Title: PSI - Relativity (PHYS 604) - 15

Date: Sep 23, 2009 10:30 AM

URL: http://pirsa.org/09090074

Abstract:

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha{}_{\beta\mu} V^\beta$$

→ yields tensor

reduces to $\partial_\alpha V^\beta$ in flat space

(or inertial frame)

with Cartesian coord's

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha{}_{\beta\mu} V^\beta$$

→ yields tensor

reduces to $\partial_\alpha V^\beta$ in flat space (or inertial frame) with Cartesian coord's

→ Leibniz rule + scalar rule $\nabla_\mu f = \partial_\mu f$

$$\nabla_\mu S_\alpha = \partial_\mu S_\alpha - \Gamma^\beta{}_{\alpha\mu} S_\beta$$

Covariant derivative

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma^\alpha{}_{\beta\mu} V^\beta$$

→ yields tensor

reduces to $\partial_\alpha V^\beta$ in flat space (or inertial frame) with Cartesian coord's

→ Leibniz rule + scalar rule $\nabla_\mu f = \partial_\mu f$

$$\nabla_\mu S_\alpha = \partial_\mu S_\alpha - \Gamma^\beta{}_{\alpha\mu} S_\beta$$

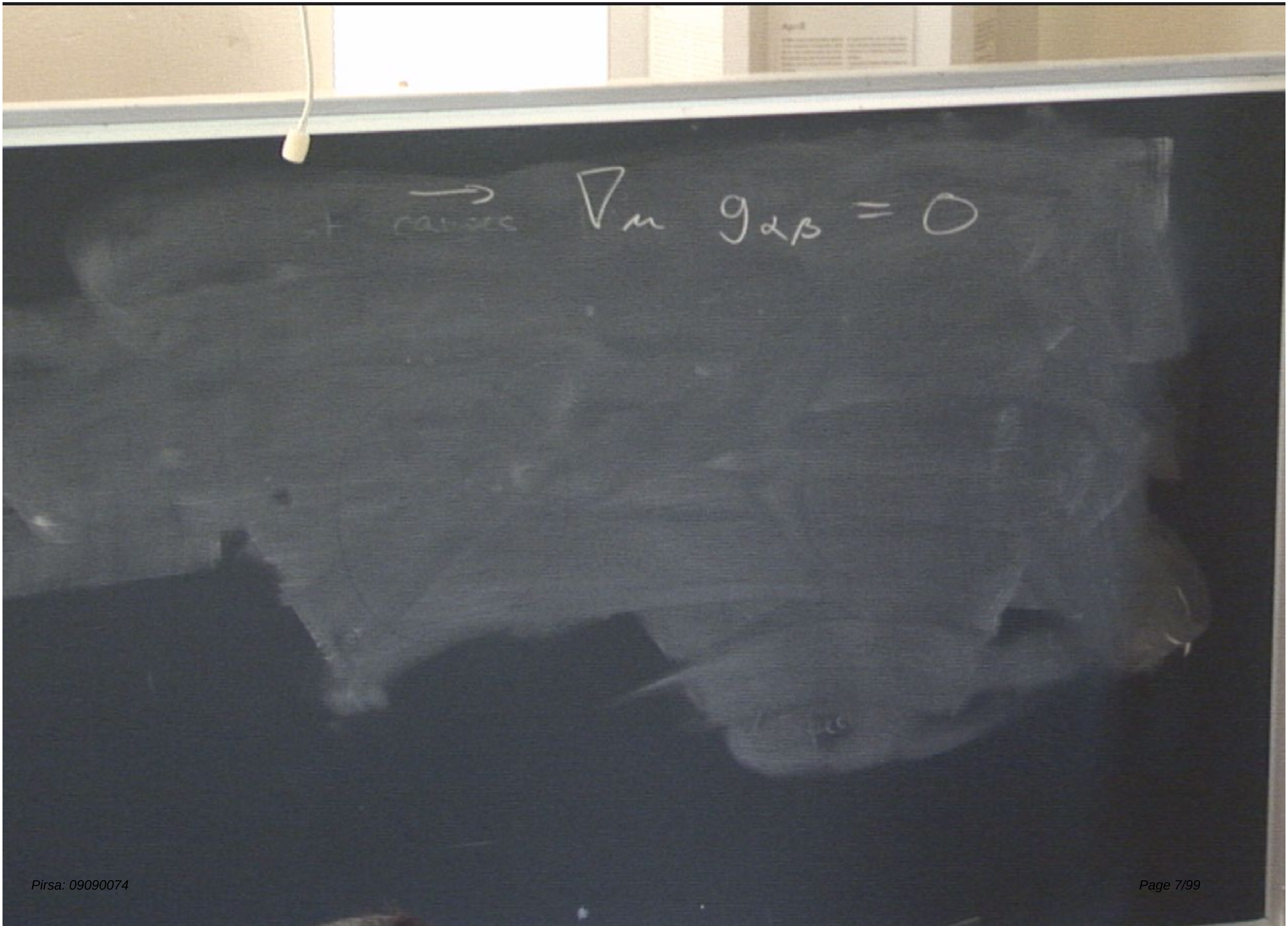
$$\nabla_\mu T^{\alpha_1 \cdots \alpha_n}{}_{\beta_1 \cdots \beta_m} = \partial_\mu T^{\alpha_1 \cdots \alpha_n}{}_{\beta_1 \cdots \beta_m} + \sum_{j=1}^{n} \Gamma^{\alpha_j}{}_{\gamma \mu} T^{\alpha_1 \cdots \gamma \cdots \alpha_n}{}_{\beta_1 \cdots \beta_m} - \sum_{j=1}^{m} \Gamma^{\gamma}{}_{\beta_j \mu} T^{\alpha_1 \cdots \alpha_n}{}_{\beta_1 \cdots \gamma \cdots \beta_m}$$

→ can "bootstrap" up to general
<u>rule</u>:

$$\nabla_\mu T^{\alpha_1 \cdots \alpha_n}{}_{\beta_1 \cdots \beta_m} = \partial_\mu T^{\alpha_1 \cdots \alpha_n}{}_{\beta_1 \cdots \beta_m}$$

$$+ \sum_{j=1}^{n} \Gamma^{\alpha_j}{}_{\gamma\mu} T^{\alpha_1 \cdots \gamma \cdots \alpha_n}{}_{\beta_1 \cdots \beta_m}$$

$$- \sum_{j=1}^{m} \Gamma^{\gamma}{}_{\beta_j \mu} T^{\alpha_1 \cdots \alpha_n}{}_{\beta_1 \cdots \gamma \cdots \beta_m}$$



$\longrightarrow \nabla_\mu g_{\alpha\beta} = 0$

$\xrightarrow{\text{cancels}}$ $\nabla_\mu g_{\alpha\beta} = 0$

$\rightarrow$ comes using def. of $\Gamma^\alpha{}_{\beta\mu}$

$\longrightarrow$ causes $\nabla_\mu g_{\alpha\beta} = 0$

$\longrightarrow$ comes using def. of $\Gamma^\alpha{}_{\beta\mu}$

$T_\mu{}^\alpha = \nabla_\mu v^\alpha \longrightarrow$

$$\longrightarrow \nabla_\mu g_{\alpha\beta} = 0$$

$\rightarrow$ comes using def. of $\Gamma^\alpha{}_{\beta\mu}$

$$T_\mu{}^\alpha = \nabla_\mu v^\alpha \longrightarrow T_{\mu\beta} = g_{\beta\alpha} T_\mu{}^\alpha$$

$\longrightarrow$ causes $\nabla_\mu \, g_{\alpha\beta} = 0$

$\rightarrow$ comes using def. of $\Gamma^\alpha{}_{\beta\mu}$

$T_\mu{}^\alpha = \nabla_\mu V^\alpha \longrightarrow T_{\mu\beta} = g_{\beta\alpha} T_\mu{}^\alpha$

$= g_{\beta\alpha} \nabla_\mu V^\alpha$

$$\xrightarrow{\text{comes}} \nabla_\mu g_{\alpha\beta} = 0$$

$\rightarrow$ comes using def. of $\Gamma^\alpha{}_{\beta\mu}$

$$T_\mu{}^\alpha = \nabla_\mu V^\alpha \longrightarrow T_{\mu\beta} = g_{\beta\alpha} T_\mu{}^\alpha$$
$$= g_{\beta\alpha} \nabla_\mu V^\alpha$$
$$= \nabla_\mu (g_{\beta\alpha} V^\alpha)$$
$$= \nabla_\mu V_\beta$$

$\longrightarrow$ causes $\nabla_\mu \, g_{\alpha\beta} = 0$

$\longrightarrow$ comes using def. of $\Gamma^\alpha{}_{\beta\mu}$

$$T_\mu{}^\alpha = \nabla_\mu V^\alpha \longrightarrow T_{\mu\beta} = g_{\beta\alpha} T_\mu{}^\alpha$$
$$= g_{\beta\alpha} \nabla_\mu V^\alpha$$
$$= \nabla_\mu \left(g_{\beta\alpha} V^\alpha \right)$$
$$= \nabla_\mu V_\beta$$

$\longrightarrow$ can raise and lower indices at will inside covariant deriv.

geodesic eq $\Rightarrow$ $u^\alpha \nabla_\alpha u^\beta = 0$

similar expression for basis (co)vectors

geodesic eq $\Rightarrow$ $u^\alpha \nabla_\alpha u^\beta = 0$

similar expression for basis (co)vectors

$$\frac{d}{d\tau}(e^a)_\alpha$$

geodesic eq $\Rightarrow$ $u^\alpha \nabla_\alpha u^\beta = 0$

similar expression for basis (co)vectors

$$\frac{d}{d\tau}(e^a)_\alpha - \Gamma^\mu{}_{\alpha\gamma}\, u^\gamma (e^a)_\mu = 0$$

geodesic eq $\Longrightarrow$ $u^\alpha \nabla_\alpha u^\beta = 0$

similar expression for basis (co)vectors

$$\frac{d}{d\tau}(e^a)_\alpha - \Gamma^\beta{}_{\alpha\gamma}\, u^\gamma (e^a)_\mu = 0$$

$$\longrightarrow \quad u^\beta \nabla_\beta \left[(e^a)_\alpha\right] = 0$$

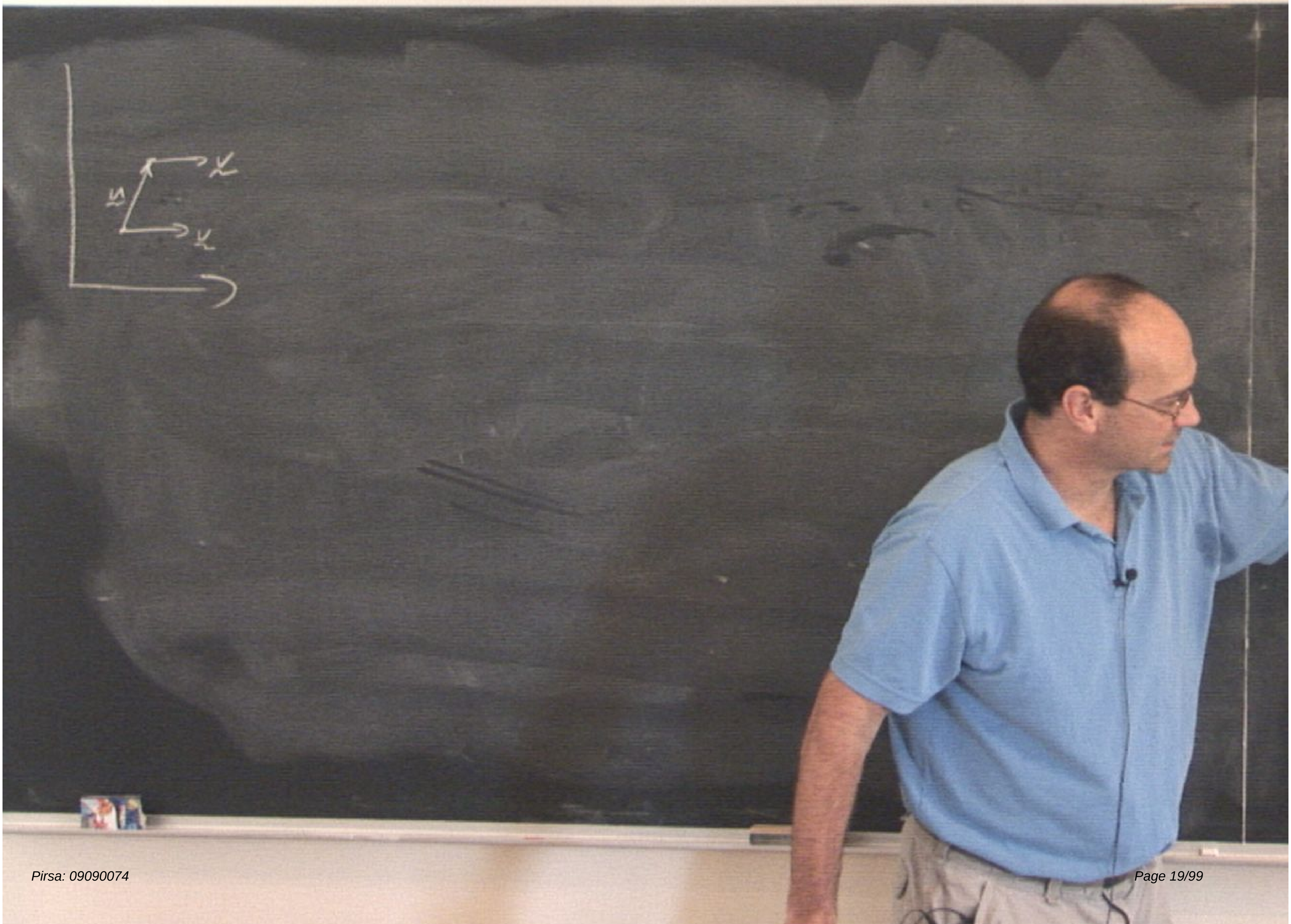
geodesic eq $\Rightarrow$ $u^\alpha \nabla_\alpha u^\beta = 0$

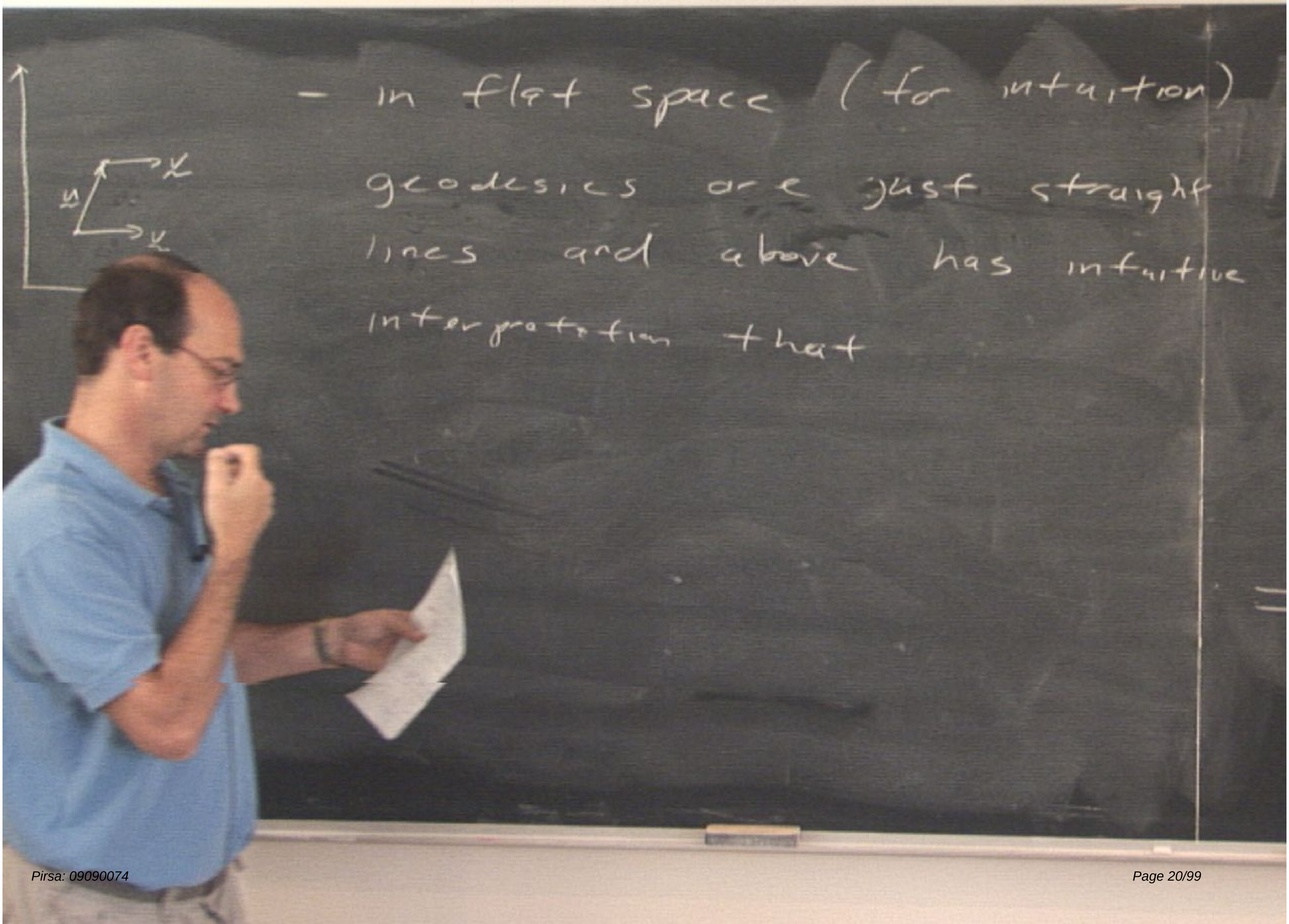
similar expression for basis (co)vectors

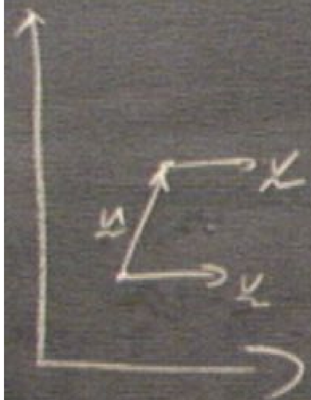
$$\frac{d}{d\tau}(e^a)_\alpha - \Gamma^{\rho}{}_{\alpha\gamma}\, u^\gamma (e^a)_\mu = 0$$

$\rightarrow$ $u^\beta \nabla_\beta \left[(e^a)_\alpha \right] = 0$

$\Rightarrow$ "parallel transport"

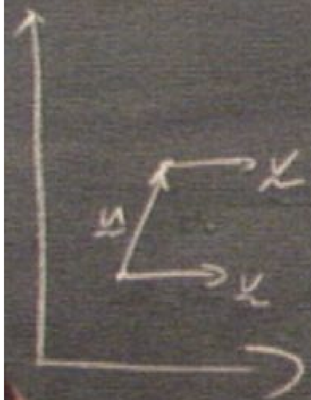



- in flat space (for intuition)
geodesics are just straight
lines and above has intuitive
interpretation that



– in flat space (for intuition)

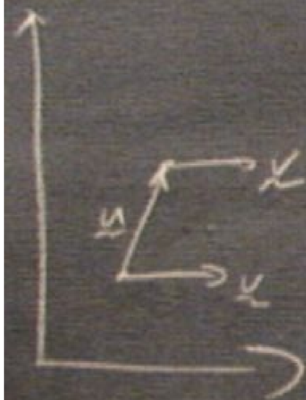
geodesics are just straight lines and above has intuitive interpretation that the basis vectors remain constant



– in flat space (for intuition)

geodesics are just straight lines and above has intuitive interpretation that the basis vectors remain constant

→ idea is extended to curved space and arbitrary paths $x^\alpha(\lambda)$ with tangent =

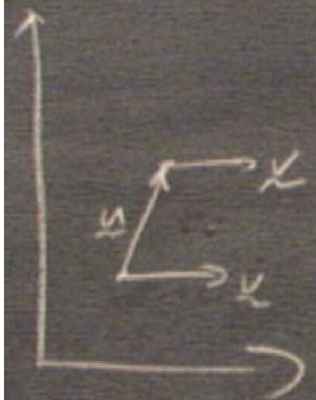


– in flat space (for intuition)

geodesics are just straight lines and above has intuitive interpretation that the basis vectors remain constant

en is extended to curved space and arbitrary paths $x^\alpha(\lambda)$ with tangent $t^\alpha = \frac{dx^\alpha}{d\lambda}$ $\Longrightarrow$ parallel transport:

$$t^\alpha \nabla_\alpha V^\beta = 0$$



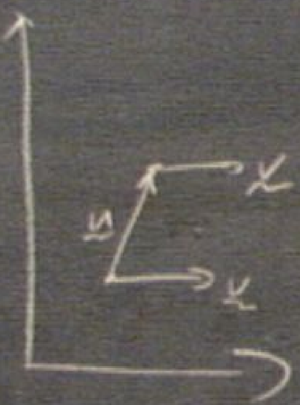
- in flat space (for intuition)

geodesics are just straight lines and above has intuitive interpretation that the basis vectors remain constant

→ idea is extended to curved space and arbitrary paths $x^\alpha(\lambda)$ with tangent

$t^\alpha = \frac{dx^\alpha}{d\lambda} \Longrightarrow$ parallel transport:

$$t^\alpha \nabla_\alpha V^\beta = 0$$

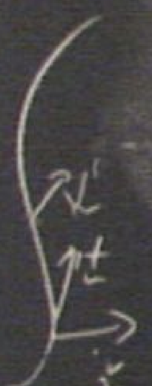


- in flat space (for intuition)

geodesics are just straight lines and above has intuitive interpretation that the basis vectors remain constant

→ idea is extended to curved space and arbitrary paths $x^\alpha(\lambda)$ with tangent $t^\alpha = \frac{dx^\alpha}{d\lambda} \Rightarrow$ parallel transport:

$$t^\alpha \nabla_\alpha V^\beta = 0$$



intuition: that with this rule $\underset{\sim}{V}$ is constant for an observer following the path $x^\alpha(\lambda)$

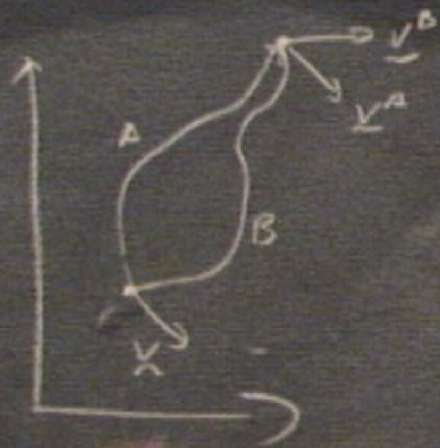
<u>intuition</u>: that with this rule $\underset{\sim}{V}$ is constant for an observer following the path $x^\alpha(\lambda)$

- in flat space we don't give this much thought because we can find vectors satisfy $\nabla_\alpha V^\beta = 0$

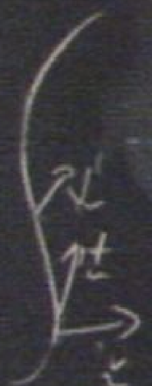
intuition: that with this rule $\underset{\sim}{V}$ is constant for an observer following the path $x^\alpha(\lambda)$

– in flat space we don't give this much thought because we can find vectors satisfy $\nabla_\alpha V^\beta = 0$

→ so don't worry about path

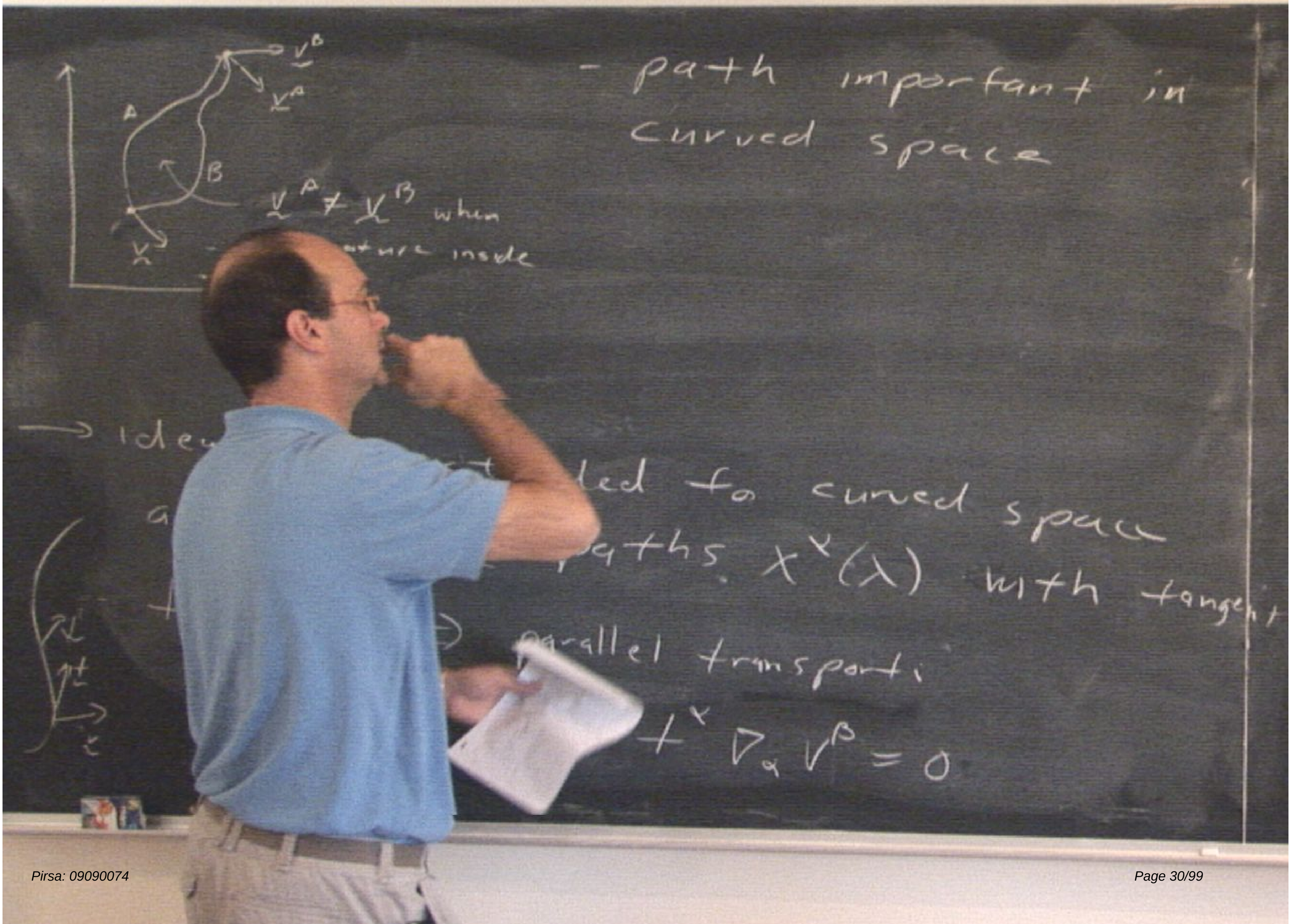


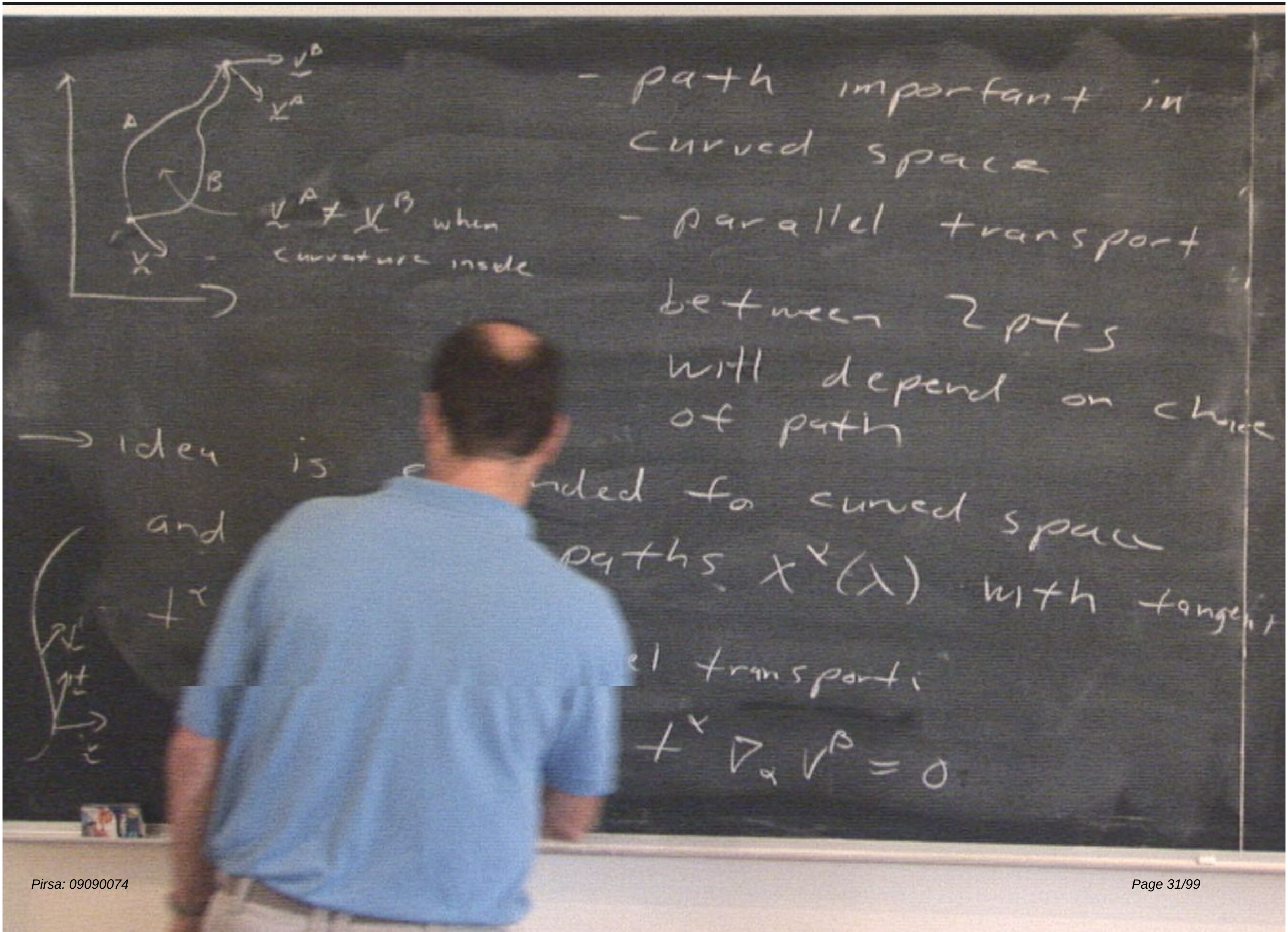
→ idea is extended to curved space and arbitrary paths $x^\alpha(\lambda)$ with tangent

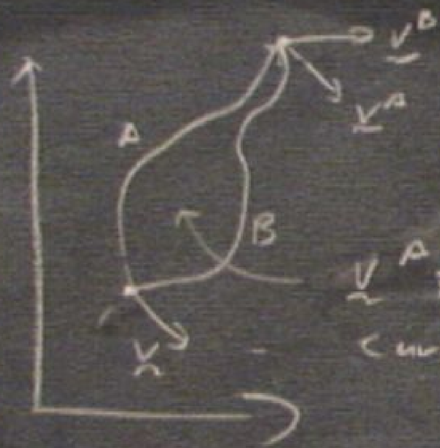


$t^\alpha = \dfrac{dx^\alpha}{d\lambda} \Rightarrow$ parallel transport:

$$t^\alpha \nabla_\alpha V^\beta = 0$$

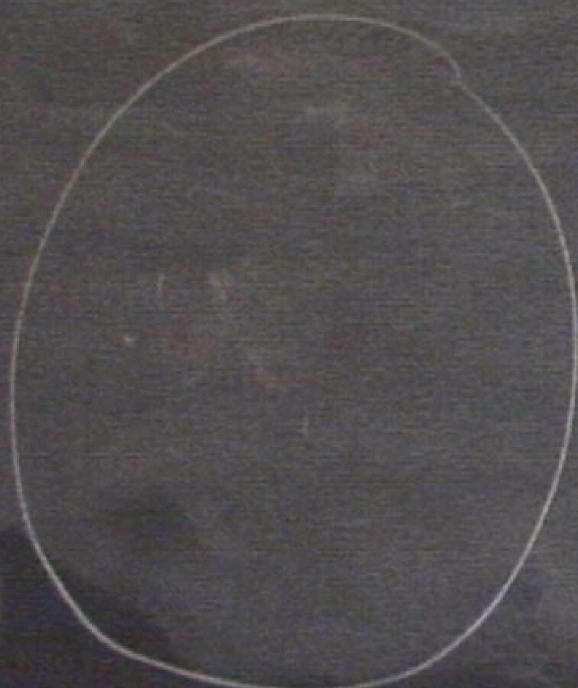

- path important in curved space
V^A ≠ V^B when
ature inside
led to curved space
paths x^α(λ) with tangent
parallel transport
t^α ∇_α V^β = 0


- path important in curved space
- parallel transport between 2 pts will depend on choice of path
$V^A \neq V^B$ when curvature inside
A
B
→ idea is ... ded to curved space
and ... paths $x^\alpha(\lambda)$ with tangent
t^α
... el transport:
$t^\alpha \nabla_\alpha V^\beta = 0$



$\underset{\sim}{V}^A \neq \underset{\sim}{V}^B$ when curvature inside

- path important in curved space
- parallel transport between 2 pts will depend on choice of path



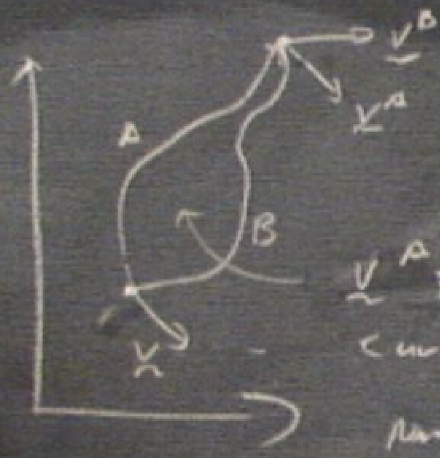
$V^A \neq V^B$ when curvature inside

North Pole

- path important in curved space
- parallel transport between 2 pts will depend on choice of path

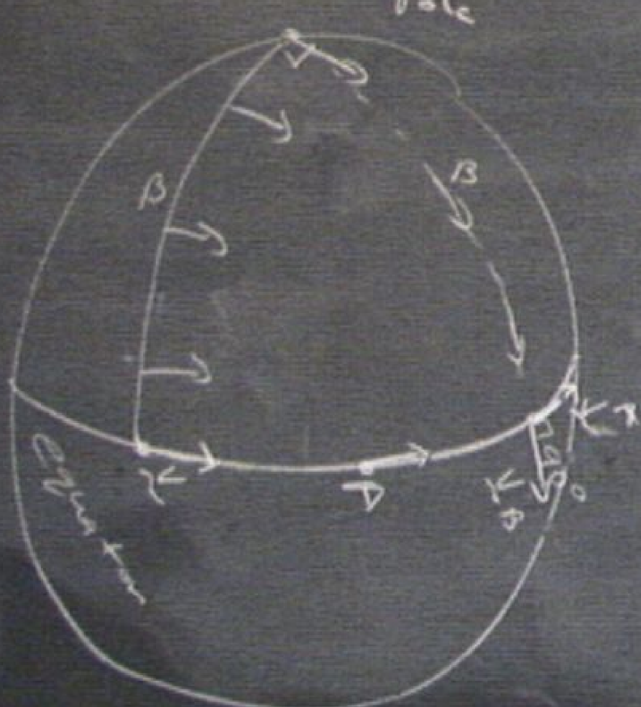
$\underset{\sim}{V}^A \neq \underset{\sim}{V}^B$ when curvature inside

- path important in curved space
- parallel transport between 2 pts will depend on choice of path

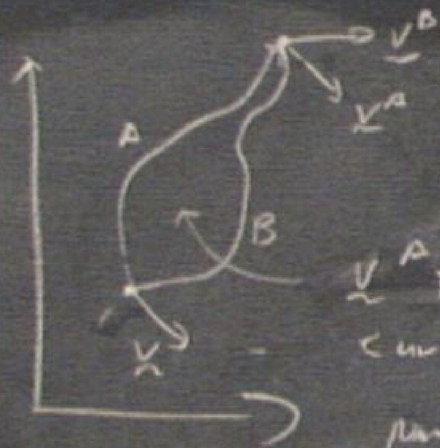


$V^A \neq V^B$ when curvature inside

North Pole

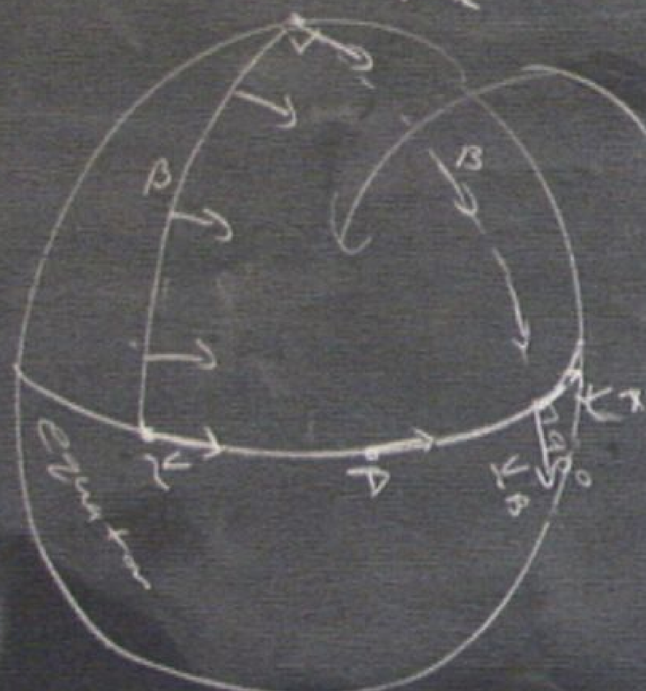


- path important in curved space
- parallel transport between 2 pts will depend on choice of path



$v^A \neq v^B$ when curvature inside

North Pole



contains curvature

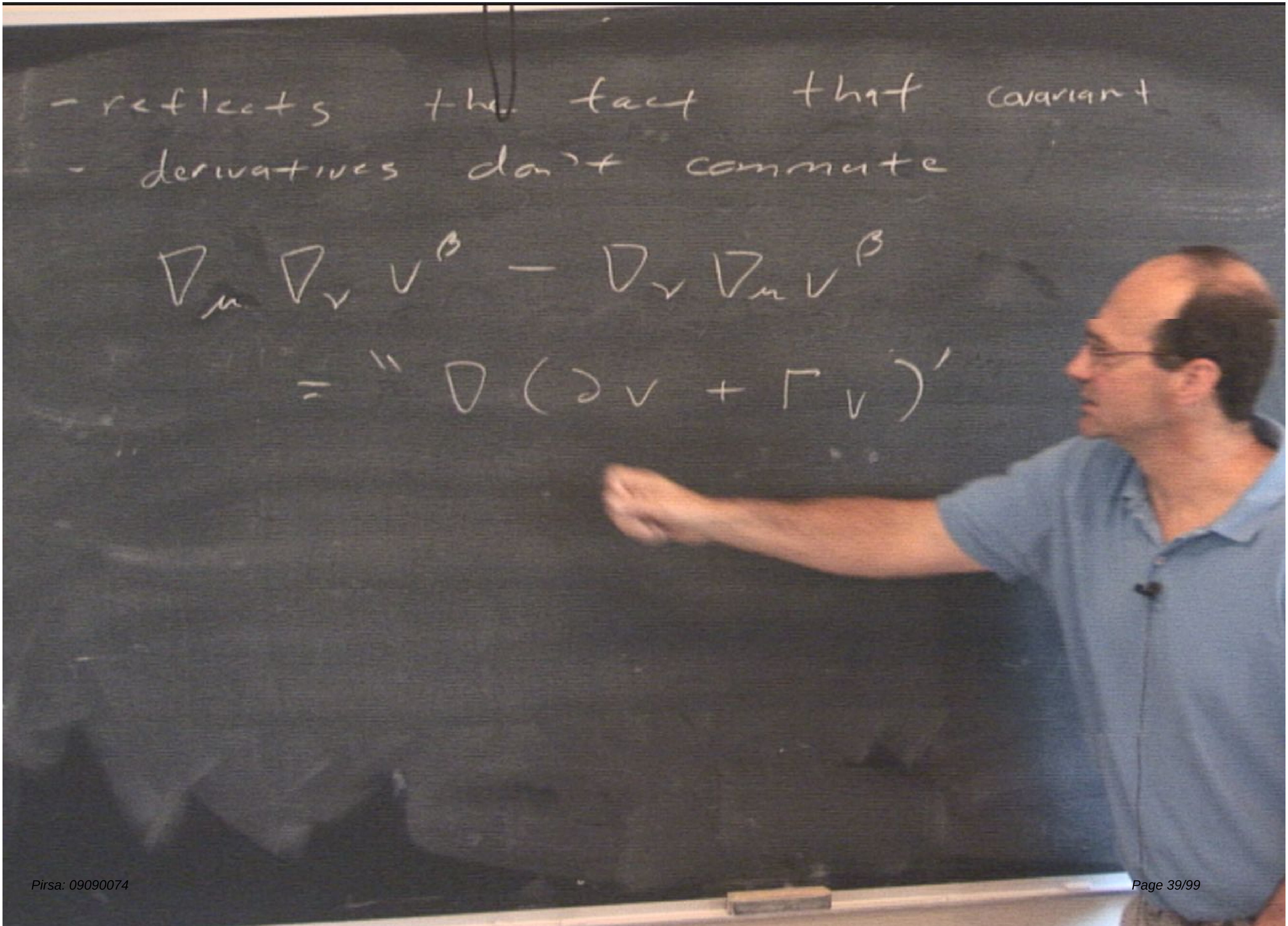
- path important in curved space
- parallel transport between 2 pts will depend on choice of path

← globe or 2d sphere gives a simple example

- reflects the fact that covariant
- derivatives don't commute

- reflects the fact that covariant
- derivatives don't commute

$$\nabla_\mu \nabla_\nu V^\beta - \nabla_\nu \nabla_\mu V^\beta$$



- reflects the fact that covariant
- derivatives don't commute
$\nabla_\mu \nabla_\nu V^\beta - \nabla_\nu \nabla_\mu V^\beta$
$= \text{“} \nabla (\partial V + \Gamma V) \text{”}$

- reflects the fact that covariant
- derivatives don't commute

$$\nabla_\mu \nabla_\nu V^\beta - \nabla_\nu \nabla_\mu V^\beta$$

$$= \text{"} \nabla (\partial V + \Gamma V) \text{"}$$

$$= \text{"} \cancel{(\partial_\mu \partial_\nu - \partial_\nu \partial_\mu)} V^\beta + \Gamma \partial V$$

- reflects the fact that covariant
- derivatives don't commute

$$\nabla_\mu \nabla_\nu V^\beta - \nabla_\nu \nabla_\mu V^\beta$$

$$= \text{" } \nabla (\partial V + \Gamma V) \text{"}$$

$$= \text{" } (\cancel{\partial_\mu \partial_\nu - \partial_\nu \partial_\mu}) V^\beta + \cancel{\Gamma} \partial V$$

$$+ (\partial \Gamma + \Gamma\Gamma) V \text{"}$$

- reflects the fact that covariant
- derivatives don't commute

$$\nabla_\mu \nabla_\nu V^\beta - \nabla_\nu \nabla_\mu V^\beta$$

$$= \text{``}\, \nabla (\partial V + \Gamma V) \,\text{''}$$

$$= \text{``}\, \cancel{(\partial_\mu \partial_\nu - \partial_\nu \partial_\mu)} V^\beta + \cancel{\Gamma \partial V} + (\partial \Gamma + \Gamma \Gamma) V \,\text{''}$$

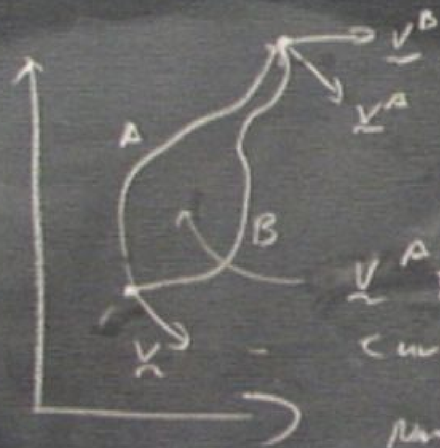
- reflects the fact that covariant
- derivatives don't commute

$$\nabla_\mu \nabla_\nu V^\beta - \nabla_\nu \nabla_\mu V^\beta$$

$$= \text{"} \nabla (\partial V + \Gamma V) \text{"}$$

$$= \text{"} \cancel{(\partial_\mu \partial_\nu - \partial_\nu \partial_\mu)} V^\beta + \cancel{\Gamma \partial V} + (\partial \Gamma + \Gamma \Gamma) V$$

$$= R^\beta{}_{\alpha\mu\nu} V^\alpha$$



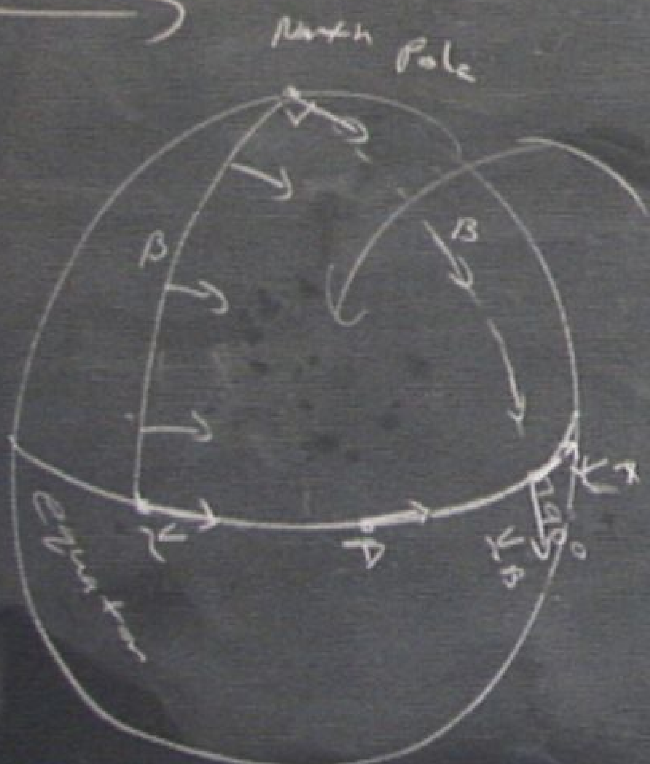
$V^A \neq V^B$ when curvature inside

So if looked for $\nabla_\alpha V^\beta$

$\Longrightarrow R^\beta{}_{\alpha\mu\nu} V^\alpha = 0$

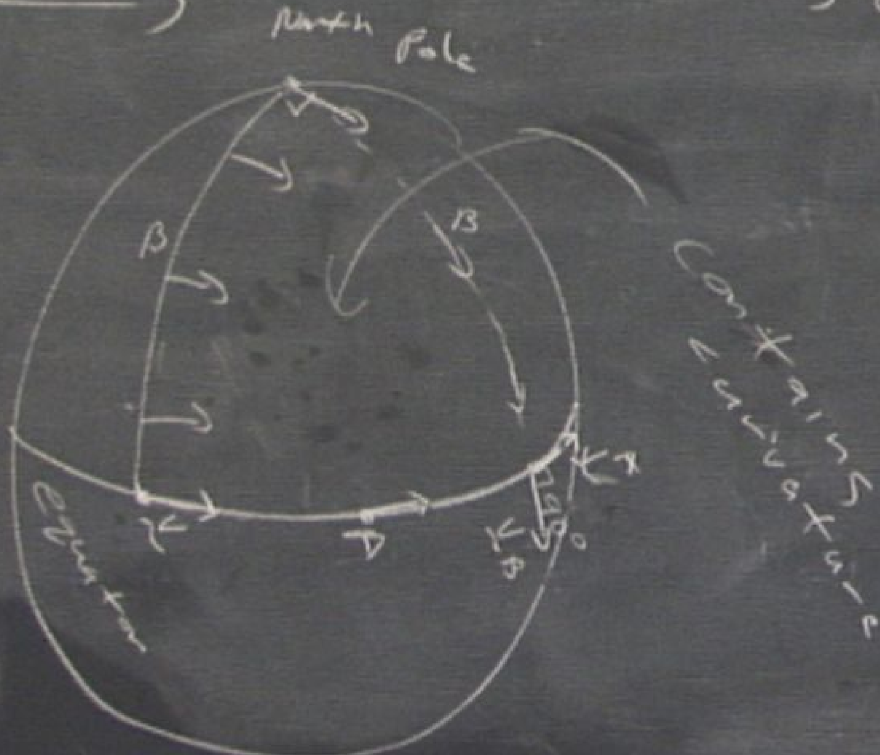
algebraic eq

$\longrightarrow$ generically no solution



contains curvature

$\underset{\sim}{V}^A \neq \underset{\sim}{V}^B$ when curvature inside



So if looked for $\nabla_\alpha V^\beta$

$\longrightarrow R^\beta{}_{\alpha\mu\nu} V^\alpha = 0$

algebraic eq

$\longrightarrow$ <u>generically</u>, no solution other than the trivial solution $V^\beta = 0$

Back to Einstein's eq's

$R_{ab} = 0 \longleftarrow$ complicated nonlinear 2nd order PDE's

Back to Einstein's eq's

$R_{\alpha\beta} = 0 \leftarrow$ complicated nonlinear 2nd order PDE's

Back to Einstein's eq's

$R_{\alpha\beta} = 0 \leftarrow$ complicated nonlinear

2nd order PDE's

$\rightarrow$ hard to solve

Back to Einstein's eq's

$R_{\alpha\beta} = 0 \Leftarrow$ complicated nonlinear 2nd order PDE's

→ hard to solve

→ typically in solving impose various symmetries

eg – spherical symmetry

– time independent

- some discussion in book

Weak field Einstein eq

- some discussion in book

Weak field Einstein eq

- working in nearly flat space

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

- some discussion in book

Weak field Einstein eq

- working in nearly flat space

$$g_{\alpha\beta} = \underbrace{\eta_{\alpha\beta}}_{\operatorname{diag}(-1,1,1,1)} + h_{\alpha\beta}$$

metric perturb

- some discussion in book

Weak field Einstein eq

- working in nearly flat space

$$g_{\alpha\beta} = \underbrace{\eta_{\alpha\beta}}_{\text{diag}(-1,1,1,1)} + h_{\alpha\beta}$$

$\leftarrow$ metric perturb[...]

each $|h_{\alpha\beta}| << 1$

- some discussion in book

Weak field Einstein eq

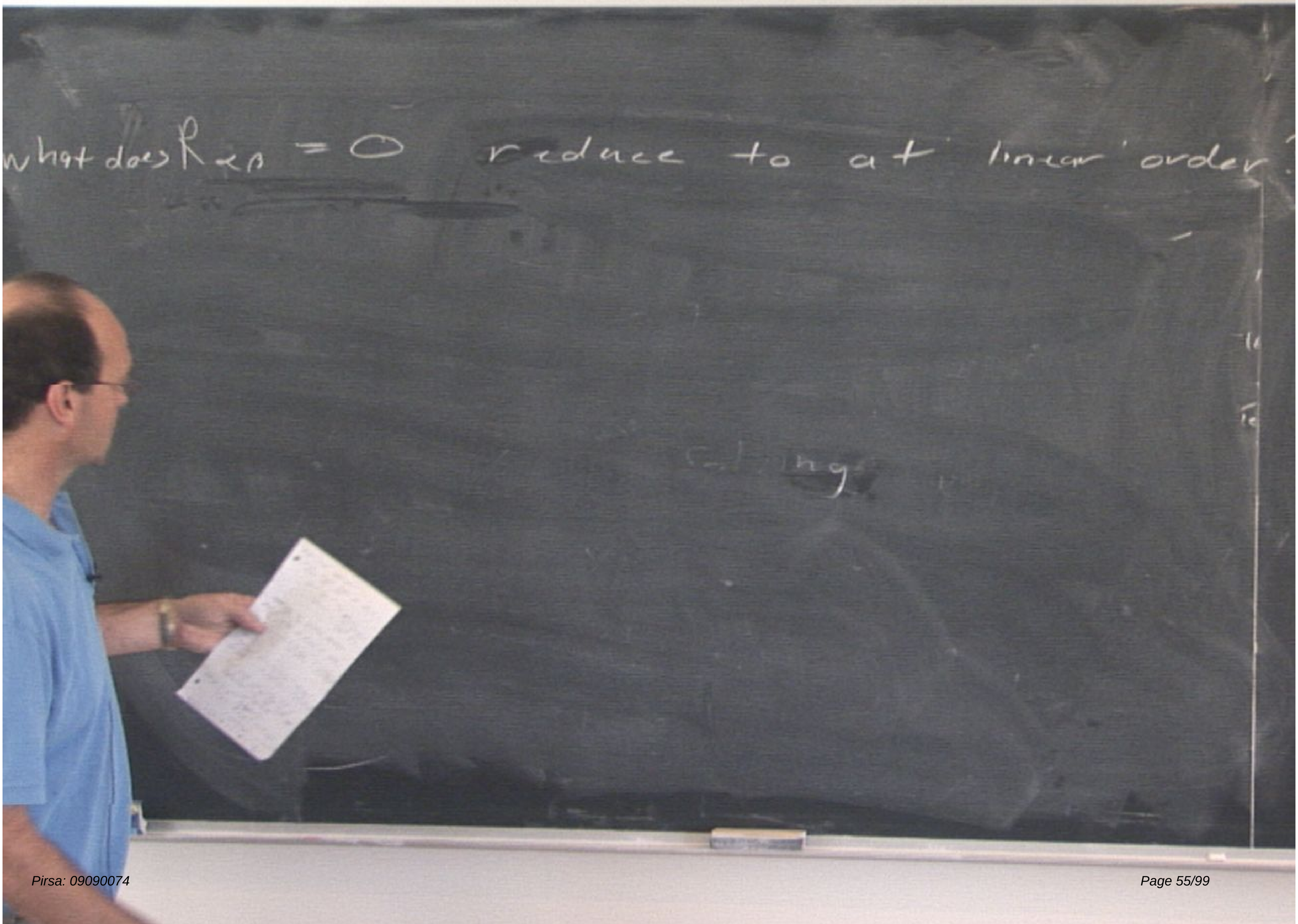
- working in nearly flat space

$$g_{\alpha\beta} = \underbrace{\eta_{\alpha\beta}}_{\mathrm{diag}(-1,1,1,1)} + h_{\alpha\beta}$$

metric perturbations

each $|h_{\alpha\beta}| \ll 1$

$\longrightarrow$ linear order h


what does $R_{\alpha\beta} = 0$ reduce to at linear order?

...at does $R_{\alpha\beta} = 0$ reduce to at linear order?

...short cut : recall in inertial frame

$$R_{\mu\nu\alpha\beta} = \frac{1}{2}\left(\partial_\nu \partial_\alpha g_{\mu\beta} + \partial_\mu \partial_\beta g_{\nu\alpha} - \partial_\nu \partial_\beta g_{\mu\alpha} - \partial_\mu \partial_\alpha g_{\nu\beta}\right)$$

at does $R_{\alpha\beta} = 0$ reduce to at linear order?

short cut : recall in inertial frame

$$R_{\mu\nu\alpha\beta} = \frac{1}{2}\left(\partial_\nu \partial_\alpha g_{\mu\beta} + \partial_\mu \partial_\beta g_{\nu\alpha} - \partial_\nu \partial_\beta g_{\mu\alpha} - \partial_\mu \partial_\alpha g_{\nu\beta}\right.$$

...t does $R_{\alpha\beta} = 0$ reduce to at 'linear' order?

...hort cut : recall in inertial frame

$$R_{\mu\nu\alpha\beta} = \frac{1}{2}\left(\partial_\nu \partial_\alpha g_{\mu\beta} + \partial_\mu \partial_\beta g_{\nu\alpha} - \partial_\nu \partial_\beta g_{\mu\alpha} - \partial_\mu \partial_\alpha g_{\nu\beta} \right.$$

claim is in present problem:

$$R_{\mu\nu\alpha\beta} = \frac{1}{2}\left(\partial_\nu \partial_\alpha h_{\mu\beta} + \partial_\mu \partial_\beta h_{\nu\alpha} - \partial_\nu \partial_\beta h_{\mu\alpha} - \partial_\mu \partial_\alpha h_{\nu\beta} \right)$$

what does $R_{\alpha\beta} = 0$ reduce to at linear order?

short cut : recall in inertial frame

$$R_{\mu\nu\alpha\beta} = \frac{1}{2}\left(\partial_\nu \partial_\alpha g_{\mu\beta} + \partial_\mu \partial_\beta g_{\nu\alpha} - \partial_\nu \partial_\beta g_{\mu\alpha} - \partial_\mu \partial_\alpha g_{\nu\beta}\right)$$

claim is in present problem,

Working to linear order

$$\delta R_{\mu\nu\alpha\beta} = \frac{1}{2}\left(\partial_\nu \partial_\alpha h_{\mu\beta} + \partial_\mu \partial_\beta h_{\nu\alpha} - \partial_\nu \partial_\beta h_{\mu\alpha} - \partial_\mu \partial_\alpha h_{\nu\beta}\right)$$

→ terms we discarded are all higher order in h

$\delta R_{\nu\beta}$

β)

$\rightarrow$ terms we discarded are all higher order in h

;

$$\delta R_{\nu\beta} = \eta^{\mu\alpha}\, \delta R_{\mu\nu\alpha\beta}$$

in general $g^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta} + O(h^2)$]

$$\delta R_{\nu\beta} = \frac{1}{2}\Big($$

$\beta\,)$

$\rightarrow$ terms we discarded are all higher order in h

$$\delta R_{\nu\beta} = \eta^{\mu\alpha}\, \delta R_{\mu\nu\alpha\beta}$$

$$\uparrow$$

$\Big[$ in general $g^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta} + O(h^2) \Big]$

$$\delta R_{\nu\beta} = \tfrac{1}{2}\left(- \Box h_{\nu\beta} + \partial_\nu \partial^\alpha h_{\alpha\beta} \right.$$

→ terms we discarded are all higher order in h

$$\delta R_{\nu\beta} = \eta^{\mu\alpha}\, \delta R_{\mu\nu\alpha\beta}$$

in general $g^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta} + O(h^2)$

$$\delta R_{\nu\beta} = \frac{1}{2}\left(-\Box h_{\nu\beta} + \partial_\nu \partial^\alpha h_{\alpha\beta}\right.$$

$$- \partial_\nu \partial_\beta g_{\mu\alpha} - \partial_\mu \partial_\alpha g_{\beta\nu}$$

problem:

$$\partial_\alpha h_{\mu\beta} \quad \partial_\mu \partial_\beta h_{\nu\alpha}$$

$$\partial_\nu \partial_\beta h_{\mu\alpha} - \partial_\mu \partial_\alpha h_{\nu\beta}$$

$\longrightarrow$ terms we discarded are all higher order in h

$$\delta R_{\nu\beta} = \eta^{\mu\alpha}\, \delta R_{\mu\nu\alpha\beta}$$

in general $g^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta} + O(h^2)$

$$\delta R_{\nu\beta} = \frac{1}{2}\Big(- \Box h_{\nu\beta} + \partial_\nu (\partial^\alpha h_{\alpha\beta}) + \partial_\beta (\partial^\alpha h_{\alpha\nu}) \Big)$$

→ terms we discarded are all higher order in h

$$\delta R_{\nu\beta} = \eta^{\mu\alpha}\, \delta R_{\mu\nu\alpha\beta}$$

in general $g^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta} + O(h^2)$

$$\delta R_{\nu\beta} = \frac{1}{2}\Big[-\Box h_{\nu\beta} + \partial_\nu (\partial^\alpha h_{\alpha\beta}) + \partial_\beta (\partial^\alpha h_{\alpha\nu}) - \partial_\nu \partial_\beta (h_\alpha{}^\alpha) \Big]$$

$$\delta R_{\nu\beta} = \frac{1}{2}\left(-\Box h_{\nu\beta} + \partial_\nu V_\beta + \partial_\beta V_\nu\right)$$

with $V_\beta = \partial^\alpha h_{\alpha\beta} - \frac{1}{2}\partial_\beta h_\alpha{}^\alpha$

$\longrightarrow$ terms we discarded are all higher order in h

$$\delta R_{\nu\beta} = \eta^{\mu\alpha}\, \delta R_{\mu\nu\alpha\beta}$$

in general $g^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta} + O(h^2)$

$$\delta R_{\nu\beta} = \frac{1}{2}\Big[-\Box h_{\nu\beta} + \partial_\nu (\partial^\alpha h_{\alpha\beta}) + \partial_\beta (\partial^\alpha h_{\alpha\nu}) - \partial_\nu \partial_\beta (h_\alpha{}^\alpha)$$

→ terms we discarded are all higher order in h

$$\left[g^{\alpha\beta} g_{\beta\mu} = \delta^{\alpha}{}_{\mu} \right]$$

$$\delta R_{\nu\beta} = \eta^{\mu\alpha}\, \delta R_{\mu\nu\alpha\beta}$$

In general $g^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta} + O(h^2)$

$$\delta R_{\nu\beta} = \frac{1}{2}\Big[-\Box h_{\nu\beta} + \partial_\nu(\partial^\alpha h_{\alpha\beta}) + \partial_\beta(\partial^\alpha h_{\alpha\nu}) - \partial_\nu \partial_\beta (h_\alpha{}^\alpha) \Big]$$

$$\delta R_{\nu\beta} = \frac{1}{2}\left(-\Box h_{\nu\beta} + \partial_\nu V_\beta + \partial_\beta V_\nu\right) =$$

with $V_\beta = \partial^\alpha h_{\alpha\beta} - \frac{1}{2}\partial_\beta h_\alpha{}^\alpha$

Note: that we have weak fields but no mention of small velocities

How do fix up our metric:

$$ds^2 = -(1+2\Phi(x))\,dt^2 + \quad dx^2 + dx^2$$

$- \partial_\nu \partial_\mu$

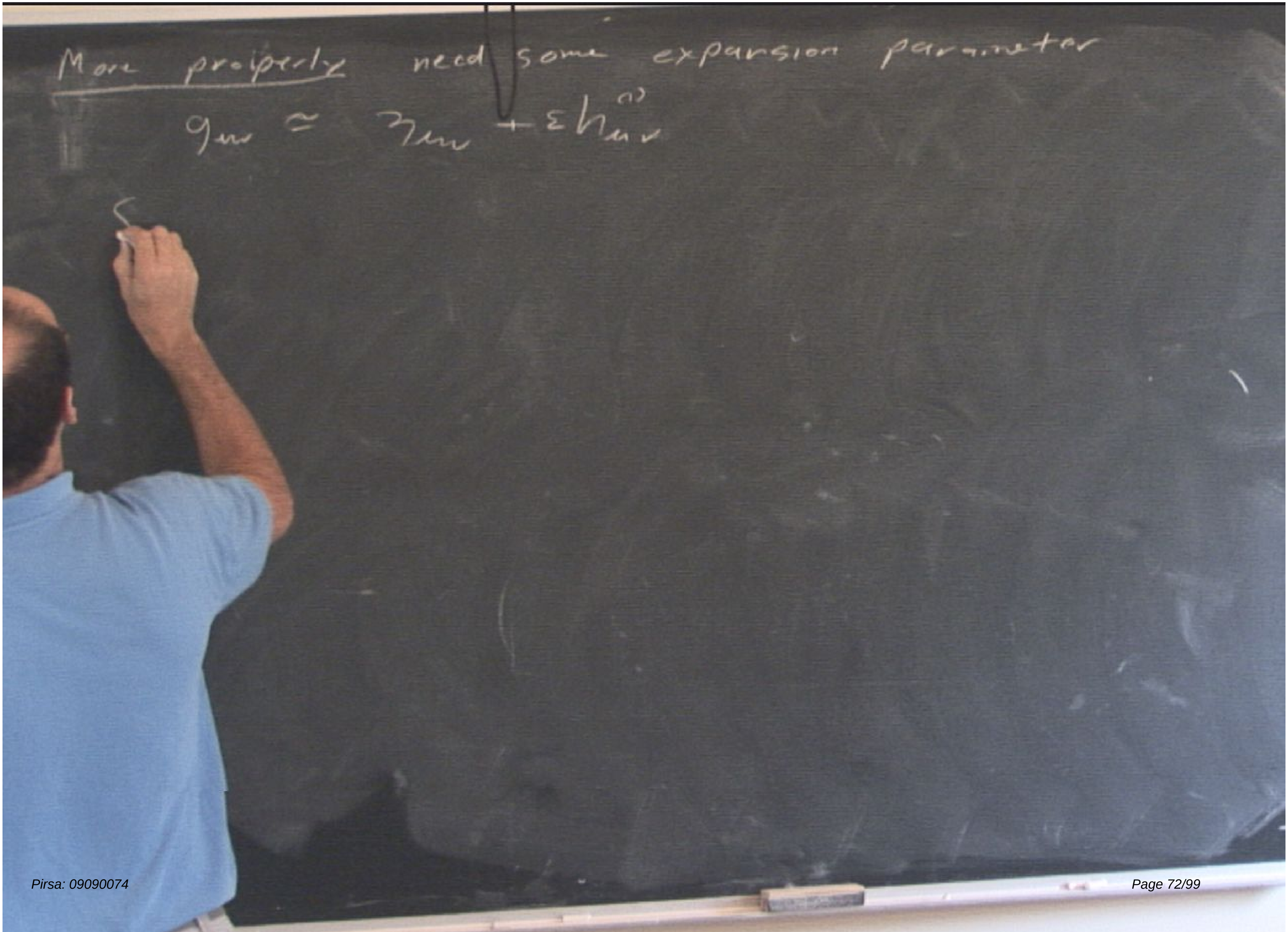
$$\delta R_{\nu\beta} = \frac{1}{2}\left(-\Box h_{\nu\beta} + \partial_\nu V_\beta + \partial_\beta V_\nu\right) = 0$$

with $V_\beta = \partial^\alpha h_{\alpha\beta} - \frac{1}{2}\partial_\beta h_\alpha{}^\alpha$

Note: that we have weak fields but no mention of small velocities

How do fix up our metric:

$$ds^2 = -(1+2\Phi(x))\,dt^2 + dx^2 + dy^2 + dz^2$$


More properly need some expansion parameter
$g_{\mu\nu} \simeq \eta_{\mu\nu} + \varepsilon h^{(1)}_{\mu\nu}$

More properly need some expansion parameter

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + \varepsilon h^{(1)}_{\mu\nu} + \varepsilon^2 h^{(2)}_{\mu\nu} + \cdots\cdots$$

ε^1: $\delta R_{\mu\nu}(h^{(1)}) = 0 \longrightarrow$ solve for $h^{(1)}$

ε^2: $\delta R_{\mu\nu}(h^{(2)}) =$ stuff $h^{(1)2} \longrightarrow$ solve $h^{(2)}$

dz^2

$$\delta R_{\nu\beta} = \frac{1}{2}\left(-\Box h_{\nu\beta} + \partial_\nu V_\beta + \partial_\beta V_\nu\right) =$$

with $V_\beta = \partial^\alpha h_{\alpha\beta} - \frac{1}{2}\partial_\beta h_\alpha{}^\alpha$

Note: that we have weak fields but no mention of small velocities

How do fix up our metric for Φ

$$ds^2 = -(1+2\Phi(x))\,dt^2 + (dx^2 + dx^2 +$$

Hartle suggest $(1+a\Phi$

$$= \frac{1}{2}\left(-\Box h_{\gamma\beta} + \partial_\gamma V_\beta + \partial_\beta V_\gamma\right) = 0$$

$$V_\beta = \partial^\alpha h_{\alpha\beta} - \frac{1}{2}\partial_\beta h_\alpha{}^\alpha$$

that we have weak fields but no mention of small velocities

ow do fix up our metric for $\Phi << 1$, $\underline{v^2 << 1}$)

$$ds^2 = -(1 + 2\Phi(x))\,dt^2 + \left(dx^2 + dy^2 + dz^2\right)$$

Hartle suggest $(1 + a\Phi)$

More prop

$g_{\mu\nu}$

ε^1: $\delta R_{\mu\nu}($

ε^2: $\delta R_{\mu\nu}($

fix

More properly need some expansion parameter

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + \varepsilon h^{(1)}_{\mu\nu} + \varepsilon^2 h^{(2)}_{\mu\nu} + \cdots$$

ε^1: $\delta R_{\mu\nu}(h^{(1)}) = 0 \longrightarrow$ solve for $h^{(1)}$

ε^2: $\delta R_{\mu\nu}(h^{(2)}) =$ stuff $h^{(1)2} \longrightarrow$ solve $h^{(2)}$

---

fix a using $\delta R_{\alpha\beta} = 0$

$$h_{++} = -2\Phi \qquad h_{+i} = 0$$

$$h_{ij} = a\,\Phi\,\delta_{ij}$$

$\ll 1,\ v^2 \ll 1)$

$d z^2)$

More properly need some expansion parameter

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + \varepsilon h^{(1)}_{\mu\nu} + \varepsilon^2 h^{(2)}_{\mu\nu} + \cdots\cdots$$

ε^1: $\delta R_{\mu\nu}(h^{(1)}) = 0 \longrightarrow$ solve for $h^{(1)}$

ε^2: $\delta R_{\mu\nu}(h^{(2)}) =$ stuff $h^{(1)2} \longrightarrow$ solve $h^{(2)}$

fix a using $\delta R_{\alpha\beta} = 0$

$$h_{++} = -2\Phi \qquad h_{+i} = 0$$

$$h_{ij} = a\,\Phi\,\delta_{ij}$$

$<1, v^2 \ll 1)$

Newton: $\nabla^2 \Phi = 0$

$dz^2)$

More properly need some expansion parameter

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + \varepsilon h^{(1)}_{\mu\nu} + \varepsilon^2 h^{(2)}_{\mu\nu} + \cdots\cdots$$

ε^1: $\delta R_{\mu\nu}(h^{(1)}) = 0 \longrightarrow$ solve for $h^{(1)}$

ε^2: $\delta R_{\mu\nu}(h^{(2)}) =$ stuff $h^{(1)2} \longrightarrow$ solve $h^{(2)}$

fix a using $\delta R_{\alpha\beta} = 0$

$$h_{tt} = -2\Phi \qquad h_{ti} = 0$$

$$h_{ij} = a\,\Phi\,\delta_{ij}$$

$<1,\ v^2 \ll 1)$

$d z^2)$

Newton: $\nabla^2 \Phi = 0$ $\quad\Phi$ is independen[t] of t

More properly need some expansion parameter

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + \varepsilon h^{(1)}_{\mu\nu} + \varepsilon^2 h^{(2)}_{\mu\nu} + \cdots$$

ε^1: $\delta R_{\mu\nu}(h^{(1)}) = 0 \longrightarrow$ solve for $h^{(1)}$

ε^2: $\delta R_{\mu\nu}(h^{(2)}) = \text{stuff } h^{(1)2} \longrightarrow$ solve $h^{(2)}$

fix a using $\delta R_{\alpha\beta} = 0$

$$h_{++} = -2\Phi \qquad h_{+i} = 0$$

$$h_{ij} = a\,\Phi\,\delta_{ij}$$

$<1,\ v^2 \ll 1)$

Newton: $\nabla^2 \Phi = 0$ $\quad\Phi$ is independent of t

$d z^2)$

$$\Longrightarrow \Box h_{\alpha\beta} = 0$$

$$\delta R_{\nu\beta} = \frac{1}{2}\left(-\Box h_{\nu\beta} + \partial_\nu V_\beta + \partial_\beta V_\nu\right) =$$

with $V_\beta = \partial^\alpha h_{\alpha\beta} - \frac{1}{2}\partial_\beta h_\alpha{}^\alpha$

~~to~~ set last two terms $= 0$
we try setting $V_\alpha = 0$

$$h_\alpha{}^\alpha = \eta^{\alpha\beta} h_{\alpha\beta} = \Phi(2$$

$$\delta R_{\nu\beta} = \frac{1}{2}\left(-\Box\, h_{\nu\beta} + \partial_\nu V_\beta + \partial_\beta V_\nu\right) =$$

with $V_\beta = \partial^\alpha h_{\alpha\beta} - \frac{1}{2}\partial_\beta h_\alpha{}^\alpha$

~~for~~ to set last two terms $= 0$

we try setting $V_\alpha = 0$

$$h_\alpha{}^\alpha = \eta^{\alpha\beta} h_{\alpha\beta} = \Phi\,(2+3a)$$

diagonal h

$\beta = t \qquad V_t = 0 \longleftarrow \Phi(\vec{x})$

$x \qquad V_x$

Newton: $\nabla^2 \Phi = 0$ $\qquad \Phi$ is independen[t] of t

$\Longrightarrow \Box h_{\alpha\beta} = 0$

diagonal h

$\beta = t \qquad V_t = 0 \longleftarrow \Phi(\vec{x})$

$x \qquad V_x = \partial_x h_{xx} - \partial_x (h_\alpha{}^\alpha)$

$=$

Newton: $\nabla^2 \Phi = 0$ $\qquad \Phi$ is independent of t

$\Longrightarrow \Box h_{\alpha\beta} = 0$

diagonal h

$\beta = t \qquad V_t = 0 \longleftarrow \Phi(\vec{x})$

$x \qquad V_x = \partial_x h_{xx} - \frac{1}{2}\partial_x (h_\alpha{}^\alpha)$

$= \partial_x \Phi \left(a - \tfrac{1}{2}(2 + 3a)\right)$

$= \partial_x \Phi \left(-\tfrac{a}{2} - 1\right)$

Newton: $\nabla^2 \Phi = 0$ $\quad$ Φ is independent of t

$\Longrightarrow \Box\, h_{\alpha\beta} = 0$

diagonal h

$\beta = t \qquad V_t = 0 \longleftarrow \Phi(\vec{x})$

$x \qquad V_x = \partial_x h_{xx} - \frac{1}{2}\partial_x (h_\alpha{}^\alpha)$

$= \partial_x \Phi \left(a - \frac{1}{2}(2 + 3a) \right)$

$= \partial_x \Phi \left(-\frac{a}{2} - 1 \right)$

$y \qquad V_y = \partial_y \Phi \left(-\frac{a}{2} - 1 \right)$

Same for z

$\Longrightarrow V_\rho = 0$ iff $a = -2$

Newton: $\nabla^2 \Phi = 0 \qquad \Phi$ is independent of t

$\Longrightarrow \Box h_{\alpha\beta} = 0$

diagonal h

$\beta = t \qquad V_t = 0 \leftarrow \Phi(\vec{x})$

$x \qquad V_x = \partial_x h_{xx} - \frac{1}{2}\partial_x (h_\alpha{}^\alpha)$

$= \partial_x \Phi \left(a - \frac{1}{2}(2 + 3a) \right)$

$= \partial_x \Phi \left(-\frac{a}{2} - 1 \right)$

$y \qquad V_y = \partial_y \Phi \left(-\frac{a}{2} - 1 \right)$

Same for z

$\Longrightarrow V_\rho = 0$ iff $a = -2$

$$ds^2 = -(1 + 2\Phi)\,dt^2 + (1 - 2\Phi)(dx^2 + dy^2 + dz^2)$$

diagonal h

$\beta = t \qquad V_t = 0 \leftarrow \Phi(\vec{x})$

$x \qquad V_x = \partial_x h_{xx} - \frac{1}{2}\partial_x (h_\alpha{}^\alpha)$

$= \partial_x \Phi \left(a - \frac{1}{2}(2 + 3a) \right)$

$= \partial_x \Phi \left(-\frac{a}{2} - 1 \right)$

$y \qquad V_y = \partial_y \Phi \left(-\frac{a}{2} - 1 \right)$

same for z

$\Longrightarrow V_\rho = 0$ iff $a = -2$

$$ds^2 = -(1 + 2\Phi)\,dt^2 + (1 - 2\Phi)(dx^2 + dy^2 + dz^2)$$

- solves linearized Einstein eq's

diagonal h

$\beta = t \qquad V_t = 0 \leftarrow \Phi(\vec{x})$

$x \qquad V_x = \partial_x h_{xx} - \frac{1}{2}\partial_x (h_\alpha{}^\alpha)$

$= \partial_x \Phi \left(a - \frac{1}{2}(2 + 3a) \right)$

$= \partial_x \Phi \left(-\frac{a}{2} - 1 \right)$

$y \qquad V_y = \partial_y \Phi \left(-\frac{a}{2} - 1 \right)$

same for z

$\Longrightarrow V_\rho = 0$ iff $a = -2$

$$ds^2 = -(1 + 2\Phi)\,dt^2 + (1 - 2\Phi)(dx^2 + dy^2 + dz^2)$$

- solves linearized Einstein eq's

$\hookrightarrow$ agrees with Schwarzschild metric

diagonal h

$\beta = t \qquad V_t = 0 \longleftarrow \Phi(\vec{x})$

$x \qquad V_x = \partial_x h_{xx} - \frac{1}{2}\partial_x (h_\alpha{}^\alpha)$

$= \partial_x \Phi \left(a - \frac{1}{2}(2+3a) \right)$

$= \partial_x \Phi \left(-\frac{a}{2} - 1 \right)$

$y \qquad V_y = \partial_y \Phi \left(-\frac{a}{2} - 1 \right)$

same for z

$\Longrightarrow V_\rho = 0$ iff $a = -2$

$$ds^2 = -(1+2\Phi)\,dt^2 + (1-2\Phi)(dx^2+dy^2+dz^2)$$

- solves linearized Einstein eq's

$\hookrightarrow$ agrees with Schwarzschild metric in isotropic coord's

diagonal h

$\beta = t \qquad V_t = 0 \leftarrow \Phi(\vec{x})$

$x \qquad V_x = \partial_x h_{xx} - \frac{1}{2}\partial_x (h_\alpha{}^\alpha)$

$= \partial_x \Phi \left(a - \frac{1}{2}(2 + 3a)\right)$

$= \partial_x \Phi \left(-\frac{a}{2} - 1\right)$

$y \qquad V_y = \partial_y \Phi \left(-\frac{a}{2} - 1\right)$

same for z

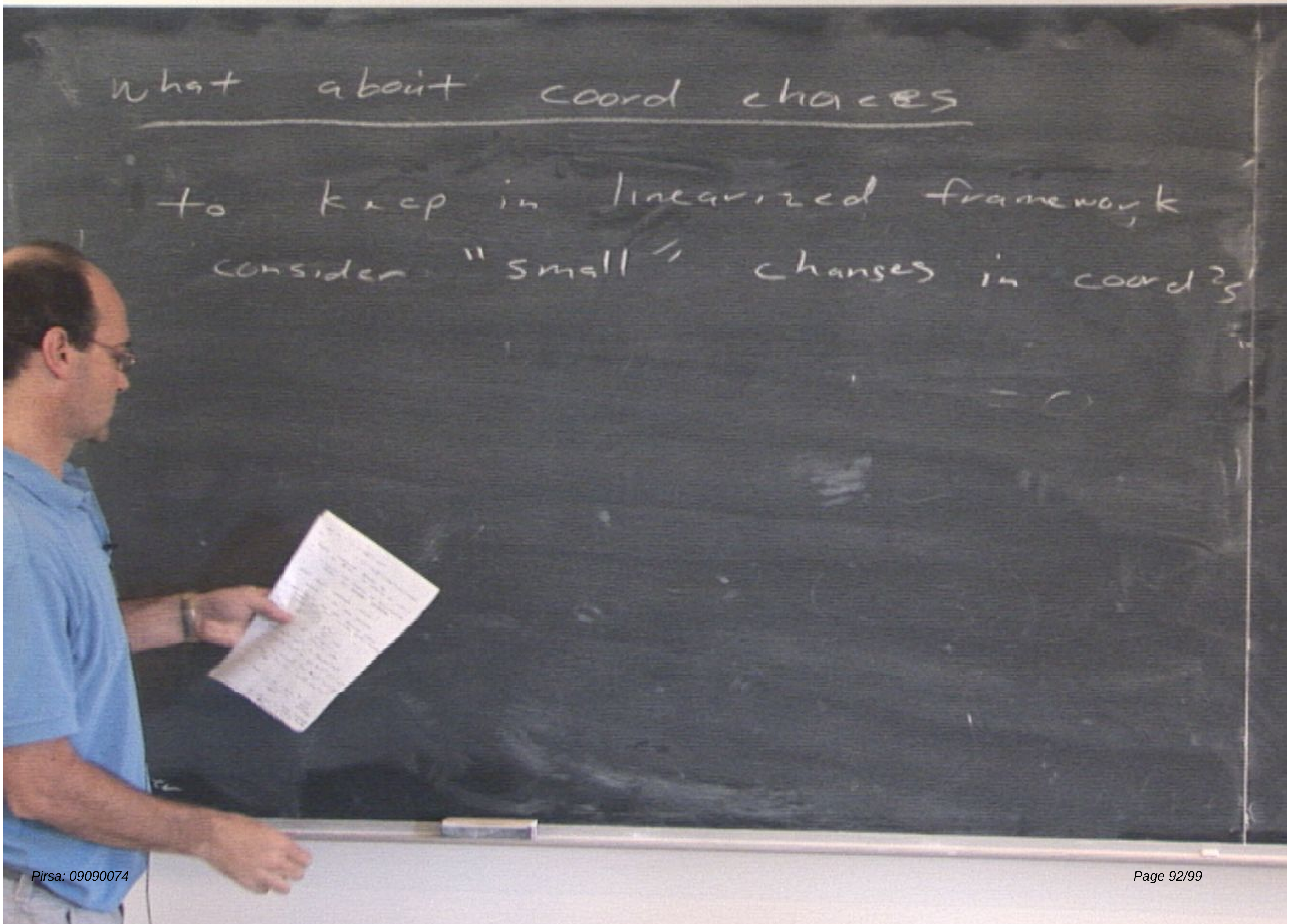
$\Longrightarrow V_\beta = 0$ iff $a = -2$

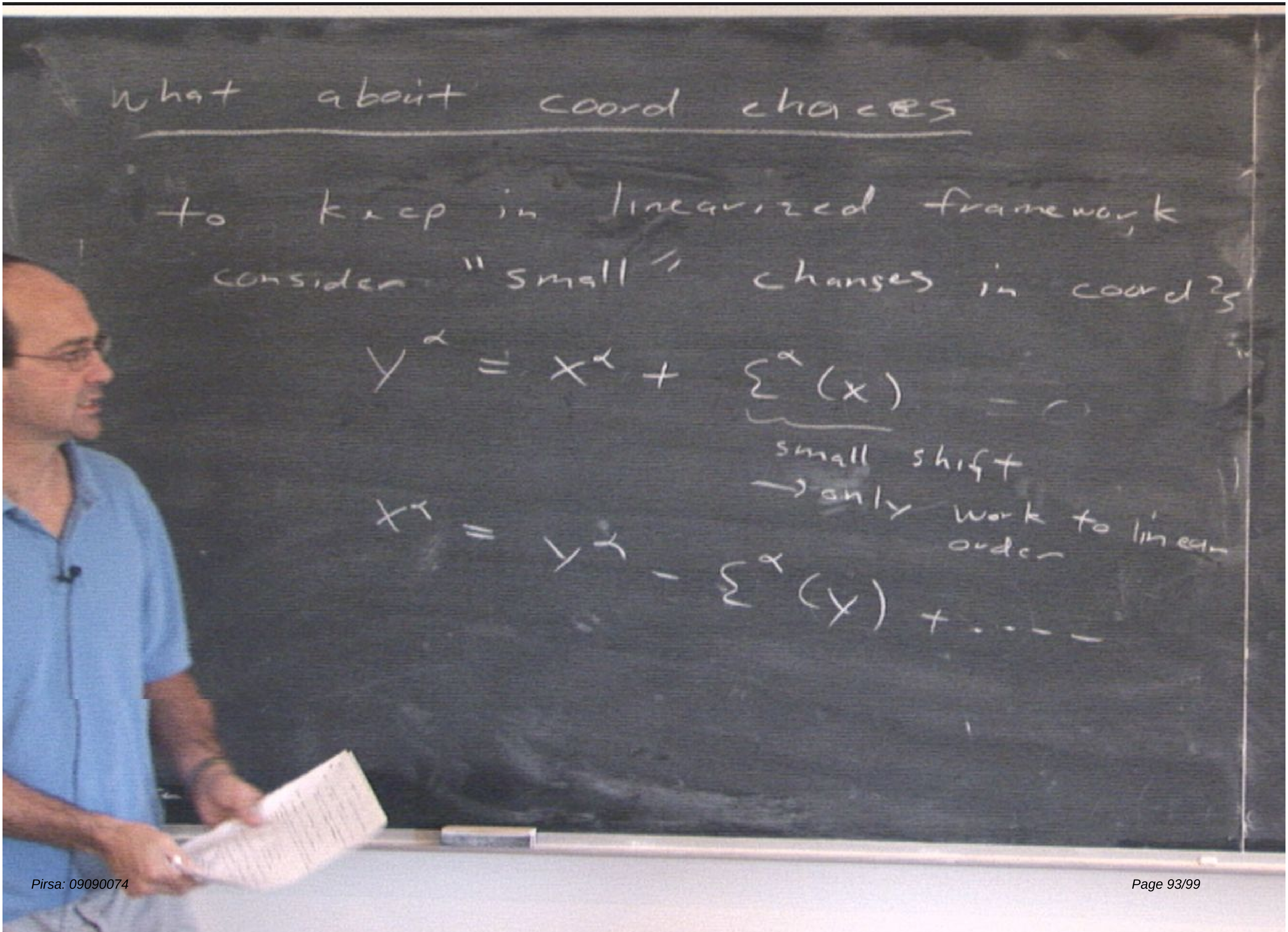
$$ds^2 = -(1 + 2\Phi)\,dt^2 + (1 - 2\Phi)(dx^2 + dy^2 + dz^2)$$

- solves linearized Einstein eq's

$\hookrightarrow$ agrees with Schwarzschild metric in isotropic coord's

What about coord choices


what about coord changes
to keep in linearized framework
consider "small" changes in coord's


what about coord chaces
to keep in linearized framework
consider "small" changes in coord's
$y^\alpha = x^\alpha + \xi^\alpha(x)$
small shift
→ only work to linear order
$x^\alpha = y^\alpha - \xi^\alpha(y) + \ldots$

$$ds^2 = (\eta_{\alpha\beta} + h_{\alpha\beta})\, dx^\alpha dx^\beta$$

$$ds^2 = (\eta_{\alpha\beta} + h_{\alpha\beta})\, dx^\alpha dx^\beta$$

$$= (\eta_{\alpha\beta} + h_{\alpha\beta})(dy^\alpha + \partial_\mu \xi^\alpha dy^\mu)(dy^\beta + \partial_\nu \xi^\beta dy^\nu)$$

$$ds^2 = (\eta_{\alpha\beta} + h_{\alpha\beta})\,dx^\alpha dx^\beta$$

$$= (\eta_{\alpha\beta} + h_{\alpha\beta})(dy^\alpha + \partial_\mu \xi^\alpha dy^\mu)(dy^\beta + \partial_\nu \xi^\beta dy^\nu)$$

$$= (\eta_{\alpha\beta} + h_{\alpha\beta} + \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha)\,dy^\alpha dy^\beta$$

$$ds^2 = (\eta_{\alpha\beta} + h_{\alpha\beta})\, dx^\alpha dx^\beta$$

$$= (\eta_{\alpha\beta} + h_{\alpha\beta})(dy^\alpha + \partial_\mu \xi^\alpha dy^\mu)(dy^\beta + \partial_\nu \xi^\beta dy^\nu)$$

$$= (\eta_{\alpha\beta} + h_{\alpha\beta} + \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha)\, dy^\alpha dy^\beta$$

$$= (\eta_{\alpha\beta} + \tilde{h}_{\alpha\beta})\, dy^\alpha dy^\beta$$

$$ds^2 = (\eta_{\alpha\beta} + h_{\alpha\beta})\, dx^\alpha dx^\beta$$

$$= (\eta_{\alpha\beta} + h_{\alpha\beta})(dy^\alpha + \partial_\mu \xi^\alpha dy^\mu)(dy^\beta + \partial_\nu \xi^\beta dy^\nu)$$

$$= (\eta_{\alpha\beta} + h_{\alpha\beta} + \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha)\, dy^\alpha dy^\beta$$

$$= (\eta_{\alpha\beta} + \tilde{h}_{\alpha\beta})\, dy^\alpha dy^\beta$$

$$\tilde{h}_{\alpha\beta} = h_{\alpha\beta} + \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha$$

in y coords   in x coords

$$ds^2 = (\eta_{\alpha\beta} + h_{\alpha\beta})\, dx^\alpha dx^\beta$$

$$= (\eta_{\alpha\beta} + h_{\alpha\beta})(dy^\alpha + \partial_\mu \xi^\alpha dy^\mu)(dy^\beta + \partial_\nu \xi^\beta dy^\nu)$$

$$= (\eta_{\alpha\beta} + h_{\alpha\beta} + \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha)\, dy^\alpha dy^\beta$$

$$= (\eta_{\alpha\beta} + \tilde{h}_{\alpha\beta})\, dy^\alpha dy^\beta$$

$$\tilde{h}_{\alpha\beta} = h_{\alpha\beta} + \underbrace{\partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha}_{\text{gauge transformations}}$$

in y coords

in x coords