Title: Relativity - Core (PHYS 604) - Lecture 11

Date: Sep 17, 2009 10:30 AM

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Abstract:

# Gravitational Collapse + Black Holes

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Gravitational Collapse + Black Holes

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$M_{fin} \gtrsim 2\, M_{sun} \longrightarrow$ no known physics can overcome gravitational forces

$\longrightarrow$

Gravitational Collapse + Black Holes

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$M_{fin} \gtrsim 2 M_{sun} \longrightarrow$ no known physics can overcome gravitational forces

$\longrightarrow$ object collapses

$\longrightarrow$ "black hole"

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$\rightarrow$ during collapse more and more of
Sch spacetime is revealed and
so we have to come to grips with

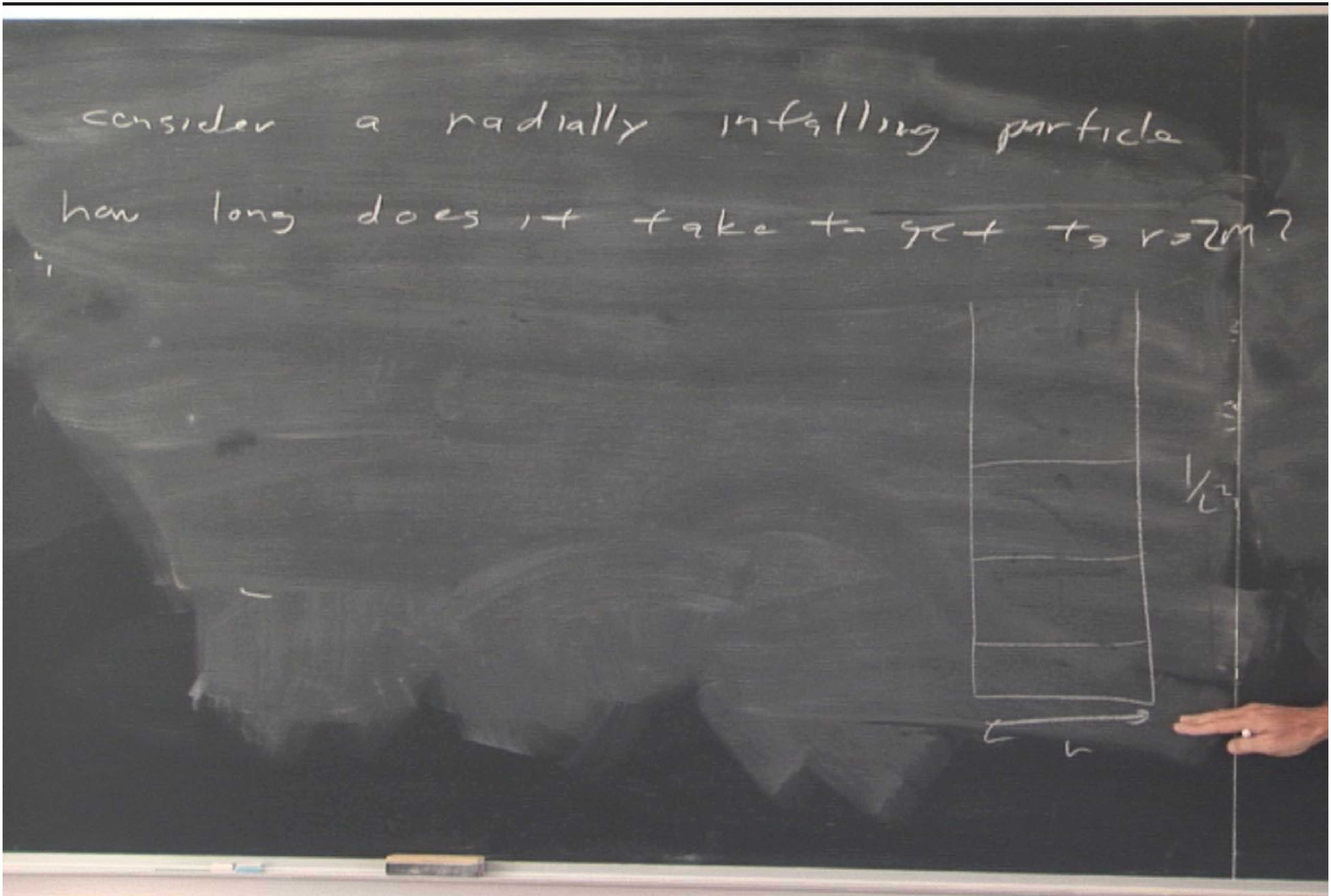
$$r = 2M \text{ and } r = 0$$

consider a radially infalling particle

how long does it take to get to $r=2M$?

consider a radially infalling particle

how long does it take to get to $r=2m$?


consider a radially infalling particle
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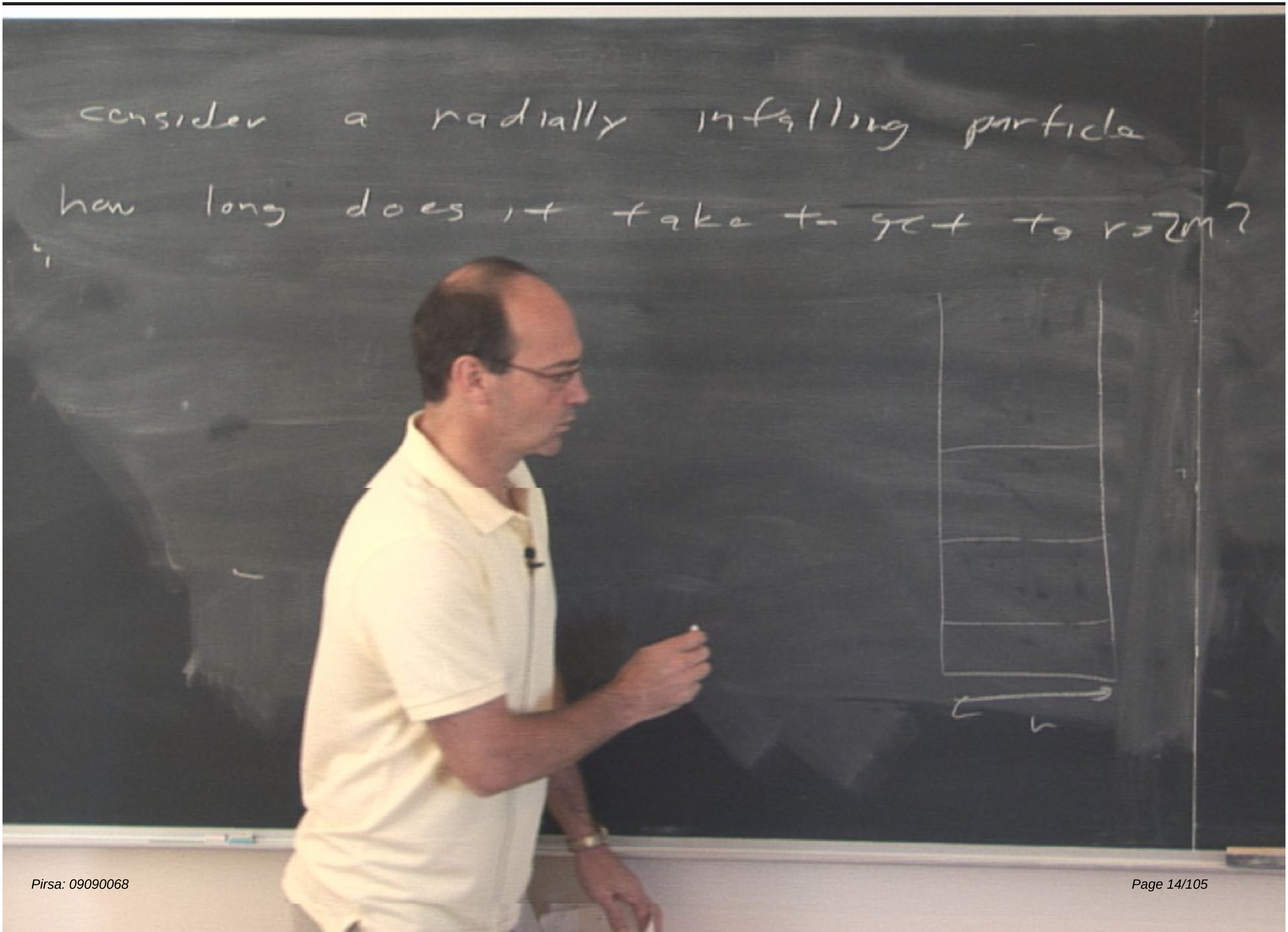
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how long does it take to get to $r=2M$

1.

$$\underset{\sim}{u}\cdot\underset{\sim}{u}=-1 \Rightarrow \frac{dr}{d\tau}=\left[e^2-\left(1-2M/r\right)\right]^{1/2}$$

$$\frac{dt}{d\tau}= e$$

consider a radially infalling particle

how long does it take to get to $r = 2M$

infalling

$$\underset{\sim}{u} \cdot \underset{\sim}{u} = -1 \Rightarrow \frac{dr}{d\tau} = -\left[ e^2 - \left(1 - \frac{2M}{r}\right) \right]^{1/2}$$

$$\frac{dt}{d\tau} = \frac{e}{1 - \frac{2M}{r}}$$

$$\Delta \tau = -\int_R^{r_{fin}} \frac{dr}{\left[ e^2 - 1 + \frac{2M}{r} \right]^{1/2}}$$

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$$\Delta\tau = -\int_R^{r_{fin}} \frac{dr}{\left[e^2 - 1 + \frac{2M}{r}\right]^{1/2}}$$

$\longrightarrow$ finite answer even if $r_{fin} = 2M$

consider $e^2 = 1$

$$\delta\tau = \int_R^{r_{fin}} \frac{\sqrt{r}\,dr}{\sqrt{2m}}$$

we

Sch

consider $e^2 = 1$

$$\Delta\tau = \int_R^{r_{fin}} \frac{\sqrt{r}\,dr}{\sqrt{2M}}$$

$$\Delta\tau = \frac{2M}{3}\left[\left(\frac{R}{2M}\right)^{3/2} - \left(\frac{r_{fin}}{2M}\right)^{3/2}\right]$$

Sch

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$$\Delta\tau = \frac{2M}{3}\left[\left(\frac{R}{2M}\right)^{3/2} - \left(\frac{r_{fin}}{2M}\right)^{3/2}\right]$$

in fact see here we can consider

$r_{fin} < 2M \longrightarrow \Delta\tau$ remains finite

consider a radially infalling particle

how long does it take to get to $r=2m$?

what about coord time $t$?

$$d\tau^2 = -ds^2 \implies -\left(1-\frac{2m}{r}\right) + \frac{\left(\frac{dr}{dt}\right)^2}{1-2m/r^2} = -\left(\frac{d\tau}{dt}\right.$$

consider a radially infalling particle

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$$d\tau^2 = -ds^2 \Longrightarrow -\left(1-\frac{2m}{r}\right) + \frac{\left(dr/dt\right)^2}{1-2m/r^2} = -\left(\frac{d\tau}{dt}\right)^2$$

$$\frac{dt}{d\tau} = \frac{e}{1-\frac{2m}{r}}$$

$$= -\left(1-2m/r\right)^2$$

consider a radially infalling particle

how long does it take to get to $r=2m$?

what about coord time $t$?

$$\frac{d\tau}{dt} \times \left( \frac{dr}{d\tau} = -\left[e^2 - \left(1 - \frac{2m}{r}\right)\right]^{1/2} \right) \times \frac{\left(1 - \frac{2m}{r}\right)}{e}$$

consider a radially infalling particle

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$$\frac{d\tau}{dt}\times\left(\frac{dr}{d\tau} = -\left[e^2-\left(1-\frac{2M}{r}\right)\right]^{1/2}\right)\times\frac{\left(1-\frac{2M}{r}\right)}{e}$$

$$= -\left\{\left(1-\frac{2M}{r}\right)^2 - \frac{1}{e^2}\left(1-\frac{2M}{r}\right)^3\right\}^{1/2}$$

$$\Delta t = -\int_R^{r_{fin}} \frac{dr}{\left[1-\frac{1}{e^2}\left(1-\frac{2M}{r}\right)\right]^{1/2}\left(1-\frac{2M}{r}\right)}$$

consider a radially infalling particle

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$\frac{dr}{dt}$

$$= -\left\{ \left(1-\frac{2m}{r}\right)^2 - \frac{1}{e^2}\left(1-\frac{2m}{r}\right)^3 \right\}^{1/2}$$

$$\Delta t = -\int_R^{r_{fin}} \frac{dr}{\left[1 - \frac{1}{e^2}\left(1-\frac{2m}{r}\right)\right]^{1/2}\left(1-\frac{2m}{r}\right)} \longrightarrow \text{get log-div as approach}$$

consider $e^2 = 1$

$\Delta t / 2M$

$\frac{M}{r})$

- divergence

-ough $r = 2M$

consider $e^2 = 1$

$$\frac{\Delta t}{2M} = \text{constant} - \frac{2}{3}\left(\frac{r_{fin}}{2M}\right)^{3/2} - 2\left(\frac{r_{fin}}{2M}\right)^{1/2} + \log\left|\frac{\left(\frac{r_f}{2M}\right)^{1/2} + 1}{\left(\frac{r_f}{2M}\right)^{1/2} - 1}\right|$$

-divergence

rough $r = 2M$

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a curiosity? for $r_{fin} < 2M$, get finite answer $\longrightarrow$

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- divergence

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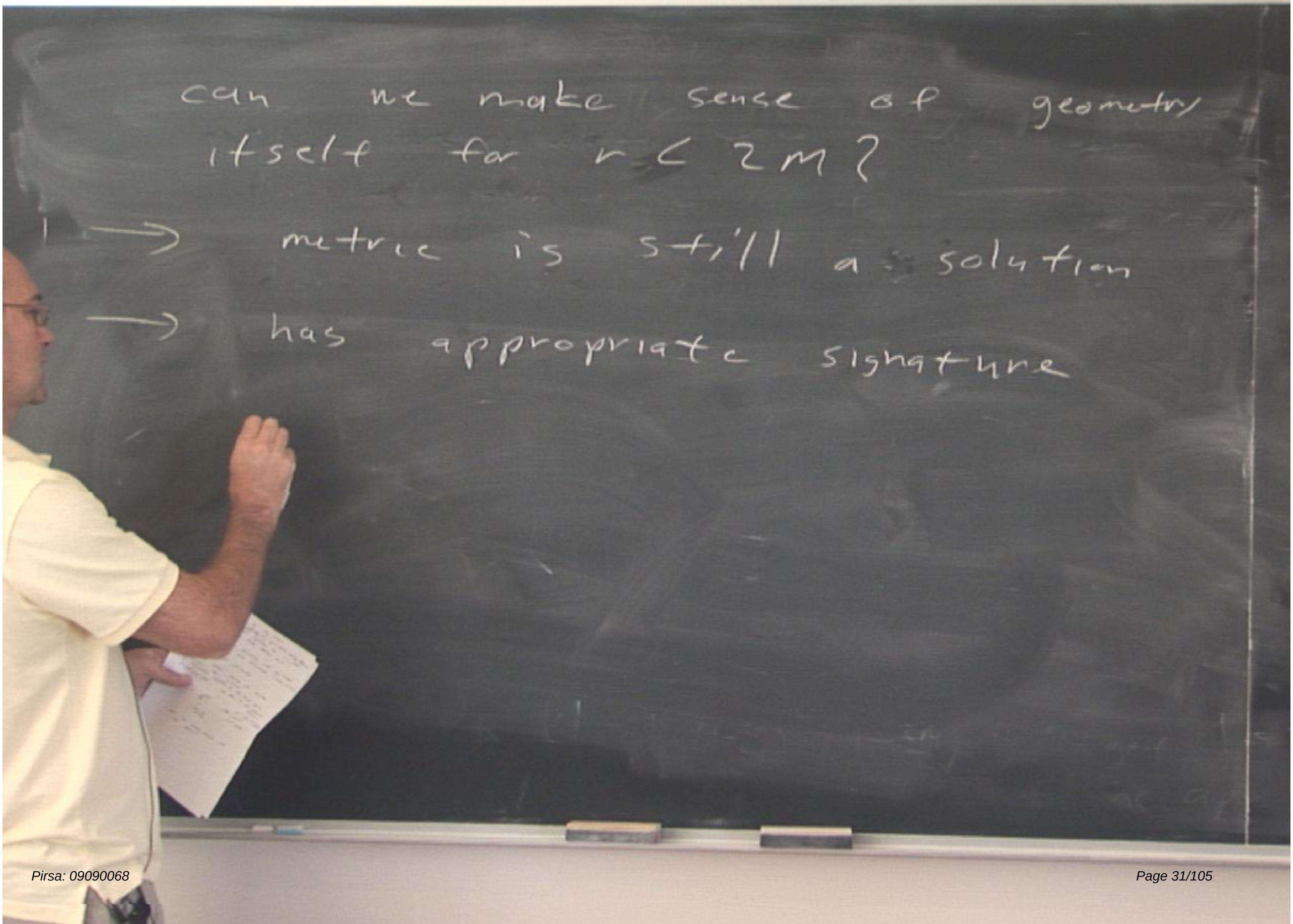
a curiosity? : for $r_{fin} < 2M$, get finite answer $\longrightarrow$ $r_f$ decreases past $2M$, $t$ is also decreasing

does it make sense?


can we make sense of geometry itself for r < 2m ?
→ metric is still a solution
→ has appropriate signature

can we make sense of geometry itself for $r < 2M$?

$\longrightarrow$ metric is still a solution

$\longrightarrow$ has appropriate signature

$0 < r < 2M$

$$\left.\begin{aligned} g_{tt} &> 0 \\ g_{rr} &< 0 \end{aligned}\right\} \longleftarrow$$

metric still has 3 positive eigenvalues and 1 negative eigenvalue

can we make sense of geometry itself for $r < 2M$?

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$$\frac{dr^2}{2M/r - 1} + \left(\frac{2M}{r} - 1\right) dt^2$$

$$+ r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

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$0 < r < 2M$

$g_{tt} > 0$

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timelike direction

spacelike direction

$$ds^2 = -\frac{dr^2}{\frac{2M}{r}-1} + \left(\frac{2M}{r}-1\right)dt^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

get intuition. What do light cones

look like in r - t plane

$ds^2 =$

get intuition: what do light cones look like in $r-t$ plane

$$ds^2 = 0 = -\left(1-\frac{2m}{r}\right)dt^2 + \frac{dr^2}{1-\frac{2m}{r}}$$

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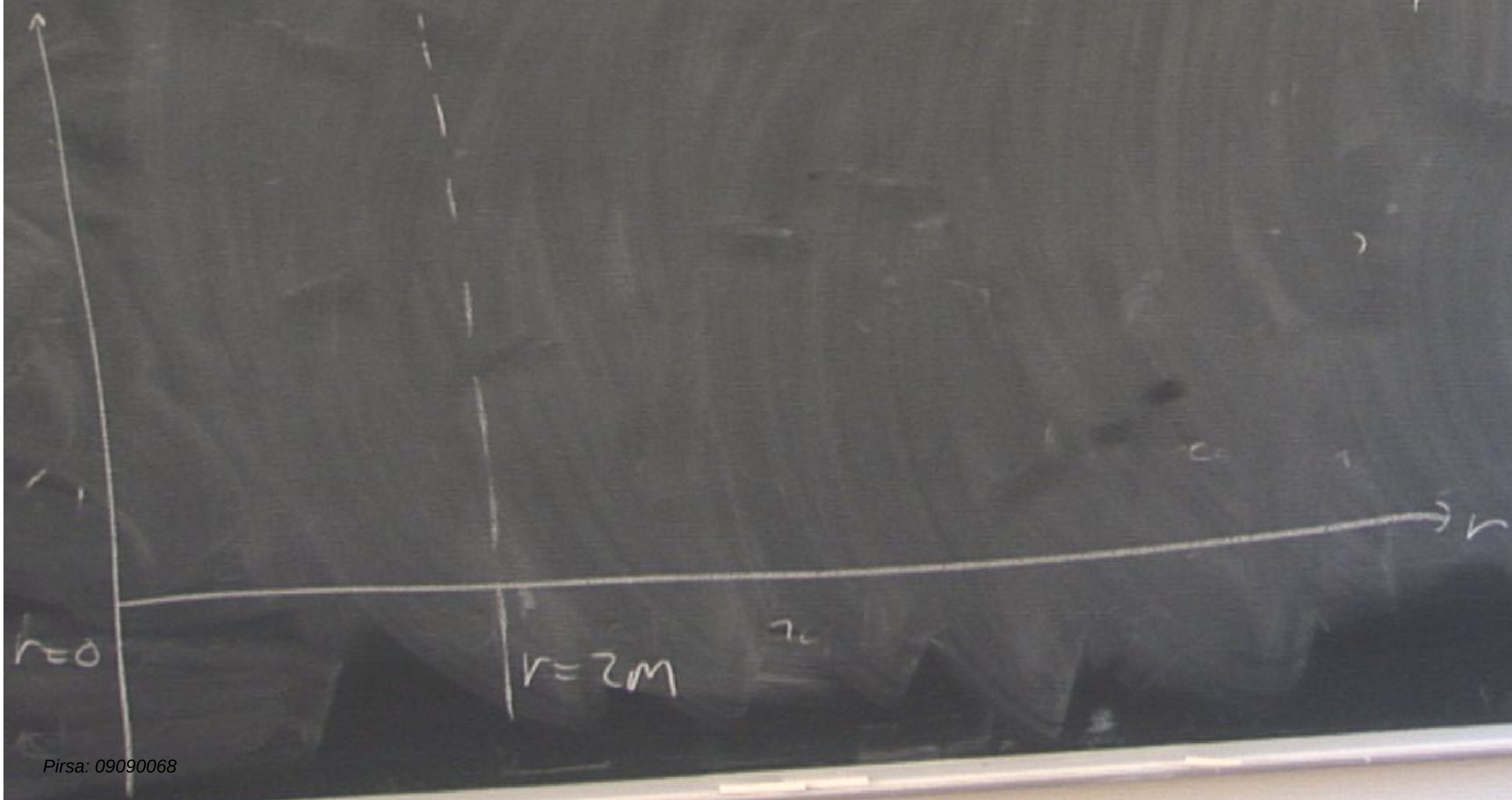

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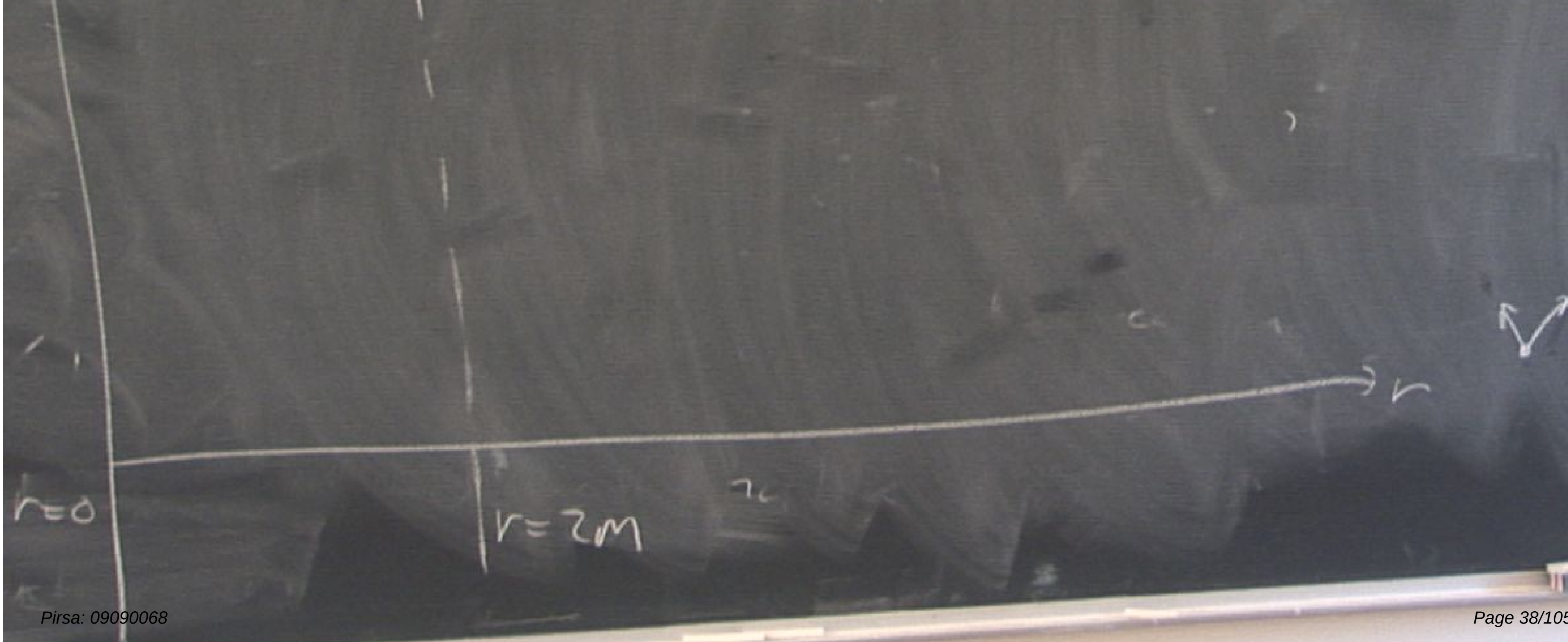

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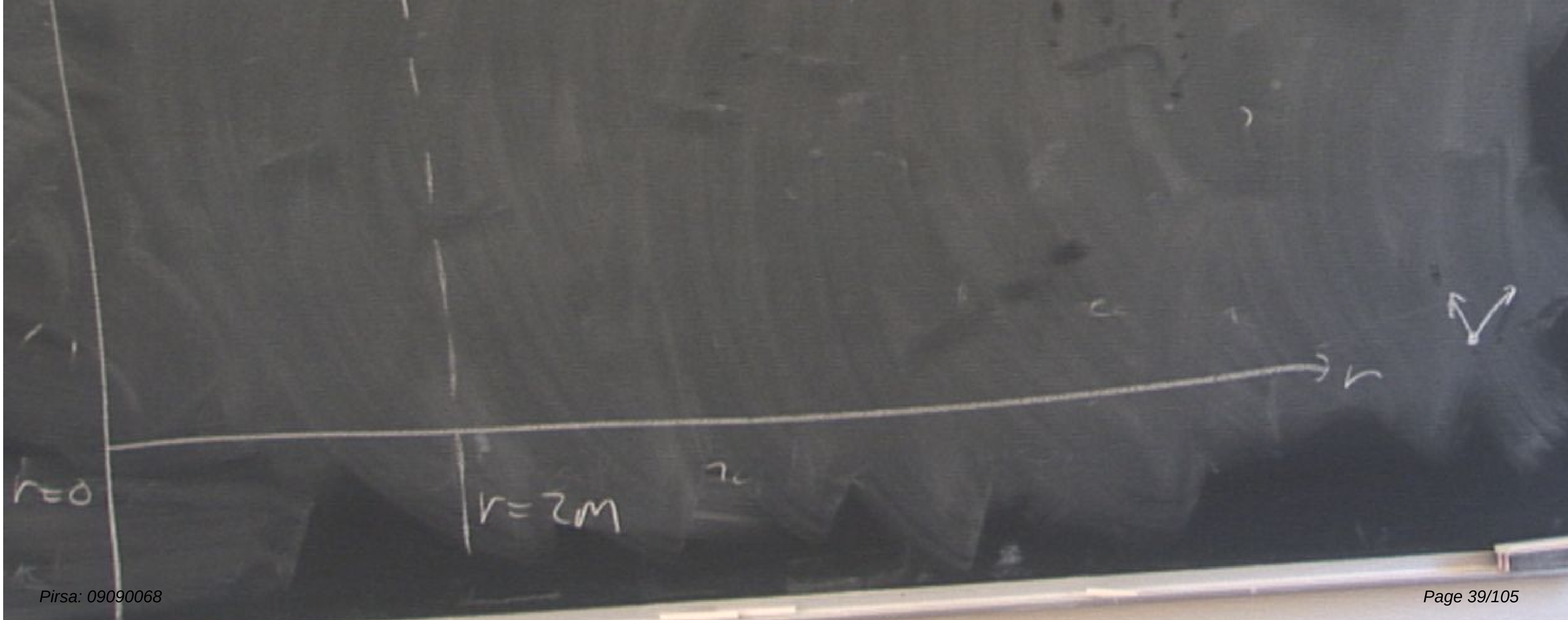

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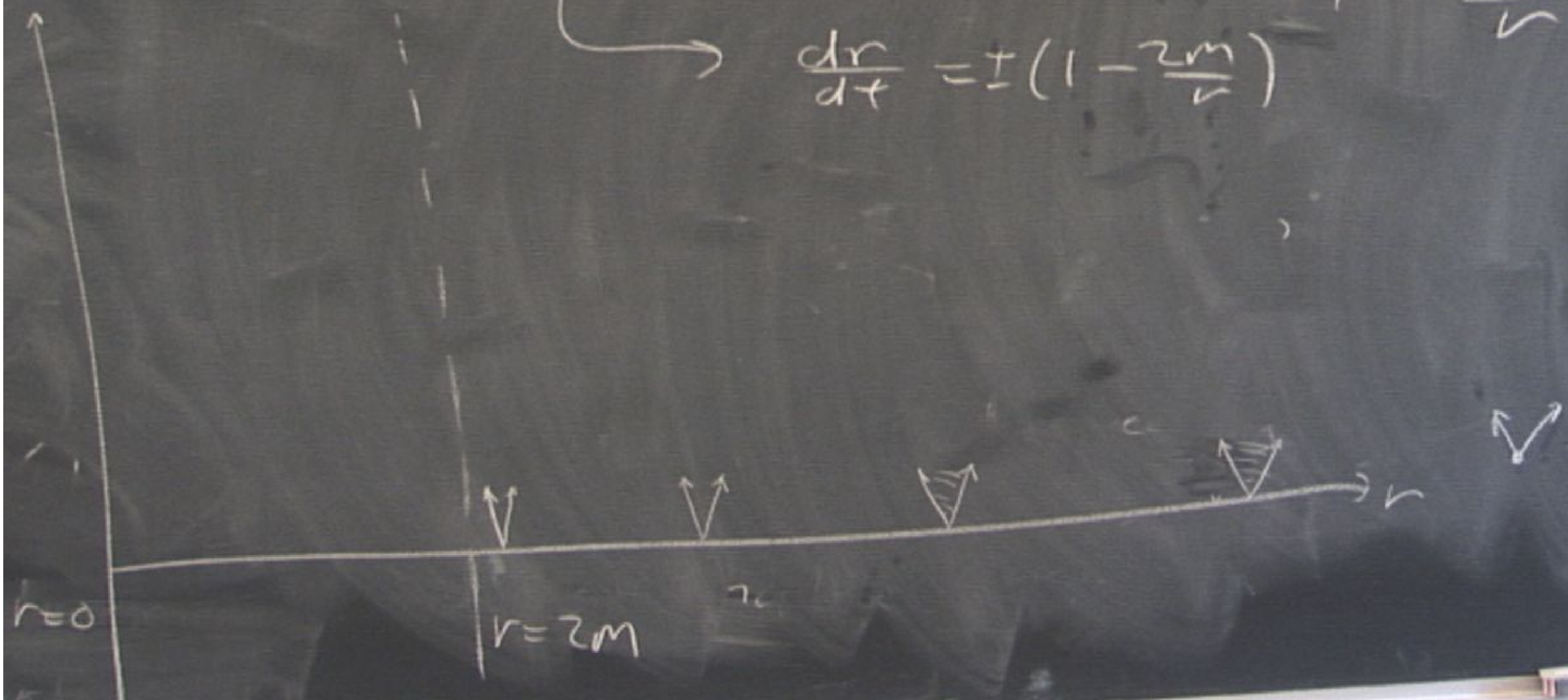

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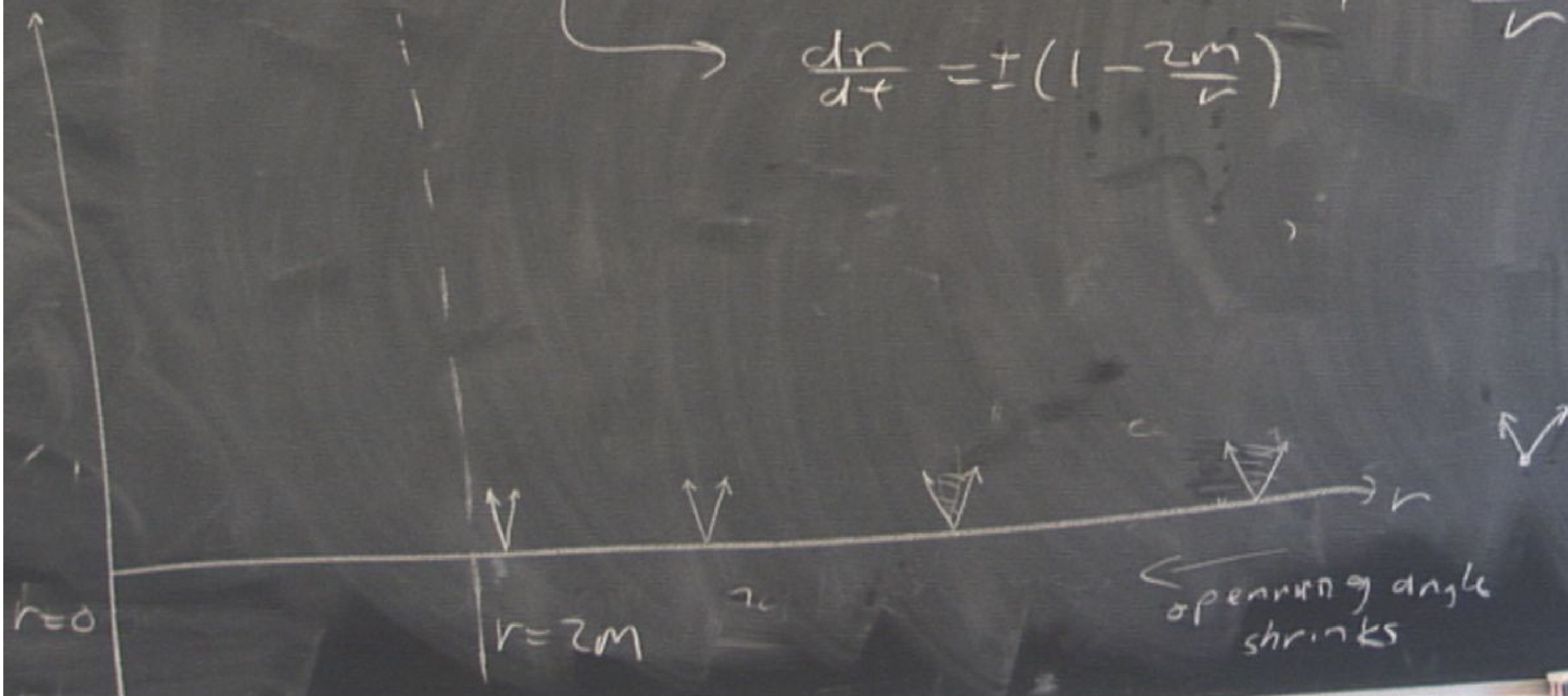

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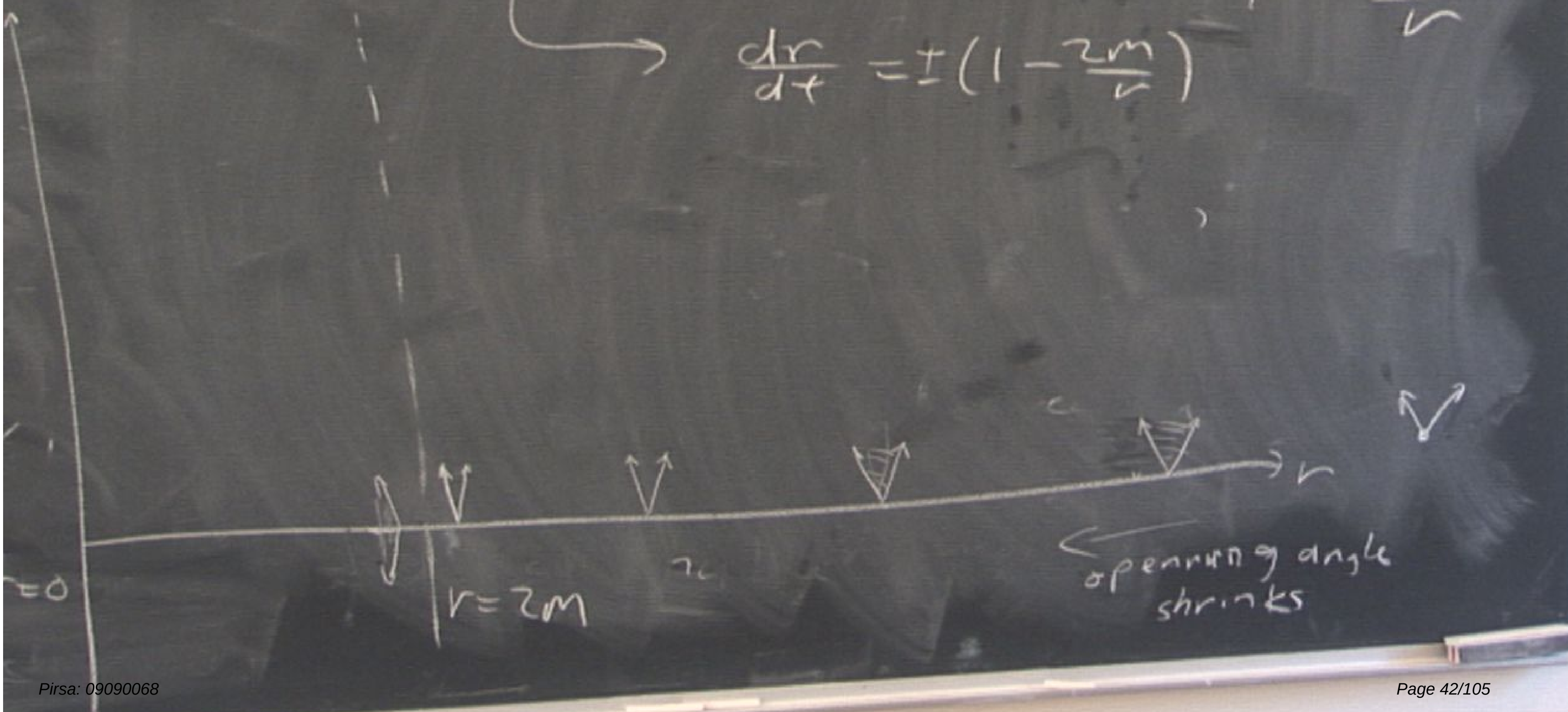

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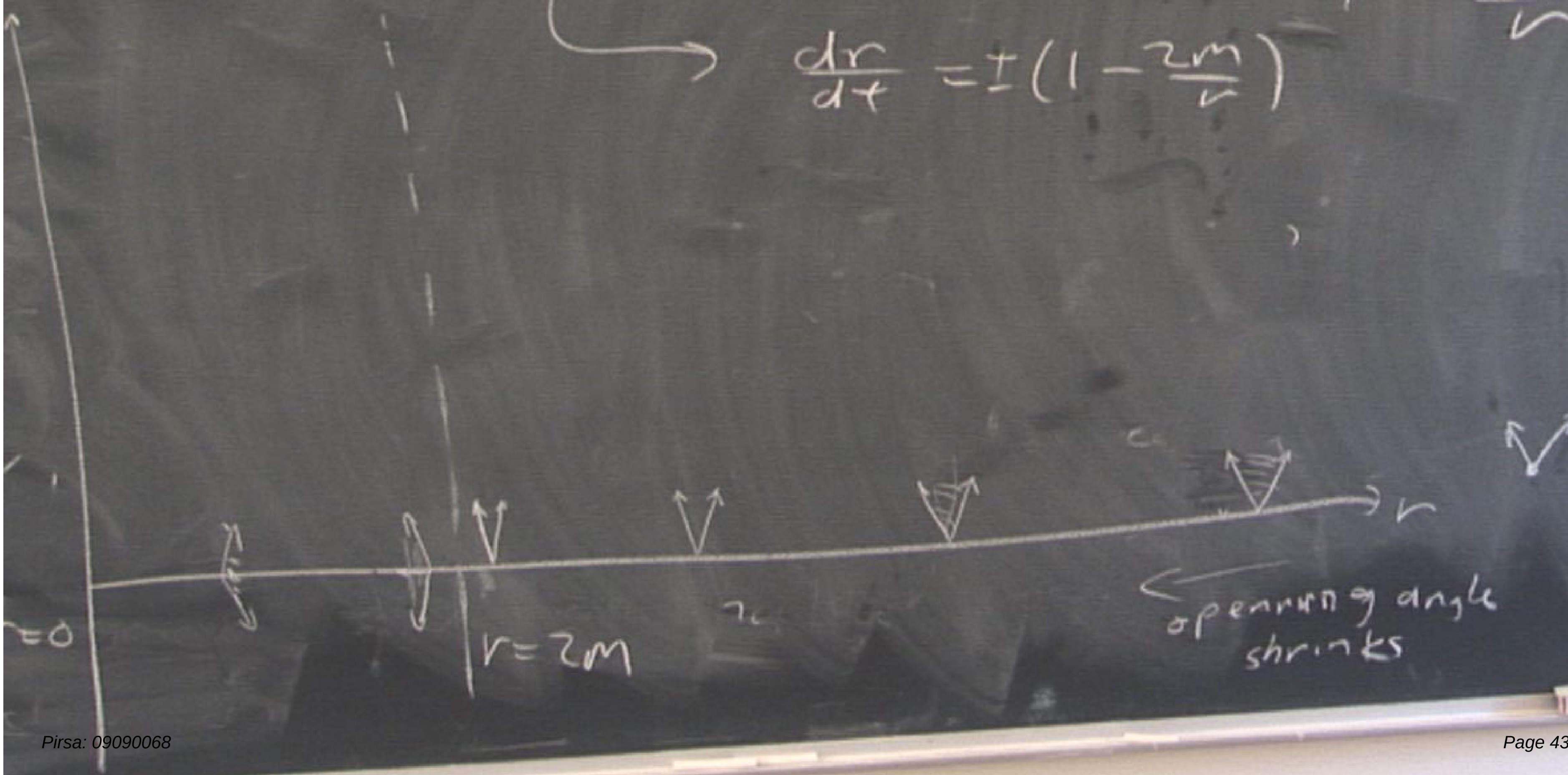

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get intuition: what do light

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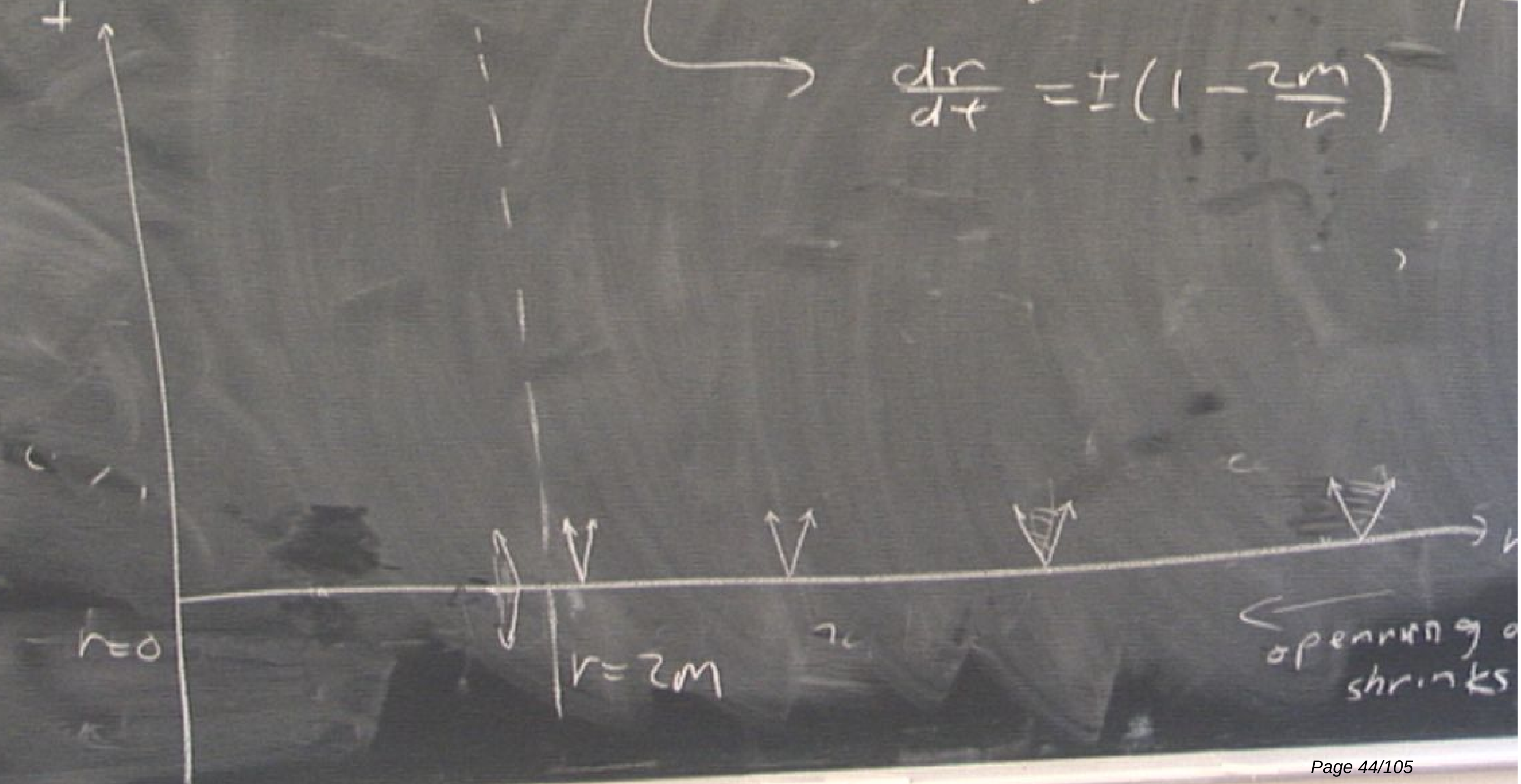

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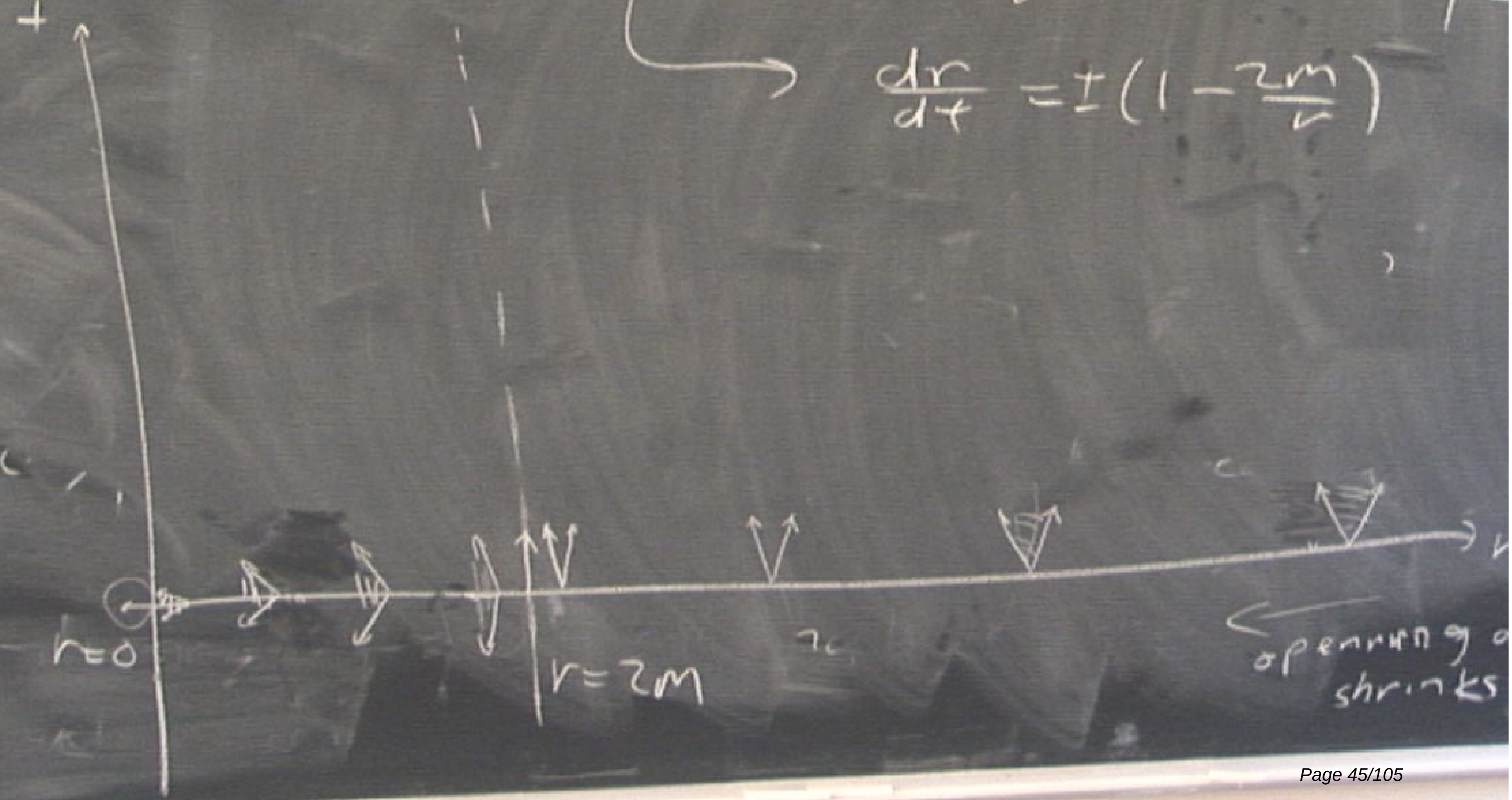

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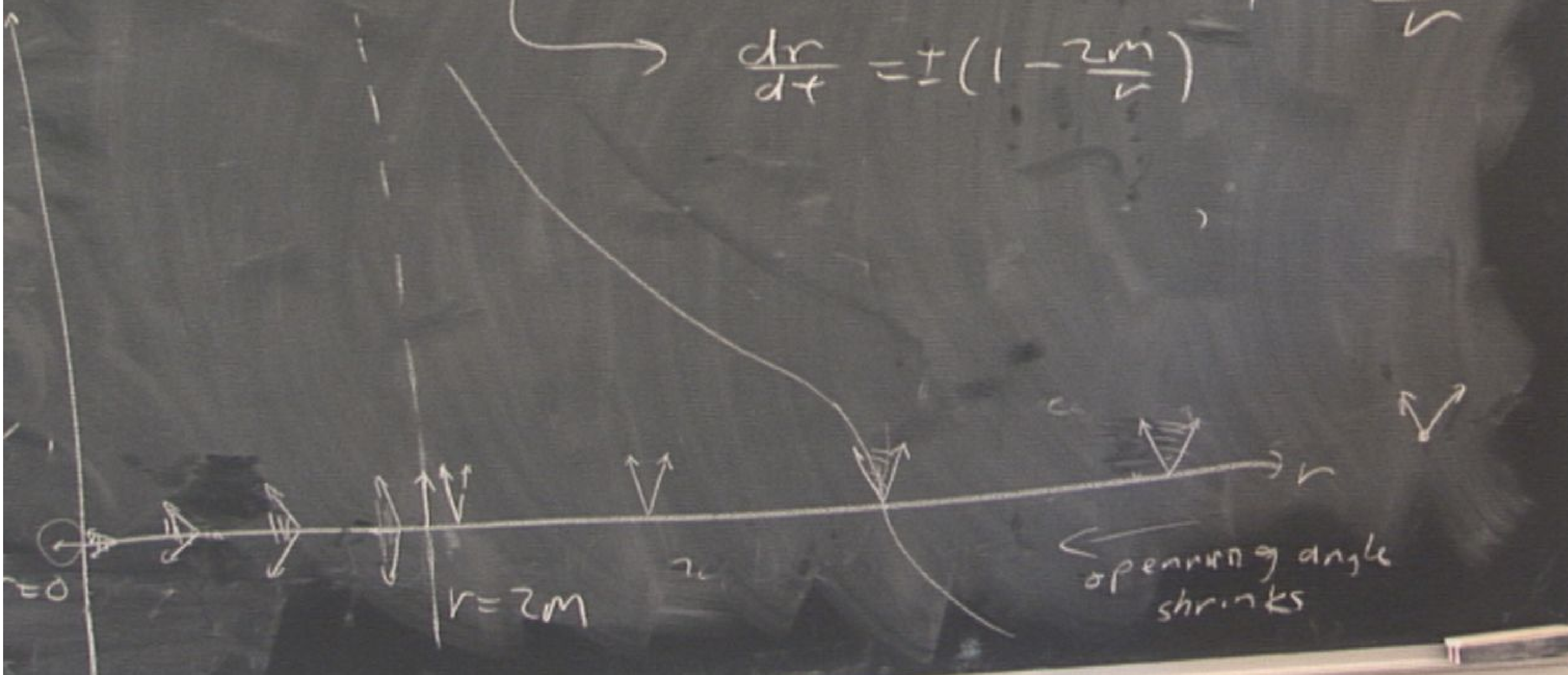

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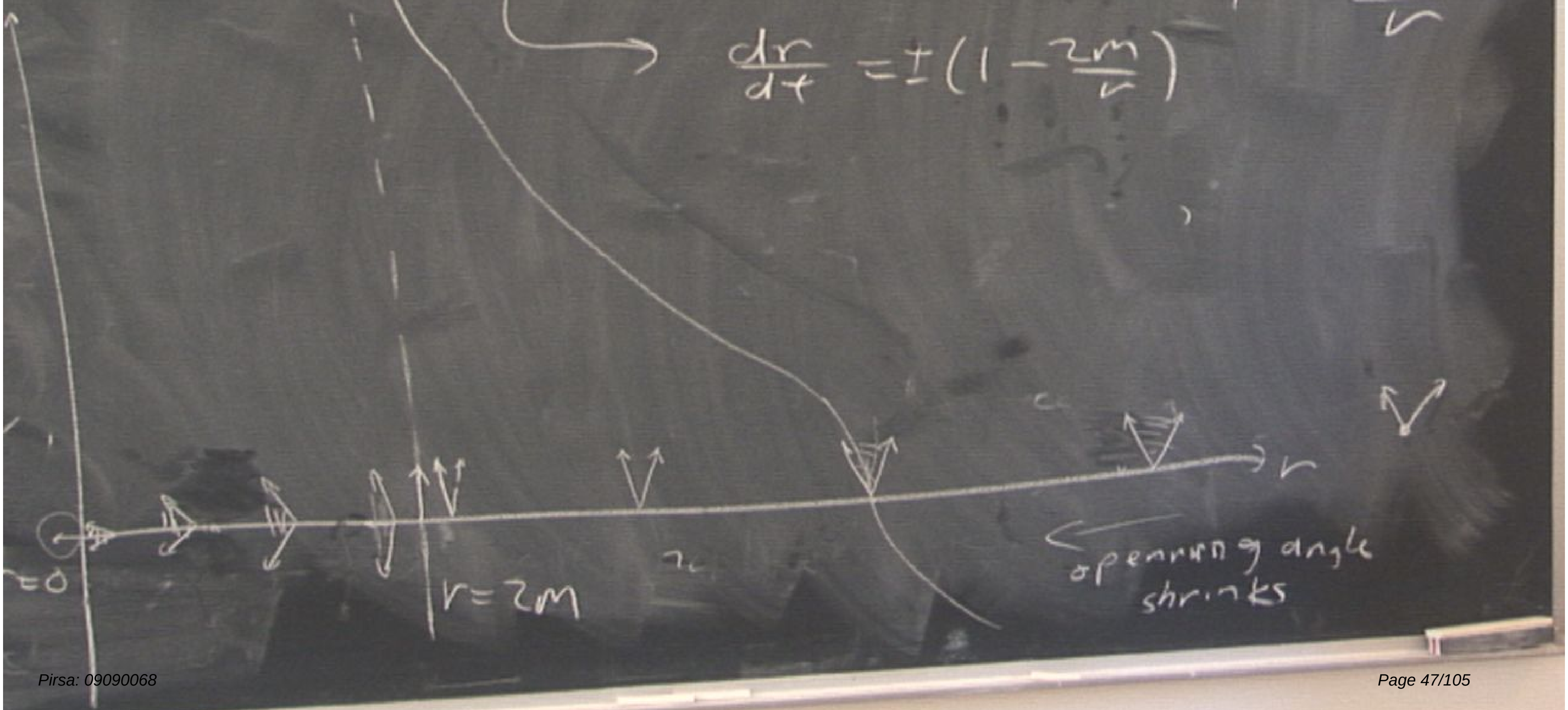

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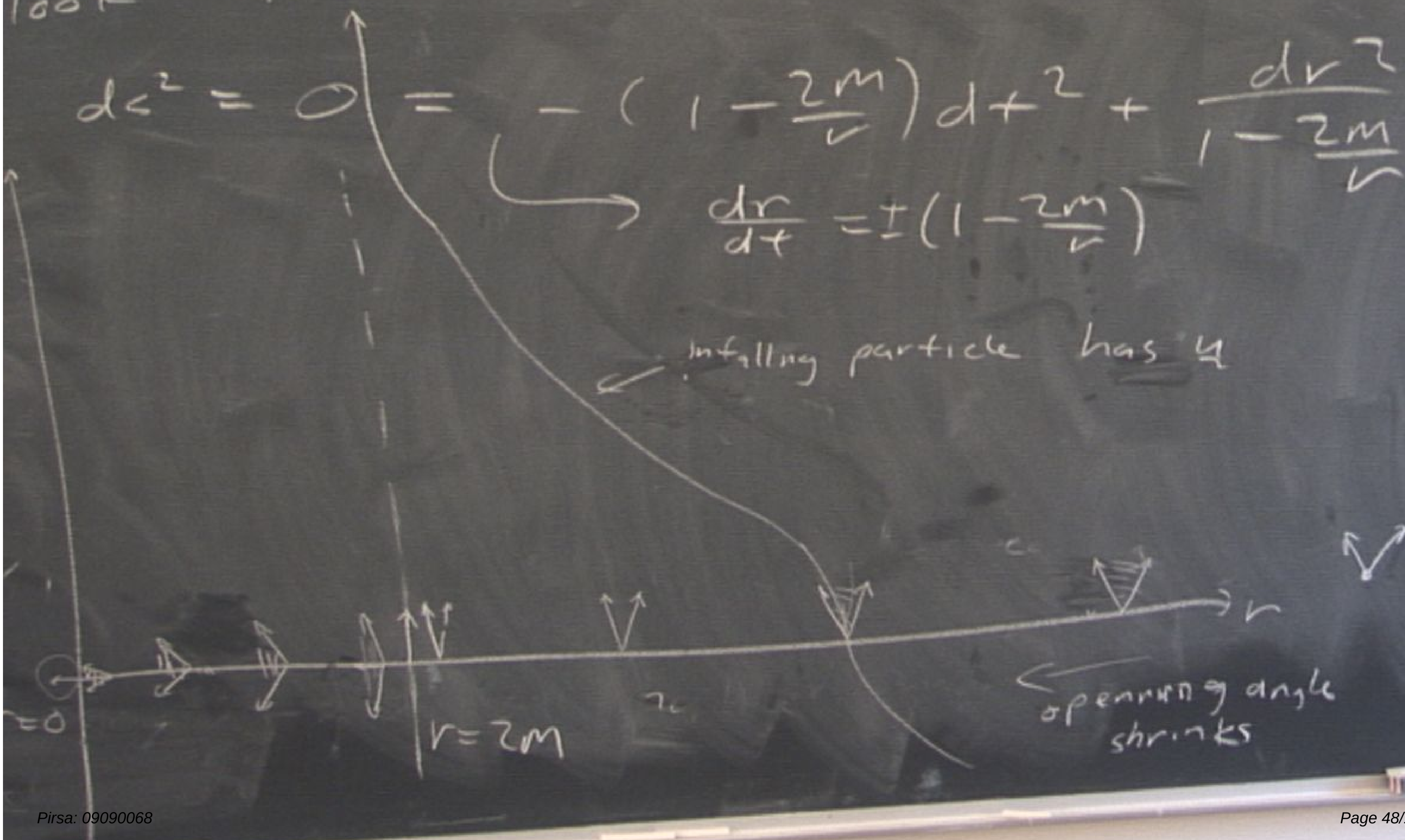

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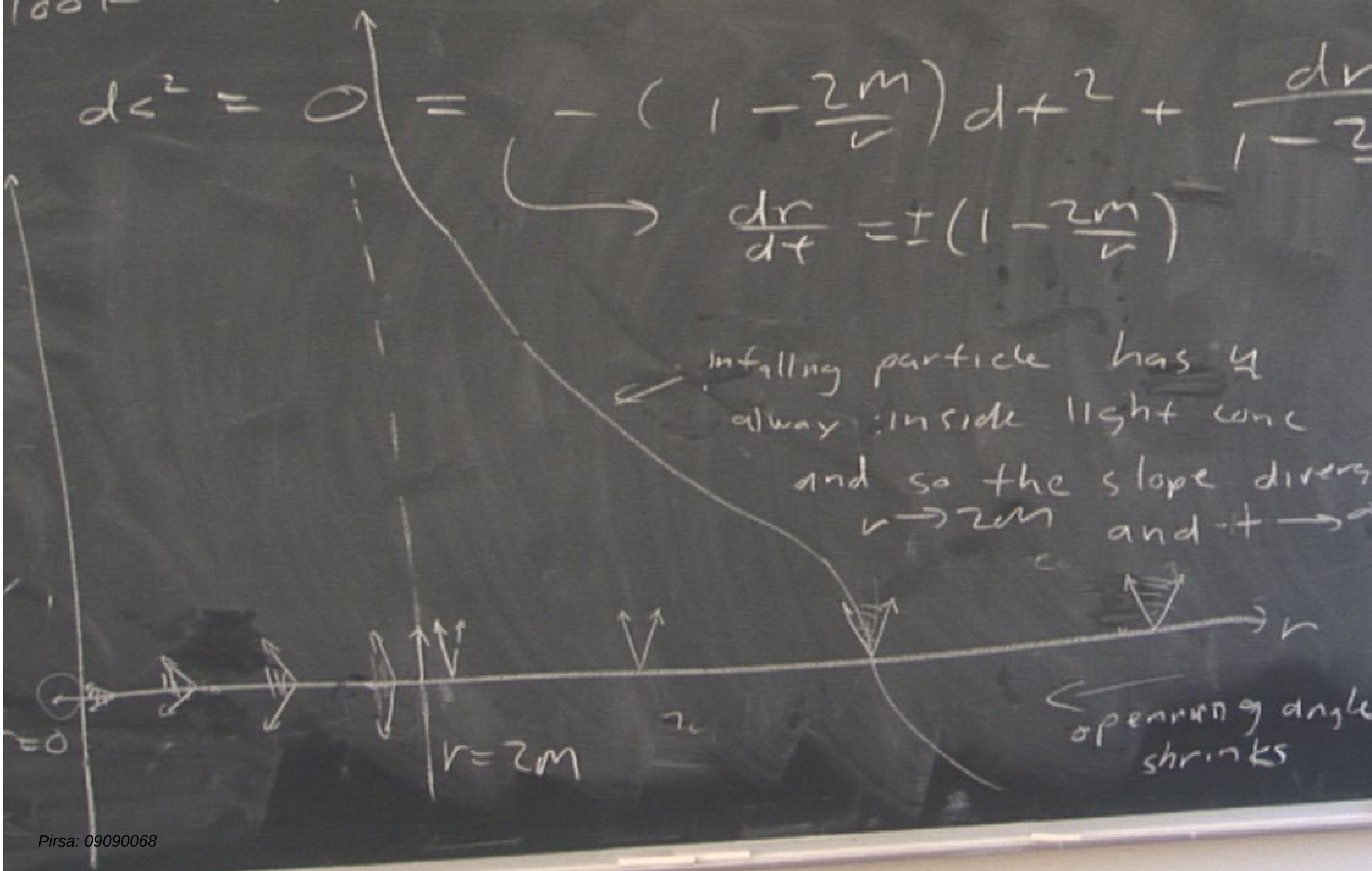
infalling particle has $u$ always inside light cone

and so the slope diverges as $r \to 2m$ and $t \to \infty$



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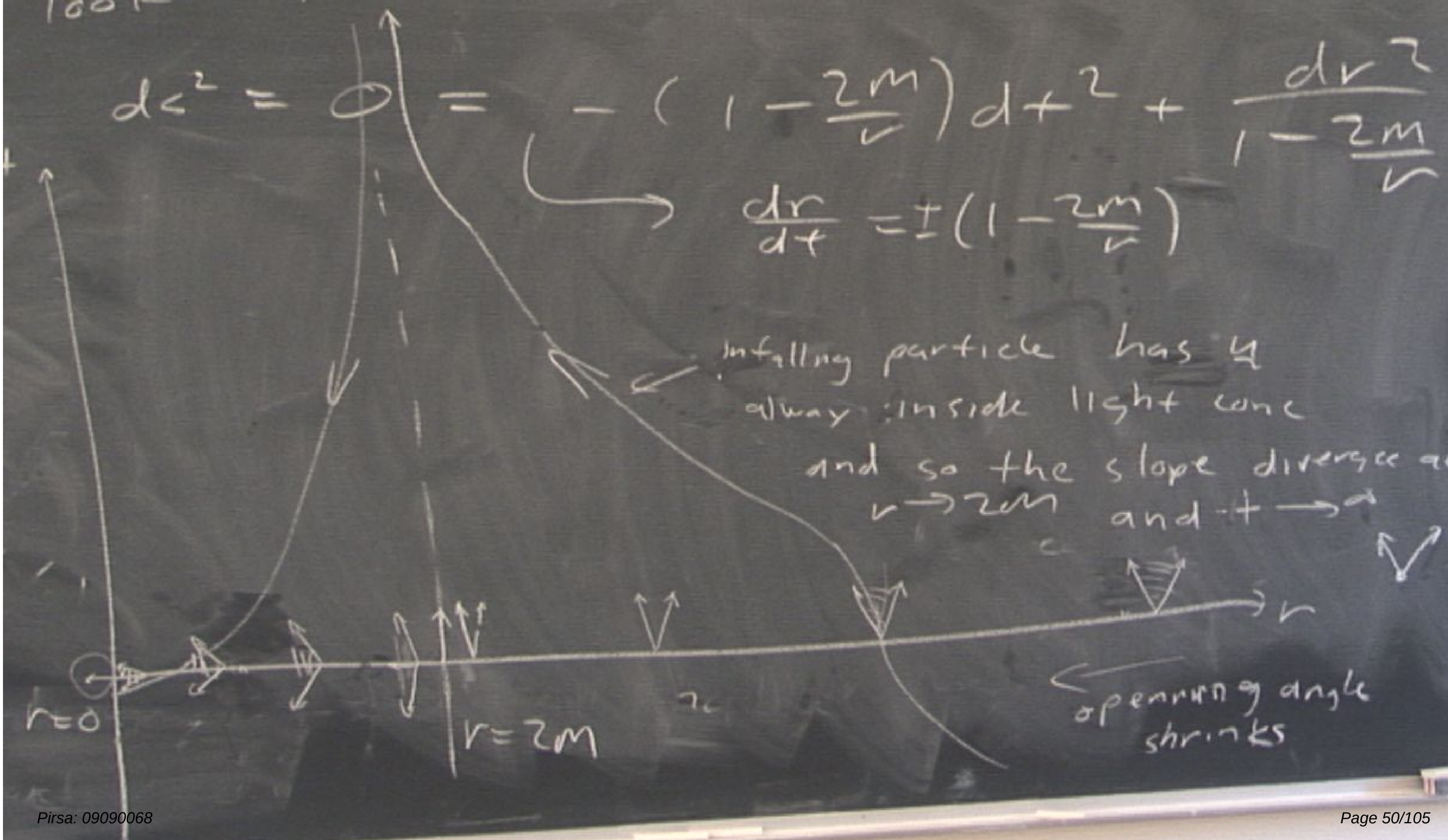

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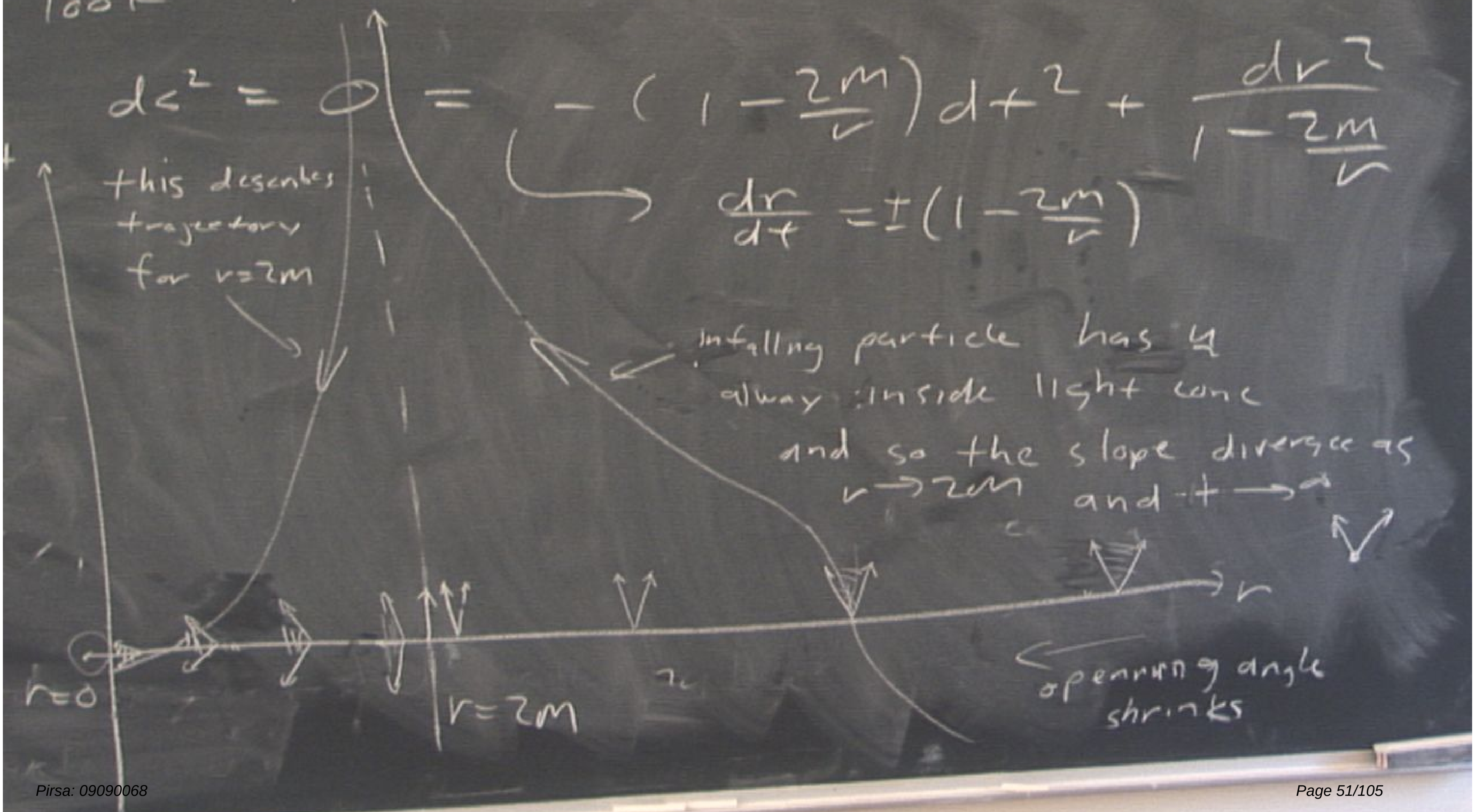


this describes trajectory for r=2M

infalling particle has u always inside light cone

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opening angle shrinks

get intuition. what do light cones look like in r - t plane

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in inside the local light cone ]

infalling particle has u always inside light cone

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opening angle shrinks

$t$

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$r=2m$

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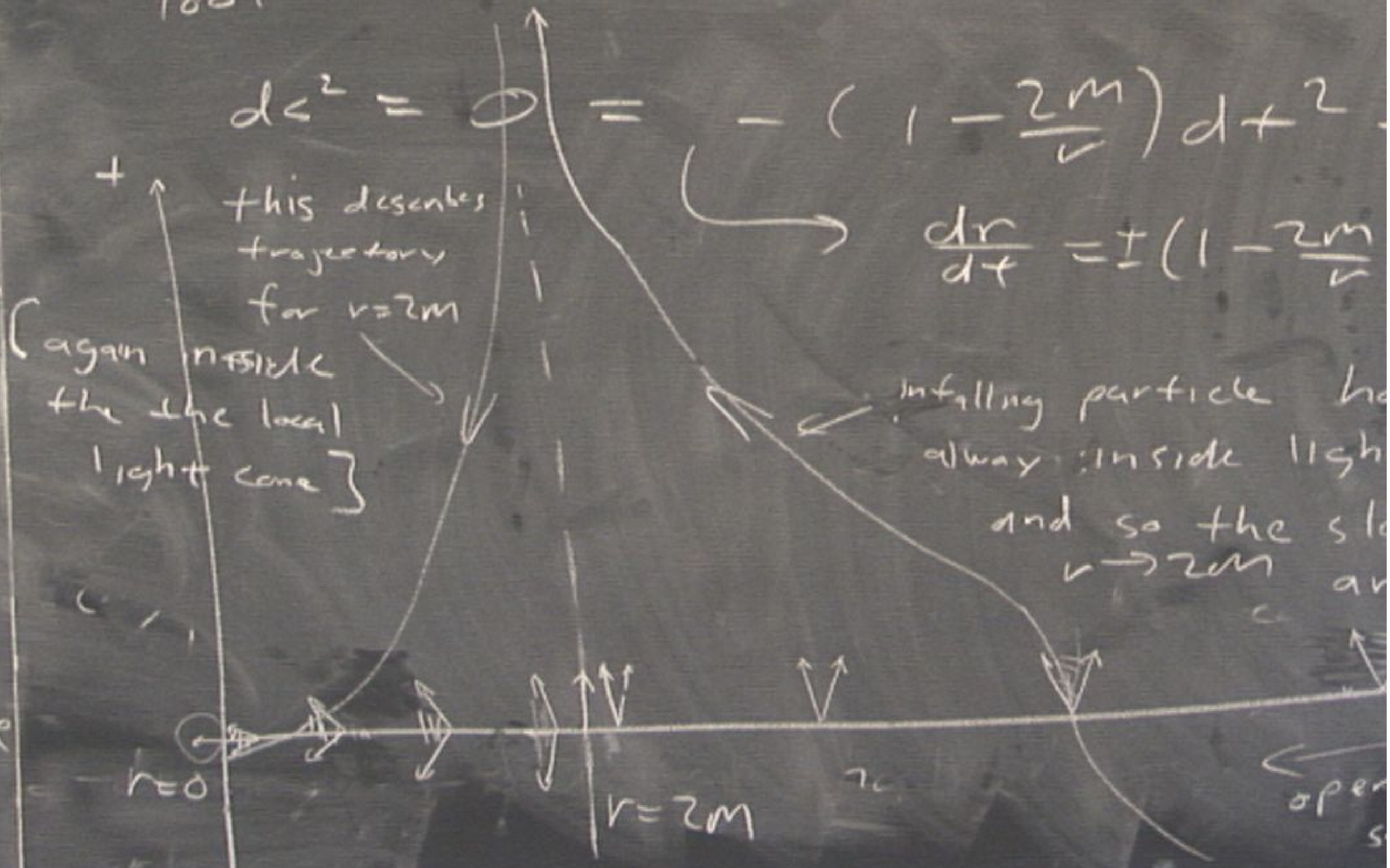

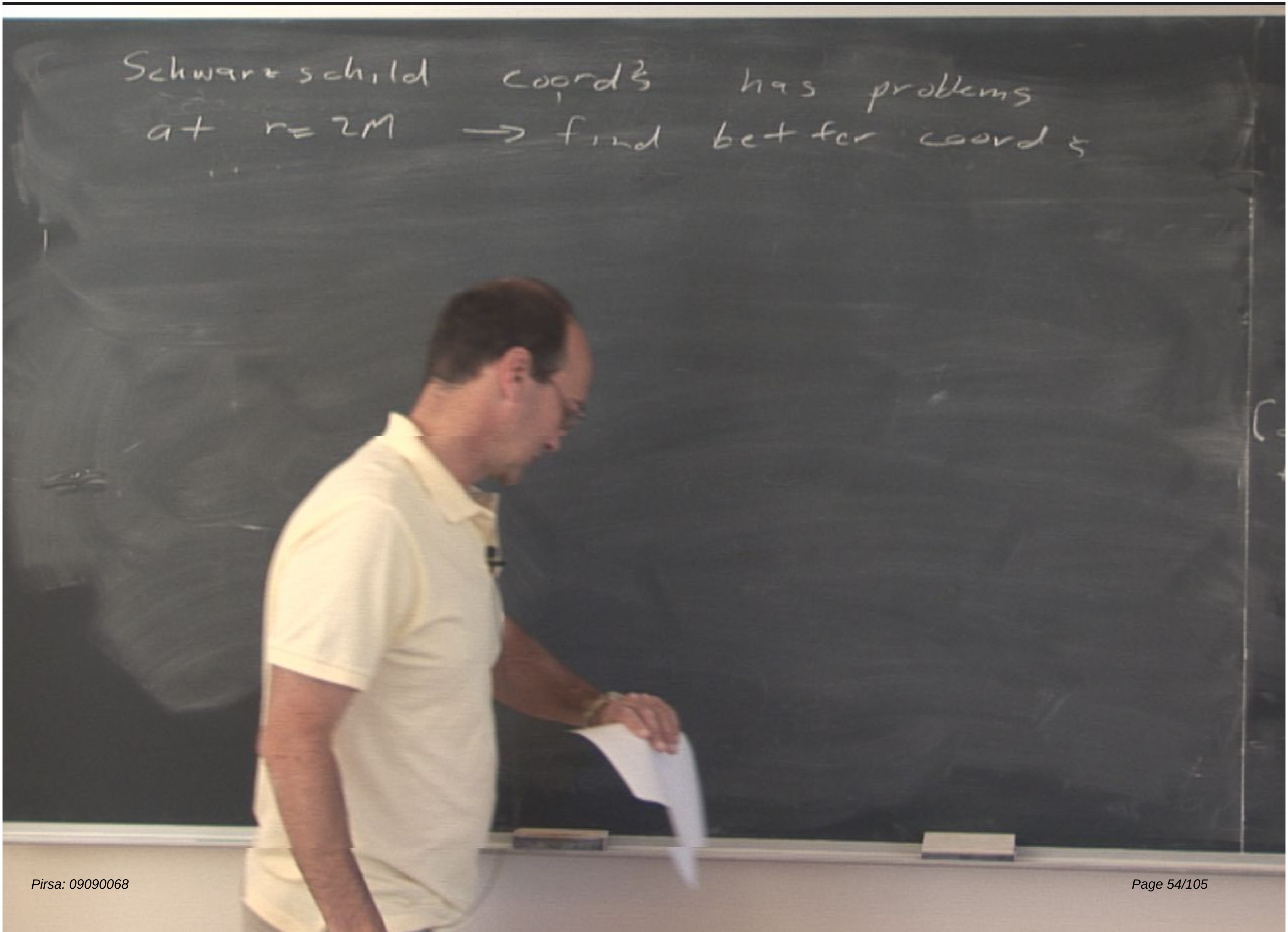
Schwarzschild coord's has problems
at r=2M → find better coord's

Schwarzschild coords has problems at $r = 2M$ $\rightarrow$ find better coords

(infalling) Eddington – Finkelstein coords

replace $t$ with

$$v = t + r + 2M \log \left| \frac{r}{2M} - 1 \right|$$

Schwarzschild coord's has problems at $r = 2M$ $\rightarrow$ find better coord's

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where from?

$$ds^2 = -\left(1 - 2M/r\right) dt^2 + \frac{dr^2}{1 - 2M/r} + \cdots$$

Schwarzschild coords has problems at $r = 2M$ $\longrightarrow$ find better coords

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$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + \cdots$$

not so bad

but get rid of divergence

try

$$ds^2 = -\left(1-\frac{2m}{r}\right)(dv + f(r)\,dr)^2 + \frac{dr^2}{1-\frac{2m}{r}} + \cdots$$

try

$$ds^2 = -\left(1-\frac{2m}{r}\right)\left(dv + f(r)\,dr\right)^2 + \frac{dr^2}{1-\frac{2m}{r}} + \cdots$$

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$$+ dr^2\left(\frac{1}{1-\frac{2m}{r}} - \left(1-\frac{2m}{r}\right)f^2\right)$$

try

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would like this to be finite at $r=2M$

- a simple choice is to set $g_{rr} = 0$ everywh

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$$ds^2 = -\left(1-\frac{2M}{r}\right)(dv + f(r)\,dr)^2 + \frac{dr^2}{1-\frac{2M}{r}} + \cdots$$

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Would like this to be finite at $r=2M$

- a simple choice is to set $g_{rr} = 0$ everywh

$$f = \pm \frac{1}{1-\frac{2M}{r}}$$

$$ds^2 = -\left(1-\frac{2M}{r}\right)dv^2 \pm 2\,dv\,dr + \cdots$$

try

$$ds^2 = -\left(1-\frac{2M}{r}\right)(dv + f(r)\,dr)^2 + \frac{dr^2}{1-\frac{2M}{r}} + \ldots$$

$$= -\left(1-\frac{2M}{r}\right)dv^2 - 2\left(1-\frac{2M}{r}\right)f(r)\,dv\,dr$$

$$+\, dr^2 \underbrace{\left(\frac{1}{1-\frac{2M}{r}} - \left(1-\frac{2M}{r}\right)f^2\right)}_{\text{would like this to be finite at } r=2M}$$

- a simple choice is to set $g_{rr} = 0$ everywh

$$f = \pm \frac{1}{1-\frac{2M}{r}}$$

$$ds^2 = -\left(1-\frac{2M}{r}\right)dv_{\mp}^2 \pm 2\,dv_{\mp}\,dr + \cdots$$

as $r \to 2M$ $\quad ds^2 = \pm 2\, dv \mp dr + \cdots$

reminiscent of null coord's in Mink space

$$u = t + x$$
$$v = t - x$$

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choices

$$dt = dv_{\mp} \pm \frac{dr}{1 - \frac{2m}{r}}$$

$$dv_{\pm} = dt \pm \frac{dr}{1 - \frac{2m}{r}}$$

focuses on

$$\to v_{\pm} = t \pm \left(r + 2m \log \left| r/2m - 1 \right|\right)$$

Where is lightcone in $(v, r)$

$$ds^2 = -\left(1-\frac{2M}{r}\right) dv_{\mp}^2 \pm 2\, dv_{\mp}\, dr + \cdots$$

Where is lightcone in $(v_+, r)$

$$ds^2 = 0 = -\left(1 - \frac{2m}{r}\right) dv^2 + 2\, dv\, dr$$

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$\longrightarrow$ fix $dv = 0$ but move in $r$

or

$$dr = \frac{1}{2}\left(1 - \frac{2m}{r}\right) dv$$

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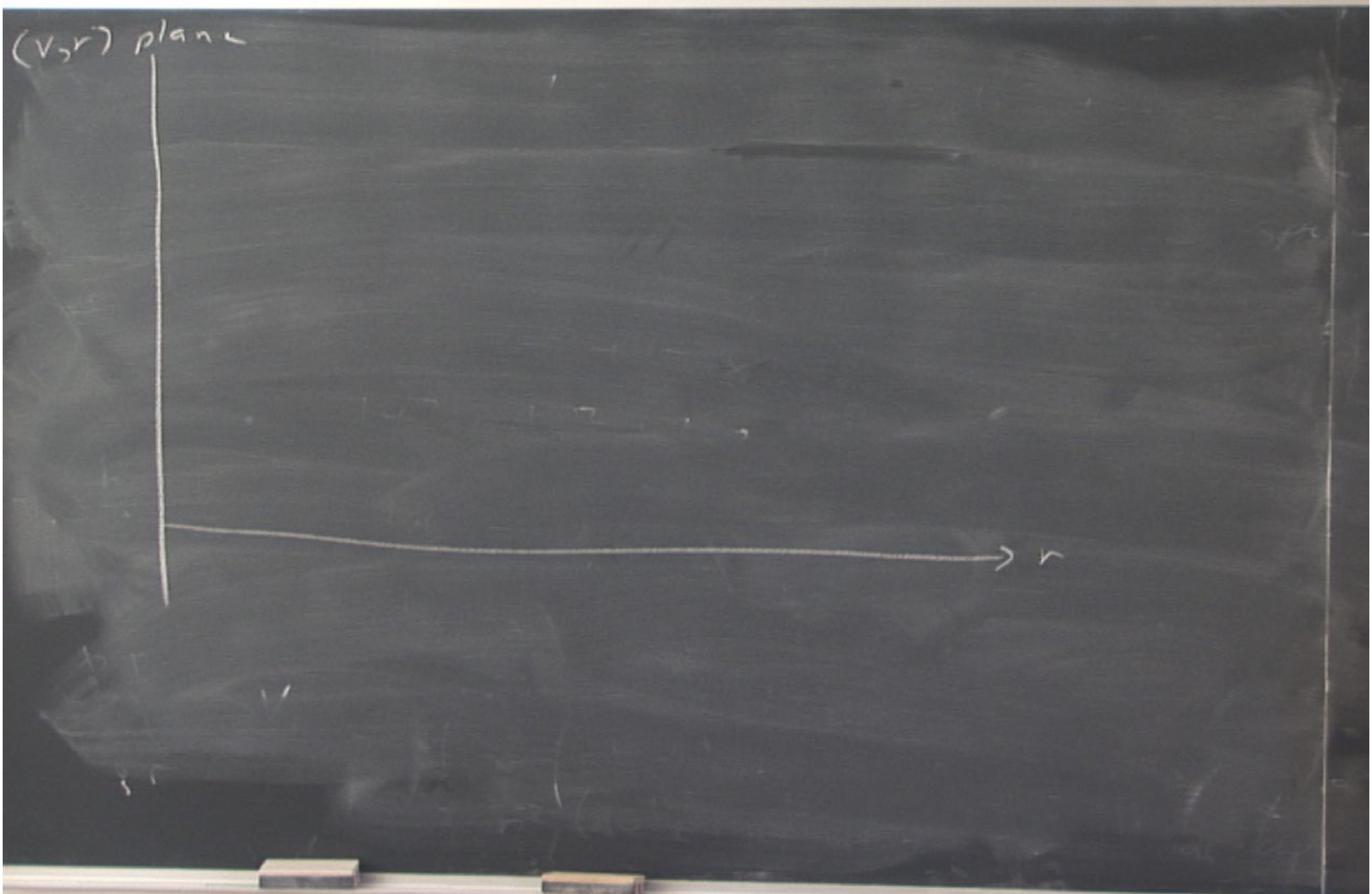
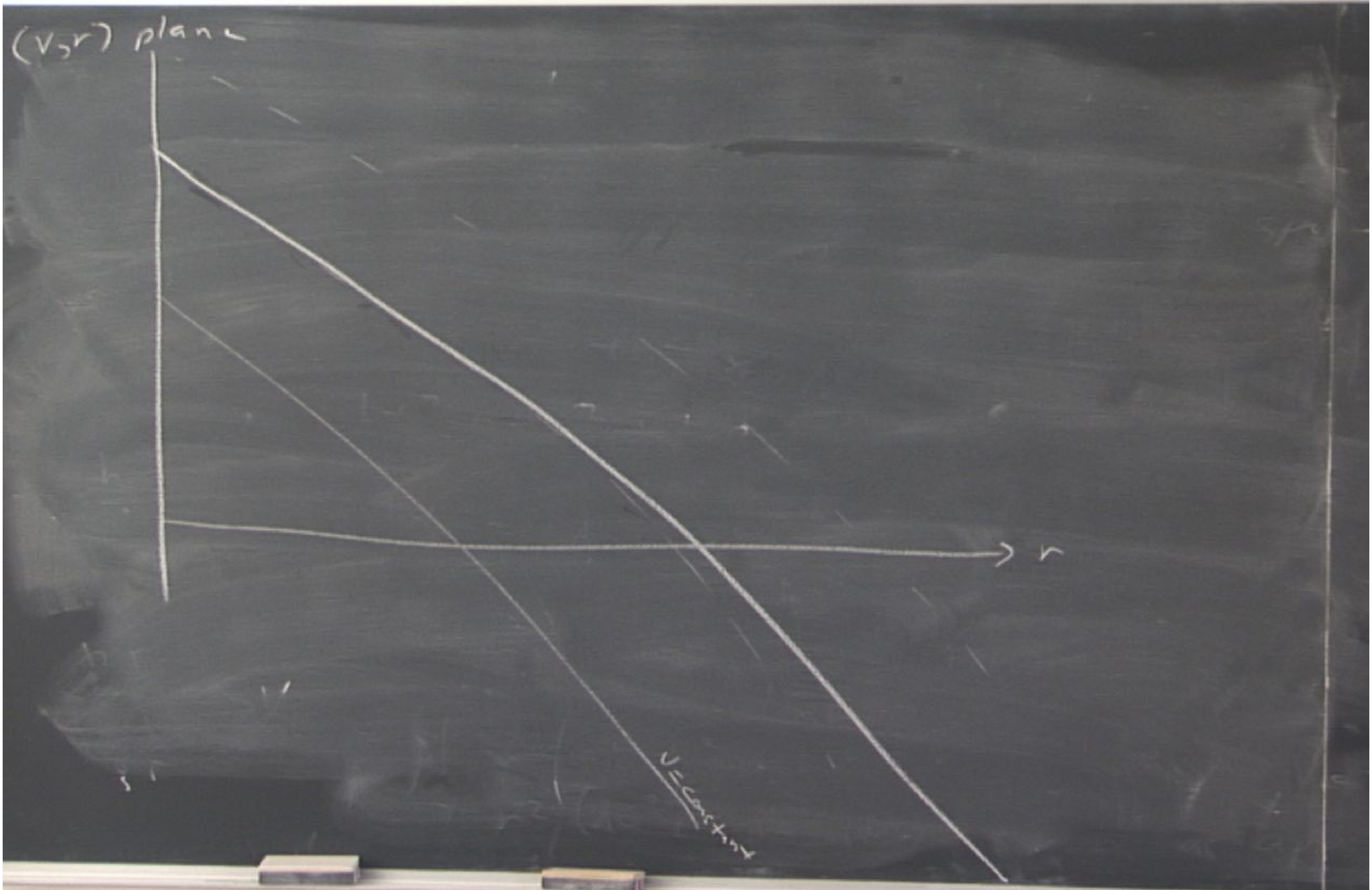
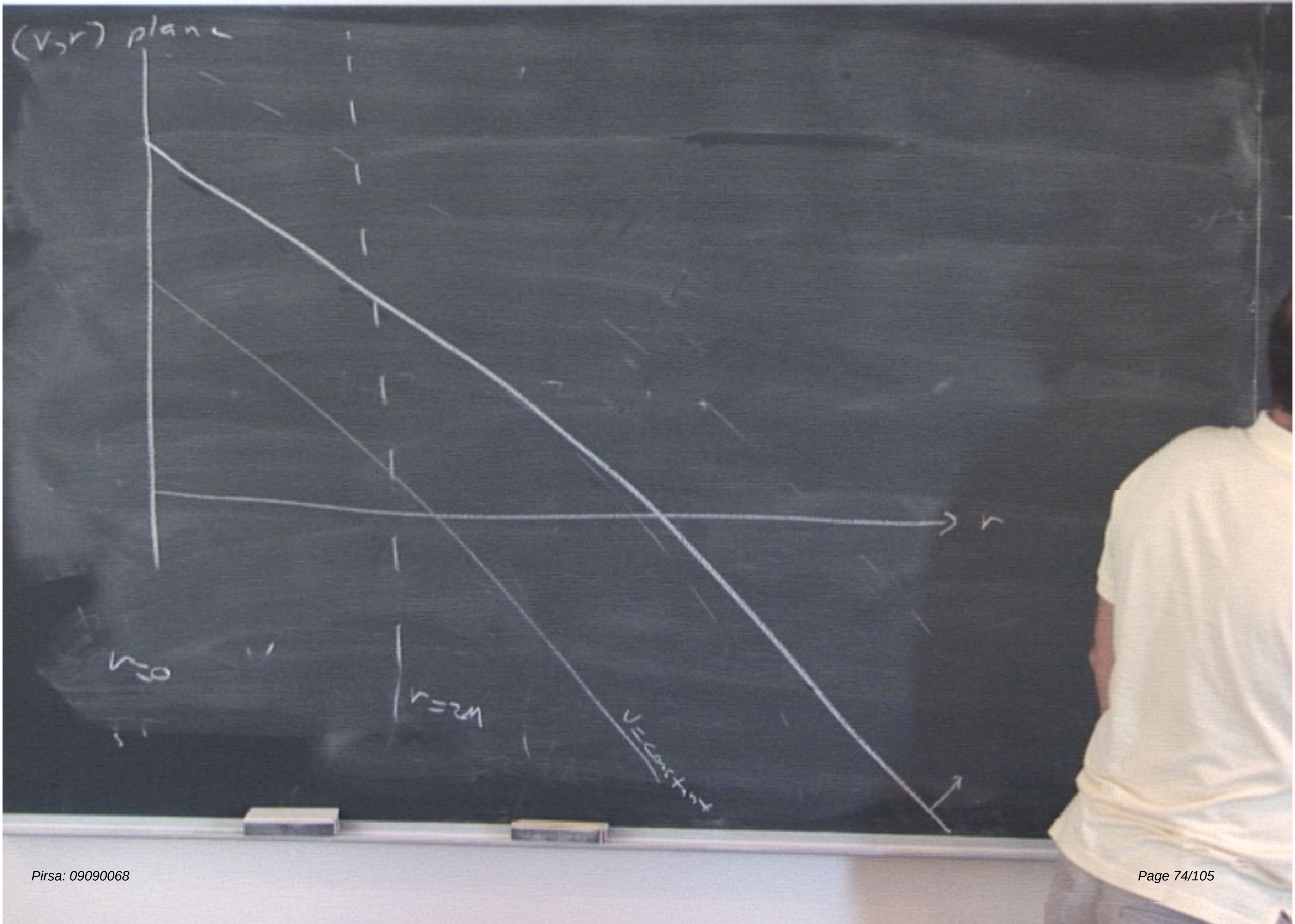
$> 0$ $\quad r > 2M$ $(dv > 0)$

$< 0$ $\quad r < 2M$ $(dv > 0)$


(v,r) plane
r


(v, r) plane
r
v=constant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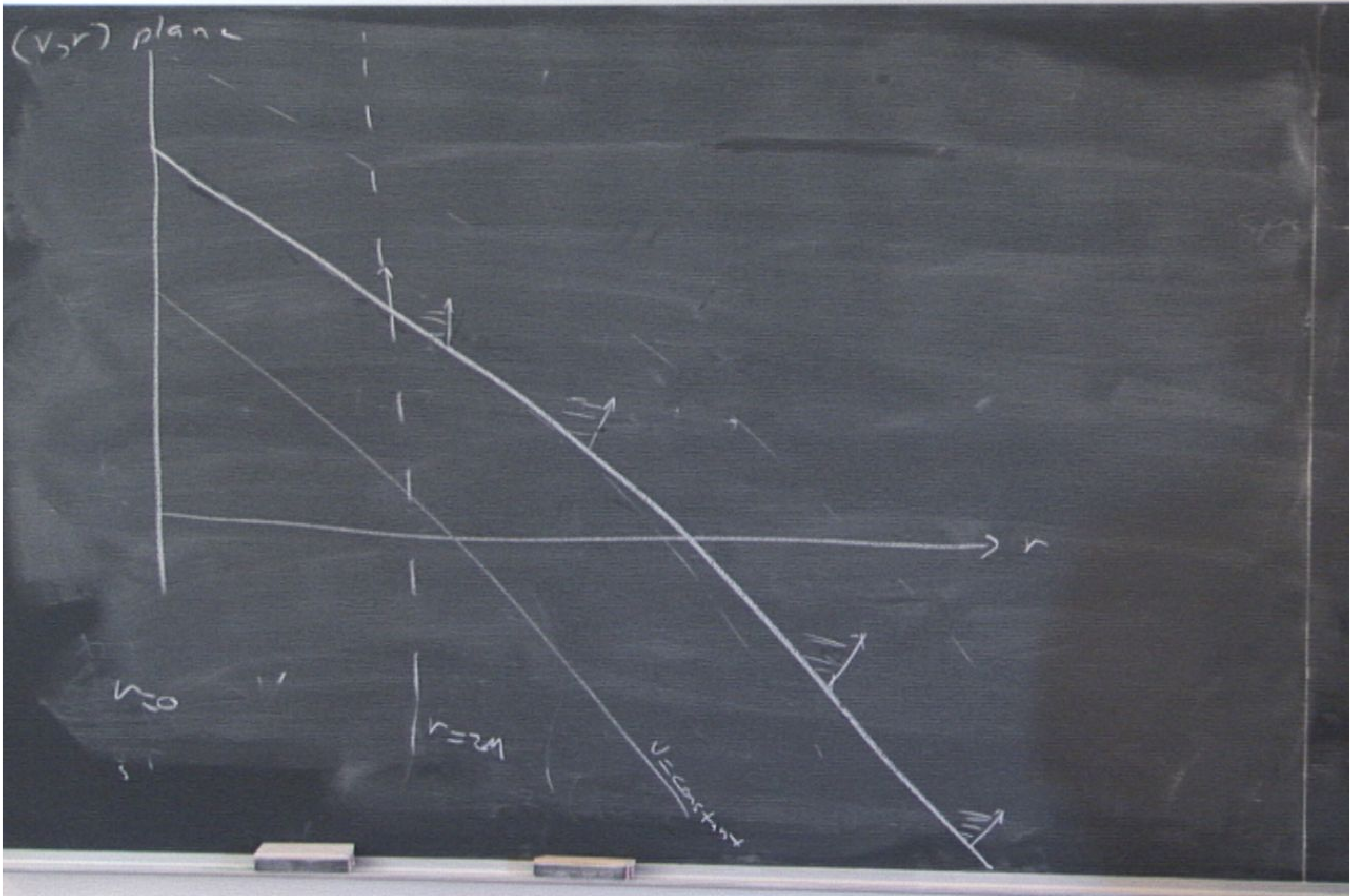
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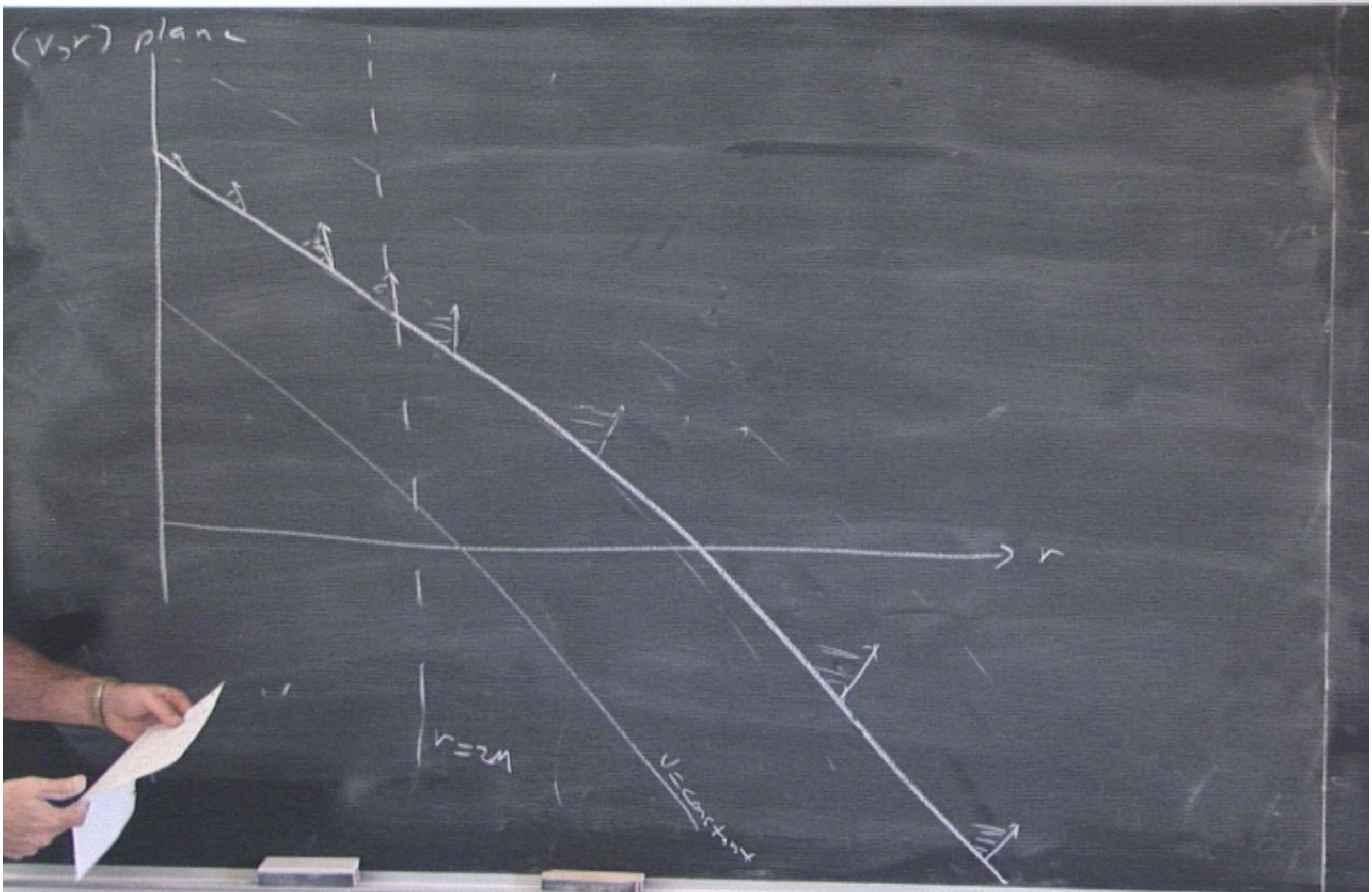
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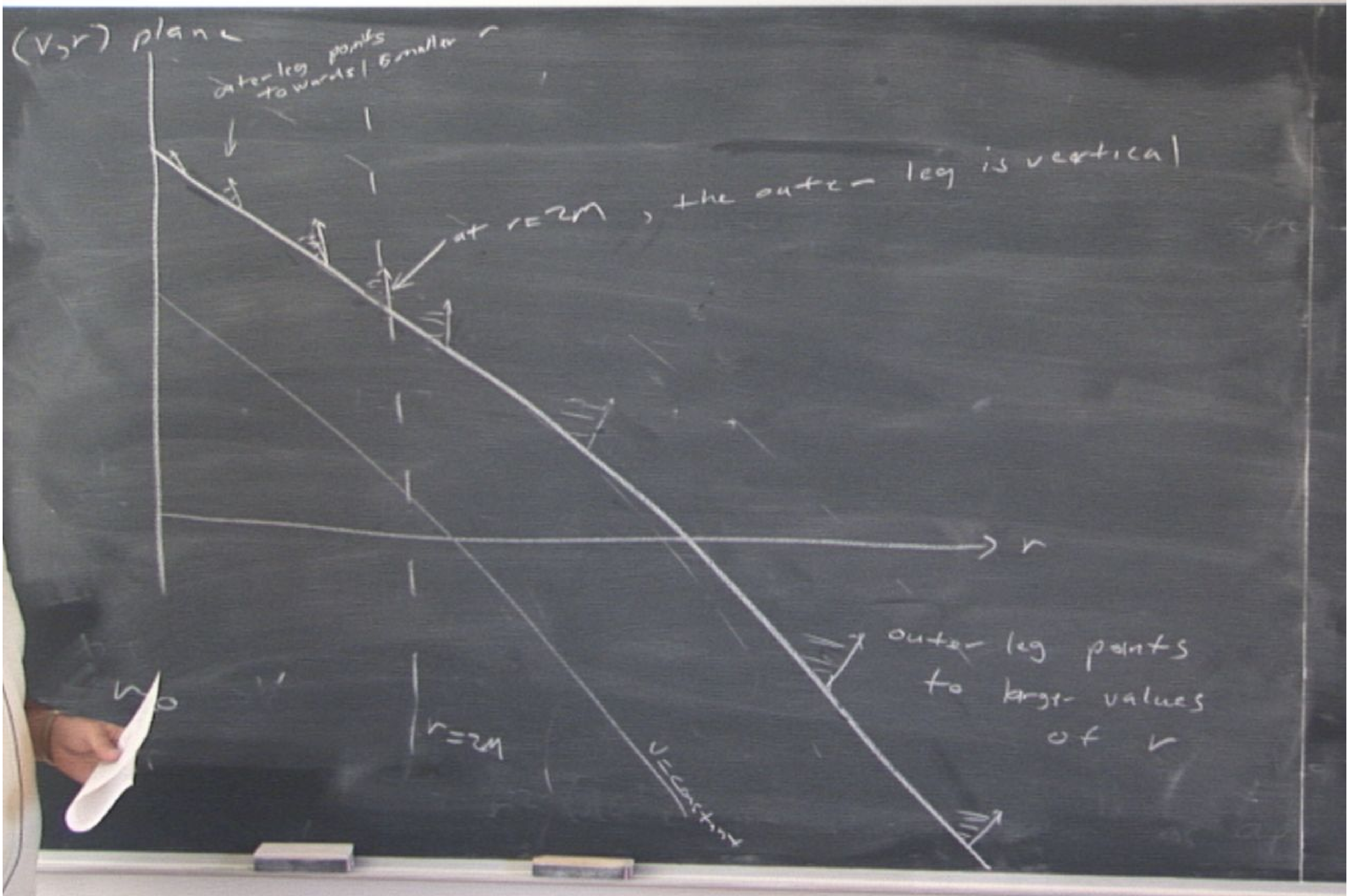
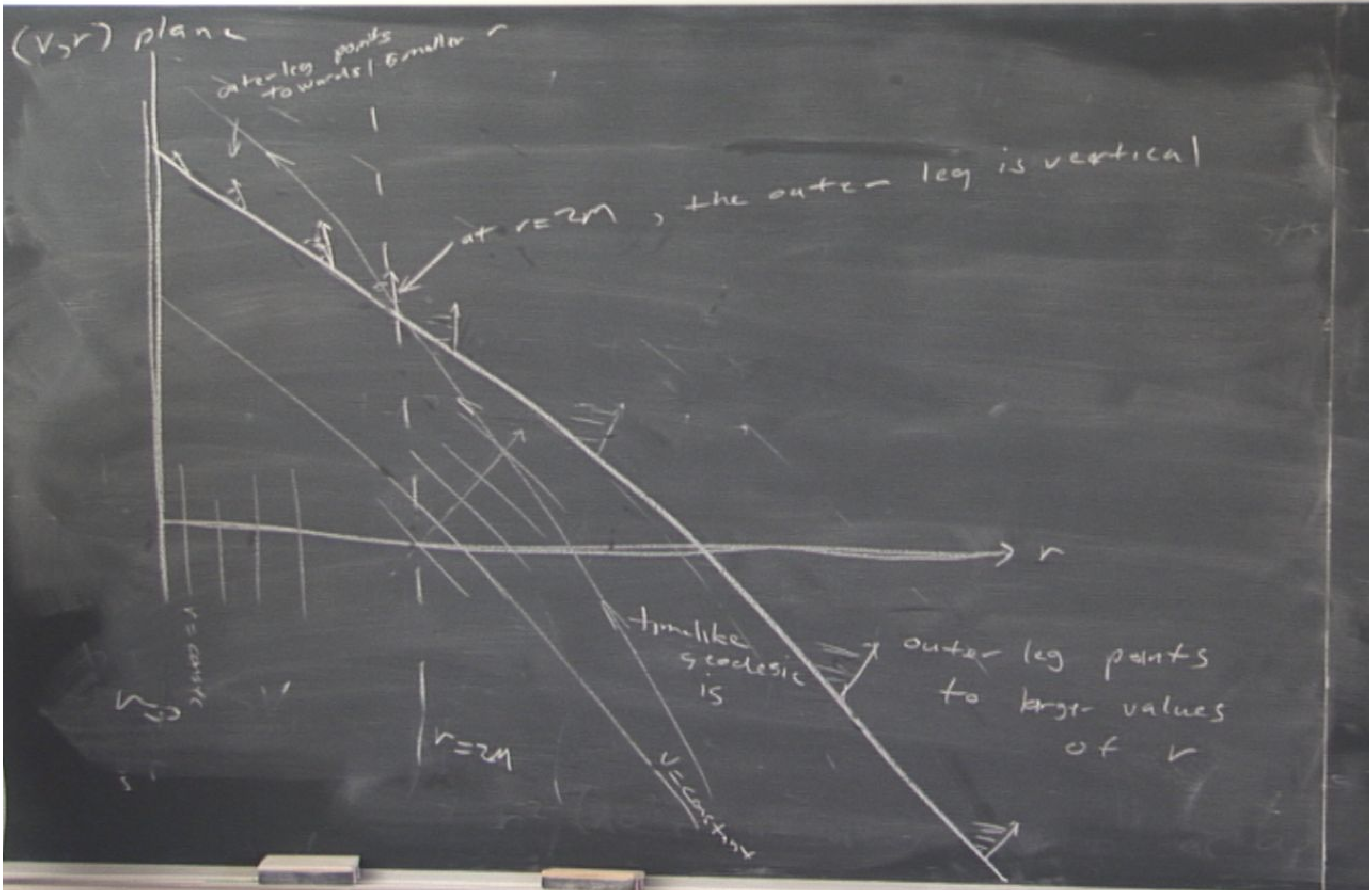

(v, r) plane
outer leg points towards smaller r
at r=2M, the outer leg is vertical
r
outer leg points to larger values of r
r=2M
v=constant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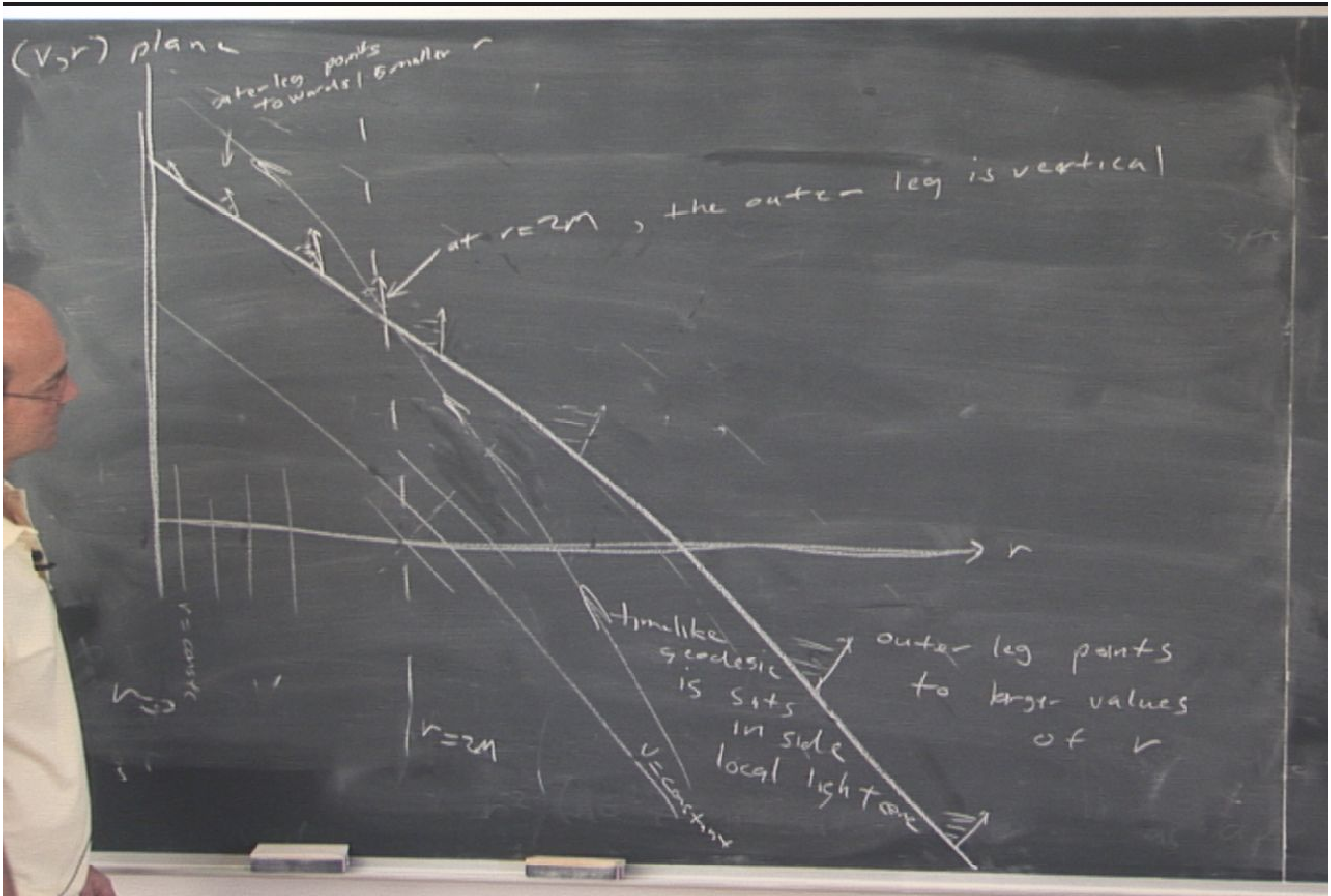
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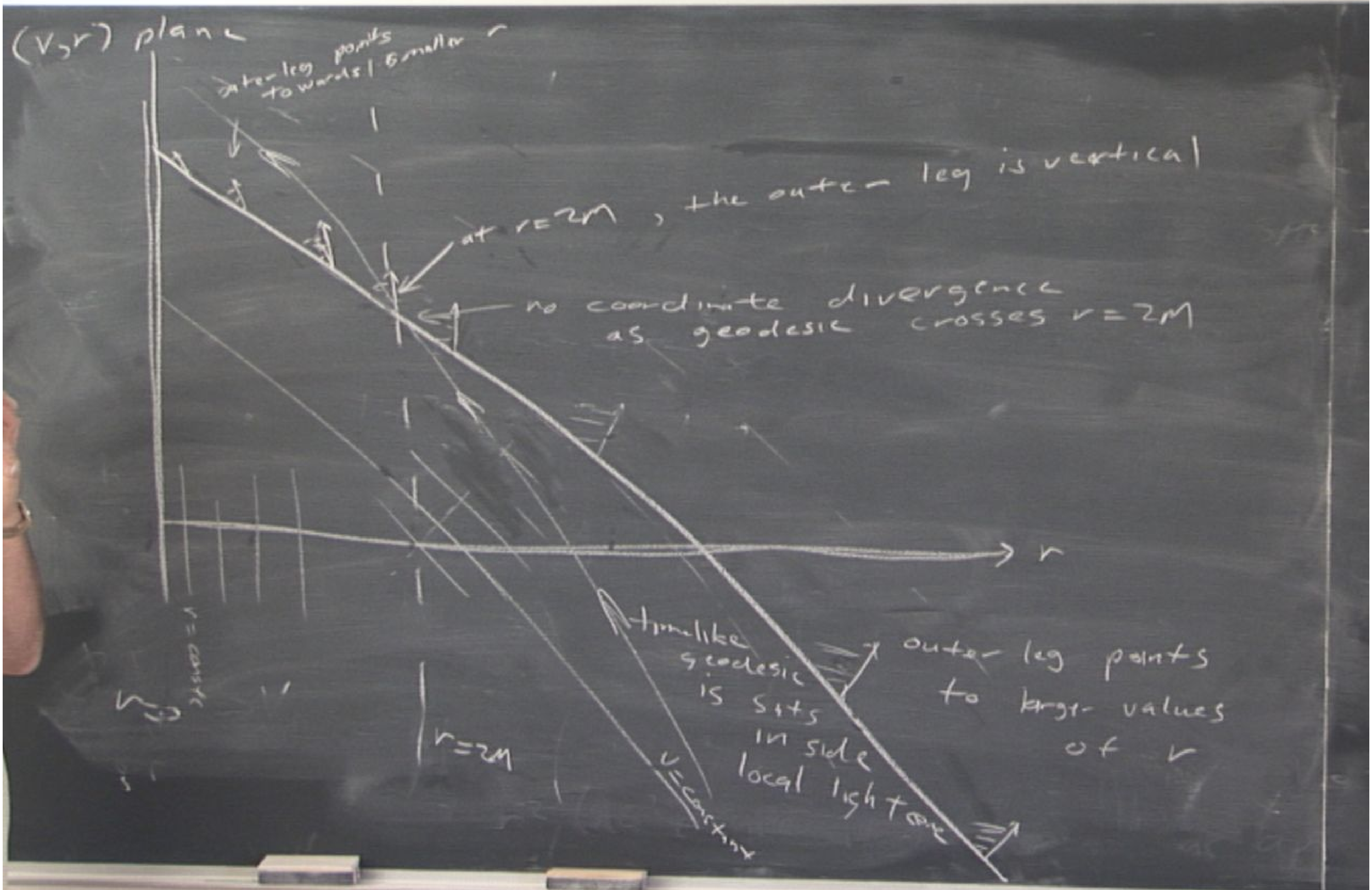
timelike geodesic is sits inside local lightcone
outer leg points to larger values of r
r=2M
v=constant
r = const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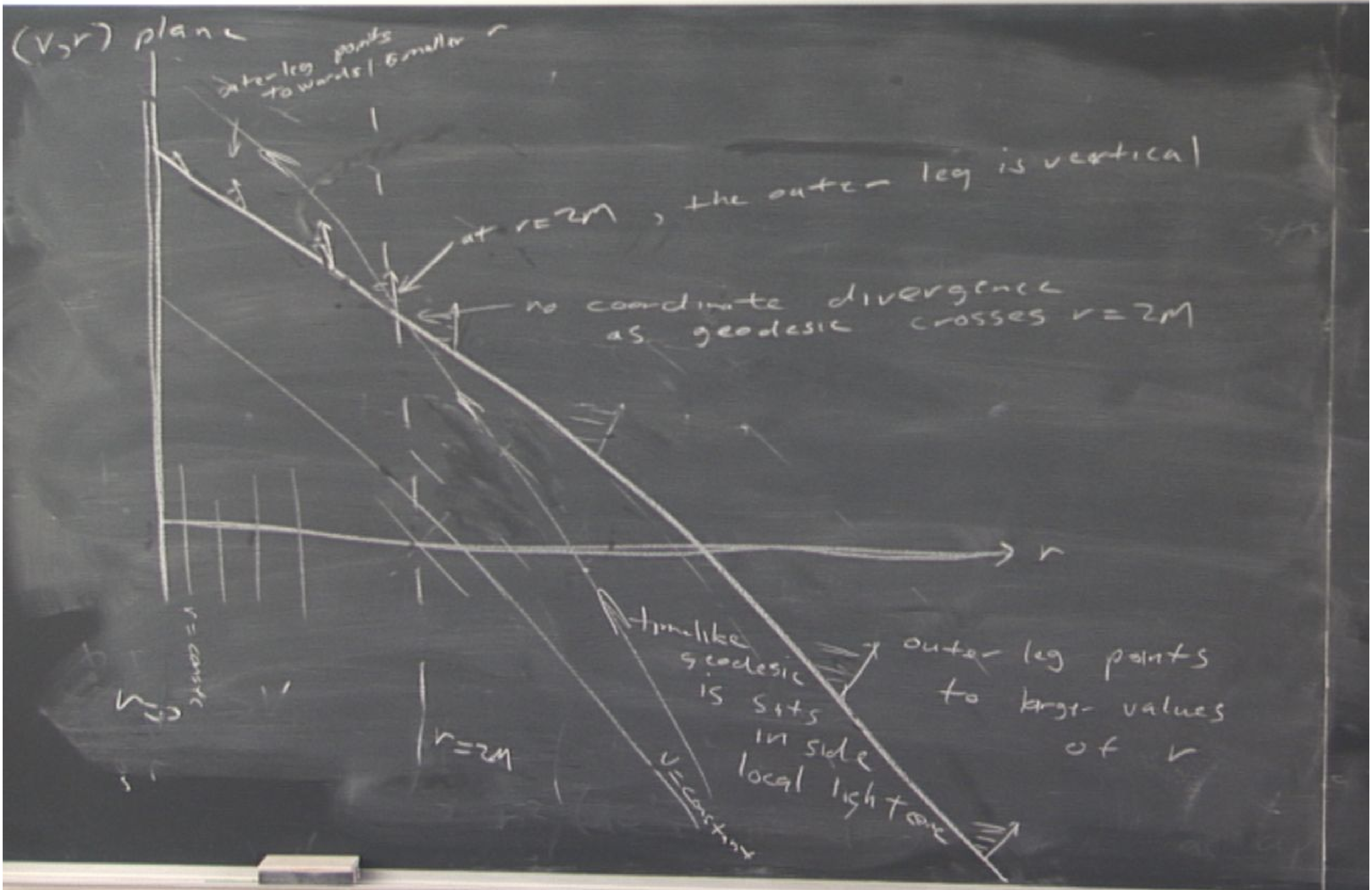
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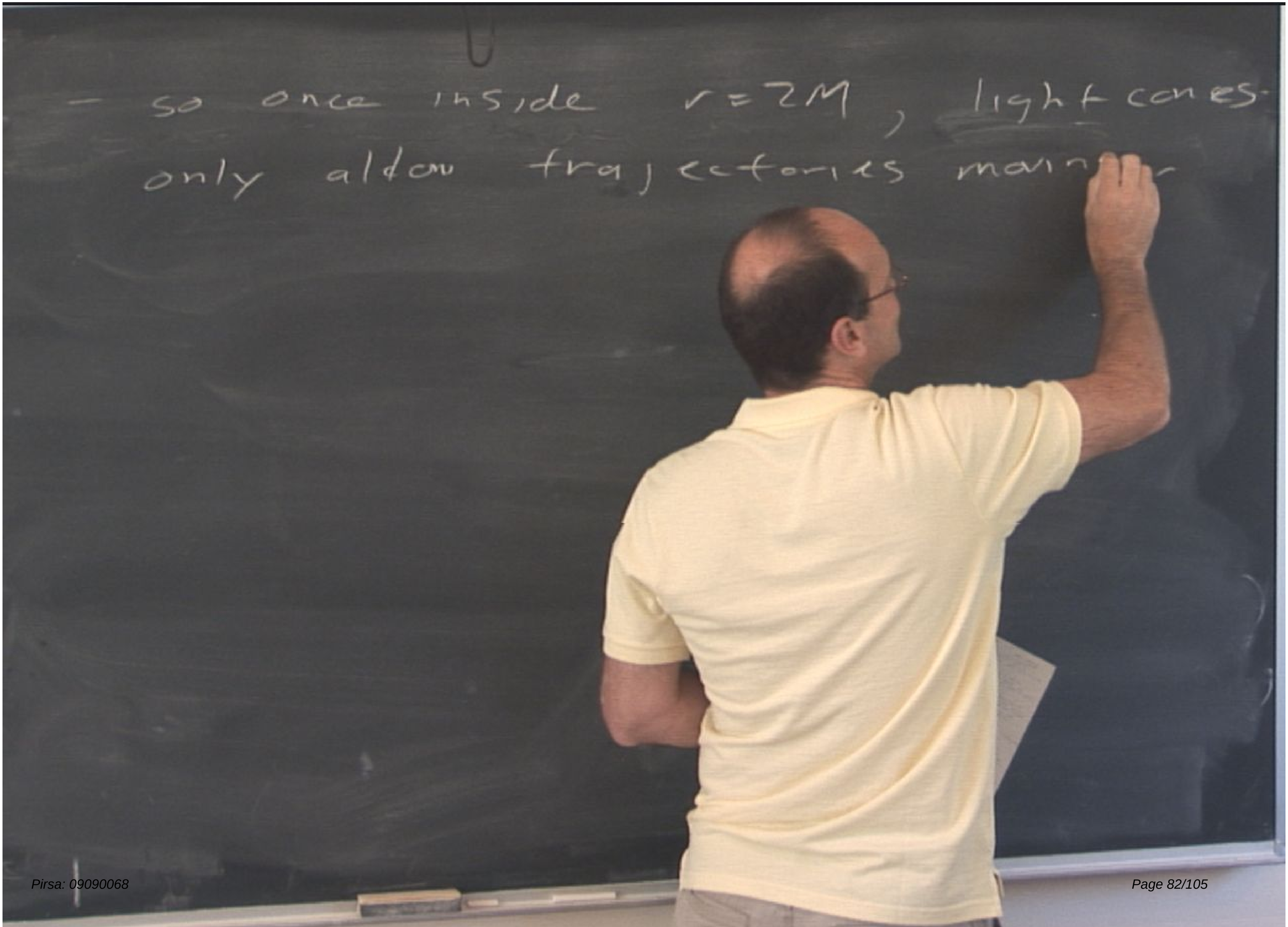
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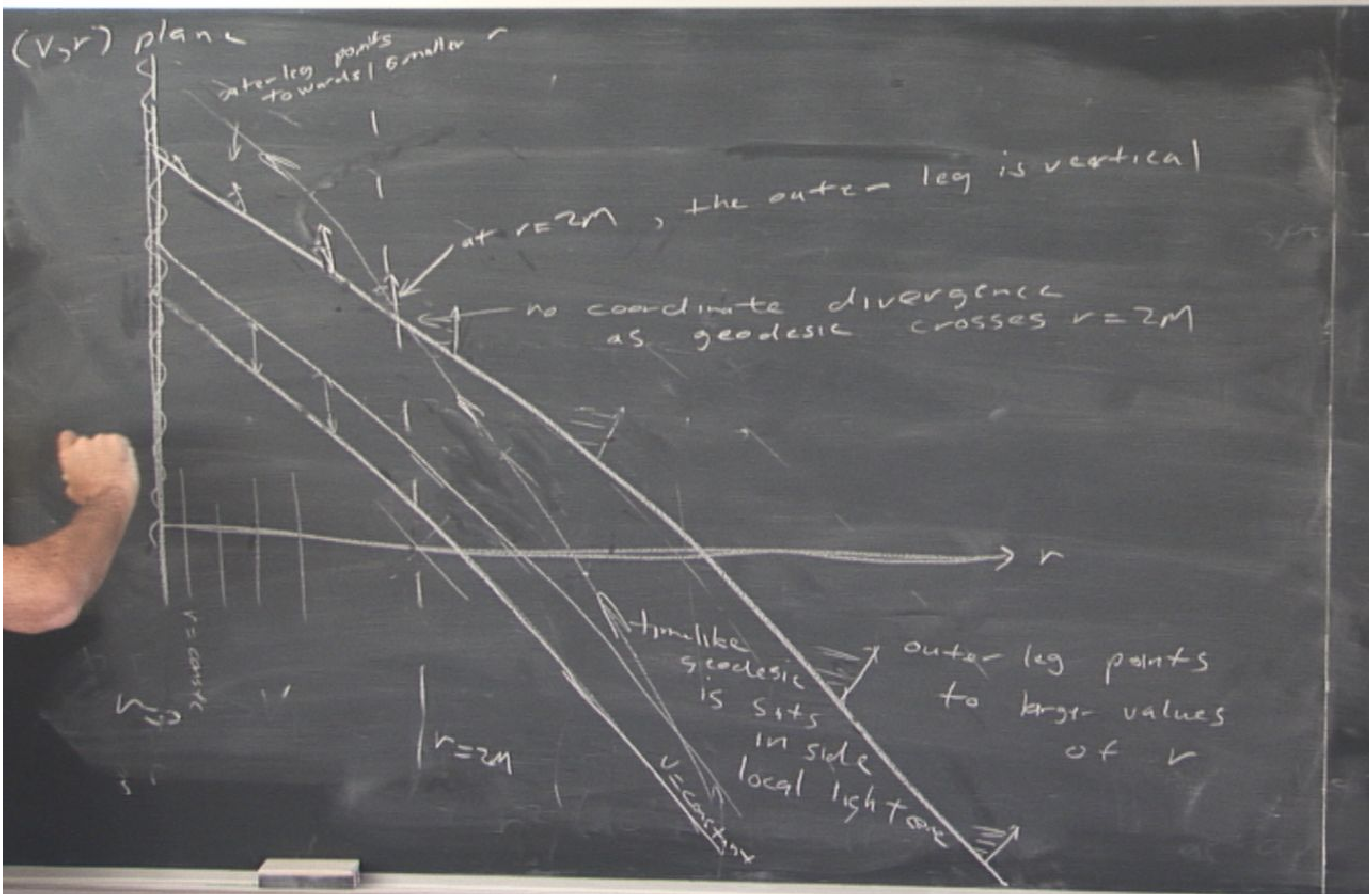
– so once inside $r = 2M$, light cones only allow trajectories moving to smaller values of $r \longrightarrow$ all signals / particles inevitably end up at $r = 0$

$\Longrightarrow g_{vv} \longrightarrow \infty$ @ $r \longrightarrow 0$

is this a coord singularity? No!!

real divergence in curvature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Kruskal coords

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$r, t$ replaced in terms of $U, V$

$$\left.\begin{aligned} U &= \left(\tfrac{r}{2M}-1\right)^{1/2} e^{r/4M} \cosh \tfrac{t}{4M} \\ V &= \left(\tfrac{r}{2M}-1\right)^{1/2} e^{r/4M} \sinh \tfrac{t}{4M} \end{aligned}\right\} r > 2M$$

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$$\left.\begin{aligned} U &= \left(1-\frac{r}{2M}\right)^{1/2} e^{r/4M} \sinh\left(\frac{t}{4M}\right) \\ V &= \left(1-\frac{r}{2M}\right)^{1/2} e^{r/4M} \cosh\left(\frac{t}{4M}\right) \end{aligned}\right\} r<2M$$

Schwarzschild metric becomes

$$ds^2 = \frac{32M^3}{r} e^{-r/2M}\left(-dV^2 + dU^2\right) + r^2\left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

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think of $r$ as a function of $U$ and $V$

$$U^2 - V^2 = \left(r/2m - 1\right) e^{r/2m}$$

$r = 2m \longrightarrow U = \pm V$

$r = 0 \longrightarrow U^2 - V^2 = -1$

time: $\tanh(t/4M) = V/U$ for $r > 2M$

$= U/V$ for $r < 2M$

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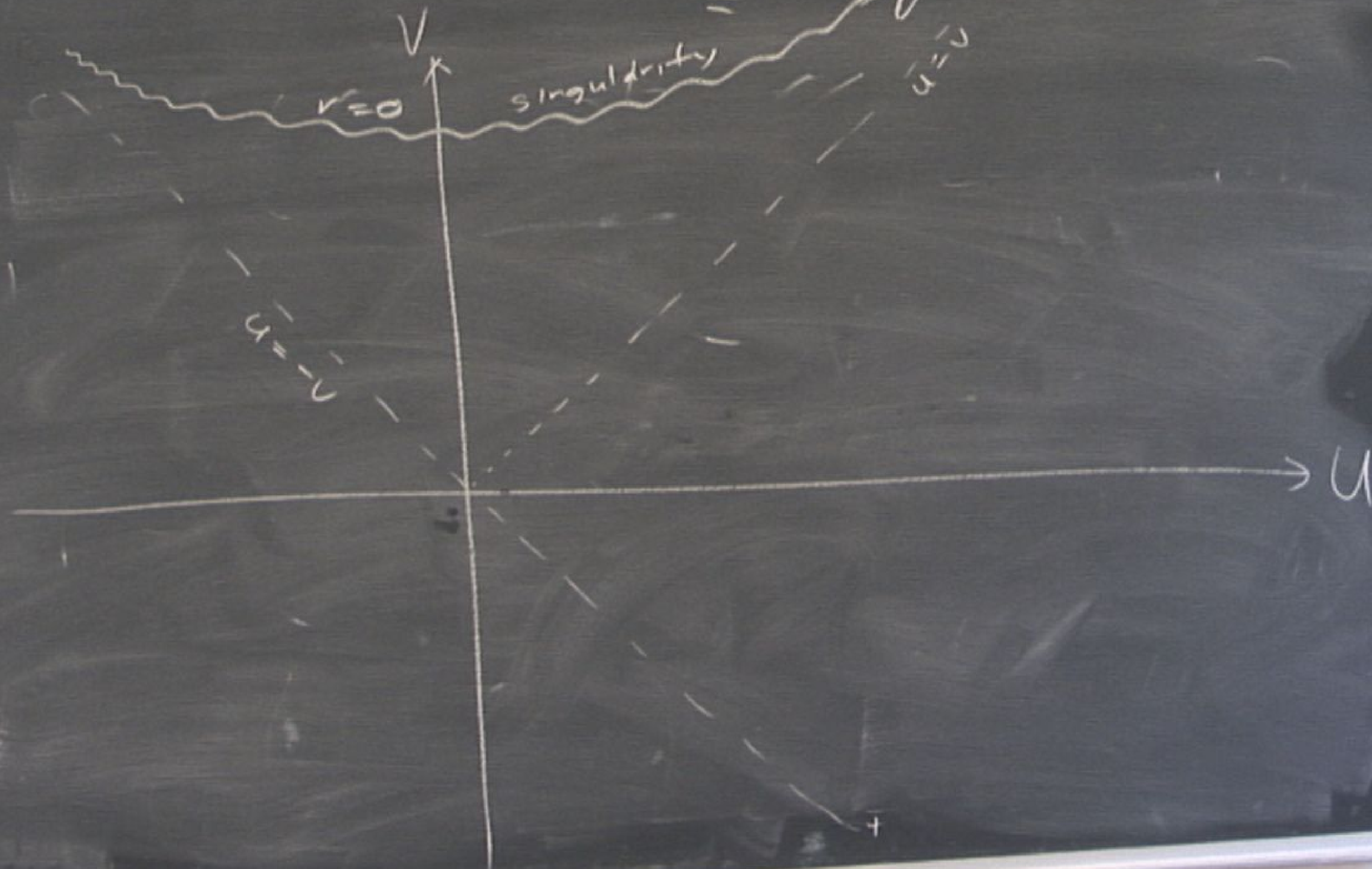

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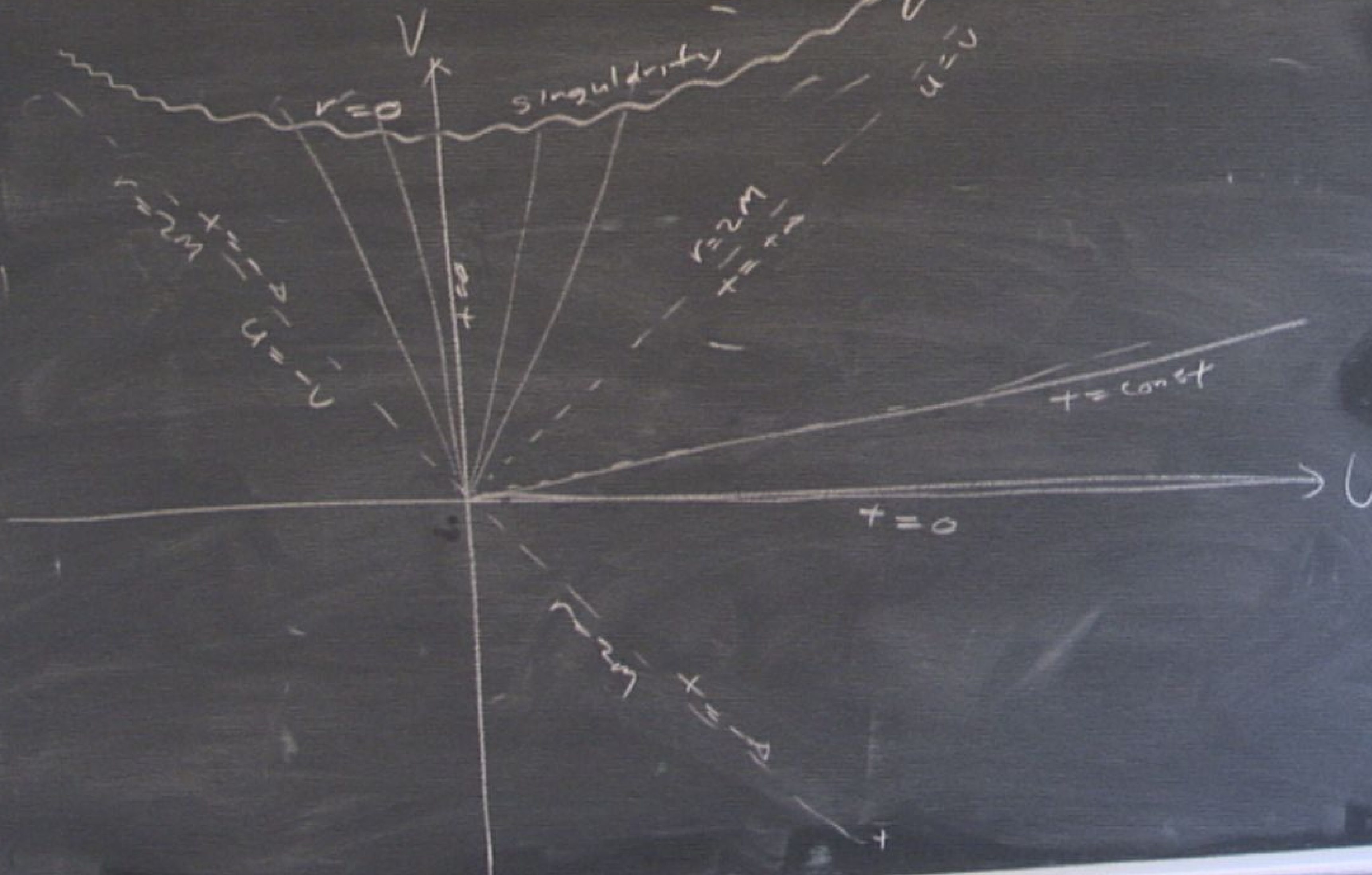

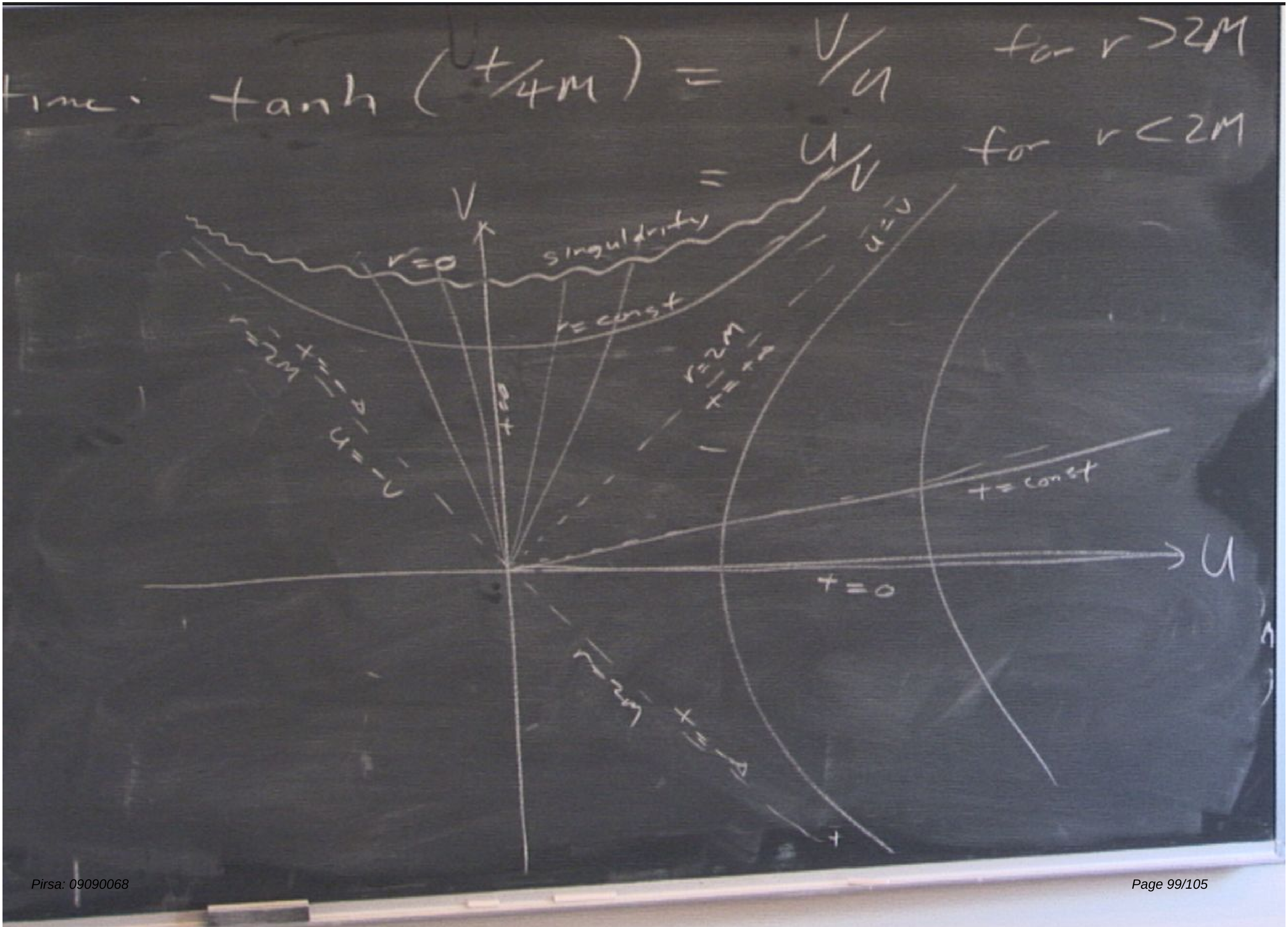
time: tanh(t/4M) = V/U for r>2M
= U/V for r<2M
V
U
r=0
singularity
r = const
u=v
r=2M
t=+∞
r=2M
t=−∞
u=−v
t
t = const
t=0
r=2M
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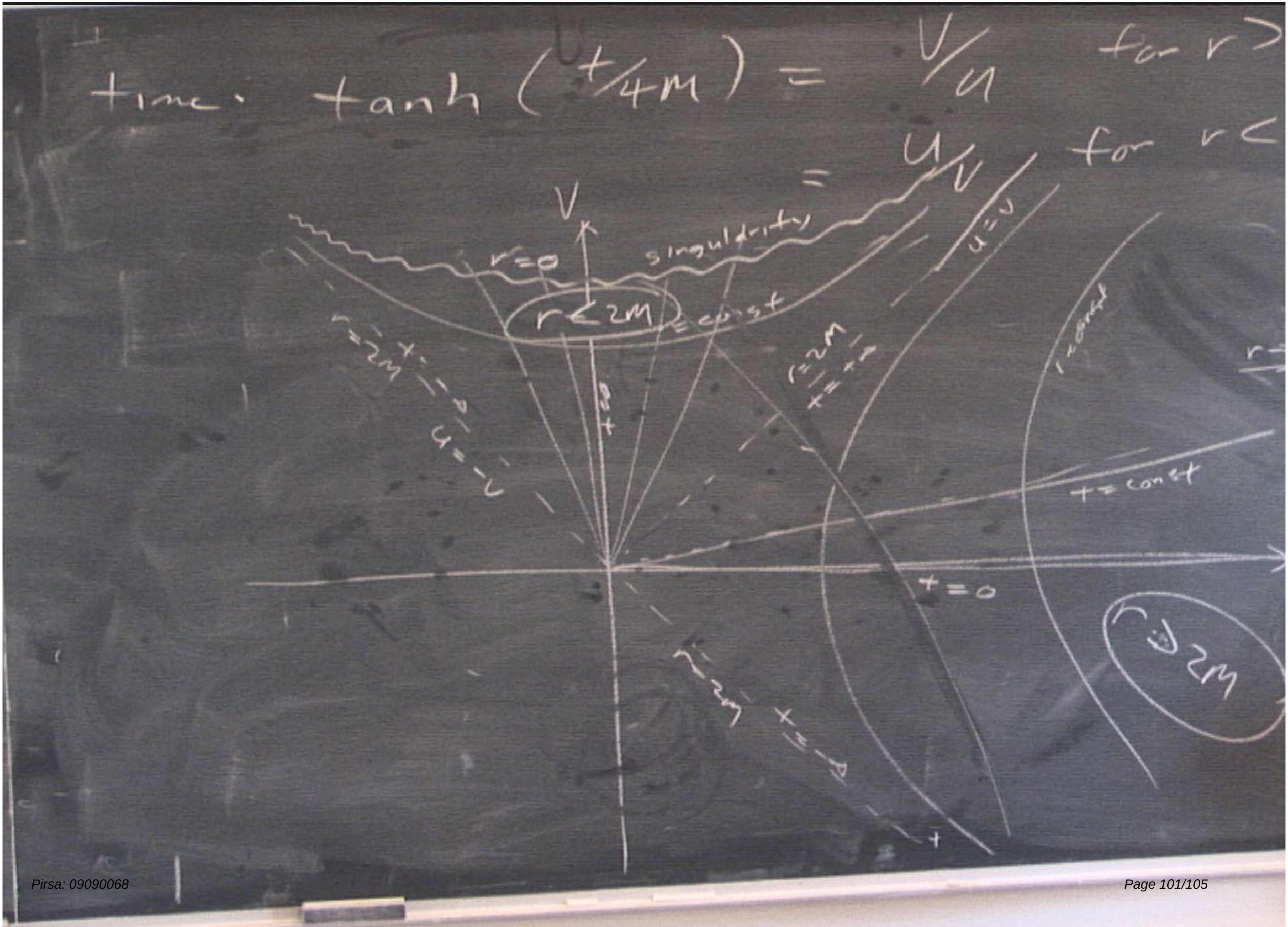
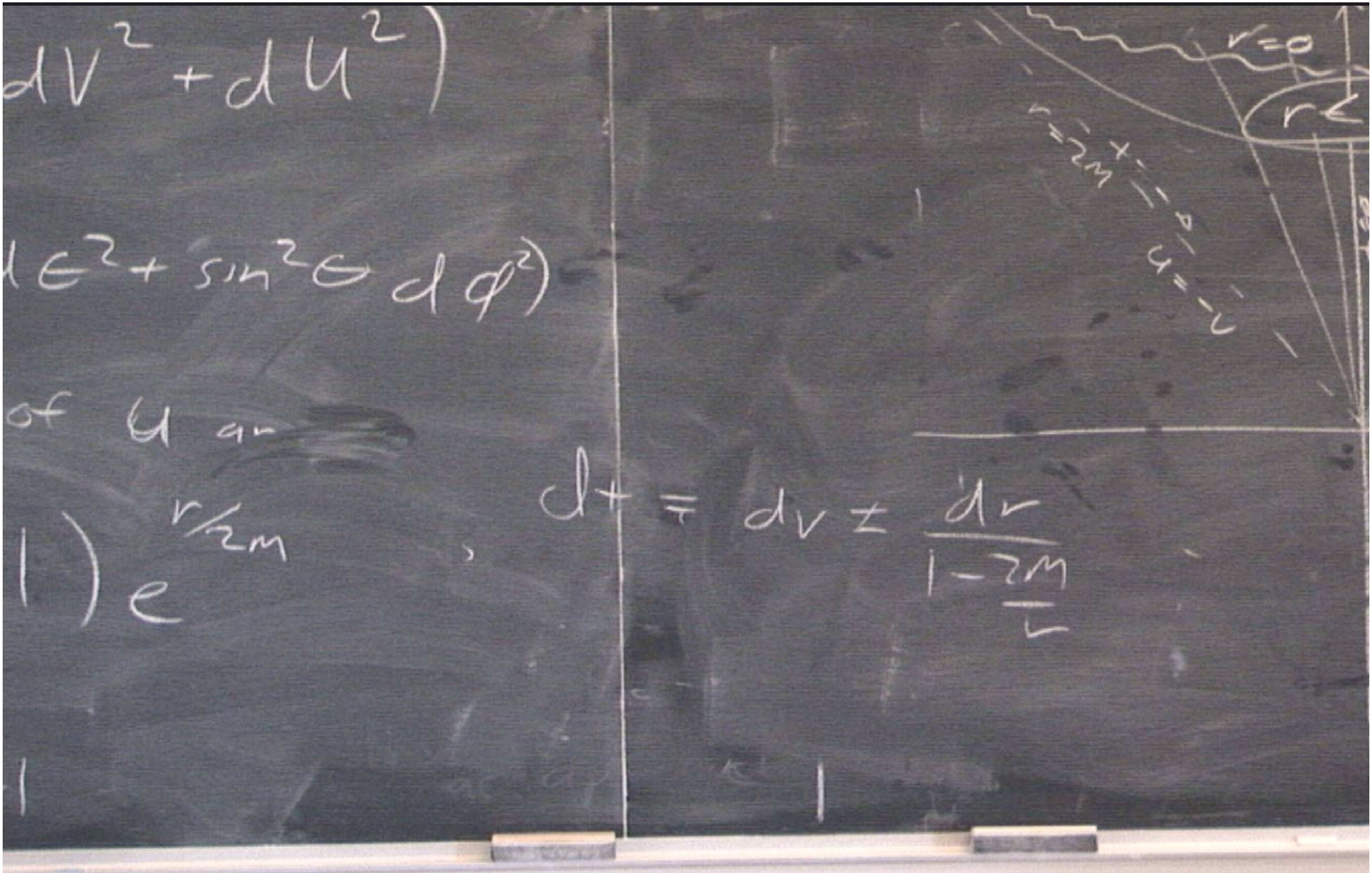

time: tanh (t/4M) = V/U for r >
= U/V for r <
V
singularity
r = 0
r < 2M
= const
U = V
r = 2M
t = +∞
r = 2M
t = −∞
U = −V
t = const
t = 0
r > 2M
r = 2M
t = −∞


dV² + dU²)
dθ² + sin²θ dφ²)
of U an
1) e^{r/2m}
dt ∓ = dv ± dr / (1 − 2m/r)
r=0

$^{r/2m}\left(-dV^2+dU^2\right)$

$+r^2\left(d\theta^2+\sin^2\theta\, d\phi^2\right)$

function of U an

$\sqrt{1-\frac{2m}{r}}\, dt = dv \pm \frac{dr}{1-\frac{2m}{r}}$

$\left(r/2m - 1\right)e^{r/2m}$

$= \pm V$

$^2 - V^2 = -1$

$$\frac{\ }{2m}\left(-dV^2+dU^2\right)$$

$$+r^2\left(d\theta^2+\sin^2\theta\, d\phi^2\right)$$

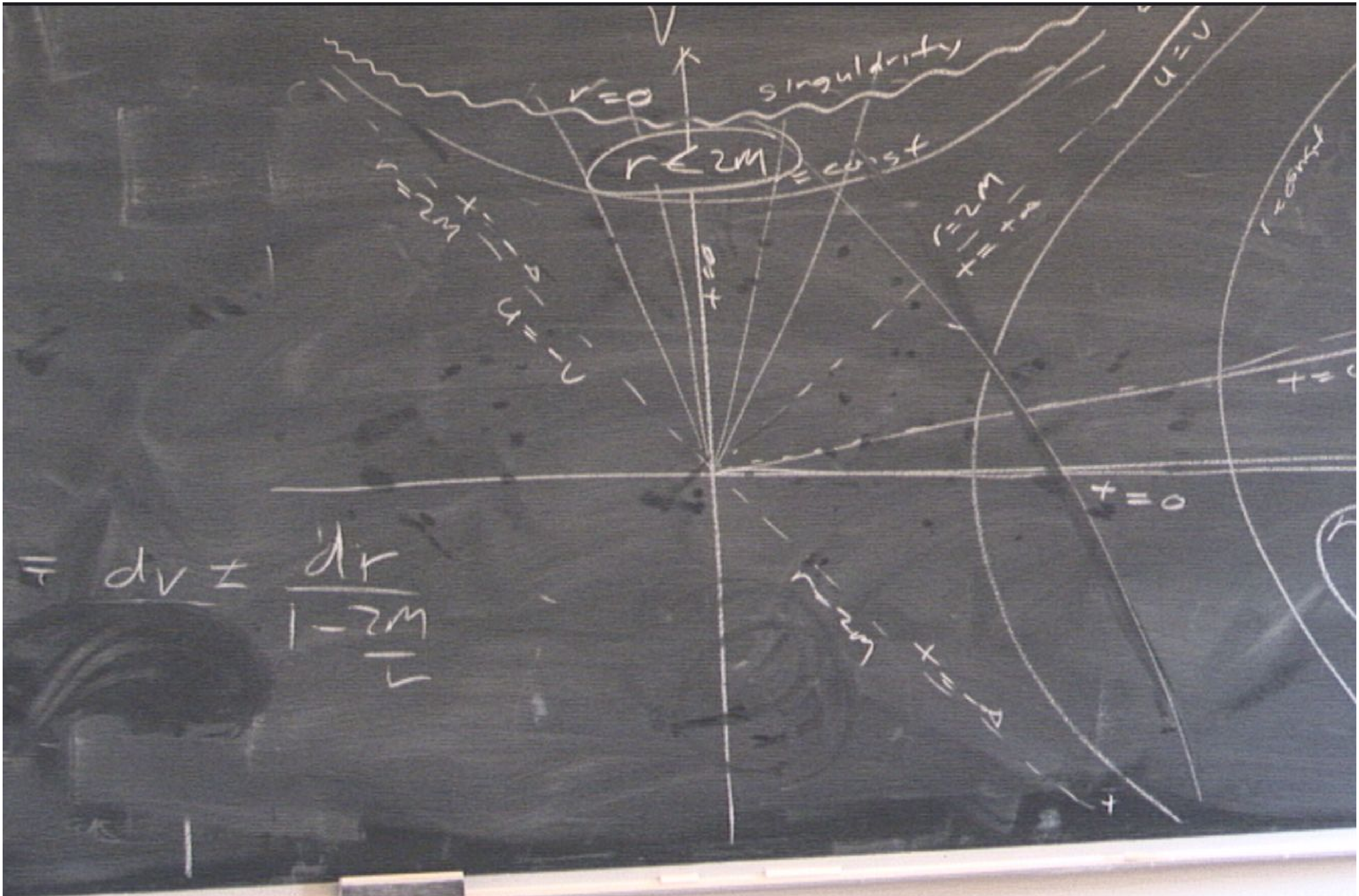
function of $U$ an

$$\sqrt{1-\frac{2m}{r}}\, dt \mp dv \pm \frac{dr}{1-\frac{2m}{r}}$$

$$\left(\frac{r}{2m}-1\right)e^{r/2m}$$

$$\pm V$$

$$-V^2=-1$$


singularity
r=0
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$\mp\, dv \pm \frac{dr}{1-\frac{2M}{r}}$