Title: Gravitational waves and the neutron-star equation of state

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Abstract: The uncertainty in the equation of state of cold matter above nuclear density is notorious. Despite four decades of neutron-star observations, recent observational estimates of neutron-star radii still range from 8 to 16 km; the pressure above nuclear density is not known to better than a factor of 5; and one cannot yet rule out the possibility that the ground state of cold matter at zero pressure might be strange quark matter -- that the term "neutron star" is a misnomer for strange quark stars.

The last few orbits of binary inspiral are sensitive to the stars' distortion, and a major goal of the next generation of gravitational wave detectors is to extract parameters characterizing the high-density equation of state from inspiral waveforms. This talk reports a first study that uses numerical simulations to estimate the accuracy with which the equation of state can be measured.

Measuring the Neutron-Star EOS with Gravitational Waves from Binary Inspiral

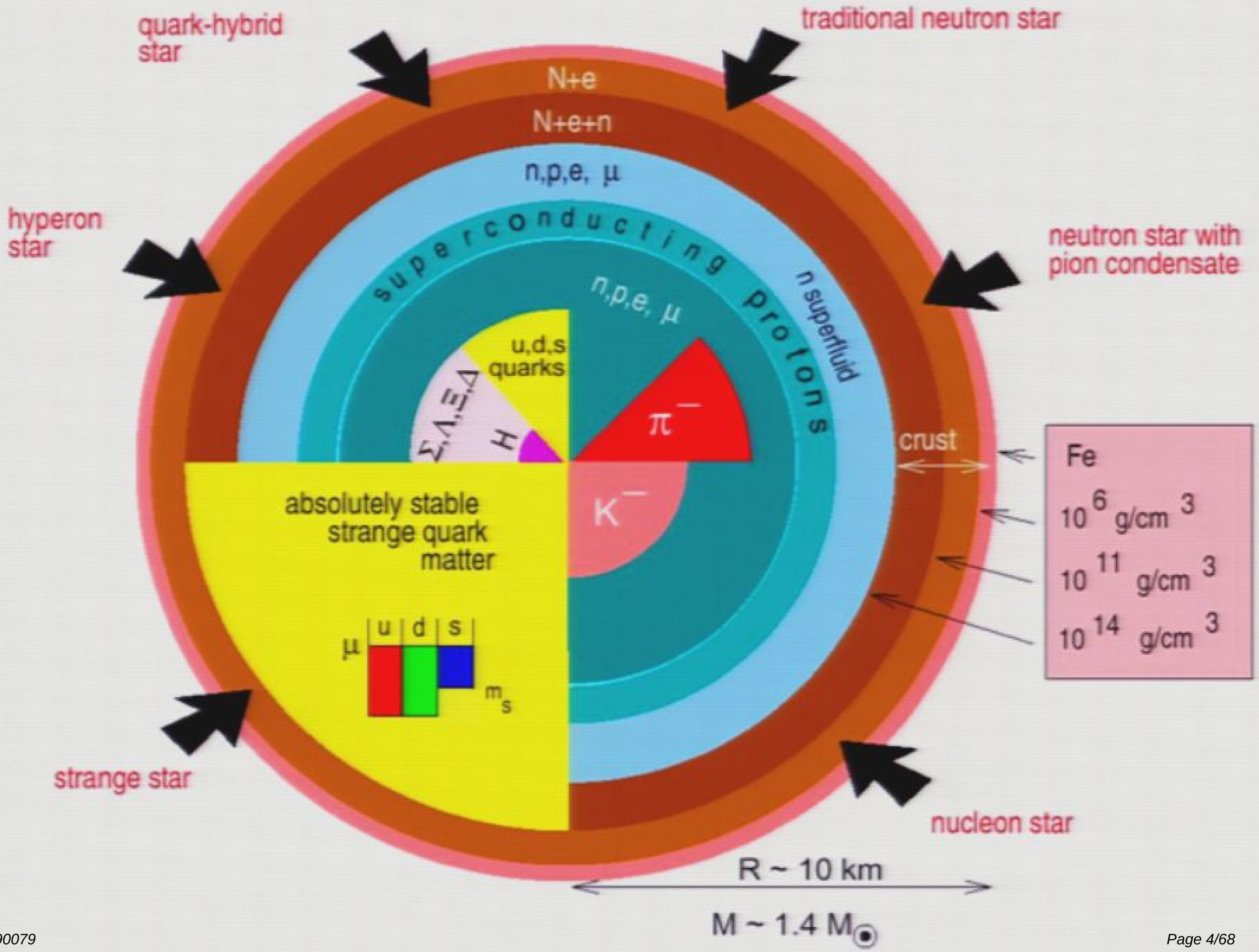
NS-NS: Jocelyn Read, Charalampos Markakis, Masaru Shibata, Koji Uryu, JF

Tanja Hinderer, Lackey, Read;

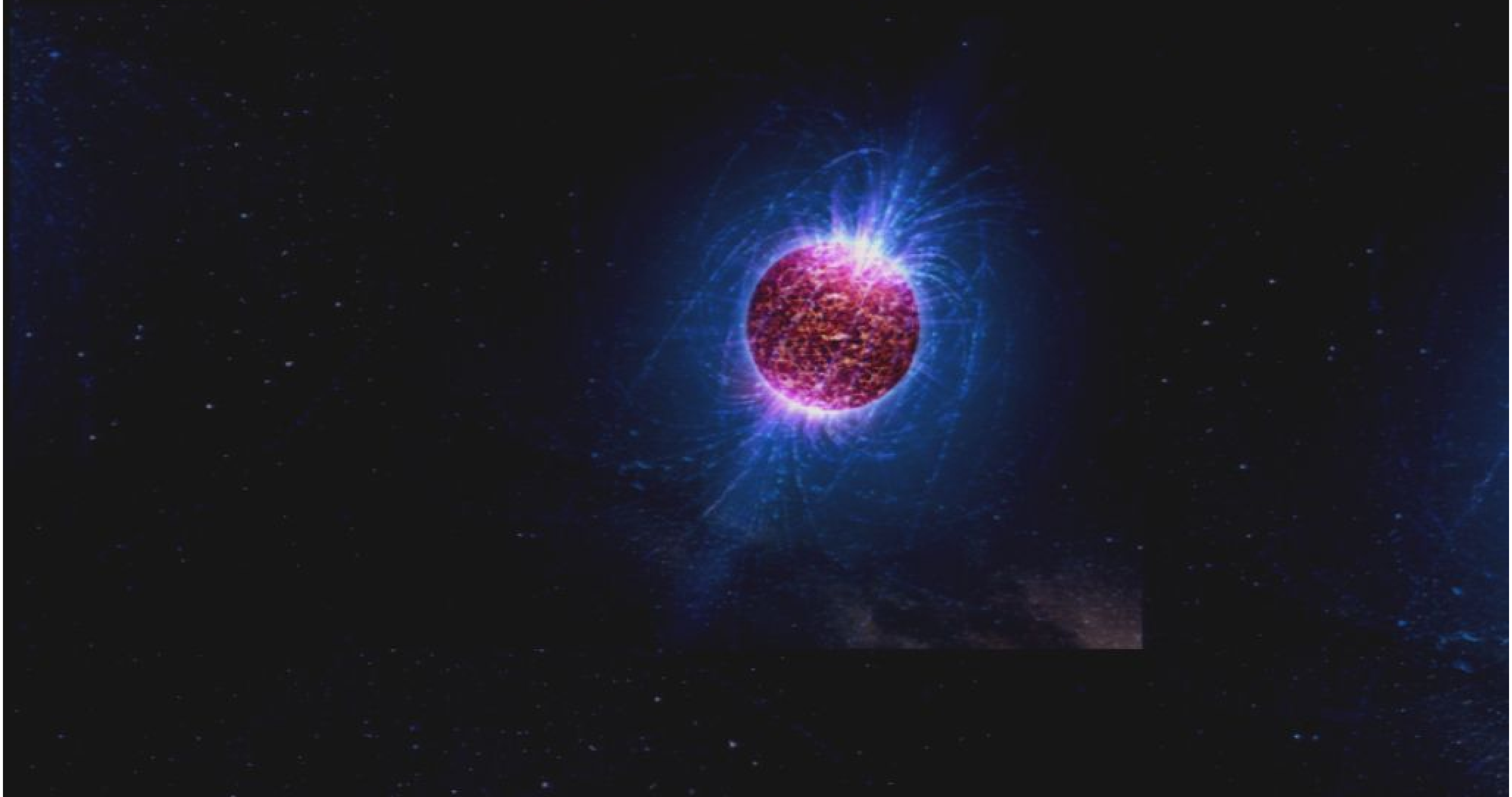
BH-NS: Shibata, Keisuke Taniguchi

Matter above nuclear density

- Far from an ideal gas of neutrons
- Pressure above nuclear density uncertain by a factor of 10.
- Hundreds of candidate equations of state relying on different underlying models
non-relativistic mean field;
relativistic mean field,
hyperons;
pion or kaon condensates;
up, down and strange quarks


quark-hybrid star
traditional neutron star
N+e
N+e+n
n,p,e, μ
hyperon star
superconducting protons
n superfluid
neutron star with pion condensate
n,p,e, μ
u,d,s quarks
Σ,Λ,Ξ,Δ
H
π⁻
crust
Fe
10^6 g/cm^3
10^11 g/cm^3
10^14 g/cm^3
K⁻
absolutely stable strange quark matter
u d s
μ
m_s
strange star
nucleon star
R ~ 10 km
M ~ 1.4 M☉

Radius uncertain to about a factor of 2:
8-16 km for slowly rotating neutron stars



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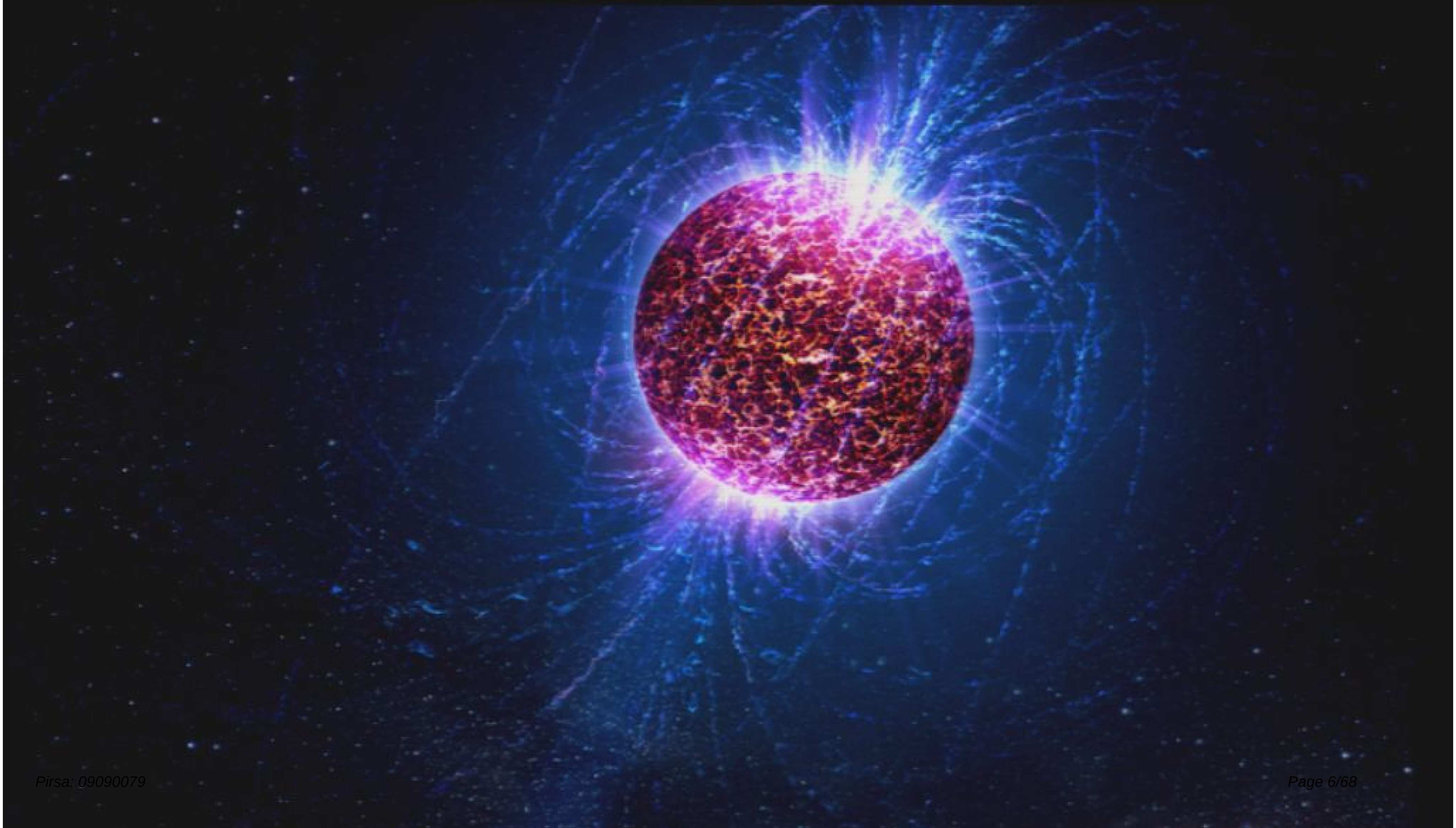


Systematizing Constraints on EOS

Want parameterized EOS $p=p(\rho)$

$p =$ pressure

$\rho =$ rest-mass density

$\epsilon =$ energy density

- Number of parameters
  $<$ number of astrophysical observables
- Number of parameters large enough to accurately model the universe of candidate EOSs. Test by accuracy in modeling the universe of candidate equations of state.

Systematizing Constraints on EOS

Difficulty:

No small, model-independent set of fundamental parameters.

Systematizing Constraints on EOS

Difficulty:

No small, model-independent set of fundamental parameters.

Instead, try phenomelogical parametrization

Systematizing Constraints on EOS

Nonrelativistic and highly relativistic ideal Fermi gases have EOS

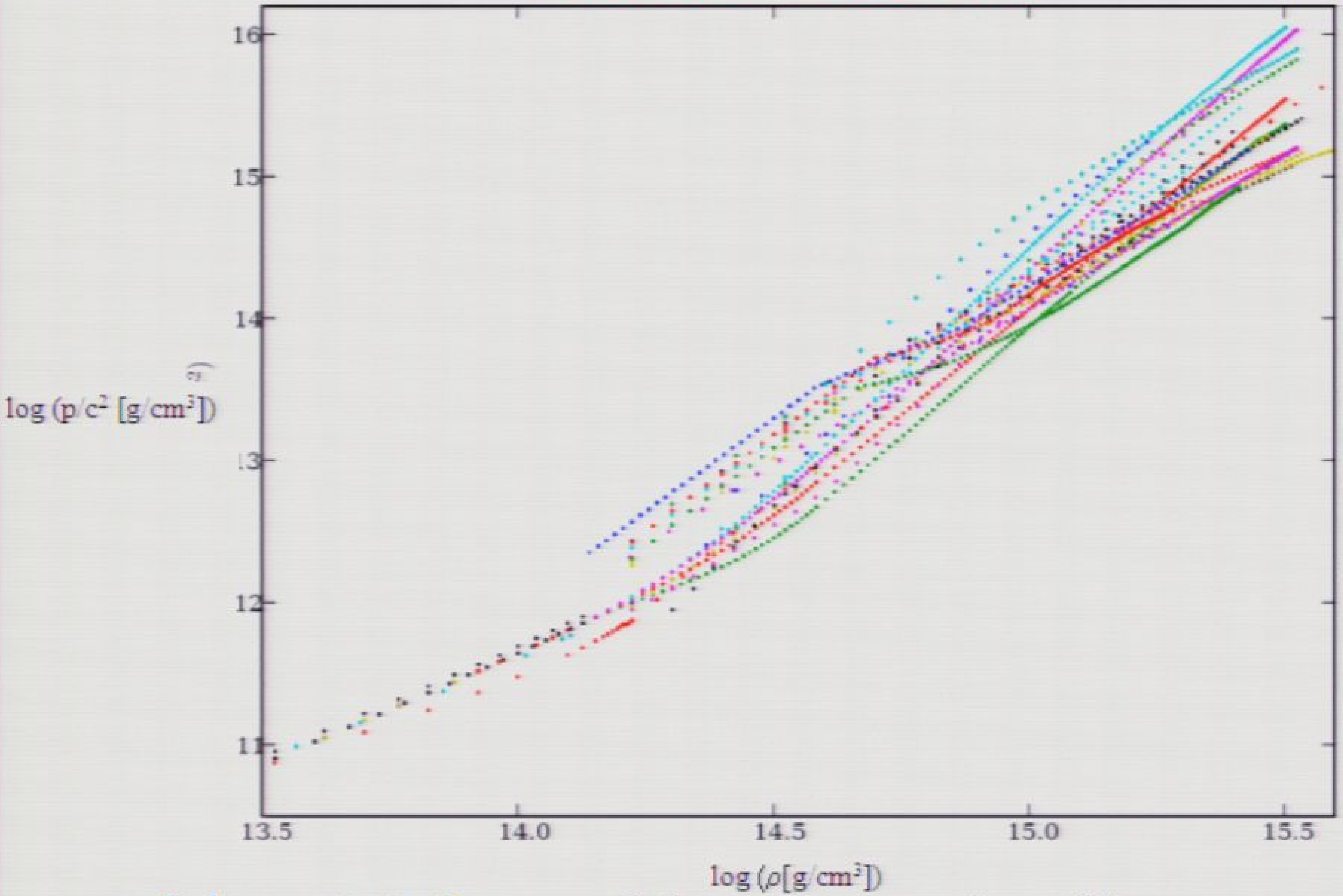
$$pV^{\Gamma} = \text{constant}$$

or

$$p = K\rho^{\Gamma}$$

with $\Gamma = \dfrac{5}{3}$ and $\Gamma = \dfrac{4}{3}$, respectively.

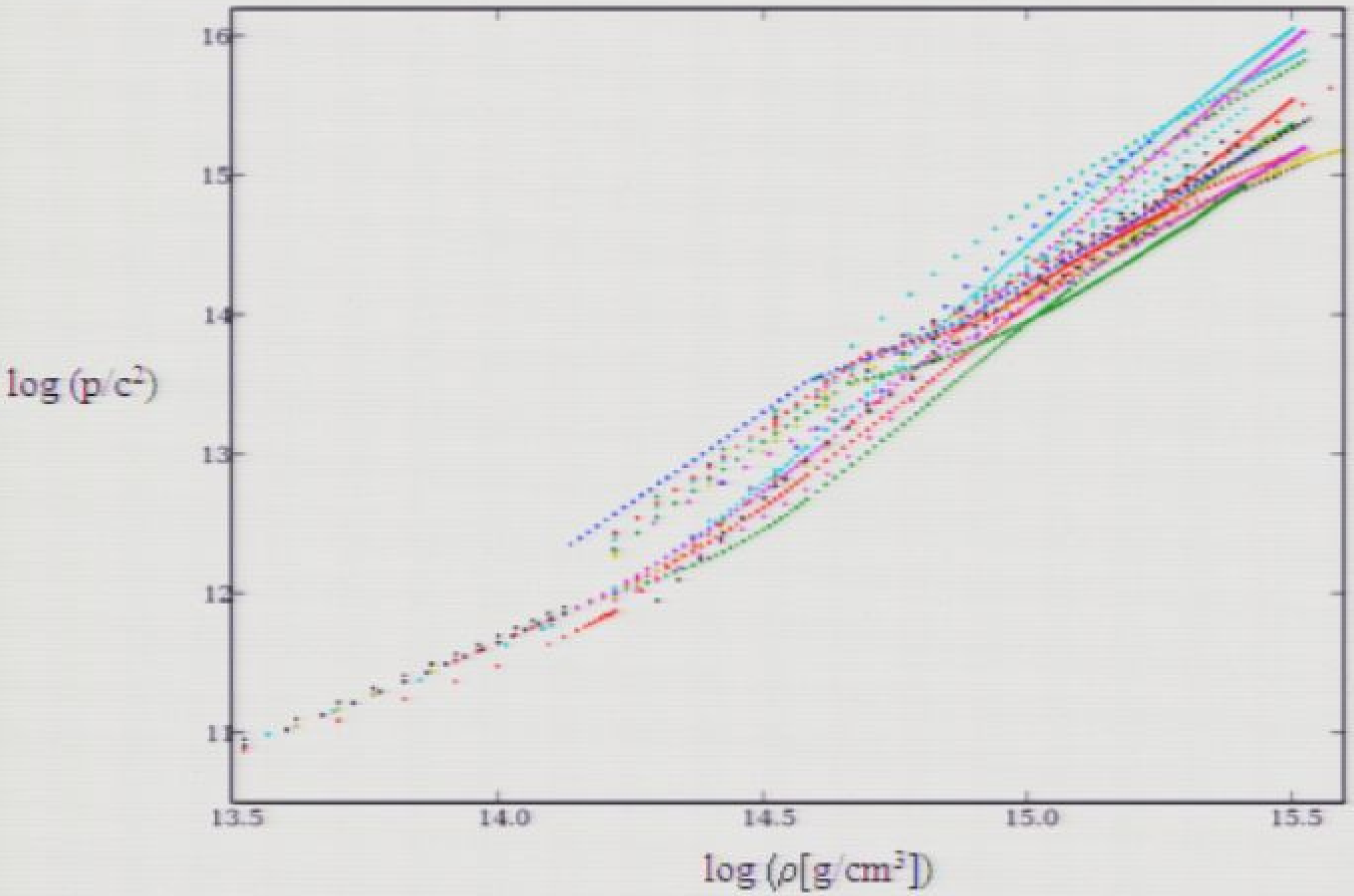
Candidate EOSs



log p [log (ρ)] roughly piecewise linear

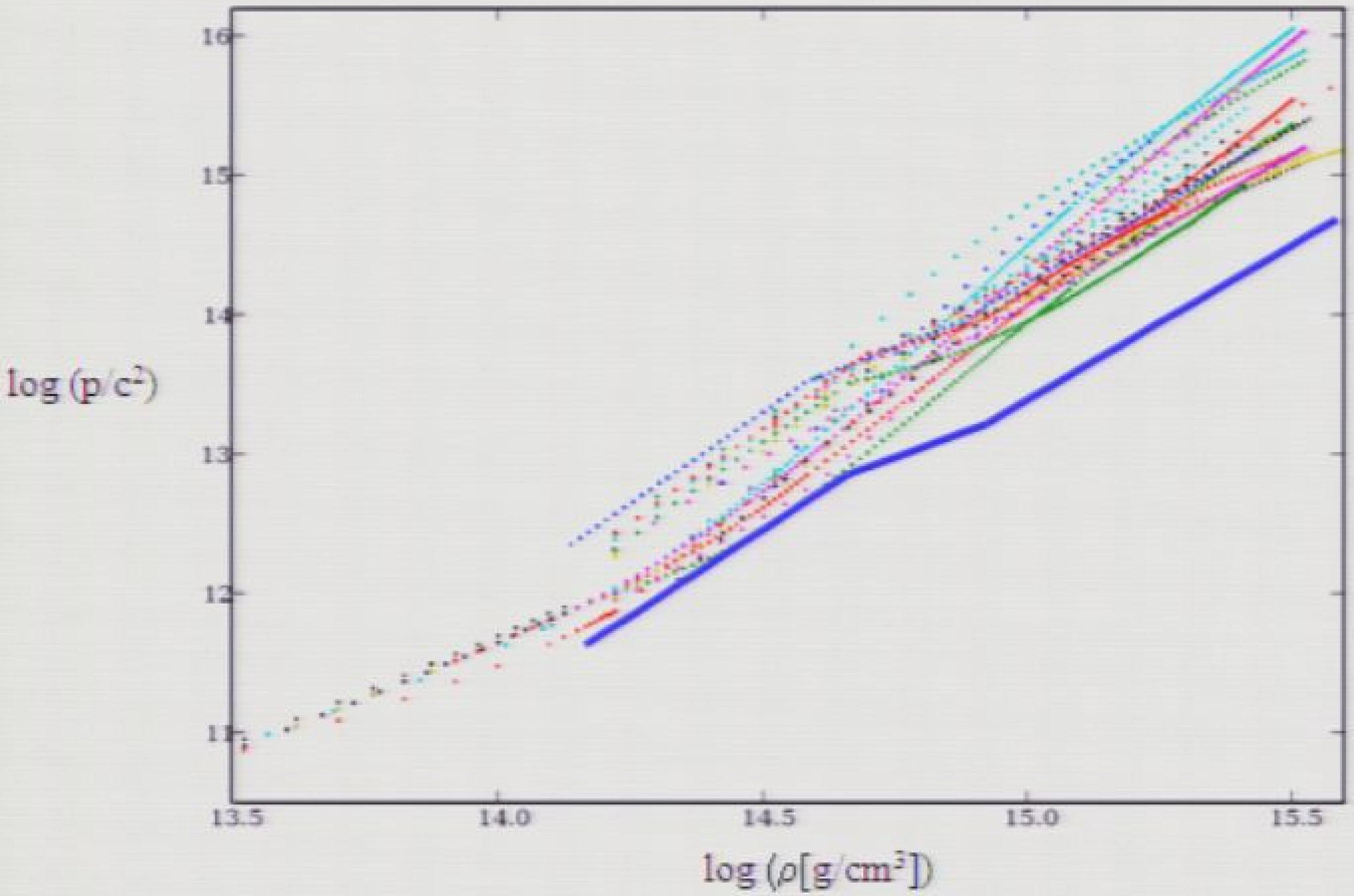
(Vuille'99)

Candidate EOSs



Specify $\dfrac{d\log p}{d\log\rho}$ at 3 different densities

Candidate EOSs



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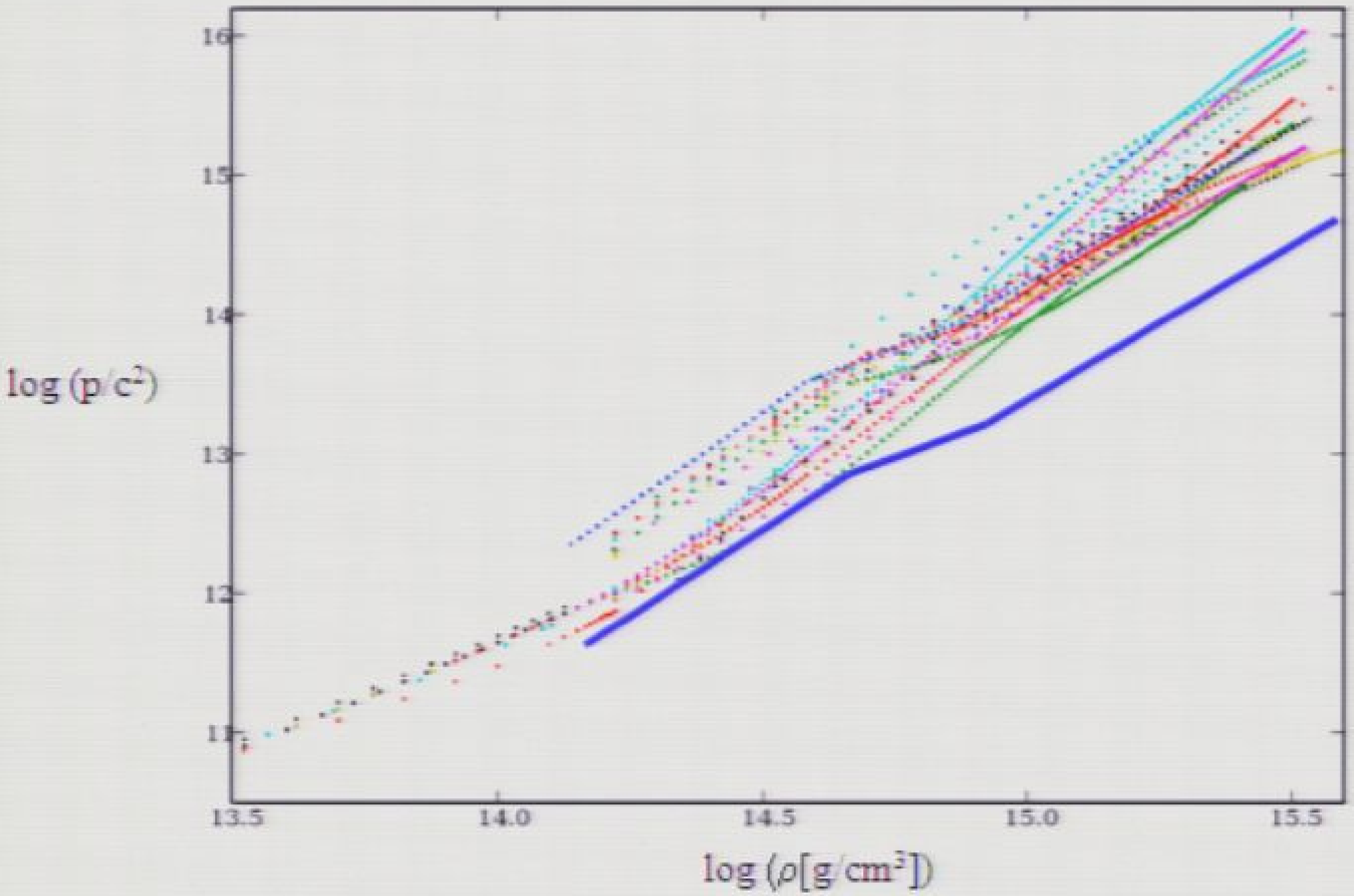
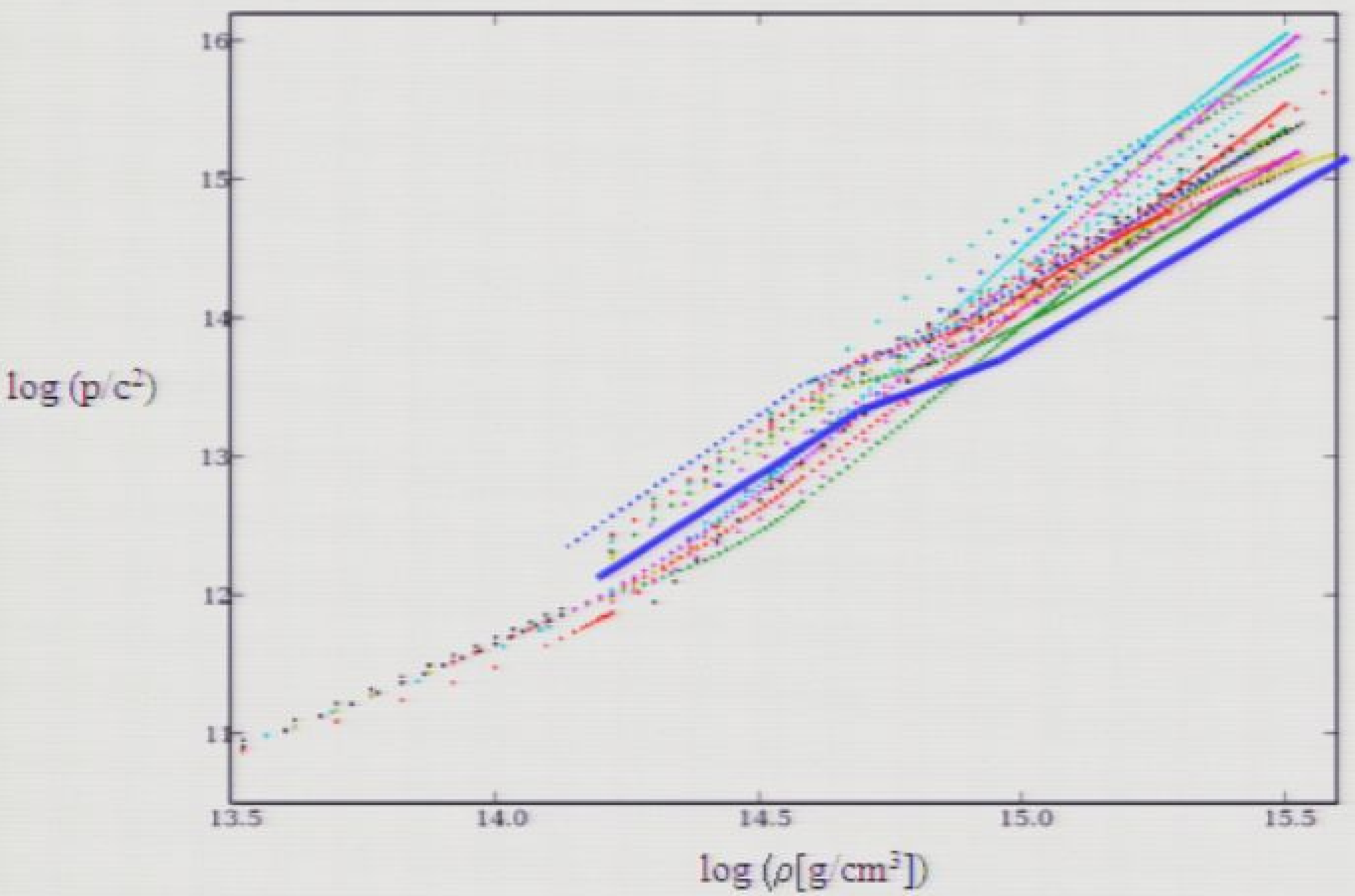


Specify $\frac{d\log p}{d\log \rho}$ at 3 different densities

Translate $p(\rho)$ curve up or down

Candidate EOSs



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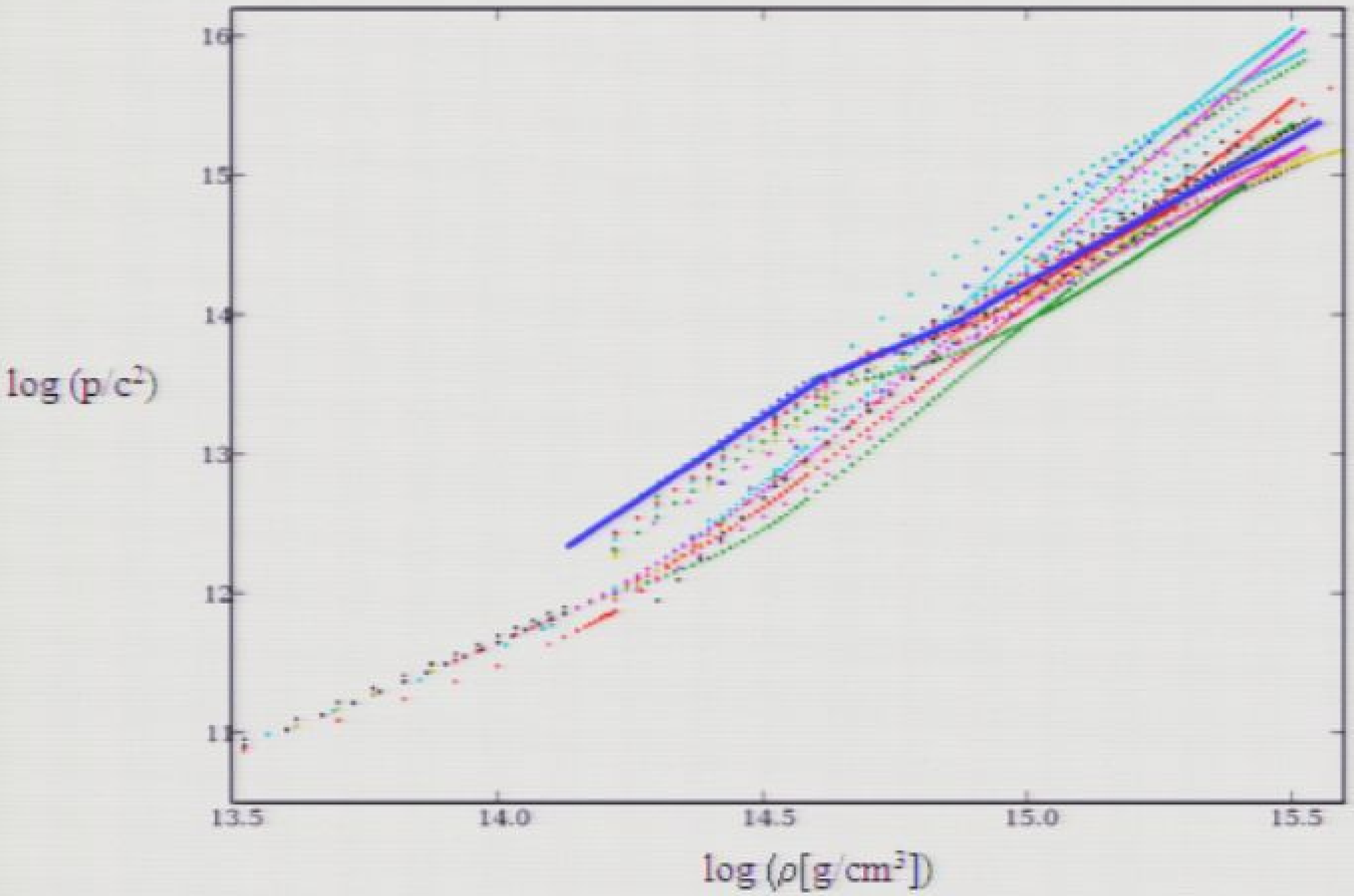
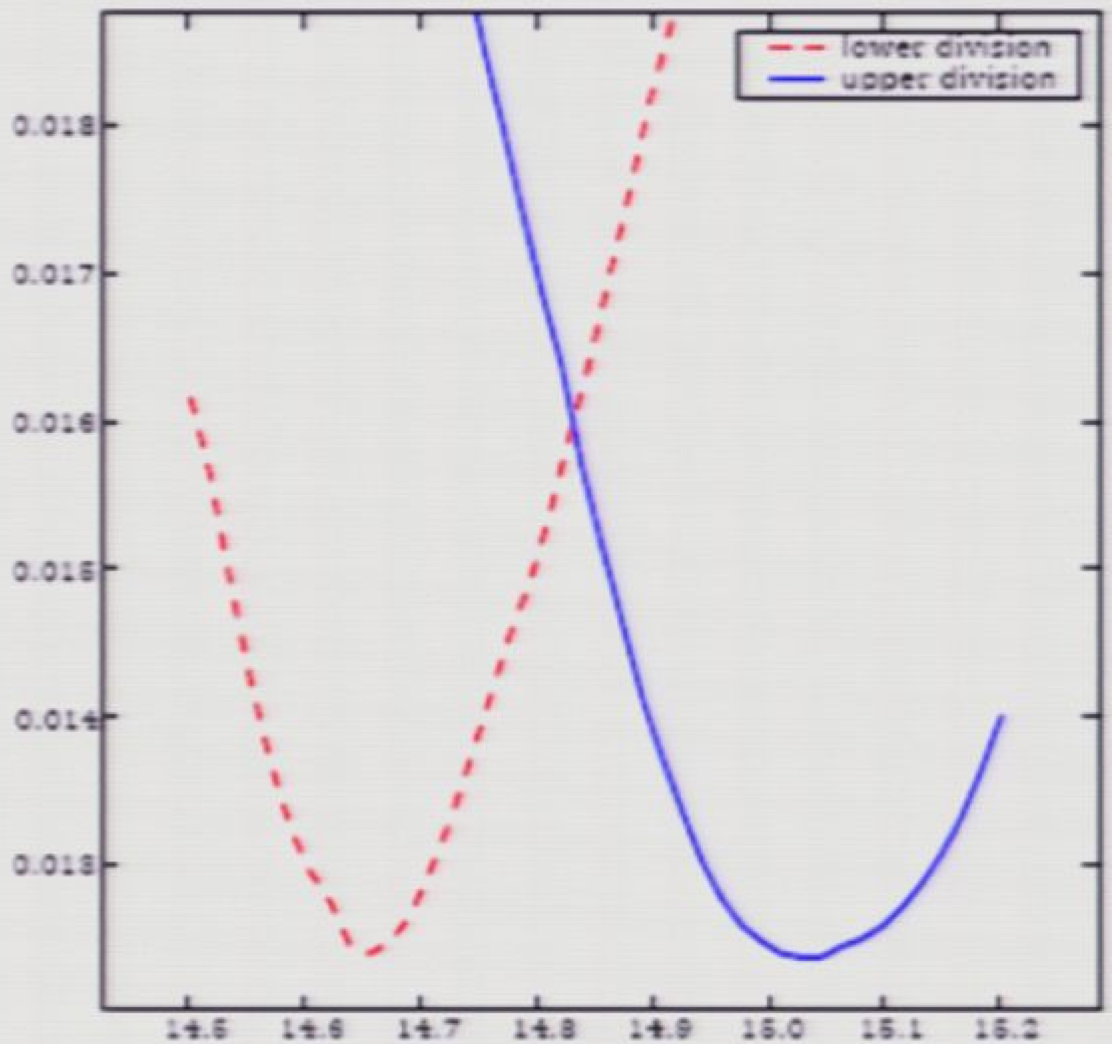


Specify $\dfrac{d\log p}{d\log \rho}$ at 3 different densities

Translate $p(\rho)$ curve up or down

Happily, universe of candidates has preferred dividing densities:

Average rms error in match to universe of candidate EOSs



$\log \rho_{\text{dividing}}\, [\text{g/cm}^3]$

These are piecewise polytropes

The parameters: p_*, Γ_1, Γ_2, Γ_3

$$p_* = p(2.5 \times 10^{14}\,\mathrm{g/cm^3})$$

$$p(\rho) = \begin{cases} \text{Known} & \rho < \rho_{\text{nuclear}} \equiv \rho_1 \\ K_1 \rho^{\Gamma_1} & \rho_1 < \rho < \rho_2 \\ K_2 \rho^{\Gamma_2} & \rho_2 < \rho < \rho_3 \\ K_m \rho^{\Gamma_3} & \rho > \rho_3 \end{cases}$$

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K_i fixed by continuity

Given $p = p(\rho)$, energy density $\epsilon = \epsilon(\rho)$ determined by the first law of thermodynamics:

$$d\epsilon = \left(\epsilon + p\right)\frac{d\rho}{\rho}$$

Given $p = p(\rho)$, energy density $\epsilon = \epsilon(\rho)$ determined by the first law ... dynamics:

$$\frac{d\rho}{\rho} = -\frac{dV}{V}$$

$$d\epsilon = \left(\epsilon + p\right)\frac{d\rho}{\rho}$$

Given $p = p(\rho)$, energy density $\epsilon = \epsilon(\rho)$ determined by the first law of thermodynamics:

$$d\epsilon = \left(\epsilon + p\right)\frac{d\rho}{\rho}$$

$-\epsilon \frac{dV}{V}$ Change in E/V from change in V

These are piecewise polytropes

See also Chris Vuille & Jim Ipser, 1999)

Given $p = p(\rho)$, energy density $\epsilon = \epsilon(\rho)$ determined by the first law of thermodynamics:

$$d\epsilon = \left(\epsilon + p\right)\frac{d\rho}{\rho}$$

$-p\,\frac{dV}{V}$ Change in E/V from work -pdV

Hard theoretical constraints:

- EOS known below nuclear density
- Causality: $v_{\text{sound}} < 1 \;\Rightarrow\; \dfrac{dp}{d\epsilon} < 1$

Hard astrophysical constraints:

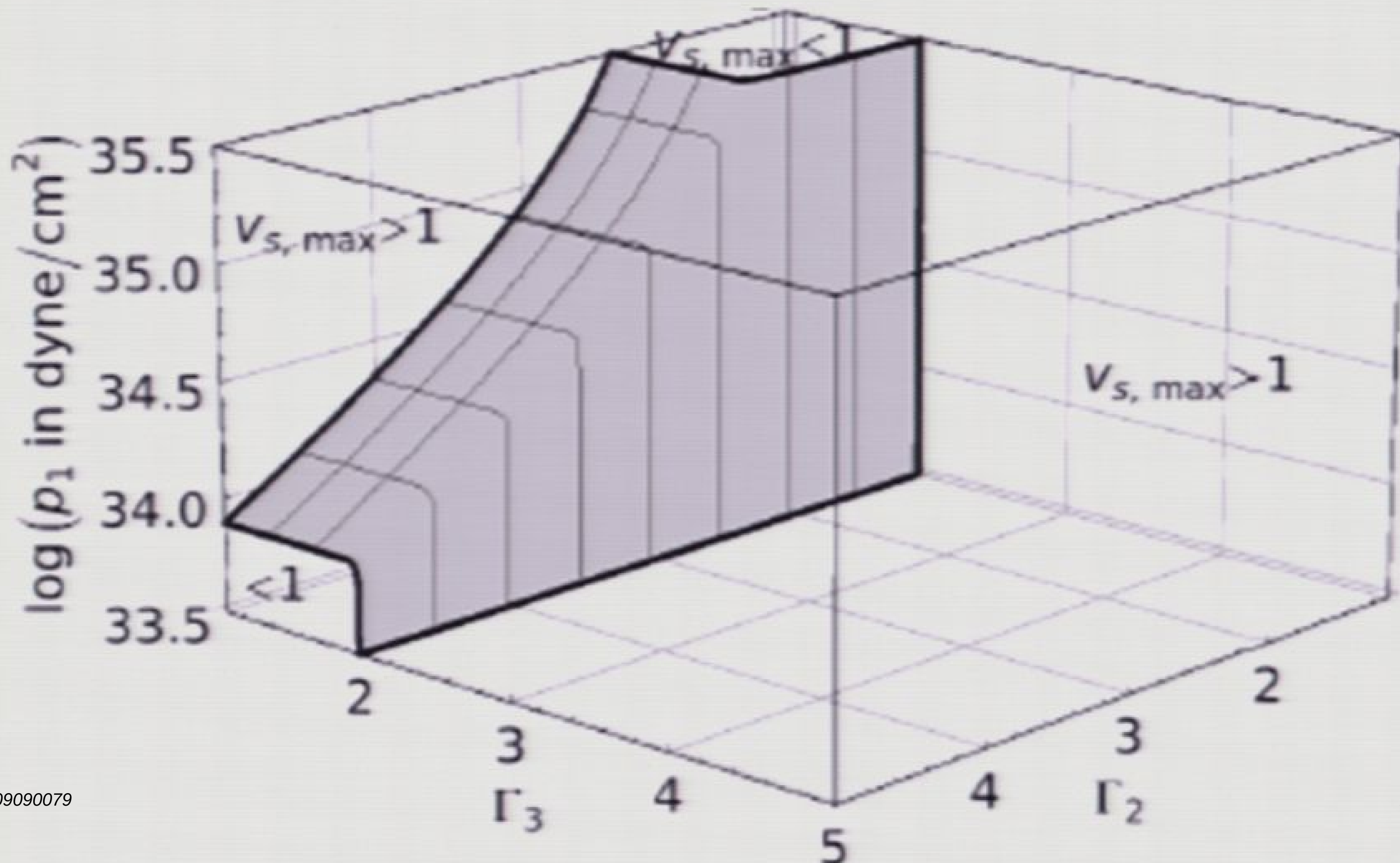
$M_{\max} > 1.7\, M_{\odot}$ (95% confidence J075-1907)

$\Omega_{\max} > 716\,\mathrm{Hz}$ (PSR J1748-2446)

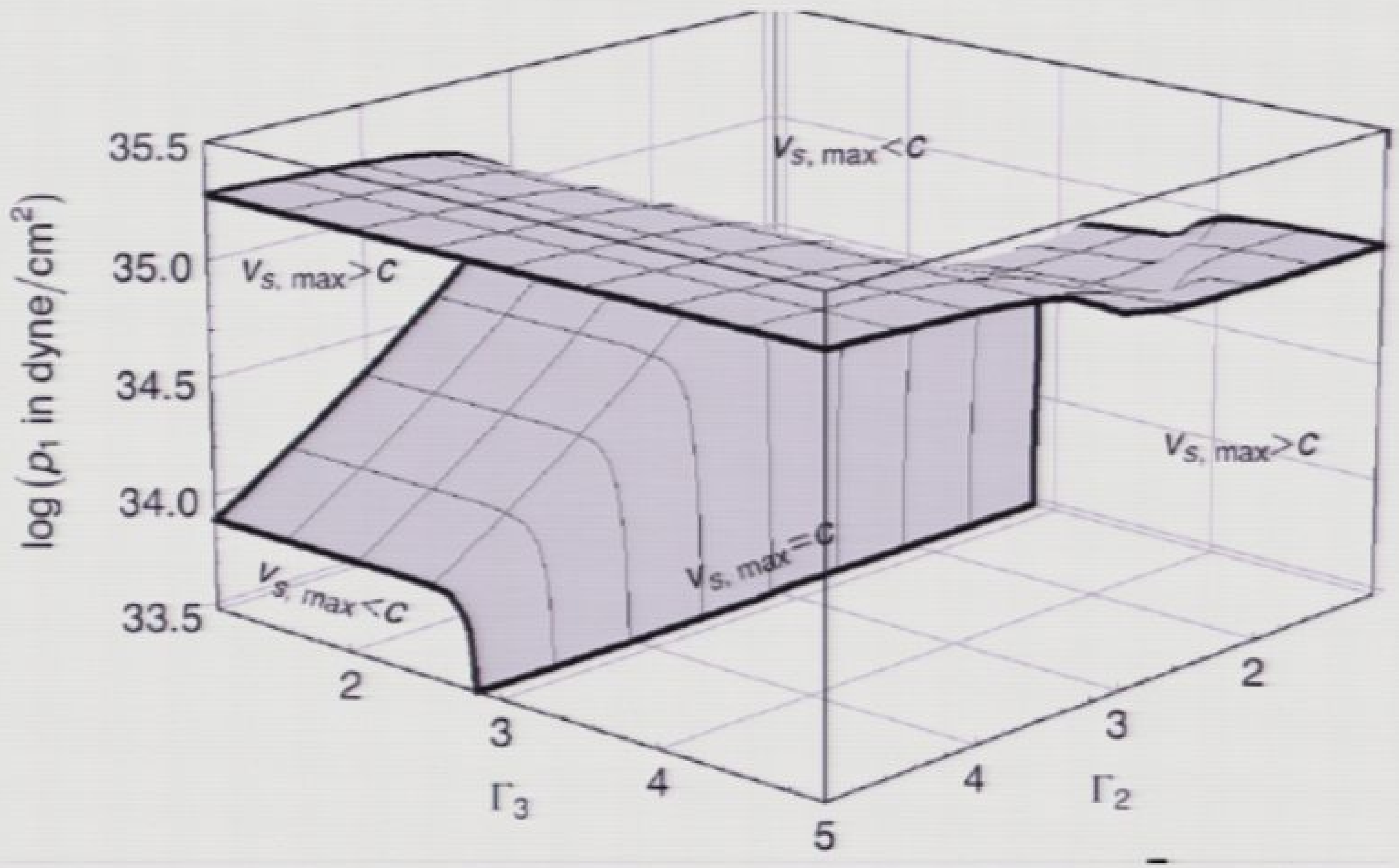
Of these, only the first –the largest observed mass –significantly constrains the EOS parameter space.

It only constrains the parameters in medium-to-high-density part of core EOS.

Causality, $v_{sound} < c$, removes the region above and in front of the surface, where the EOS is too stiff



The constraint is weaker when one demands causality only for densities below the largest density occurring in stars based on the EOS: the central density of the maximum mass star



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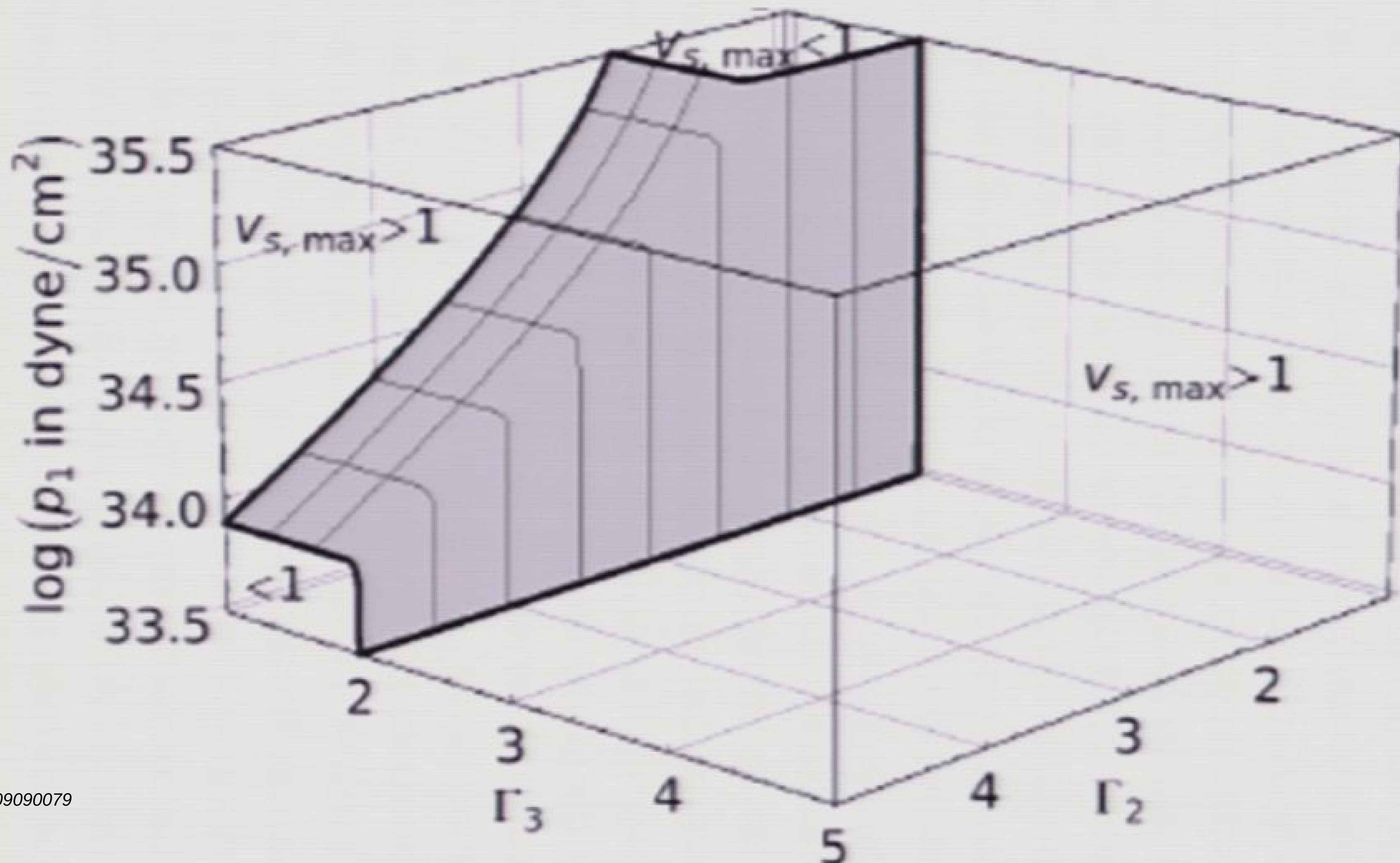

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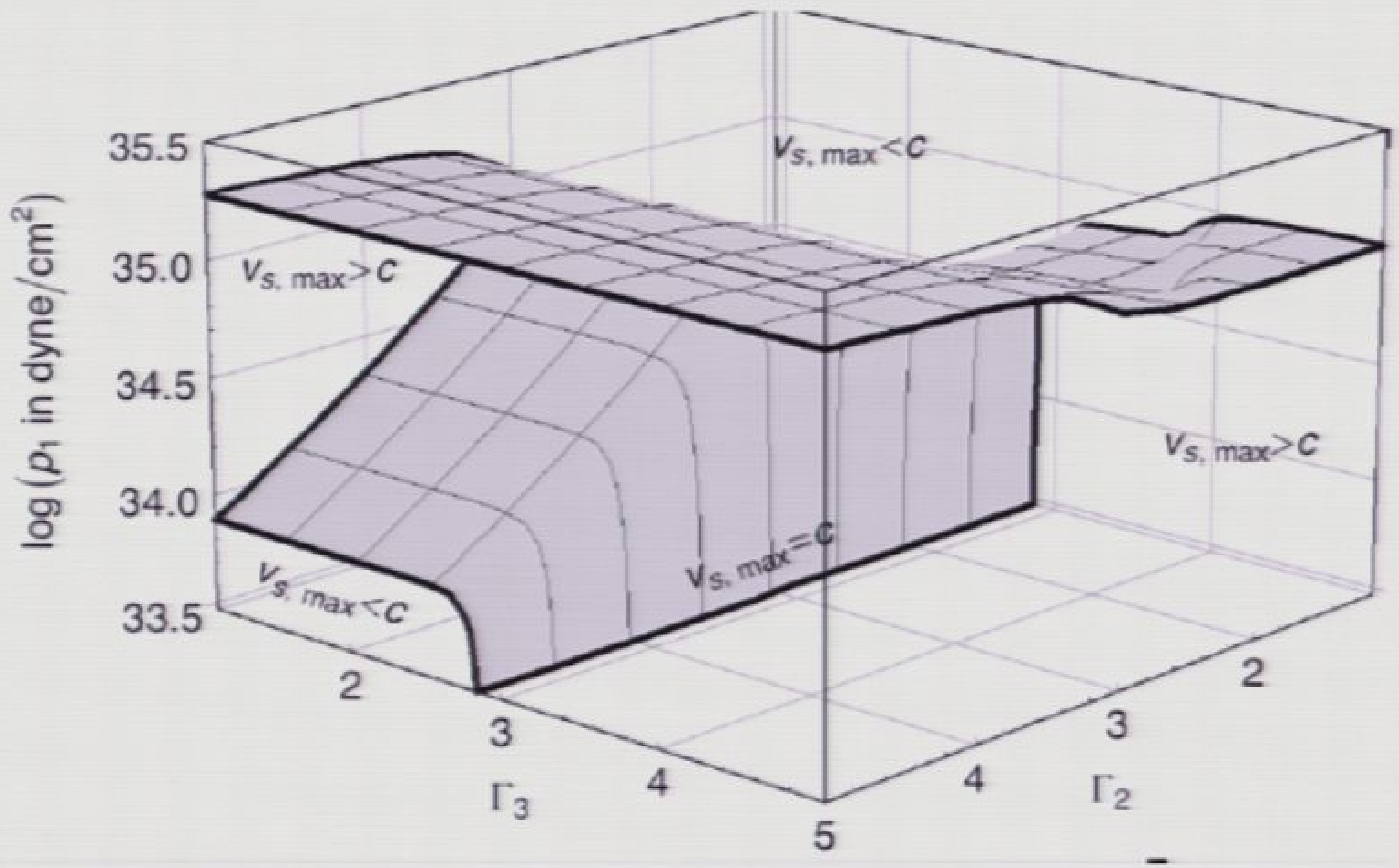
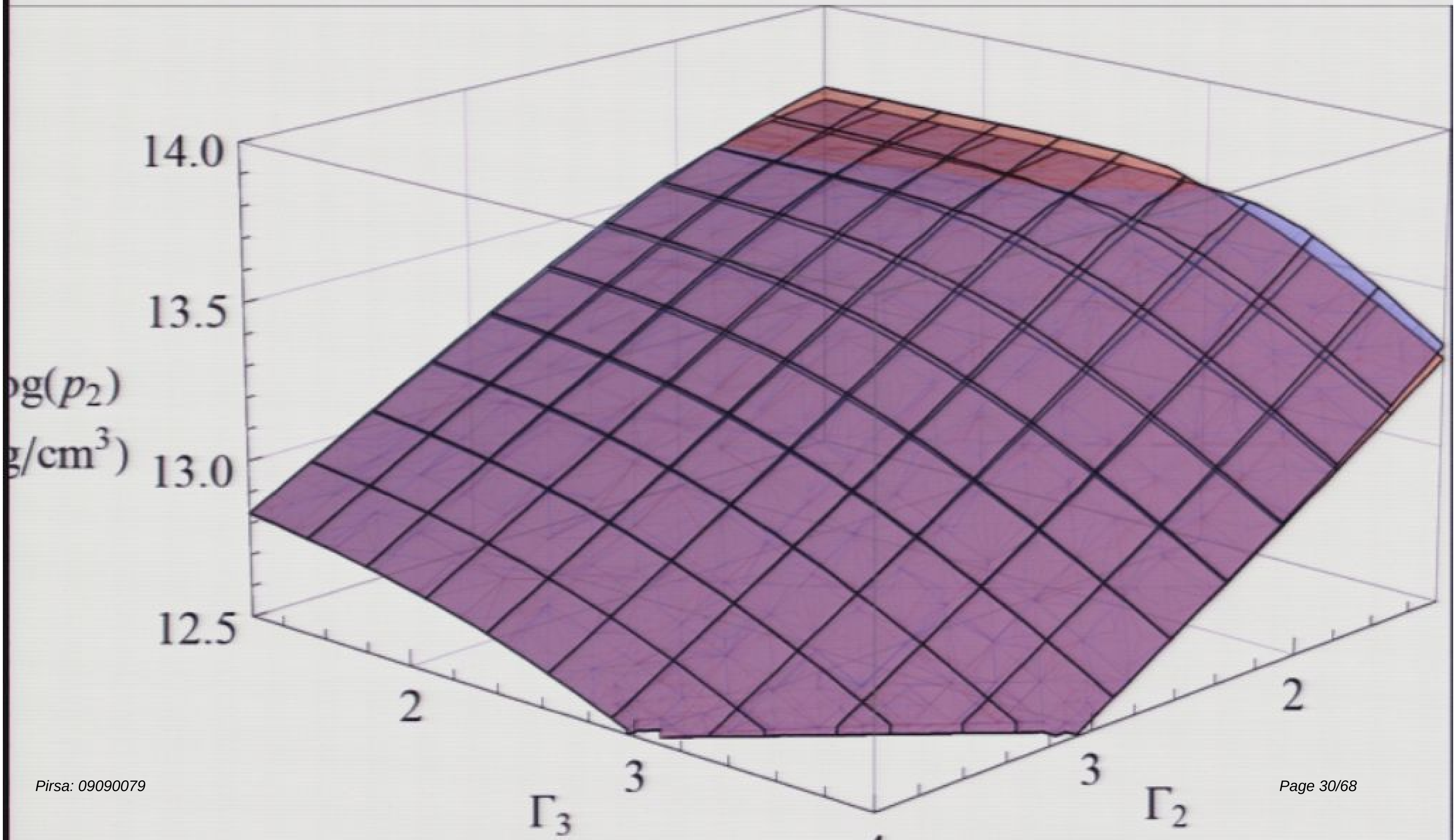


$M_{max} > 1.7\, M_{\odot}$ allows region above surface where EOS stiff enough to prevent collapse



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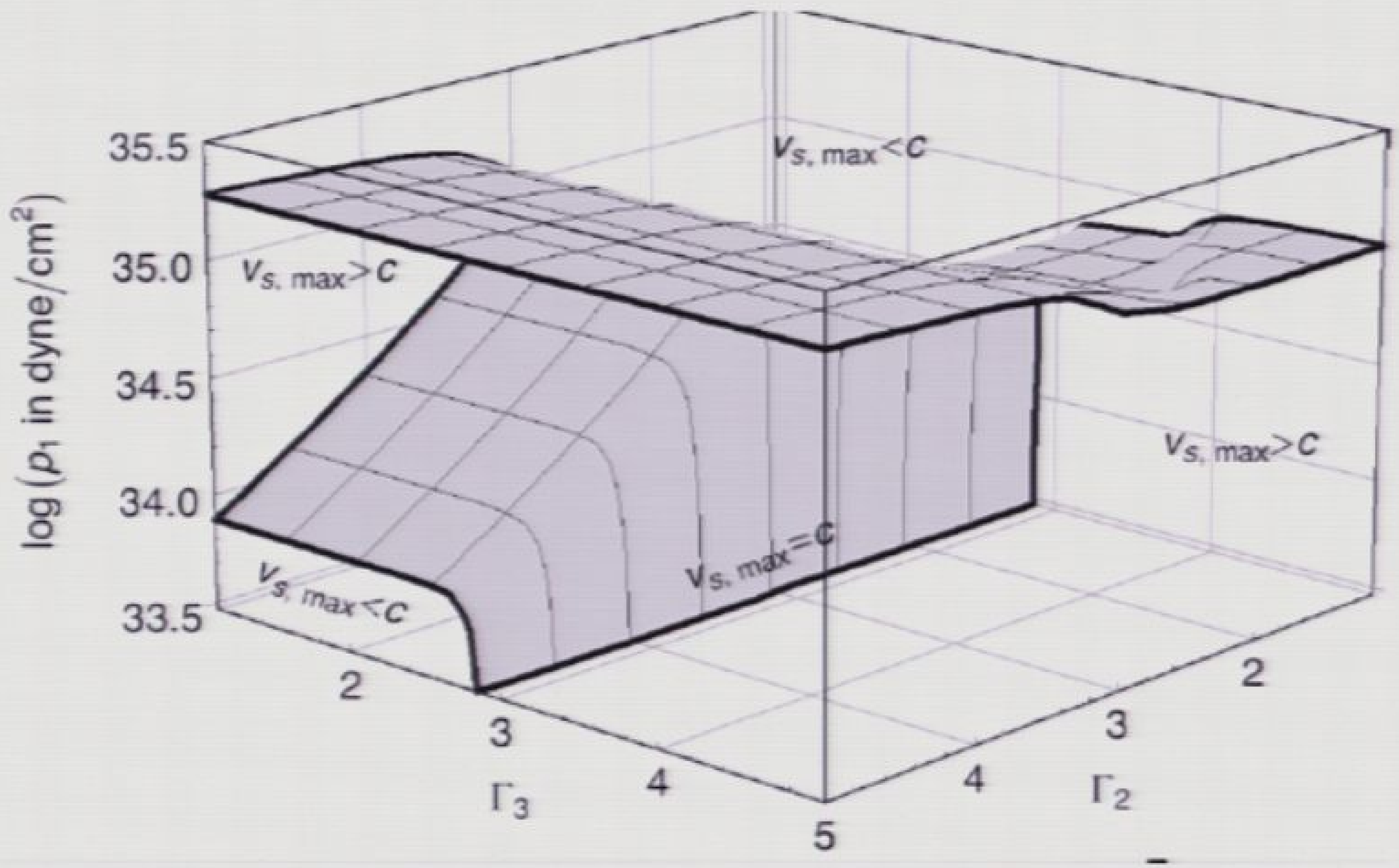

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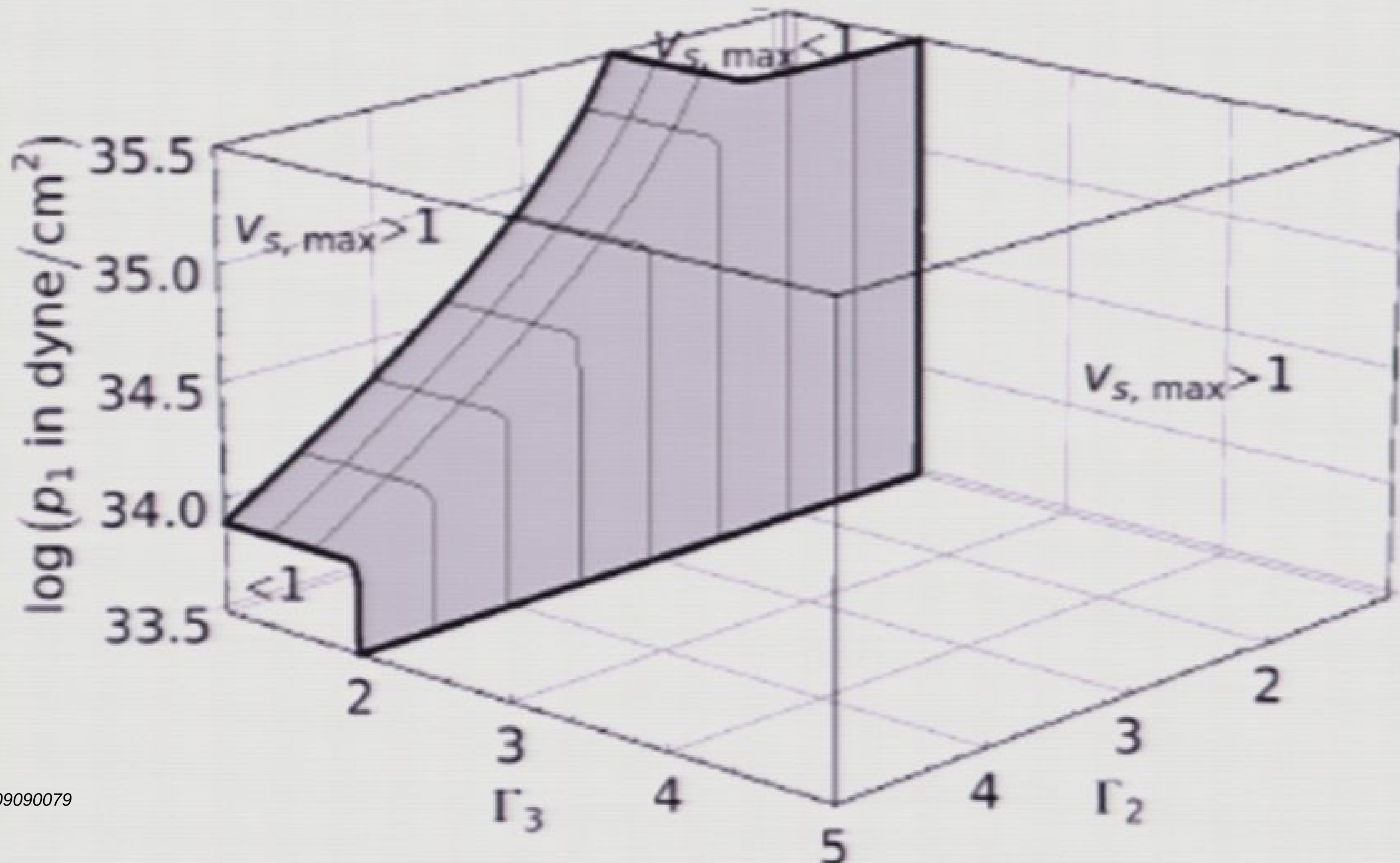

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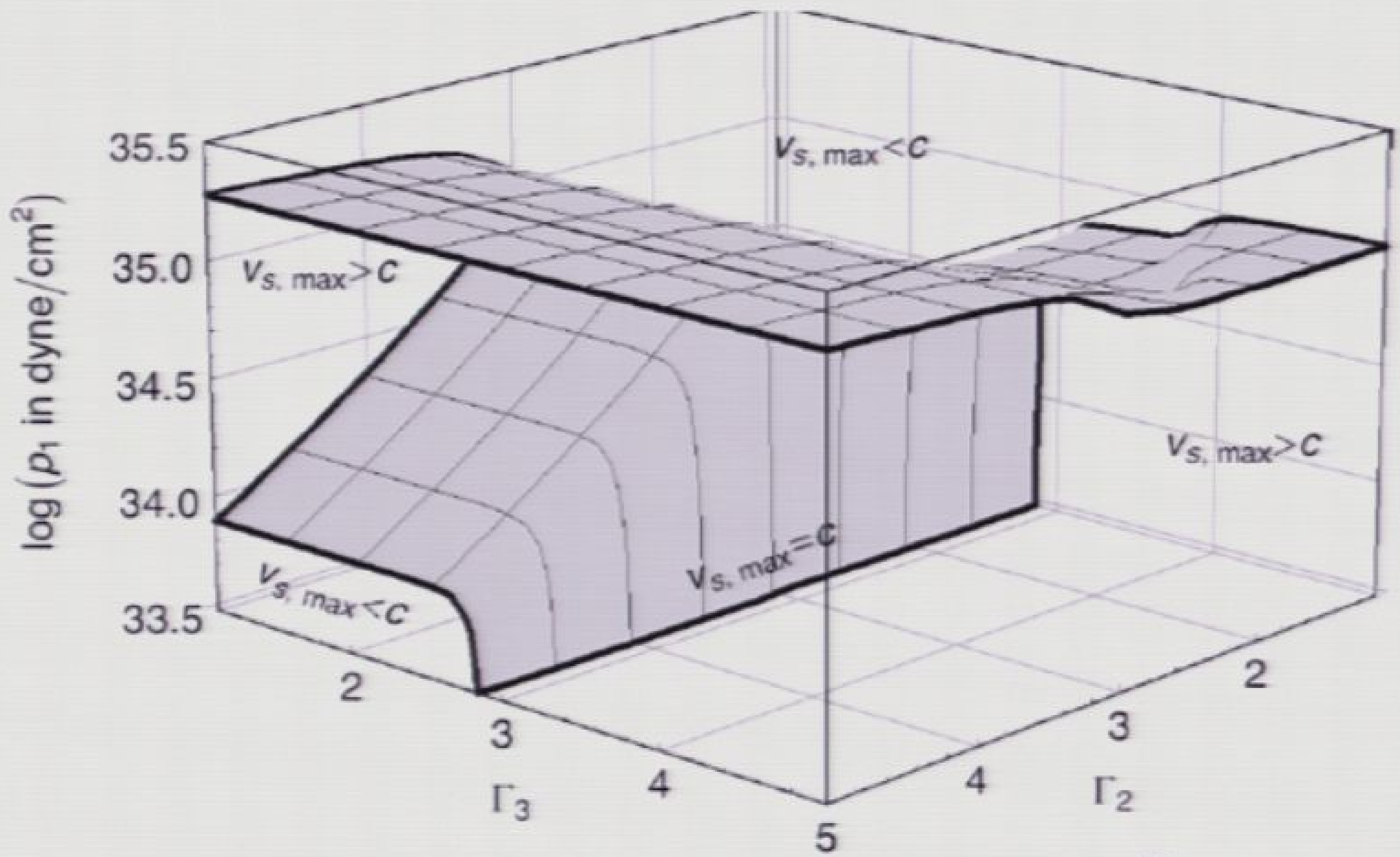

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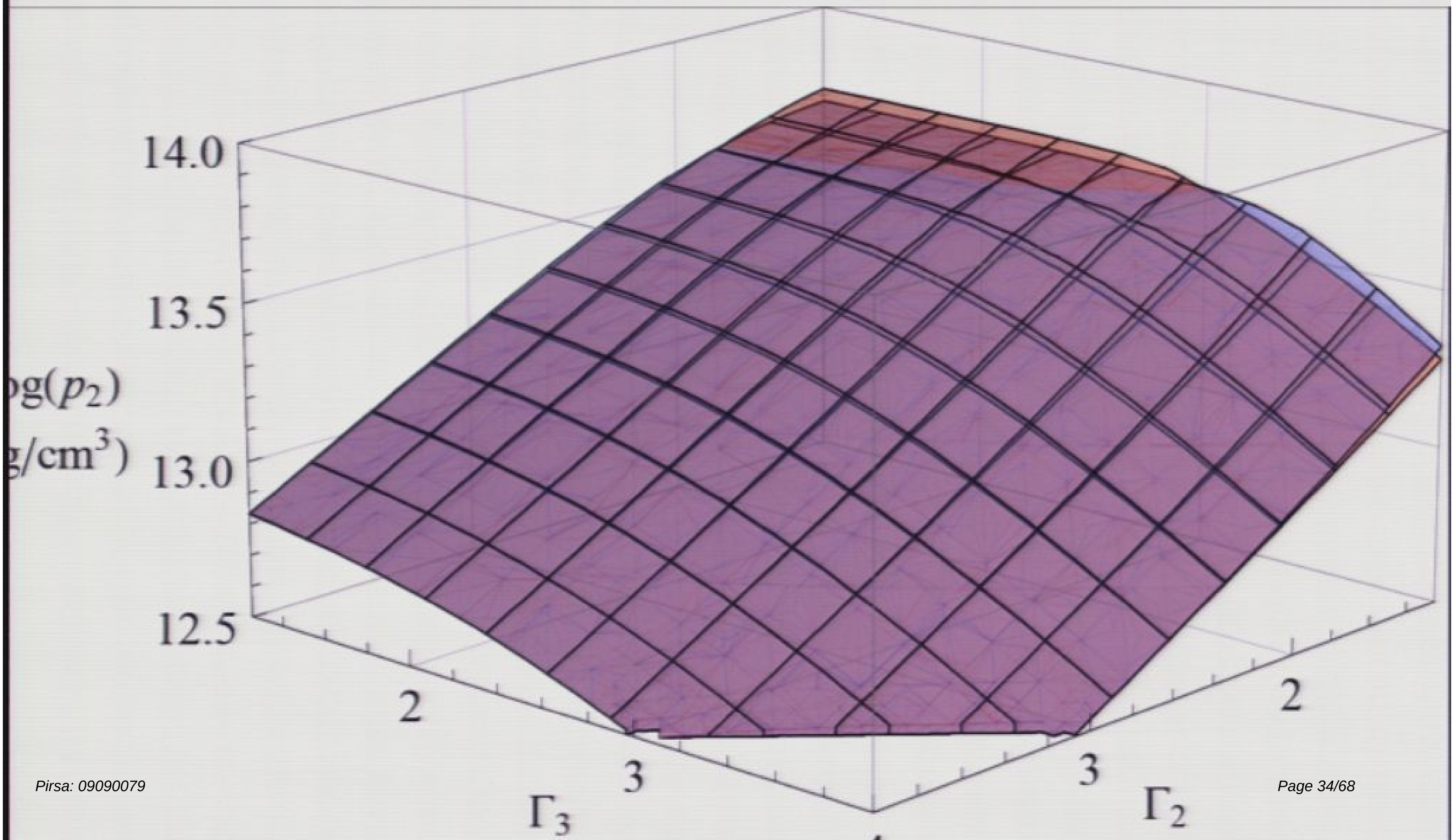

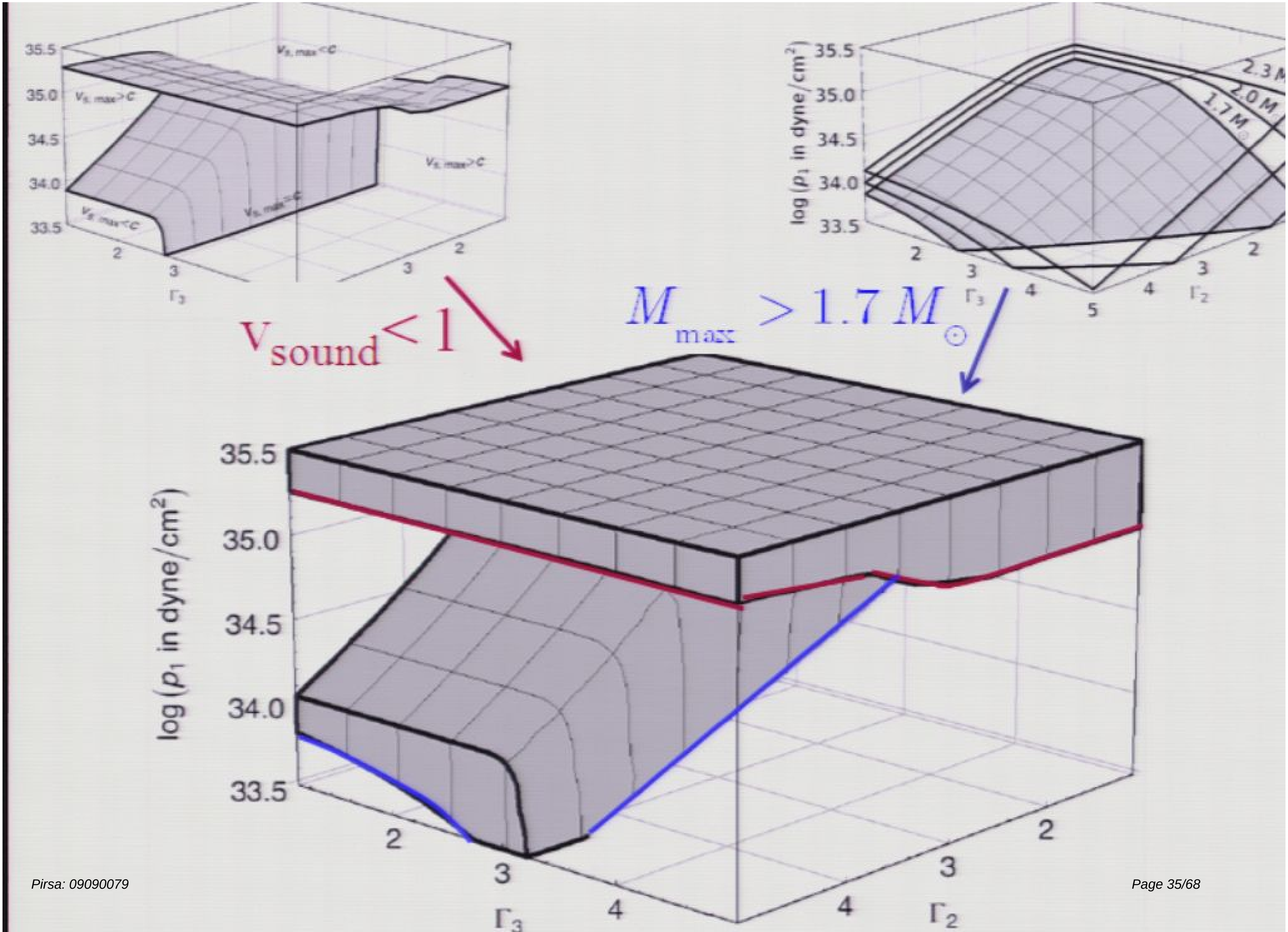
$v_{sound} < 1$
$M_{max} > 1.7\,M_\odot$
$\log(p_1 \text{ in dyne/cm}^2)$
35.5
35.0
34.5
34.0
33.5
Γ_3
Γ_2
$v_{s,max} < c$
$v_{s,max} > c$
2.3 M
2.0 M
$1.7\,M_\odot$

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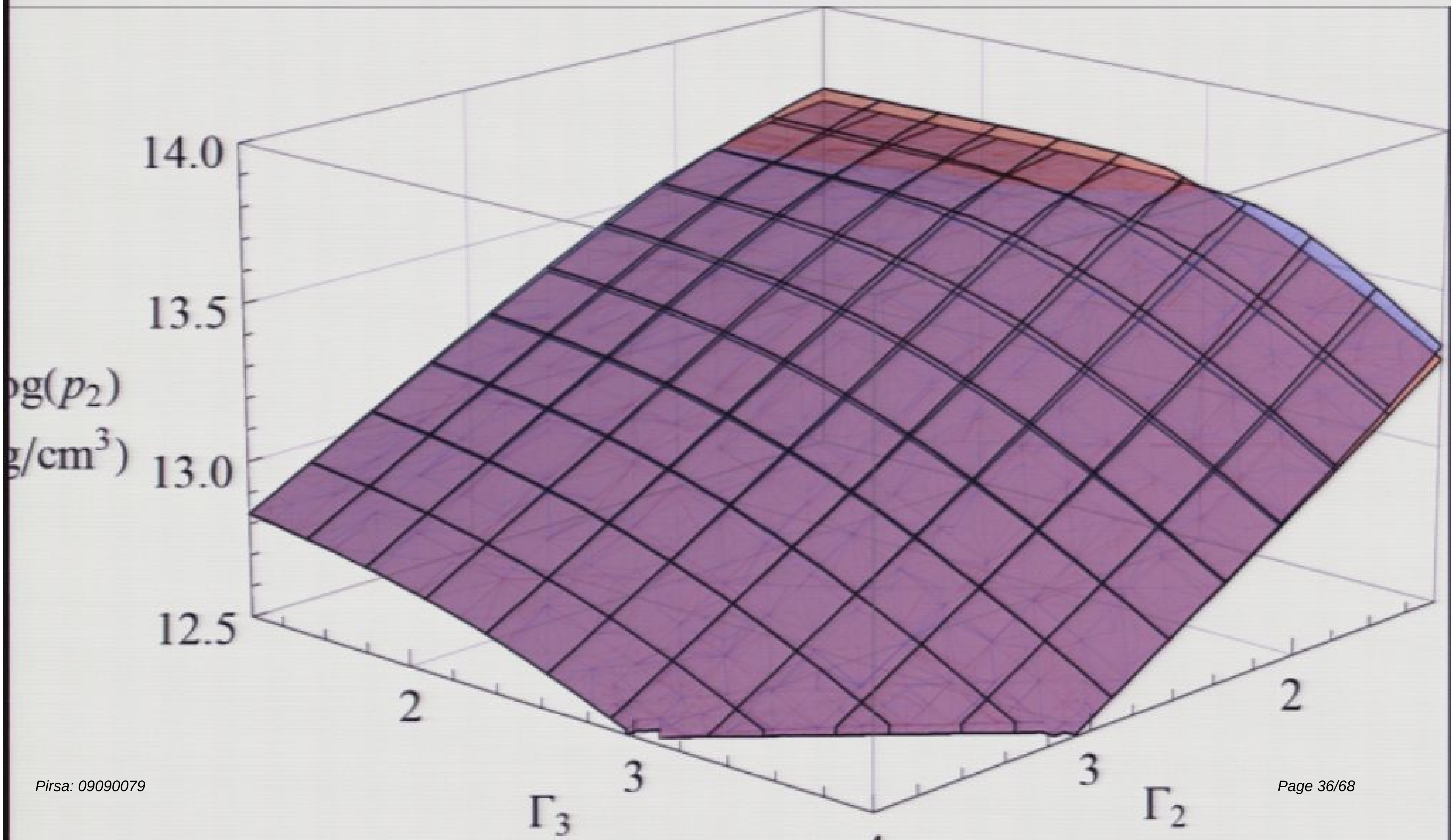

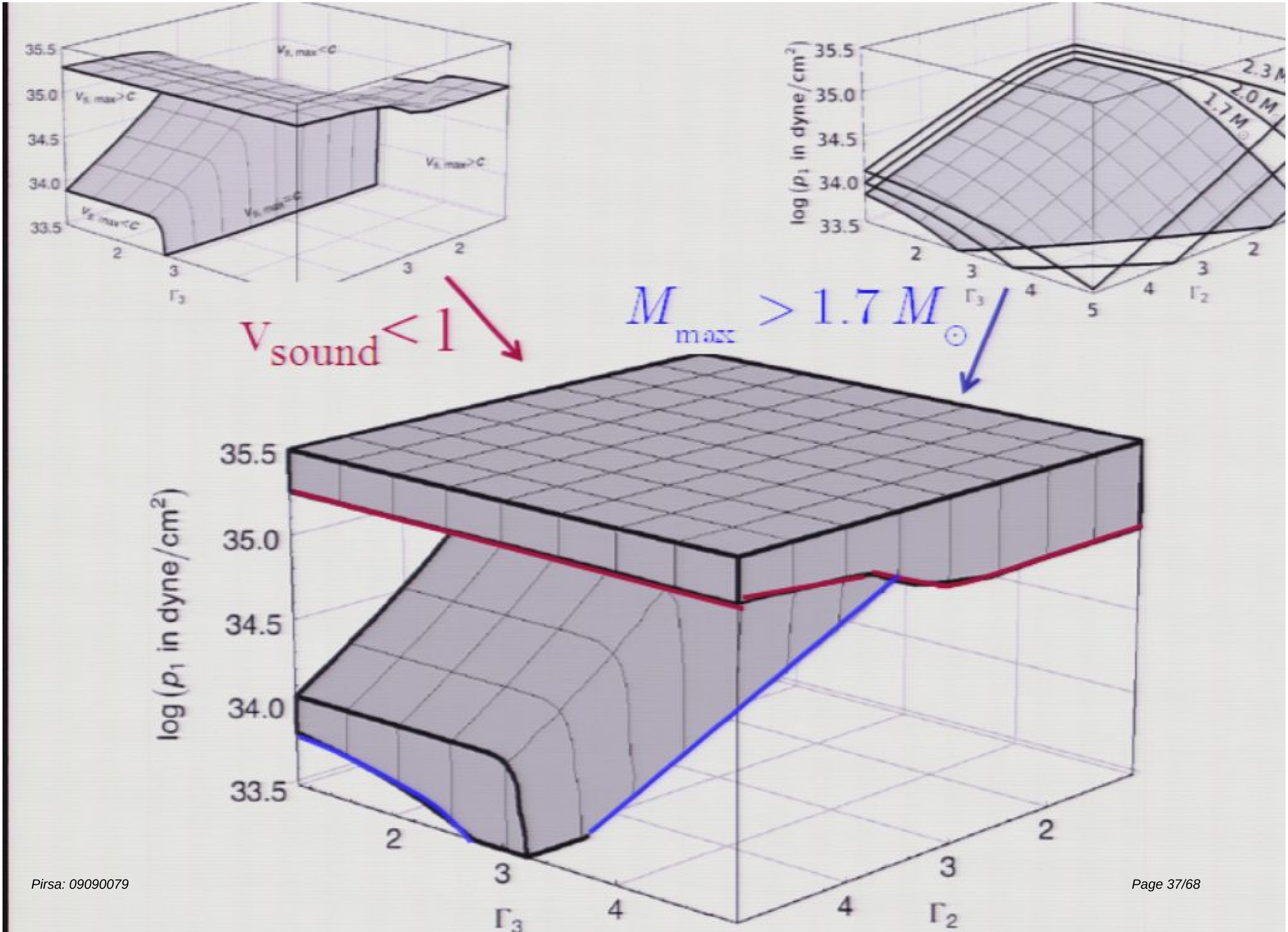

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35.5
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34.5
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33.5
Γ_3
Γ_2
$v_{s,max} < c$
$v_{s,max} > c$
$v_{s,max} = c$
2.3 M
2.0 M
1.7 M

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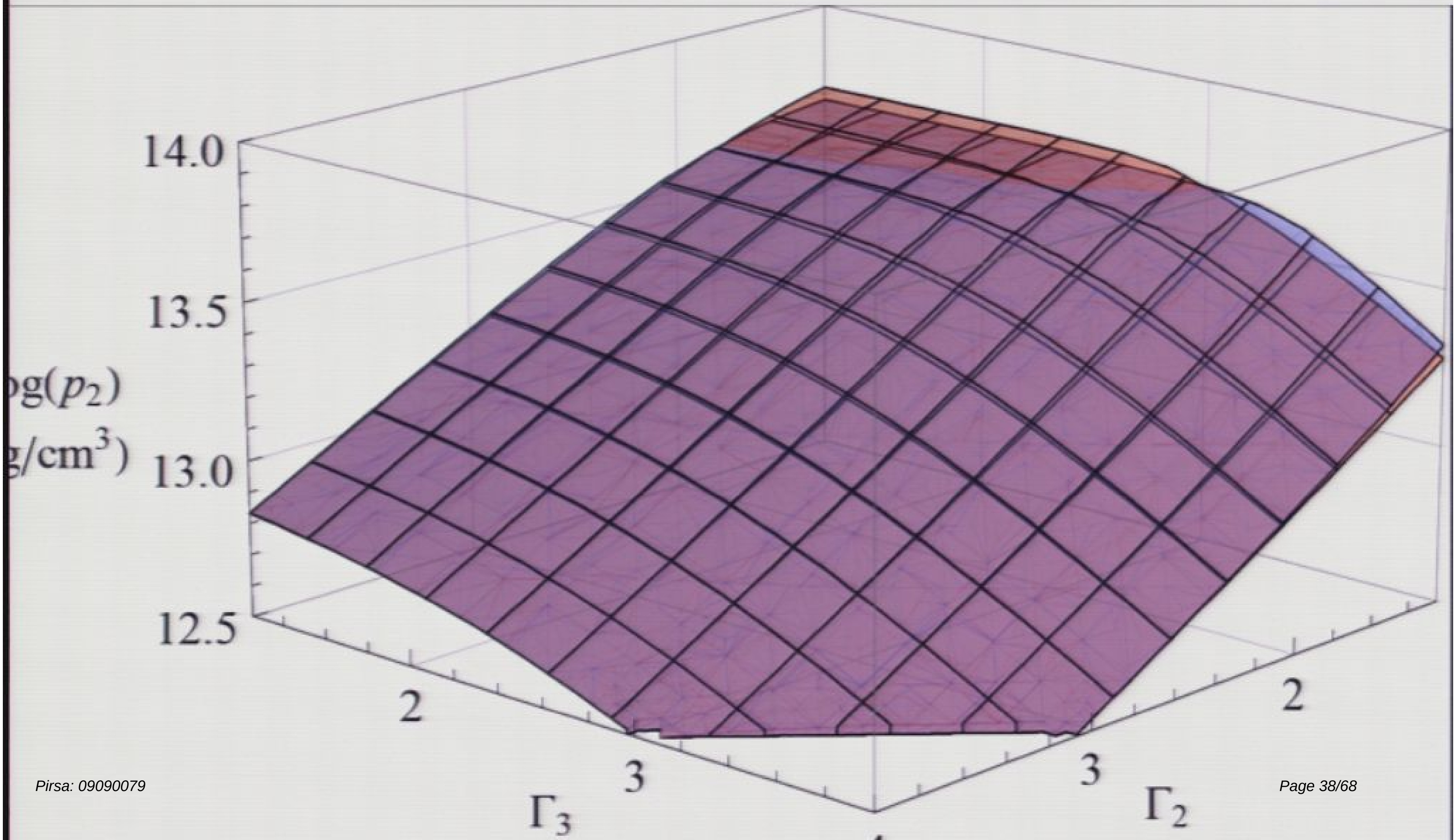

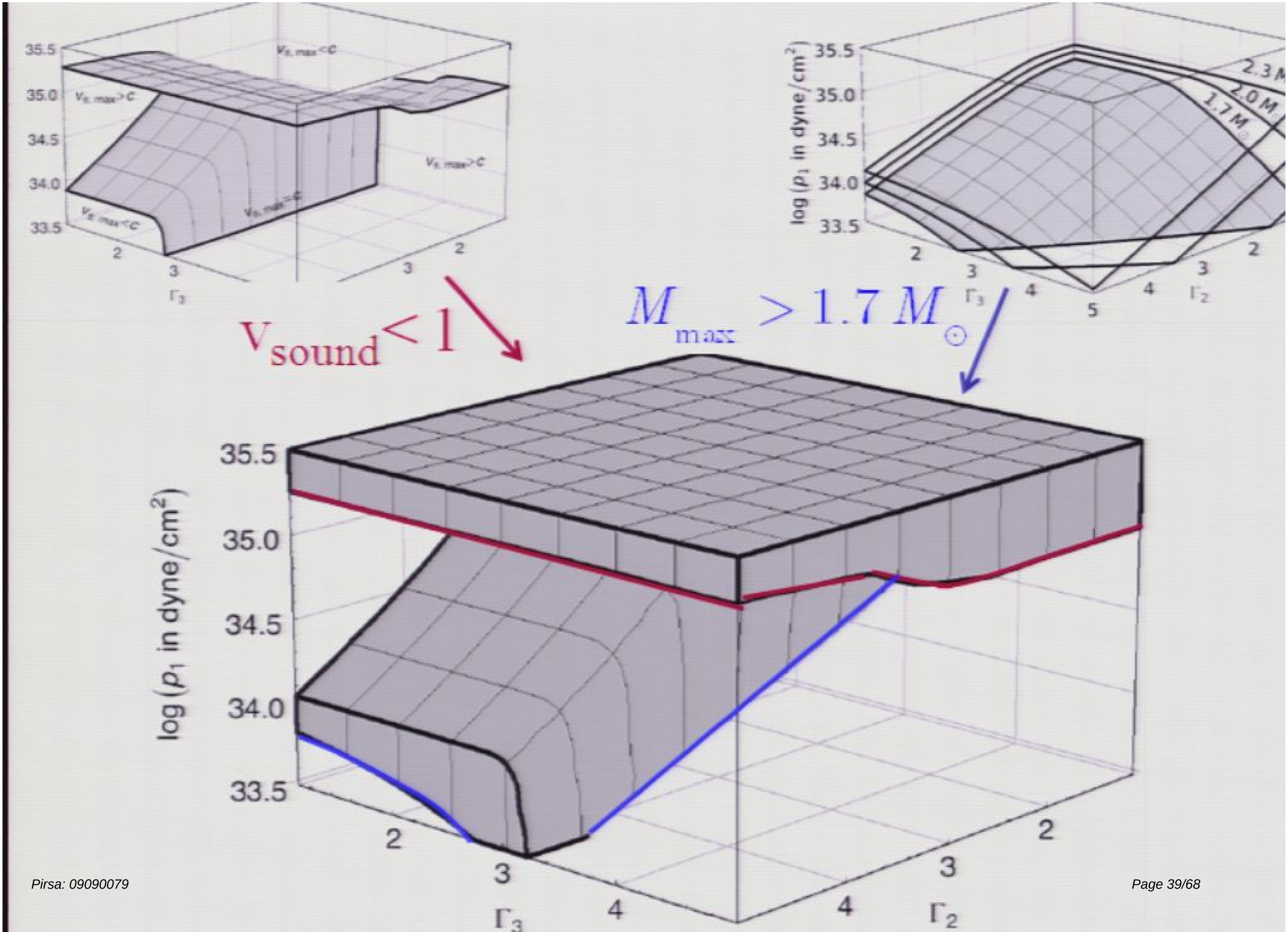
$v_{sound} < 1$
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log (ρ_1 in dyne/cm^2)
35.5
35.0
34.5
34.0
33.5
2
3
4
Γ_3
2
3
4
Γ_2

Measuring two properties of a single star restricts the EOS to a single surface (thickened by the error bars of the measurement)

Example:
Observing mass and radius restricts EOS to the surface

$$R(p_*, \Gamma_1, \Gamma_2, \Gamma_3) = R_{\text{observed}}$$

Measuring two properties of a single star restricts the EOS to a single surface

(thickened by the error bars of the measurement)

Example:

Observing mass and radius restricts EOS to the surface

$$R(p_*, \Gamma_1, \Gamma_2, \Gamma_3) = R_{\text{observed}}$$

For $M < 1.4 M_\odot$

$$R(p_*, \Gamma_1, \Gamma_2) = R_{\text{observed}}$$

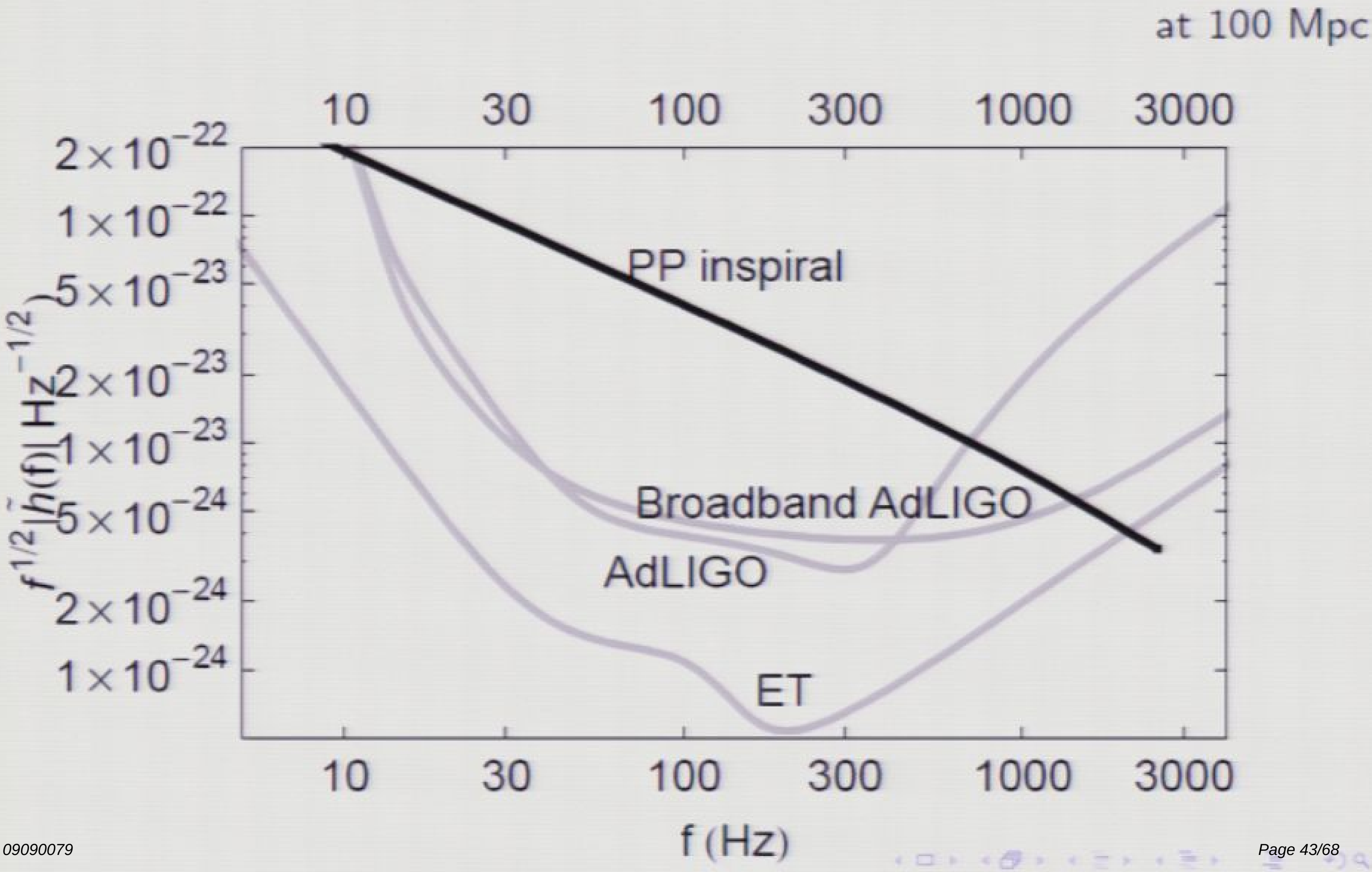
Future Constraints:

I and M for the same star, PSR J0737-3039A, in fastest binary pulsar system

M and measure of departure of NS-NS waveform from point-particle inspiral

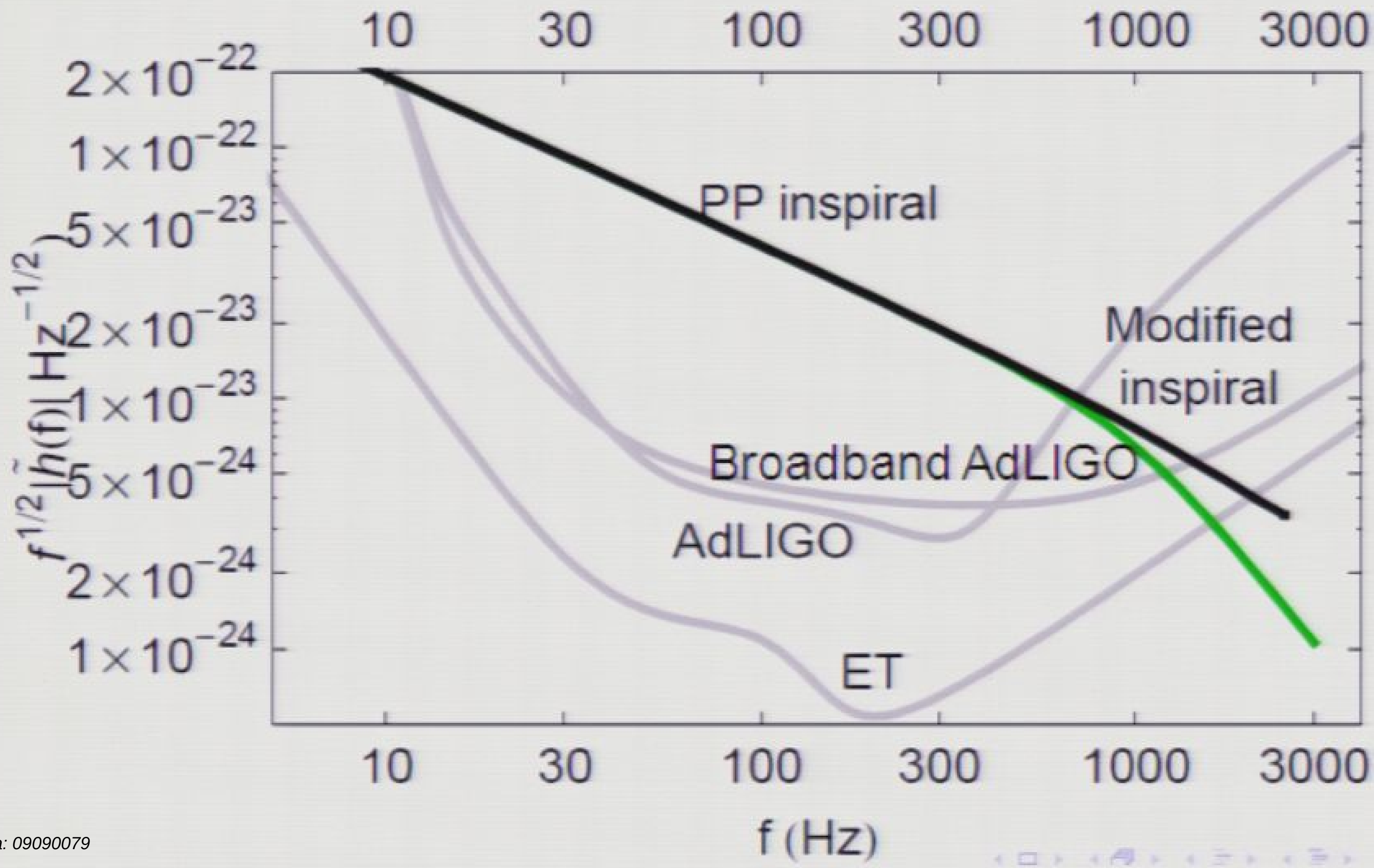
σ_{mode}: Mode frequencies for merged star prior to becoming black hole. Simultaneously know M and inspiral waveform, but EOS hot, and theoretical σ model dependent

Overview of signal from binary neutron stars

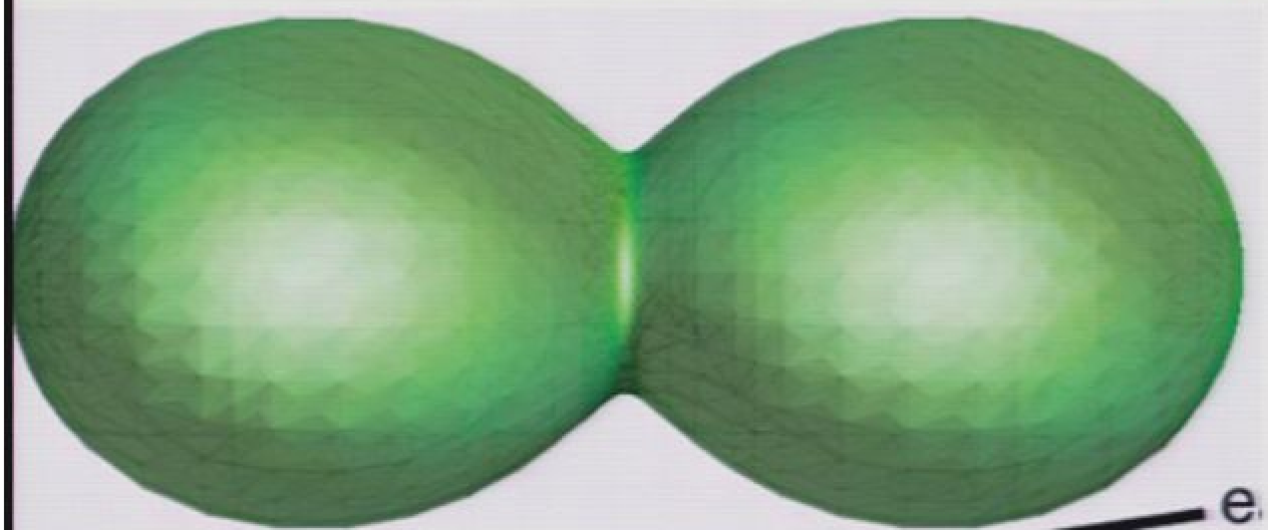


Overview of signal from binary neutron stars

at 100 Mpc:



Departure from point-particle inspiral at lower frequency for NS with larger radius, because the tidal force is greater.



Stiffer EOS

$p(1.7\,\rho_{nuclear}) = 10^{34.9}$ dyne/cm^2

Softer EOS

$p(1.7\,\rho_{nuclear}) = 10^{34.1}$ dyne/cm^2

Simulations by Markakis, Shibata, Uryu – a first cut, using a few orbits before merger

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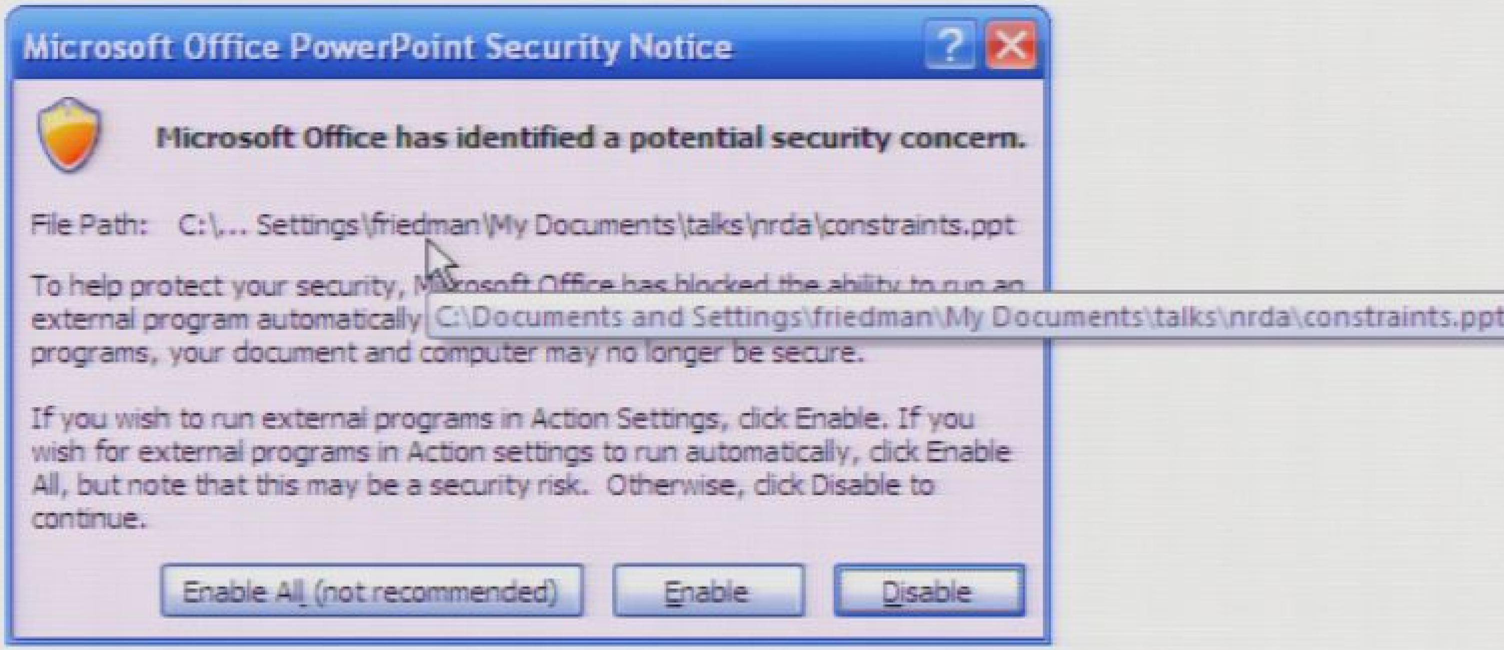

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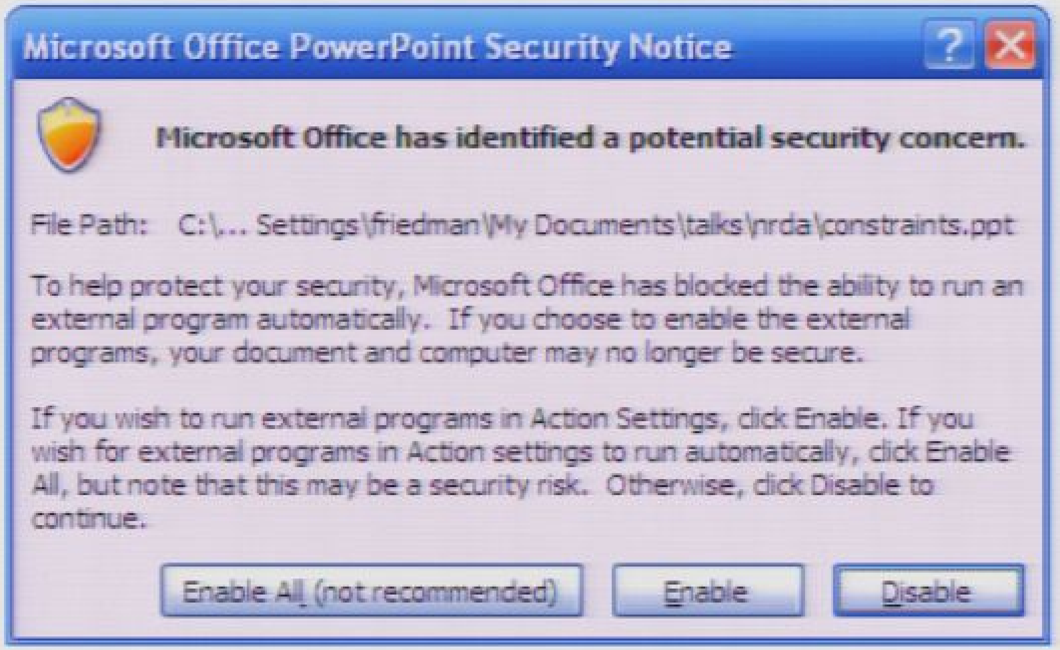

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BNSEOS

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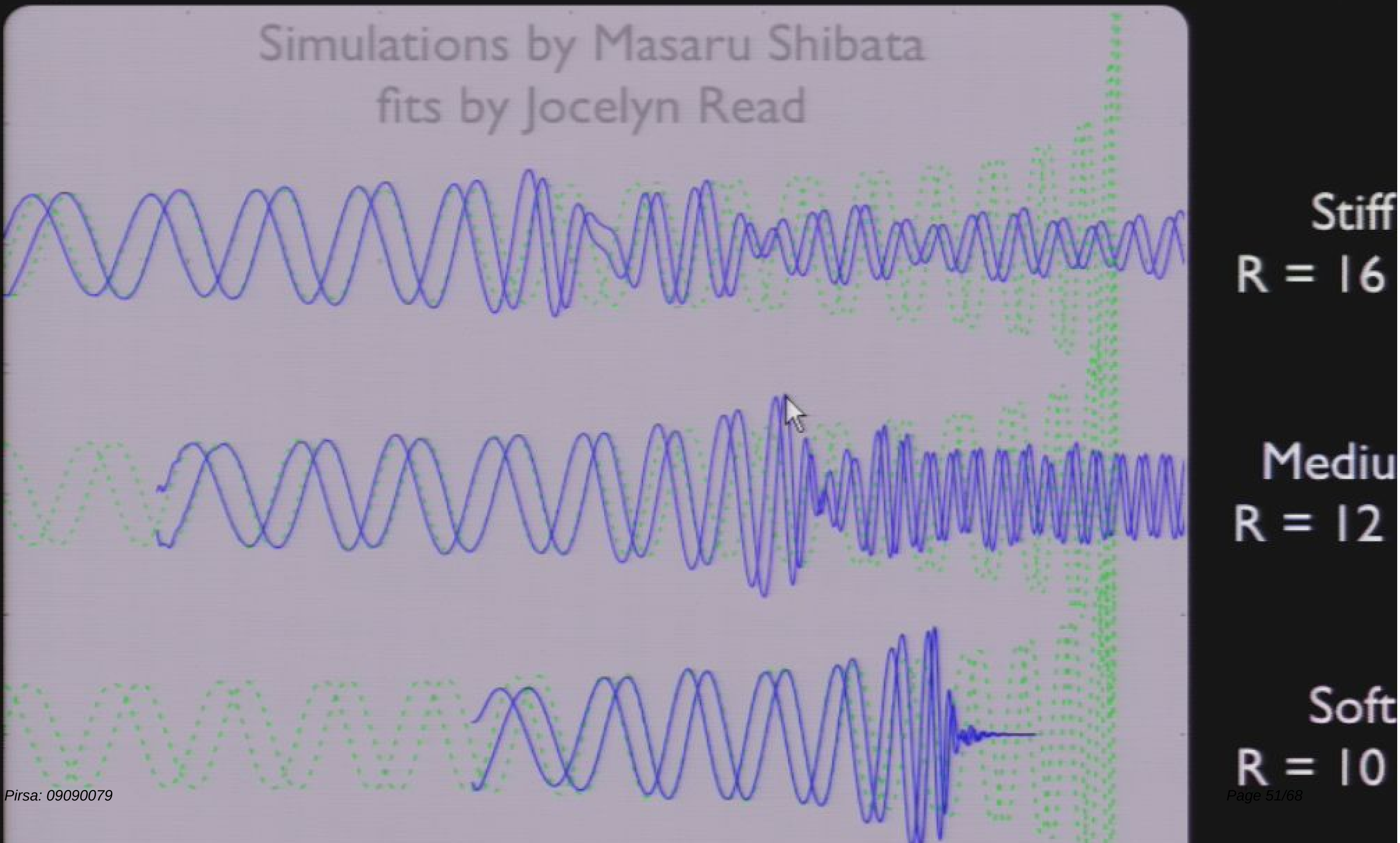
Binary Neutron Star Coalescence Wave

Simulations by Masaru Shibata
fits by Jocelyn Read

strain

BNSEOS

ary Neutron Star Coalescence Waveforms



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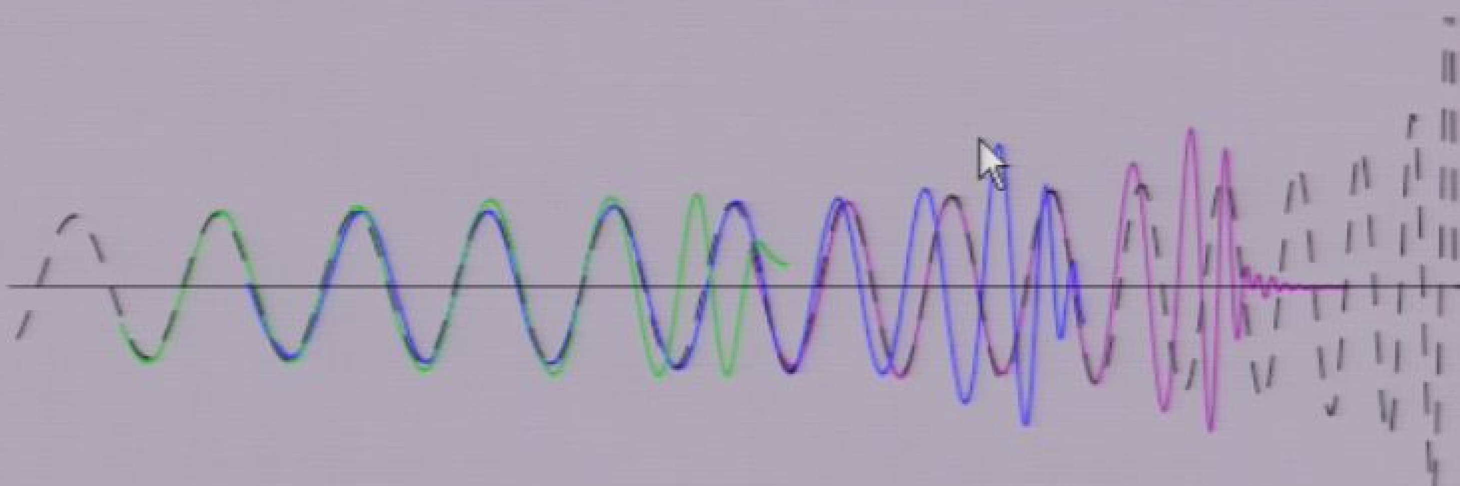
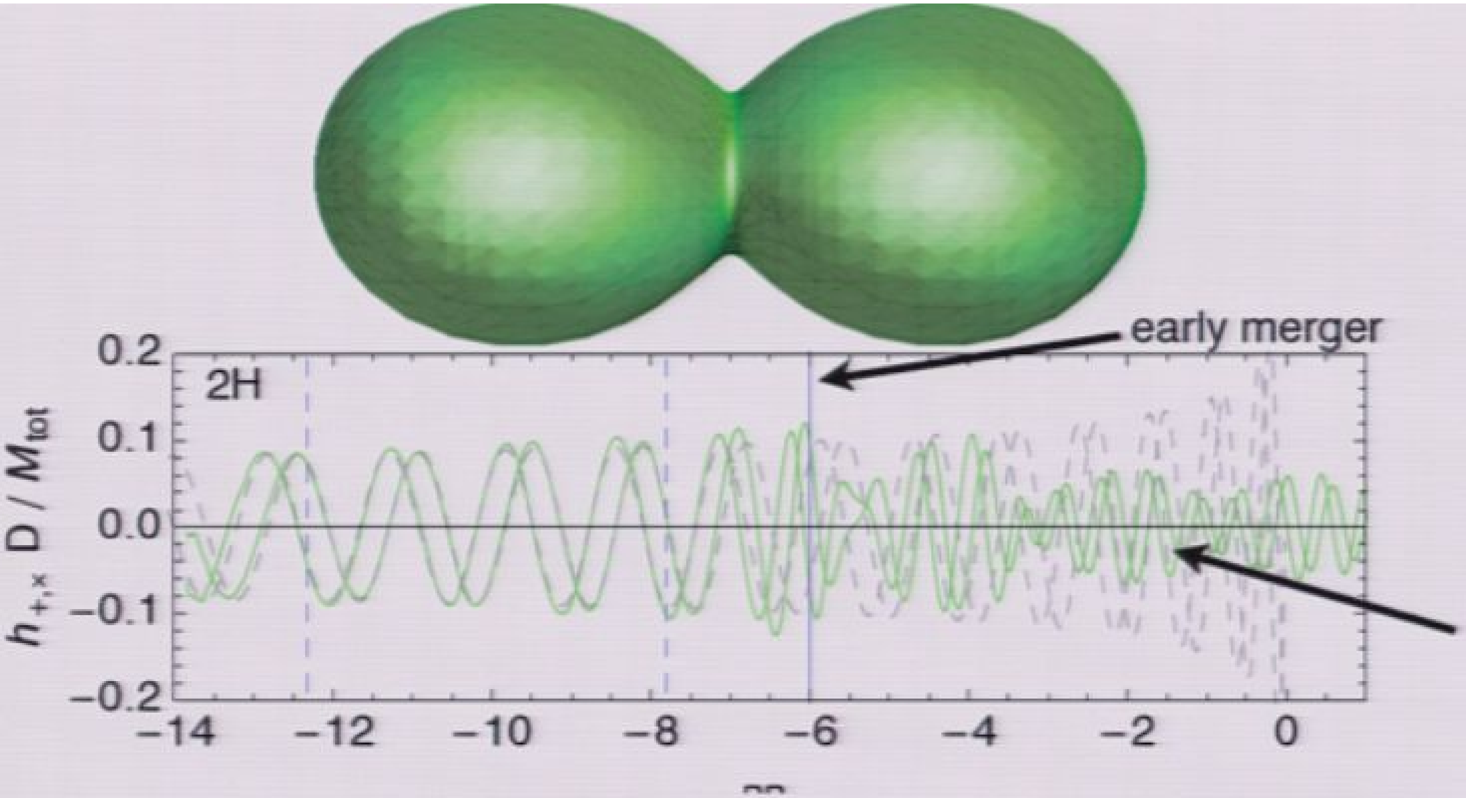


Stiff
R = 16

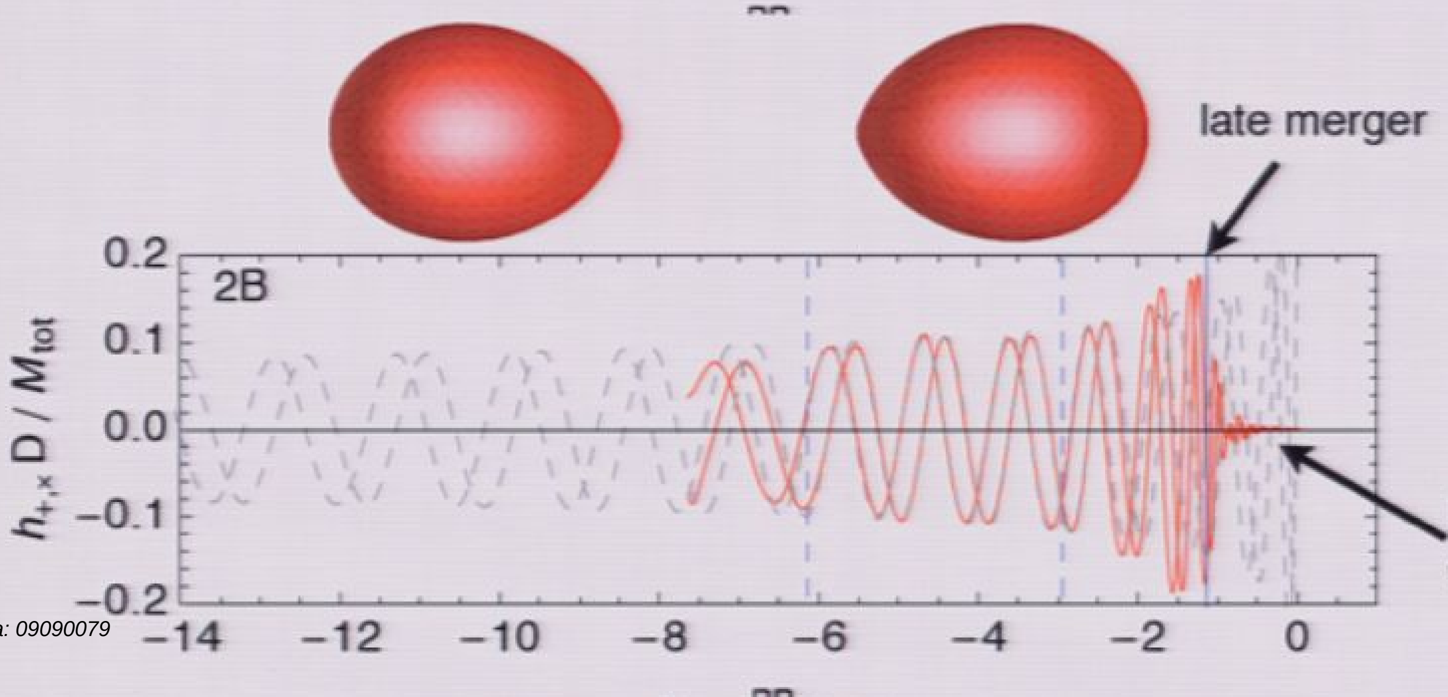
Mediu
R = 12

Soft
R = 10

Model 2H (stiff)
$p_1 = 10^{34.9}$ dyne cm^{-3}
$\Gamma_1 = \Gamma_2 = \Gamma_3 = 3$
Isolated radius: 15.2 km



Model 2B (soft)
$p_1 = 10^{34.1}$ dyne cm^{-3}
$\Gamma_1 = \Gamma_2 = \Gamma_3 = 3$
Isolated radius: 9.7 km



A larger R requires a stiffer EOS – larger pressure p_* at average density of star.

$\log p_*$ (10^{13} g/cm^3)	Broadband	Narrow band 1150 Hz
13.25	±.10	±.10
13.4	±.11	±.11
13.7	±.24	±.30

Criterion: $$\langle p \rangle|_{p_{ave}} = \frac{p_1 - p_2}{\langle h(p_1) - h(p_2) \mid h(p_1) - h(p_2) \rangle^{1/2}}$$

Estimated accuracy with which radius can be extracted from inspiral waveform for two $1.4\, M_{\odot}$ stars at 100 Mpc

R (km)	Broadband	Narrow band 1150 Hz
R=10.3	$\pm.61$	$\pm.57$
R=11.25	$\pm.78$	$\pm.78$
R=13.4	±1.75	±2.13

The deviation from point-particle inspiral is dominated by the quadrupole tidal deformation of each star.

For large separations this is given by the Love number k: An external quadrupole gravitational field

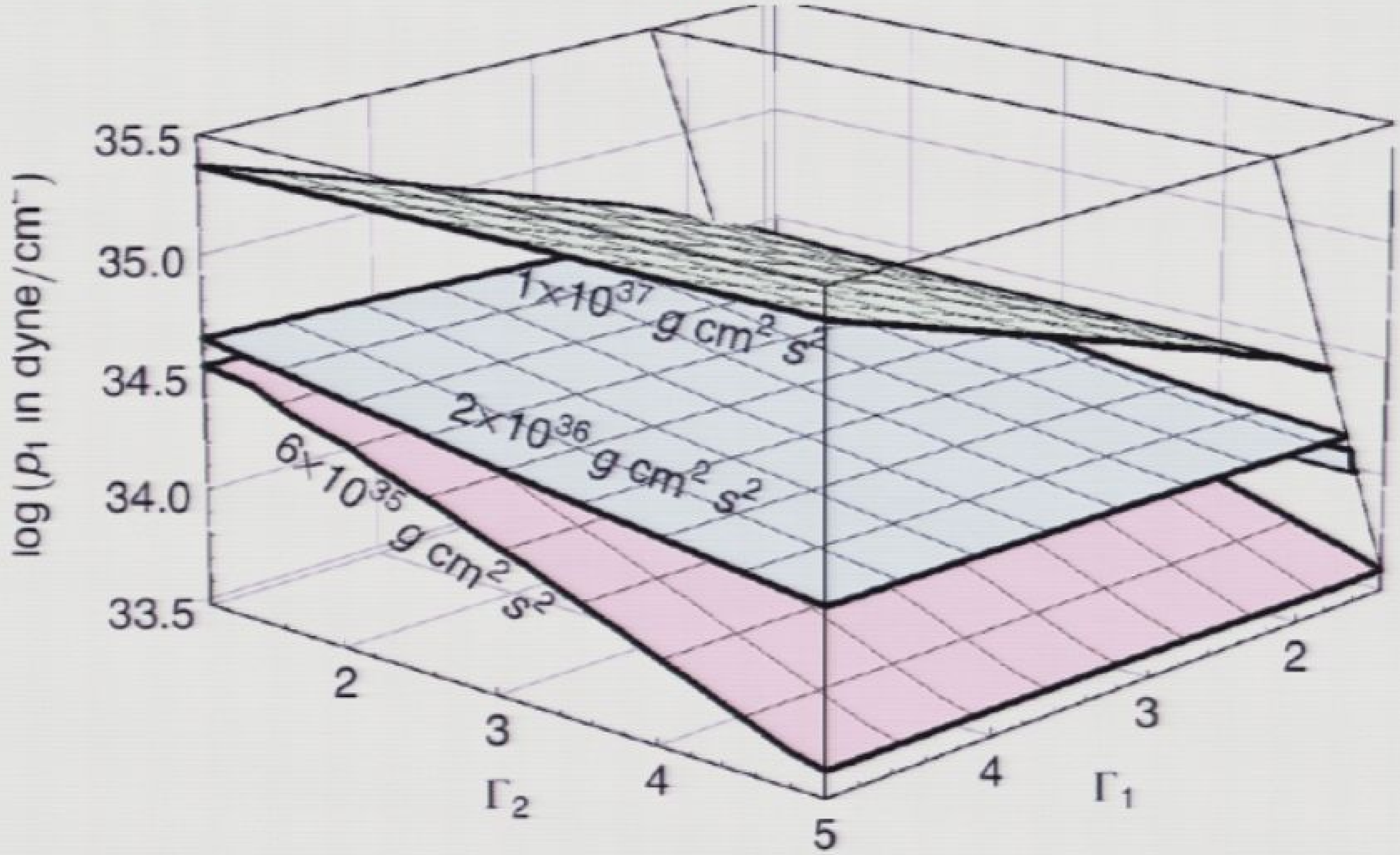
$$E_{ij}x^i x^j$$

induces a quadruple moment tensor

$$Q_{ij} = k\, R^5\, E_{ij}$$

(See Flanagan&Hinderer, Binnington&Poisson, Damour&Nagar)

Surfaces of constant induced quadrupole moment (constant kR^5) for fixed tidal force:



provided by Ben Lackey)

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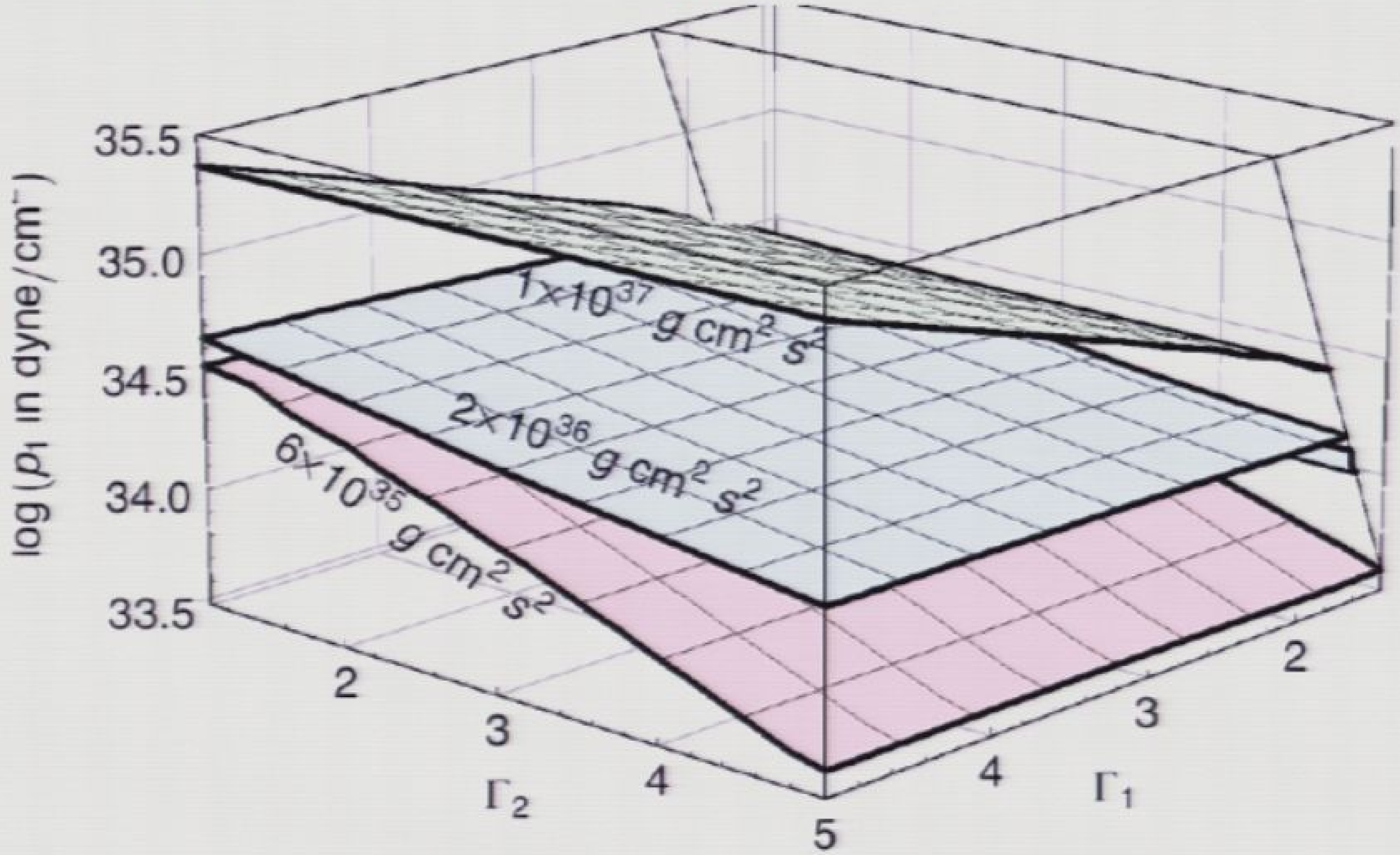
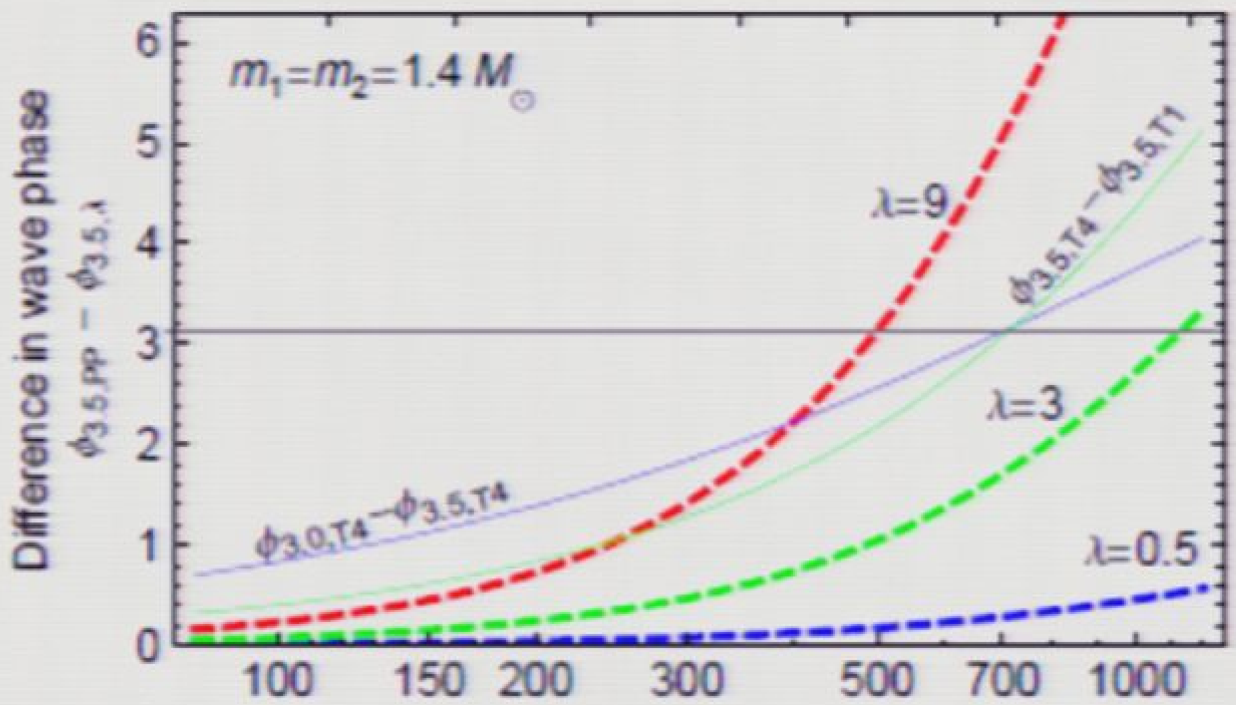


provided by Ben Lackey)

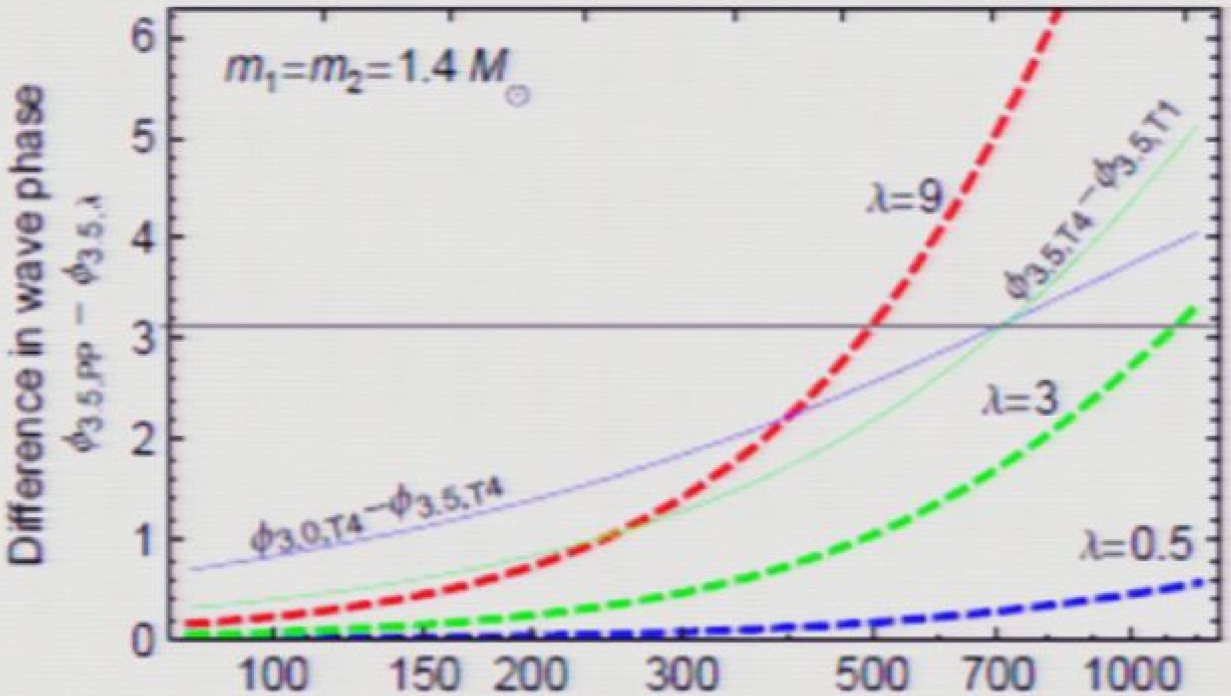
Observability in early inspiral



Advanced LIGO

M ($M_\odot$)	m_2/m_1	$\Delta\ln\mathcal{M}(\%)$	$\Delta\ln\mu(\%)$	$\Delta\tilde{\lambda}(\times10^{36})$	ρ
2.4	1.0	0.0027	0.19	9.9	42
2.6	1.0	0.0029	0.18	12.1	44
2.8	1.0	0.0031	0.17	14.6	47
3.0	1.0	0.0033	0.16	17.3	51
2.9	0.7	0.0031	0.16	15.5	48

Observability in early inspiral



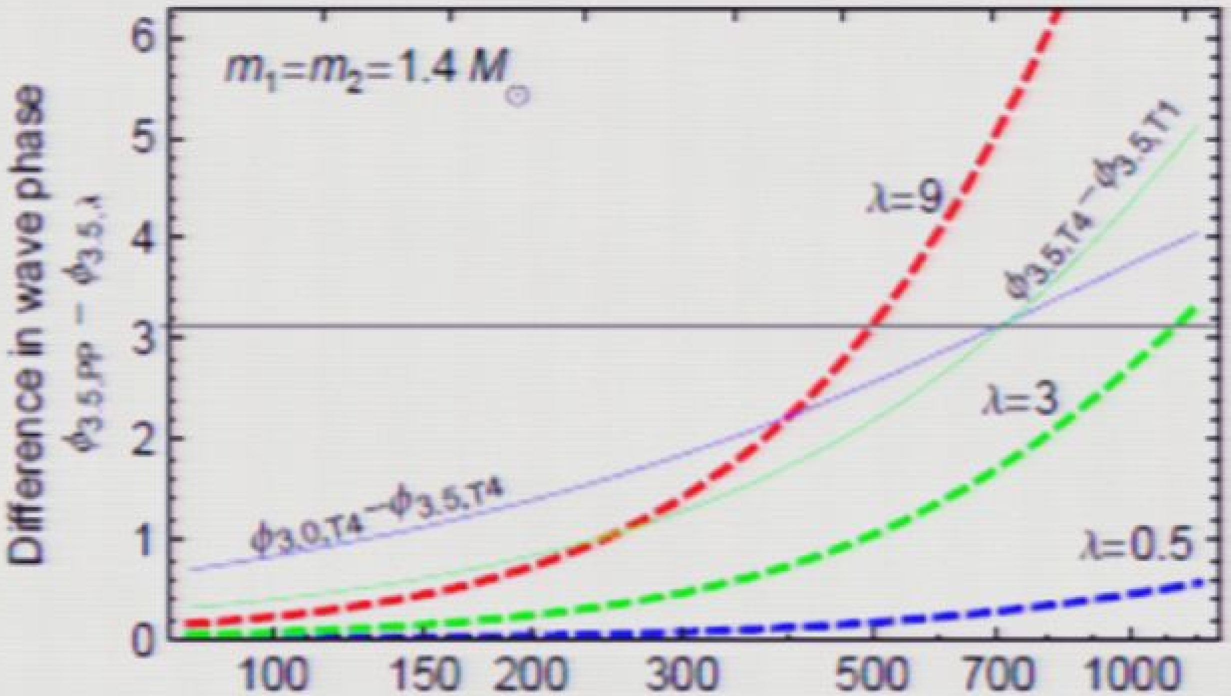
Einstein Telescope

M ($M_\odot$)	m_2/m_1	$\Delta \ln \mathcal{M}(\%)$	$\Delta \ln \mu(\%)$	$\Delta \ln \lambda(\times 10^{36})$	ρ
2.4	1.0	0.00018	0.017	0.93	404
2.6	1.0	0.0002	0.016	1.14	432
2.8	1.0	0.00021	0.015	1.37	459
3.0	1.0	0.00022	0.014	1.62	487

Black-hole –neutron-star inspiral

Again obtain surface in EOS space by measuring departure of NS-BH waveform from point-particle inspiral

Observability in early inspiral



Einstein Telescope

M ($M_\odot$)	m_2/m_1	$\Delta \ln \mathcal{M}(\%)$	$\Delta \ln \mu(\%)$	$\Delta \ln \bar{\lambda}(\times 10^{36})$	ρ
2.4	1.0	0.00018	0.017	0.93	404
2.6	1.0	0.0002	0.016	1.14	432
2.8	1.0	0.00021	0.015	1.37	459
3.0	1.0	0.00022	0.014	1.62	487

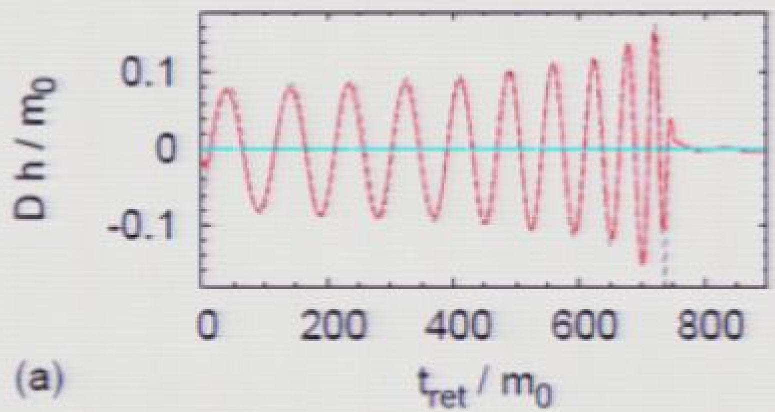
Black-hole –neutron-star inspiral

Again obtain surface in EOS space by measuring departure of NS-BH waveform from point-particle inspiral

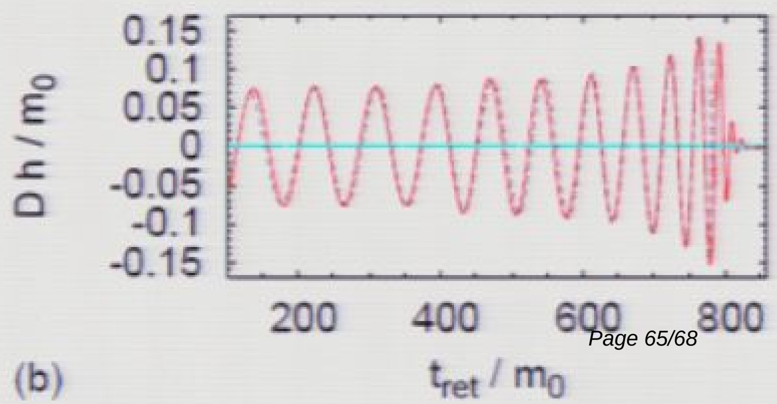
A wide range of possible mass ratios, orbits, BH spins, implies a constraint surface thickened by uncertainty.

Shibata, (Kyoto) Kyutoku, Yamamoto (Tokyo) Taniguchi (UWM)

Waveforms for nonrotating BHs, mass ratios 2 and 5.



(a)

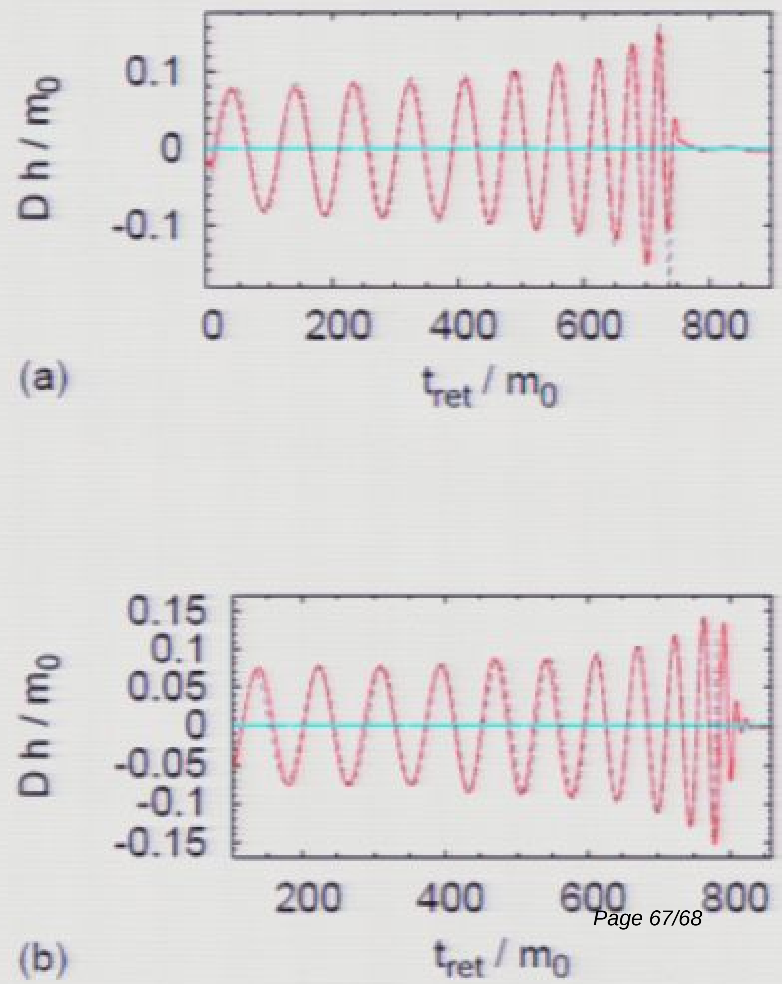


(b)

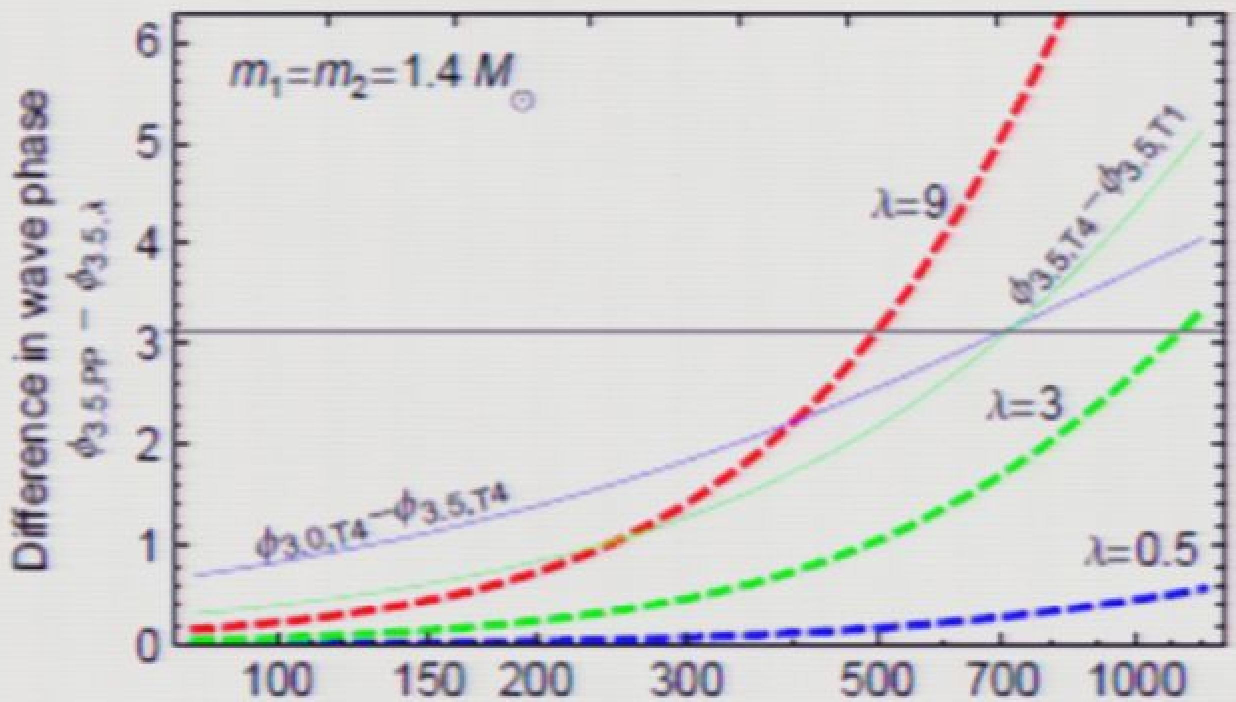
A wide range of possible mass ratios, orbits, BH spins, implies a constraint surface thickened by uncertainty.

Shibata, (Kyoto)
Kyutoku, Yamamoto
(Tokyo) Taniguchi
(UWM)

Waveforms for nonrotating BHs, mass ratios 2 and 5.



Observability in early inspiral



Advanced LIGO

M ($M_\odot$)	m_2/m_1	$\Delta \ln \mathcal{M}(\%)$	$\Delta \ln \mu(\%)$	$\Delta\tilde{\lambda}(\times 10^{36})$	ρ
2.4	1.0	0.0027	0.19	9.9	42
2.6	1.0	0.0029	0.18	12.1	44
2.8	1.0	0.0031	0.17	14.6	47
3.0	1.0	0.0033	0.16	17.3	51
2.9	0.7	0.0031	0.16	15.5	48