Title: Texas-Bavarian Home Cooking: A Quantum Bayesian Reply to Bell's (and Norsen's) la Nouvelle Cuisine

Date: Sep 29, 2009 09:30 AM

URL: http://pirsa.org/09090087

Abstract: TBA

QBist Home Cooking:

A Serving for the Nonlocalists

Christopher Fuchs
$\hat{P}I$

Thoughts w/ Ruediger Schack
Royal Holloway

"QBism" – the quantum Bayesian program of

C. M. Caves

R. Schack

D. M. Appleby

myself

See arXiv.org.

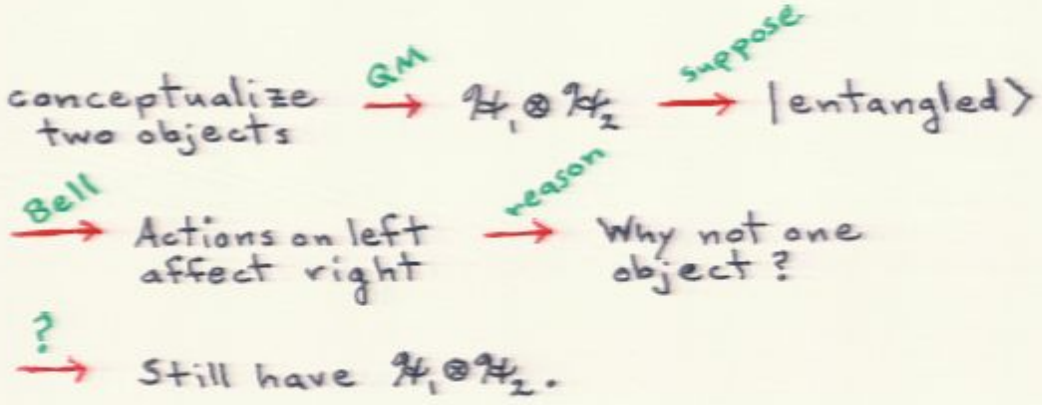
See also:

C. G. Timpson,
"Quantum Bayesianism: A Study"
and
pirsa.org/09080010
09090029

Vague Worry

with failure of "local causality"
i.e., that "the direct causes (and effects) of events are near by ..."
Bell's la nouvelle cuisine

- not that it contradicts special relativity
- but the conceptual difficulty of how to draw lines in nature
- and the nonetheless stubborn resilience of the appearance of lines in QM



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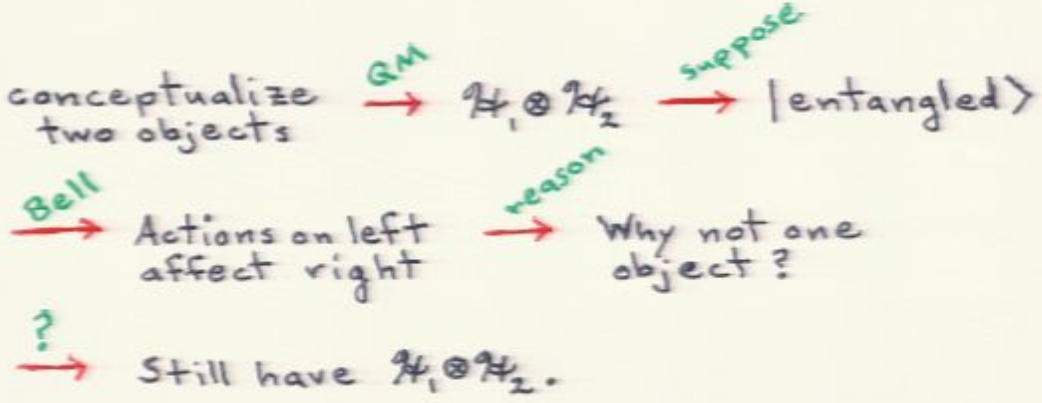


The hypothesis that there is an external world, not dependent on human minds, made of something, is so obviously useful and so strongly confirmed by experience down through the ages that we can say without exaggerating that it is better confirmed than any other empirical hypothesis.

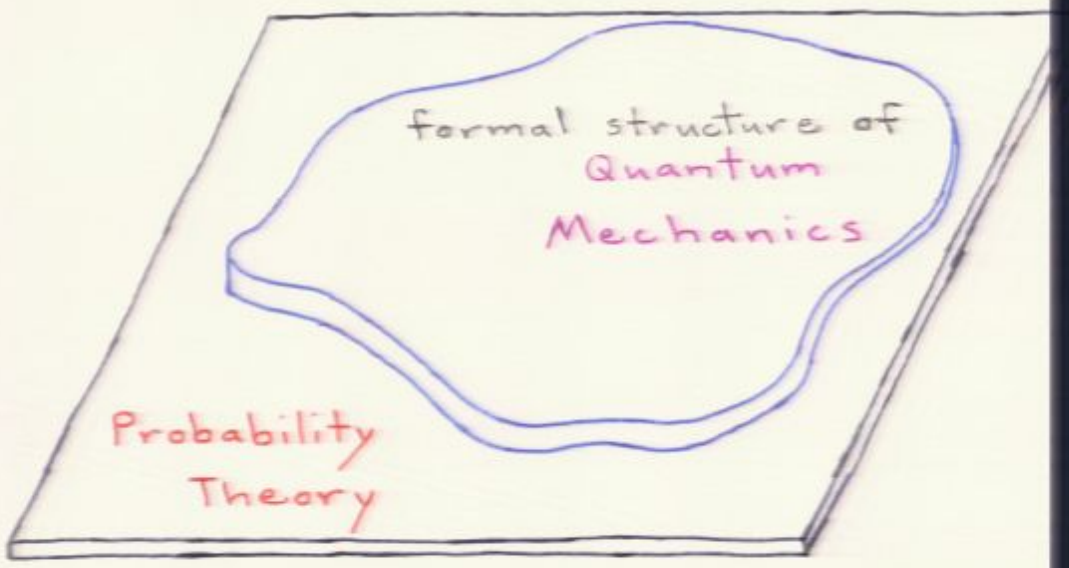
— Martin Gardner

A Single-User Theory

- probability theory
- quantum theory

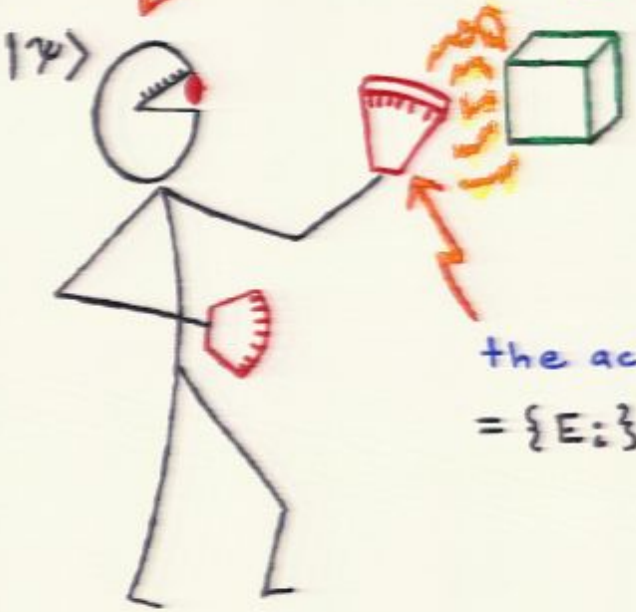
"The Bayesian, subjectivist, or coherent, paradigm is egocentric. It is a tale of one person contemplating the world and not wishing to be stupid (technically incoherent). He realizes that to do this his statements of uncertainties must be probabilistic."

— D. V. Lindley


formal structure of
Quantum
Mechanics
Probability
Theory

the consequence
= an experience, E_k

the catalyst
= quantum system, $\mathcal{H}_d$



the action
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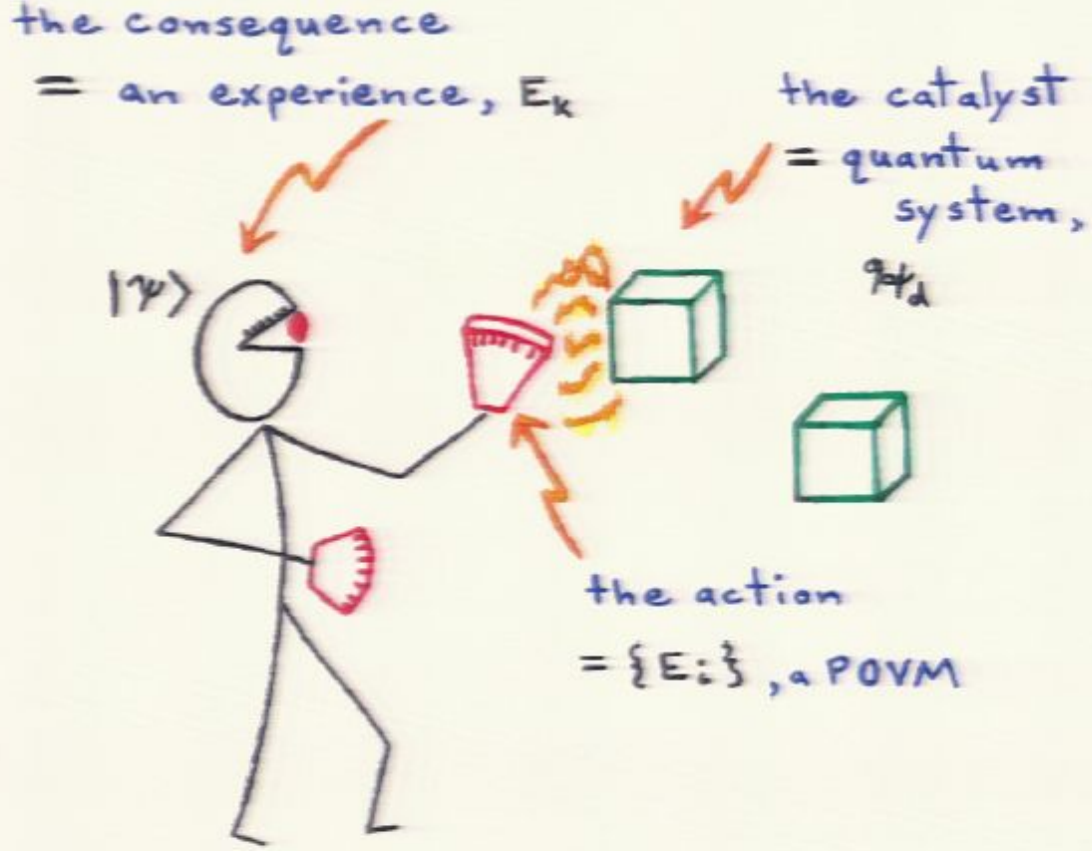
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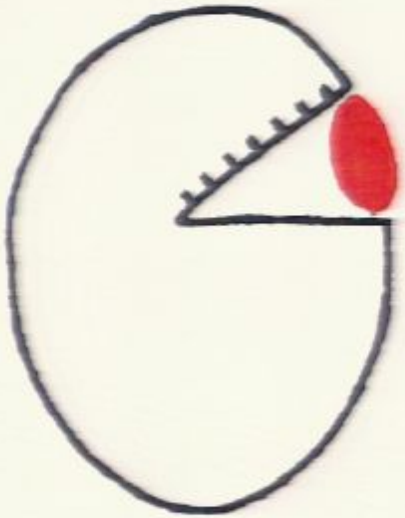

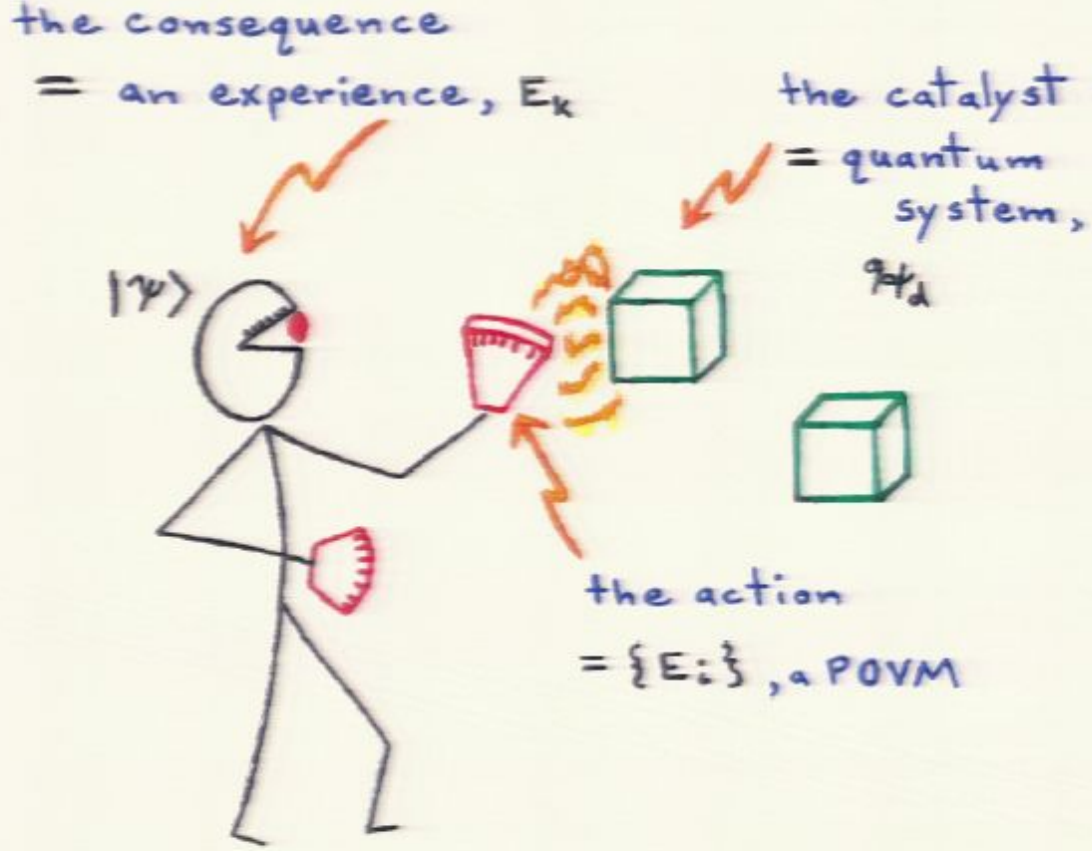
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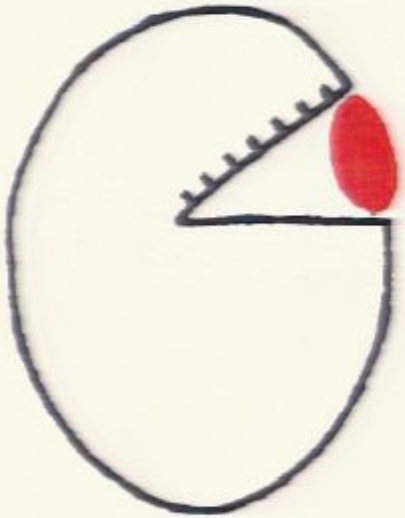

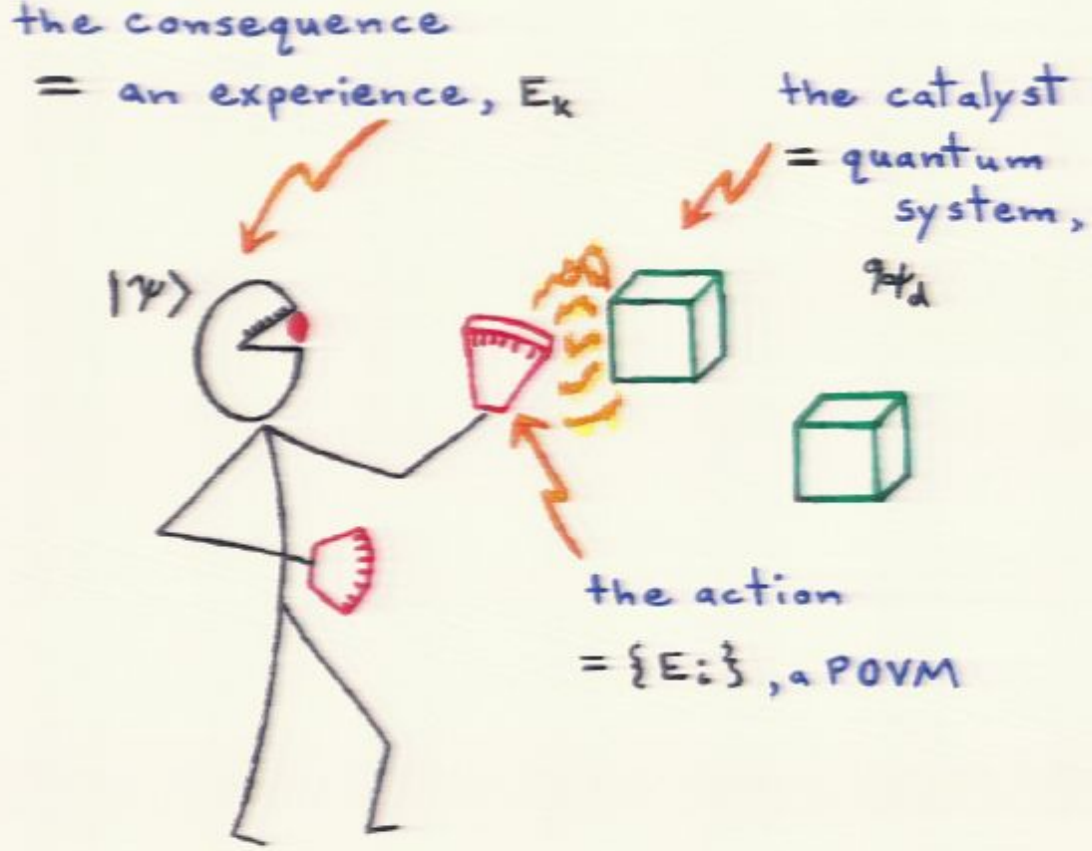
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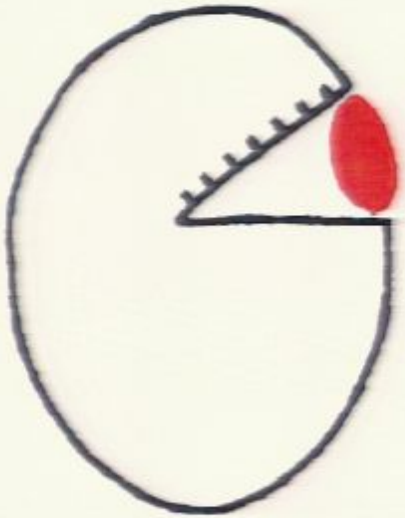

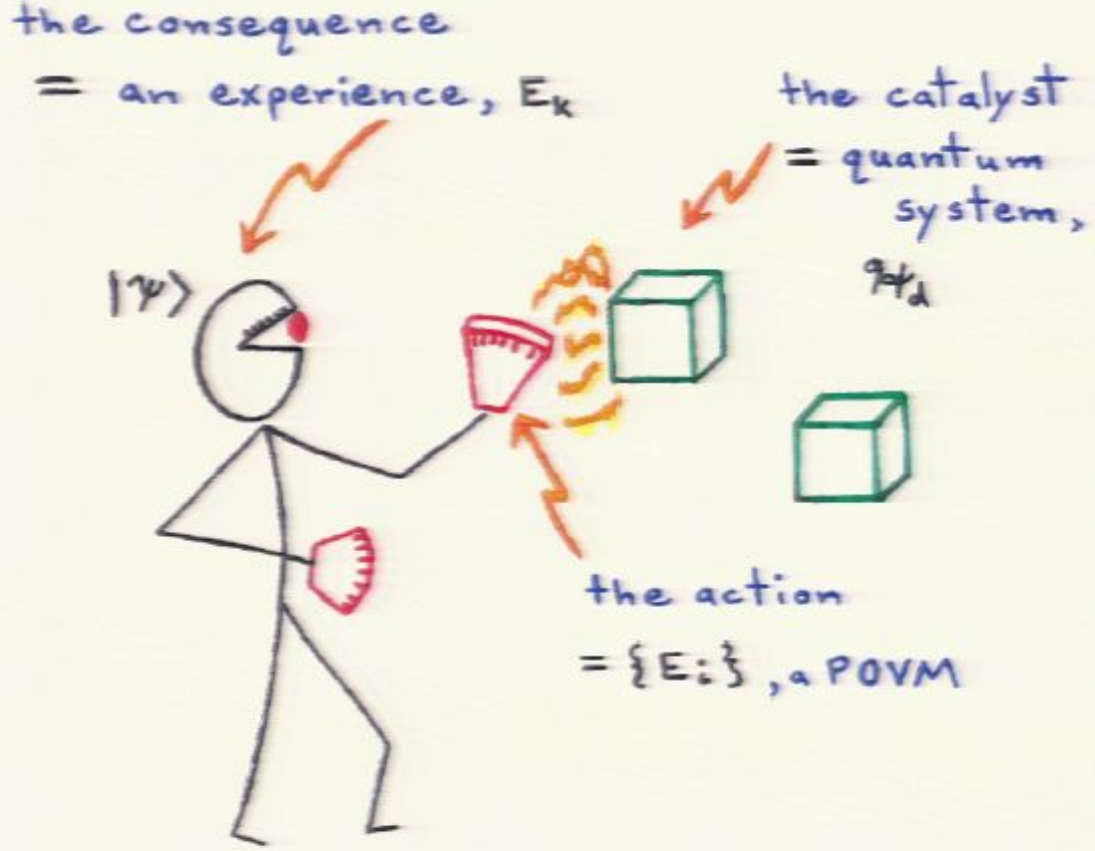
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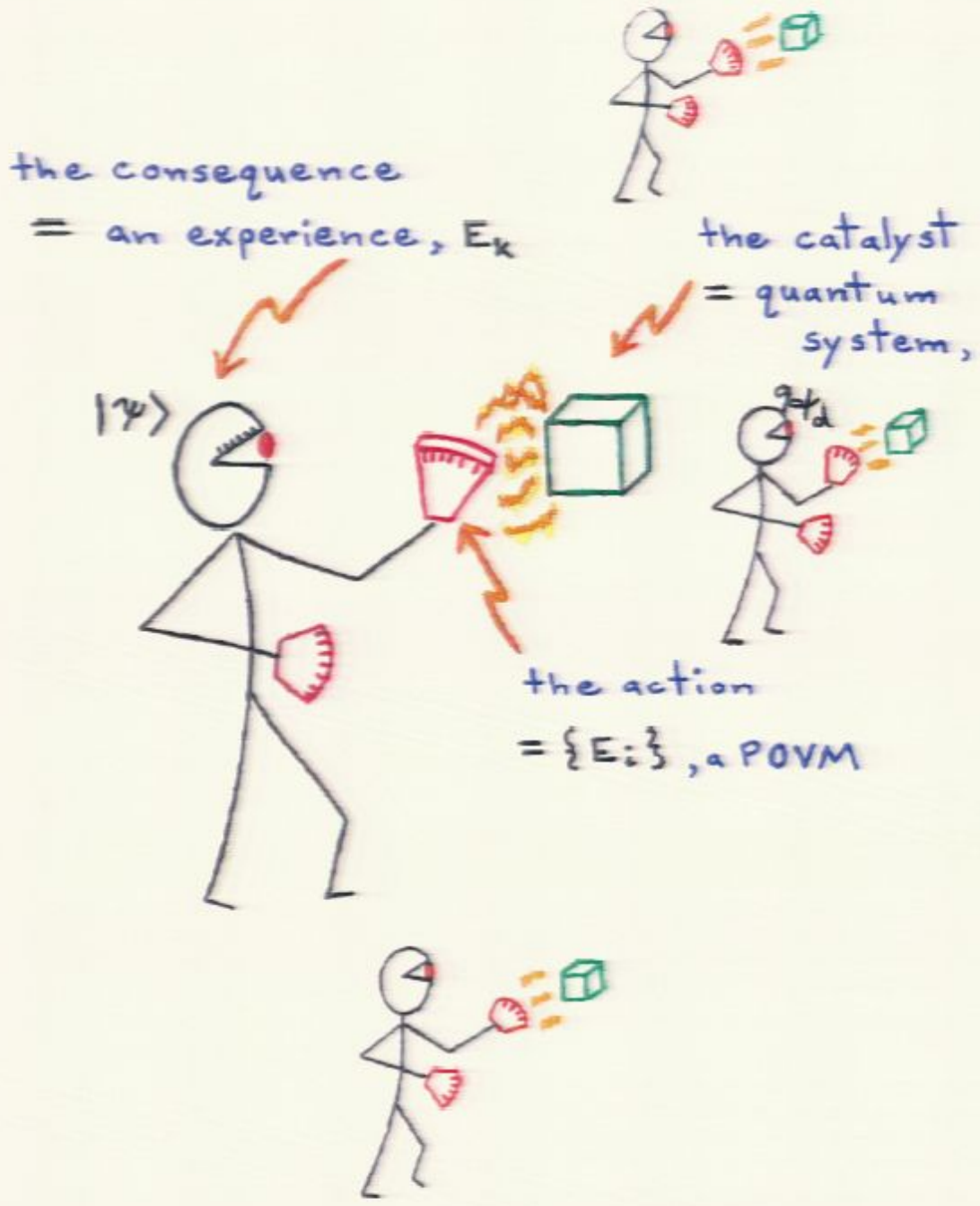
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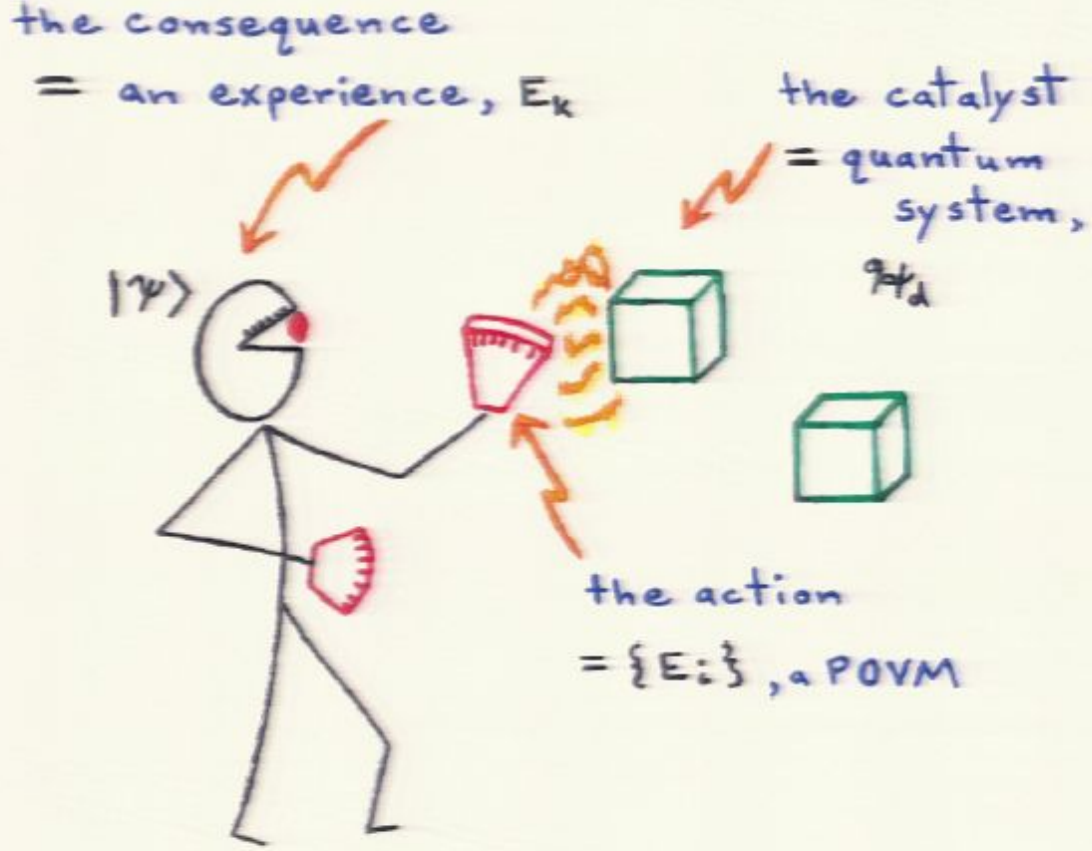
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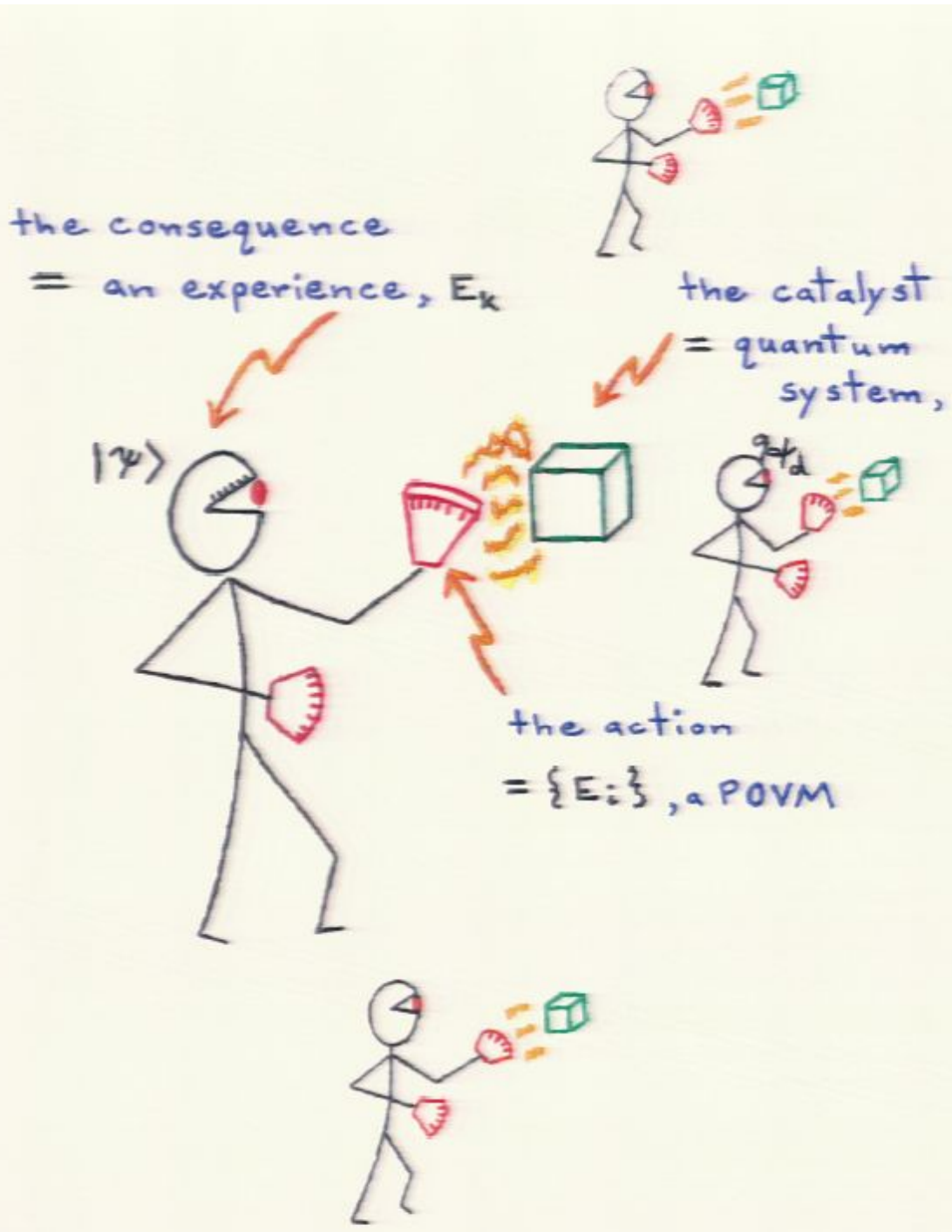
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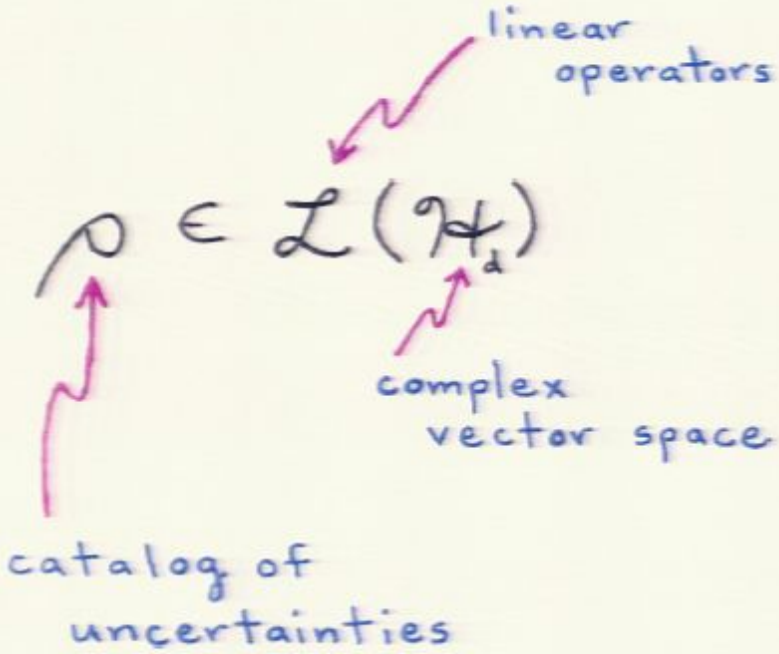
Density Operators



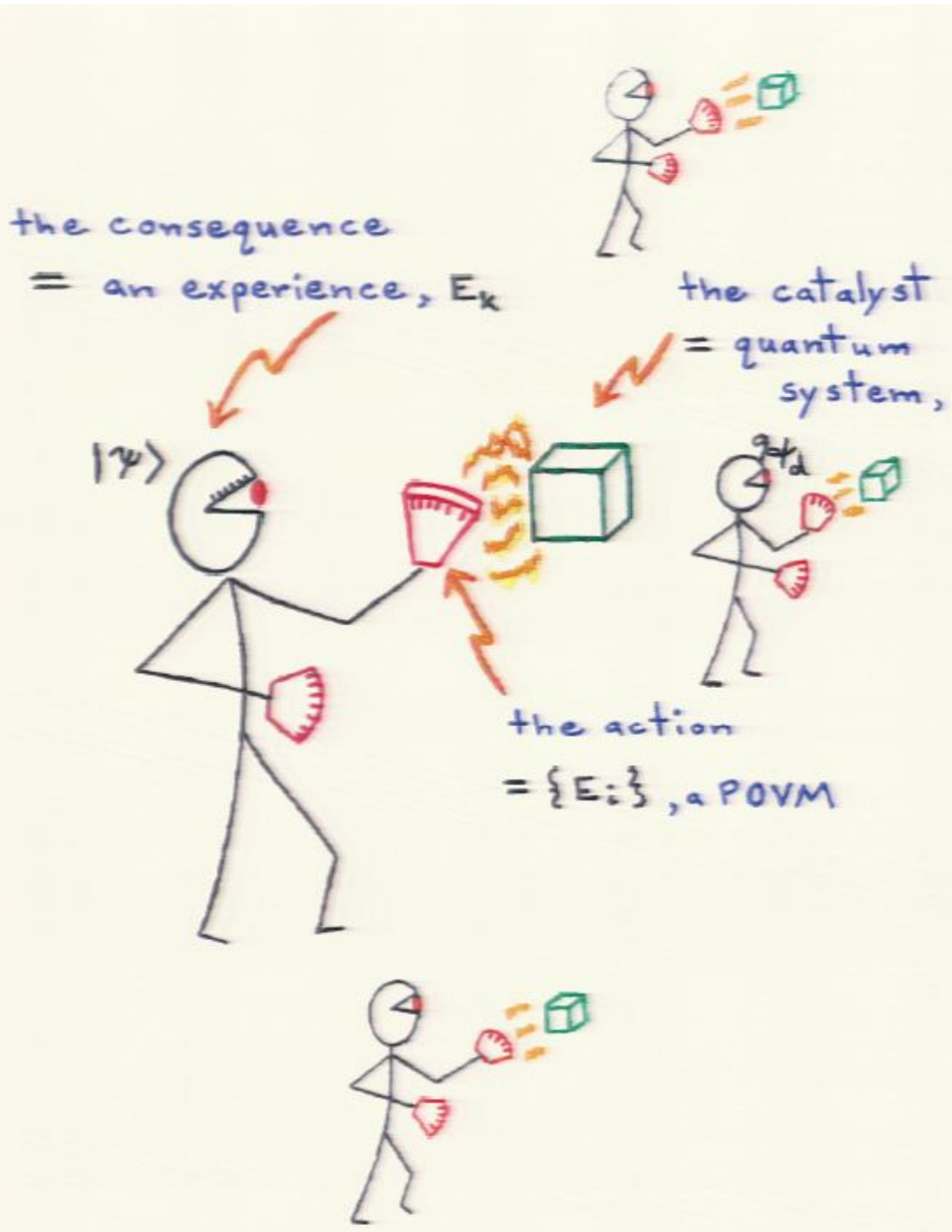
1) $\rho^\dagger = \rho$

2) $\mathrm{tr}\,\rho = 1$

3) $\lambda_i(\rho) \geq 0$

eigenvalues

convex hull of the set $\{|\psi\rangle\langle\psi| : |\psi\rangle \in \mathcal{H}_d\}$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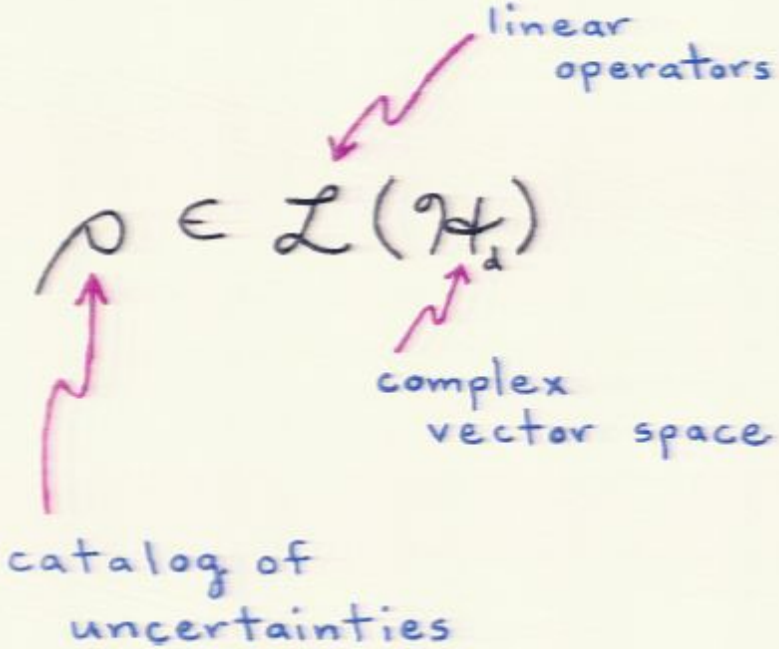

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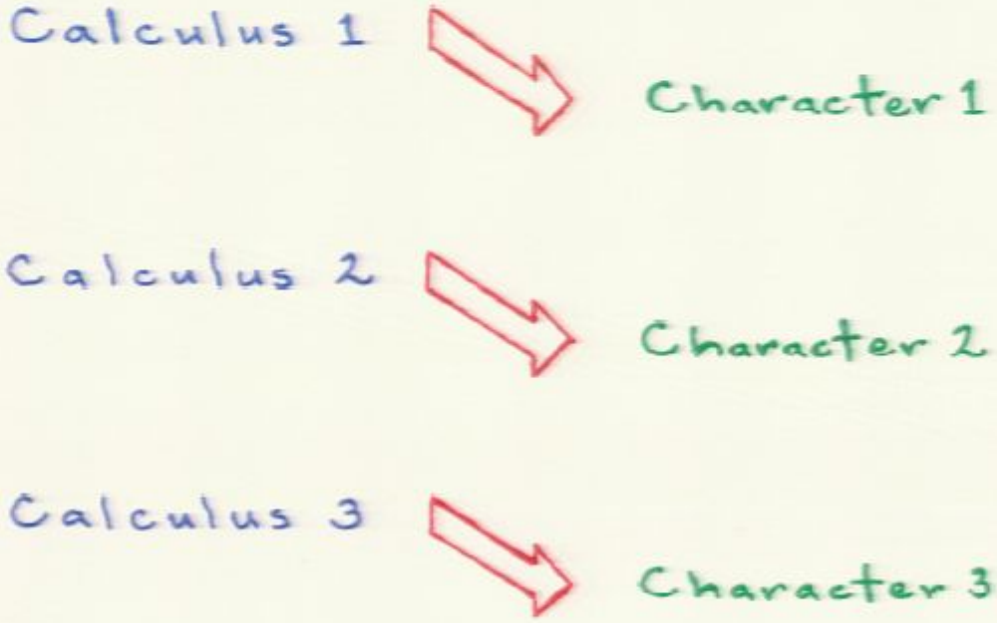
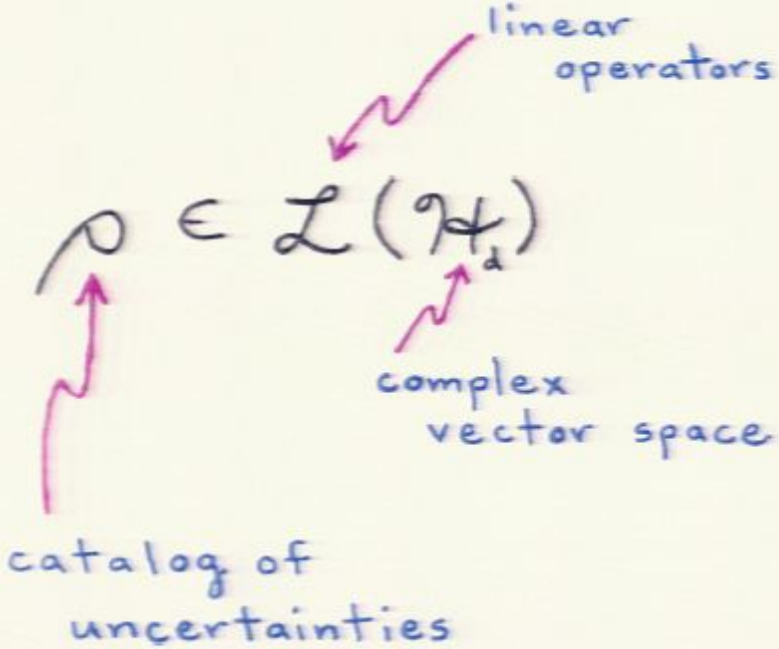

Calculus 1
Character 1
Calculus 2
Character 2
Calculus 3
Character 3

Density Operators



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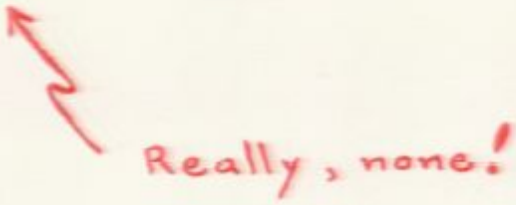
> convex hull of the set $\{|\psi\rangle\langle\psi| : |\psi\rangle \in \mathcal{H}_d\}$

A superior statement about the objective characteristics of our quantum world, of the things in it, would contain no $|\psi\rangle$'s at all.



I think there are professional problems [with quantum mechanics]. That is to say, I'm a professional theoretical physicist and I would like to make a clean theory. And when I look at quantum mechanics I see that its a dirty theory. The formulations of quantum mechanics that you find in the books involve dividing the world into an observer and an observed, and you are not told where that division comes ... So you have a theory which is fundamentally ambiguous ...

— J. S. Bell

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The Born Rule

Given ρ and $\{E_i\}$,

ρ: quantum state

$\{E_i\}$: POVM measurement

$$p(i) = \mathrm{tr}\, \rho E_i$$

"The Born Rule"

NOT a law of nature.

RATHER something we should strive for.

Not like

$$\vec{F} = m\vec{a}$$

Not like

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\epsilon_0}\rho \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} \qquad \vec{\nabla}\cdot\vec{B} = 0$$

Not like

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

THE TEN COMMANDMENTS

- Thou shalt not kill.
- Thou shalt not steal.
- Thou shalt not covet thy neighbor's wife.

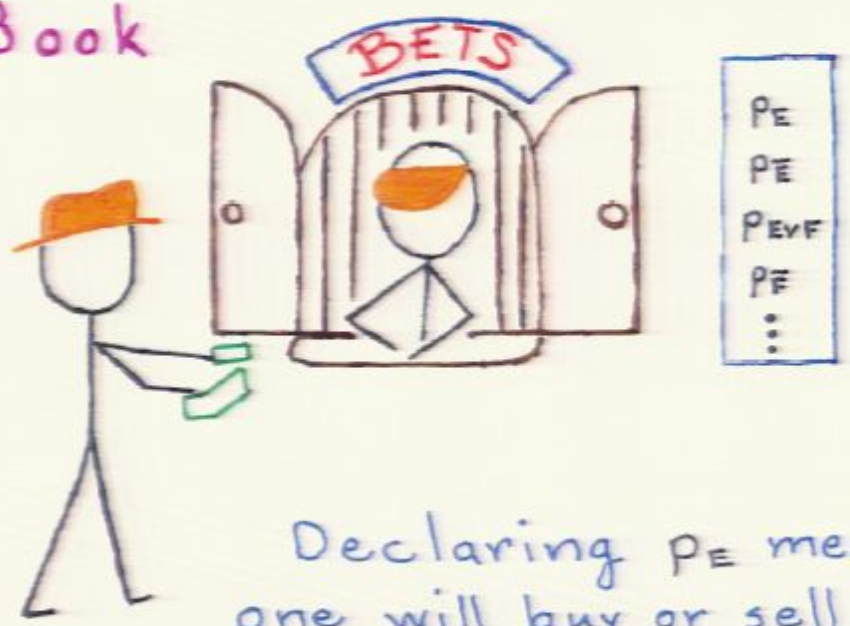
⋮

- The firstling of an ass thou shalt redeem with a lamb.

⋮

Defining Probability

Dutch Book



Declaring p_E means one will buy or sell a lottery ticket

Worth $1 if E

for $\$p_E$.

Dutch Book

Normative Rule:

Never declare p_E, $p_{\bar{E}}$, $p_{E \vee F}$, etc. that will lead to sure loss.

Example 1:

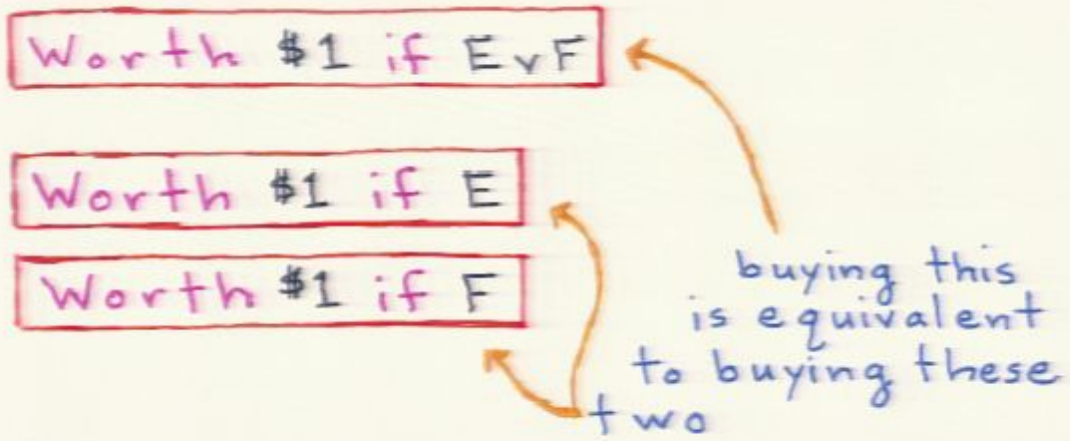
If $p_E < 0$, bookie will sell ticket for negative money. Sure loss!

Example 2:

If $p_E > 1$, bookie will buy ticket for more than it is worth in best case. Sure loss.

Example 3:

Suppose E and F mutually exclusive.



So must have $p_{E \vee F} = p_E + p_F$.

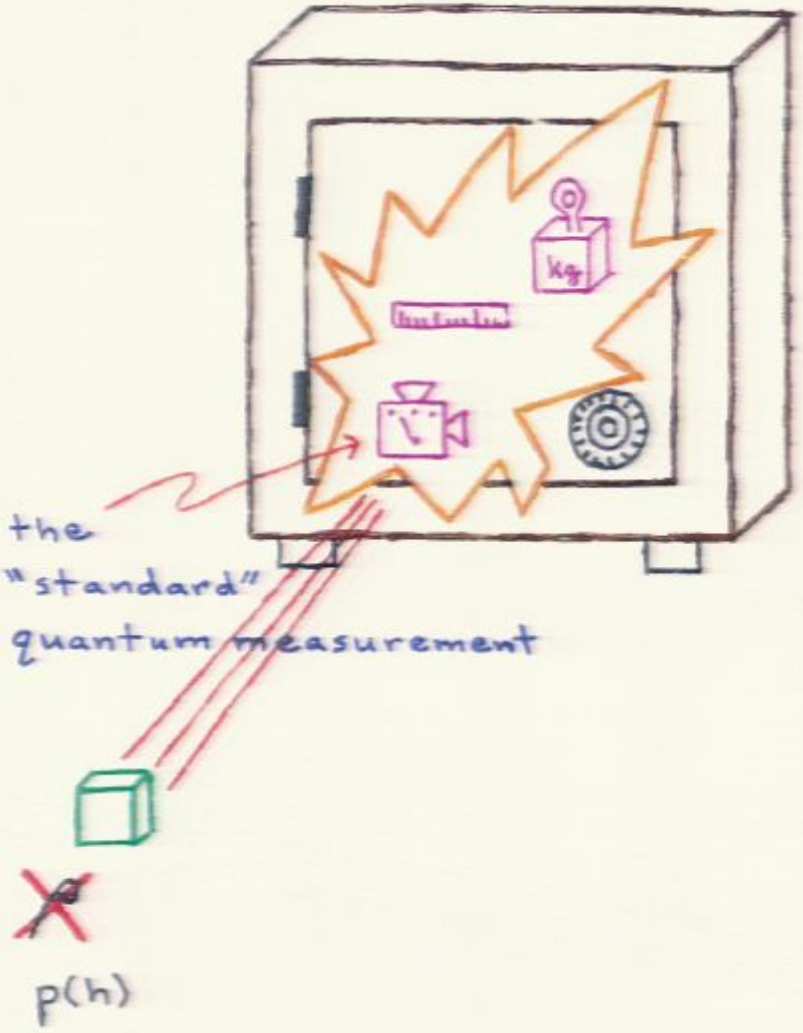
Example 4:

Worth $\$\frac{m}{n}$ if E

Price? $\$\frac{m}{n} p_E$ of course.


ρ
p(h)

Bureau of Standards



A Very Fundamental Mmt ?

Caves, 1999
Zauner

Suppose d^2 projectors $\Pi_i = |\psi_i\rangle\langle\psi_i|$ satisfying

$$\mathrm{tr}\,\Pi_i\Pi_j = \frac{1}{d+1} \quad , \quad i \neq j$$

exist. ← Called SIC.

Can prove:

1) the Π_i linearly independent

2) $\sum_i \frac{1}{d}\Pi_i = I$

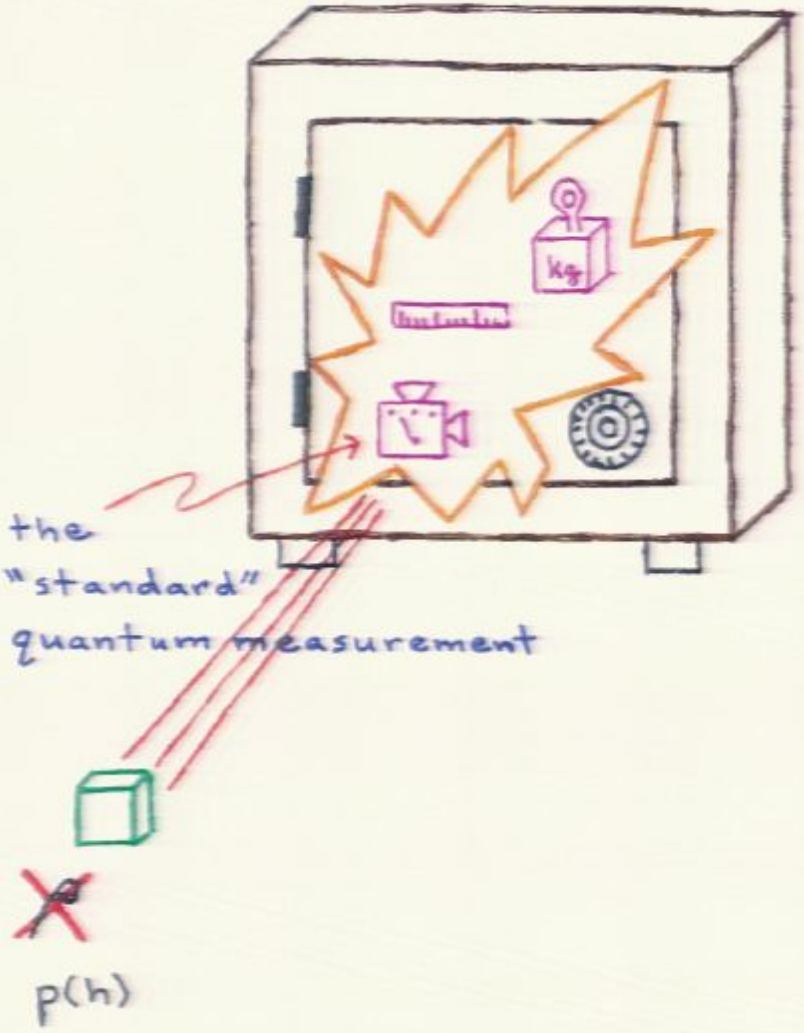
So good for Bureau of Standards.

Also

$$p(i) = \frac{1}{d}\mathrm{tr}\,\rho\Pi_i$$

$$\boxed{\rho = \sum_i [(d+1)p(i) - \tfrac{1}{d}]\Pi_i}$$

Bureau of Standards



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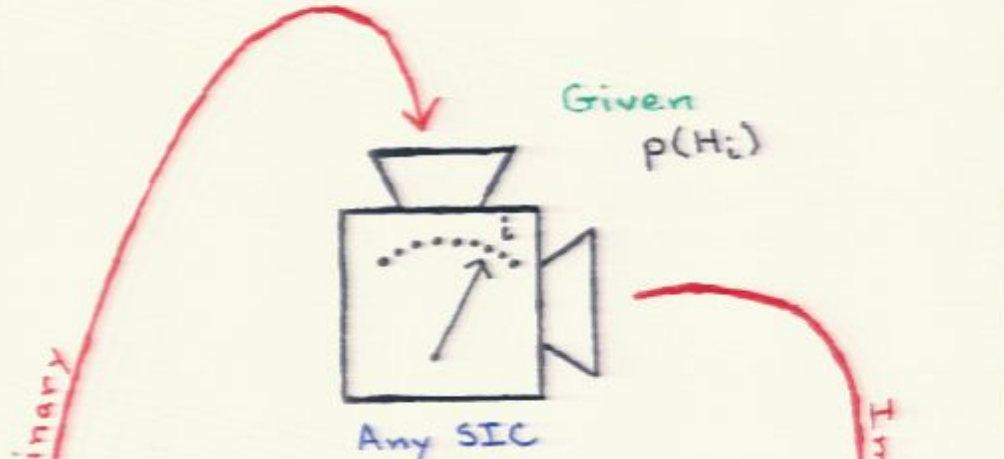

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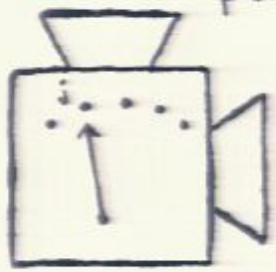


But really going to do this.

Any von Neumann measurement

What $p(D_j)$?

Laws of Probability

H_i – various hypotheses one might have

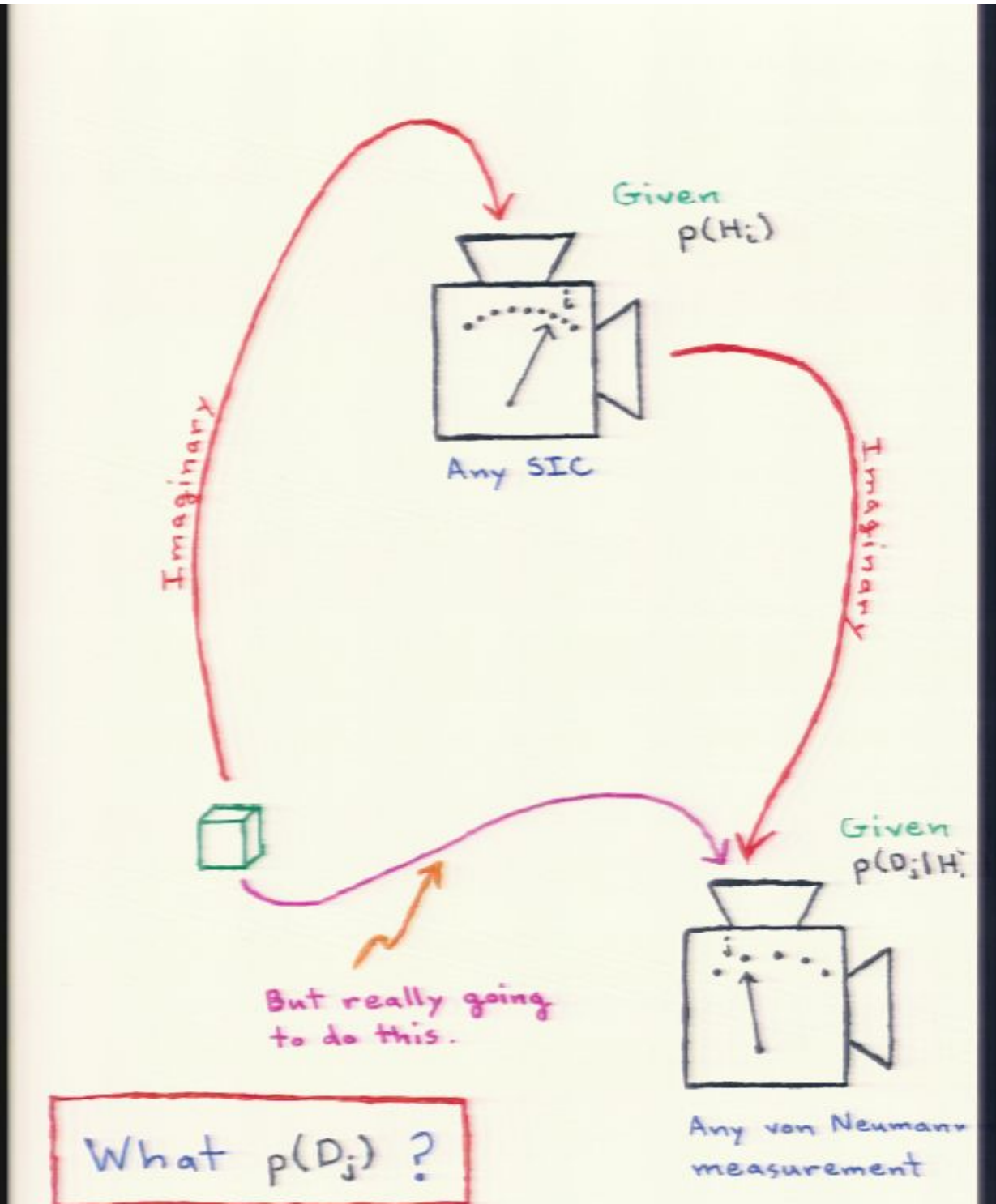
D_j – data values one might gather

Given: $p(D_j|H_i)$ ← expectations for data given hypothesis

$p(H_i)$ ← expectations for hypotheses themselves

Question: What expectations should one have for the D_j?

Answer: $P(D_j) = \sum_i p(H_i)\, p(D_j|H_i)$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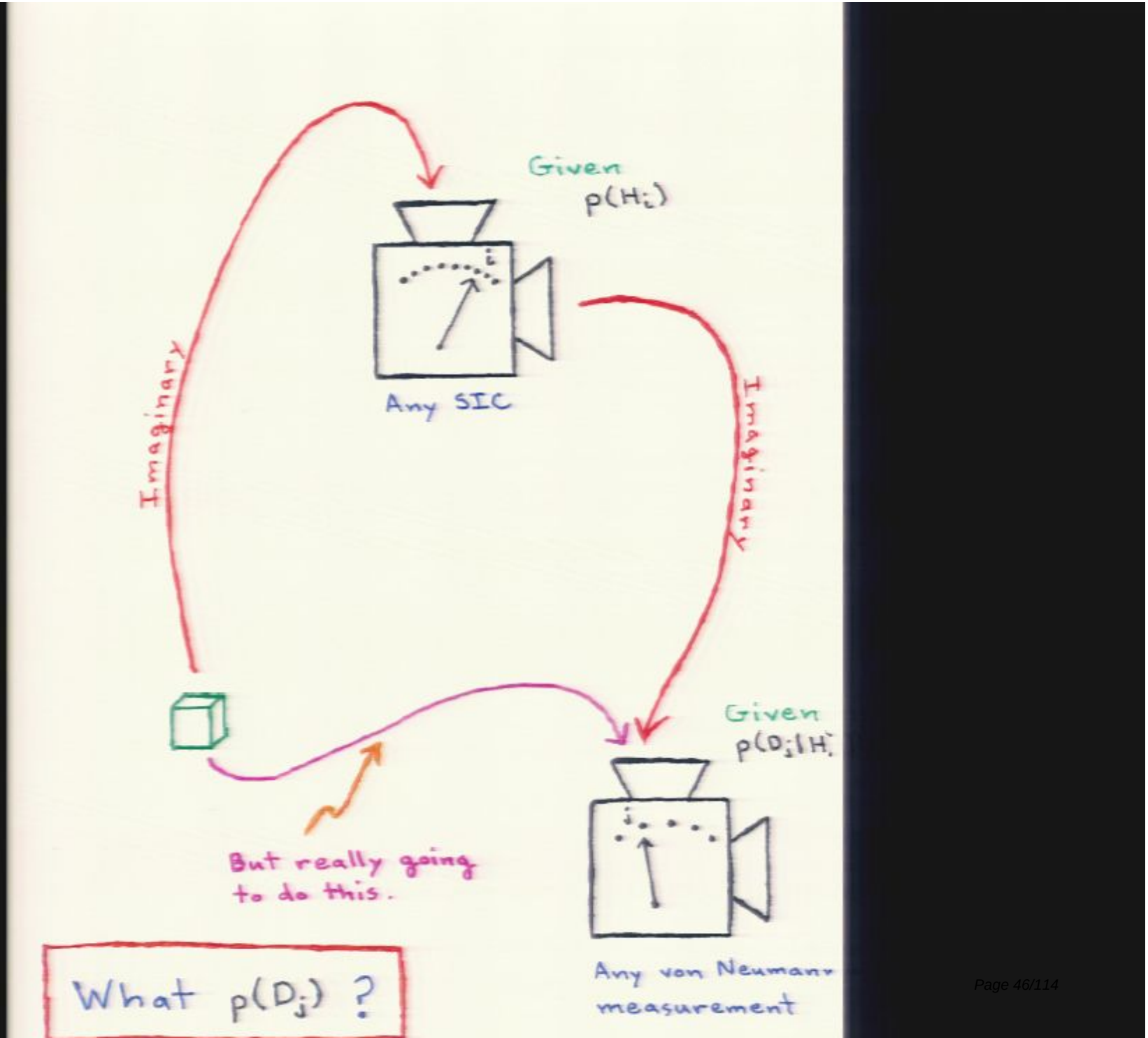
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In this case,

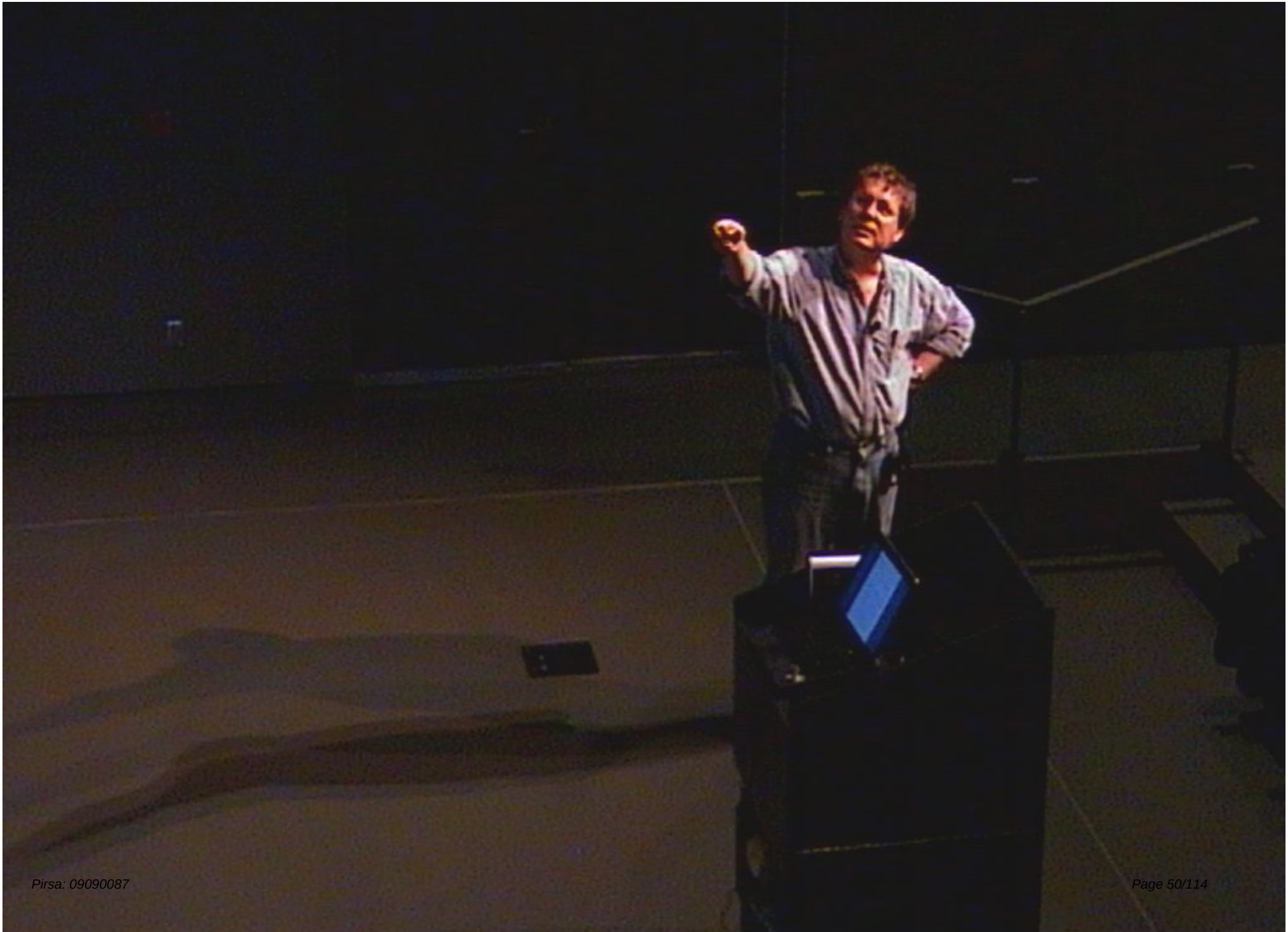
$$p(D_j) \neq \sum_i p(H_i)\, p(D_j|H_i) .$$

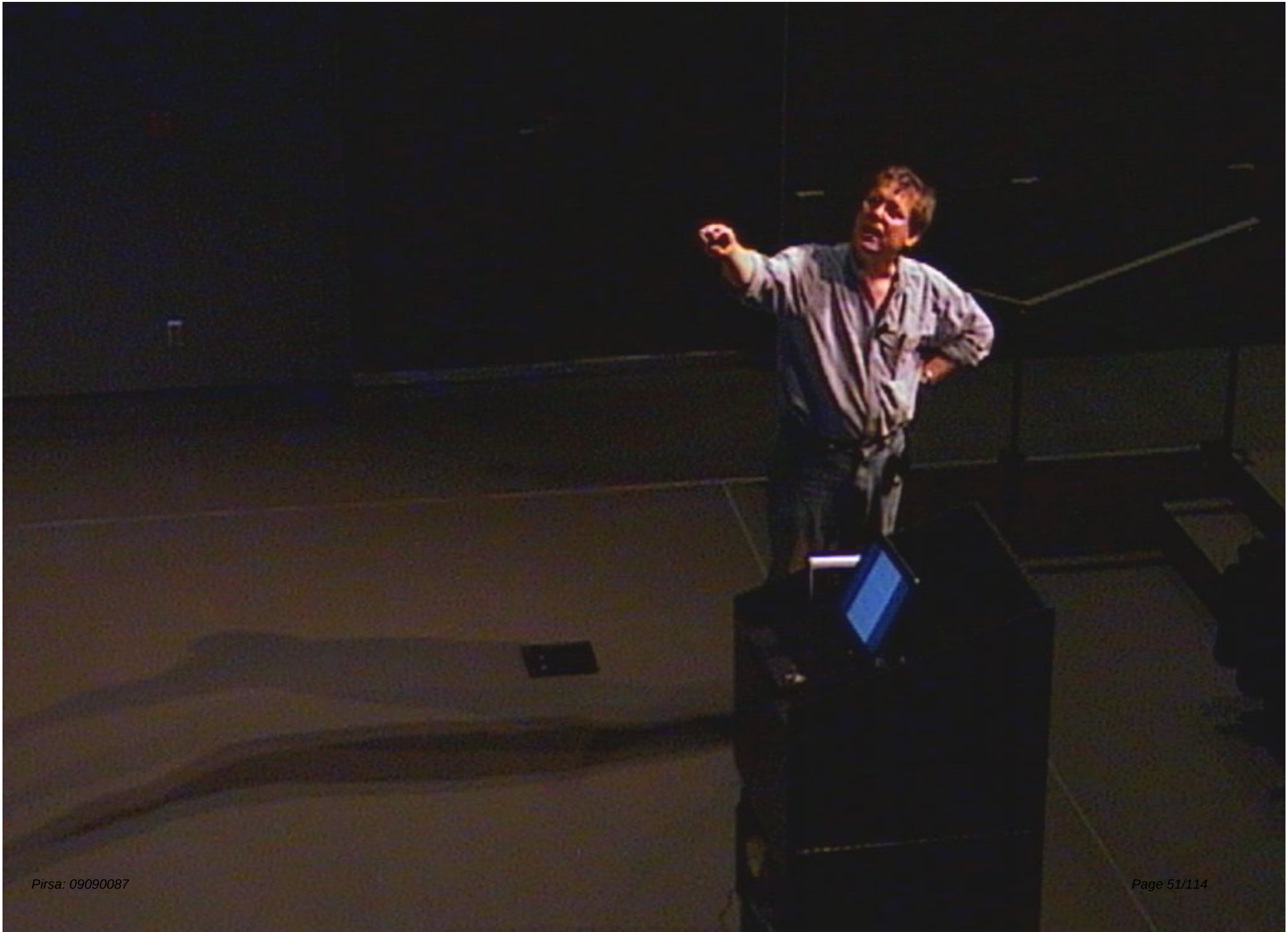
As Ballentine (1986) points out, there are hidden conditionals

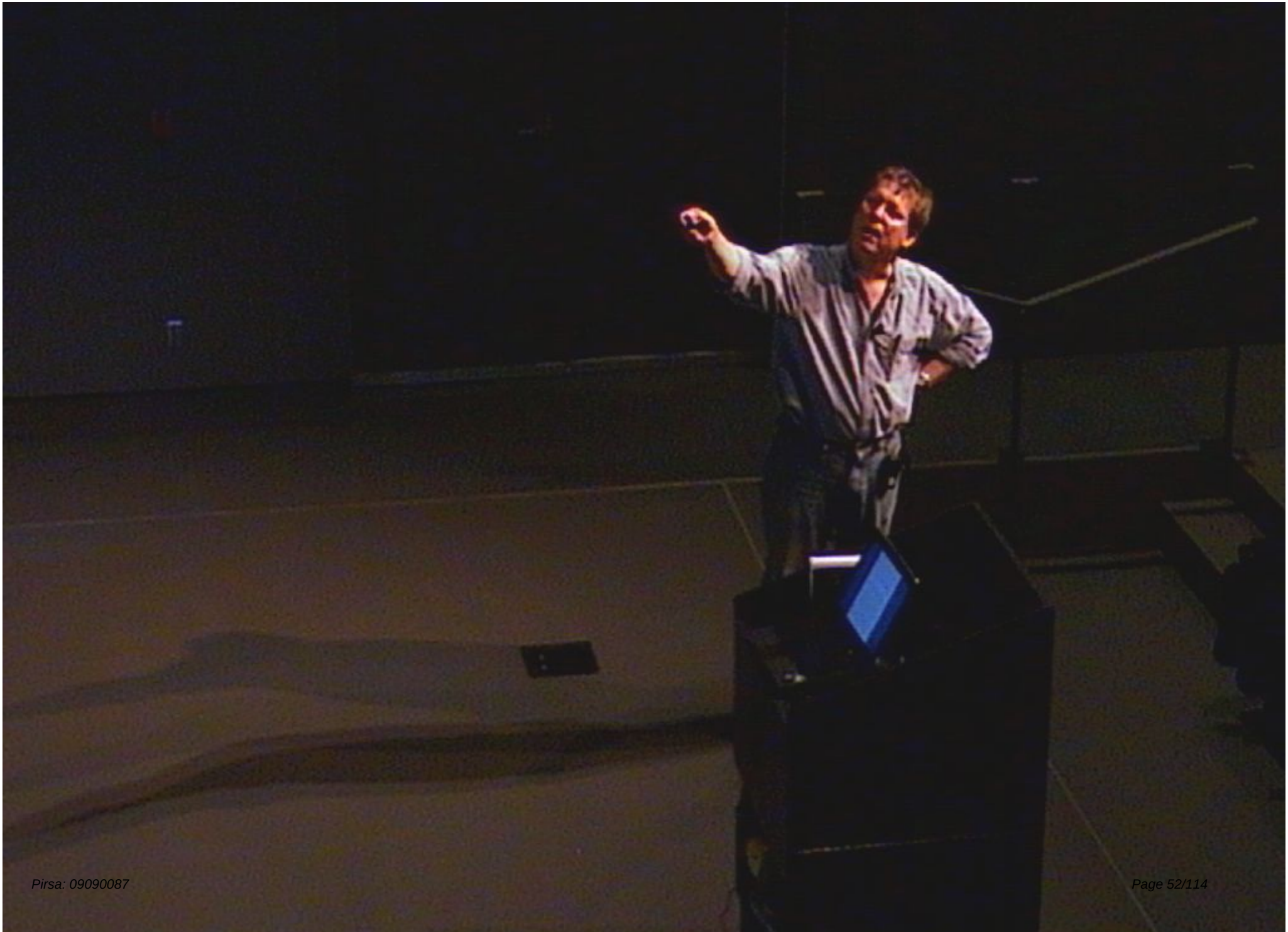
$p(D_j)$ really $p(D_j|C_1)$

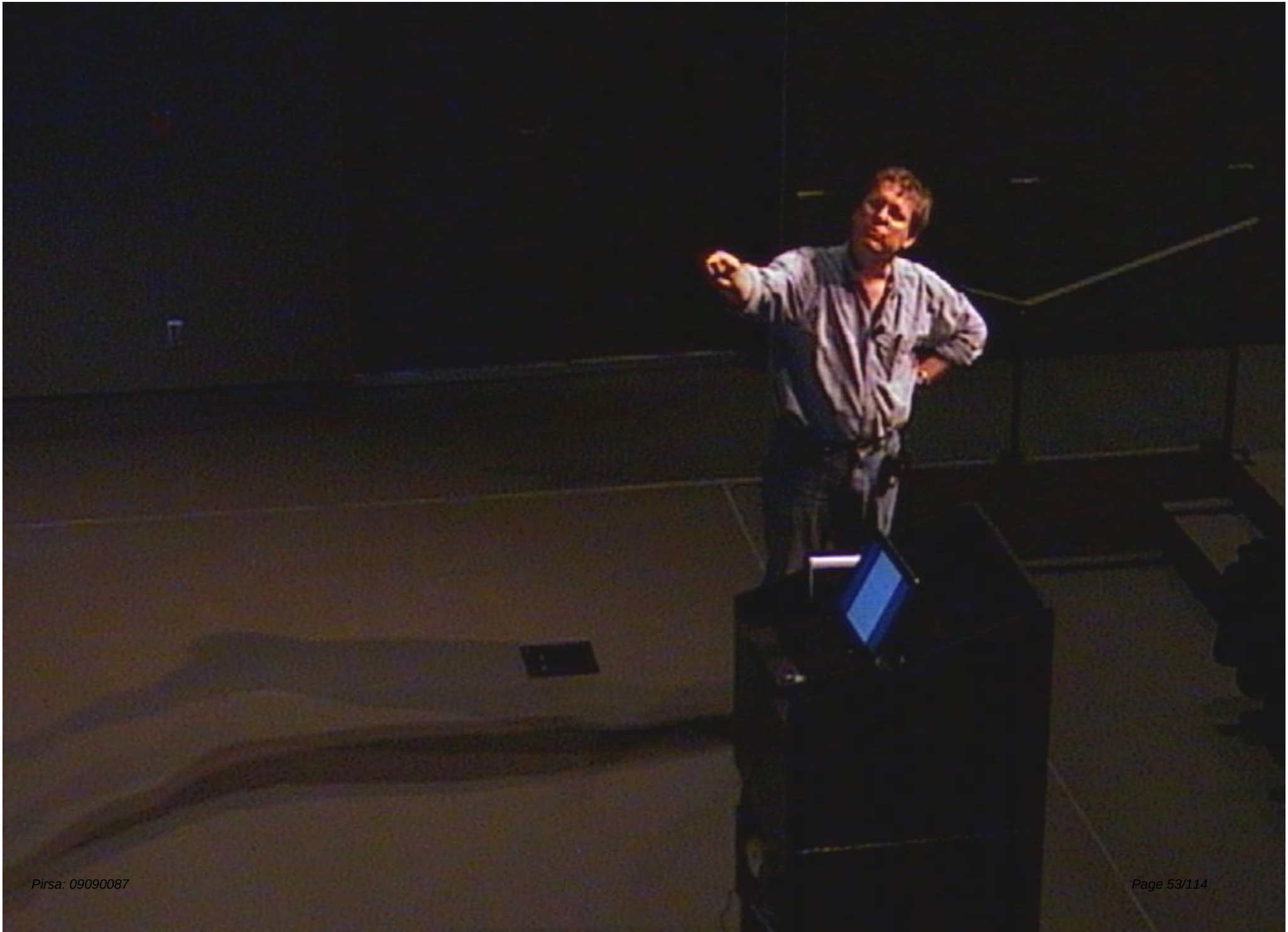
$p(H_i)$ really $p(H_i|C_2)$

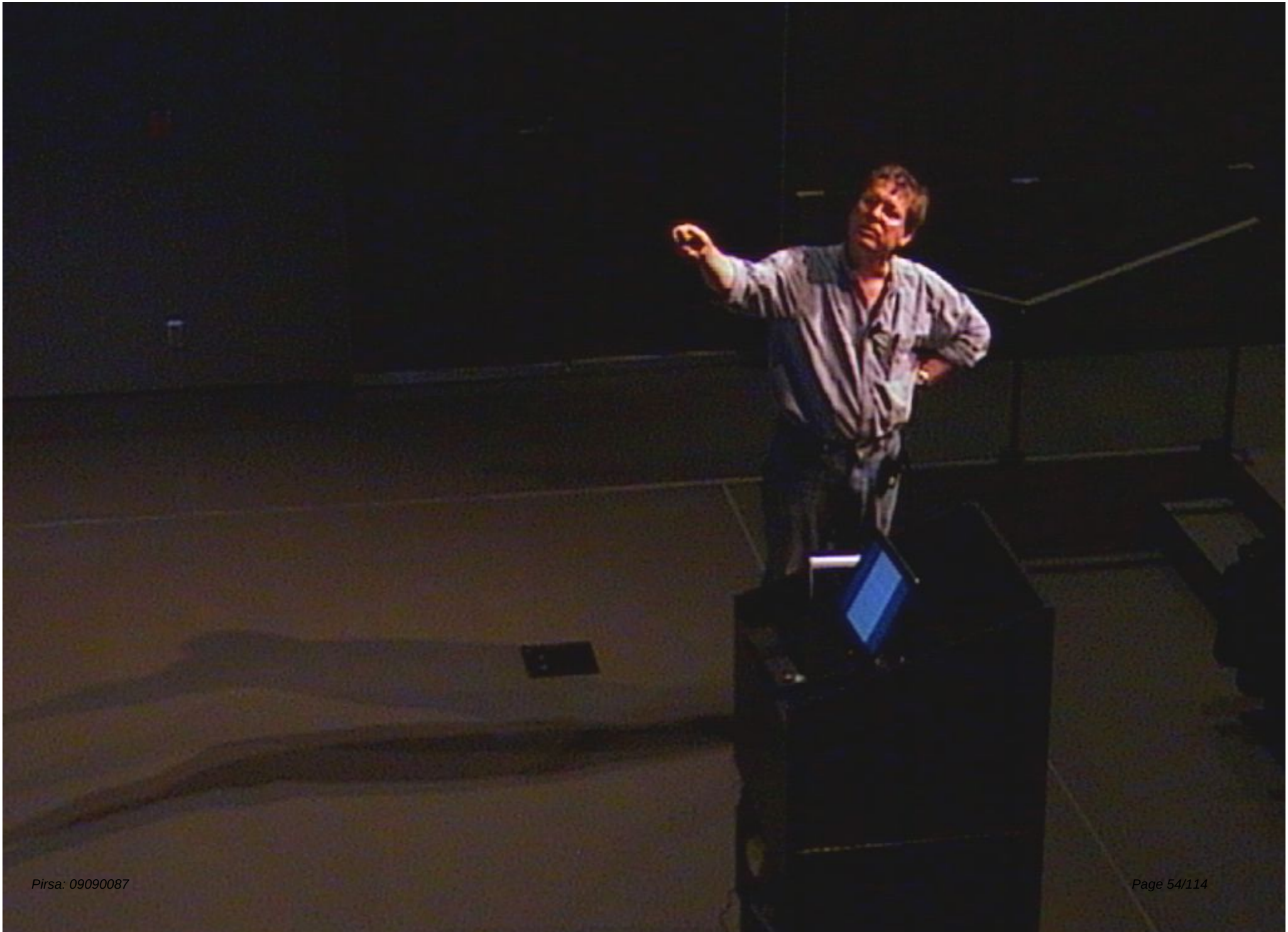
$p(D_j|H_i)$ really $p(D_j|H_i,C_2)$

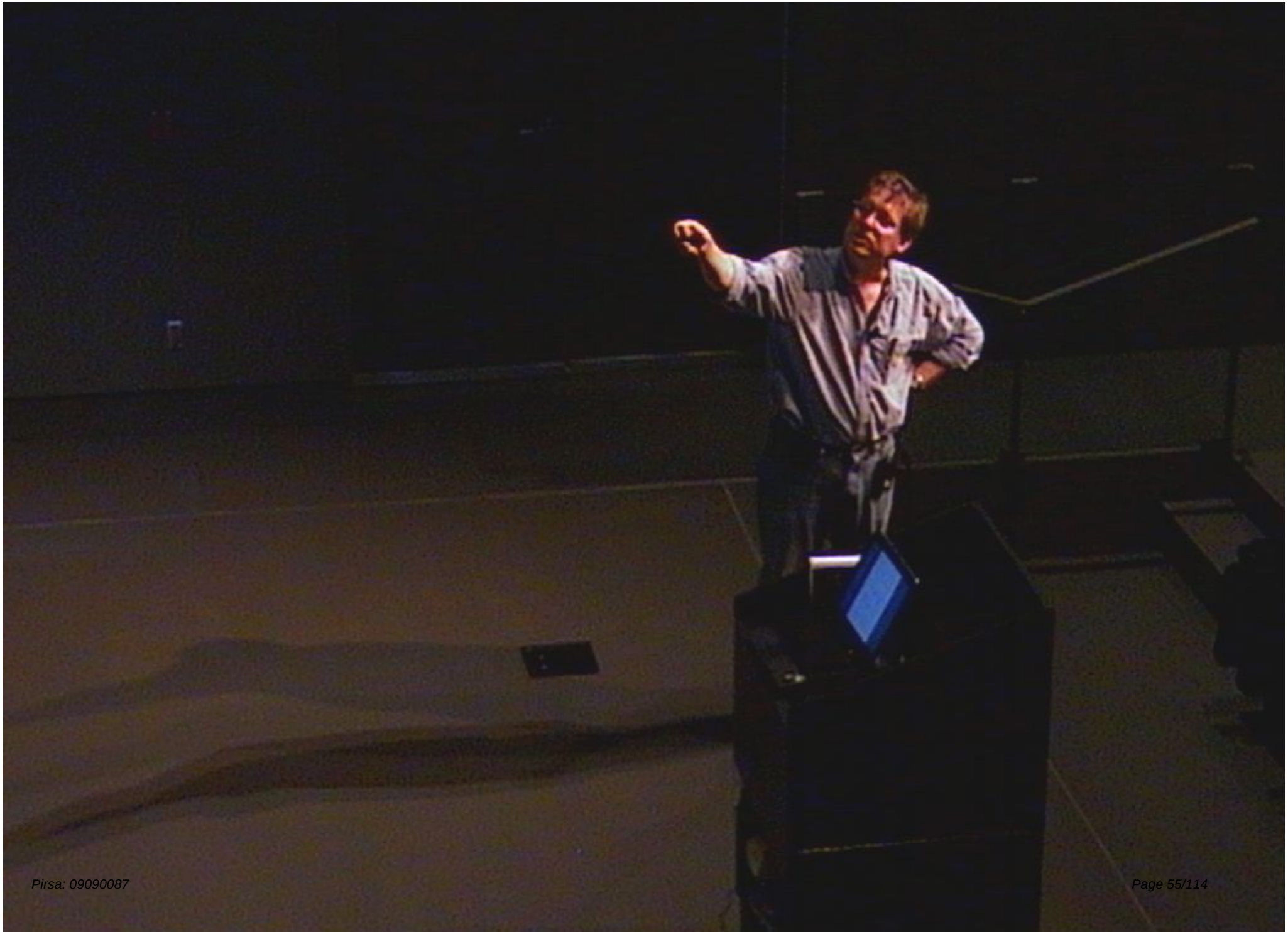


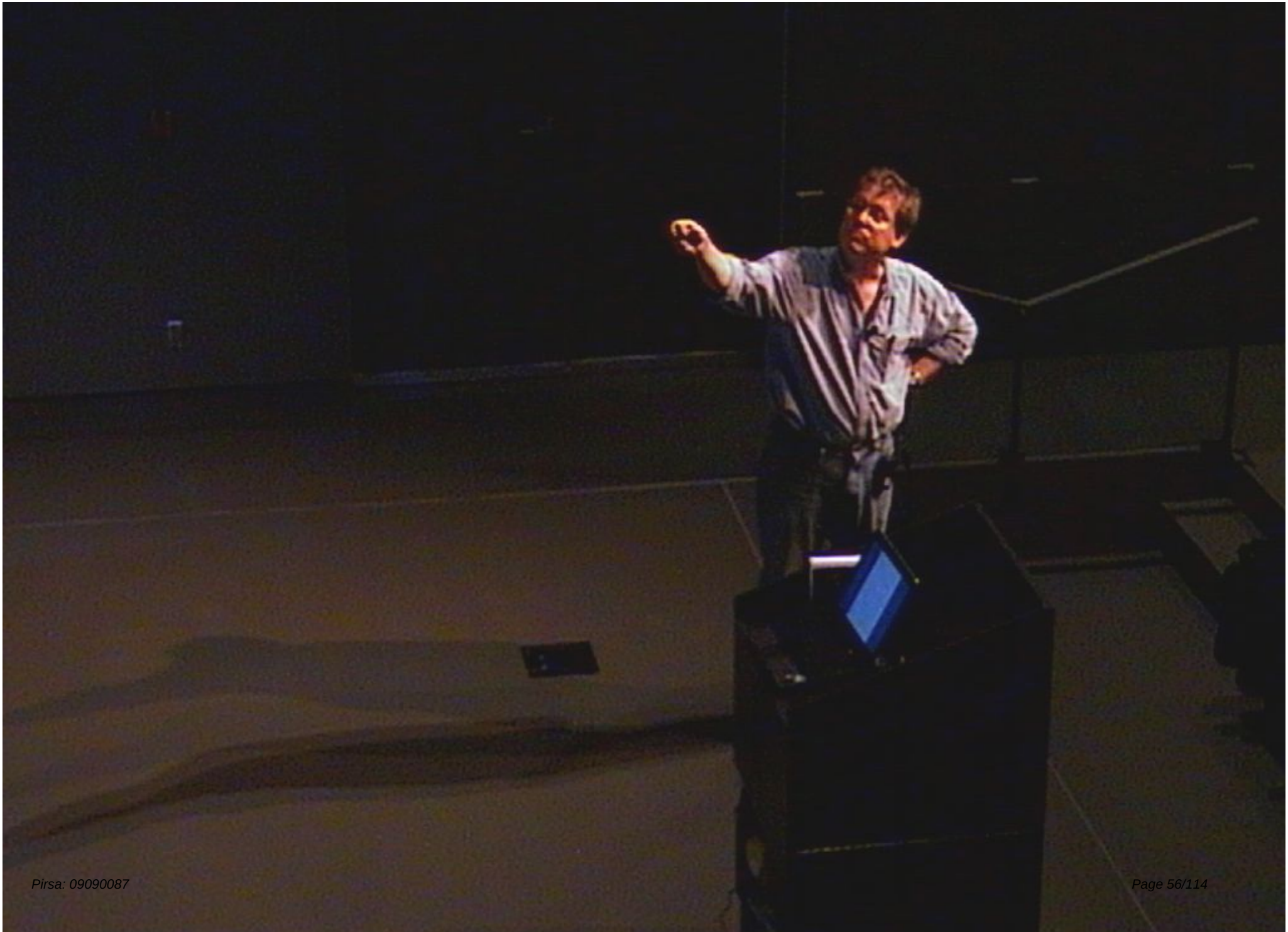


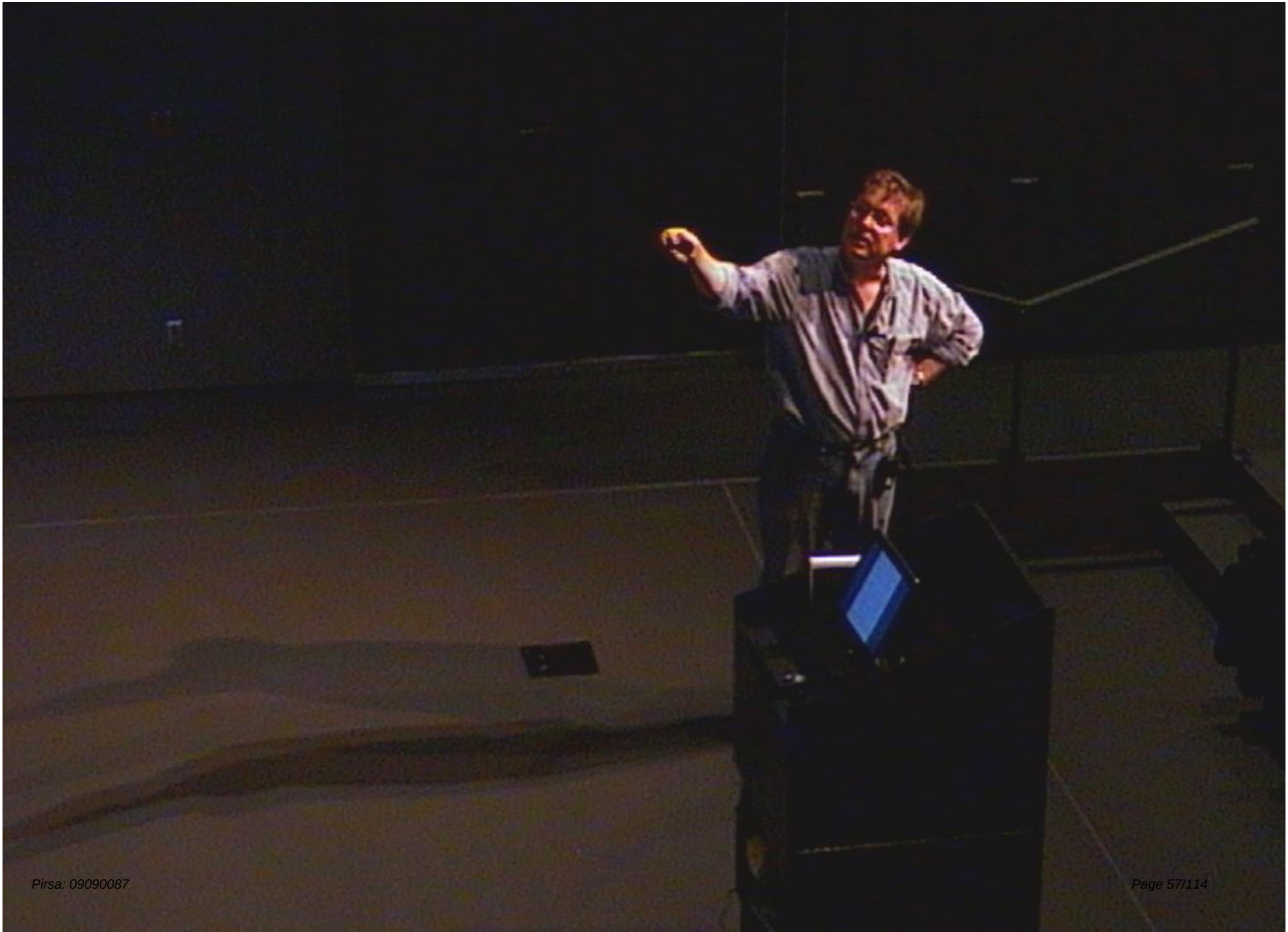


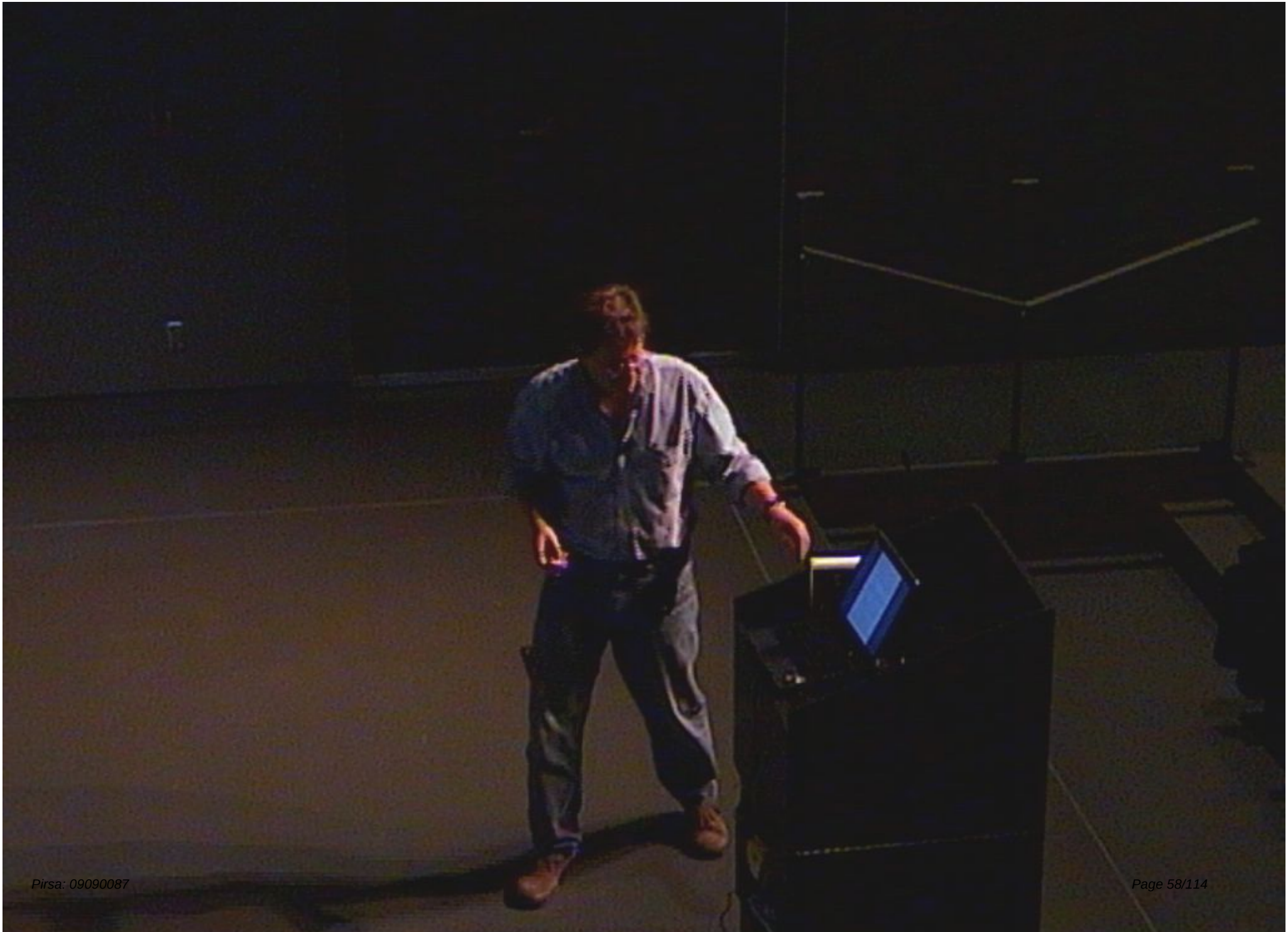


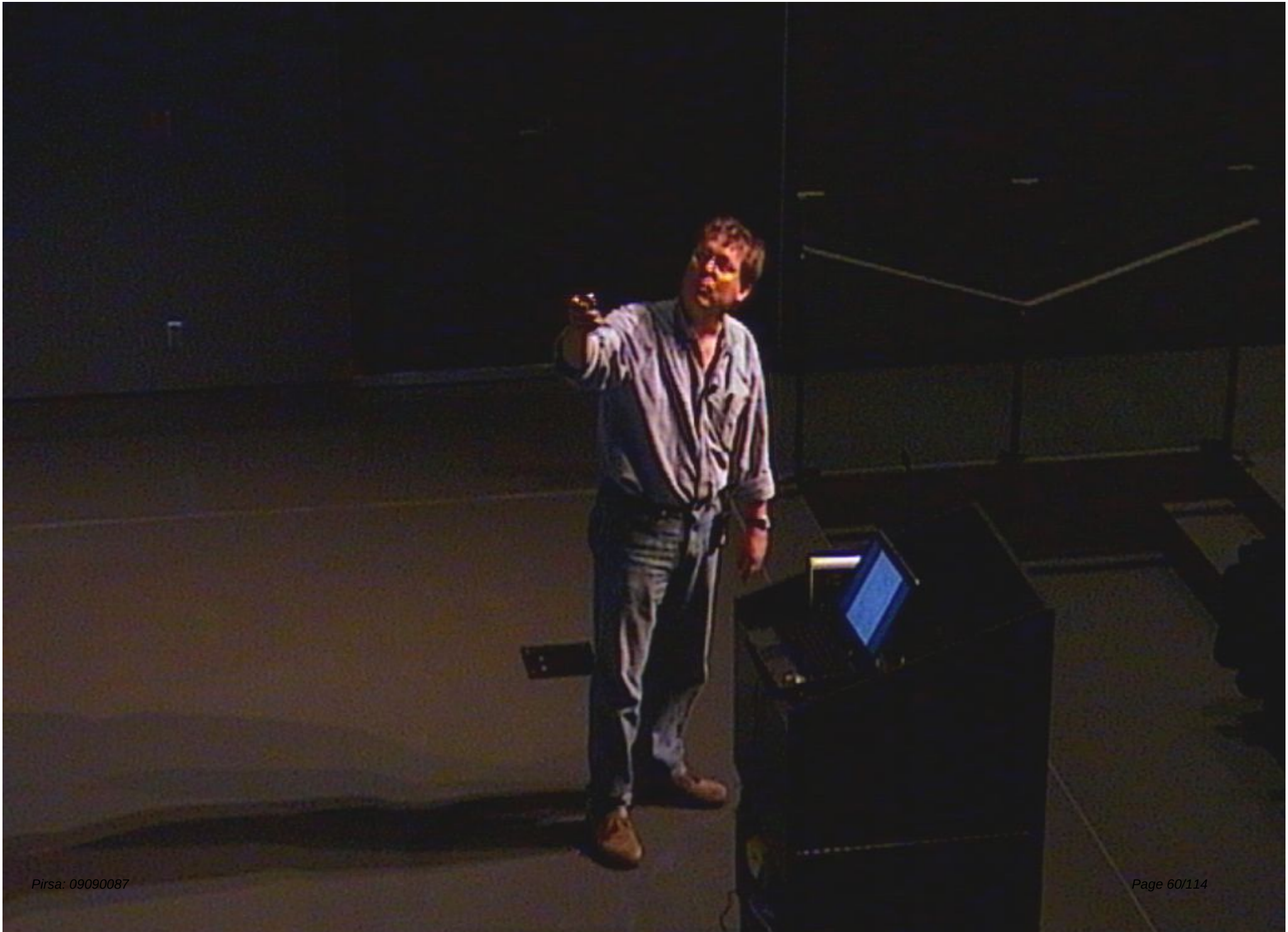


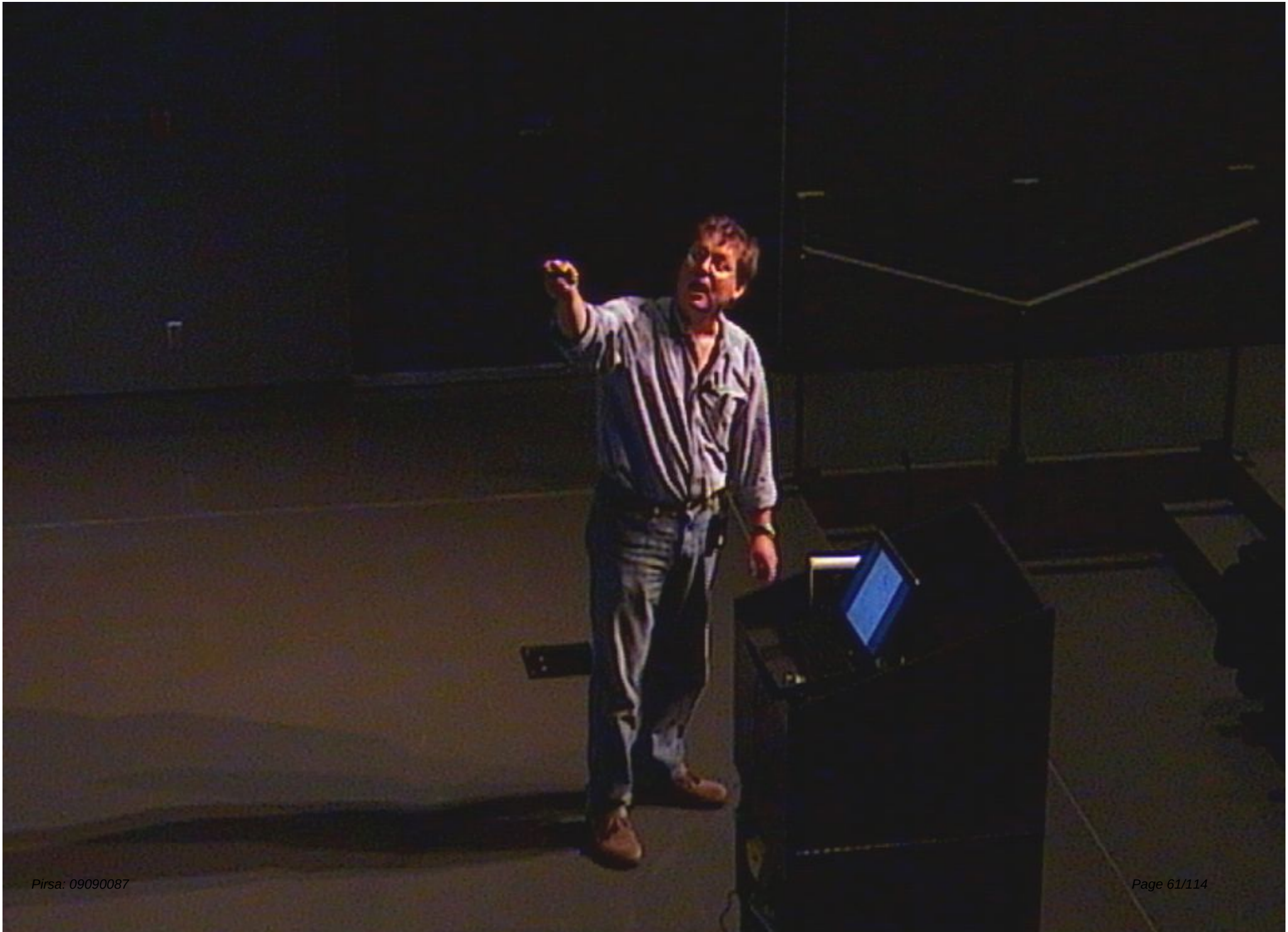


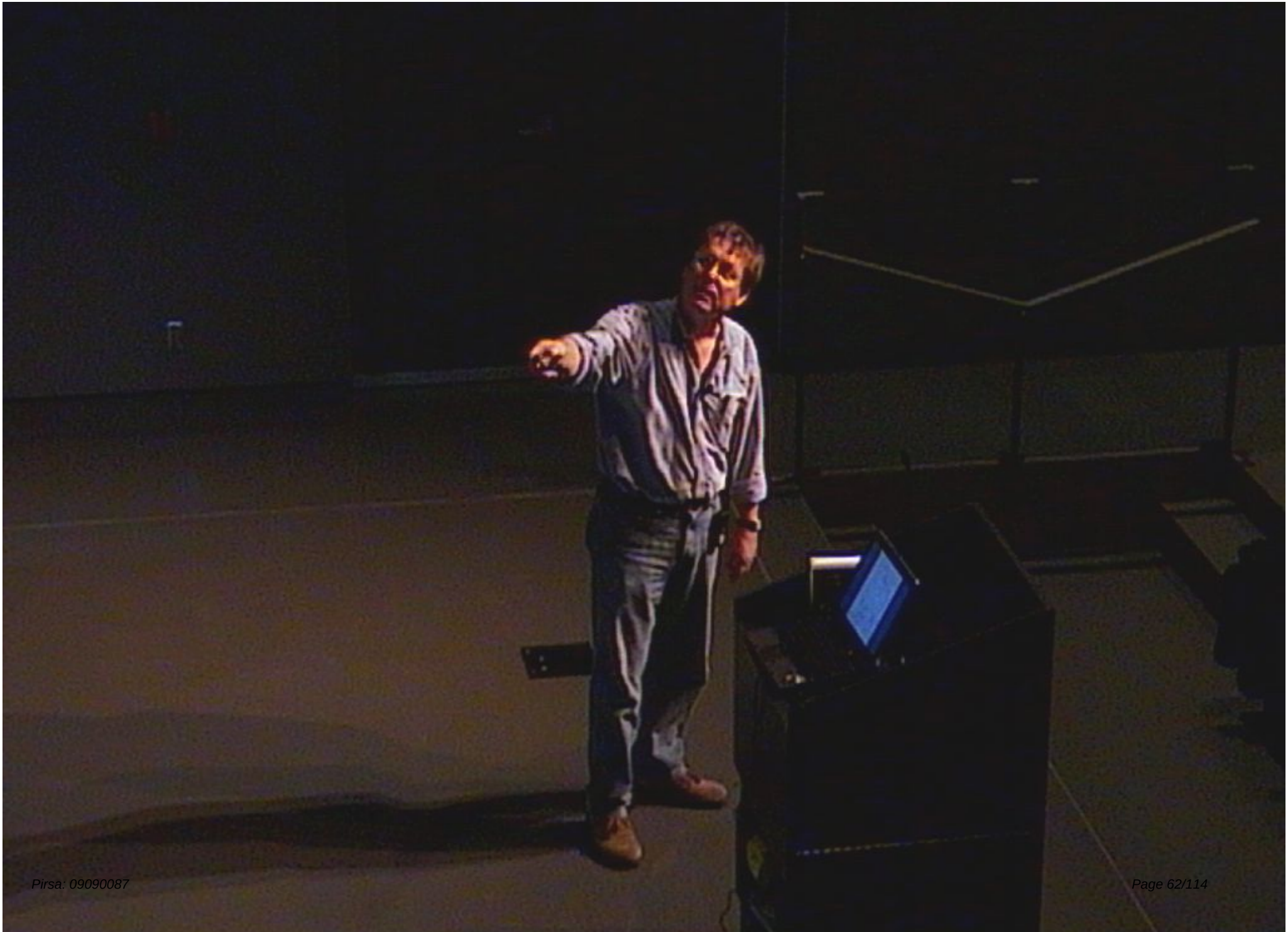


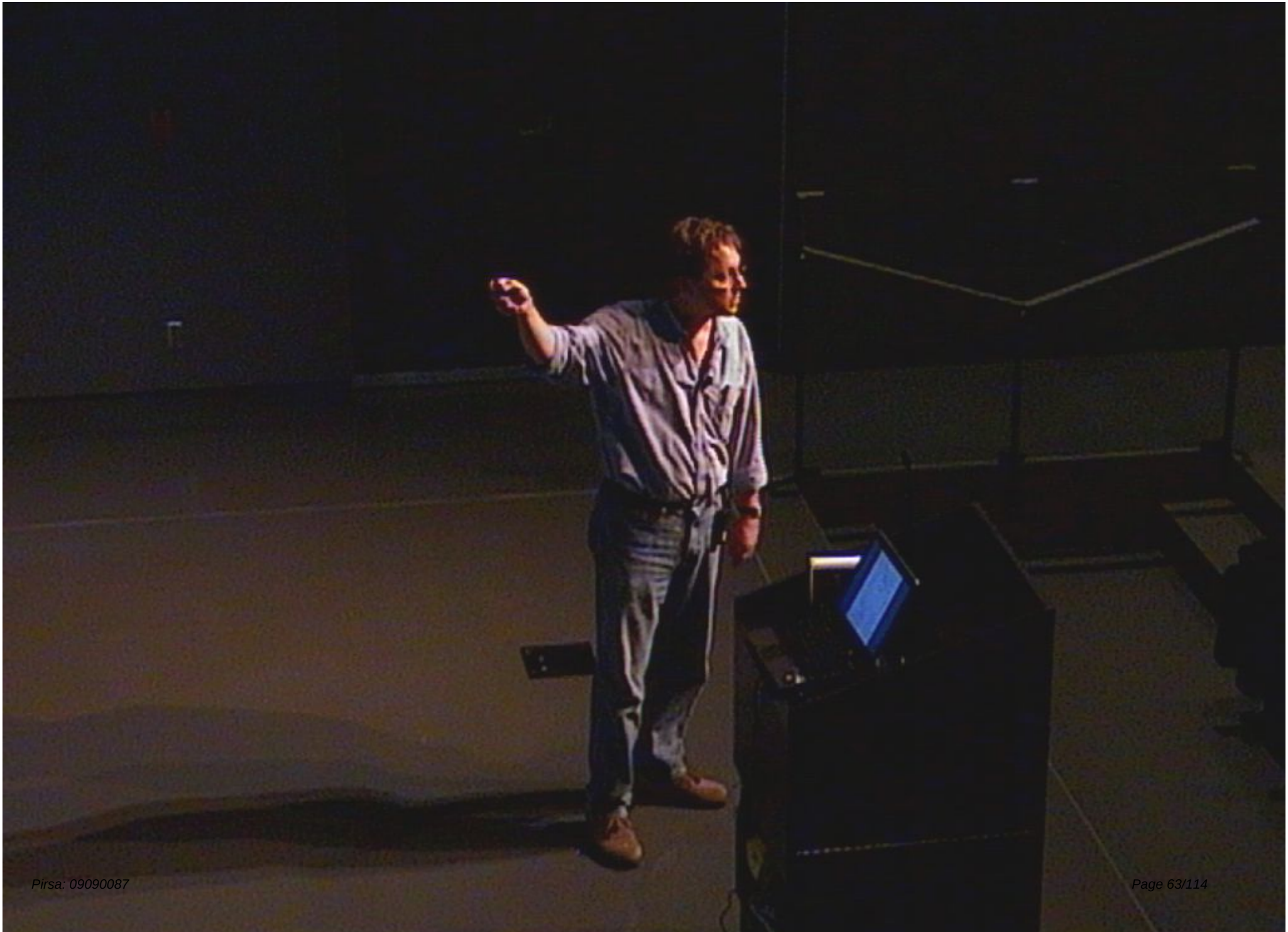


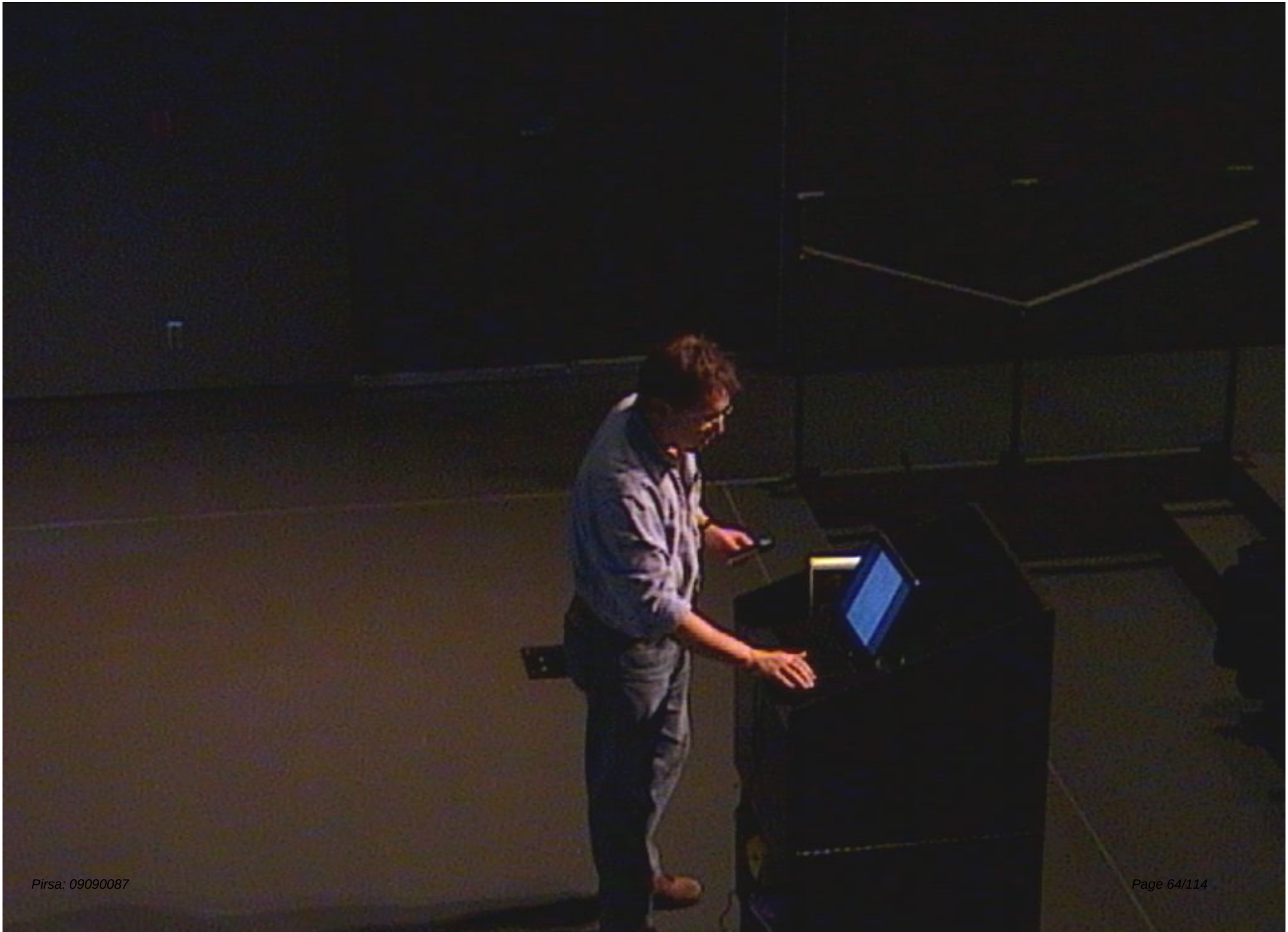


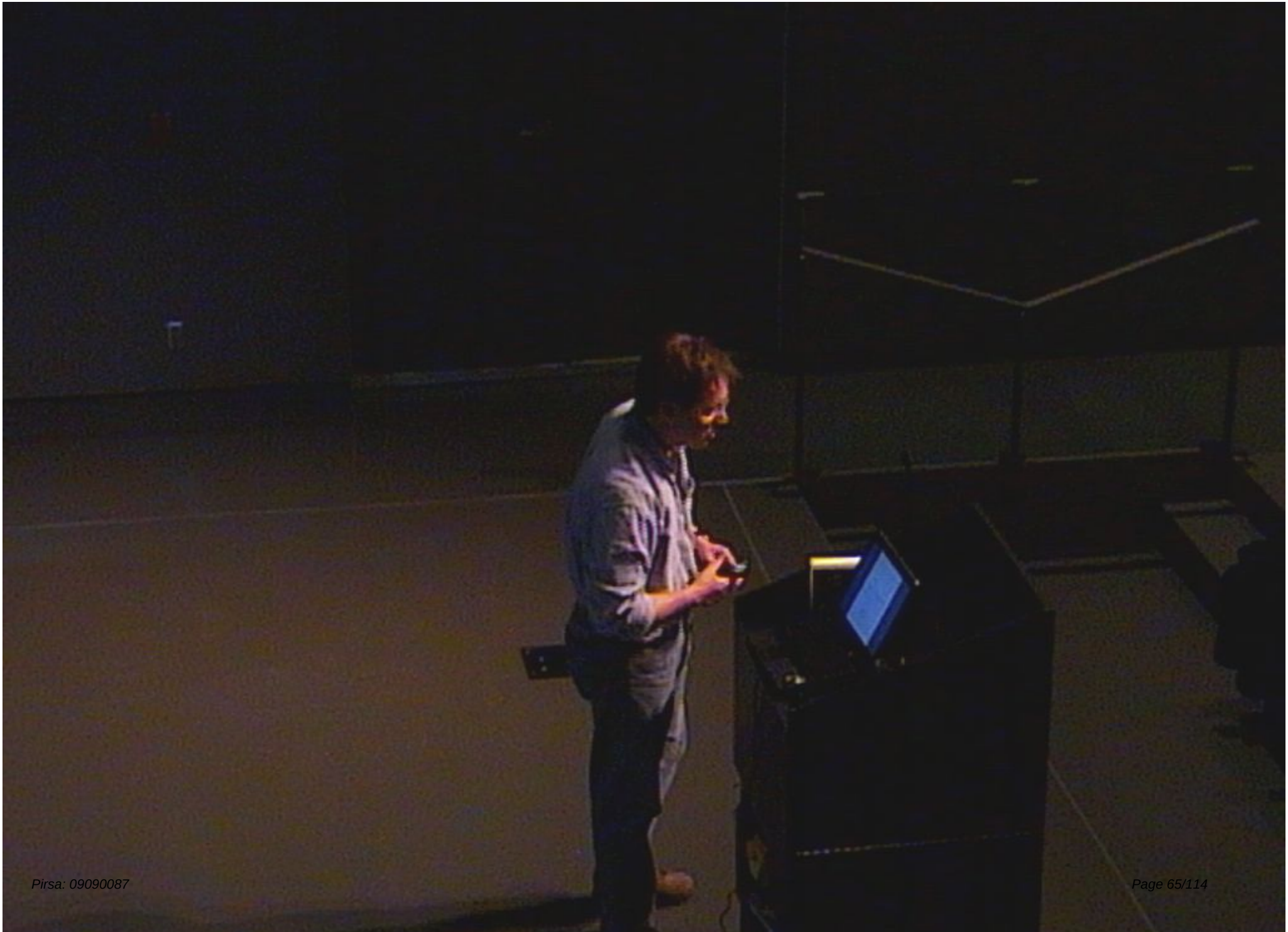


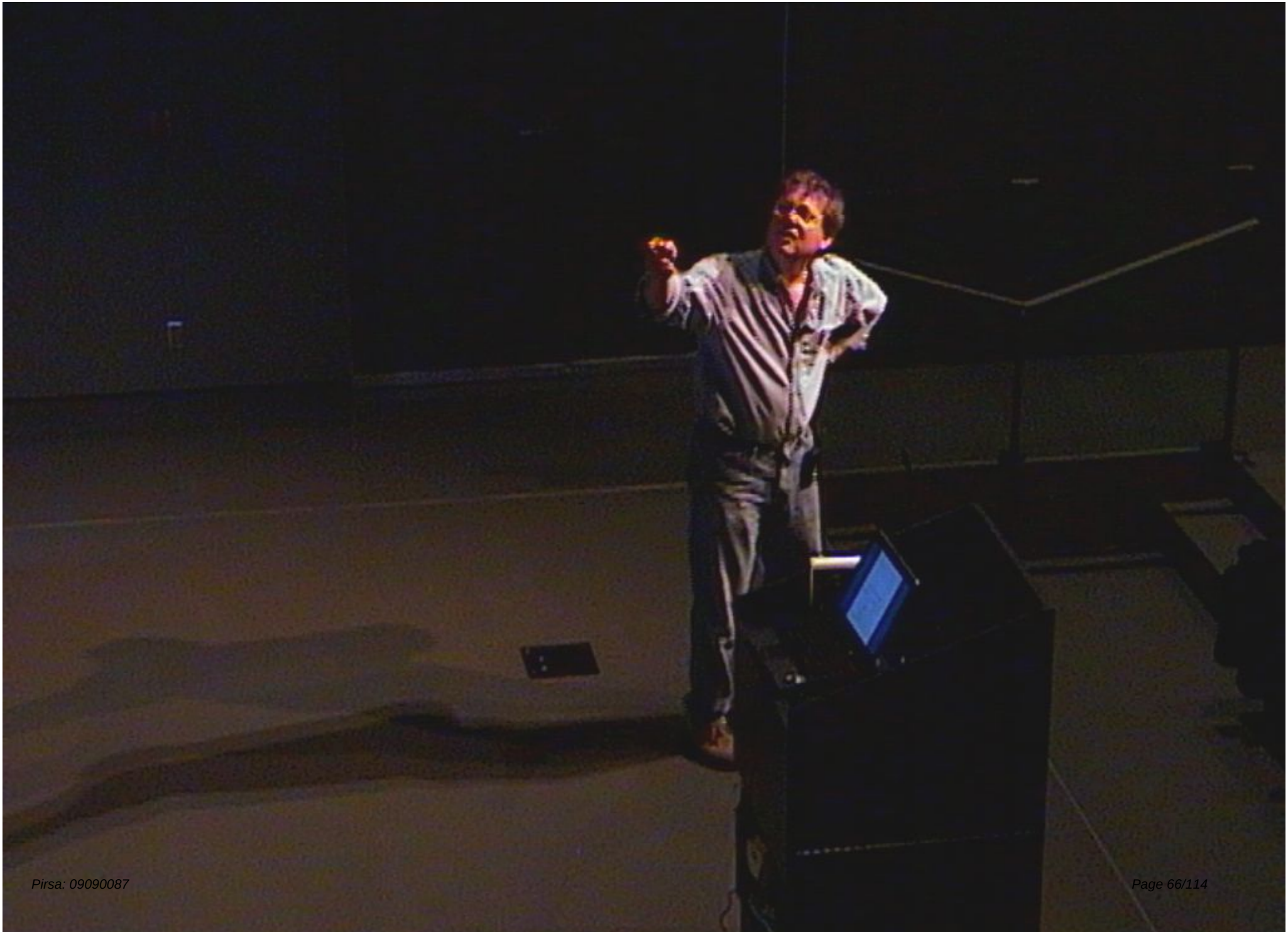


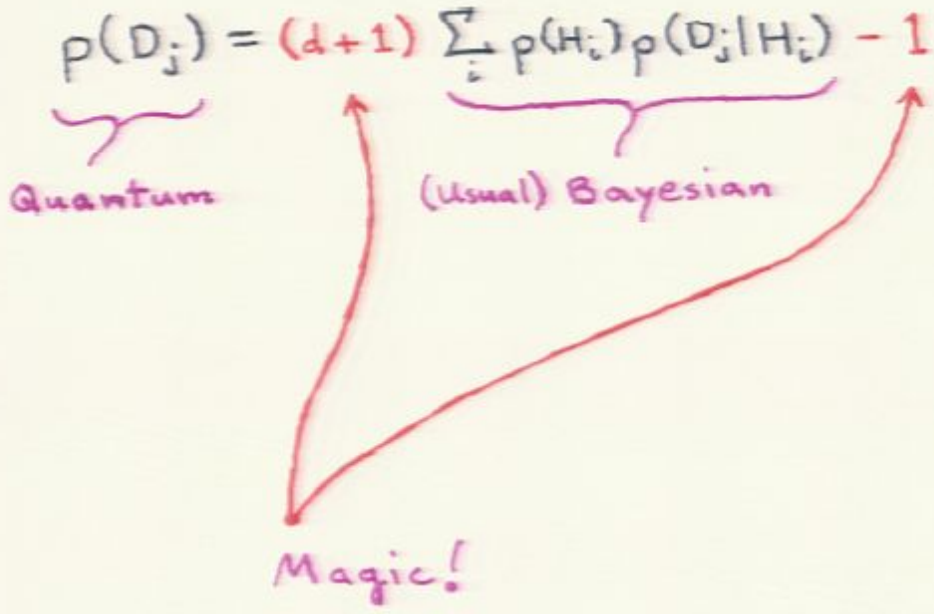


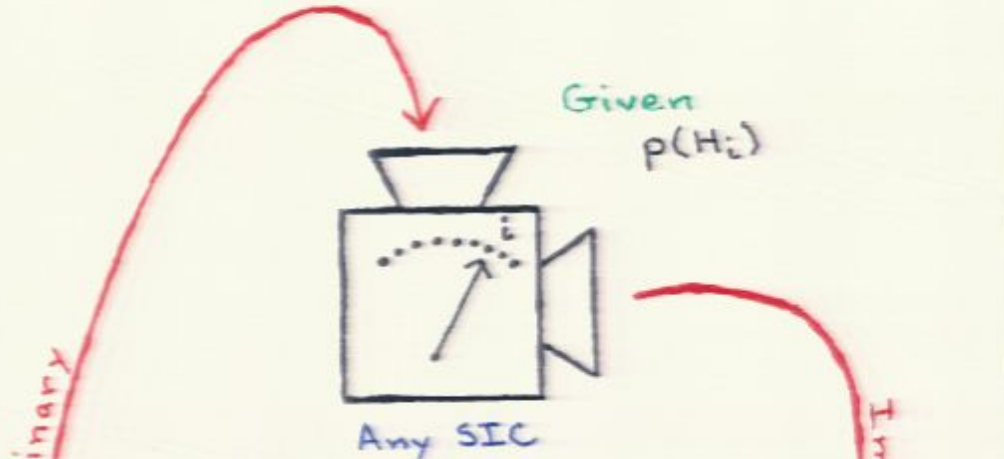









$p(D_j) = (d+1) \sum_i p(H_i) p(D_j|H_i) - 1$
Quantum
(Usual) Bayesian
Magic!


Given
$p(H_i)$
Any SIC
Imaginary
Imaginary


Given
$p(D_j|H_i)$
But really going
to do this.
Any von Neumann
measurement

What $p(D_j)$?

In this case,

$$p(D_j) \neq \sum_i p(H_i)\, p(D_j|H_i) .$$

As Ballentine (1986) points out, there are hidden conditionals

$p(D_j)$ really $p(D_j|C_1)$

$p(H_i)$ really $p(H_i|C_2)$

$p(D_j|H_i)$ really $p(D_j|H_i,C_2)$

Rather, we see the Born rule as an (empirical) addition to Dutch-book coherence.

What is real about a system?

Laws of Probability

H_i – various hypotheses one might have

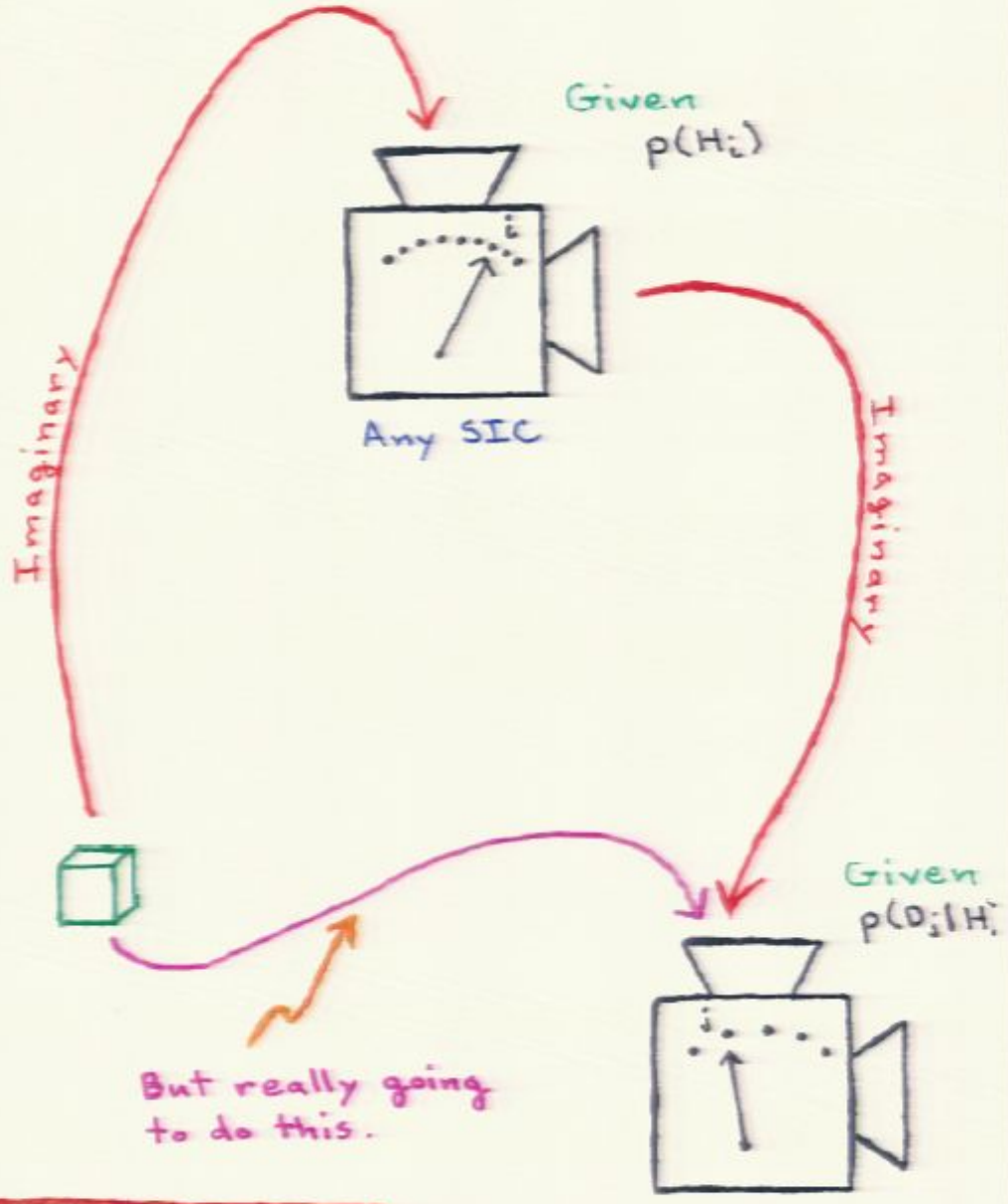
D_j – data values one might gather

Given: $p(D_j|H_i)$ ← expectations for data given hypothesis

$p(H_i)$ ← expectations for hypotheses themselves

Question: What expectations should one have for the D_j?

Answer: $P(D_j) = \sum_i p(H_i)\, p(D_j|H_i)$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Generalized considerations give

$$p(D_j) = (1 + \tfrac{1}{2}qd) \sum_i p(H_i) p(D_j|H_i) - \tfrac{1}{2}q$$

$q = 0, 1, 2, \ldots$
character of the zing
(For QM, $q=2$.)

$d = 2, 3, 4, \ldots$
value of a (local) beable, how much zing

Certainty

What means probability 1?

It means one will buy or sell
a lottery ticket

Worth #1 if E

for in fact #1, full stop.

That is all it means.

Probability 1 $\not\Rightarrow$ { truth
pre-existent truth
isolated truth

Certainty

"Certainty is as it were a tone of voice in which one declares how things are, but one does not infer from the tone of voice that one is justified."

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EPR Criterion of Reality

"If, without in any way disturbing a system [one can gather the information required to] predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

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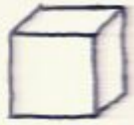
Motivated by EPR

Consider two spatially separated qutrits in a maximally entangled state:

$$|EPR\rangle = \sum_{i=1}^{3} |i\rangle|i\rangle$$



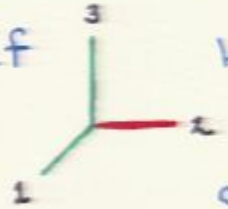
Assume locality.

Now measure the left one any way you like. Say with A or B, two nondegenerate noncommuting observables.

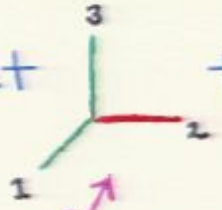
EPR Criterion of Reality

"If, without in any way disturbing a system [one can gather the information required to] predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

So measurement is simple revelation after all?

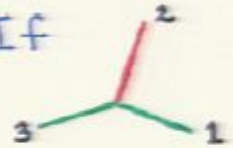
If 

here,

can predict 

there.

element of reality

If 

here,

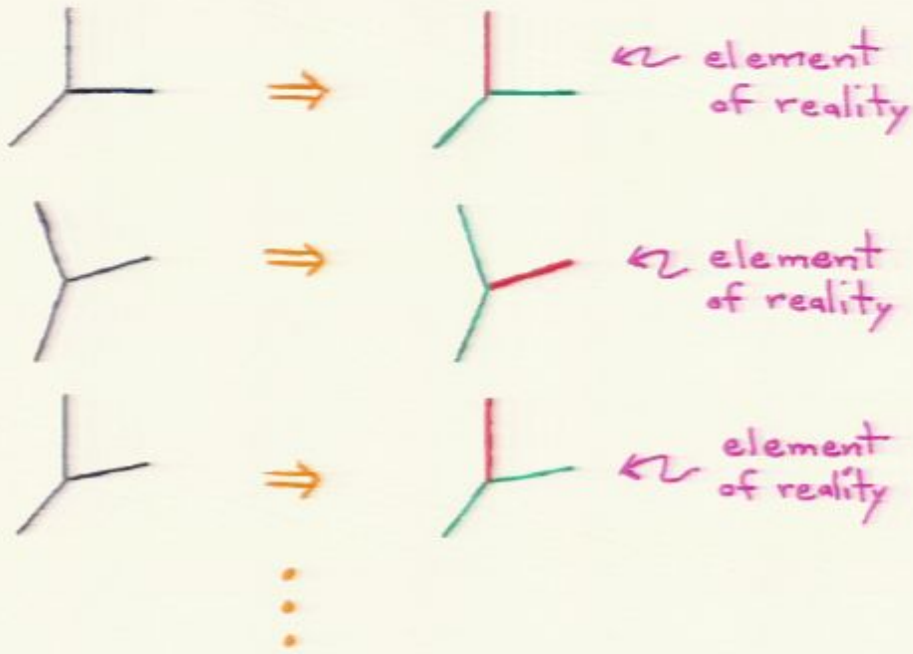
can predict 

there.

element of reality

EPR Still Implodes

But must consider many more bases than two. (~44-46)

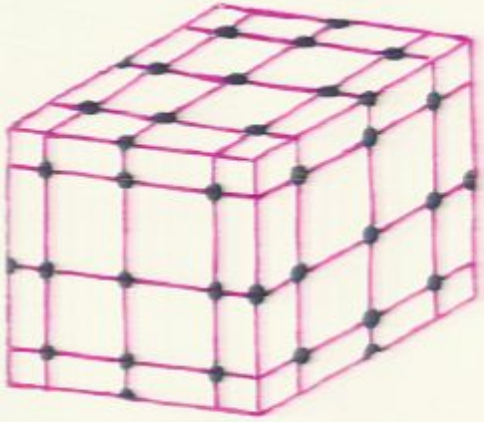


Until contradiction.

(Hint, think of Kochen-Specker.)

Kochen - Specker

Cannot be colored:



33 rays, Peres

(when completed into full triads, consists of 40 triads made from 57 rays)

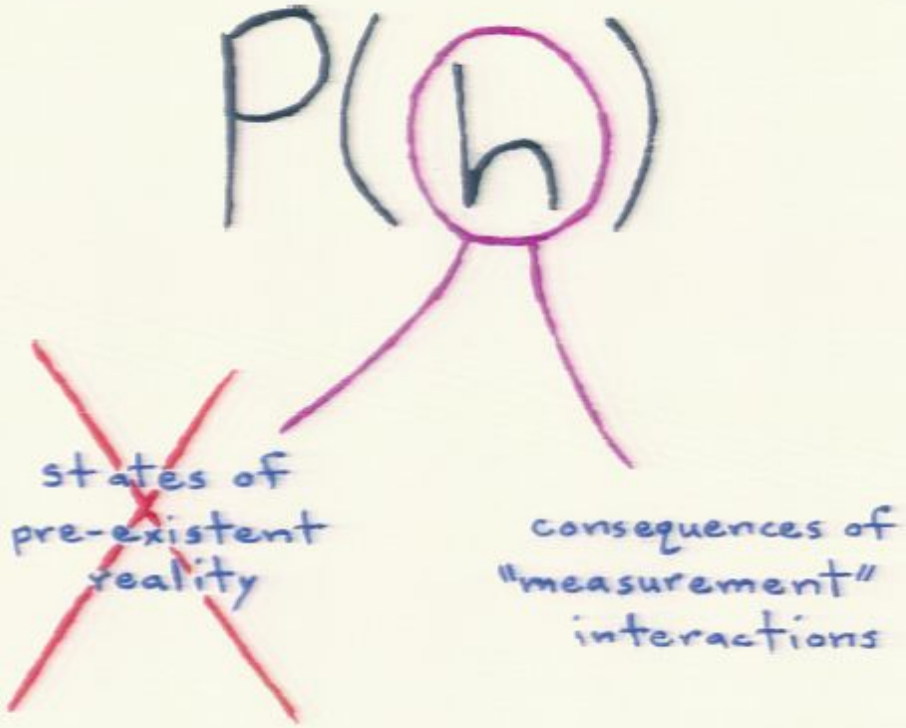
"Measurement"

Does it reveal a pre-existing, but unknown, value?

or

Does it in some sense go toward creating the very value?

$P(h)$


P(h)
states of pre-existent reality
consequences of "measurement" interactions

Example 1: The Pure Einstein

Alice measures one half of an EPR pair, updating to $|\psi\rangle$ for the other side.

All it means is if she were to walk to the other side and measure $\{|\psi\rangle\langle\psi|, I - |\psi\rangle\langle\psi|\}$ she would gamble her life on getting outcome $|\psi\rangle\langle\psi|$.

Example 2: Scenario of Bell Inequality Tests

Alice and Bob set out to demonstrate Bell inequality violations.

Alice believes quantum mechanics.

Alice's beliefs evolve:

initial $|EPR\rangle\langle EPR| \otimes \rho_{Bob} \equiv \rho_0$

believing Bob interacts with his qubit

$\rightarrow (I \otimes U_{QB})\, \rho_0\, (I \otimes U_{QB}^{\dagger})$

she measures, updates Bob & his qubit

$\rightarrow$ ρ_{QB} generally entangled

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Example 2 cont.

Where in Alice's beliefs (i.e. quantum states) is any notion of clicks on Bob's side?

Her quantum states do not pierce into those systems.

They only refer to what she believes will be the consequences of her later interactions with Bob.

This work was supported in part by the U. S. Office of Naval Research,

Grant N00014-09-1-0247.

What are you trying to accomplish?

The value of a [pluriverse], as compared with a universe, lies in this, that where there are cross-currents and warring forces our own strength and will may count and help decide the issue; it is a world where nothing is irrevocably settled, and all action matters. A monistic world is for us a dead world; in such a universe we carry out, willy-nilly, the parts assigned to us by an omnipotent deity or a primeval nebula; and not all our tears can wipe out one word of the eternal script. In a finished universe individuality is a delusion; "in reality," the monist assures us, we are all bits of one mosaic substance. But in an unfinished world we can write some lines of the parts we play, and our choices mould in some measure the future in which we have to live. In such a world we can be free; it is a world of chance, and not of fate; everything is "not quite"; and what we are or do may alter everything.

— Will Durant, on my favorite philosophical vision

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A Very Fundamental Mmt ?

Caves, 1999
Zauner

Suppose d^2 projectors $\Pi_i = |\psi_i\rangle\langle\psi_i|$ satisfying

$$\mathrm{tr}\,\Pi_i\Pi_j = \frac{1}{d+1} \quad , \quad i \neq j$$

exist. ← Called SIC.

Can prove:

1) the Π_i linearly independent

2) $\sum_i \frac{1}{d}\Pi_i = I$

So good for Bureau of Standards.

Also

$$p(i) = \frac{1}{d}\,\mathrm{tr}\,\rho\Pi_i$$

$$\boxed{\rho = \sum_i \left[(d+1)p(i) - \frac{1}{d}\right]\Pi_i}$$

The Born Rule

Given ρ and $\{E_i\}$,

ρ: quantum state

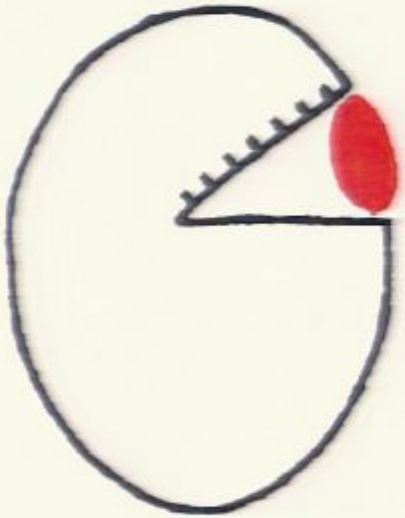
$\{E_i\}$: POVM measurement

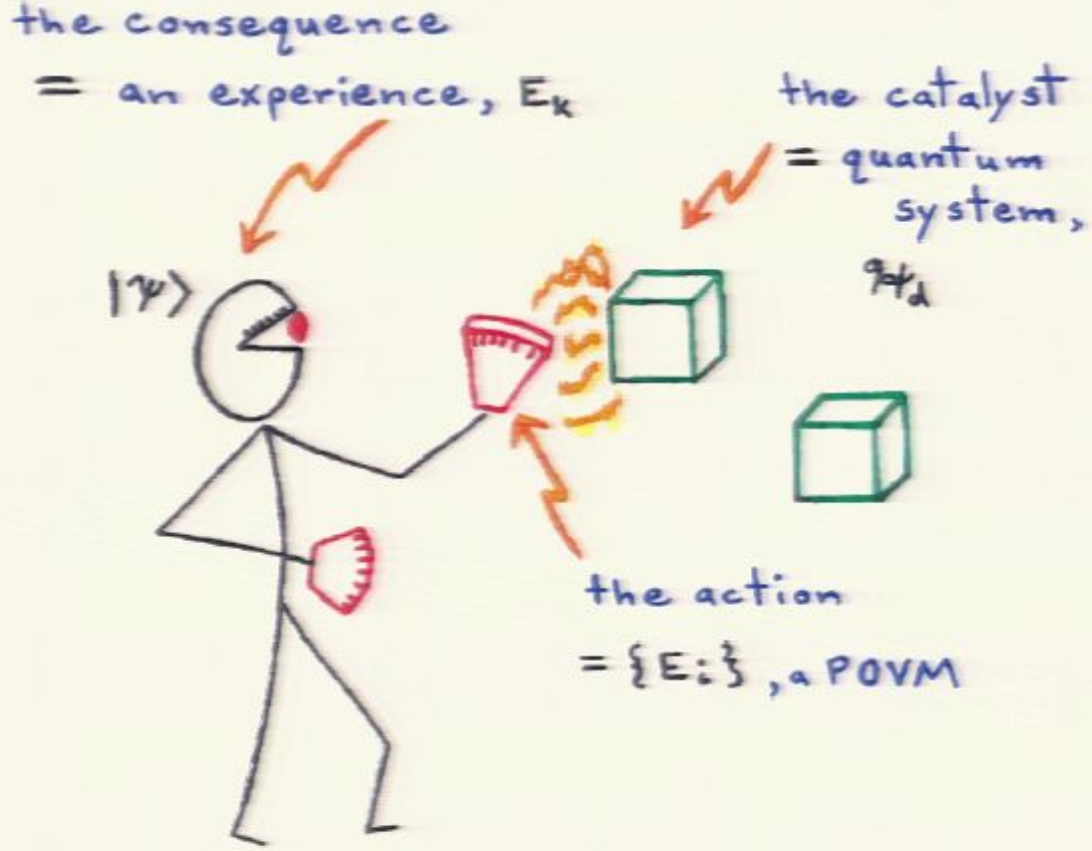
$$p(i) = \mathrm{tr}\, \rho E_i$$

"The Born Rule"

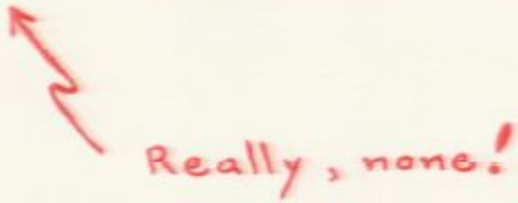
NOT a law of nature.

RATHER something we should strive for.




the consequence
= an experience, E_k
the catalyst
= quantum system, $\mathcal{H}_d$
$|\psi\rangle$
the action
= $\{E_i\}$, a POVM

A superior statement about the objective characteristics of our quantum world, of the things in it, would contain no $|\psi\rangle$'s at all.

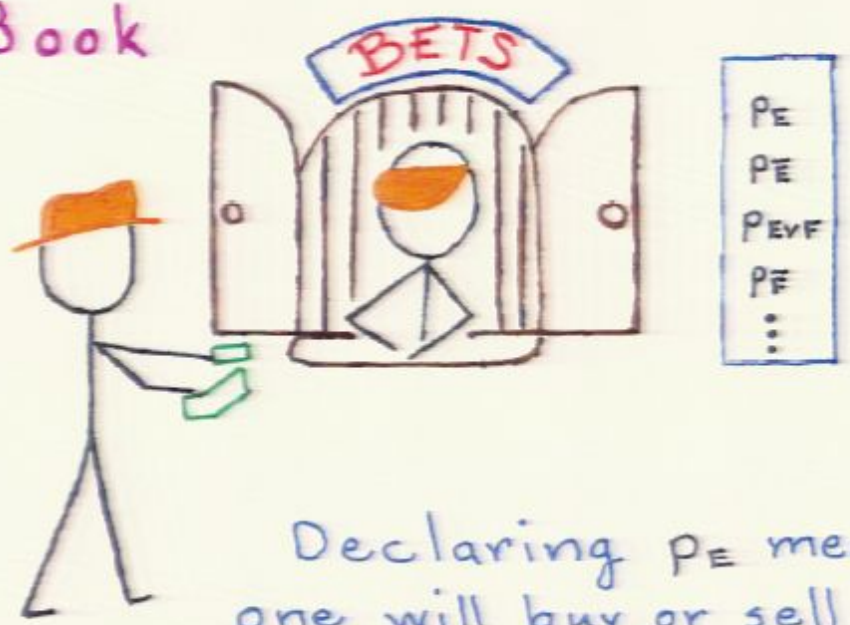


I think there are professional problems [with quantum mechanics]. That is to say, I'm a professional theoretical physicist and I would like to make a clean theory. And when I look at quantum mechanics I see that its a dirty theory. The formulations of quantum mechanics that you find in the books involve dividing the world into an observer and an observed, and you are not told where that division comes ... So you have a theory which is fundamentally ambiguous ...

— J. S. Bell

Defining Probability

Dutch Book



Declaring p_E means one will buy or sell a lottery ticket

Worth $1 if E

for $\$p_E$.

Laws of Probability

H_i – various hypotheses one might have

D_j – data values one might gather

Given: $p(D_j|H_i)$ ← expectations for data given hypothesis

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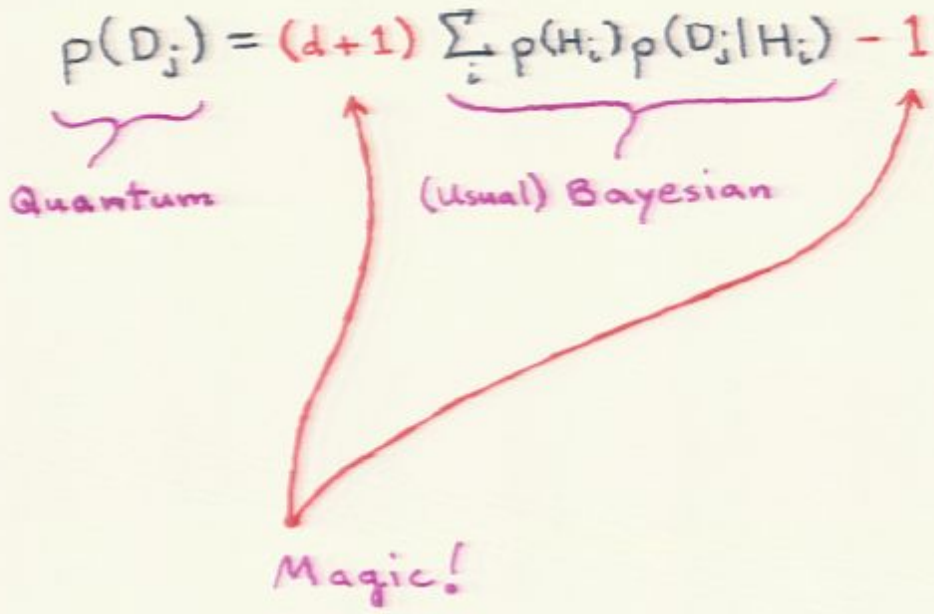
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Quantum
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