Title: Relativity - Core (PHYS 604) - Lecture 14

Date: Sep 22, 2009 10:30 AM

URL: http://pirsa.org/09090073

Abstract:

Riemann curvature:

$$R^{\alpha}{}_{\beta\gamma\delta} = \text{"}\,\partial\Gamma + \Gamma\Gamma\,\text{"}$$

Riemann curvature.

$$R^{\alpha}{}_{\beta\gamma\delta} = \text{"} \partial\Gamma + \Gamma\Gamma \text{"}$$

- tensor description spacetime curvature

Riemann curvature.

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→ contains 2nd deriv's of metric

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→ but in fact R satisfies a number of symmetries

– find these with trick

→ at a point choose coord's

such that $g_{\alpha\beta} = \eta_{\alpha\beta}$, $\partial_\gamma g_{\alpha\beta} = 0$

ents

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→ at a point choose coord's such that $g_{\alpha\beta} = \eta_{\alpha\beta}$, $\partial_\gamma g_{\alpha\beta} = 0$

$$\downarrow$$

$$\Gamma^{\alpha}_{\beta\mu} = 0$$

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also lower first index

ents

$R_{\alpha\beta\mu\nu} = g_\alpha$

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$R_{\alpha\beta\mu\nu} = g_{\alpha\lambda} R^{\lambda}{}_{\beta\mu\nu}$

– find these with trick

→ at a point choose coord's

such that $g_{\alpha\beta} = \eta_{\alpha\beta}$, $\partial_\gamma g_{\alpha\beta} = 0$

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also lower first index

ents

$$R_{\alpha\beta\mu\nu} = g_{\alpha\lambda} R^{\lambda}{}_{\beta\mu\nu}$$

$$= \tfrac{1}{2}\left(\partial_\beta \partial_\mu g_{\alpha\nu} + \partial_\alpha \partial_\nu g_{\beta\mu} - \partial_\beta \partial_\nu g_{\alpha\mu} - \partial_\alpha \partial_\mu g_{\beta\nu} \right)$$

Riemann curvature:

$$R_{\alpha\beta\mu\nu} = R_{\alpha\beta\nu\mu}$$

but in fact R satisfies a number of symmetries

Riemann curvature.

$$R_{\alpha\beta\mu\nu} = -R_{\alpha\beta\nu\mu}$$

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Riemann curvature

$$R_{\alpha\beta\mu\nu} = -R_{\alpha\beta\nu\mu}$$

→ this conclusion is a tensor eq which applies in all coord systems (at all pts)

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also find $$R_{\alpha\beta\mu\nu} = -R_{\beta\alpha\mu\nu}$$

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Riemann curvature.

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→ this conclusion is a tensor eq which applies in all coord systems (at all pts)

also find $$R_{\alpha\beta\mu\nu} = -R_{\beta\alpha\mu\nu}$$

$$R_{\alpha\beta\mu\nu} = +R_{\mu\nu\alpha\beta}$$

cyclic identity: $$R_{\alpha\beta\mu\nu} + R_{\beta\mu\alpha\nu} + R_{\mu\alpha\beta\nu} = 0$$

$\longrightarrow$ reduces # of independent components to 20 (which matches our previous counting)

→ reduces # of independent components to 20 (which matches our previous counting)

→ also have Bianchi identity involving

→ reduces # of independent components to 20 (which matches our previous counting)

→ also have Bianchi identity involving derivatives of R

More on bases

More on bases

$$\underset{\sim}{e}_\alpha : \quad \underset{\sim}{V} = V^\alpha \underset{\sim}{e}_\alpha$$

dual basis: e

More on bases

$\underset{\sim}{e}_\alpha$: $\underset{\sim}{V} = V^\alpha \underset{\sim}{e}_\alpha$

dual basis: $\hat{e}^\alpha$

$$\hat{e}^\alpha(\underset{\sim}{e}_\beta) = \delta^\alpha{}_\beta$$

More on bases

vectors → $\underset{\sim}{e}_\alpha$: $\underset{\sim}{V} = V^\alpha \underset{\sim}{e}_\alpha$ $\qquad \hat{e}^\alpha \cdot \underset{\sim}{e}_\beta =$

dual basis: $\hat{e}^\alpha$ $\qquad \hat{e}^\alpha(\underset{\sim}{e}_\beta) = \delta^\alpha{}_\beta$

$\hat{S} = S_\alpha \hat{e}^\alpha$

one-forms
co-vectors

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vectors $\rightarrow \underset{\sim}{e}_\alpha$: $\underset{\sim}{V} = V^\alpha \underset{\sim}{e}_\alpha$ $\qquad \hat{e}^\alpha \cdot \underset{\sim}{e}_\beta =$

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→ can use metric to relate 4vectors
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$V_\alpha = g_{\alpha\beta} V^\beta$

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one-forms
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$\rightarrow$ can use metric to relate 4vectors + co-vectors

$V_\alpha = g_{\alpha\beta} V^\beta$

$\hat{S} \cdot V = S_\alpha V^\alpha$

More on bases

vectors → $\underset{\sim}{e}_\alpha$: $\boxed{\underset{\sim}{V}} = V^\alpha \underset{\sim}{e}_\alpha$ $\quad \hat{e}^\alpha \cdot \underset{\sim}{e}_\beta =$

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$\hat{S} = S_\alpha \hat{e}^\alpha$ ← one-forms co-vectors

→ can use metric to relate 4 vectors + co-vectors

$V_\alpha = g_{\alpha\beta} V^\beta$

$\hat{S} \cdot \underset{\sim}{V} = S_\alpha V^\alpha$ ← invariant under coord trans's

More on bases

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orthonormal basis

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$\underset{\sim}{e}_a$

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$$\underset{\sim}{e}_a \cdot \underset{\sim}{e}_b = \eta_{ab}$$

orthonormal basis

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dual basis for this choice

e^b

orthonormal basis

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converting back and forth.

$$\underset{\sim}{e}_a = (e_a)^\alpha \underset{\sim}{e}_\alpha$$

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$$\boxed{V^\alpha = V^a (e_a)^\alpha}$$

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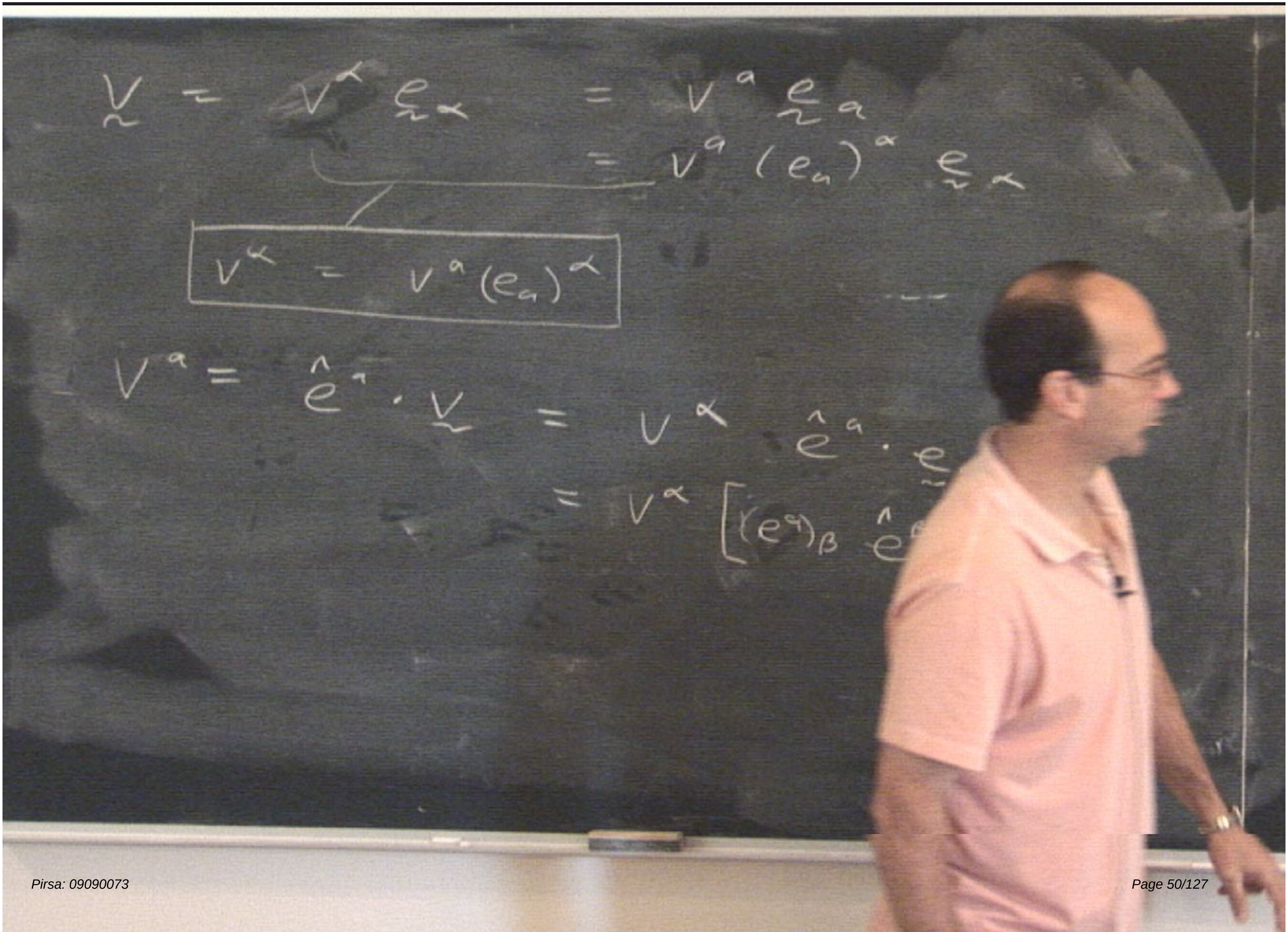
$$V^a = \hat{e}^a \cdot \underset{\sim}{V} = V^\alpha \, \hat{e}^a \cdot \underset{\sim}{e}_\alpha$$



$\underset{\sim}{V} = V^{\alpha} \underset{\sim}{e}_{\alpha} = V^{a} \underset{\sim}{e}_{a}$
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$\boxed{V^{\alpha} = V^{a} (e_{a})^{\alpha}}$
$V^{a} = \hat{e}^{a} \cdot \underset{\sim}{V} = V^{\alpha} \hat{e}^{a} \cdot \underset{\sim}{e}$
$= V^{\alpha} [(e^{a})_{\beta} \hat{e}$

$$\underset{\sim}{V} = V^{\alpha}\,\underset{\sim}{e}_{\alpha} = V^{a}\,\underset{\sim}{e}_{a}$$

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$$\boxed{V^{\alpha} = V^{a}(e_{a})^{\alpha}}$$

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$$= V^{\alpha}\left[(e^{a})_{\beta}\,\hat{e}^{\beta}\right]\cdot\underset{\sim}{e}_{\alpha}$$

$$\underset{\sim}{V} = V^\alpha \underset{\sim}{e}_\alpha = V^a \underset{\sim}{e}_a = V^a (e_a)^\alpha \underset{\sim}{e}_\alpha$$

$$\boxed{V^\alpha = V^a (e_a)^\alpha}$$

$$V^a = \hat{e}^a \cdot \underset{\sim}{V} = V^\alpha \hat{e}^a \cdot \underset{\sim}{e}_\alpha = V^\alpha \left[(e^a)_\beta \underbrace{\hat{e}^\beta \right] \cdot \underset{\sim}{e}_\alpha}_{\delta^\beta{}_\alpha} = (e^a)_\beta V^\beta$$

$$\underset{\sim}{V} = V^{\alpha} \underset{\sim}{e}_{\alpha} = V^{a} \underset{\sim}{e}_{a}$$

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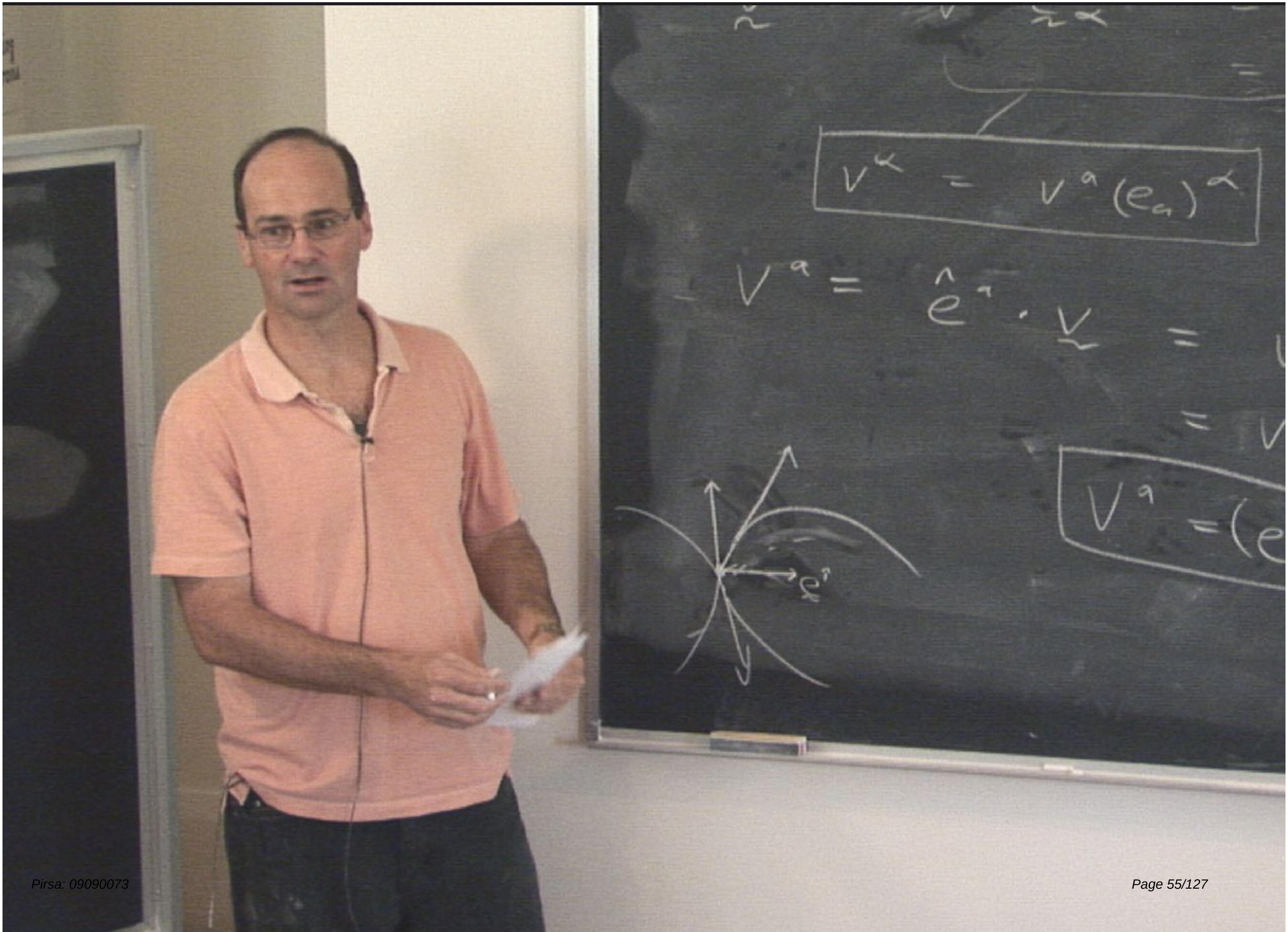
$$V^a = \hat{e}^a \cdot \underset{\sim}{V} = V^\alpha \hat{e}^a \cdot \underset{\sim}{e}_\alpha$$

e^a

e_b

$$= V^\alpha \underbrace{\left[(e^a)_\beta \hat{e}^\beta\right] \cdot \underset{\sim}{e}_\alpha}_{\delta^\beta{}_\alpha}$$

$$\boxed{V^a = (e^a)_\beta V^\beta}$$

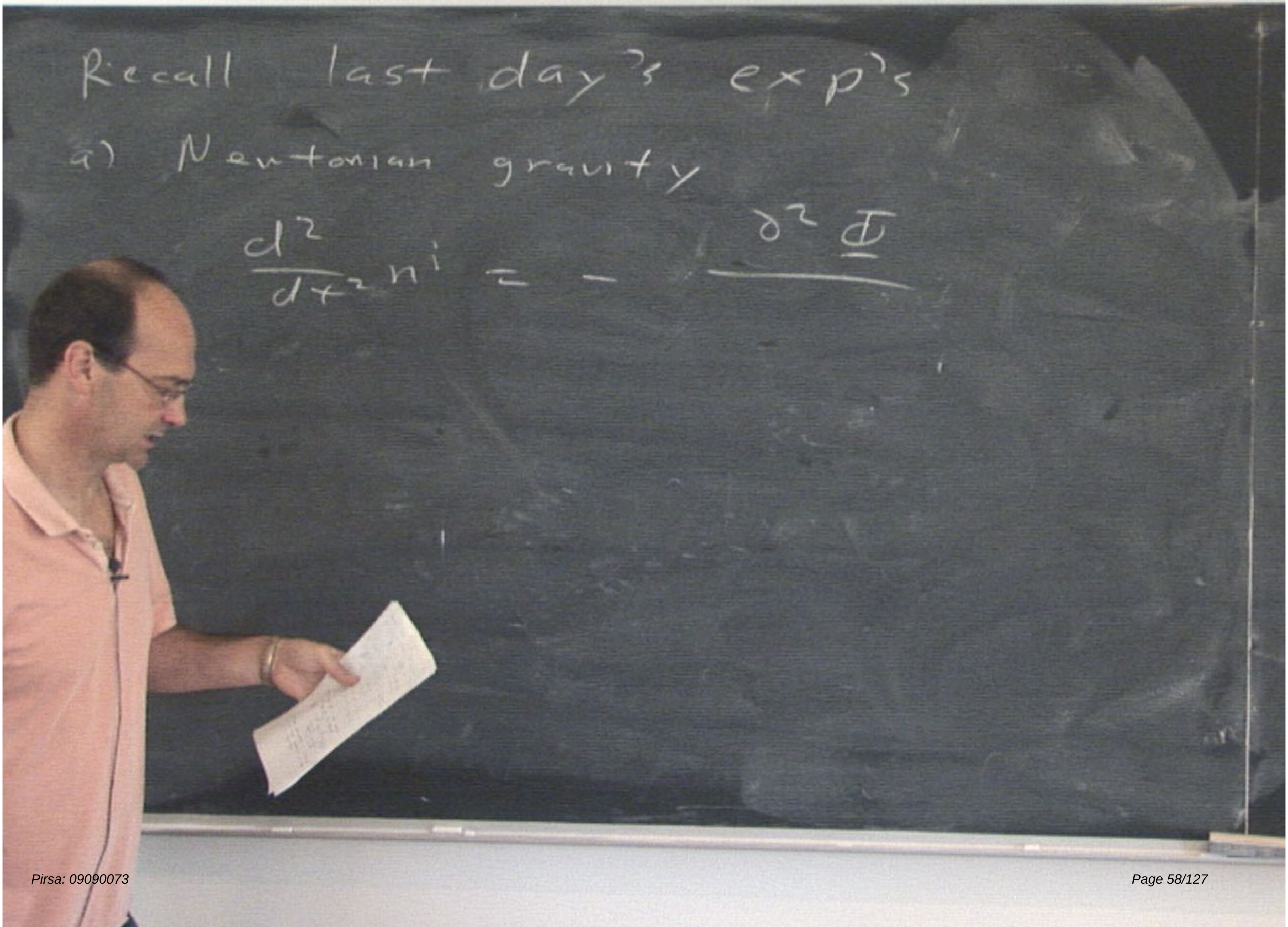

$V^\alpha = V^a (e_a)^\alpha$
$V^a = \hat{e}^a \cdot \underset{\sim}{V} =$
$V^a = (e$

Recall last day's exp's

Recall last day's exp's

a) Newtonian gravity

$$\frac{d^2}{dt^2} n^i$$


Recall last day's exp's
a) Newtonian gravity
$\frac{d^2}{dt^2} n^i = - \frac{\partial^2 \Phi}{}$

Recall last day's exp's

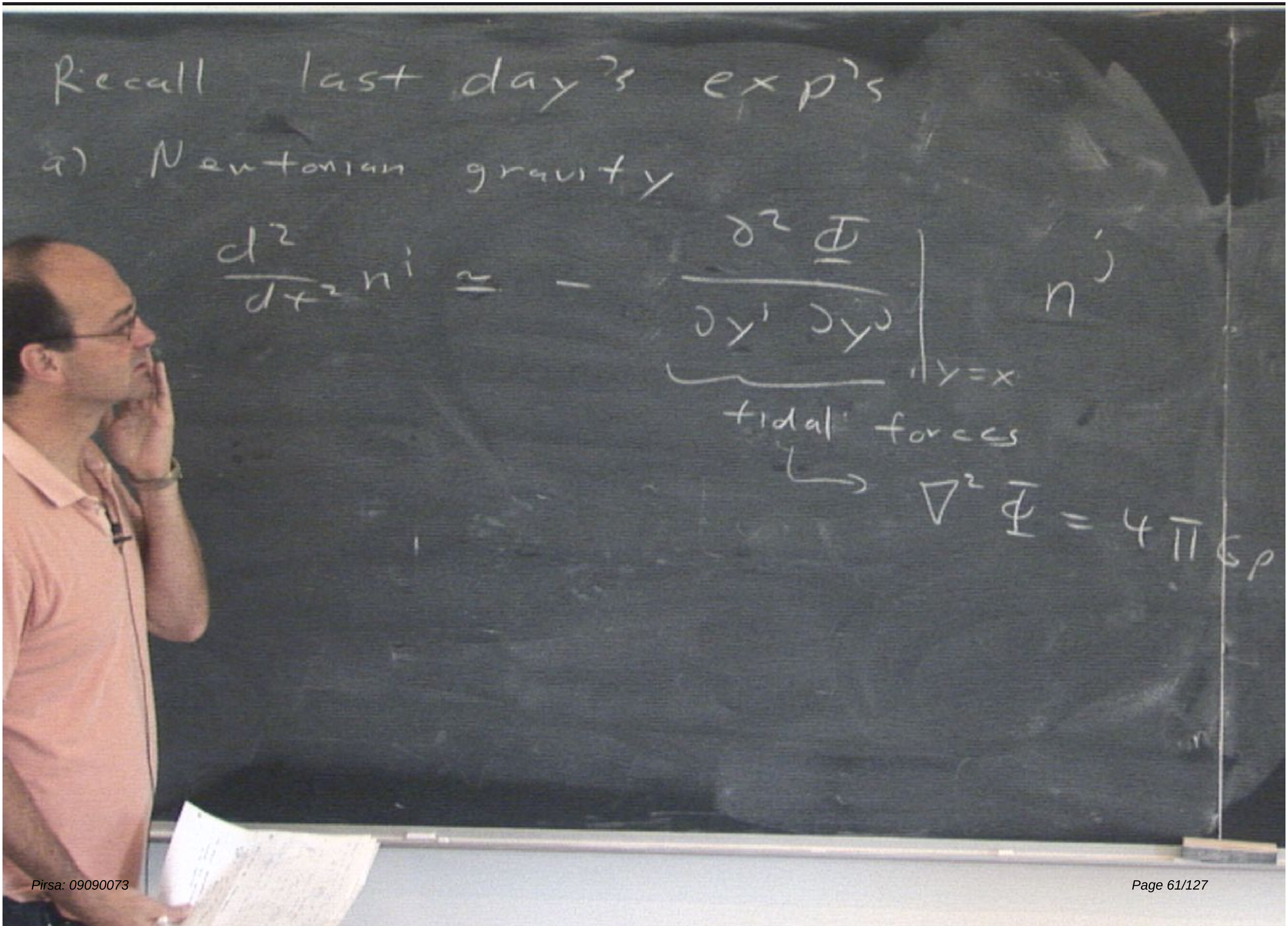
a) Newtonian gravity

$$\frac{d^2}{dt^2} n^i \simeq - \left. \frac{\partial^2 \Phi}{\partial y^i \, \partial y^j} \right|_{y=x} n^j$$

Recall last day's exp's

a) Newtonian gravity

$$\frac{d^2}{dt^2} n^i \simeq - \underbrace{\left.\frac{\partial^2 \Phi}{\partial y^i \partial y^j}\right|_{y=x}}_{\text{tidal forces}} n^j$$


Recall last day's exp's
a) Newtonian gravity
$\frac{d^2}{dt^2} n^i \simeq - \underbrace{\frac{\partial^2 \Phi}{\partial y^i \partial y^j}}_{\text{tidal forces}} \Big|_{y=x} n^j$
$\nabla^2 \Phi = 4 \pi G \rho$

Recall last day's exp's

a) Newtonian gravity

$$\frac{d^2}{dt^2} n^i \simeq - \underbrace{\left.\frac{\partial^2 \Phi}{\partial y^i \partial y^j}\right|_{y=x}}_{\text{tidal forces}} n^j$$

$\hookrightarrow \nabla^2 \Phi = 4\pi G \rho$

b)

equation of geodesic deviation

Recall last day's exp's

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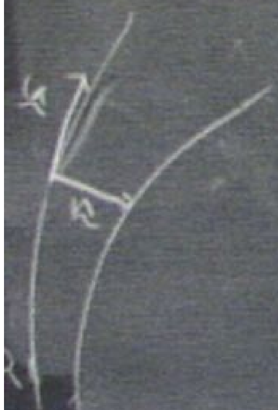
b) equation of geodesic deviation



call last day's exp's

Newtonian gravity

$$\frac{d^2}{dt^2} n^i \simeq - \underbrace{\frac{\partial^2 \Phi}{\partial y^i \, \partial y^j}\Bigg|_{y=x}}_{\text{tidal forces}} n^j$$

$$\nabla^2 \Phi = 4 \pi G \rho$$

equation of geodesic deviation

$$\frac{d^2 n_a}{d\tau^2} = - \underbrace{R^{\beta}{}_{\gamma \alpha \delta}}\, u^{\gamma} u^{\delta} (e_a)_{\beta} \, n^{\alpha}$$

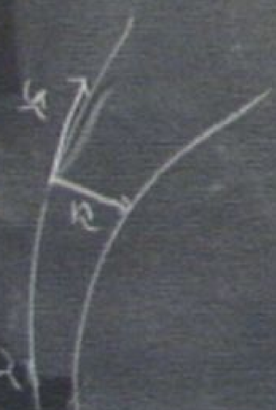
Recall last day's exp's

a) Newtonian gravity

$$\frac{d^2}{dt^2} n^i \simeq - \underbrace{\left.\frac{\partial^2 \Phi}{\partial y^i \, \partial y^j}\right|_{y=x}}_{\text{tidal forces}} n^j$$

$\longrightarrow \nabla^2 \Phi = 4$

b) equation of geodesic deviation



$$\frac{d^2 n_\alpha}{d\tau^2} = - \underbrace{R^{\beta}{}_{\gamma\alpha\delta}}_{\text{Riemann curvature}} u^\gamma u^\delta \; (e$$

exp's

$$\underbrace{\frac{\partial^2 \Phi}{\partial y^i \partial y^j}\bigg|_{y=x}}_{\text{tidal forces}} n^j$$

tidal forces $\hookrightarrow \nabla^2 \Phi = 4\pi G\rho$

geodesic deviation

$$\underbrace{R^{\beta}{}_{\gamma\alpha\delta}}_{\text{Riemann curvature}} u^{\gamma} u^{\delta} (e^a)_{\beta}\, n^{\alpha}$$

Riemann curvature

orthonormal basis

$\underset{\sim}{e}_a \quad \underset{\sim}{e}_a \cdot \underset{\sim}{e}_b$

dual basis for the

$\Longrightarrow \partial^b \qquad \hat{e}^a(\underset{\sim}{e}_b)$

$\underset{\sim}{V} = V^a \underset{\sim}{e}_a \longrightarrow$

$\hat{S} = S_a \hat{e}^b$

$\hookrightarrow S_a = \hat{S} \cdot \underset{\sim}{e}_a$

converting back and f

$\underset{\sim}{e}_a = (e_a)^{\alpha} \underset{\sim}{e}_{\alpha} \qquad \hat{e}^a$

ecall last day's exp's

Newtonian gravity

$$\frac{d^2}{dt^2} n^i \simeq - \underbrace{\frac{\partial^2 \Phi}{\partial y^i \, \partial y^j}\Bigg|_{y=x}}_{\text{tidal forces}} n^j$$

$\hookrightarrow \nabla^2 \Phi = 4\pi G \rho$

equation of geodesic deviation

$$\frac{d^2 n^a}{d\tau^2} = - \underbrace{R^{\beta}{}_{\gamma\alpha\delta}}_{\text{Riemann curvature}} u^\gamma u^\delta (e^a)_\beta \, n^\alpha$$

would like to apply $n^{\alpha} = (c_b)^{\alpha} n^b$

would like to apply $n^\alpha = (e_b)^\alpha n^b$

$$\Rightarrow \quad \frac{d^2 n^a}{d\tau^2} = - K^a{}_b \, n^b$$

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$$K^a{}_b = R$$

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$$K^a{}_b = R^\beta{}_{\gamma\alpha\delta} \, u^\gamma u^\delta (e^a)_\beta (e_b)^\alpha$$

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following Newtonian result, propose what enters Einstein eq is $K^a{}_a$

proposed eq in vaccuum is $\boxed{K^a{}_a = 0}$

$$K^{\alpha}{}_{\alpha} = 0 = R^{\alpha}{}_{\gamma\alpha\delta}\, u^{\gamma} u^{\delta}$$

convert invariant contraction back to coord. indices

$$= R^{\alpha}{}_{\gamma\alpha\delta}\, u^{\gamma} u^{\delta}$$

$$= \underbrace{R_{\gamma\delta}}_{\text{Ricci tensor}}\, u^{\gamma} u^{\delta}$$

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want ~~this~~ to apply independent of reference geodesic (ie u^{α}) so natural eq is

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want this to apply independent of reference geodesic (ie u^{α}) so natural eq is

$$\boxed{R_{\alpha\beta} = 0}$$

can verify that:

$$R_{\alpha\beta} = g^{\lambda\gamma} R_{\lambda\alpha\gamma\beta}$$

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symmetric

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$$= g^{\lambda\gamma} R_{\gamma\beta\lambda\alpha}$$

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$$= g^{\lambda\gamma} R_{\lambda\beta\gamma\alpha} = R_{\beta\alpha}$$

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$\longrightarrow$ $R_{\alpha\beta} = 0 \longrightarrow$ gives use a symmetric tensor's worth of eq's to solve for $g_{\alpha\beta}$

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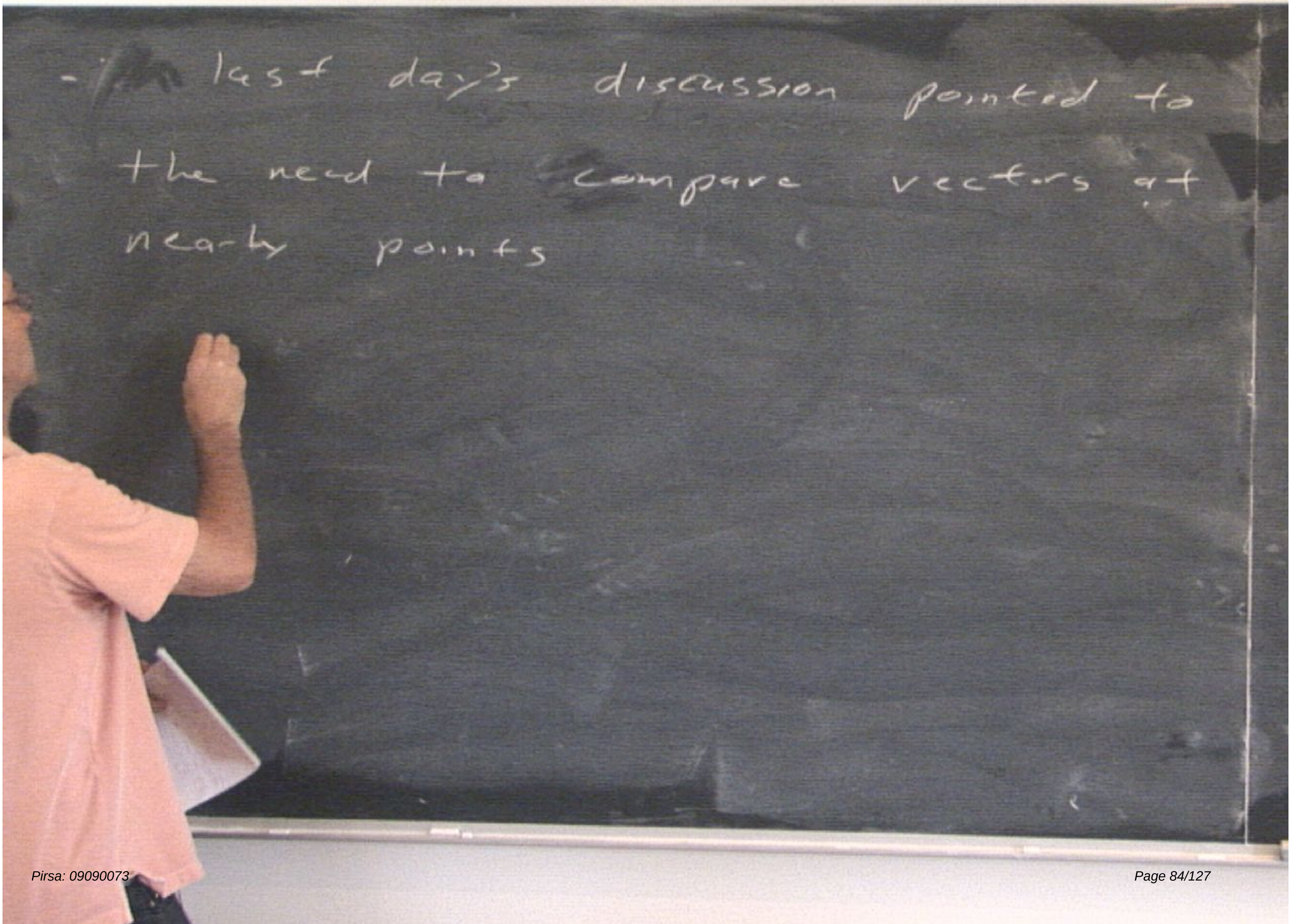
$$= g^{\lambda\gamma} R_{\lambda\beta\gamma\alpha} =$$

$\nabla^2 \Phi = 0$

reference $\longrightarrow$ $R_{\alpha\beta} = 0 \longrightarrow$ gives use a sy…

tensor's worth

to solve for ga…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$V^\alpha(x)$; $V^\alpha(x+\delta x) = V^\alpha(x) + \delta x^\beta \dfrac{\partial V^\alpha}{\partial x^\beta} \cdots$

<u>but</u> $\dfrac{\partial V^\alpha}{\partial x^\beta}$ is <u>not</u> a useful tool since

- last day's discussion pointed to the need to compare vectors at near-by points

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but $\dfrac{\partial V^\alpha}{\partial x^\beta}$ is not a useful tool since it doesn't transform nicely under

$(t, x\,y, z)$

that

$\underset{\sim}{V} = (0, 1, 0, 0)$

rd

anc

(t, x, y, z) $\longrightarrow$ (t, r, θ, ϕ)

$\underset{\sim}{V} = (0, 1, 0, 0)$ $\longrightarrow$ $\left(0, \sin\theta\cos\phi, \frac{-\cos\theta\cos\phi}{r}, -\frac{\sin\phi}{r\sin\theta}\right)$

rd

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(t, x, y, z) $\longrightarrow$ (t, r, θ, ϕ)

$\underset{\sim}{V} = (0, 1, 0, 0)$

$V^\mu = \left(0,\ \sin\theta\cos\phi,\ -\frac{\cos\theta\cos\phi}{r},\ -\frac{\sin\phi}{r\sin\theta}\right)$

$\frac{\partial V^\mu}{\partial x^\mu} = 0$

rd

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(t, x, y, z) $\longrightarrow$ (t, r, θ, ϕ)

$$\underset{\sim}{V} = (0, 1, 0, 0) \qquad V = \left(0, \sin\theta\cos\phi, \frac{-\cos\theta\cos\phi}{r}, -\frac{\sin\phi}{r\sin\theta}\right)$$

$$\frac{\partial V^\mu}{\partial x^\nu} = 0 \longrightarrow \frac{\partial V^\alpha}{\partial x^\beta} \neq 0$$

(t, x, y, z) $\longrightarrow$ (t, r, θ, ϕ)

$\underset{\sim}{V} = (0, 1, 0, 0)$ $\quad V^\mu = \left(0, \sin\theta\cos\phi, -\frac{\cos\theta\cos\phi}{r}, -\frac{\sin\phi}{r\sin\theta}\right)$

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rd
nc

(t, x, y, z) $\longrightarrow$ (t, r, θ, ϕ)

$\underset{\sim}{V} = (0, 1, 0, 0)$ $\longrightarrow$ $V^{\alpha} = \left(0,\ \sin\theta\cos\phi,\ -\frac{\cos\theta\cos\phi}{r},\ -\frac{\sin\phi}{r\sin\theta}\right)$

$\frac{\partial V^{\alpha}}{\partial x^{\alpha}} = 0$ $\longrightarrow$ $\frac{\partial V^{\alpha}}{\partial x^{\beta}} \neq 0$

$\longrightarrow$ reflects the fact that when we move around the new basis vectors are changing

rd

ne

$(t, x, y, z) \longrightarrow (t, r, \theta, \phi)$

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board

tranc

$(t, x, y, z) \longrightarrow (t, r, \theta, \phi)$

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$\frac{\partial V^{\nu}}{\partial x^{\mu}} = 0 \longrightarrow \frac{\partial V^{\alpha}}{\partial x^{\beta}} \neq 0$

$\longrightarrow$ reflects the fact that when we move around the new basis vectors are changing

would like a new derivative operator with tensor tranc. properties

ord
tranc

$$\nabla_a V^b \longrightarrow \nabla_{\hat a} V^{\hat b} = \frac{\partial x^\gamma}{\partial y^{\hat a}} \nabla_\gamma V^\delta \frac{\partial y^{\hat b}}{\partial x^\delta}$$

$$\nabla_\alpha V^\beta \longrightarrow \nabla_{\hat\alpha} V^{\hat\beta} = \frac{\partial x^\gamma}{\partial y^{\hat\alpha}} \nabla_\gamma V^\delta \frac{\partial y^{\hat\beta}}{\partial x^\delta}$$

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$\longrightarrow$ would like $\nabla_\alpha V^\beta = \partial_\alpha V^\beta$ in

flat space with Cartesian coord's

$$\nabla_\alpha V^\beta \longrightarrow \nabla_{\hat\alpha} V^{\hat\beta} = \frac{\partial x^\gamma}{\partial y^{\hat\alpha}} \nabla_\gamma V^\delta \frac{\partial y^{\hat\beta}}{\partial x^\delta}$$

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$\Longrightarrow$ recall discussion of geodesics

$\frac{d^2 x^\alpha}{d\lambda^2} = 0 \leftarrow$ in flat space, doesn't naively extend to curved space

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produced a 4 vector as $\frac{d^2x^\alpha}{d\lambda^2} + \Gamma^\alpha$

$$\nabla_\alpha V^\beta \longrightarrow \nabla_{\hat{\alpha}} V^{\hat{\beta}} = \frac{\partial x^\gamma}{\partial y^{\hat{\alpha}}} \nabla_\gamma V^\delta \frac{\partial y^{\hat{\beta}}}{\partial x^\delta}$$

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$\frac{d^2 x^\alpha}{d\lambda^2} = 0 \leftarrow$ in flat space, doesn't naively extend to curved space

produced a 4 vector as $\frac{d^2 x^\alpha}{d\lambda^2} + \Gamma^\alpha_{\beta\mu} \frac{dx^\beta}{d\lambda} \frac{dx^\mu}{d\lambda} = 0$

coord trans

$$\nabla_\alpha V^\beta \longrightarrow \nabla_{\hat\alpha} V^{\hat\beta} = \frac{\partial x^\gamma}{\partial y^{\hat\alpha}} \nabla_\gamma V^\delta \frac{\partial y^{\hat\beta}}{\partial x^\delta}$$

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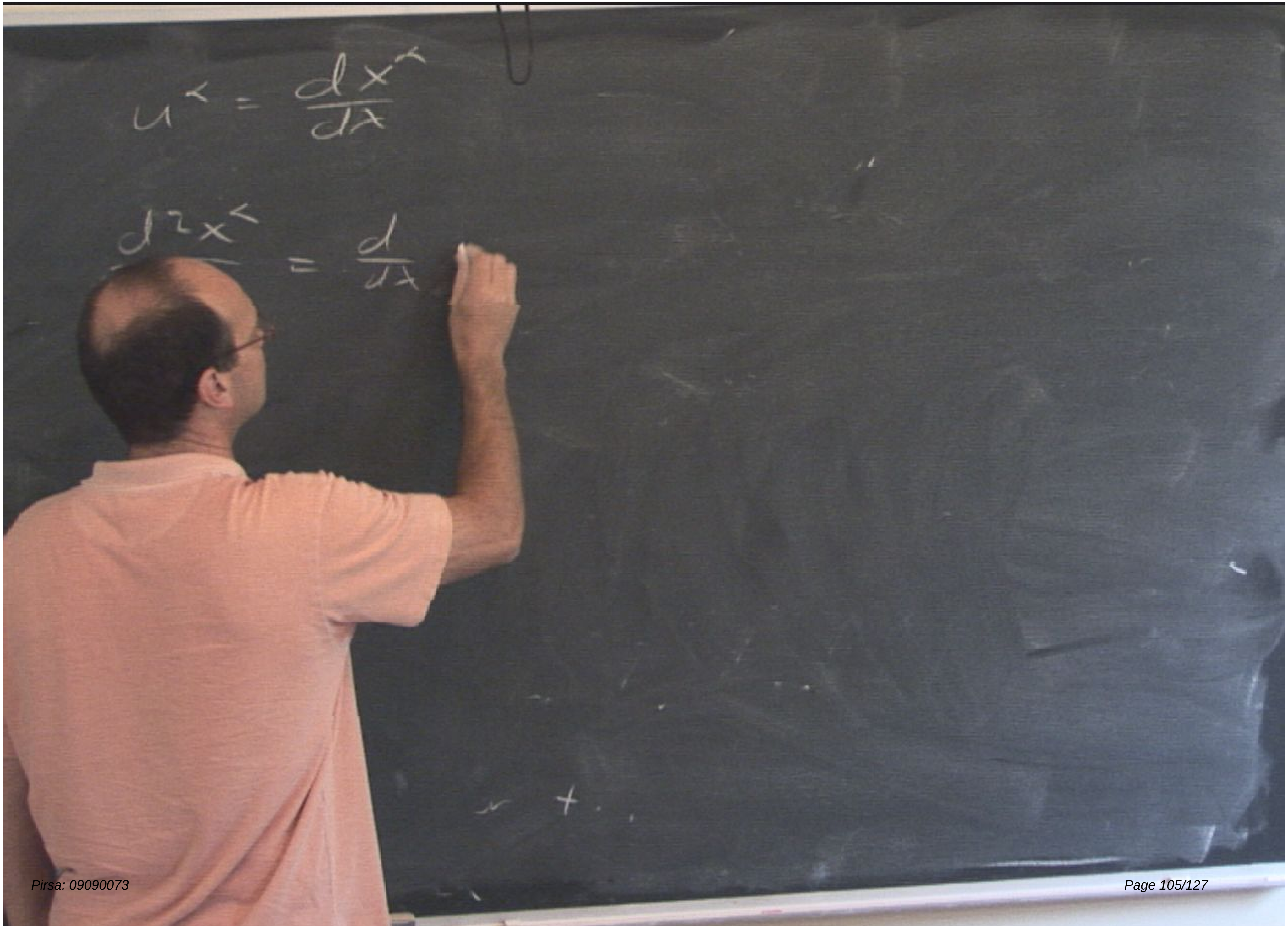
produced a 4 vector as $\frac{d^2x^\alpha}{d\lambda^2} + \Gamma^\alpha_{\beta\mu} \frac{dx^\beta}{d\lambda}\frac{dx^\mu}{d\lambda} = a^\alpha$

$V =$

$\frac{\partial V}{\partial x}$


$u^\alpha = \frac{dx^\alpha}{d\lambda}$
$\frac{d^2x^\alpha}{\quad} = \frac{d}{d\lambda}$

$$u^\alpha = \frac{dx^\alpha}{d\lambda}$$

$$\frac{d^2 x^\alpha}{d\lambda} = \frac{d}{d\lambda}\left(u^\alpha\right)$$

W

$+ \ldots$

$$u^\alpha = \frac{dx^\alpha}{d\lambda}$$

$$\frac{d^2x^\alpha}{d\lambda} = \frac{d}{d\lambda}(u^\alpha) = \frac{\partial x^\alpha}{\partial \lambda}\frac{\partial u^\alpha}{\partial x^\alpha}$$

$$u^\alpha = \frac{dx^\alpha}{d\lambda}$$

$$\frac{d^2x^\alpha}{d\lambda^2} = \frac{d}{d\lambda}\left(u^\alpha\right) = \frac{\partial x^\beta}{\partial \lambda}\frac{\partial u^\alpha}{\partial x^\beta}$$

$$= u^\beta\, \partial_\beta u^\alpha$$

$$u^\alpha = \frac{dx^\alpha}{d\lambda}$$

$$\frac{d^2x^\alpha}{d\lambda} = \frac{d}{d\lambda}(u^\alpha) = \frac{\partial x^\beta}{\partial \lambda}\frac{\partial u^\alpha}{\partial x^\beta}$$

$$= u^\beta \partial_\beta u^\alpha$$

$$a^\alpha = u^\beta \partial_\beta u^\alpha + \Gamma^\alpha{}_{\beta\mu} u^\beta u^\mu$$

$$= u^\beta \left[\partial_\beta u^\alpha + \Gamma^\alpha{}_{\beta\mu} u^\mu\right]$$

w

a^α

$$u^\alpha = \frac{dx^\alpha}{d\lambda}$$

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w

λ + ...

$= a^\alpha$

$$u^\alpha = \frac{dx^\alpha}{d\lambda}$$

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$$= \underbrace{u^\beta} \underbrace{\left[\partial_\beta u^\alpha + \Gamma^\alpha{}_{\beta\mu} u^\mu\right]}$$

w... must be a tensor under coordinate trans.s

$$\nabla_\alpha u^\beta = \partial_\alpha u^\beta + \underline{\Gamma^\beta{}_{\alpha\mu} u^\mu}$$

$$\nabla_\alpha u^\beta = \partial_\alpha u^\beta + \underline{\Gamma^\beta{}_{\alpha\mu} u^\mu}$$

↳ tells us about variation of coord basis

$$\nabla_\alpha u^\beta = \partial_\alpha u^\beta + \underbrace{\Gamma^\beta{}_{\alpha\mu}}\, u^\mu$$

↳ tells us about variation of coord basis

$$\nabla_\alpha \underline{u}^\beta = \partial_\alpha u^\beta + \underline{\Gamma^\beta{}_{\alpha\mu} u^\mu}$$

↳ tells us about variation of coord basis

we would like to extend this

$$\nabla_\alpha \underline{u}^\beta = \partial_\alpha u^\beta + \underline{\Gamma^\beta{}_{\alpha\mu} u^\mu}$$

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we would like to extend this to general tensors:

$$\nabla_\alpha \underline{u}^\beta = \partial_\alpha u^\beta + \underline{\Gamma^\beta{}_{\alpha\mu} u^\mu}$$

↳ tells us about variation of coord basis

we would like to extend this to general tensors:

$$\nabla_\alpha \phi$$

$$\nabla_\alpha \underline{u}^\beta = \partial_\alpha u^\beta + \underline{\Gamma^\beta{}_{\alpha\mu} u^\mu}$$

↳ tells us about variation of coord basis

we would like to extend this to general tensors:

$$\nabla_\alpha \phi = \partial_\alpha \phi$$

$$\nabla_\alpha \underline{u}^\beta = \partial_\alpha u^\beta + \underline{\Gamma^\beta{}_{\alpha\mu} u^\mu}$$

↳ tells us about variation of coord basis

we would like to extend this to general tensors:

$$\nabla_\alpha \phi = \partial_\alpha \phi \leftarrow \text{scalar function.}$$

$$\nabla_\alpha \underline{u}^\beta = \partial_\alpha u^\beta + \underbrace{\Gamma^\beta{}_{\alpha\mu} u^\mu}_{}$$

↳ tells us about variation of coord basis

we would like to extend this to general tensors:

$$\nabla_\alpha \phi = \partial_\alpha \phi \leftarrow \text{scalar function}$$

covariant vector: $\nabla_\beta V_\alpha = ?$

$$\nabla_\alpha \underline{u}^\beta = \partial_\alpha u^\beta + \underline{\Gamma^\beta{}_{\alpha\mu} u^\mu}$$

↳ tells us about variatoon of coord basis

we world like to extend this to general tensors:

$$\nabla_\alpha \phi = \partial_\alpha \phi$$ ← scalar function

covariant vector: $\nabla_\beta V_\alpha = ?$

$$\nabla_\beta (t^\alpha V_\alpha)$$

$$\nabla_\alpha \underline{u}^\beta = \partial_\alpha u^\beta + \underline{\Gamma^\beta{}_{\alpha\mu}} u^\mu$$

↳ tells us about variation of coord basis's

would like to extend this general tensors:

$$\nabla_\alpha \phi = \partial_\alpha \phi \leftarrow \text{scalar function}$$

nt vector: $\nabla_\beta V_\alpha = ?$

$$\nabla_\beta (t^\alpha V_\alpha) = \partial_\beta (t^\alpha V_\alpha) = (\partial_\beta t^\alpha) V_\alpha + t^\alpha \partial_\beta V_\alpha$$

u

$\frac{d^2}{d\lambda}$

$\underline{a}^\alpha$

w

$$= (\nabla_\beta t^\alpha) V_\alpha + t^\alpha (\nabla_\beta V_\alpha)$$

$$t^\alpha \partial_\beta V_\alpha$$

$$= (\nabla_\beta t^\alpha) V_\alpha + t^\alpha (\nabla_\beta V_\alpha)$$

also demand
Leibniz rule for ∇_α

$t^\alpha \partial_\beta V_\alpha$

$$= (\nabla_\beta t^\alpha) V_\alpha + t^\alpha (\nabla_\beta V_\alpha)$$

also demand
Leibniz rule for ∇_α

$$= (\partial_\beta t^\alpha + \Gamma^\alpha{}_{\beta\mu} t^\mu) V_\alpha + t^\alpha (\nabla_\beta V_\alpha)$$

$t^\alpha \partial_\beta V_\alpha$

$$= (\nabla_\beta t^\alpha) V_\alpha + t^\alpha (\nabla_\beta V_\alpha)$$

also demand

Leibniz rule for ∇_α

$$= (\partial_\beta t^\alpha + \Gamma^\alpha{}_{\beta\mu} t^\mu) V_\alpha + t^\alpha (\nabla_\beta V_\alpha)$$

$$0 = -(\partial_\beta V_\alpha - \Gamma^\gamma{}_{\beta\alpha} V_\gamma) t^\alpha + t^\alpha (\nabla_\beta V_\alpha)$$

$t^\alpha \partial_\beta V_\alpha$

$$= (\nabla_\beta t^\alpha) V_\alpha + t^\alpha (\nabla_\beta V_\alpha)$$

also demand
Leibniz rule for ∇_α

$$= (\partial_\beta t^\alpha + \Gamma^\alpha{}_{\beta\mu} t^\mu) V_\alpha + t^\alpha (\nabla_\beta V_\alpha)$$

$$0 = -(\partial_\beta V_\alpha - \Gamma^\gamma{}_{\beta\alpha} V_\gamma) t^\alpha + t^\alpha (\nabla_\beta V_\alpha)$$

$$\longrightarrow \boxed{\nabla_\beta V_\alpha = \partial_\beta V_\alpha - \Gamma^\gamma{}_{\beta\alpha} V_\gamma}$$

$t^\alpha \partial_\beta V_\alpha$