Title: The Statistical Interpretation of Quantum Mechanics and its Relation to Probability Theory

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Abstract: TBA

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by

Leslie Ballentine
Simon Fraser University

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(a.k.a. the quantum measurement problem)

System:

unstable atom + cat + environment

state:

$$|undecayed\rangle|live\rangle|e_1\rangle + |decayed\rangle|dead\rangle|e_2\rangle$$

Problem:

What is the meaning of ... te vector that is a coherent superposition ... macroscopically different terms?

Decoherence is irrelevant to this problem!

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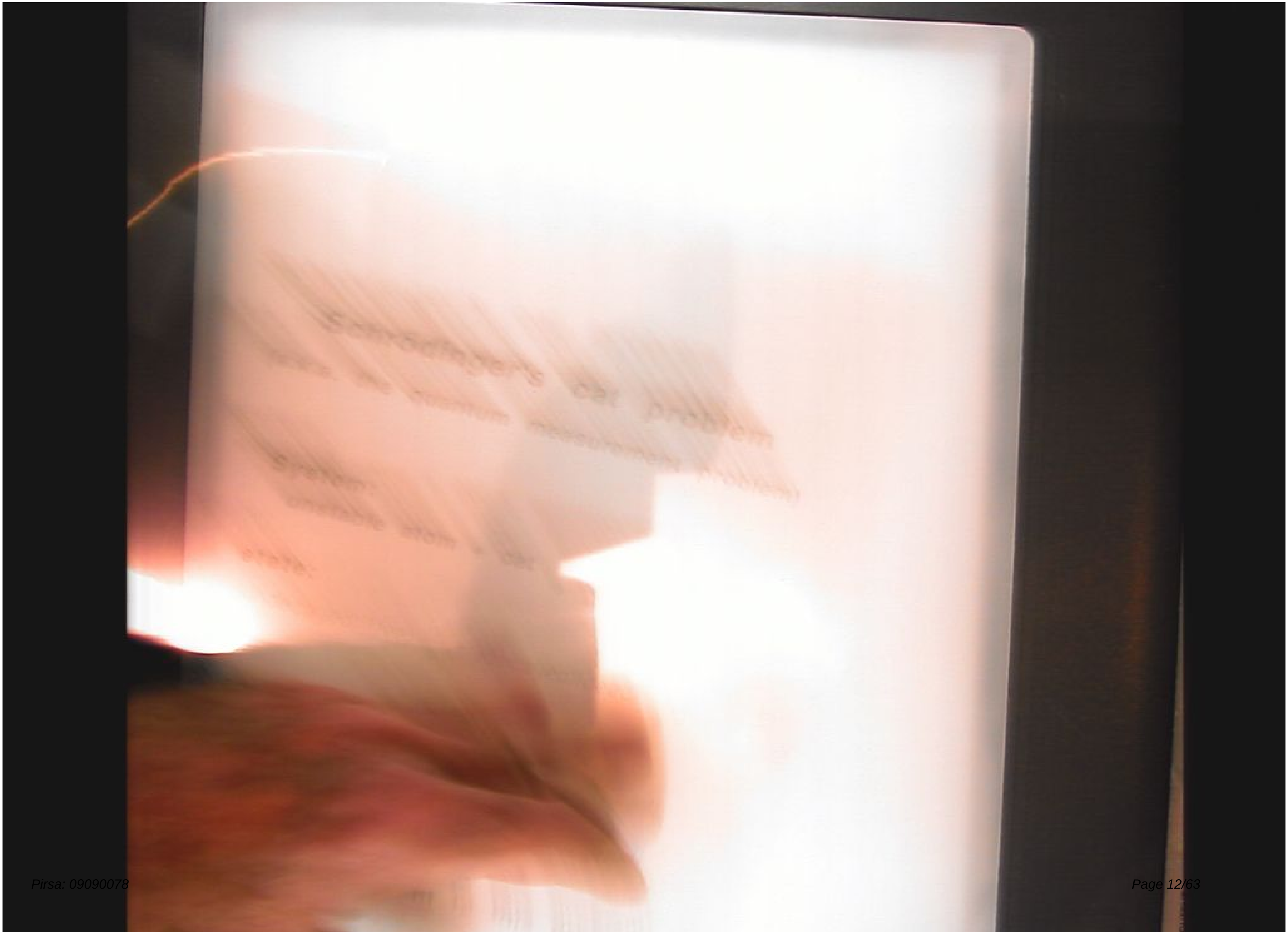

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Design the *interaction* between them to have the effect

$$|q_1\rangle \otimes |A_0\rangle \rightarrow |q_1\rangle \otimes |A_1\rangle \quad \text{(if value of q unchanged)}$$

or

$$|q_1\rangle \otimes |A_0\rangle \rightarrow |\phi_1\rangle \otimes |A_1\rangle \quad \text{(if value of q changed)}$$

Theorem:

For an *initial state* of the object that is a *superposition* , $|\Psi\rangle = (|q_1\rangle + |q_2\rangle)/\sqrt{2}$,
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"But we never observe macroscopic objects to be in a superposition state!" ~~says a supporter of (i).~~

"How do you know? Have you ever looked?" ~~says a supporter of (ii).~~

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Suppose the state is a **coherent superposition of two non-overlapping wave packets:**

$$\psi(\vec{x}) = \phi(\vec{x}) + \phi(\vec{x}+\vec{a})$$

The <u>**position**</u> **probability density** is

$$|\psi(\vec{x})|^2 = |\phi(\vec{x}) + \phi(\vec{x}+\vec{a})|^2$$

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-- No interference because the w[illegible] packets do not overlap.
-- No difference from an *incoher*[illegible] (classical) mixture.

The <u>**momentum**</u> **probability density** is

$$|\langle\vec{p}|\psi\rangle|^2 = |\langle\vec{p}|\psi\rangle + \exp(i\vec{p}\cdot\vec{a}/\hbar)\langle\vec{p}|\psi\rangle|^2$$

-- It contains a very **fine-grained interference pattern.**
-- Observable in principle, but very difficult to detect if the separation a is macroscopic.

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The Ensemble is

the set of all systems that have been, or can be, prepared in the appropriate state.
It may involve separate preparations of different copies of the system, or repeated preparations of a single copy of the system.

The function of the State

(vector, operator, or wave function) is

to generate probability distributions for any and all observables.

It may even be identified with this collection of probability distributions.

Some misinterpretations:

- If $|\Psi\rangle = \sum_i c_i |E_i\rangle$ then the ensemble is <u>not</u> an ensemble of systems in eigenstates $\{|E_i\rangle\}$.

- The mixed state $\rho = \sum_i |\phi_i\rangle\langle\phi_i| c_i$ is <u>not</u> an ensemble of systems each of which belongs to one of the pure states $\{|\phi_i\rangle\}$.

- The state vector is <u>not</u> an "element of reality". (reject the extreme ontic position)

- The state vector does <u>not</u> represent knowledge. (reject the extreme epistemic position)

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  (reject the extreme epistemic position)

Ehrenfest's Theorem

Let $\hat{H} = \hat{p}^2/2m + V(\hat{q})$. Then

$$d\hat{q}/dt = (i/\hbar)\,[H,\hat{q}] = \hat{p}/m$$

$$d\hat{p}/dt = (i/\hbar)\,[H,\hat{p}] = F(\hat{q}),$$

where $F(x) = -\nabla V(x)$

Take the average in some state:

$$d\langle\hat{q}\rangle/dt = \langle\hat{p}\rangle/m$$

$$d\langle\hat{p}\rangle/dt = \langle F(\hat{q})\rangle$$

If $\langle F(\hat{q})\rangle \approx F(\langle\hat{q}\rangle)$ then $\langle\hat{q}\rangle$ will follow a classical orbit.

Corrections to Eherenfest's Theorem

Expand in powers of deviation operators,

$$\delta\hat{q} = \hat{q} - \langle\hat{q}\rangle$$

$$\delta\hat{p} = \hat{p} - \langle\hat{p}\rangle$$

Write $P = \langle\hat{p}\rangle$ and $Q = \langle\hat{q}\rangle$.

Then

$$dQ/dt = P/m$$

$$\frac{dP}{dt} = F(Q) + \frac{1}{2}\langle(\delta\hat{q})^2\rangle \frac{\partial^2}{\partial Q^2}F(Q) + \ldots$$

classical orbit — corrections

not all due to GM

Classical Ensemble, Liouville equation:

$$\frac{\partial}{\partial t}\rho(q,p,t) = -\frac{p}{m}\frac{\partial}{\partial q}\rho(q,p,t) - F(q)\frac{\partial}{\partial p}\rho(q,p,t)$$

$$\langle q\rangle_c = \iint q\,\rho(q,p,t)\,dq\,dp$$

$$\langle p\rangle_c = \iint p\,\rho(q,p,t)\,dq\,dp$$

Differentiate w.r.t. t:

$$d\langle q\rangle_c/dt = \langle p\rangle_c/m$$

$$d\langle p\rangle_c/dt = \iint F(q)\,\rho(q,p,t)\,dq\,dp$$

Expand in powers of $\delta q = q - \langle q\rangle_c$:

$$\frac{d}{dt}\langle p\rangle_c = F(\langle q\rangle_c) + \frac{1}{2}\langle(\delta q)^2\rangle_c\,\frac{\partial^2}{\partial\langle q\rangle_c^2}F(\langle q\rangle_c) + \ldots$$

The centroid of a classical ensemble need not follow a classical trajectory.

Weaknesses of Ehrenfest correspondence

(motion of the centroid of a QM state compared to an individual classical trajectory)

as a criterion of classicality:

- Corrections to Ehrenfest's theorem depend only on the *width* of the state.

- The corrections have no systematic dependence on $\hbar$. (So not QM in origin)

- Ehrenfest's theorem fails when the state-width grows sufficiently large, which can happen on an observable time scale for chaotic macroscopic systems.

  (ex.: 20 years for tumbling of Hyperion)

- This conclusion holds for both *pure* and *mixed* quantum states, so *decoherence* (which changes pure states into mixed states) is irrelevant.

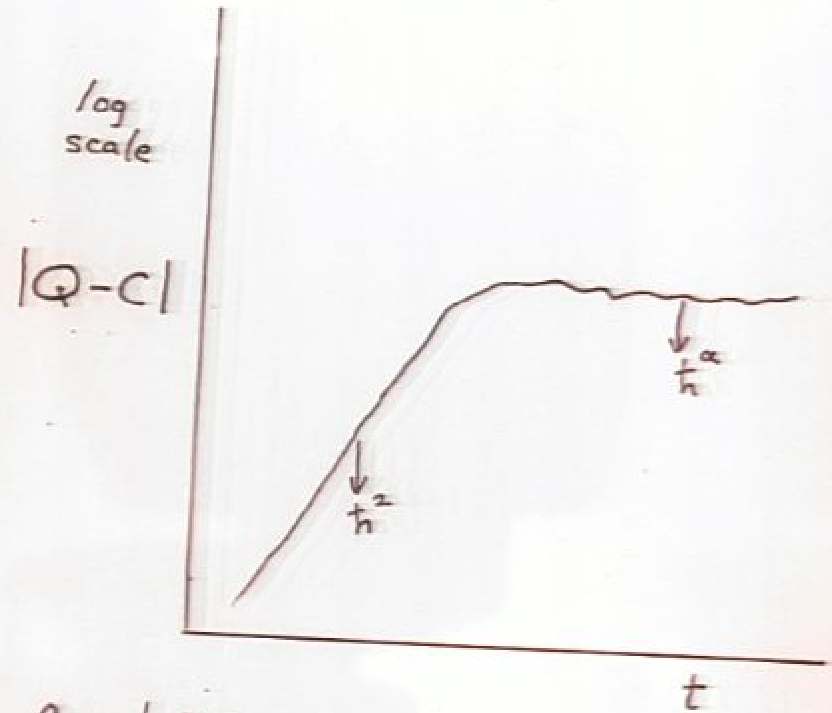
Advantages of Liouville correspondence

(compare quantum probabilities to classical <u>ensemble</u> probabilities):

- Ensemble quantum-classical differences are not sensitive to the width of the initial state.

- Ensemble QC differences scale with $\hbar$ (indicating a quantum origin).

- Ensemble QC differences $\rightarrow 0$ as $\hbar \rightarrow 0$ (decoherence is not essential).



for $\left| \langle A \rangle_Q - \langle A \rangle_C \right| \qquad \alpha \approx \frac{2}{3}$

for $\int \left| P_Q(A) - P_C(A) \right| dA \qquad \alpha \approx \frac{1}{3}$

Conclusions:

The classical limit of a quantum state is an ensemble of classical trajectories.

The role of the quantum state is as a generator of probabilities for an ensemble of similarly prepared systems.

But this raises another question:

What kind of probability is quantum probability?

(1) Frequency and Ensemble theories

Probability identified with a *limit frequency* in an ordered sequence.

--(early frequency theory)

Probability identified with a measure on a set (which need not be ordered).

--(Kolmogorov)

(2) Inferential probability

--(E.T. Jaynes, "Probability Theory: The Logic of Science", 2003)

(2.1) *Objective version*

- Probability as Inductive Inference;
- a logical relation among propositions that is weaker than entailment.

(2.2) *Subjective version* :

- Incomplete knowledge,
- Degrees of reasonable belief.

(3) Propensity:

- a form of causality weaker than determinism.

-- (K.R. Popper)

Some differences between *Propensity* and *Inferential* probability

Propensity:

P(A | S) is the *propensity* for **A** to occur under the physical condition **S**.

The second argument **S** must describe the *state of the system*, or a sequence of *events* that correspond to the *preparation of the state.*

Consequently, the applicability of *Bayes theorem* is subject to restrictions.

Inferential probability:

In **P(A | B)**, the argum... **A** and **B** may be *any propositions*.

Bayes theorem applies without restriction.

Quantum Probability is defined, for purposes of this talk, as that which is calculated from familiar expressions such as:

$|\psi(x)|^2$

$|\langle a_n|\psi\rangle|^2$, where a_n is an eigenvalue of some observable.

$Tr(\rho E_\omega)$, where E_ω projects onto the subset ω of the spectrum of an observable.

Some attributes of Quantum Probability:

- always conditional on the preparation of a *state*.
- asserts the probability of occurence of *events*, such as individual measurement results.
- These events are *indeterministic* and/or *unpredictable*, not merely unknown.
- Since the above characteristics describe the world, not merely someone's knowledge of it, Quantum probability is *objective* rather than *subjective*.

Hence, it is natural to interpret Quantum probability as as *propensity*.

Ex: Objective and Subjective states

(1) Prepare a photon in polarization state $|\psi\rangle = |\updownarrow\rangle$ (vertical)

(2) Tell Alice the polarization is either $\updownarrow$ or $\leftrightarrow$.

(3) Tell Bob it is either

$\updownarrow, \leftrightarrow, \swarrow\!\!\!\!\nearrow \ \nwarrow\!\!\!\!\searrow$.

In the basis $\{|\updownarrow\rangle, |\leftrightarrow\rangle\}$
the density matrices are:

$$\rho_{\updownarrow} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \rho_{\leftrightarrow} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\rho_{\swarrow\!\nearrow} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \quad \rho_{\nwarrow\!\searrow} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Both Alice and Bob will assign the same subjective density matrix,

$$\rho_A = \rho_B = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}.$$

But Alice and Bob have different states of knowledge.

(4) Using a h-v fiter, determine the polarization to be ↕ (not ↔).

Alice correctly infers that the initial state must have been $\rho_{\updownarrow} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

Bob can only make a probalistic estimate of the initial state, by using Bayes' theorem.

Bayes' theorem

$$P(X \mid Y \wedge C) = P(Y \mid X \wedge C)\,\frac{P(X \mid C)}{P(Y \mid C)}$$

Denote Y_i = (possible measurement results)

i.e. $\{\updownarrow$ or $\leftrightarrow\}$

Prior probabilities are $P(Y_i \mid C) = \frac{1}{2}$.

Denote X_j = (possible initial states)

i.e. $\{|\updownarrow\rangle, |\leftrightarrow\rangle, |\nearrow\rangle, |\nwarrow\rangle\}$

Prior probabilities are $P(X_j \mid C) = \frac{1}{4}$.

Likelihoods (probability of the observed data, assuming a particular state):

$P(\updownarrow \mid |\updownarrow\rangle) = 1$, $P(\updownarrow \mid |\leftrightarrow\rangle) = 0$

$P(\updownarrow \mid |\nearrow\rangle) = \frac{1}{2}$, $P(\updownarrow \mid |\nwarrow\rangle) = \frac{1}{2}$.

Thus, after the measurement, Bob's posterior (subjective) probabilities for the 4 possible initial states are

$\lvert\updownarrow\rangle$	$\lvert\leftrightarrow\rangle$	$\lvert\nearrow\!\!\!\swarrow\rangle$	$\lvert\nwarrow\!\!\!\searrow\rangle$
$\frac{1}{2}$	0	$\frac{1}{4}$	$\frac{1}{4}$

Bob's final <u>subjective</u> <u>density</u> <u>matrix</u> is

$$\rho_B' = \tfrac{1}{2}\rho_{\updownarrow} + \tfrac{1}{4}\rho_{\swarrow\!\!\!\nearrow} + \tfrac{1}{4}\rho_{\nwarrow\!\!\!\searrow}$$

$$= \begin{bmatrix} 3/4 & \text{[illegible]} \\ 0 & 1/4 \end{bmatrix}$$

(d) Interpretation of Bobs' final state $\rho_B' = \begin{bmatrix} 3/4 & 0 \\ 0 & 1/4 \end{bmatrix}$

(d.1) It is NOT an estimate of the final state of the system after measurement.

(d.2) It is Bob's final "best estimate" of what the initial (objective) state was, in the sense that predictions from it will have the lowest probable error.

(d.3) But it does not represent Bob's state of belief. Bob does not believe the initial state was ρ_B'; rather, he believes it was either $\rho_{\updownarrow}$, $\rho_{\nearrow}$, or $\rho_{\searrow}$ (with probabilities 50%, [illegible], and 25%, respectively).

(d.4) It is not correct to interpret ρ_B' as "75% $\updownarrow$, 25% $\leftrightarrow$", because Bob knows from the measurement that the polarization $\leftrightarrow$ has zero probability.

Conclusions

(a) There is an objective state, determined by the operation of state preparation.

In the example it is $\rho = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) There may be both objective and subjective probabilities/information in the same problem. They are both conceptually and quantitatively different.

(c) Observers who have *different* states of knowledge may, nevertheless, assign the *same* (subjective) density matrix.
Therefore, *it is not correct to say that the density matrix represents a state of knowledge.*

General Summary

- The <u>Quantum State</u> describes an *ensemble* of similarly prepared systems.

- The <u>state vector</u> (or density matrix) is a generator of *probabilities* for all of the observables of the system.

- The specifically <u>Quantum Probabilities</u> (like $|\Psi|^2$) should be interpreted as *propensities*. Their connection to frequencies arises via the Law of Large Numbers.

- <u>Inferential probabilities</u> (either objective or subjective) may be used in quantum theory, as elsewhere in science.

  ex: information theory (classical or quantum),
  cryptography (classical or quantum),
  ... ,

  but they should not be confused with the specifically quantum probabilities.