matching: $\lambda k^2 = a''/a \Longrightarrow l_{phys} = \lambda^{1/2}\, l_c$

- The two key ingredients:
1) The fluid that dominates when crossing the potential.
2) The mixing of the modes when crossing the bounce.

THE RESULTING SPECTRAL INDICES:

$$n_s = 1 + \frac{12\lambda}{1+3\lambda}$$

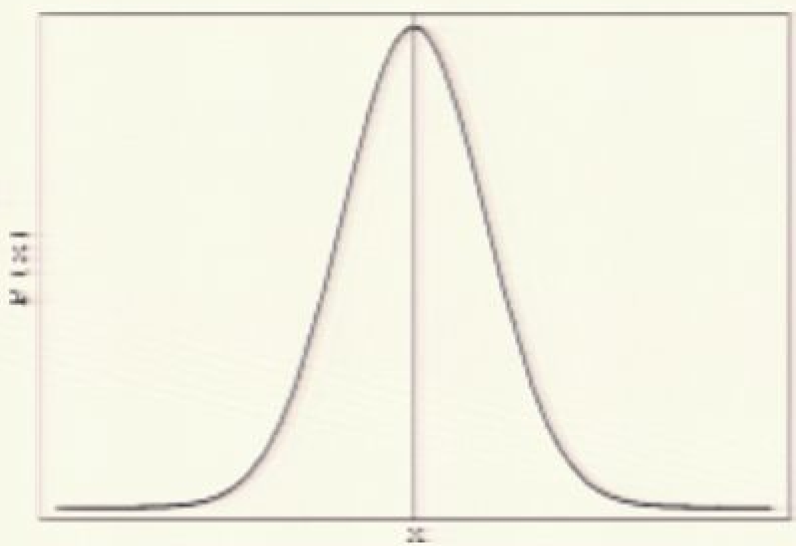
$$n_T = \frac{12\lambda}{1+3\lambda}$$

FOR A PERFECT FLUID: $p = \lambda \varepsilon$

$$v''_k + \left(\lambda k^2 - \frac{a''}{a}\right) v_k = 0.$$

$$a(\eta) = a_0 (b^2 \eta^2 + 1)^{1/2}$$

Before the bounce, in the contracting phase



After the bounce, In the expanding phase.

Point of matching: $\lambda k^2 = a''/a \Longrightarrow l_{phys} = \lambda^{1/2} l_c$

- The two key ingredients:
1) The fluid that dominates when crossing the potential.
2) The mixing of the modes when crossing the bounce.

THE RESULTING SPECTRAL INDICES:

$$n_s = 1 + \frac{12\lambda}{1+3\lambda}$$

$$n_T = \frac{12\lambda}{1+3\lambda}$$

- **Ordinary matter (dark matter?): scale invariant**

It is not necessary to have ordinary matter dominating all along; just when entering the potential.

Another fluid or field may dominate at the bounce (radiation), but the bounce must mix the modes.

FEATURES OF THE MODEL

1) No horizon problem.
2) No singularity.
3) Perturbations of quantum mechanical origin.
4) Enhancement of perturbations at the bounce.
5) Flatness problem: if the contraction phase is much longer then the expansion phase, then the Universe is almost flat because it has not expanded enough!
6) Prediction of n_s not less than 1.
7) One fundamental parameter: the curvature radius L_0 at the bounce, which must have the reasonable value $10^3\, l_{pl}$.
8) Transplanckian problem can be solved.
9) Homogeneity problem may be less severe.

$$|\Omega_T - 1| = -2\frac{\ddot{a}}{\dot{a}^3}$$

- **Ordinary matter (dark matter?): scale invariant**

It is not necessary to have ordinary matter dominating all along; just when entering the potential.

Another fluid or field may dominate at the bounce (radiation), but the bounce must mix the modes.

FEATURES OF THE MODEL

1) No horizon problem.
2) No singularity.
3) Perturbations of quantum mechanical origin.
4) Enhancement of perturbations at the bounce.
5) Flatness problem: if the contraction phase is much longer then the expansion phase, then the Universe is almost flat because it has not expanded enough!
6) Prediction of n_s not less than 1.
7) One fundamental parameter: the curvature radius L_0 at the bounce, which must have the reasonable value $10^3\, l_{pl}$.
8) Transplanckian problem can be solved.
9) Homogeneity problem may be less severe.

$$|\Omega_T - 1| = -2\frac{\ddot{a}}{\dot{a}^3}$$

-- Basic General Relativity and Quantum Mechanics (within the Bohm-de Broglie interpretation) yield a sensible bouncing model which can explain the origin of cosmological perturbations similarly to inflationary models.

-- Complete different perspective concerning initial conditions which solve many of the cosmological puzzles.

-- There are no observational reasons for a beginning of the Universe, so why not exploring the consequences of bouncing models?

-- Bouncing models may be competitive with usual inflation.

-- Notable advances in observational cosmology: one can test primordial cosmological models, eg

Inflationary models with $V(\varphi)=\varphi^p$ with $p > 3.1$ discarded with 95% of confidence level;

String motivated bouncing models: (Phys.Rev. D73 (2006) 123513).

More precise observations of anisotropies (spectral indices, gravitational waves and polarization, superimposed oscillations in the spectrum, ...) may decide these questions:

inflation: $T/S = C\,(n_s - 1)$; bounce: $T/S = C\,(n_s - 1)^{1/2}$

P. Peter, E. Pinho and NPN, JCAP 07, 014 (2005).

P. Peter, E. Pinho and NPN, Phys. Rev. D 73, 0104017 (2006).

P. Peter, E. Pinho and NPN, Phys. Rev. D 75, 023516 (2007).

E. Pinho and NPN, Phys. Rev. D 76, 023506 (2007).

CONCLUSION

Bohm-de Broglie interpretation is very suitable for quantum cosmology:

Allows precise answers.
Allows calculations of potentially observational effects.

What about the other interpretations?

$$\frac{\partial \bar{R}_2^2}{\partial t} + \int \frac{d^3x}{\sqrt{\gamma}} \frac{\delta}{\delta v}\left(\bar{R}_2^2 \frac{\delta \bar{S}_2}{\delta v} d^3x\right) = 0, \tag{47}$$

$$\frac{\partial \bar{S}_2}{\partial t} + \frac{1}{2}\int d^3x\left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta \bar{S}_2}{\delta v}\right)^2 + \sqrt{\gamma} e^{4\alpha(t)} v^{,i} v_{,i}\right) - \frac{1}{2}\int \frac{d^3x}{\bar{R}_2\sqrt{\gamma}} \frac{\delta^2 \bar{R}_2}{\delta v^2} = 0, \tag{48}$$

where $\bar{R}_2(t, v) \equiv \exp(\bar{A}_2(t, v))$. In order to obtain these equations we used that

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right) + \left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right) = \frac{\partial \bar{S}_2}{\partial t}, \tag{49}$$

and the same for R_2 and $\bar{R}_2$.

These two equations can be grouped into a single Schrödinger equation

$$i\frac{\partial \bar{\chi}}{\partial t} = \hat{H}_2 \bar{\chi}, \tag{50}$$

Quantization of perturbation AND background

$$(\hat{H}_0^{(0)} + \hat{H}_2)\Psi = 0,$$

where

$$\hat{H}_0^{(0)} = -\frac{\hat{P}_\alpha^2}{2} + \frac{\hat{P}_\varphi^2}{2},$$

$$\hat{H}_2 = \frac{1}{2}\int d^3x \left(\frac{\hat{\pi}^2}{\sqrt{\gamma}} + \sqrt{\gamma}e^{4\hat{\alpha}}\hat{v}^{,i}\hat{v}_{,i}\right).$$

Hamilton-Jacobi like equation

1) Background

$$-\frac{1}{2}\left(\frac{\partial S_0}{\partial \alpha}\right)^2+\frac{1}{2}\left(\frac{\partial S_0}{\partial \varphi}\right)^2+\frac{1}{2R_0}\left(\frac{\partial^2 R_0}{\partial \alpha^2}-\frac{\partial^2 R_0}{\partial \varphi^2}\right)\approx 0.$$

2) Perturbation

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right)+\left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right)+\ \frac{1}{2}\int d^3x\left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta S_2}{\delta v}\right)^2+\sqrt{\gamma}e^{4\alpha}v^{,i}v_{,i}\right)$$

$$+\ \frac{1}{2R_2}\left(\frac{\partial^2 R_2}{\partial \alpha^2}-\frac{\partial^2 R_2}{\partial \varphi^2}\right)-\frac{1}{2}\int\frac{d^3x}{\sqrt{\gamma}}\frac{1}{R_2}\frac{\delta^2 R_2}{\delta v^2}=0\,.$$

From the background equations one can obtain the Bohmian trajectories α(t) and φ(t) ---- insert into the perturbation equation

Assume wave function which can yield semi-classical gravity.

$$\frac{\partial \bar{R}_2^2}{\partial t} + \int \frac{d^3x}{\sqrt{\gamma}} \frac{\delta}{\delta v}\left(\bar{R}_2^2 \frac{\delta \bar{S}_2}{\delta v} d^3x\right) = 0, \tag{47}$$

$$\frac{\partial \bar{S}_2}{\partial t} + \frac{1}{2}\int d^3x \left(\frac{1}{\sqrt{\gamma}}\left(\frac{\delta \bar{S}_2}{\delta v}\right)^2 + \sqrt{\gamma} e^{4\alpha(t)} v^{,i} v_{,i}\right) - \frac{1}{2}\int \frac{d^3x}{\bar{R}_2\sqrt{\gamma}} \frac{\delta^2 \bar{R}_2}{\delta v^2} = 0, \tag{48}$$

where $\bar{R}_2(t, v) \equiv \exp(\bar{A}_2(t, v))$. In order to obtain these equations we used that

$$-\left(\frac{\partial S_0}{\partial \alpha}\right)\left(\frac{\partial S_2}{\partial \alpha}\right) + \left(\frac{\partial S_0}{\partial \varphi}\right)\left(\frac{\partial S_2}{\partial \varphi}\right) = \frac{\partial \bar{S}_2}{\partial t}, \tag{49}$$

and the same for R_2 and $\bar{R}_2$.

These two equations can be grouped into a single Schrödinger equation

$$i\frac{\partial \bar{\chi}}{\partial t} = \hat{H}_2 \bar{\chi}, \tag{50}$$