Title: The reflection principle and quantum Bayesian decoherence

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Abstract: TBA

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The reflection principle and quantum Bayesian decoherence

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Reality versus Locality

EPR criterion of reality

"If, without in any way disturbing a system [one can gather the information required to] predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

Reality versus Locality

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Principle of locality

A measurement at A does not in any way disturb the system at B.

EPR

EPR criterion of reality plus principle of locality leads to a contradiction.

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EPR

EPR criterion of reality plus principle of locality leads to a contradiction.

Scylla or Charybdis:

abandon locality

or

abandon the EPR criterion of reality.

Charybdis

Quantum Bayesianism rejects the EPR criterion of reality.

Quantum Bayesianism also rejects the eigenvector/eigenvalue link:

If $\hat{O}|\psi\rangle = \lambda|\psi\rangle$, measuring $\hat{O}$ gives λ with certainty, but there is no element of reality corresponding to λ prior to the measurement.

> "Unperformed experiments have no outcomes." (Asher Peres)

Quantum Bayesianism

A quantum state ρ represents an agent's personalist probabilities, or Bayesian degrees of belief, concerning the results of measurements.

Quantum states are probabilities

With respect to an informationally complete POVM, $\{E_1, \ldots, E_n\}$, a quantum state ρ is represented by a vector of probabilities, $\{p_1, \ldots, p_n\}$, defined by $p(i) = \mathrm{tr}(E_i \rho)$.

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The prepared spin state depends on the prior state of the device.

Quantum mechanics does not provide a rule for choosing the prior quantum state.

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Frequencies, chances, credences

In a repeated trial, the prior is a sequence of states:

$\rho^{(1)}$ on $\mathcal{H}$
$\rho^{(2)}$ on $\mathcal{H} \otimes \mathcal{H}$
$\rho^{(3)}$ on $\mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H}$
$\cdots\cdots$

Quantum de Finetti theorem: If the prior is exchangeable, then

$$\rho^{(n)} = \int d\rho \, p(\rho) \, \rho \otimes \rho \otimes \cdots \otimes \rho \quad (n = 1, 2, \ldots)$$

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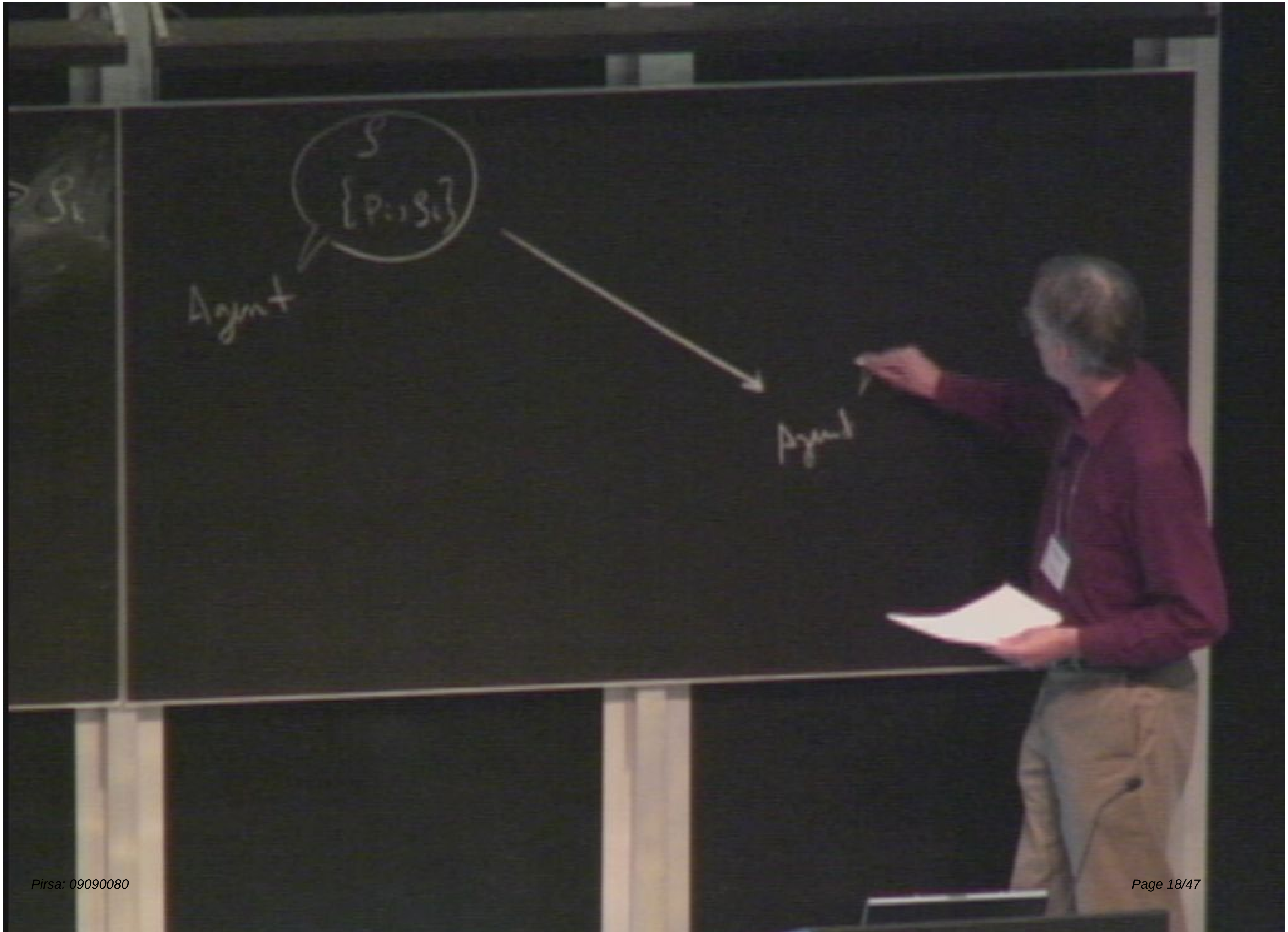
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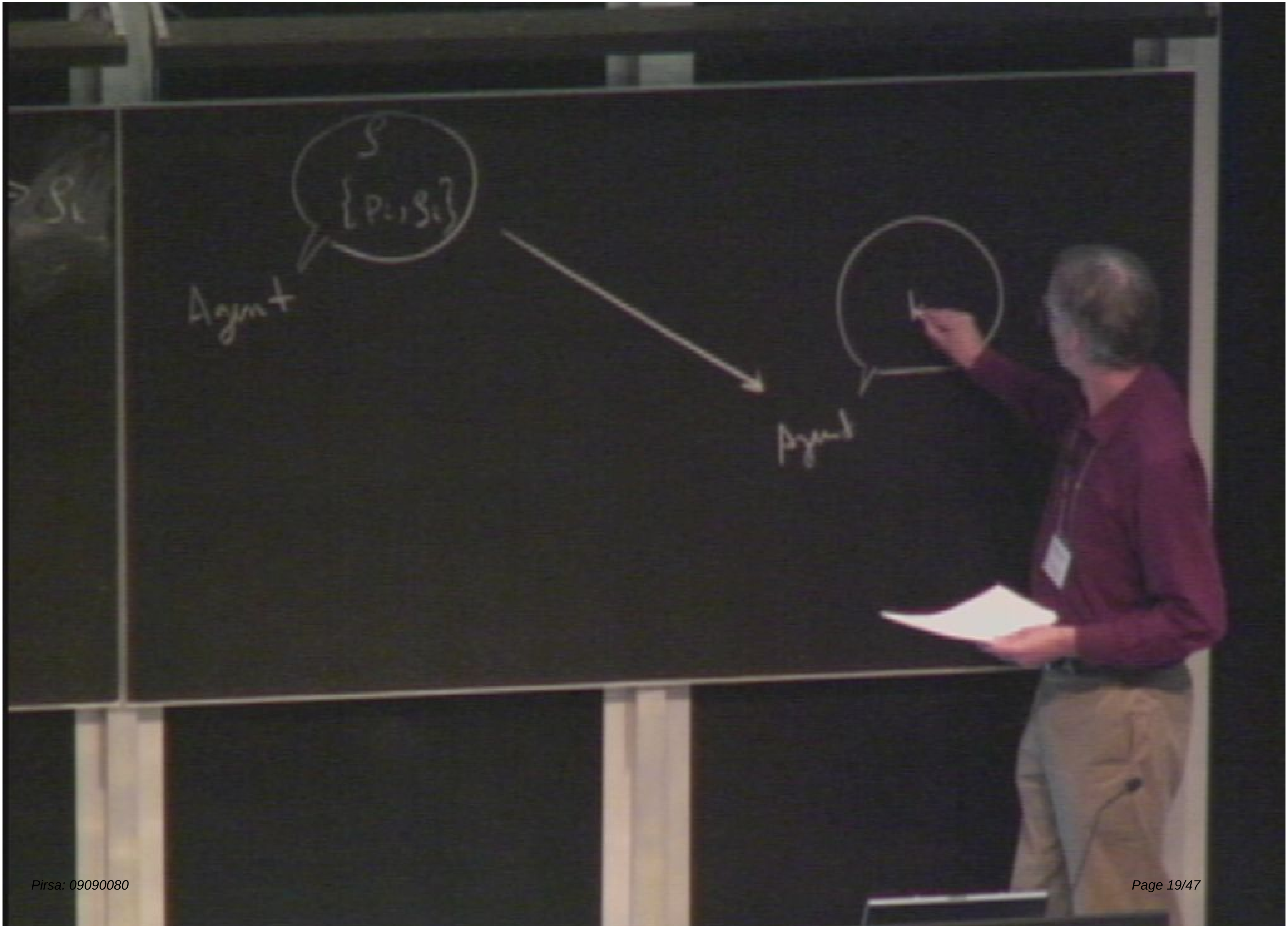
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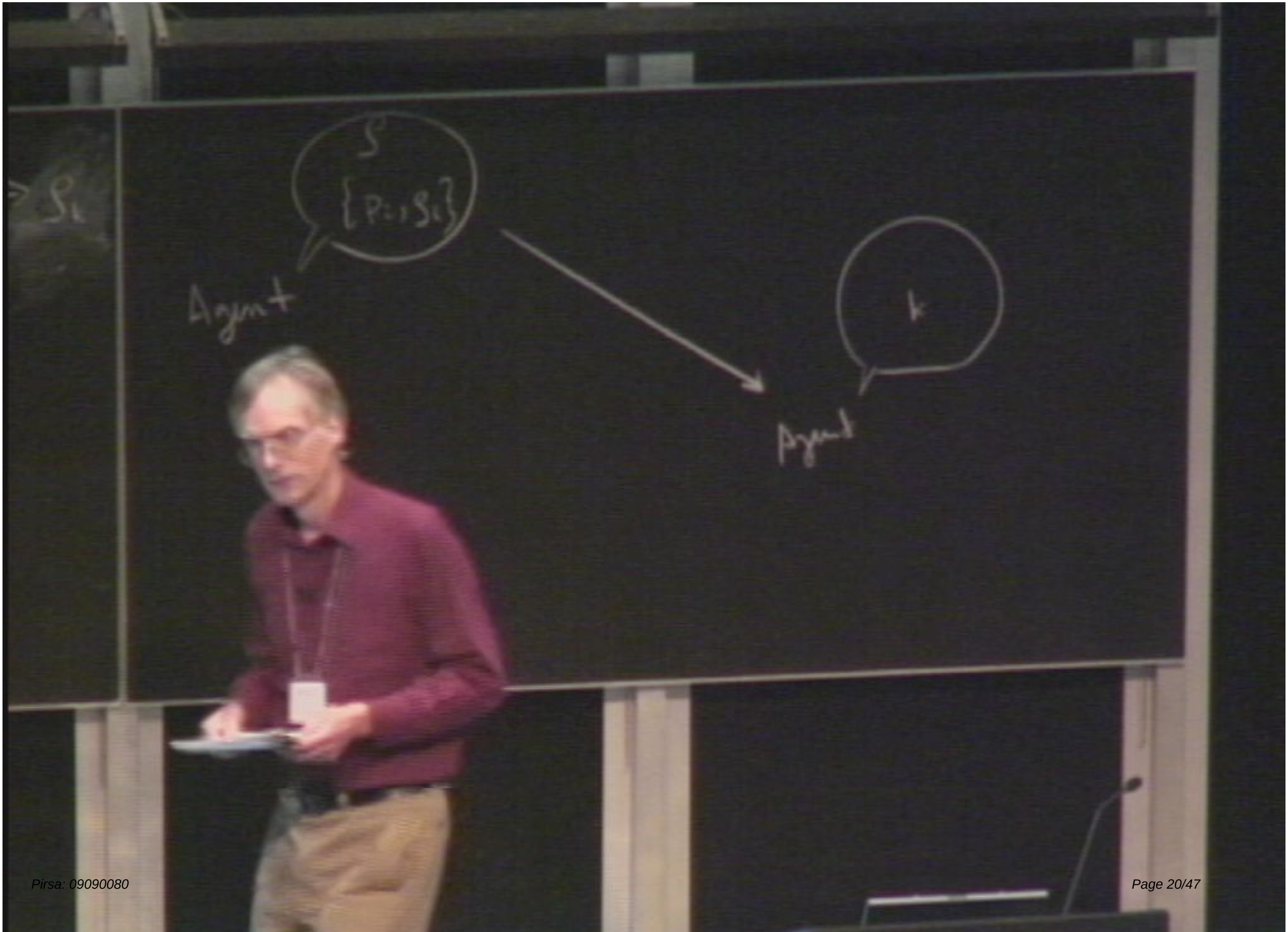
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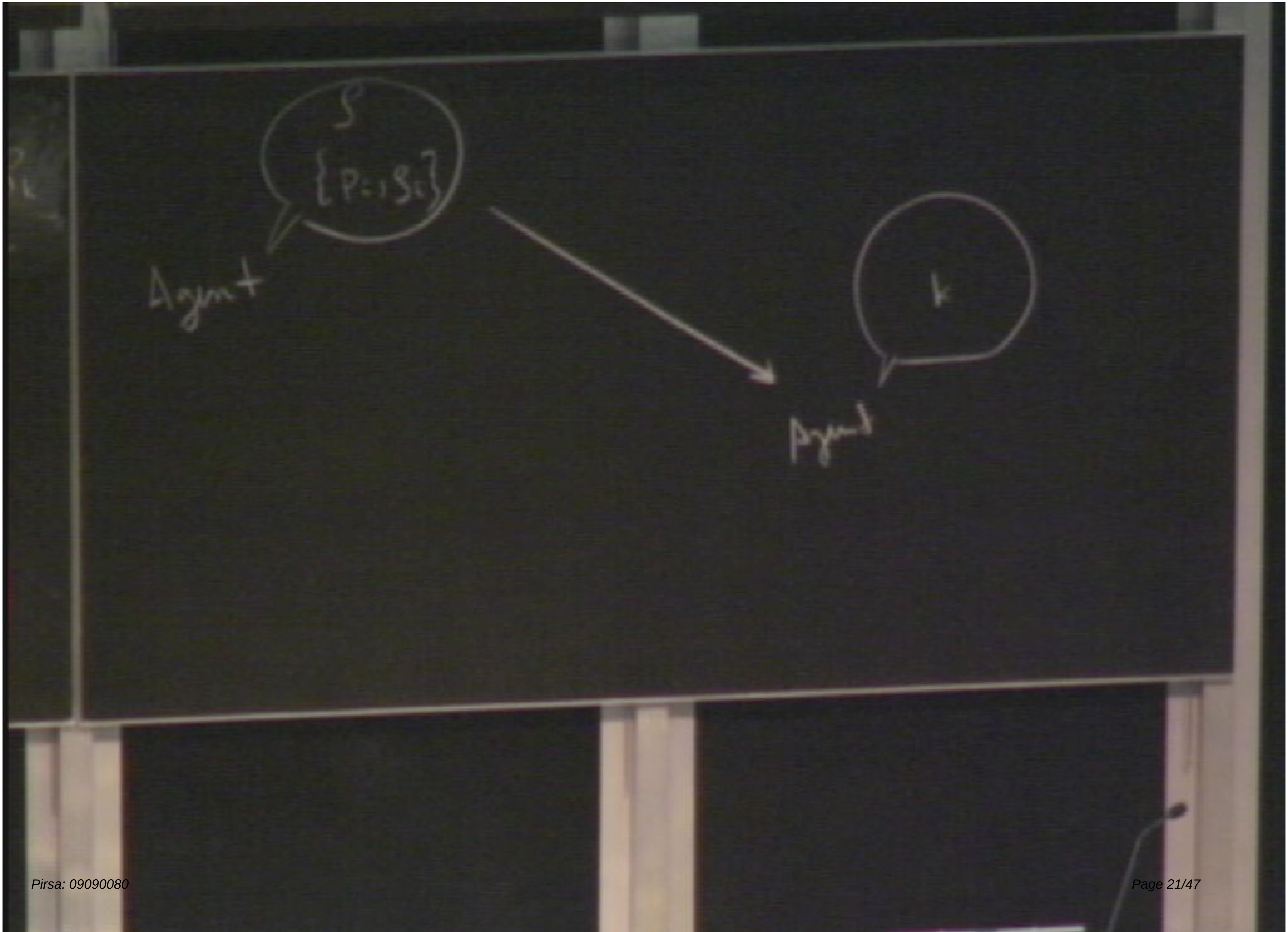
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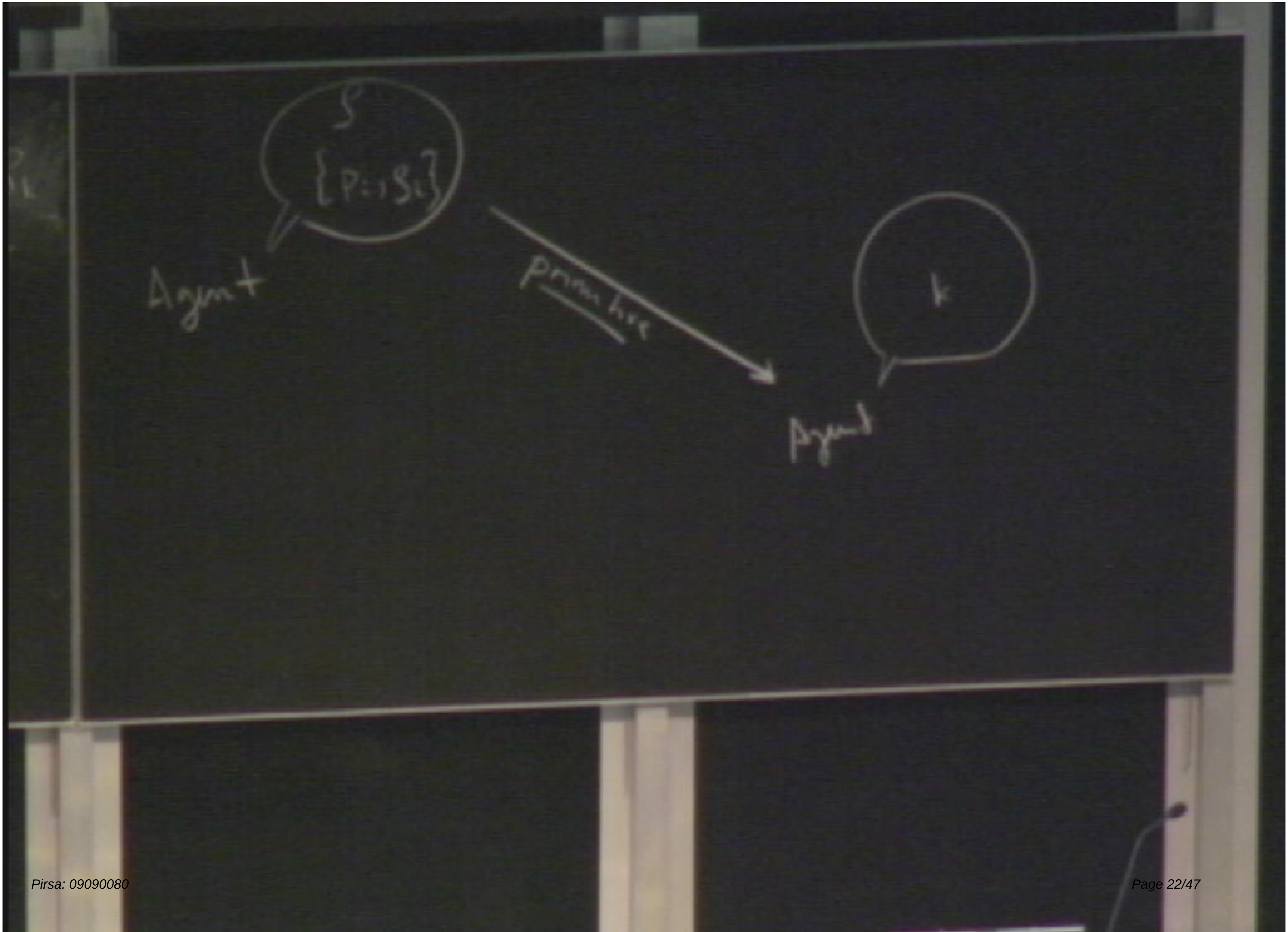
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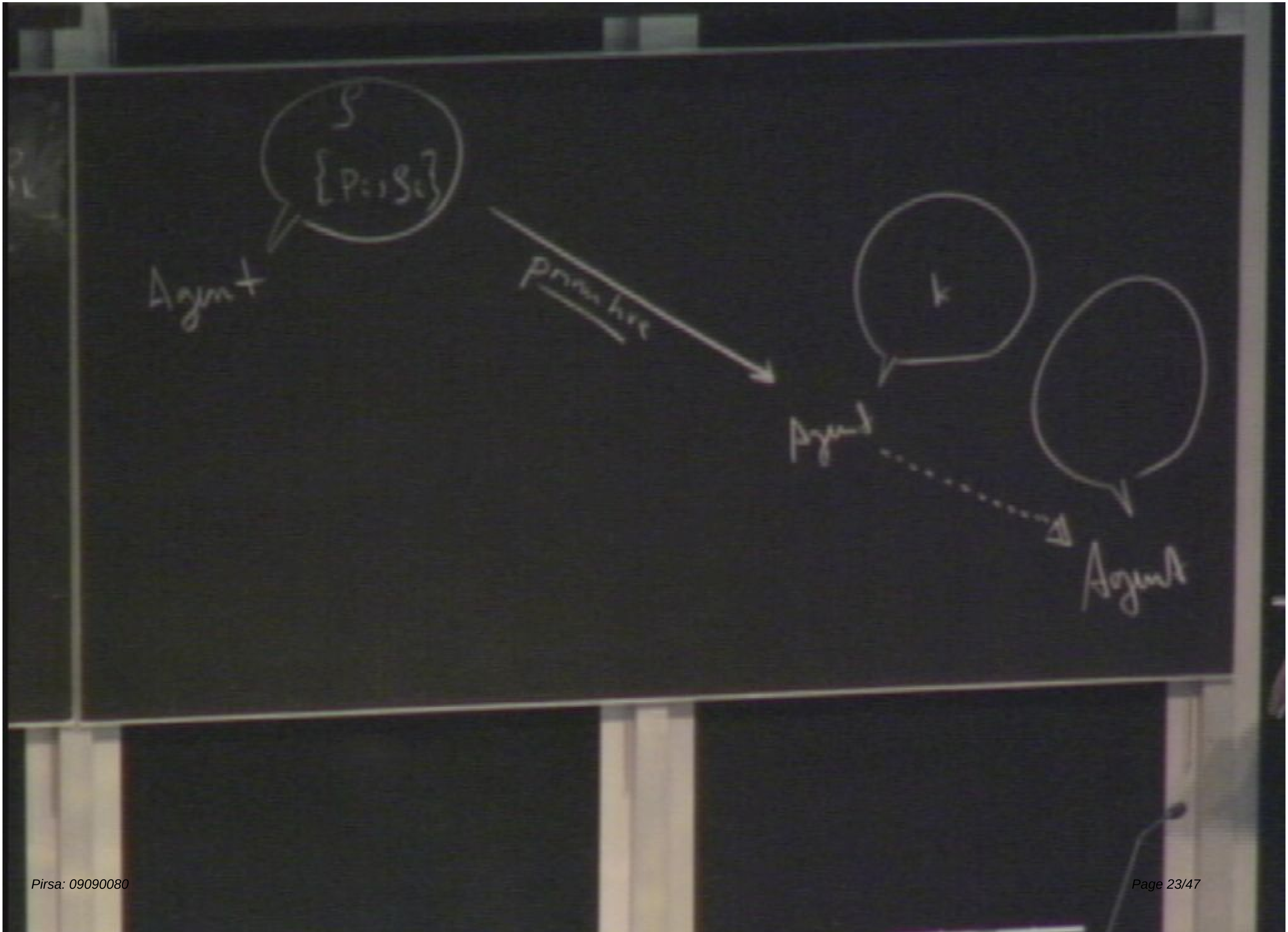
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We think it unlikely that the role of probability in quantum theory will be understood until it is generally understood in classical theory and in applications outside of physics.
E. T. Jaynes (1984)

Bayesian Theory

Bayesian Statistics offers a rationalist theory of personalistic beliefs in contexts of uncertainty, with the central aim of characterising how an individual should act in order to avoid certain kinds of undesirable behavioural inconsistencies. … The goal, in effect, is to establish rules and procedures for individuals concerned with disciplined uncertainty accounting. The theory is not descriptive, in the sense of claiming to model actual behaviour. Rather, it is prescriptive, in the sense of saying 'if you wish to avoid the possibility of these undesirable consequences you must act in the following way.'

Bernardo and Smith (1994)

Buy or Sell?

| Worth \$1 if E is true |

ticket price $\$q$

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Buy or Sell?

Worth \$1 if E is true

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Decision-theoretic definition of probability:

An agent assigns $\Pr(E) = q$ to the event E

$\Longleftrightarrow$

the agent regards $\$q$ as the fair price for the ticket.

Dutch book coherence

An agent's probability assignments (i.e., ticket valuations) are incoherent if they can lead to a sure loss; otherwise they are coherent.

Coherence alone implies

(i) $\Pr \geq 0$

(ii) $\Pr(E) = 1$ if the agent believes that E is certain to occur.

(iii) $\Pr(E \vee D) = \Pr(E) + \Pr(D)$ if the agent believes that E and D are mutually exclusive.

(iv) $\Pr(E \wedge D) = \Pr(E|D)\Pr(D)$

From Dutch book coherence to quantum decoherence

Plan for the remainder of this talk:

1. What does Dutch book coherence imply for Bayesian updating?

2. What does Dutch book coherence imply for quantum decoherence?

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A scenario

D is the event “the atmospheric pressure is below 980mbar tomorrow at 9am”

E is the event “it rains tomorrow at 6pm”

Today the agent has probabilities $\Pr(E)$, $\Pr(D)$, $\Pr(E \wedge D)$, $\Pr(E|D)$.

Tomorrow at 9am, the agent looks at the barometer and concludes that D is true. He updates his probability for rain at 6pm to $\Pr_{\mathrm{new}}(E)$.

Assuming the agent is coherent, how are the above probabilities related?

The standard Dutch book argument

Worth \$1 if $E \wedge D$ Worth $\$\,\mathrm{Pr}(E|D)$ if $\neg D$	price $\$\,\mathrm{Pr}(E|D)$

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some combination of these tickets will lead to a sure loss.

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some combination of these tickets will lead to a sure loss.

This says nothing about $\Pr_{\mathrm{new}}(E)$. (Hacking, 1967)

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Impasse?

Dutch book coherence does not put any constraints on tomorrow's probabilities in terms of today's probabilities...

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Impasse?

Dutch book coherence does not put any constraints on tomorrow's probabilities in terms of today's probabilities...

but it constrains today's probabilities in terms of tomorrow's.

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Our scenario revisited

D is the event "the atmospheric pressure is below 980mbar tomorrow at 9am"

E is the event "it rains tomorrow at 6pm"

Today the agent has probabilities $\Pr(E)$, $\Pr(D)$, $\Pr(E|D)$ ...

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Our scenario revisited

D is the event "the atmospheric pressure is below 980mbar tomorrow at 9am"

E is the event "it rains tomorrow at 6pm"

Today the agent has probabilities $\Pr(E)$, $\Pr(D)$, $\Pr(E|D)$ ...

... and $\Pr(\Pr_{\text{new}}(E) = q)$.

Assuming the agent is coherent, how are the above probabilities related?

Van Fraassen's reflection principle

Let Q be the proposition $\mathrm{Pr}_{\mathrm{new}}(E) = q$, and assume that $\mathrm{Pr}(Q) > 0$.

Then Dutch book coherence implies

$$\mathrm{Pr}(E|Q) = q \; .$$

This is an example of a diachronic Dutch-book argument (David Lewis).

It concerns today's beliefs about tomorrow's probabilities.

Quantum measurement

$\{E_i\}$ a measurement tomorrow morning.

$\{F_j\}$ an informationally complete measurement tomorrow evening.

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Reflection principle $\Longrightarrow$

$$\Pr(j) = \sum_i p_i \text{tr}(F_j \rho_i) = \text{tr}(F_j \sum_i p_i \rho_i)$$