

Title: What are the costs of dealing with "states of reality" in quantum theory?

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Abstract: Bell and experimental tests of his inequality showed that it is impossible to explain all of the predictions of quantum mechanics using a theory which satisfies the basic concepts of locality and realism, but which (if not both) is violated is still an open question. As it seems impossible to resolve this question experimentally, one can ask how plausible realism -- the idea that external properties of systems exist prior to and independent of observations -- is, by considering the amount of resources consumed by itself and its non-local features. I will construct an explicit realistic model in which the number of hidden-variable states scales polynomially with the number of possible quantum measurements. In the limit of a large number of measurements, the model recovers the result of Montina, that no hidden-variable theory that agrees with quantum predictions could use less hidden-variable states than the straightforward model in which every quantum state is associated with one such hidden state. Thus, for any given system size, realistic theories cannot describe nature more efficiently than quantum theory itself. I will then turn to the problem of "non-locality" in realistic theories showing that every such theory that agrees with quantum predictions allows superluminal signaling at the level of hidden variable states.



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Faculty of Physics, University of Vienna
& Institute of Quantum Optics and Quantum Information (IQOQI)

What are the costs of dealing with „states of reality“ in quantum theory?

Časlav Brukner

MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in

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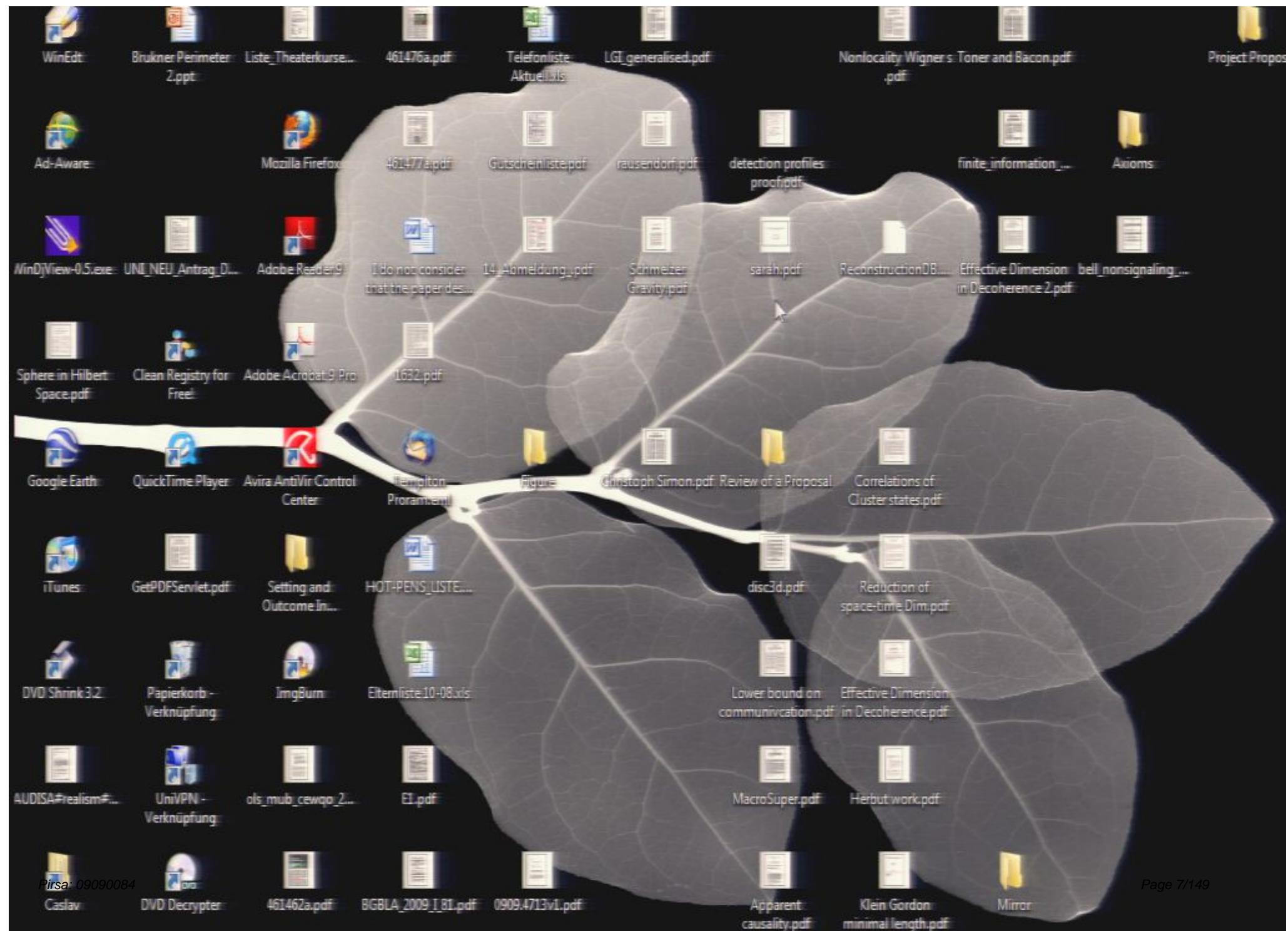


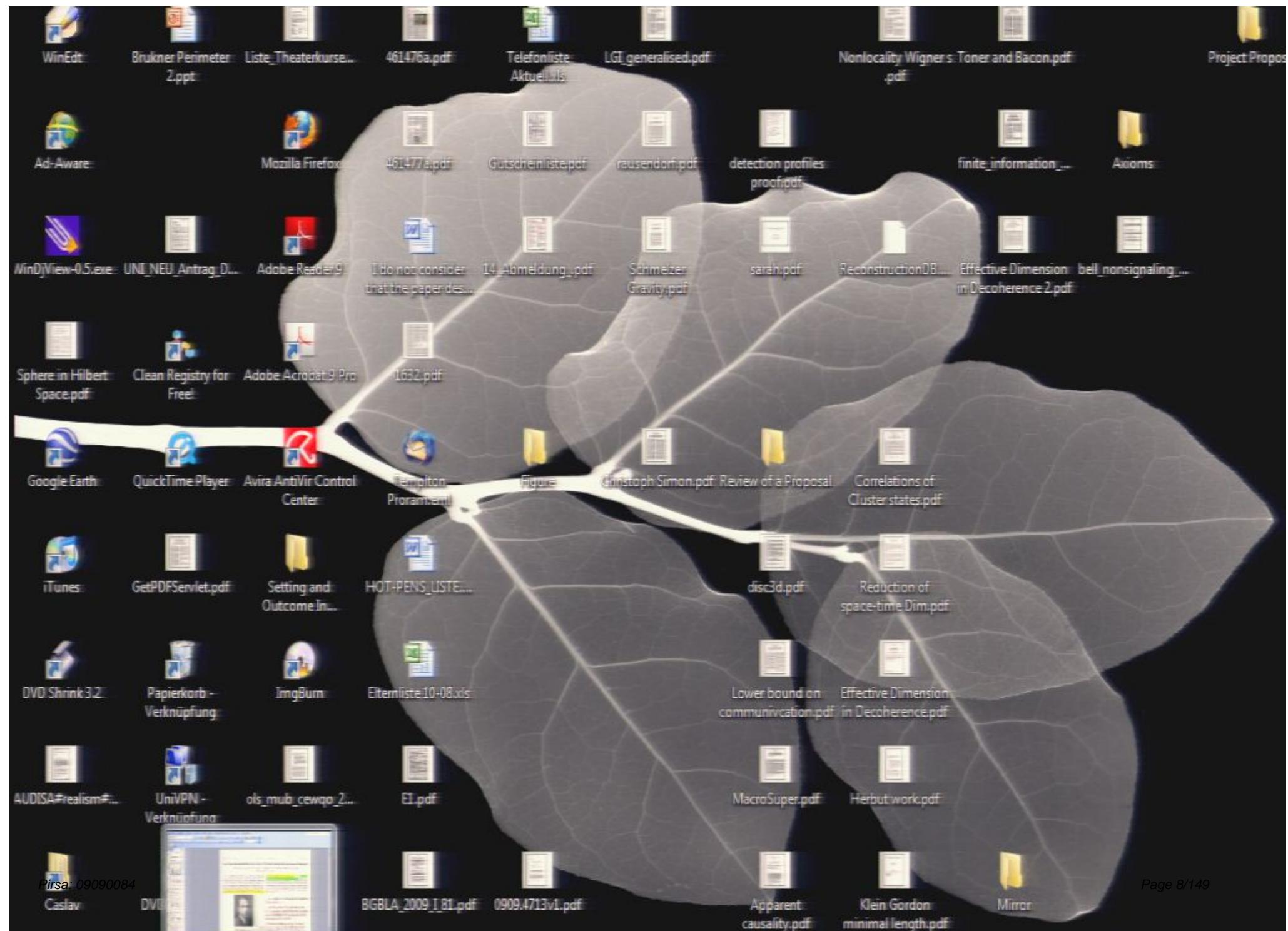
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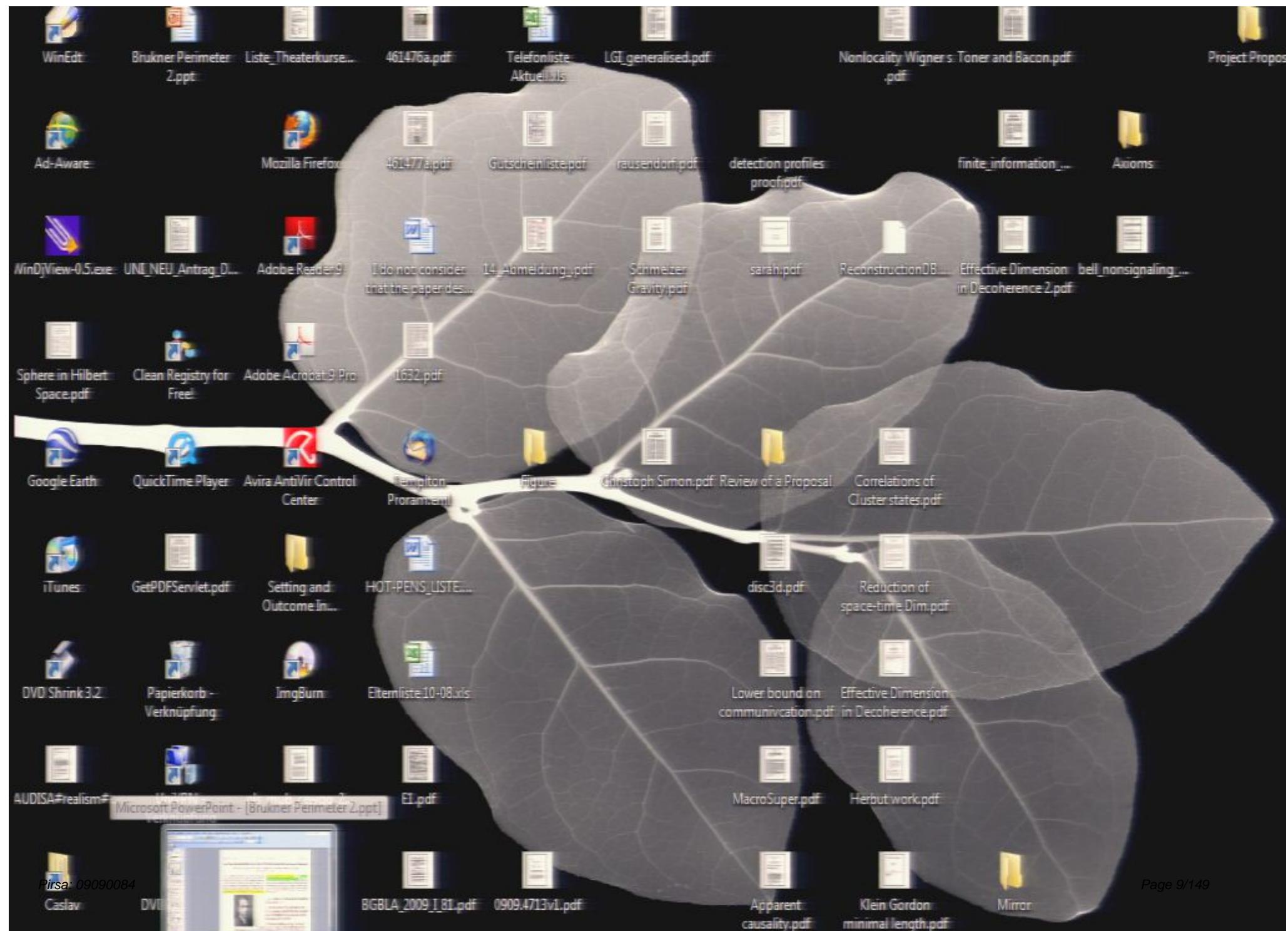
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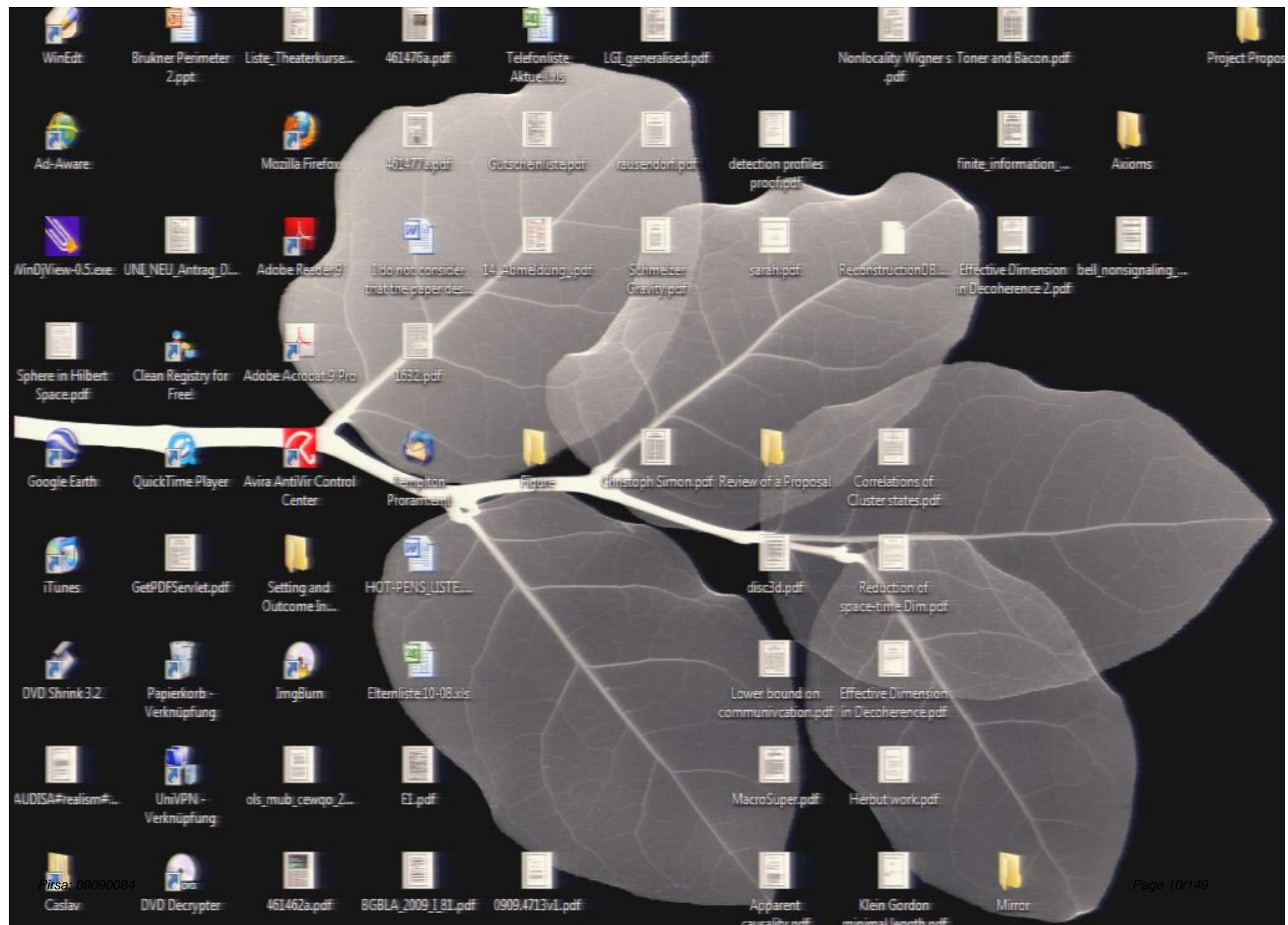
„Truth and clarity are complementary.“

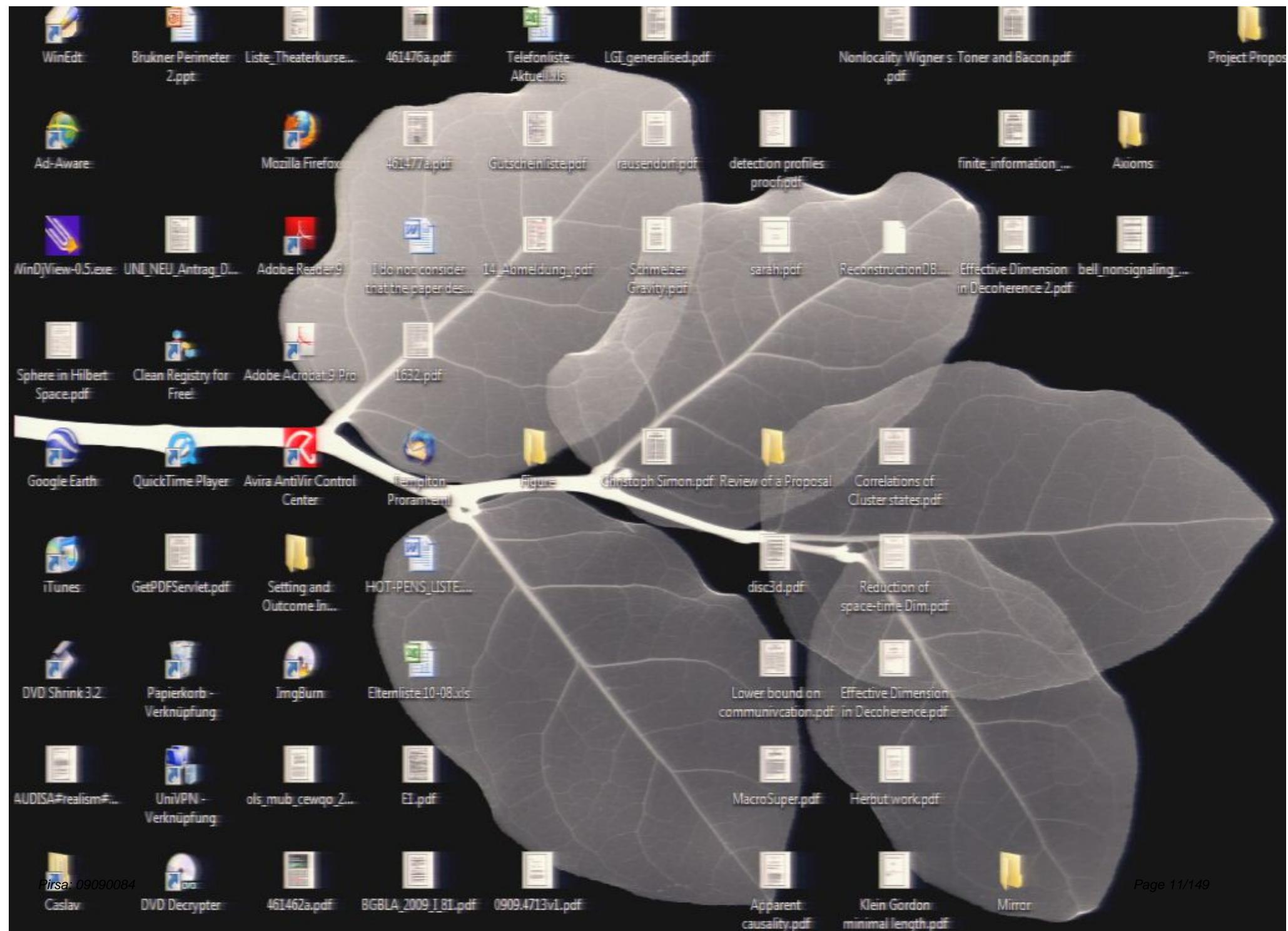
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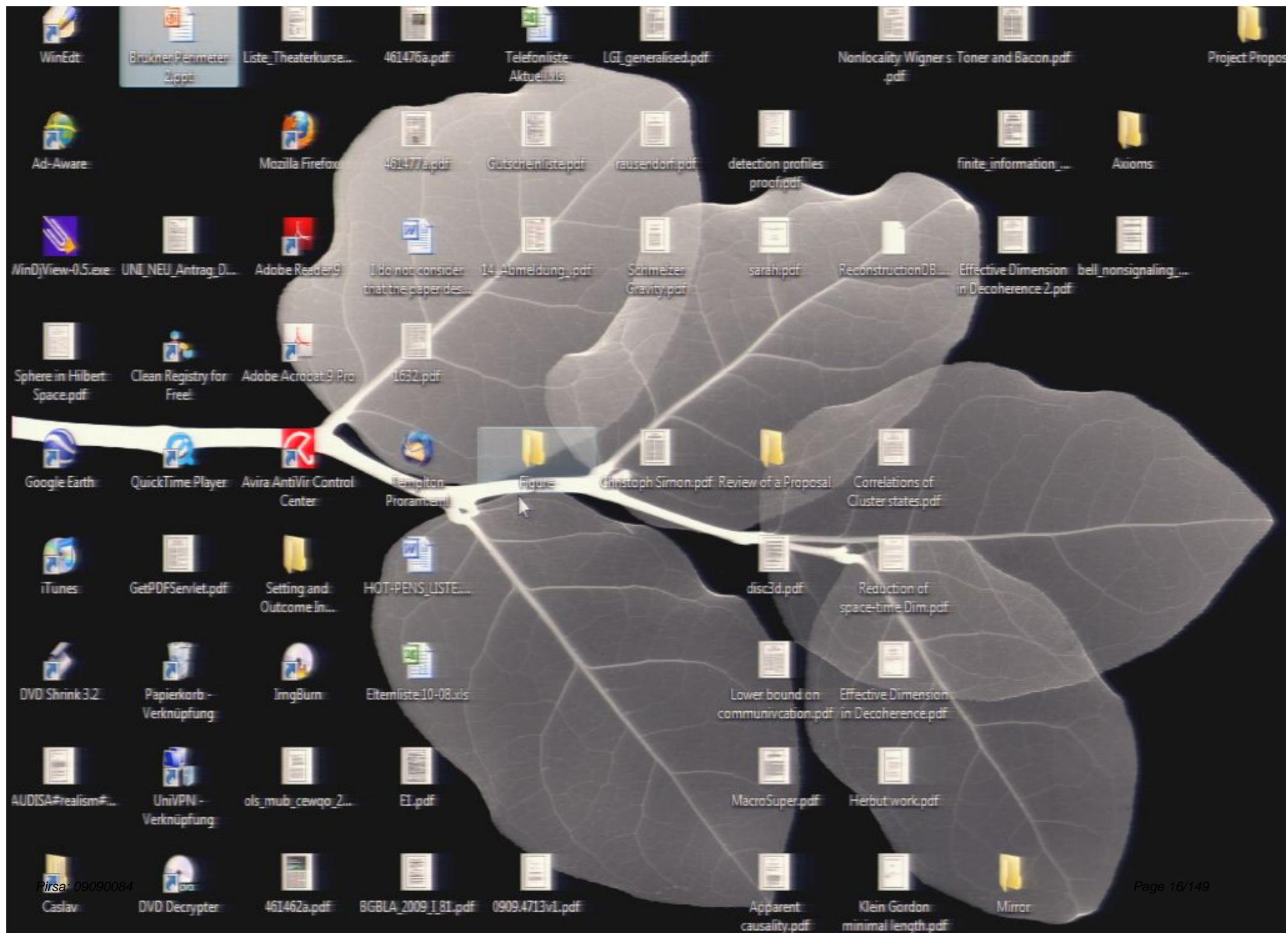
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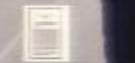
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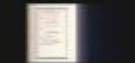
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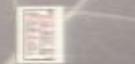
Axioms



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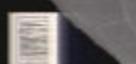
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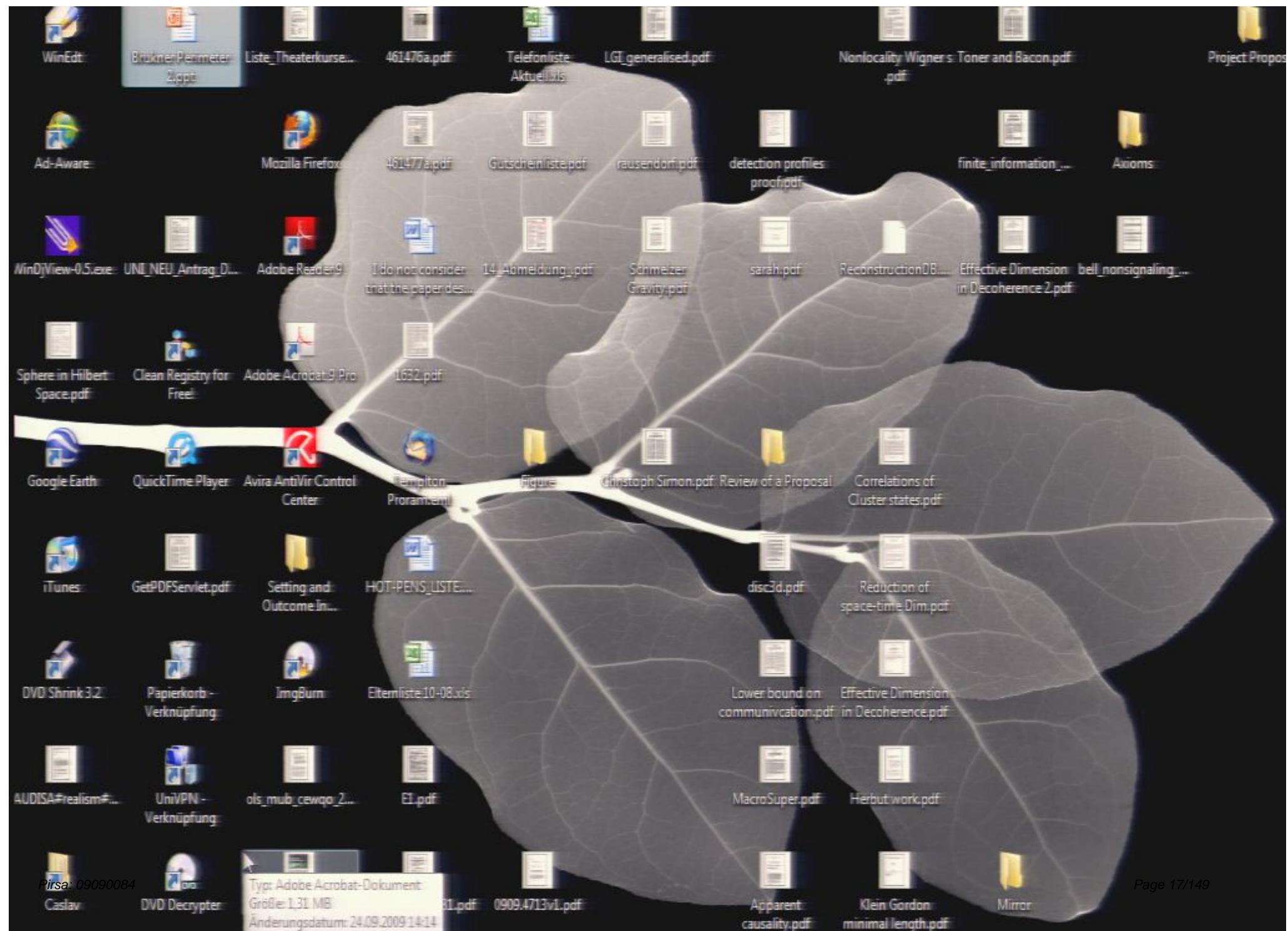
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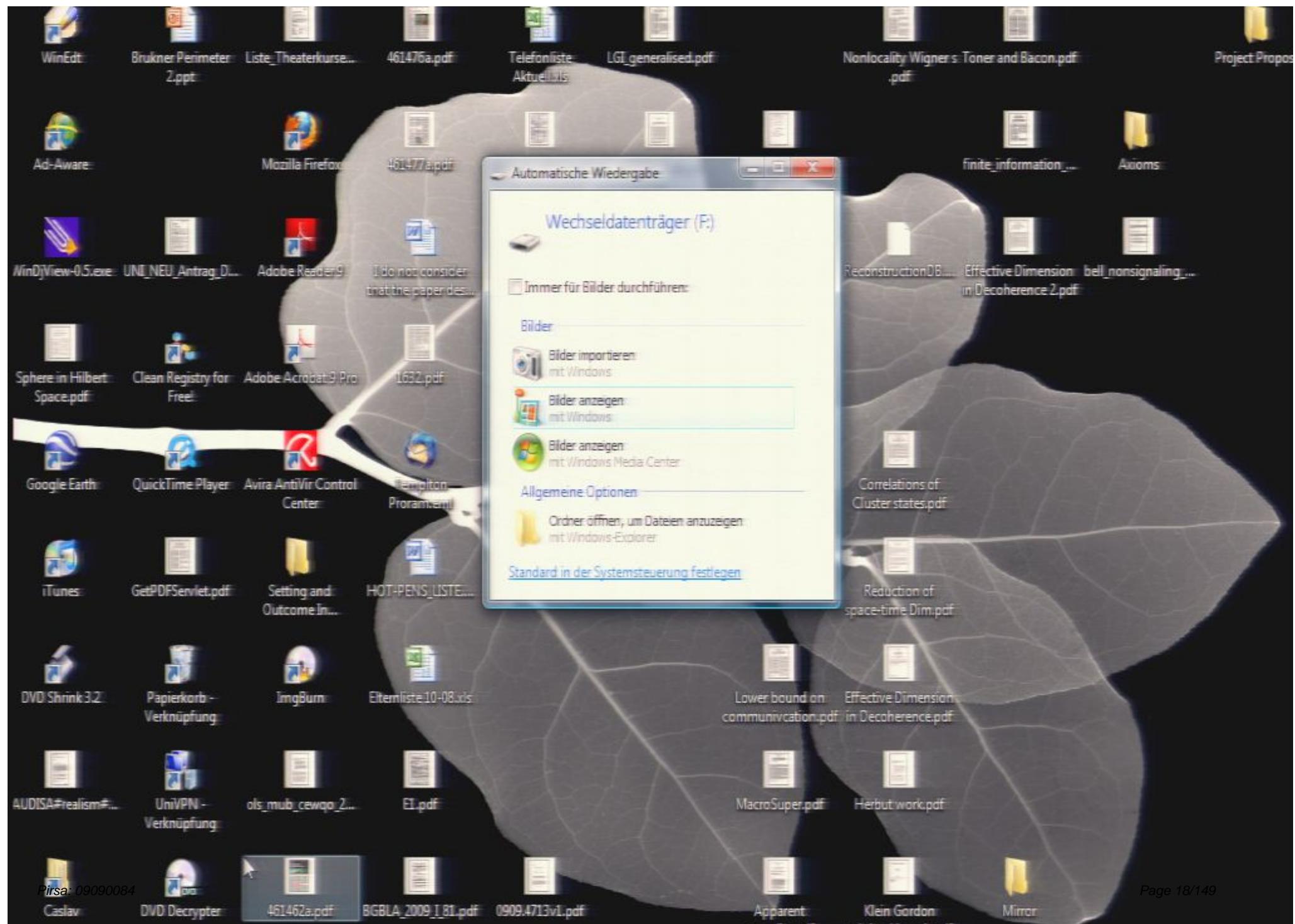


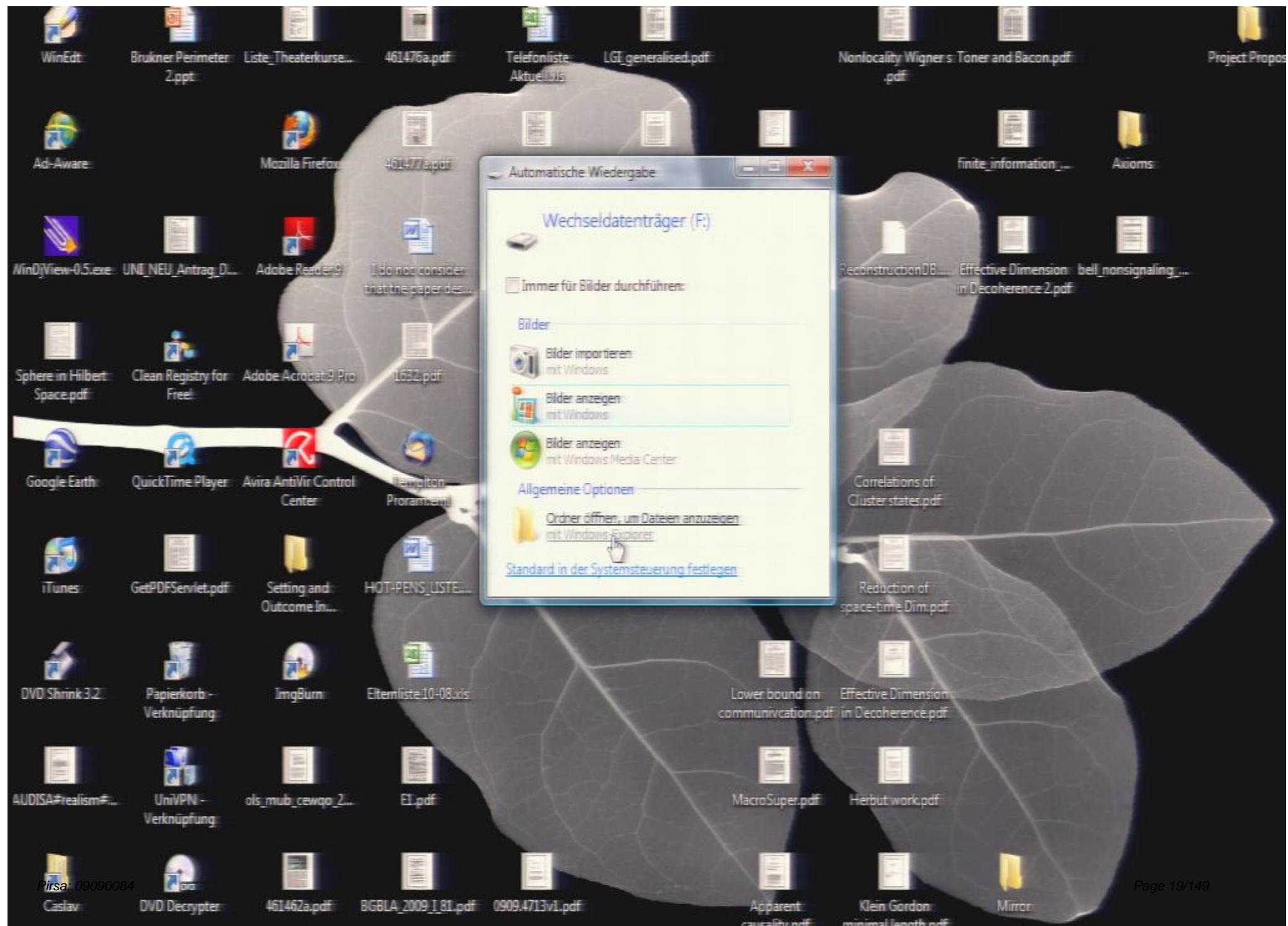
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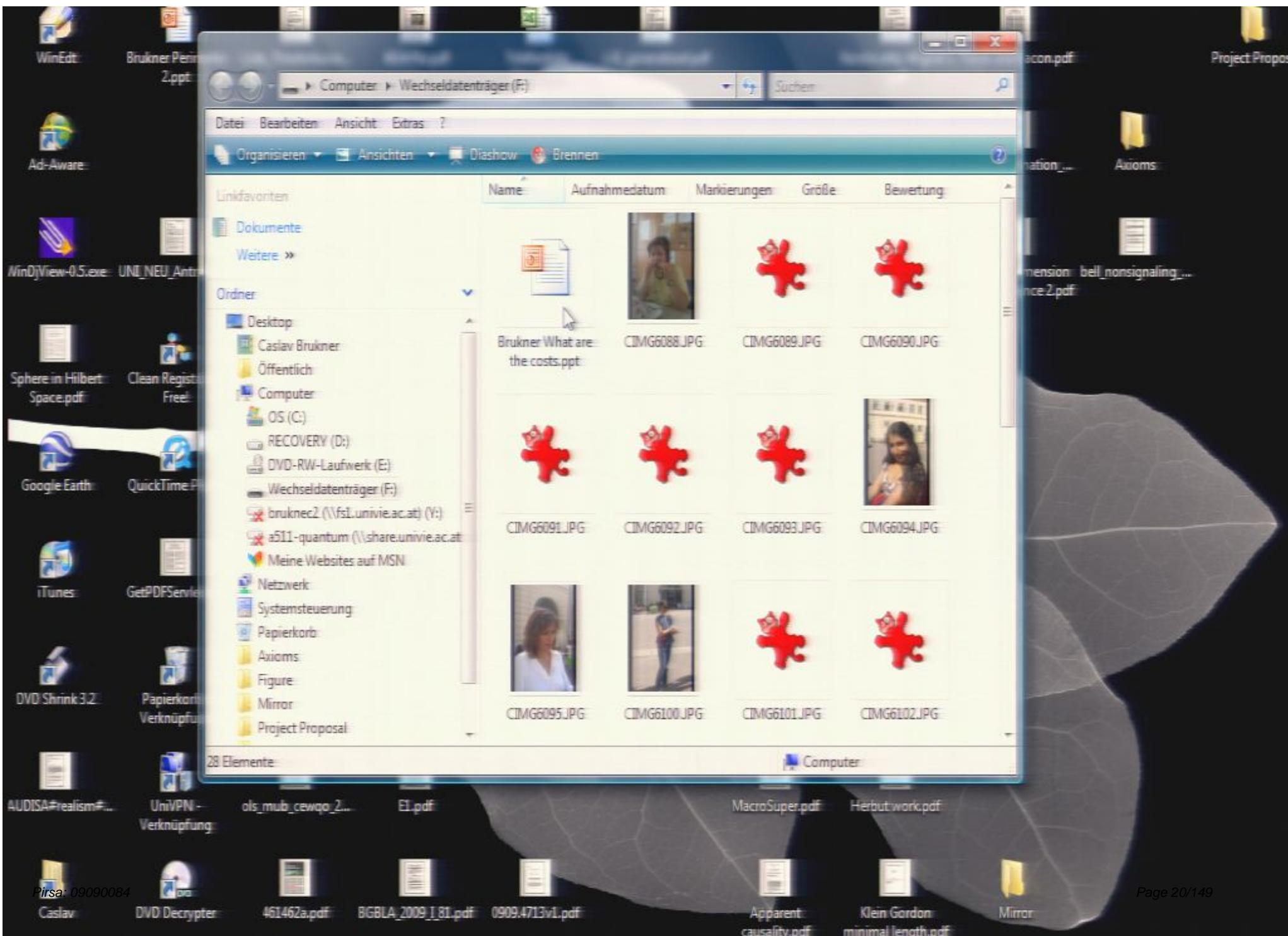
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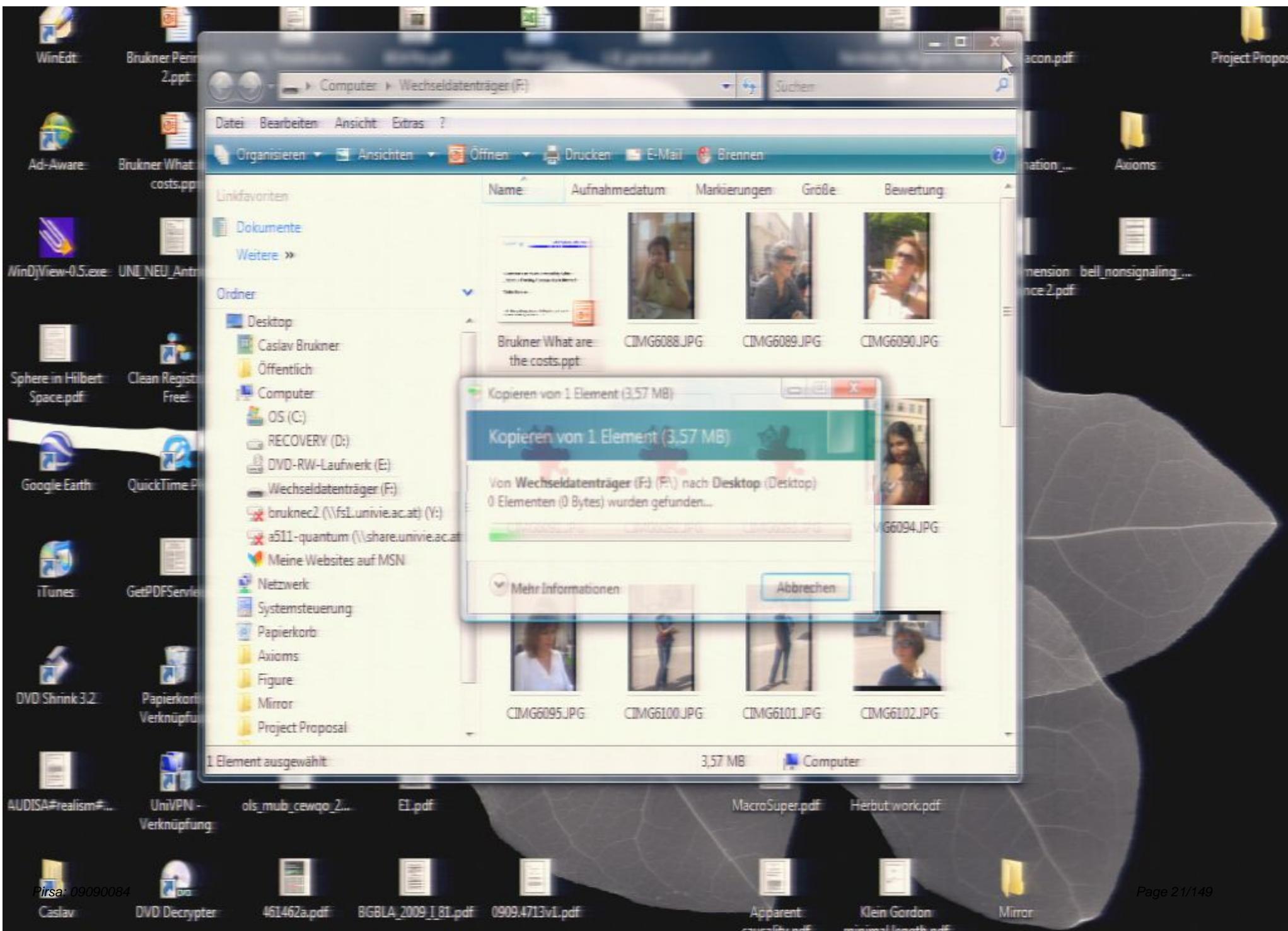
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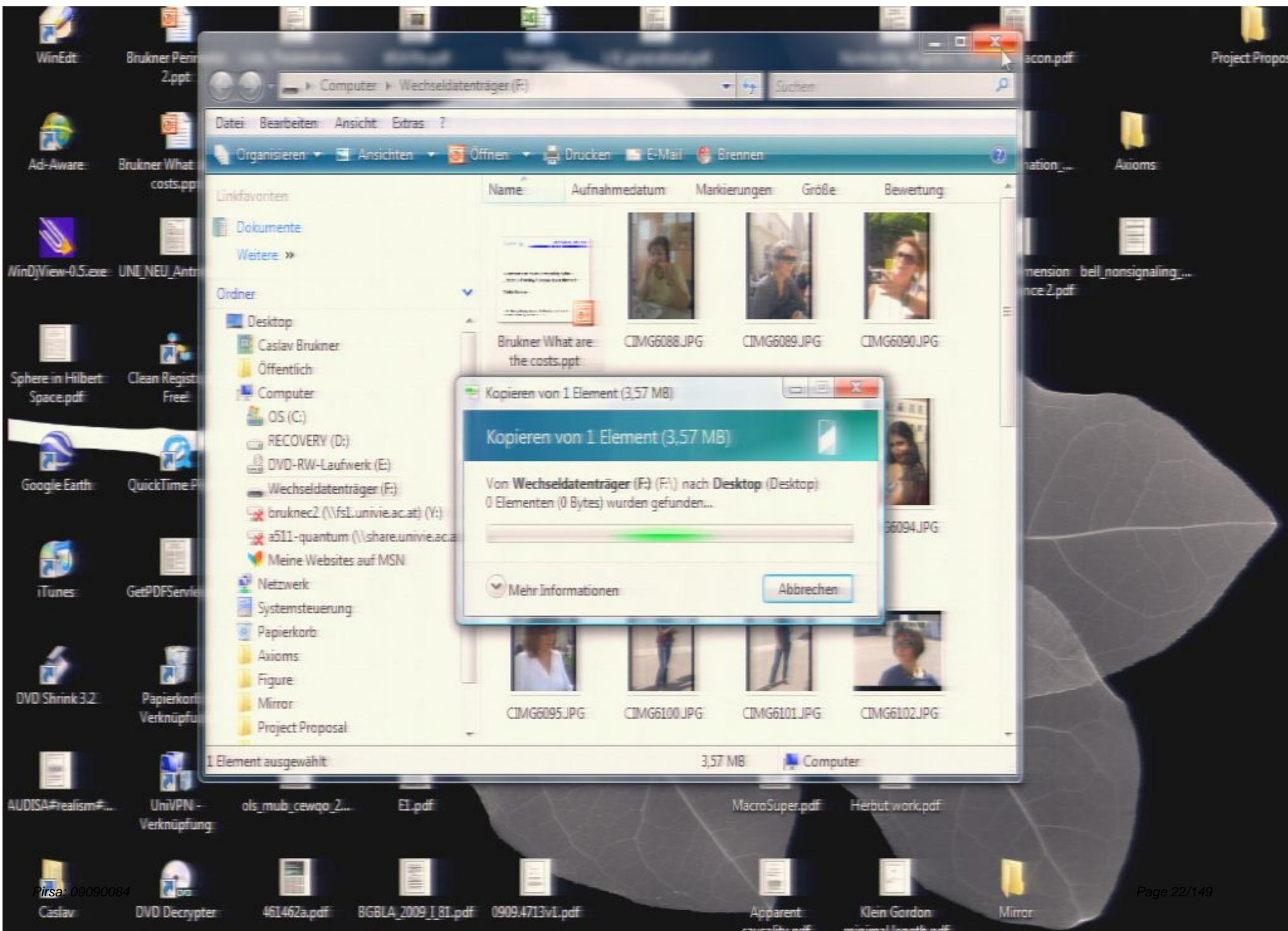


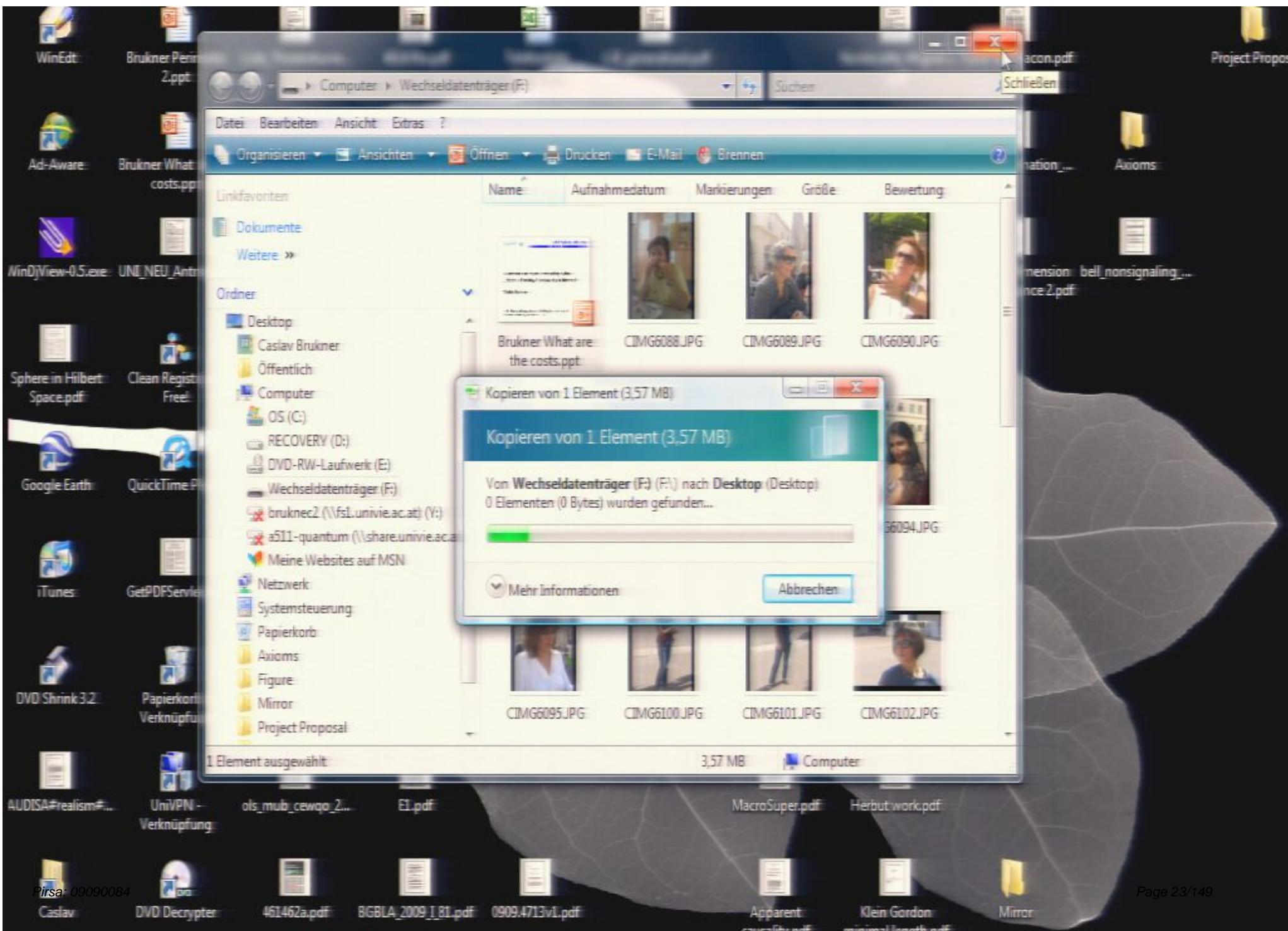


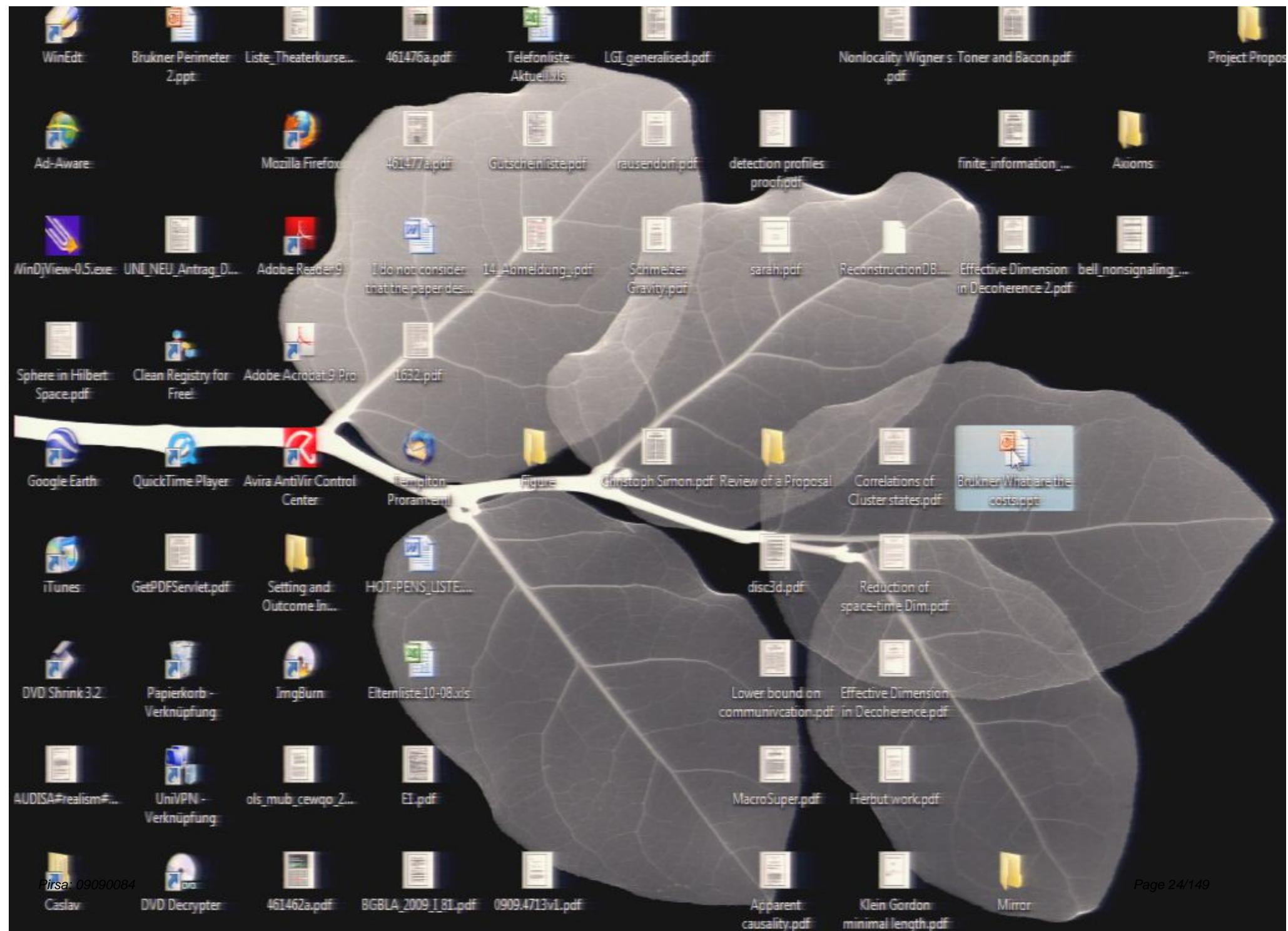














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with Borivoje Dakic, Johannes Kofler, Tomasz Paterek,
Marcin Pawłowski, Milovan Šuvakov

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„Epistemic“

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But ...

Questions for „ontic“ interpretation

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- What is the meaning of complex multidimensional wave function?

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- „Collaps“ of the wave function

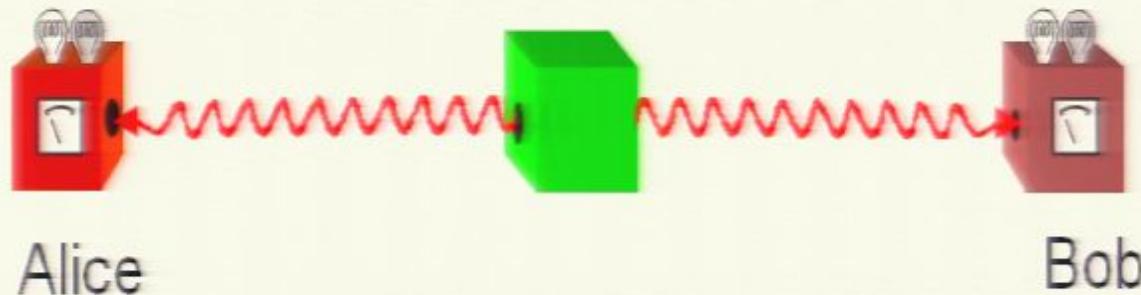
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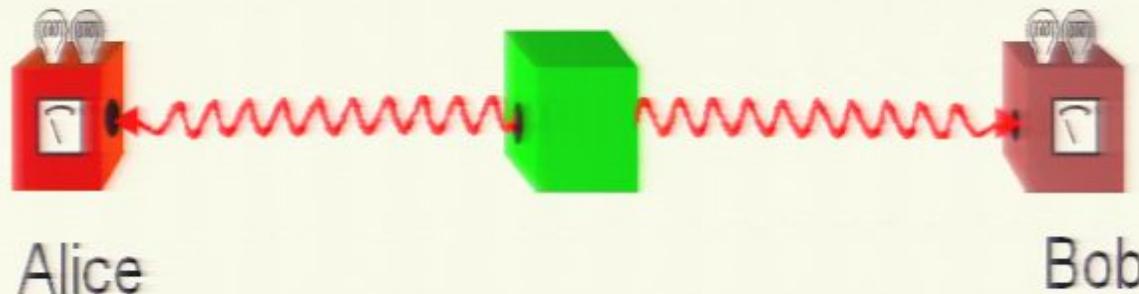
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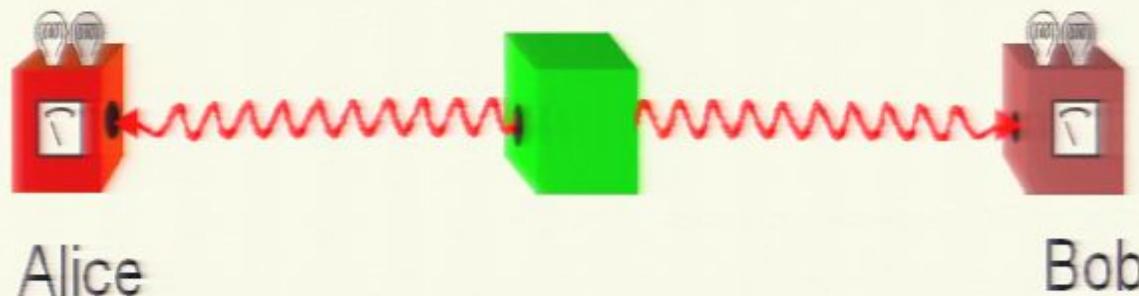


Ref. Frame 1: A's choice of setting before B's outcome
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- Who „really“ influenced whom?
- And in which reference frame?

Questions for „probabilistic“ interpretation

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Questions for „probabilistic“ interpretation



„What exactly qualifies some physical systems to play the role of „measurer“? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system ... with a PhD?“

John. S. Bell, Against "Measurement"

Questions for „epistemic“ interpretation

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What is distributed here?

What are states of some „deeper reality“?

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Hidden Variable Program

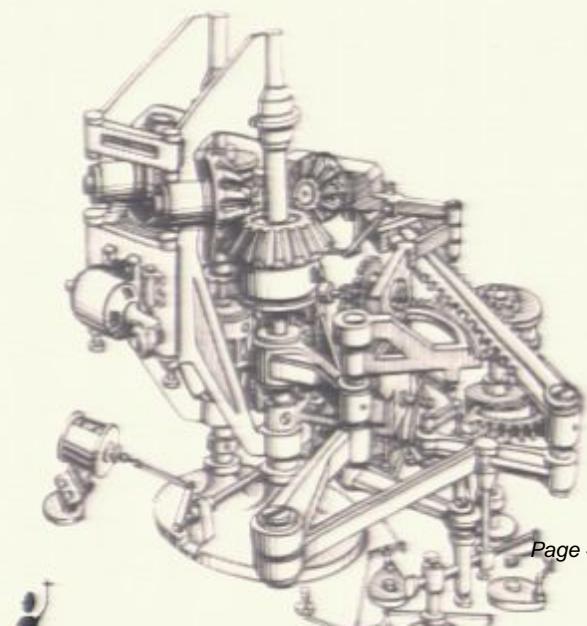
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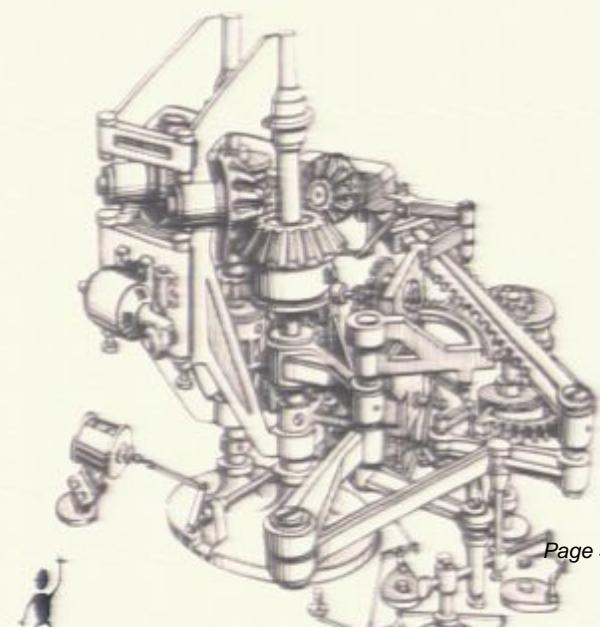
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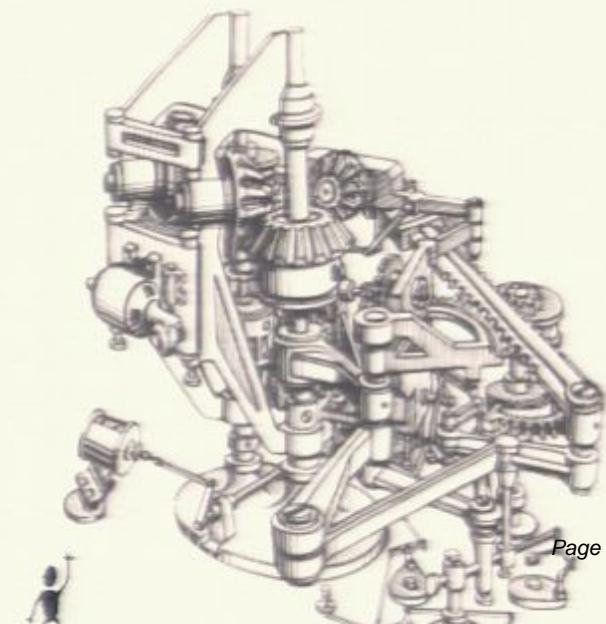
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→ End of the road?



This talk

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Less than $\text{Exp}(N)$?

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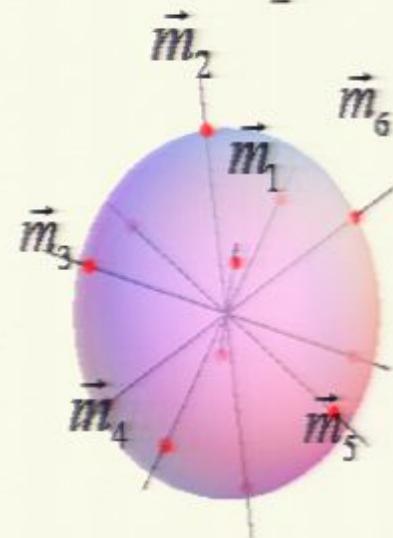
Does the number $\Omega(N)$ of HV scale with the number of measurements N ?
Less than $\text{Exp}(N)$?

2. How “non-local” is a non-local realistic theory?

“Bell’s Non-locality” \neq Superluminal signalling?

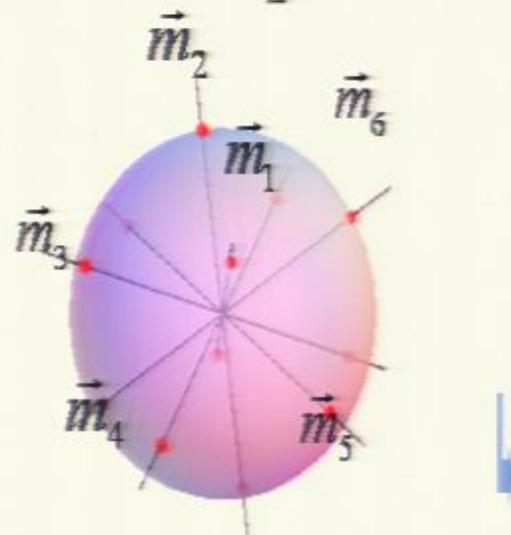
Single Qubit

$$\vec{x} \in S^2, \quad p_i = \frac{1}{2}(1 + \vec{x}\vec{m}_i)$$



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Measurement directions

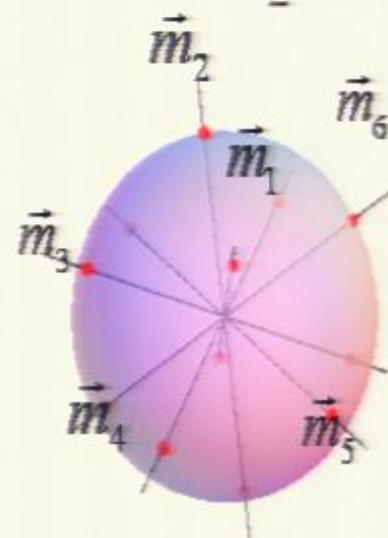
$$\vec{m}_1 \vec{m}_2 \vec{m}_3 \dots \vec{m}_N$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$\vec{p} = \begin{pmatrix} p_1 & p_2 & p_3 & \dots & p_N \end{pmatrix}^T$$

$$\vec{O}_{101\dots 1} = \begin{pmatrix} 1 & 0 & 1 & \dots & 1 \end{pmatrix}^T$$

$\Omega=2^N$ Hidden-Variable (HV) States

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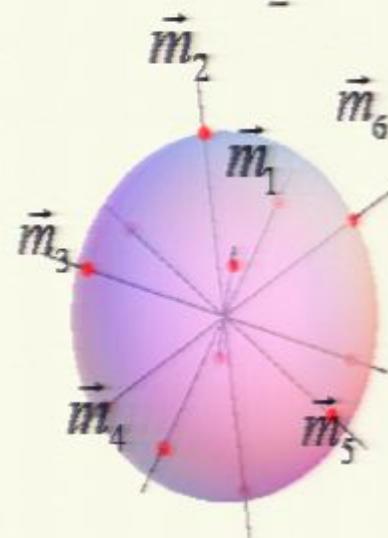
$\Omega=2^N$ Hidden-Variable (HV) States

Any quantum state is
a mixture over HVs:

$$\forall \vec{x} \in S^2, \quad \vec{p}(\vec{x}) = \sum_{i=1}^{\Omega} \alpha_i \vec{O}_i$$

Single Qubit

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Measurement directions

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↓ ↓ ↓ ↓

$$\vec{p} = \begin{pmatrix} p_1 & p_2 & p_3 & \dots & p_N \end{pmatrix}^T$$

$$\vec{O}_{101\dots 1} = \begin{pmatrix} 1 & 0 & 1 & \dots & 1 \end{pmatrix}^T$$

$\Omega=2^N$ Hidden-Variable (HV) States

Any quantum state is
a mixture over HVs:

$$\forall \vec{x} \in S^2, \quad \vec{p}(\vec{x}) = \sum_{i=1}^{\Omega} \alpha_i \vec{O}_i$$

What is the minimal number Ω_{\min} of HVs for all
quantum states (*preparation-universal*) for a
given set of N measurements?

$\Omega_{\min} = ?$

Caratheodory's theorem

A point in a convex polytope in \mathbb{R}^N can be written as a convex combination of $N+1$ vertices

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$$\begin{matrix} \vec{m}_1 & \vec{m}_2 & \vec{m}_3 & \dots & \vec{m}_N \\ \downarrow & \downarrow & \downarrow & & \downarrow \\ p_1 & \leq p_2 & \leq p_3 & \dots & \leq p_N \end{matrix}$$

0	0	0	0	$1 - p_N$
1	1	1	1	p_1
0	1	1	1	$p_2 - p_1$
0	0	1	1	$p_3 - p_2$
⋮	⋮	⋮	⋮	⋮	⋮
0	0	0	1	$p_N - p_{N-1}$

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$$\begin{array}{ccccccc} \vec{m}_1 & \vec{m}_2 & \vec{m}_3 & \dots & \vec{m}_N & \xleftarrow{\text{Measurement Directions}} \\ \downarrow & \downarrow & \downarrow & & \downarrow & \\ p_1 & \leq p_2 & \leq p_3 & \dots & \leq p_N & \end{array}$$

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0	0	0	0
1	1	1	1
0	1	1	1
0	0	1	1
⋮	⋮	⋮	⋮	⋮
0	0	0	1

$$\begin{aligned} 1 - p_N & \quad \uparrow \\ p_1 & \\ p_2 - p_1 & \quad \leftarrow \text{Probabilities for } N+1 \text{ HVs} \\ p_3 - p_2 & \quad \downarrow \\ & \\ p_N - p_{N-1} & \end{aligned}$$

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$$1 - p_N$$
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Probabilities for $N+1$ HVs

Spekkens' toy model

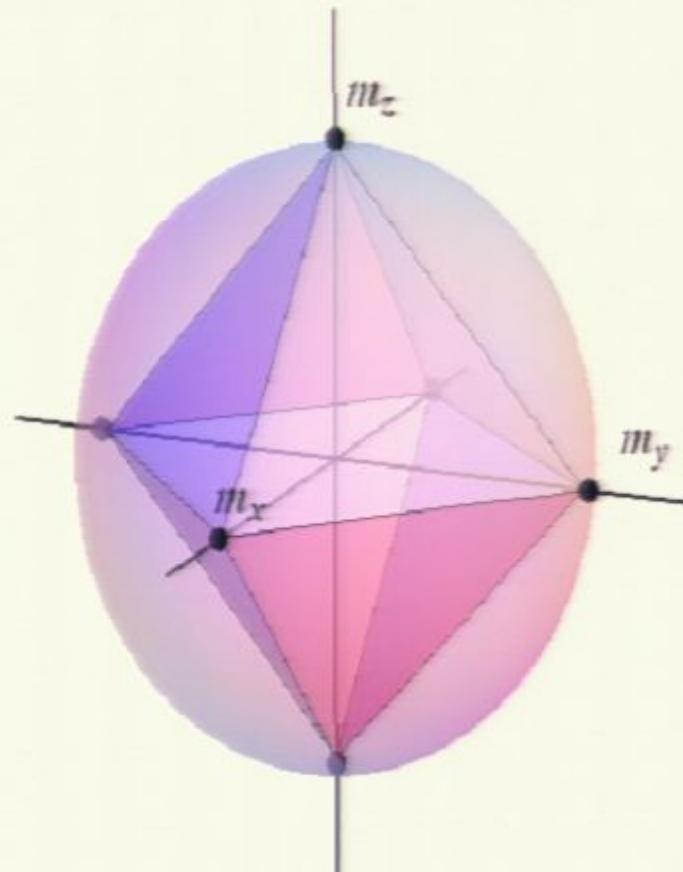
(R. W. Spekkens, Phys. Rev. A, 75 0323110 (2007))

- N=3 complementary measurements (x, y, z)
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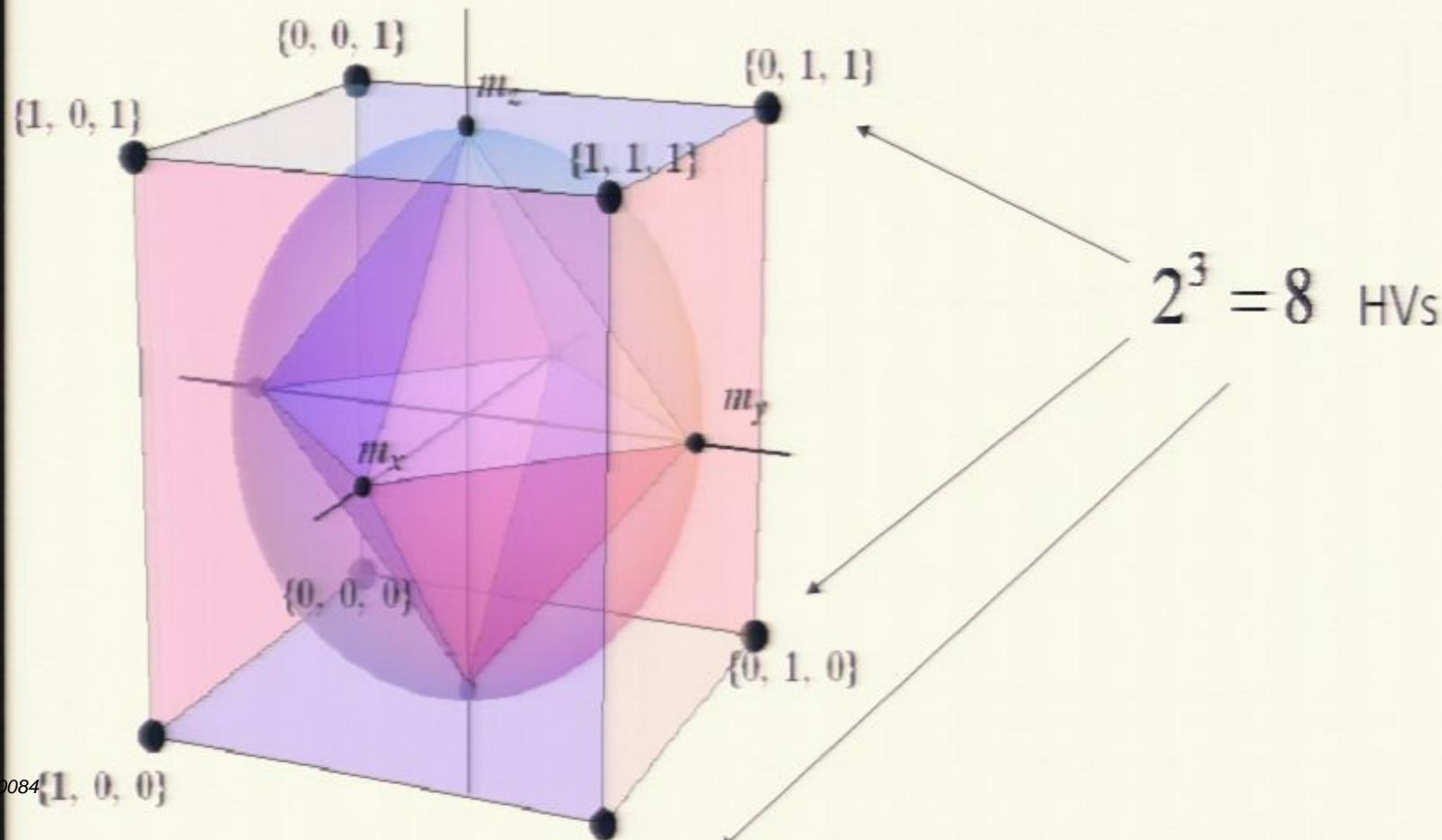
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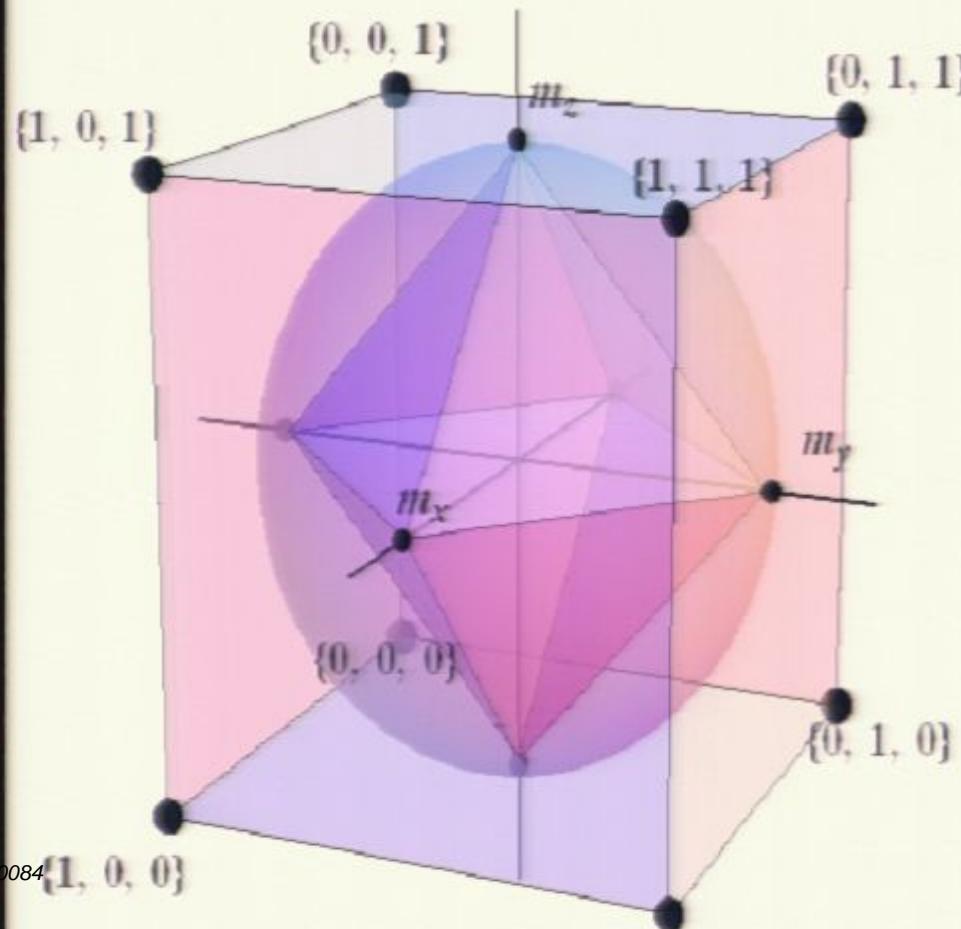
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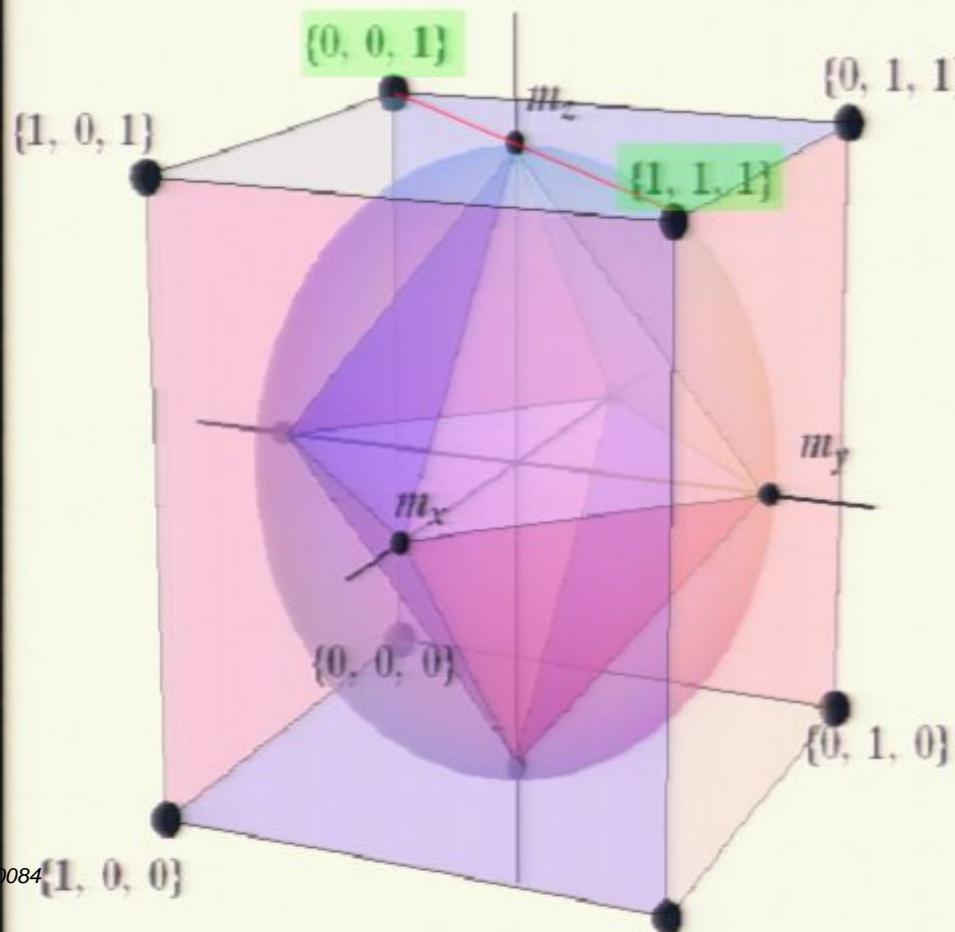
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$\vec{p}(\vec{m}_z)$ as 50-50% mixture

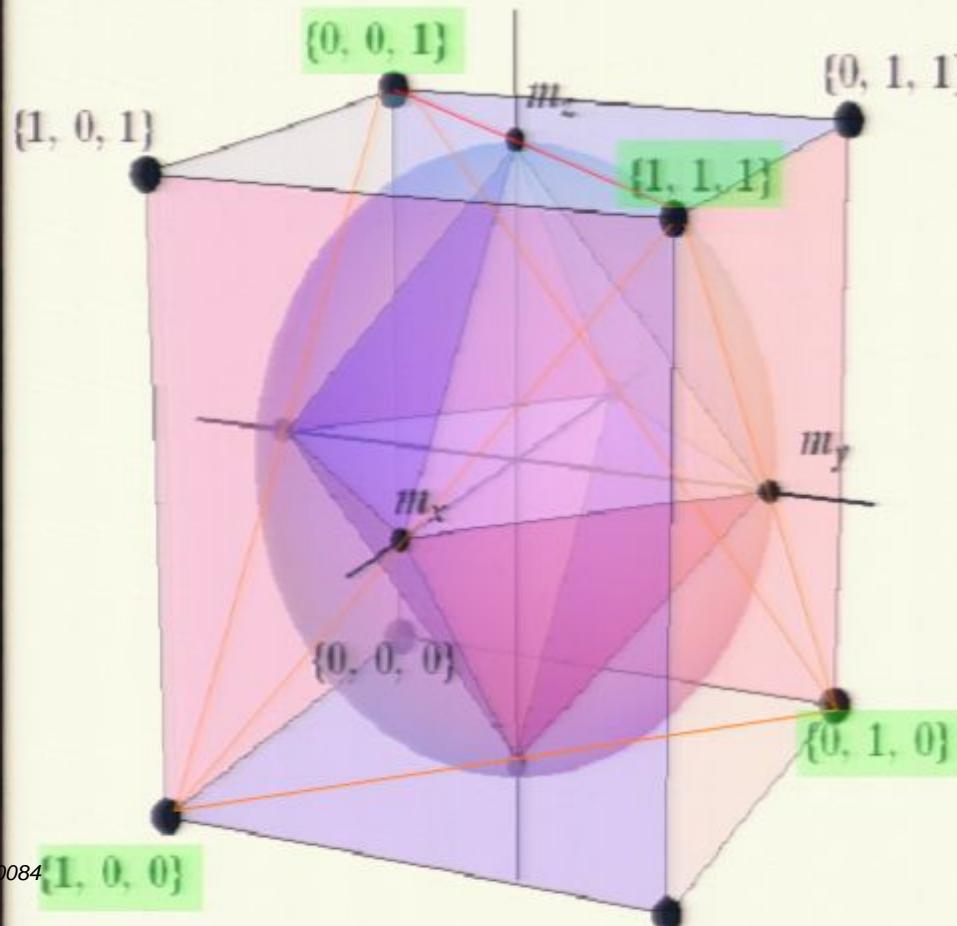
$$\vec{p}(\vec{m}_z) = (p_x(\vec{m}_z), p_y(\vec{m}_z), p_z(\vec{m}_z))$$

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4 HVs needed

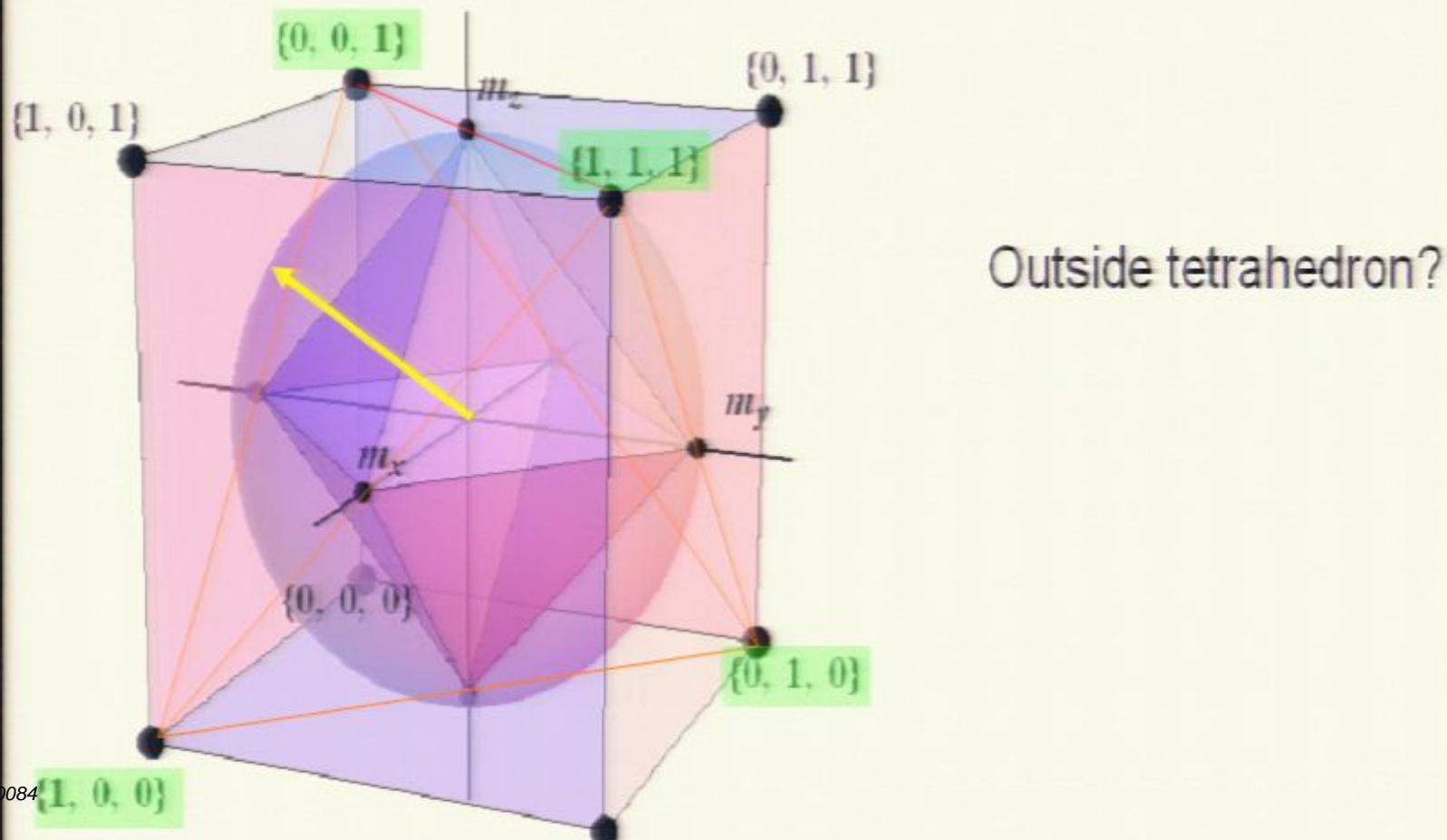
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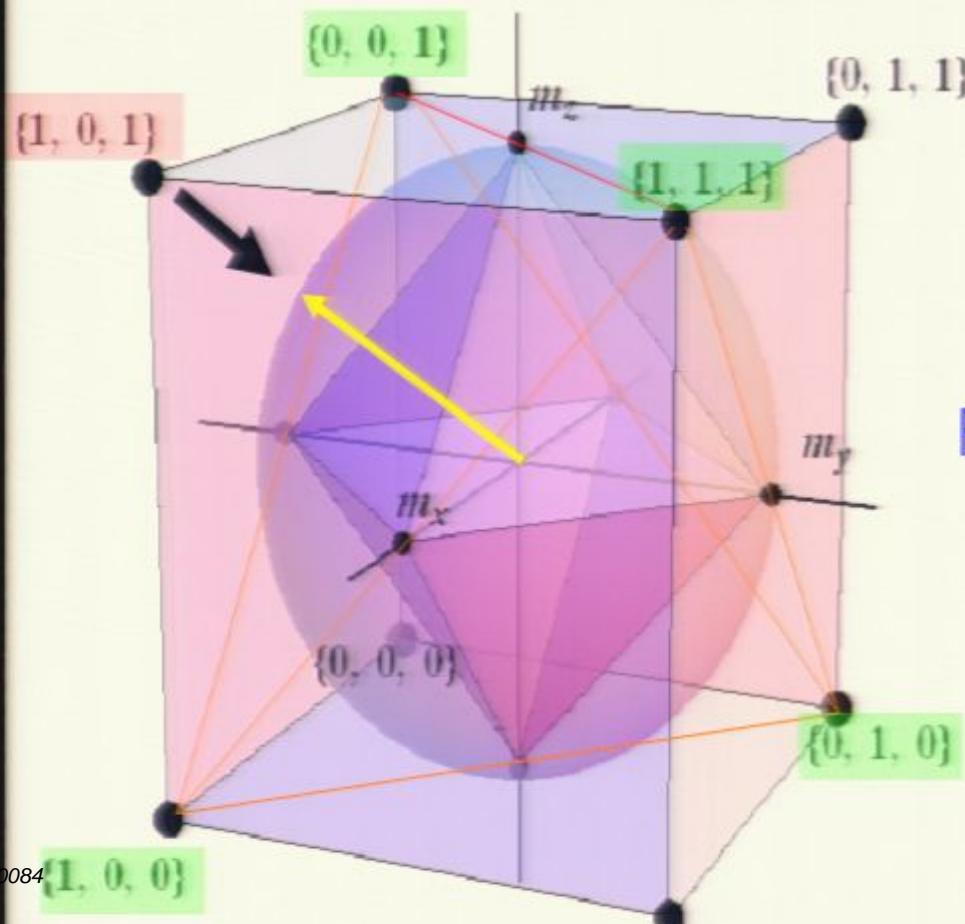
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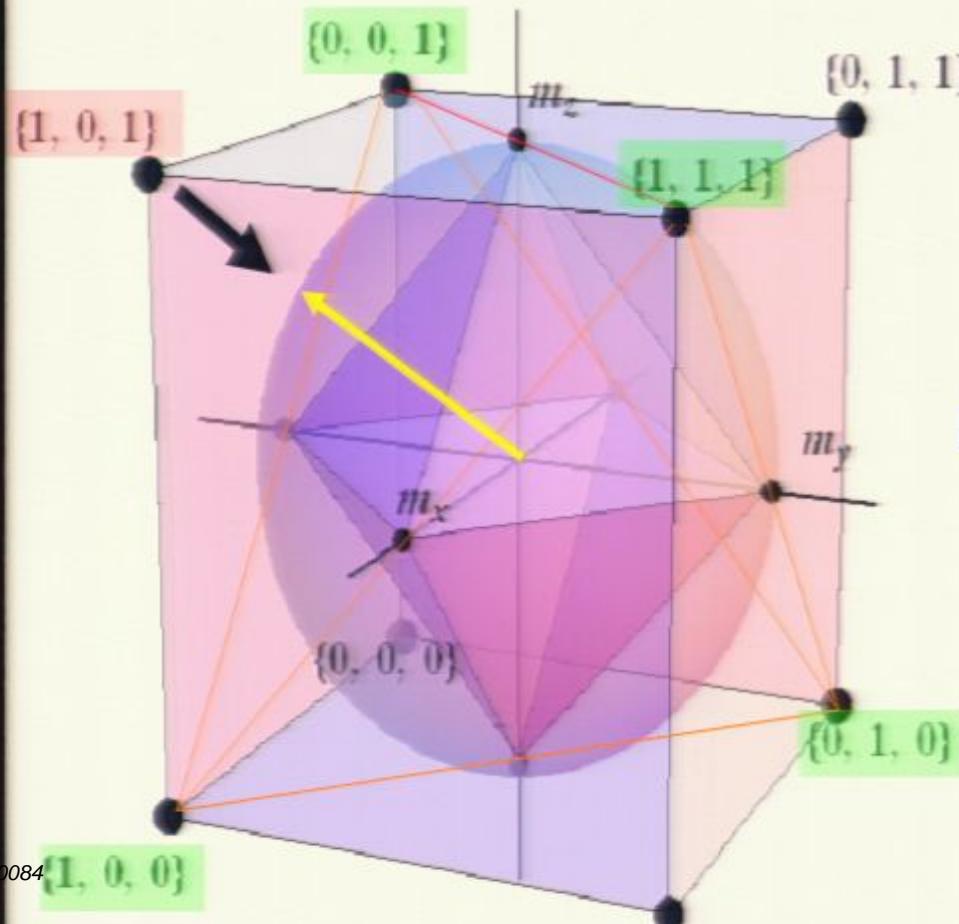
Outside tetrahedron?

For states outside of tetrahedron
All $2^3=8$ HVs are needed!

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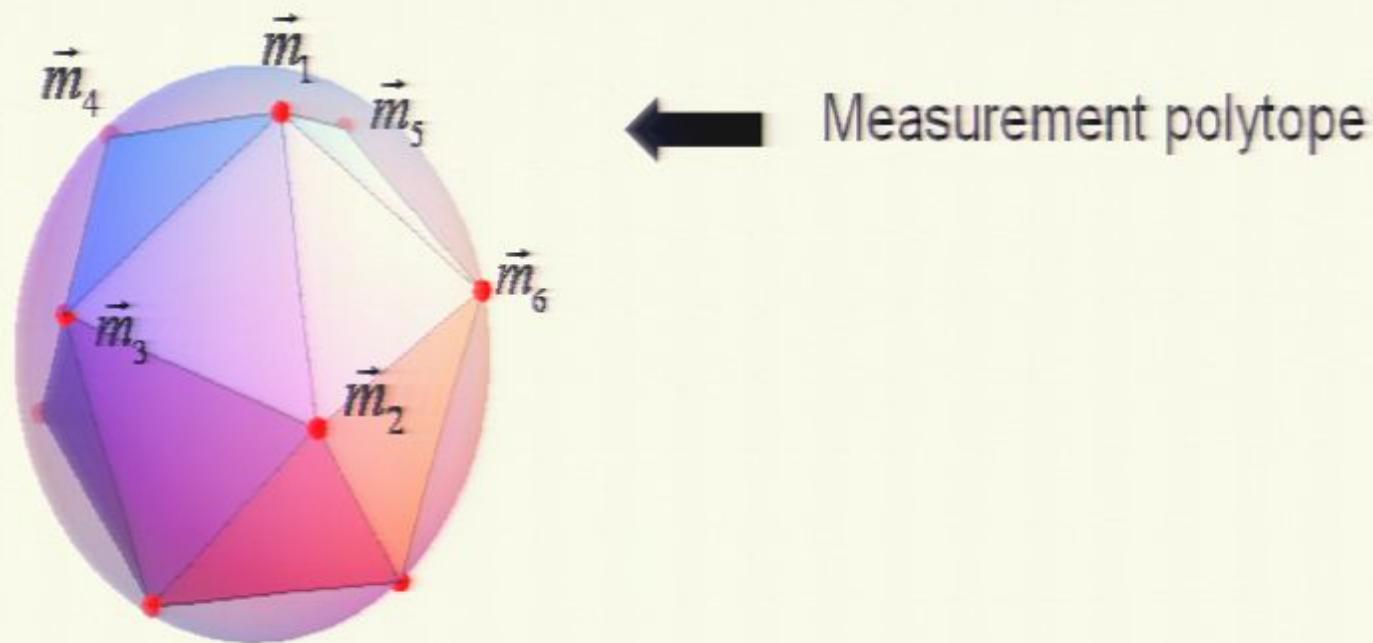
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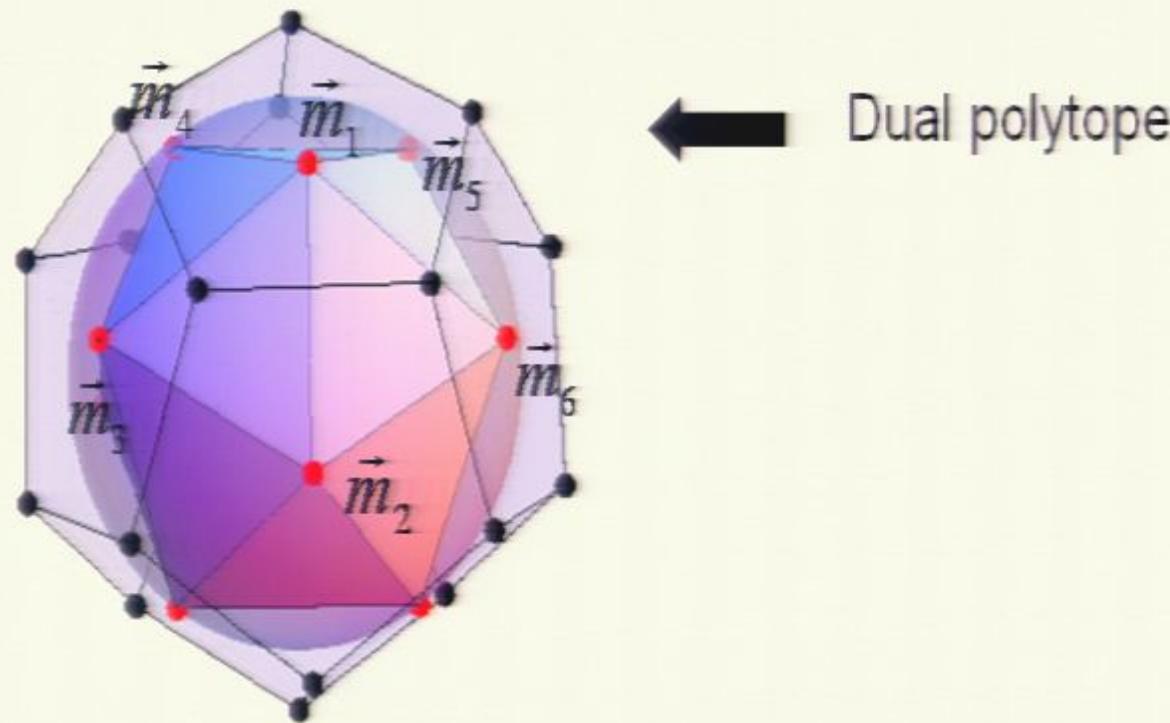
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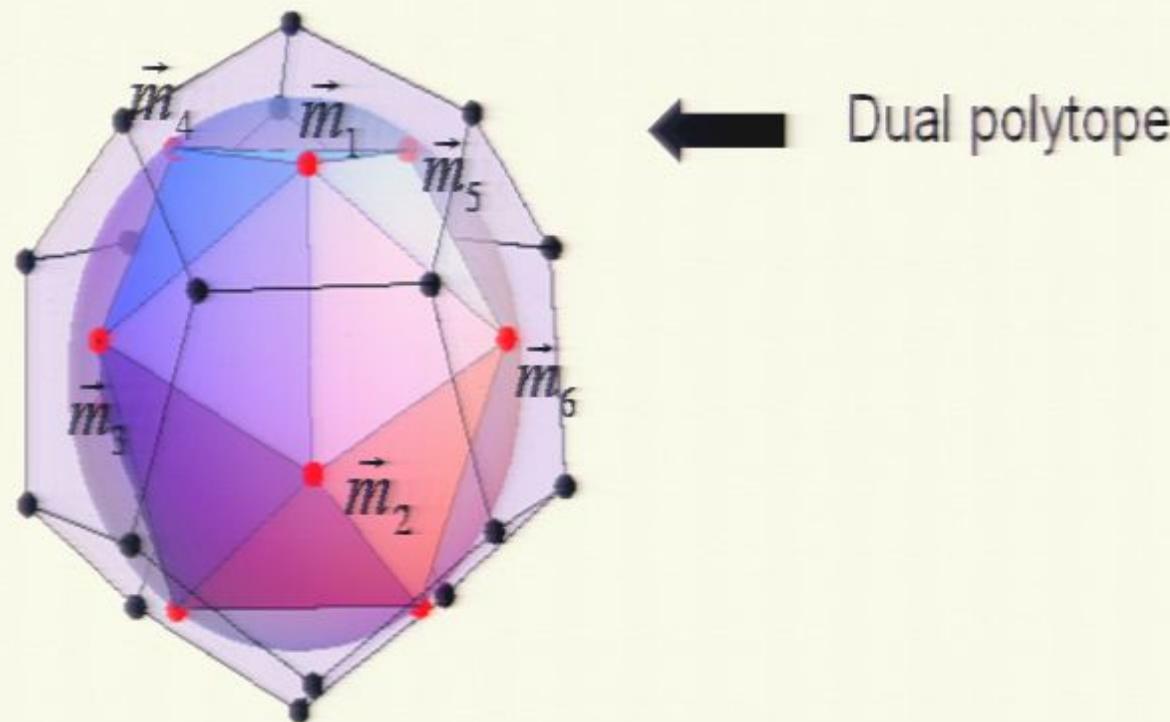
HV-Model Based on Dual Polytope

Dual Polytopes, $\tilde{P}_{\text{dual}} = \{y \in \mathbb{R}^6 \mid 1 \leq \text{wt}_n y \leq 1, n = 1 \dots N\}$



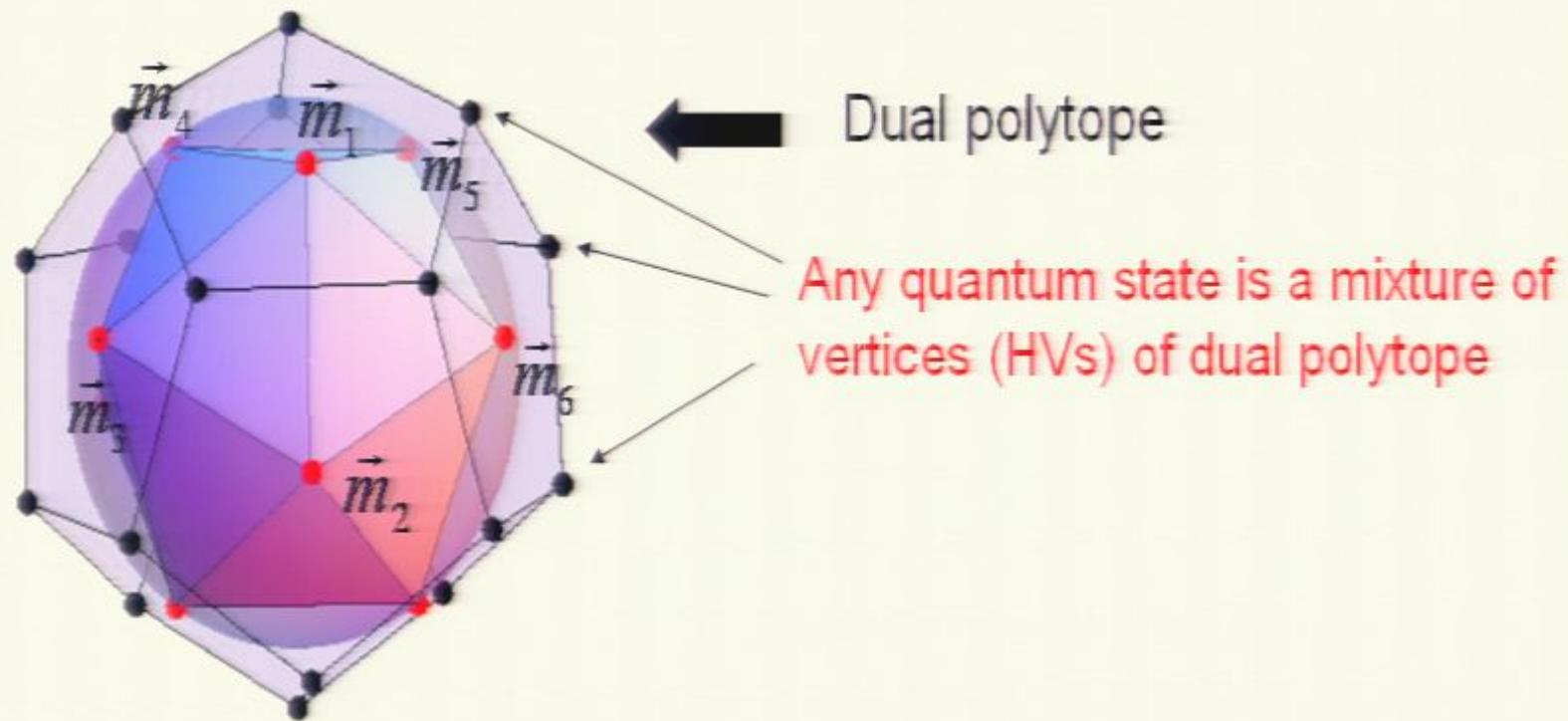
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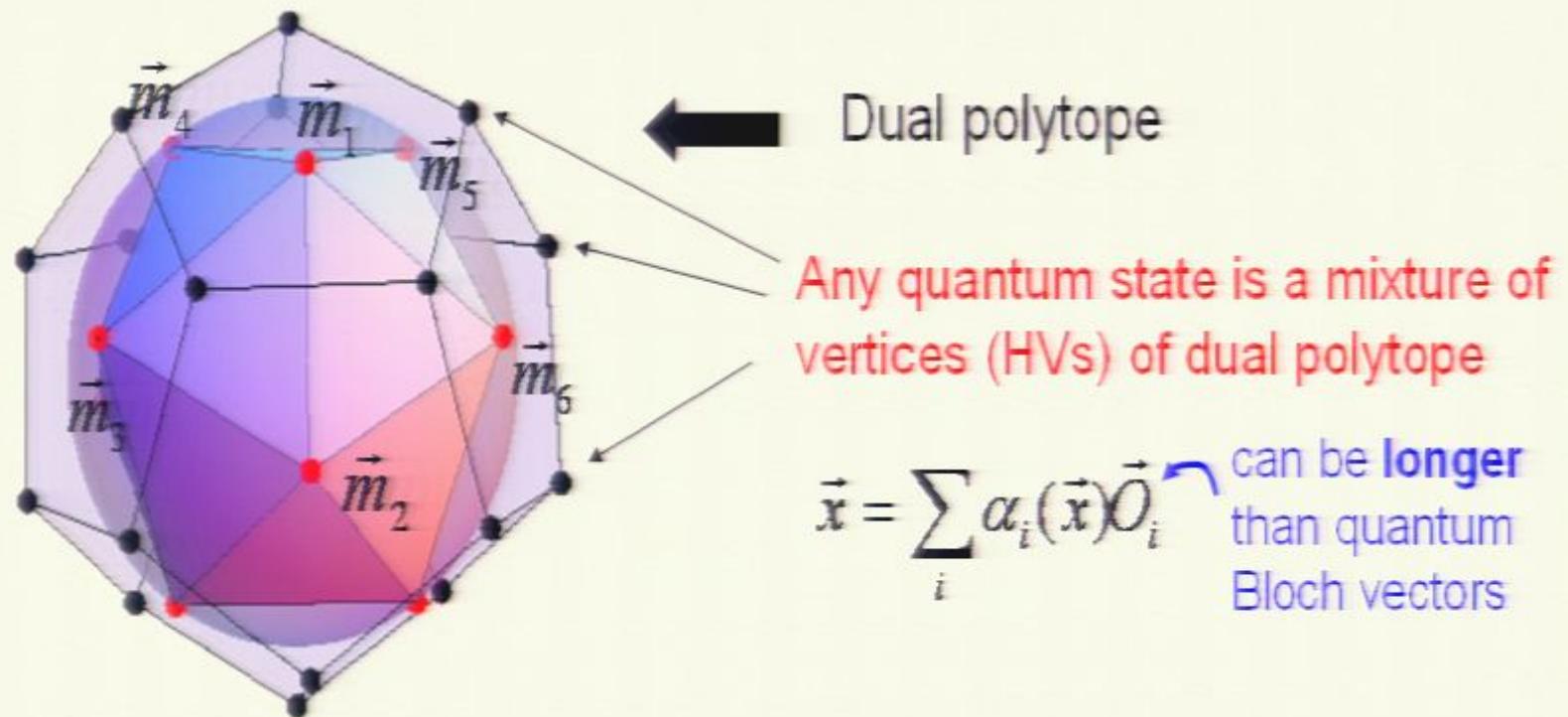
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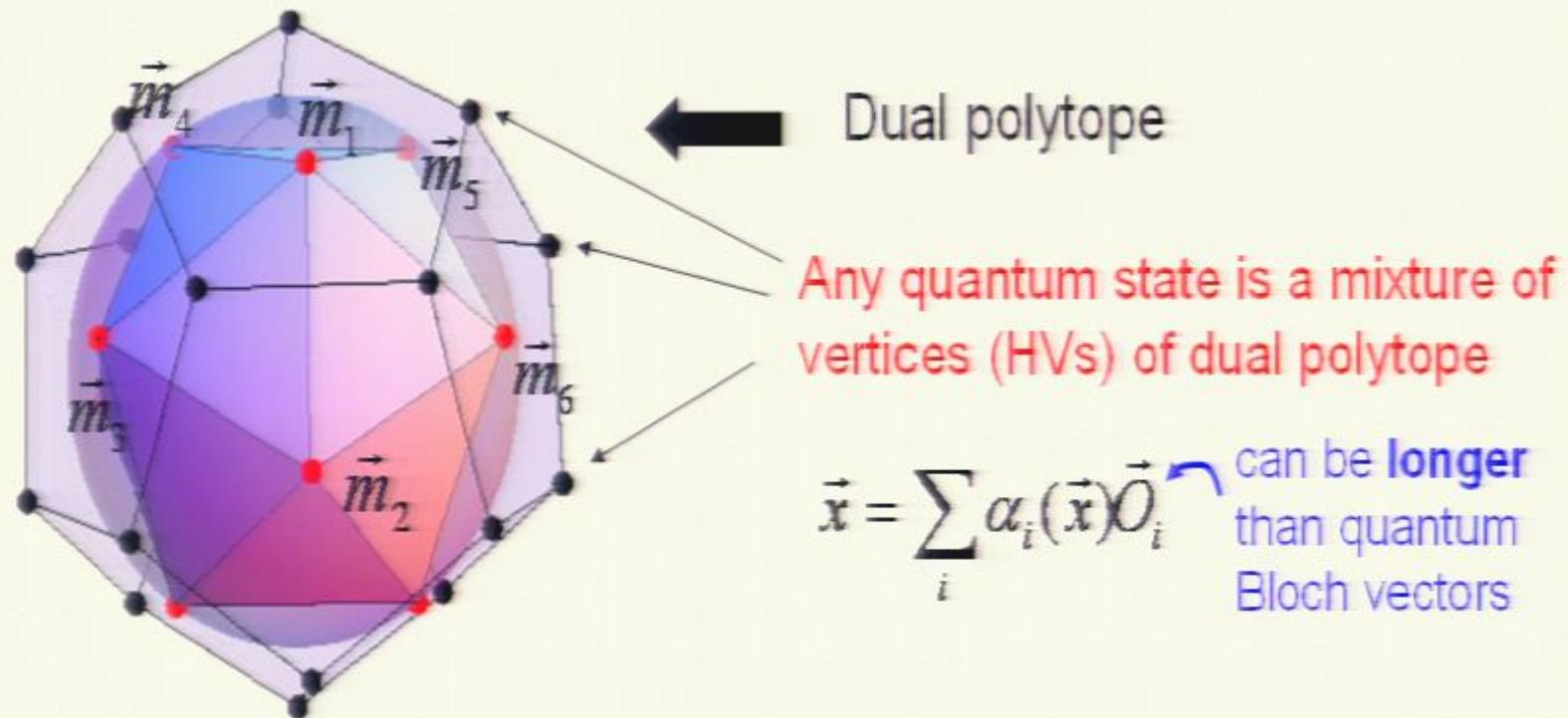
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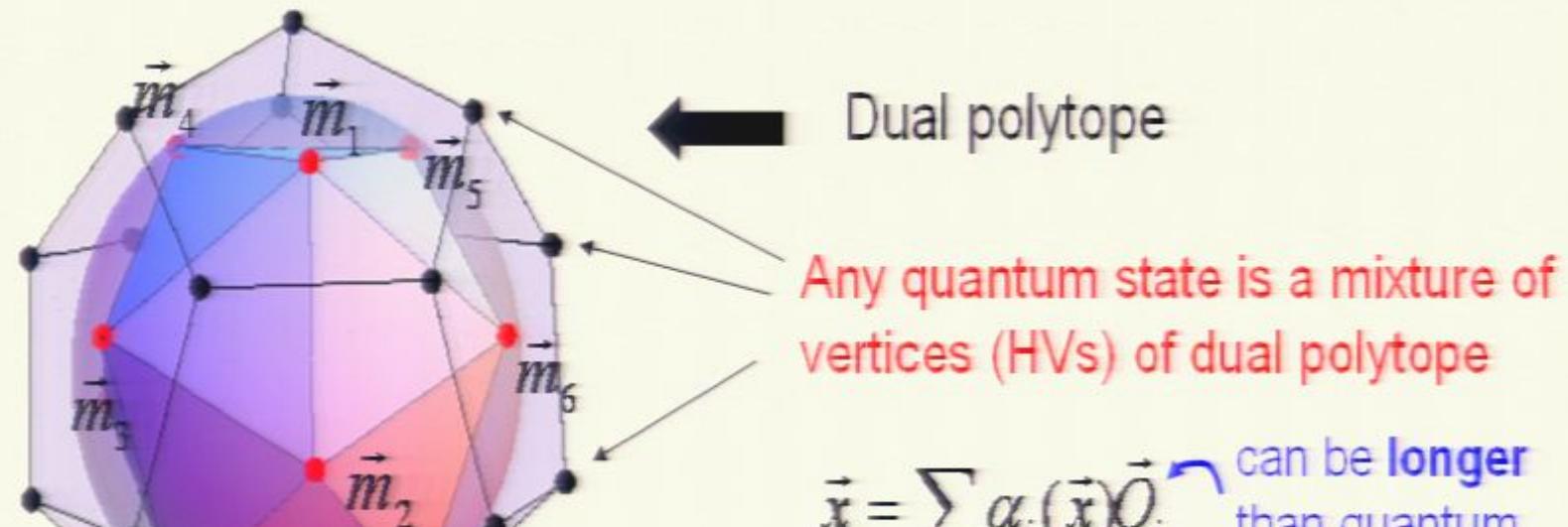
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$$p_n(\vec{x}) = \sum_i \alpha_i(\vec{x}) \frac{1}{2} (1 + \vec{O}_i \cdot \vec{m}_n)$$
$$\sum_n p_n(\vec{x}) = 1$$

Measurement Polytope: Cube

- N=4 measurement directions

MONDAY 4:30-5:00PM BRIAN LACOUR

TUESDAY 9:30-10:30AM CHRISTOPHER FUCHS
3:00-3:30PM SAMUEL COLIN



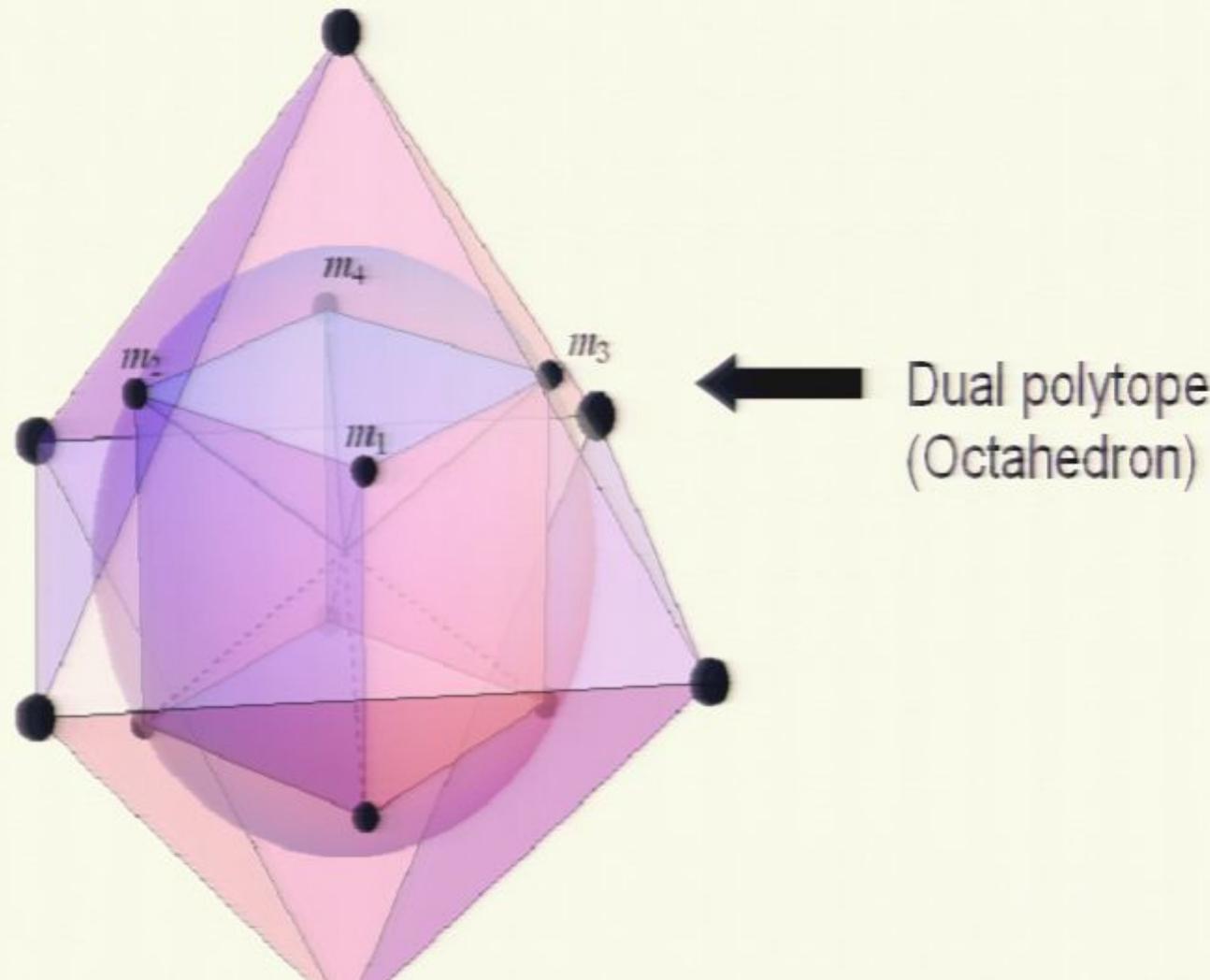
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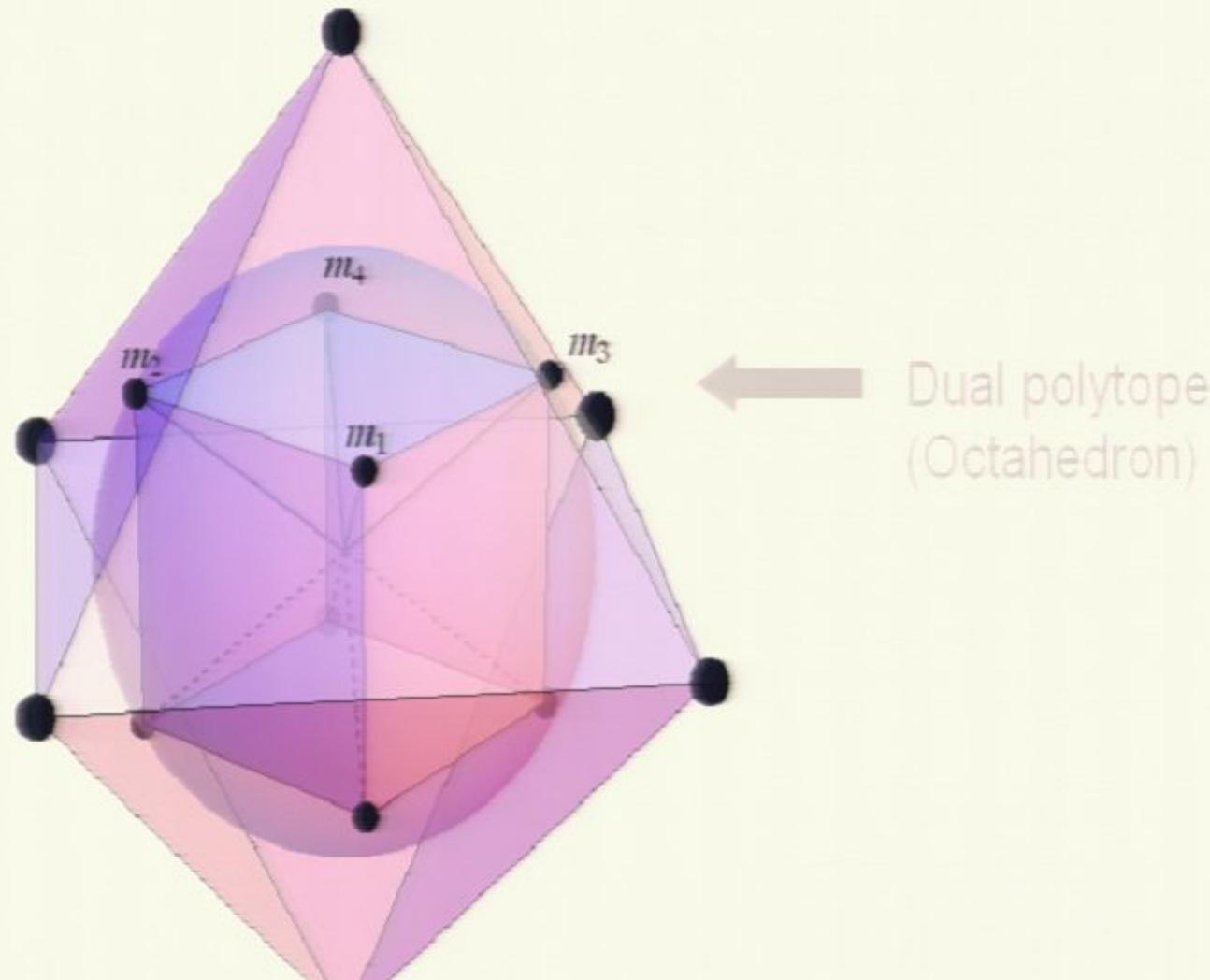
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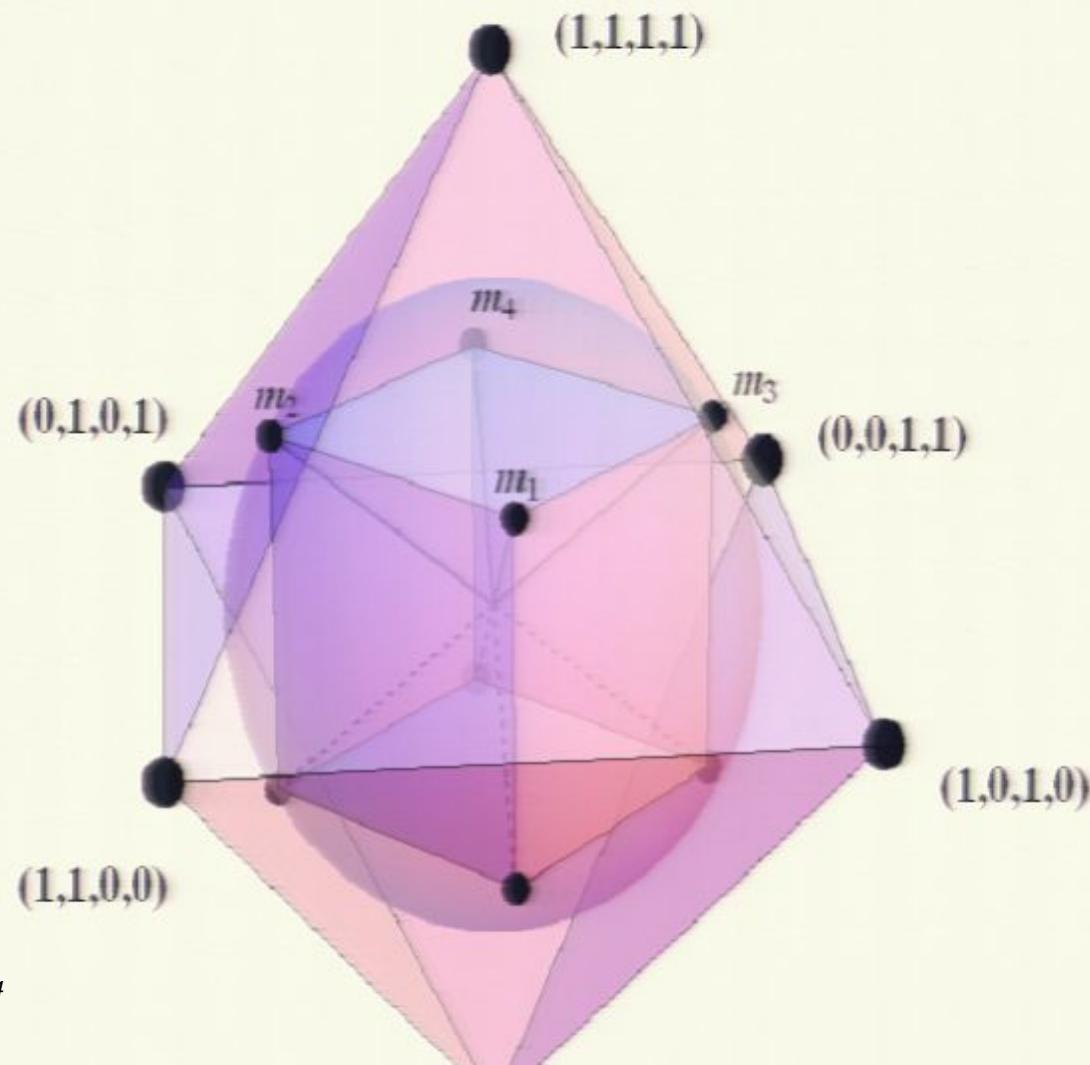
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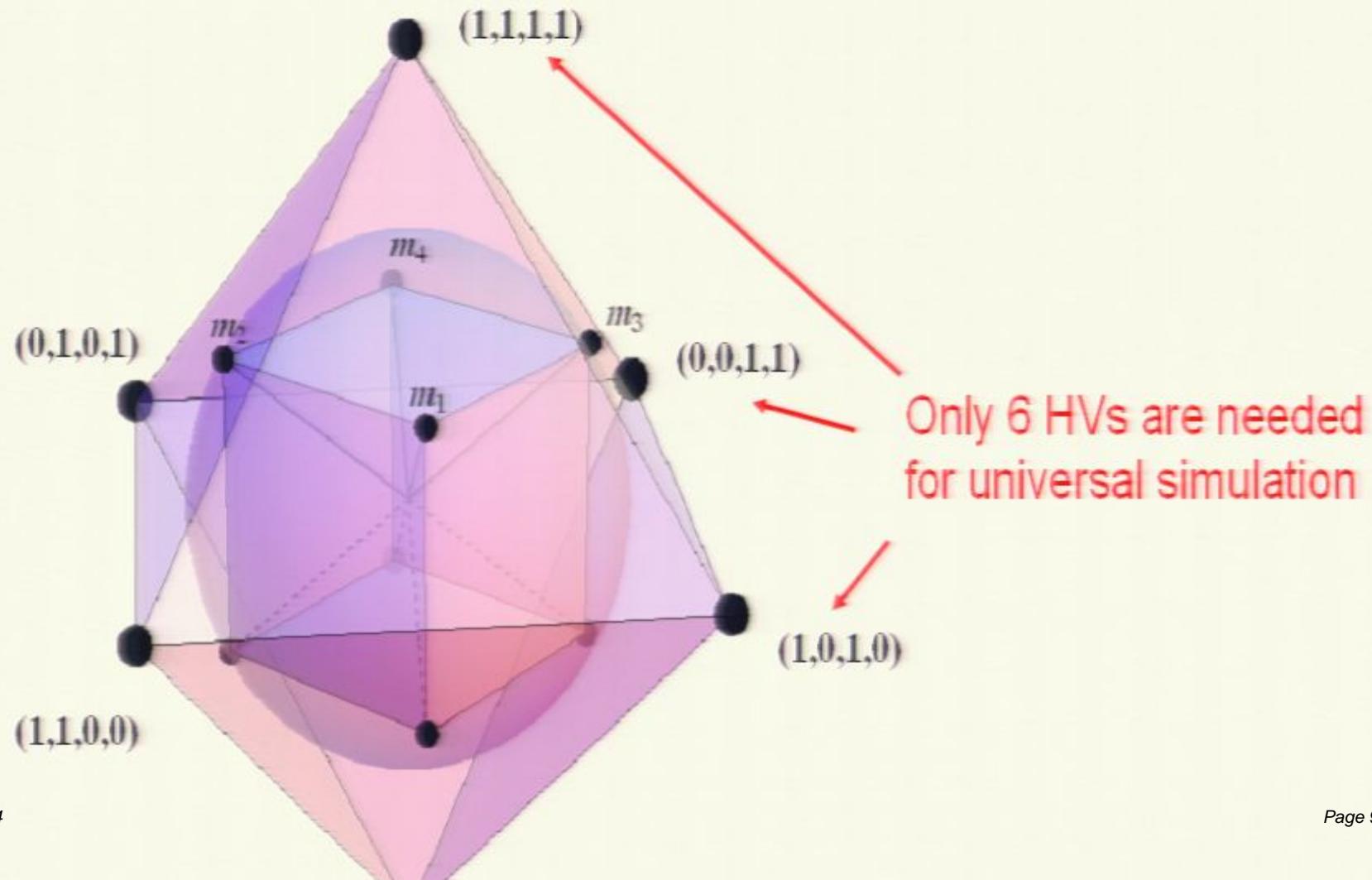
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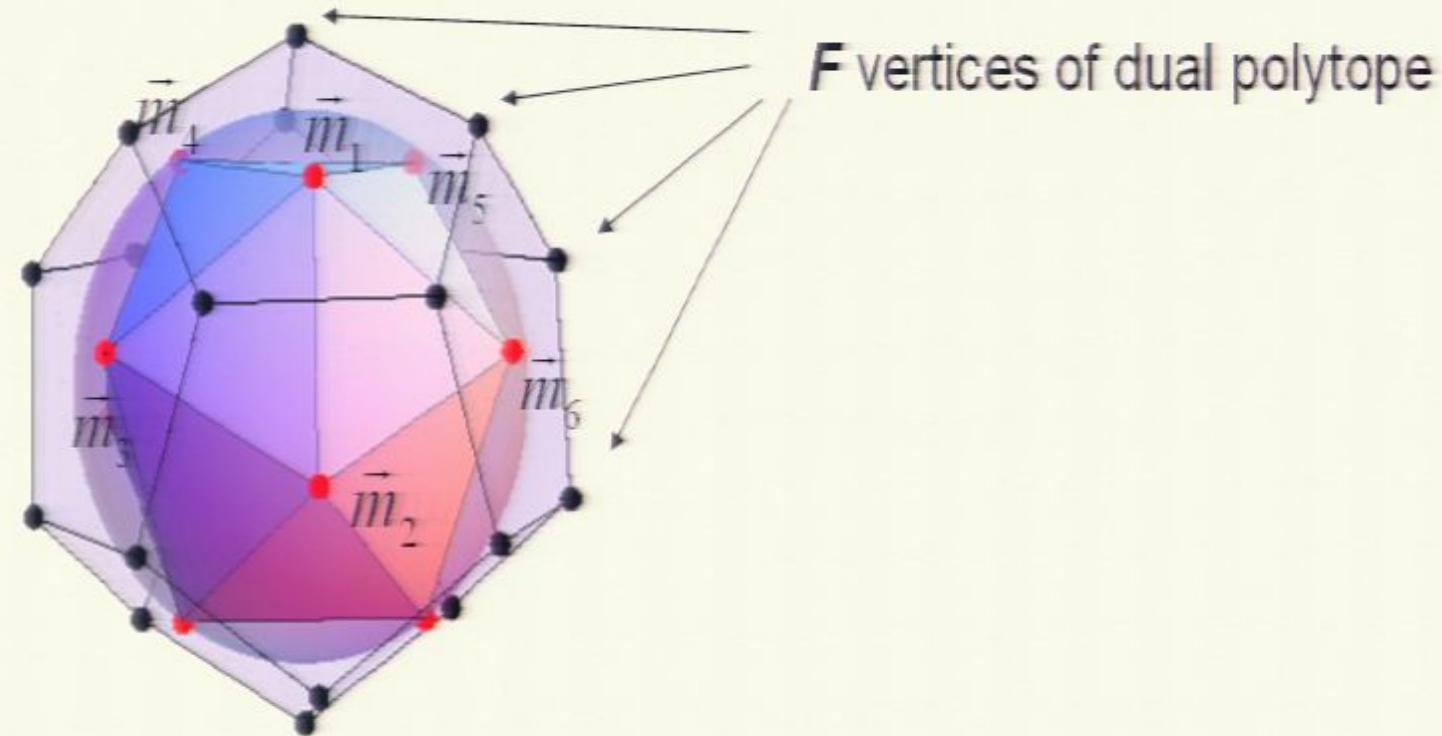


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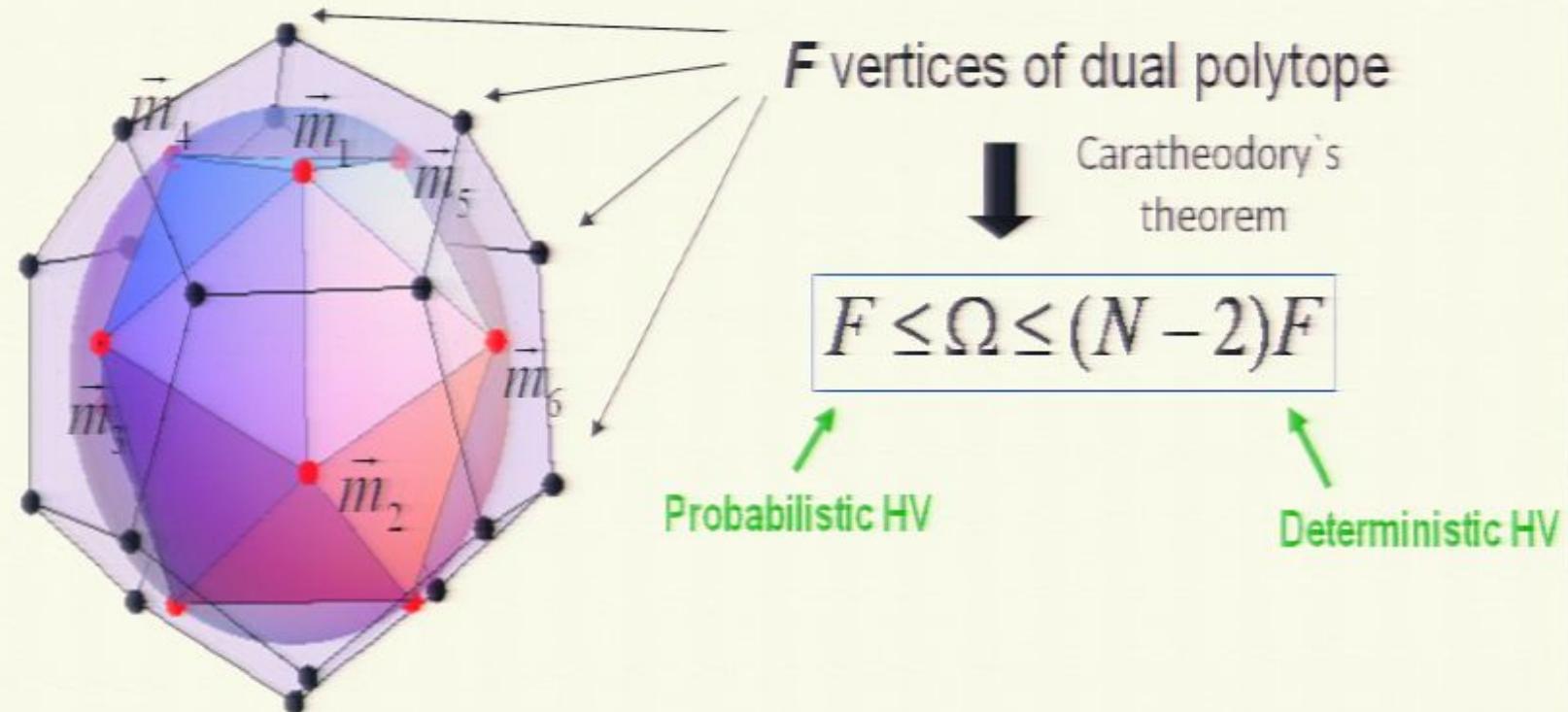
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Lower and upper bound

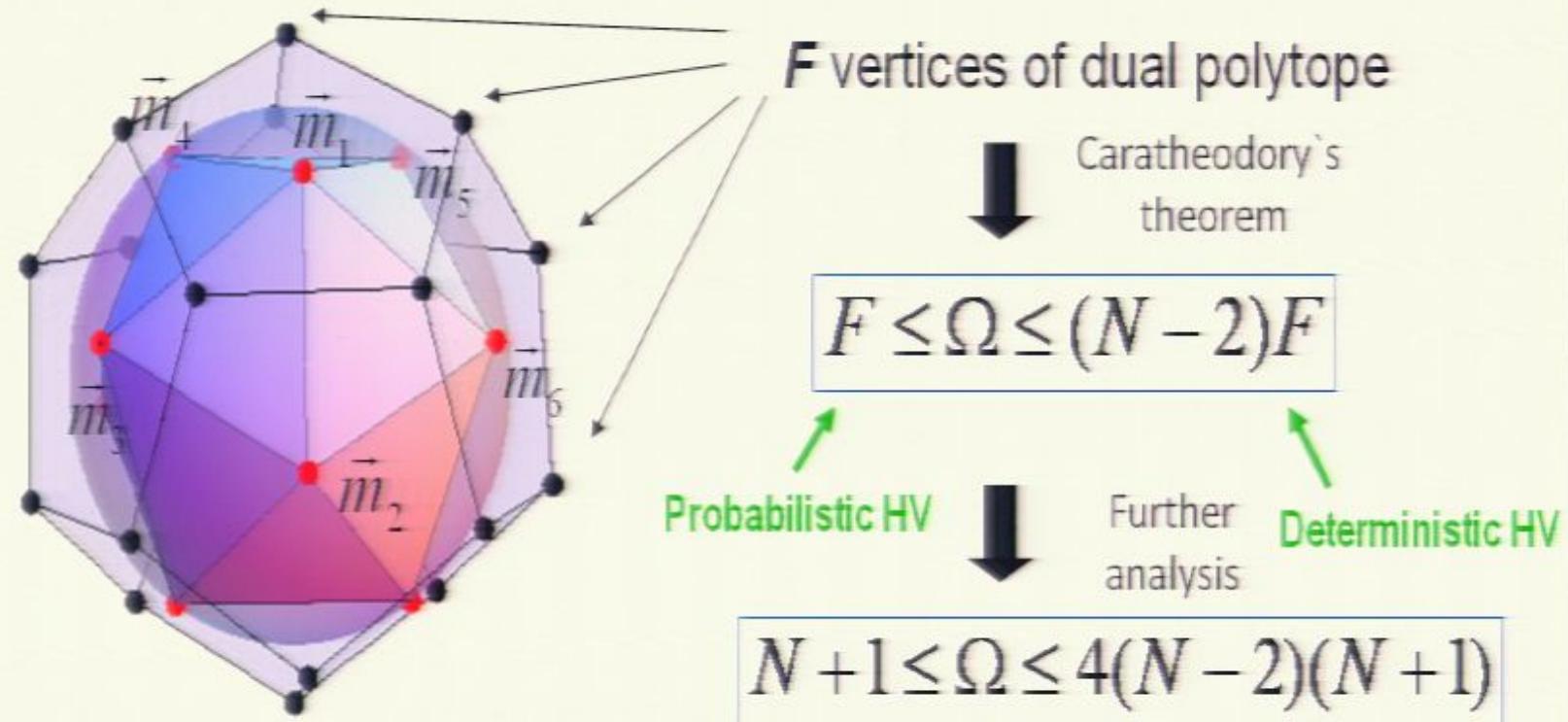


Lower and upper bound



$N - 2$ instead of $N + 1$: a vertex of the dual polytope saturates at least three of the inequalities defining the polytope

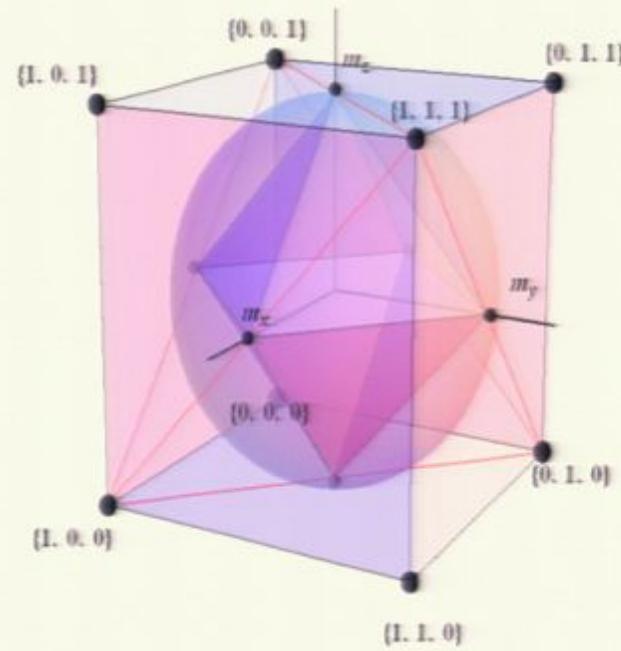
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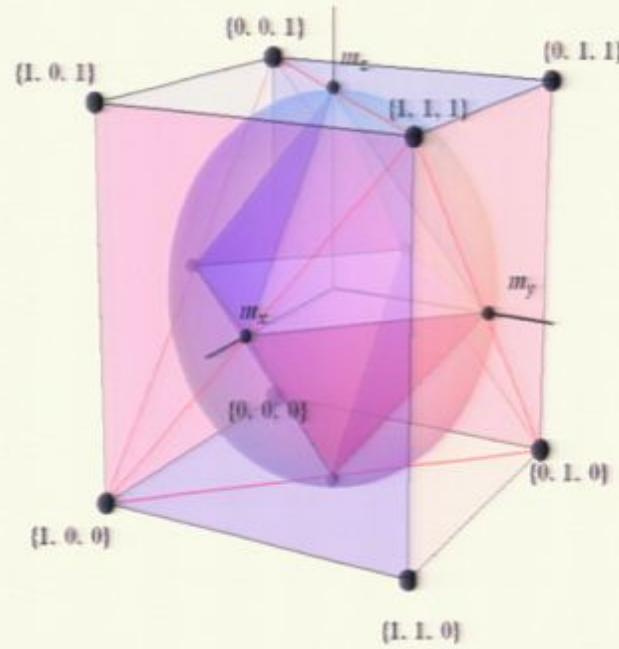
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P. McMullen, *Mathematika* 17, 179 (1970): A convex polytope with $2N$ vertices in \mathbb{R}^3 can have $N + 1 \leq F \leq 4(N - 1)$ facets

Symmetric Polytopes:

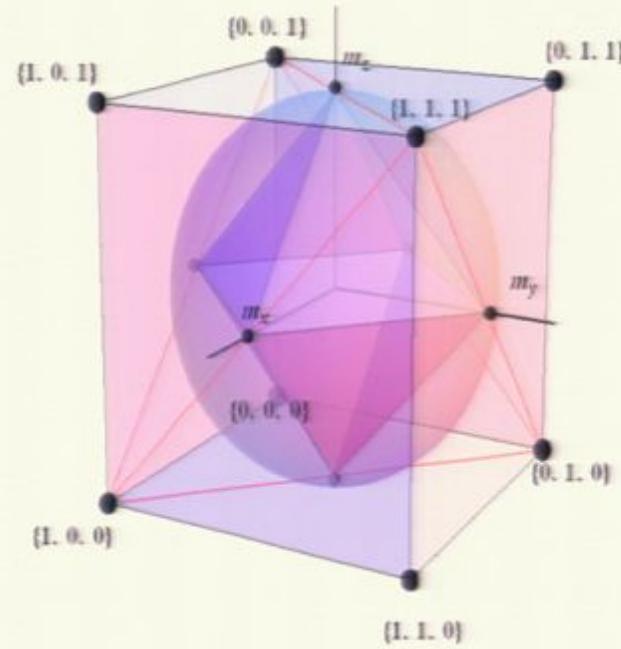


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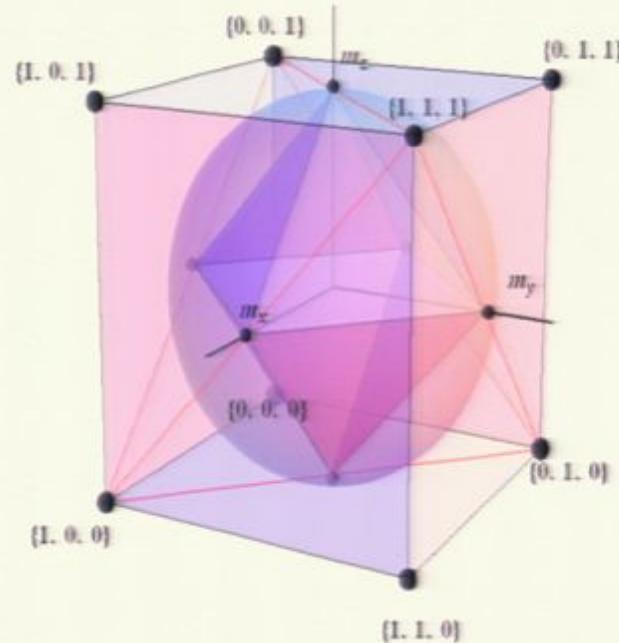


Measurement and Dual Polytope
has the same (group) symmetry

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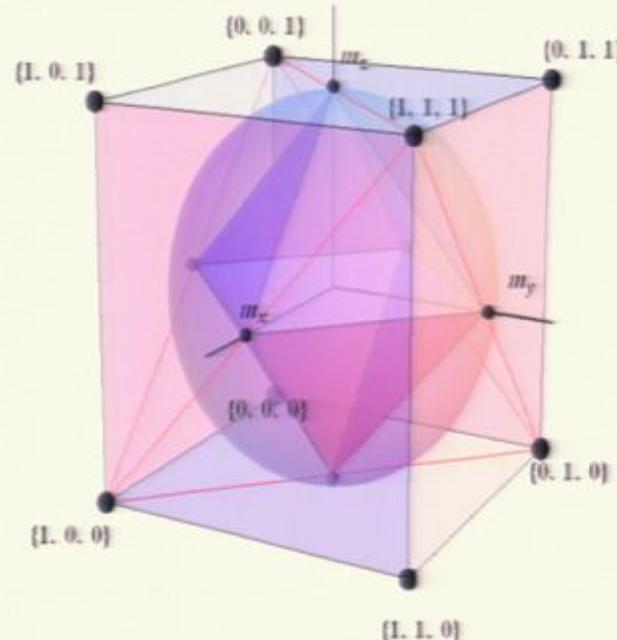
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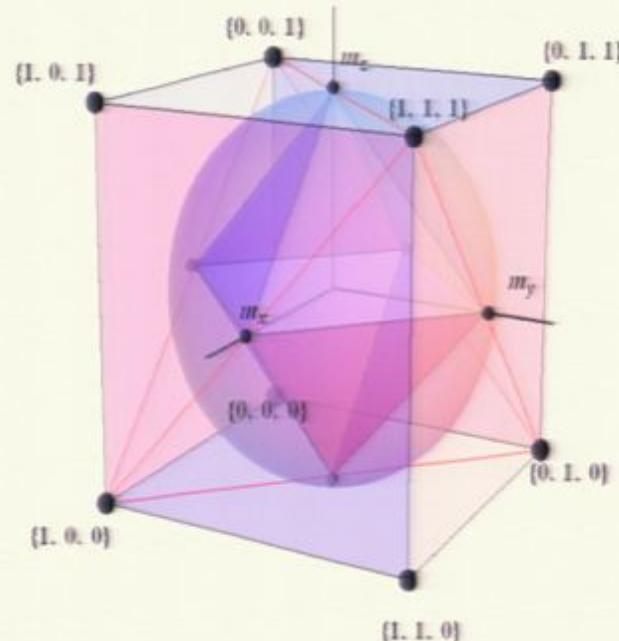
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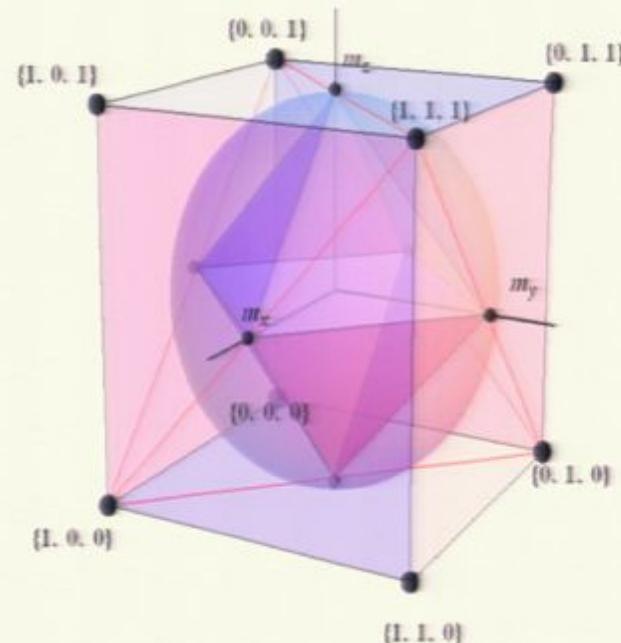
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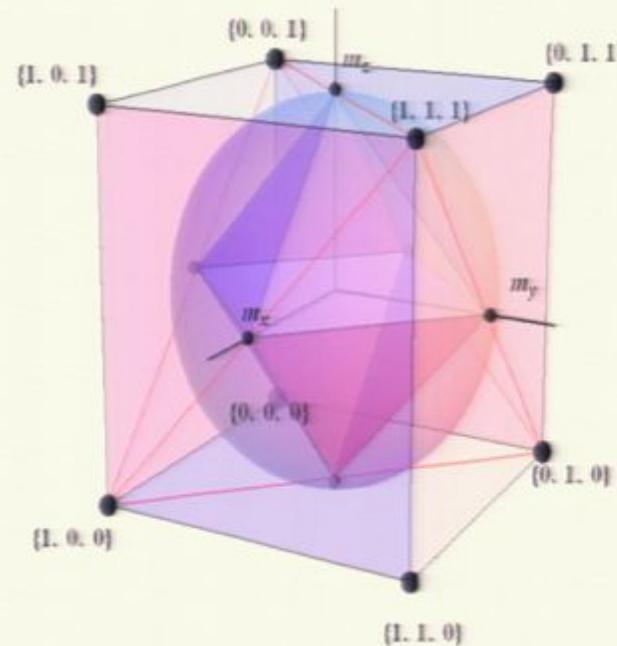
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Number of deterministic HVs cannot be smaller than
the number of elements of the smallest orbit

Platonic solids



Solid	N	2^N	Ω
Octahedron	3	8	8
Cube	4	16	6
Icosahedron	6	64	20
Dodecahedron	10	1024	24

Largest orbit

Union of two lower
orbits 12+12

Octahedron & Cube: Octahedral group, with 24 rotations

Icosahedron & Dodecahedron: Icosahedral group, with 60 rotations.

General Case: Qudit

Generalized Gell-Mann Operators

$$\hat{\rho} = \frac{1}{d} [\mathbb{1} + (d-1) \sum_{i=1}^D x_i \hat{\lambda}_i] \quad \text{Tr}(\hat{\lambda}_i \hat{\lambda}_j) = \frac{d}{d-1} \delta_{ij}$$
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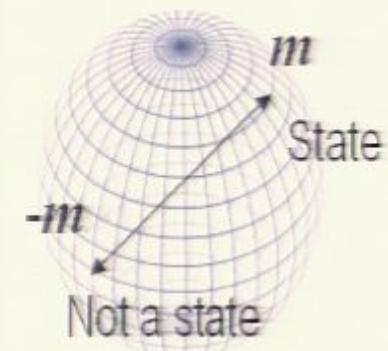
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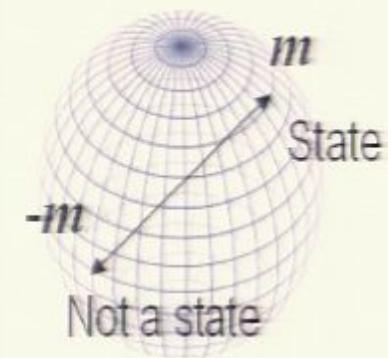
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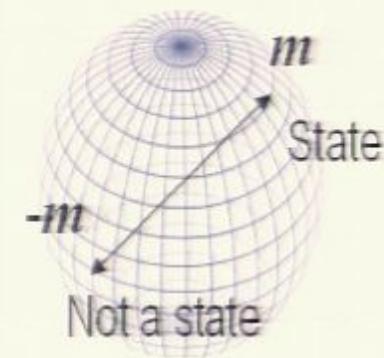
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Number of vertices F

Probabilistic HV: $\Omega = F \cong (2dN - \delta)^{D-\delta}$ $\delta \equiv \lfloor (D+1)/2 \rfloor$.



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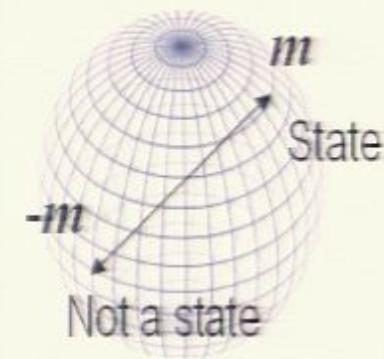
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$$\text{Probabilistic HV: } \Omega = F \cong (2dN - \delta)^{D-\delta} \quad \delta \equiv \lfloor (D+1)/2 \rfloor.$$

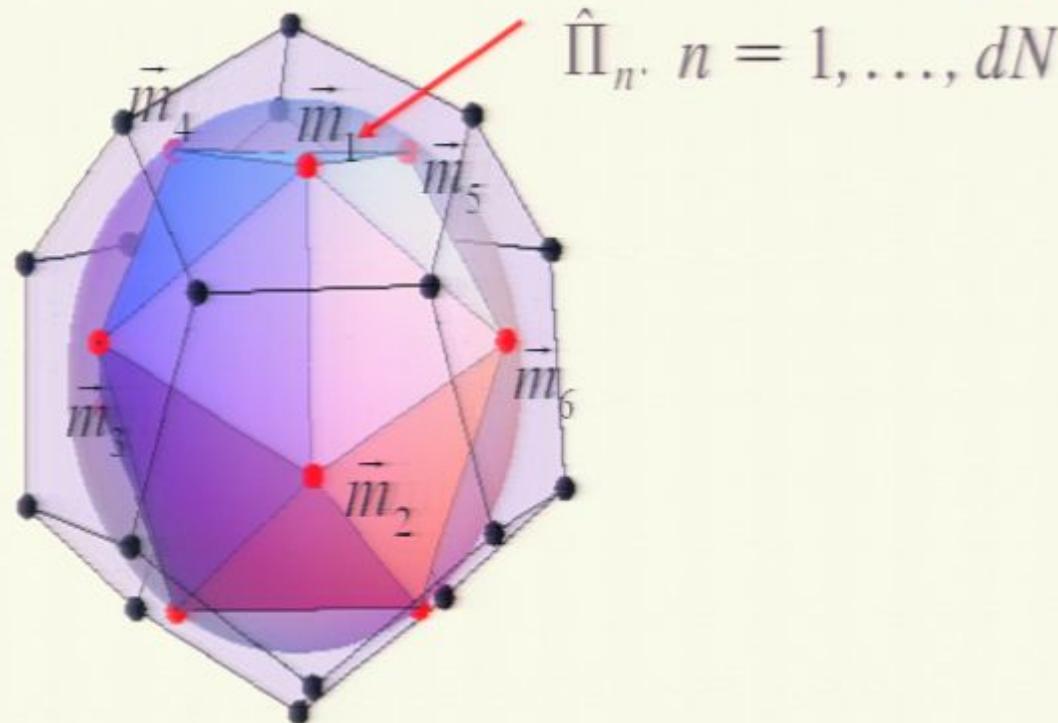
$$\text{Deterministic HV: } \Omega \cong (2dN - \delta)^{D-\delta} \cdot O(Nd)$$



Limit of large number of measurements (preparation- and measurement-universal model)

Hilbert-Schmidt Space of Hermitian operators with unit trace

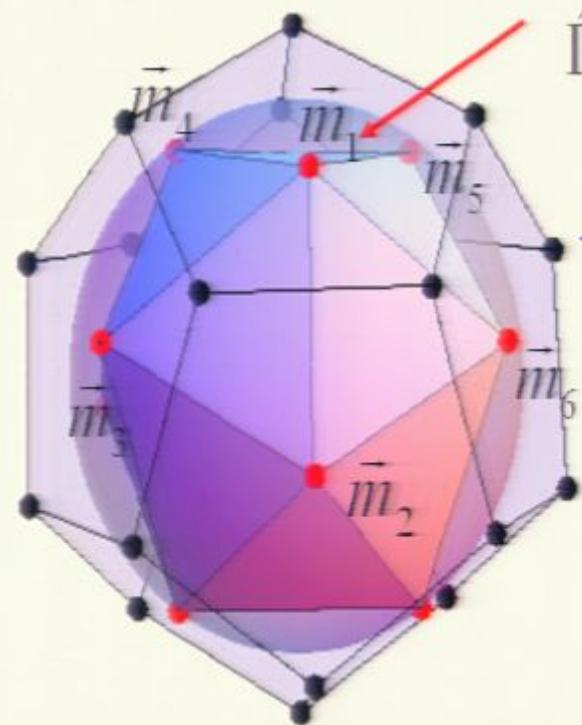
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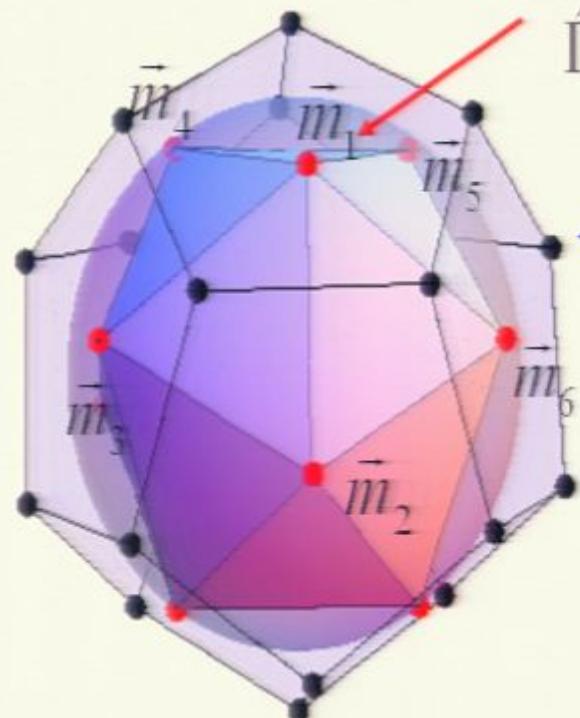
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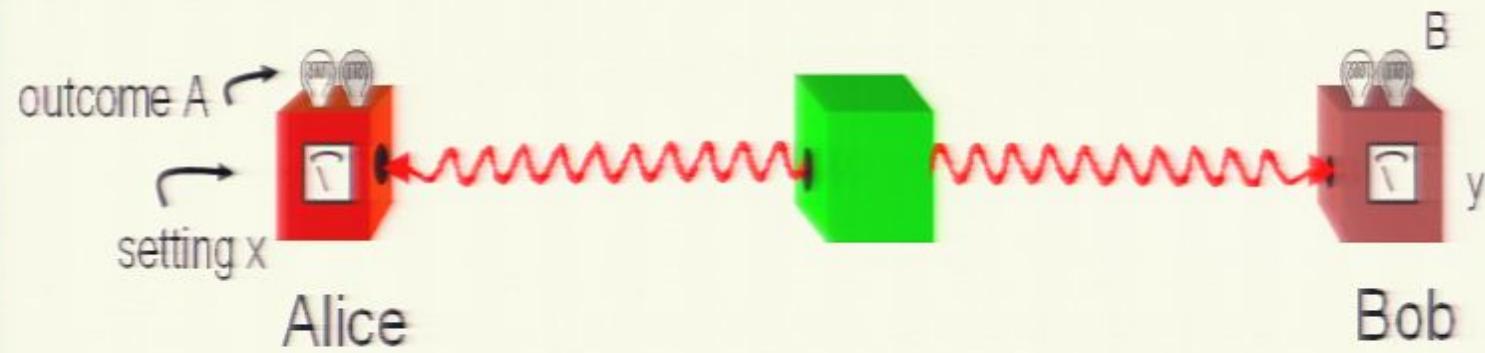
Eigenvalues between 0 and 1

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→ \hat{y}_l are quantum states

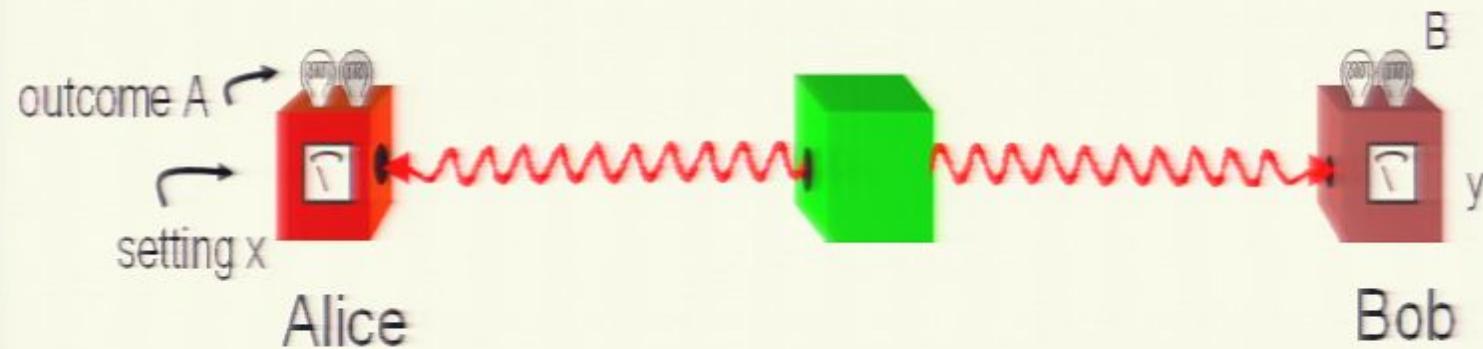
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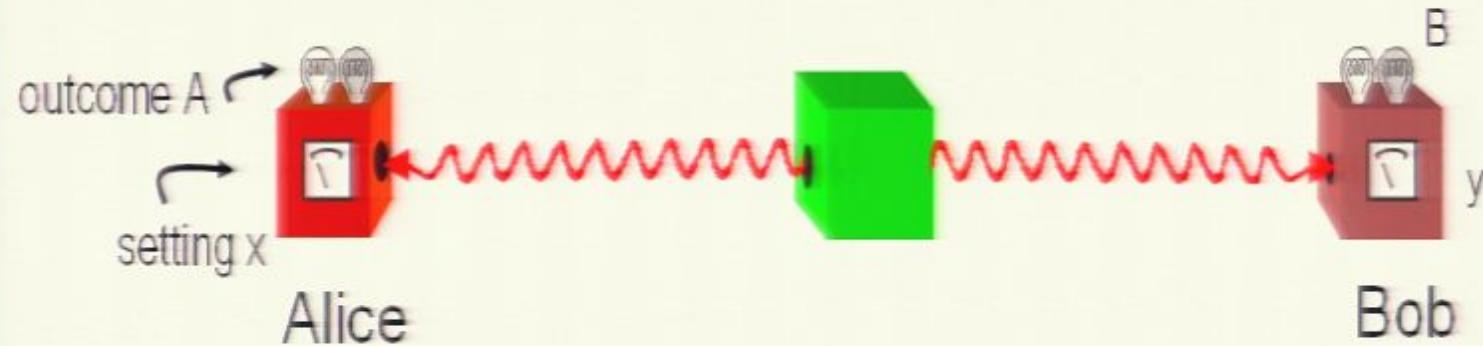
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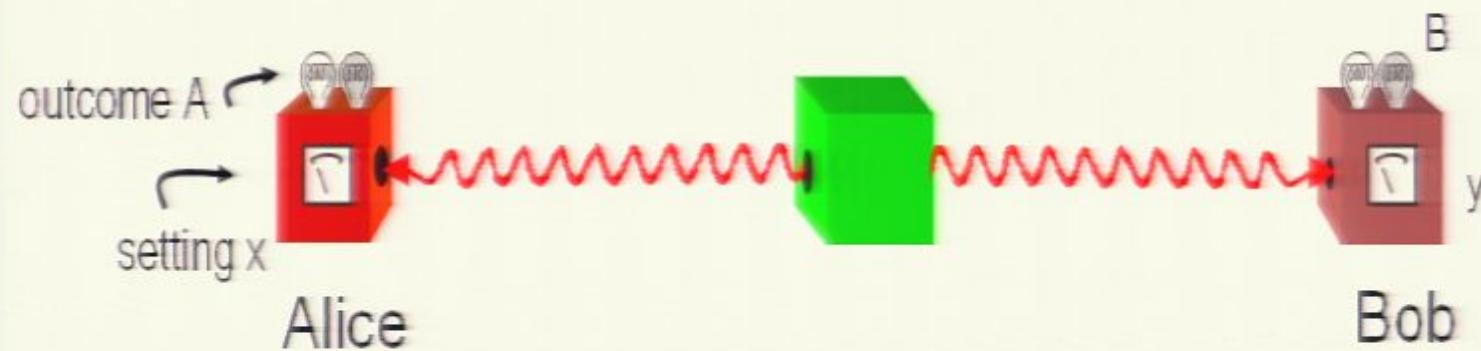


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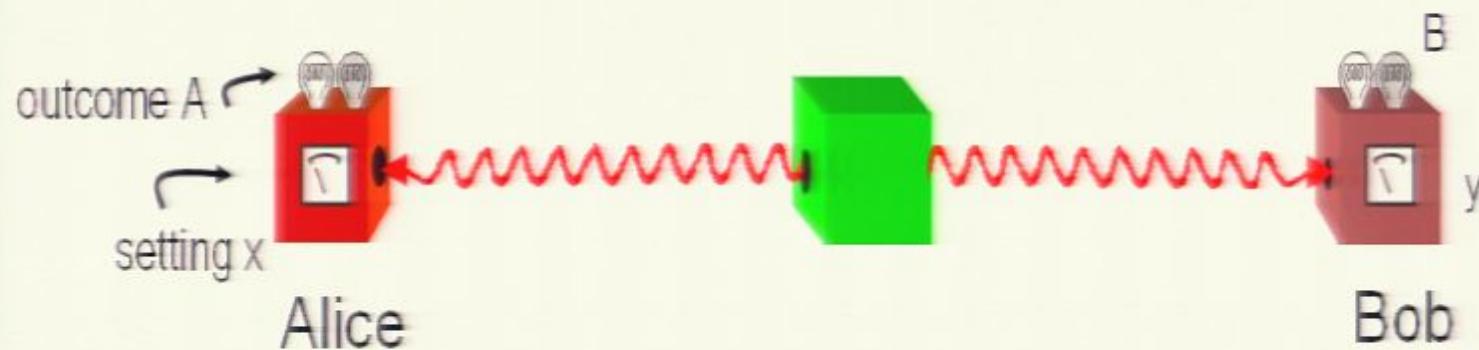
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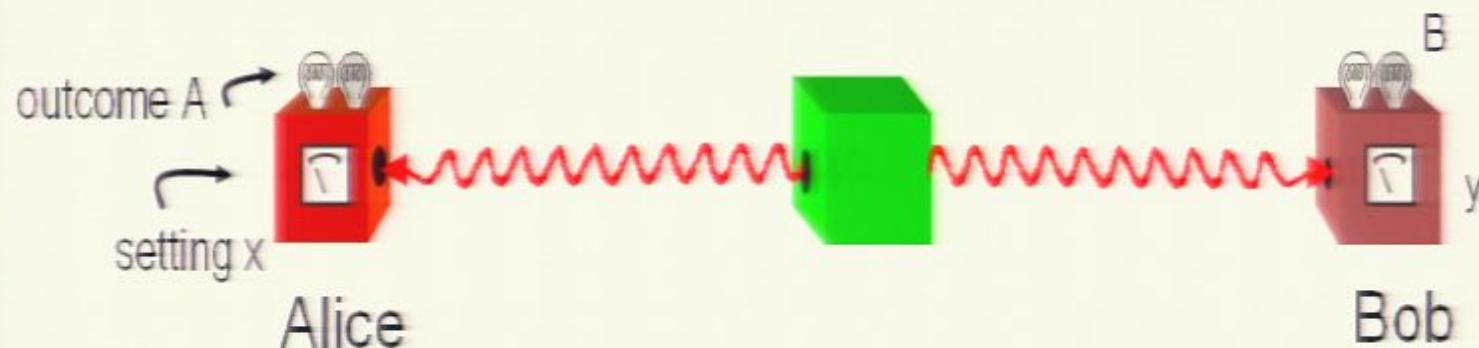
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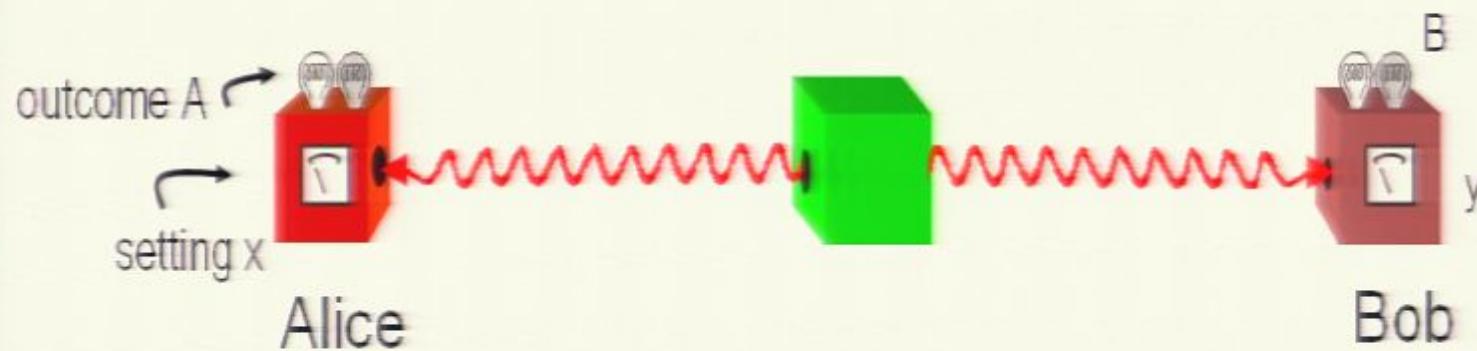
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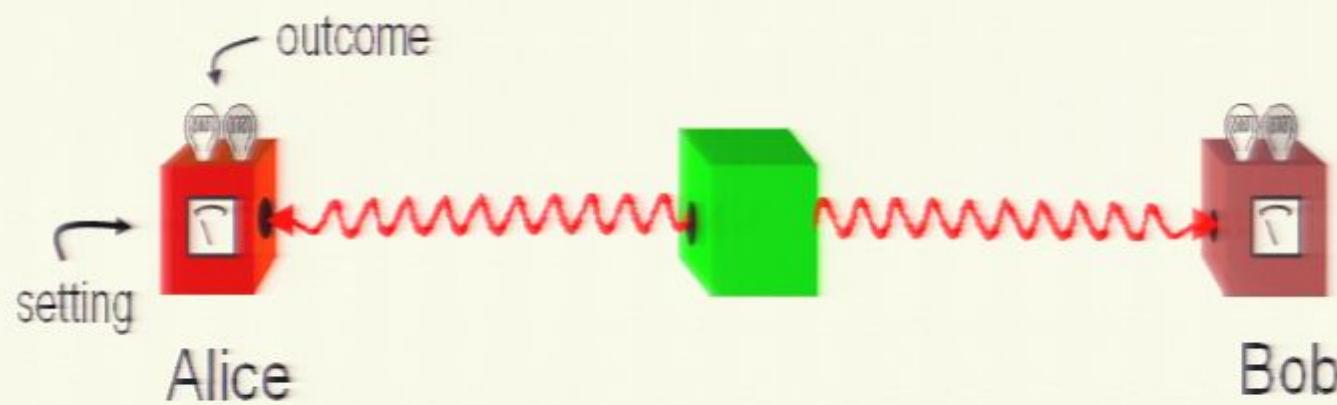
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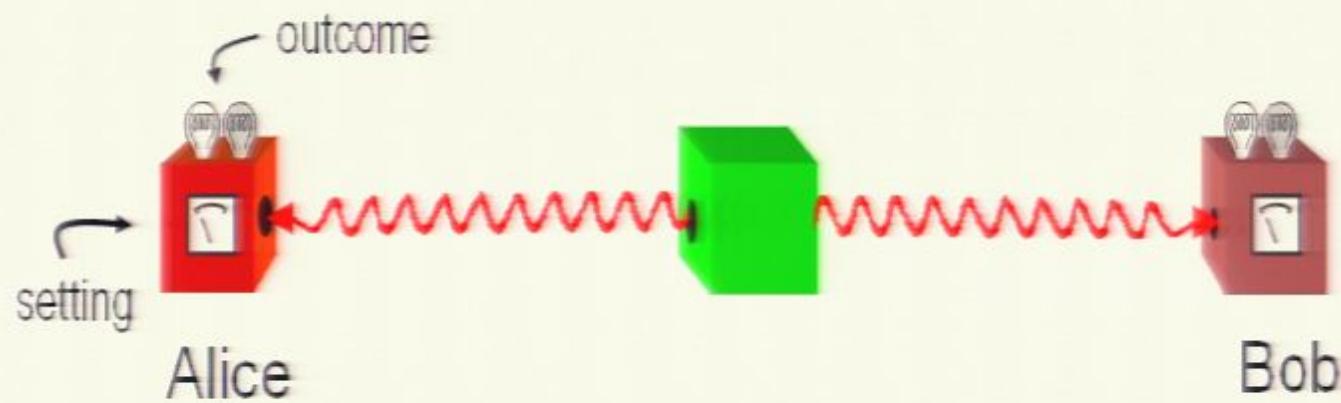
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“Half-realism” (joint probability)

Intuition

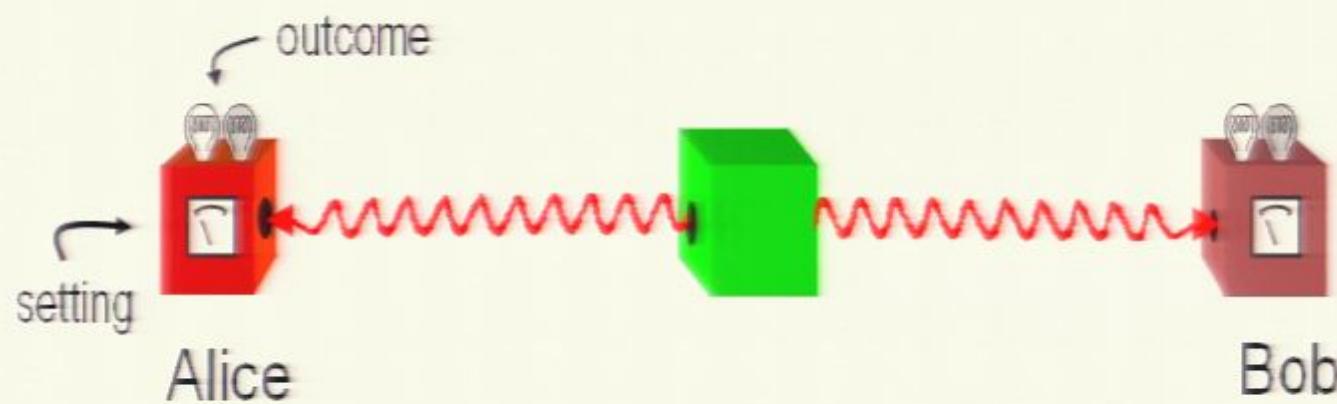


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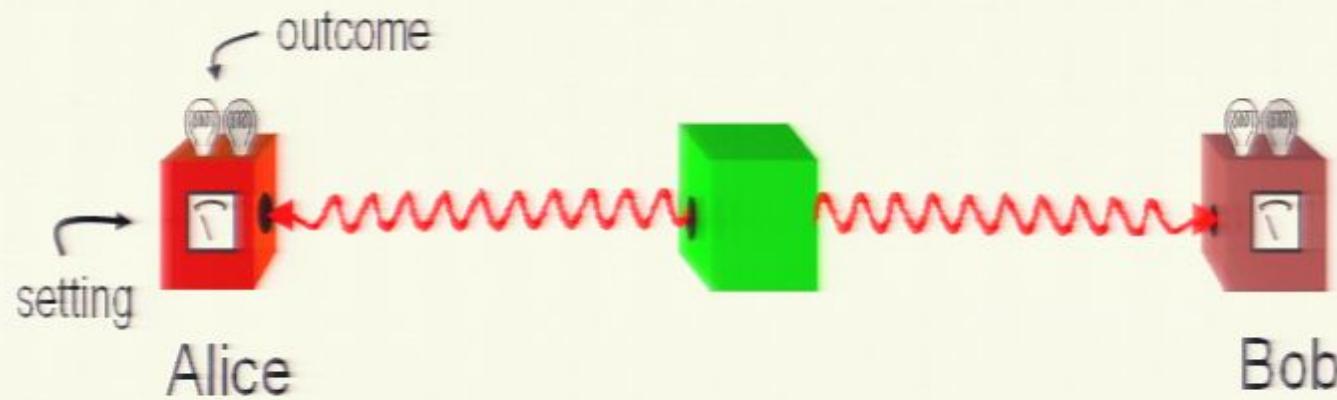
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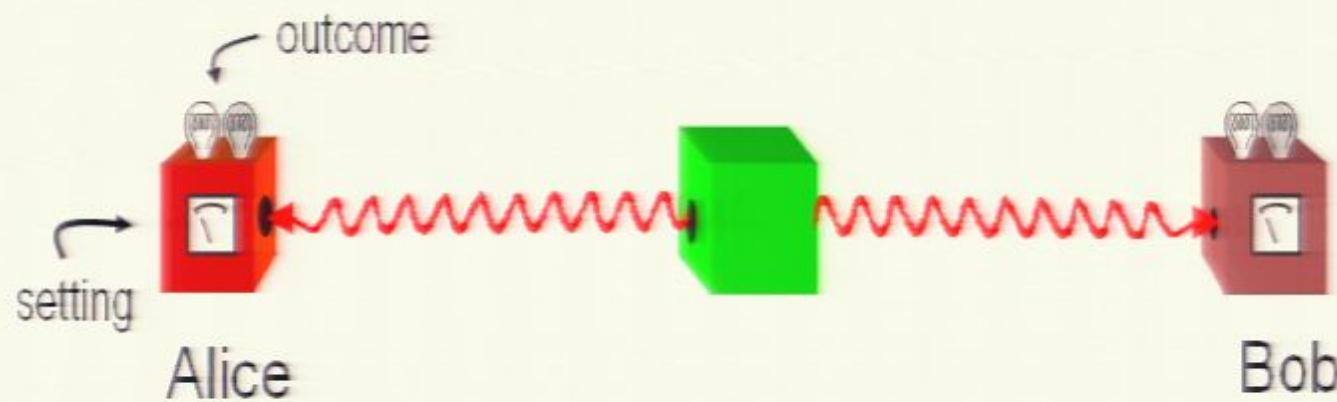


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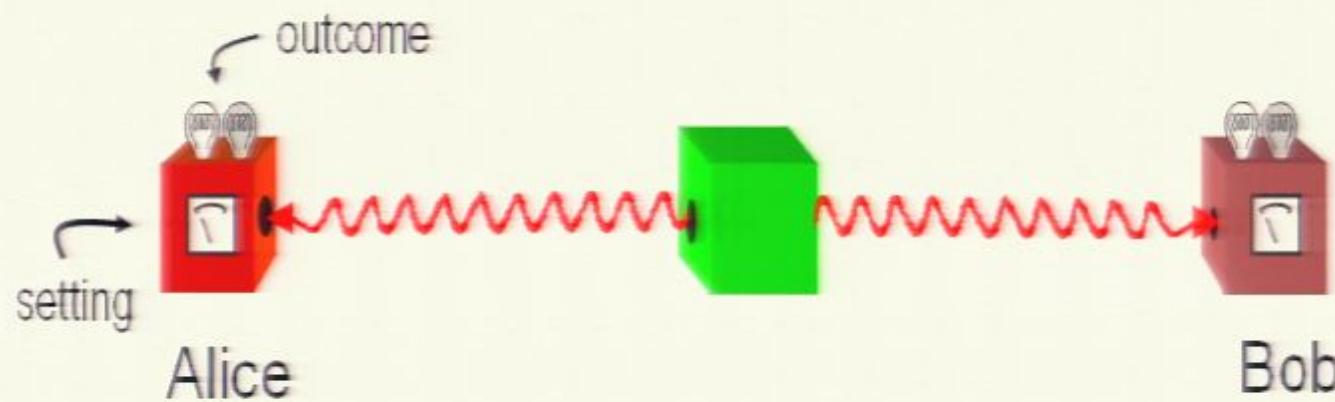
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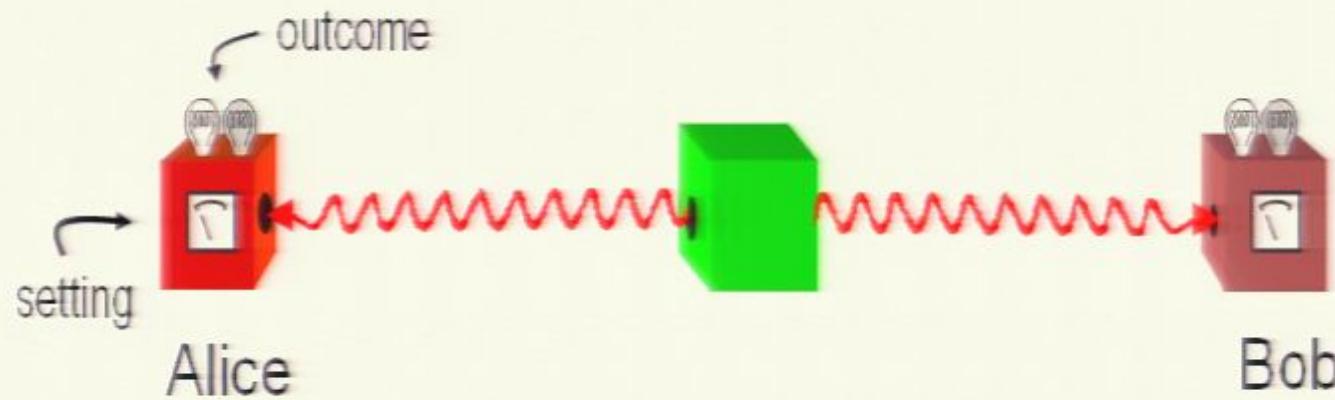
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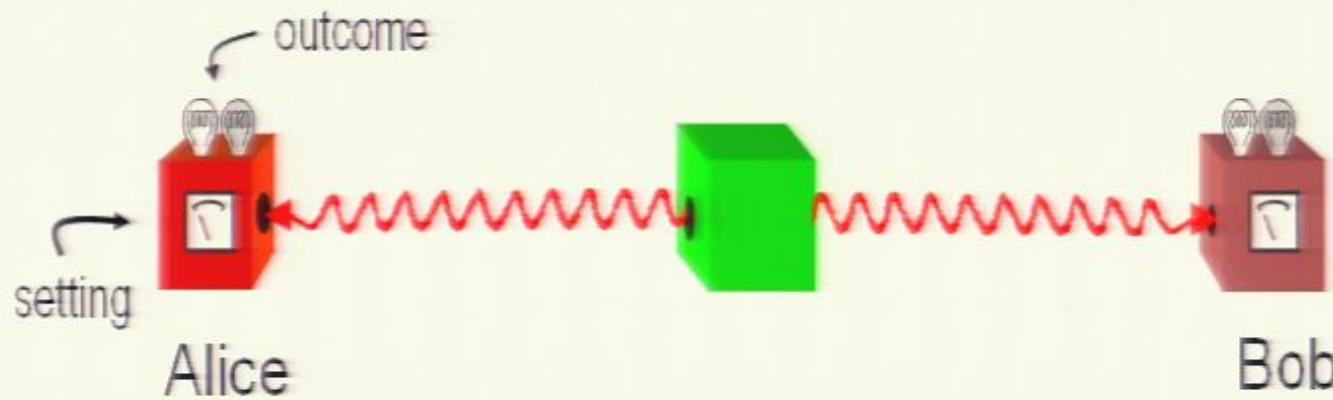
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Pauli to Born (1954)

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Against all the retrogressive endeavors (Bohm, Schrödinger etc. and in some sense also Einstein) I am sure that the statistical character of the Ψ -function and hence of nature's laws -- on which you insisted from the very beginning against Schrödinger's resistance -- will define the style of the laws at least for some centuries. It may be that later, e.g. in connection with the living processes, one will find something entirely new, but to dream of a way back, back to the classical style of Newton-Maxwell (and these are just dreams which these gentlemen are giving themselves up to) seems to me hopeless, groundless, bad taste. And, we could add, it is not even a beautiful dream.

Moral ...

The theory has been telling us for 74 years that there is a problem with naive realism.

Why not take the message?

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