Title: PSI - Relativity (PHYS 604) - 10

Date: Sep 16, 2009 10:30 AM

URL: http://pirsa.org/09090067

Abstract:

Photon orbits

$$\underset{\sim}{\xi} \cdot \underset{\sim}{u} = -e \longrightarrow \frac{dt}{d\lambda} = \frac{e}{1 - 2M/r}$$

$$\underset{\sim}{\eta} \cdot \underset{\sim}{u} = \ell \longrightarrow \frac{d\phi}{d\lambda} = \frac{\ell}{r^2 \sin^2\theta}$$

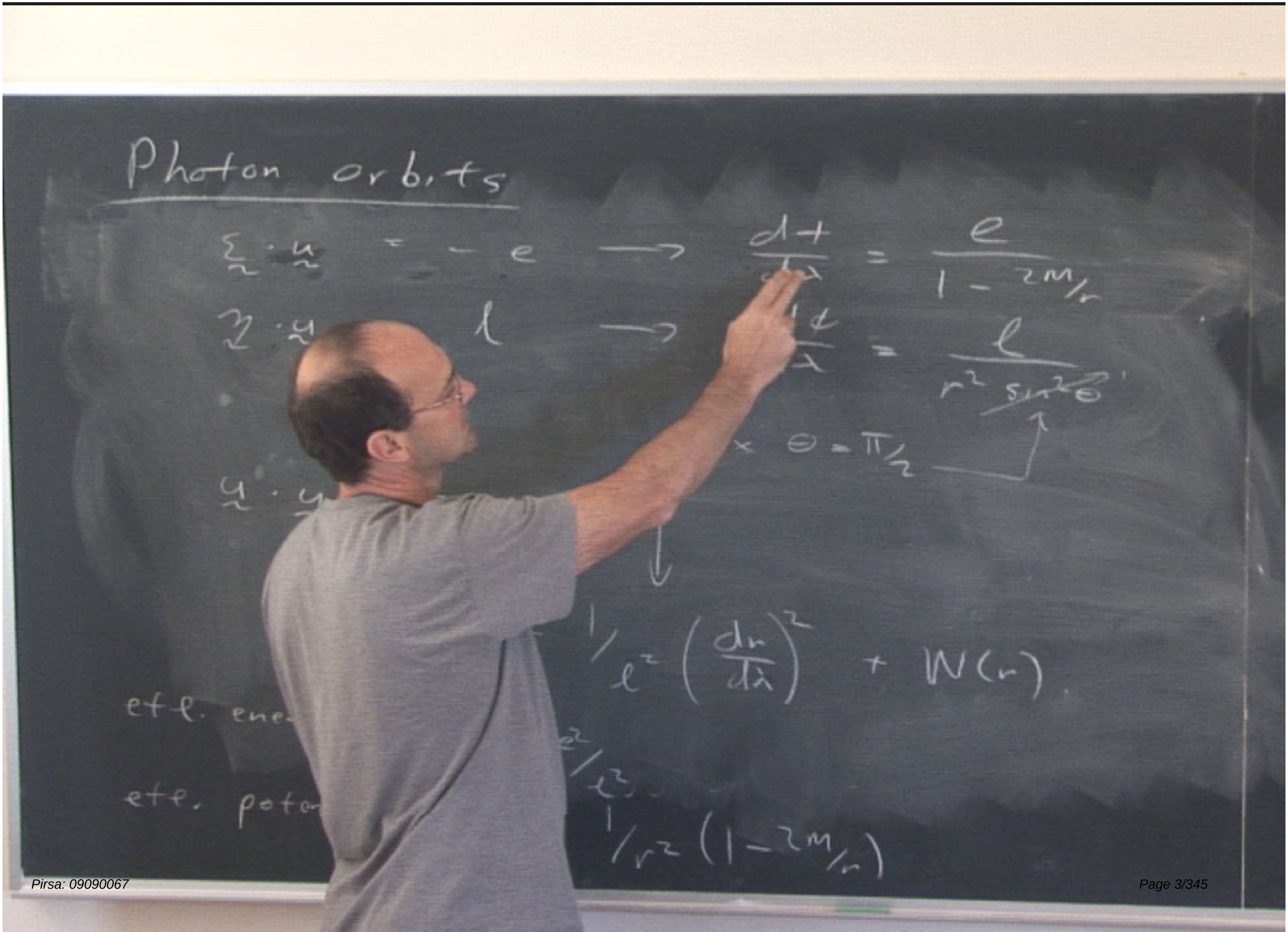
fix $\theta = \pi/2$

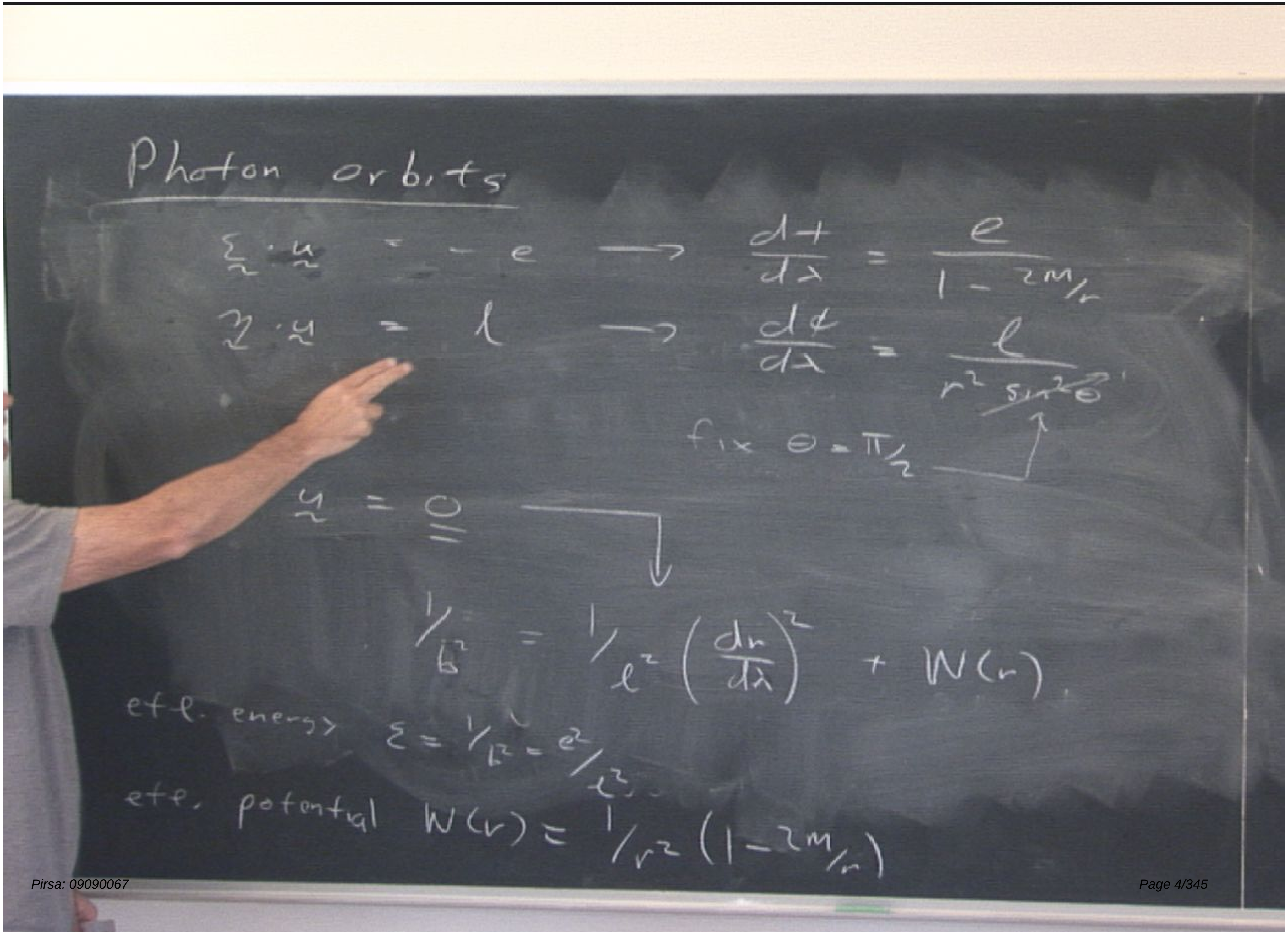
$$\underset{\sim}{u} \cdot \underset{\sim}{u} = \underline{\underline{0}}$$

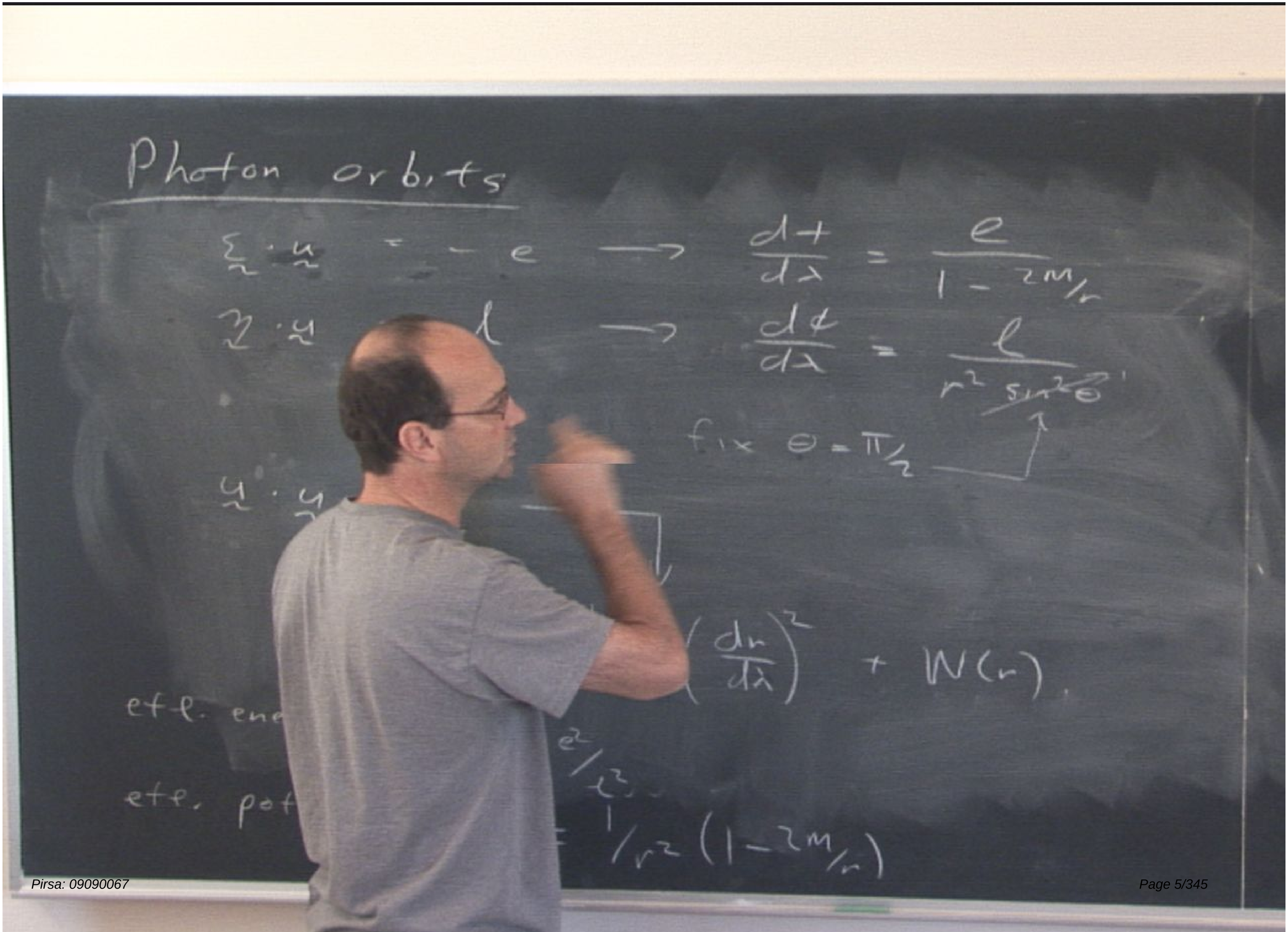
$$\frac{1}{b^2} = \frac{1}{\ell^2}\left(\frac{dr}{d\lambda}\right)^2 + W(r)$$

eff. energy $\mathcal{E} = 1/b^2 = e^2/\ell^2$

eff. potential $W(r) = \frac{1}{r^2}\left(1 - 2M/r\right)$


Photon orbits
eff. ene
eff. poten


Photon orbits
ξ · u = − e → dt/dλ = e / (1 − 2M/r)
η · u = l → dφ/dλ = l / (r² sin²θ)
fix θ = π/2
u · u = 0
1/b² = 1/l² (dr/dλ)² + W(r).
eff. energy ε = 1/b² = e²/l²
eff. potential W(r) = 1/r² (1 − 2M/r)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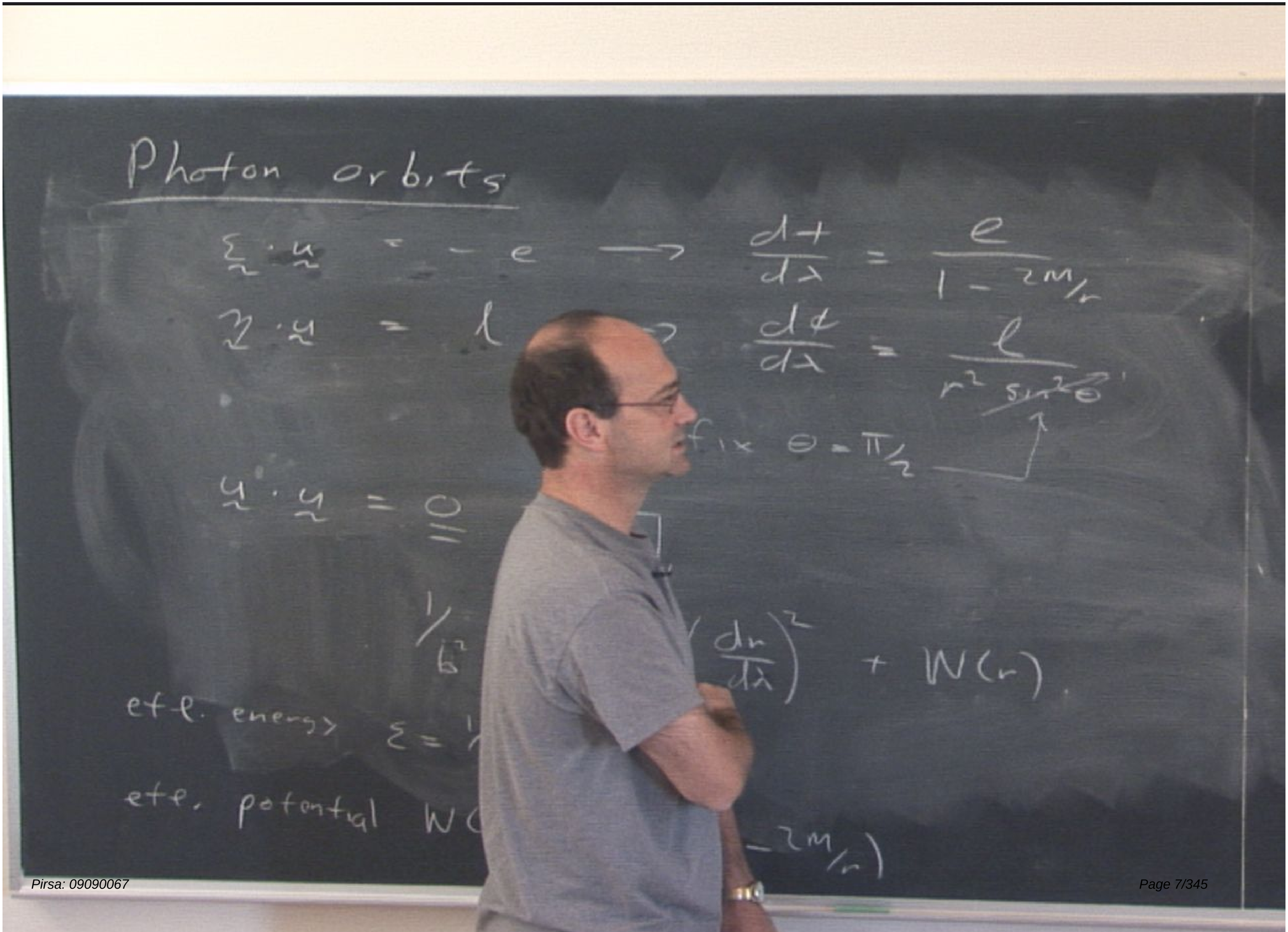
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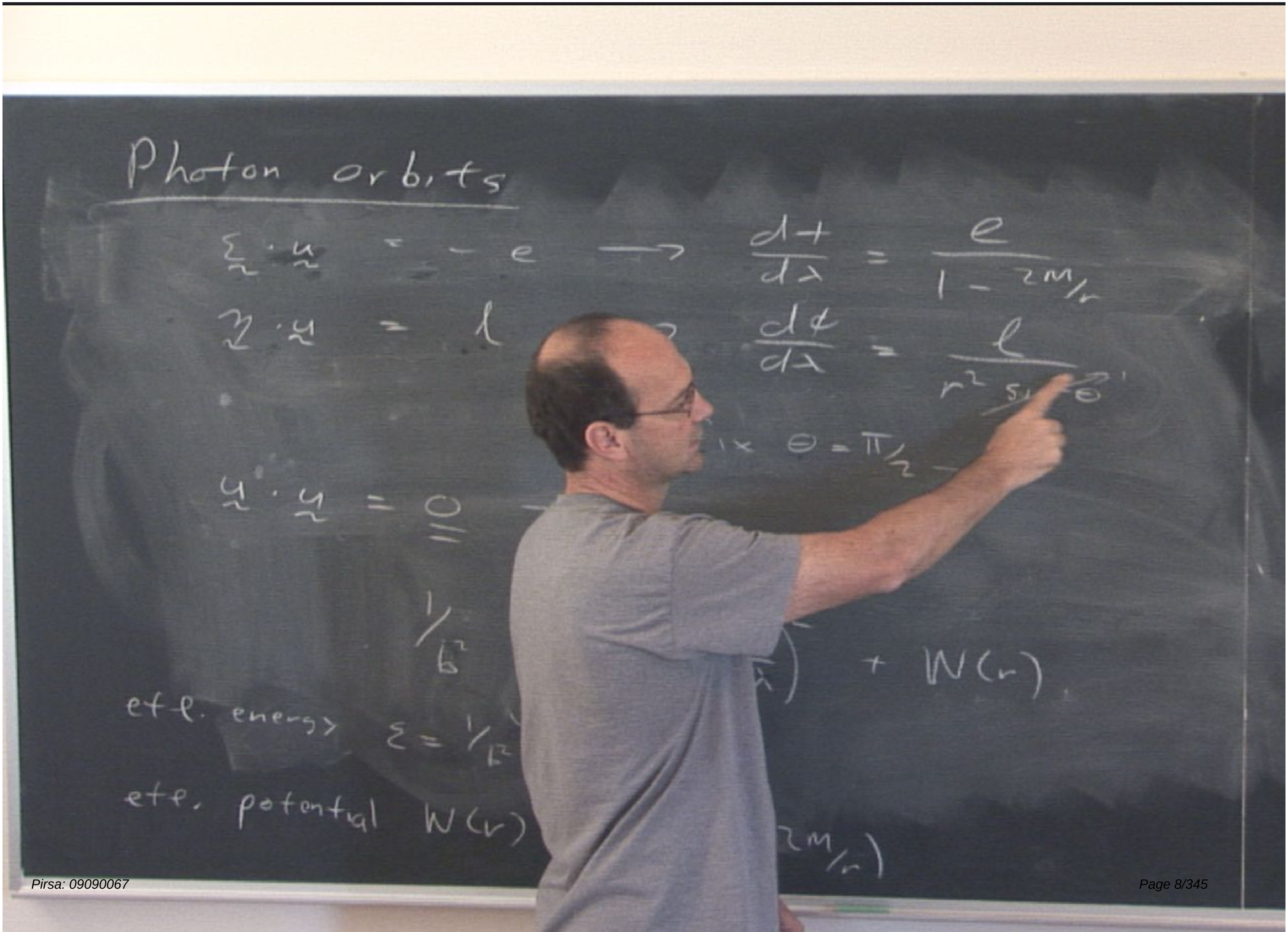
1/b²
(dr/dλ)² + W(r)
eff. energy ℰ = ½
eff. potential W
− 2M/r)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1/b²
+ W(r)
eff. energy ℰ = 1/b²
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2M/r)

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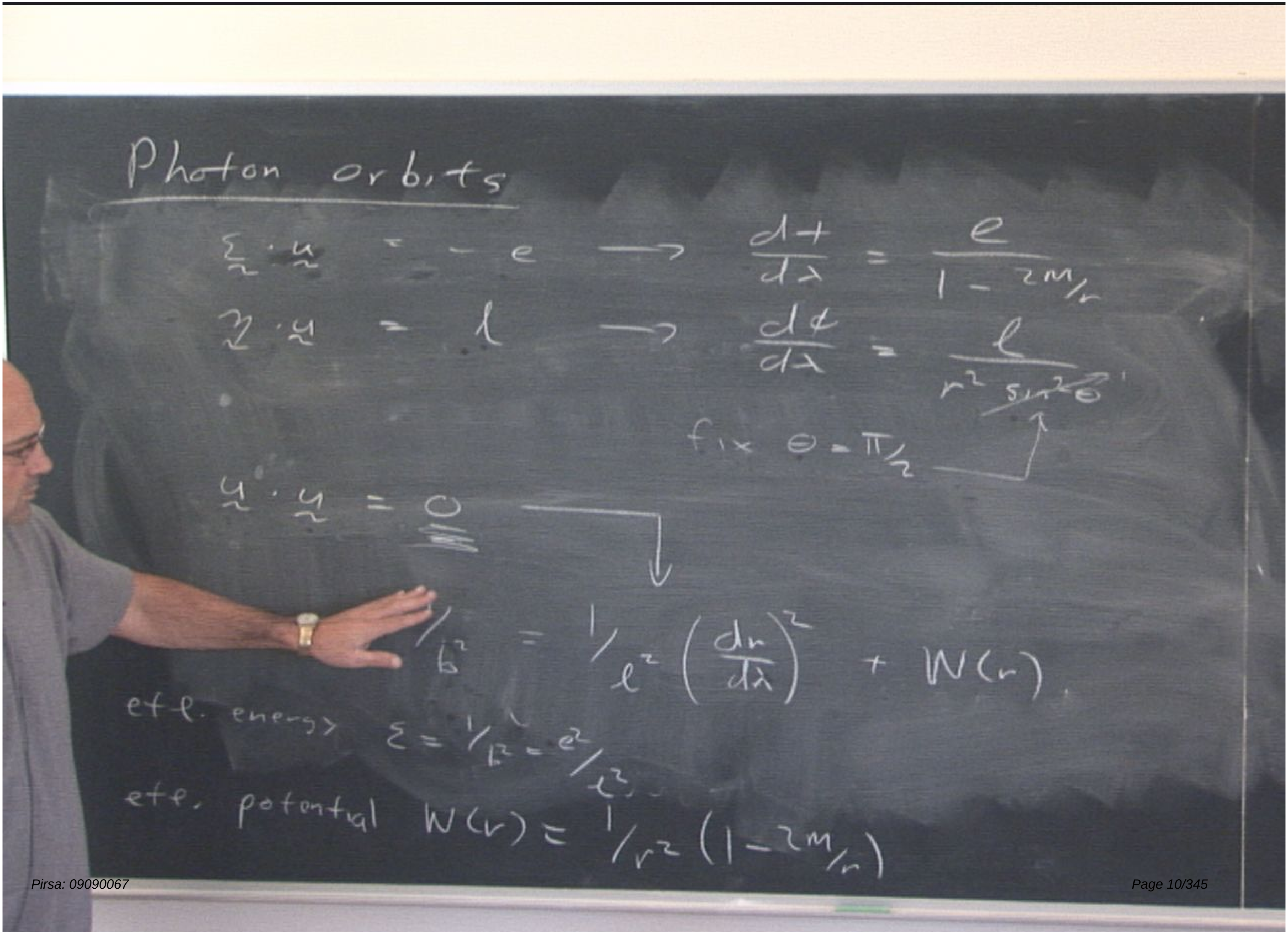
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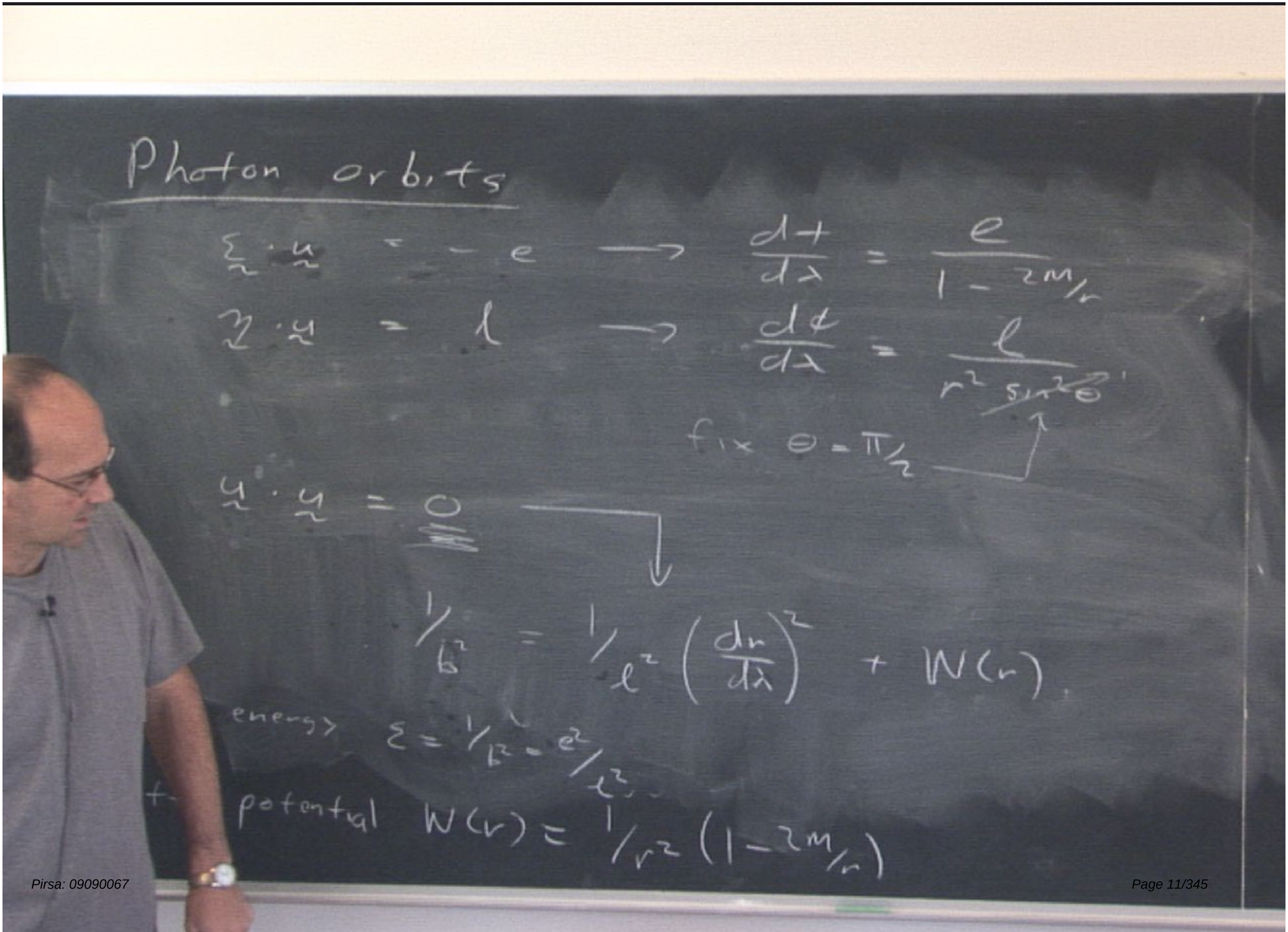
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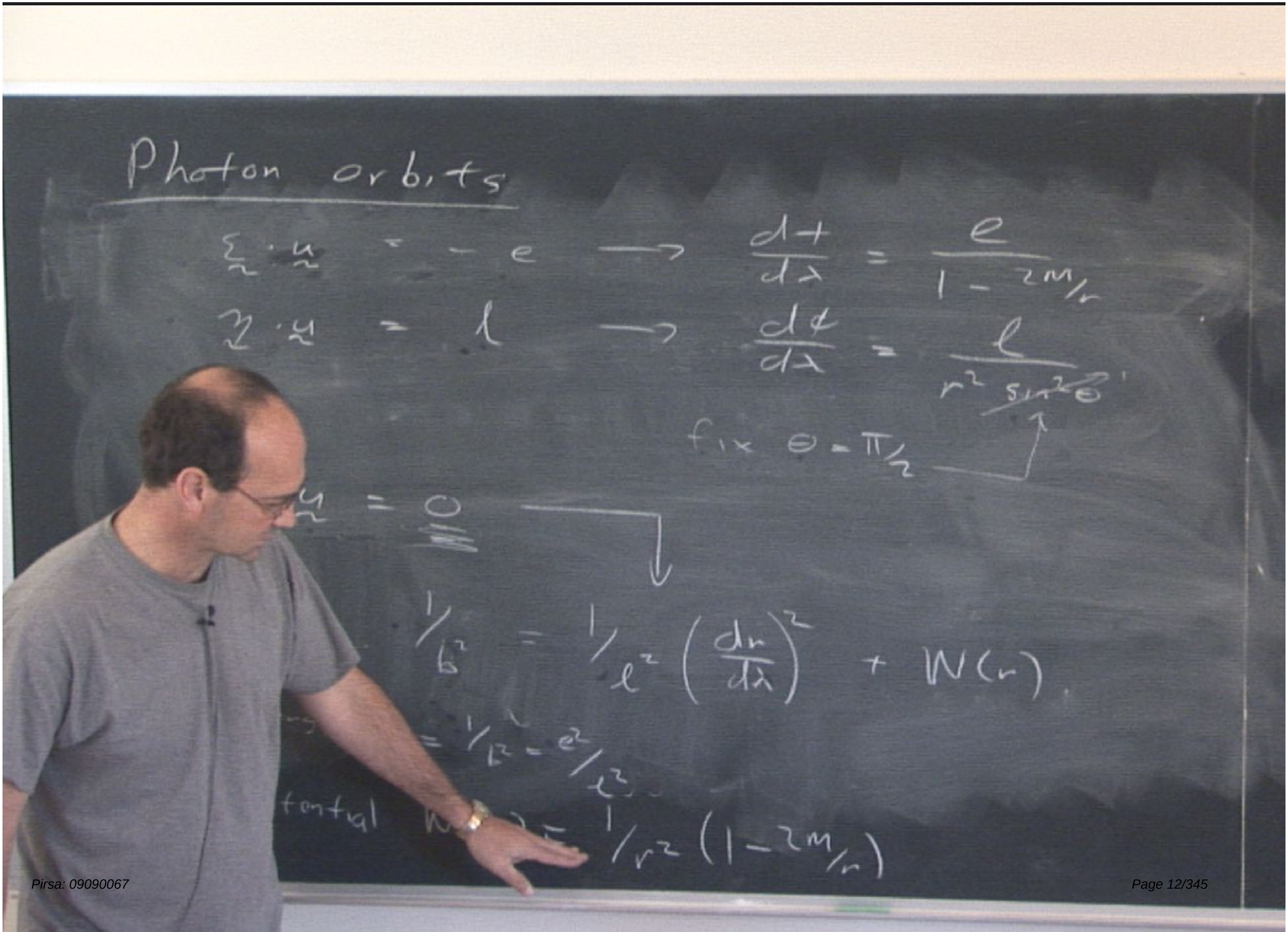
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1/b² = 1/l² (dr/dλ)² + W(r)
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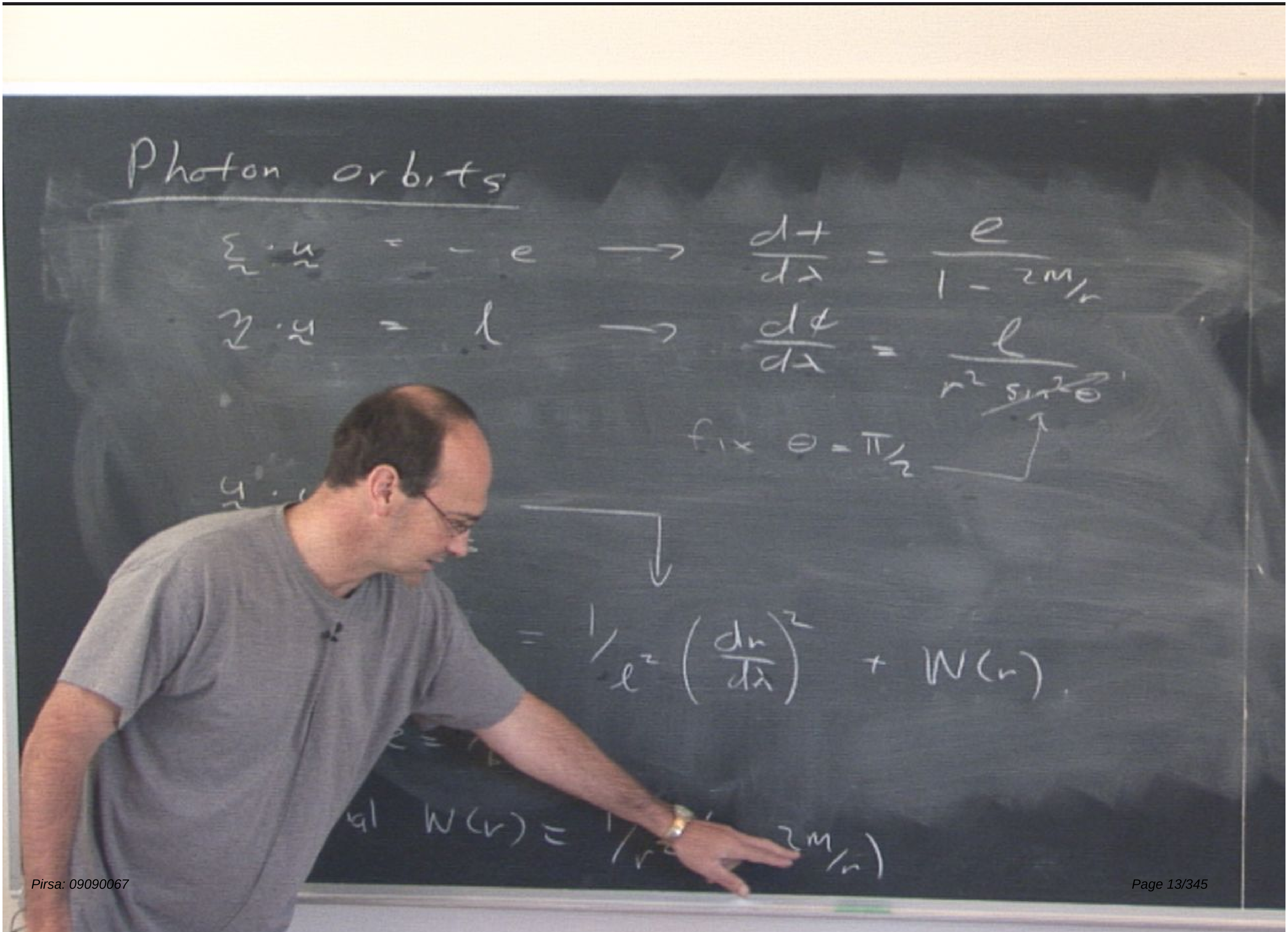
tential W(r) = 1/r² (1 − 2M/r)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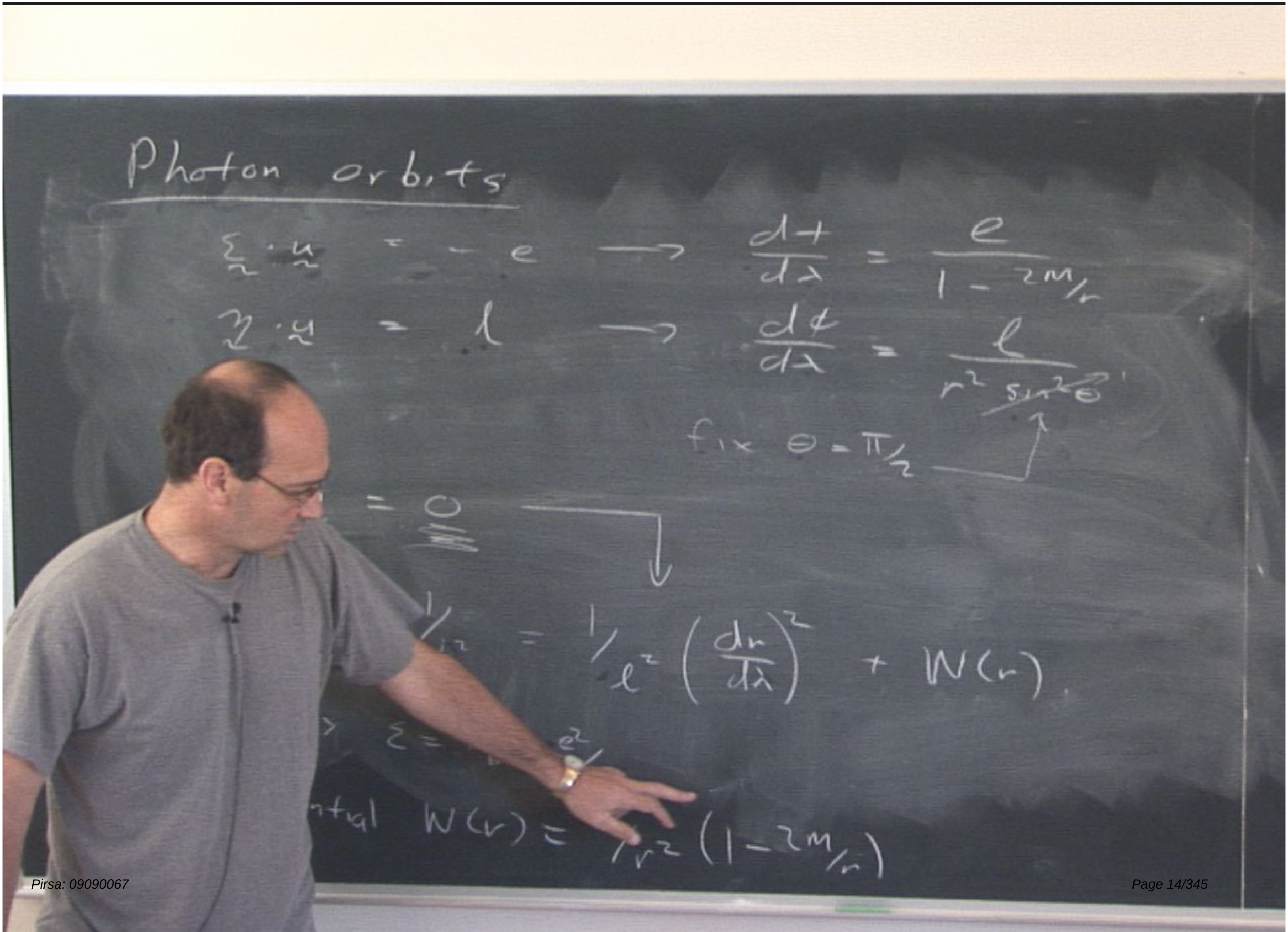
fix θ = π/2
= 1/l² (dr/dλ)² + W(r)
W(r) =


Photon orbits
fix θ = π/2

Photon orbits

$$\xi \cdot u = -e \longrightarrow \frac{dt}{d\lambda} = \frac{e}{1 - 2M/r}$$

$$\eta \cdot u = \ell \longrightarrow \frac{d\phi}{d\lambda} = \frac{\ell}{r^2 \sin^2\theta}$$

fix $\theta = \pi/2$

$$u \cdot u = 0 \longrightarrow \frac{1}{b^2} = \frac{1}{\ell^2}\left(\frac{dr}{d\lambda}\right)^2 + W(r)$$

eff. energy $\mathcal{E} = 1/b^2 = e^2/\ell^2$

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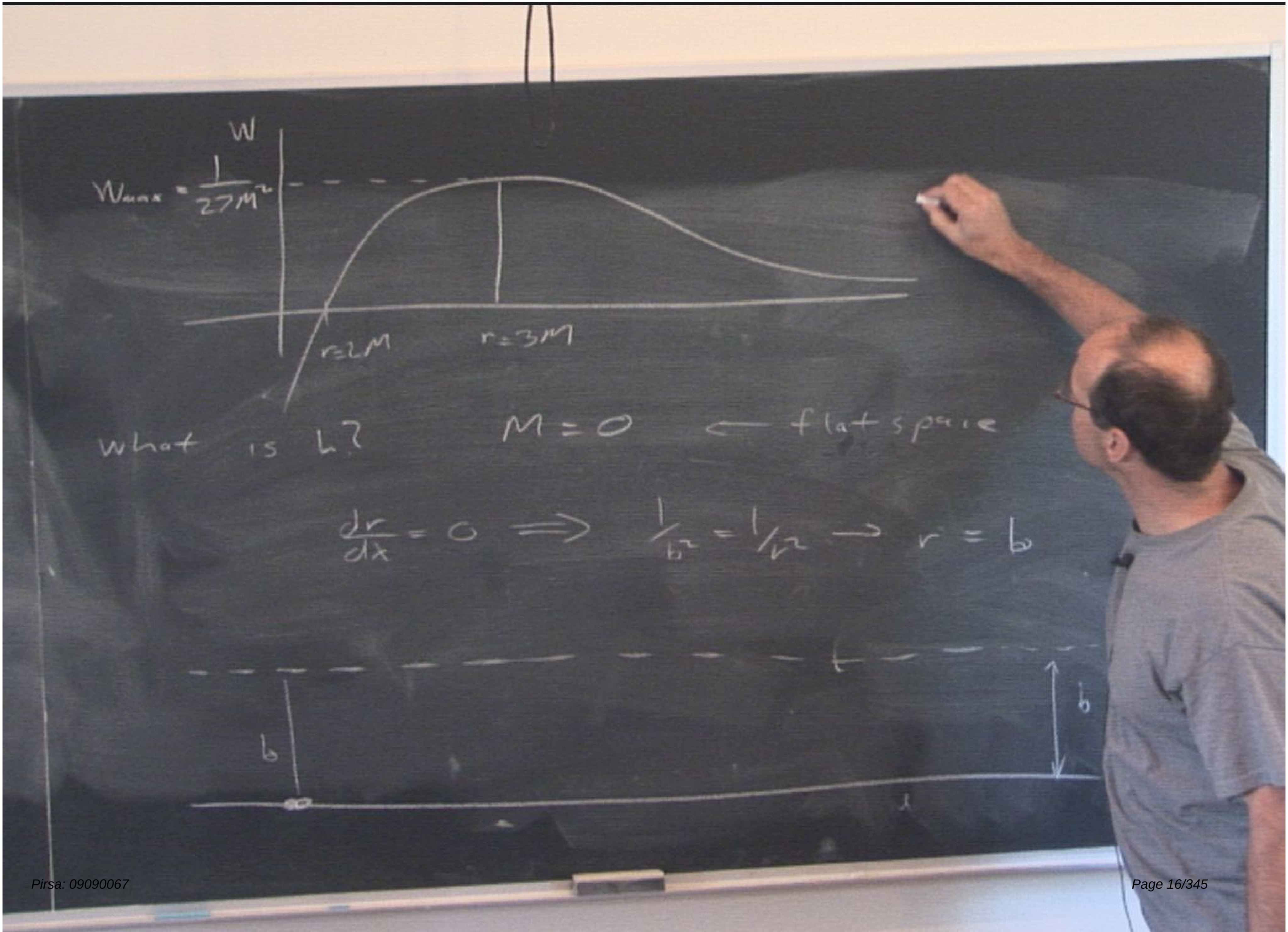
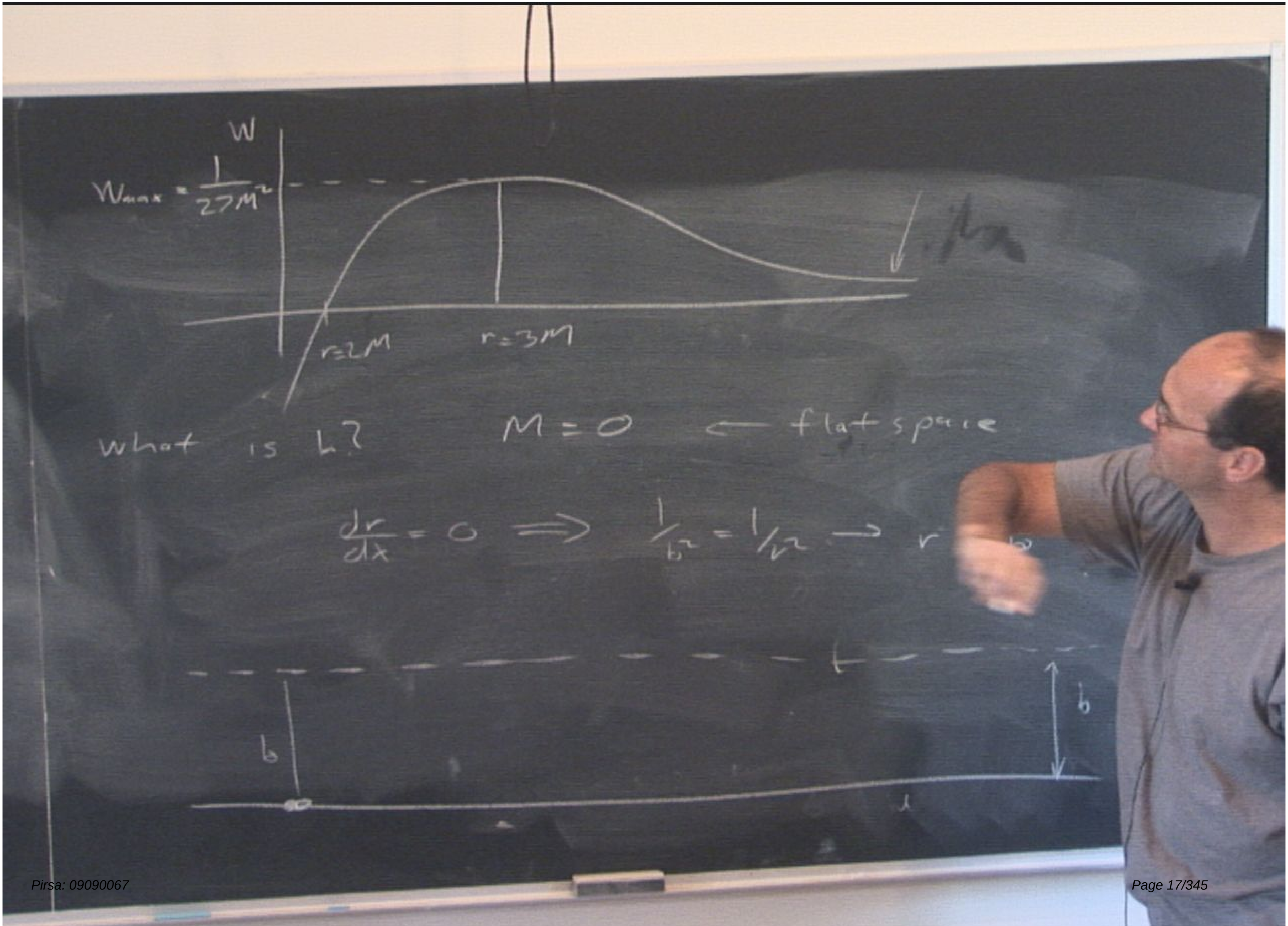
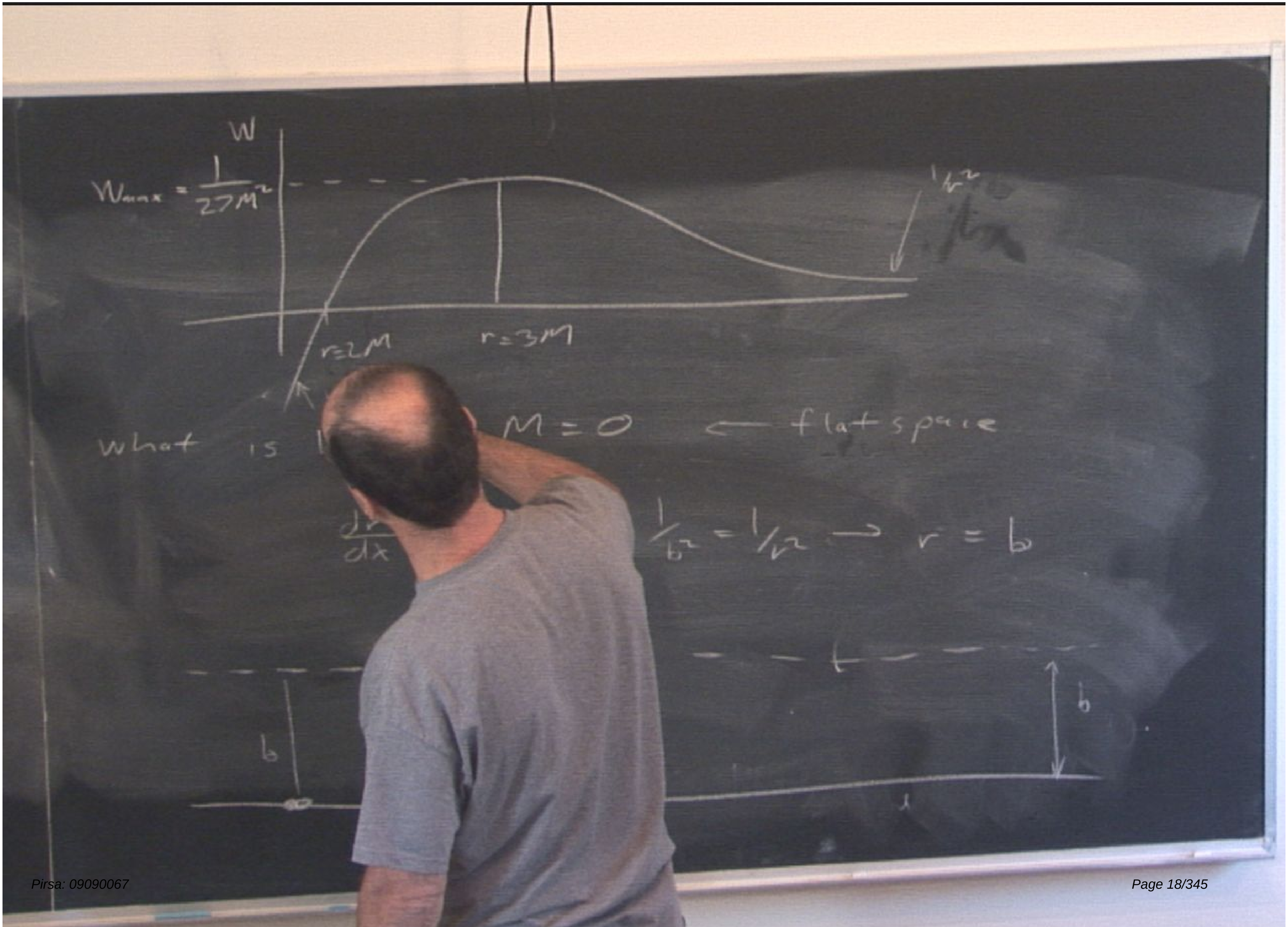

W
W max = 1/27M^2
r=2M
r=3M
what is b?
M=0 ← flat space
dr/dx = 0 ⟹ 1/b^2 = 1/r^2 → r = b
b
b


W
Wmax = 1/27M²
r=2M
r=3M
what is L?
M = 0 ← flat space
dr/dλ = 0 ⟹ 1/b² = 1/r² → r
b
b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Wmax = 1/27M²
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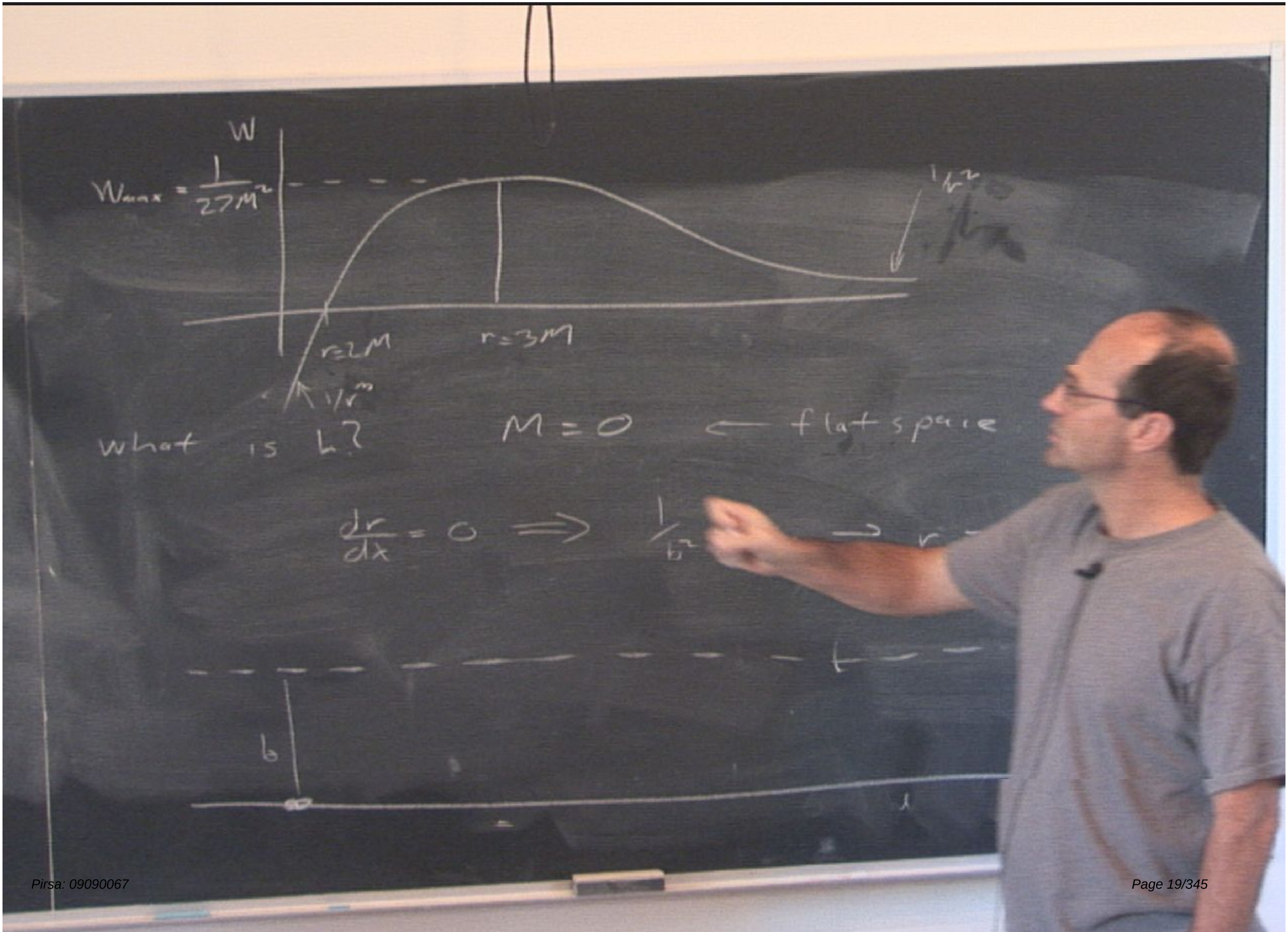
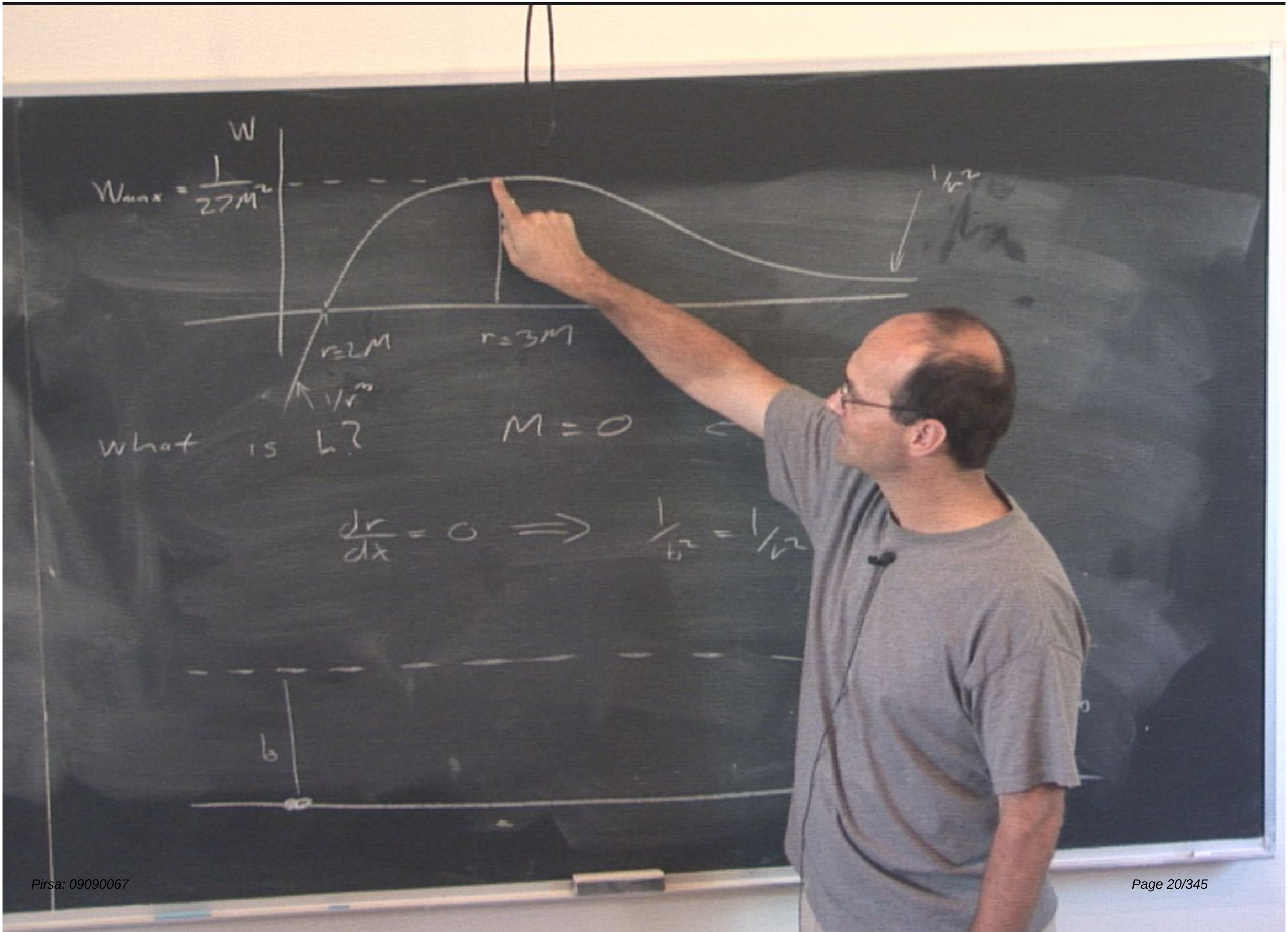
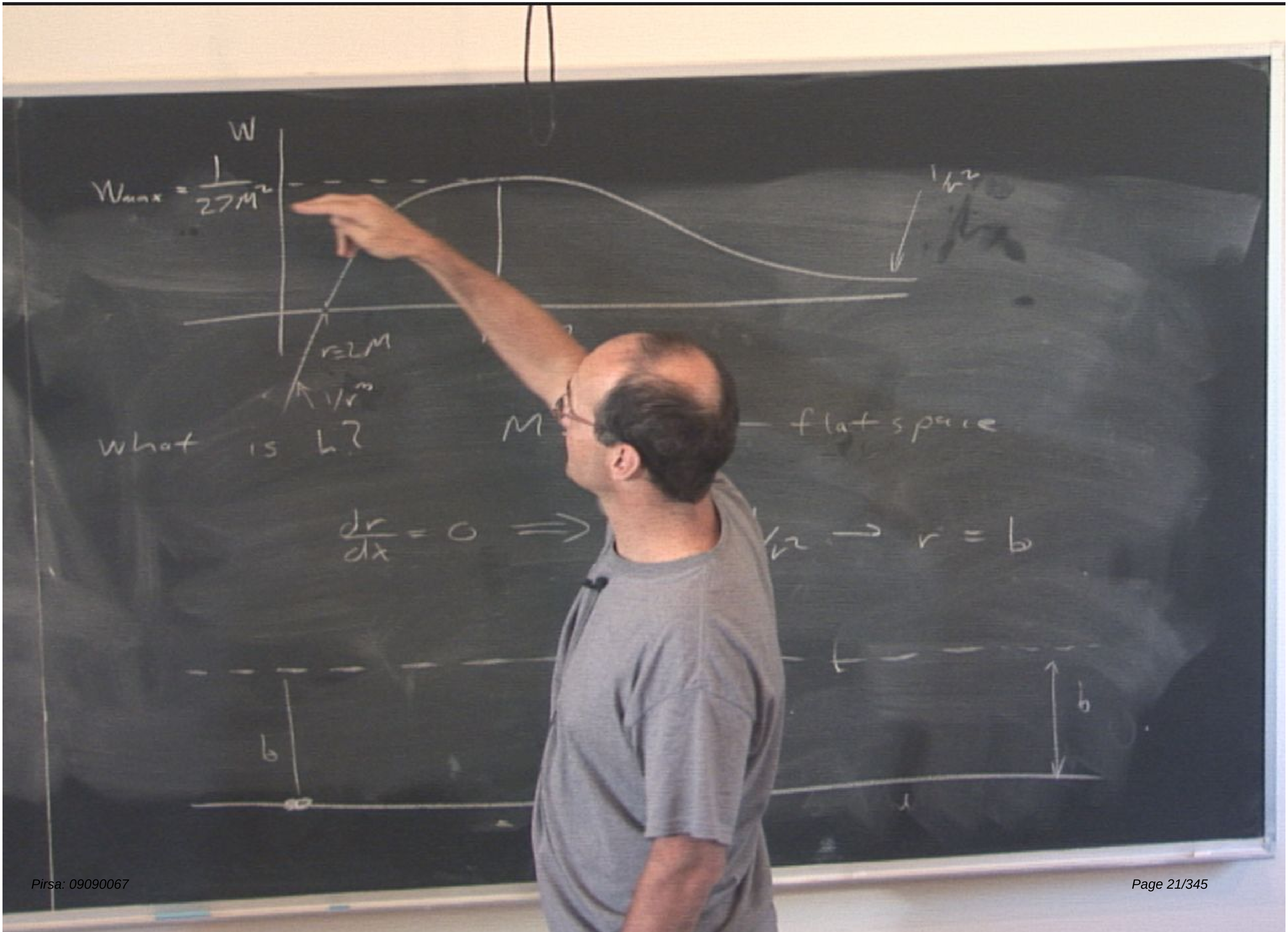
r=3M
what is
M = 0 ← flat space
1/b² = 1/r² → r = b
b
b


W
Wmax = 1/27M²
r=2M
r=3M
what is b?
M = 0 ← flat space
dr/dx = 0 ⟹ 1/b²
b
λ


W
Wmax = 1/27M²
r=2M
r=3M
1/r²
what is L?
M = 0
dr/dx = 0 ⟹ 1/b² = 1/r²
b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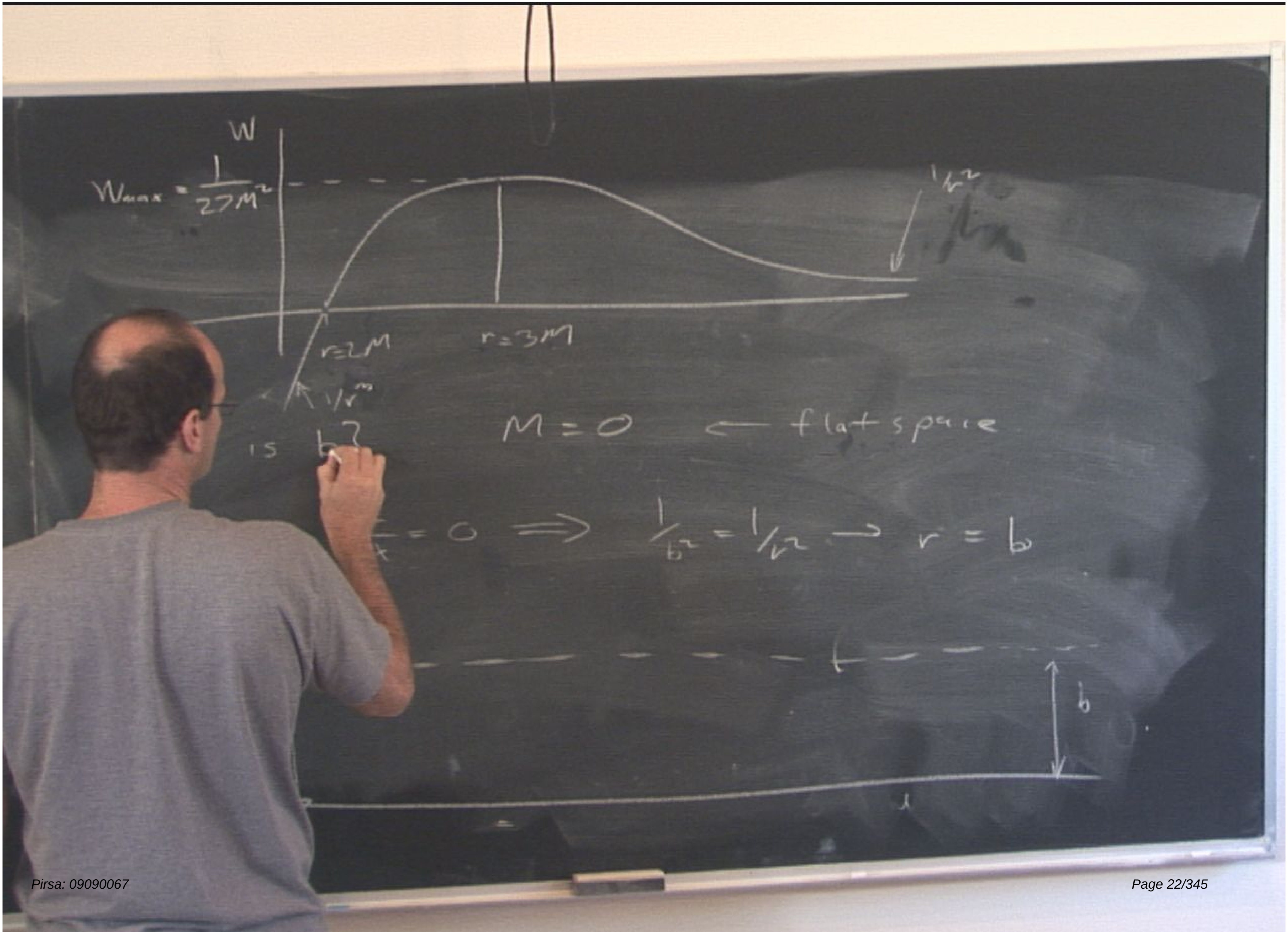
M
flat space
dr/dx = 0 ⟹
r = b
b
b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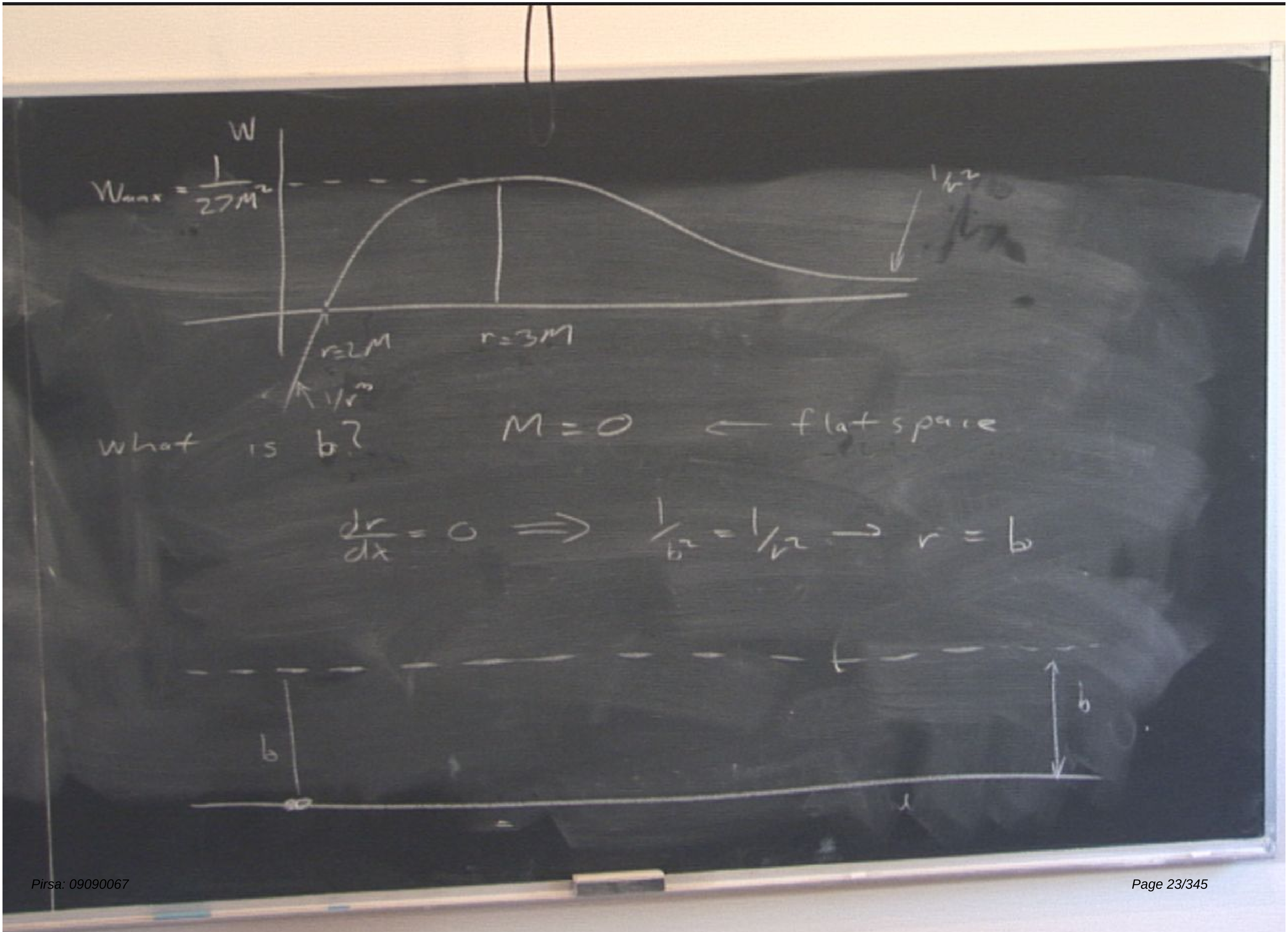
r=3M
M = 0 ← flat space
r = b
b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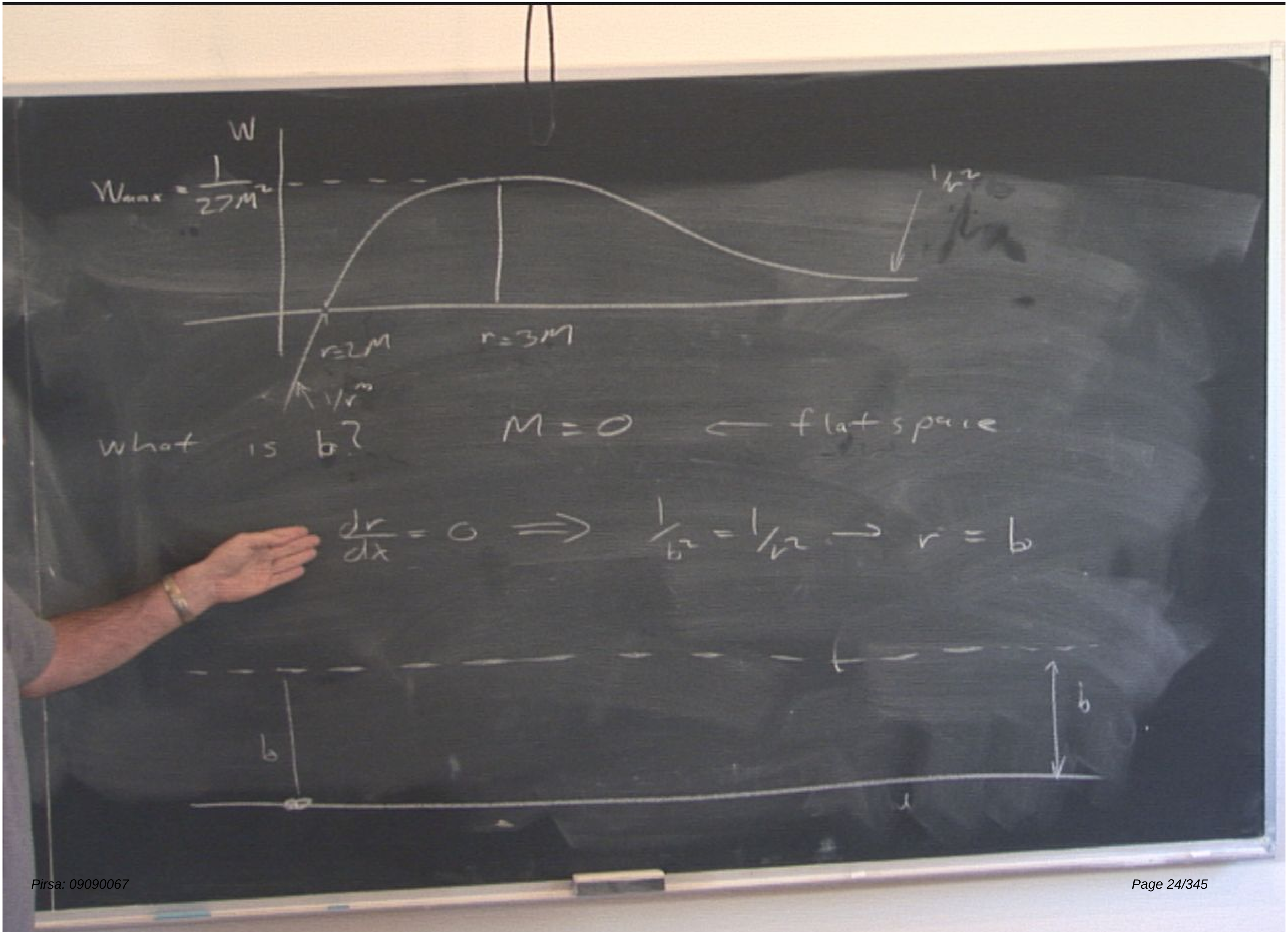
b
b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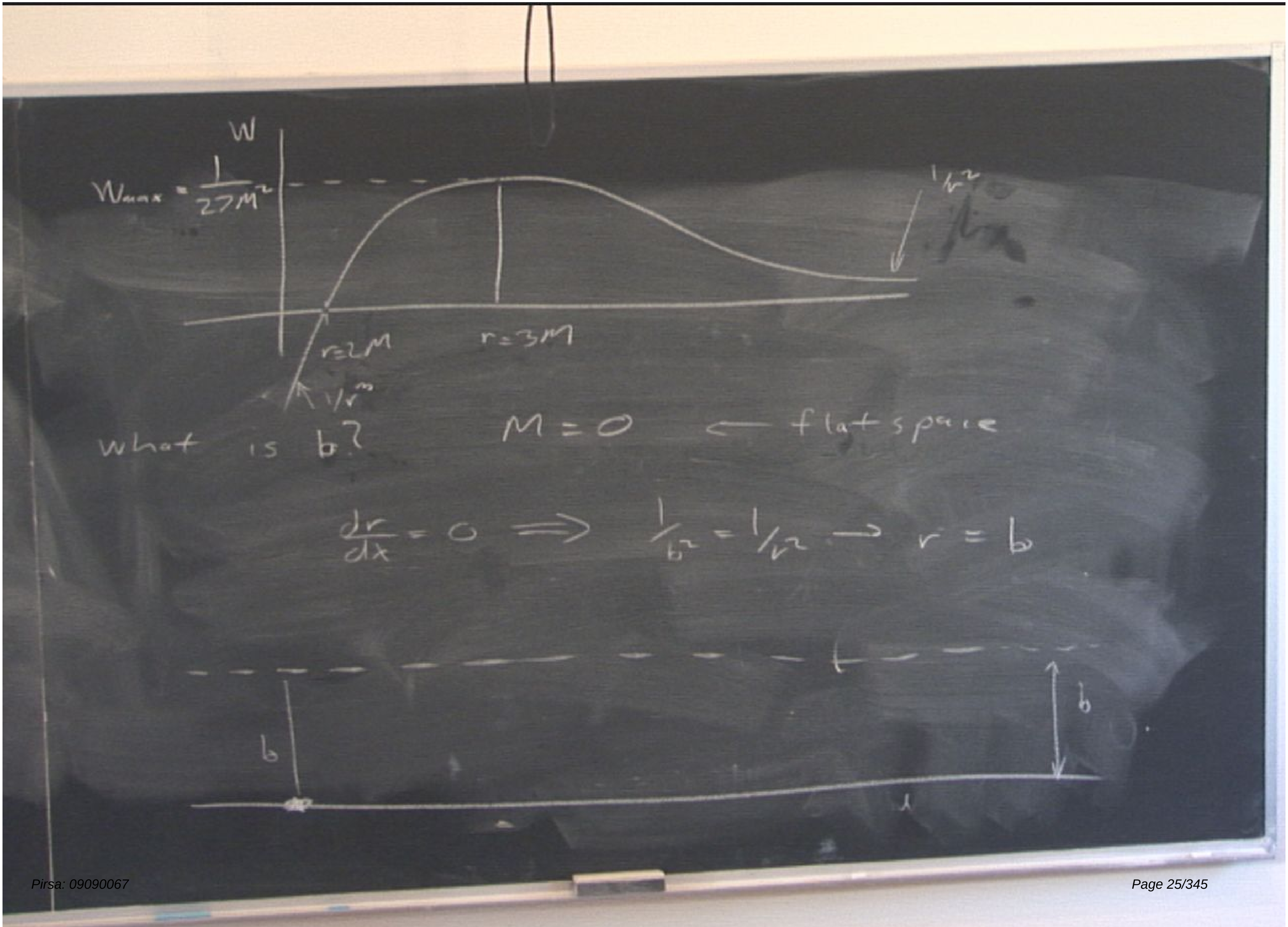
b
b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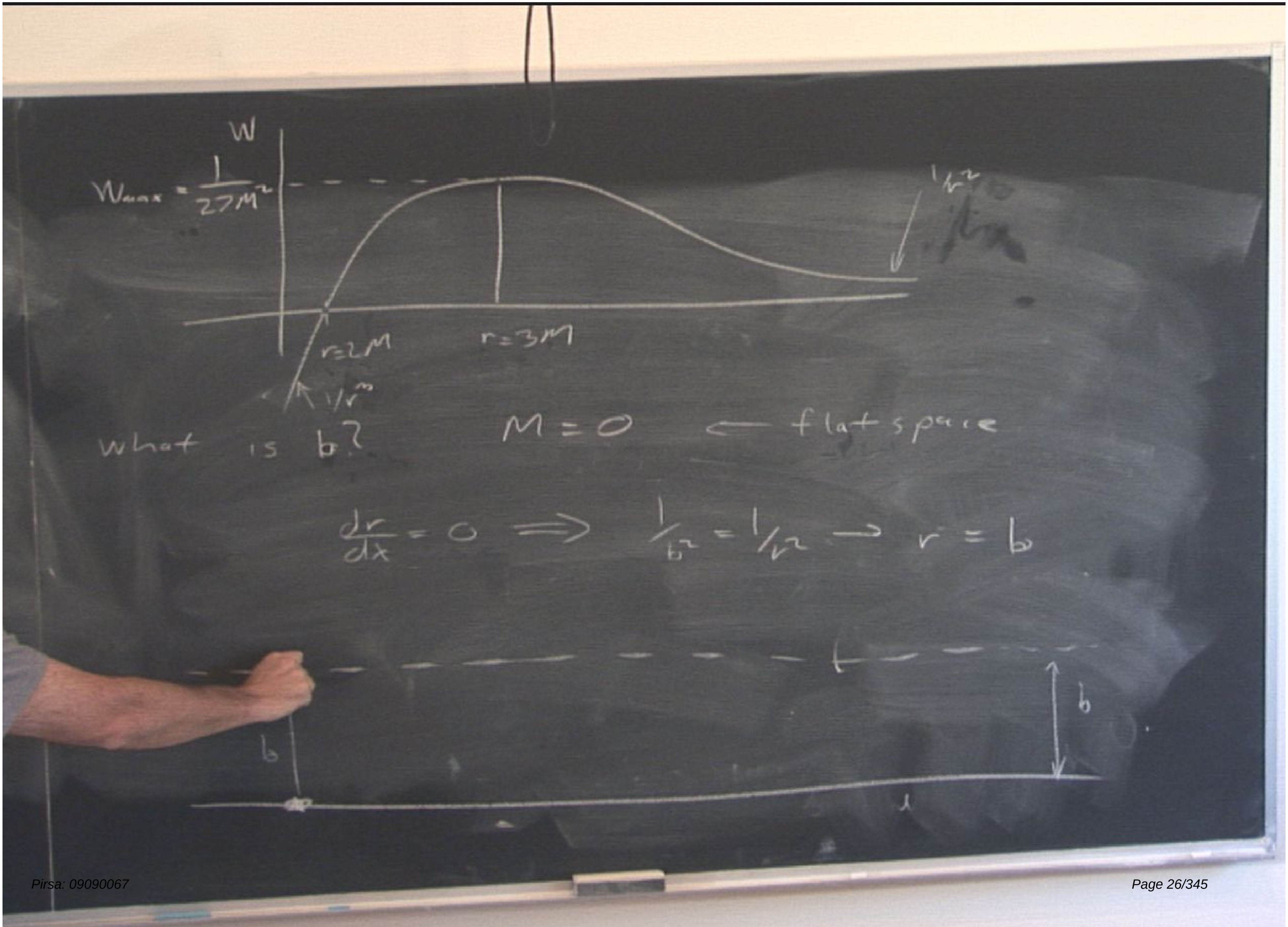
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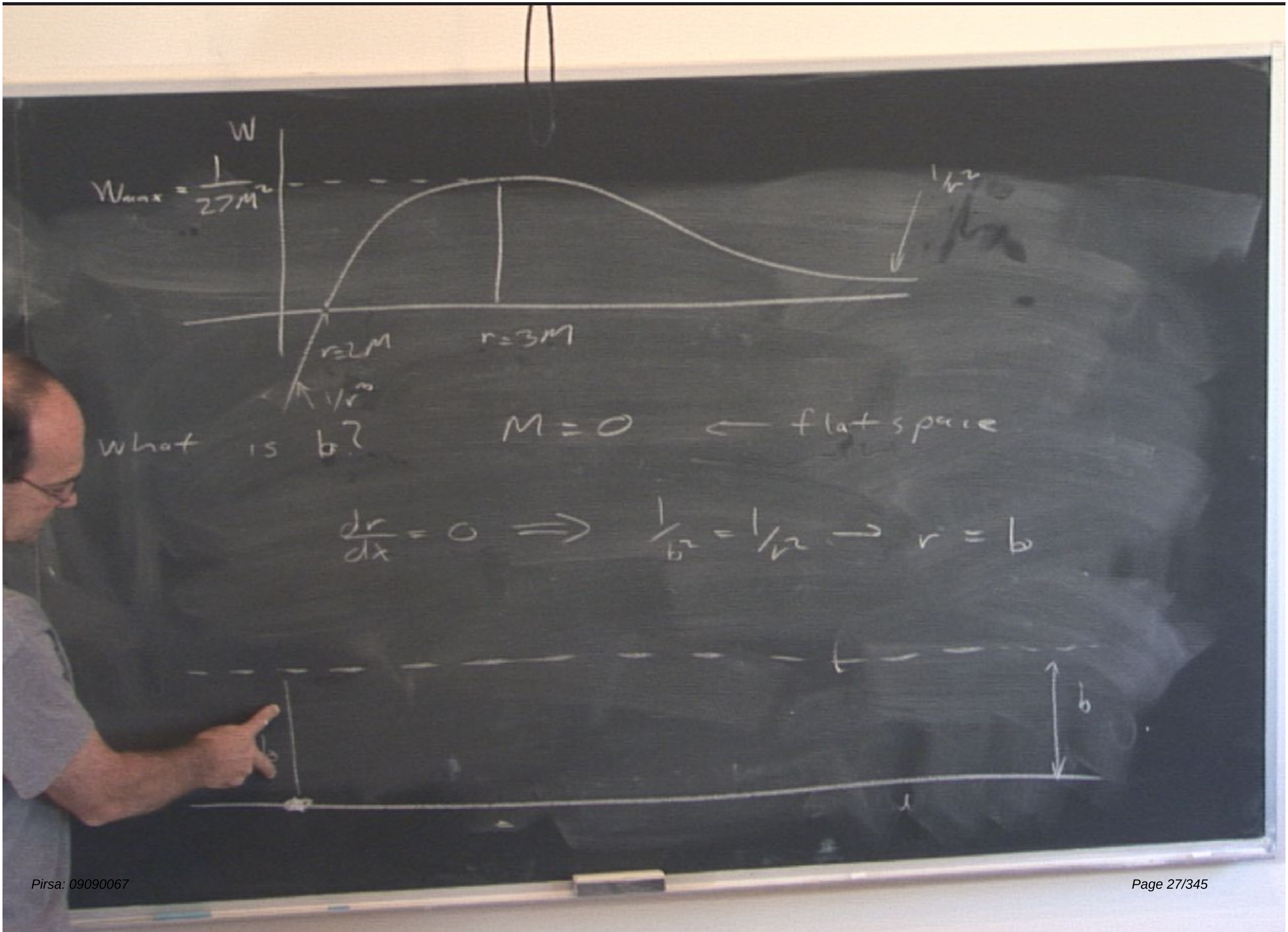
b
b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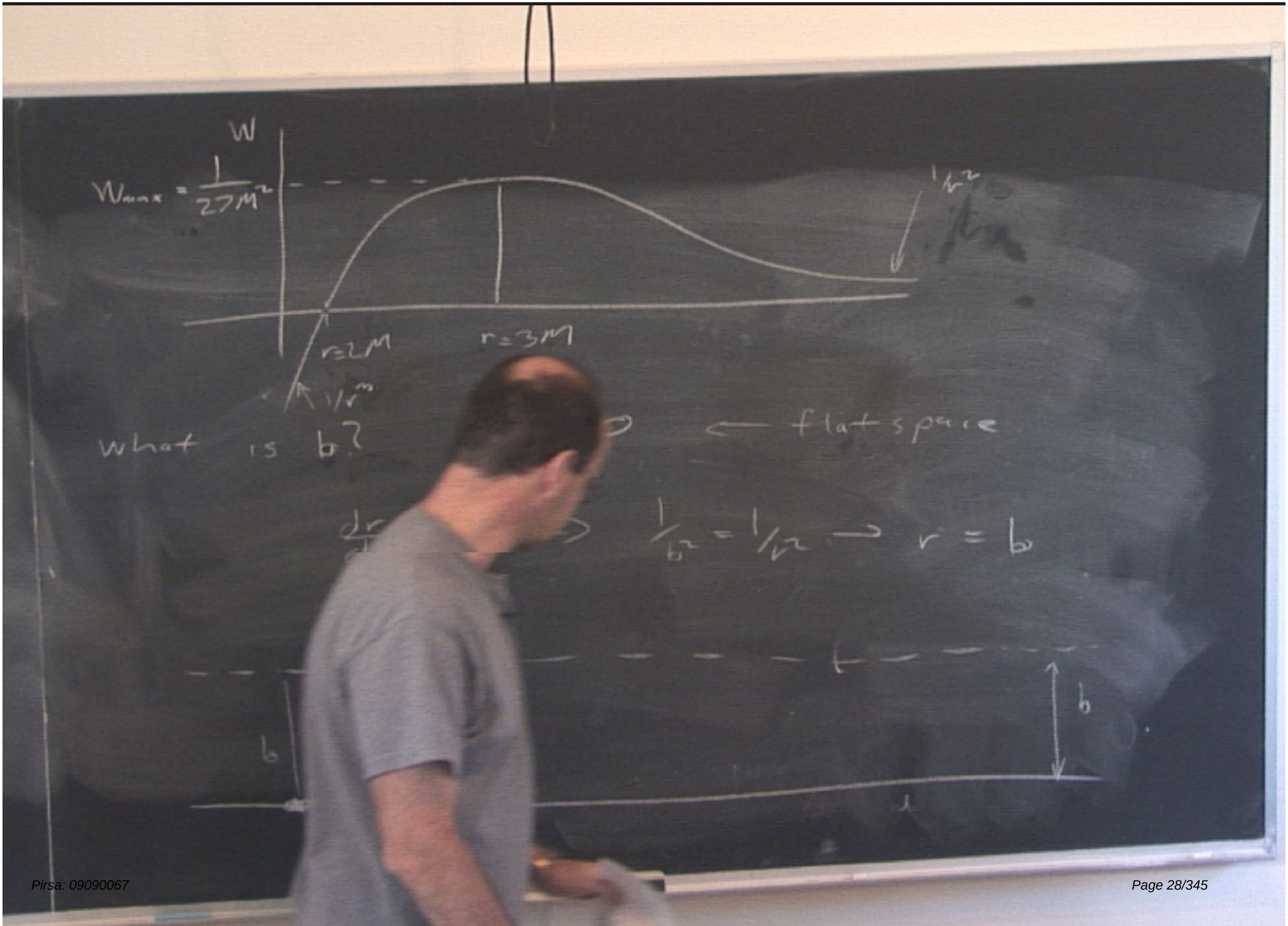
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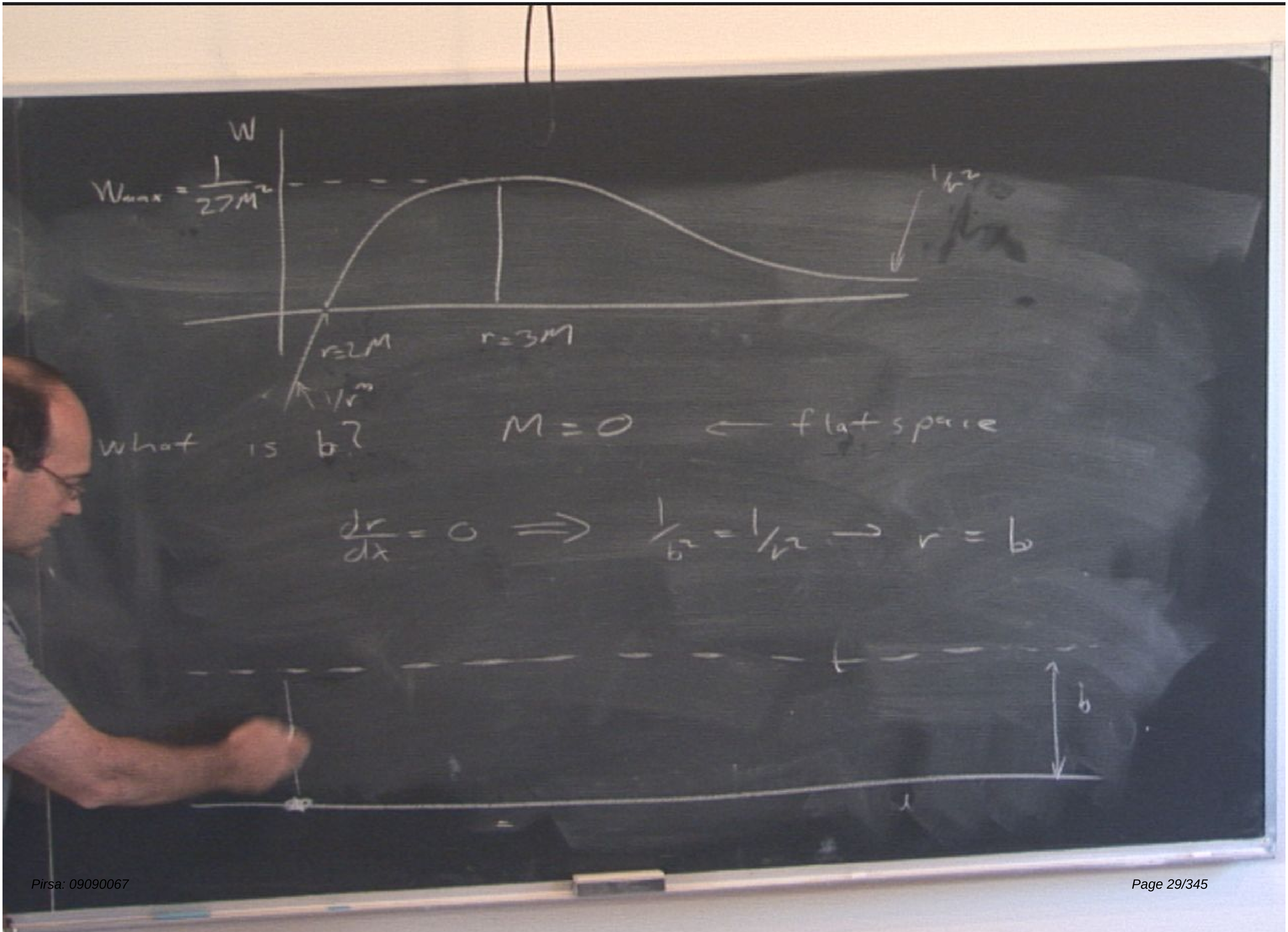
what is b?
← flat space
1/b² = 1/r² → r = b
b
b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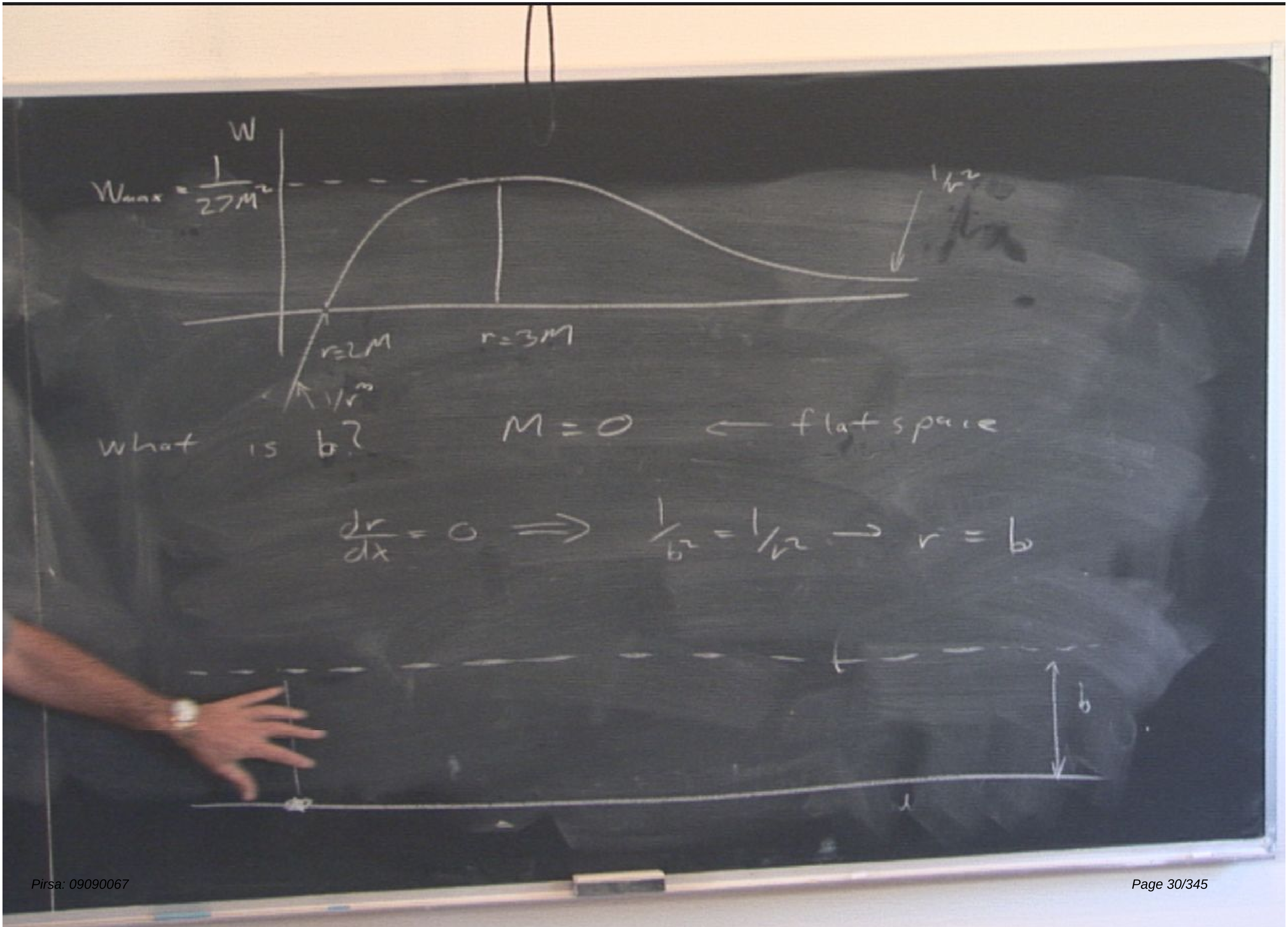
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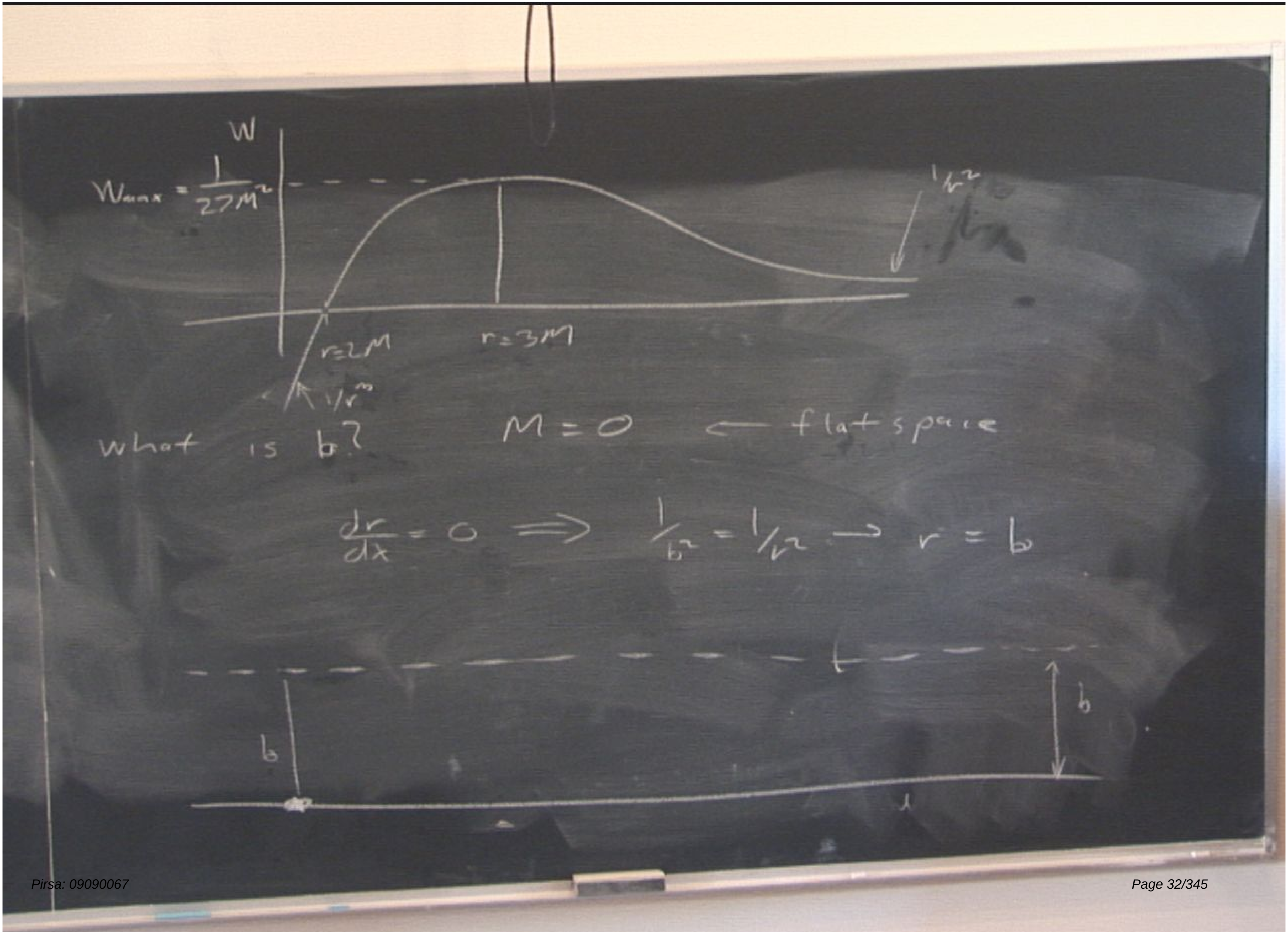
b


W
W max = 1/27M²
1/r²
r=2M
r=3M
1/rⁿ
what is b?
M = 0
← fla
dr/dx = 0 ⟹ 1/b² = 1/r²
= b
b
b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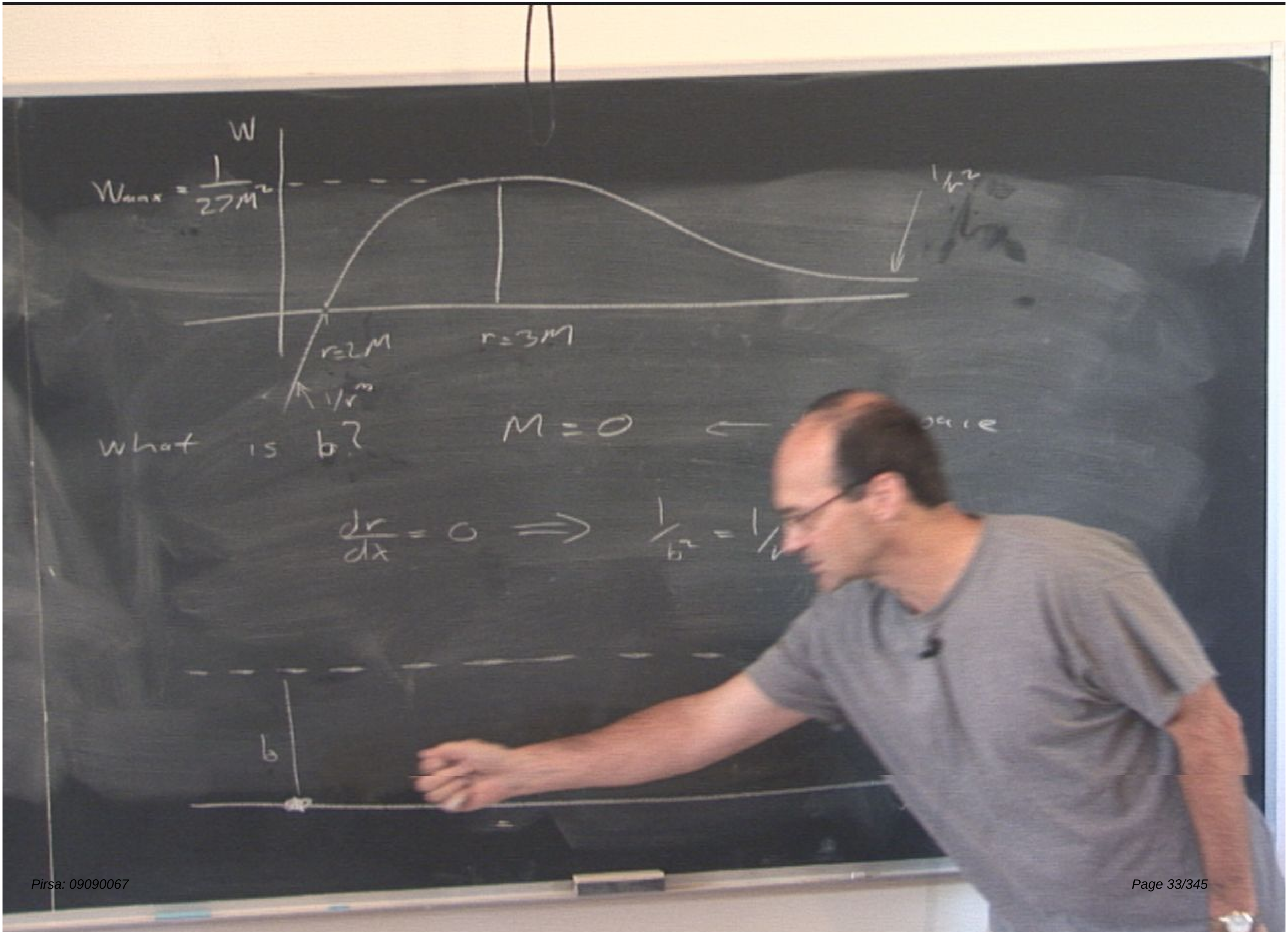
b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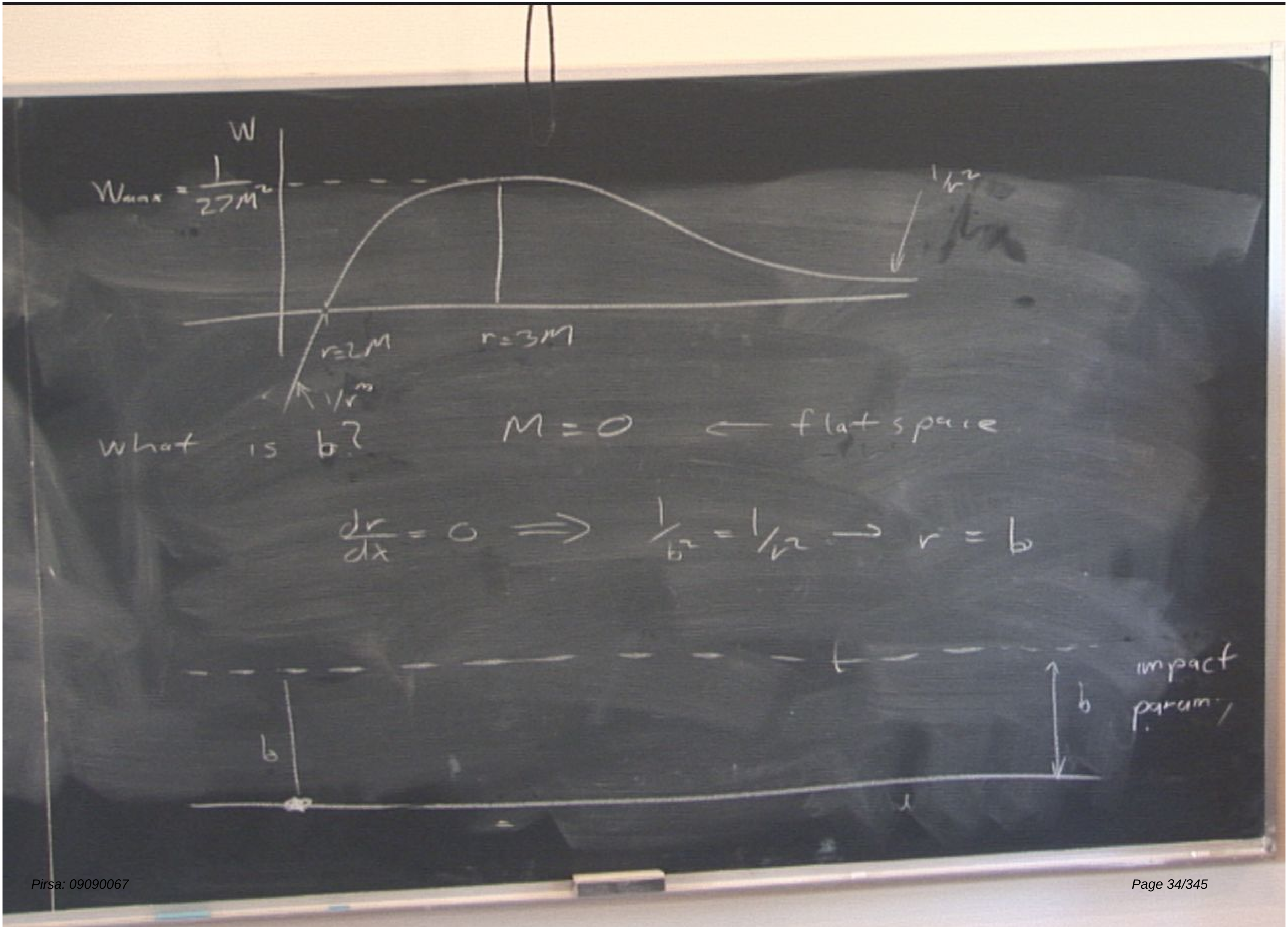
what is b?
M = 0
dr/dx = 0 ⟹ 1/b² = 1/
b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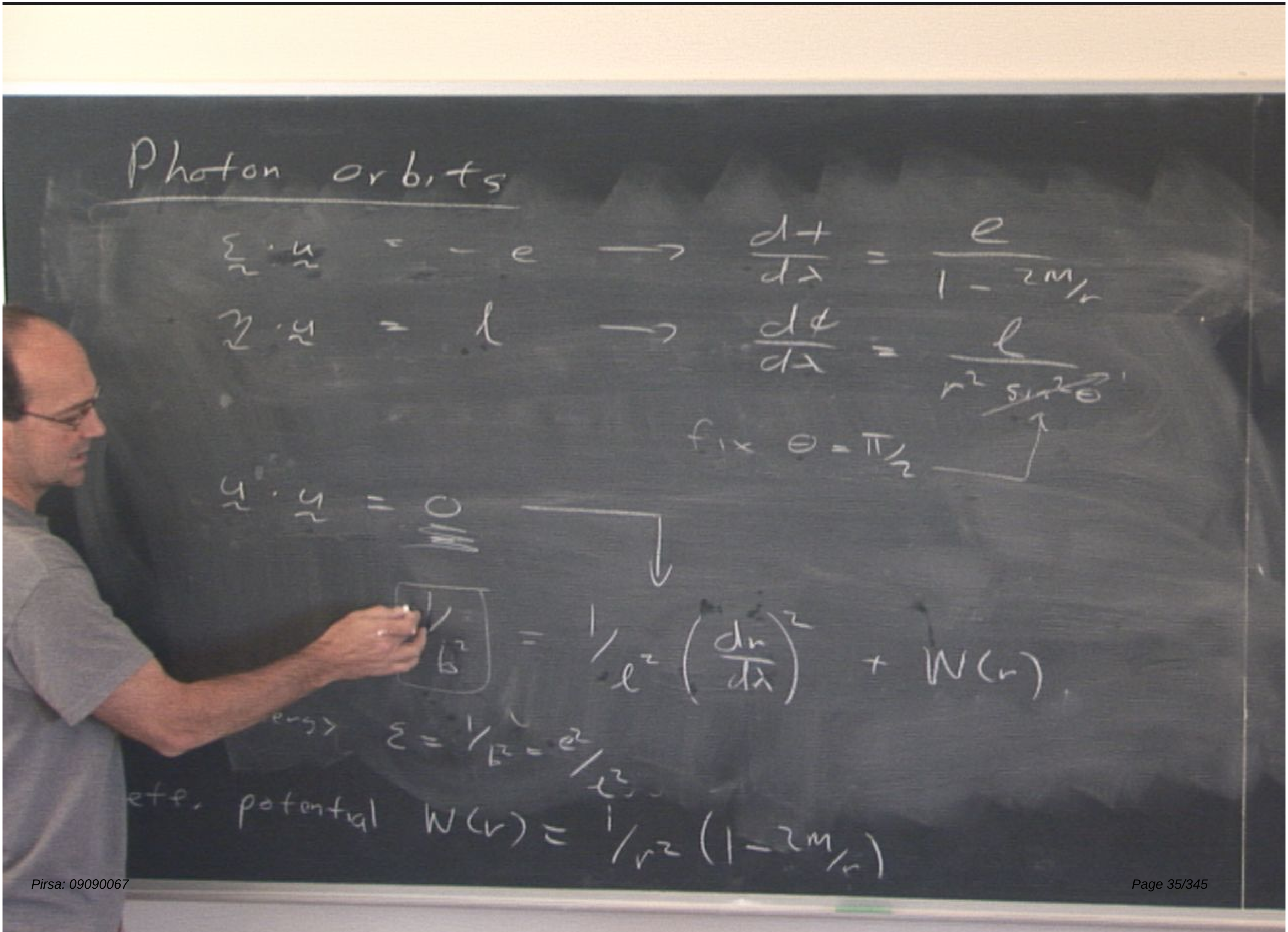
b
impact param
b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$$\eta \cdot u = \ell \longrightarrow \frac{d\phi}{d\lambda} = \frac{\ell}{r^2 \sin^2\theta}$$

fix $\theta = \pi/2$ (so $\sin^2\theta \to 1$)

$$u \cdot u = 0$$

$$\boxed{\frac{1}{b^2}} = \frac{1}{\ell^2}\left(\frac{dr}{d\lambda}\right)^2 + W(r)$$

$\frac{e^2 - 1}{\;}$

eff. energy $\mathcal{E} = 1/b^2 = e^2/\ell^2$

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Photon orbits

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fix $\theta = \pi/2$

$u \cdot u = 0$

$\frac{e^2 - 1}{2}$

$$\boxed{1/b^2} = 1/l^2 \left(\frac{dr}{d\lambda}\right)^2 + W(r)$$

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fix $\theta = \pi/2$

$$u \cdot u = 0$$

$$\frac{e^2 - 1}{2\ell^2}$$

$$\boxed{1/b^2} = 1/\ell^2 \left(\frac{dr}{d\lambda}\right)^2 + W(r)$$

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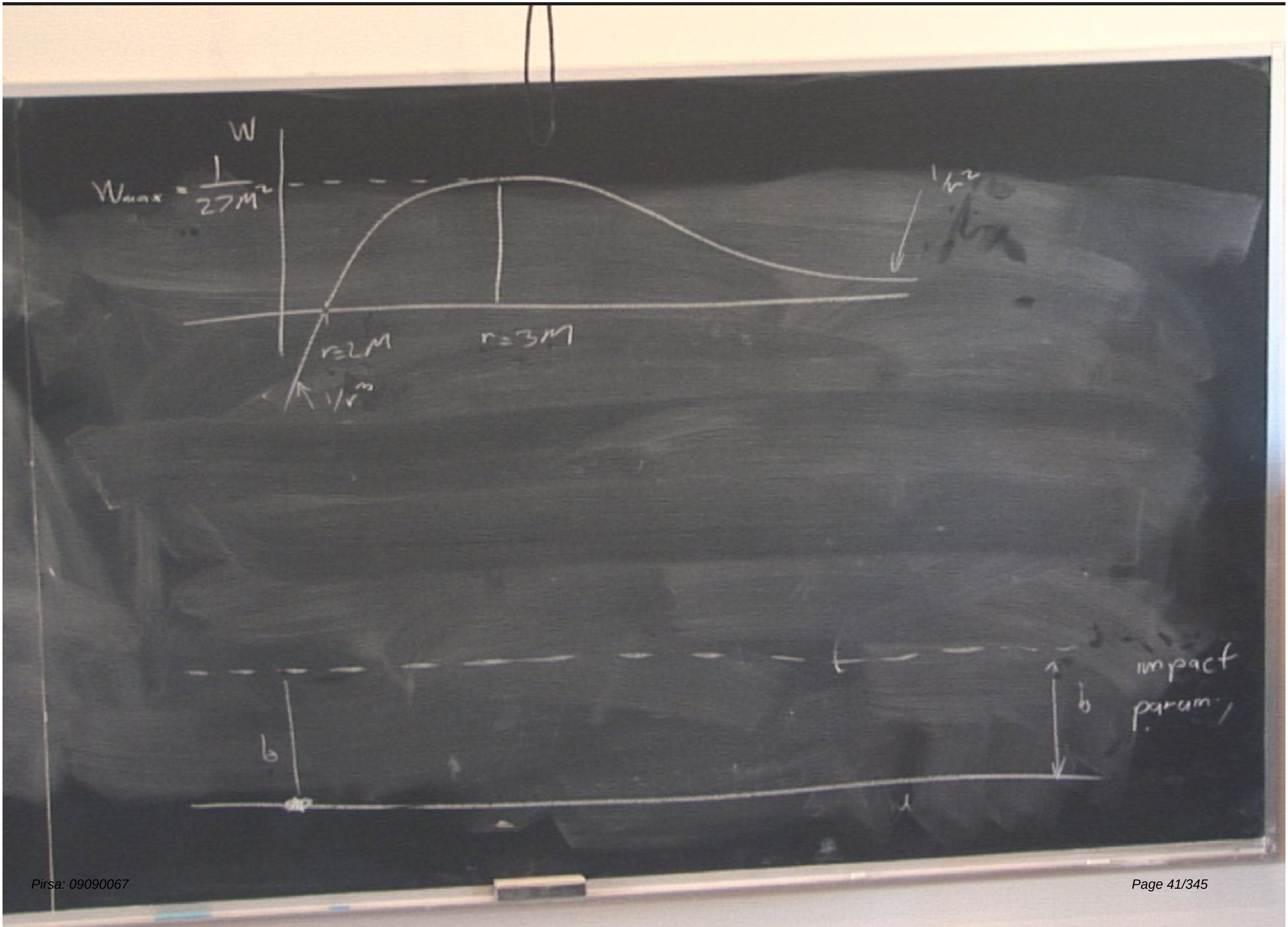
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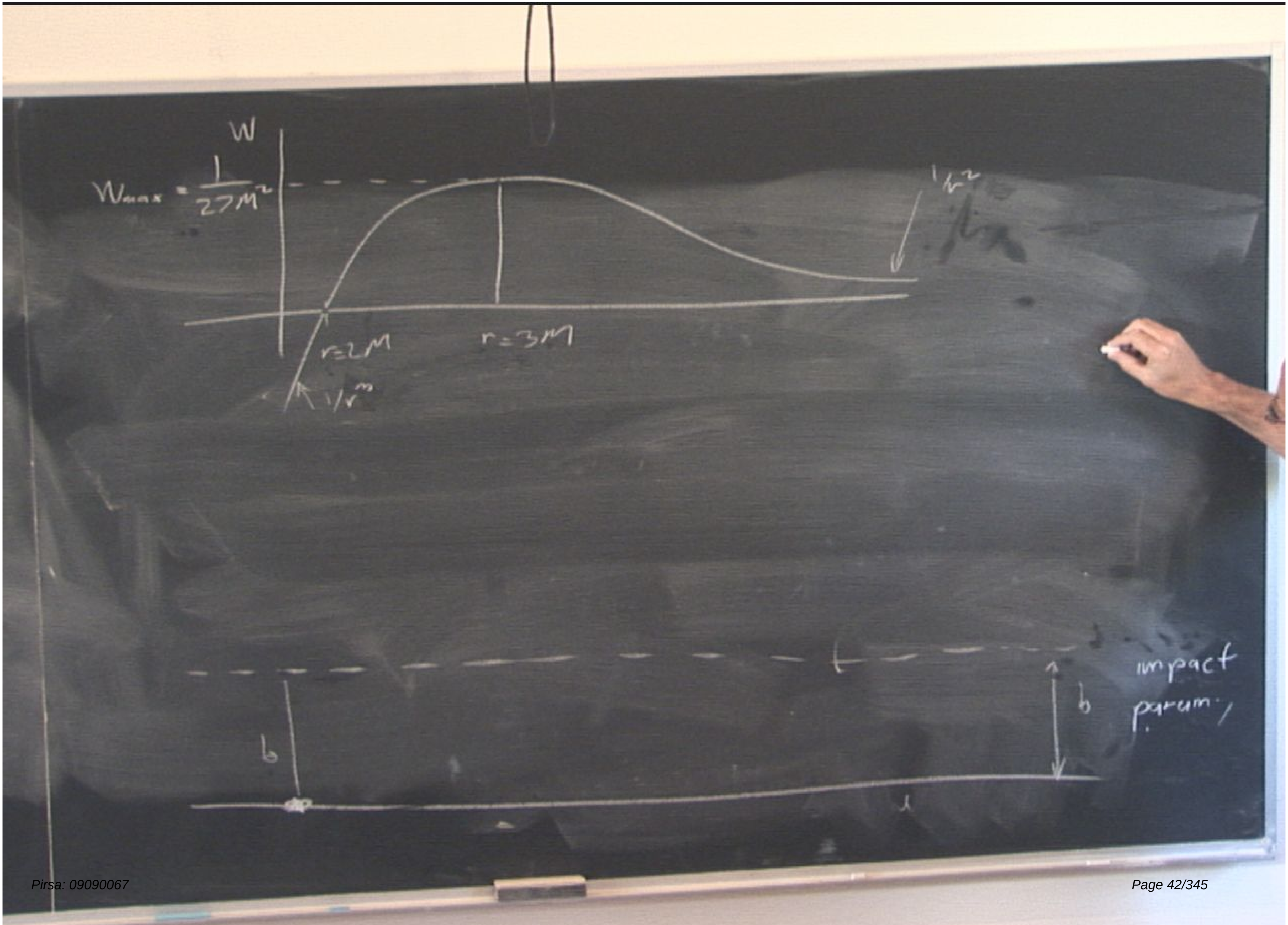
Wmax = 1/27M²
1/r²
r=2M
r=3M
1/r^m
impact param.
b
b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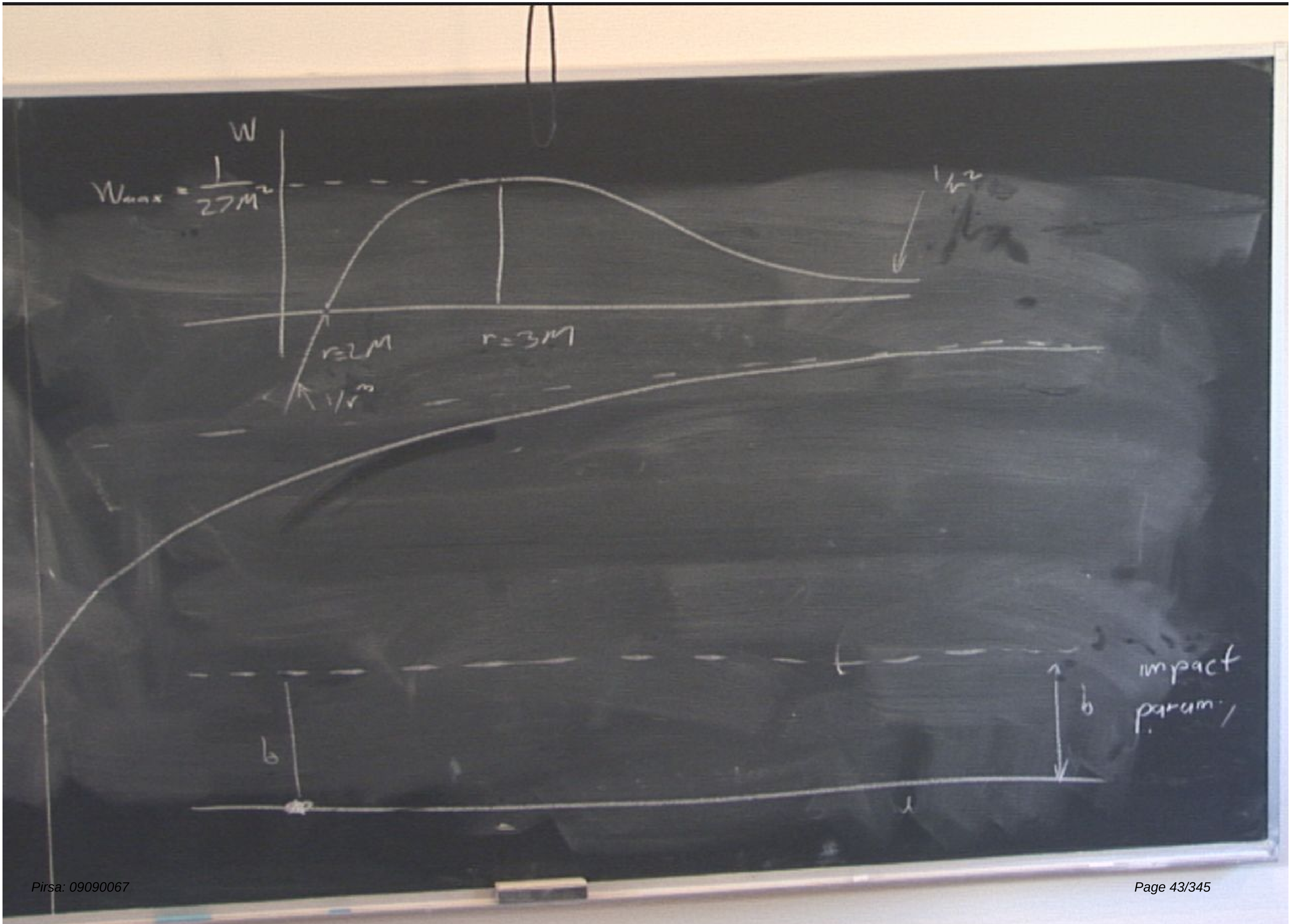
Wmax = 1/27M²
r=2M
r=3M
1/r²
b
b
impact
param.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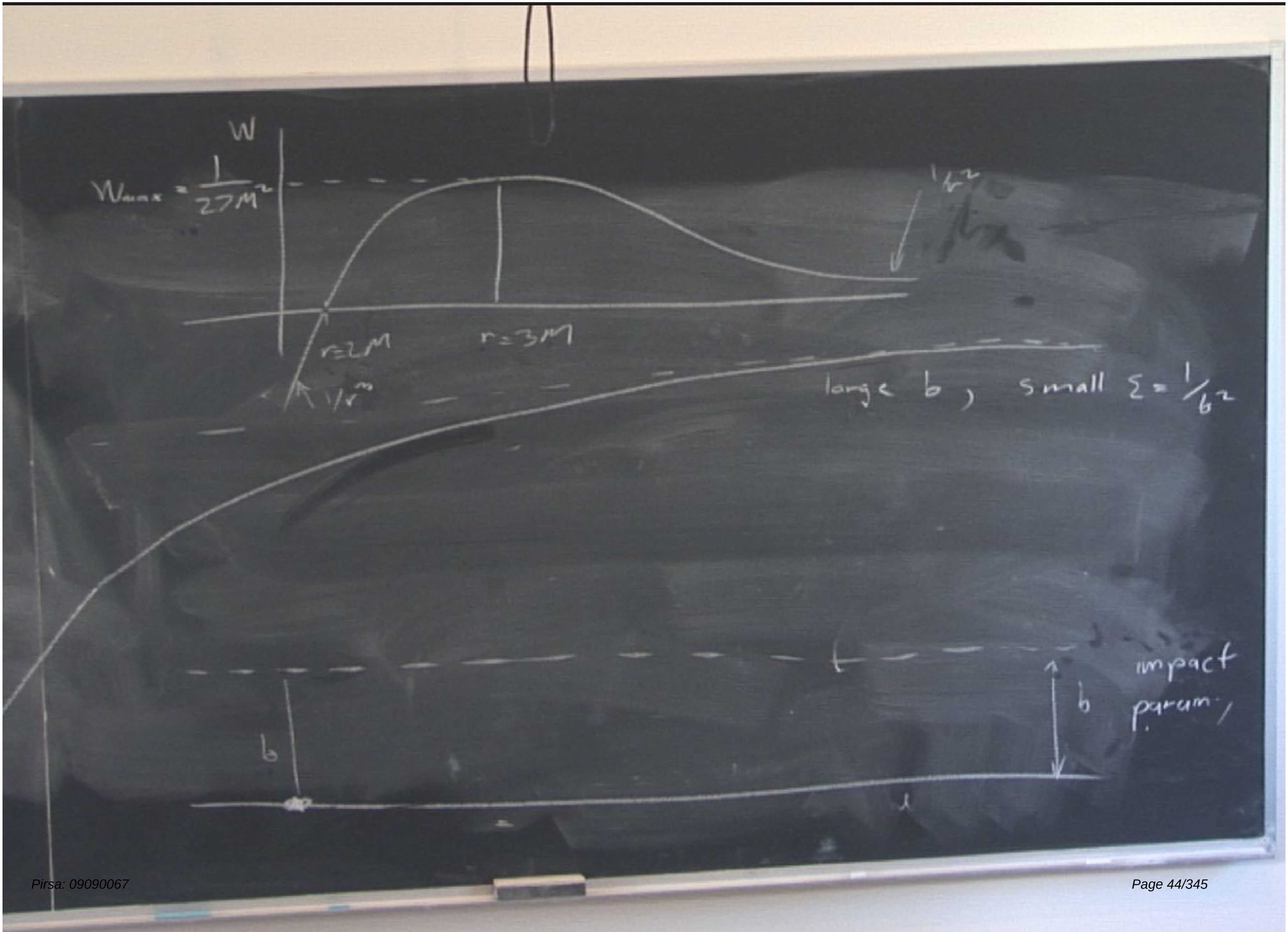
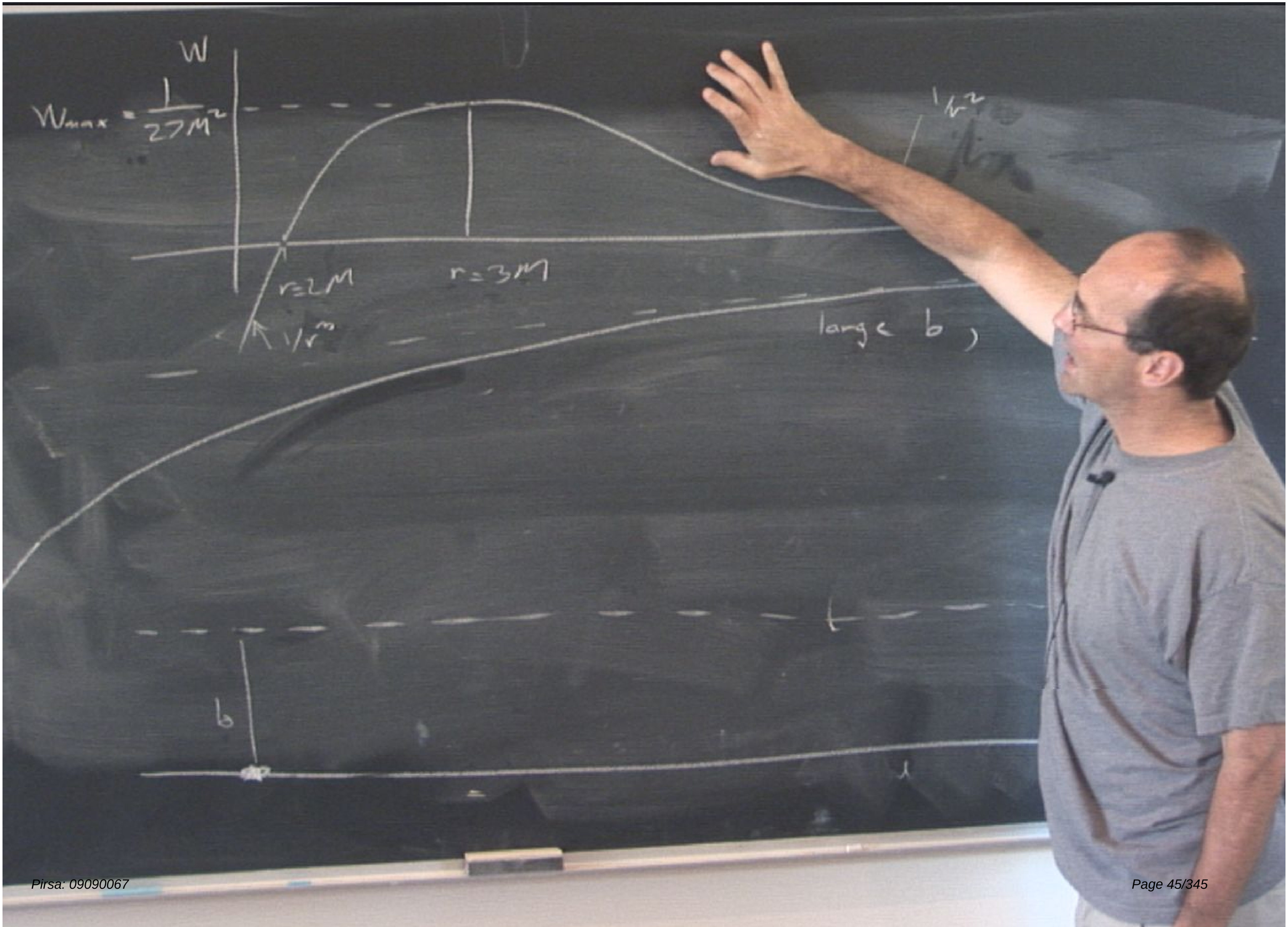
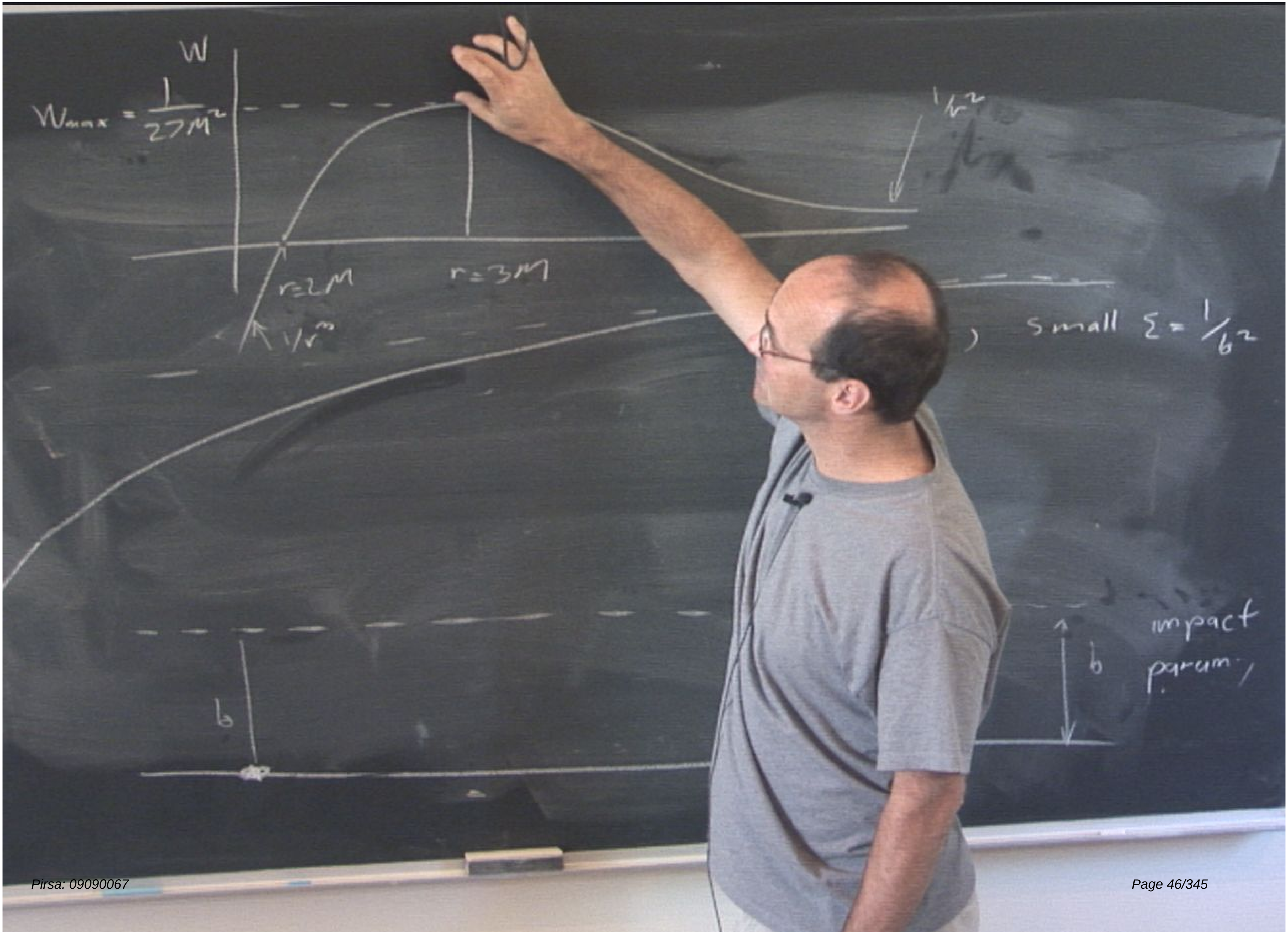
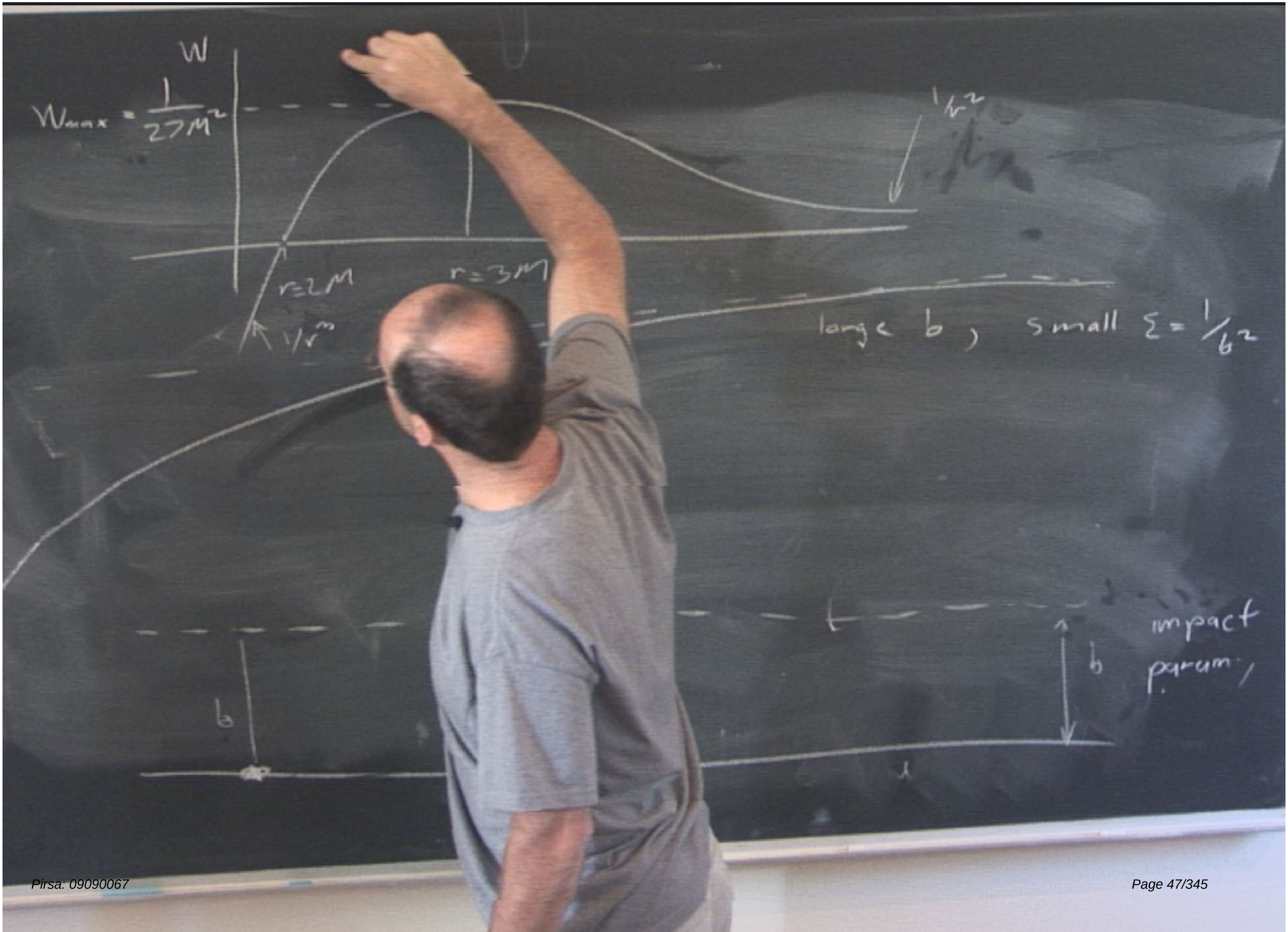
r=2M
r=3M
1/r²
impact param.
b
b

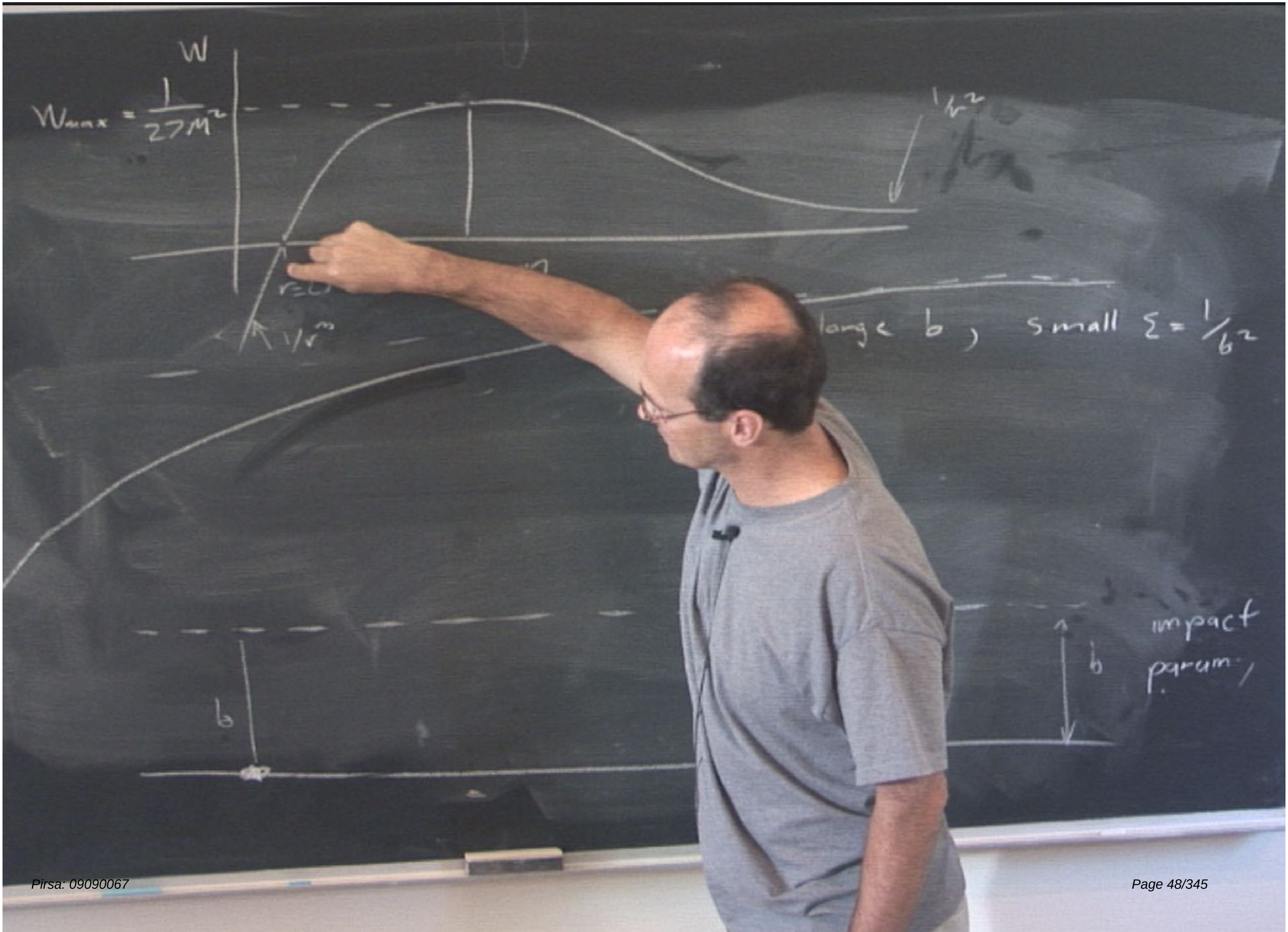

W
Wmax = 1/27M²
r=2M
r=3M
1/b²
large b, small ε = 1/b²
impact param.
b
b

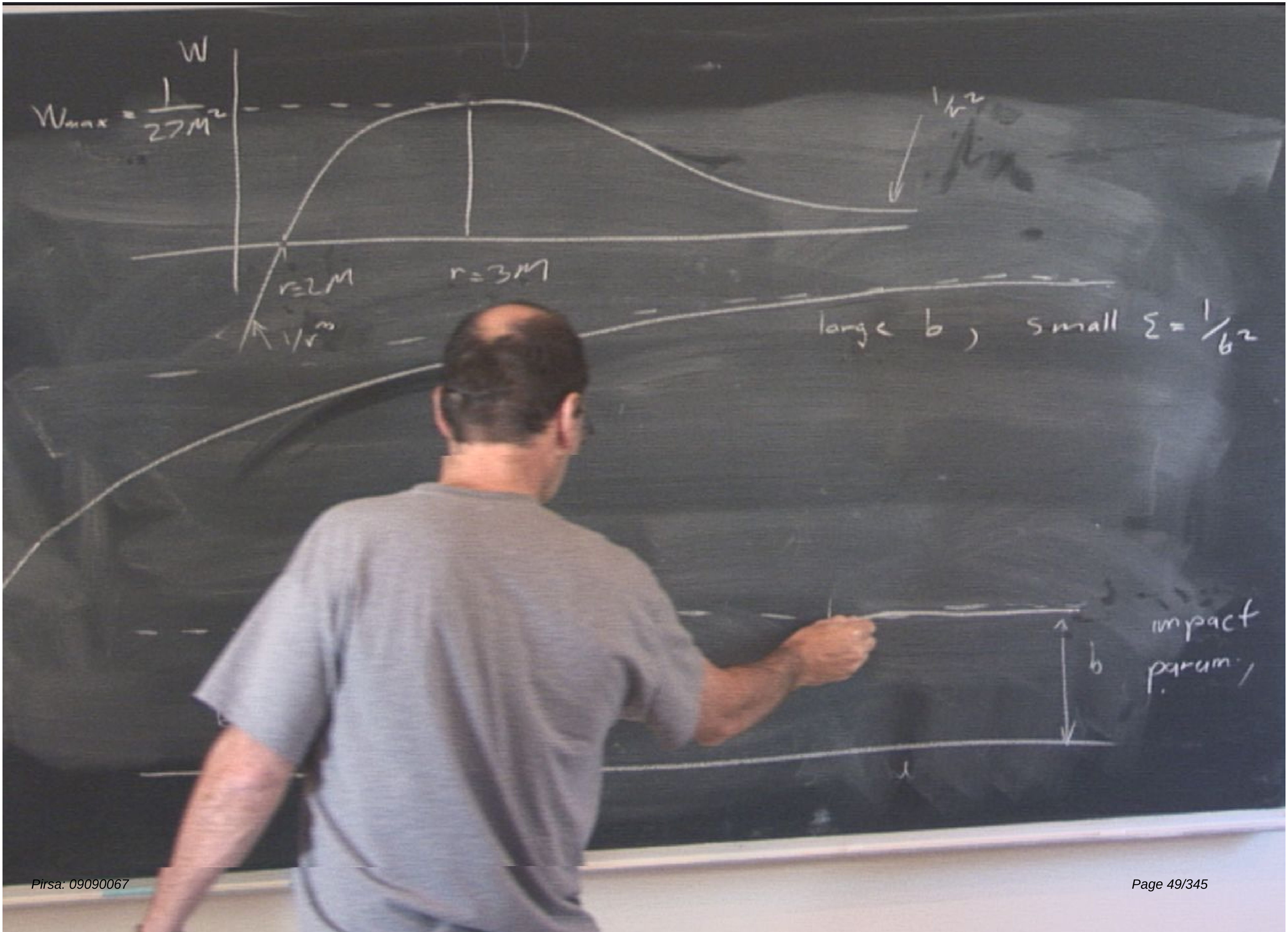

W
Wmax = 1/27M²
r=2M
r=3M
1/r²
large b,
b



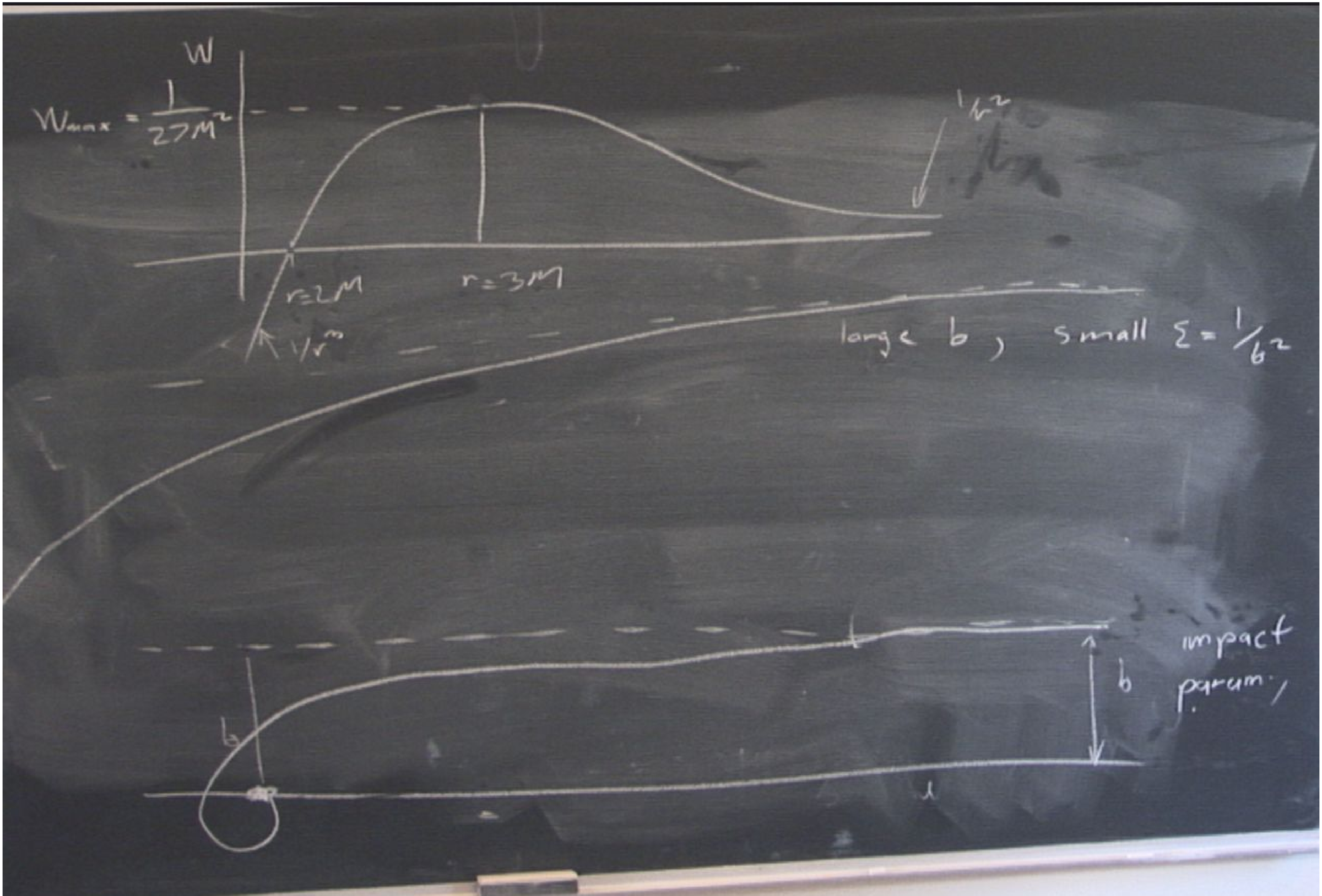
W
Wmax = 1/27M²
r=2M
r=3M
1/rⁿ
1/r²
small ξ = 1/b²
b
b
impact param.


W
Wmax = 1/27M²
r=2M
r=3M
1/r^m
1/r²
large b, small ξ = 1/b²
b
impact param.
b

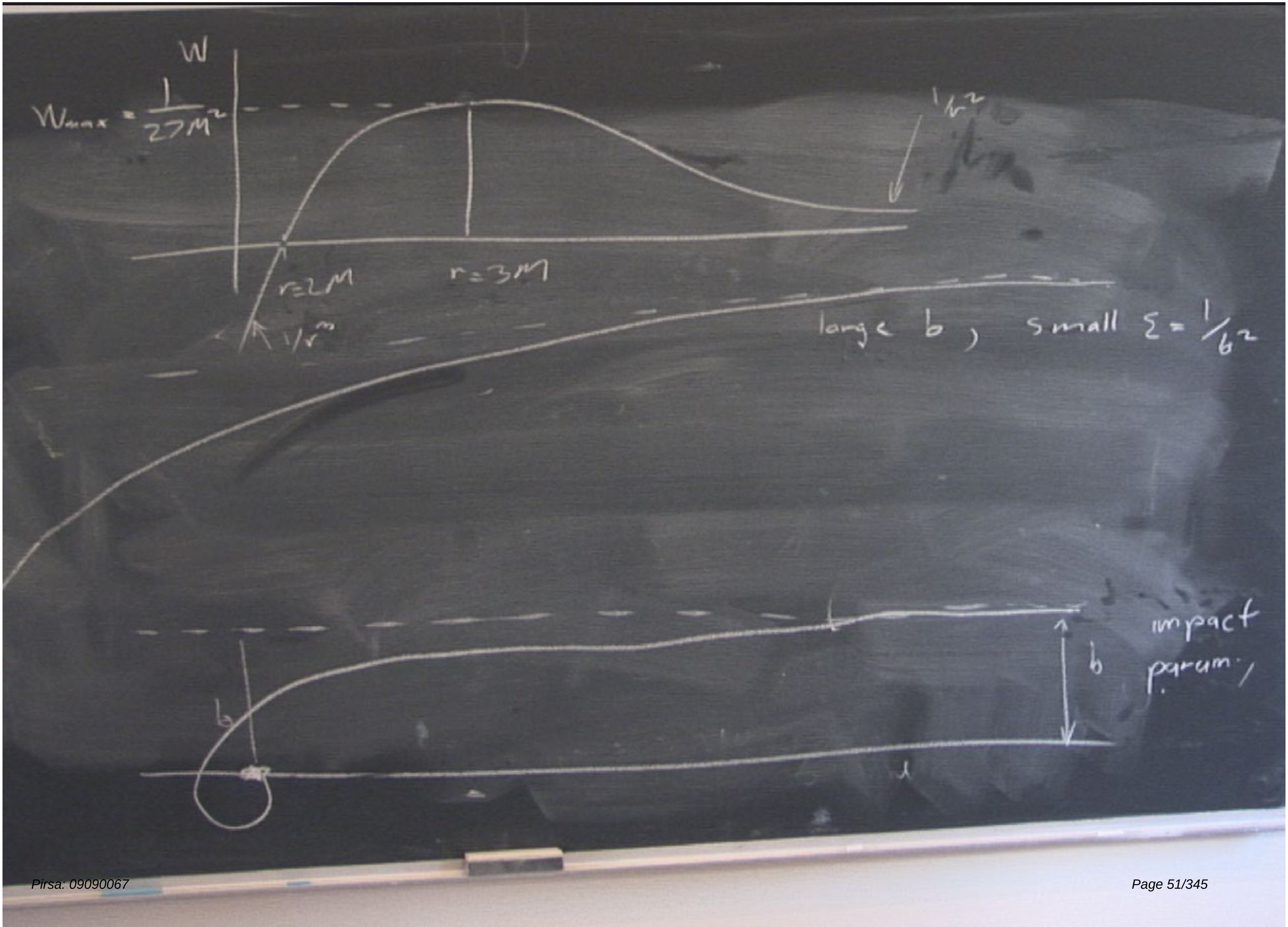

W
Wmax = 1/27M²
large b, small ξ = 1/b²
impact param.
b



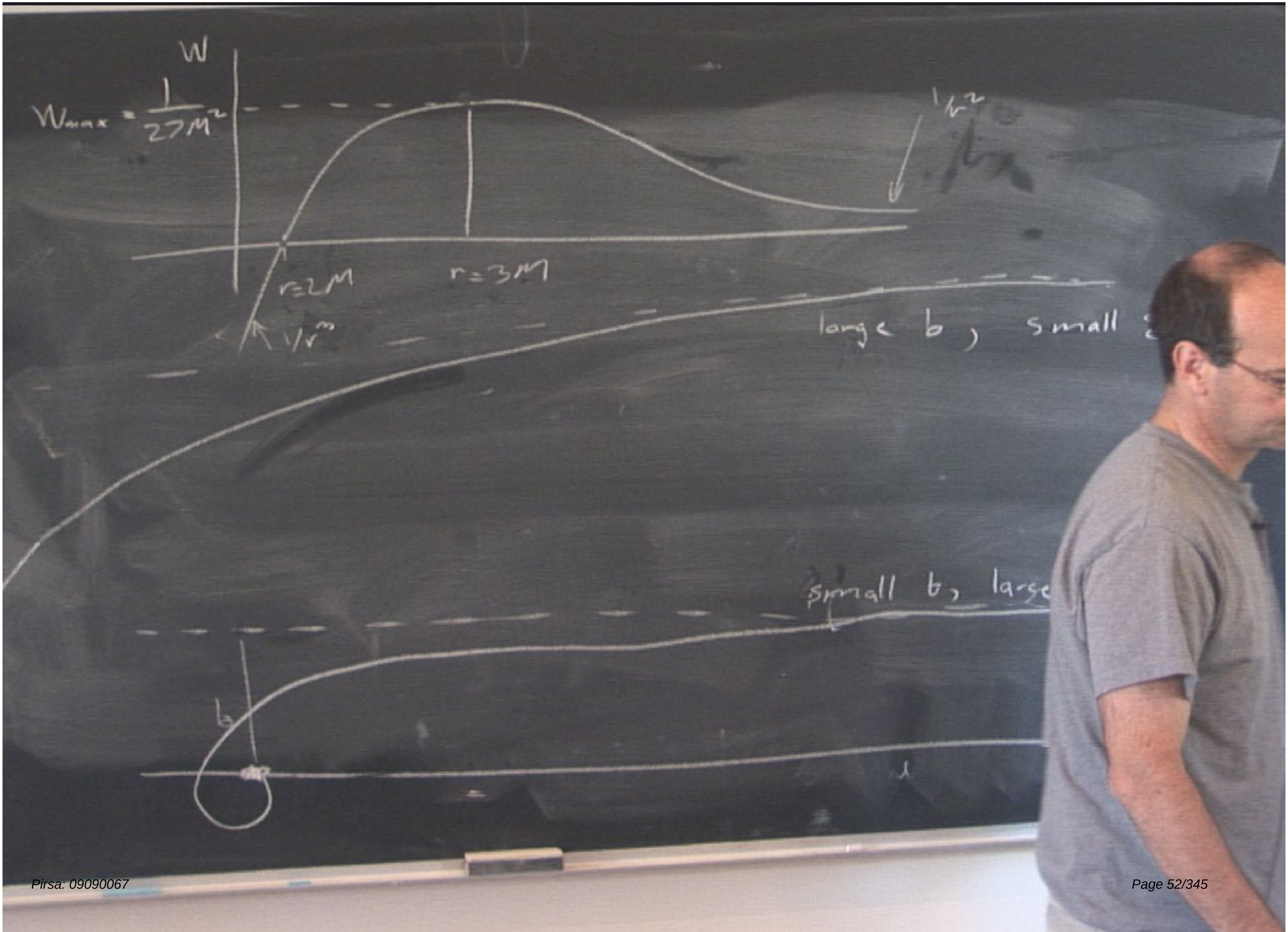
W
$W_{max} = \frac{1}{27M^2}$
$1/r^2$
r=2M
r=3M
$1/r^m$
large b, small $\xi = 1/b^2$
impact param.
b

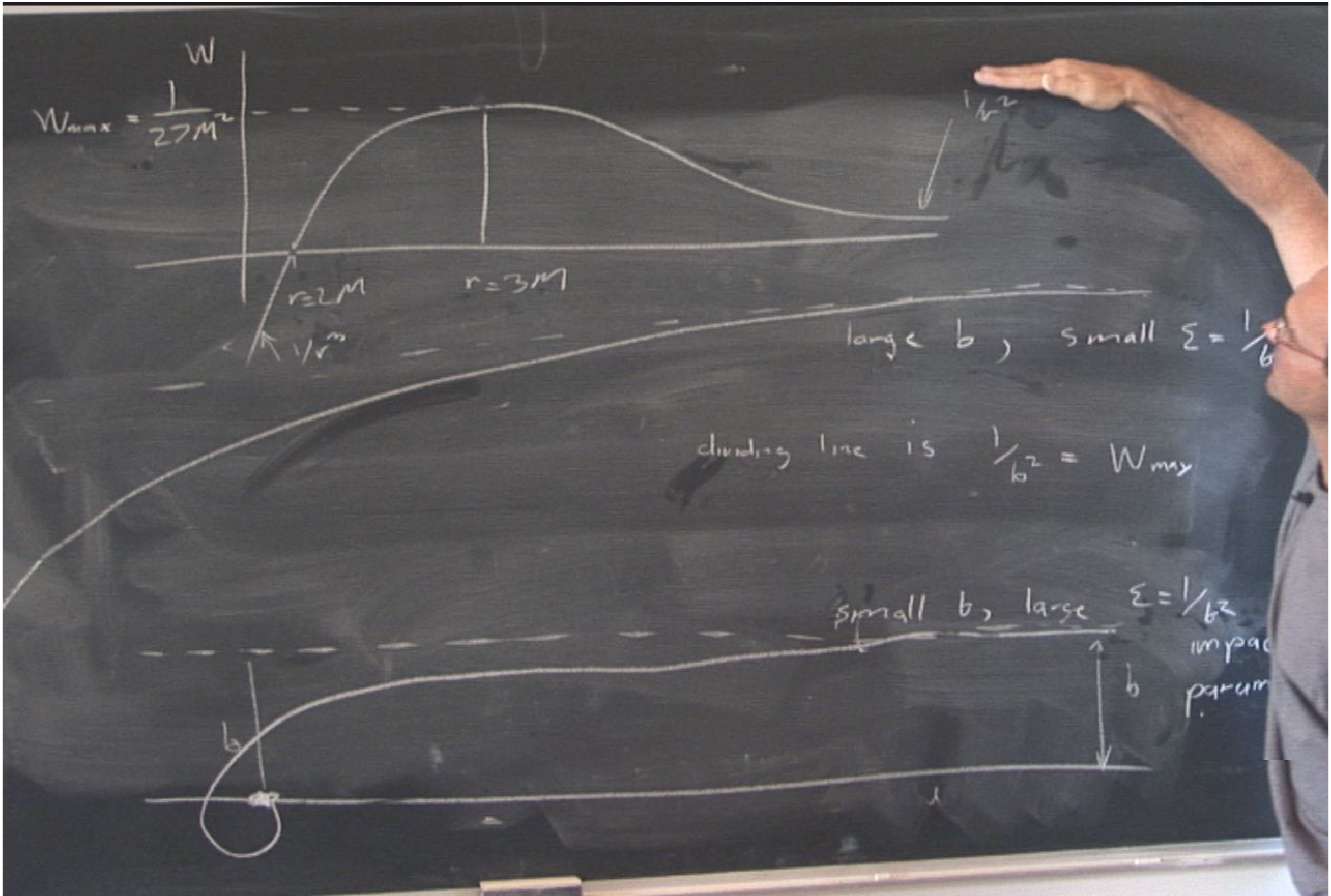


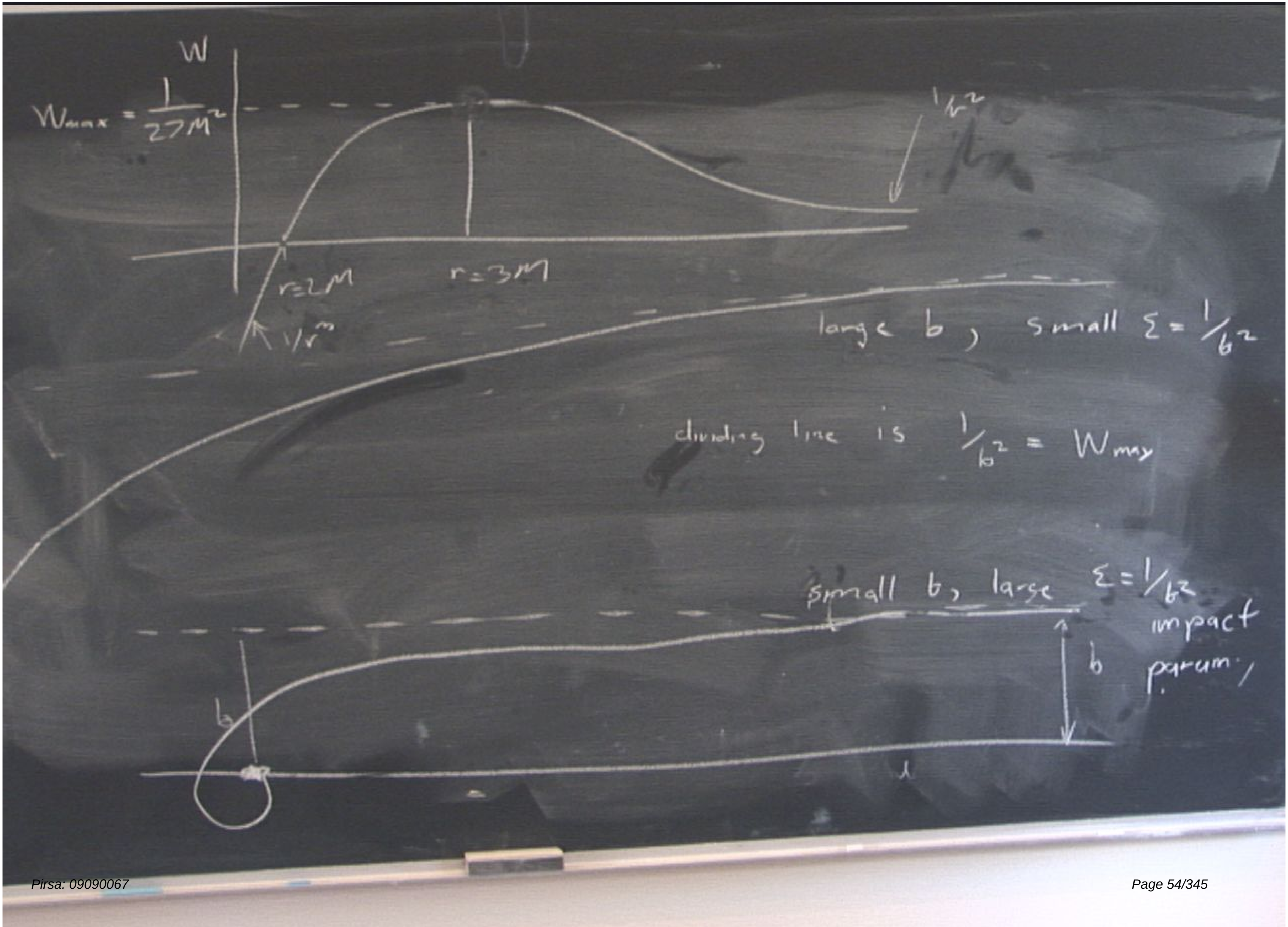
W
$W_{max} = \frac{1}{27M^2}$
$1/r^2$
$r=2M$
$r=3M$
$1/r^m$
large b, small $\xi = 1/b^2$
b
impact param.

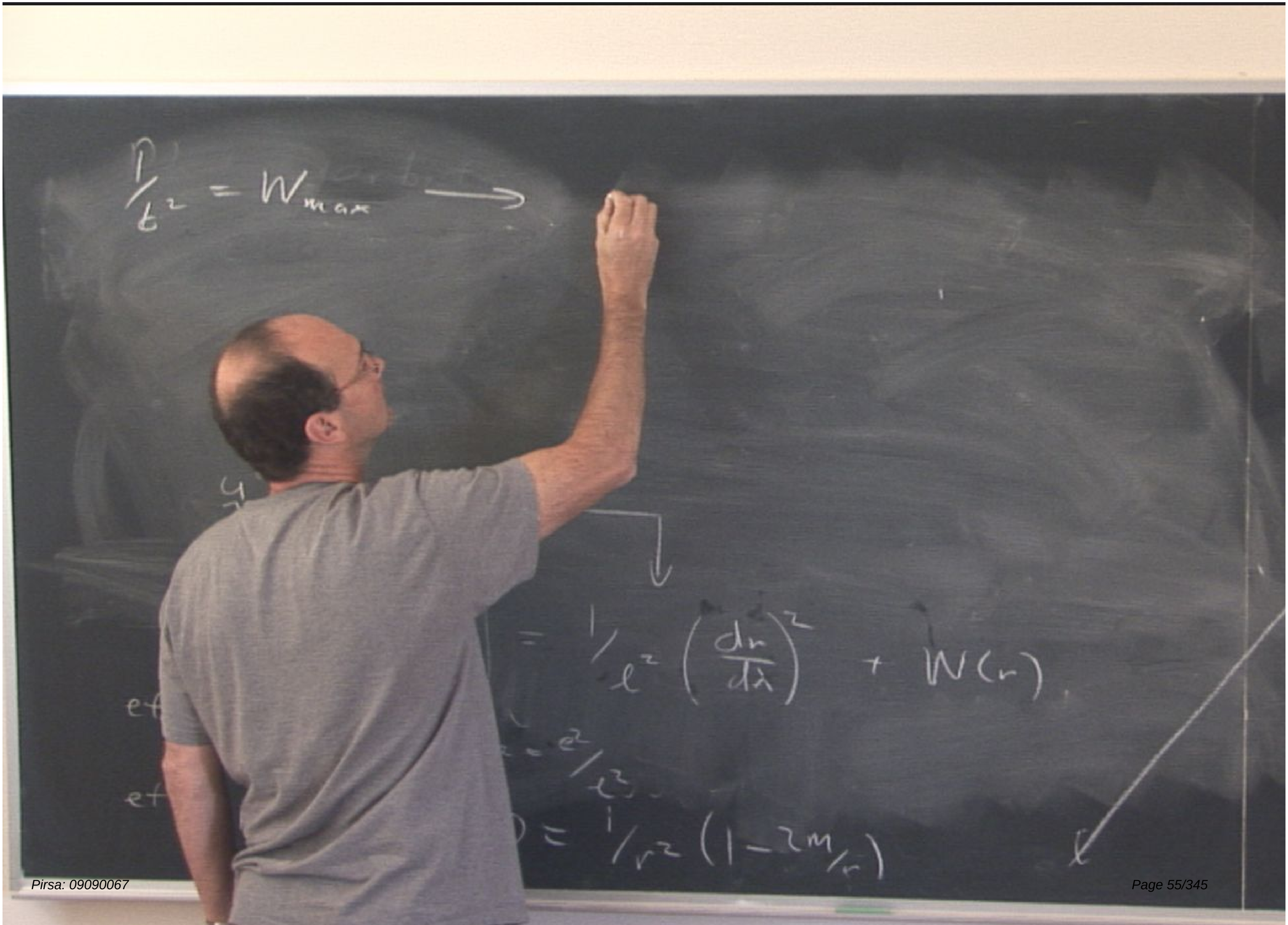


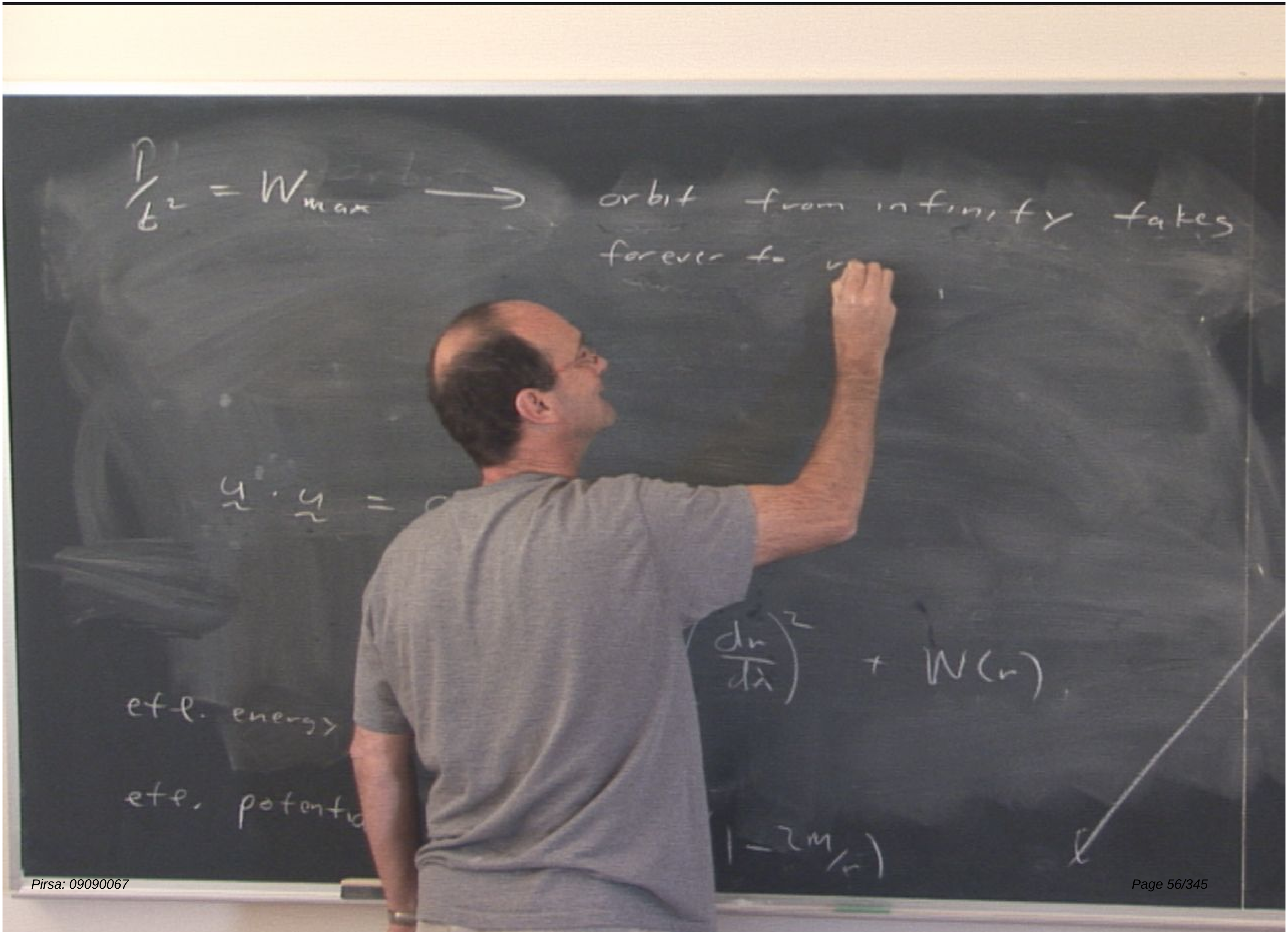
W
$W_{max} = \frac{1}{27M^2}$
$1/r^2$
r=2M
r=3M
$1/r^m$
large b, small $\xi = 1/b^2$
b
b
impact param.

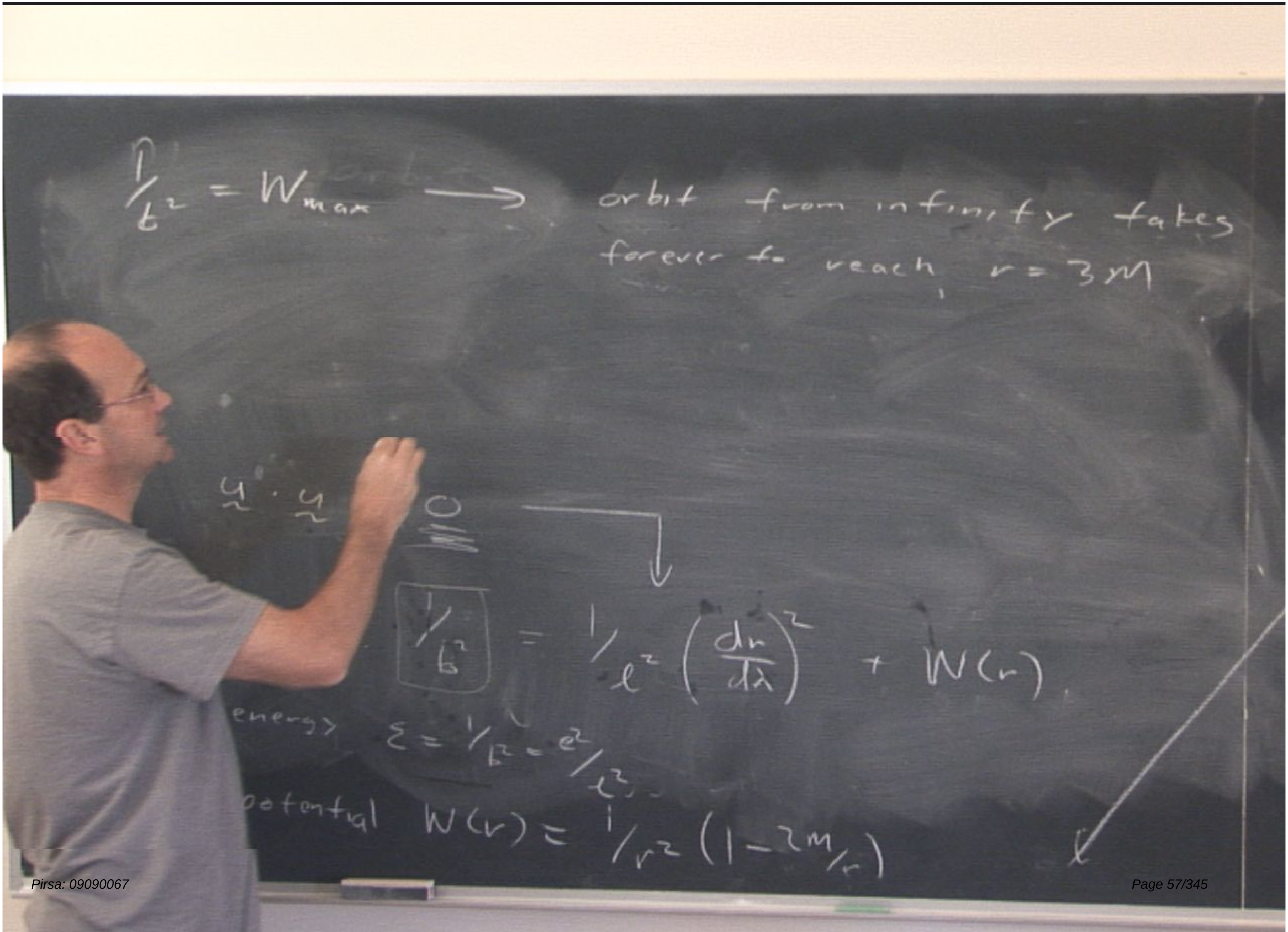

W
$W_{max} = \frac{1}{27M^2}$
r=2M
r=3M
1/r^m
1/r^2
large b, small
small b, large
b


W
Wmax = 1/27M²
1/r²
r=2M
r=3M
1/r
large b, small ε = 1/b²
dividing line is 1/b² = Wmax
small b, large ε = 1/b²
impac
param
b
b


W
Wmax = 1/27M²
1/b²
r=2M
r=3M
large b, small ε = 1/b²
dividing line is 1/b² = Wmax
small b, large ε = 1/b²
impact param.
b


1/ℓ² = W_max
= 1/ℓ² (dr/dλ)² + W(r).
= 1/r² (1 − 2M/r)


1/b² = W_max → orbit from infinity takes
forever to
u · u =
(dr/dλ)² + W(r)
eff. energy
eff. potent
(1 − 2M/r)


1/b² = W_max → orbit from infinity takes forever to reach r = 3M
u · u = 0
1/b² = 1/ℓ² (dr/dλ)² + W(r)
energy ε = 1/b² = e²/ℓ²
potential W(r) = 1/r² (1 − 2M/r)
ℓ

$1/b^2 = W_{max}$ → orbit from infinity takes forever to reach $r = 3M$

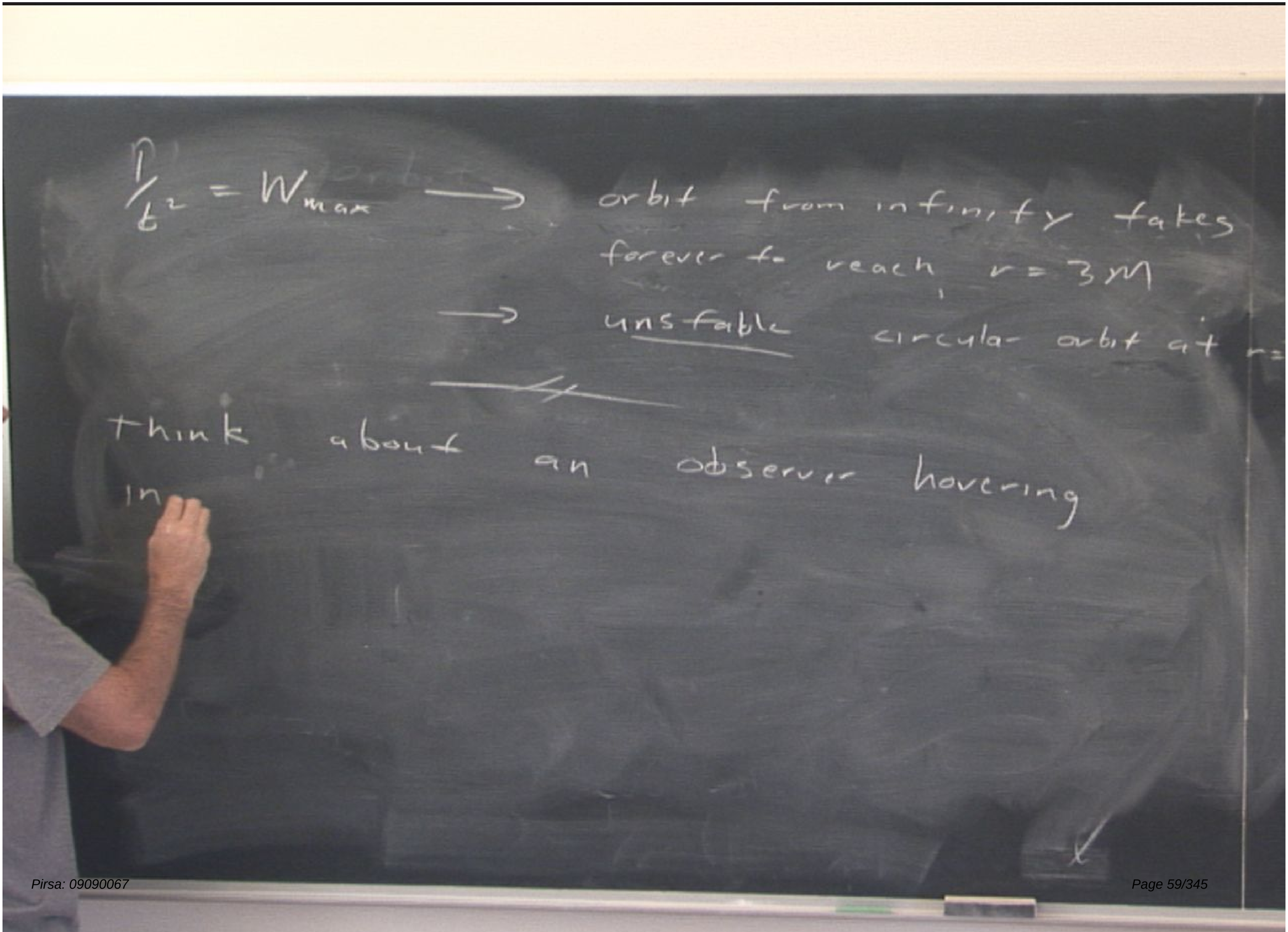
→ <u>unstable</u> circular orbit at r=

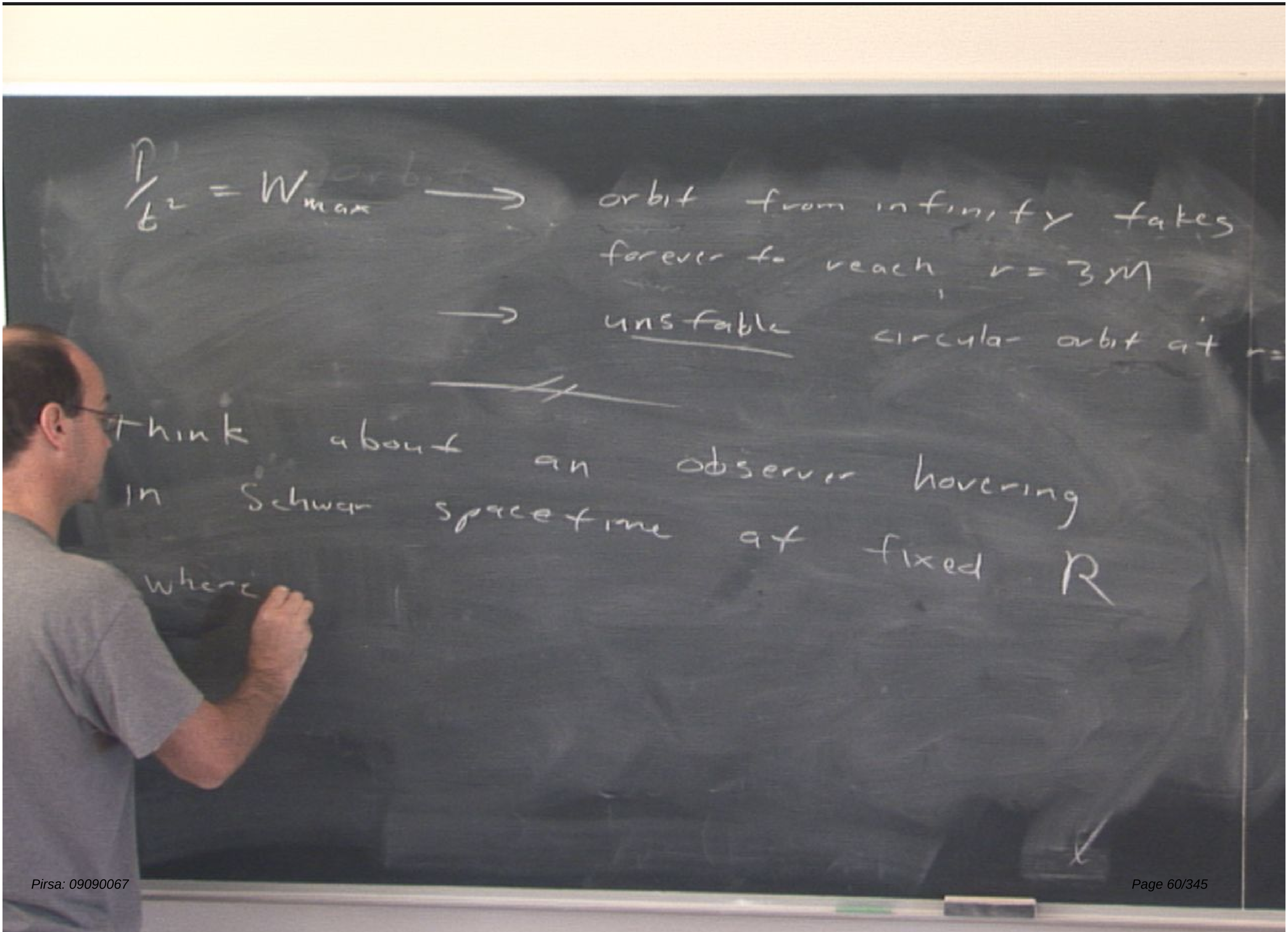
$u \cdot u = 0$

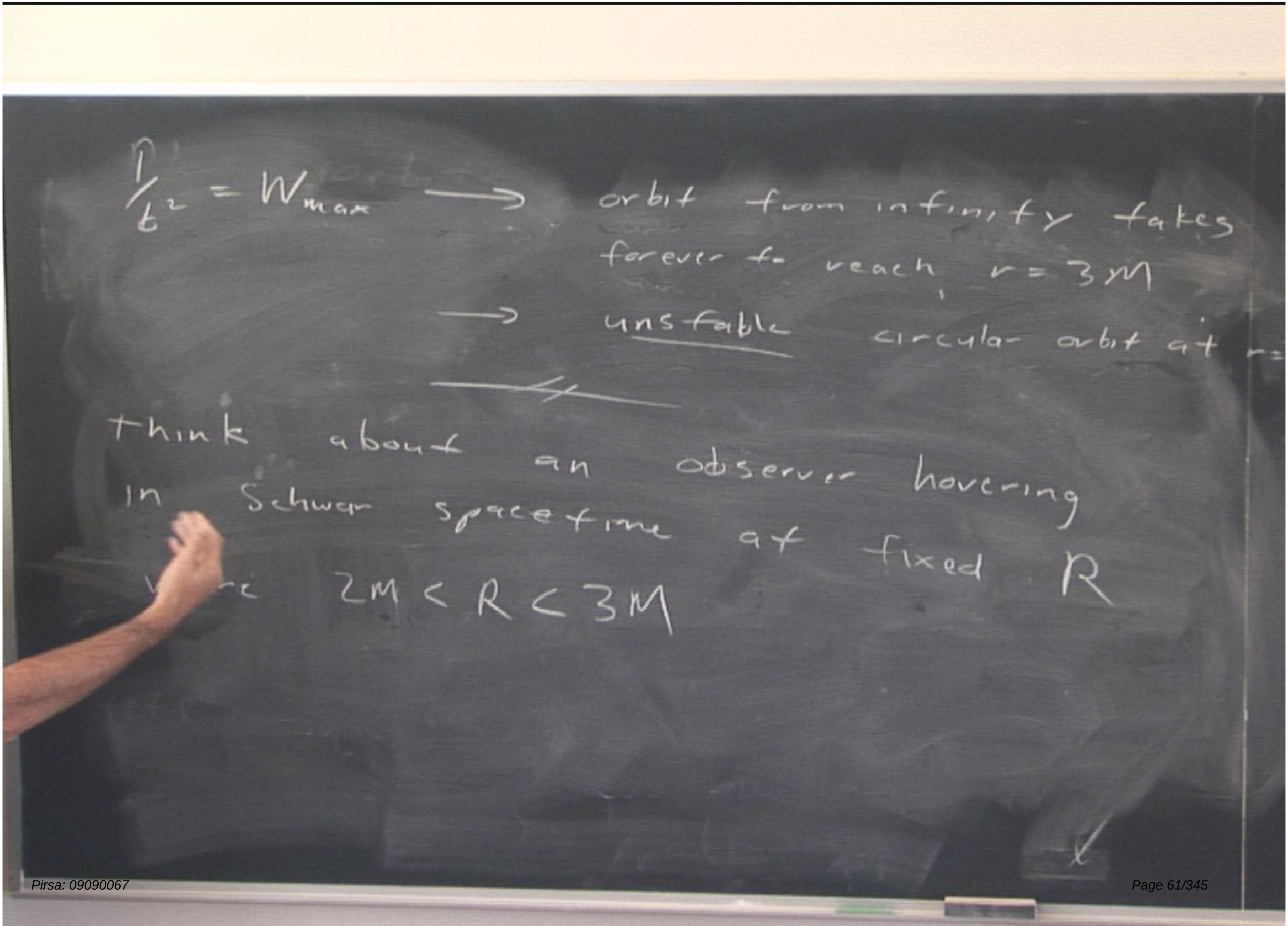
$$\boxed{1/b^2} = \frac{1}{\ell^2}\left(\frac{dr}{d\lambda}\right)^2 + W(r)$$

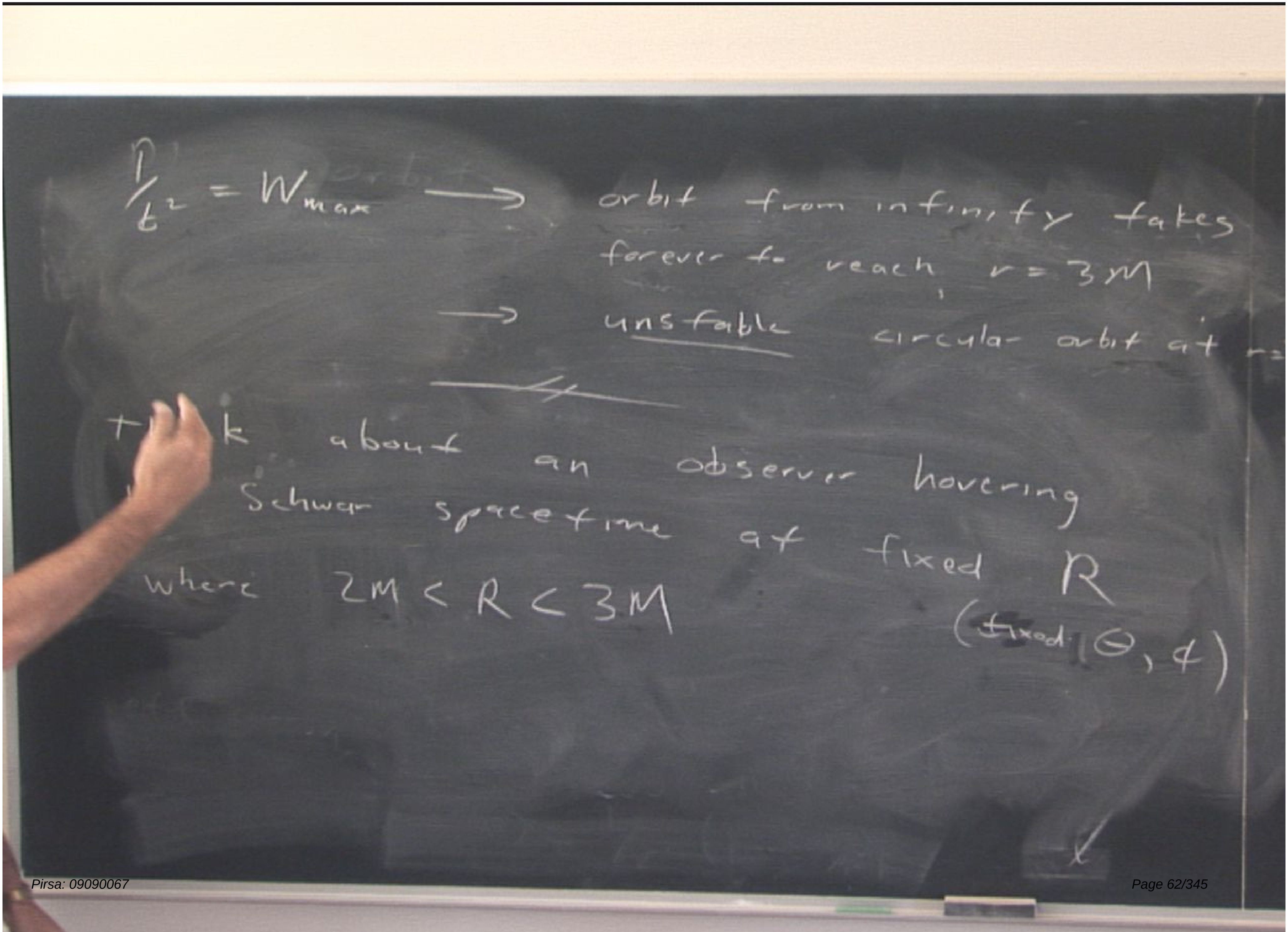
eff. energy $\mathcal{E} = 1/b^2 = e^2/\ell^2$

eff. potential $W(r) = \frac{1}{r^2}\left(1 - 2M/r\right)$


1/b² = W_max → orbit from infinity takes forever to reach r = 3M
→ unstable circular orbit at r=
think about an observer hovering
in


P/b² = W_max → orbit from infinity takes forever to reach r = 3M
→ unstable circular orbit at r=
think about an observer hovering
in Schwar spacetime at fixed R
where


p/t² = W_max → orbit from infinity takes forever to reach r = 3M
→ unstable circular orbit at r=
think about an observer hovering
in Schwar spacetime at fixed R
2M < R < 3M


1/b² = W_max → orbit from infinity takes forever to reach r = 3M
→ unstable circular orbit at r=
k about an observer hovering
Schwar spacetime at fixed R
where 2M < R < 3M
(fixed θ, φ)

$\frac{1}{b^2} = W_{max}$ $\longrightarrow$ orbit from infinity takes forever to reach $r = 3M$

$\longrightarrow$ unstable circular orbit at $r =$

$\not\longrightarrow$

think about an observer hovering in Schwar spacetime at fixed R where $2M < R < 3M$

(fixed θ, ϕ)

$1/b^2 = W_{max}$ $\longrightarrow$ orbit from infinity takes forever to reach $r = 3M$

$\longrightarrow$ unstable circular orbit at $r =$

think about an observer hovering in Schwar spacetime at fixed R where $2M < R < 3M$

(fixed θ, ϕ)

$\frac{1}{b^2} = W_{max}$ $\longrightarrow$ orbit from infinity takes forever to reach $r = 3M$

$\longrightarrow$ unstable circular orbit at $r=$

think about an observer hovering in Schwar spacetime at fixed R where $2M < R < 3M$ (fixed θ, ϕ)

$l^2/E^2 = W_{max}$ → orbit from infinity takes forever to reach $r = 3M$

→ unstable circular orbit at r=

think about an observer hovering in Schwar spacetime at fixed R

where $2M < R < 3M$

(fixed θ, ϕ)

$\frac{1}{b^2} = W_{max}$ $\longrightarrow$ orbit from infinity takes forever to reach $r = 3M$

$\longrightarrow$ unstable circular orbit at $r=$

think about an observer hovering in Schwar spacetime at fixed R where $2M < R < 3M$

(fixed θ, ϕ)

$1/b^2 = W_{max}$ ⟶ orbit from infinity takes forever to reach $r = 3M$

⟶ <u>unstable</u> circular orbit at $r =$

think about an observer hovering in Schwar spacetime at fixed R where $2M < R < 3M$

(fixed θ, ϕ)

$\frac{1}{b^2} = W_{max}$ $\longrightarrow$ orbit from infinity takes forever to reach $r = 3M$

$\longrightarrow$ unstable circular orbit at $r =$

think about an observer hovering in Schwar spacetime at fixed R where $2M < R < 3M$

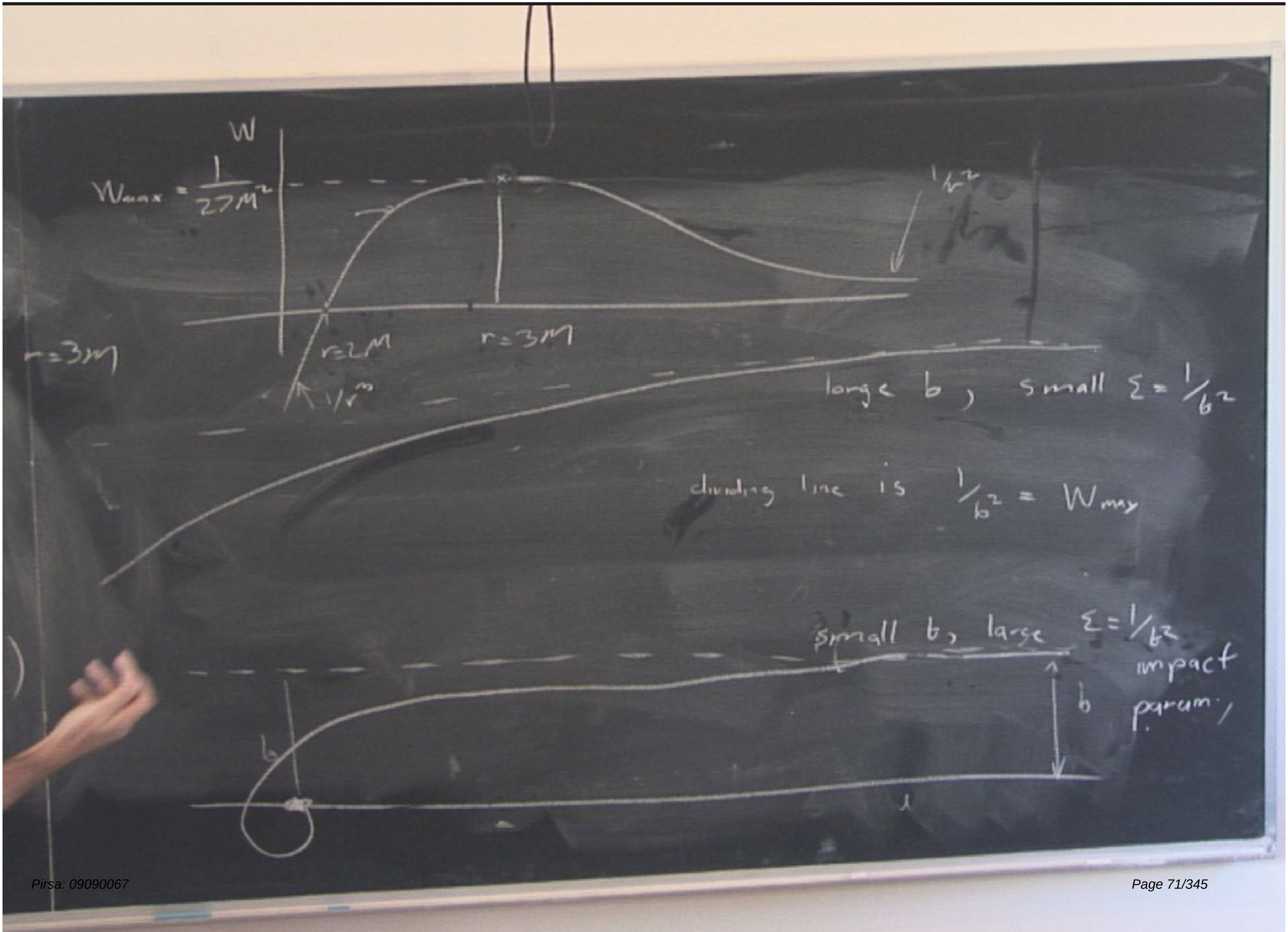
(fixed θ, ϕ)

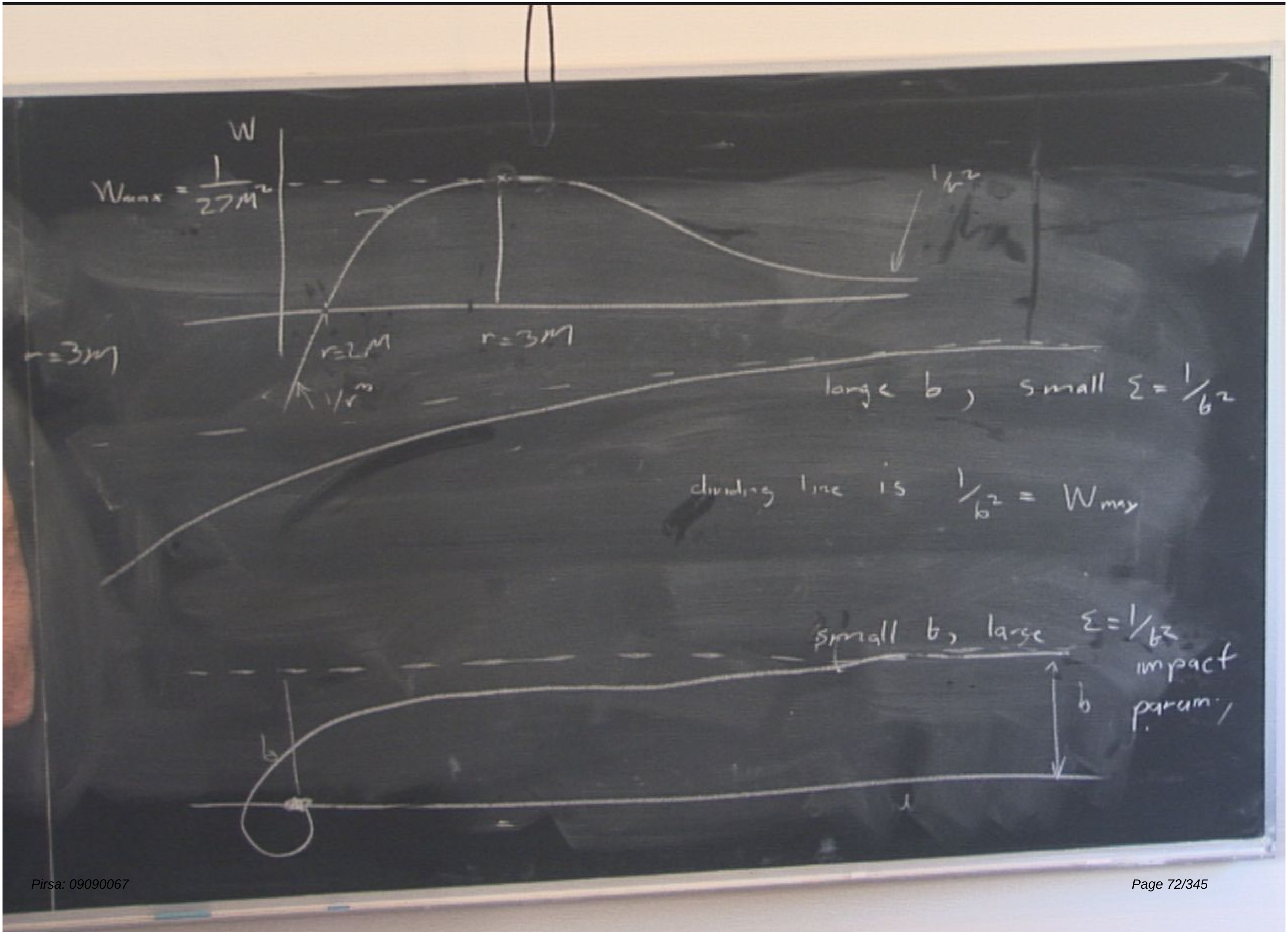
$1/b^2 = W_{max}$ ⟶ orbit from infinity takes forever to reach $r = 3M$

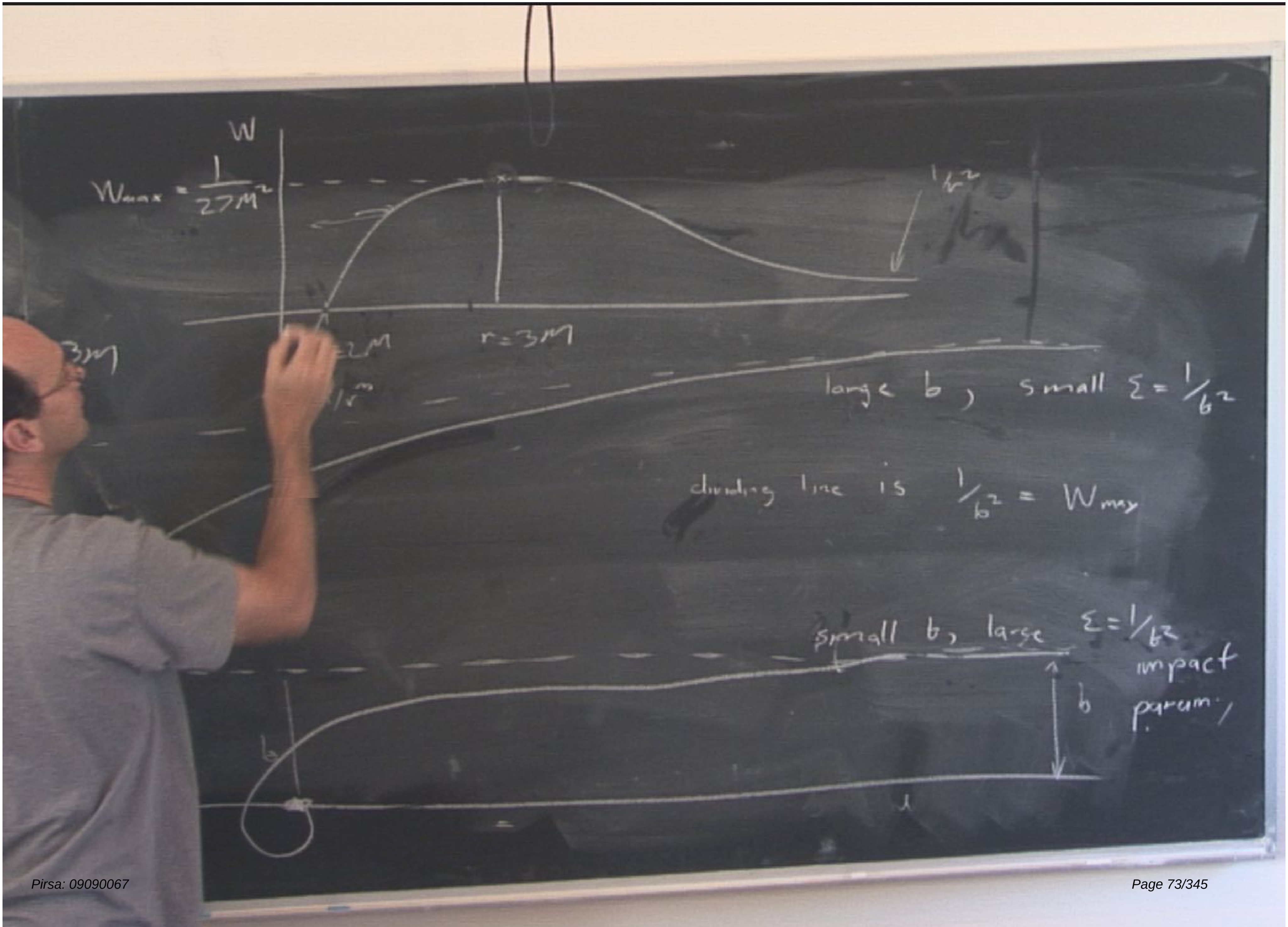
⟶ unstable circular orbit at $r=$

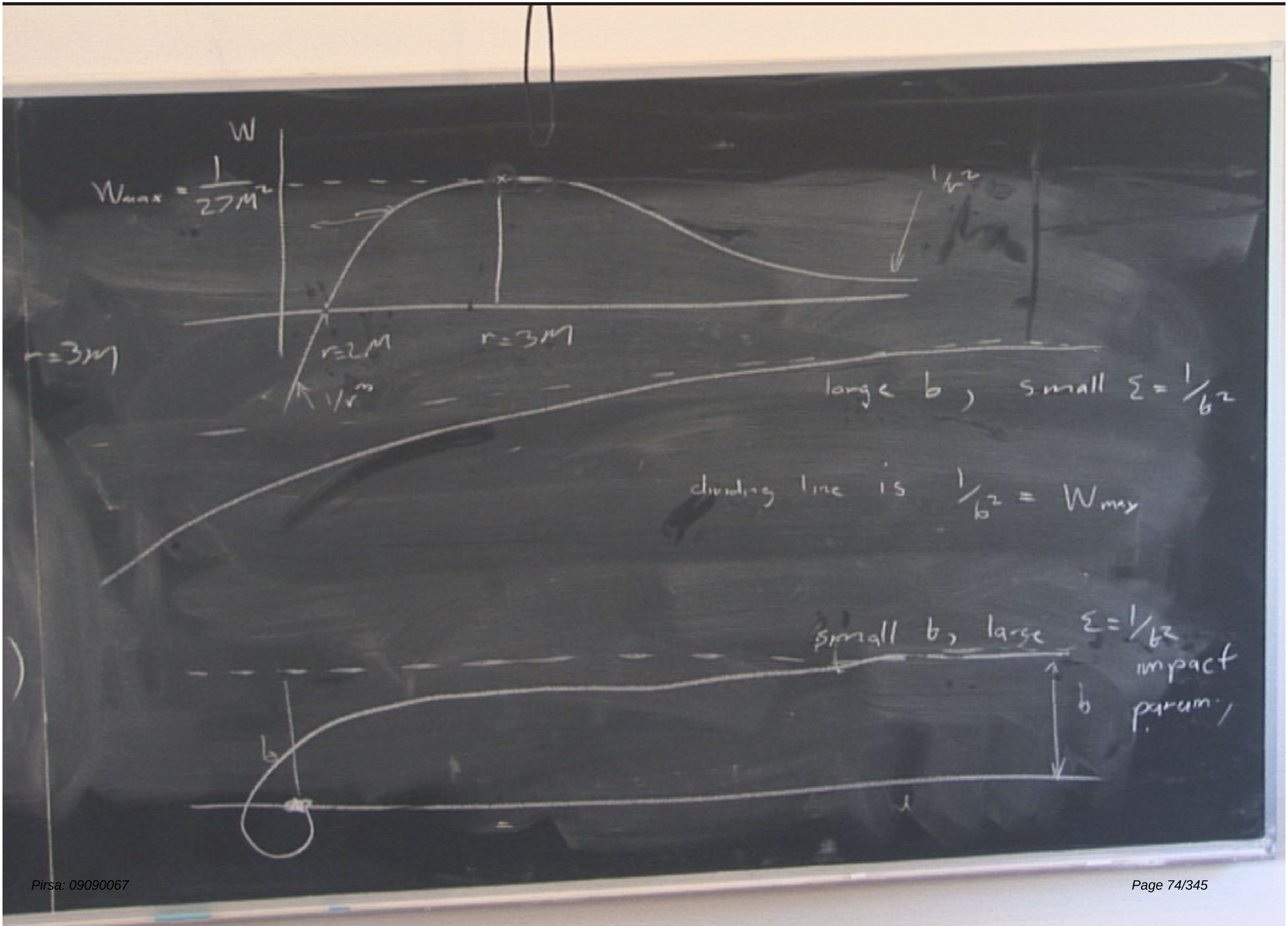
think about an observer hovering in Schwar spacetime at fixed R where $2M < R < 3M$

(fixed θ, ϕ)


W
Wmax = 1/27M²
1/b²
r=2M
r=3M
r=3M
large b, small ε = 1/b²
dividing line is 1/b² = Wmax
small b, large ε = 1/b²
impact
b
param.
b


W
Wmax = 1/27M²
1/b²
r=2M
r=3M
r=3M
large b, small ε = 1/b²
dividing line is 1/b² = Wmax
small b, large ε = 1/b²
impact param.
b


W
Wmax = 1/27M²
r=3M
1/b²
large b, small ε = 1/b²
dividing line is 1/b² = Wmax
small b, large ε = 1/b²
impact
param.
b


W
Wmax = 1/27M²
1/b²
r=2M
r=3M
r=3M
large b, small ε = 1/b²
dividing line is 1/b² = Wmax
small b, large ε = 1/b²
impact param.
b

$1/b^2 = W_{max}$ ⟶ orbit from infinity takes forever to reach $r = 3M$

⟶ <u>unstable</u> circular orbit at $r=$

think about an observer hovering in Schwar spacetime at fixed R where $2M < R < 3M$

(fixed θ, ϕ)

$\frac{1}{b^2} = W_{max}$ ⟶ orbit from infinity takes forever to reach $r = 3M$

⟶ unstable circular orbit at $r =$

think about an observer hovering in Schwar spacetime at fixed R where $2M < R < 3M$ (fixed θ, ϕ)

$1/b^2 = W_{max}$ $\longrightarrow$ orbit from infinity takes forever to reach $r = 3M$

$\longrightarrow$ unstable circular orbit at $r=$

think about an observer hovering in Schwar spacetime at fixed R where $2M < R < 3M$

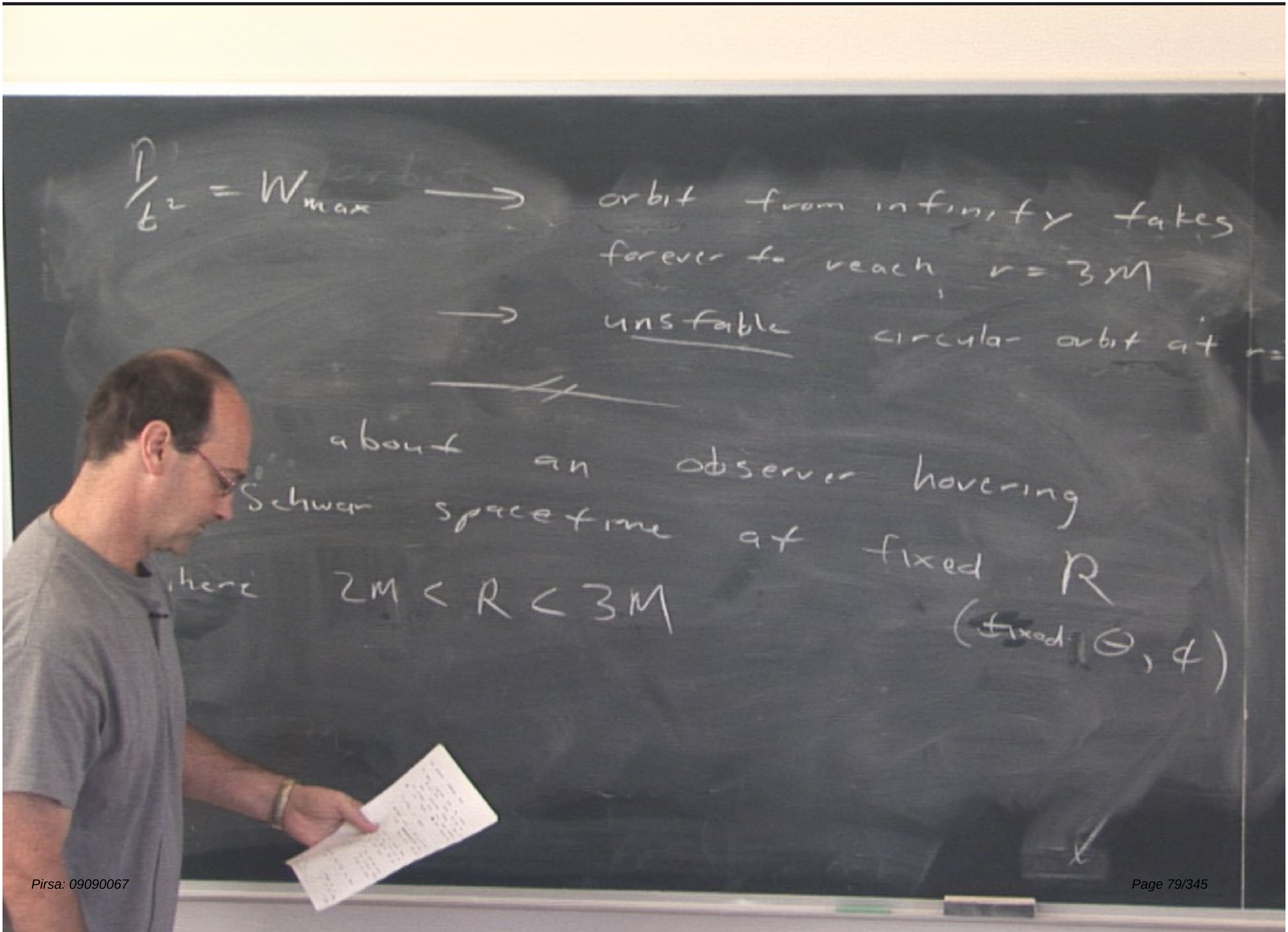
(fixed θ, ϕ)

$1/b^2 = W_{max}$ ⟶ orbit from infinity takes forever to reach $r = 3M$

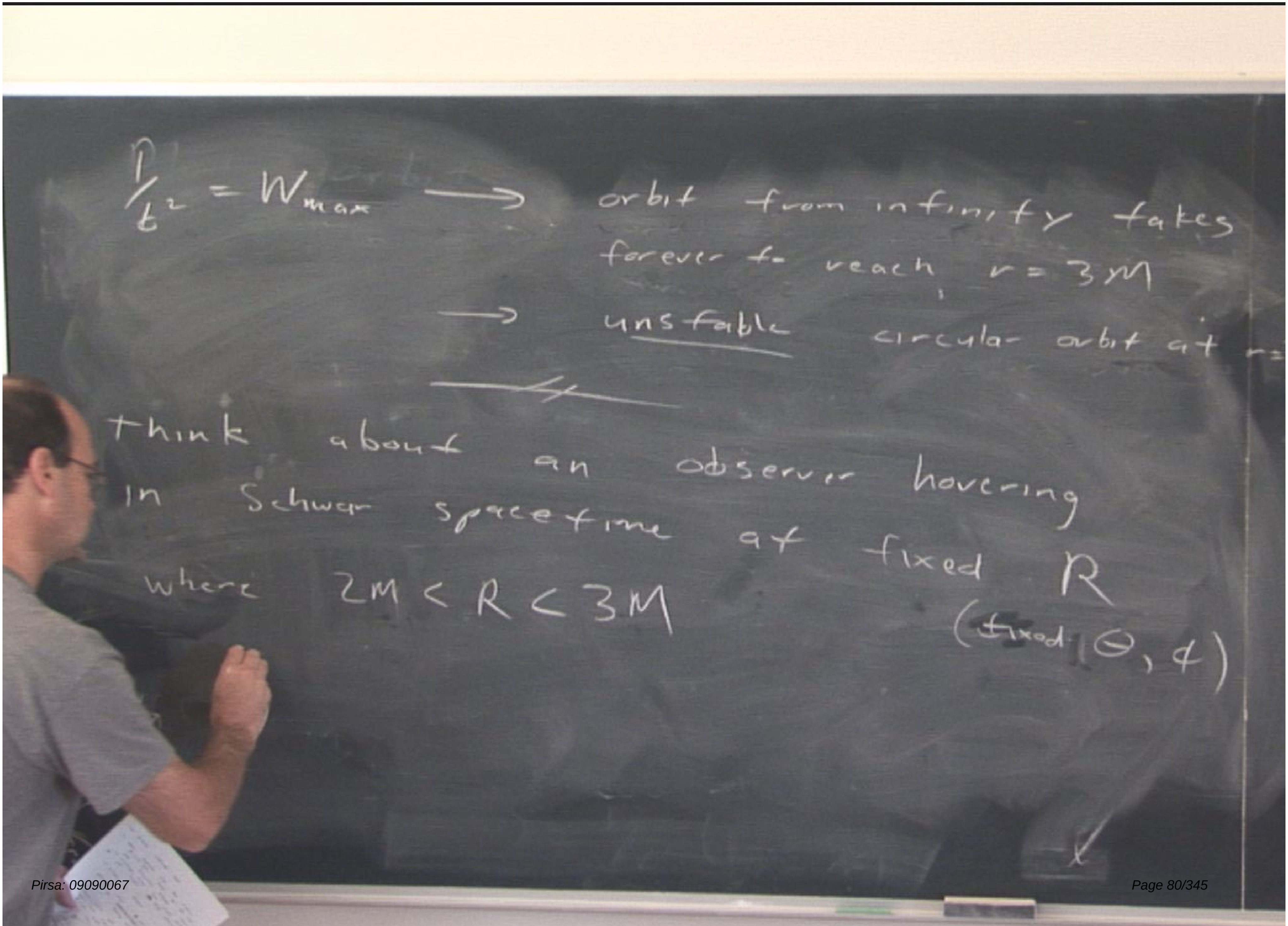
⟶ <u>unstable</u> circular orbit at $r =$

think about an observer hovering in Schwar spacetime at fixed R where $2M < R < 3M$

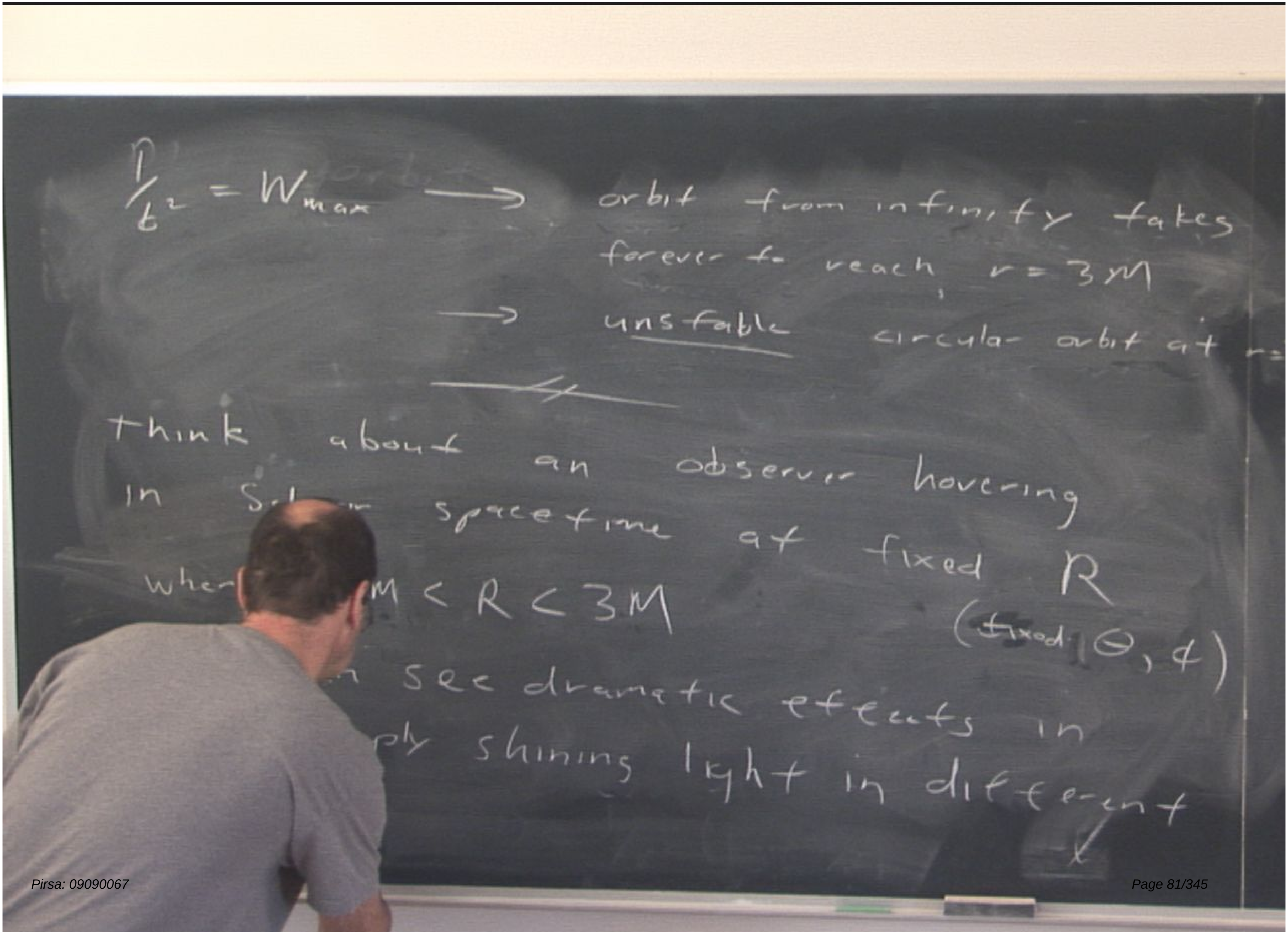
(fixed θ, ϕ)



1/b² = W_max → orbit from infinity takes
forever to reach r = 3M
→ unstable circular orbit at r=
about an observer hovering
Schwar spacetime at fixed R
here 2M < R < 3M
(fixed θ, φ)



1/b² = W_max → orbit from infinity takes
forever to reach r = 3M
→ unstable circular orbit at r=
think about an observer hovering
in Schwar spacetime at fixed R
where 2M < R < 3M
(fixed θ, φ)

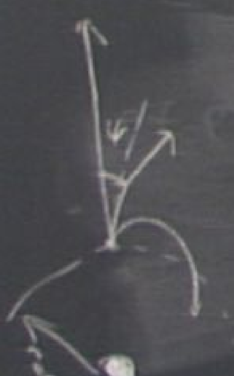

1/b² = W max
orbit from infinity takes
forever to reach r = 3M
unstable circular orbit at
think about an observer hovering
in
spacetime at fixed R
M < R < 3M
(fixed θ, φ)
see dramatic effects in
shining light in different

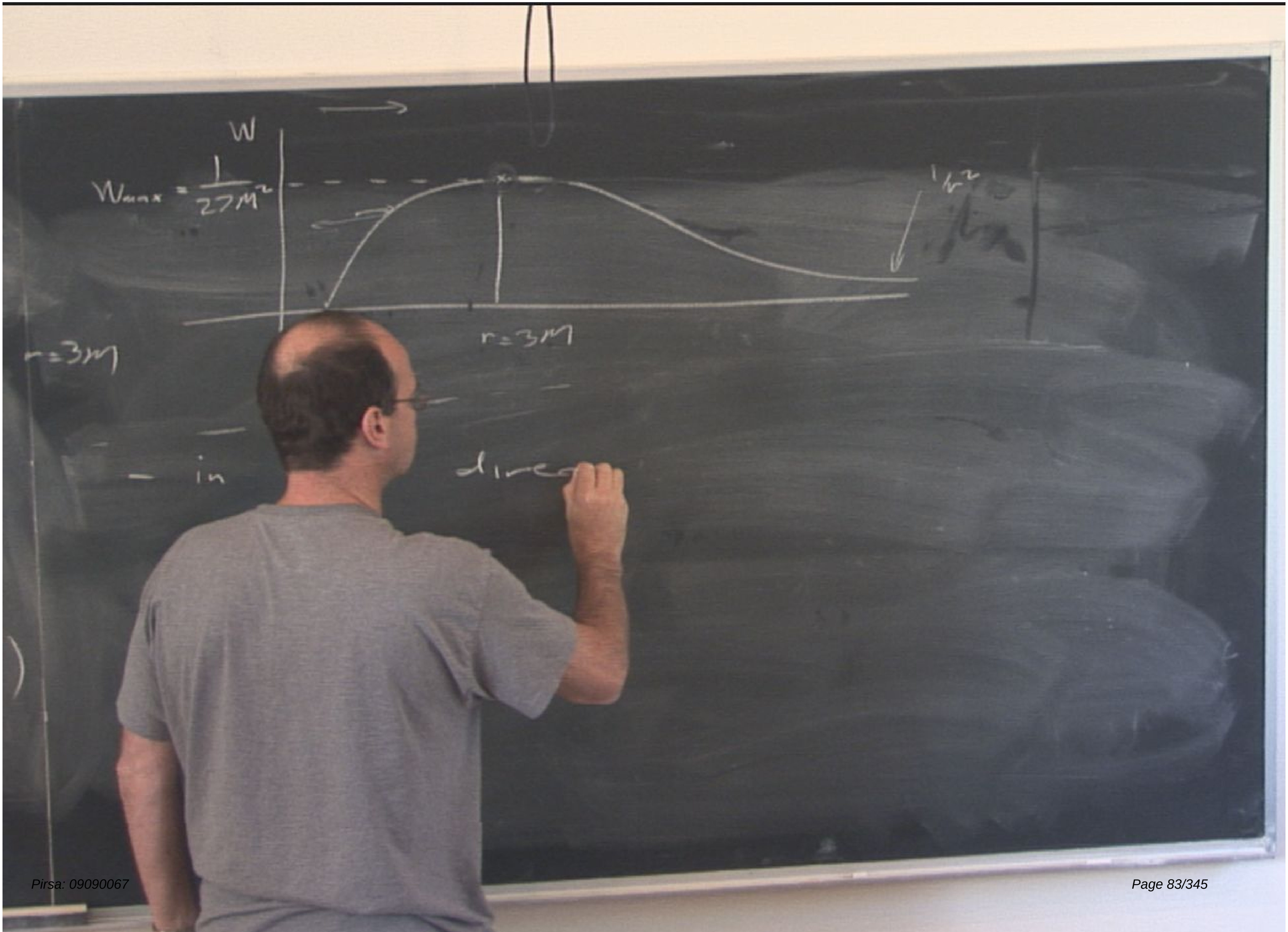
$1/b^2 = W_{max}$ $\longrightarrow$ orbit from infinity takes forever to reach $r = 3M$

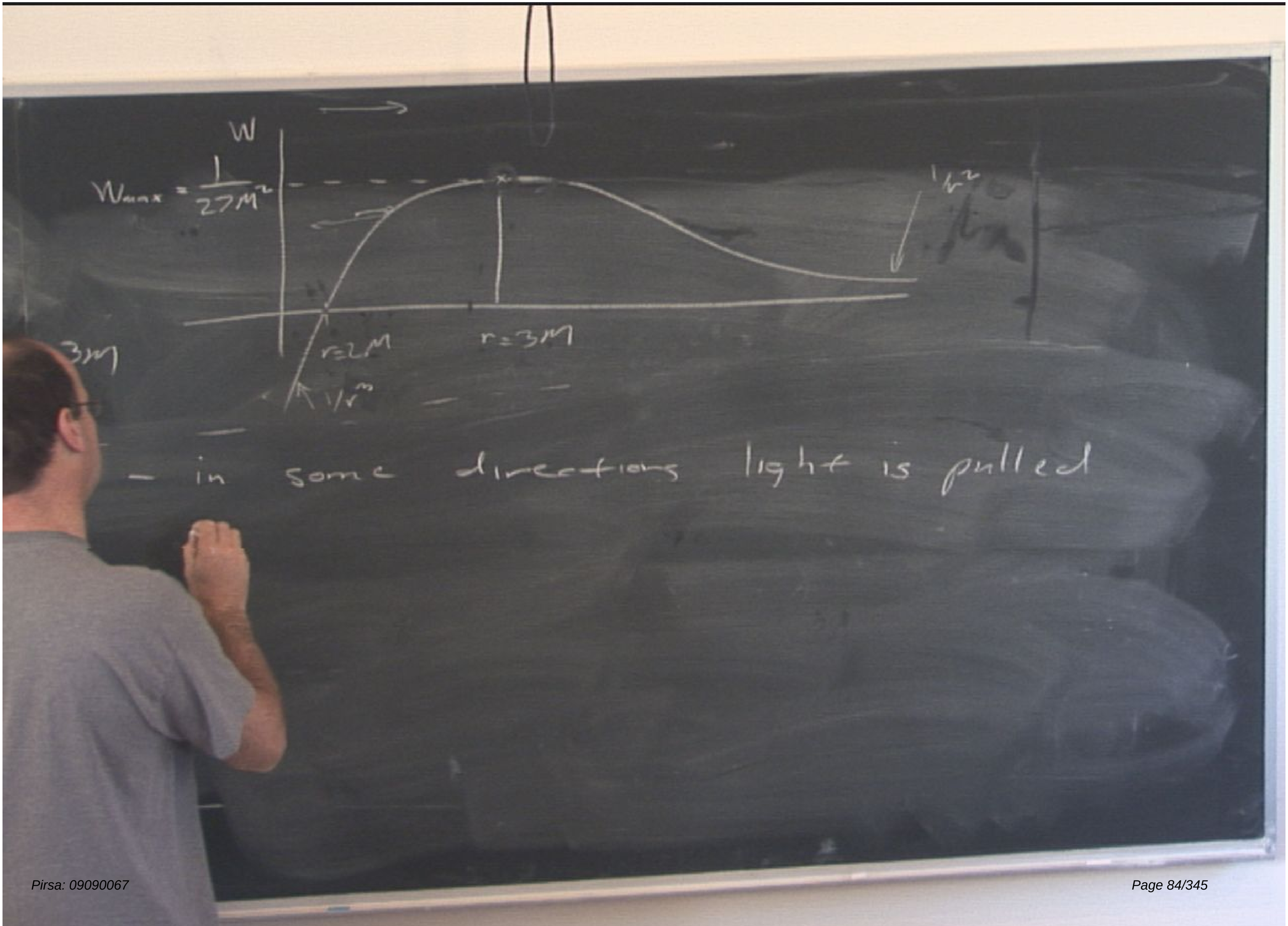
$\longrightarrow$ unstable circular orbit at $r =$

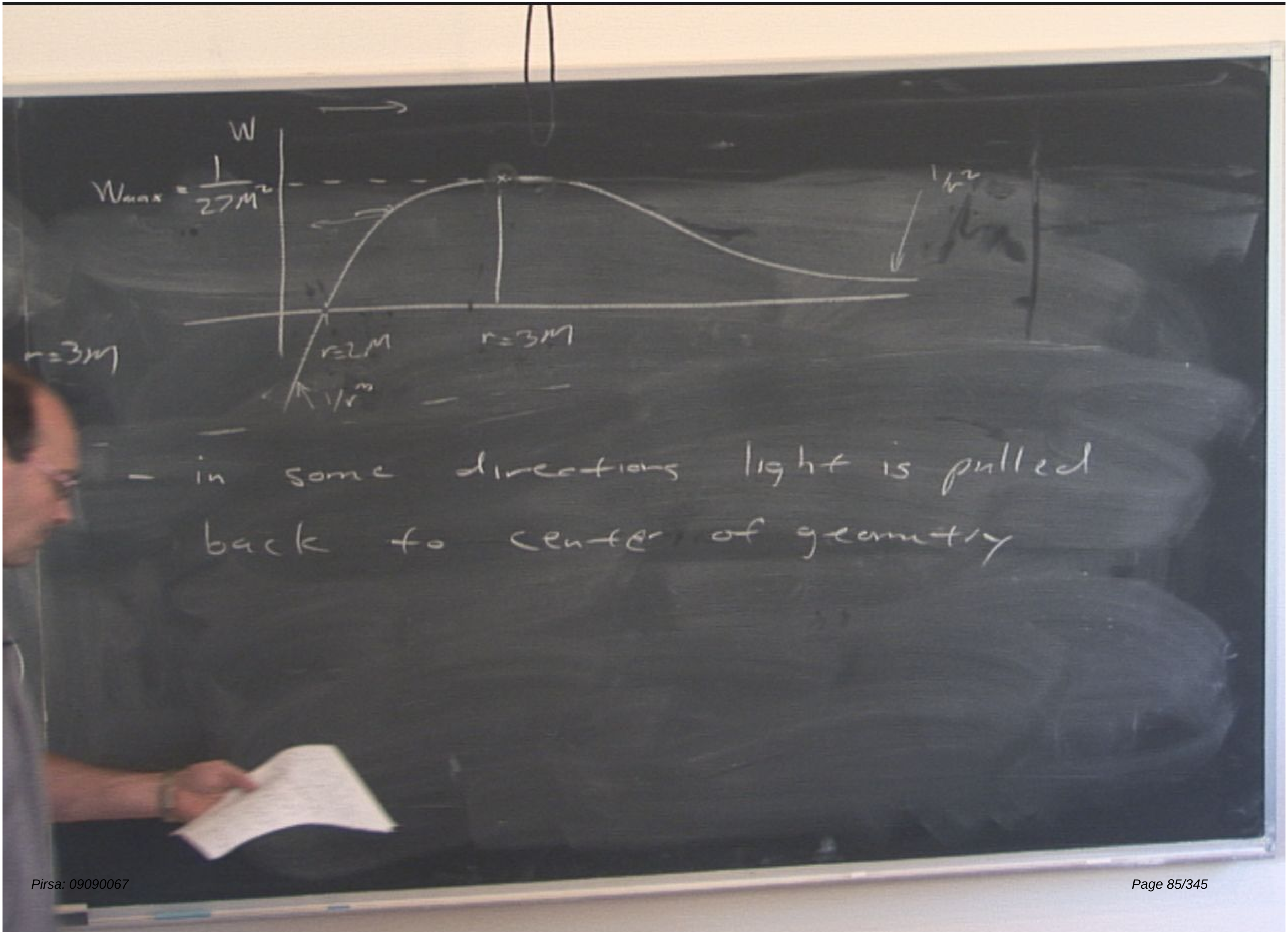
think about an observer hovering in Schwar spacetime at fixed R where $2M < R < 3M$ (fixed θ, ϕ)

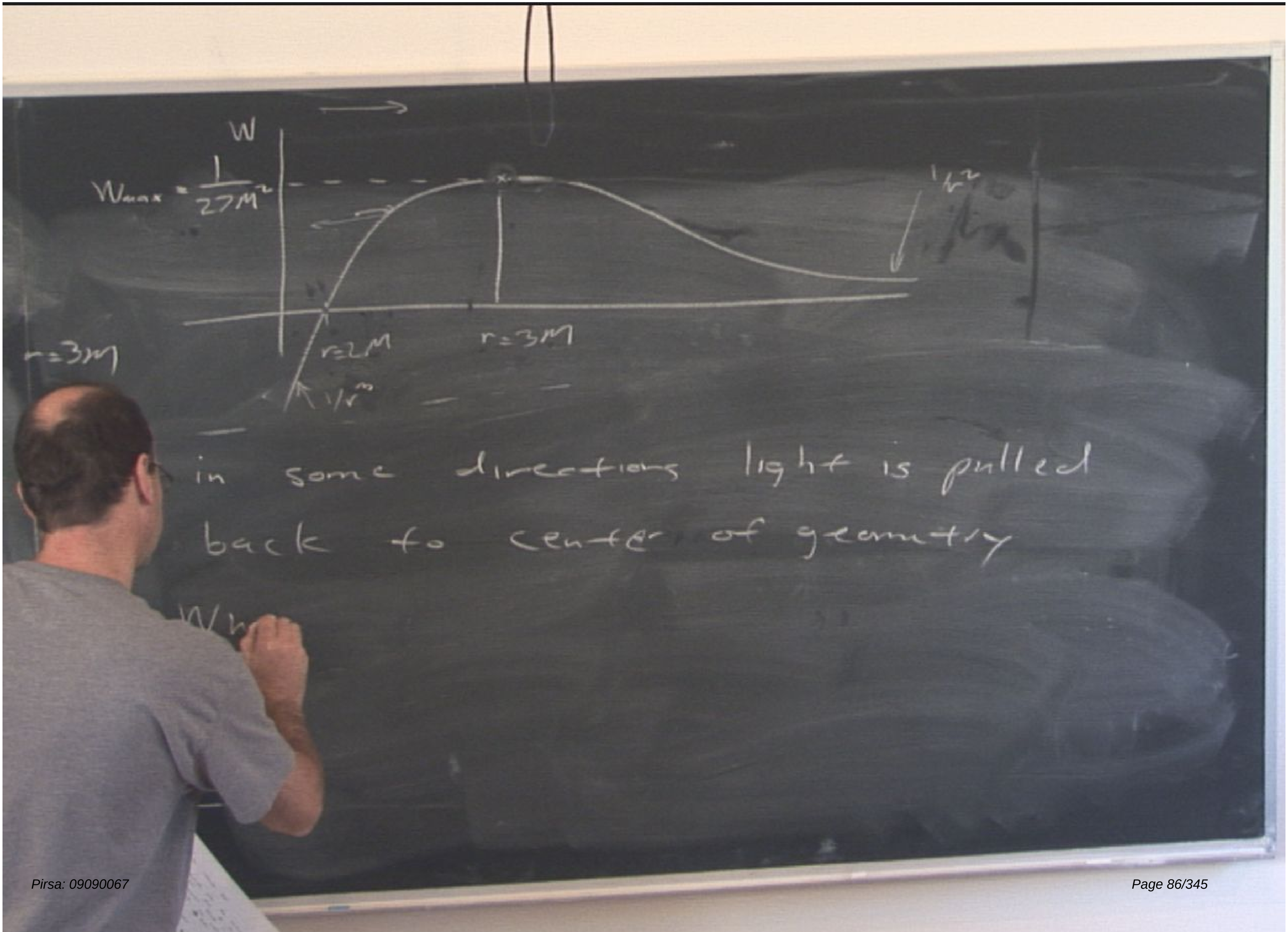
- can see dramatic effects in simply shining light in different directions

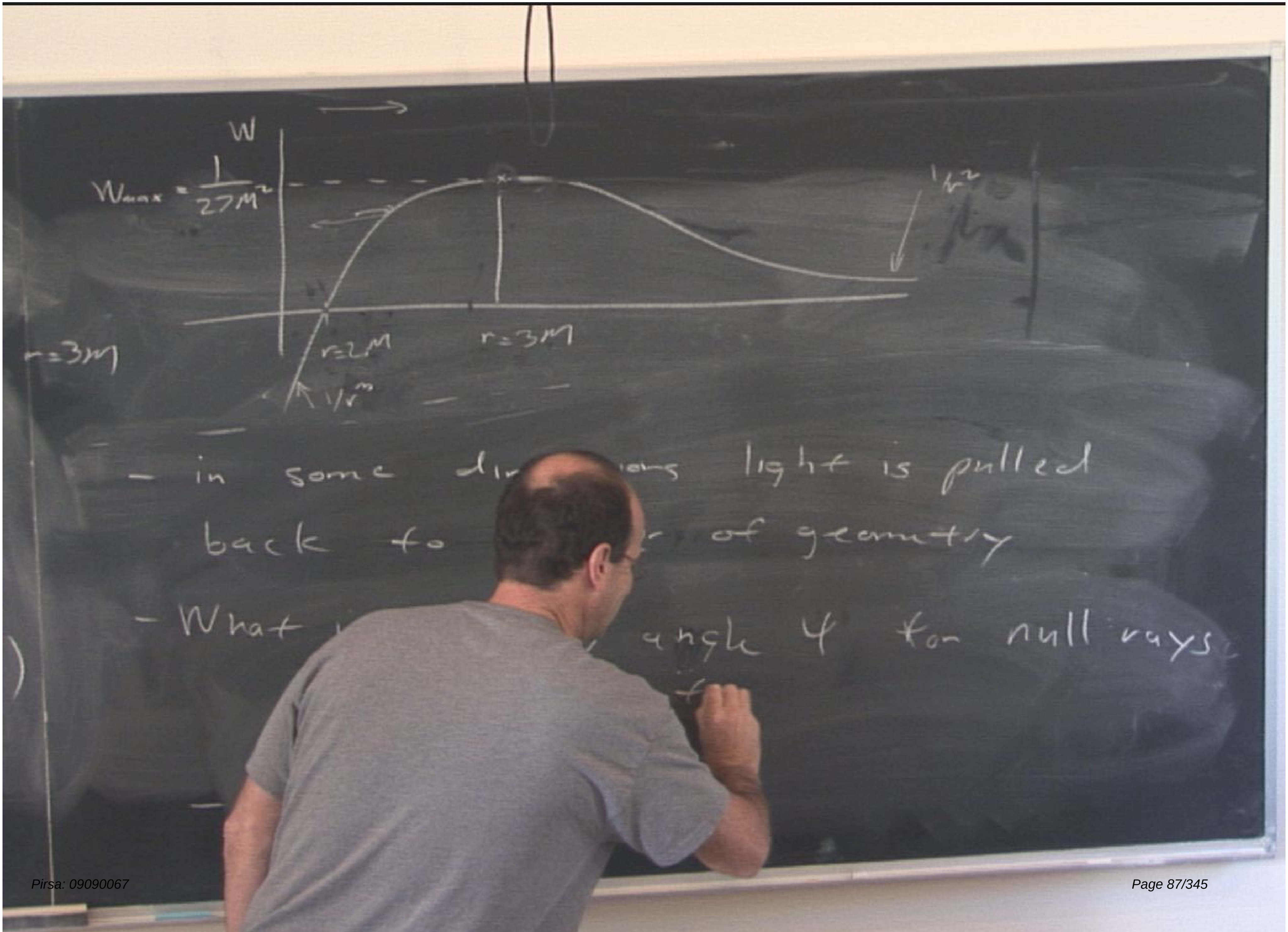


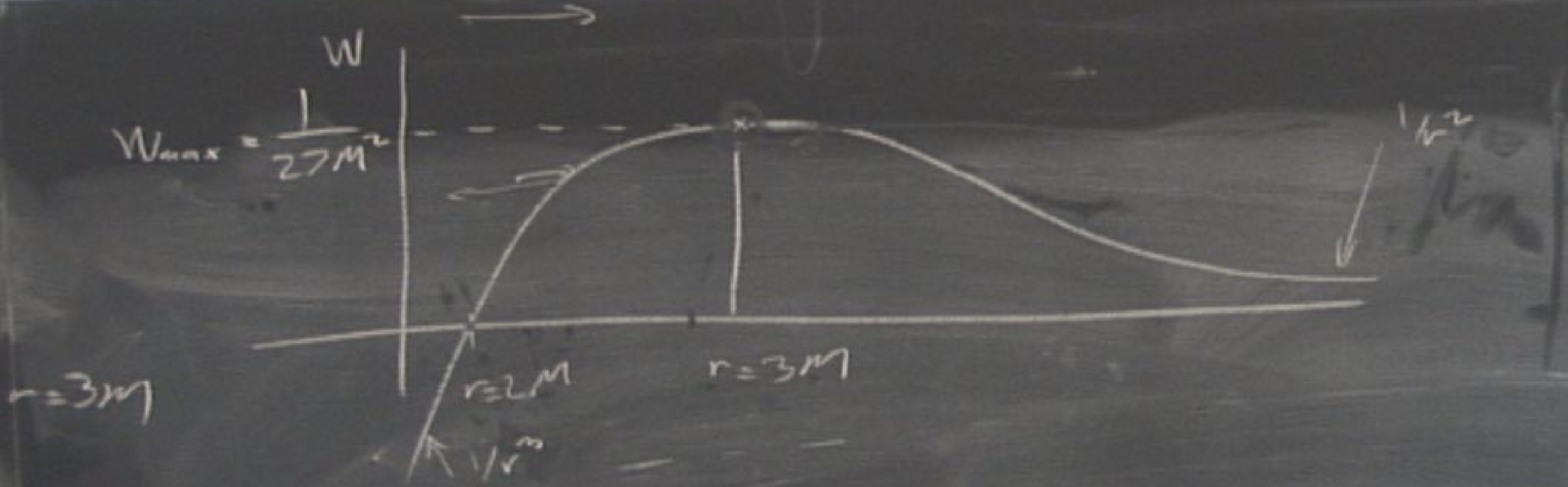

W
Wmax = 1/27M²
r=3M
r=3M
in
direc


W
Wmax = 1/27M²
r=2M
r=3M
3M
- in some directions light is pulled


W
Wmax = 1/27M²
r=2M
r=3M
r=3M
1/r²
- in some directions light is pulled
back to center of geometry

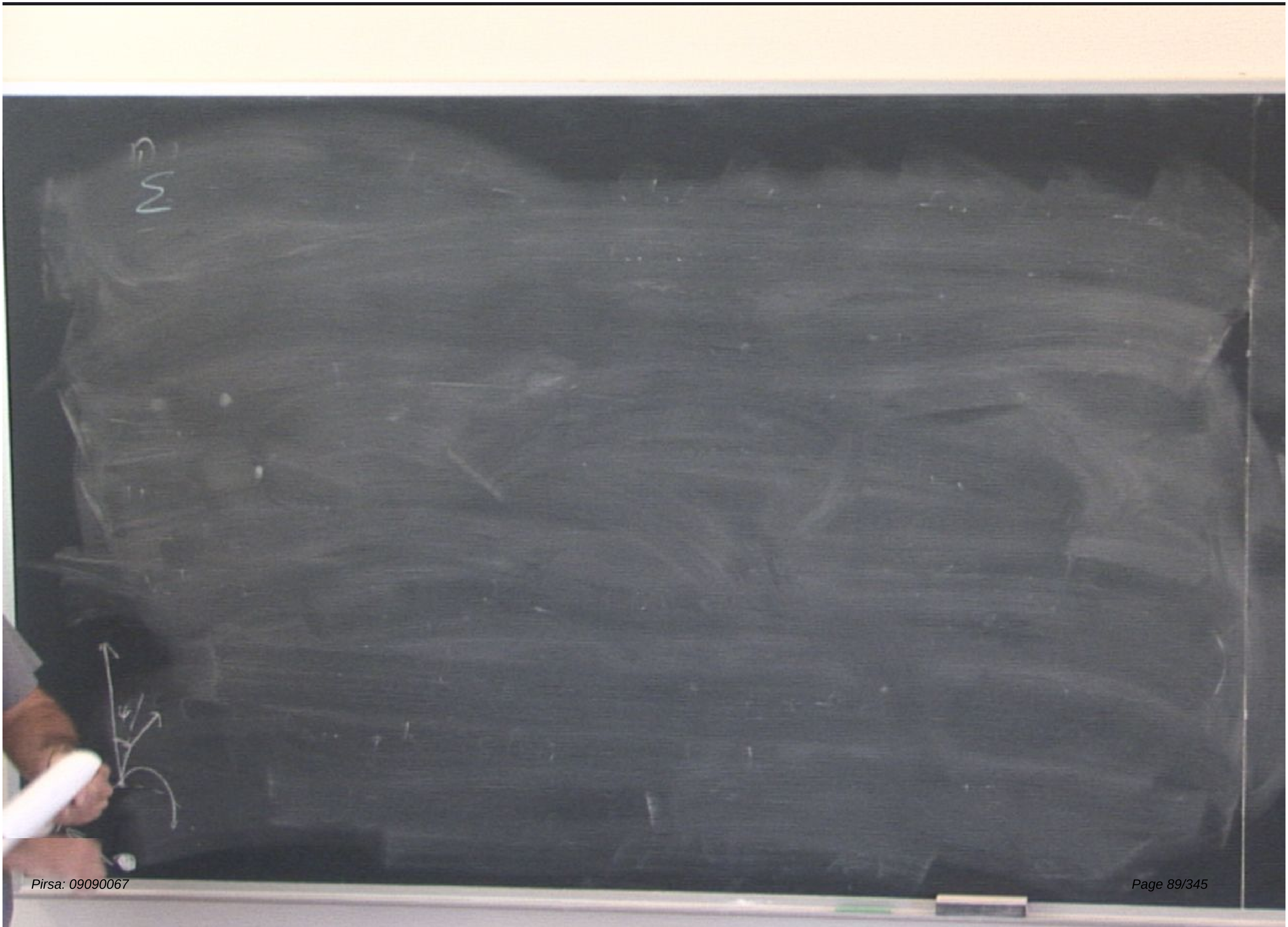

W
Wmax = 1/27M²
r=2M
r=3M
r=3M
1/r²
in some directions light is pulled
back to center of geometry

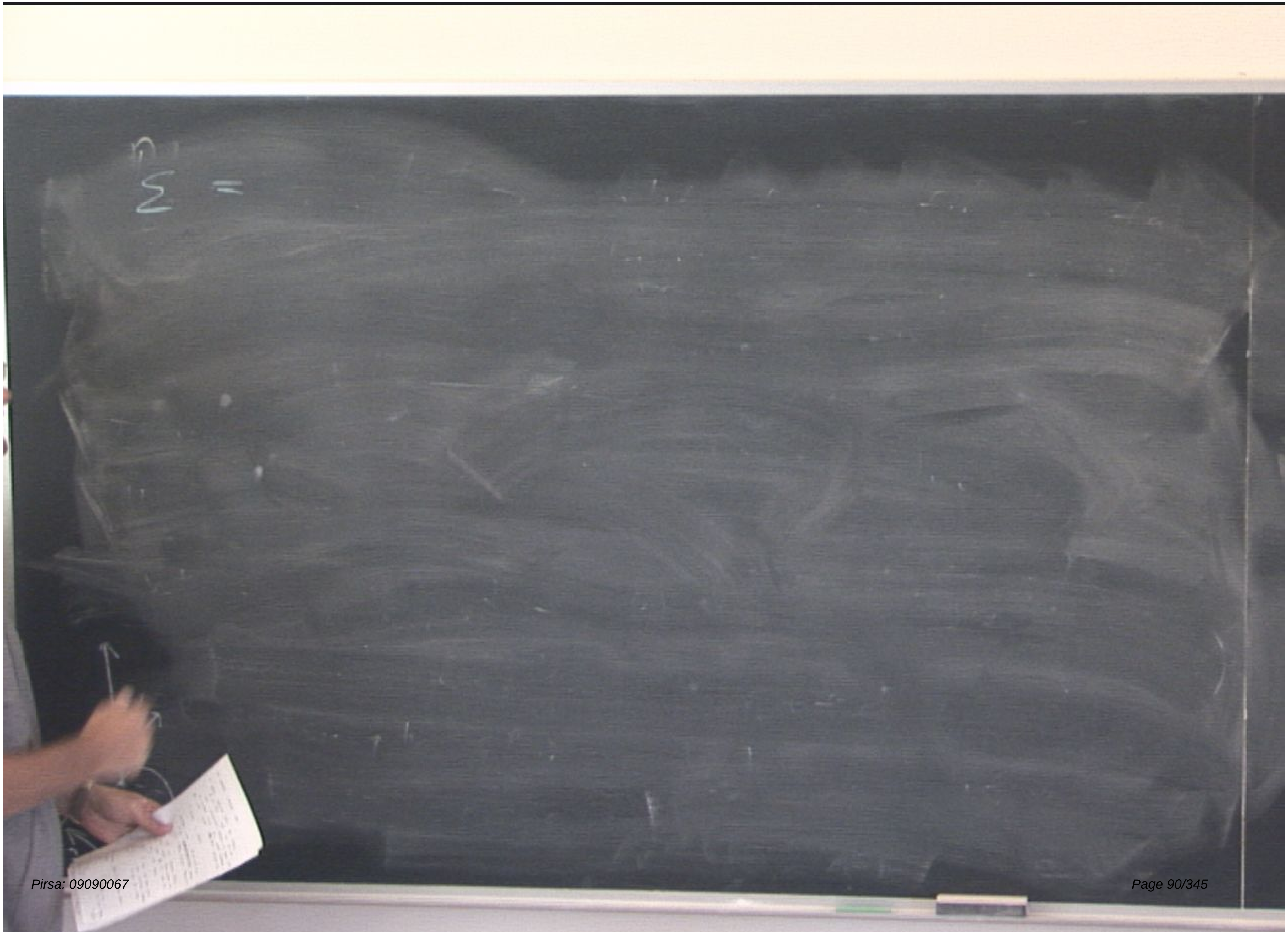

W
Wmax = 1/27M²
1/r²
r=2M
r=3M
r=3M
1/r^n
- in some di... light is pulled
back to ... of geometry
- What ... angle ψ for null rays

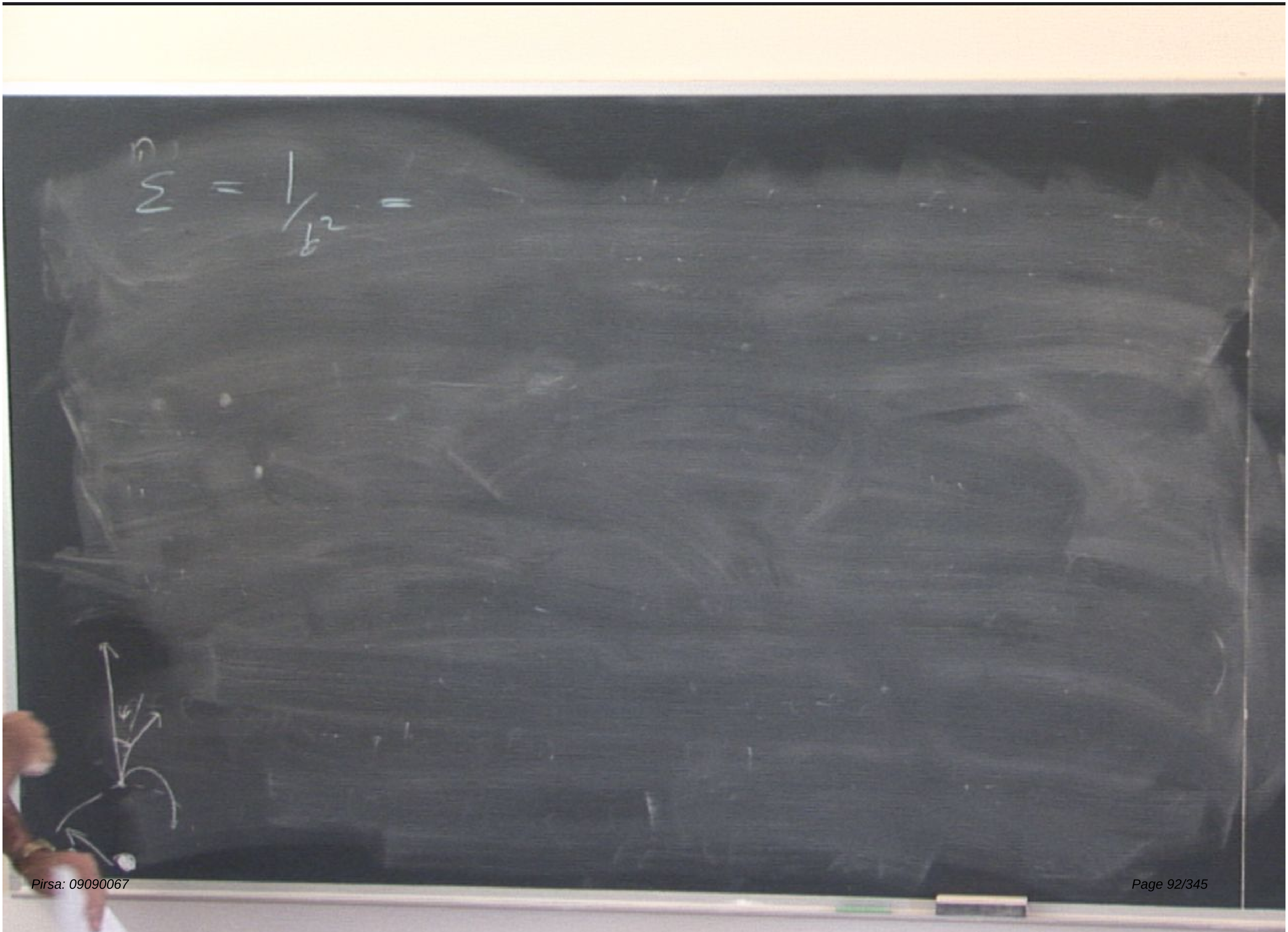


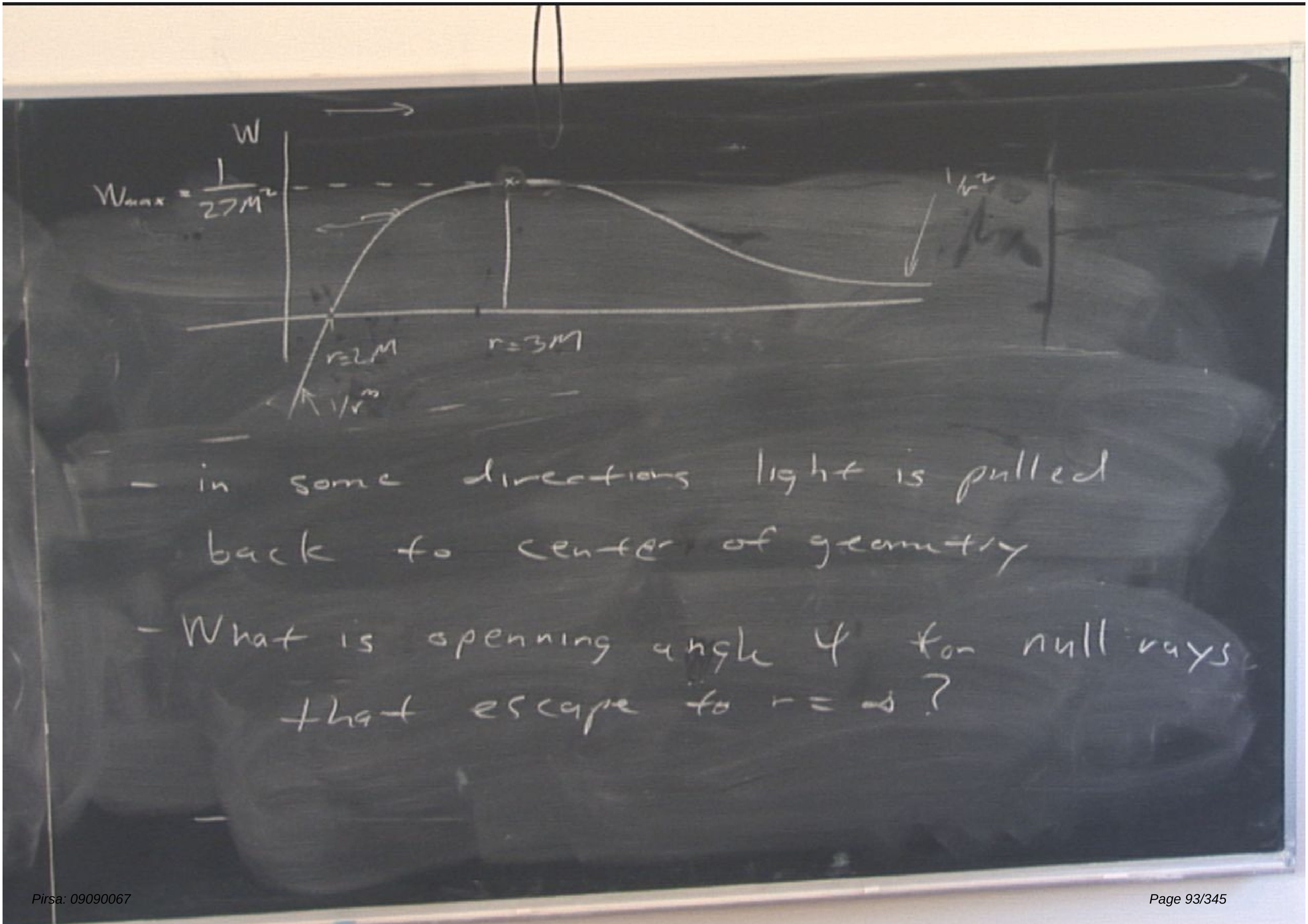
- in some directions light is pulled back to center of geometry

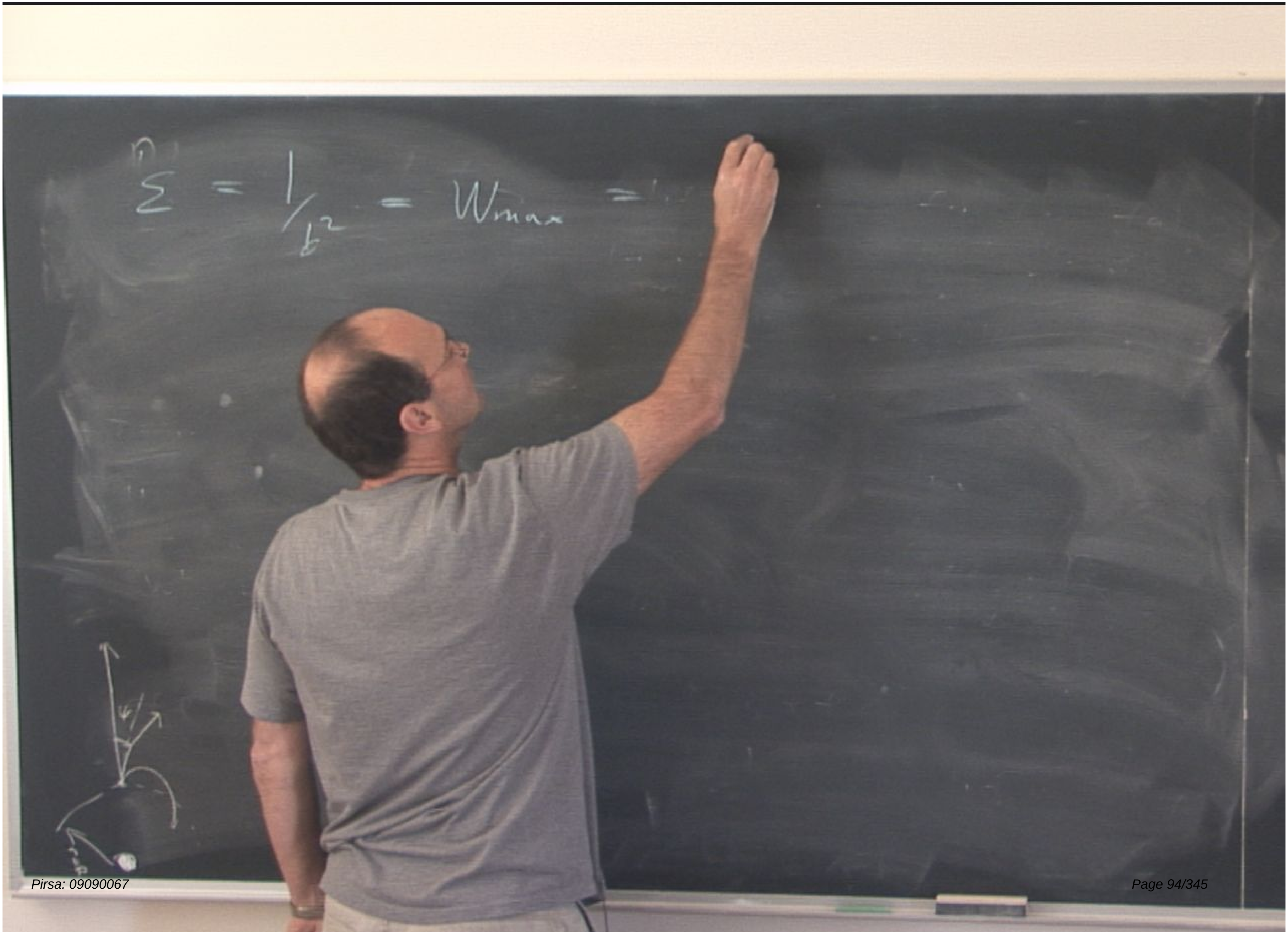
- What is openning angle ψ for null rays that escape to $r = \infty$?

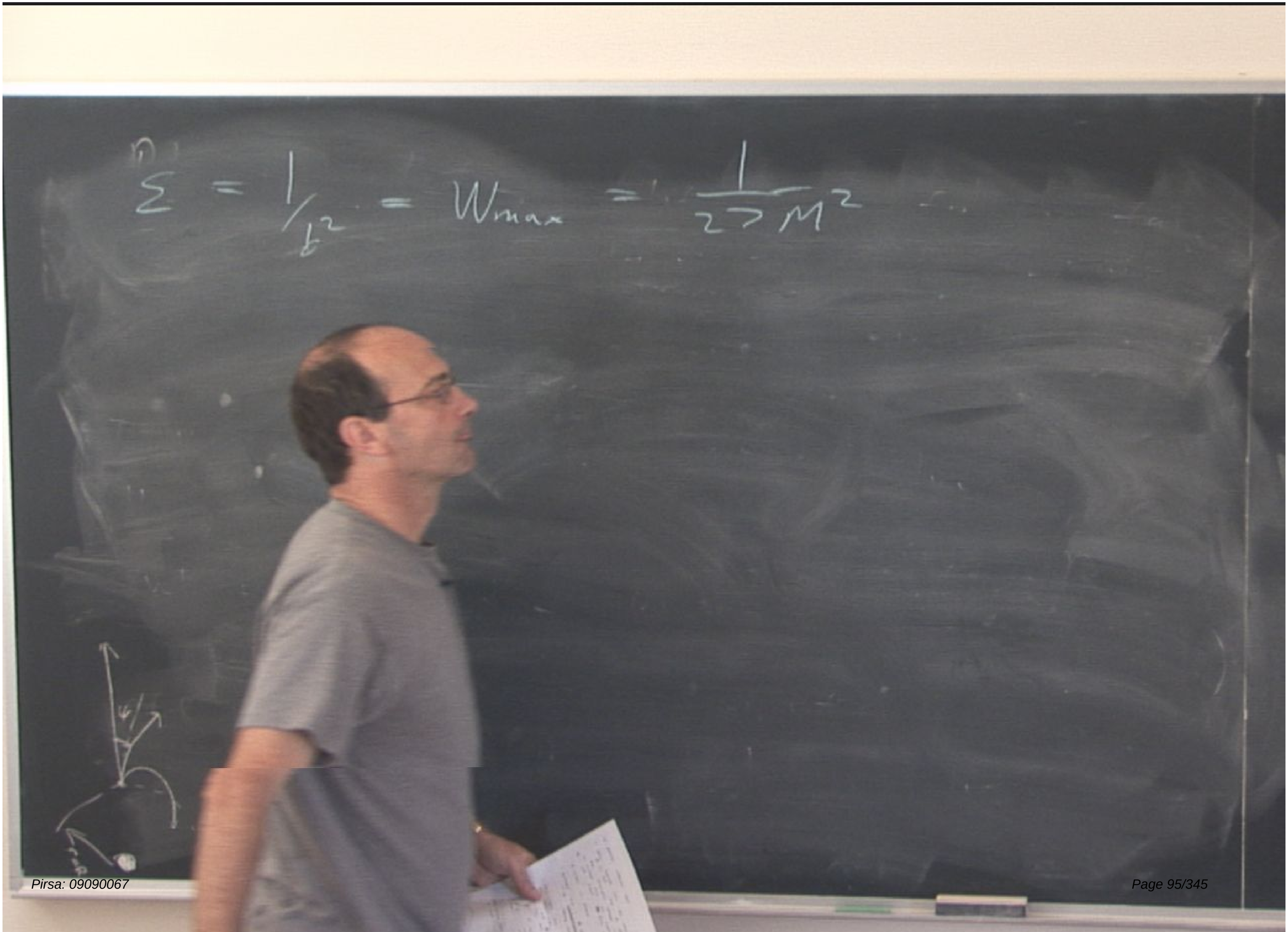




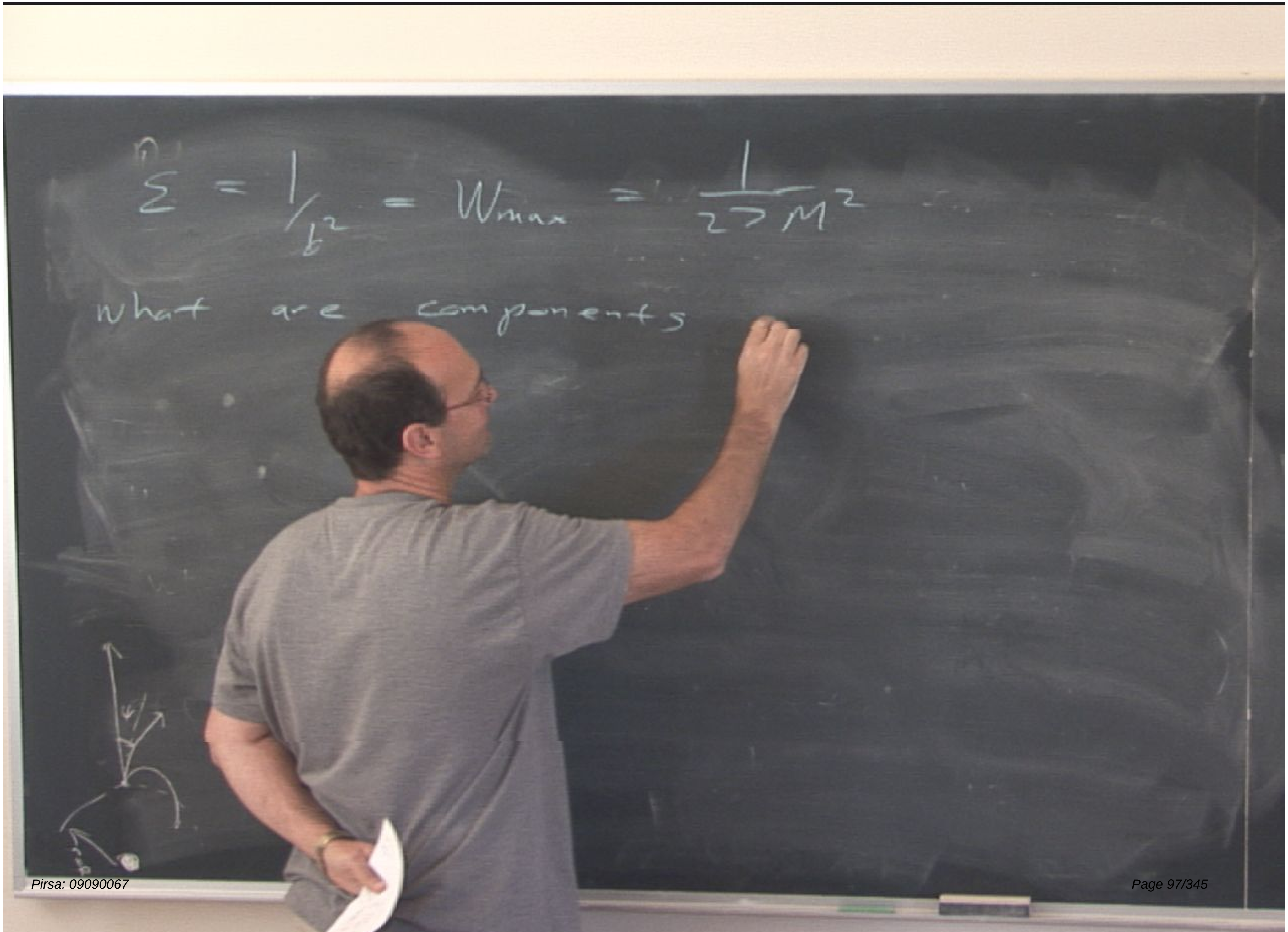

Σ = 1/b² =

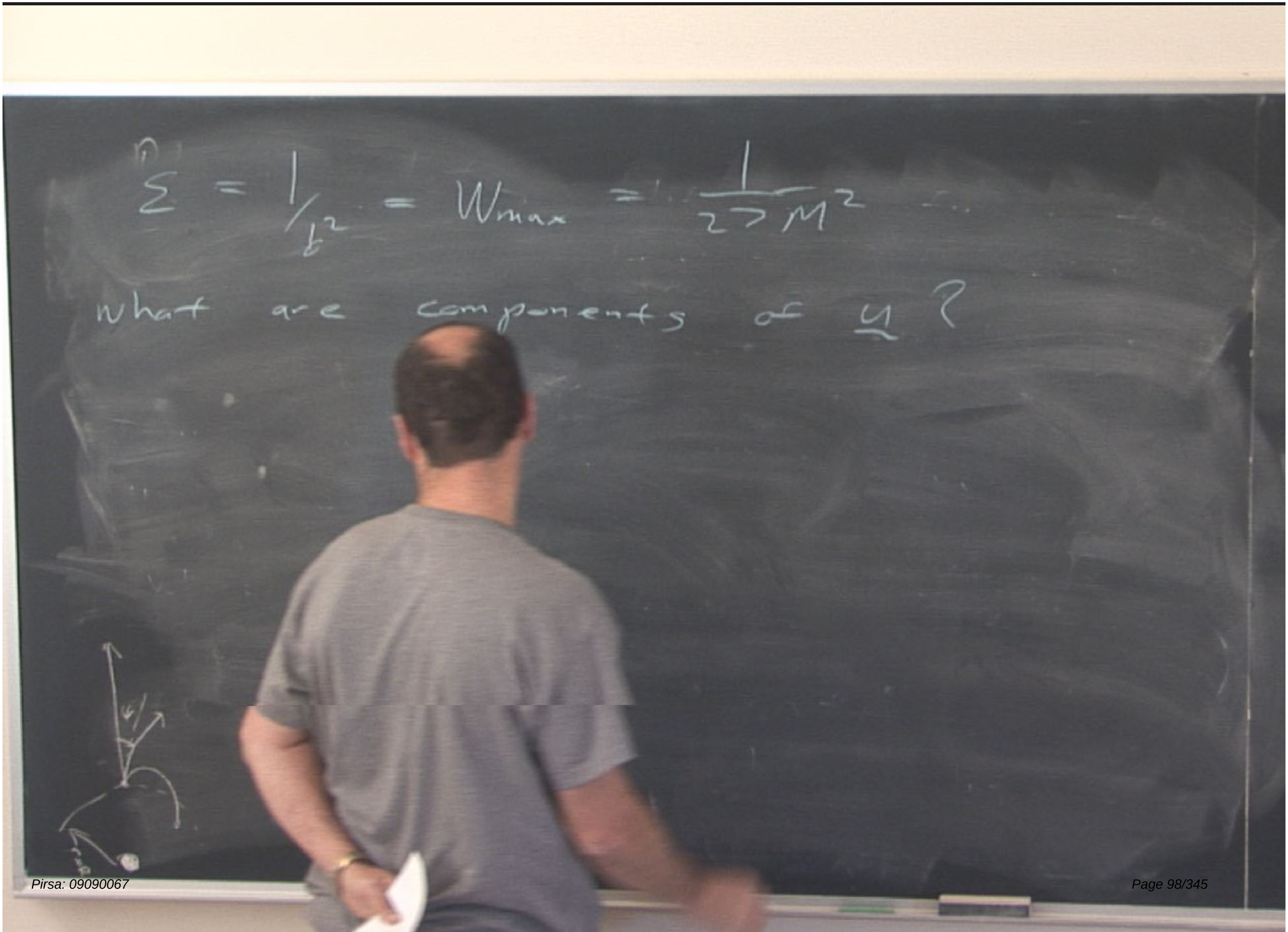

W
Wmax = 1/27M^2
r=2M
r=3M
1/r^m
1/r^2
- in some directions light is pulled back to center of geometry
- What is openning angle ψ for null rays that escape to r = ∞?


Σ = 1/b² = Wmax =


Σ = 1/ℓ² = Wmax = 1/27M²

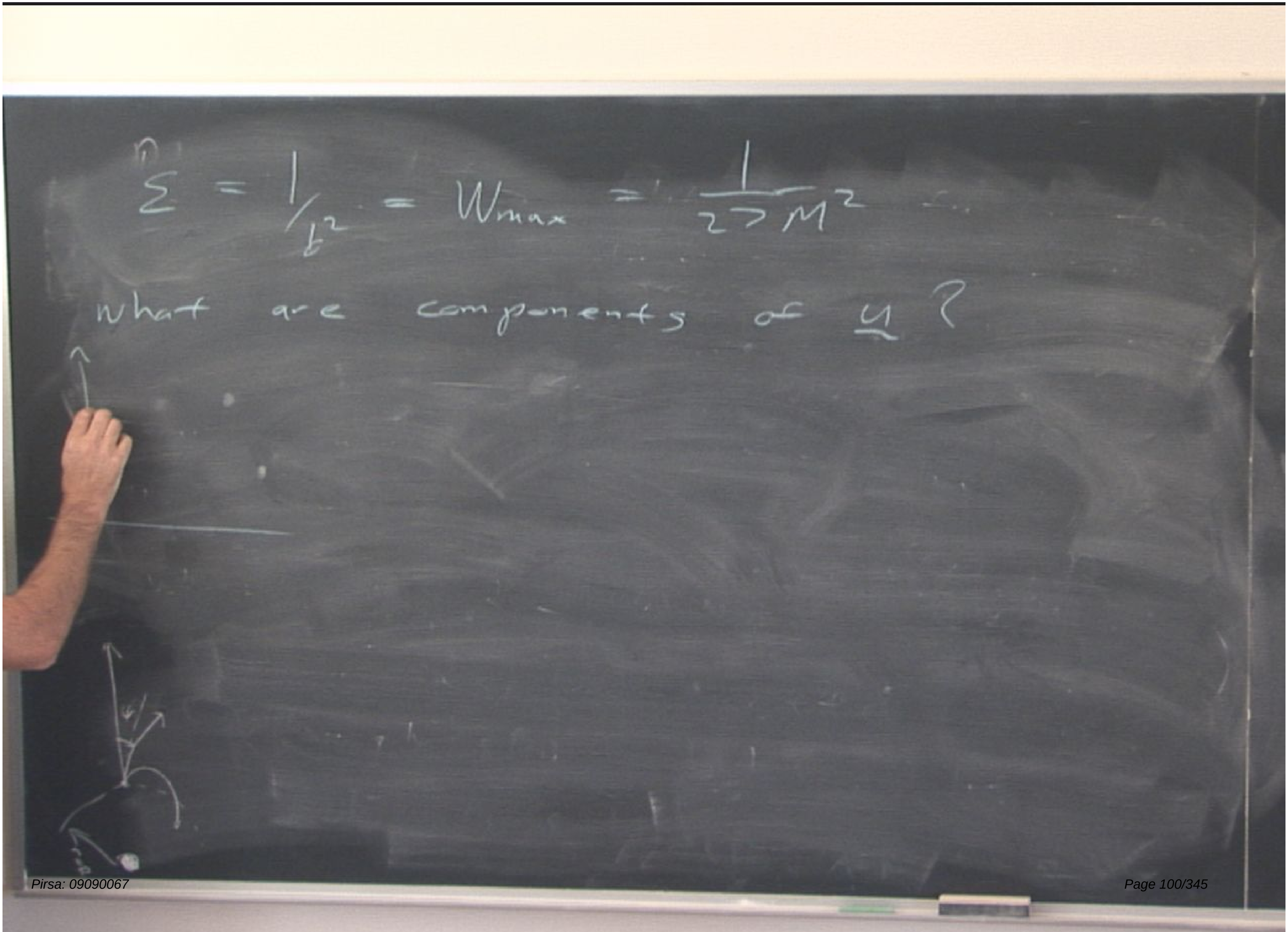
$$\Sigma = 1/b^2 = W_{max} = \frac{1}{27M^2}$$


what are components


Σ = 1/b² = Wmax = 1/27M²
what are components of u ?

$$\hat{\Sigma} = 1/b^2 = W_{max} = \frac{1}{27M^2}$$

What are components of $\underline{u}$?

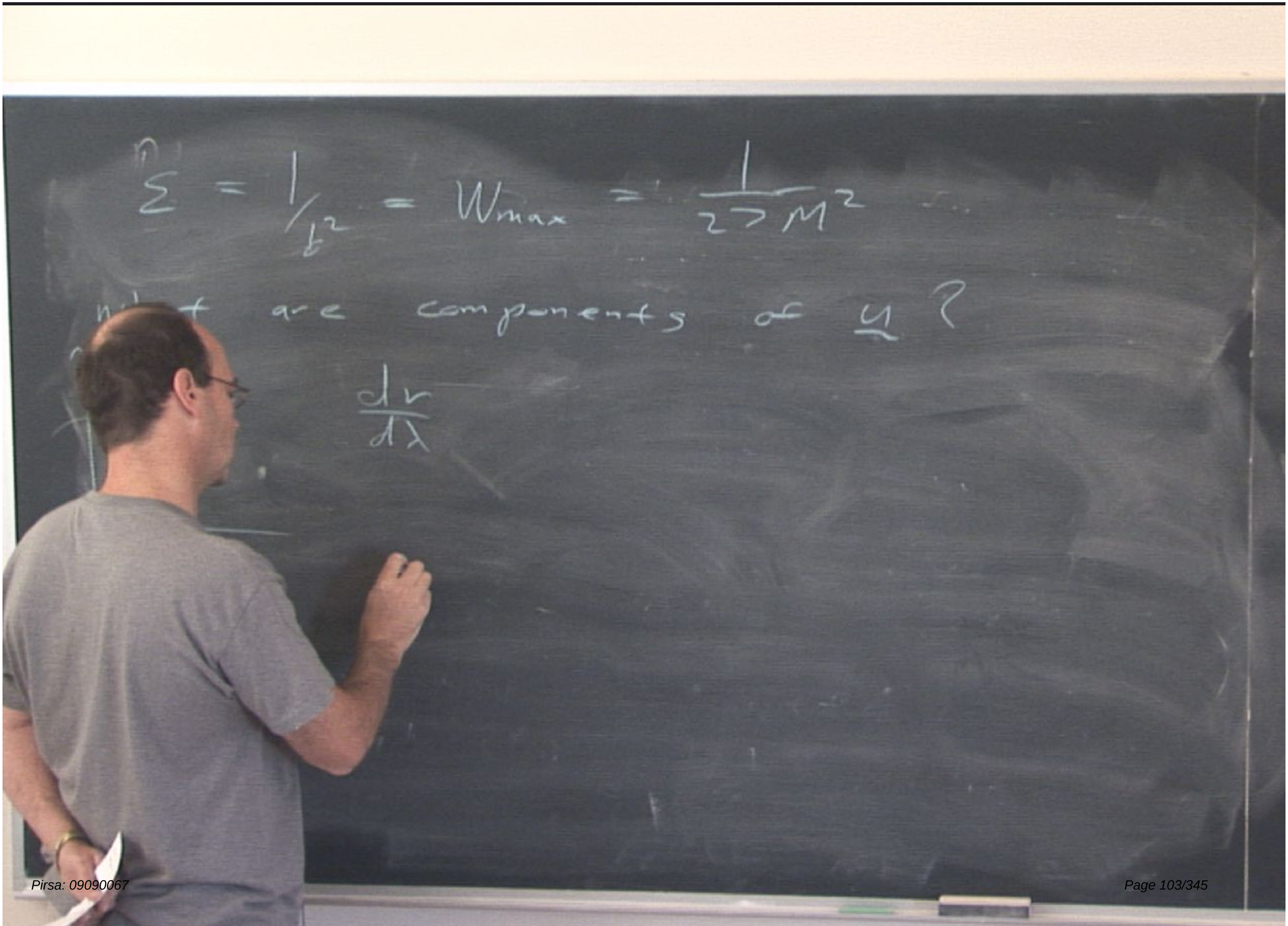

$\Sigma = 1/t^2 = W_{max} = \frac{1}{27 M^2}$
what are components of u ?

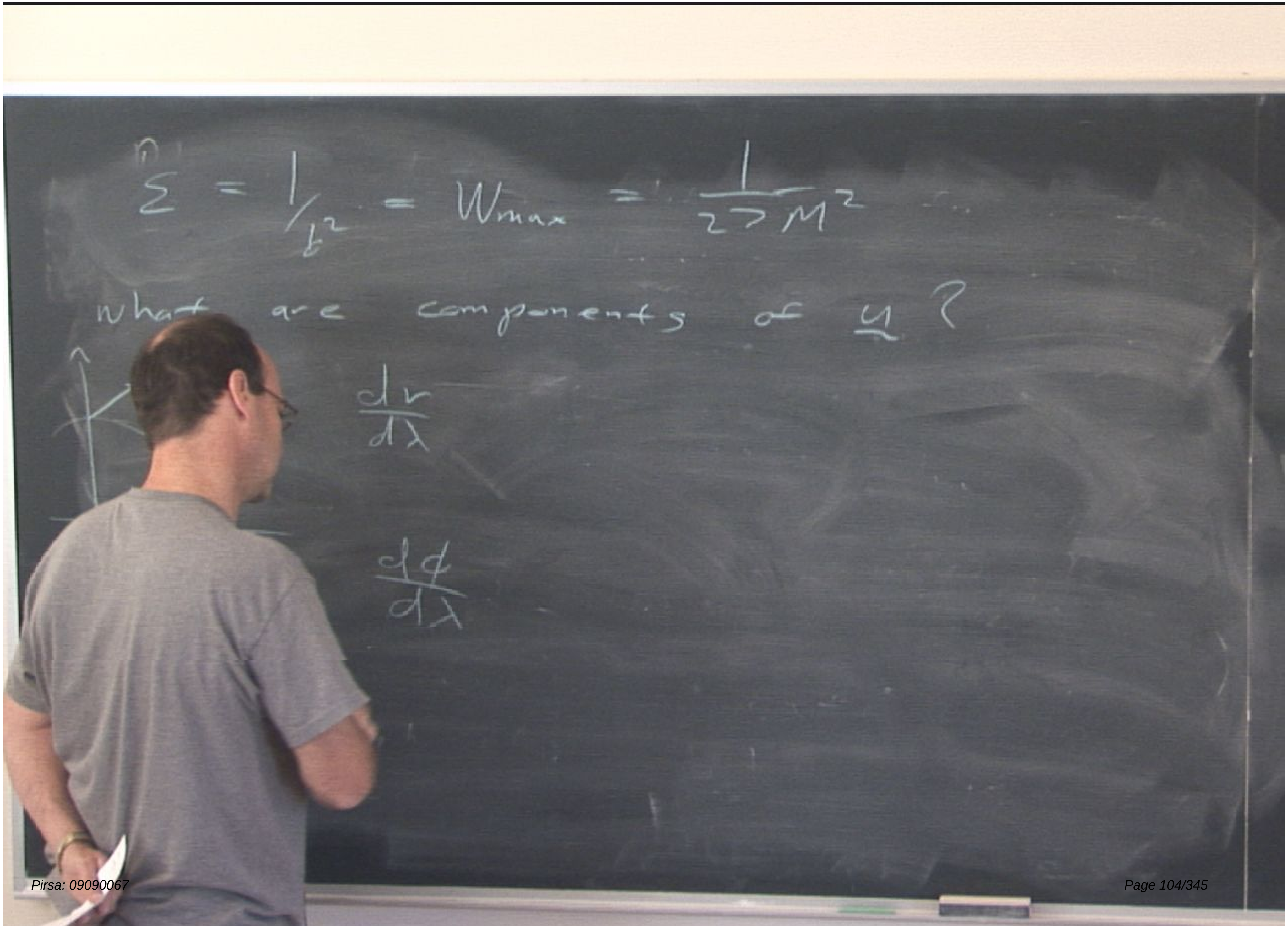
$$\hat{\Sigma} = 1/t^2 = W_{max} = \frac{1}{27 M^2}$$

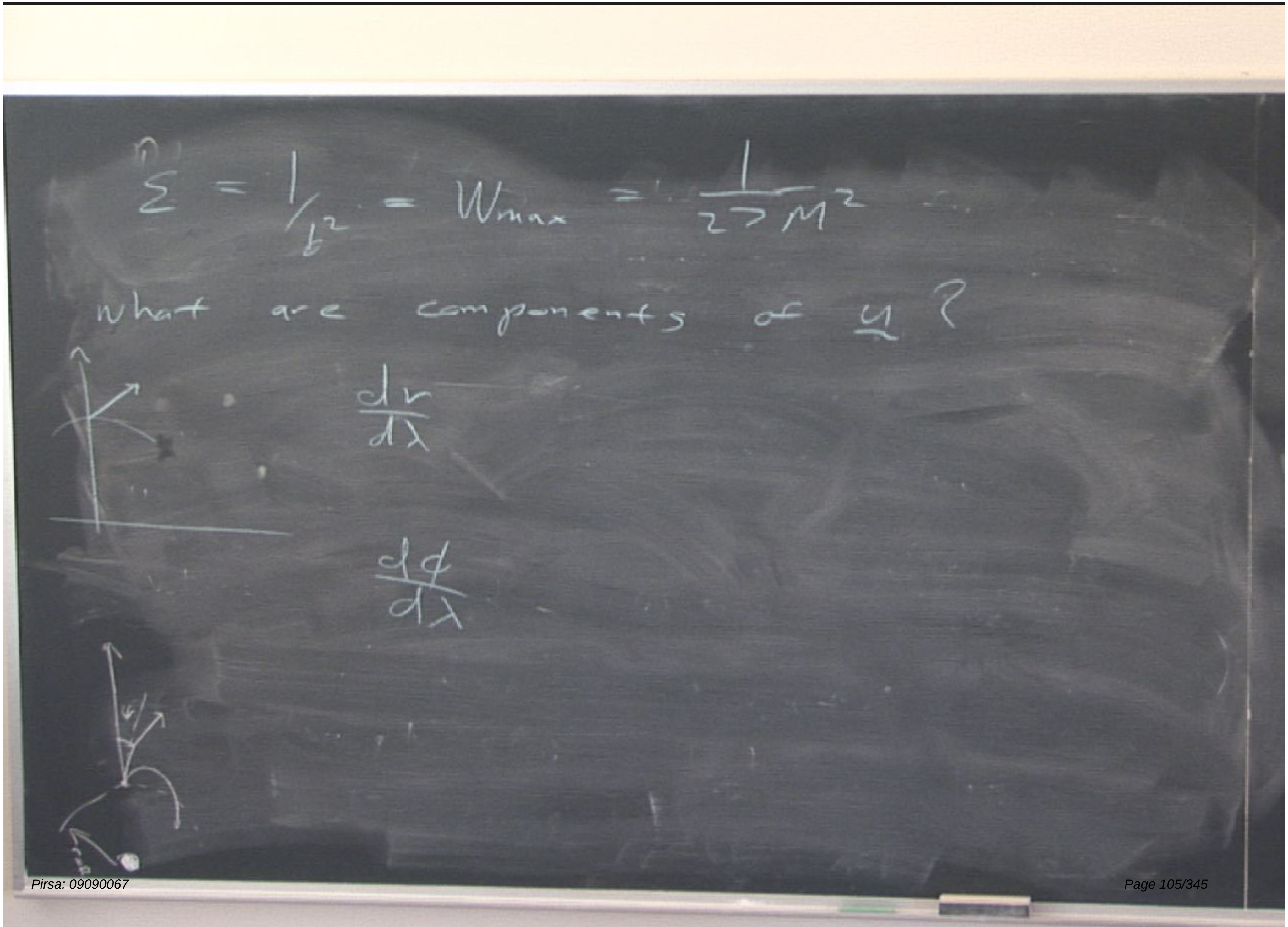
what are components of $\underline{u}$?

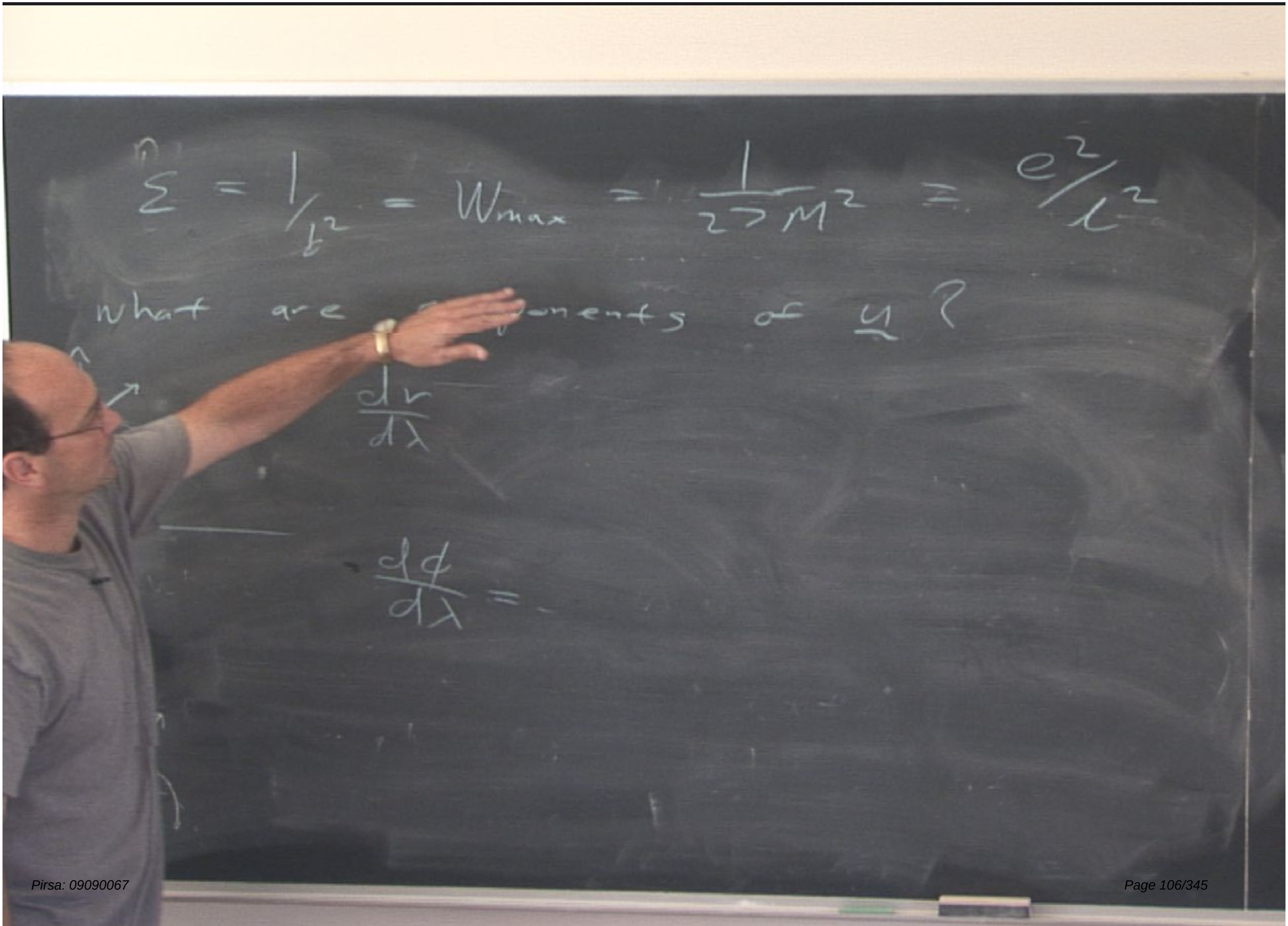
$$\hat{\Sigma} = 1/b^2 = W_{max} = \frac{1}{27M^2}$$

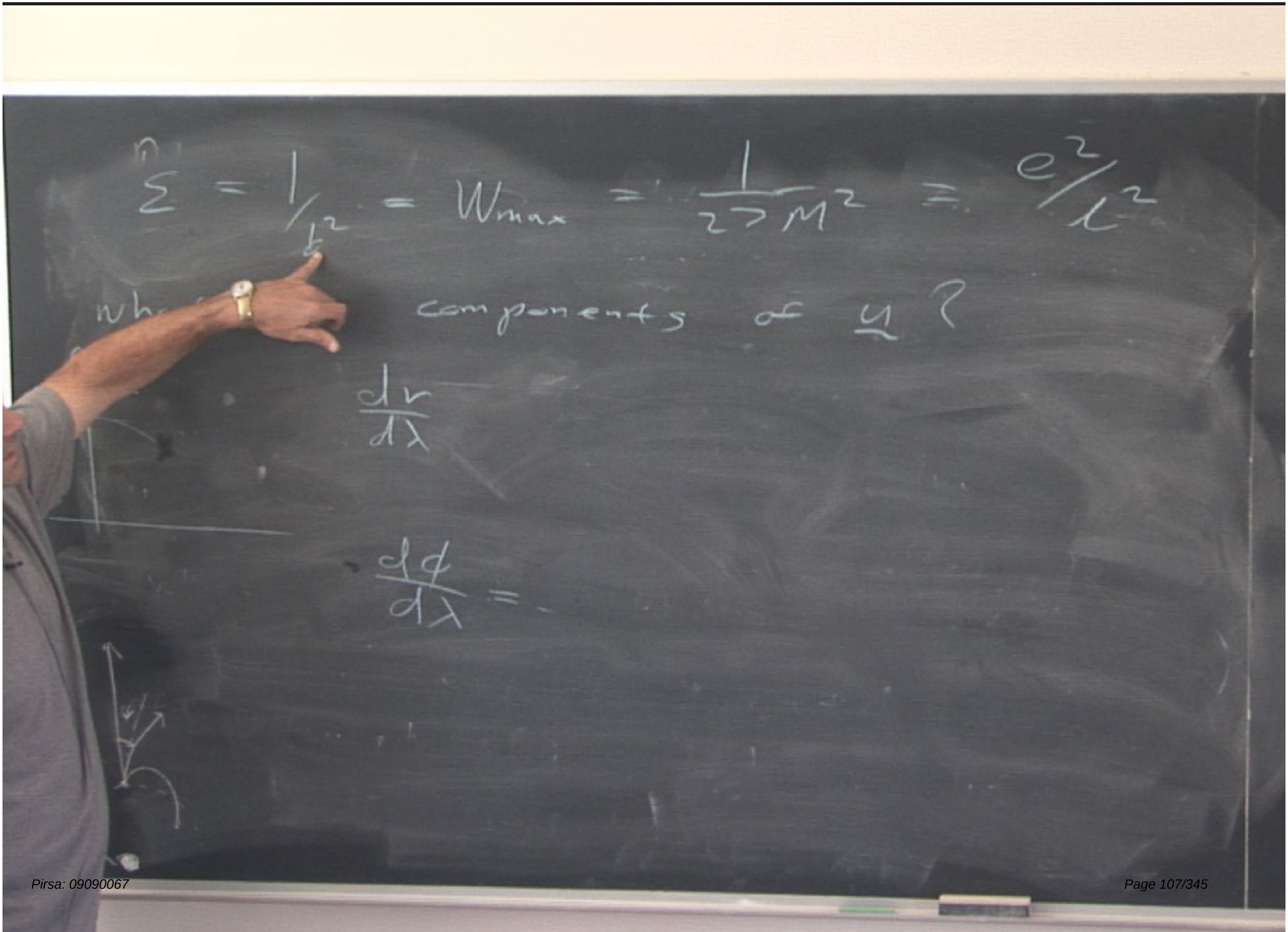
what are components of $\underset{\sim}{u}$?

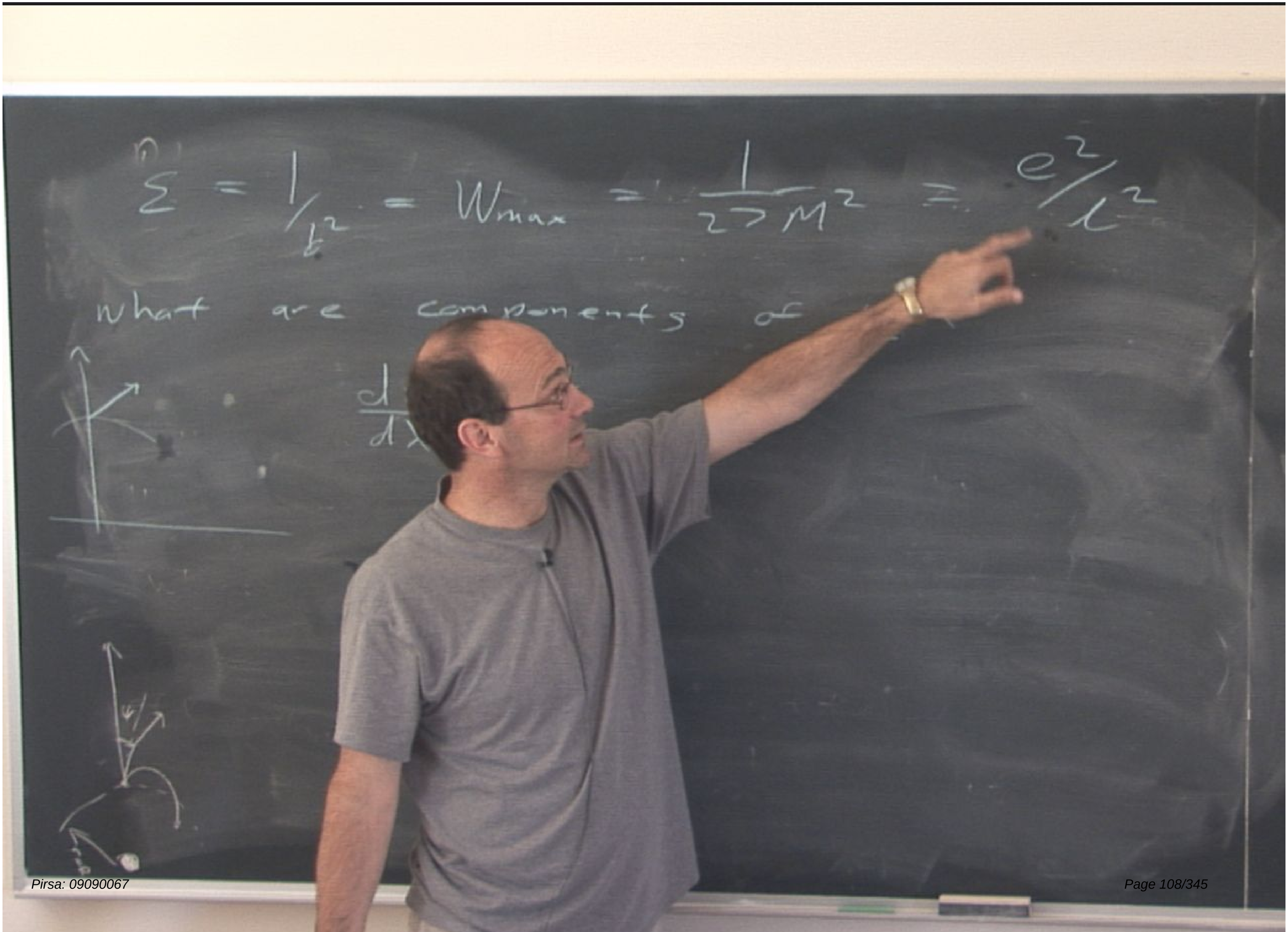

Σ = 1/b² = W_max = 1/27M²
are components of u ?
dr/dλ

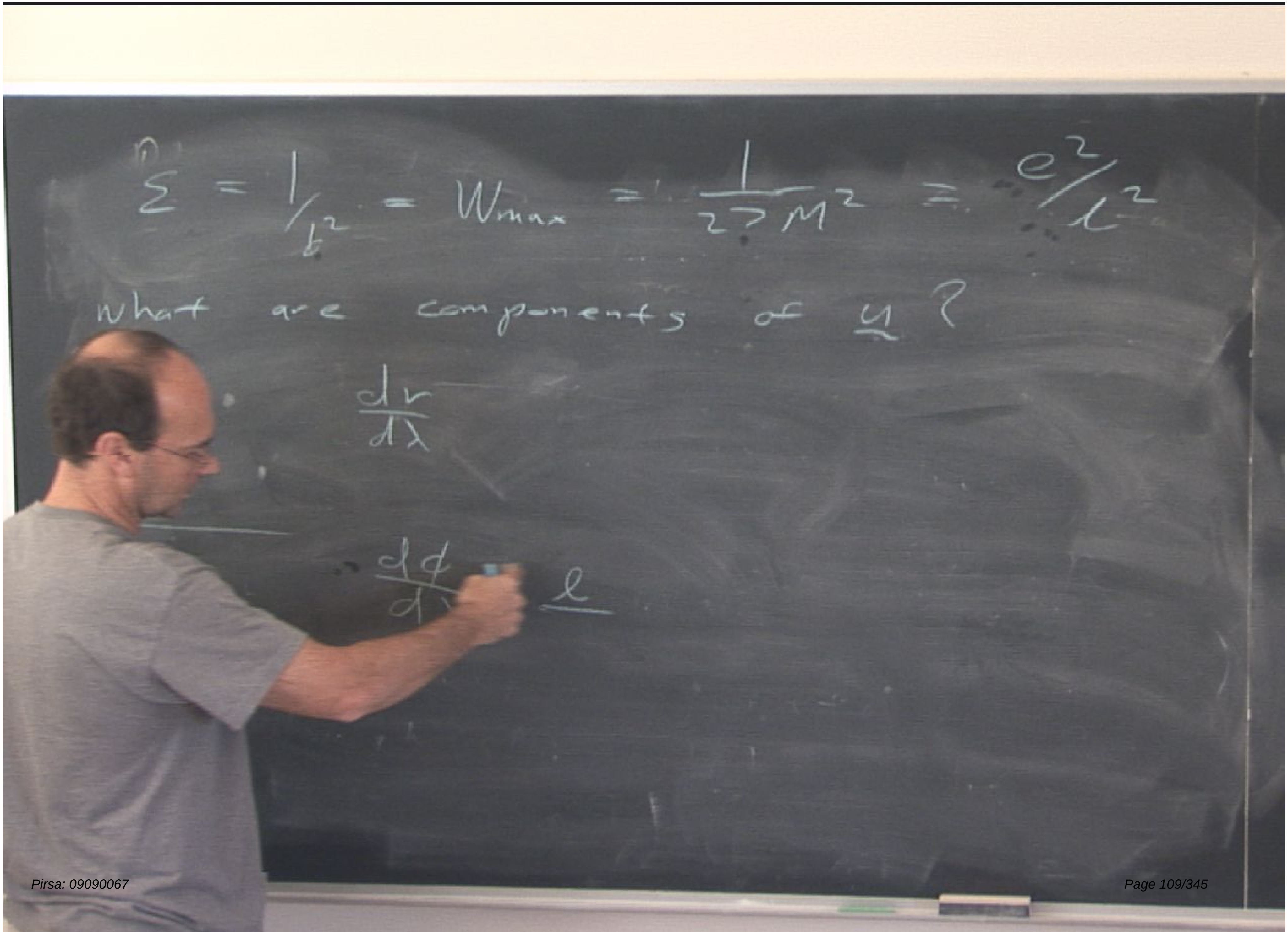

Σ = 1/b² = Wmax = 1/27M²
what are components of u ?
dr/dλ
dφ/dλ


Σ = 1/b² = Wmax = 1/27M²
what are components of u ?
dr/dλ
dφ/dλ


Σ = 1/b² = Wmax = 1/27M² = e²/l²
what are
of u ?
dr/dλ
dφ/dλ =


Σ = 1/b² = Wmax = 1/27M² = e²/ℓ²
components of u ?
dr/dλ
dφ/dλ =


what are components of


Σ = 1/b² = Wmax = 1/27M² = e²/l²
what are components of u ?
dr/dλ
dφ/dλ
l

$$\mathcal{E} = 1/b^2 = W_{max} = \frac{1}{27M^2} = e^2/\ell^2$$

what are components of $\underline{u}$?

$$\frac{dr}{d\lambda}$$

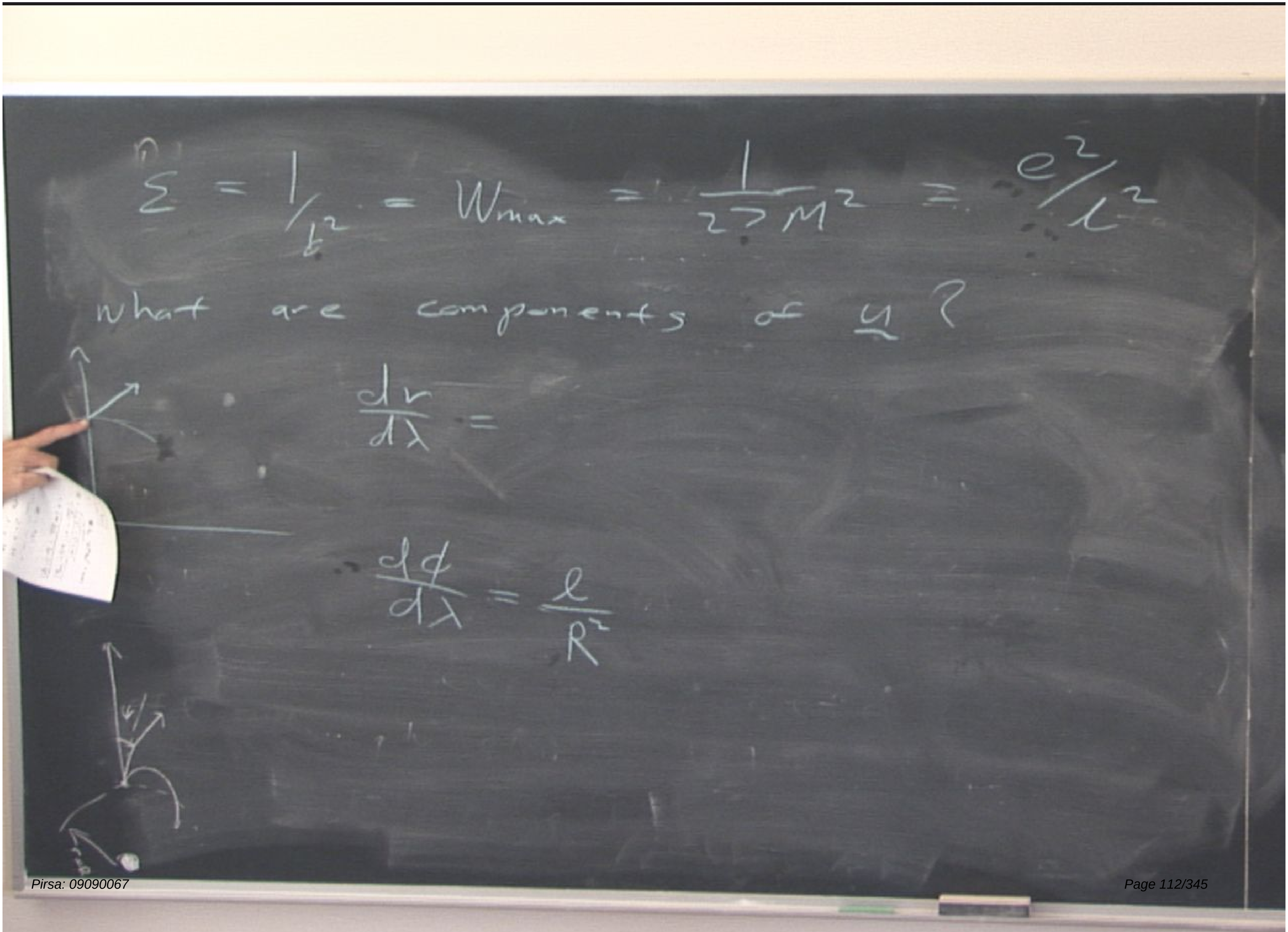
$$\frac{d\phi}{d\lambda} = \frac{\ell}{R^2}$$

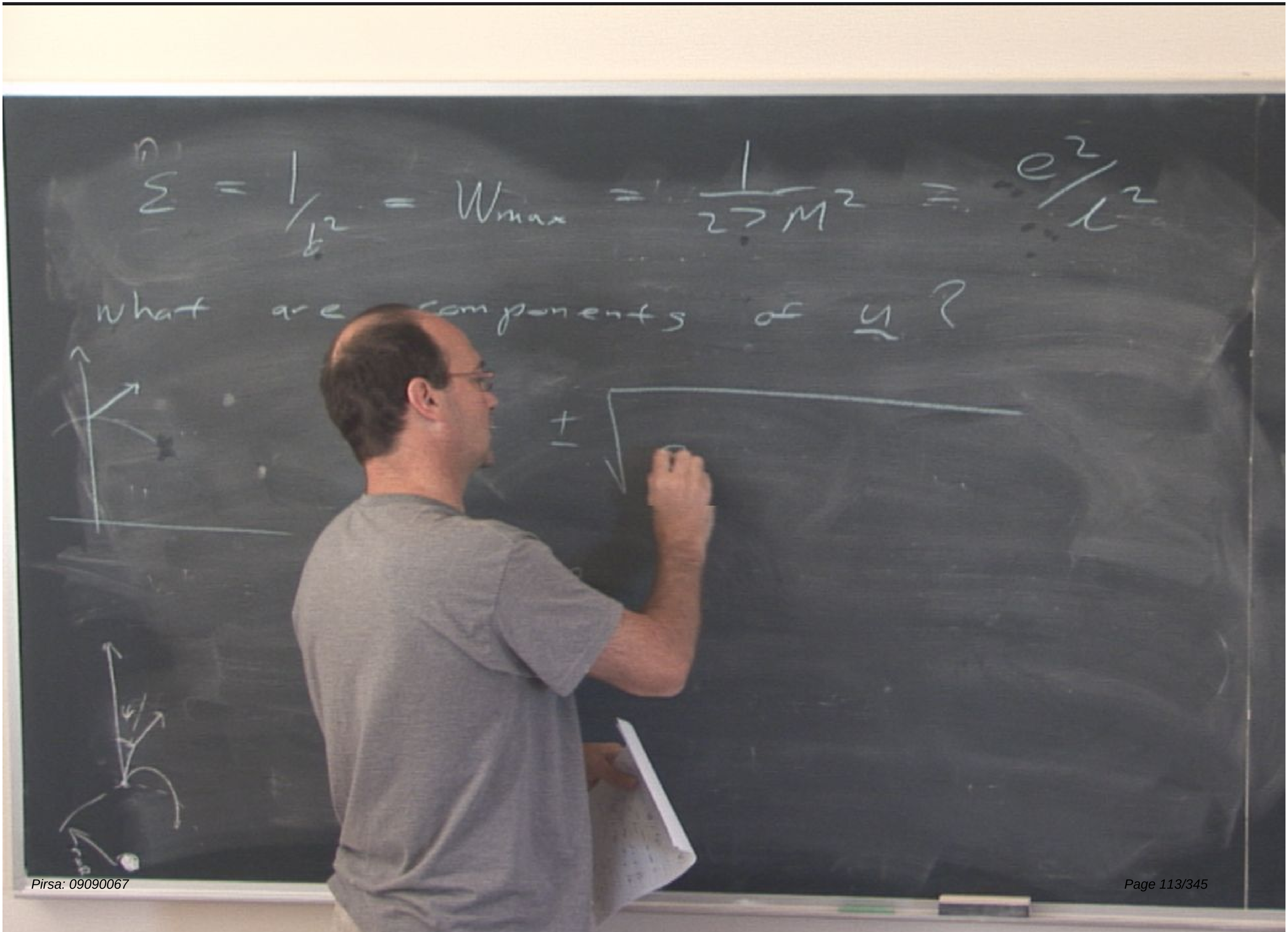
$$\hat{\mathcal{E}} = 1/b^2 = W_{max} = \frac{1}{27M^2} = e^2/\ell^2$$

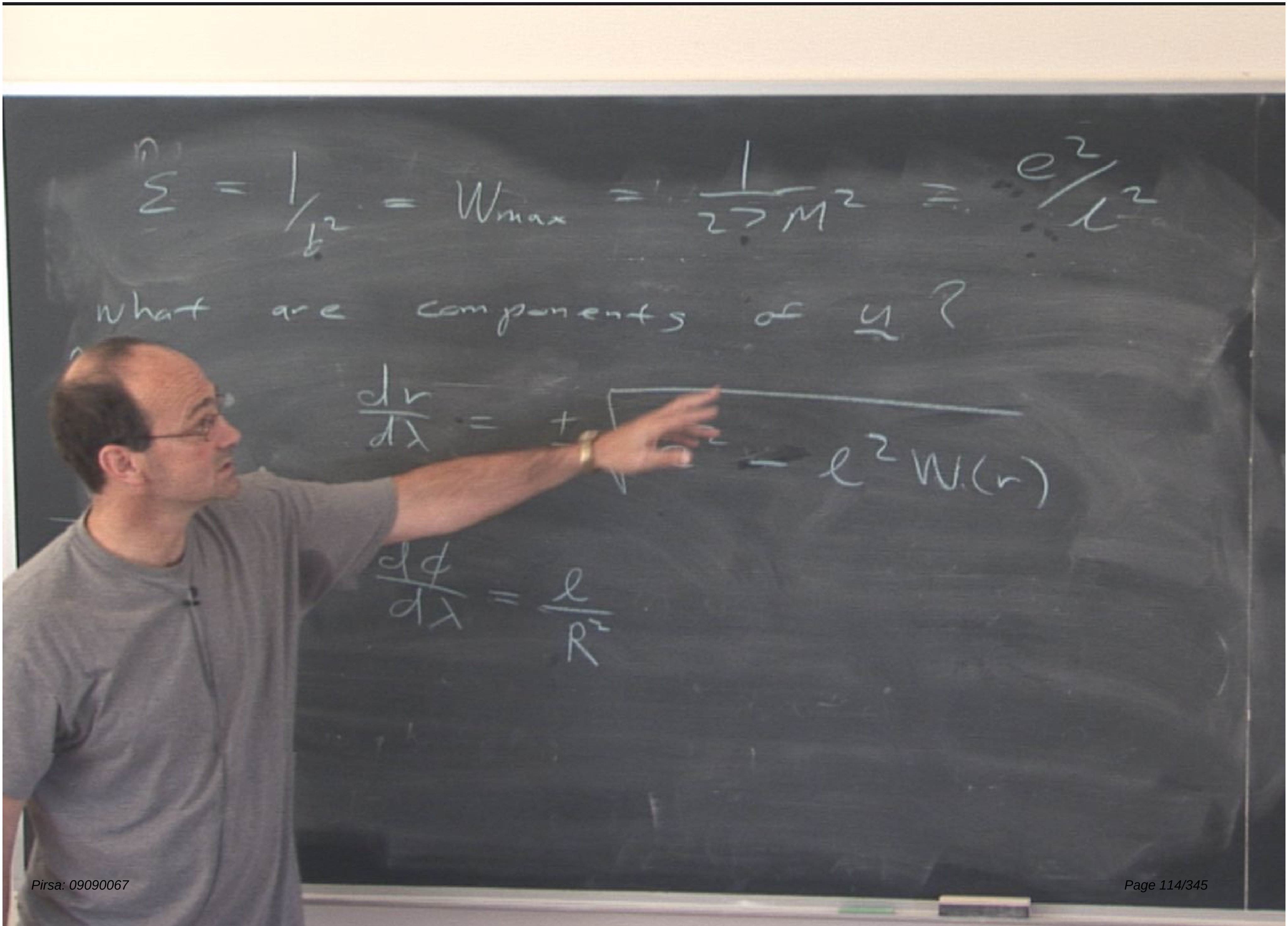
what are components of $\underline{u}$?

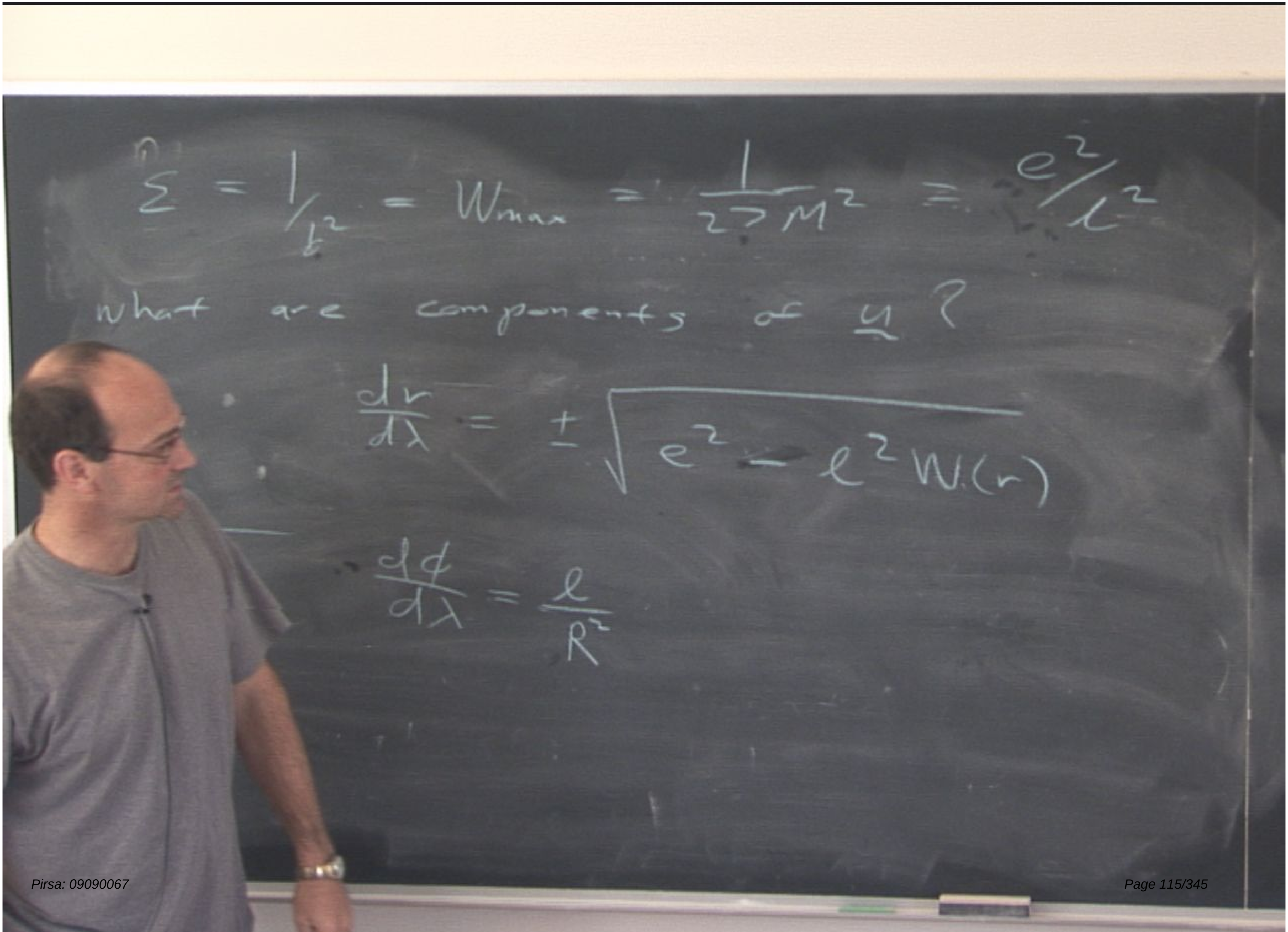
$$\frac{dr}{d\lambda} =$$

$$\frac{d\phi}{d\lambda} = \frac{\ell}{R^2}$$


Σ = 1/b² = W_max = 1/27M² = e²/ℓ²
what are components of u ?
dr/dλ =
dφ/dλ = ℓ/R²


Σ = 1/ℓ² = Wmax = 1/27M² = e²/ℓ²
What are components of u ?


Σ = 1/b² = Wmax = 1/27M² = e²/l²
what are components of u ?
dr/dλ = ± √(− l² W(r))
dφ/dλ = l/R²

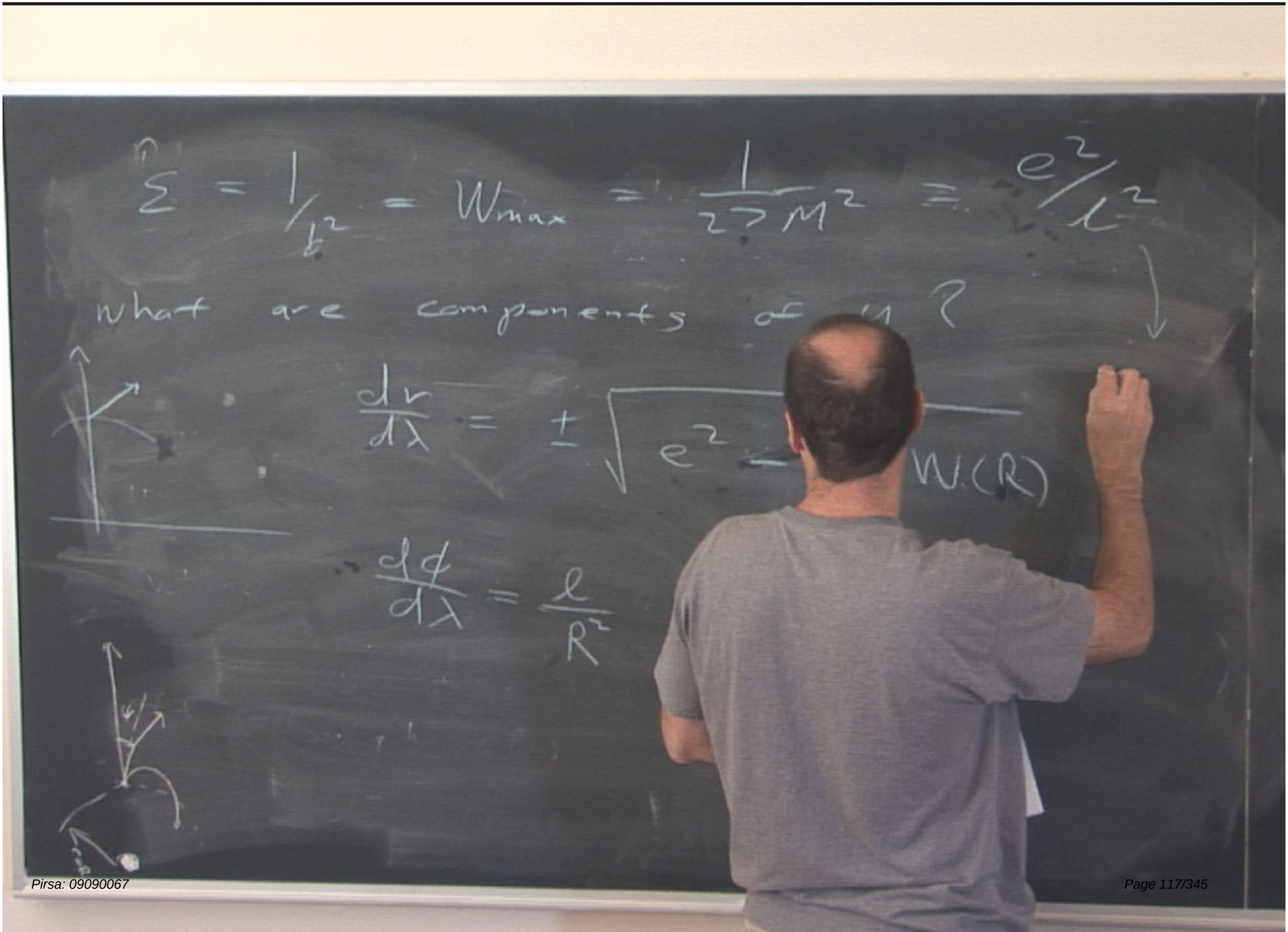

Σ = 1/b² = W_max = 1/27M² ≡ e²/ℓ²
what are components of u ?
dr/dλ = ± √(e² − ℓ² W(r))
dφ/dλ = ℓ/R²

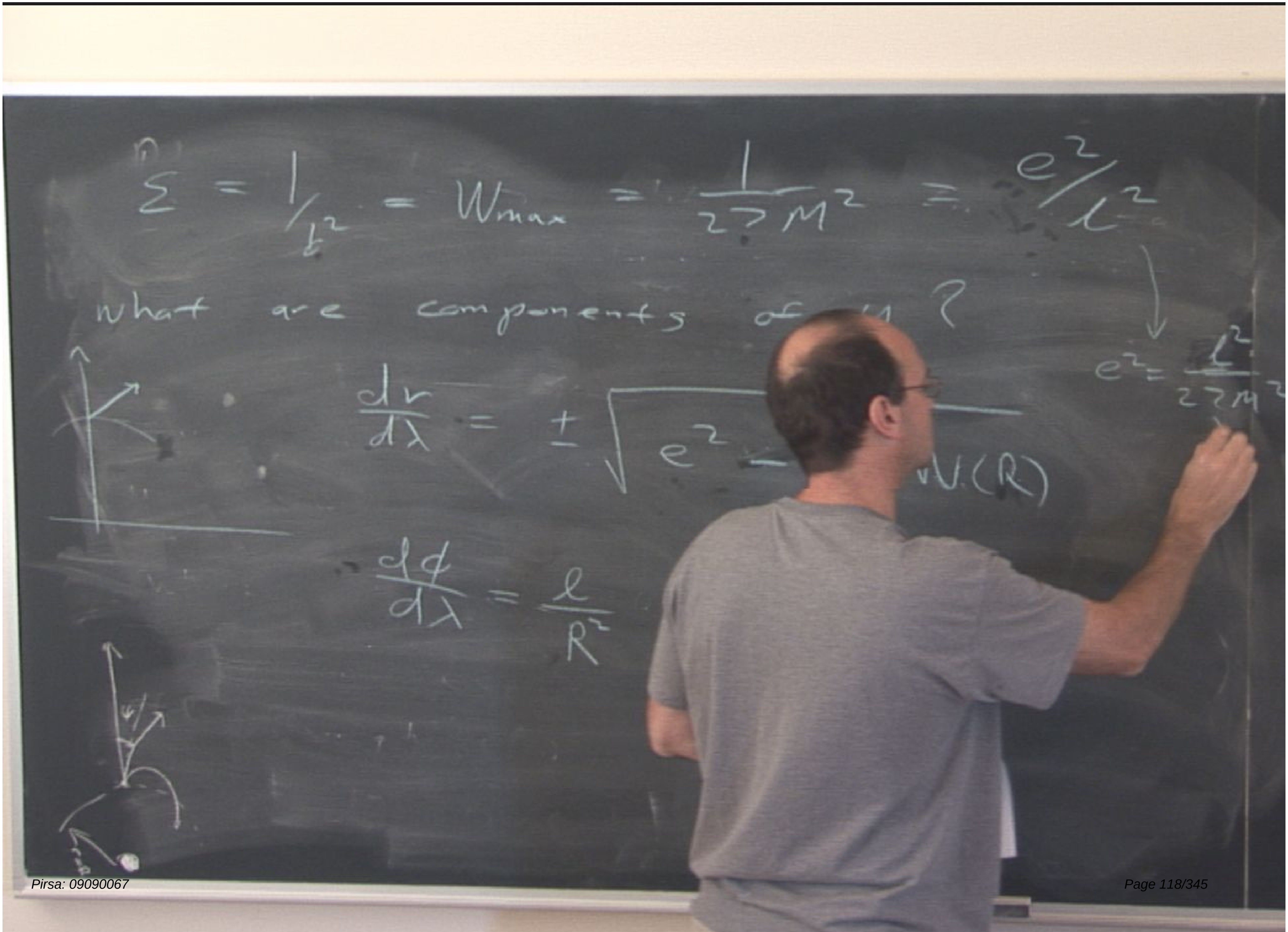
$$\hat{\Sigma} = 1/b^2 = W_{max} = \frac{1}{27M^2} = e^2/\ell^2$$

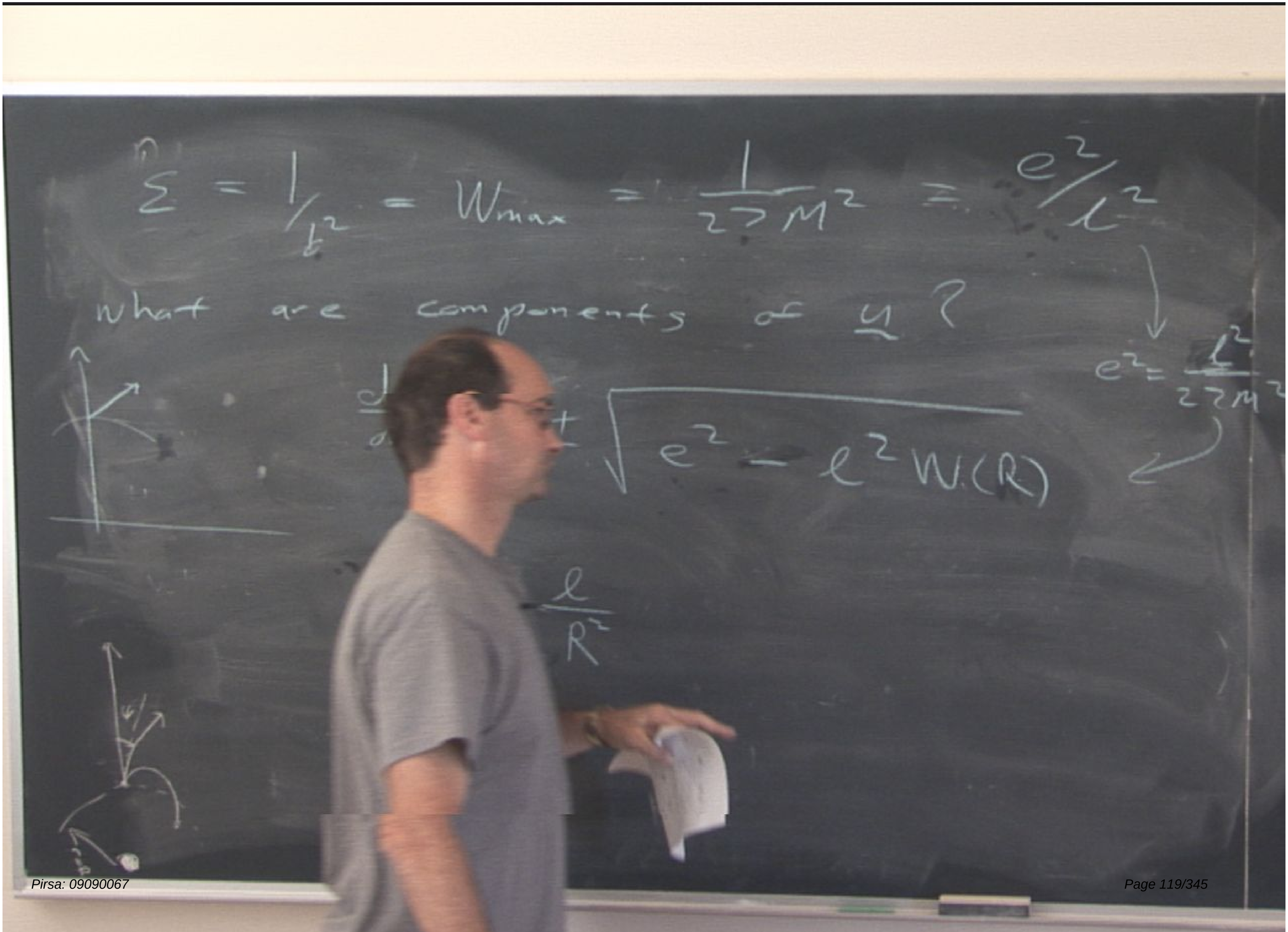
what are components of $\underline{u}$?

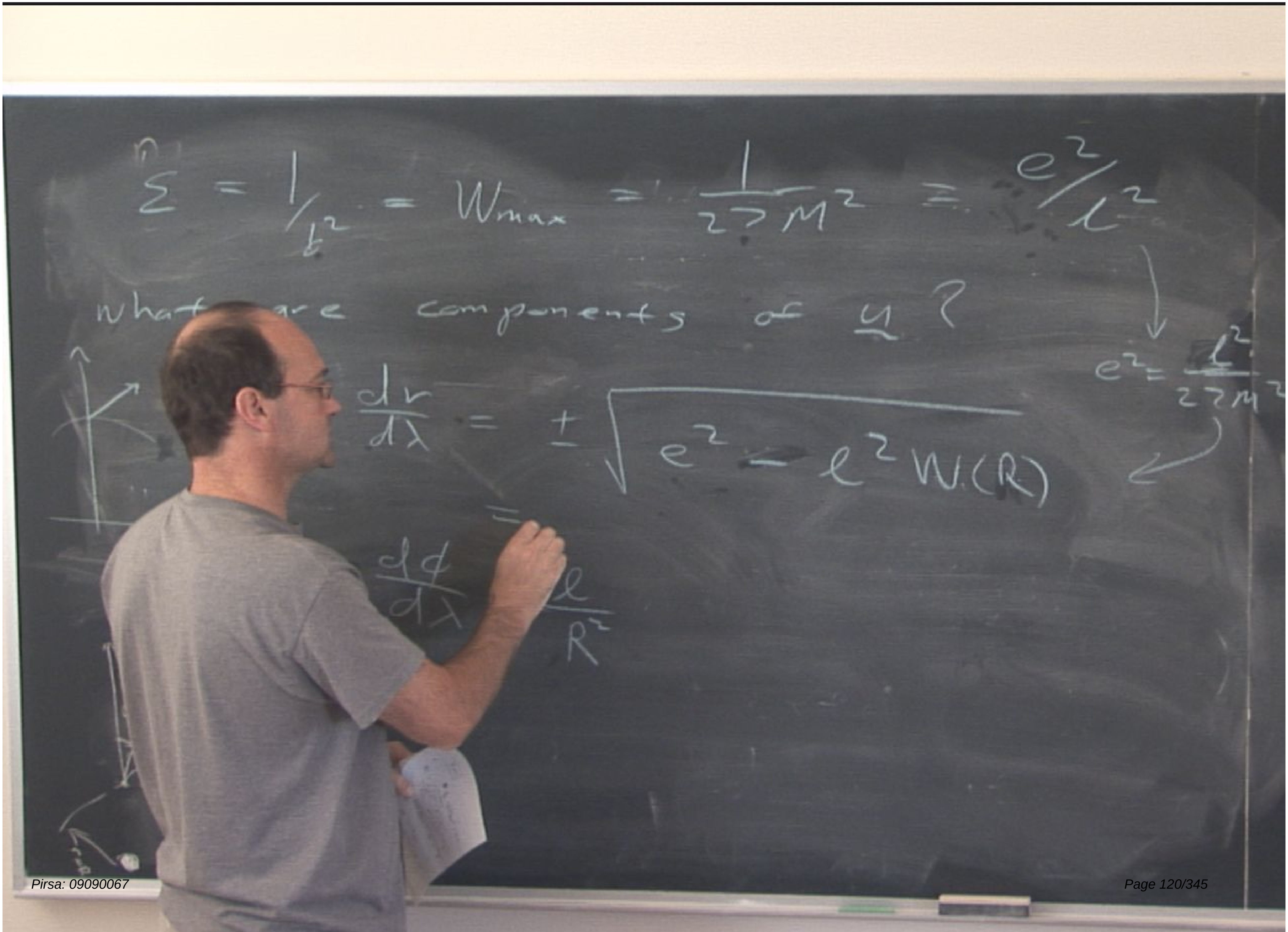
$$\frac{dr}{d\lambda} = \pm\sqrt{e^2 - \ell^2 W(R)}$$

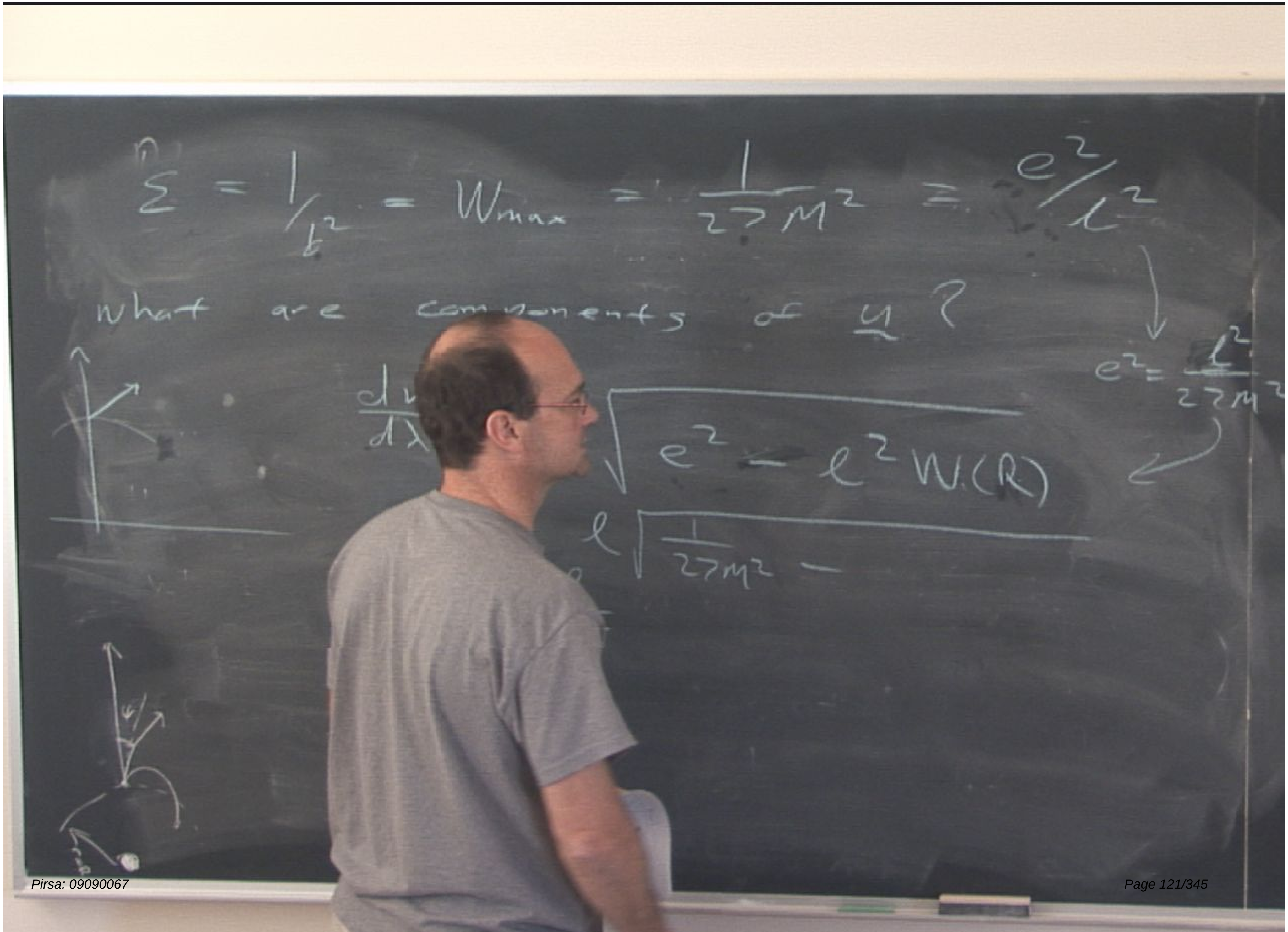
$$\frac{d\phi}{d\lambda} = \frac{\ell}{R^2}$$

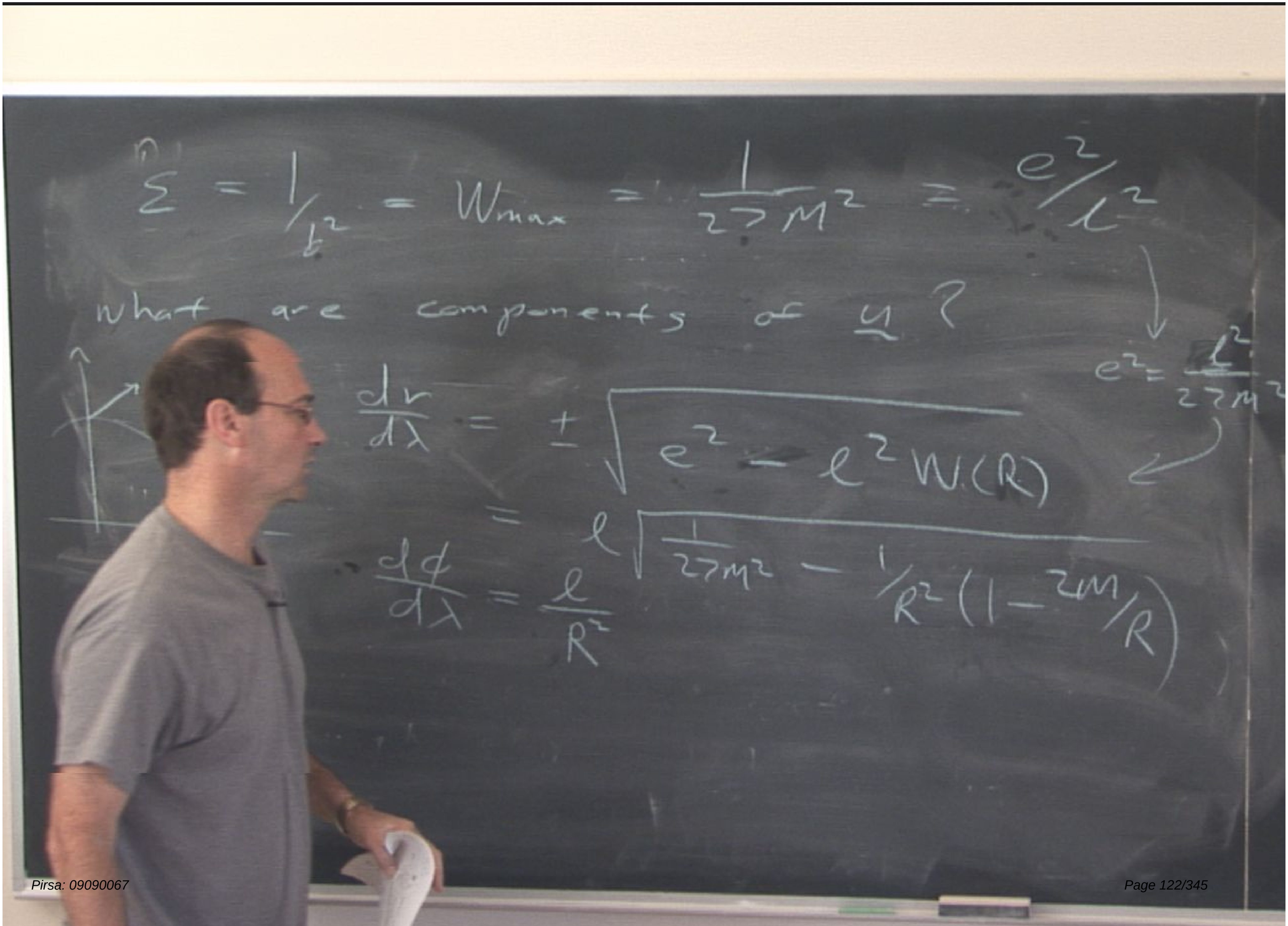

Σ = 1/b² = Wmax = 1/27M² = e²/ℓ²
what are components of
dr/dλ = ± √ e² − W(R)
dφ/dλ = ℓ/R²


Σ = 1/b² = Wmax = 1/27M² = e²/ℓ²
what are components of
dr/dλ = ± √(e² − W(R))
dφ/dλ = ℓ/R²
e² = ℓ²/27M²


Σ = 1/b² = Wmax = 1/27M² = e²/l²
What are components of u ?
e² = l²/27M²
e² − l² W(R)
l/R²


Σ = 1/b² = Wmax = 1/27M² = e²/l²
what are components of u ?
dr/dλ = ± √(e² − l² W(R))
dφ/dλ = l/R²
e² = l²/27M²


what are components of u ?
e² = ℓ² / 27M²
e² − ℓ² W(R)
ℓ √(1/27M² −


Σ = 1/b² = Wmax = 1/27M² = e²/l²
what are components of u ?
e² = l²/27M²
dr/dλ = ± √(e² − l² W(R))
= l √(1/27m² − 1/R² (1 − 2M/R))
dφ/dλ = l/R²

$$\hat{\Sigma} = 1/b^2 = W_{max} = \frac{1}{27M^2} = e^2/\ell^2$$

$$e^2 = \frac{\ell^2}{27M^2}$$

What are components of $\underline{u}$?

$$\frac{dr}{d\lambda} = \pm\sqrt{e^2 - \ell^2 W(R)}$$

$$= \ell\sqrt{\frac{1}{27M^2} - \frac{1}{R^2}\left(1 - \frac{2M}{R}\right)}$$

$$\frac{d\phi}{d\lambda} = \frac{\ell}{R^2}$$

$$\Sigma = 1/b^2 = W_{max} = \frac{1}{27M^2} = e^2/\ell^2$$

$$e^2 = \frac{\ell^2}{27M^2}$$

what are components of $\underline{u}$?

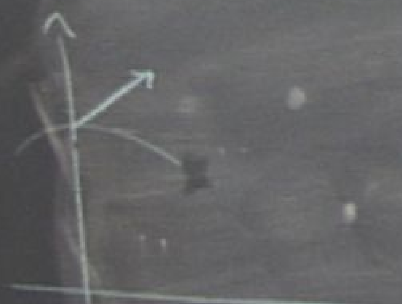
$$\frac{dr}{d\lambda} = \pm\sqrt{e^2 - \ell^2 W(R)}$$

$$= \ell\sqrt{\frac{1}{27M^2} - \frac{1}{R^2}\left(1 - 2M/R\right)}$$

$$\frac{d\phi}{d\lambda} = \frac{\ell}{R^2}$$

$$\Sigma^{n} = 1/b^2 = W_{max} = \frac{1}{27M^2} = e^2/\ell^2$$

what are components of $\underline{u}$?



for outward motion

$$\frac{dr}{d\lambda} = \pm\sqrt{e^2 - \ell^2 W(R)}$$

$$e^2 = \frac{\ell^2}{27M^2}$$

$$= \ell\sqrt{\frac{1}{27M^2} - \frac{1}{R^2}\left(1 - \frac{2M}{R}\right)}$$

$$\frac{d\phi}{d\lambda} = \frac{\ell}{R^2}$$

$$\Sigma = 1/b^2 = W_{max} = \frac{1}{27M^2} \equiv e^2/\ell^2$$

What are components of $\underline{u}$?

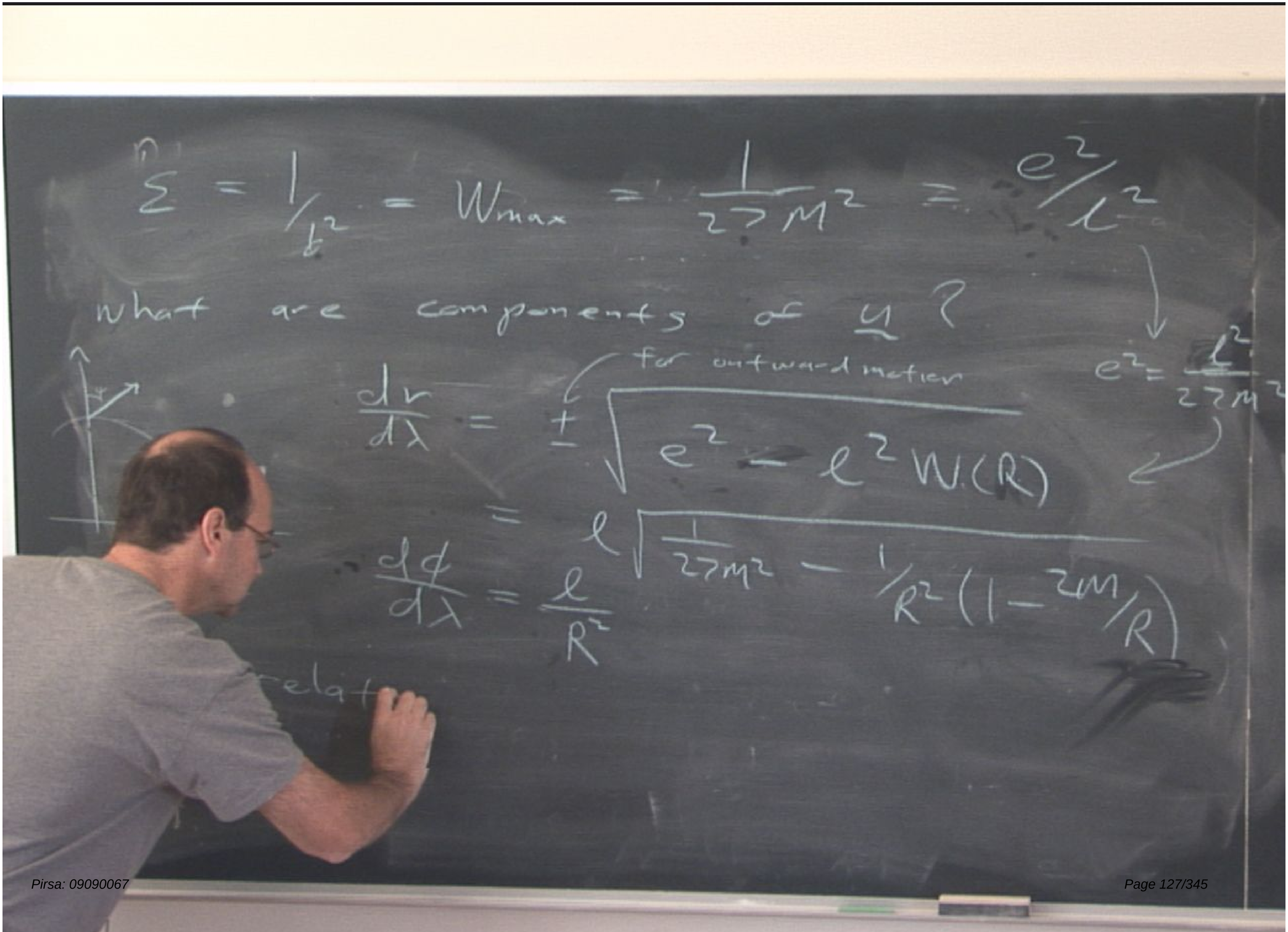
$$e^2 = \frac{\ell^2}{27M^2}$$

for outward motion

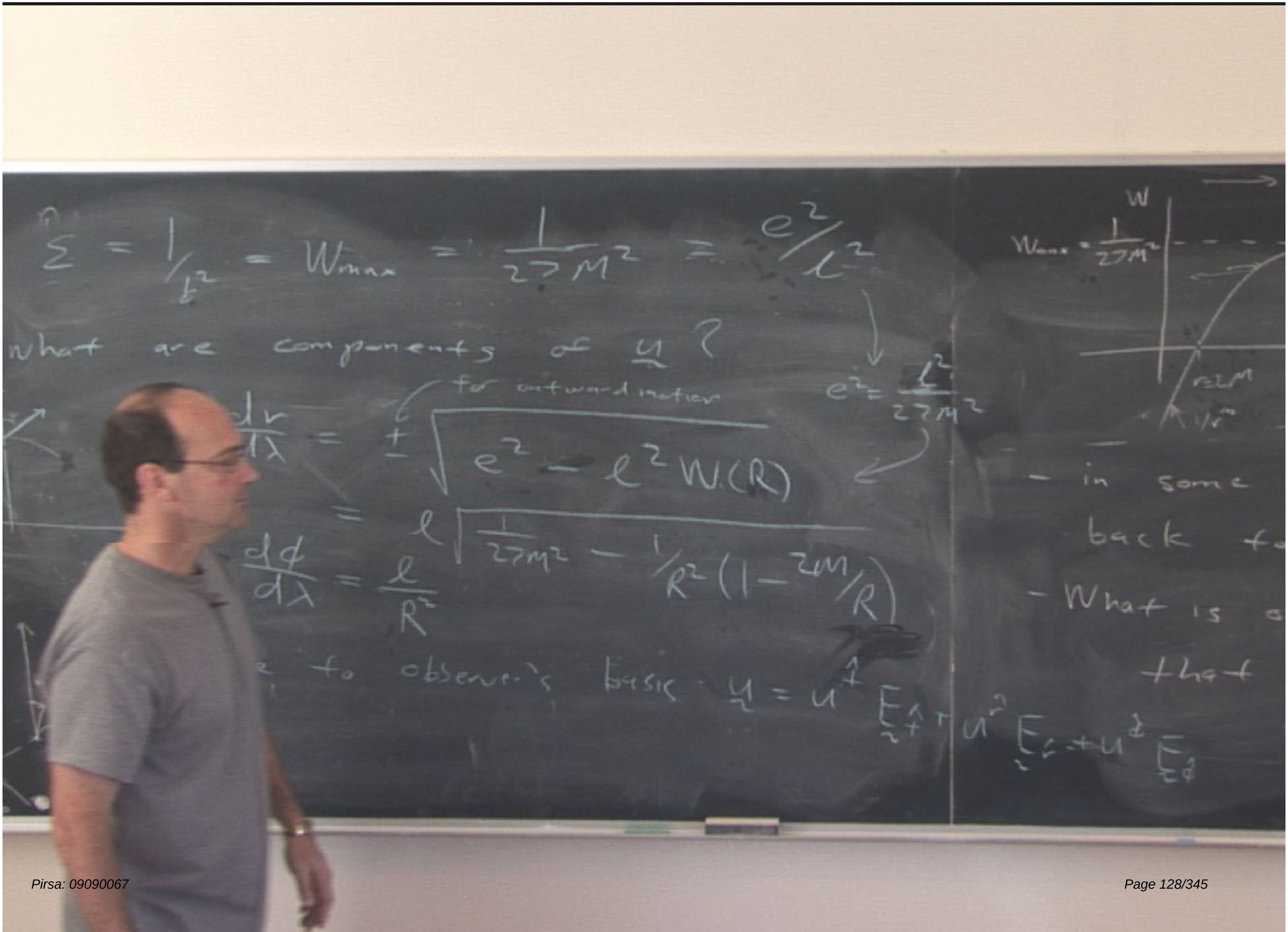
$$\frac{dr}{d\lambda} = \pm\sqrt{e^2 - \ell^2 W(R)}$$

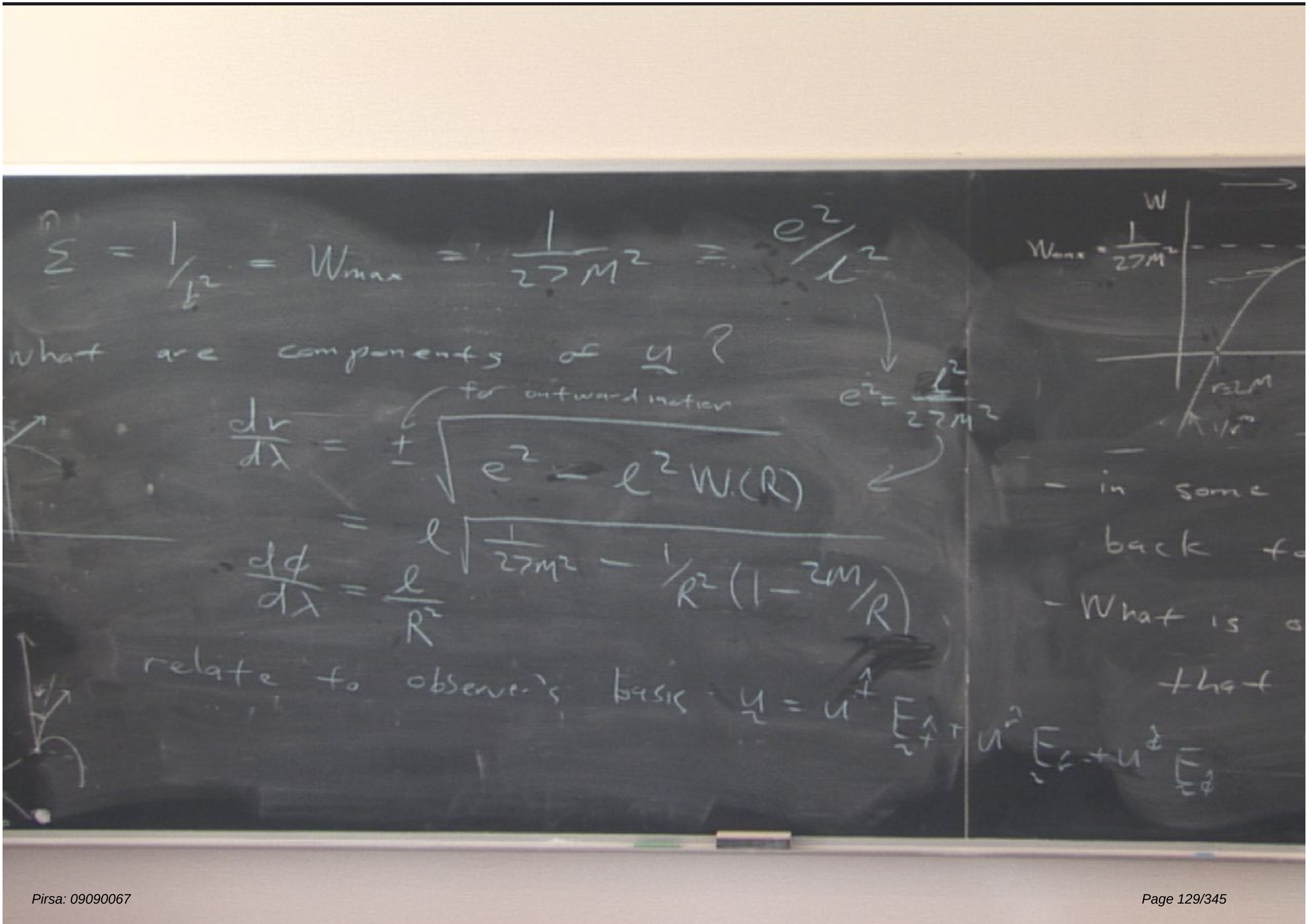
$$= \ell\sqrt{\frac{1}{27M^2} - 1/R^2\left(1 - 2M/R\right)}$$

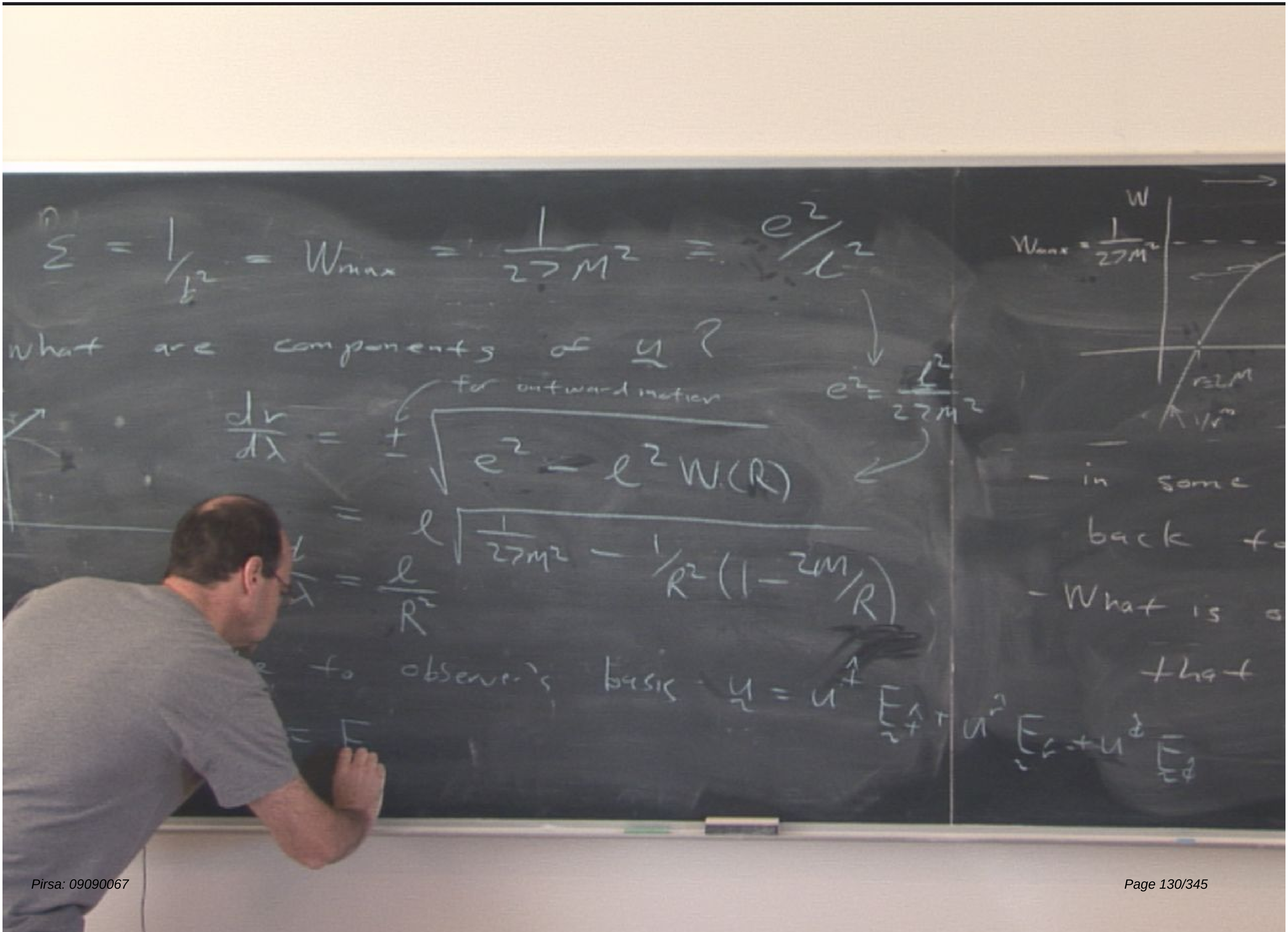
$$\frac{d\phi}{d\lambda} = \frac{\ell}{R^2}$$

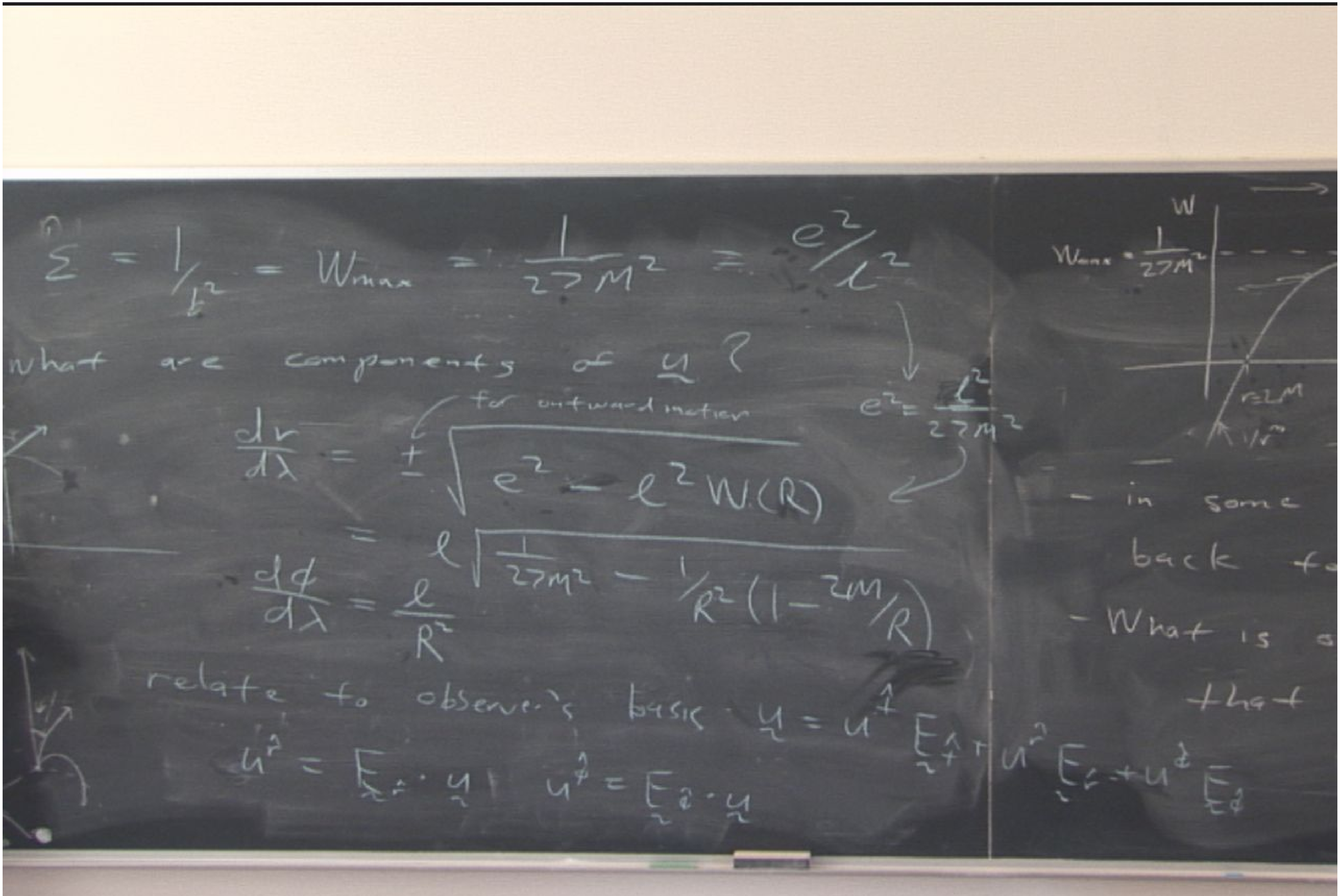


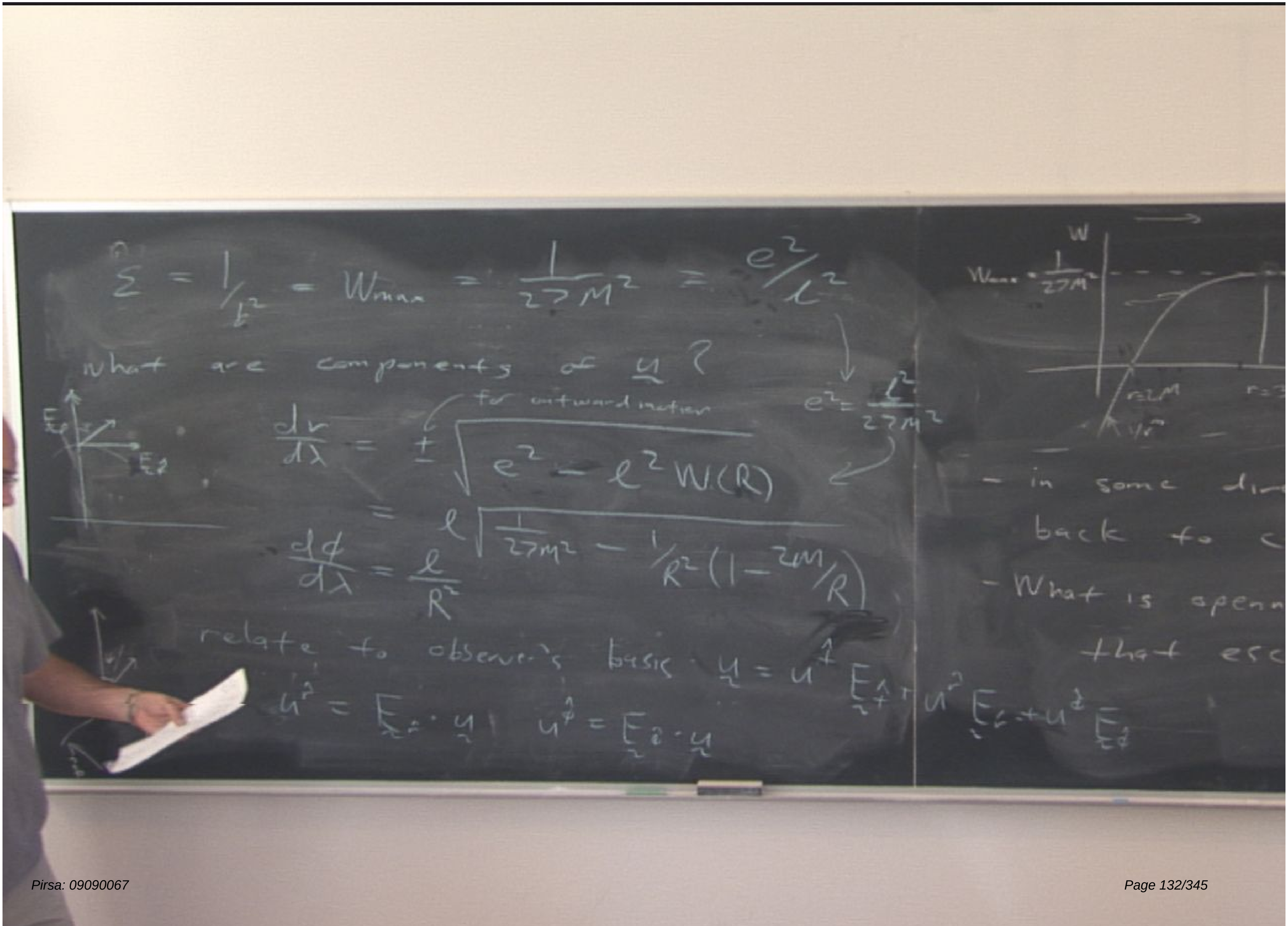
Σ = 1/b² = Wmax = 1/27M² = e²/l²
what are components of u ?
for outward motion
dr/dλ = ± √(e² − l² W(R))
= l √(1/27M² − 1/R² (1 − 2M/R))
dφ/dλ = l/R²
e² = l²/27M²


what are components of u ?
for outward motion
to observer's basis
W
in some
back
What is
that

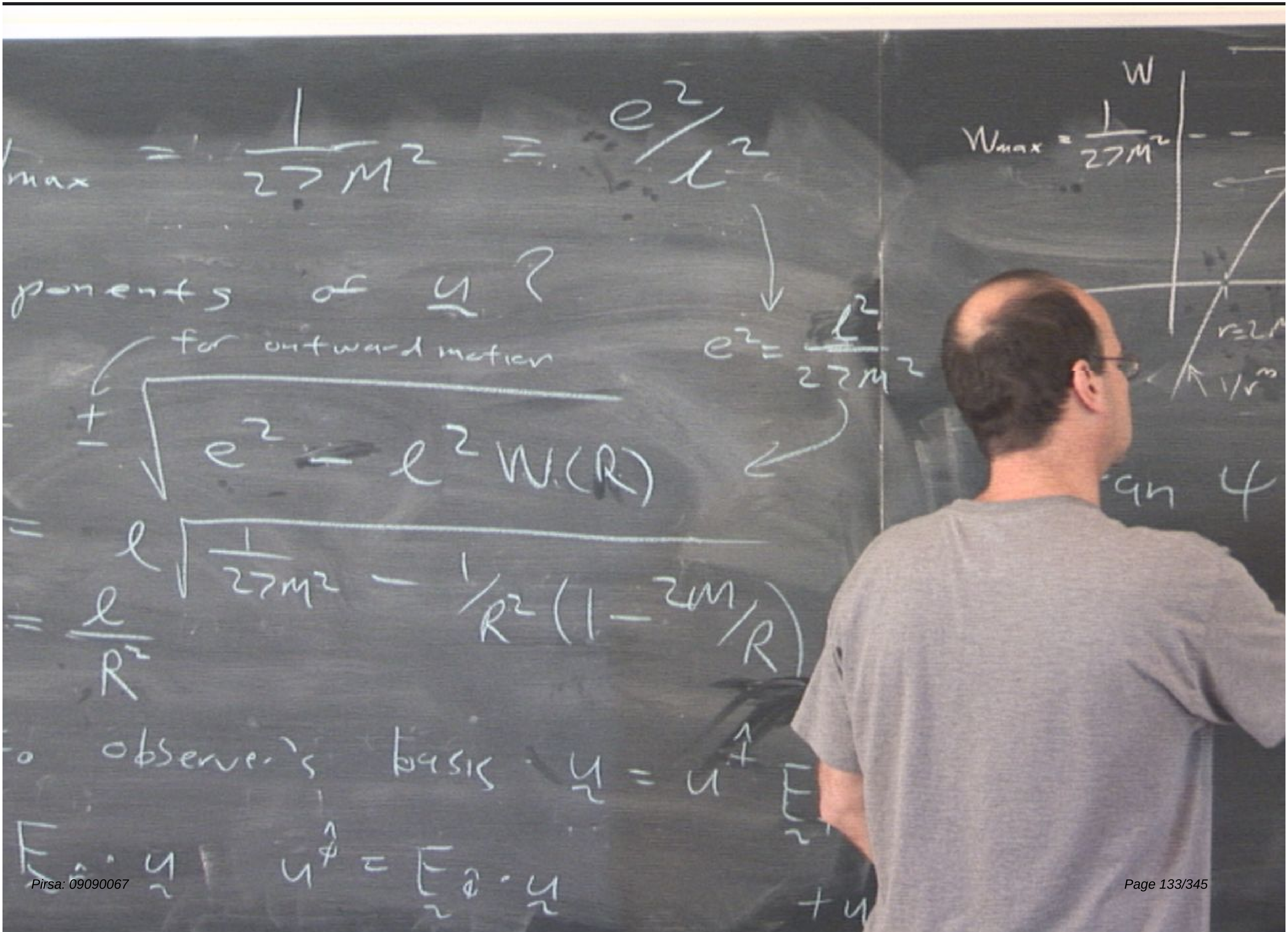

$\hat{\Sigma} = 1/l^2 = W_{max} = \frac{1}{27M^2} = e^2/l^2$
what are components of u ?
for outward motion
$\frac{dr}{d\lambda} = \pm\sqrt{e^2 - l^2 W(R)}$
$= l\sqrt{\frac{1}{27M^2} - 1/R^2(1-2M/R)}$
$\frac{d\phi}{d\lambda} = \frac{l}{R^2}$
$e^2 = \frac{l^2}{27M^2}$
relate to observer's basis $u = u^{\hat{t}} E_{\hat{t}} + u^{\hat{r}} E_{\hat{r}} + u^{\hat{\phi}} E_{\hat{\phi}}$
W
$W_{max} = \frac{1}{27M^2}$
$r = 3M$
$1/r^2$
- in some
back to
- What is
that


E = 1/b² = Wmax = 1/27M² = e²/l²
What are components of u ?
for outward motion
dr/dλ = ± √(e² − l² W(R))
= l √(1/27M² − 1/R² (1 − 2M/R))
e² = l²/27M²
dφ/dλ = l/R²
to observer's basis u = u^t̂ E_t̂ + u^r̂ E_r̂ + u^φ̂ E_φ̂
Wmax = 1/27M²
W
r=3M
1/r³
− in some
back to
− What is
that


Σ = 1/b² = Wmax = 1/27M² = e²/ℓ²
what are components of u ?
for outward motion
dr/dλ = ± √(e² − ℓ² W(R))
= ℓ √(1/27M² − 1/R² (1 − 2M/R))
dφ/dλ = ℓ/R²
e² = ℓ²/27M²
relate to observer's basis u = u^t̂ E_t̂ + u^r̂ E_r̂ + u^φ̂ E_φ̂
u^r̂ = E_r̂ · u   u^φ̂ = E_φ̂ · u
W
Wmax = 1/27M²
r=2M
− in some
back to
− What is
that



What are components of u ?
for outward motion
relate to observer's basis
- in some di
back to c
- What is open
that esc


Wmax = 1/27M² = e²/l²
ponents of u ?
for outward motion
e² = l²/27M²
± √(e² − l² W(R))
= l √(1/27M² − 1/R² (1 − 2M/R))
= l/R²
observer's basis u = u^t̂ E
W
Wmax = 1/27M²
r=2M
1/r³

$W_{max} = \frac{1}{27M^2} = e^2/\ell^2$

$e^2 = \frac{\ell^2}{27M^2}$

ponents of $\underline{u}$?

for outward motion

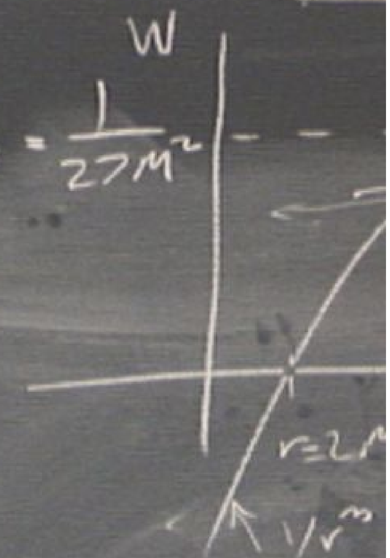
$= \pm\sqrt{e^2 - \ell^2 W(R)}$

$= \ell\sqrt{\frac{1}{27M^2} - 1/R^2\left(1 - 2M/R\right)}$

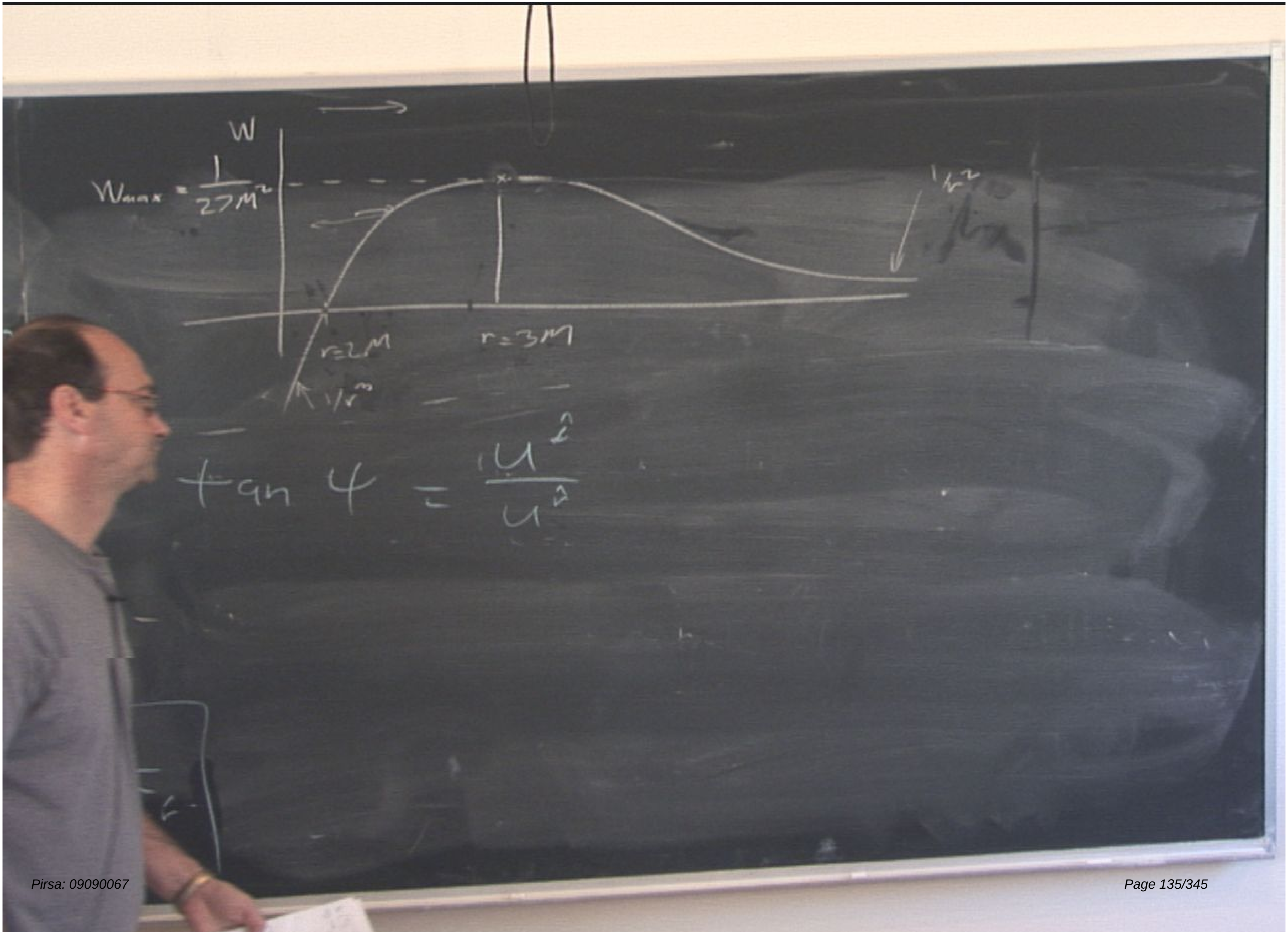
$= \frac{\ell}{R^2}$

o observer's basis $\underline{u} = u^{\hat t}\,\underline{E}_{\hat t} + u^{\hat r}\,\underline{E}_{\hat r} + u^{\hat\phi}\,\underline{E}_{\hat\phi}$

$\underline{E}_{\hat r}\cdot\underline{u}$, $u^{\hat\phi} = \underline{E}_{\hat\phi}\cdot\underline{u}$

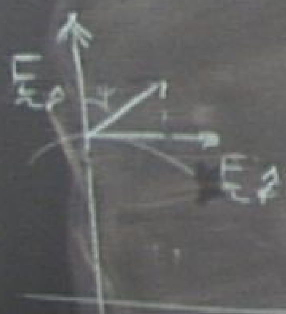


$\tan\psi$


W
Wmax = 1/27M²
r=2M
r=3M
1/r²
tan ψ = u^θ̂ / u^r̂

$$\hat{\Sigma} = 1/b^2 = W_{max} = \frac{1}{27M^2} = e^2/\ell^2$$

what are components of $\underset{\sim}{u}$?



$$e^2 = \frac{\ell^2}{27M^2}$$

for outward motion

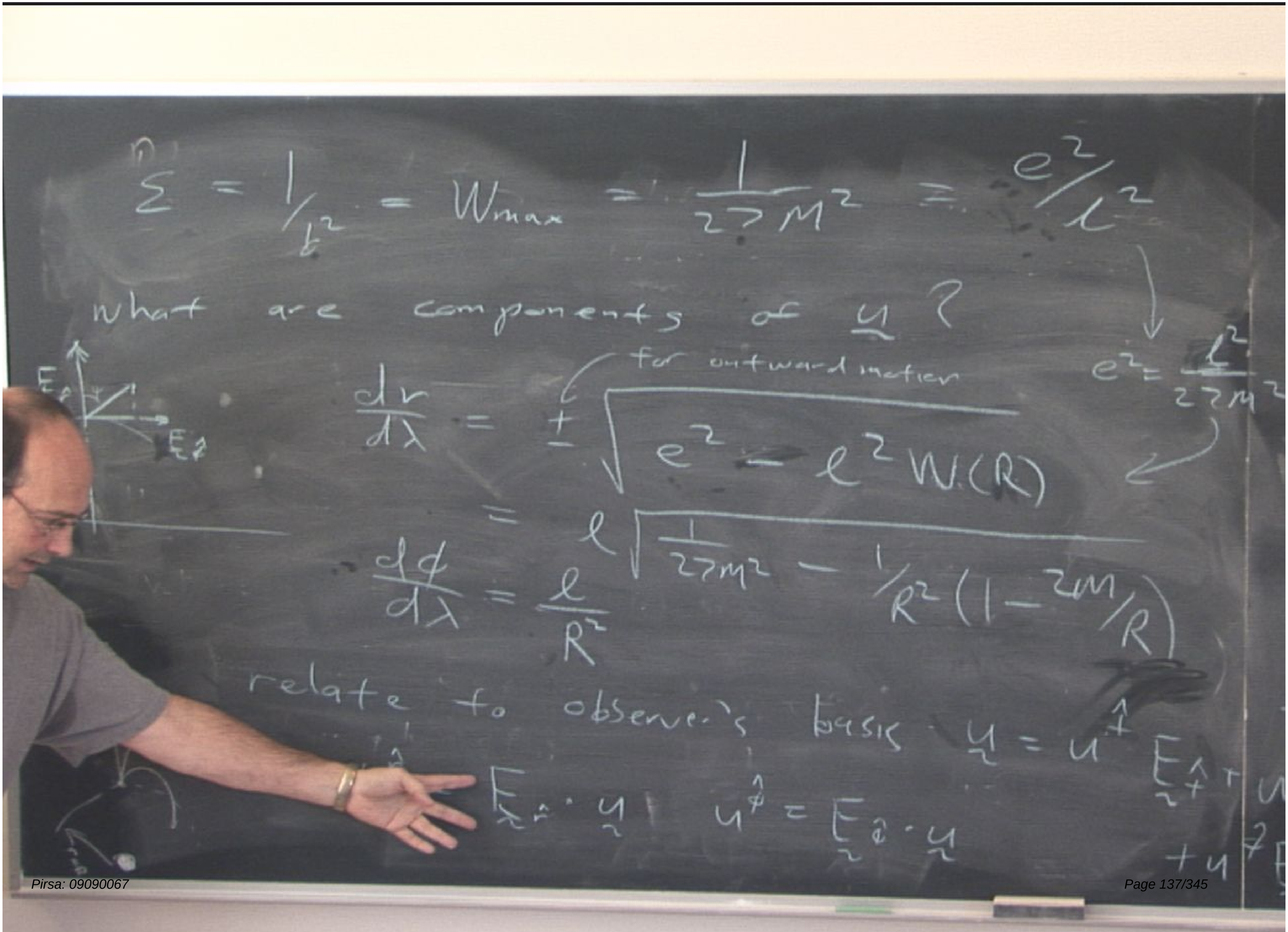
$$\frac{dr}{d\lambda} = \pm\sqrt{e^2 - \ell^2 W(R)}$$

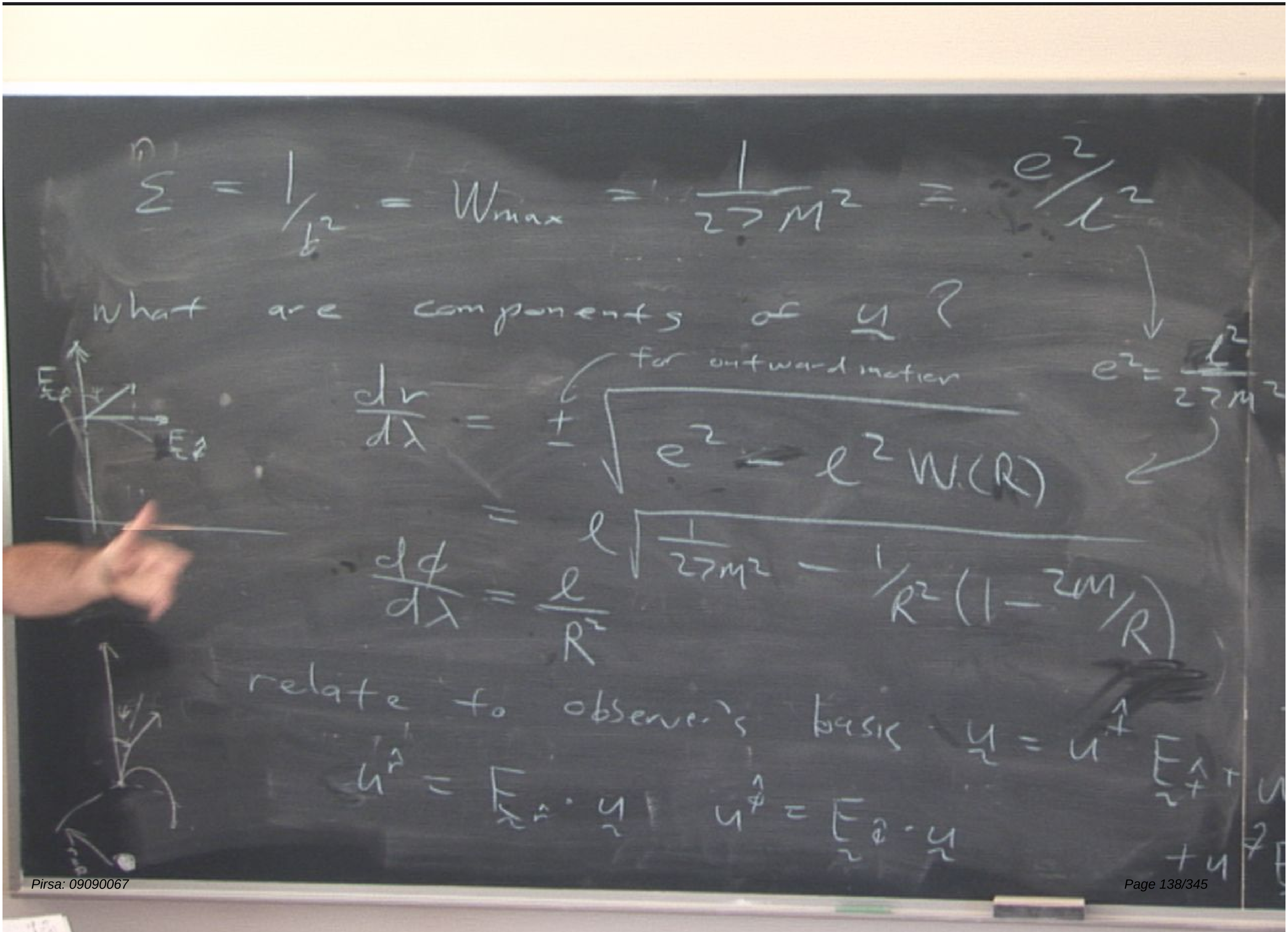
$$= \ell\sqrt{\frac{1}{27M^2} - 1/R^2\left(1 - 2M/R\right)}$$

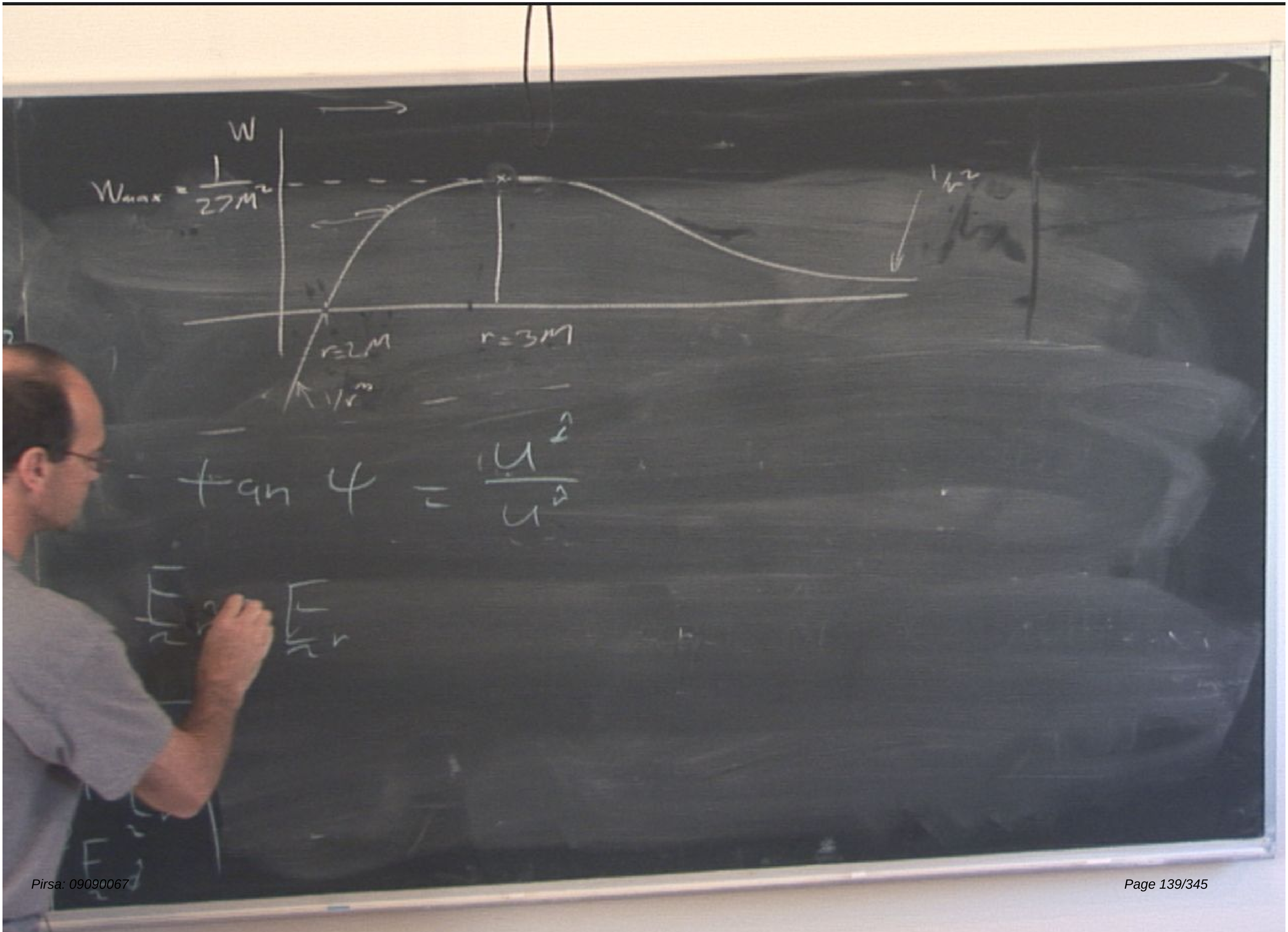
$$\frac{d\phi}{d\lambda} = \frac{\ell}{R^2}$$

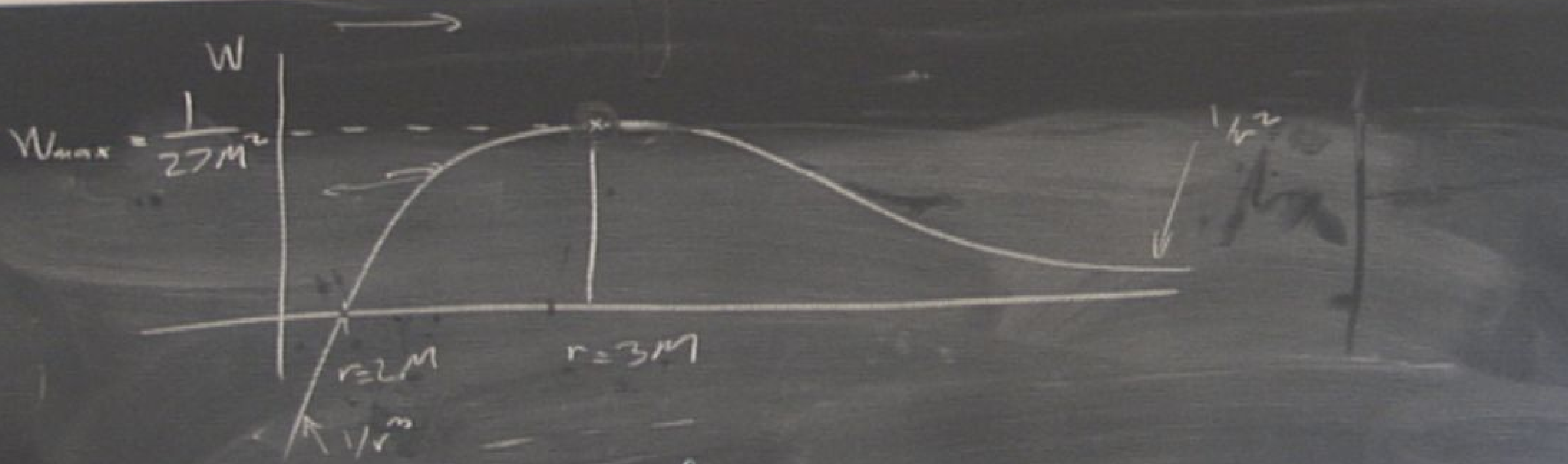
relate to observer's basis $\underset{\sim}{u} = u^{\hat{t}}\,\underset{\sim}{E}_{\hat{t}} + u \ldots + u^{\hat{\phi}} E \ldots$

$$u^{\hat{r}} = \underset{\sim}{E}_{\hat{r}} \cdot \underset{\sim}{u} \qquad u^{\hat{\phi}} = \underset{\sim}{E}_{\hat{\phi}} \cdot \underset{\sim}{u}$$

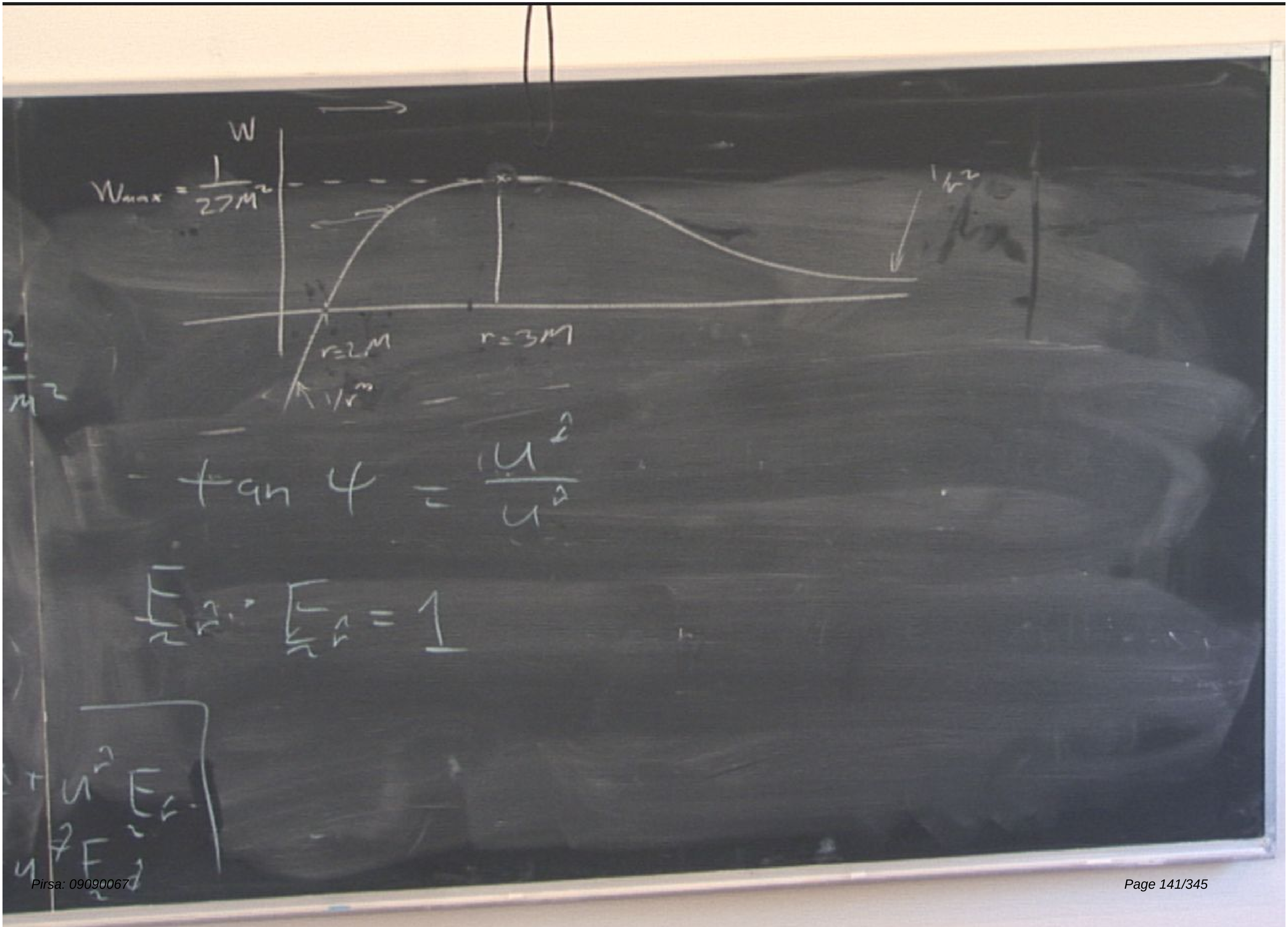

Σ = 1/b² = Wmax = 1/27M² = e²/ℓ²
what are components of u ?
for outward motion
dr/dλ = ± √(e² − ℓ² W(R))
= ℓ √(1/27M² − 1/R² (1 − 2M/R))
dφ/dλ = ℓ/R²
e² = ℓ²/27M²
relate to observer's basis u = u^t̂ E_t̂ + u^r̂ E_r̂ + u^φ̂ E_φ̂
u^φ̂ = E_φ̂ · u

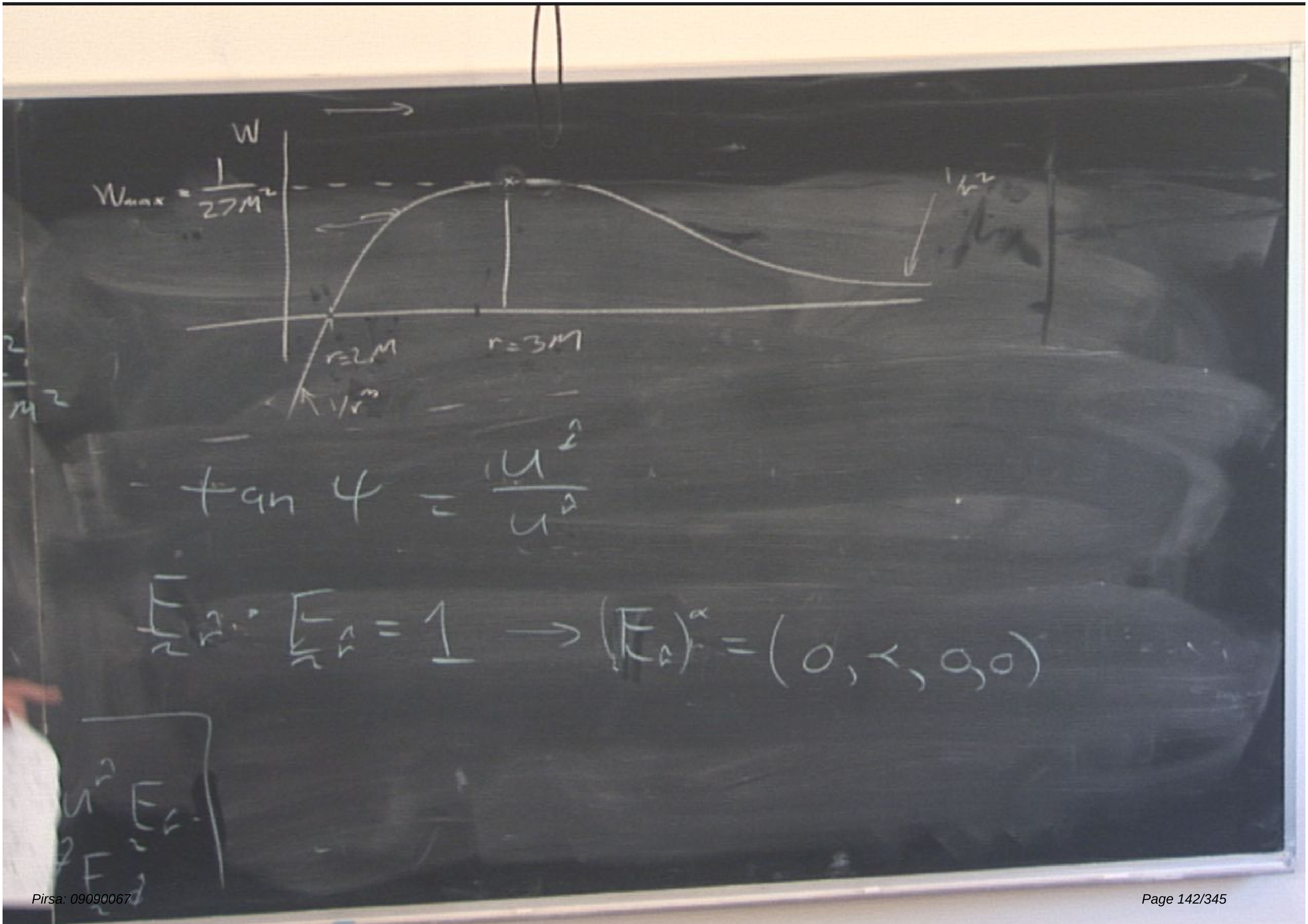

Σ = 1/b² = Wmax = 1/27M² = e²/ℓ²
what are components of u ?
for outward motion
dr/dλ = ± √(e² − ℓ² W(R))
= ℓ √(1/27M² − 1/R²(1 − 2M/R))
dφ/dλ = ℓ/R²
e² = ℓ²/27M²
relate to observer's basis u = u^t̂ E_t̂ + u
u^r̂ = E_r̂ · u   u^φ̂ = E_φ̂ · u

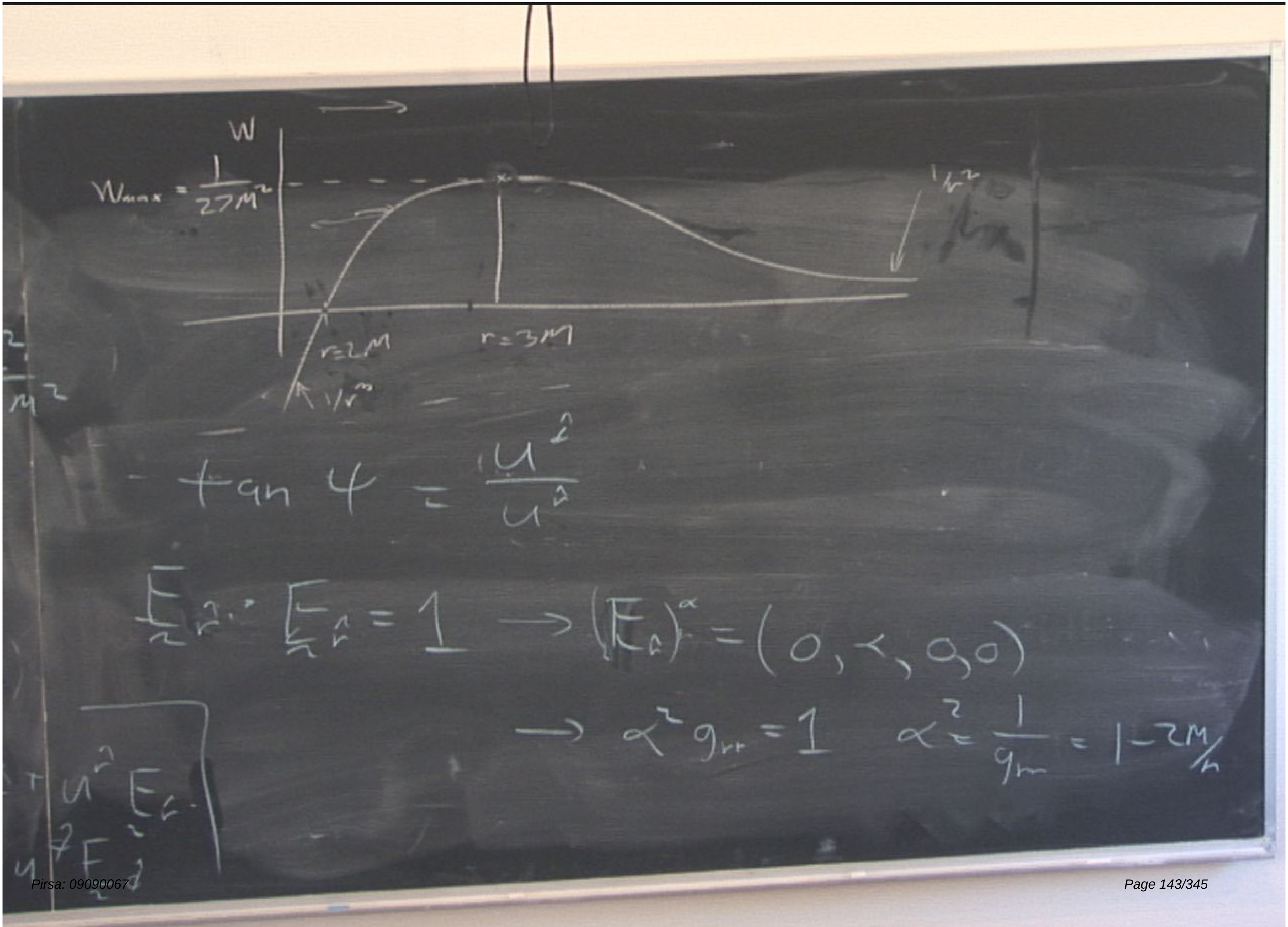

W
Wmax = 1/27M^2
r=2M
r=3M
tan ψ =

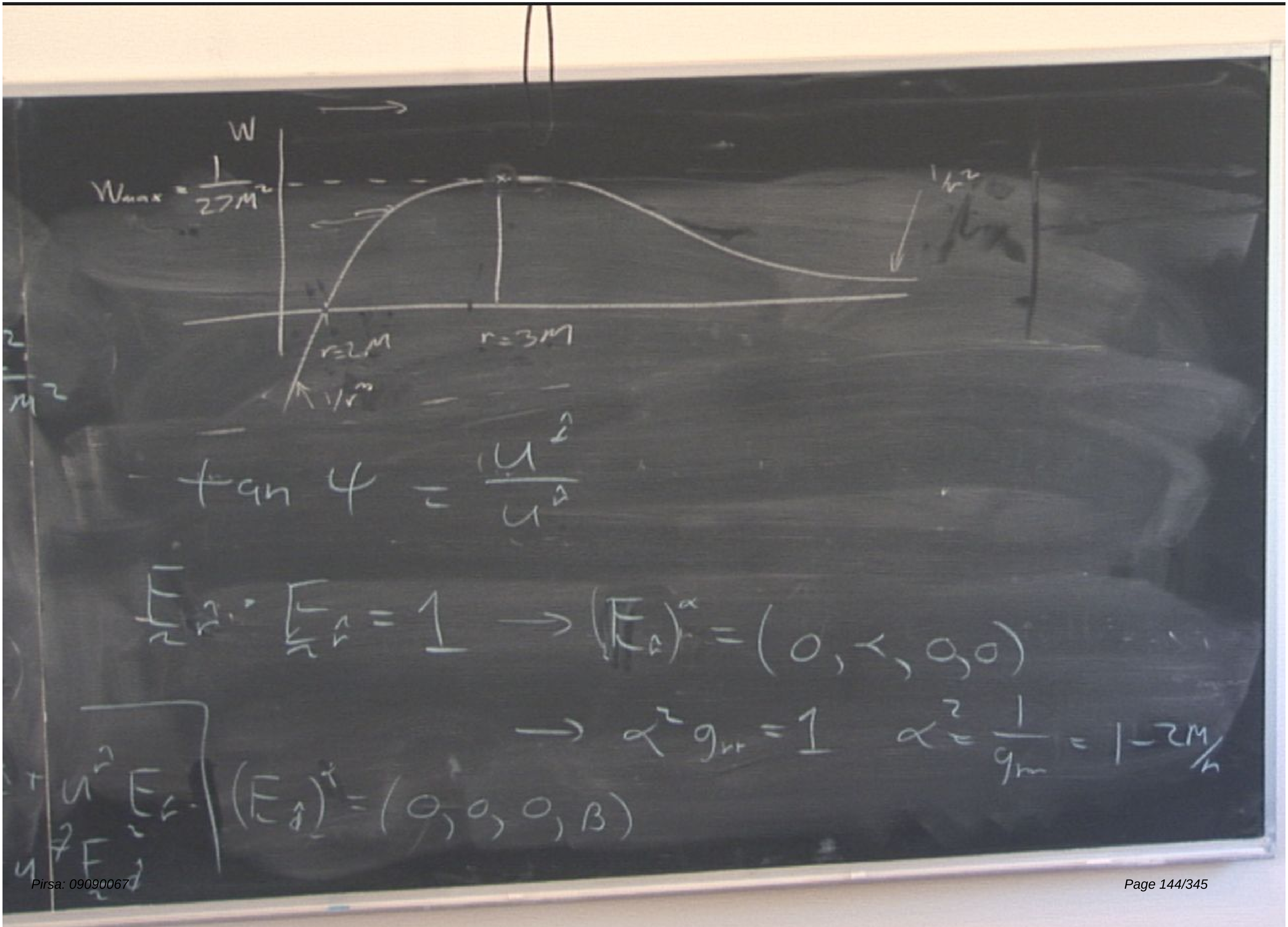


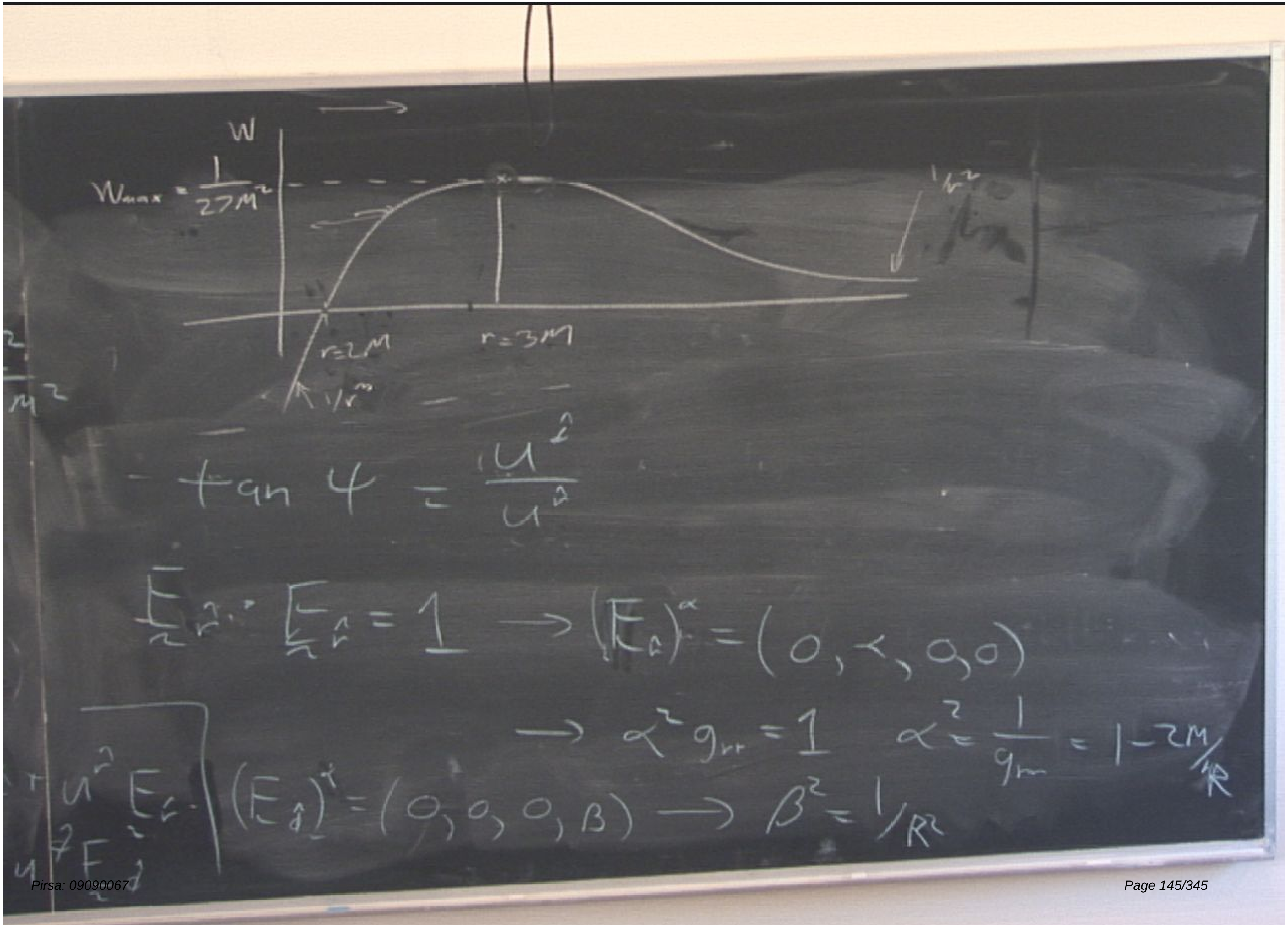
$$\tan\psi = \frac{u^{\hat{\phi}}}{u^{\hat{r}}}$$

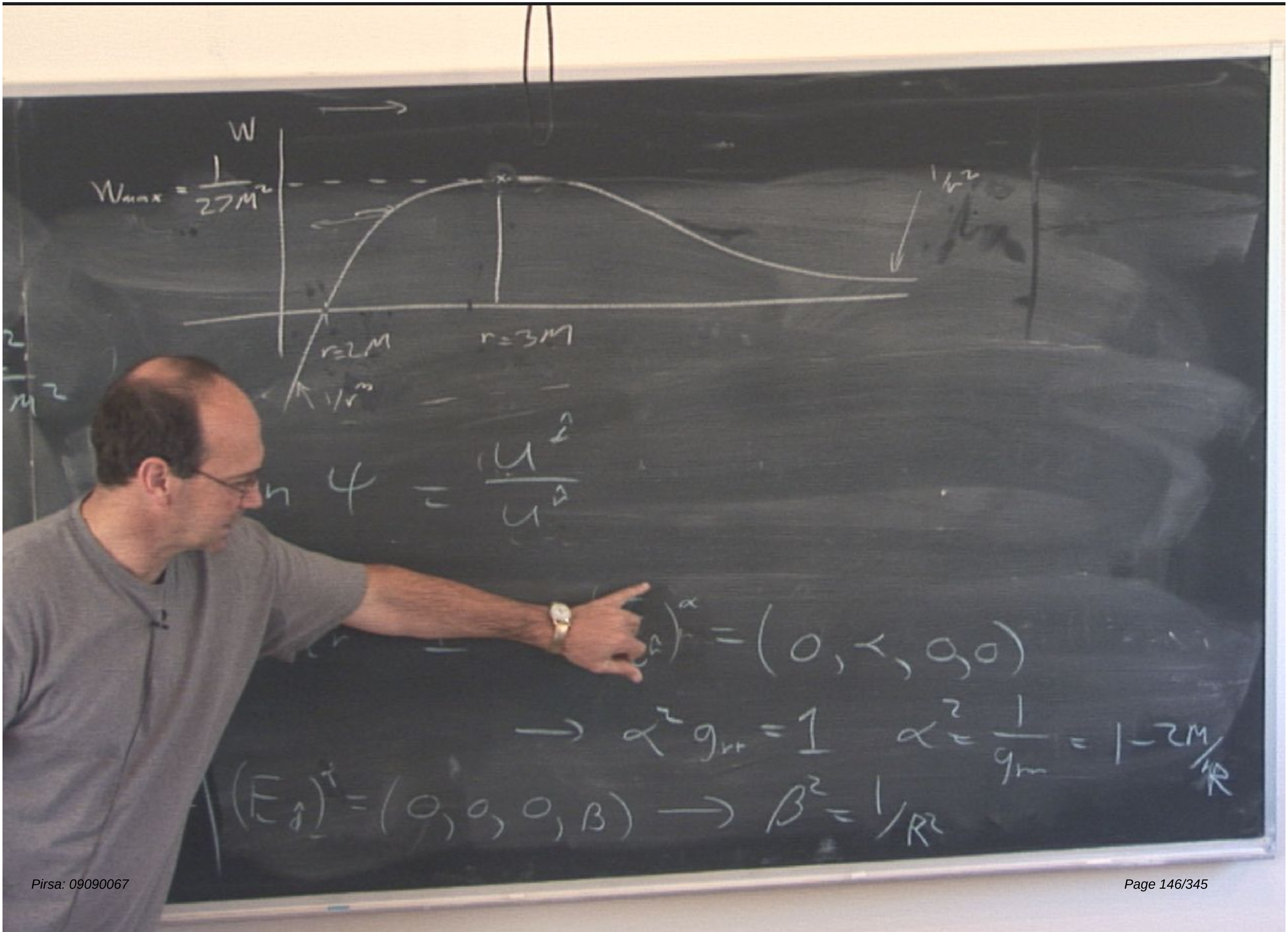

W
Wmax = 1/27M²
r=2M
r=3M
tan ψ = u^θ̂ / u^r̂
E_r̂ · E_r̂ = 1

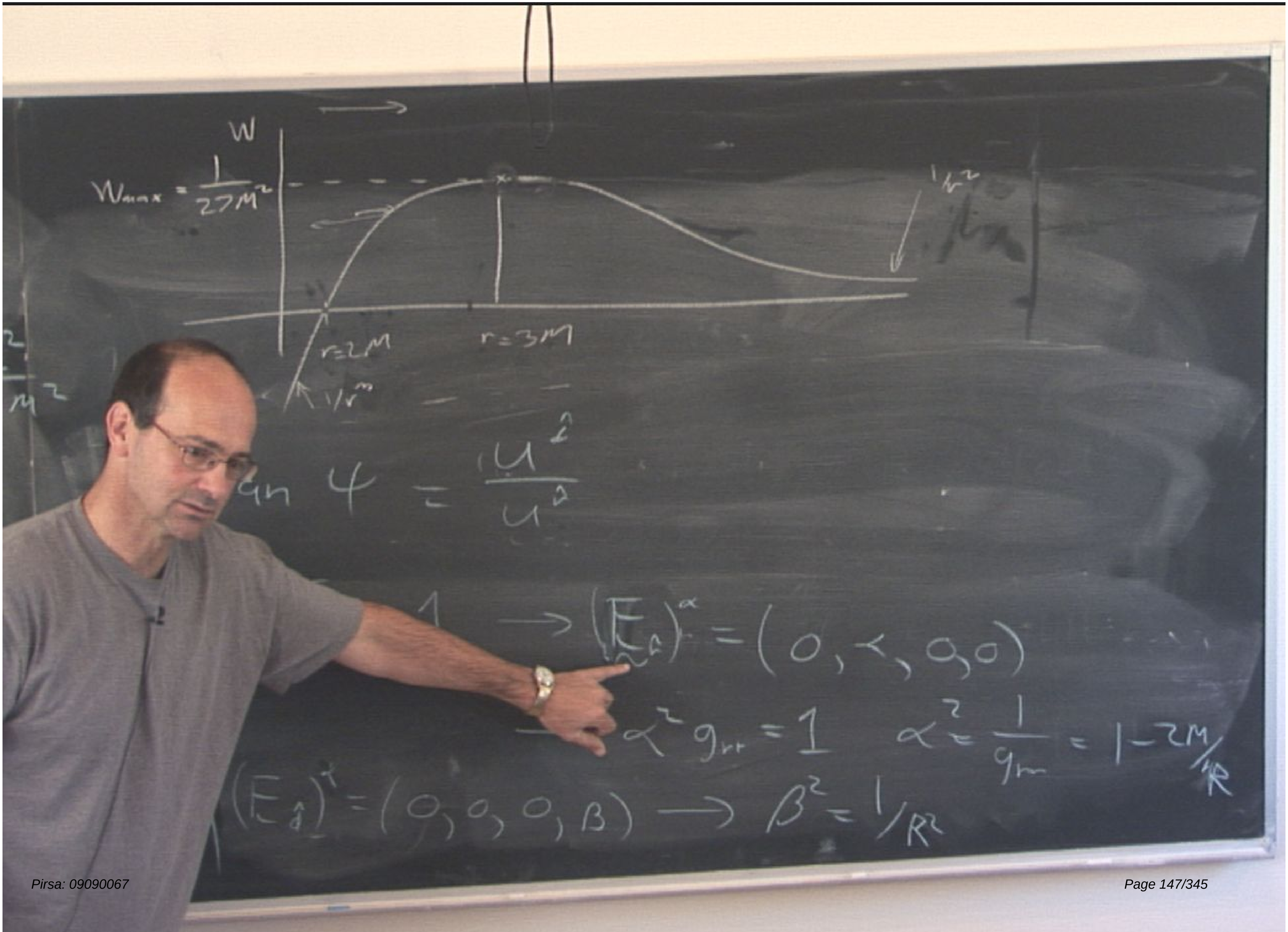

W
Wmax = 1/27M²
r=2M
r=3M
tan ψ = u/u
E · E = 1 → (E)
= (0, <, 0, 0)

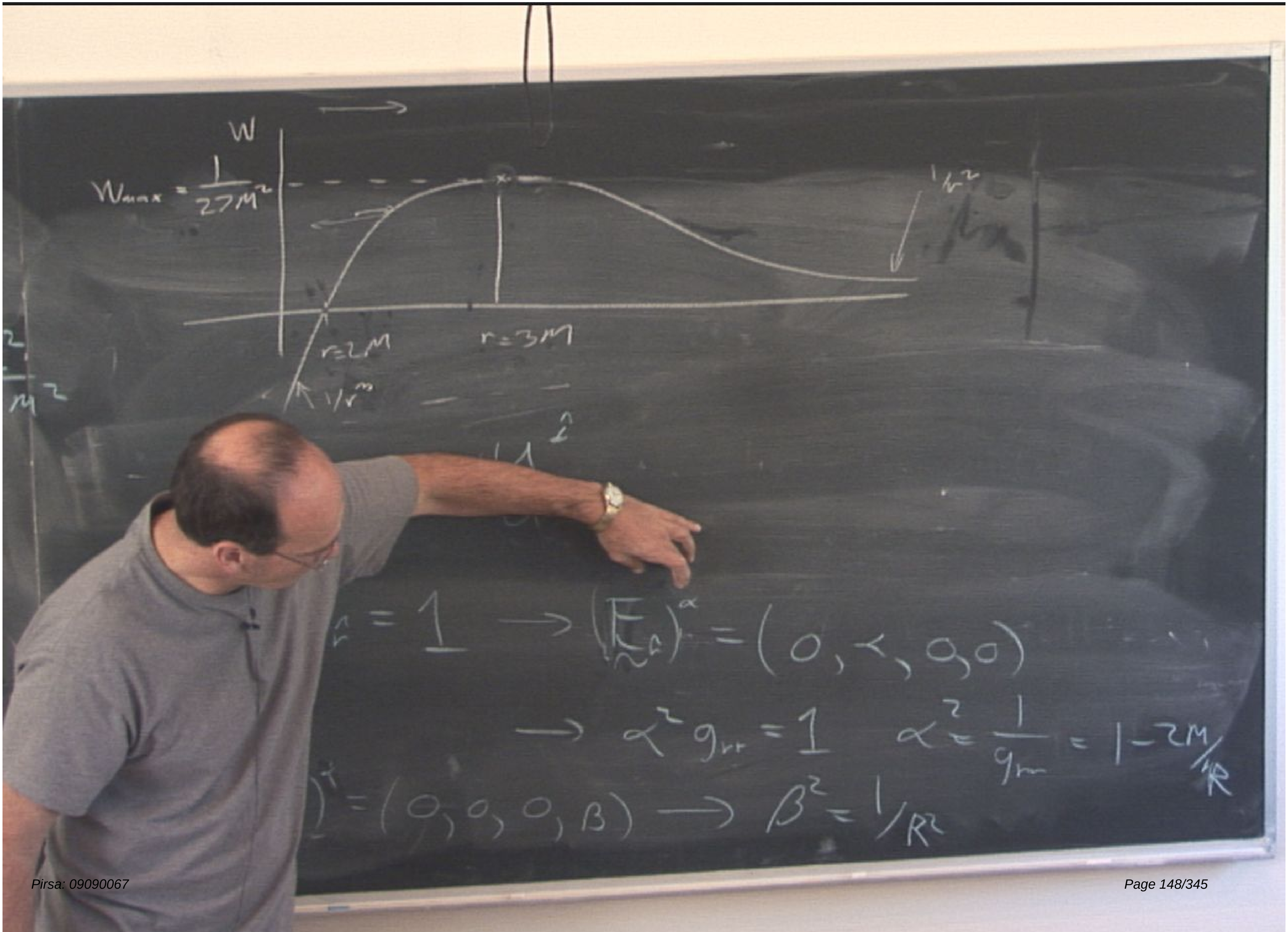

W
Wmax = 1/27M²
r=2M
r=3M
tan ψ = u^î / u^r̂
Er̂ · Er̂ = 1 → (Er̂)^α = (0, α, 0, 0)
→ α² grr = 1   α² = 1/grr = 1 − 2M/r

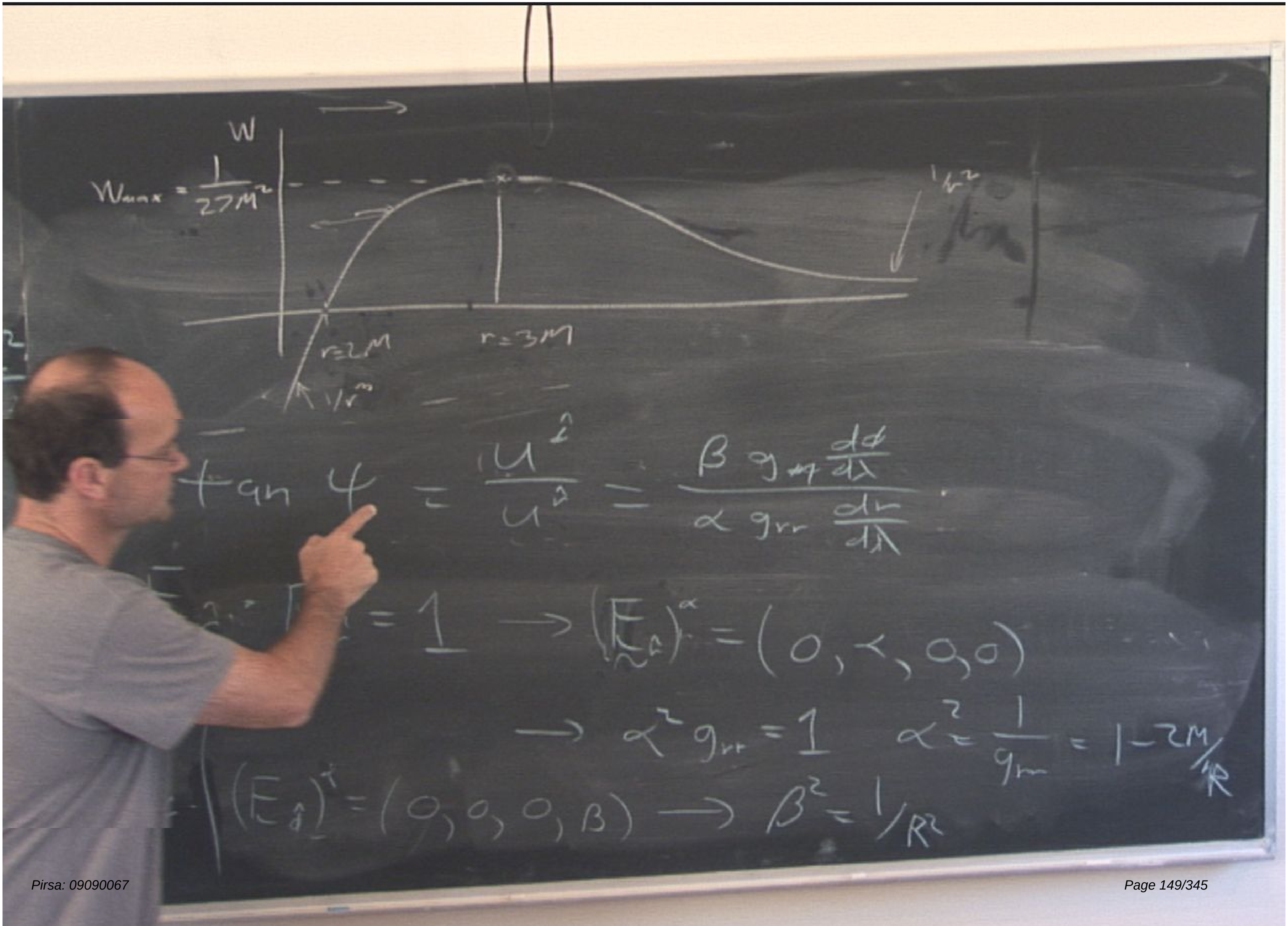

W
Wmax = 1/27M^2
r=2M
r=3M
1/r^2
tan ψ = u^î / u^t̂
E_r̂ · E_r̂ = 1 → (E_r̂)^α = (0, α, 0, 0)
→ α^2 g_rr = 1   α^2 = 1/g_rr = 1 − 2M/r
(E_t̂)^α = (0, 0, 0, β)

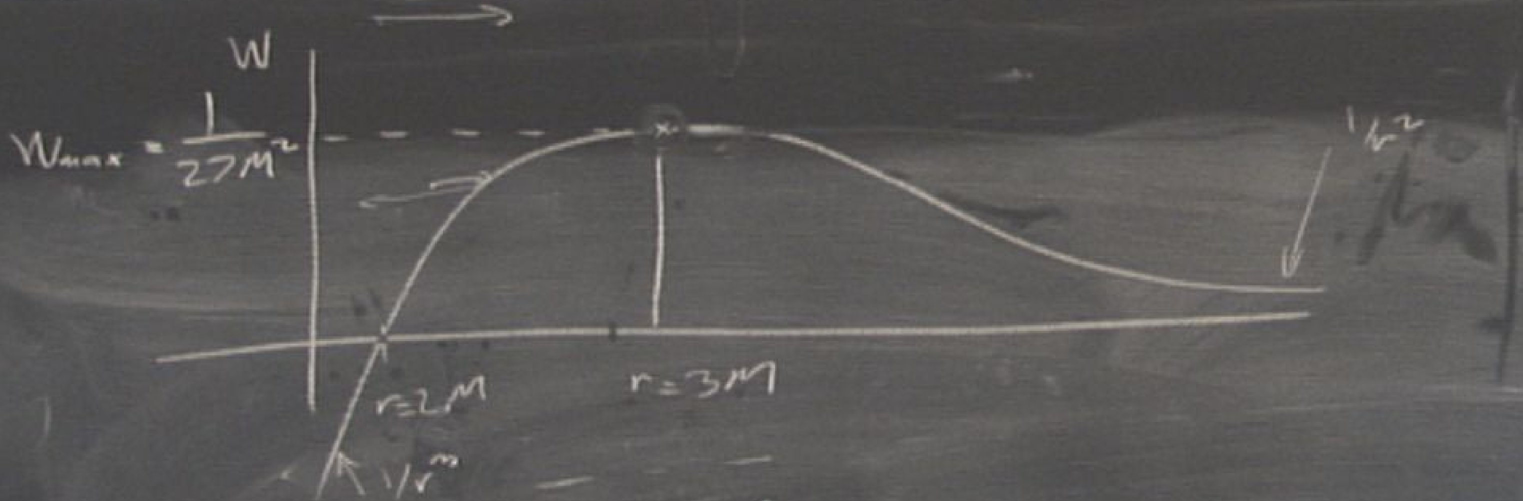

W
Wmax = 1/27M²
r=2M
r=3M
1/r^m
1/r²
tan ψ = u^î / u^r̂
(E_r̂)^α = (0, α, 0, 0)
→ α² g_rr = 1   α² = 1/g_rr = 1 − 2M/R
(E_θ̂)^θ = (0, 0, 0, β) → β² = 1/R²


W
r=2M
r=3M
=(0, α, 0, 0)
→ α²g_rr = 1
β² = 1/R²


W
Wmax = 1/27M²
r=2M
r=3M
(E_r̂)^α = (0, α, 0, 0)
α² g_rr = 1   α² = 1/g_rr = 1 − 2M/R
(E_φ̂)^α = (0, 0, 0, β) ⟶ β² = 1/R²


W
r=2M
r=3M


W
Wmax = 1/27M²
r=2M
r=3M
tan ψ = u^φ̂/u^r̂ = β gφφ dφ/dλ / α grr dr/dλ
→ α² grr = 1   α² = 1/grr = 1 − 2M/R
→ β² = 1/R²

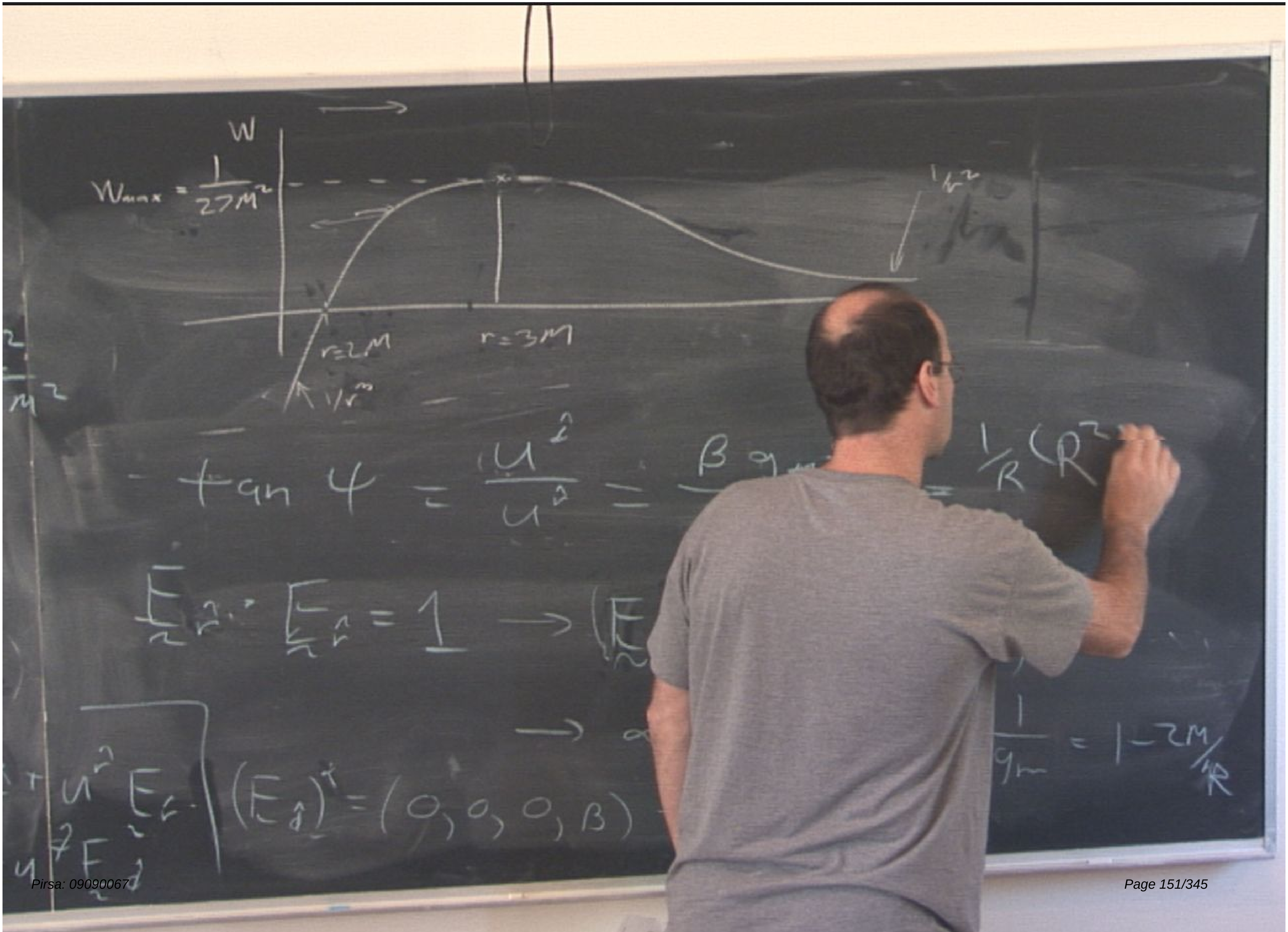


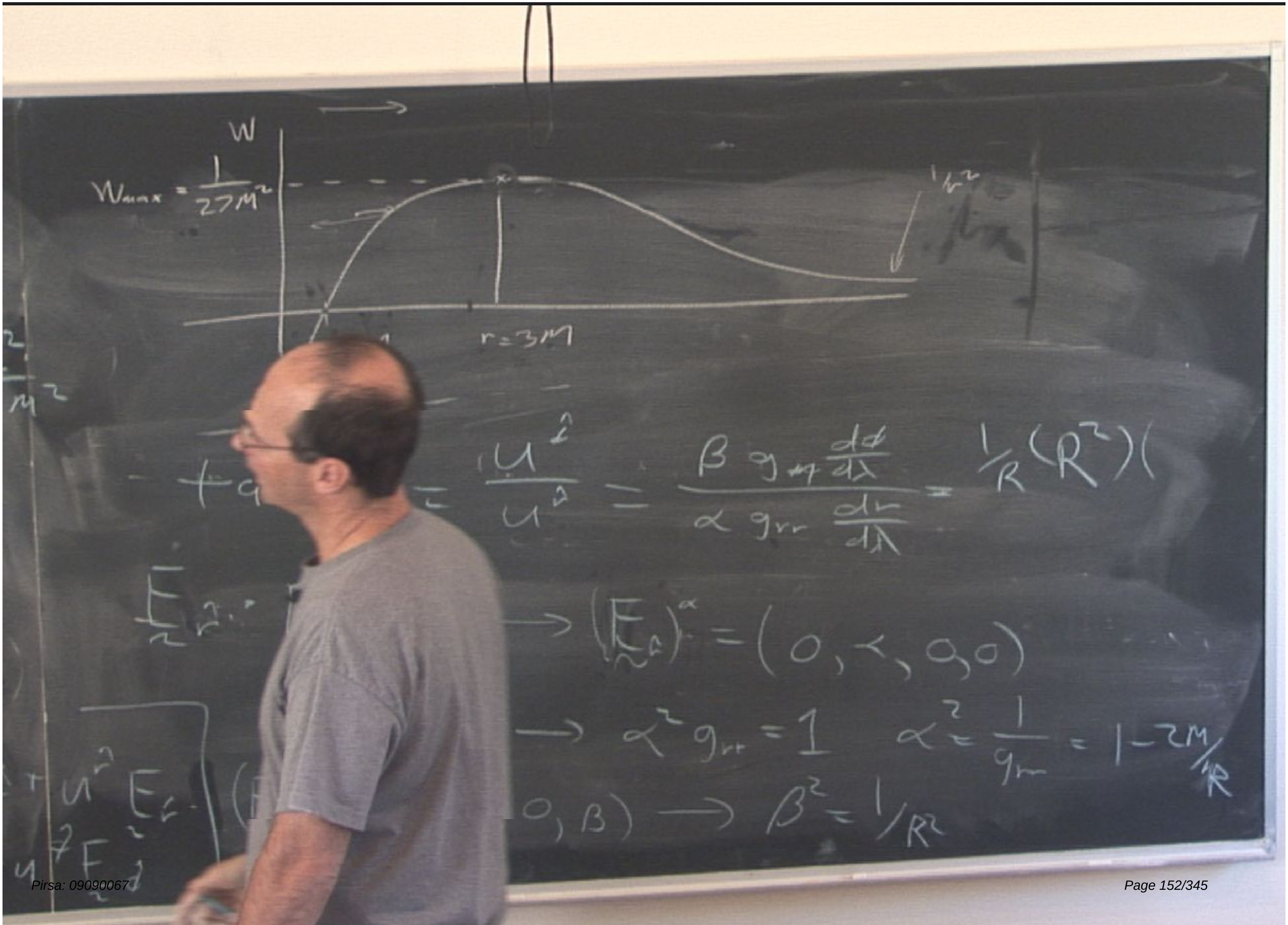
$$\tan\psi = \frac{u^{\hat{\phi}}}{u^{\hat{r}}} = \frac{\beta\, g_{\phi\phi} \frac{d\phi}{d\lambda}}{\alpha\, g_{rr} \frac{dr}{d\lambda}}$$

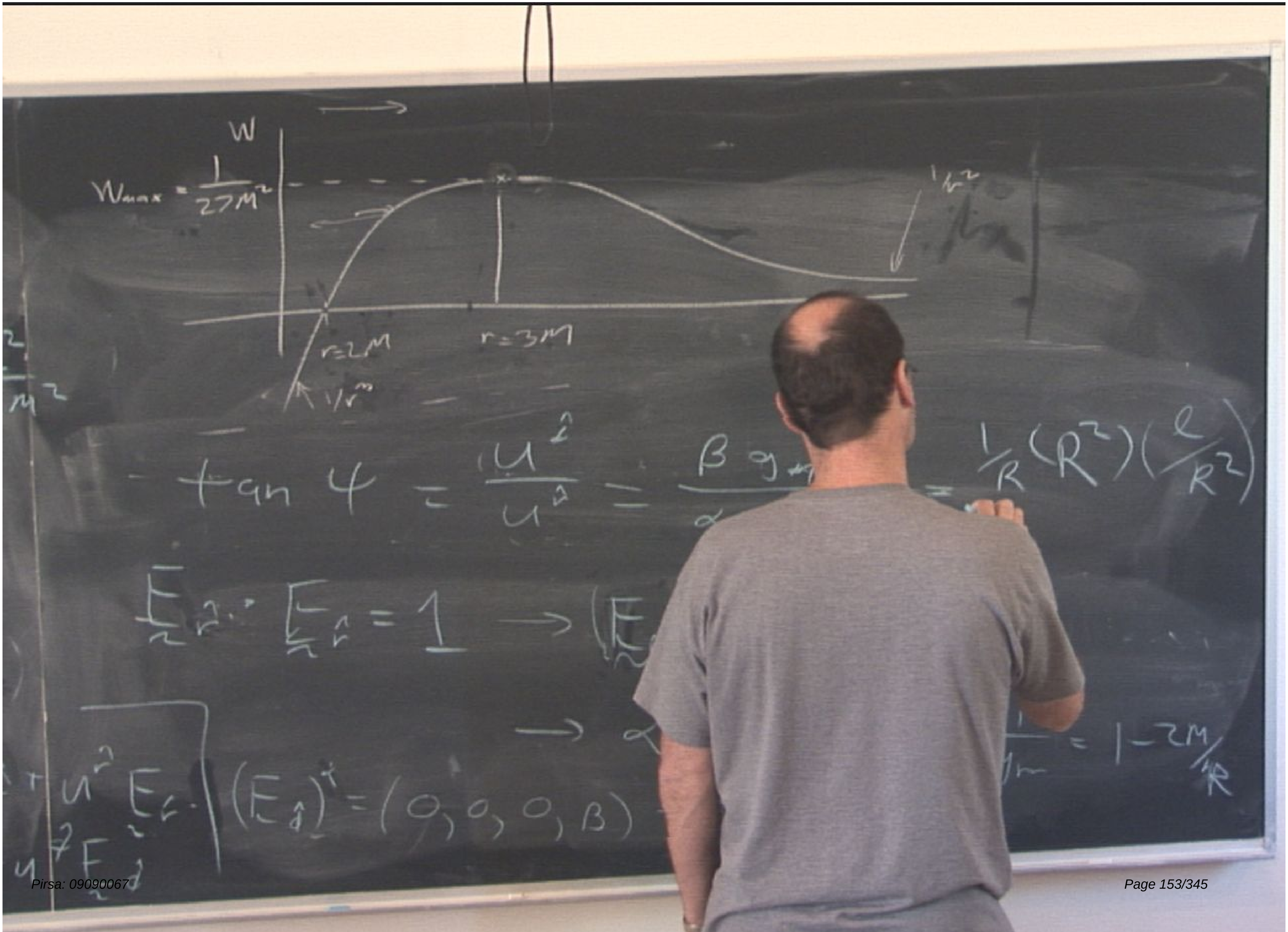
$$\underset{\sim}{E}_{\hat{r}} \cdot \underset{\sim}{E}_{\hat{r}} = 1 \longrightarrow (\underset{\sim}{E}_{\hat{r}})^{\alpha} = (0, \alpha, 0, 0)$$

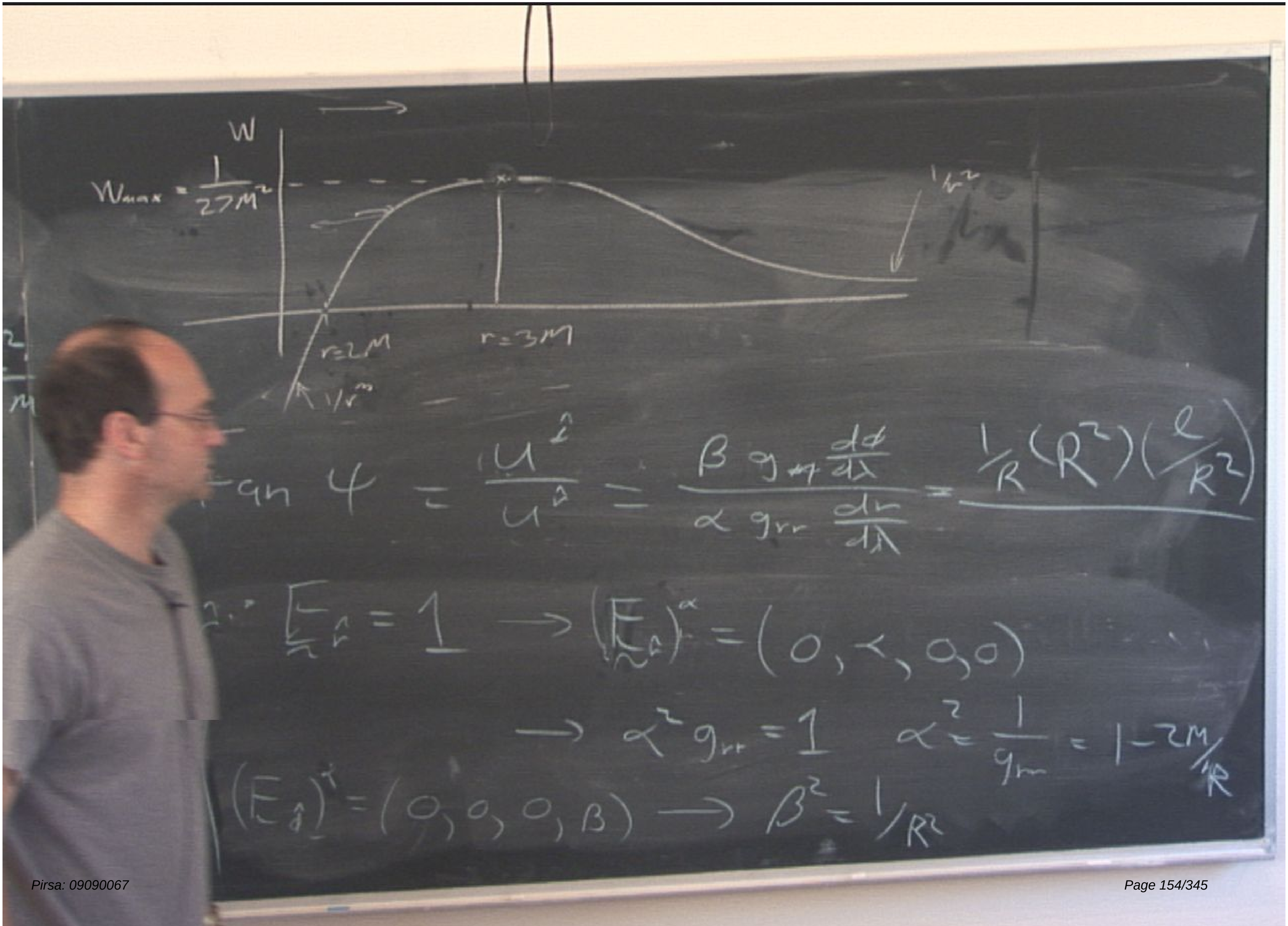
$$\longrightarrow \alpha^2 g_{rr} = 1 \quad \alpha^2 = \frac{1}{g_{rr}} = 1 - 2M/R$$

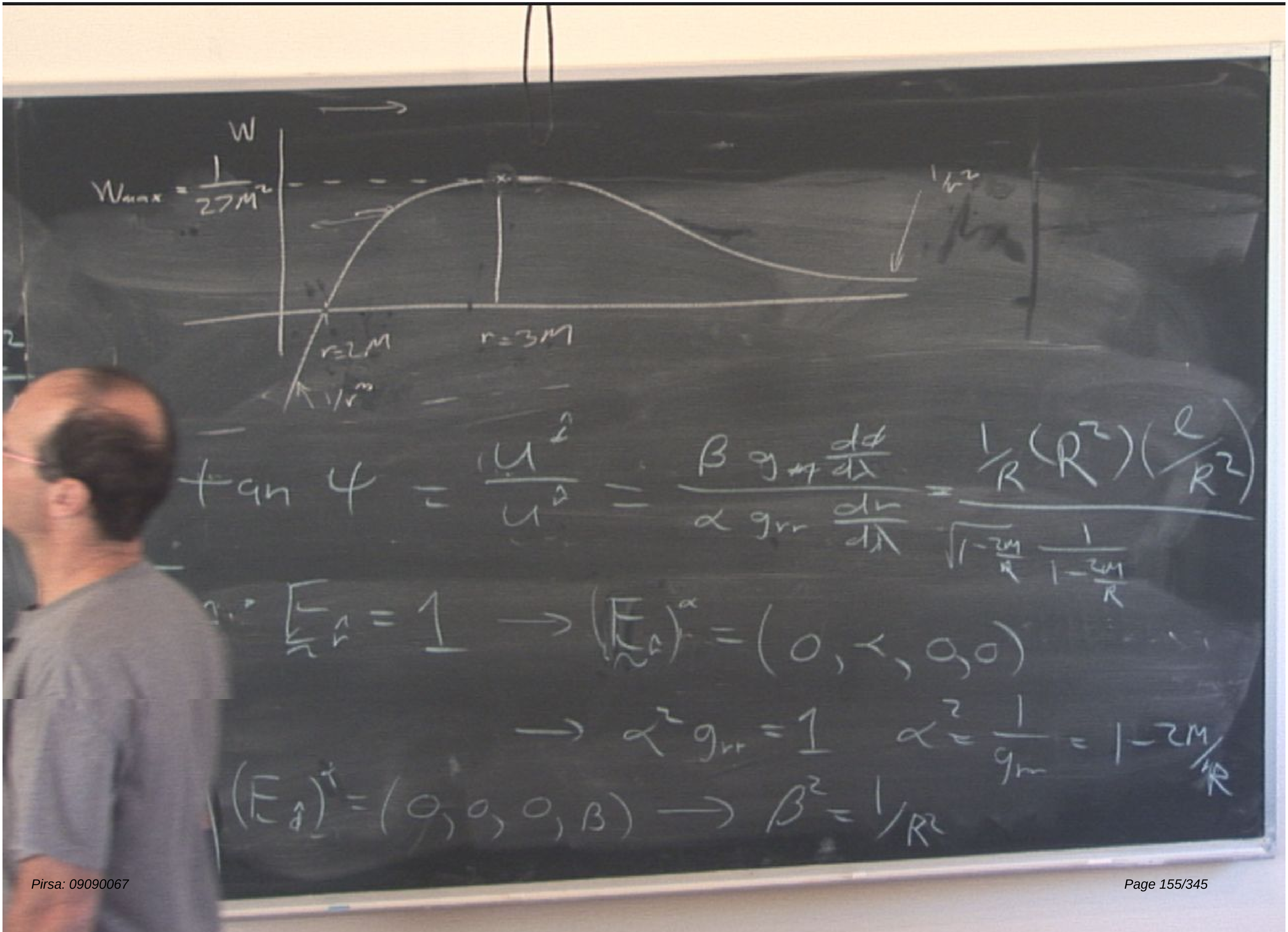
$$u^{\hat{r}} E_{\hat{r}} \quad (E_{\hat{\phi}})^{\alpha} = (0, 0, 0, \beta) \longrightarrow \beta^2 = 1/R^2$$

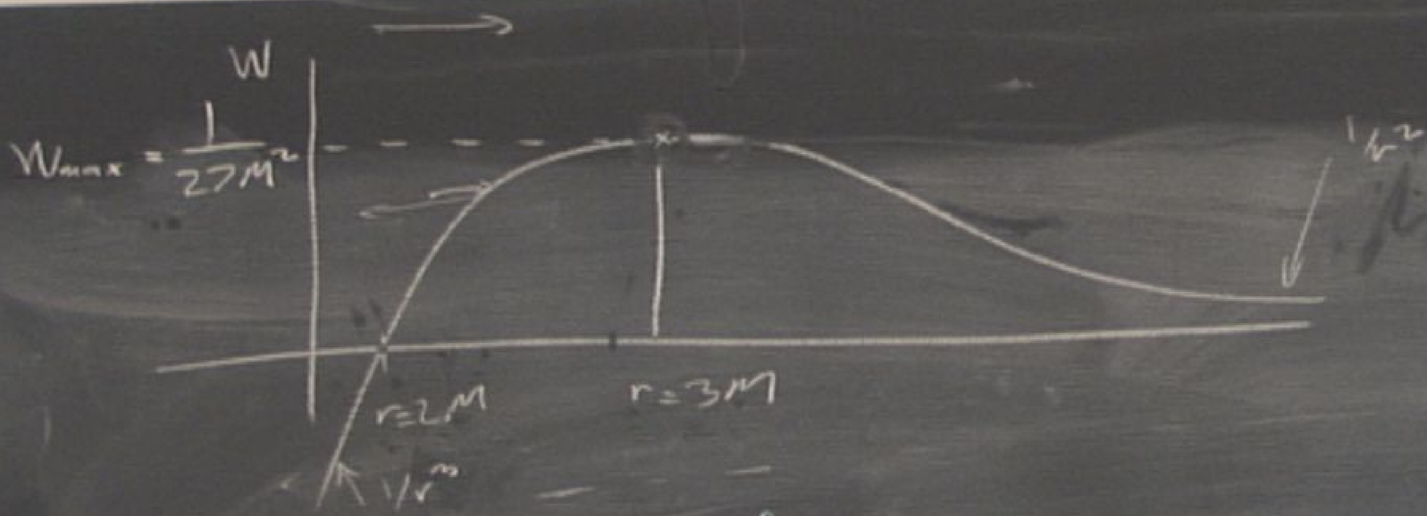

W
r=2M
r=3M


W
Wmax = 1/27M²
r = 3M


W
$W_{max} = \frac{1}{27M^2}$
r=2M
r=3M
1/r^m
1/r^2
tan ψ = u^φ / u^r =
(E_φ)^i = (0, 0, 0, β)
= 1 − 2M/R


W
Wmax = 1/27M²
r=2M
r=3M
1/r²
tan ψ = u^φ̂ / u^r̂ = β gφφ dφ/dλ / α grr dr/dλ = 1/R (R²)(ℓ/R²)
E_r̂ = 1 → (E_r̂)^α = (0, α, 0, 0)
→ α² grr = 1   α² = 1/grr = 1−2M/R
(E_φ̂)^φ = (0, 0, 0, β) → β² = 1/R²


W
$W_{max} = \frac{1}{27M^2}$
$r=2M$
$r=3M$
$1/r^2$
$1/r^m$
$\tan\psi = \frac{u^{\hat\phi}}{u^{\hat r}} = \frac{\beta\, g_{\phi\phi} \frac{d\phi}{d\lambda}}{\alpha\, g_{rr} \frac{dr}{d\lambda}} = \frac{\frac{1}{R}(R^2)\left(\frac{\ell}{R^2}\right)}{\sqrt{1-\frac{2M}{R}}\ \frac{1}{1-\frac{2M}{R}}}$
$E_{\hat r} \cdot E_{\hat r} = 1 \longrightarrow (E_{\hat r})^\alpha = (0, \alpha, 0, 0)$
$\longrightarrow \alpha^2 g_{rr} = 1 \quad \alpha^2 = \frac{1}{g_{rr}} = 1 - 2M/R$
$(E_{\hat\phi})^\alpha = (0, 0, 0, \beta) \longrightarrow \beta^2 = 1/R^2$

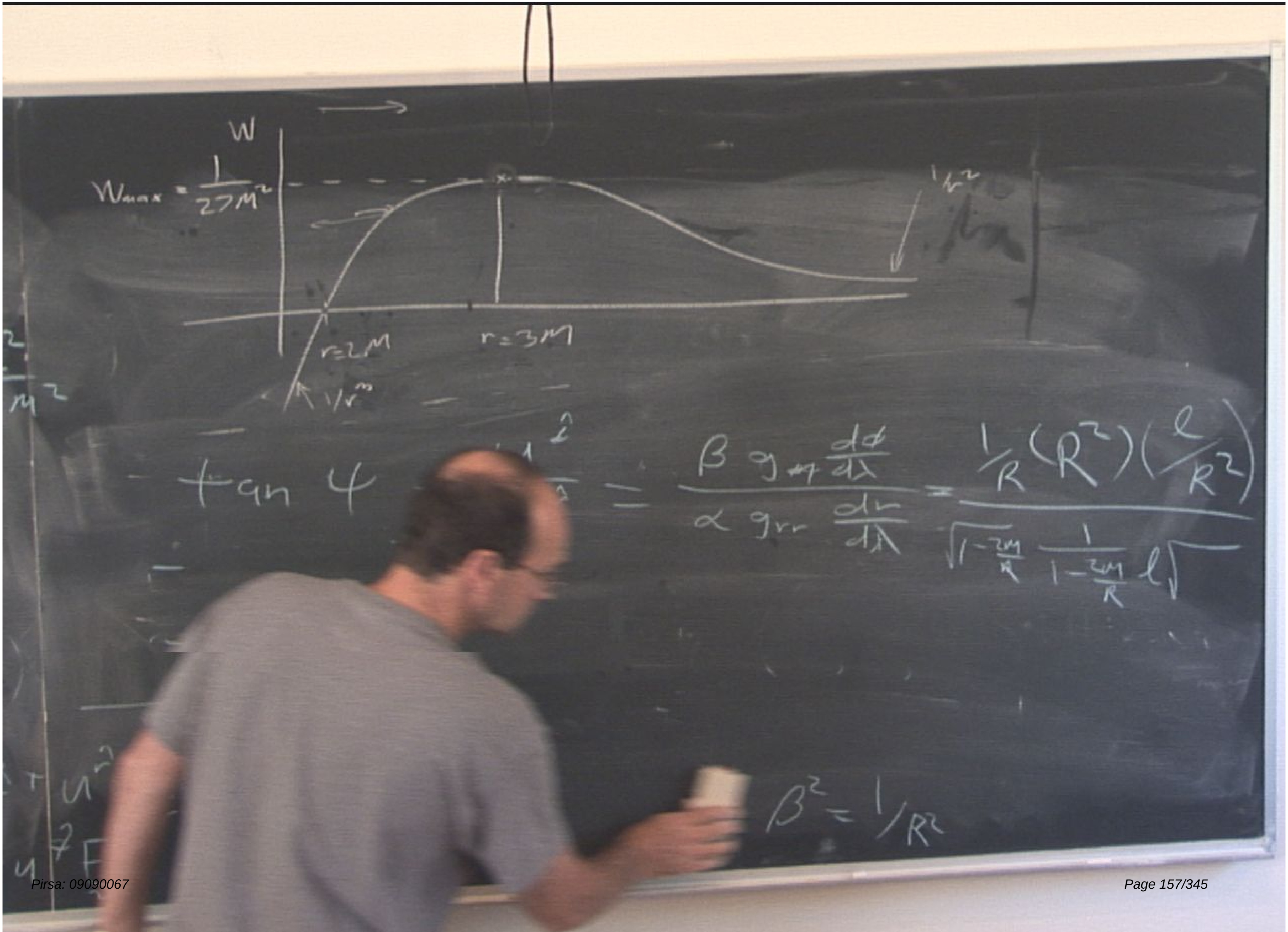


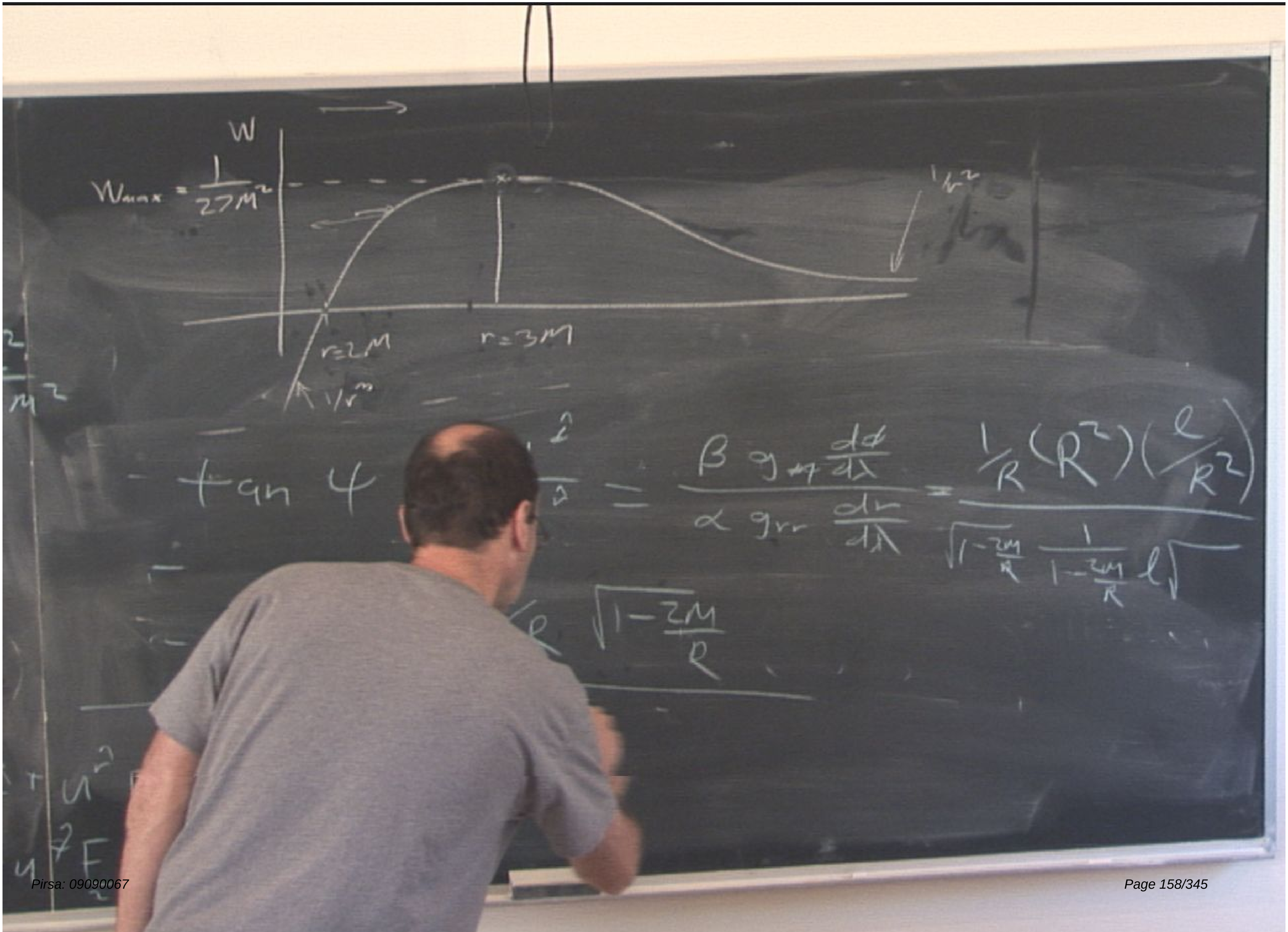
$$\tan\psi = \frac{u^{\hat\phi}}{u^{\hat r}} = \frac{\beta\, g_{\phi\phi}\frac{d\phi}{d\lambda}}{\alpha\, g_{rr}\frac{dr}{d\lambda}} = \frac{\frac{1}{R}(R^2)\left(\frac{\ell}{R^2}\right)}{\sqrt{1-\frac{2M}{R}}\;\frac{1}{1-\frac{2M}{R}}\;\ell\sqrt{}}$$

$$E_{\hat r}\cdot E_{\hat r} = 1 \longrightarrow (E_{\hat r})^{\alpha} = (0,\alpha,0,0)$$

$$\longrightarrow \alpha^2 g_{rr} = 1 \qquad \alpha^2 = \frac{1}{g_{rr}} = 1-2M/R$$

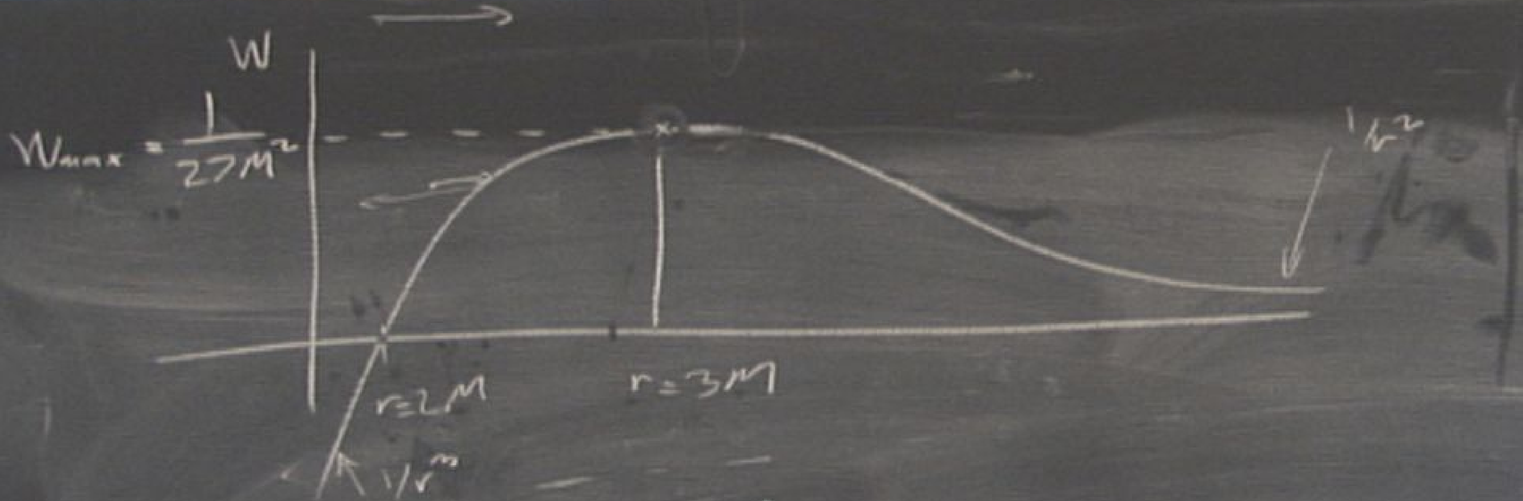
$$u^{\hat r} E_{\hat r} \qquad (E_{\hat\phi})^{\alpha} = (0,0,0,\beta) \longrightarrow \beta^2 = 1/R^2$$


W
Wmax = 1/27M²
r=2M
r=3M
tan ψ
β² = 1/R²


W
Wmax = 1/27M²
r=2M
r=3M
1/r²

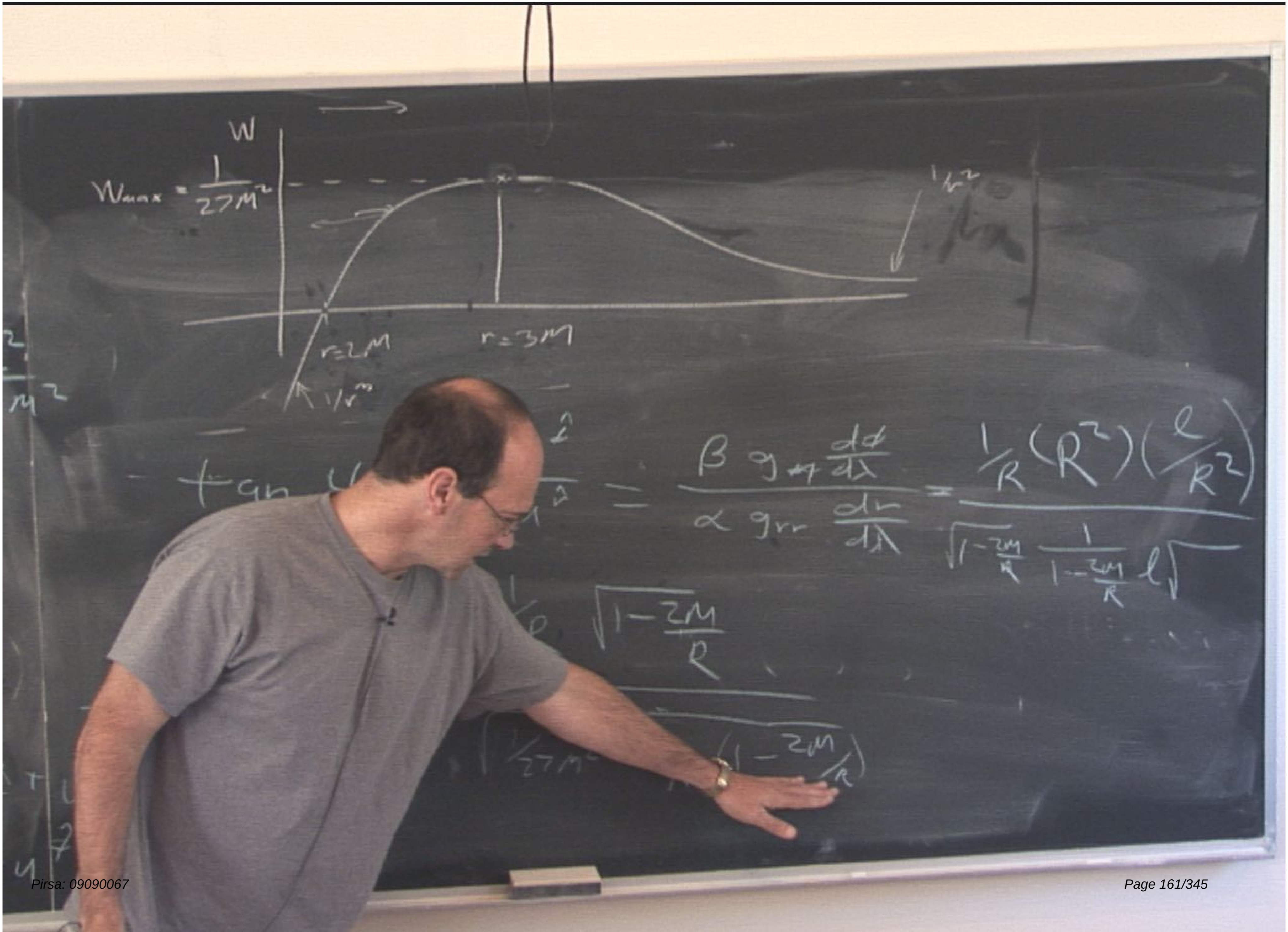
$$\tan\psi = \frac{u^{\hat\phi}}{u^{\hat r}} = \frac{\beta\, g_{\phi\phi}\frac{d\phi}{d\lambda}}{\alpha\, g_{rr}\frac{dr}{d\lambda}} = \frac{\frac{1}{R}(R^2)\left(\frac{\ell}{R^2}\right)}{\sqrt{1-\frac{2M}{R}}\;\frac{1}{1-\frac{2M}{R}}\;\ell\sqrt{}}$$

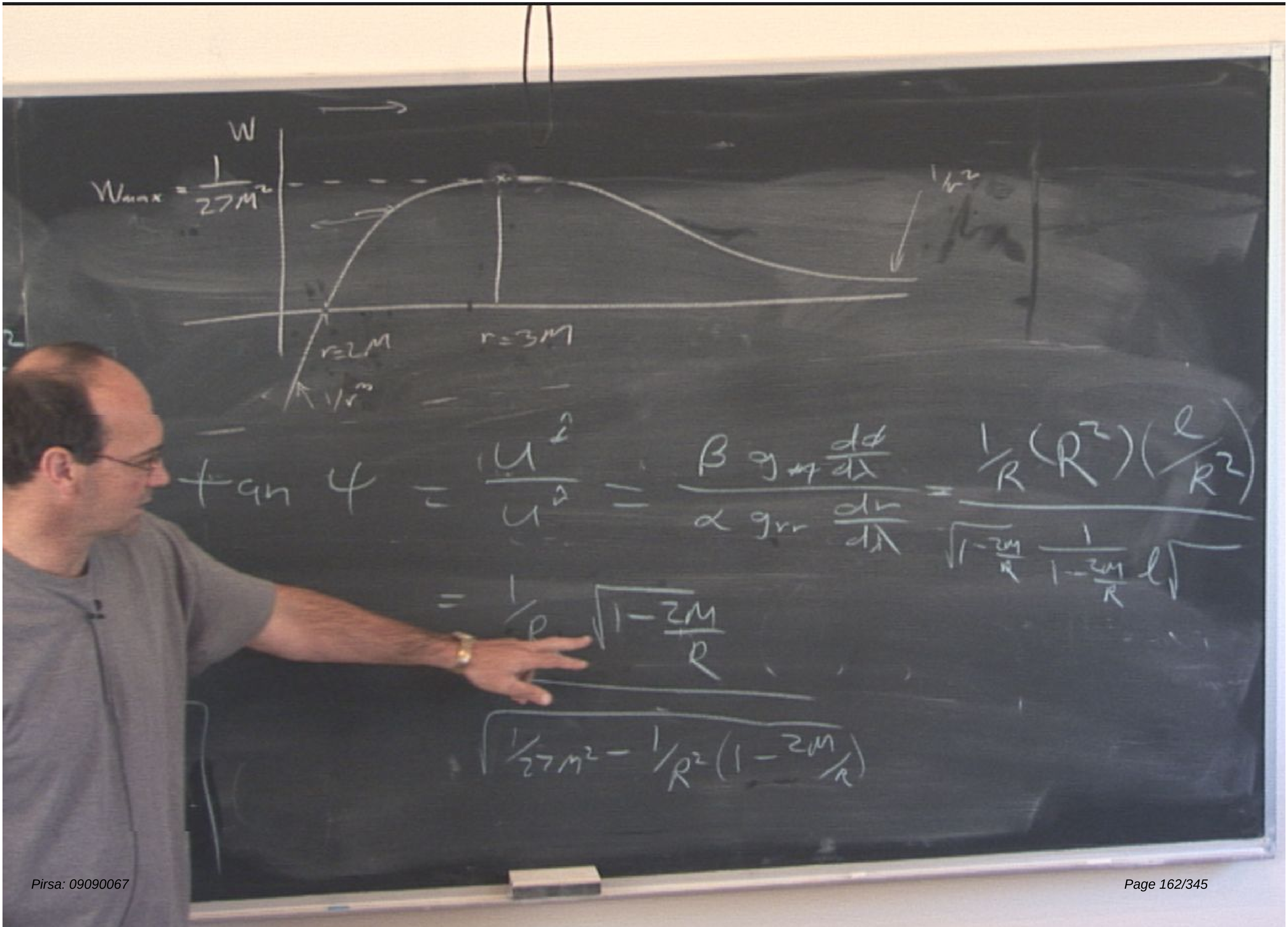
$$= \frac{\frac{1}{R}\sqrt{1-\frac{2M}{R}}}{\sqrt{\frac{1}{27M^2}-\frac{1}{R^2}\left(1-\frac{2M}{R}\right)}}$$

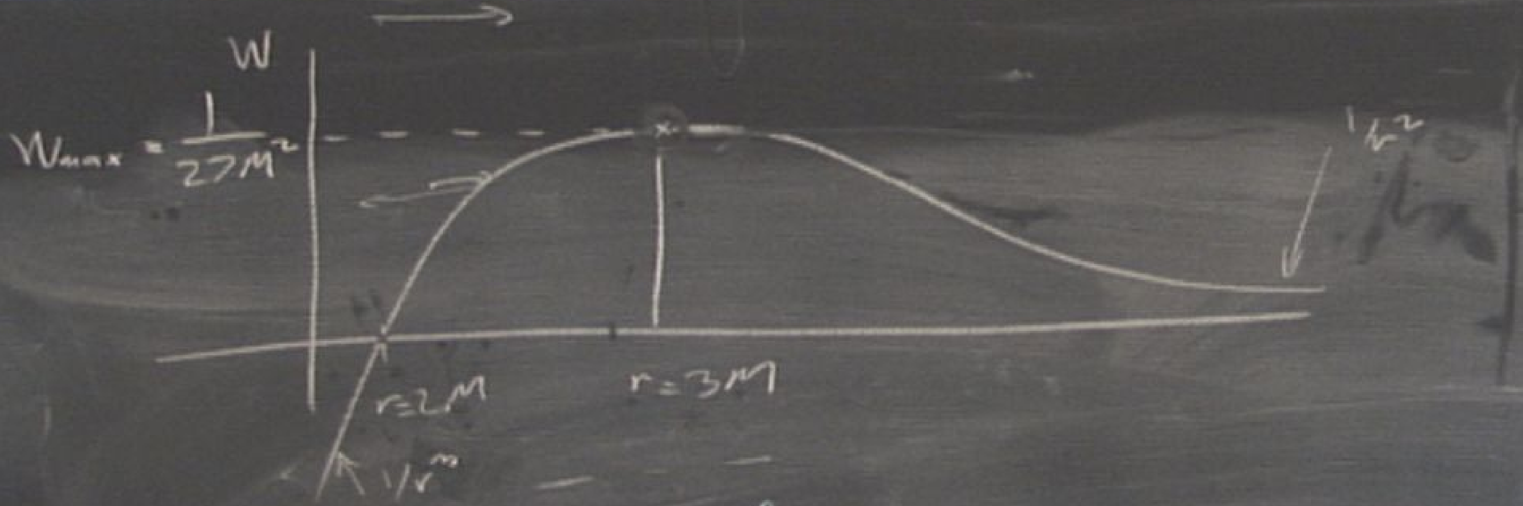


$$\tan\psi = \frac{u^{\hat\phi}}{u^{\hat r}} = \frac{\beta\, g_{\phi\phi}\frac{d\phi}{d\lambda}}{\alpha\, g_{rr}\frac{dr}{d\lambda}} = \frac{\frac{1}{R}(R^2)\left(\frac{\ell}{R^2}\right)}{\sqrt{1-\frac{2M}{R}}\;\frac{1}{1-\frac{2M}{R}}\;\ell\sqrt{\quad}}$$

$$= \frac{\frac{1}{R}\sqrt{1-\frac{2M}{R}}}{\sqrt{\frac{1}{27M^2}-\frac{1}{R^2}\left(1-\frac{2M}{R}\right)}}$$

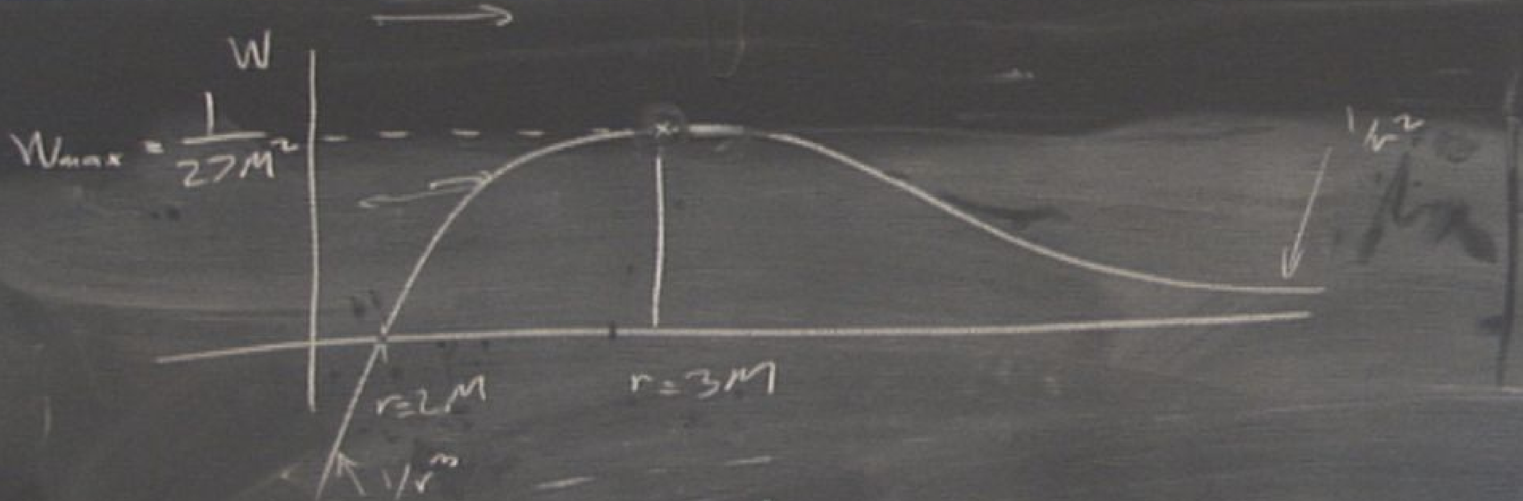

W
r=2M
r=3M


W
Wmax = 1/27M²
r=2M
r=3M
1/r^m
1/r²
tan ψ = u^φ̂ / u^r̂ = β g_φφ dφ/dλ / α g_rr dr/dλ = 1/R (R²)(ℓ/R²) / √(1−2M/R) · 1/(1−2M/R) ℓ √
= 1/ℓ √(1−2M/R) / √(1/27M² − 1/R² (1−2M/R))



$$\tan\psi = \frac{u^{\hat\phi}}{u^{\hat r}} = \frac{\beta\, g_{\phi\phi}\frac{d\phi}{d\lambda}}{\alpha\, g_{rr}\frac{dr}{d\lambda}} = \frac{\frac{1}{R}(R^2)\left(\frac{\ell}{R^2}\right)}{\sqrt{1-\frac{2M}{R}}\;\frac{1}{1-\frac{2M}{R}}\;\ell\sqrt{}}$$

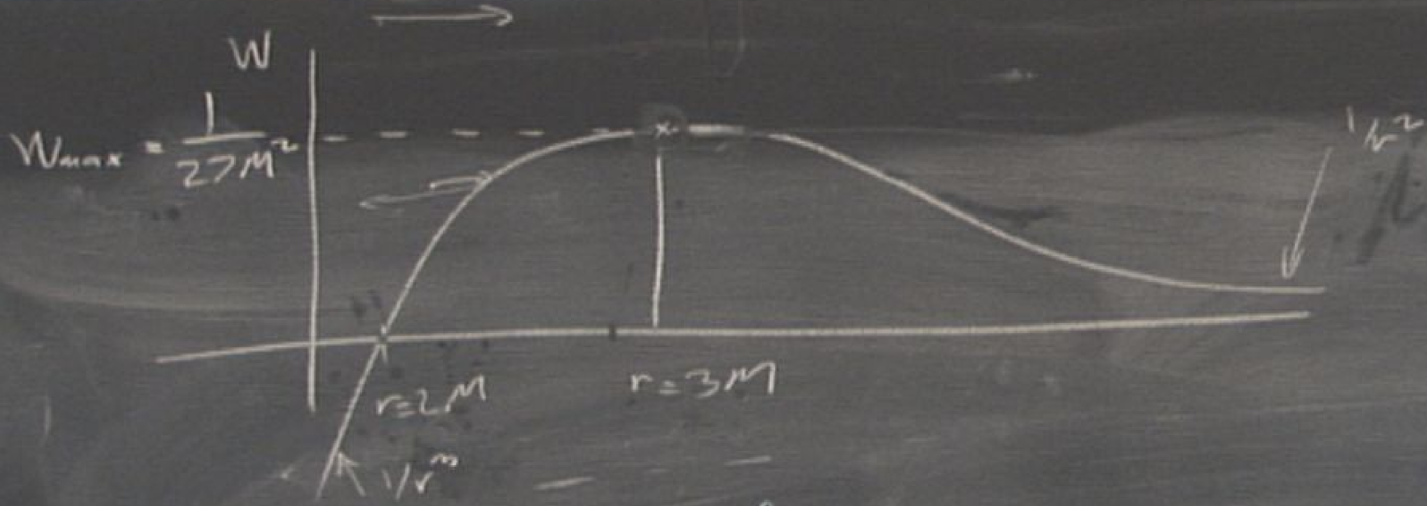
$$= \frac{\frac{1}{R}\sqrt{1-\frac{2M}{R}}}{\sqrt{\frac{1}{27M^2}-\frac{1}{R^2}\left(1-\frac{2M}{R}\right)}} \xrightarrow[R\to 2M]{} \frac{3\sqrt{3}}{2\sqrt{2}}\sqrt{\frac{R-2M}{M}}$$



$$\tan\psi = \frac{u^{\hat\phi}}{u^{\hat r}} = \frac{\beta\, g_{\phi\phi}\frac{d\phi}{d\lambda}}{\alpha\, g_{rr}\frac{dr}{d\lambda}} = \frac{\frac{1}{R}(R^2)\left(\frac{\ell}{R^2}\right)}{\sqrt{1-\frac{2M}{R}}\;\frac{1}{1-\frac{2M}{R}}\;\ell\sqrt{}}$$

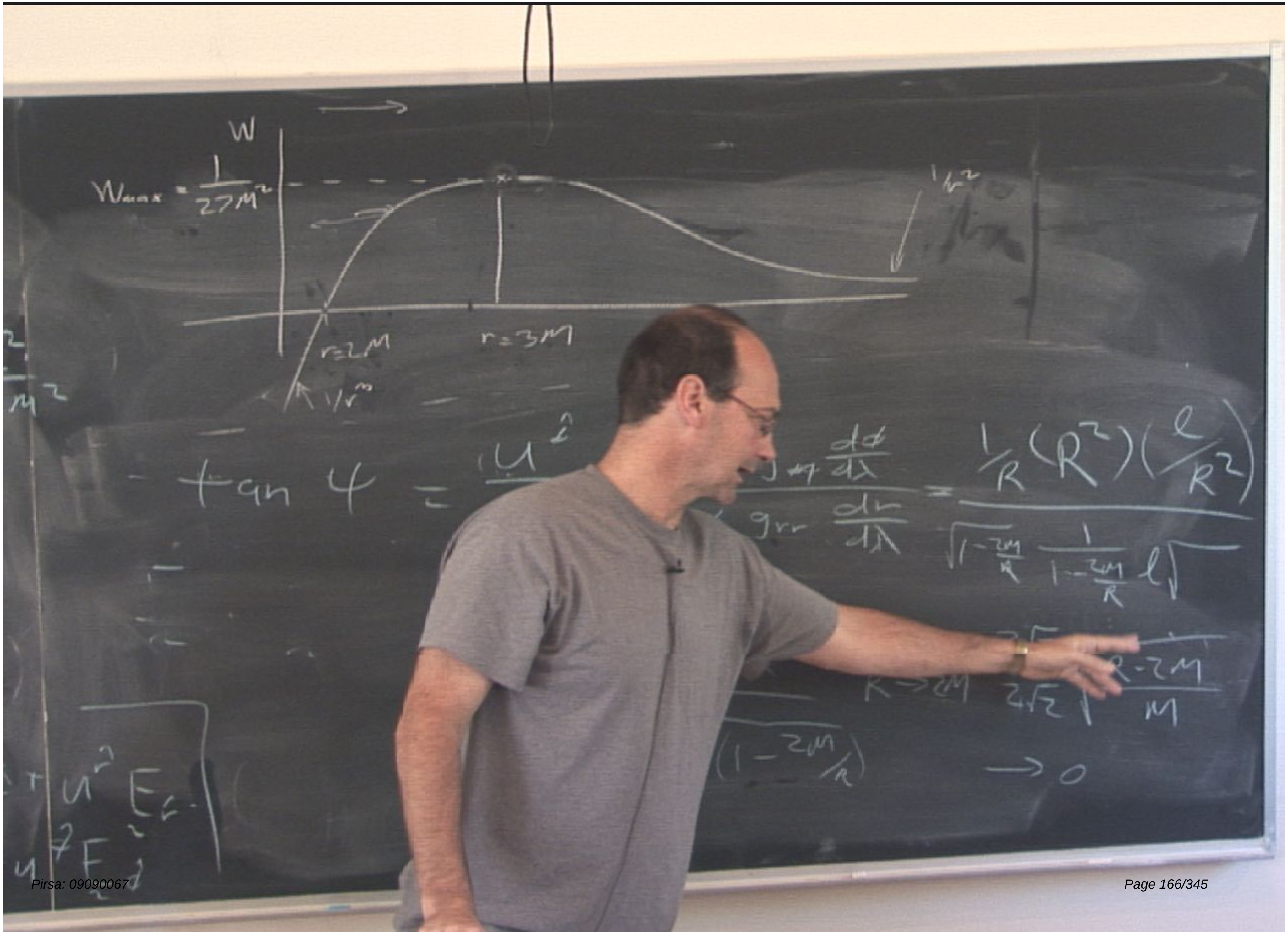
$$= \frac{\frac{1}{R}\sqrt{1-\frac{2M}{R}}}{\sqrt{\frac{1}{27M^2}-\frac{1}{R^2}\left(1-\frac{2M}{R}\right)}} \xrightarrow[R\to 2M]{} \frac{3\sqrt{3}}{2\sqrt{2}}\sqrt{\frac{R-2M}{M}} \to 0$$

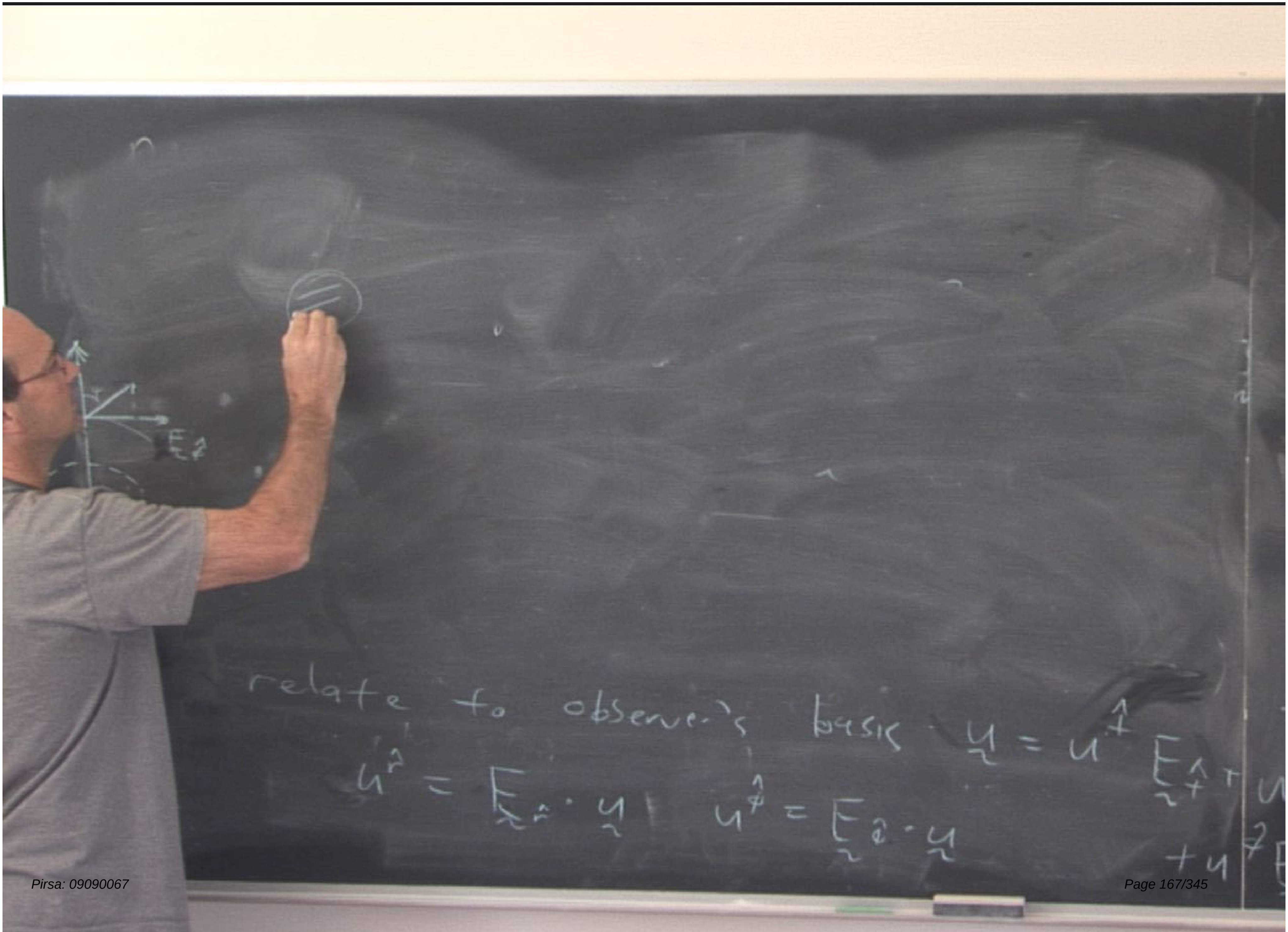
$u^{\hat r}\, E_{\hat r}$

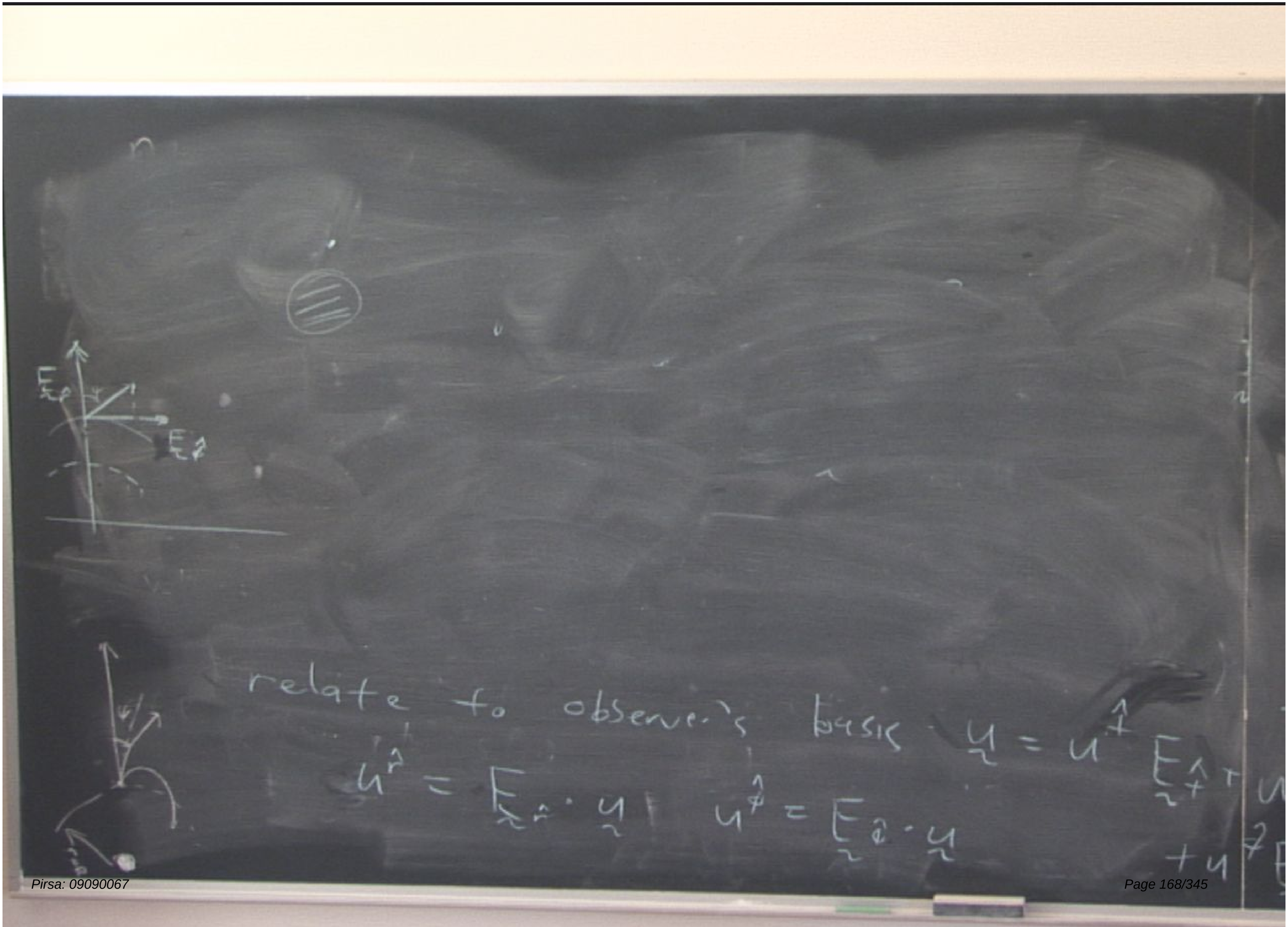


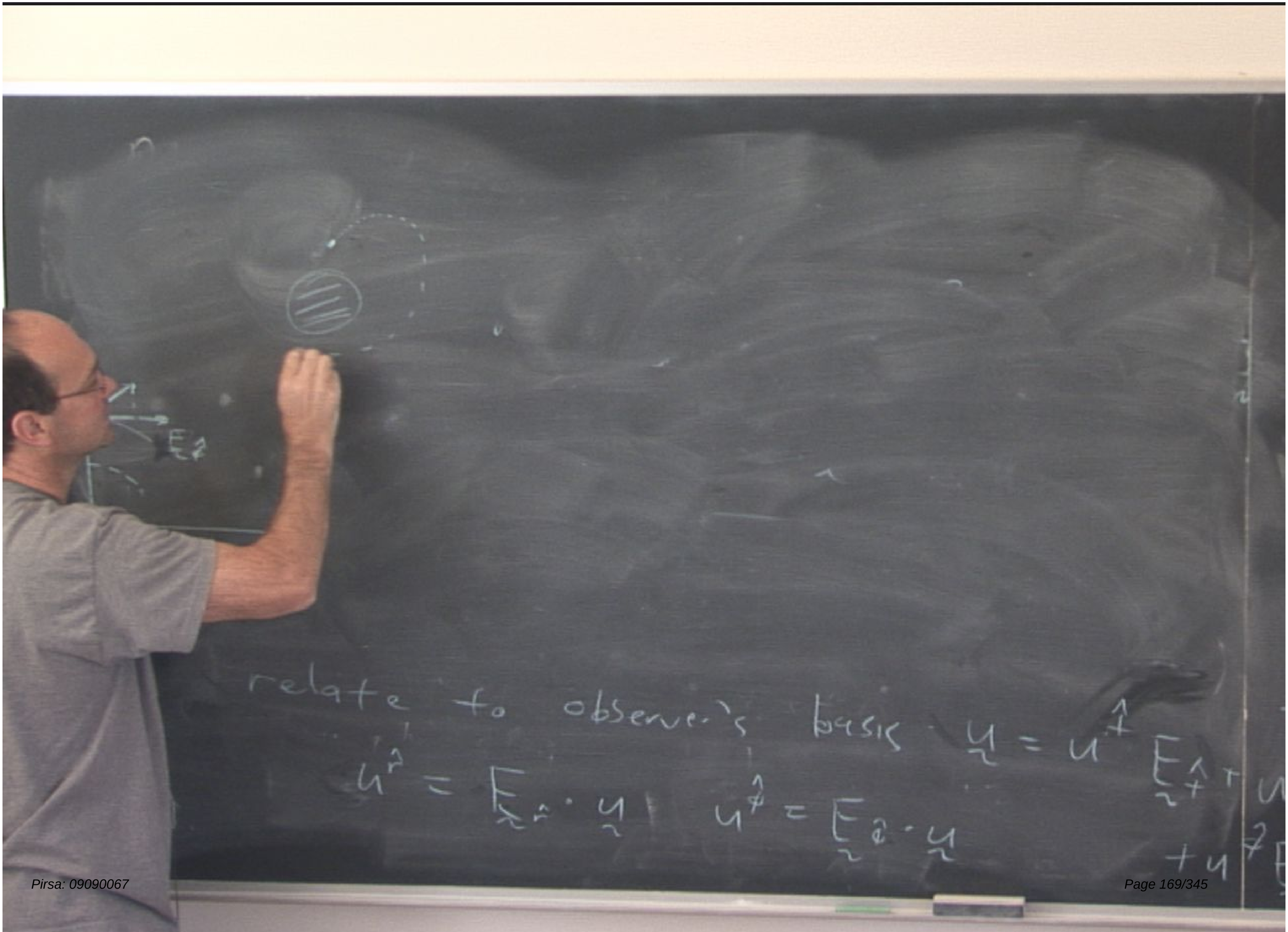
$$\tan\psi = \frac{u^{\hat\phi}}{u^{\hat r}} = \frac{\beta\, g_{\phi\phi}\frac{d\phi}{d\lambda}}{\alpha\, g_{rr}\frac{dr}{d\lambda}} = \frac{\frac{1}{R}(R^2)\left(\frac{\ell}{R^2}\right)}{\sqrt{1-\frac{2M}{R}}\;\frac{1}{1-\frac{2M}{R}}\;\ell\sqrt{\quad}}$$

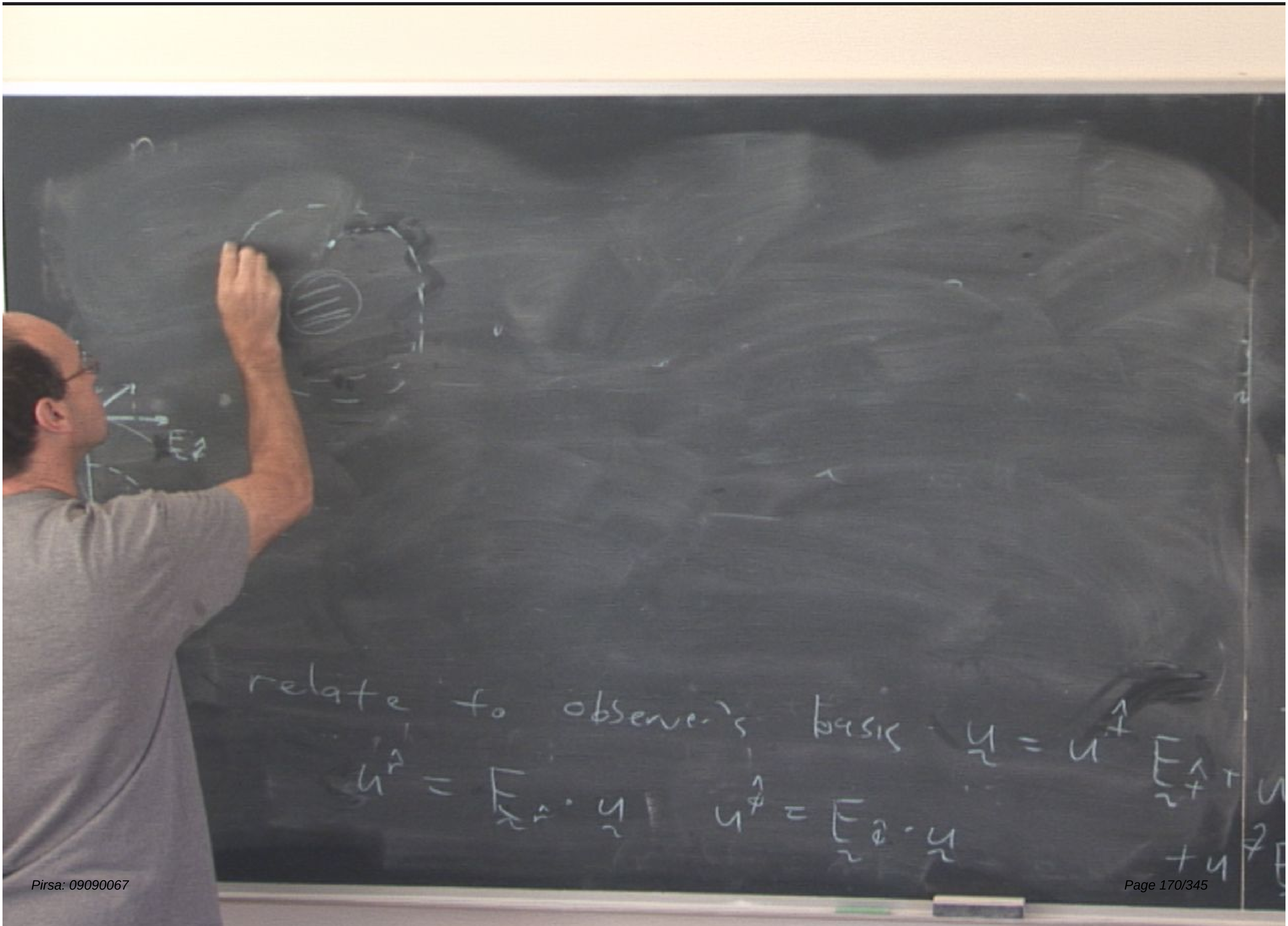
$$= \frac{\frac{1}{R}\sqrt{1-\frac{2M}{R}}}{\sqrt{\frac{1}{27M^2}-\frac{1}{R^2}\left(1-\frac{2M}{R}\right)}} \xrightarrow[R\to 2M]{} \frac{3\sqrt{3}}{2\sqrt{2}}\sqrt{\frac{R-2M}{M}} \longrightarrow 0$$

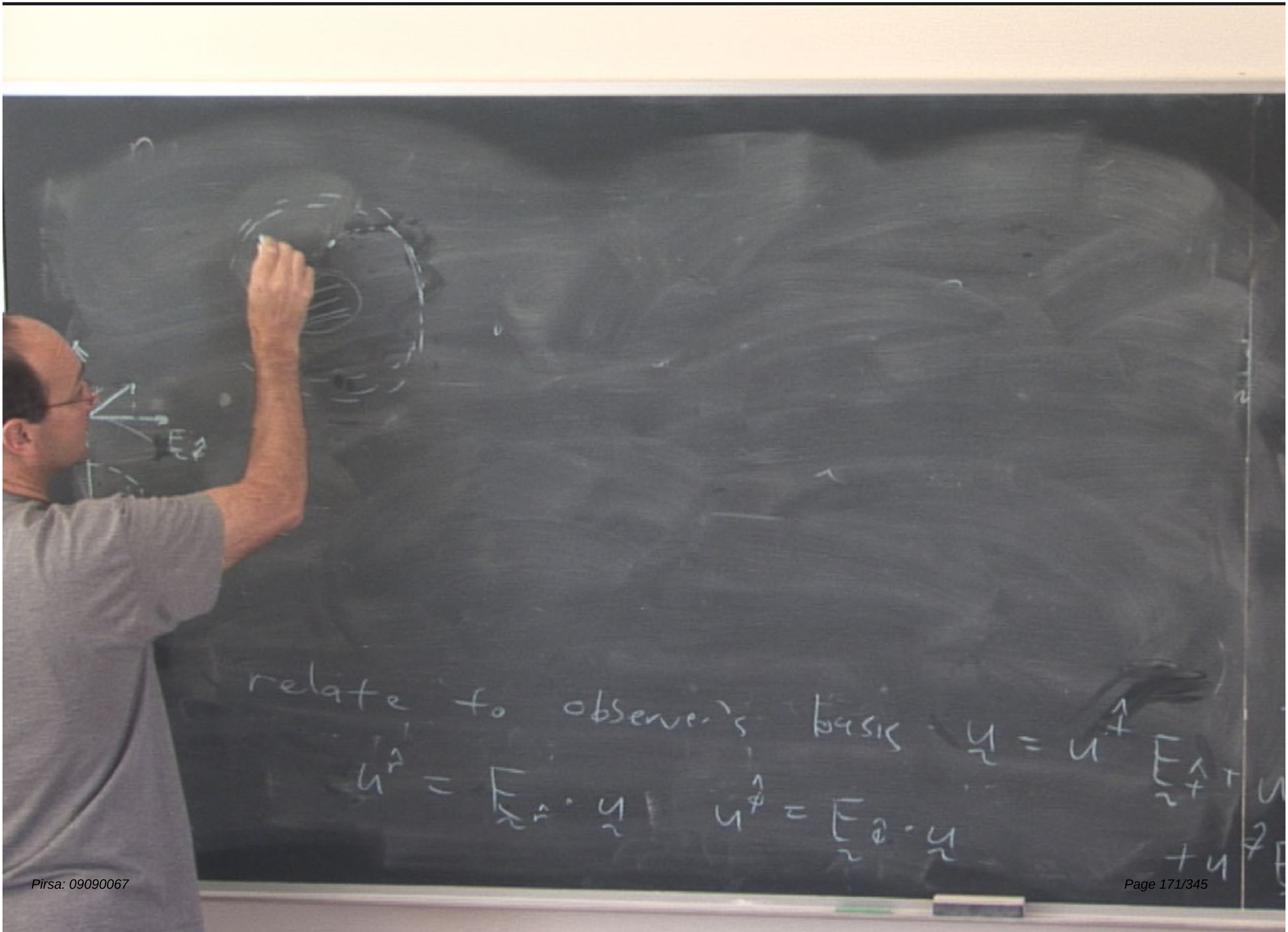

W
Wmax = 1/27M²
r=2M
r=3M
tan ψ =
→ 0

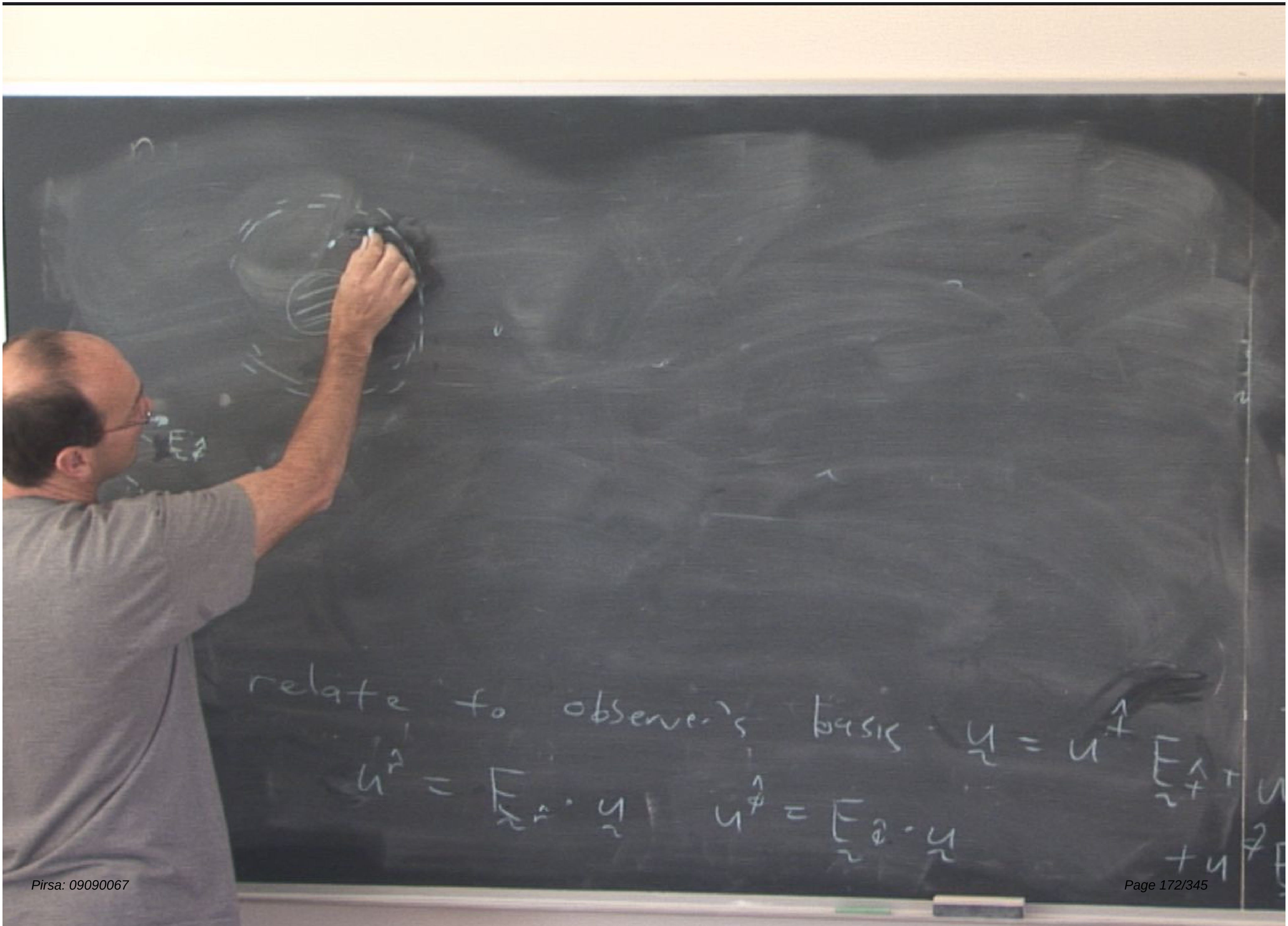

relate to observer's basis

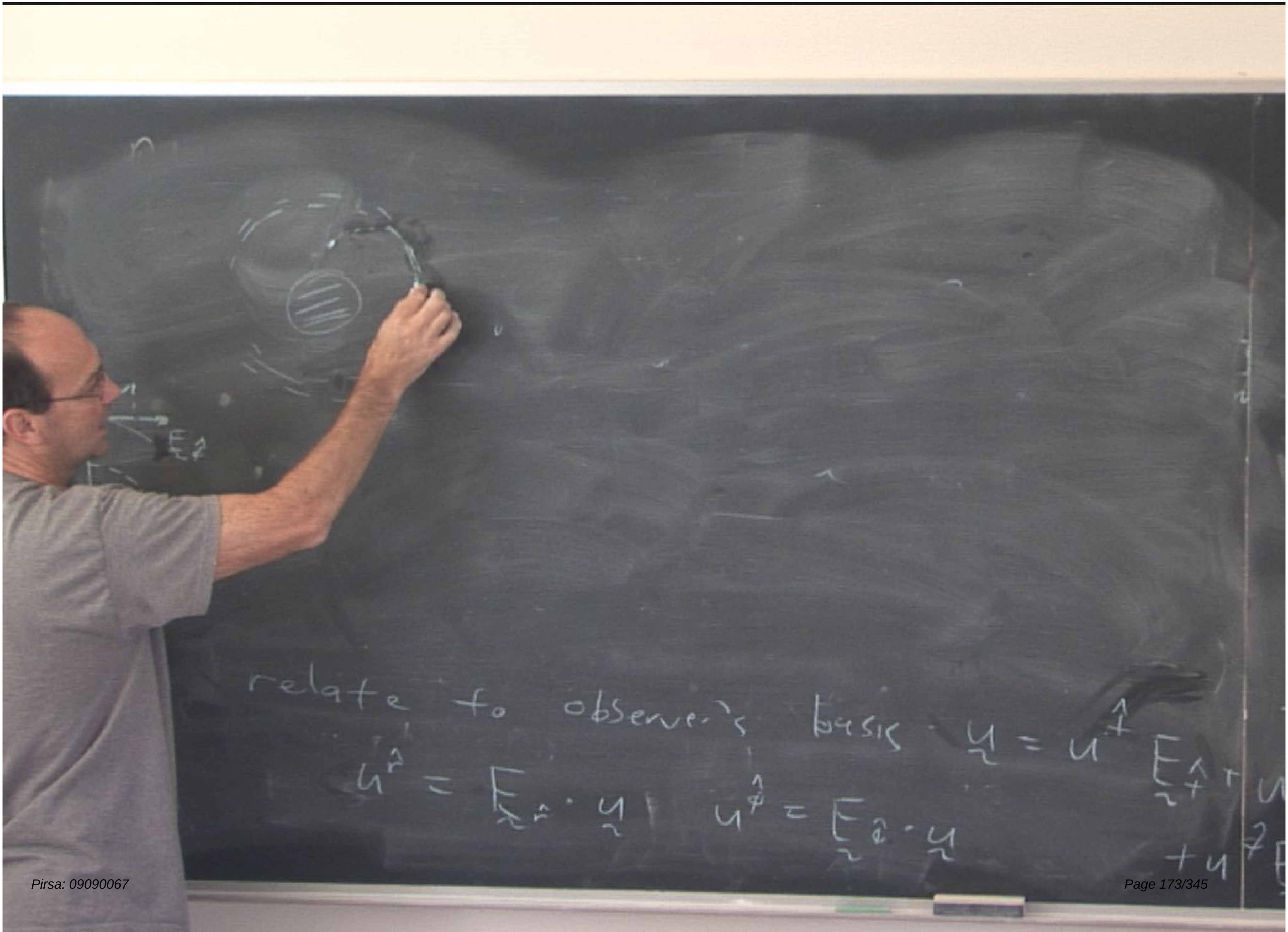

relate to observer's basis u = u^t̂ E_t̂ + u
u^r̂ = E_r̂ · u , u^φ̂ = E_φ̂ · u
+ u^φ̂ E

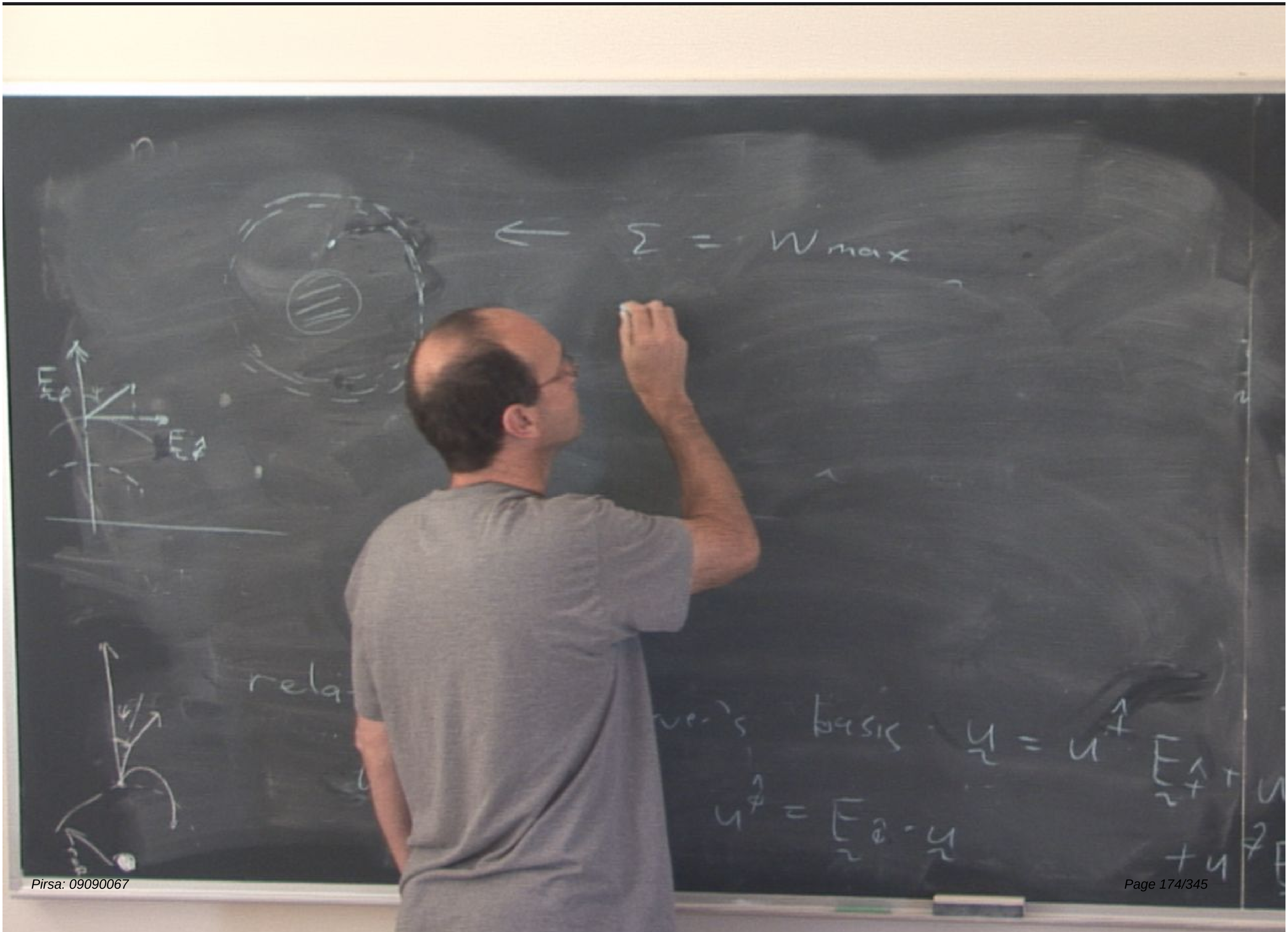

relate to observer's basis

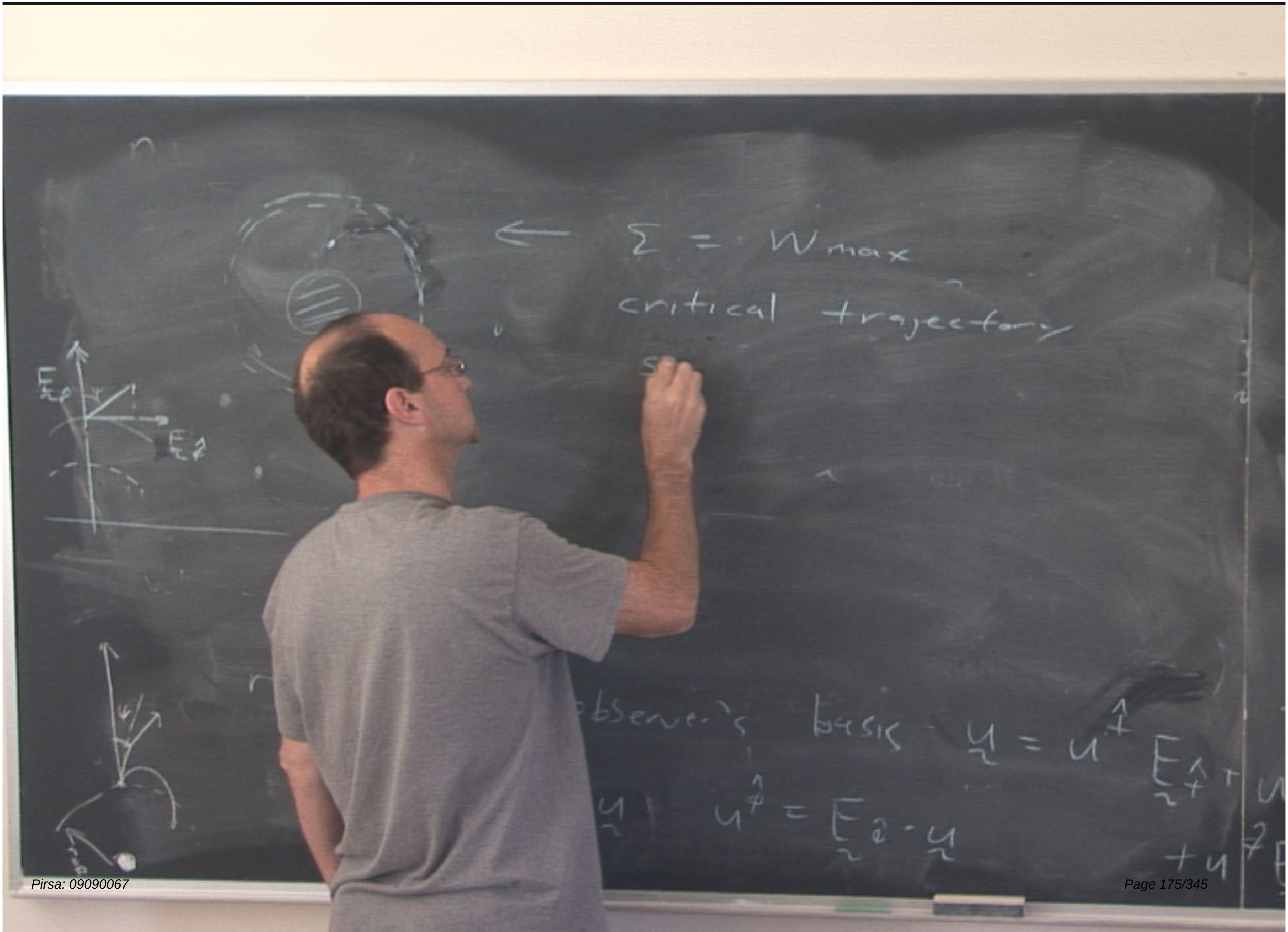

relate to observer's basis


relate to observer's basis
u = u^t̂ E_t̂ + u
u^r̂ = E_r̂ · u
u^θ̂ = E_θ̂ · u
+ u^φ̂ E


relate to observer's basis


relate to observer's basis


Σ = W max
basis


Σ = W max
critical trajectory
observer's basis

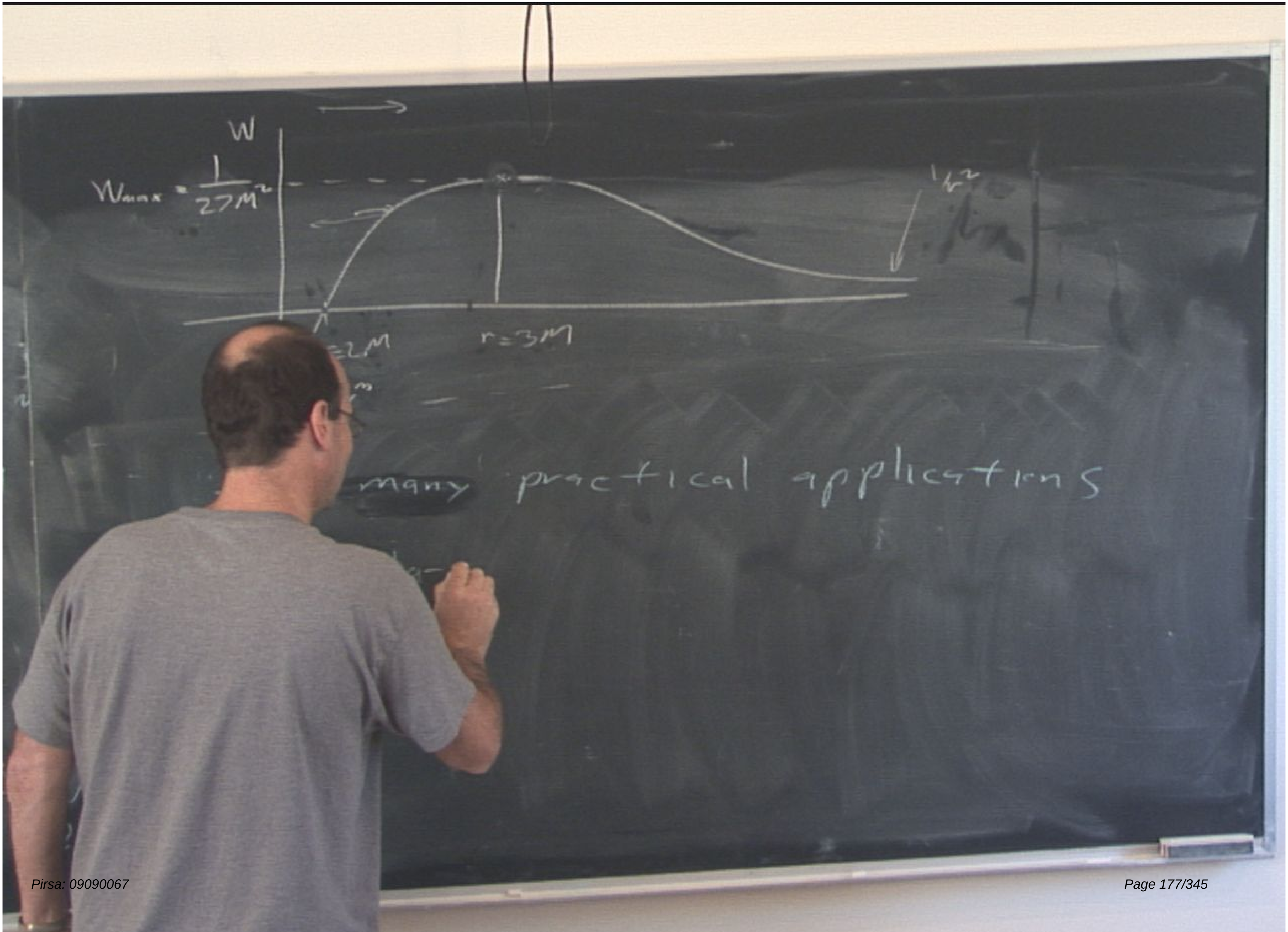
$\Leftarrow \quad \Sigma = W_{max}$

critical trajectory spirals out never quite reaching $r = 3M$

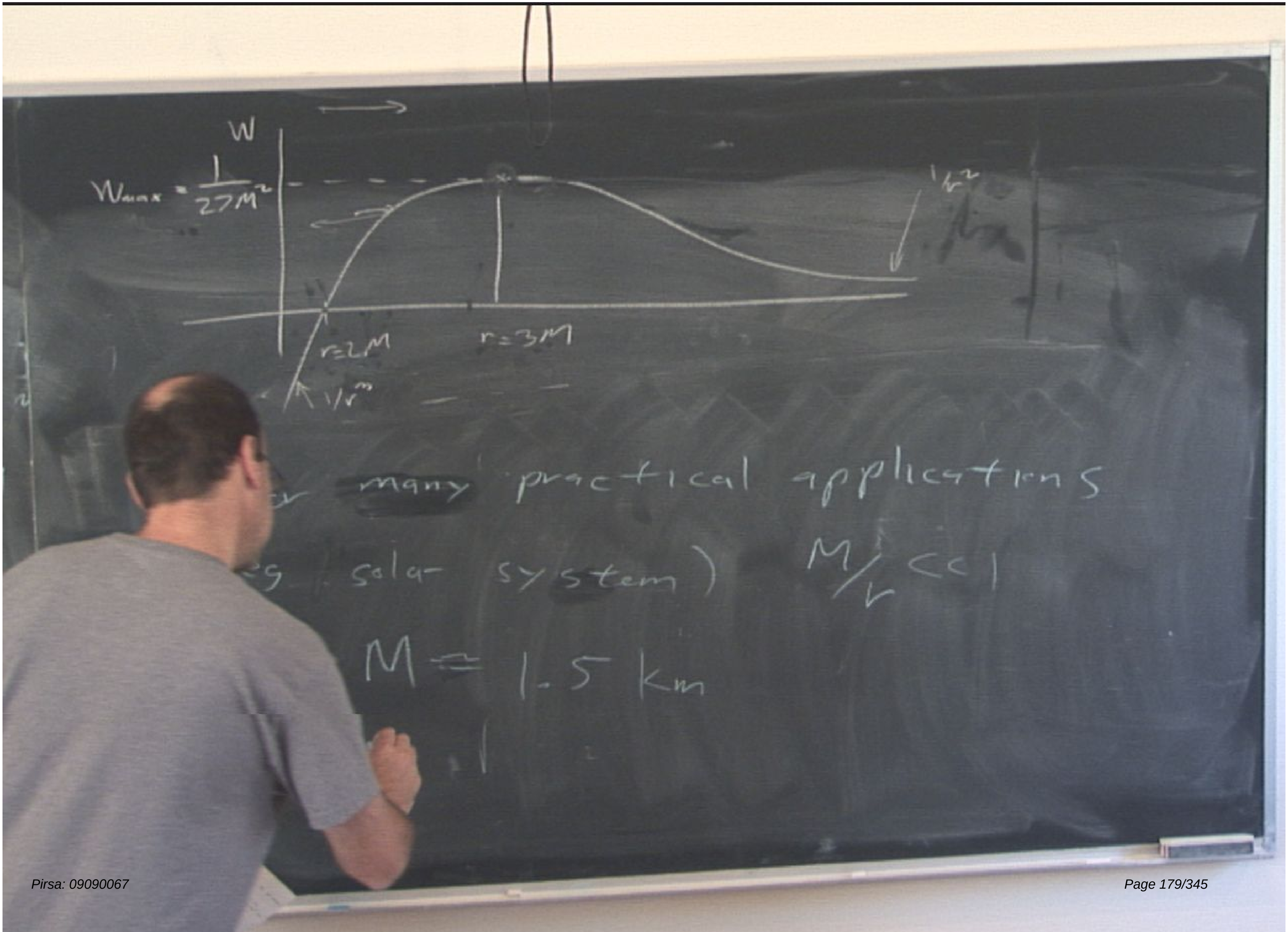
relate to observer's basis $\underset{\sim}{u} = u^{\hat t} \underset{\sim}{E}_{\hat t} + u^{\hat r} \ldots$

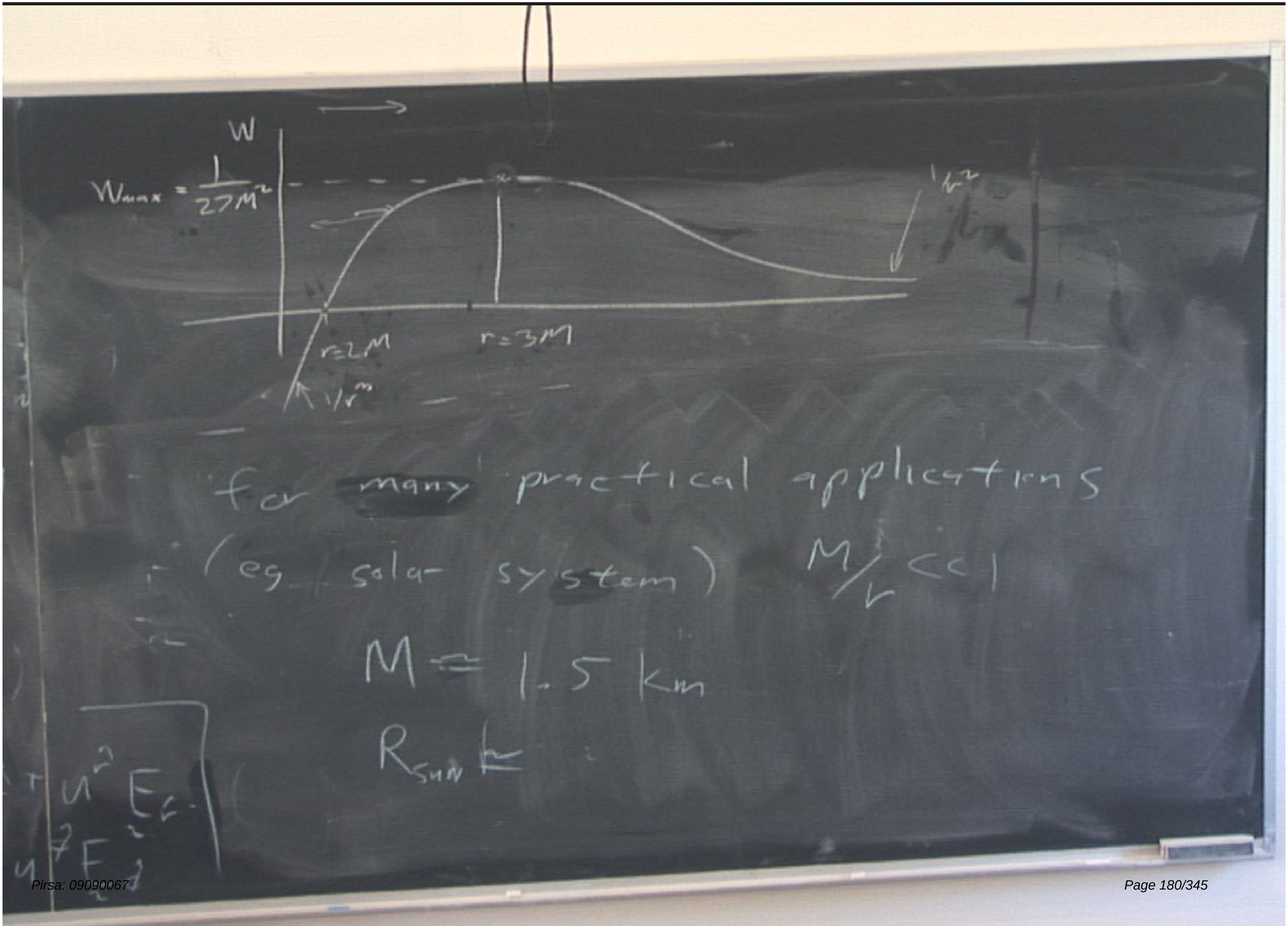
$u^{\hat r} = \underset{\sim}{E}_{\hat r} \cdot \underset{\sim}{u} \qquad u^{\hat\phi} = \underset{\sim}{E}_{\hat\phi} \cdot \underset{\sim}{u}$

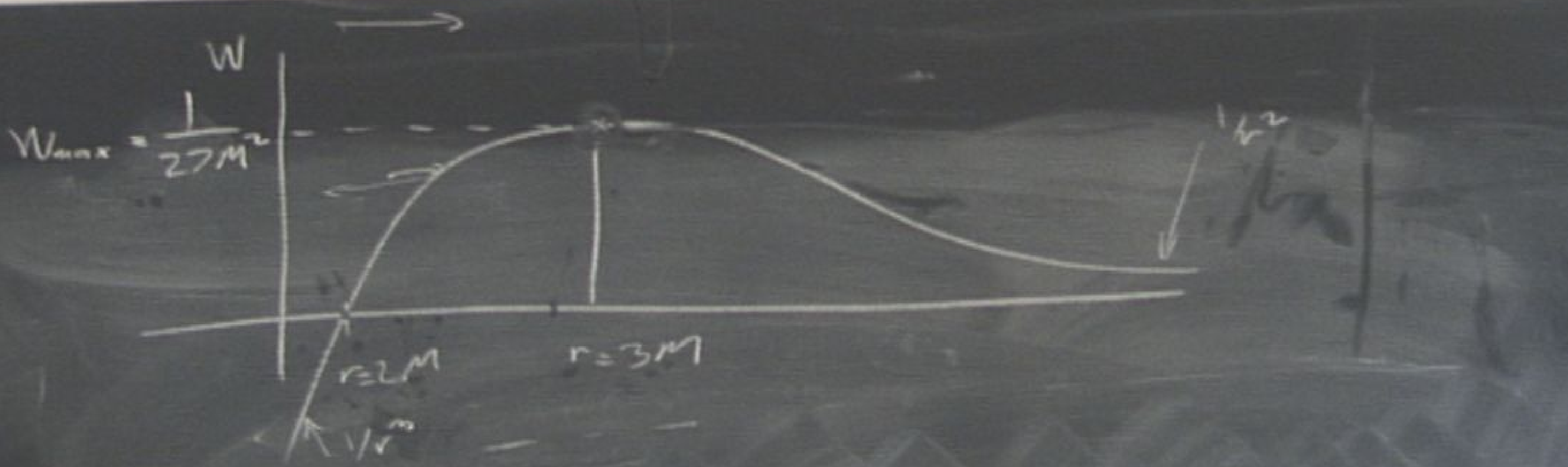
$+ u^{\hat\phi} E \ldots$


W
Wmax = 1/27M²
r=3M
many practical applications


W
Wmax = 1/27M²
r=3M
many practical applications
solar system)
M/r


W
Wmax = 1/27M²
r=2M
r=3M
1/r²
many practical applications
eg solar system)
M/r << 1
M = 1.5 km


W
Wmax = 1/27M²
r=2M
r=3M
1/r²
for many practical applications
(eg solar system) M/r << 1
M ≃ 1.5 km
Rsun ≃

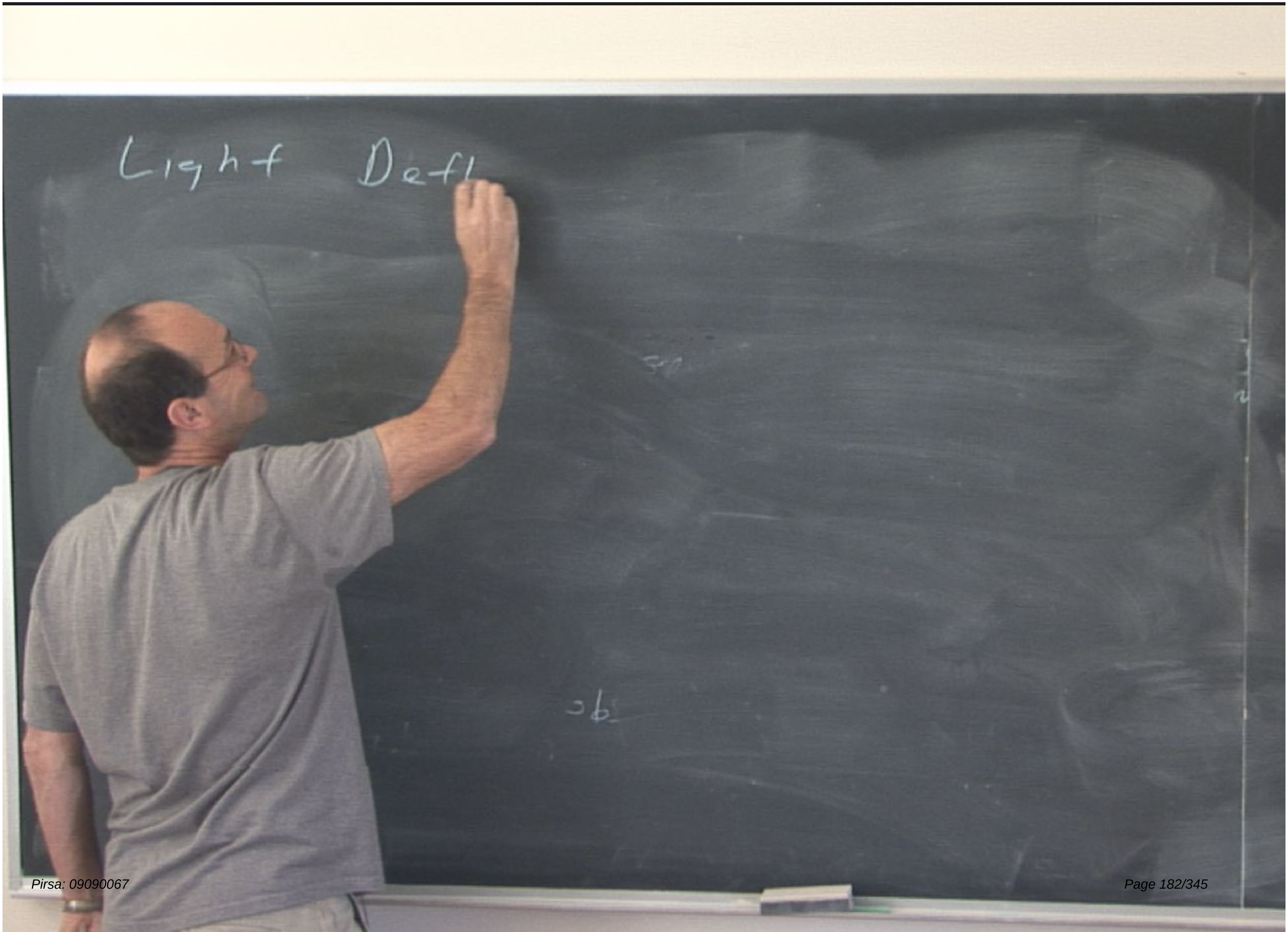


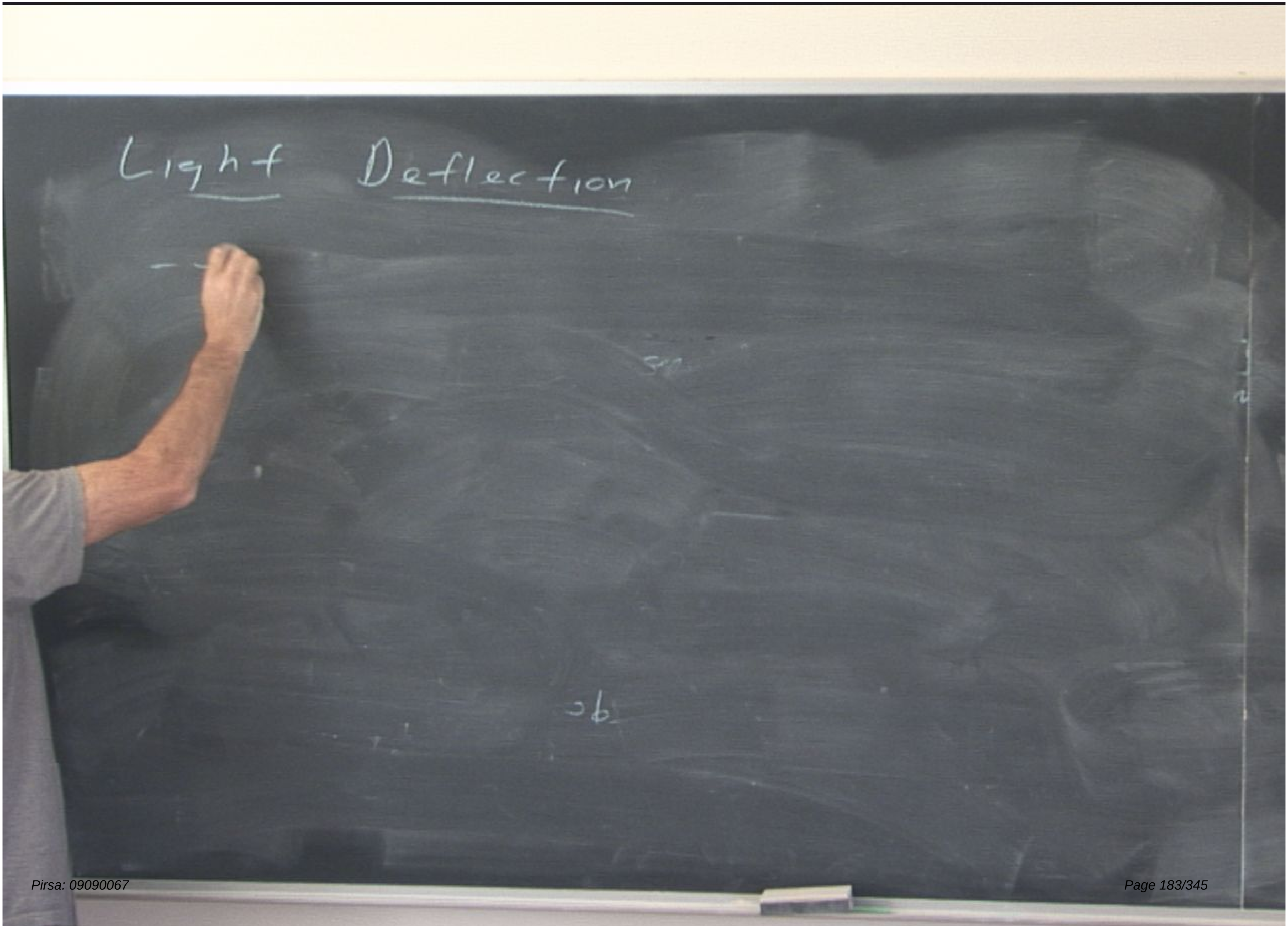
for many practical applications

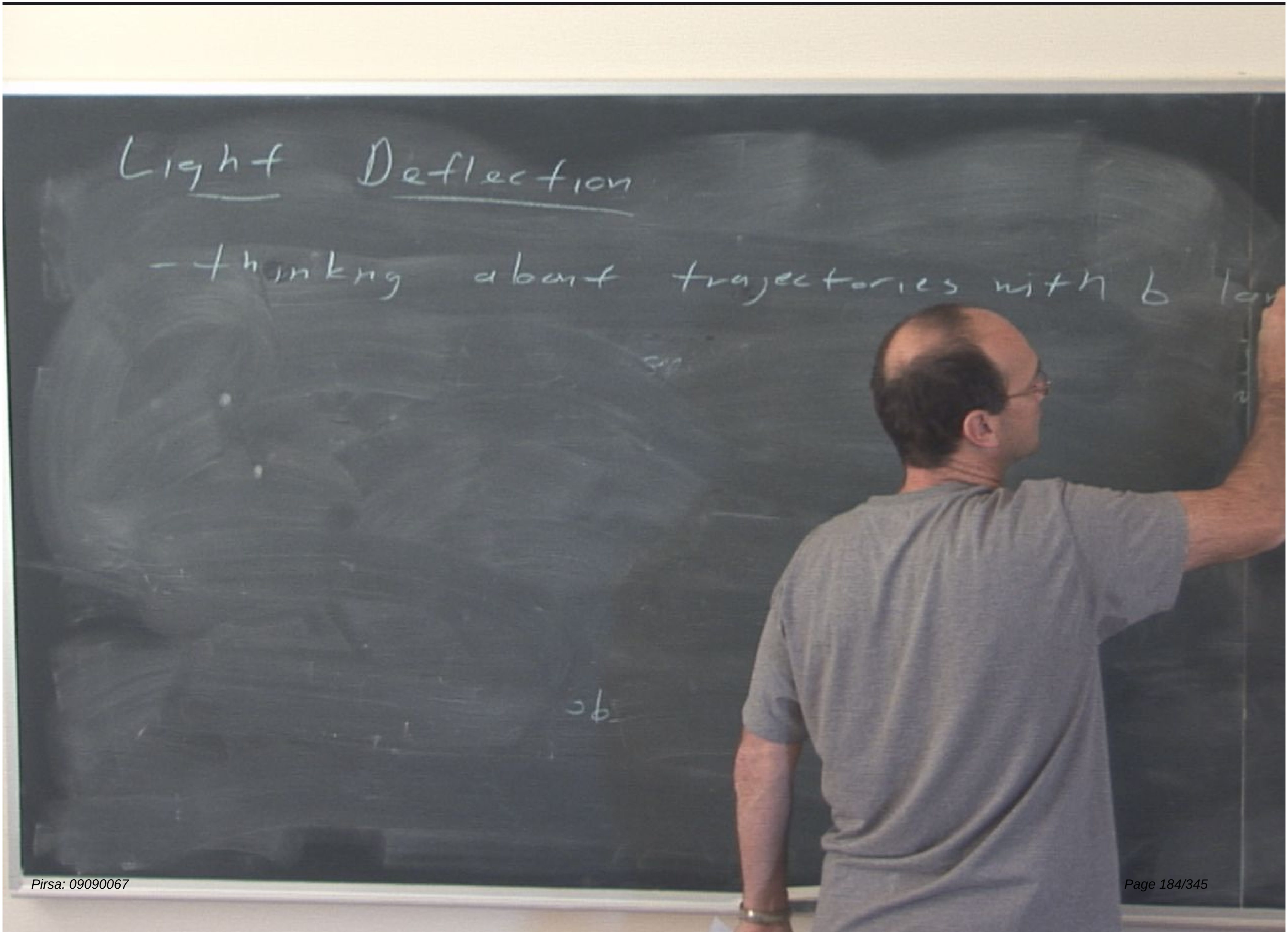
(eg solar system) $M/r \ll 1$

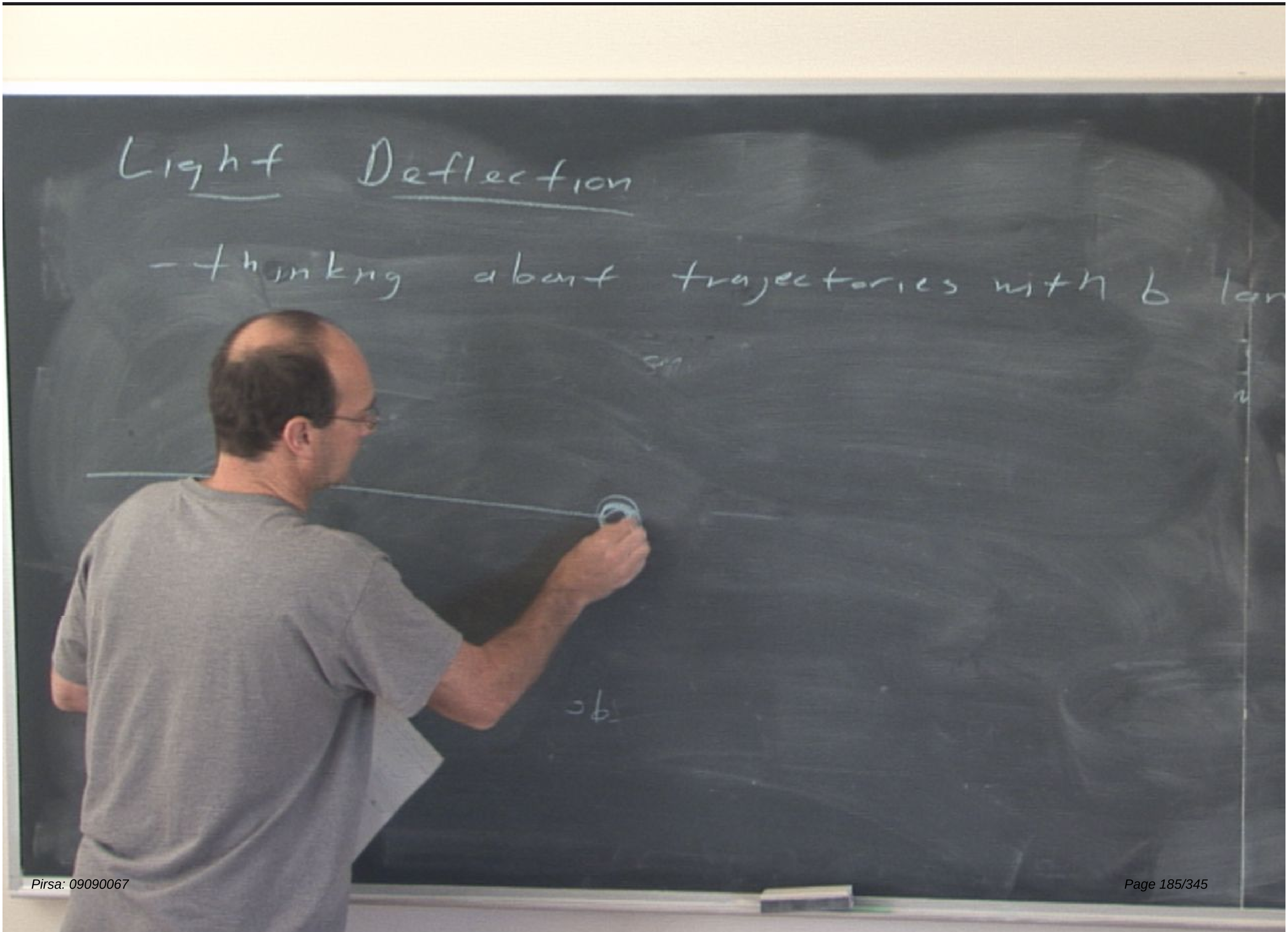
$M \simeq 1.5$ km

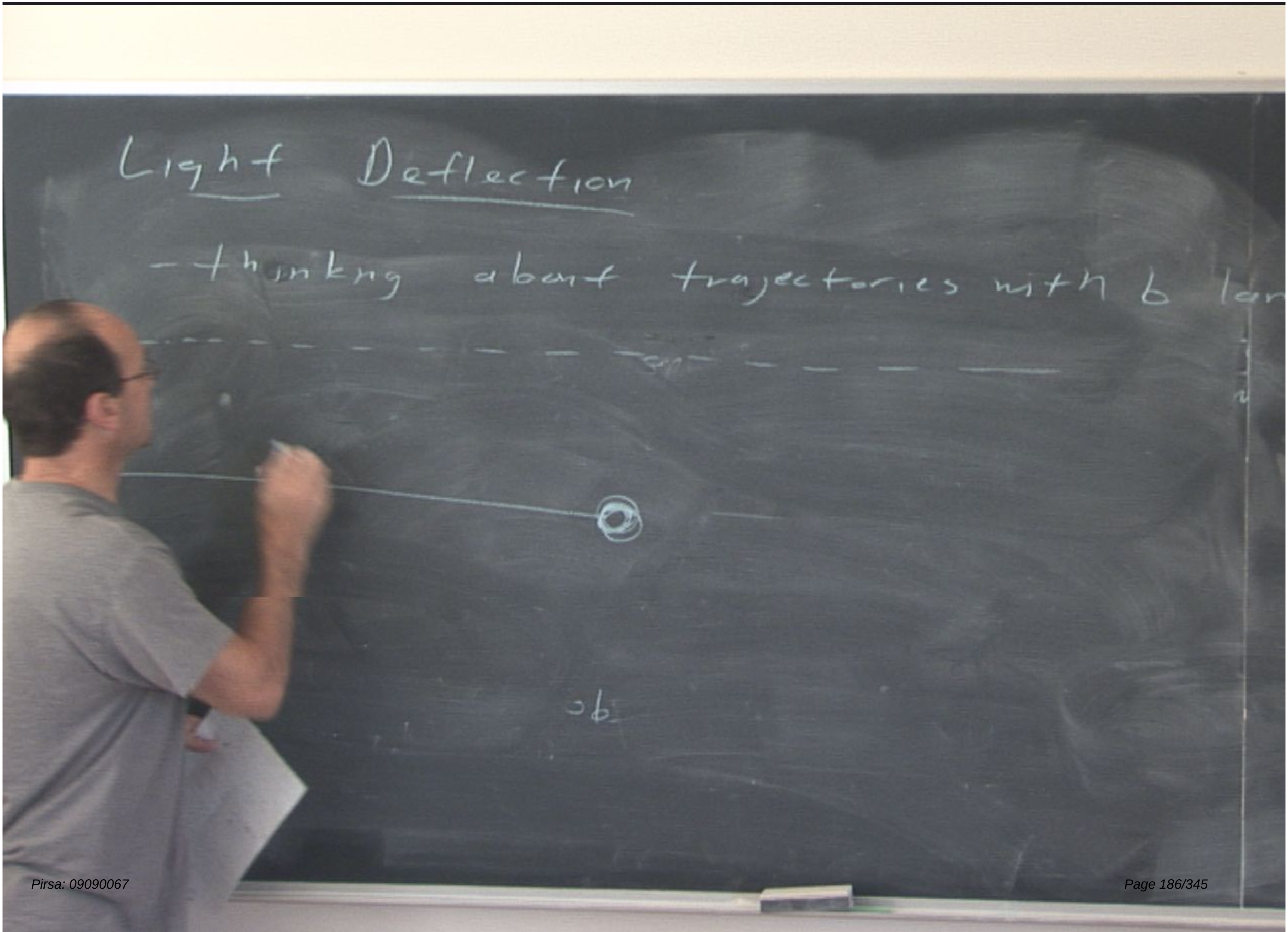
$R_{sun} \simeq 7 \times 10^5$ km

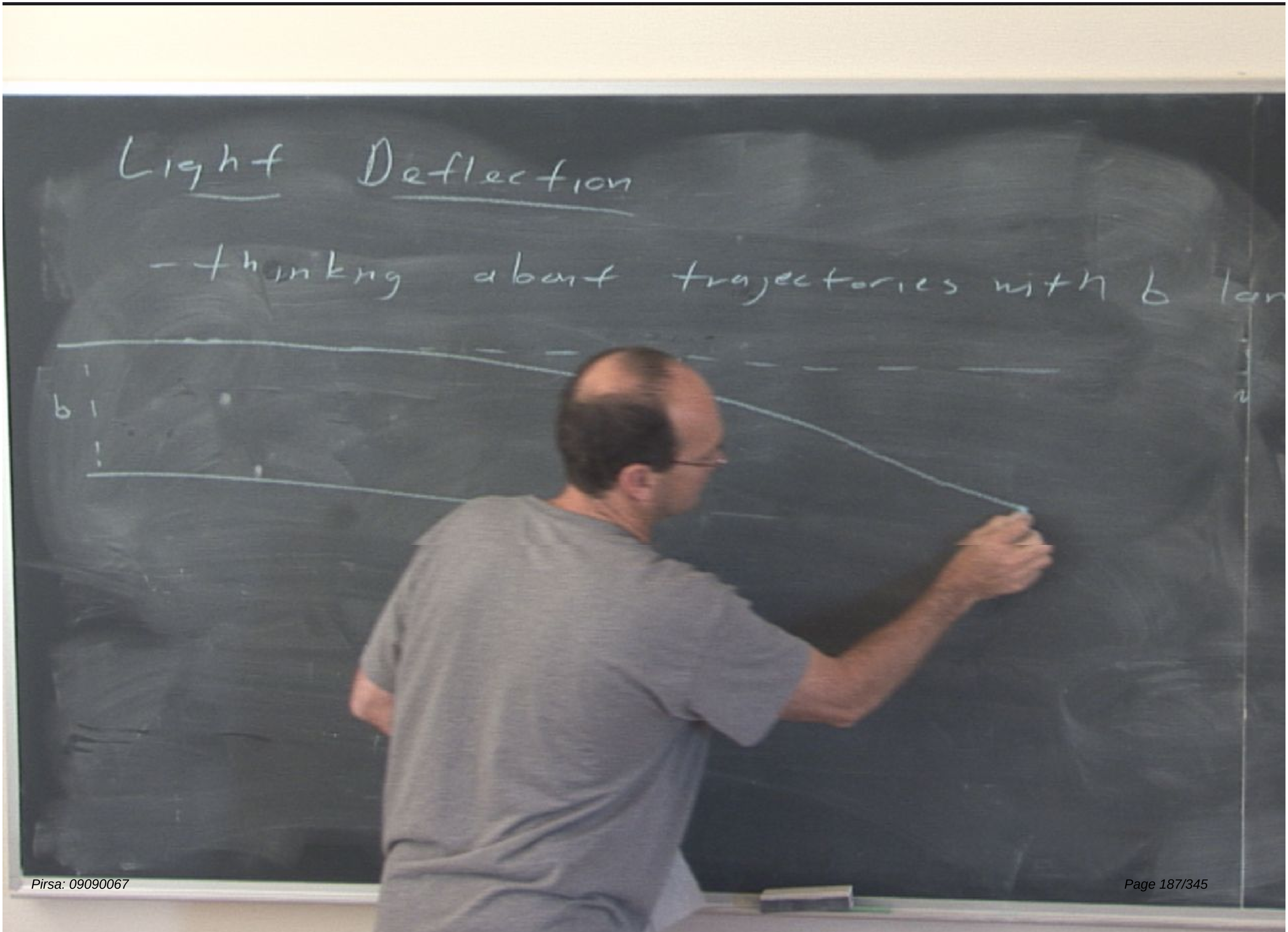

Light Defl

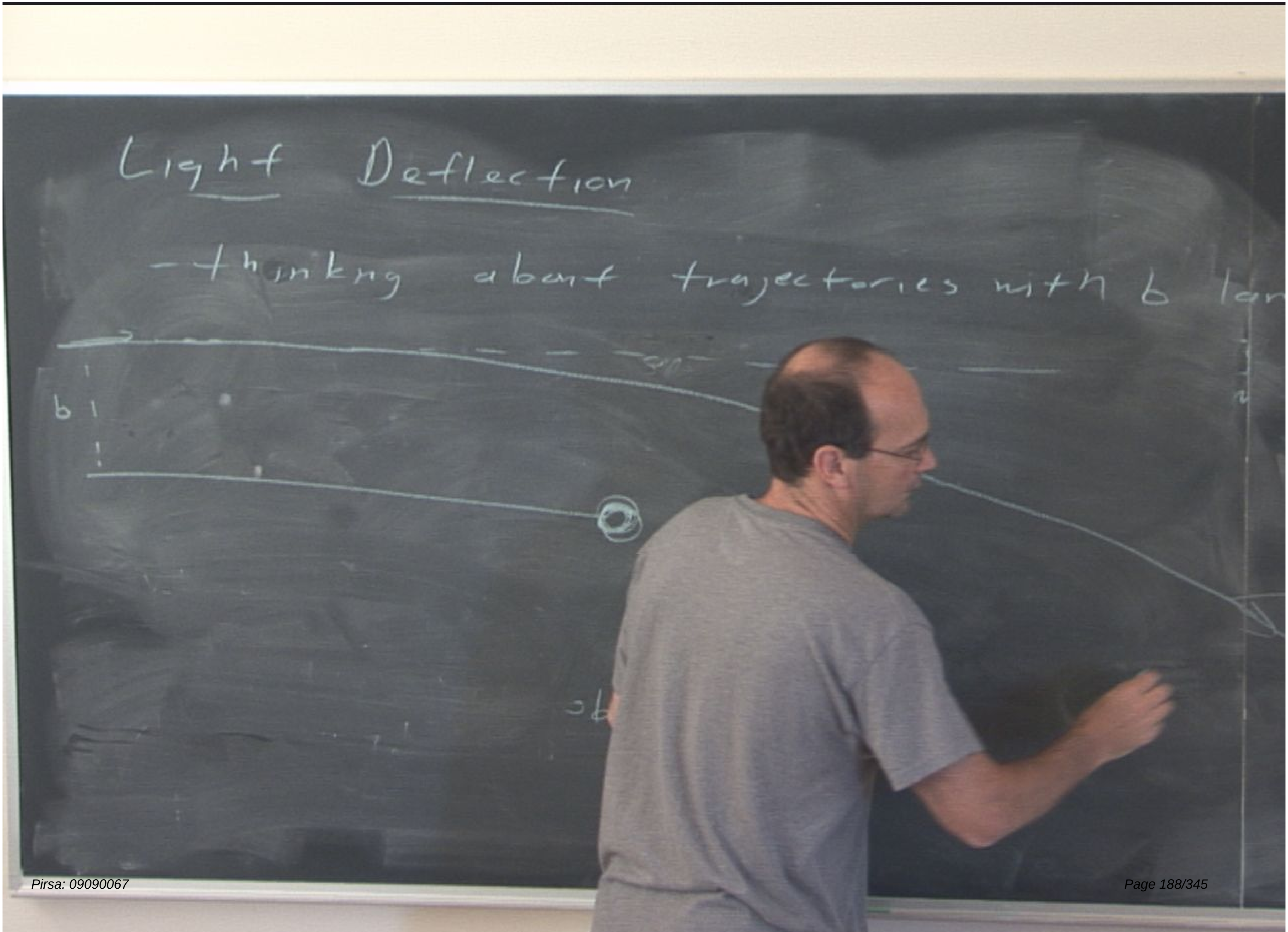

Light Deflection

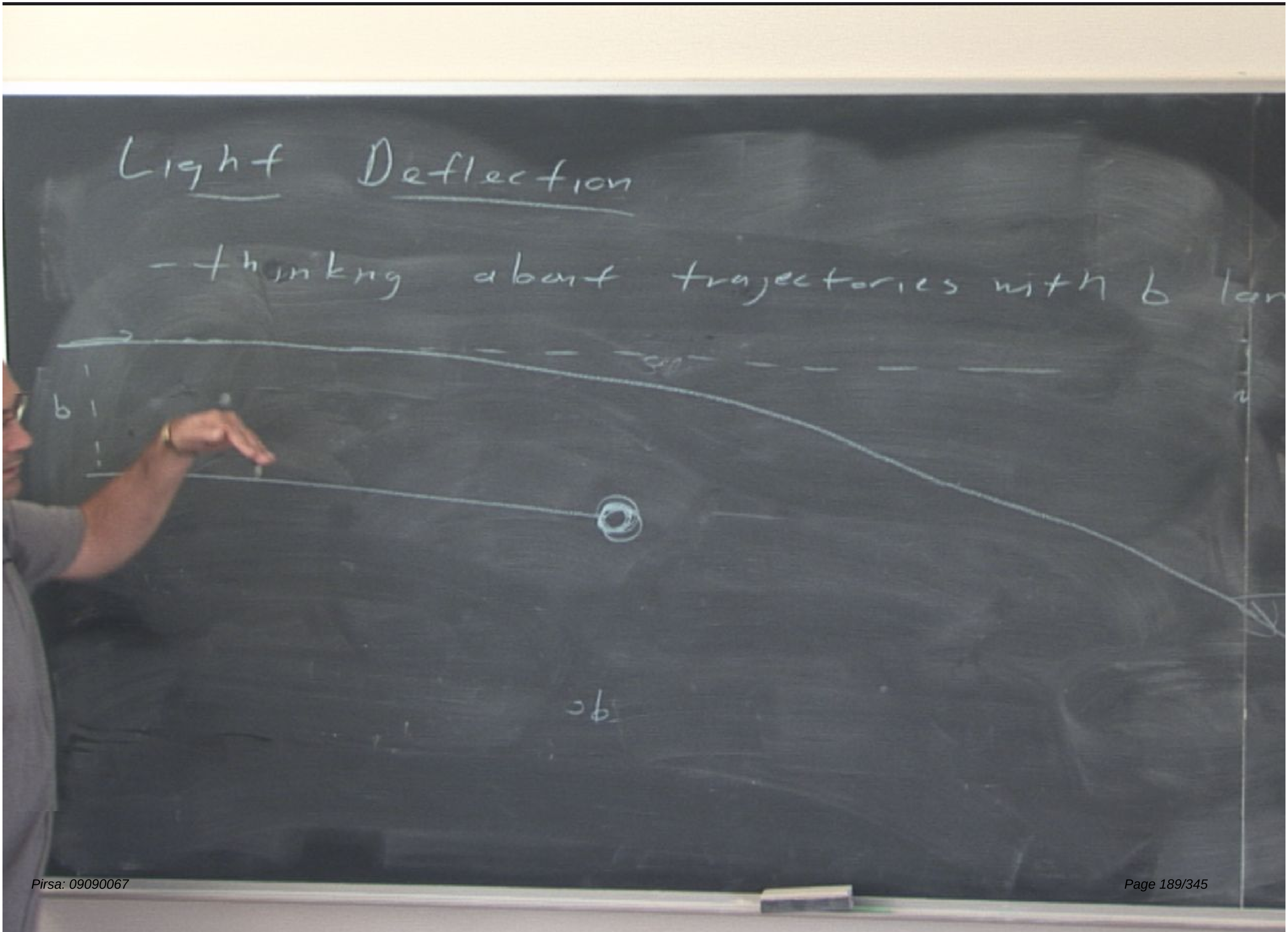

Light Deflection
- thinking about trajectories with b


Light Deflection
- thinking about trajectories with b

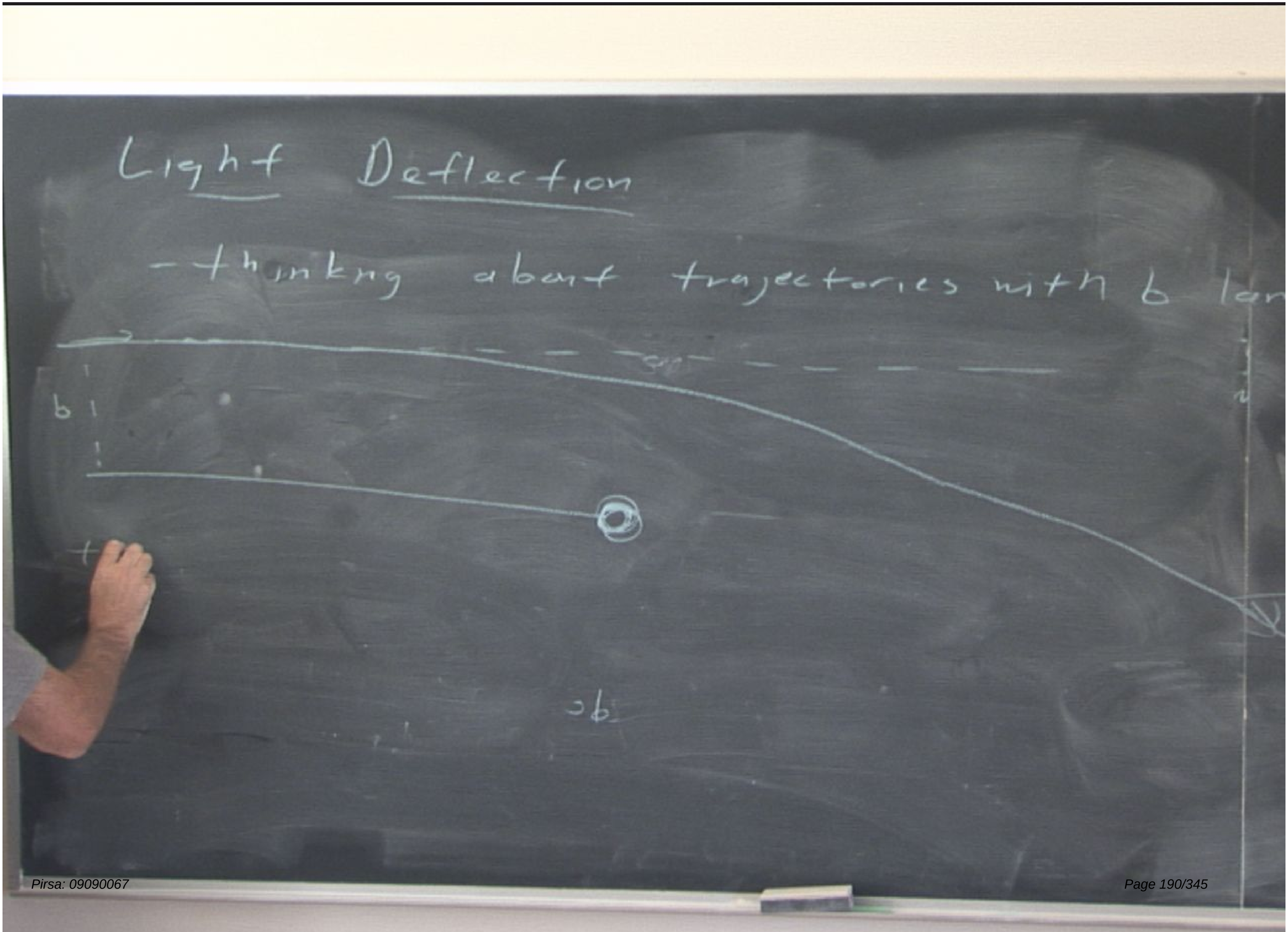

Light Deflection
- thinking about trajectories with b

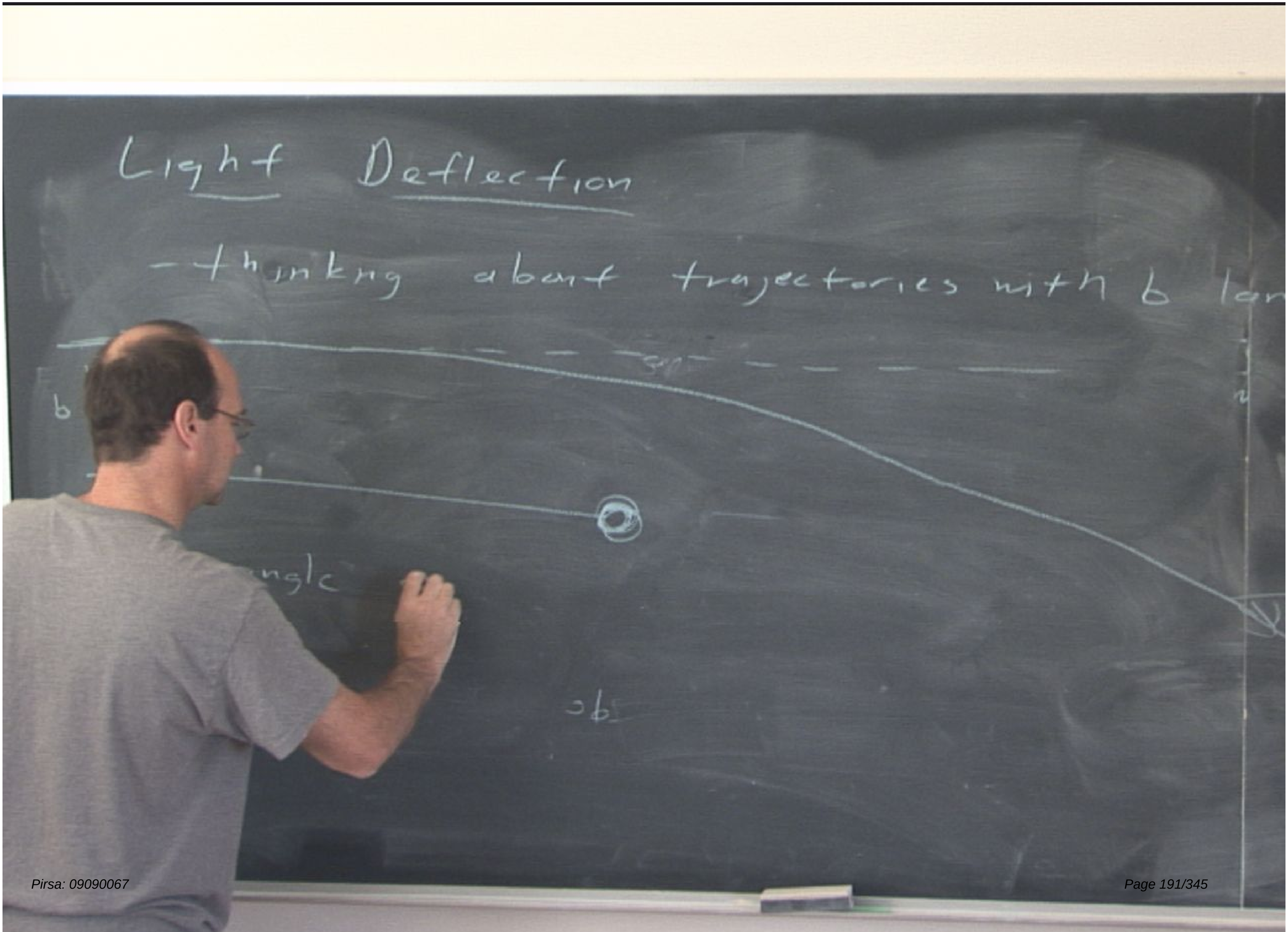

Light Deflection
- thinking about trajectories with b
b

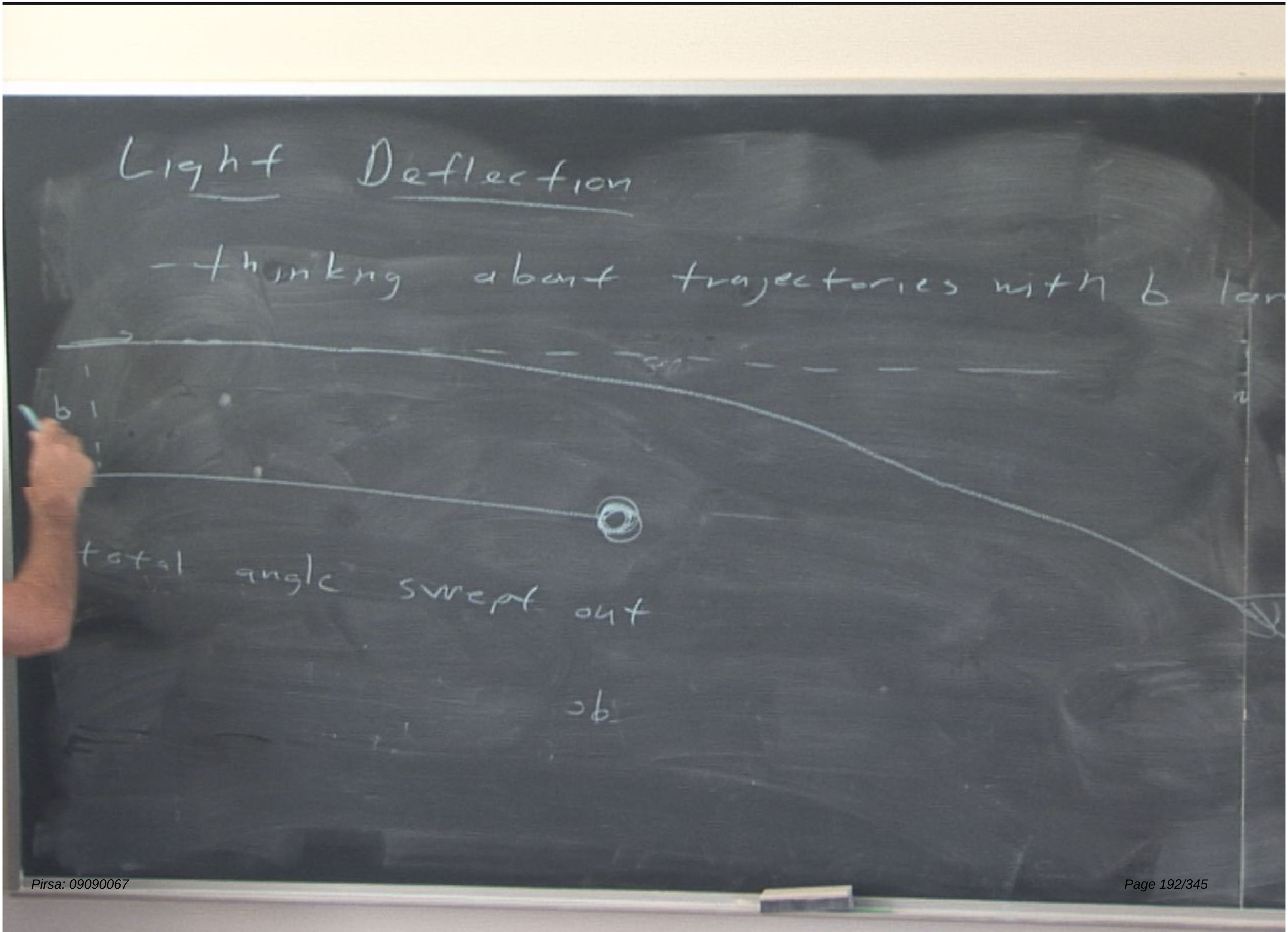

Light Deflection
- thinking about trajectories with b
b

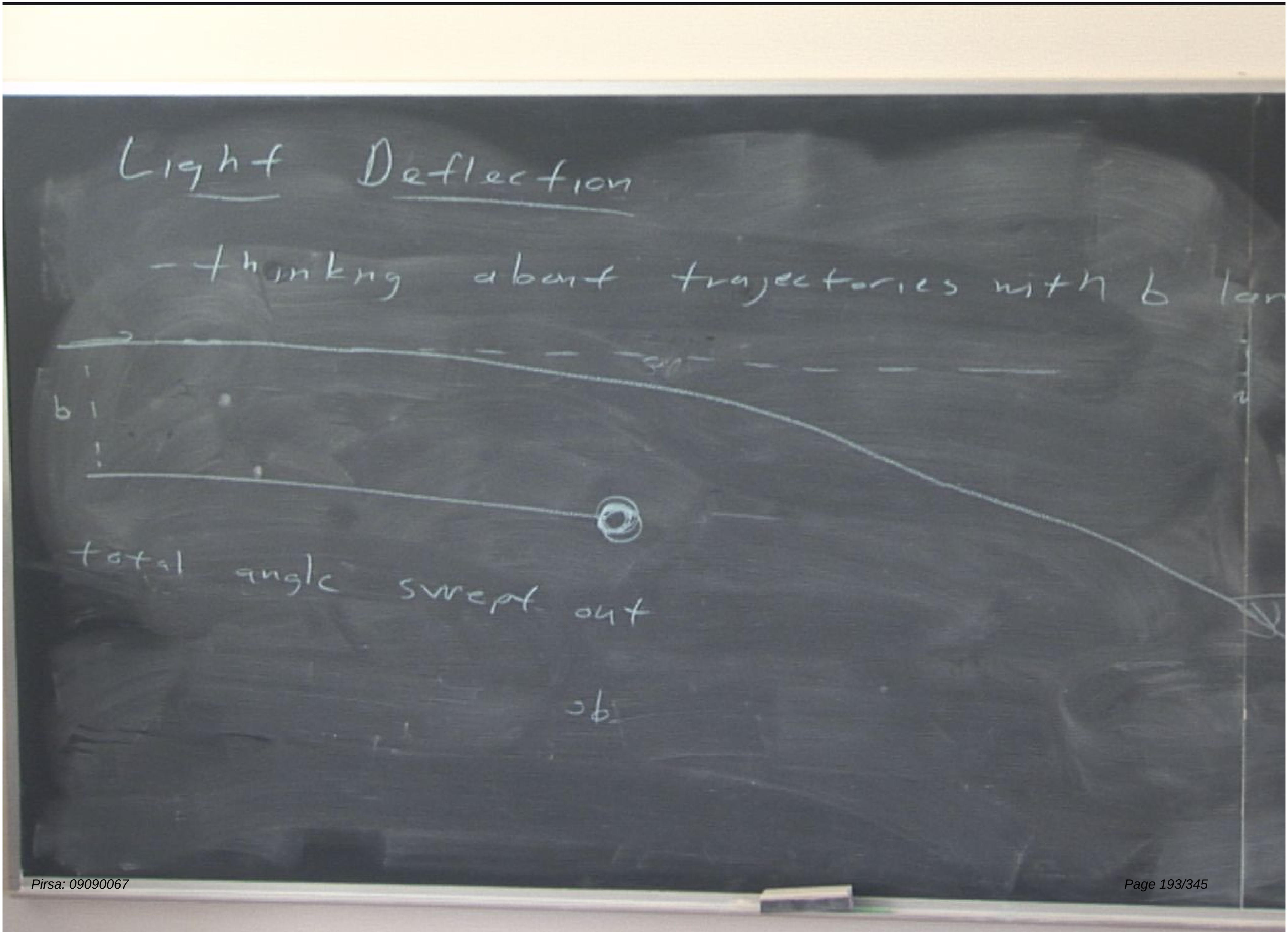


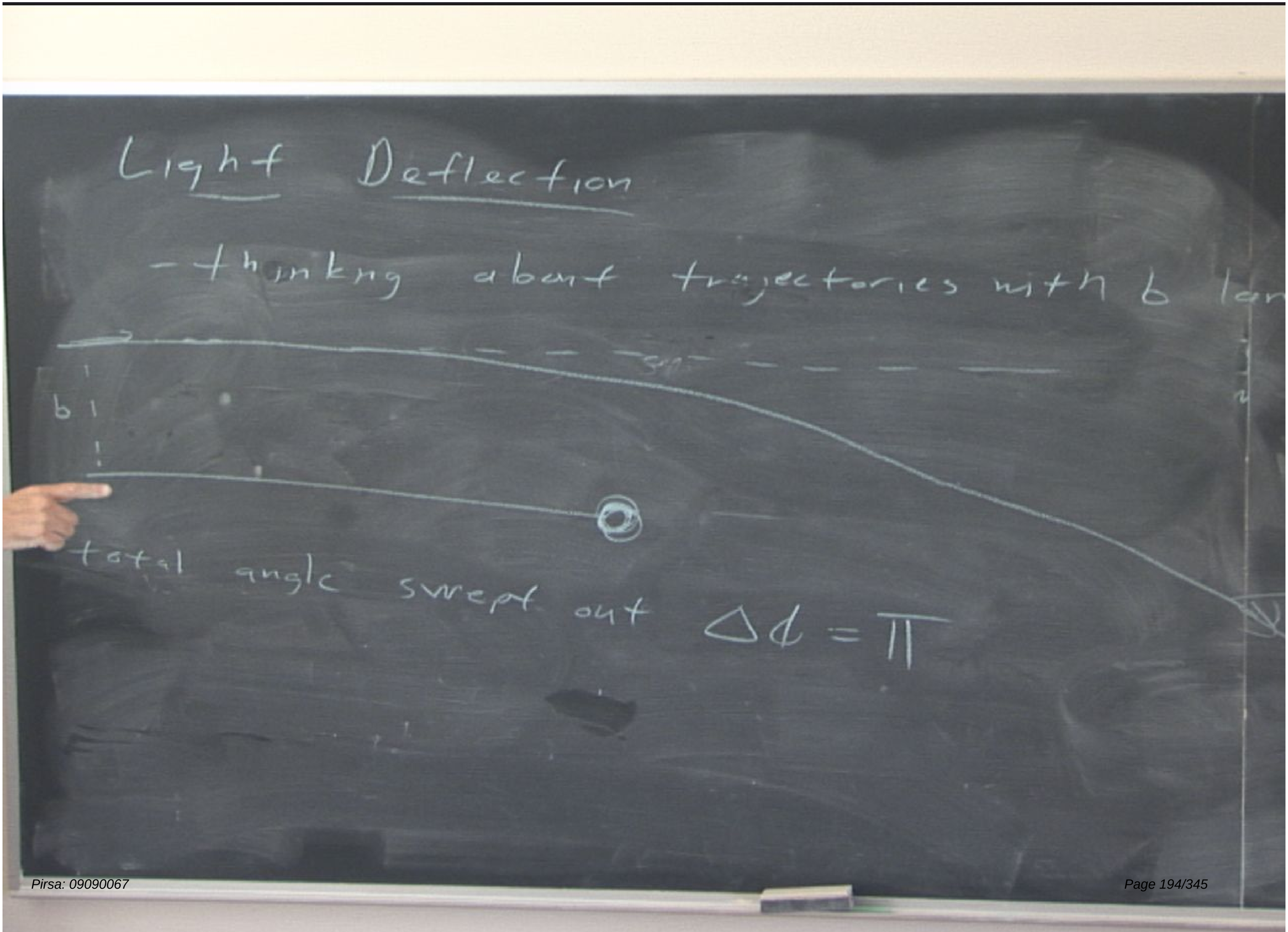
Light Deflection
- thinking about trajectories with b
b


Light Deflection
- thinking about trajectories with b lar
b


Light Deflection
- thinking about trajectories with b
b


Light Deflection
- thinking about trajectories with b lar
b
total angle swept out

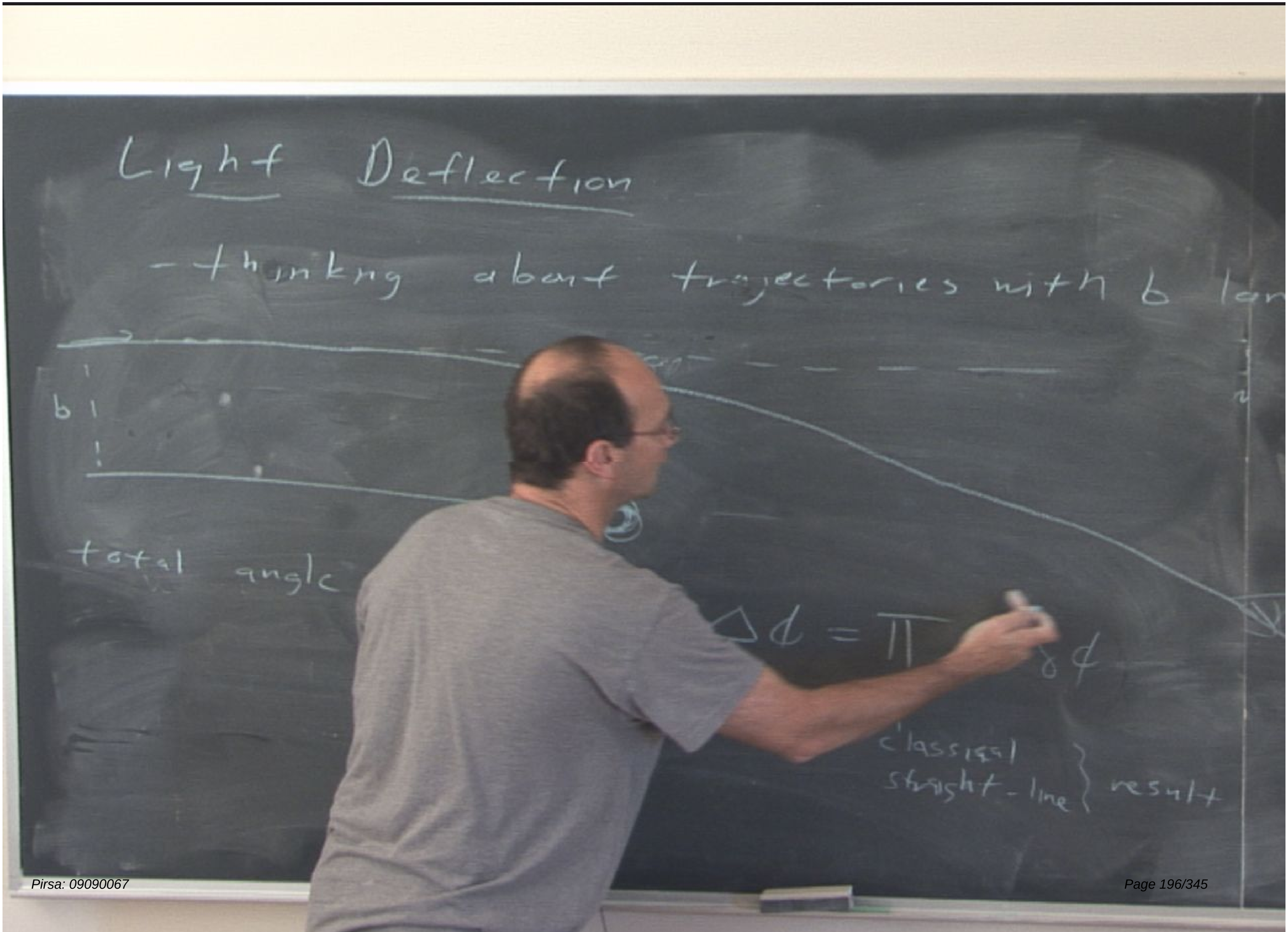

Light Deflection
- thinking about trajectories with b lar
b
total angle swept out


Light Deflection
- thinking about trajectories with b lar
b
total angle swept out Δφ = π

Light Deflection

- thinking about trajectories with b lar

total angle swept out $\Delta\phi = \pi + \delta\phi$

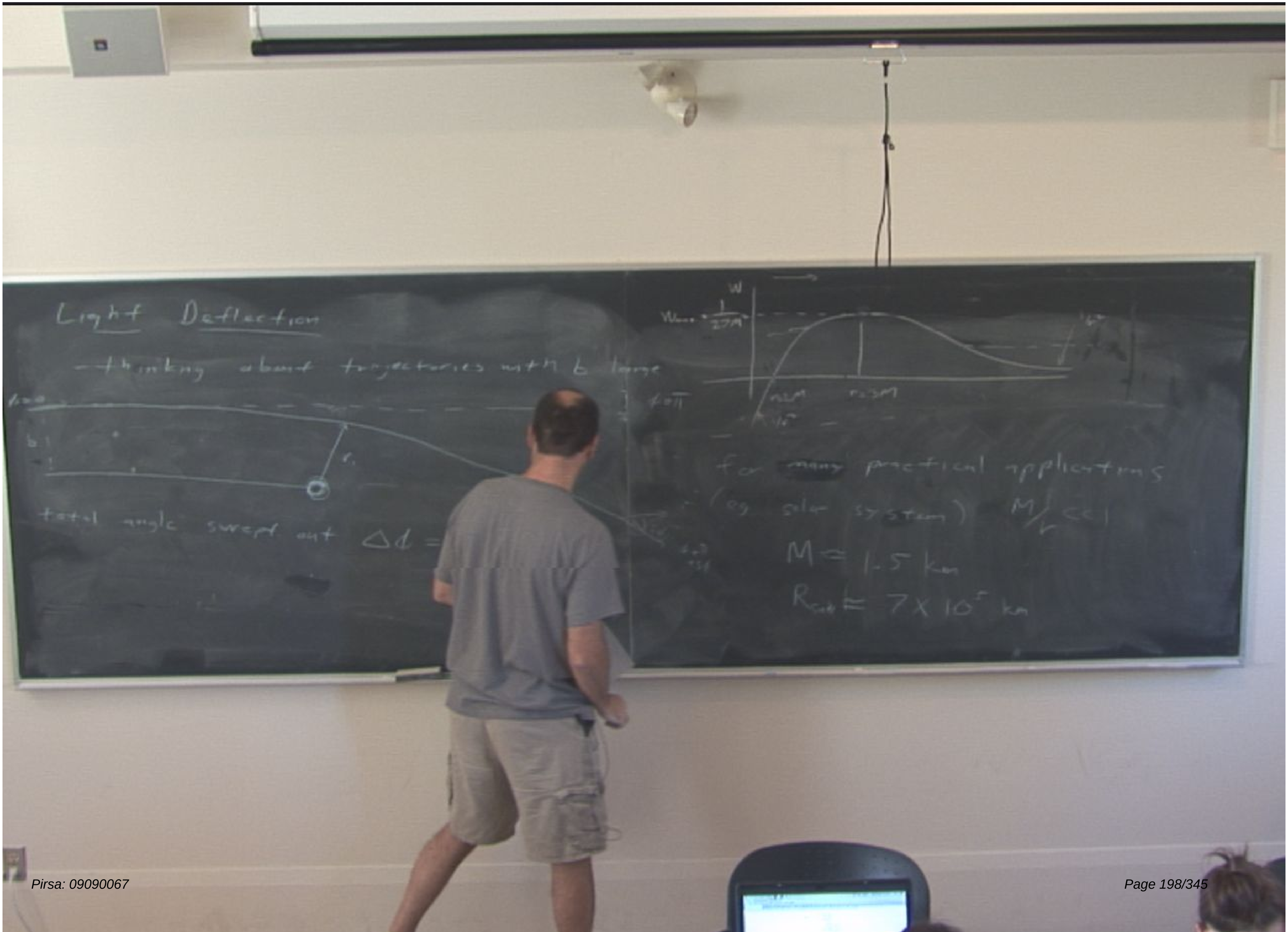

Light Deflection
- thinking about trajectories with b
b
total angle
Δϕ = π
δϕ
classical
straight-line
result

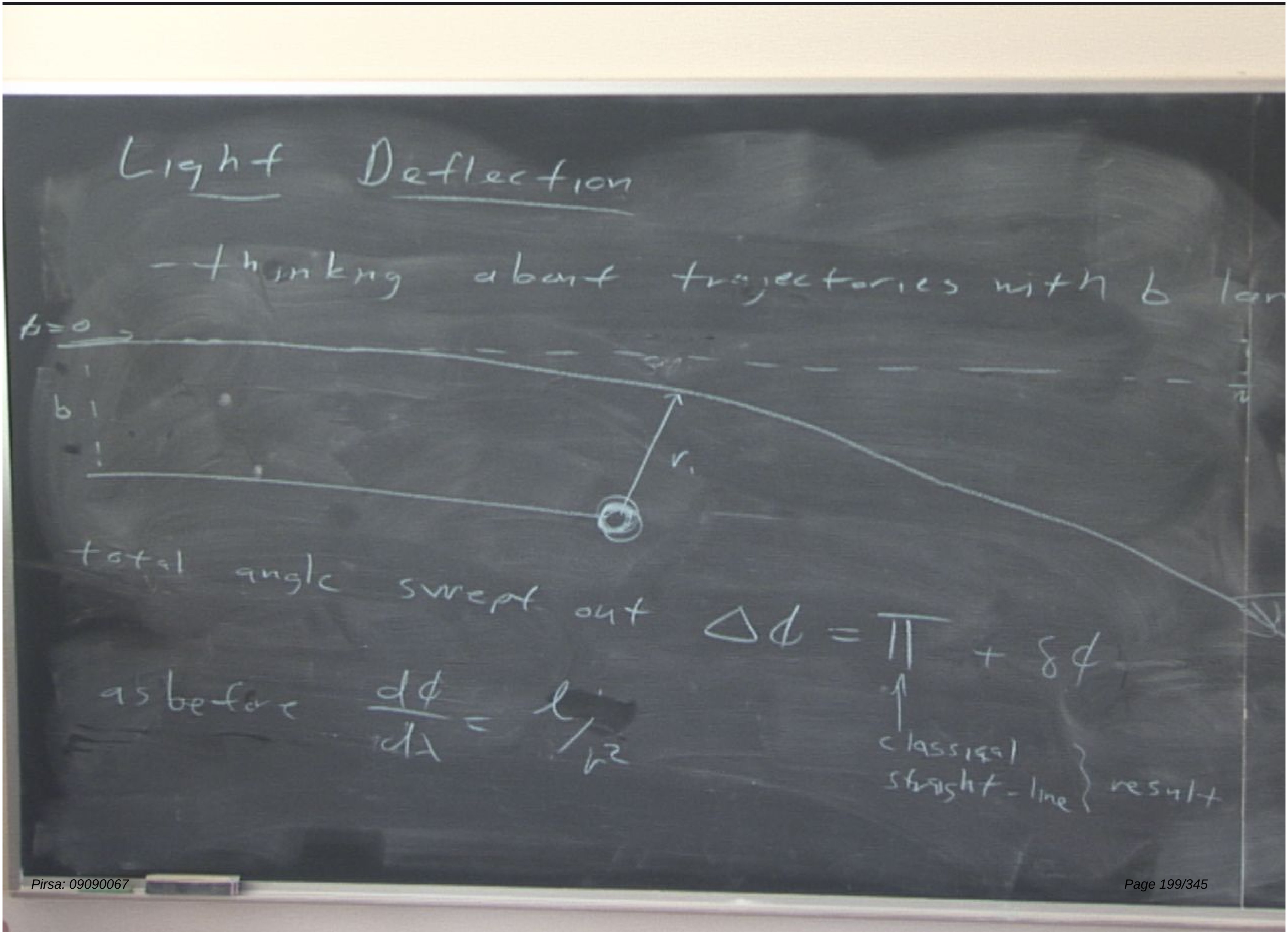
Light Deflection

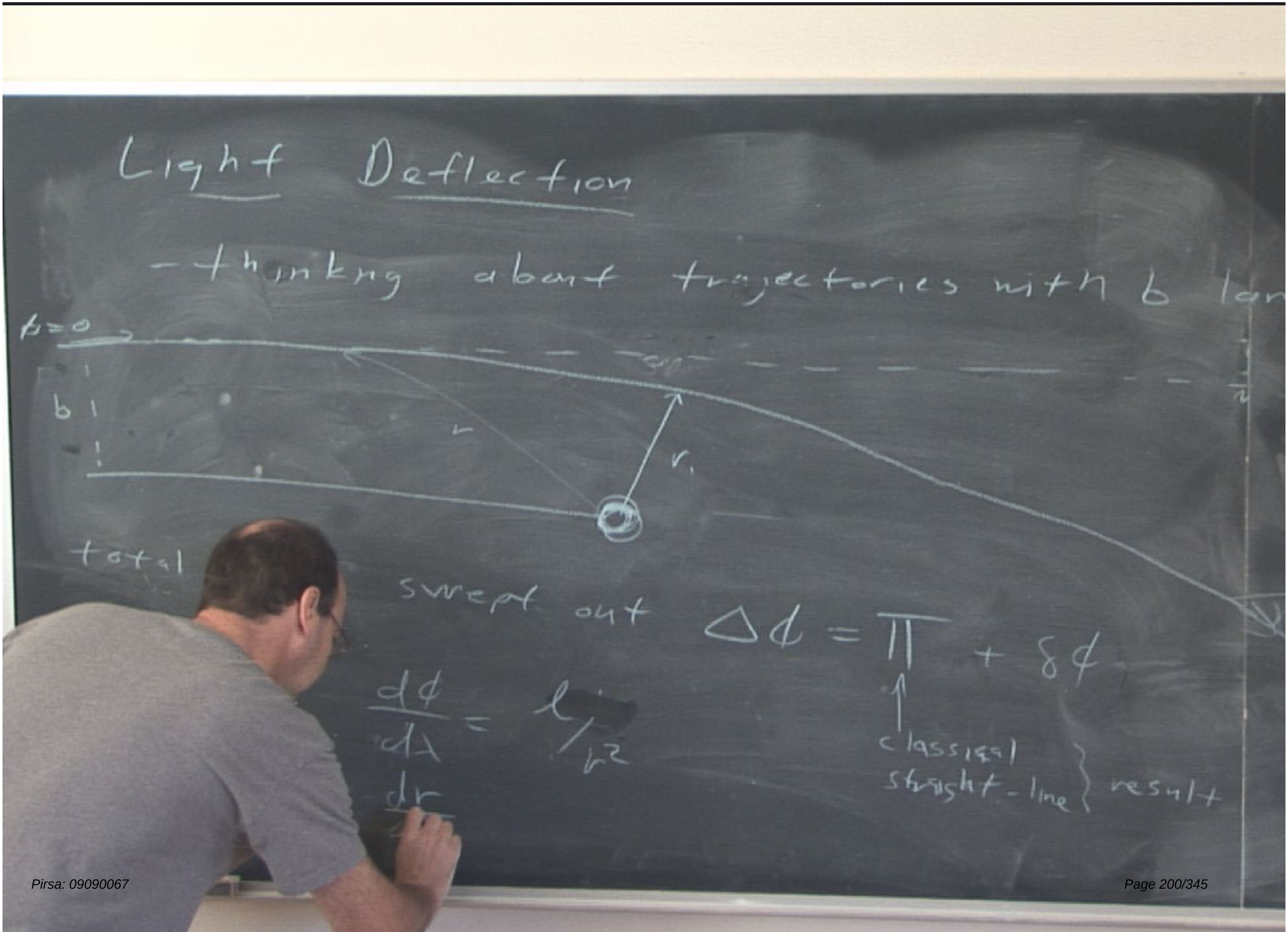
- thinking about trajectories with b lar

total angle swept out $\Delta\phi = \pi + \delta\phi$

(π: classical straight-line result)


Light Deflection
– thinking about trajectories with b large
total angle swept out Δφ =
for many practical applications
(eg. solar system) M/r << 1
M ≈ 1.5 km
R_sun ≈ 7 X 10^5 km


Light Deflection
- thinking about trajectories with b lar
ϕ=0
b
r₁
total angle swept out Δϕ = π + δϕ
as before dϕ/dλ = l/r²
classical
straight-line
result


Light Deflection
- thinking about trajectories with b lar
b=0
b
r
r1
total
swept out Δϕ = π + δϕ
dϕ/dλ = l/r²
dr
classical
straight-line
result

Light Deflection

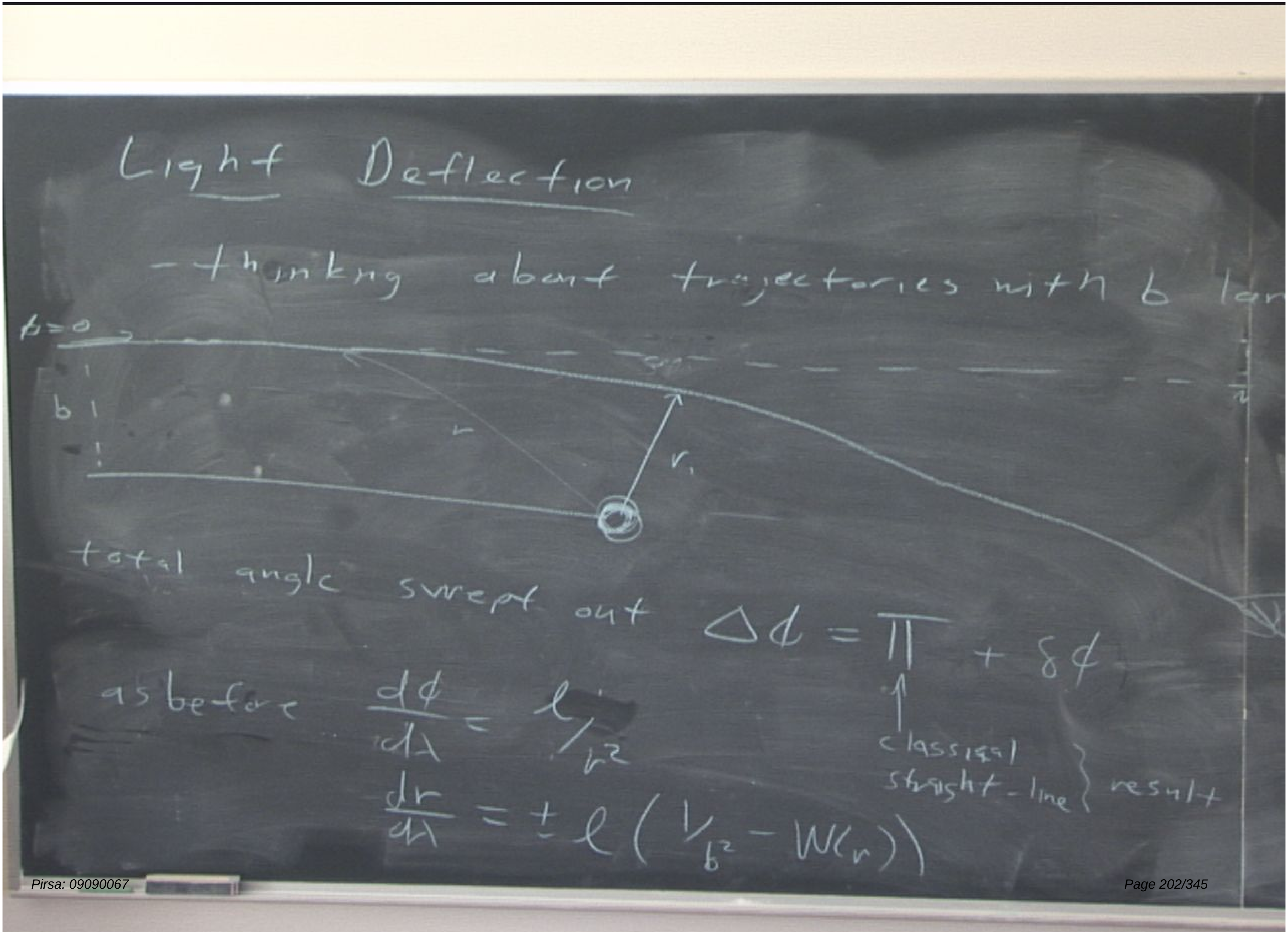
- thinking about trajectories with b lar

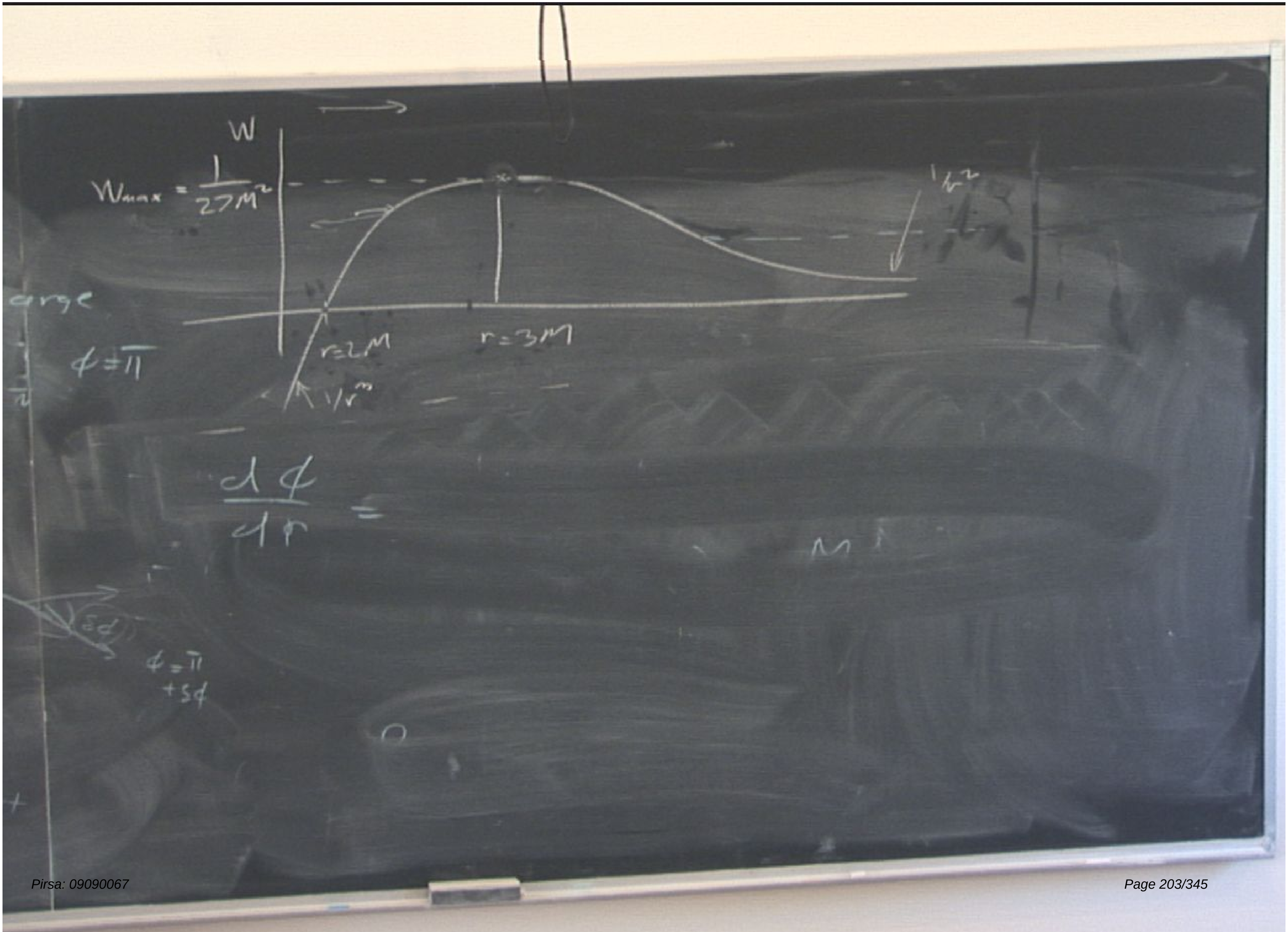
total angle swept out $\Delta\phi = \pi + \delta\phi$

(π ↑ classical straight-line result)

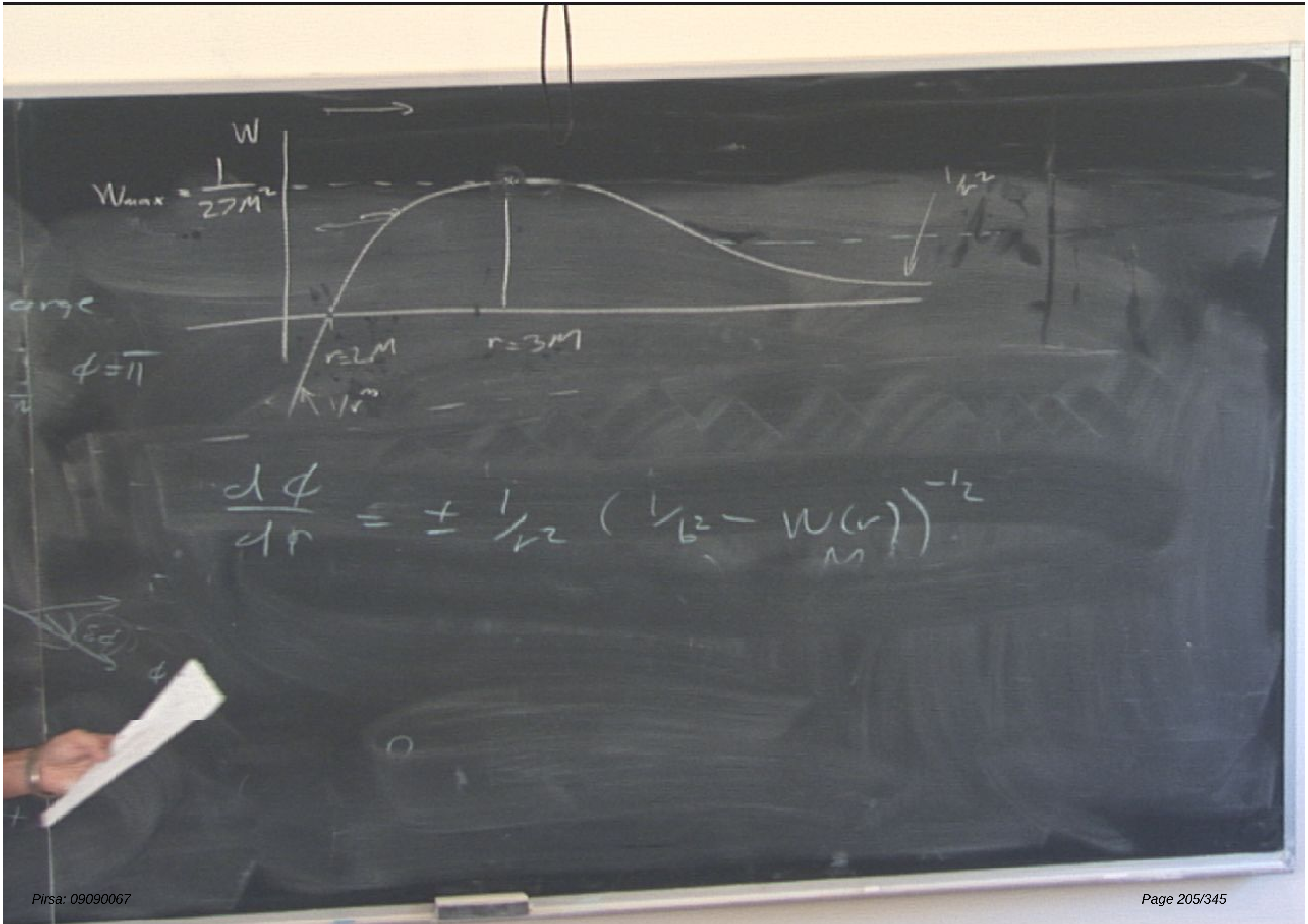
as before $\frac{d\phi}{d\lambda} = \ell / r^2$

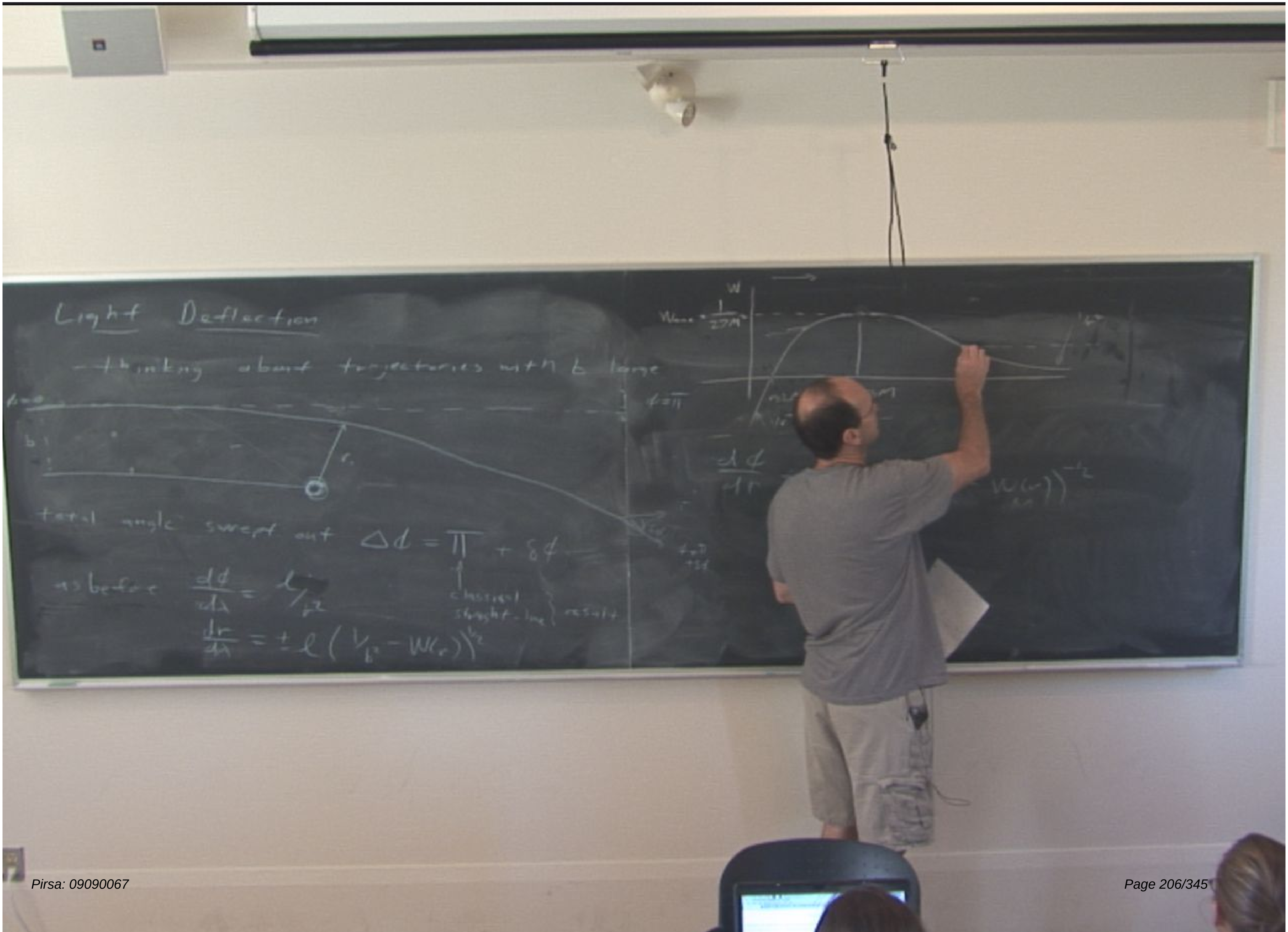
$\frac{dr}{d\lambda} =$


Light Deflection
- thinking about trajectories with b lar
b=0
b
r
r1
total angle swept out Δφ = π + δφ
as before dφ/dλ = l/r²
dr/dλ = ± l (1/b² − W(r))
classical straight-line result


W
Wmax = 1/27M²
r=2M
r=3M
1/r²
dφ/dr =
φ=π
φ=π
+δφ


W
Wmax = 1/27M²
r=3M
2M

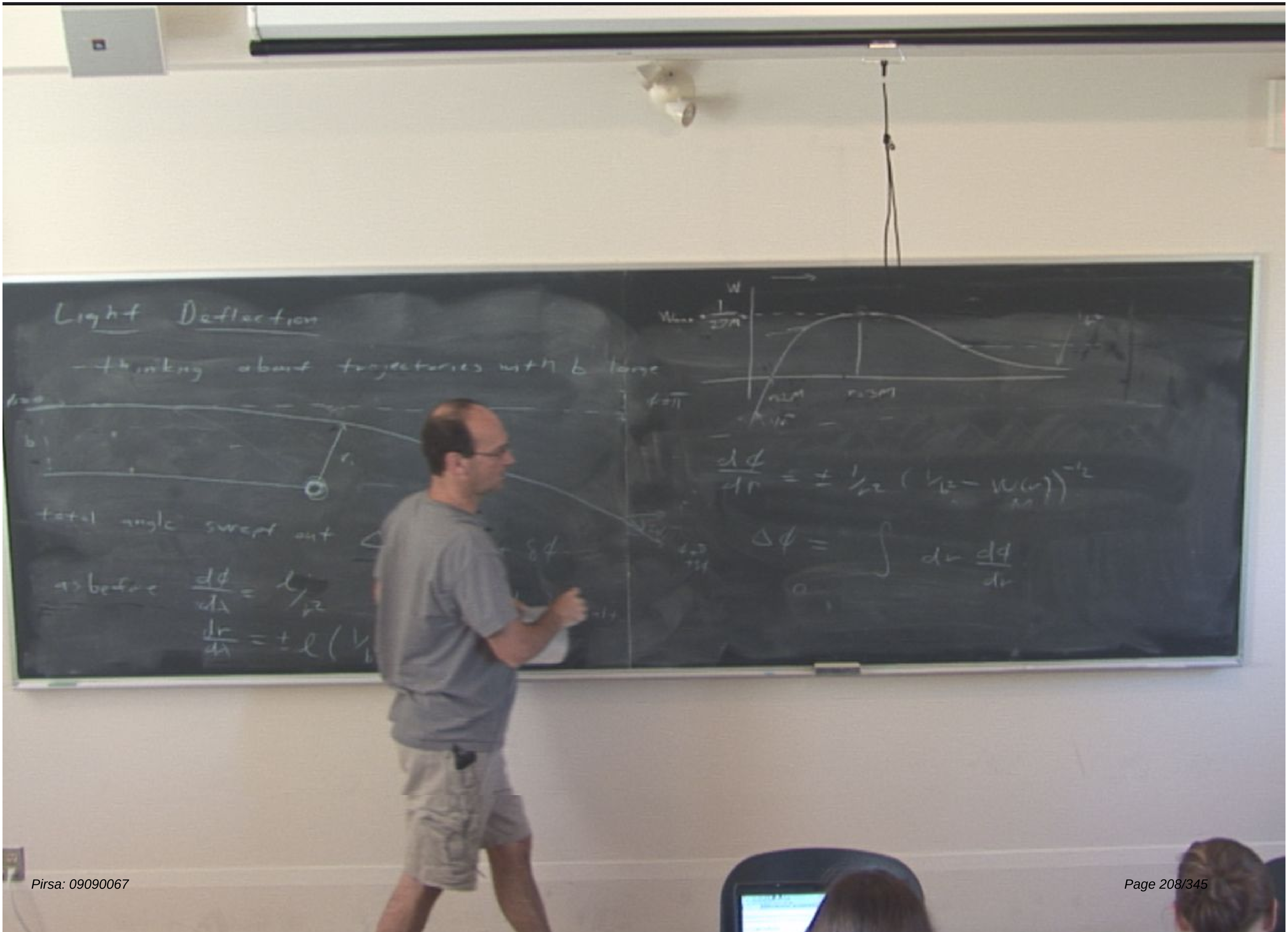

W
Wmax = 1/27M²
r=2M
r=3M
φ=π
1/r²
dφ/dr = ± 1/r² (1/b² − W(r))^(−1/2)

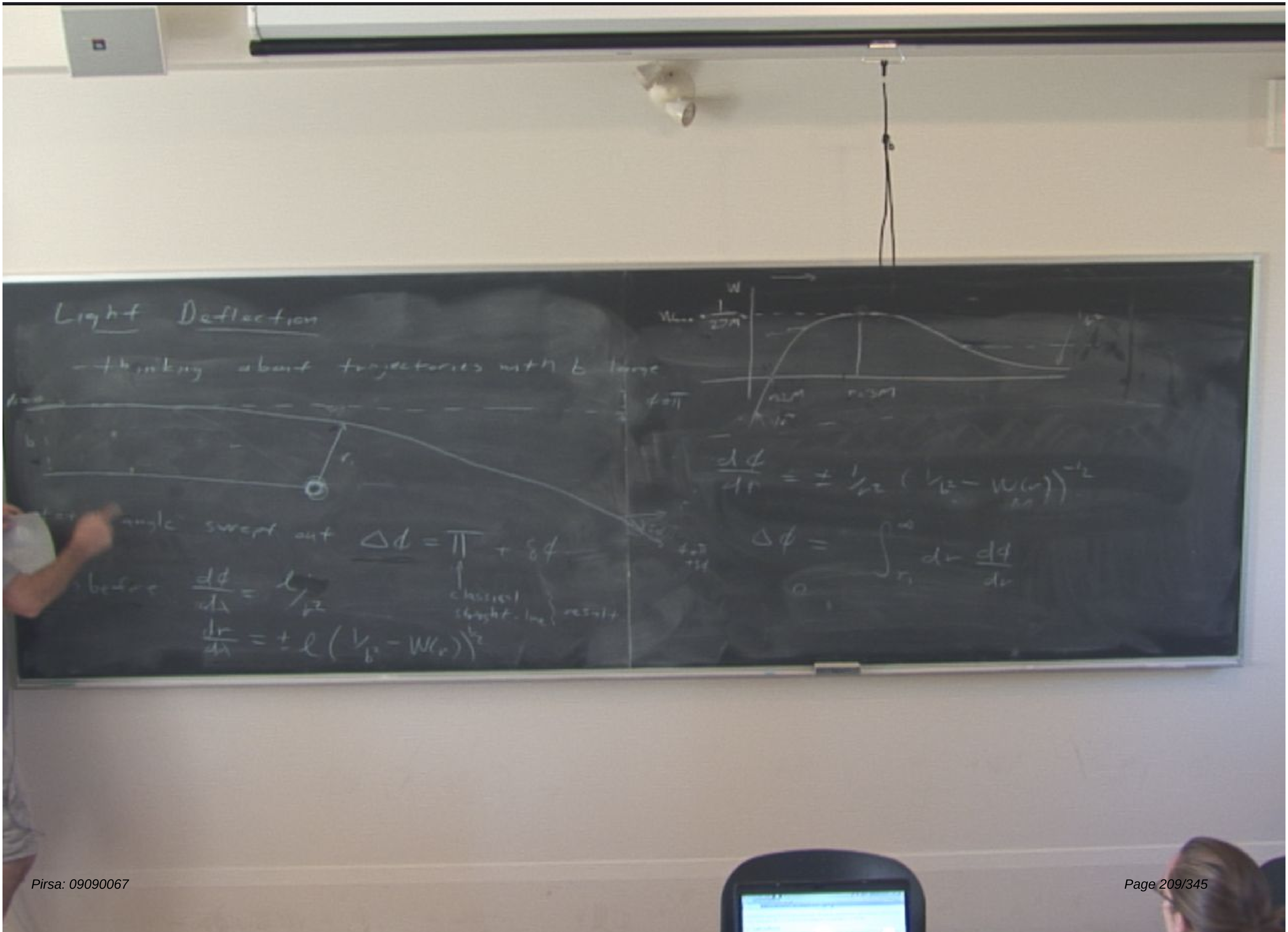

Light Deflection
thinking about trajectories with b large
total angle swept out
as before
straight-line
result

$$\frac{d\phi}{dr} = \pm \frac{1}{r^2}\left(\frac{1}{b^2} - W(r)\right)^{-1/2}$$

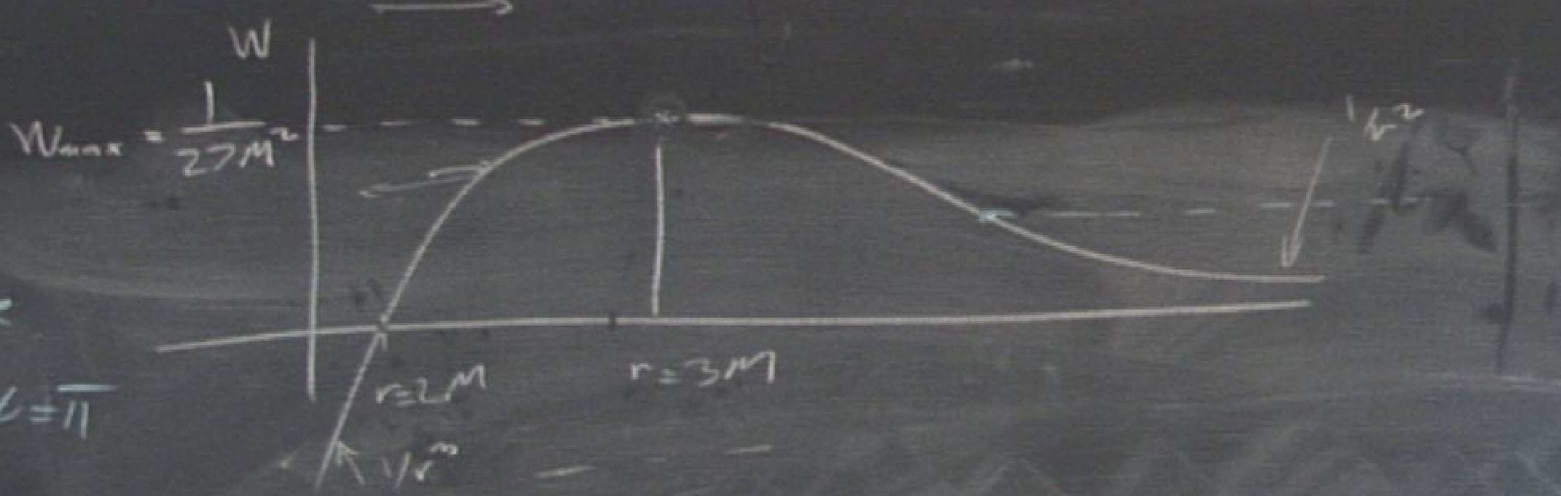
$$\Delta\phi = \int dr \, \frac{d\phi}{dr}$$

$\phi = \pi + \delta\phi$


Light Deflection
– thinking about trajectories with b large
total angle swept out
as before
$\frac{d\phi}{d\lambda} = \frac{l}{r^2}$
$\frac{d\phi}{dr} = \pm \frac{1}{r^2} \left(\frac{1}{b^2} - W(r) \right)^{-1/2}$
$\Delta\phi = \int dr \frac{d\phi}{dr}$

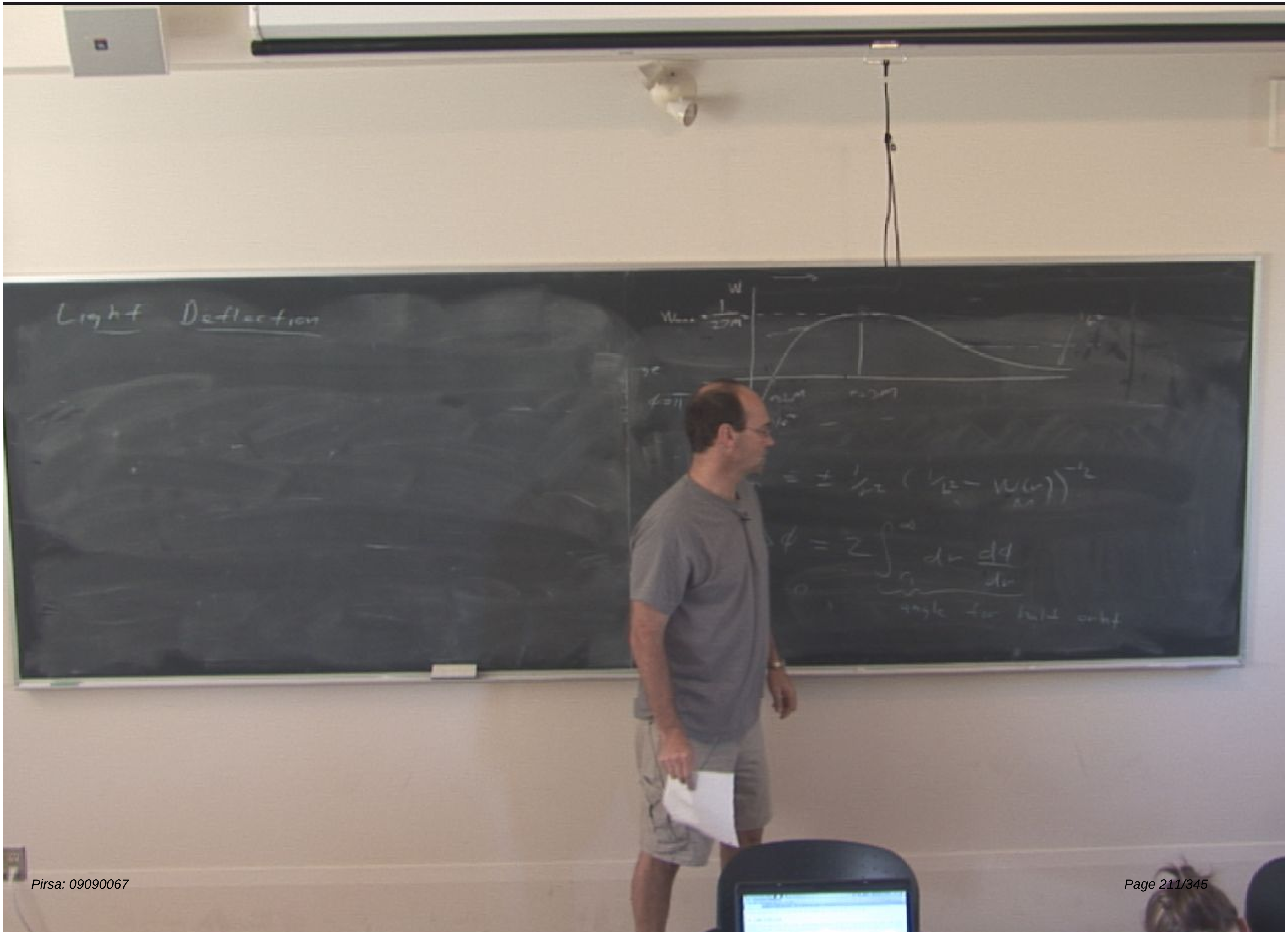


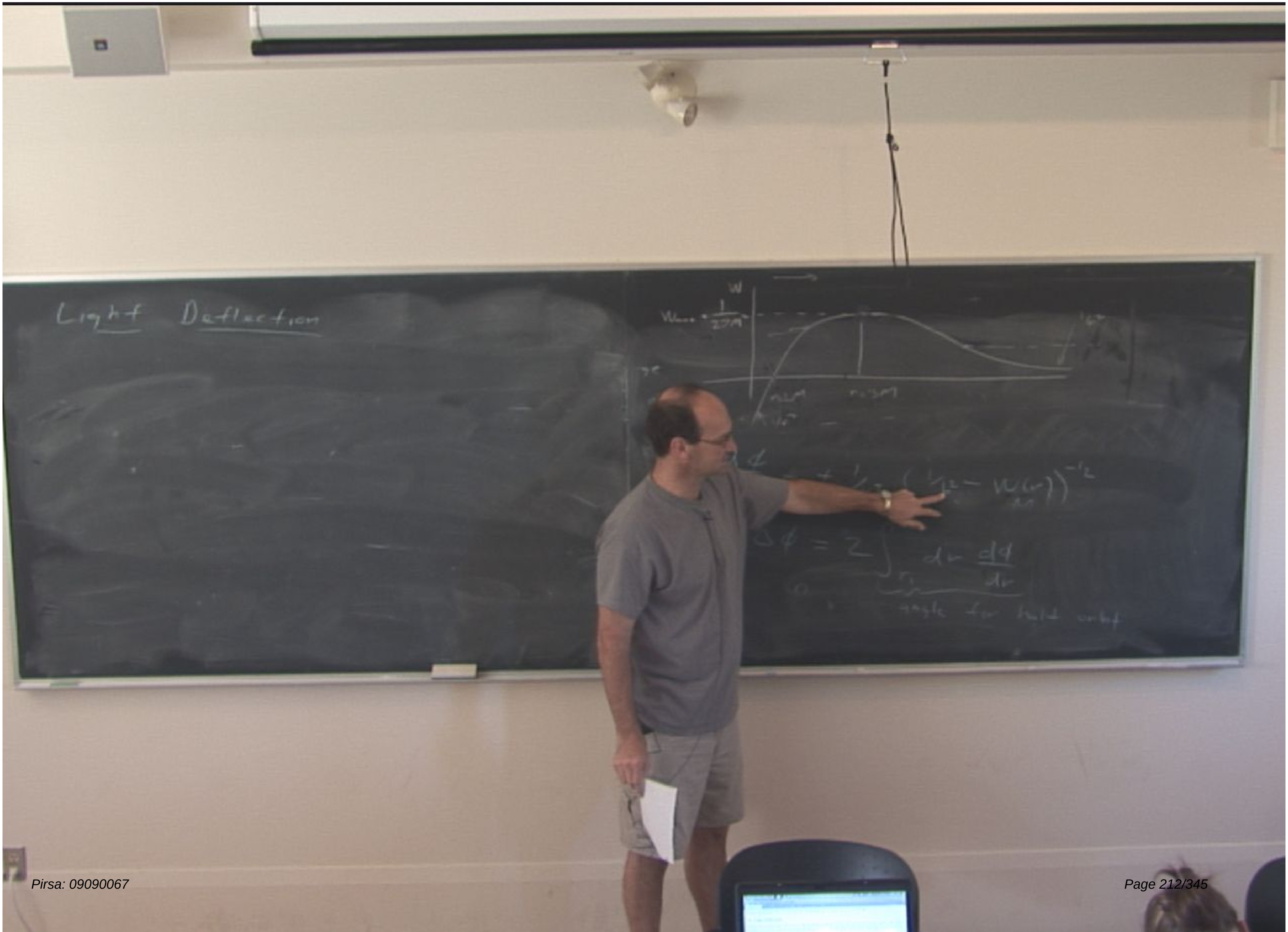
Light Deflection
– thinking about trajectories with b large
angle swept out Δφ = π + δφ
before dφ/dλ = ℓ/r²
classical straight line result
dr/dλ = ± ℓ (1/b² − W(r))^½
W
1/27M
r=3M
dφ/dr = ± 1/r² (1/b² − W(r))^-½
Δφ = ∫ from r₁ to ∞ dr dφ/dr

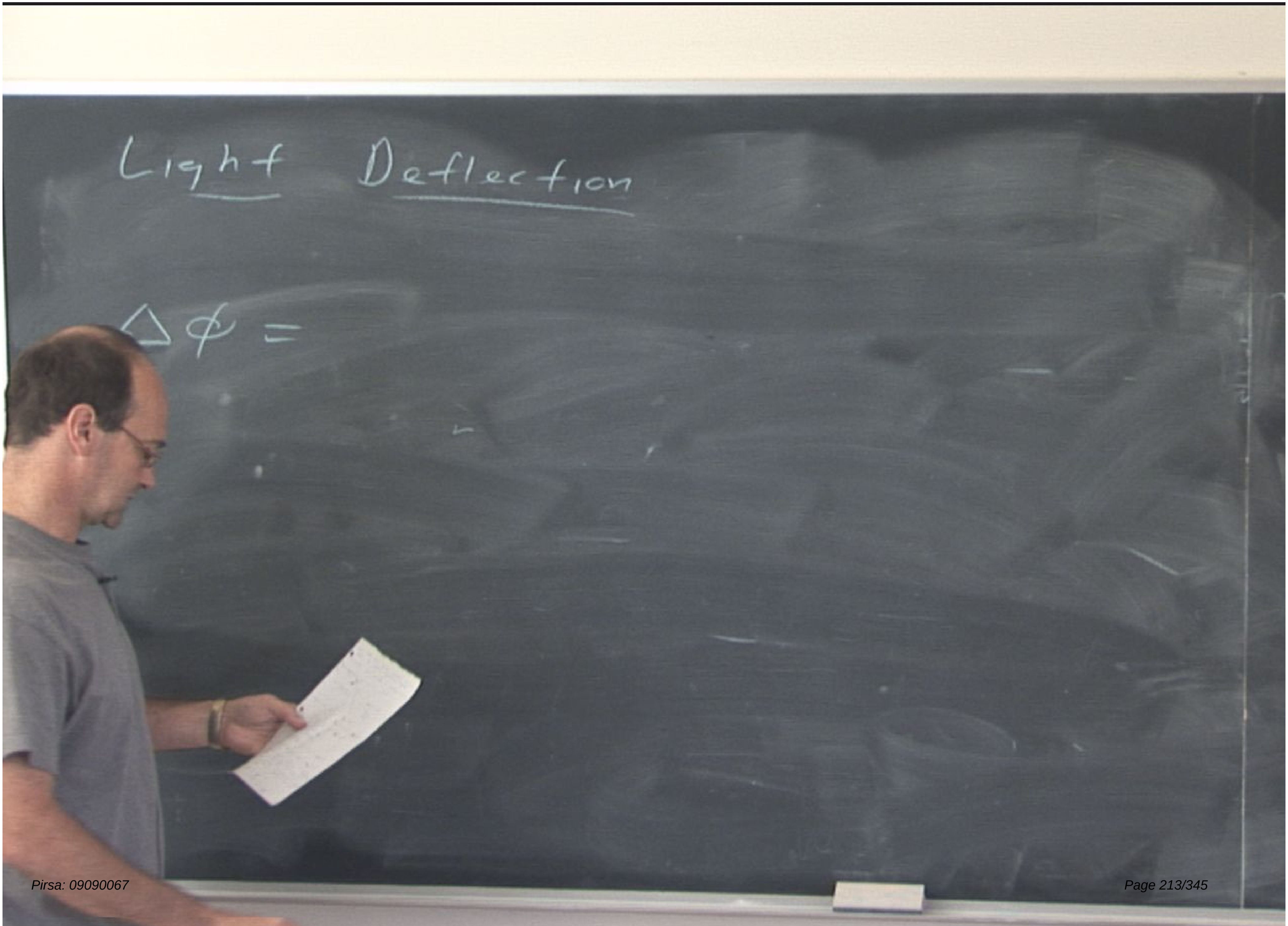


$$\frac{d\phi}{dr} = \pm \frac{1}{r^2}\left(\frac{1}{b^2} - W(r)\right)^{-1/2}$$

$$\Delta\phi = 2\underbrace{\int_{r_1}^{\infty} dr\, \frac{d\phi}{dr}}_{\text{angle for half orbit}}$$


Light Deflection


Light Deflection

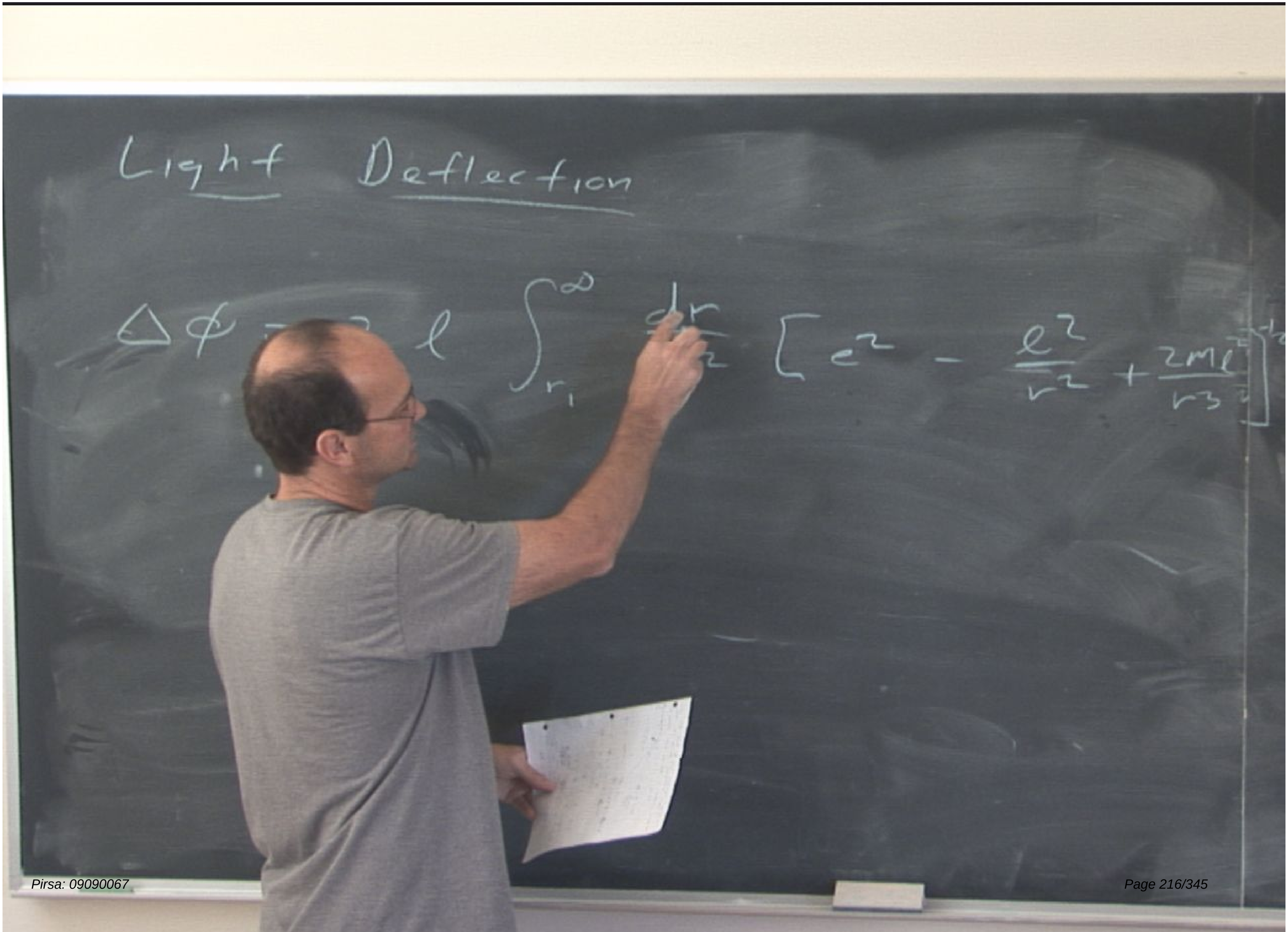

Light Deflection
Δφ =

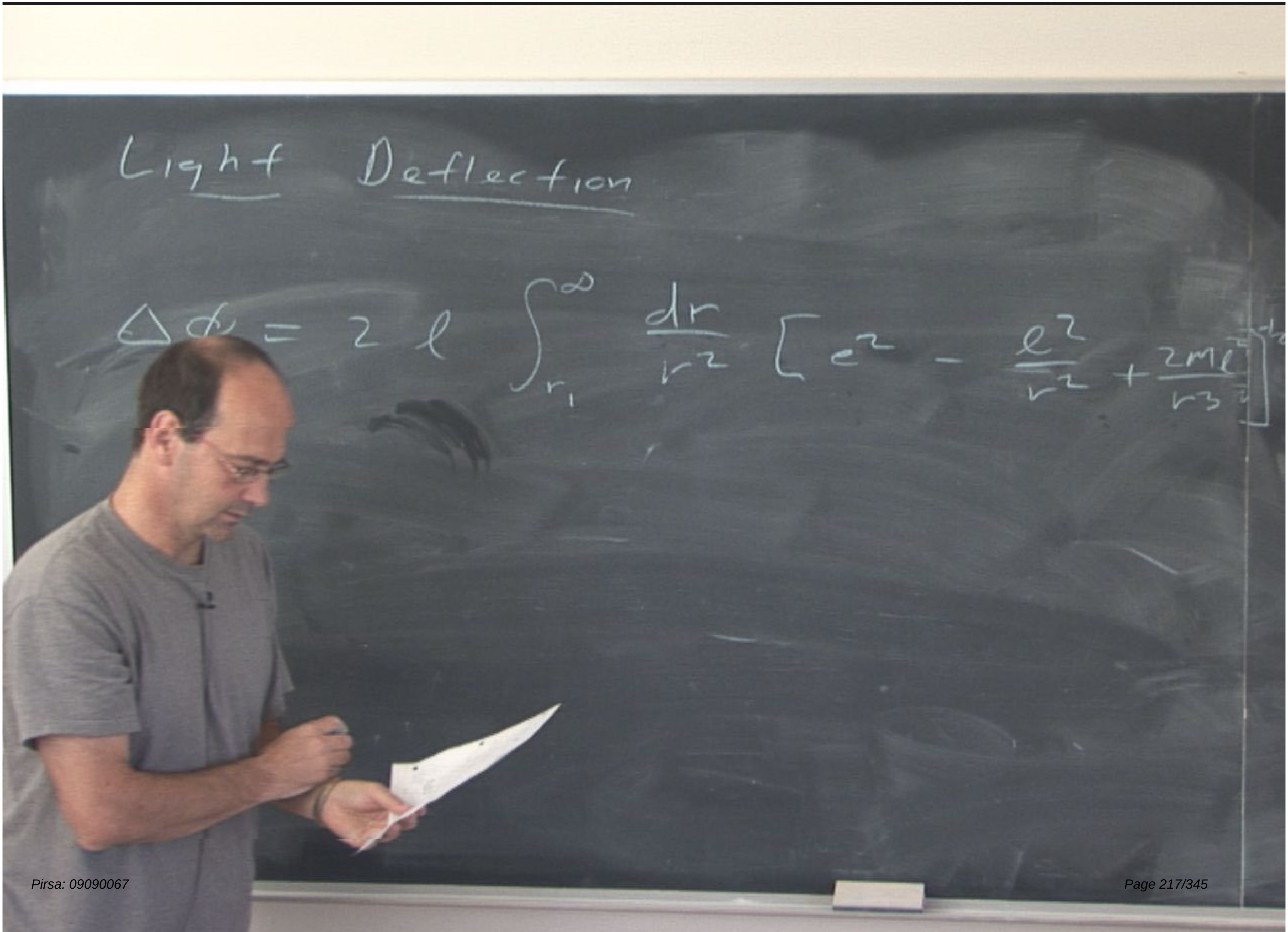
Light Deflection

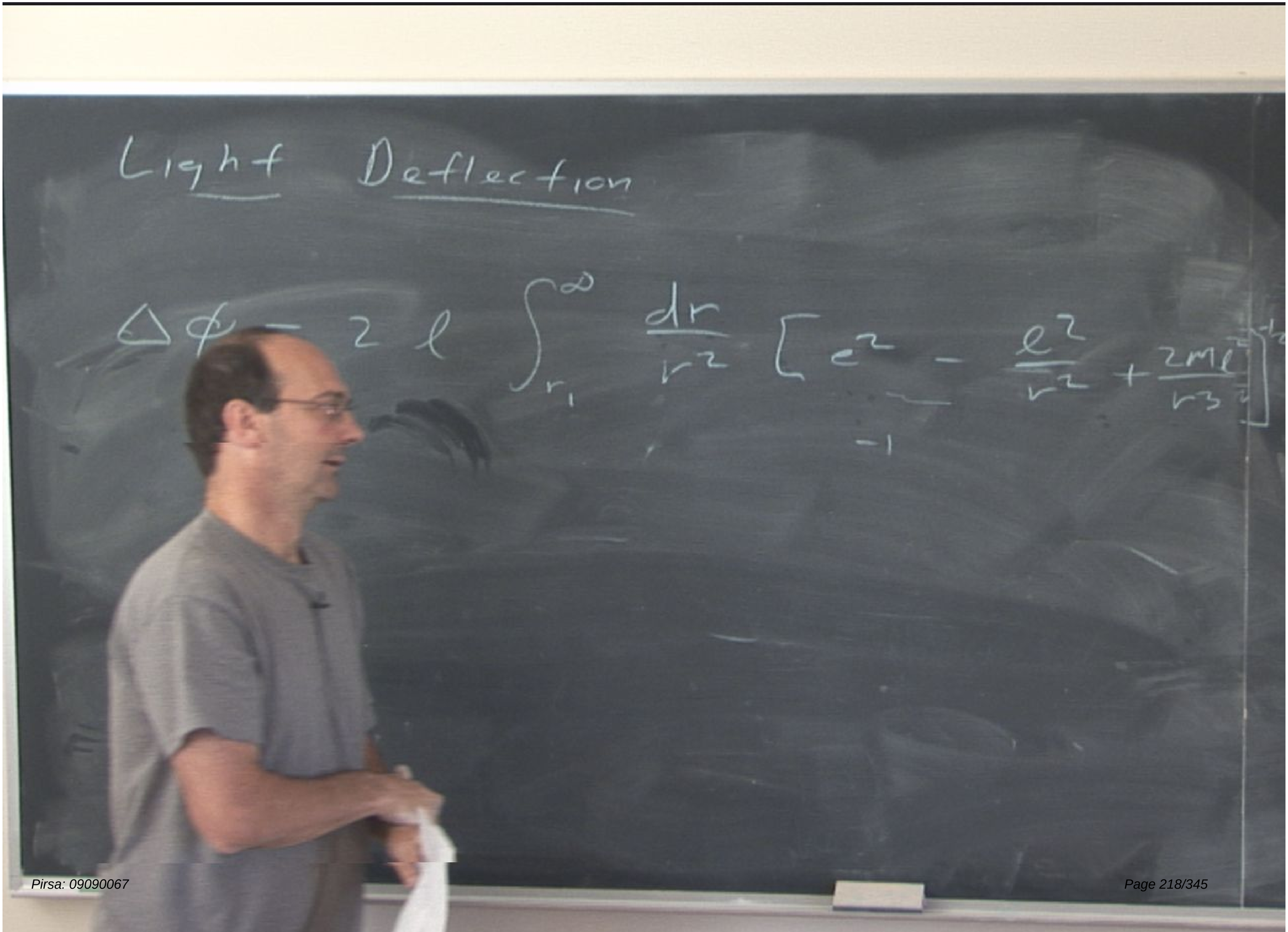
$$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[e^2 - \frac{\ell^2}{r^2} + \frac{2M\ell^2}{r^3}\right]^{-1/2}$$

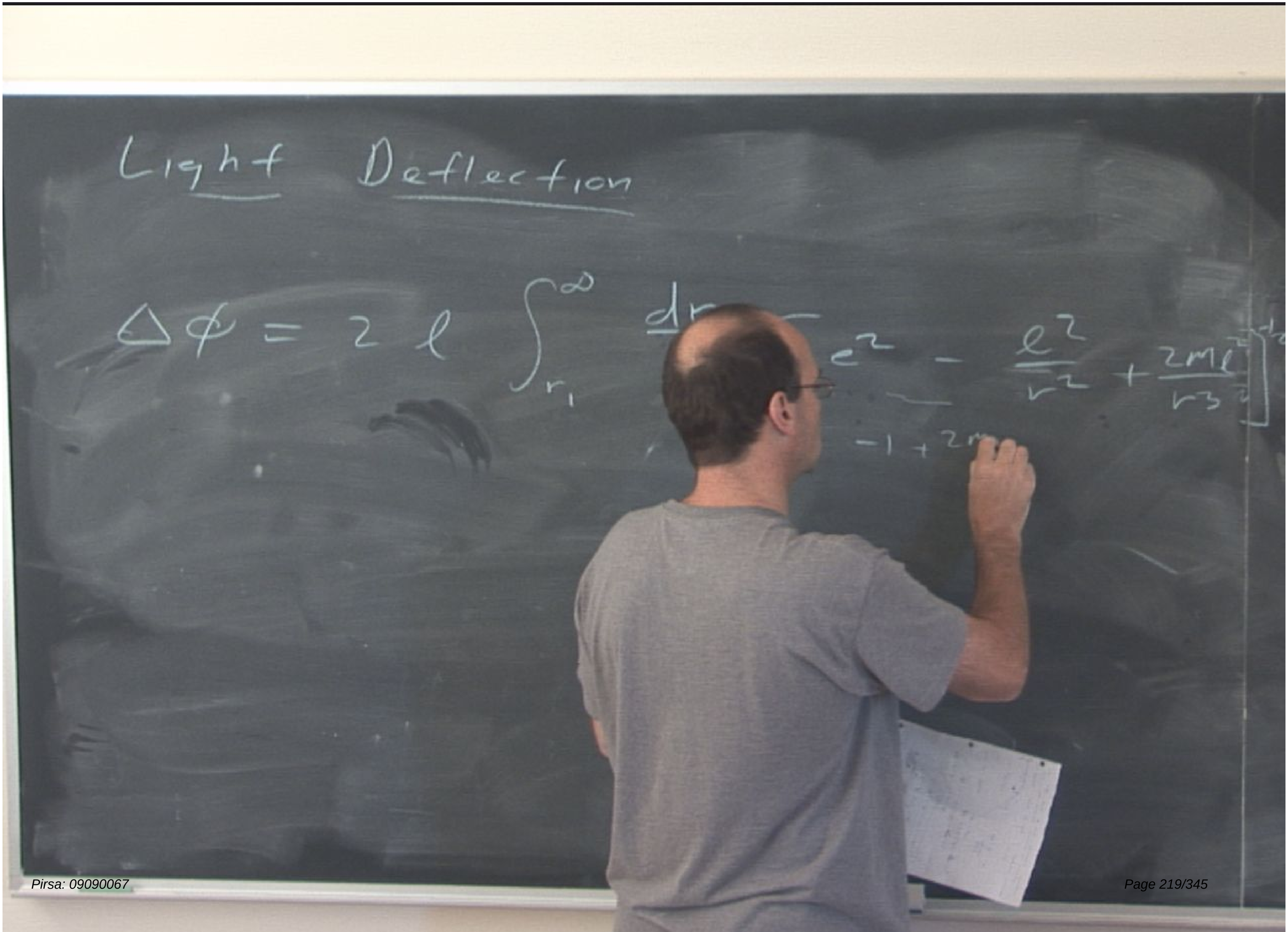
Light Deflection

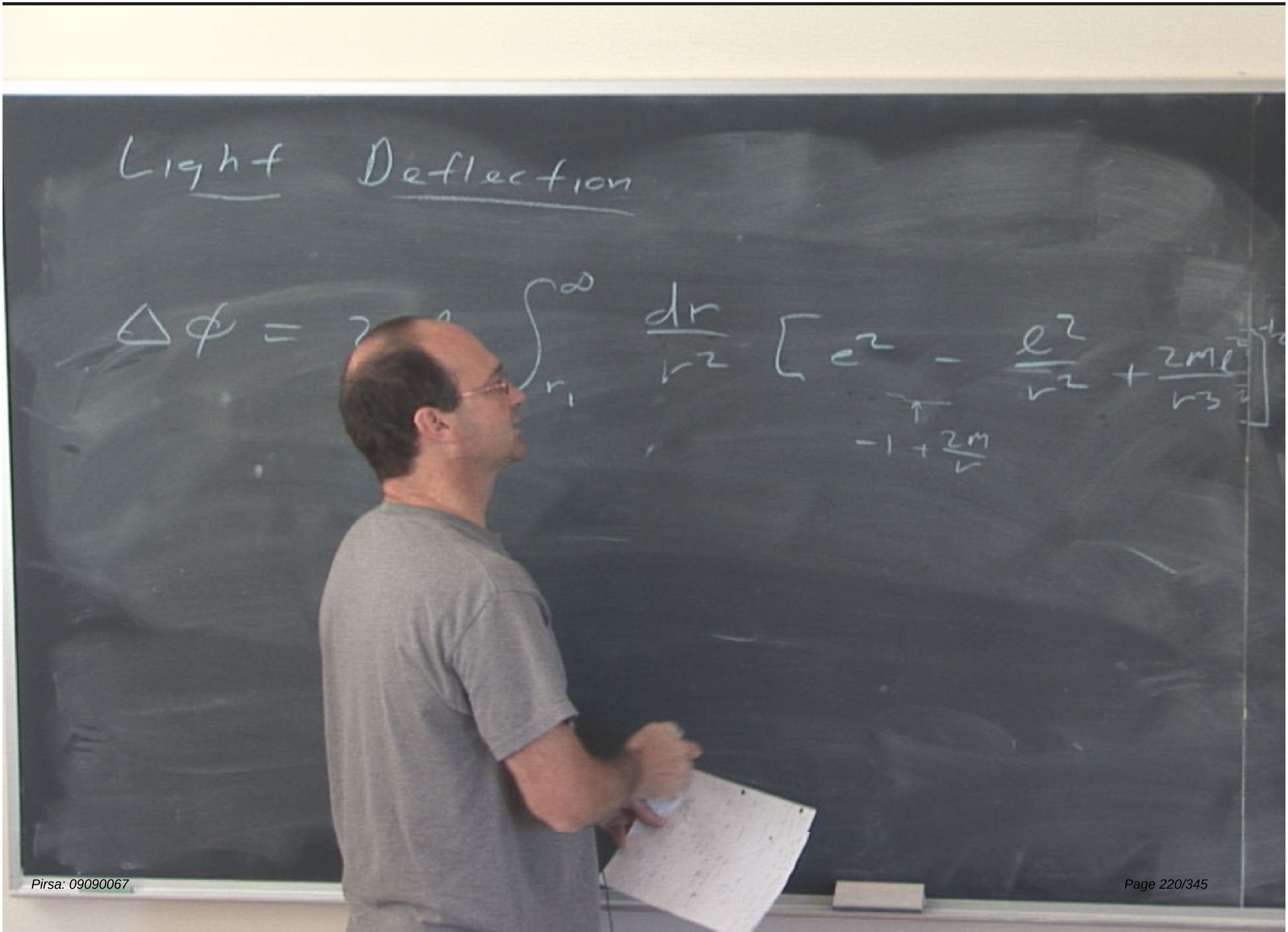
$$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[e^2 - \frac{\ell^2}{r^2} + \frac{2m\ell^2}{r^3}\right]^{-1/2}$$

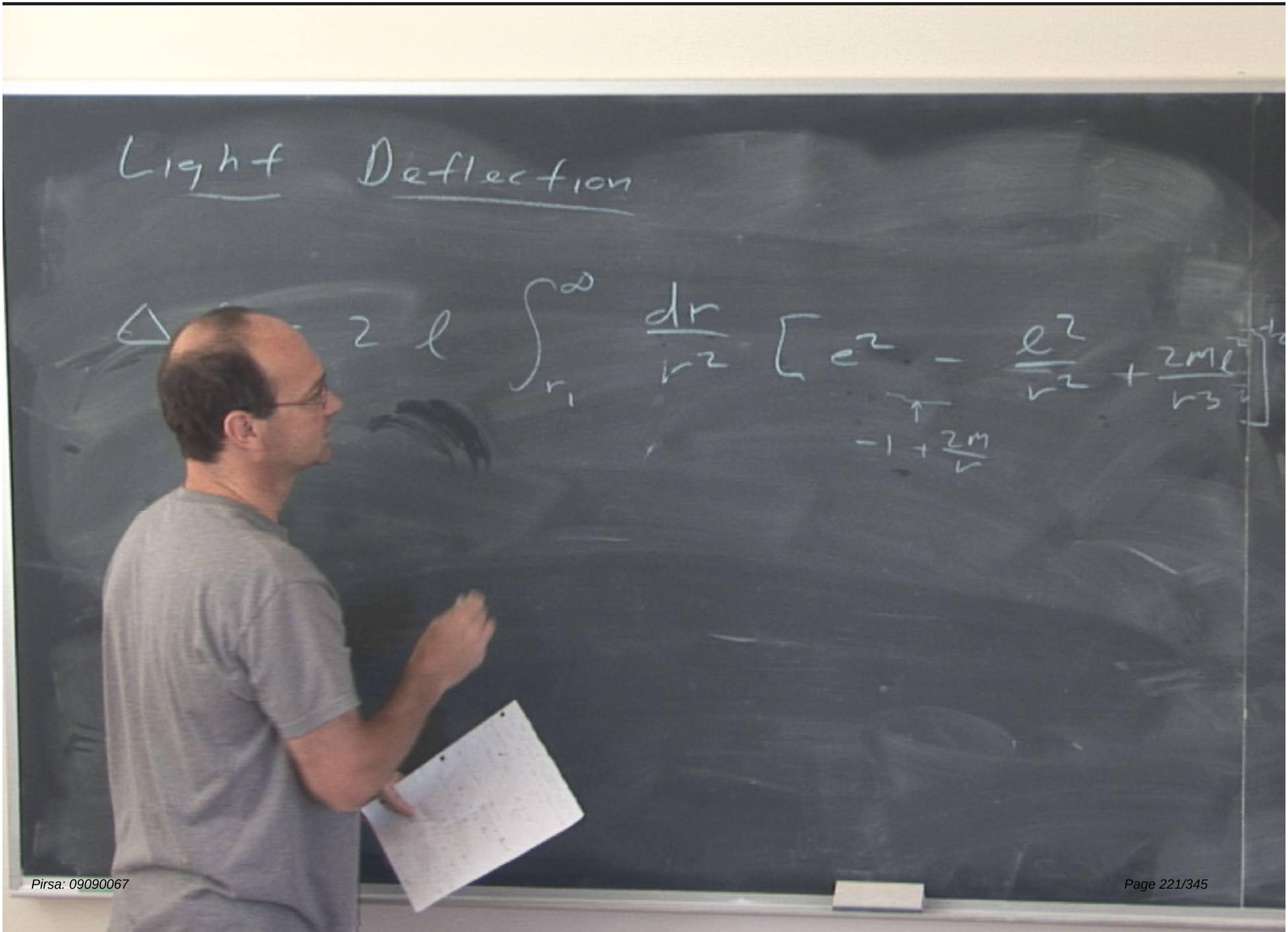

Light Deflection

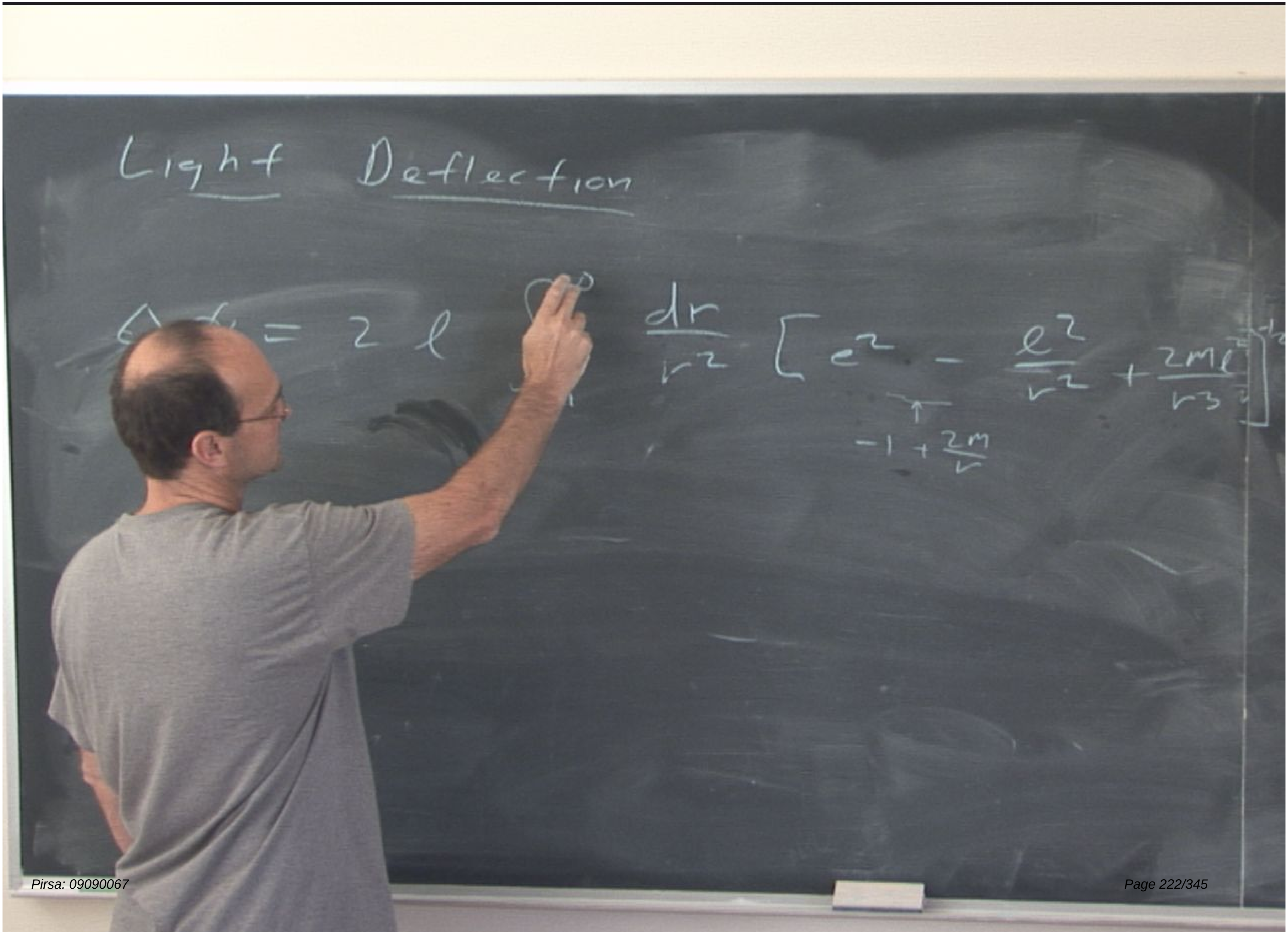

Light Deflection
$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[e^2 - \frac{\ell^2}{r^2} + \frac{2m\ell^2}{r^3}\right]^{-1/2}$


Light Deflection
$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[e^2 - \frac{\ell^2}{r^2} + \frac{2m\ell^2}{r^3}\right]^{-1/2}$
−1


Light Deflection


Light Deflection

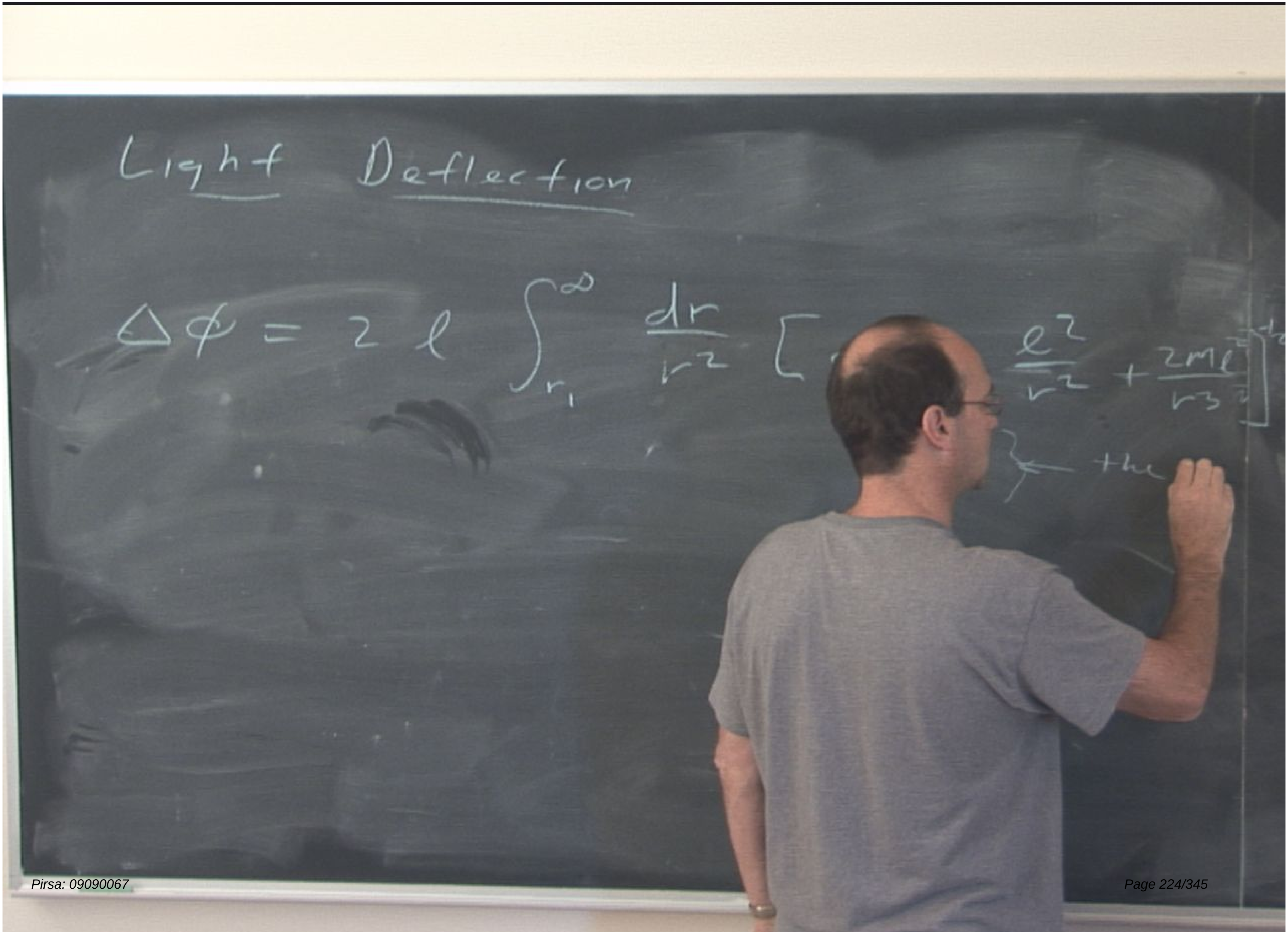

Light Deflection
$\Delta \ldots 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2} \left[e^2 - \frac{\ell^2}{r^2} + \frac{2m\ell^2}{r^3} \right]^{-1/2}$
$-1 + \frac{2m}{r}$

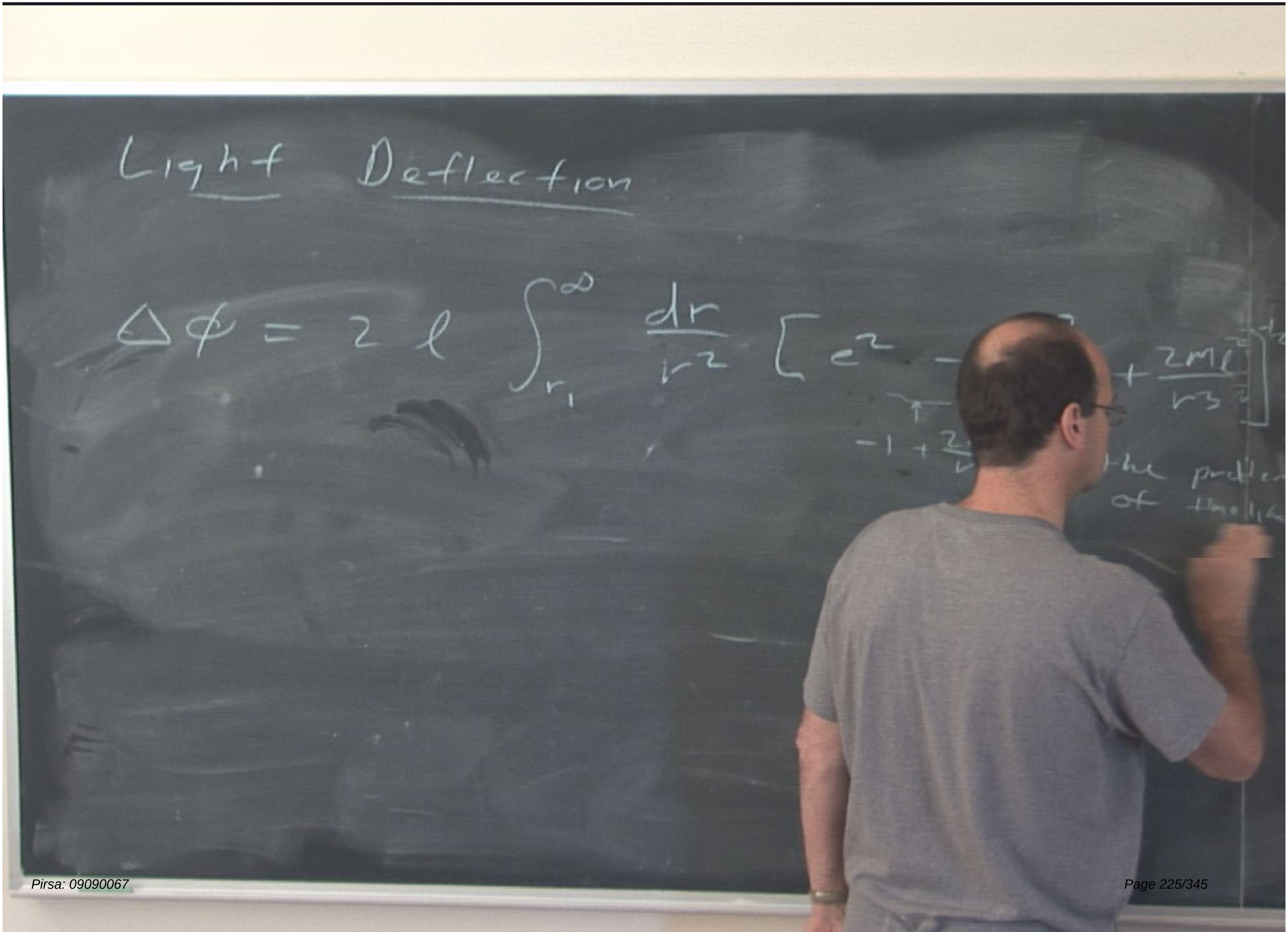

Light Deflection
= 2 ℓ
dr / r²
e² − ℓ²/r² + 2mℓ²/r³
−1 + 2m/r

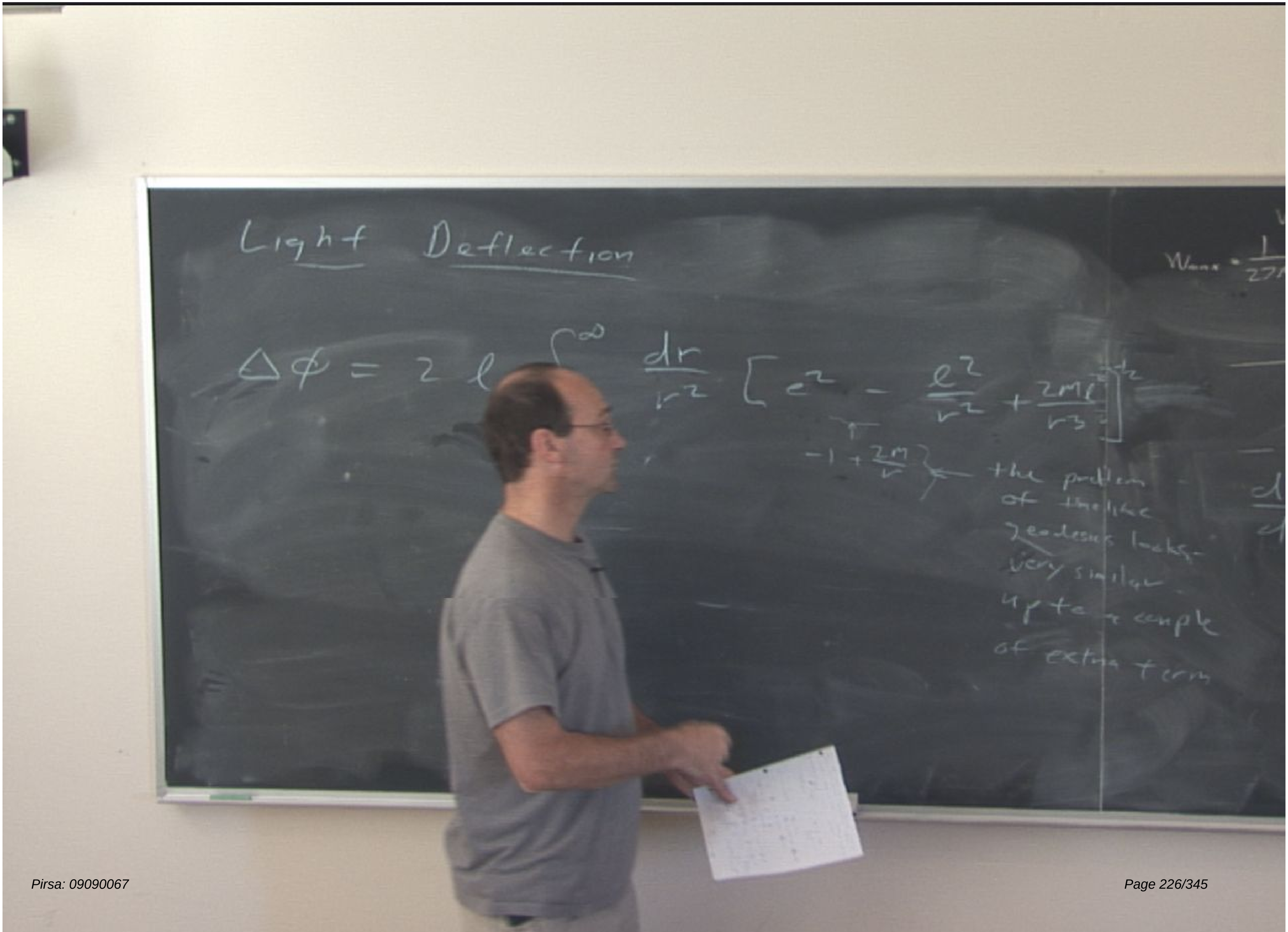
Light Deflection

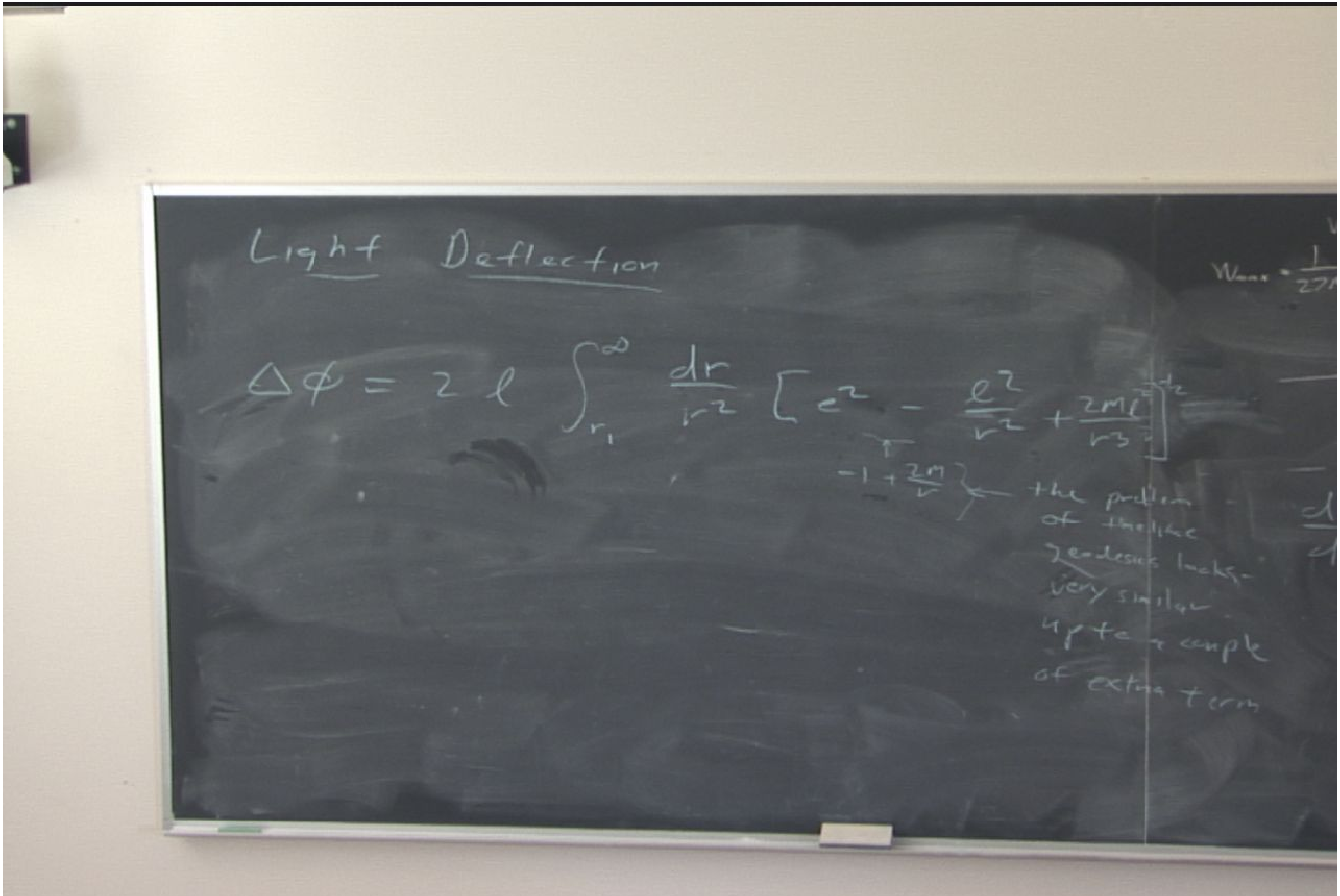
$$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[e^2 - \frac{\ell^2}{r^2} + \frac{2m\ell^2}{r^3}\right]^{-1/2}$$

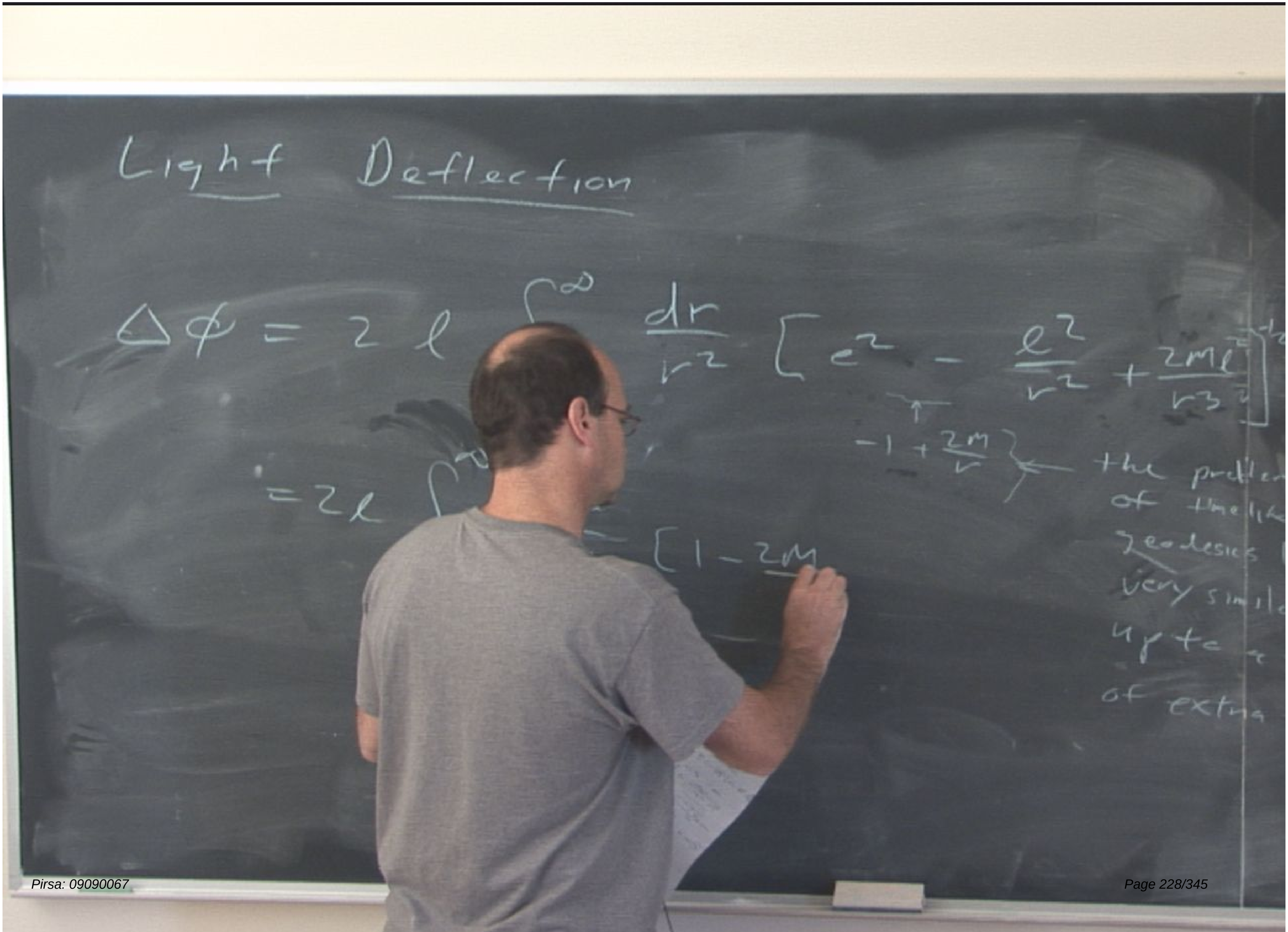
$\uparrow$ $-1 + \frac{2m}{r}$

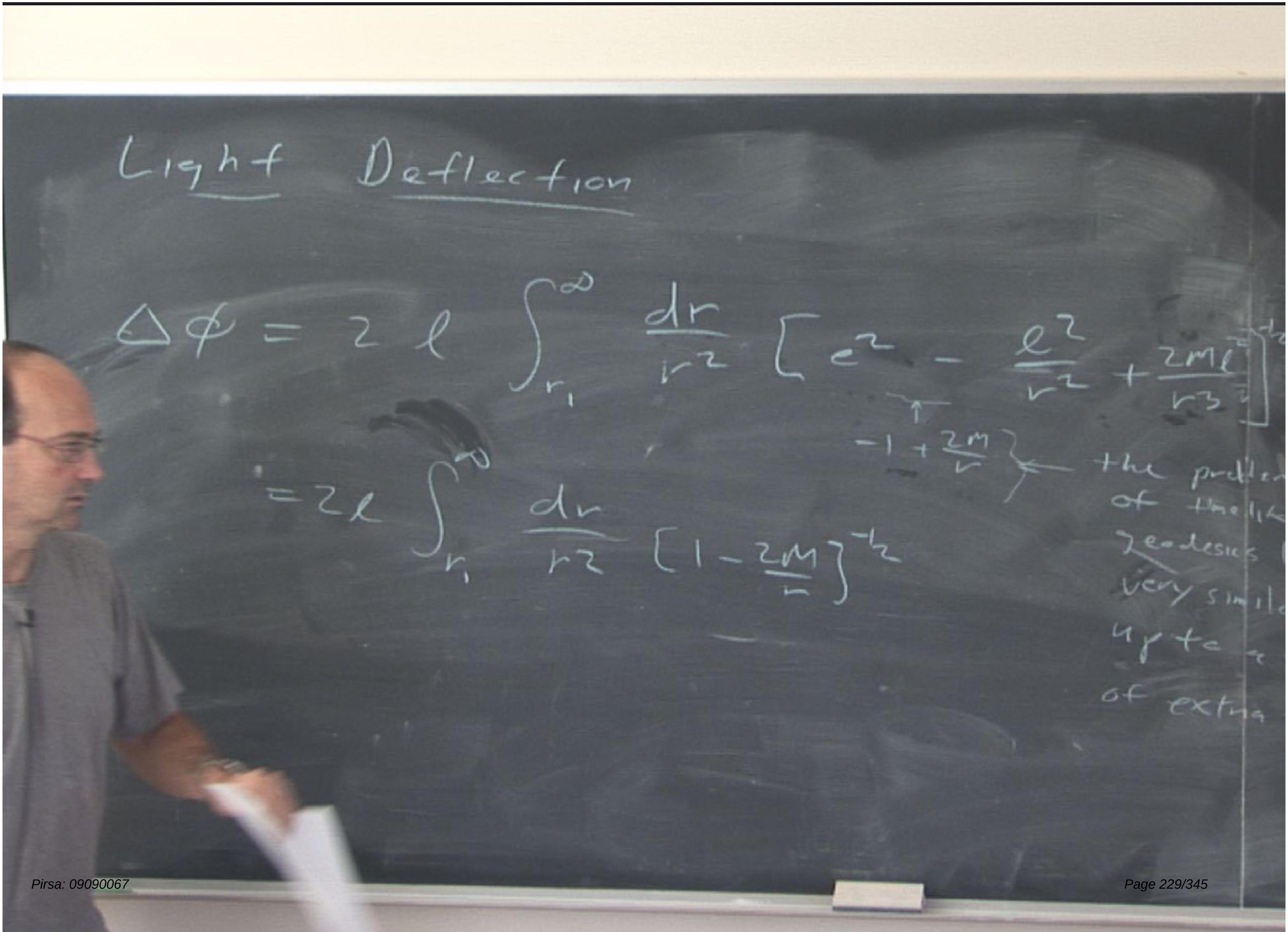

Light Deflection
$\Delta \phi = 2 \ell \int_{r_1}^{\infty} \frac{dr}{r^2} [$
$\frac{\ell^2}{r^2} + \frac{2m\ell^2}{r^3}]$
the


Light Deflection
$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2} [e^2$


Light Deflection
Δφ = 2 ℓ ∫^∞ dr/r² [e² − ℓ²/r² + 2Mℓ²/r³]^(−1/2)
−1 + 2M/r
the problem
of the like
geodesics looks
very similar
up to a couple
of extra term


Light Deflection
Δφ = 2ℓ ∫_{r_1}^∞ dr/r^2 [e^2 − ℓ^2/r^2 + 2Mℓ^2/r^3]^{-1/2}
−1 + 2M/r
the problem of the like geodesics looks very similar up to a couple of extra term


Light Deflection
the problem
of timelike
geodesics
very simil
up to
of extra

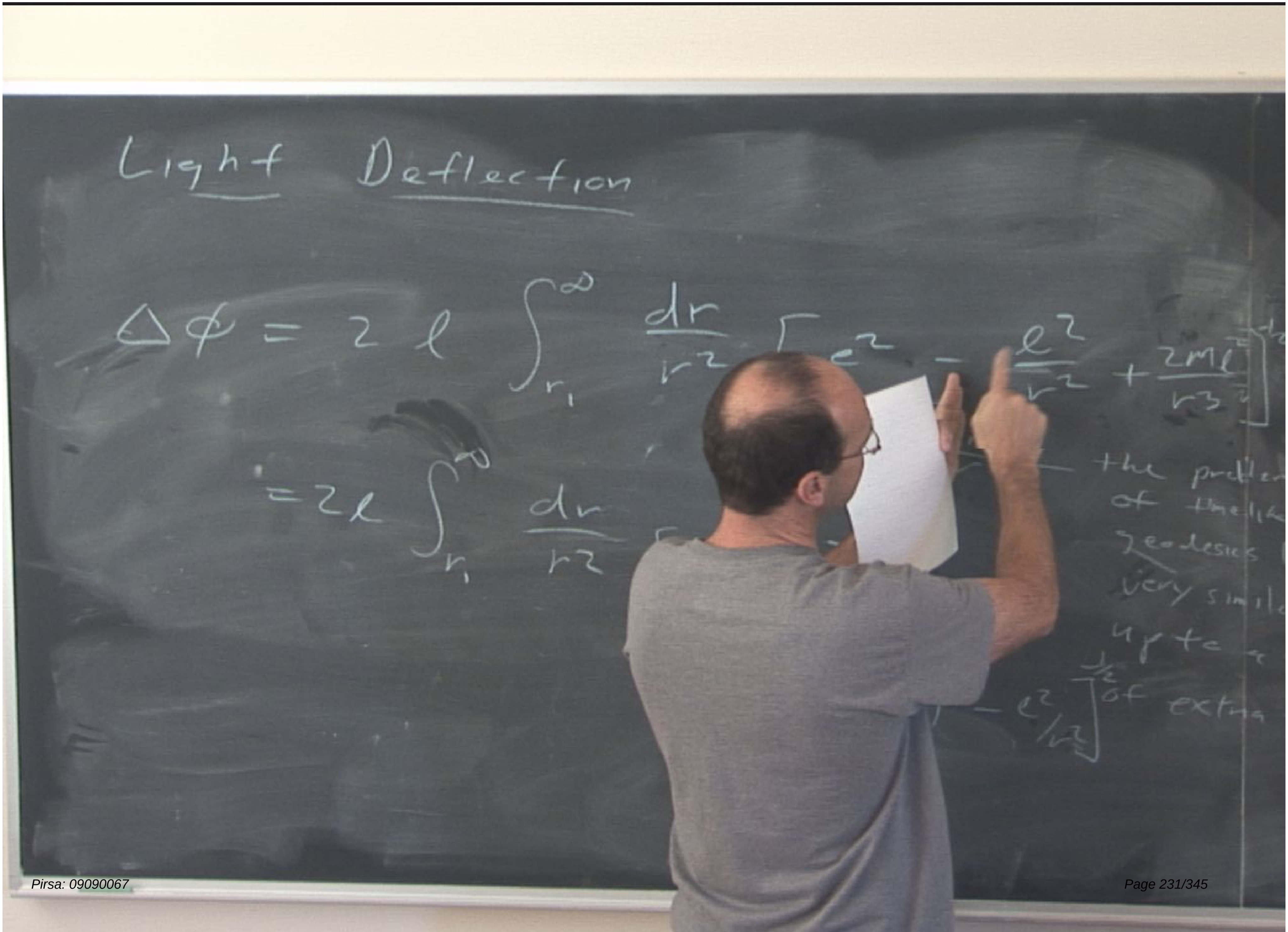

Light Deflection
$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2} \left[e^2 - \frac{\ell^2}{r^2} + \frac{2M\ell^2}{r^3} \right]^{-1/2}$
$-1 + \frac{2M}{r}$
the
of
geodesics
very
up to
of extra
$= 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2} \left[1 - \frac{2M}{r} \right]^{-1/2}$

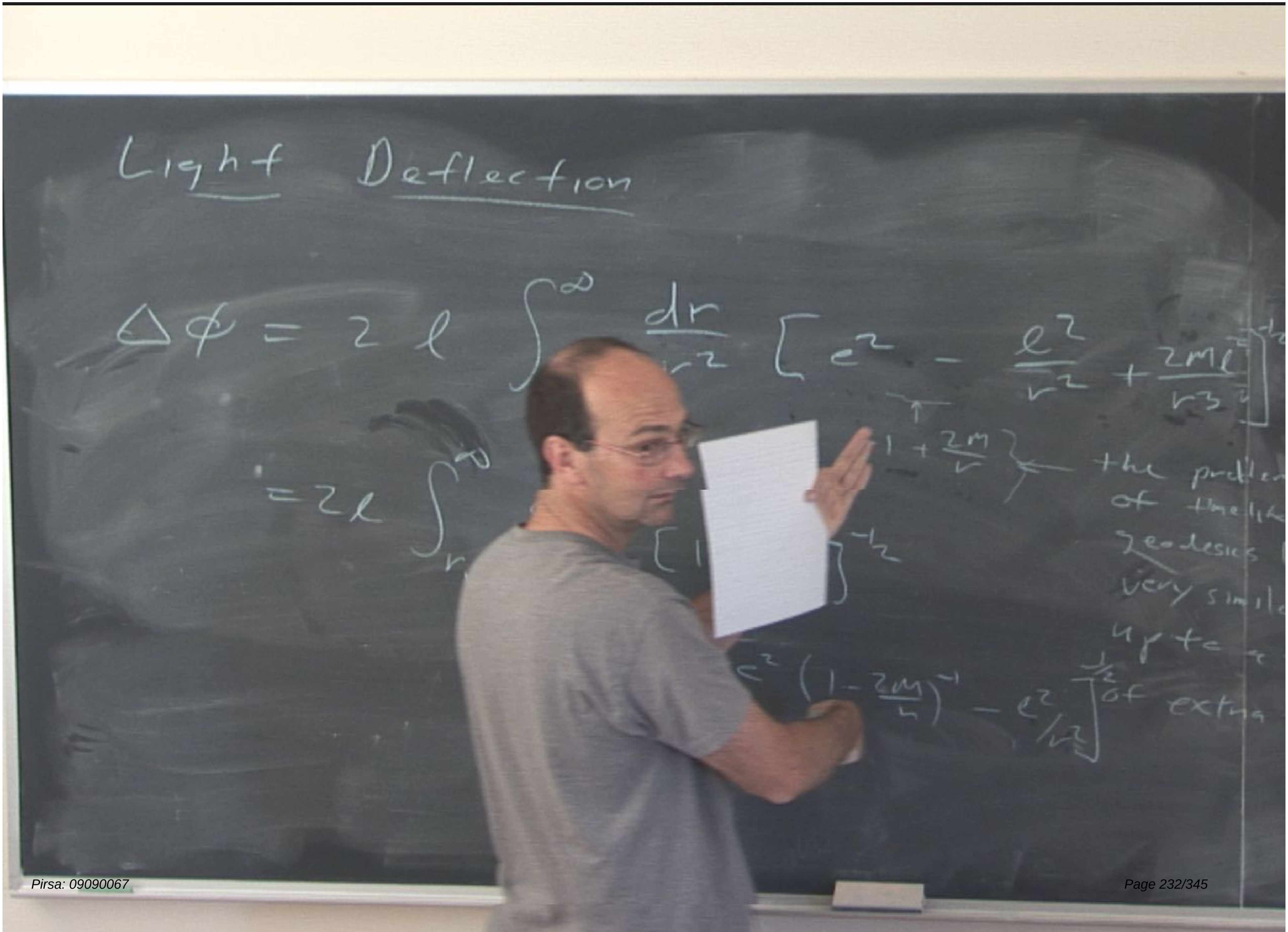
Light Deflection

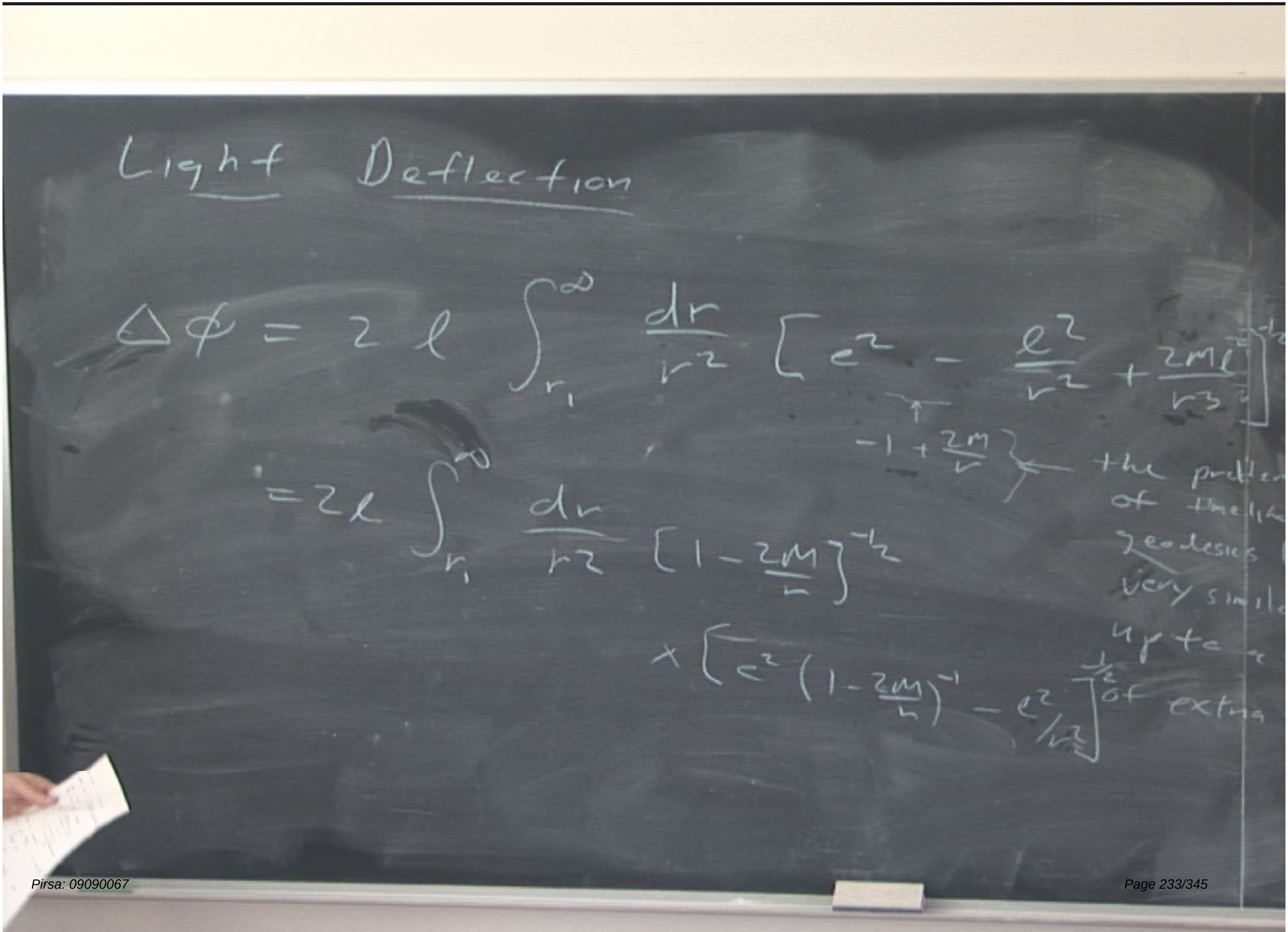
$$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[e^2 - \frac{\ell^2}{r^2} + \frac{2M\ell^2}{r^3}\right]^{-1/2}$$

$-1 + \frac{2M}{r}$ ← the problem of timelike geodesics very similar up to ... of extra

$$= 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[1 - \frac{2M}{r}\right]^{-1/2} \times \left[e^2\left(1-\frac{2M}{r}\right)^{-1} - \ell^2/r^2\right]^{-1/2}$$


Light Deflection
$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}$
$= 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}$
very simil
of extra


Light Deflection


Light Deflection
$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[e^2 - \frac{\ell^2}{r^2} + \frac{2M\ell^2}{r^3}\right]^{-1/2}$
$-1 + \frac{2M}{r}$
$= 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[1 - \frac{2M}{r}\right]^{-1/2}$
$\times \left[e^2\left(1 - \frac{2M}{r}\right)^{-1} - \ell^2/r^2\right]^{-1/2}$
the problem
of the li
geodesics
very simil
up to
of extra

Light Deflection

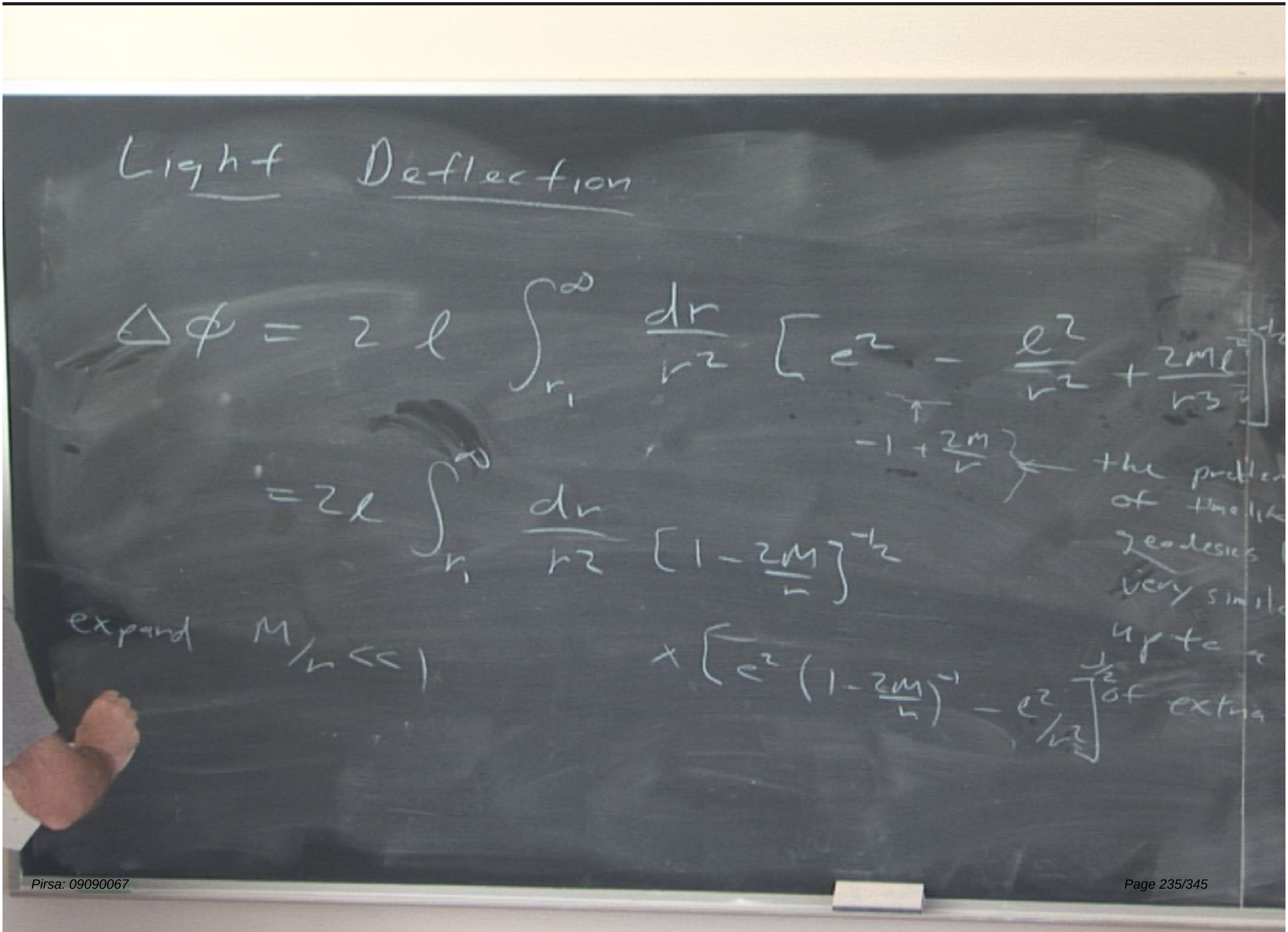
$$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[e^2 - \frac{\ell^2}{r^2} + \frac{2M\ell^2}{r^3}\right]^{-1/2}$$

e^2 ↑ $-1 + \frac{2M}{r}$ ← the probl… of timelike geodesics very simil… up to … of extra

$$= 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[1 - \frac{2M}{r}\right]^{-1/2}$$

$$\times\left[e^2\left(1-\frac{2M}{r}\right)^{-1} - e^2/r^2\right]^{-1/2}$$

ex…


Light Deflection
$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[e^2 - \frac{\ell^2}{r^2} + \frac{2M\ell^2}{r^3}\right]^{-1/2}$
$-1 + \frac{2M}{r}$
the problem of timelike geodesics very similar up to ... of extra
$= 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[1 - \frac{2M}{r}\right]^{-1/2}$
expand $M/r \ll 1$
$\times\left[e^2\left(1-\frac{2M}{r}\right)^{-1} - e^2/r^2\right]^{-1/2}$

Light Deflection

$$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[e^2 - \frac{\ell^2}{r^2} + \frac{2M\ell^2}{r^3}\right]^{-1/2}$$

$-1 + \frac{2M}{r}$ ← the problem of timelike geodesics very similar up to ... of extra

$$= 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[1 - \frac{2M}{r}\right]^{-1/2}$$

$$\times\left[e^2\left(1-\frac{2M}{r}\right)^{-1} - e^2/r^2\right]^{-1/2}$$

expand $M/r \ll 1$

change variable: $w = b/r$

Light Deflection

$$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[e^2 - \frac{\ell^2}{r^2} + \frac{2M\ell^2}{r^3}\right]^{-1/2}$$

$\uparrow$

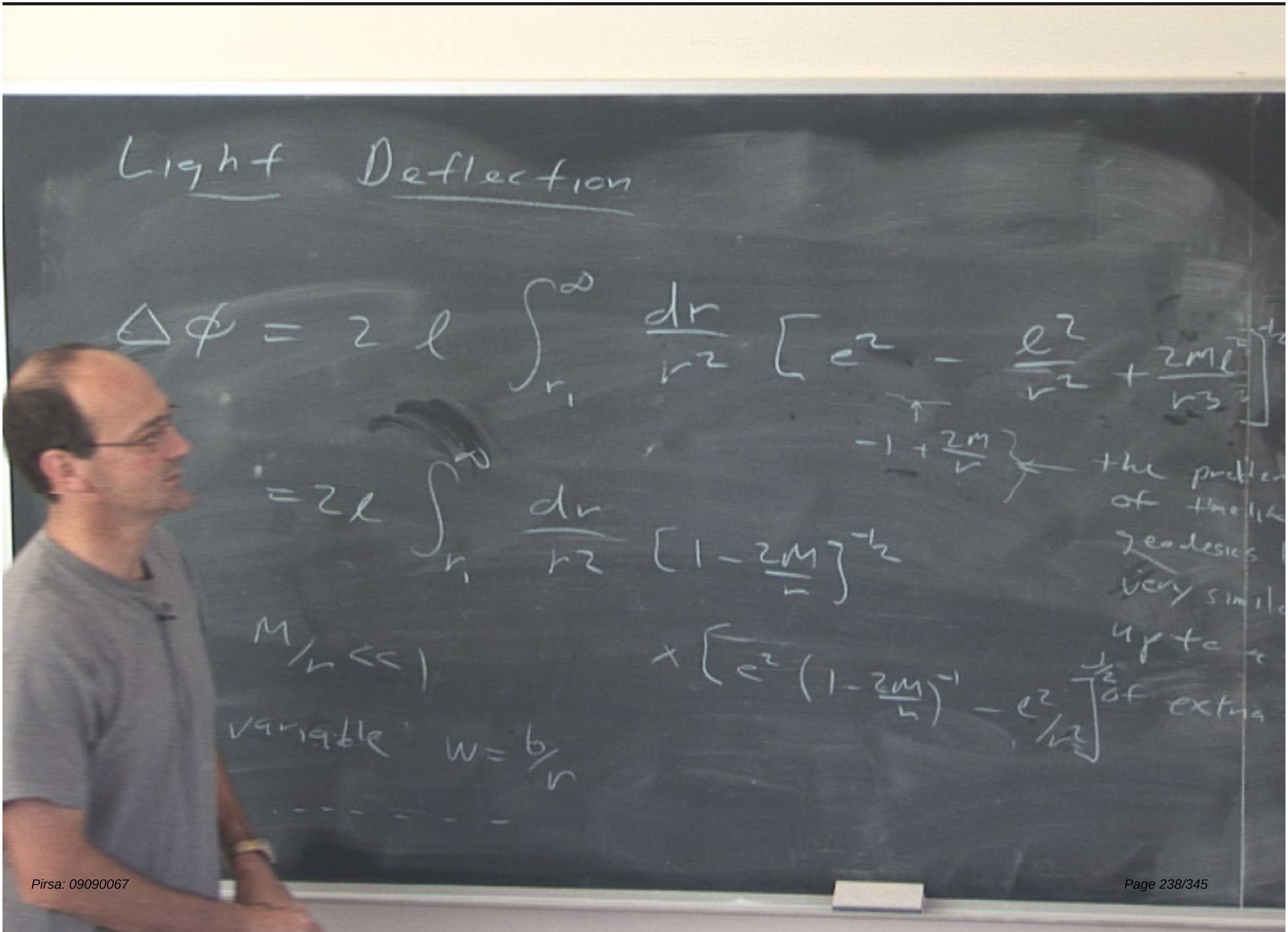
$-1 + \frac{2M}{r}$ } ← the problem of timelike geodesics very similar up to ... of extra

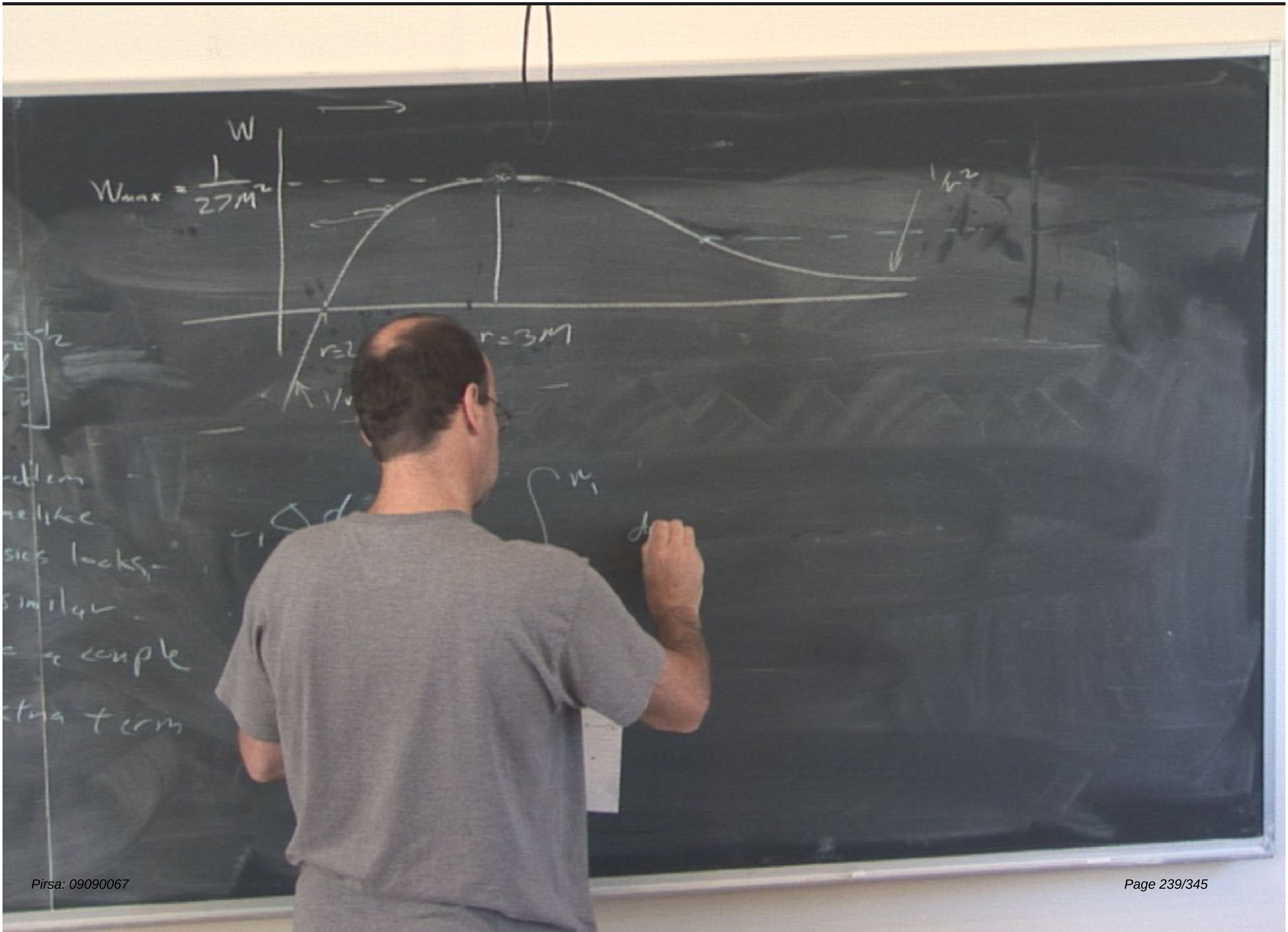
$$2\ell \int_{r_1}^{\infty} \frac{dr}{r^2}\left[1 - \frac{2M}{r}\right]^{-1/2}$$

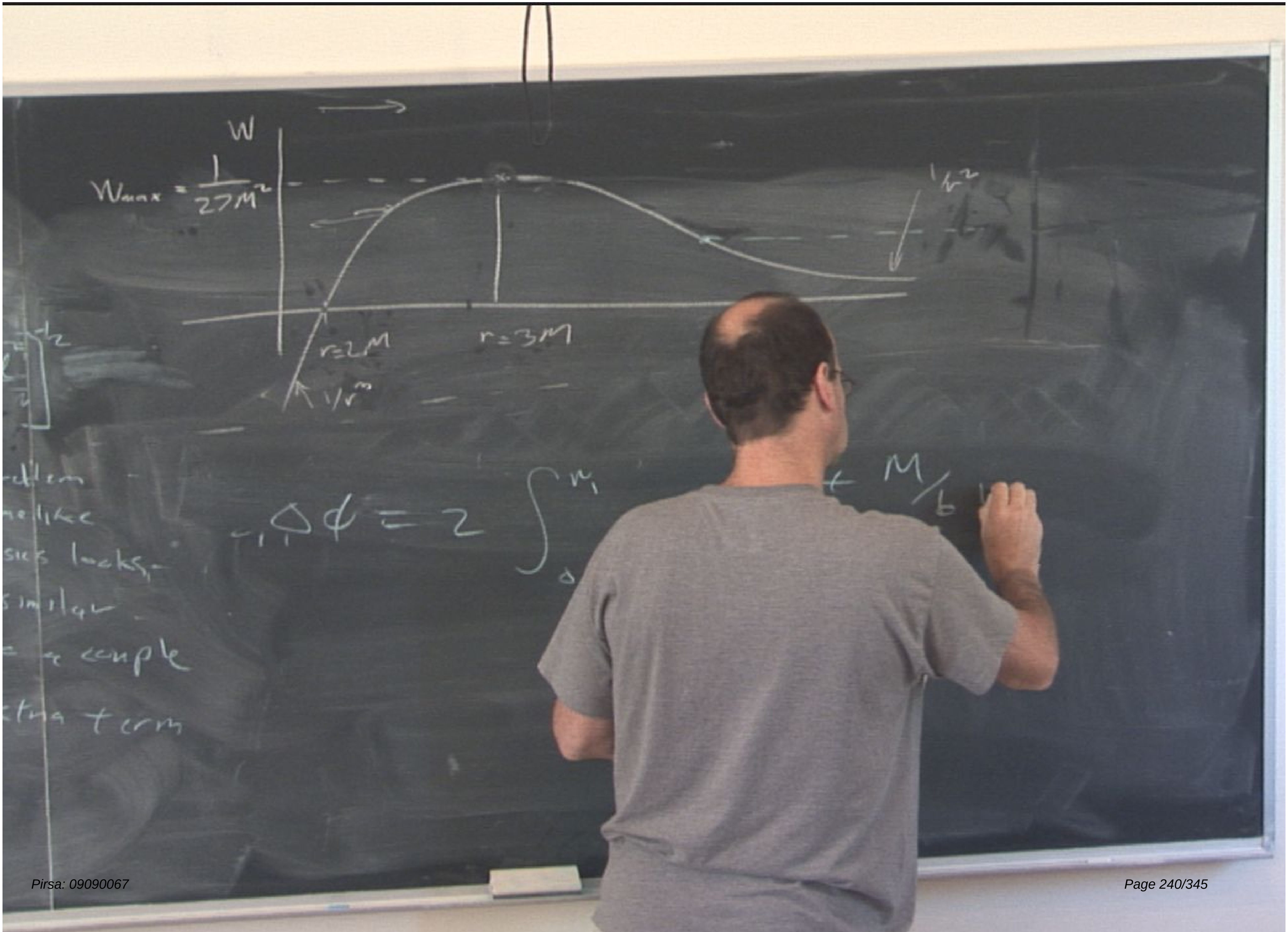
$$\times \left[e^2\left(1 - \frac{2M}{r}\right)^{-1} - e^2/r^2\right]^{-1/2}$$

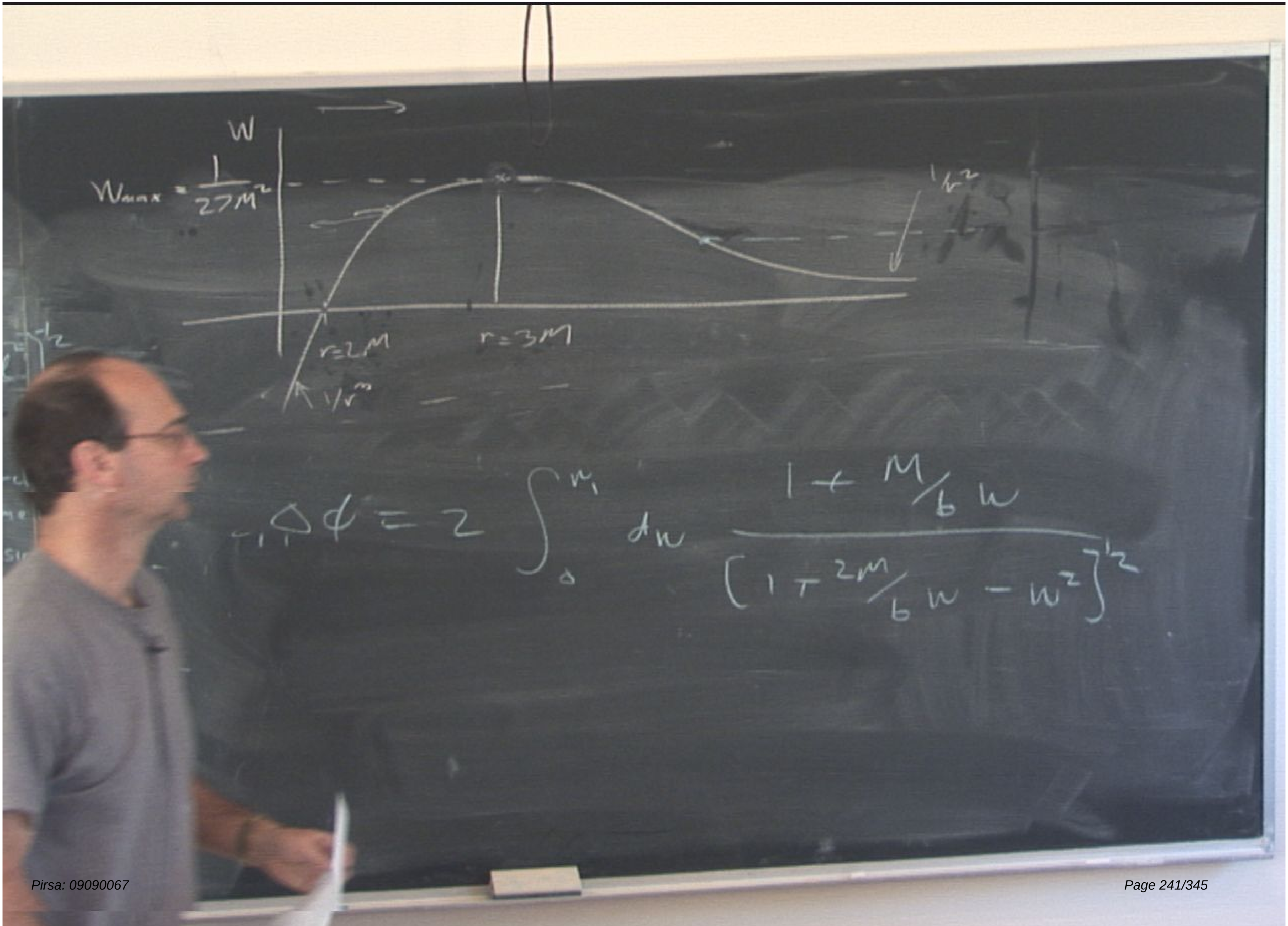
$M/r \ll 1$

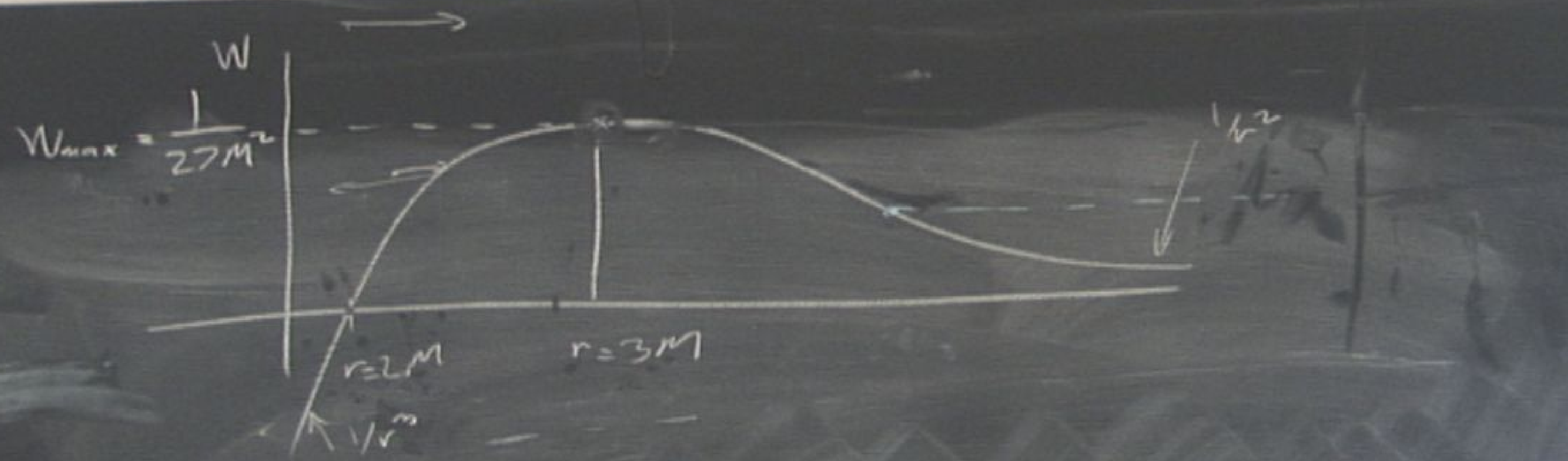
variable $w = b/r$


Light Deflection
$\Delta\phi = 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2} \left[e^2 - \frac{\ell^2}{r^2} + \frac{2M\ell^2}{r^3} \right]^{-1/2}$
$-1 + \frac{2M}{r}$
the problem of timelike geodesics very similar up to extra
$= 2\ell \int_{r_1}^{\infty} \frac{dr}{r^2} \left[1 - \frac{2M}{r}\right]^{-1/2}$
$\times \left[e^2 \left(1 - \frac{2M}{r}\right)^{-1} - \ell^2/r^2 \right]^{-1/2}$
$M/r \ll 1$
variable $w = b/r$


W
r=3M
similar
term


W
Wmax = L/27M²
r=2M
r=3M
Δφ = 2 ∫
a couple
term


W
Wmax = 1/27M²
r=2M
r=3M
1/r^m
1/r²
Δφ = 2 ∫₀^r₁ dw (1 + M/bw) / [1 + 2M/bw − w²]^½



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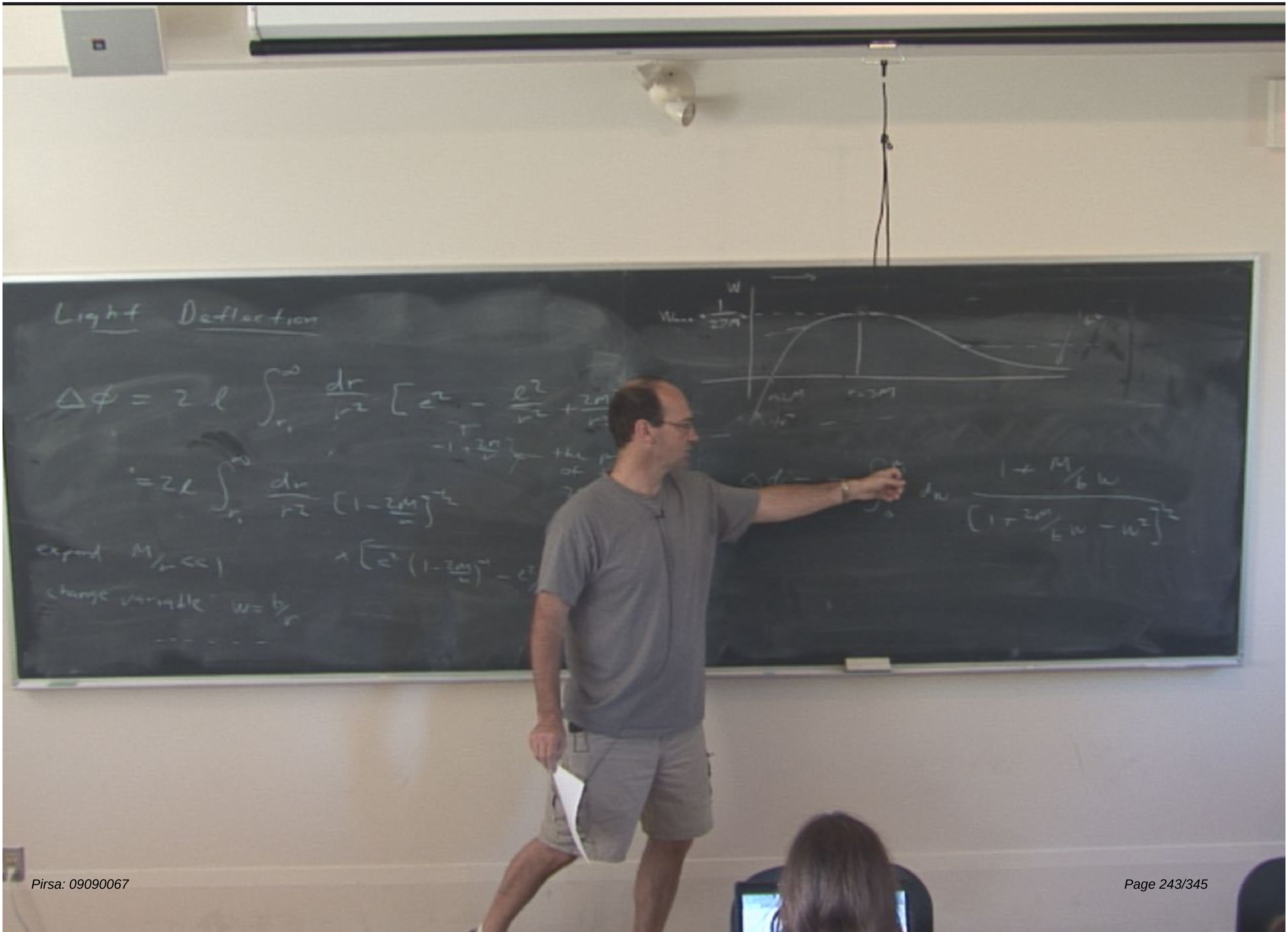
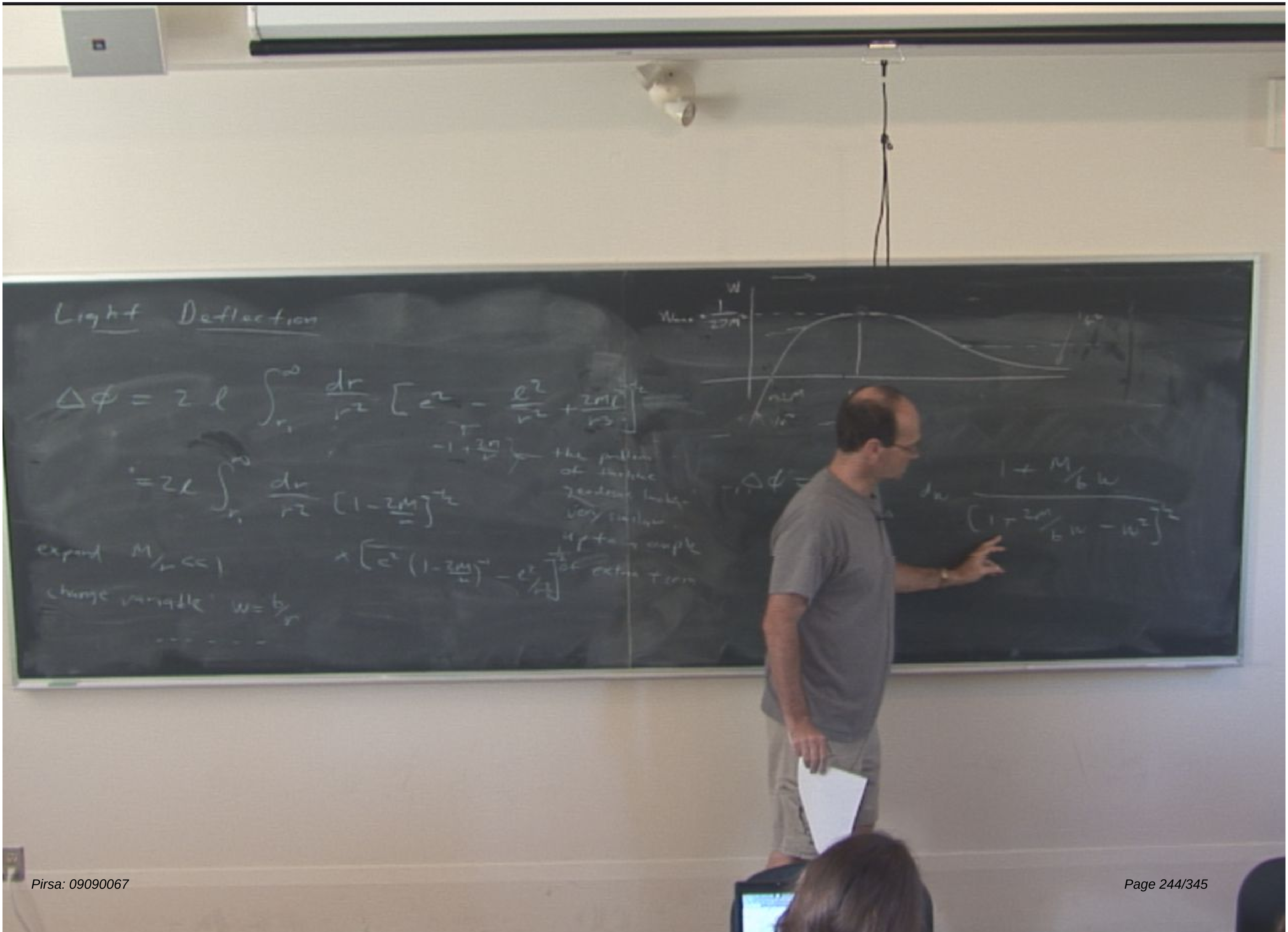
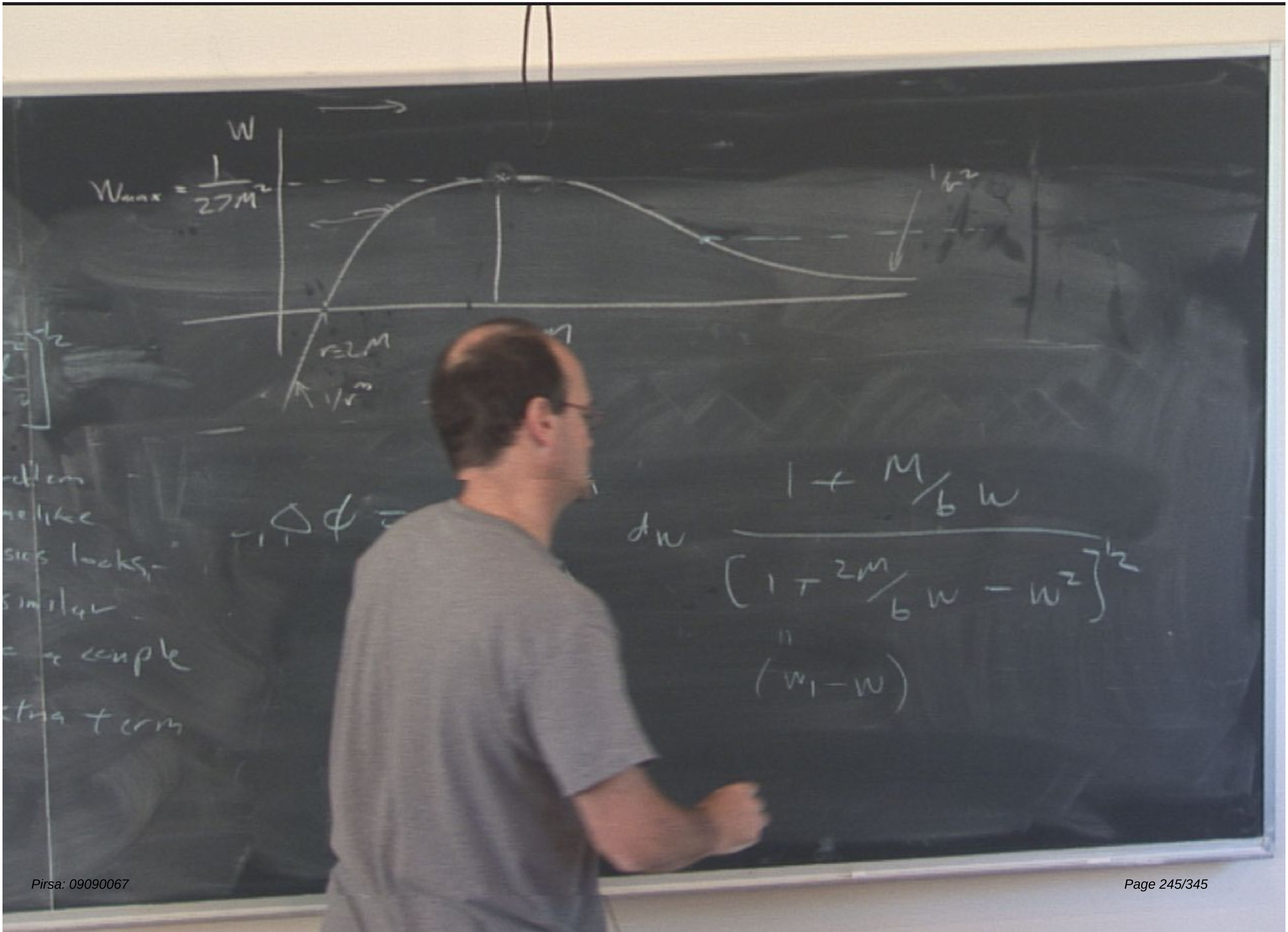
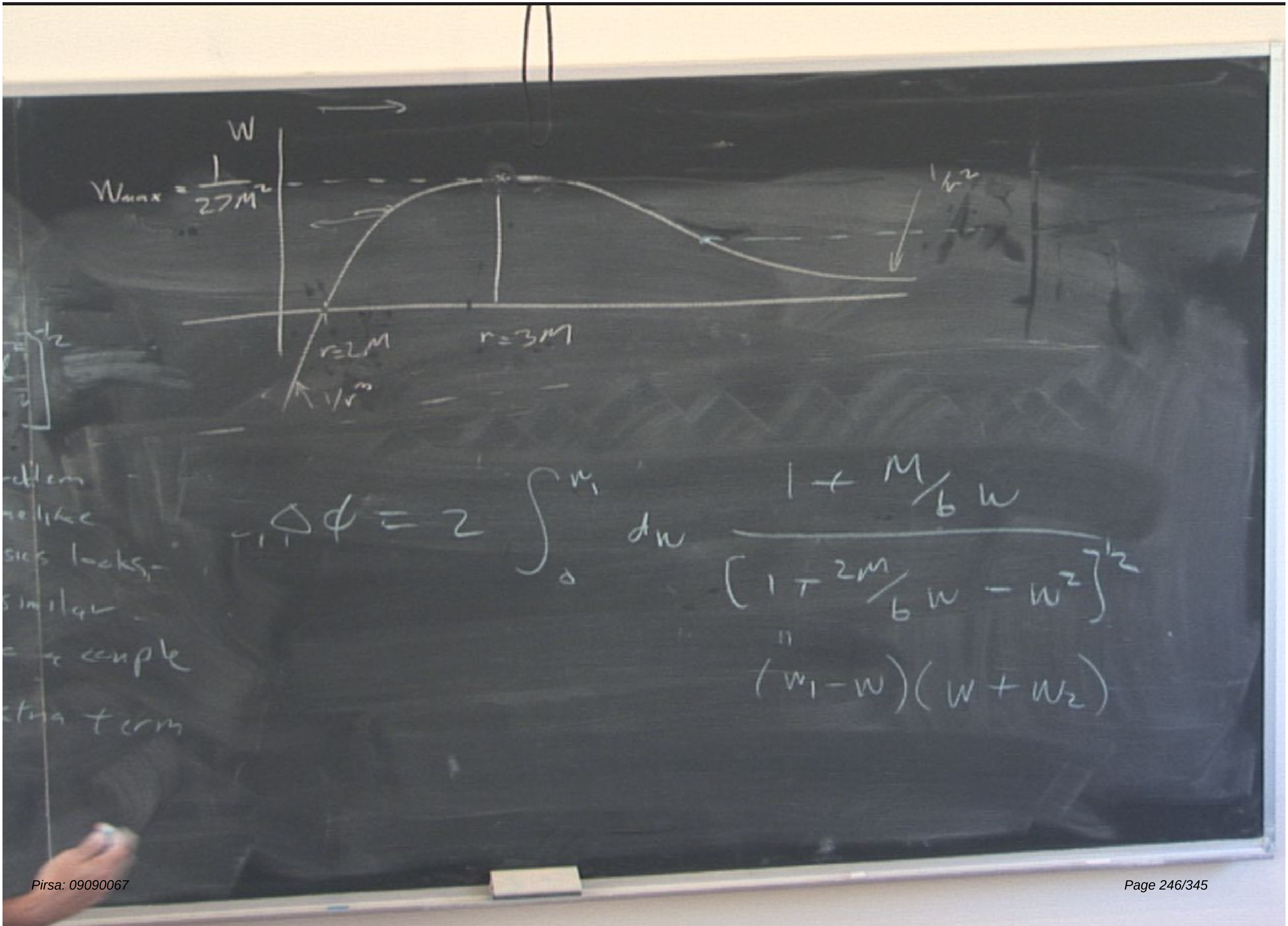
sics looks
similar.
e couple
xtra term

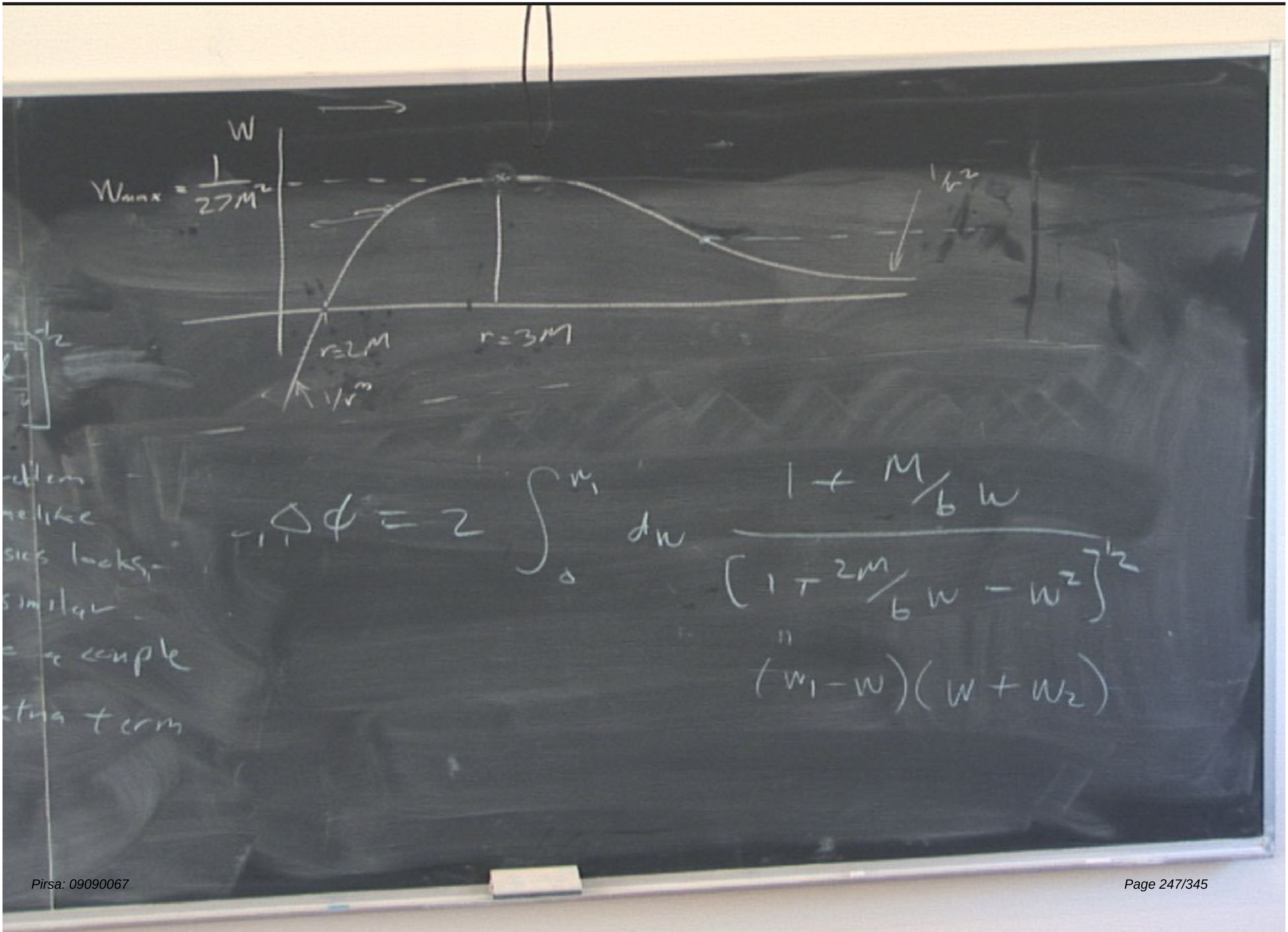
$$\Delta\phi = 2\int_0^{u_1} du \, \frac{1 + \frac{M}{b}u}{\left[1 + \frac{2M}{b}u - u^2\right]^{1/2}}$$


Light Deflection
expand M/r << 1
change variable w = b/r


Light Deflection
expand M/r << 1
change variable w = b/r


W
Wmax = 1/27M²
r=2M
similar
extra term
(w₁ − w)

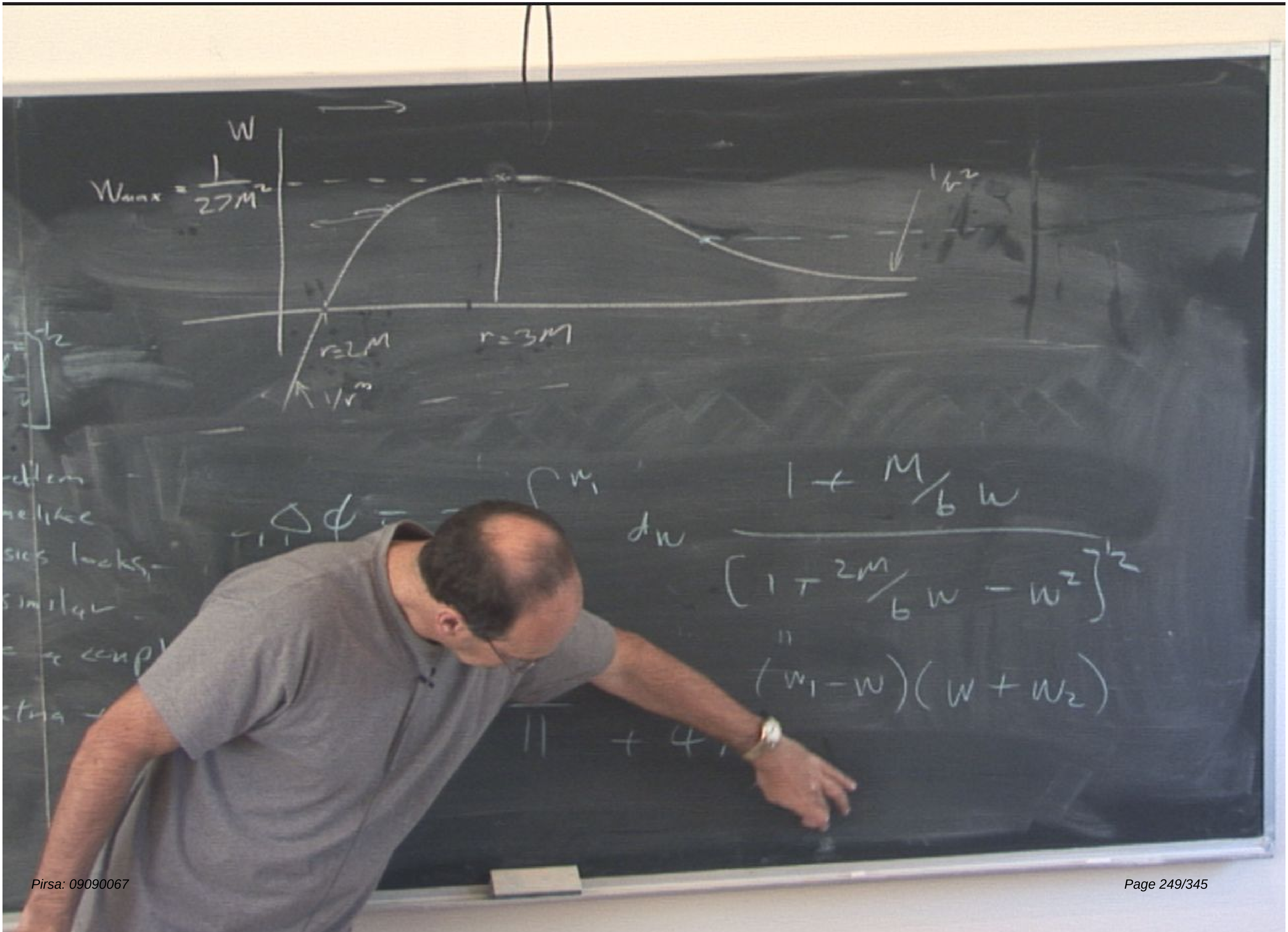

W
Wmax = 1/27M²
r=2M
r=3M
1/r²
Δφ = 2 ∫ dw (1 + M/b w) / [1 + 2M/b w − w²]^½
(w1 − w)(w + w2)
similar
extra term

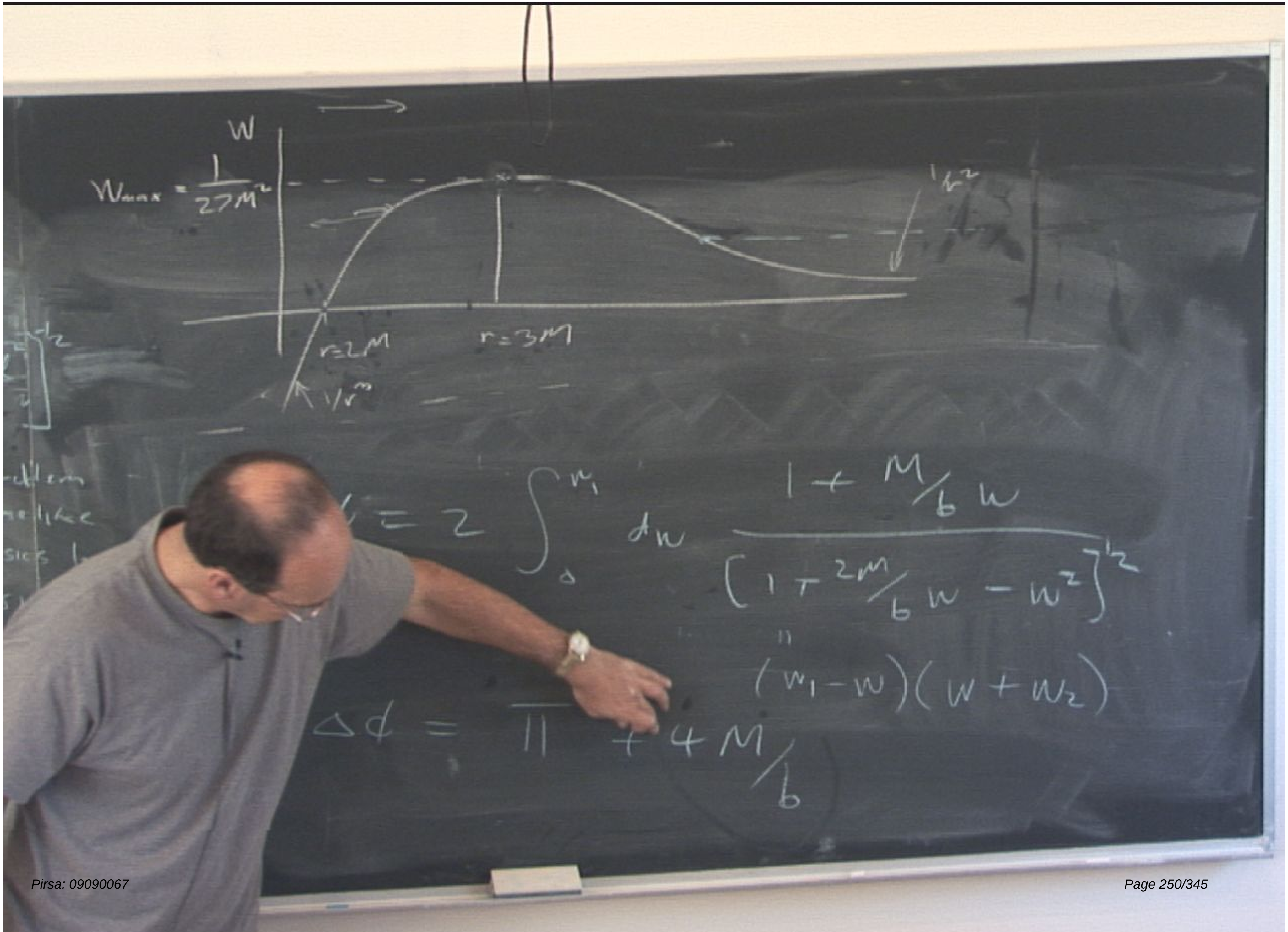

W
Wmax = 1/27M²
r=2M
r=3M
1/r²
similar
couple
extra term
Δφ = 2 ∫₀^w₁ dw (1 + M/b w) / [1 − 2M/b w − w²]^½
(w₁ − w)(w + w₂)

$$\Delta\phi = 2\int_0^{w_1} dw \, \frac{1+\frac{M}{b}w}{\left[1+\frac{2M}{b}w - w^2\right]^{1/2}}$$

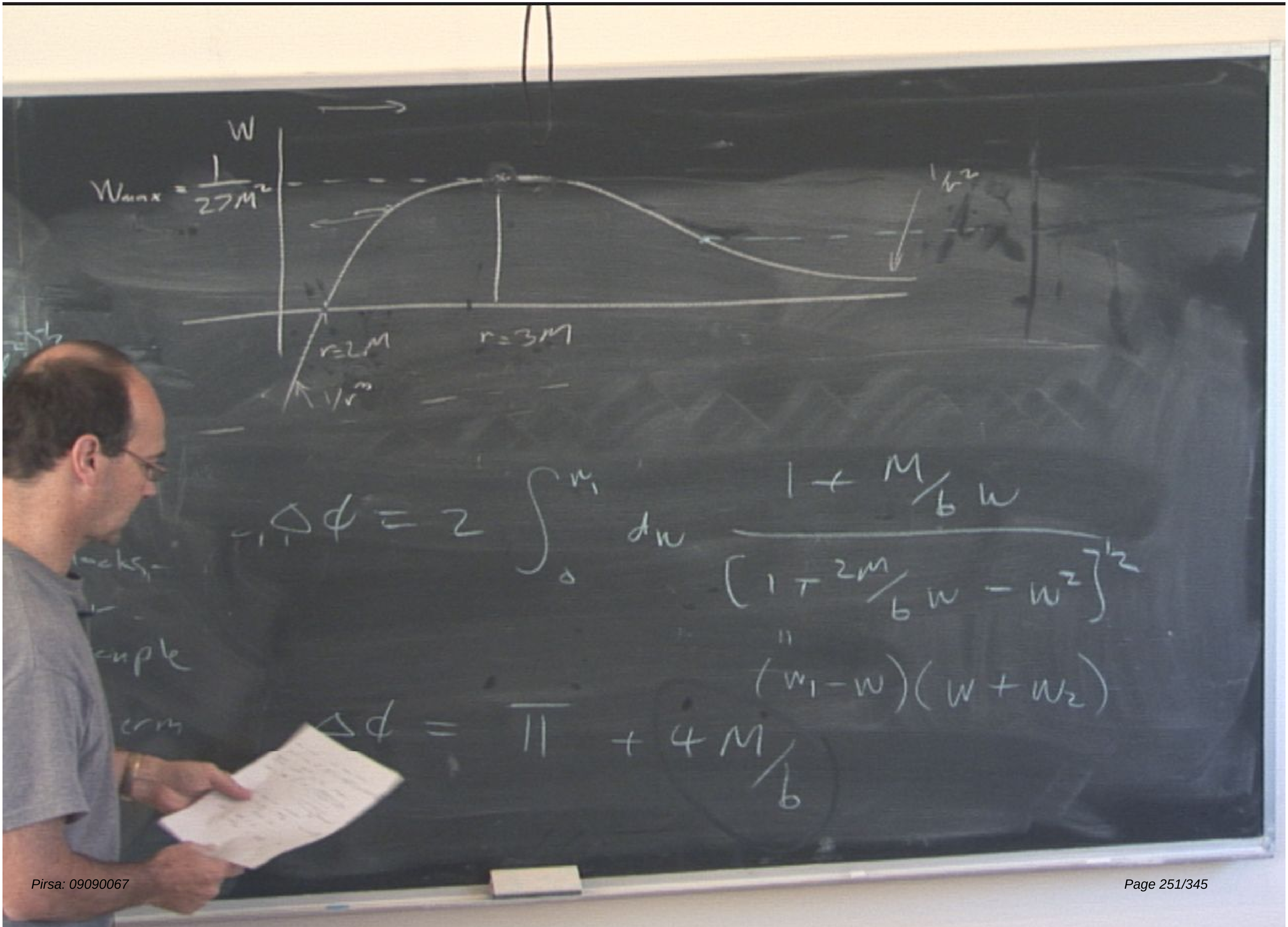
$$\left[1+\frac{2M}{b}w - w^2\right] = (w_1 - w)(w + w_2)$$

$$\Delta\phi = \pi + 4M/b$$


W
Wmax = L/27M²
r=2M
r=3M
1/r²
(w₁ − w)(w + w₂)



W
Wmax = 1/27M²
r=2M
r=3M
1/r²
1/r^m
= 2 ∫ dw
1 + M/b w
[1 + 2M/b w − w²]^½
(w1 − w)(w + w2)
Δφ = π + 4M/b

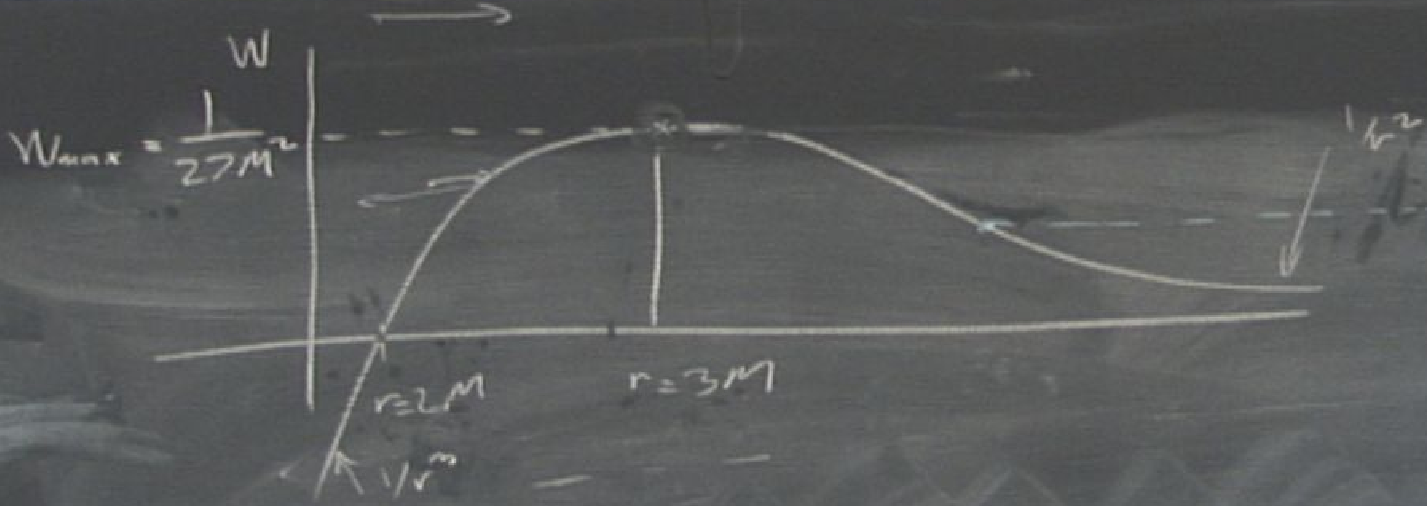

W
Wmax = 1/27M²
r=2M
r=3M
1/r²
Δφ = 2 ∫₀^{w₁} dw (1 + M/b w) / [1 + 2M/b w − w²]^{1/2}
(w₁ − w)(w + w₂)
Δφ = π + 4M/b

ctlem
ne like
sics looks
similar
a couple
xtra term

$$\Delta\phi \simeq 2\int_0^{w_1} dw \, \frac{1 + \frac{M}{b} w}{\left[1 + \frac{2M}{b} w - w^2\right]^{1/2}}$$

$$\left[1 + \frac{2M}{b} w - w^2\right] = (w_1 - w)(w + w_2)$$

$$\Delta\phi \approx \pi + \frac{4M}{b}$$



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xtra term

$$\Delta\phi \simeq 2\int_0^{w_1} dw \, \frac{1 + \frac{M}{b} w}{\left[1 + \frac{2M}{b} w - w^2\right]^{1/2}}$$

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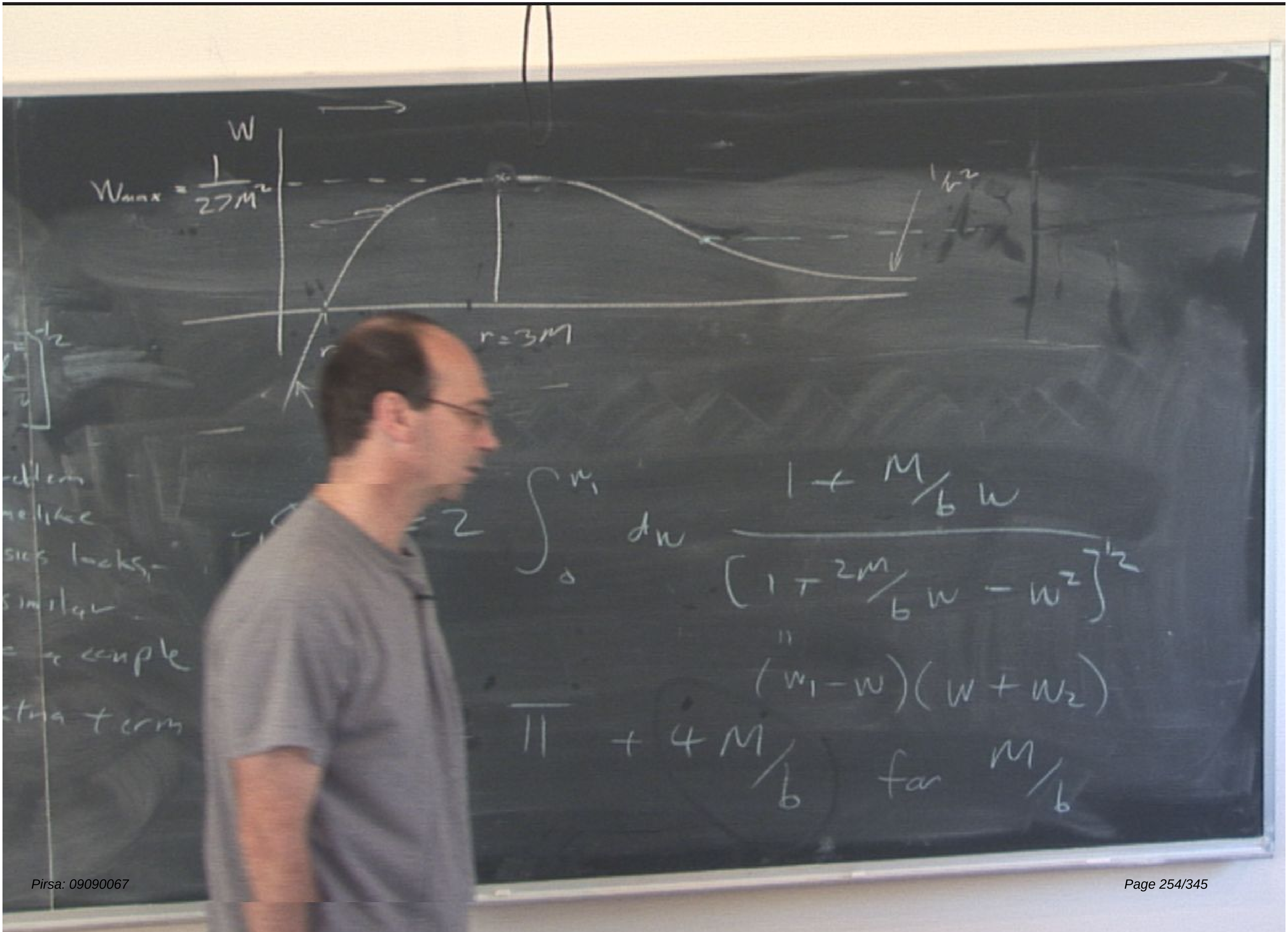
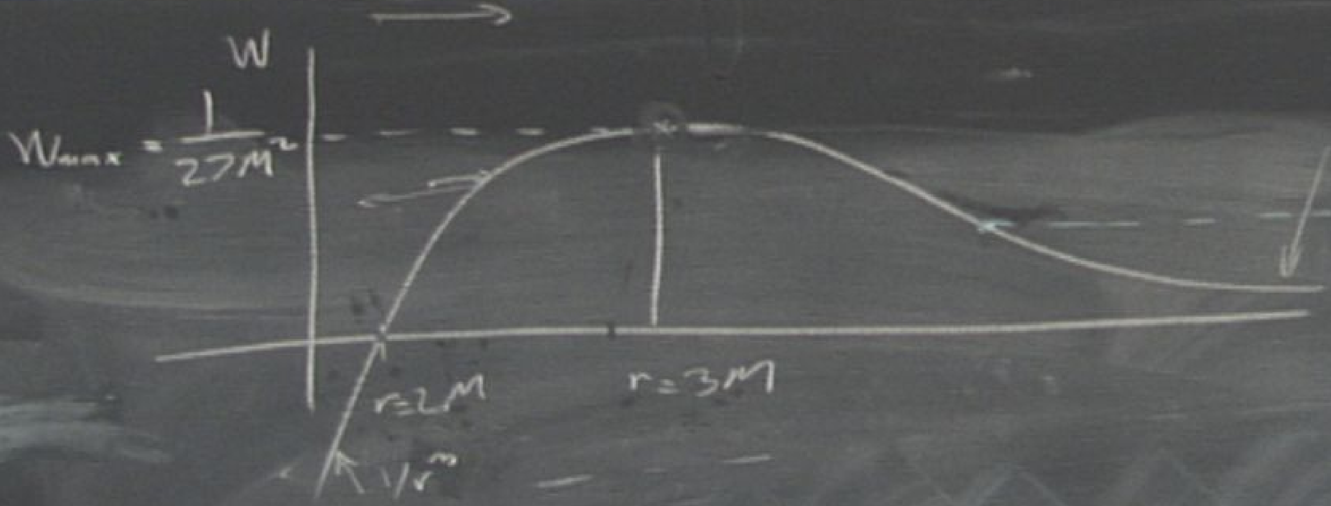
$$\Delta\phi \simeq \pi + 4M/b$$


W
Wmax = 1/27M2
r=3M
(w1 - w)(w + w2)
far M/b
extra term



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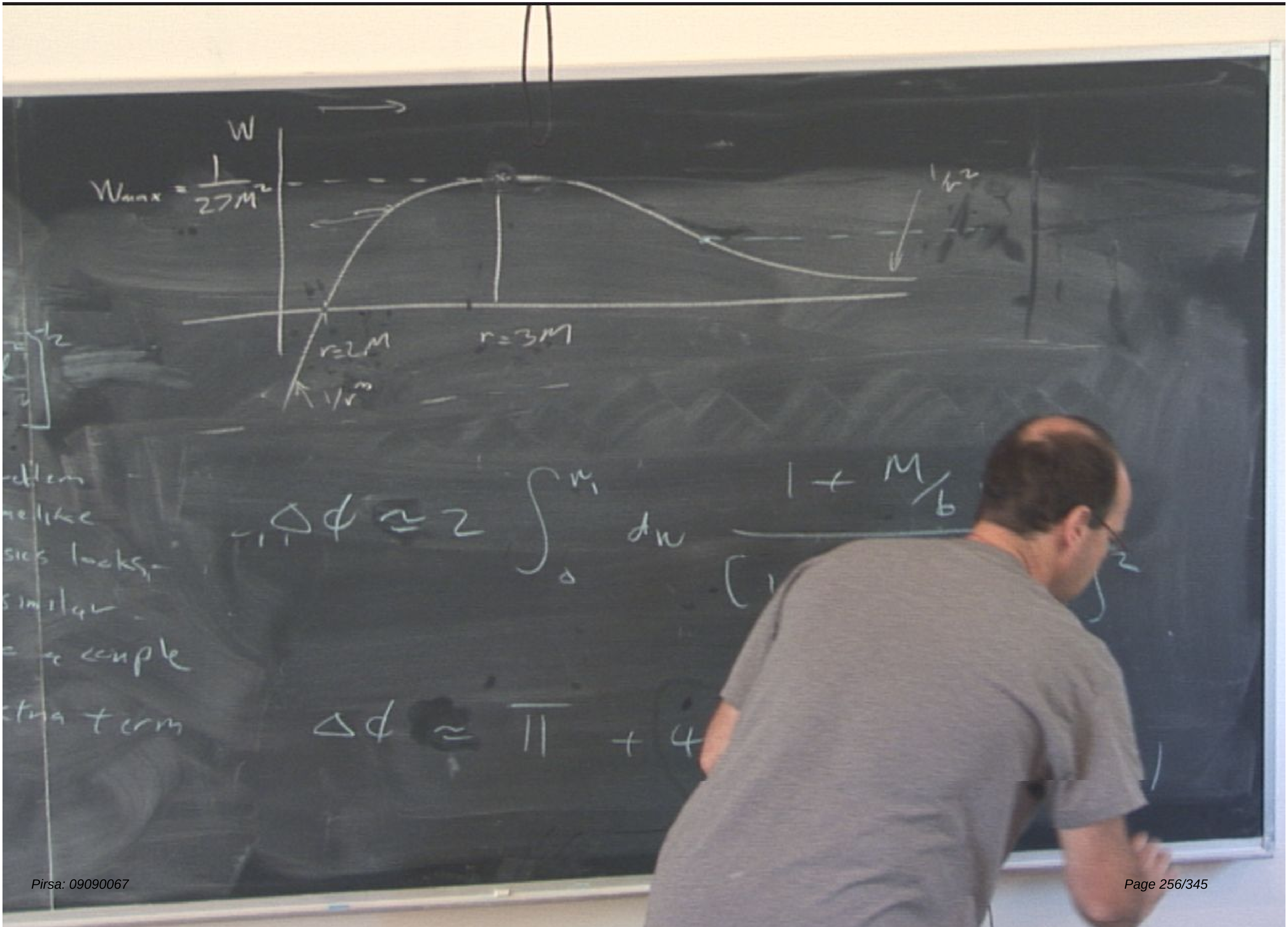
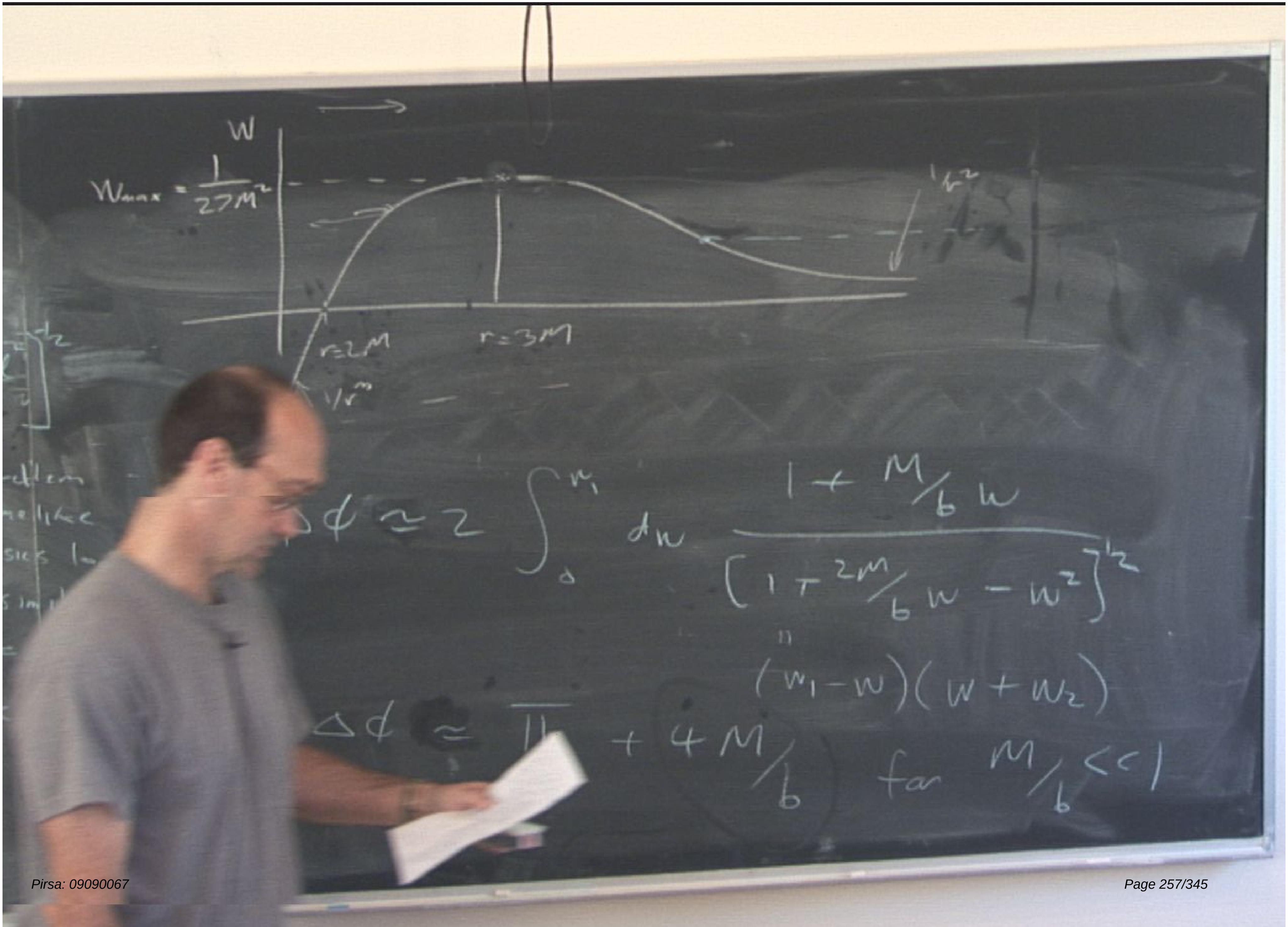
similar
… a couple
…xtra term

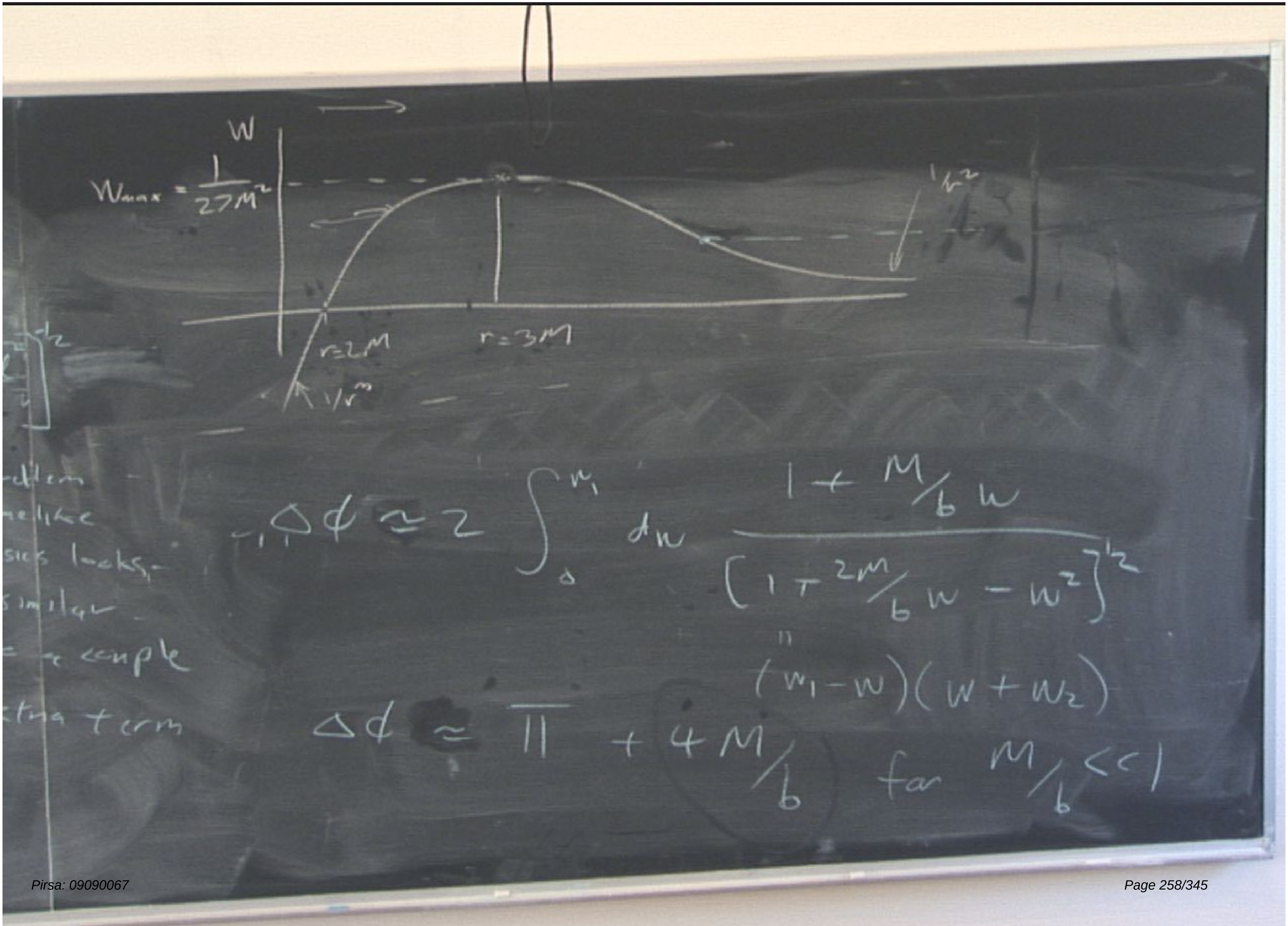
$$\Delta\phi \simeq 2\int_0^{u_1} dw \, \frac{1 + \frac{M}{b} w}{\left[1 + \frac{2M}{b} w - w^2\right]^{1/2}}$$

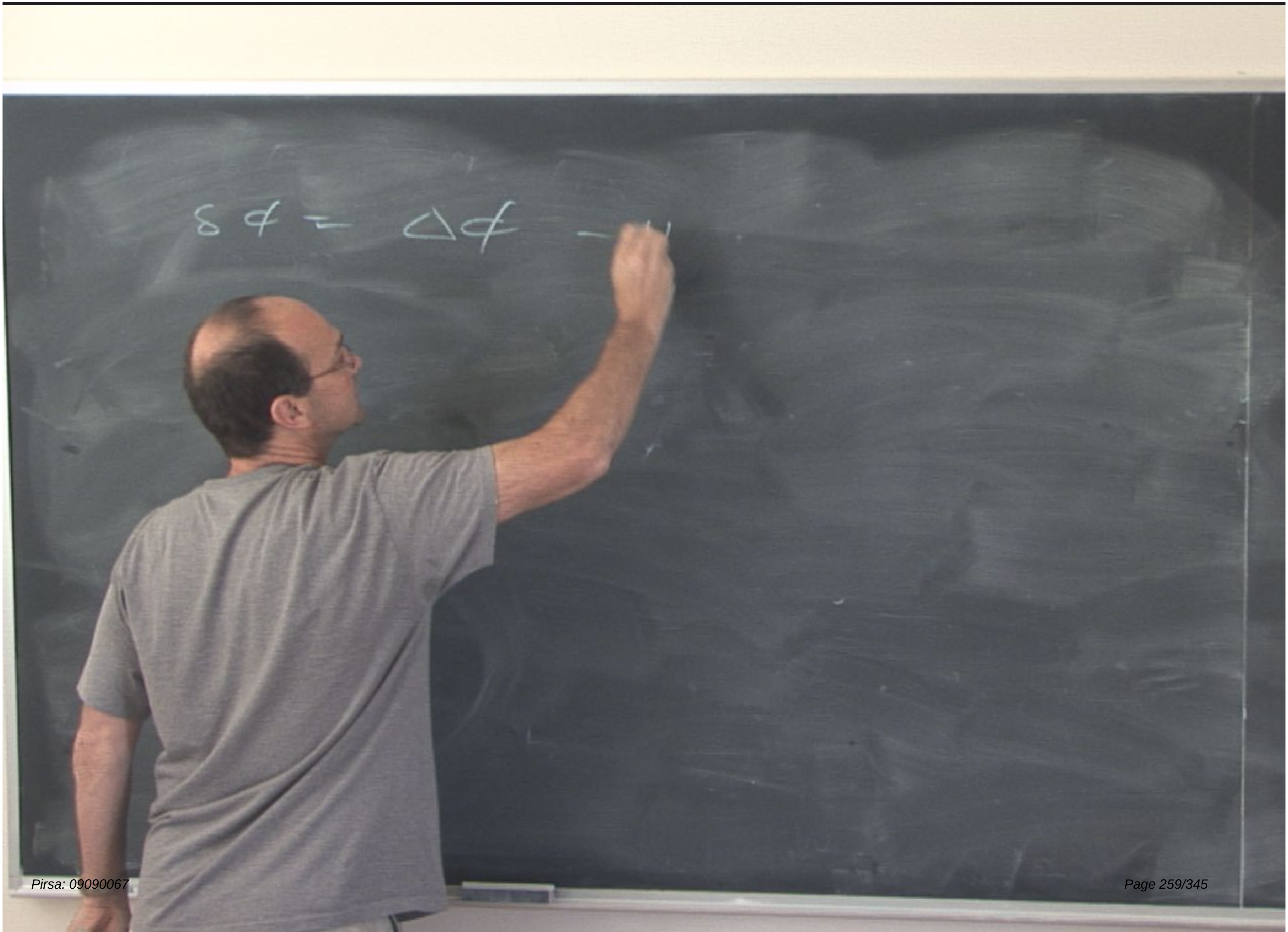
$$= (w_1 - w)(w + w_2)$$

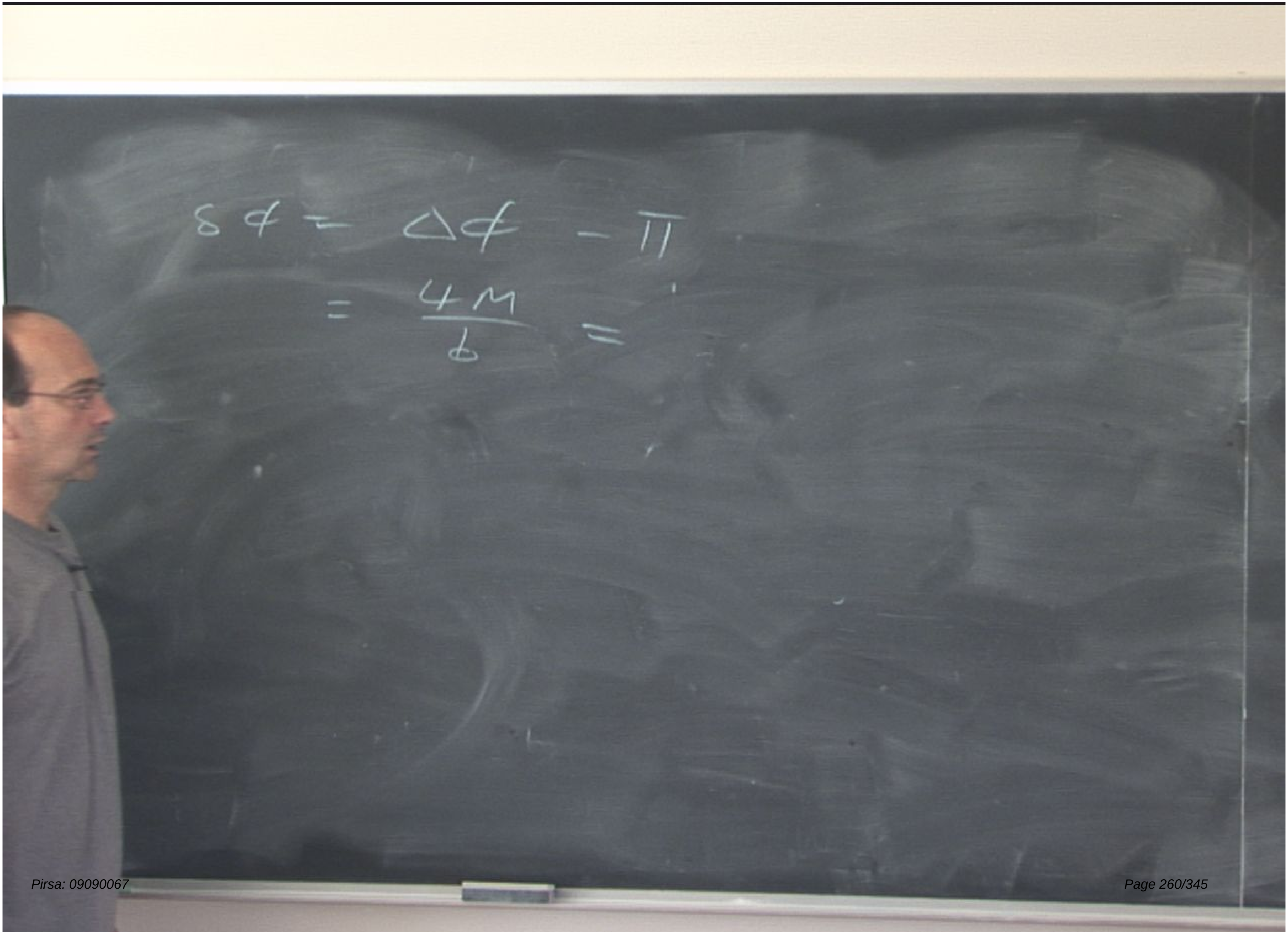
$$\Delta\phi \simeq \pi + 4M/b \quad \text{for } M/b$$

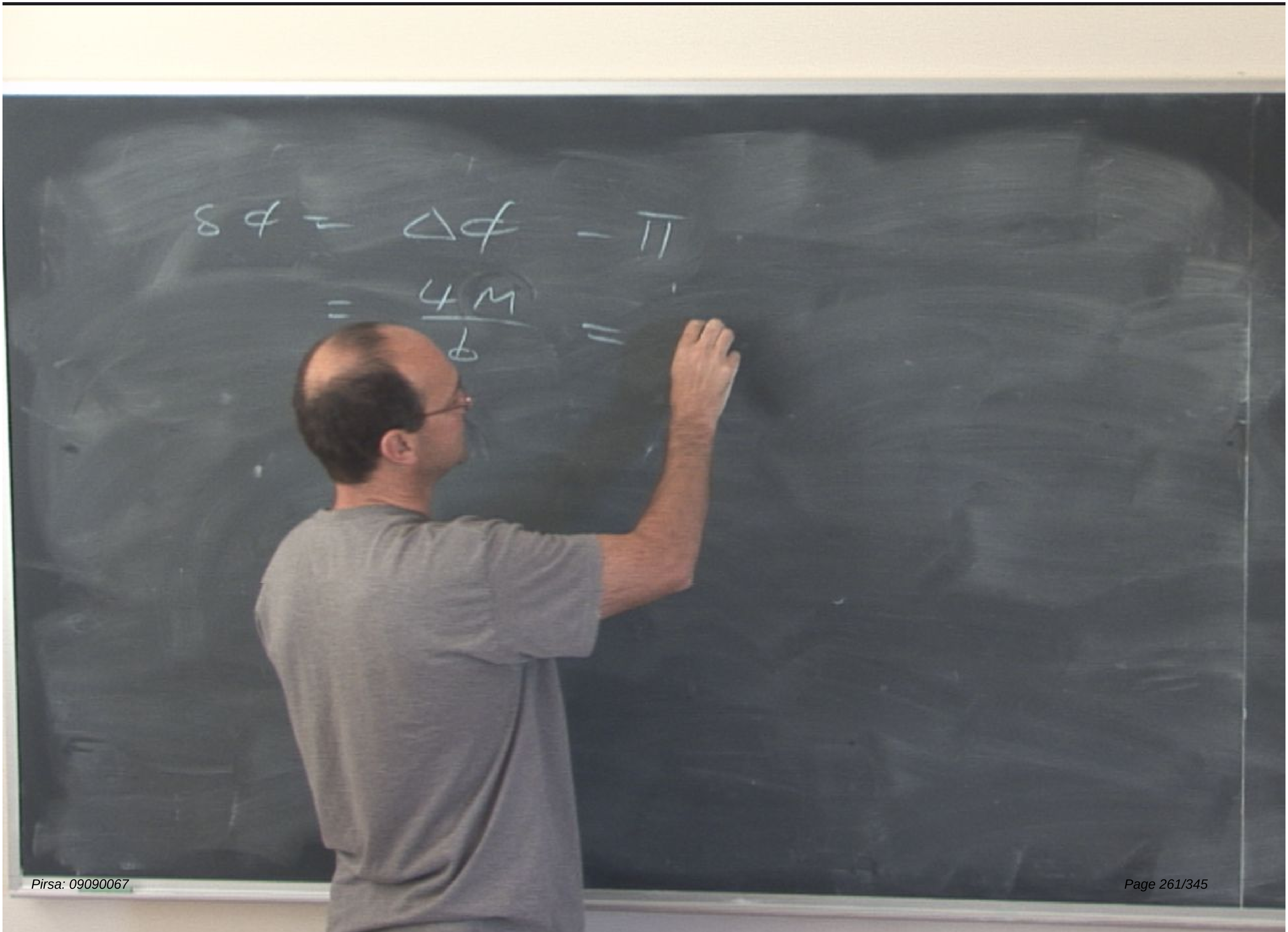

W
Wmax = 1/27M²
r=2M
r=3M
Δφ ≃ 2 ∫ dw
Δφ ≃ π + 4

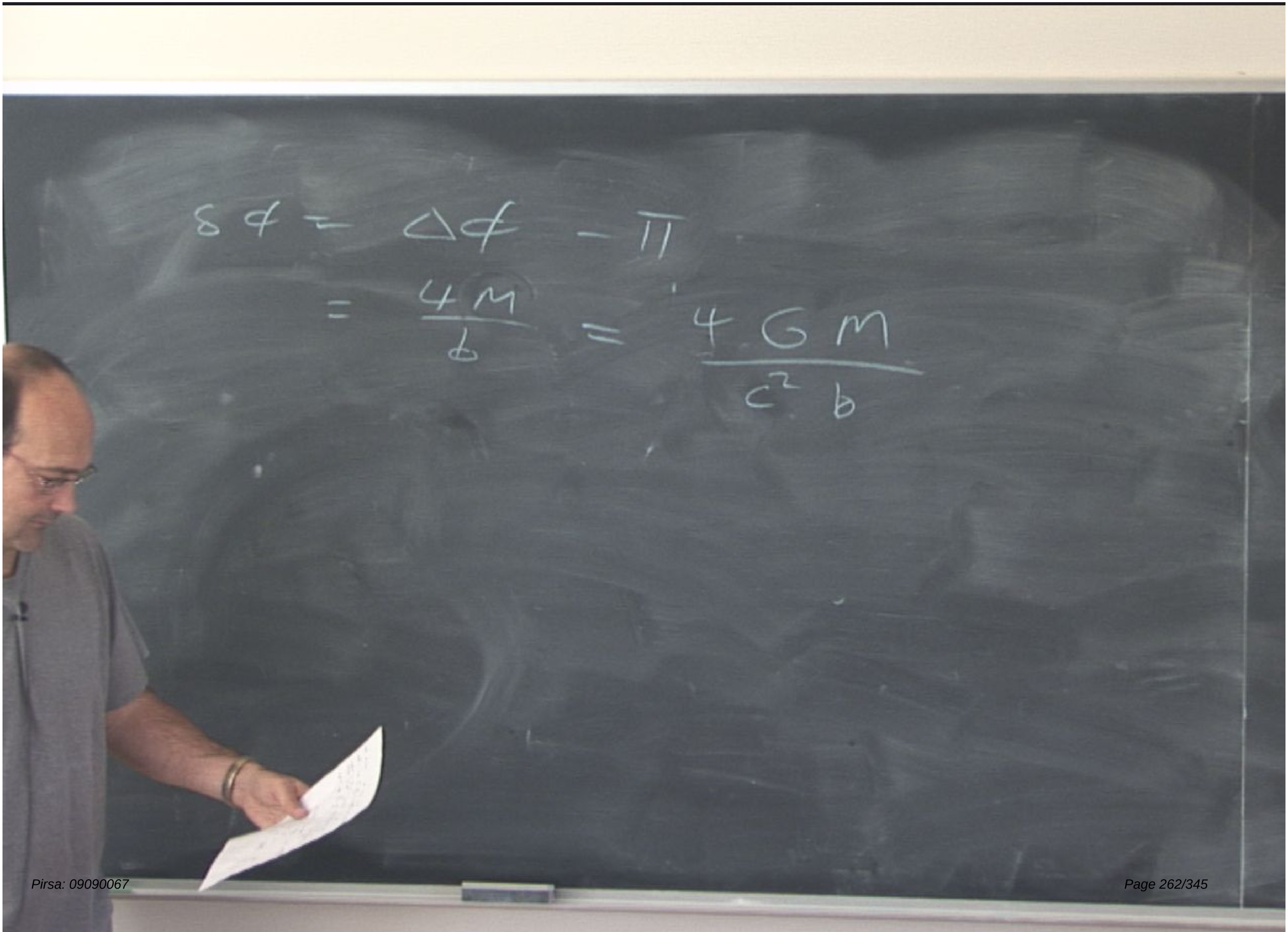

W
Wmax = 1/27M²
r=2M
r=3M
1/r²
Δφ ≃ 2 ∫₀^w₁ dw (1 + M/b w) / [1 + 2M/b w − w²]^½
(w₁ − w)(w + w₂)
Δφ ≃ π + 4M/b for M/b << 1


W
Wmax = 1/27M²
r=2M
r=3M
1/r³
1/r²
Δφ ≃ 2 ∫₀^{w₁} dw (1 + M/b w) / [1 + 2M/b w − w²]^{1/2}
(w₁ − w)(w + w₂)
Δφ ≃ π + 4M/b for M/b << 1
similar
extra term


δφ = Δφ −

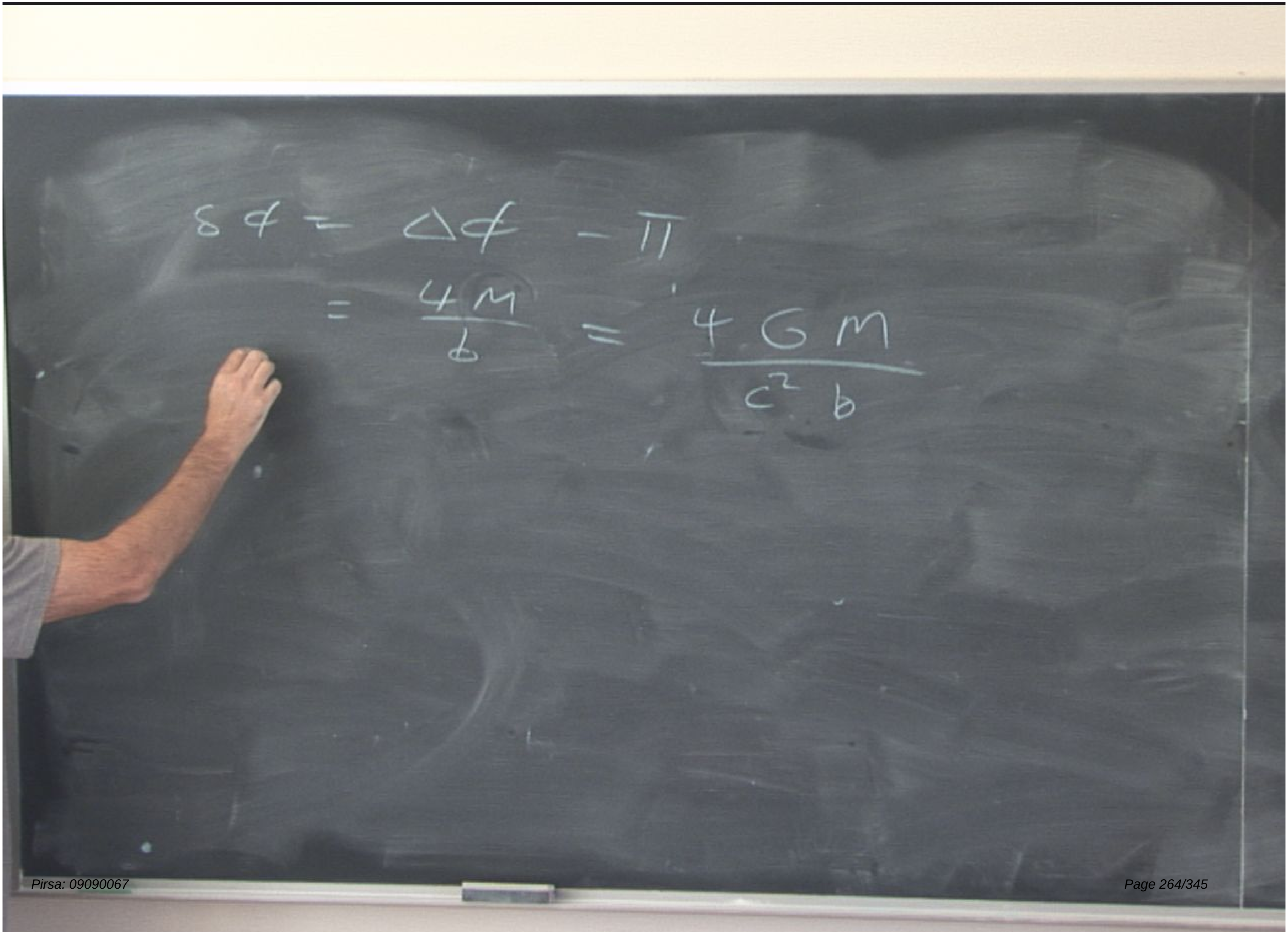

δφ = Δφ − π
= 4M / b =


δφ = Δφ − π
= 4M / b =


$\delta\phi = \Delta\phi - \pi$
$= \frac{4M}{b} = \frac{4GM}{c^2 b}$

$$\delta\phi = \Delta\phi - \pi$$

$$= \frac{4M}{b} = \frac{4GM}{c^2 b}$$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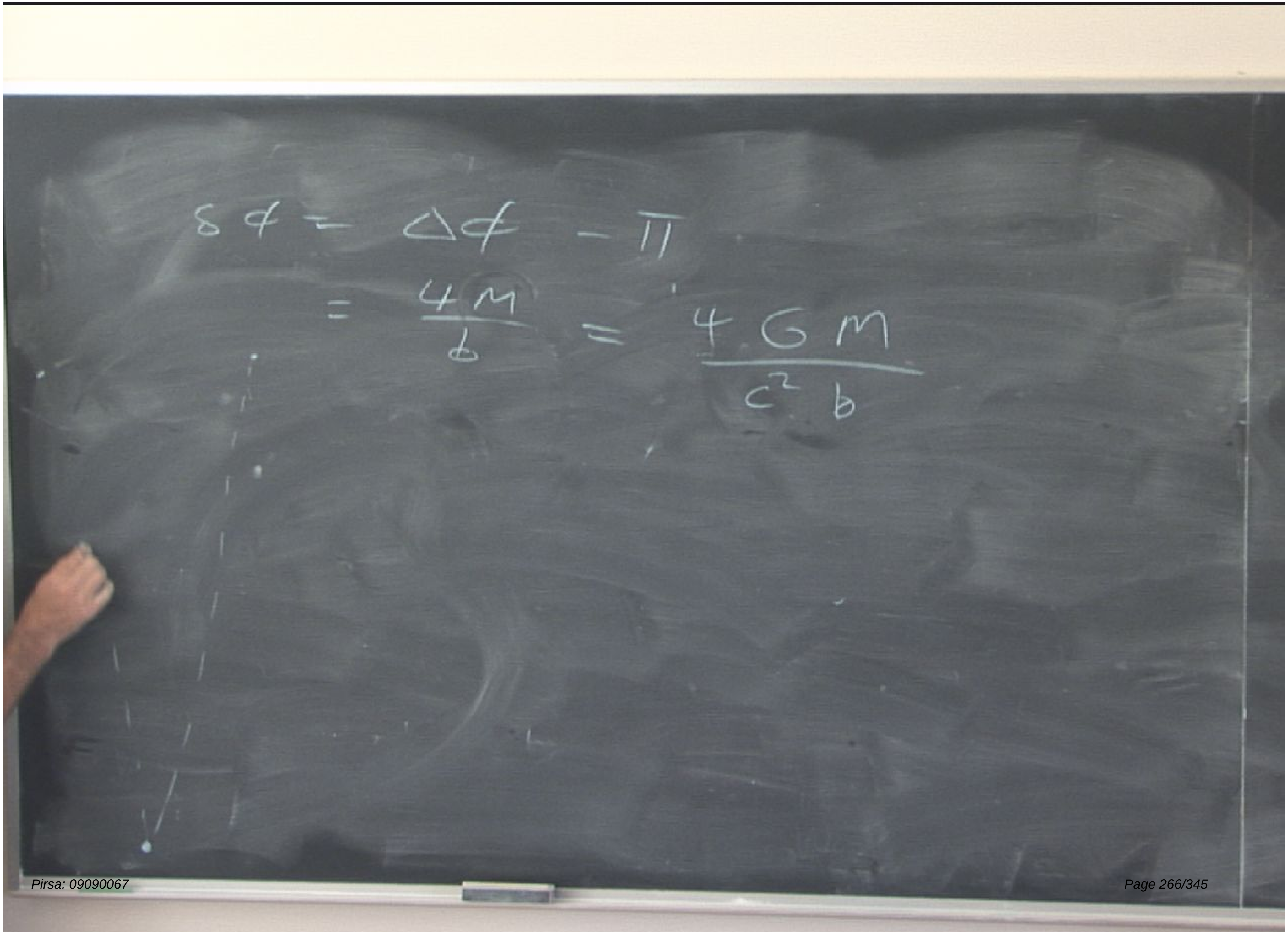
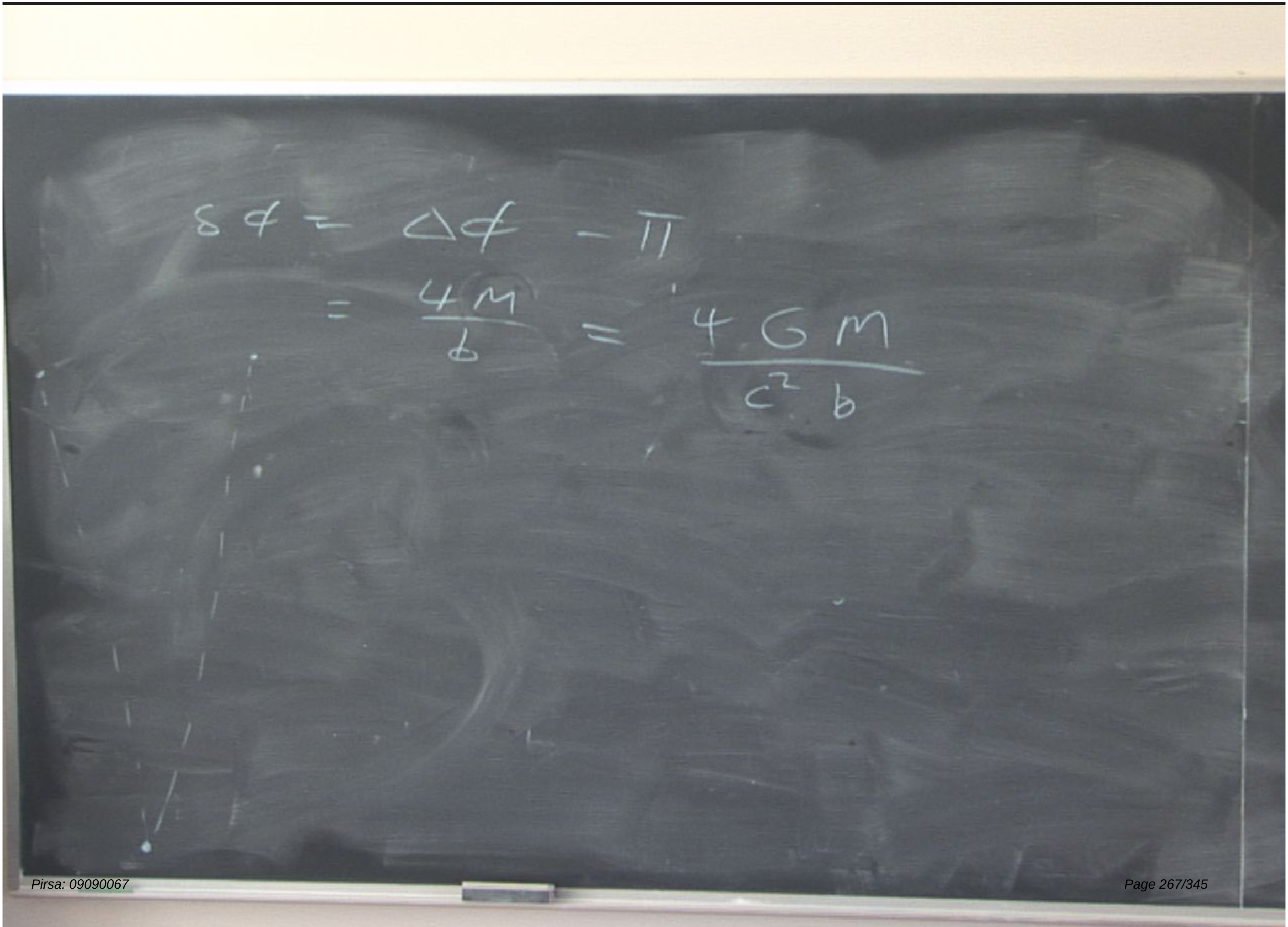
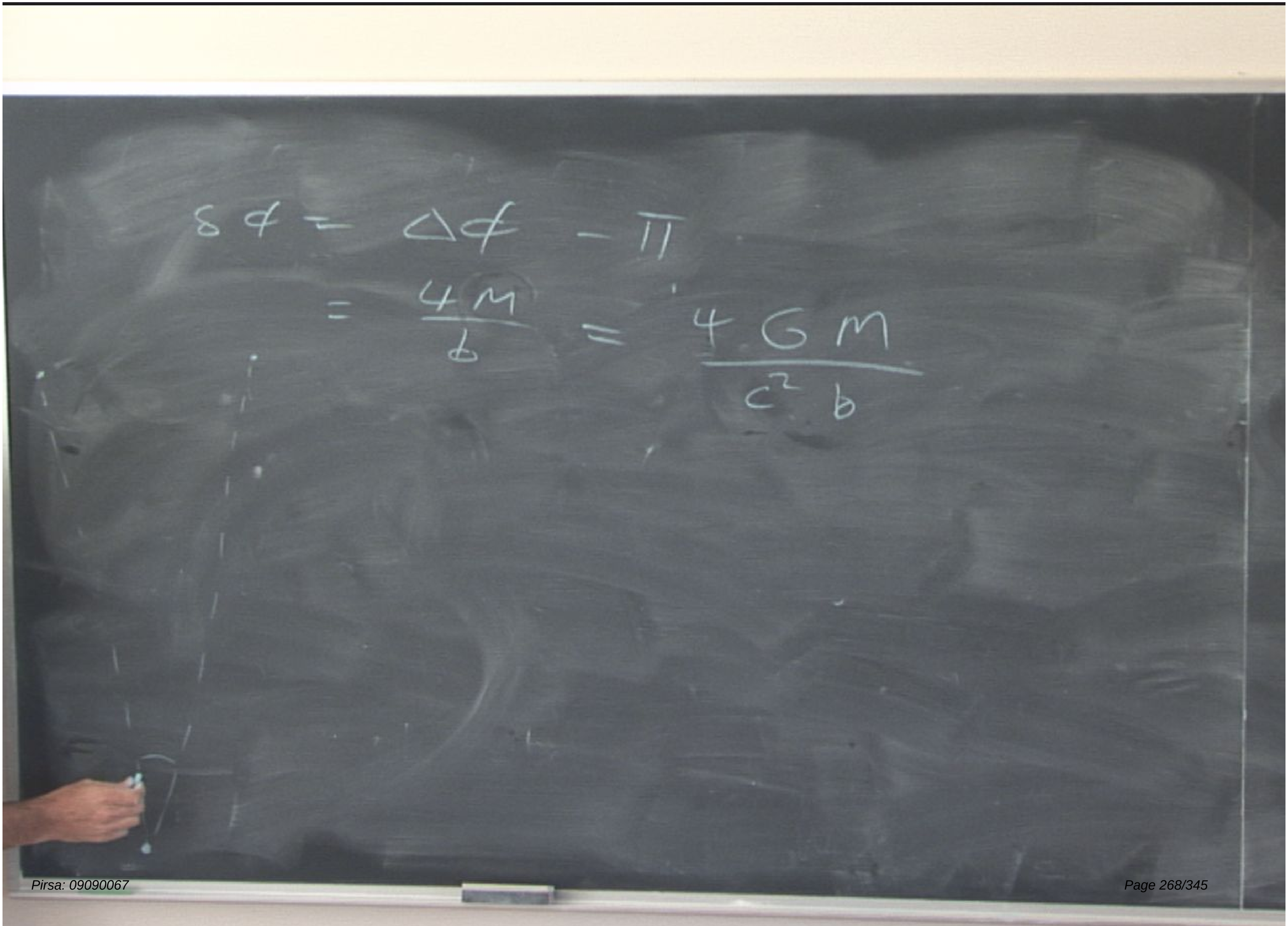

δφ = Δφ − π
= 4M / b = 4GM / c² b


$\delta\phi = \Delta\phi - \pi$
$= \frac{4M}{b} = \frac{4GM}{c^2 b}$


$\delta\phi = \Delta\phi - \pi$
$= \frac{4M}{b} = \frac{4GM}{c^2 b}$

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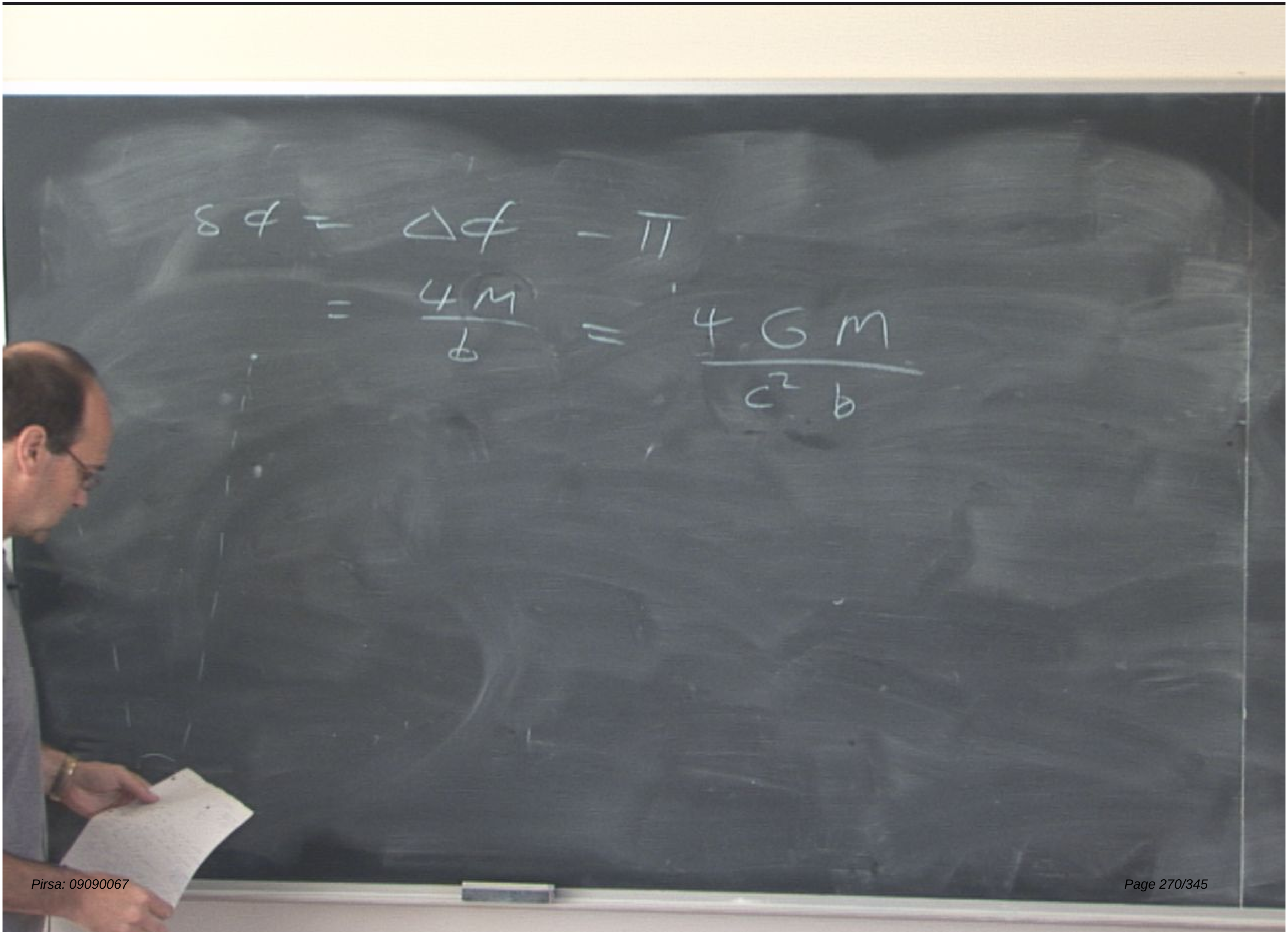
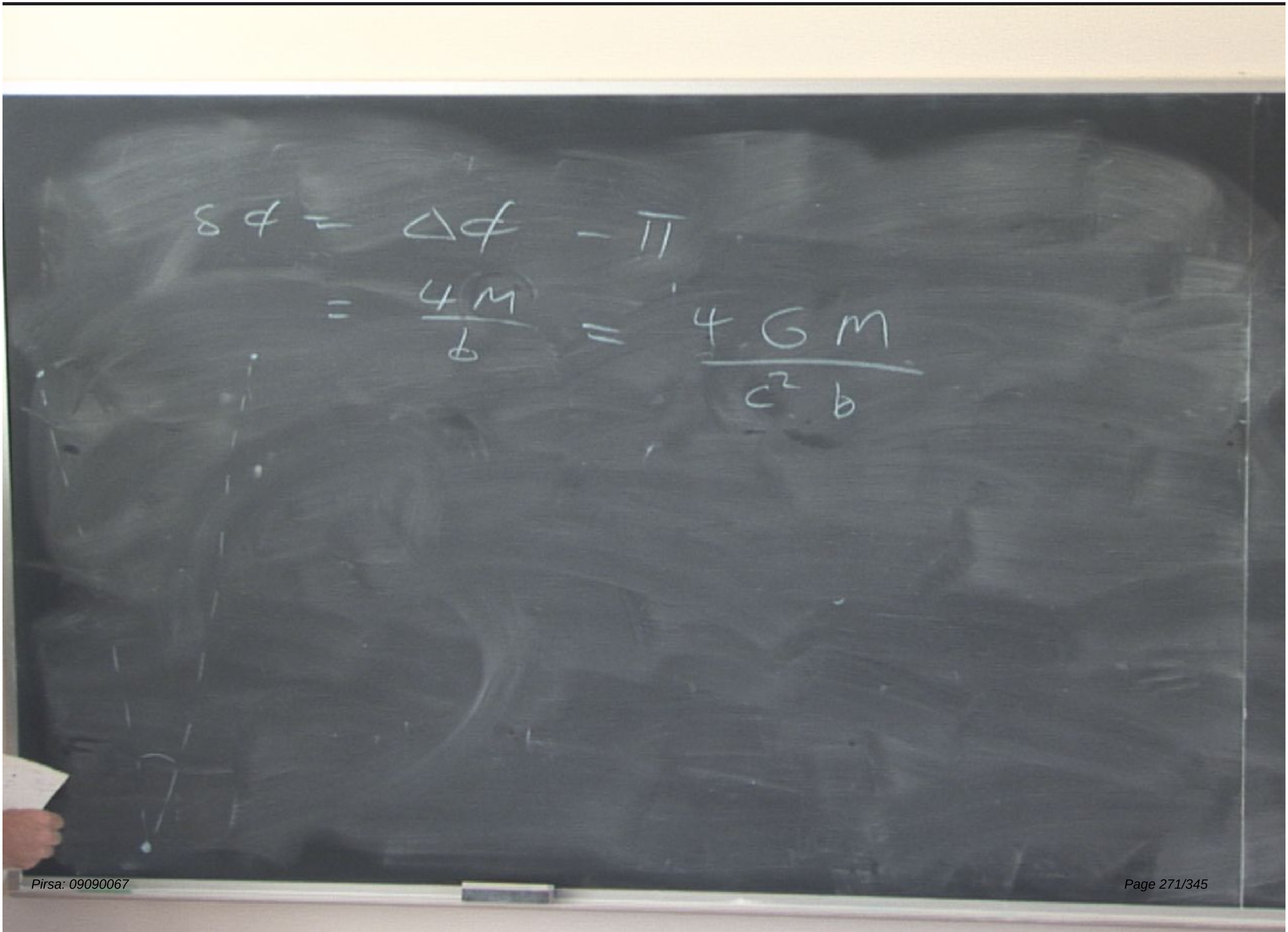
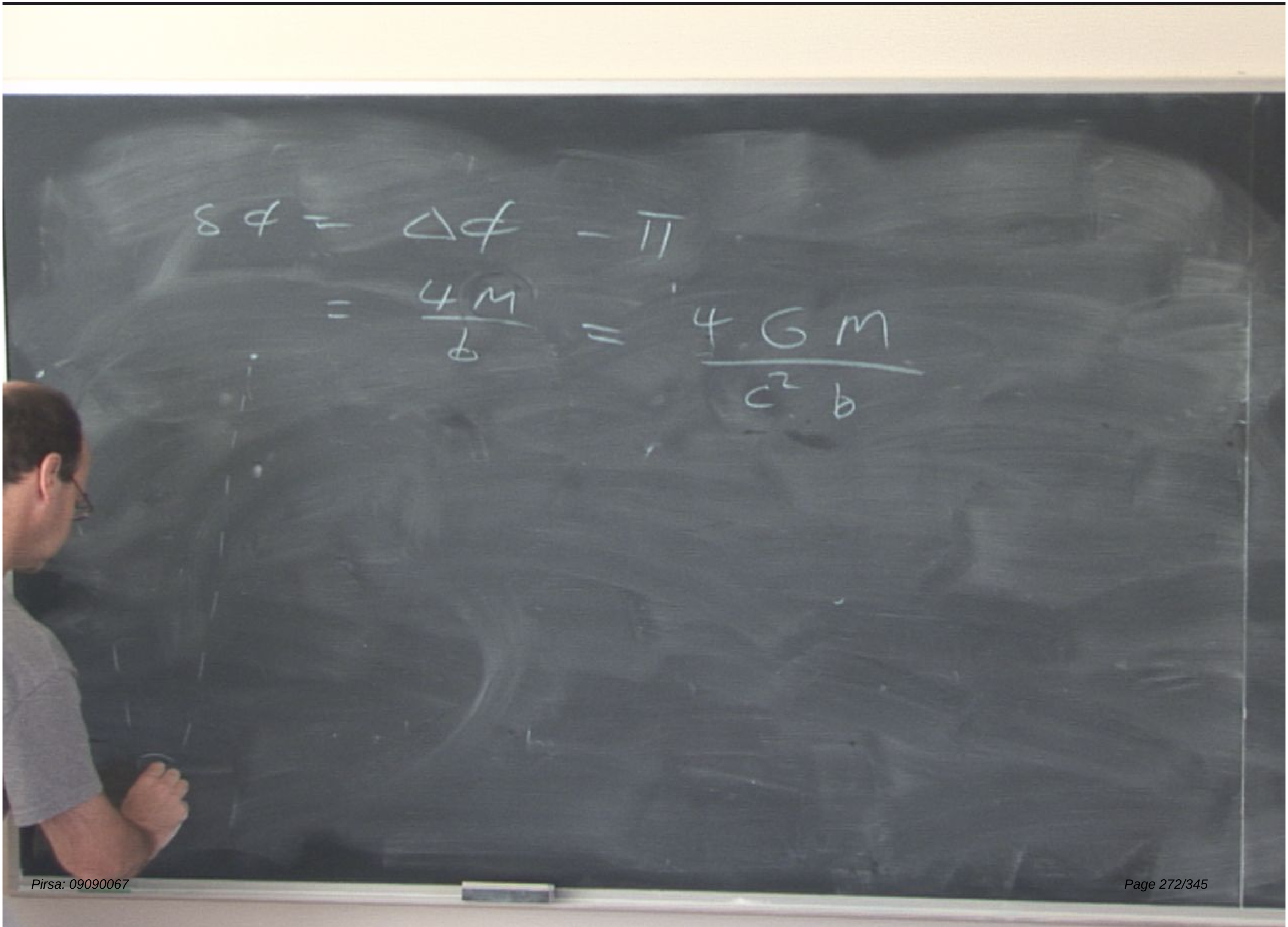
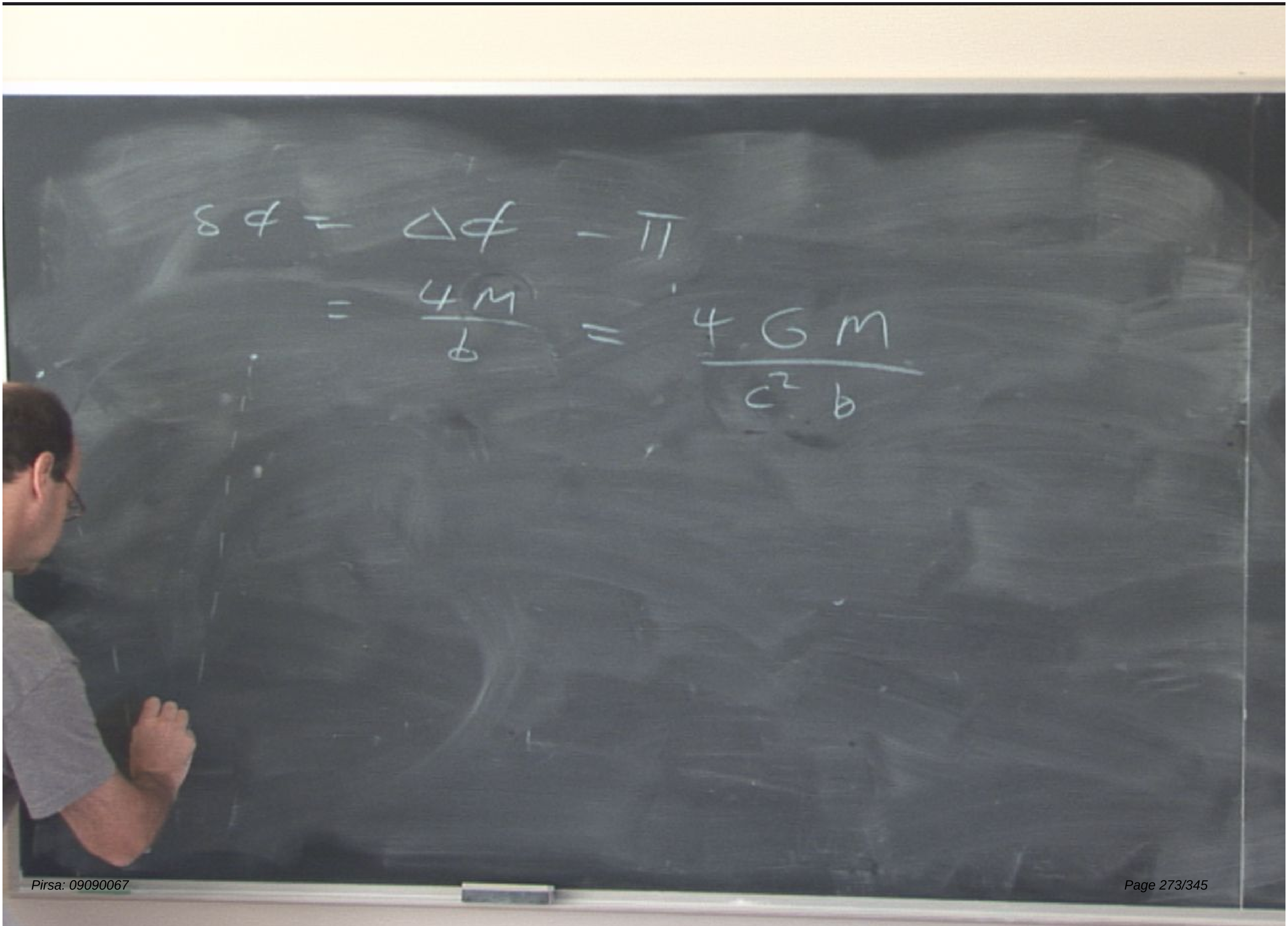
$$= \frac{4M}{b} = \frac{4GM}{c^2 b}$$


δφ = Δφ − π
= 4M/b = 4GM/c²b


$\delta \phi = \Delta \phi - \pi$
$= \frac{4M}{b} = \frac{4GM}{c^2 b}$


$\delta\phi = \Delta\phi - \pi$
$= \frac{4M}{b} = \frac{4GM}{c^2 b}$


$\delta\phi = \Delta\phi - \pi$
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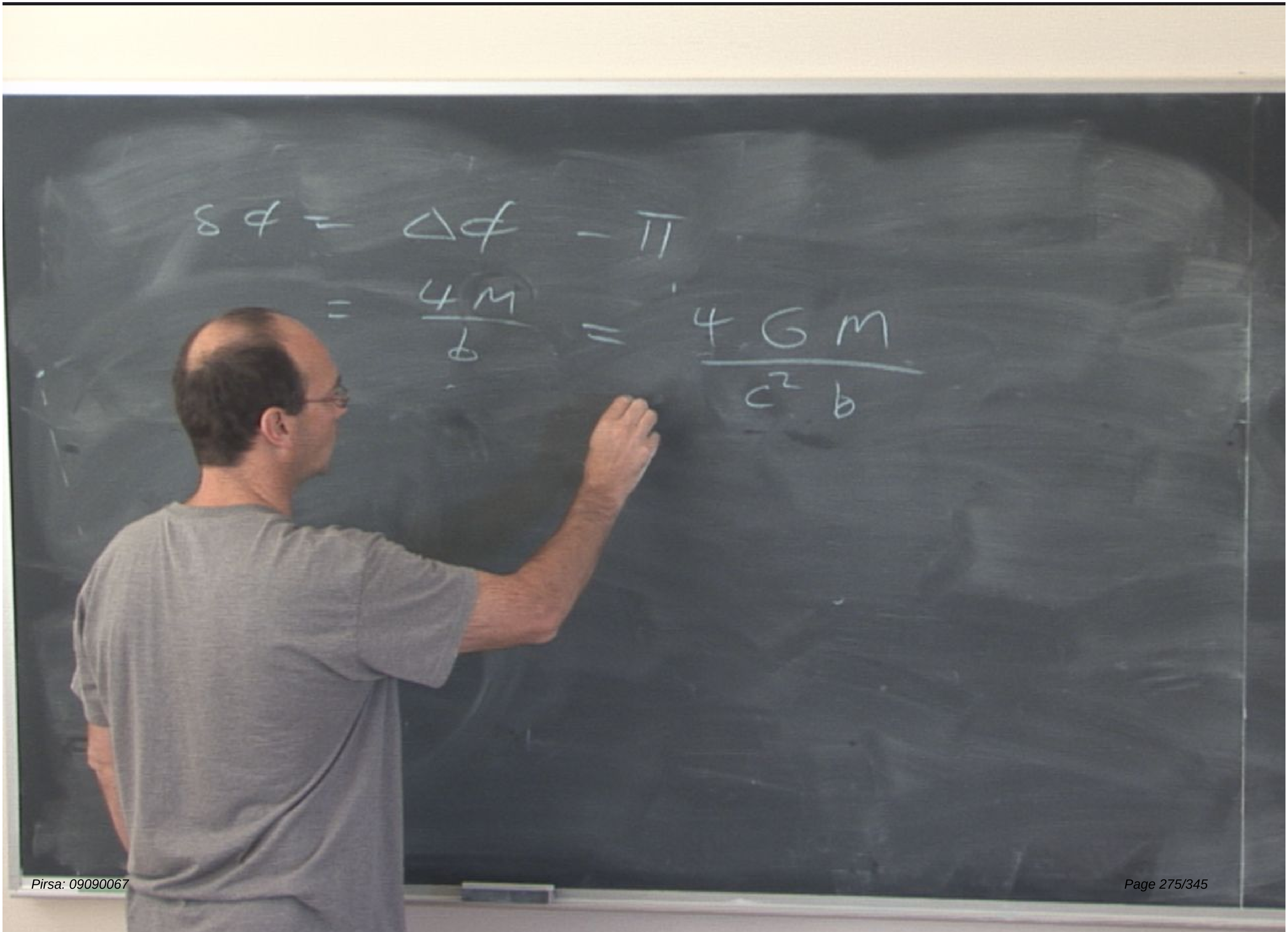
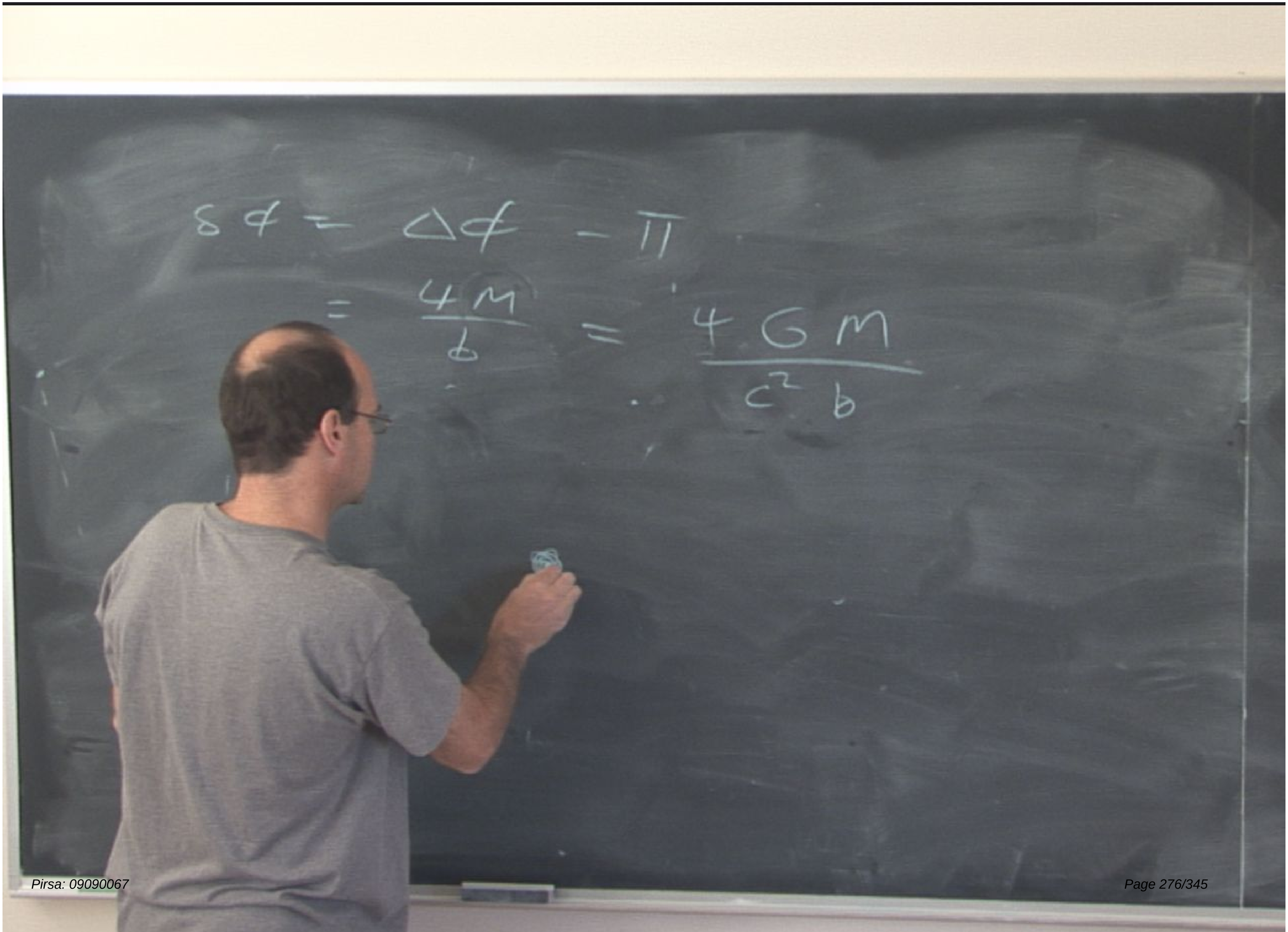
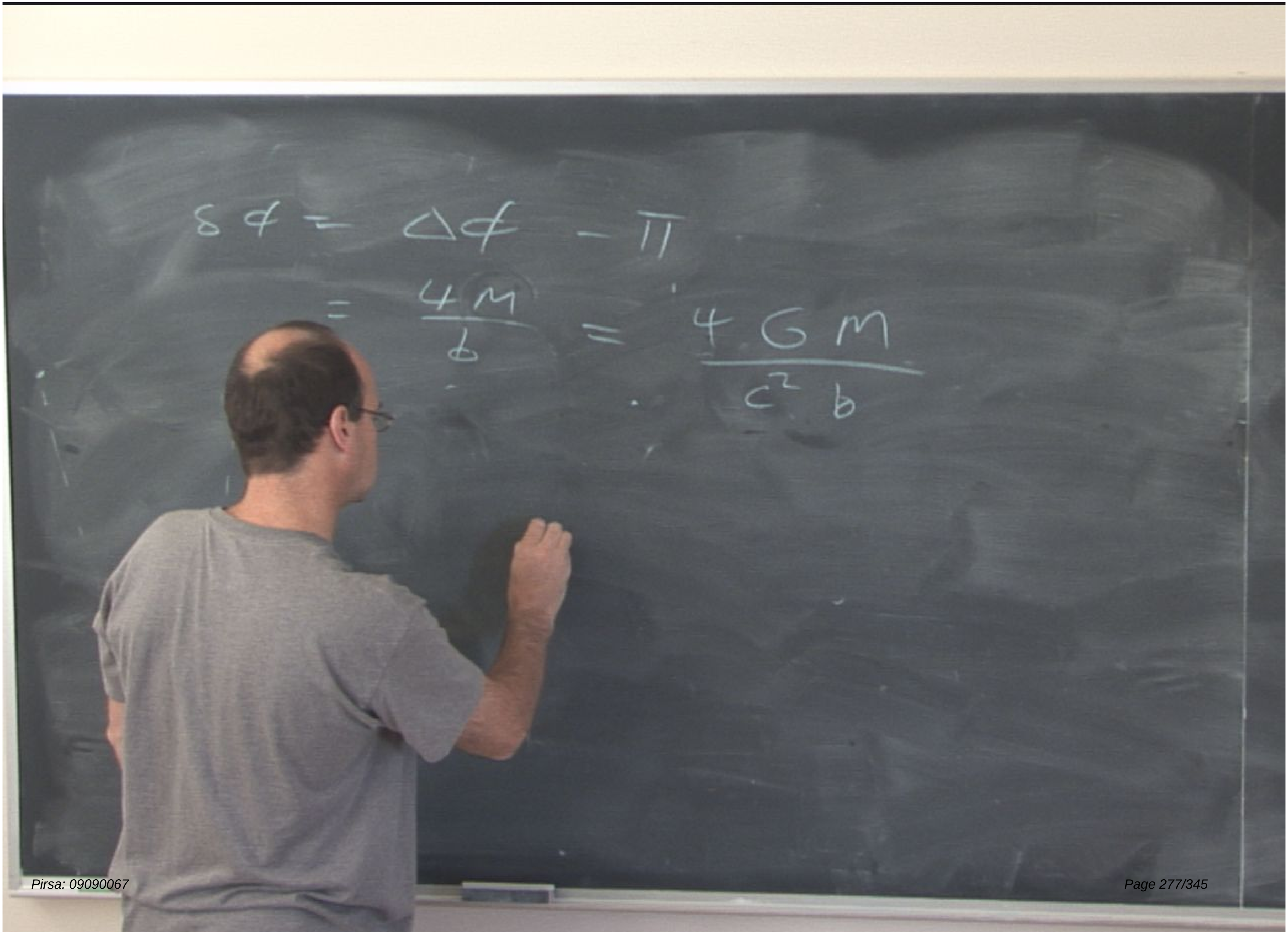
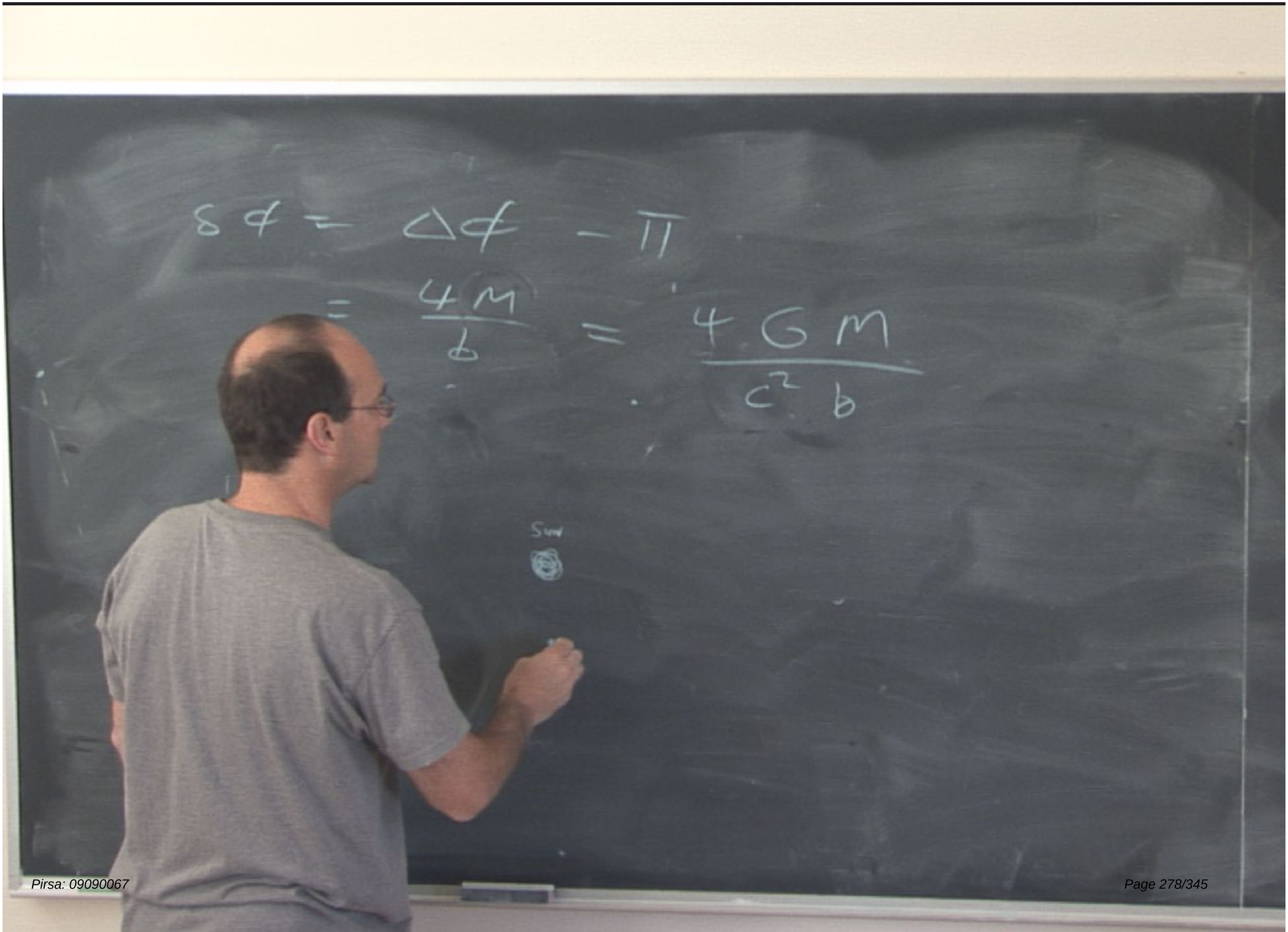
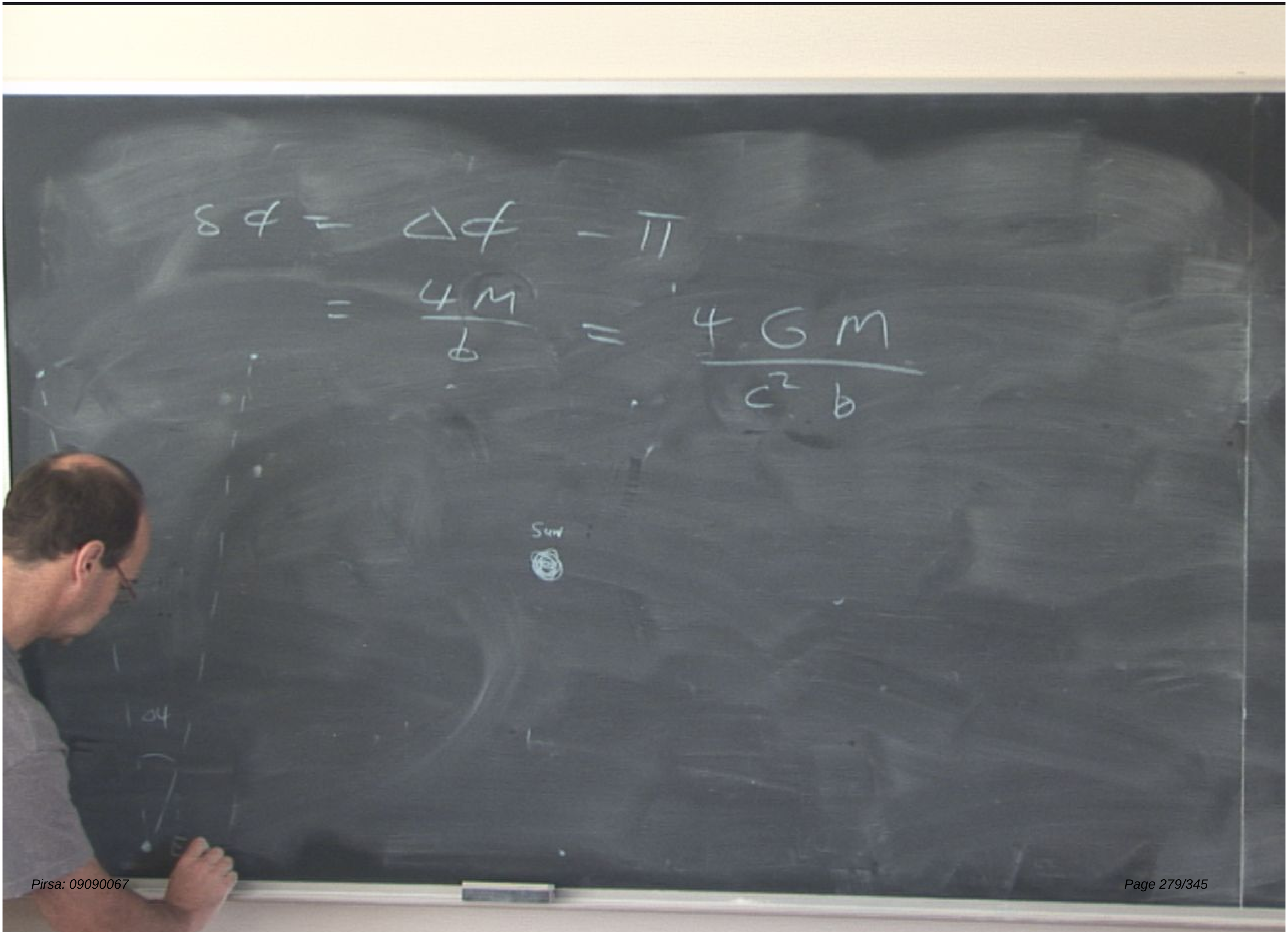
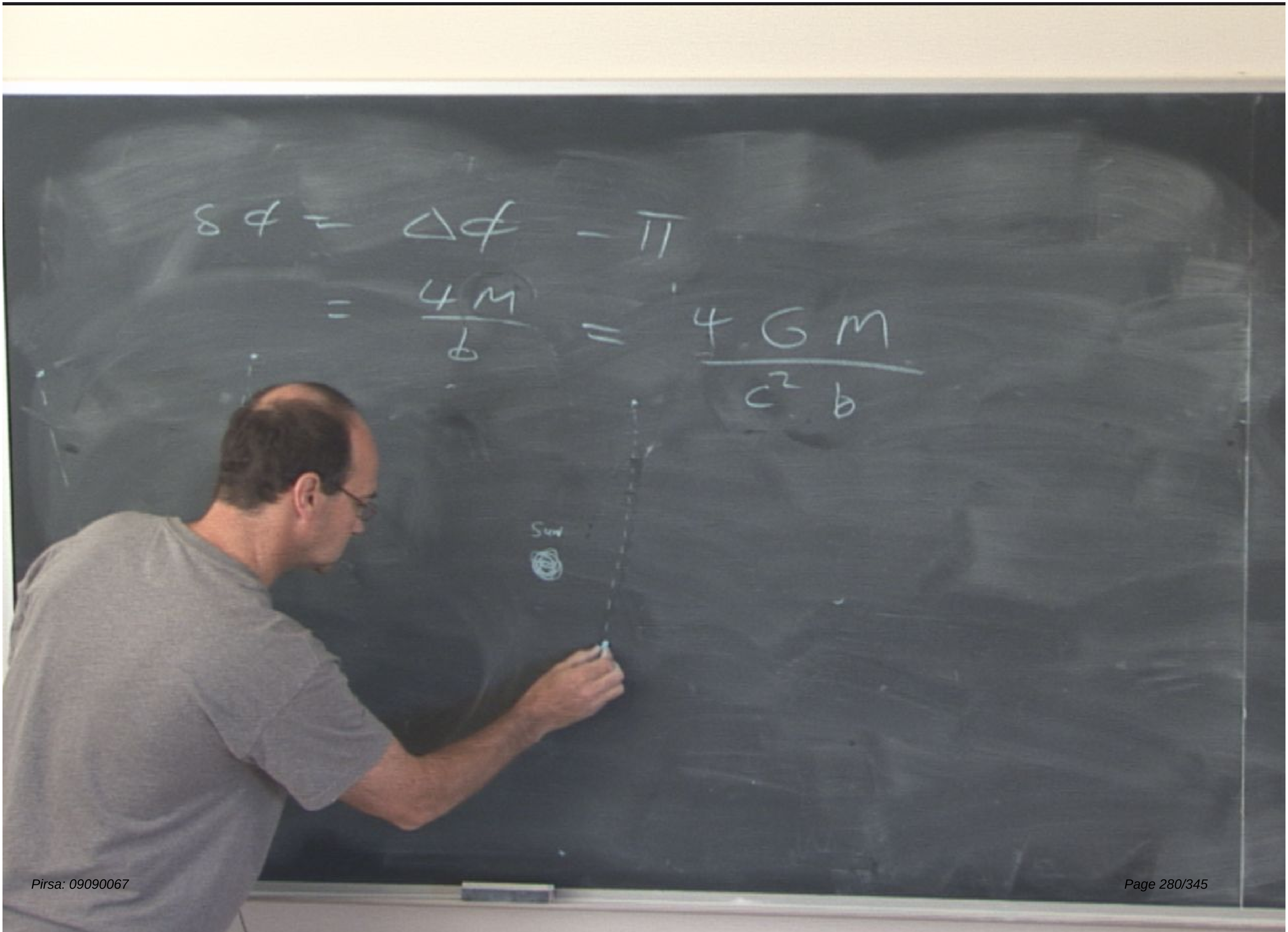

δφ = Δφ − π
= 4M/b = 4GM/c²b

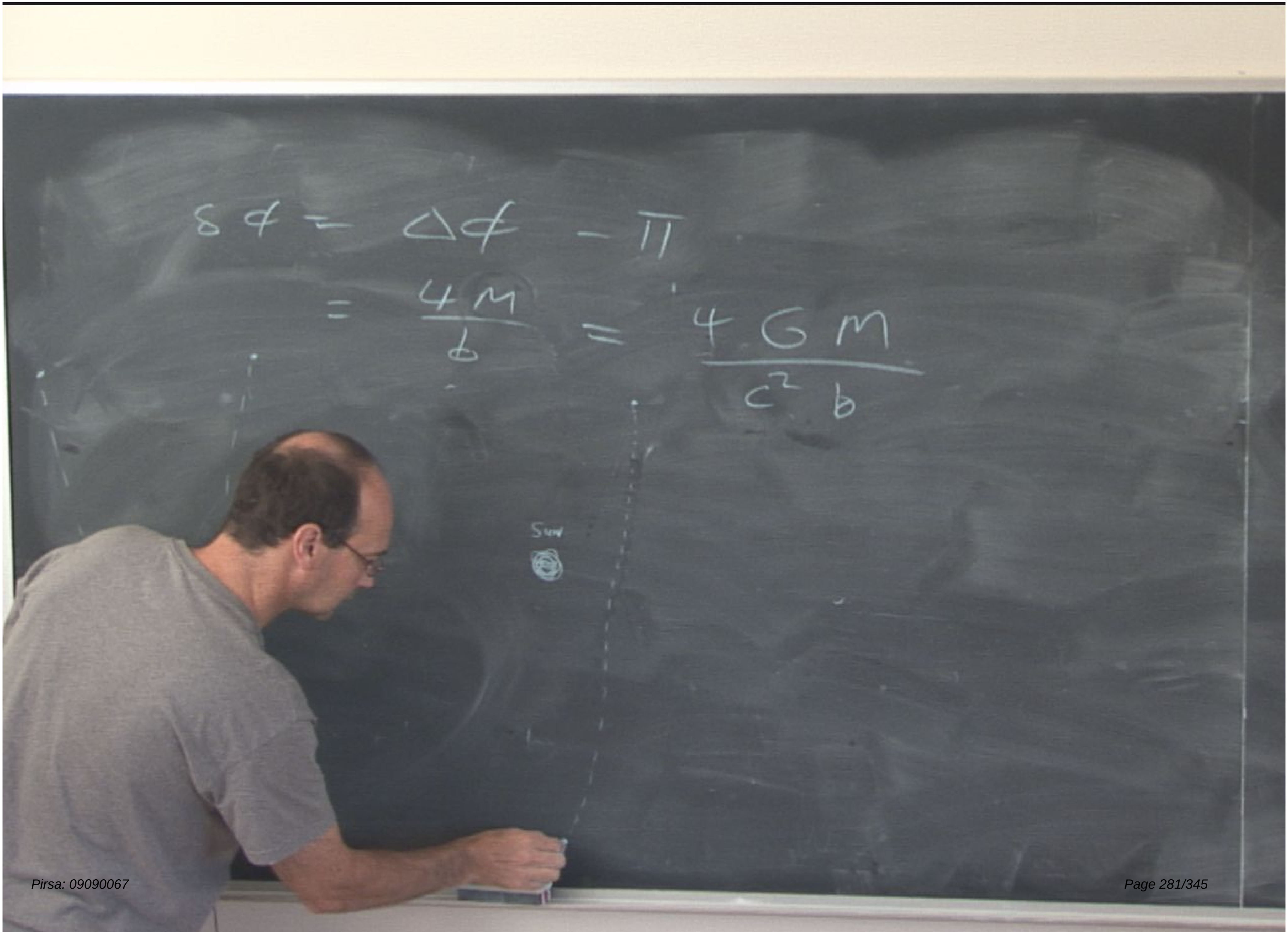

δφ = Δφ − π
= 4M/b = 4GM/(c² b)

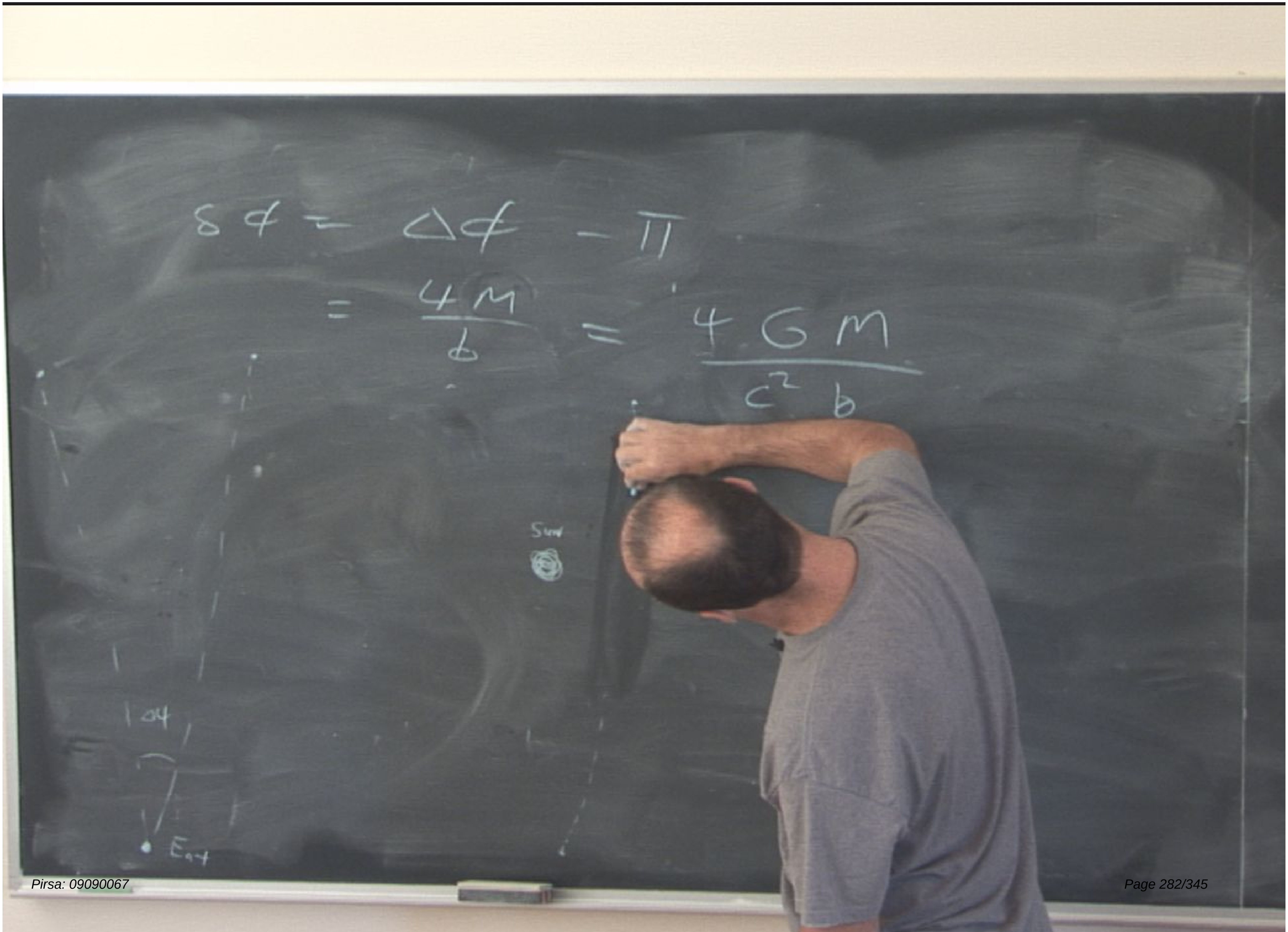

δφ = Δφ − π
= 4M/b = 4GM/(c^2 b)

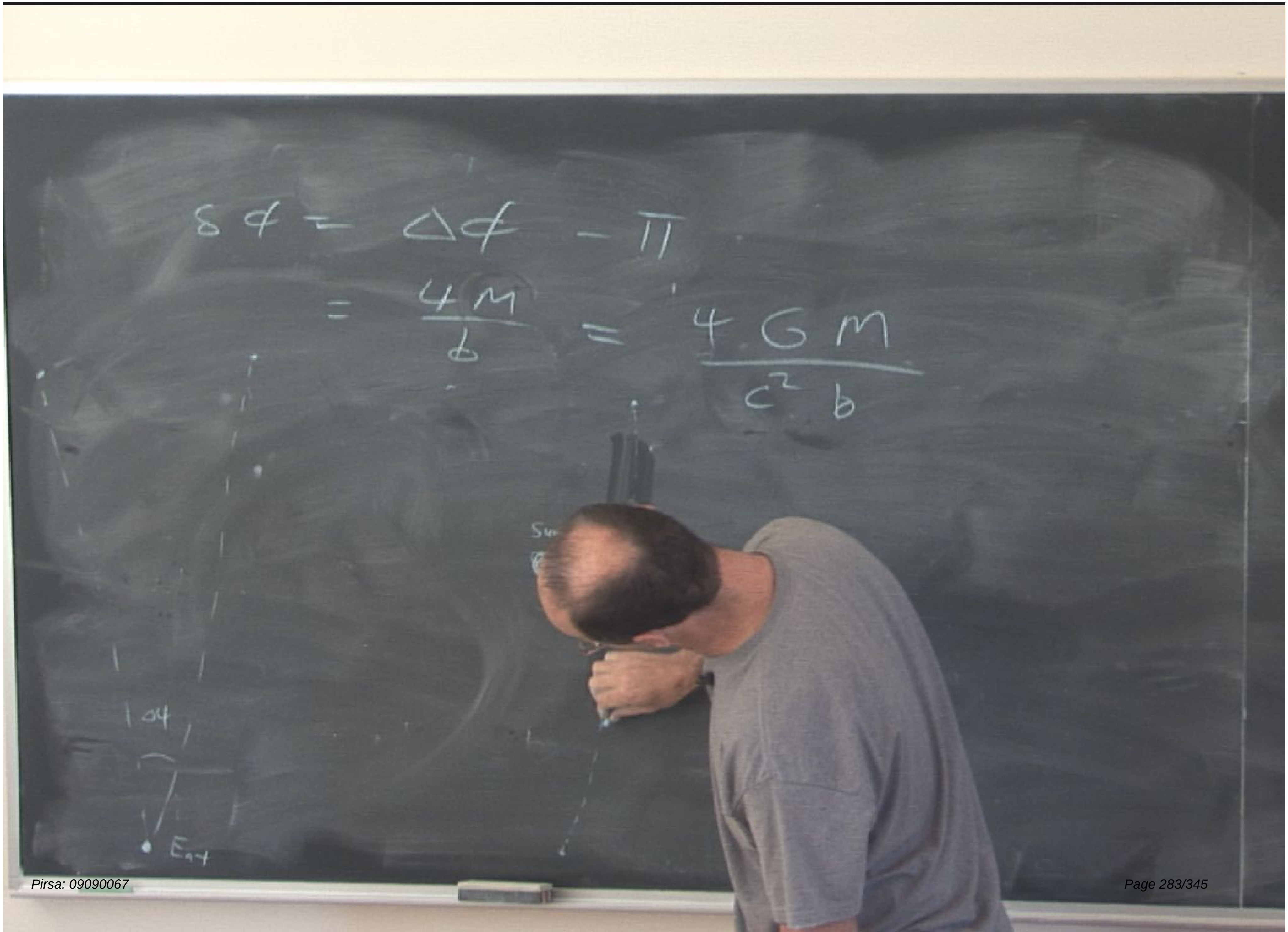

δφ = Δφ − π
= 4M/b = 4GM/c²b
Sun

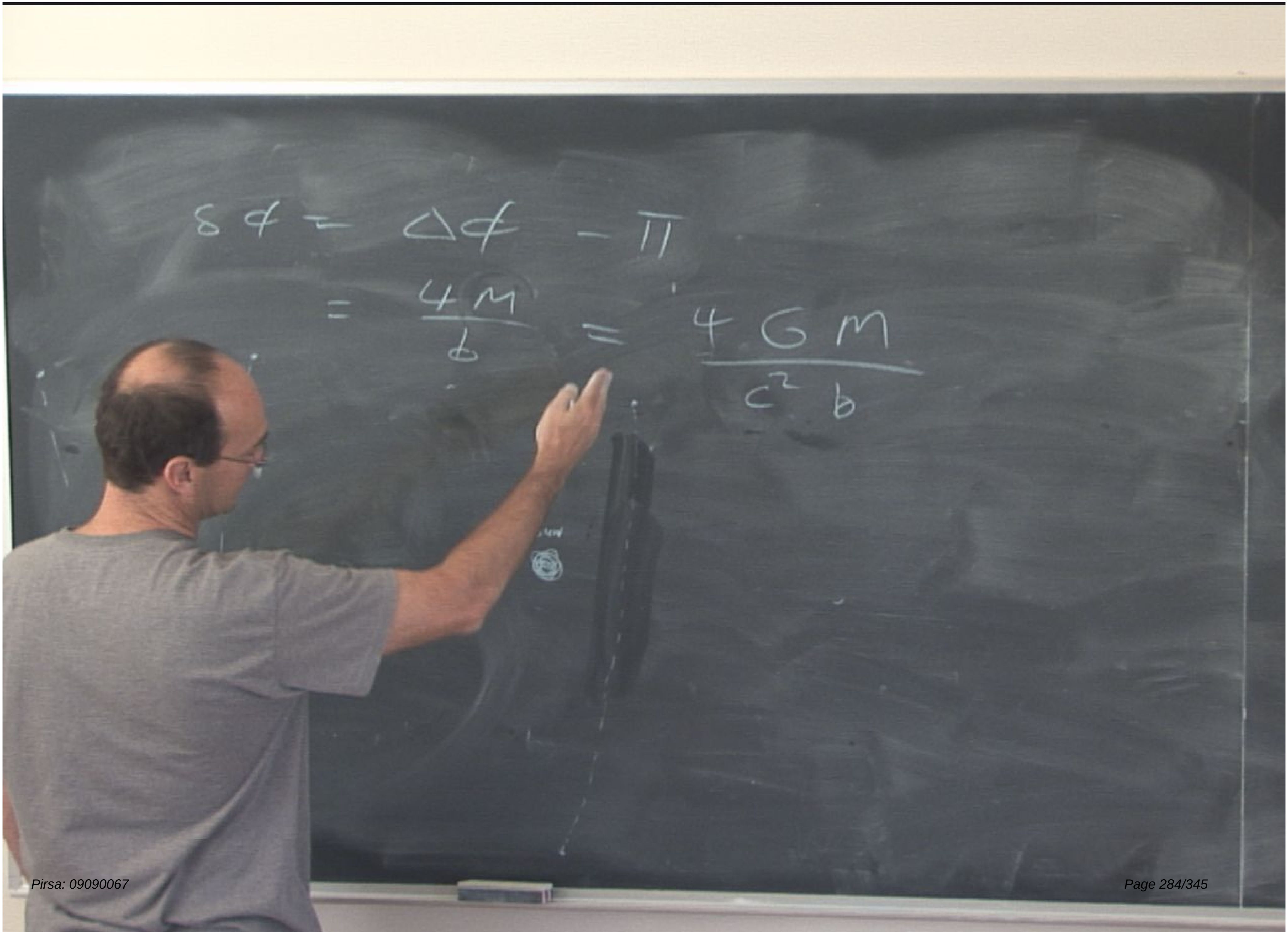

δφ = Δφ − π
= 4M/b = 4GM/(c² b)
Sun

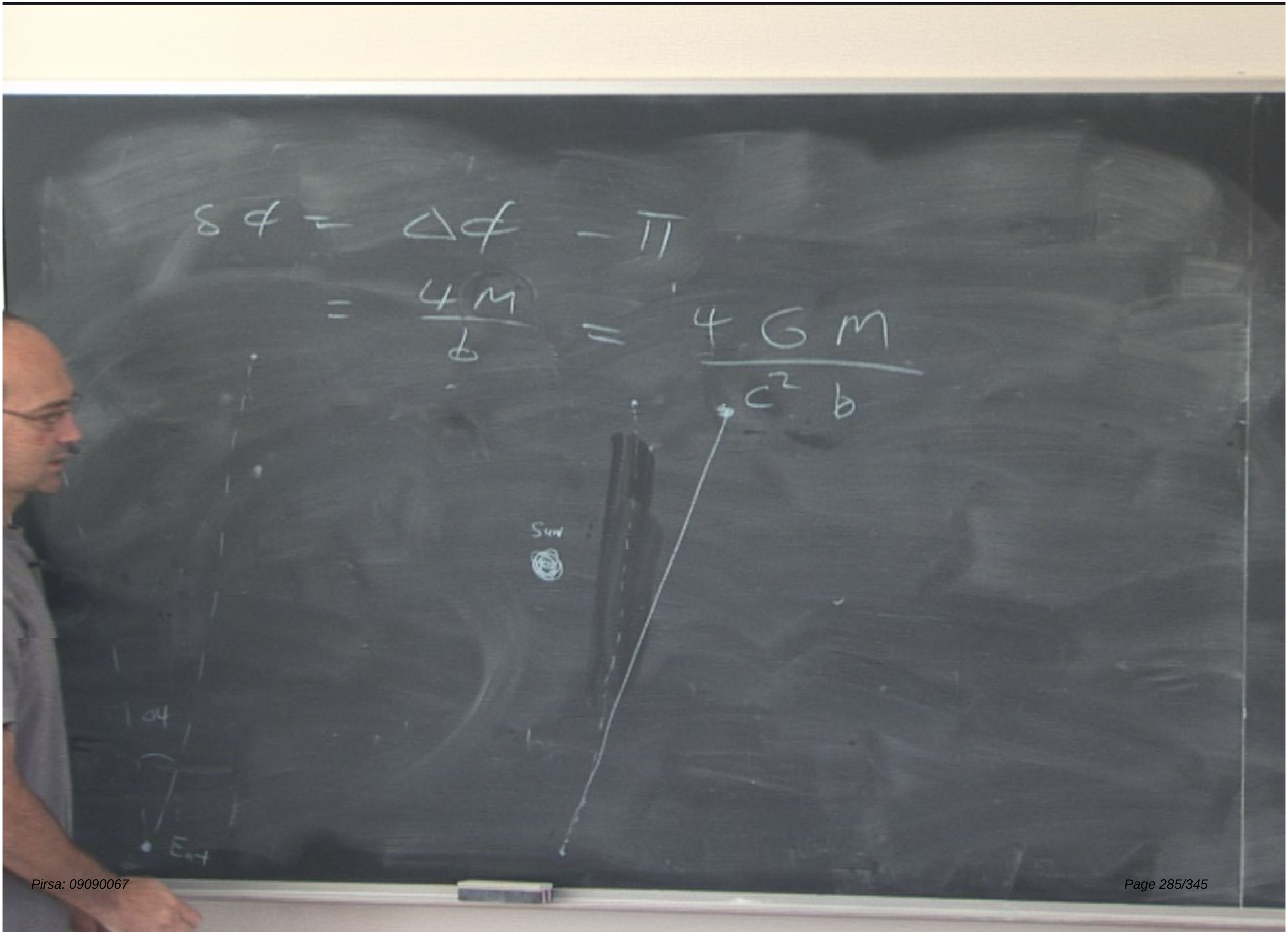

δφ = Δφ − π
= 4M/b = 4GM/c²b
Sun

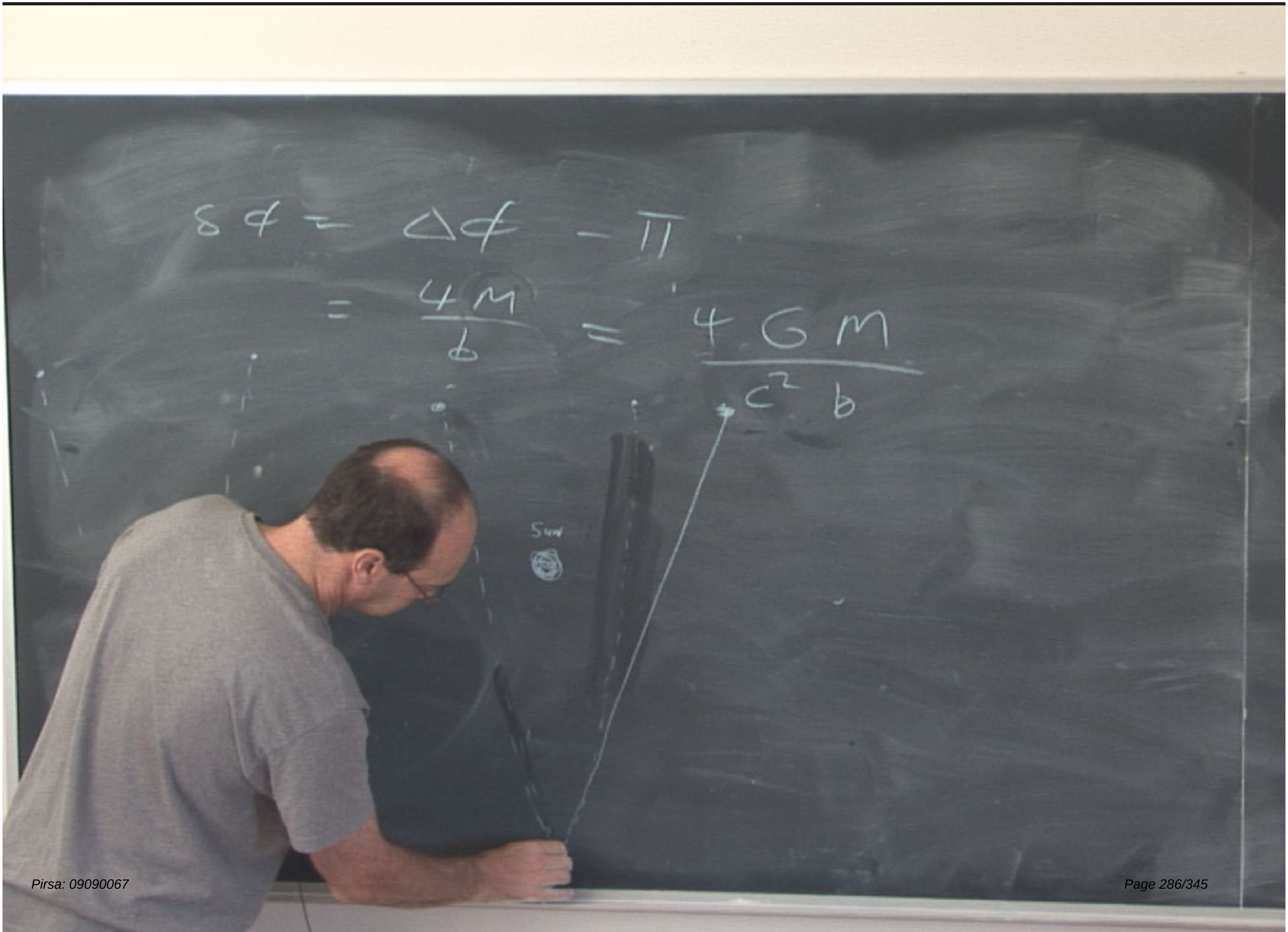

$\delta\phi = \Delta\phi - \pi$
$= \frac{4M}{b} = \frac{4GM}{c^2 b}$
Sun

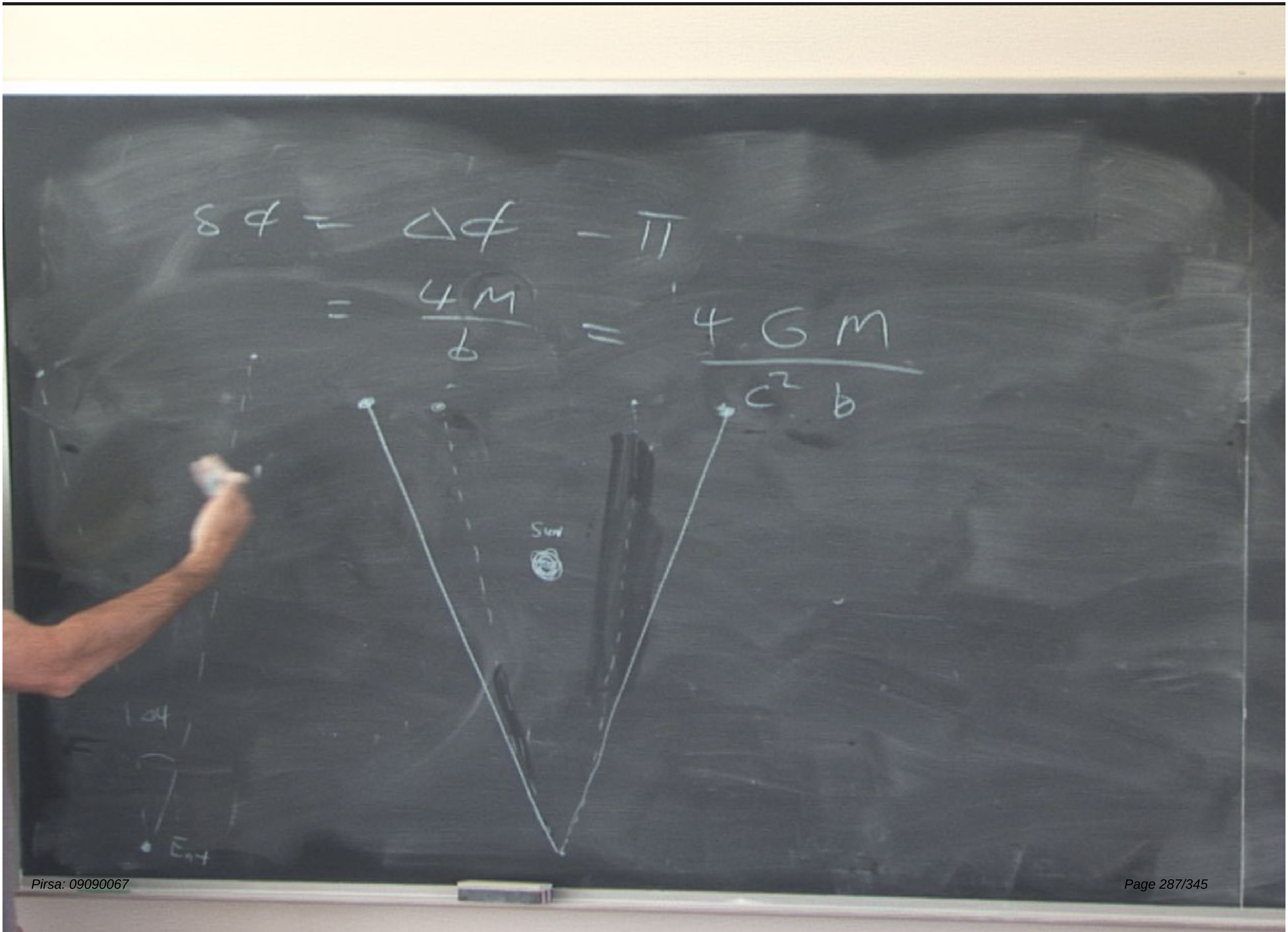

δφ = Δφ − π
= 4M/b = 4GM/c²b
Sun
Δφ
Eart


δφ = Δφ − π
= 4M / b = 4GM / c² b
1.04
Eart


$\delta \phi = \Delta \phi - \pi$
$= \frac{4M}{b} = \frac{4GM}{c^2 b}$

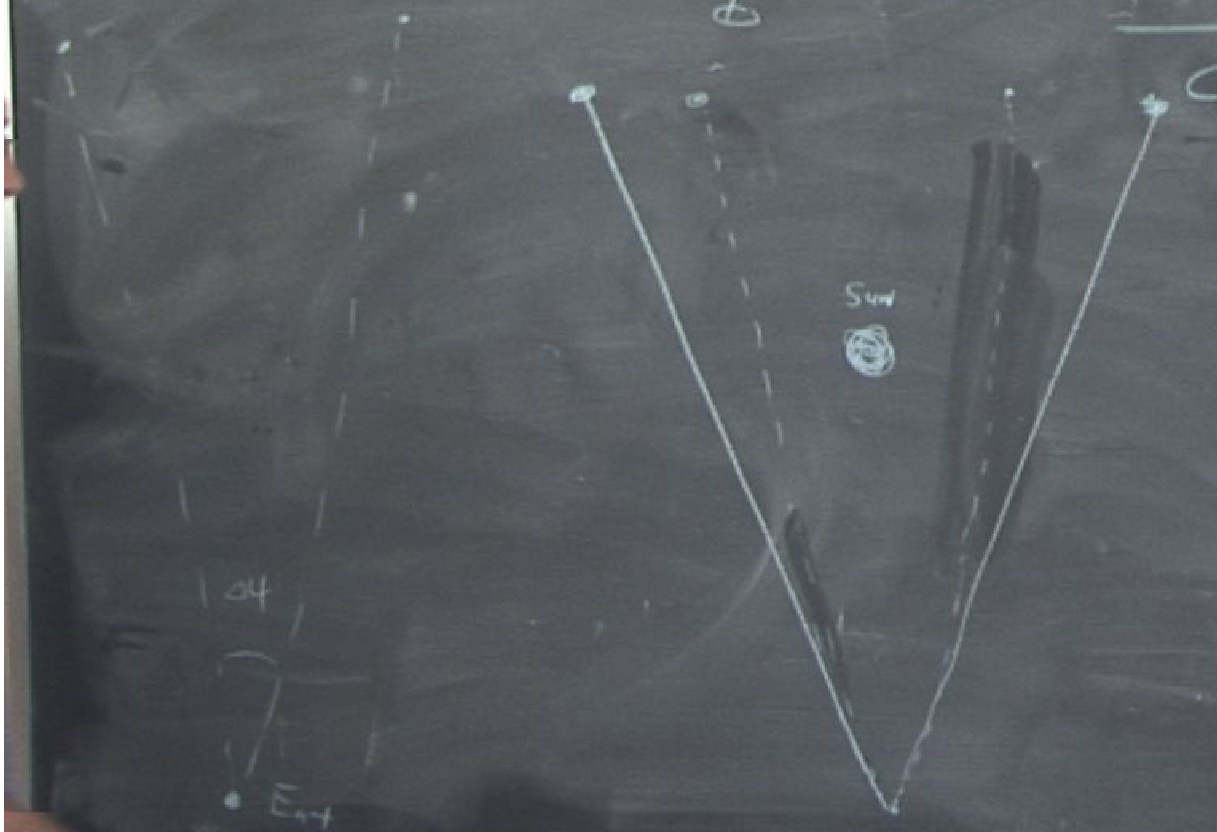

δφ = Δφ − π
= 4M / b = 4GM / c² b
Sun
Δφ

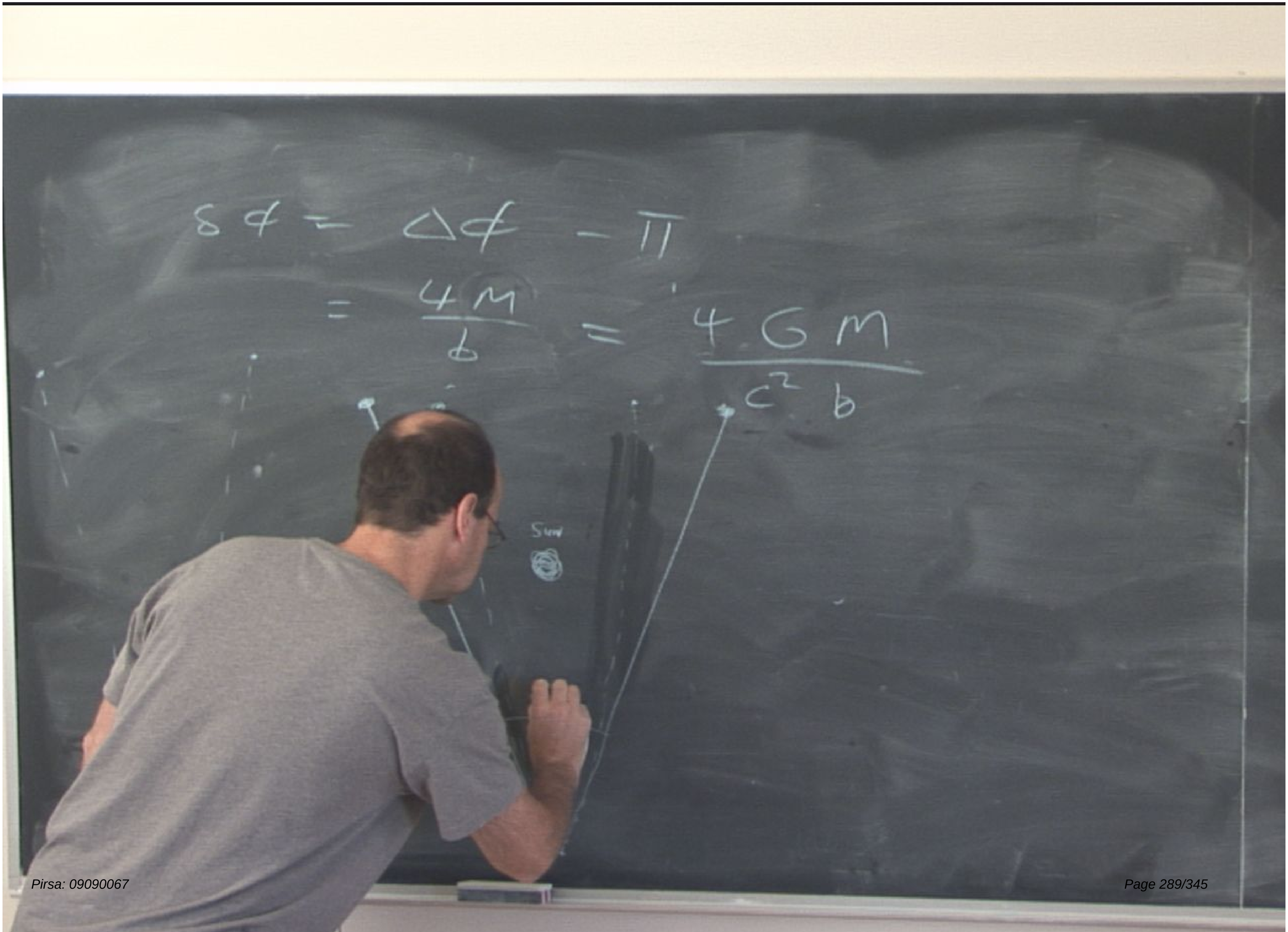

δφ = Δφ − π
= 4M/b = 4GM/c²b
Sun

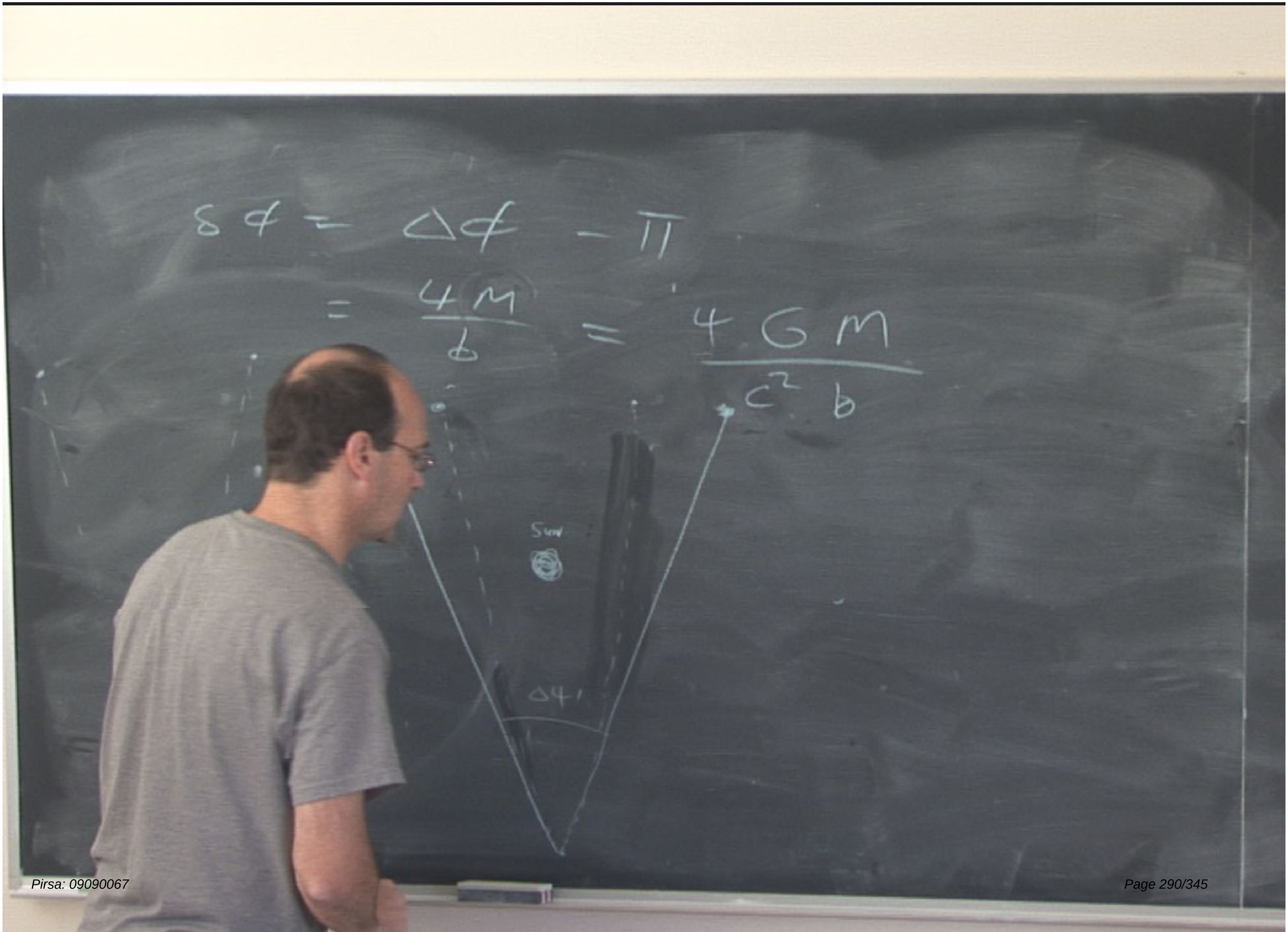

δφ = Δφ − π
= 4M/b = 4GM/c²b
Sun

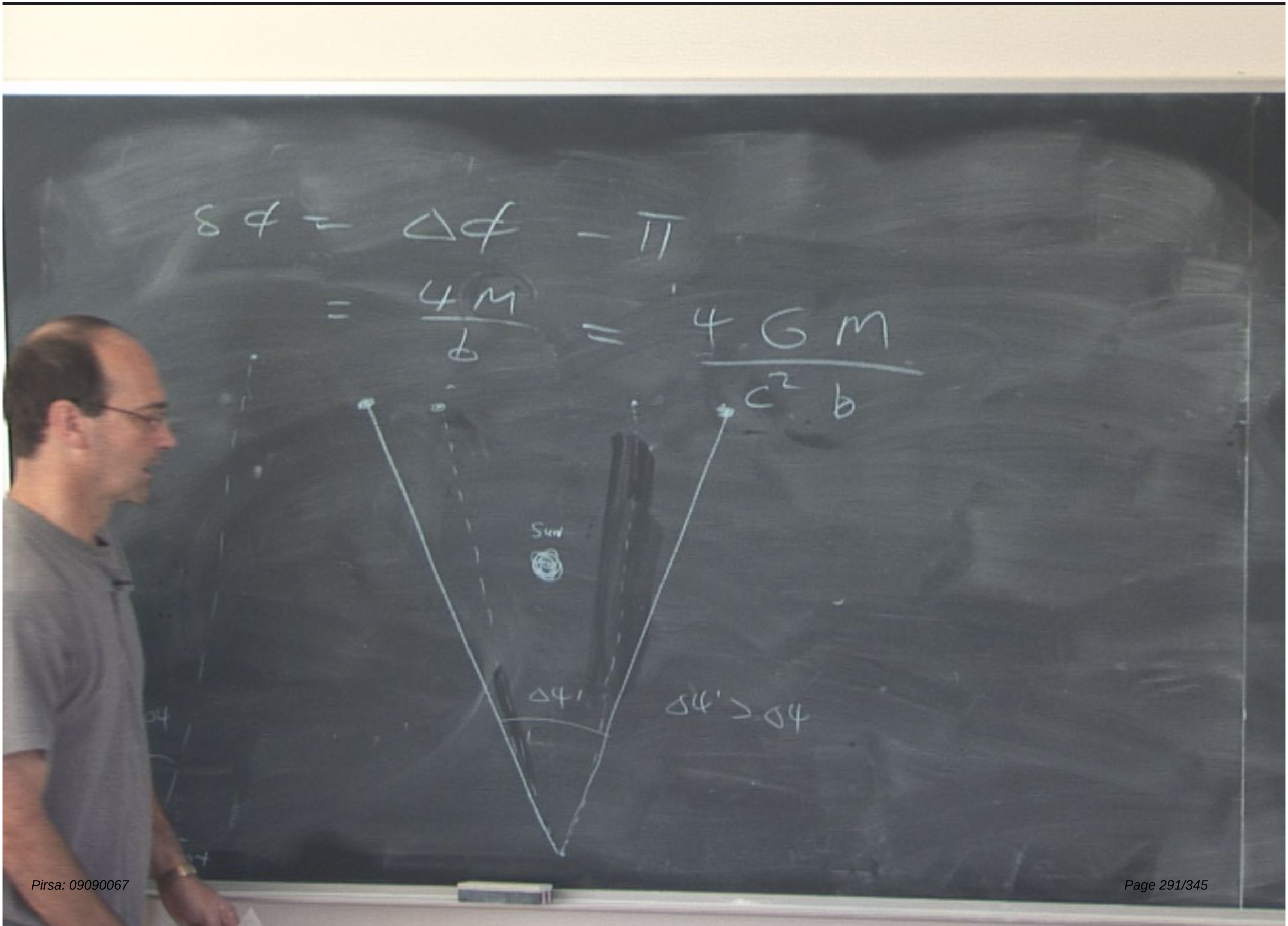
$$\delta\phi = \Delta\phi - \pi$$

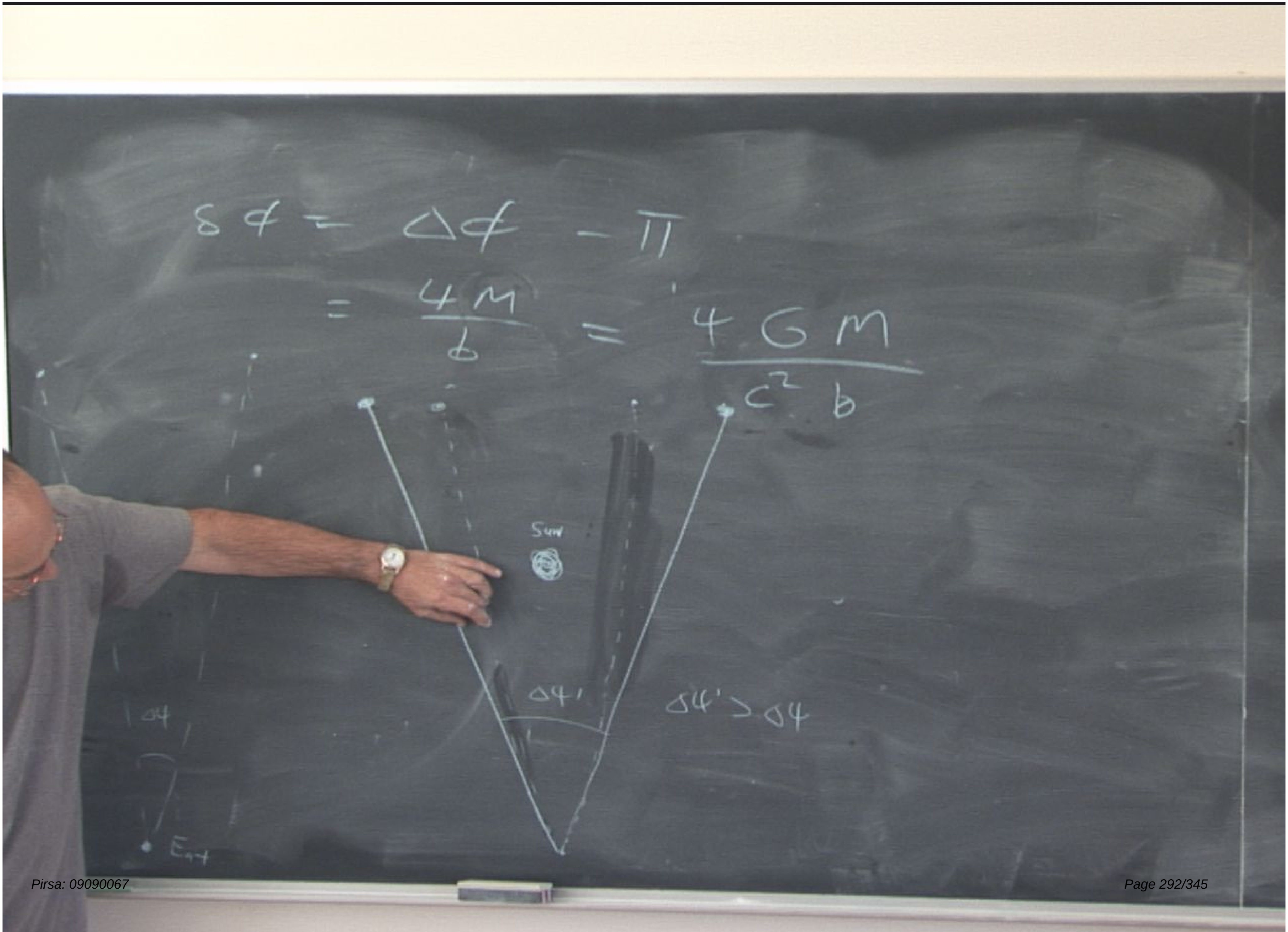
$$= \frac{4M}{b} = \frac{4GM}{c^2 b}$$

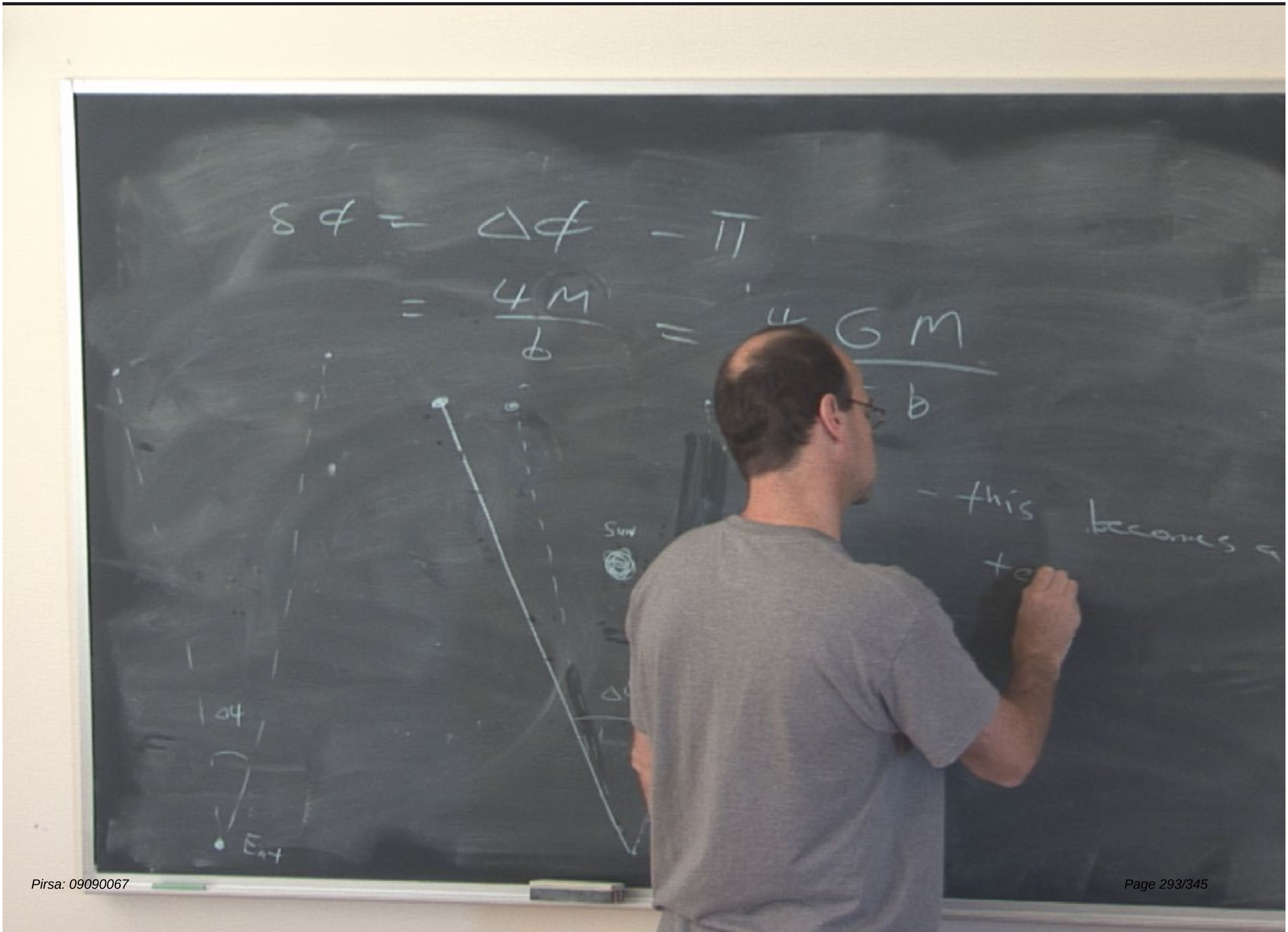


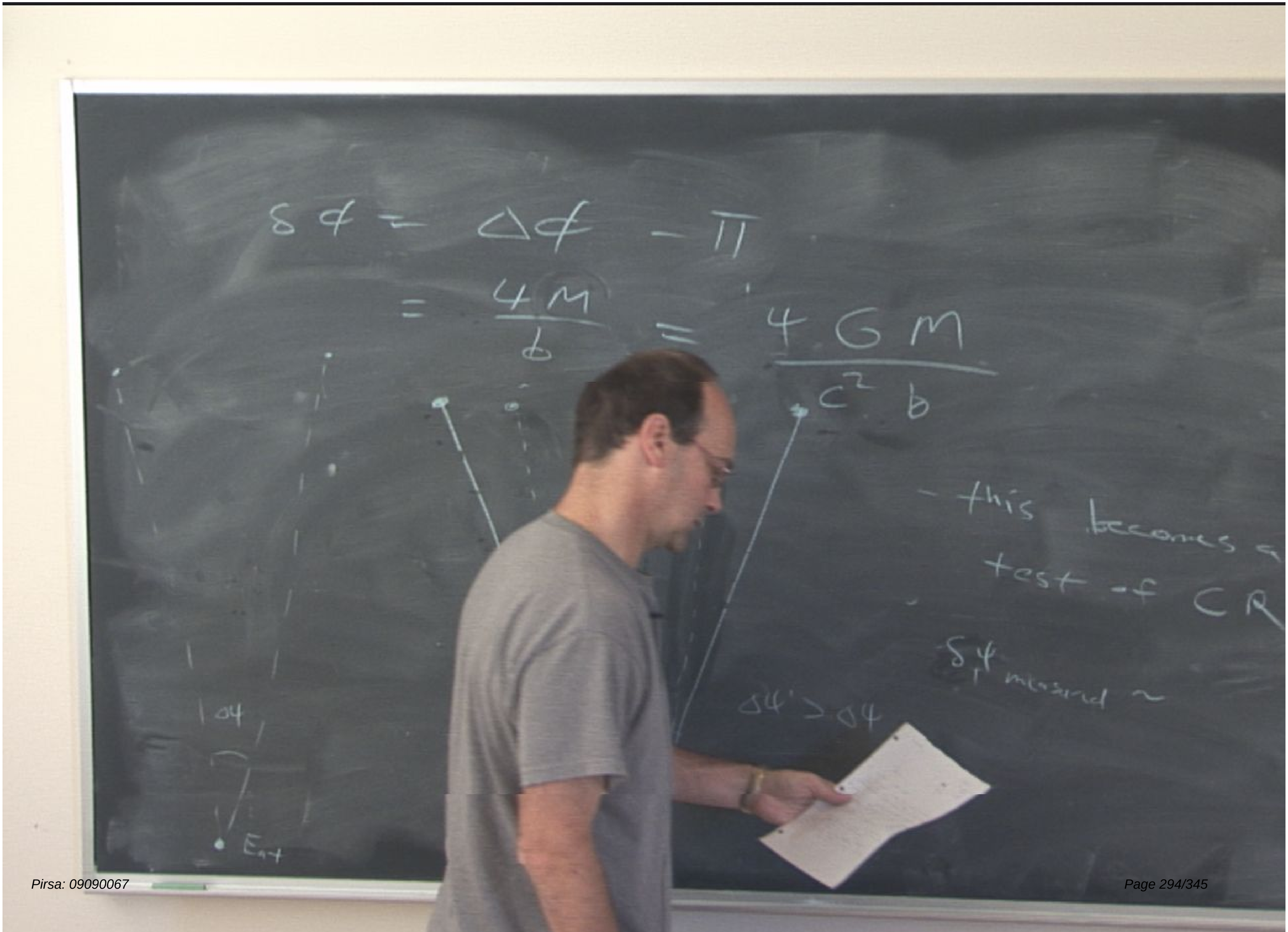

$\delta\phi = \Delta\phi - \pi$
$= \frac{4M}{b} = \frac{4GM}{c^2 b}$
Sun

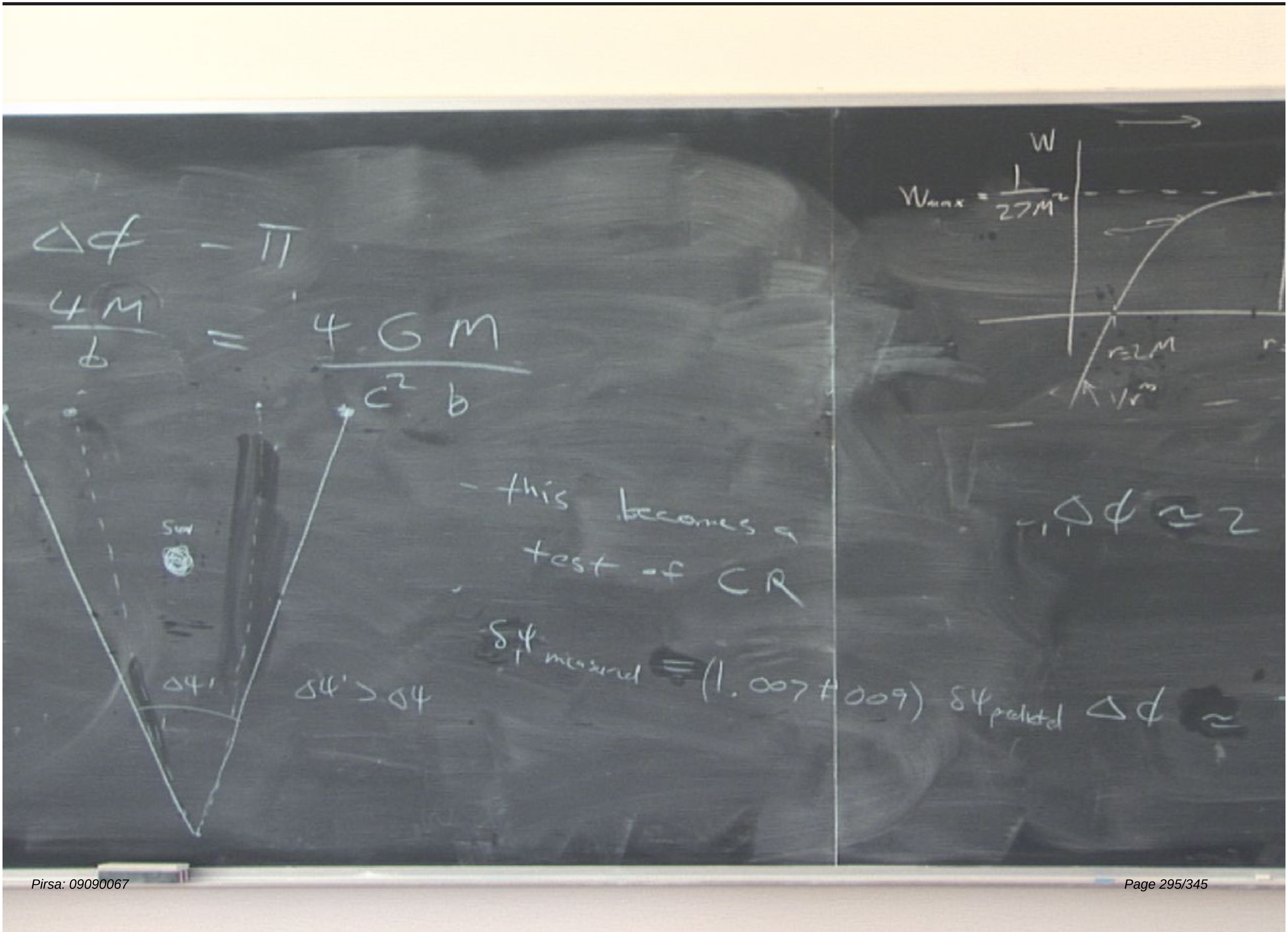

δφ = Δφ − π
= 4M / b = 4GM / c² b
Sun
Δφ

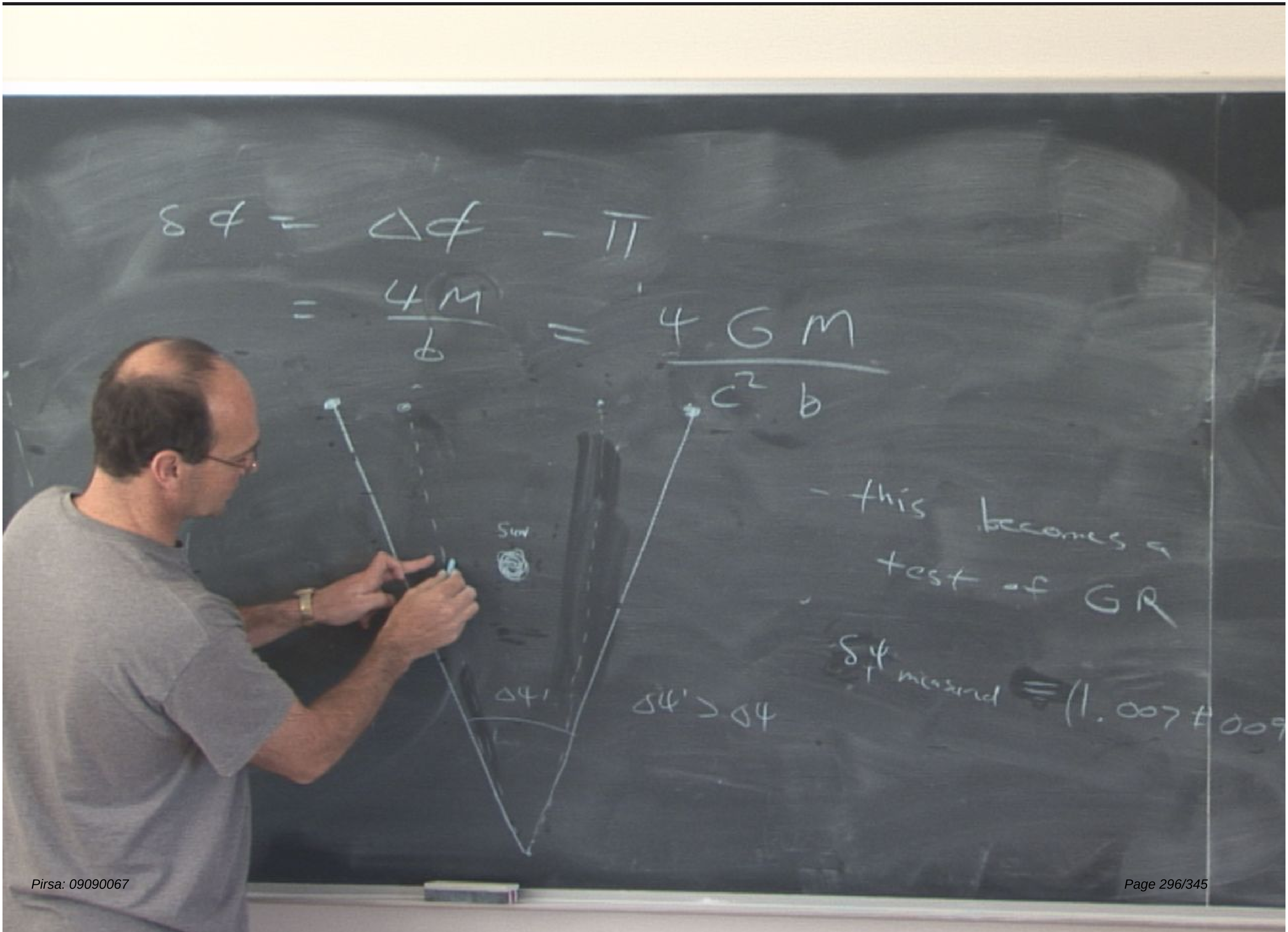

δφ = Δφ − π
= 4M/b = 4GM/c² b
Sun
Δφ'
Δφ' > Δφ

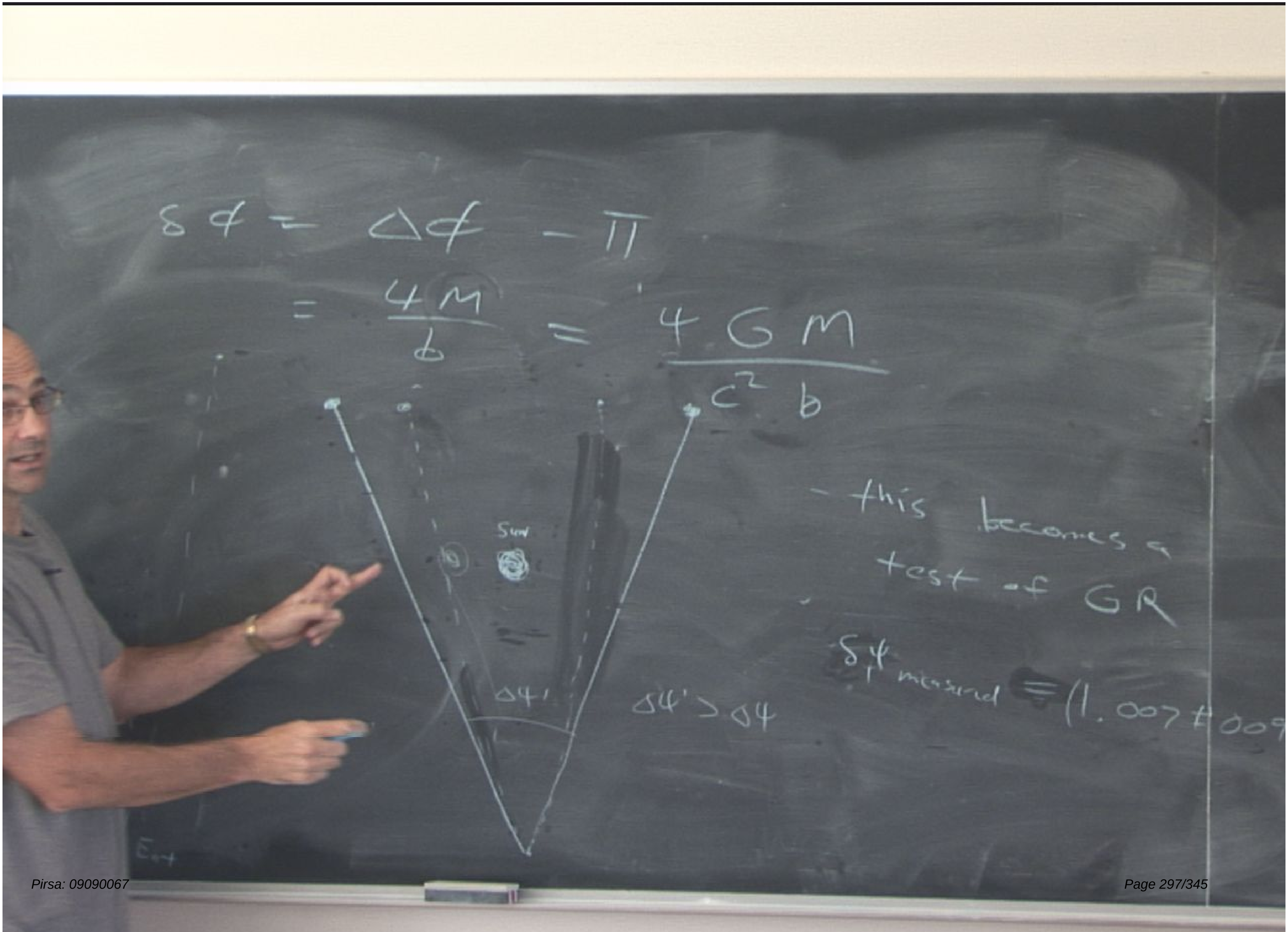

δφ = Δφ − π
= 4M/b = 4GM/(c² b)
Sun
Δφ'
Δφ' > Δφ
Δφ

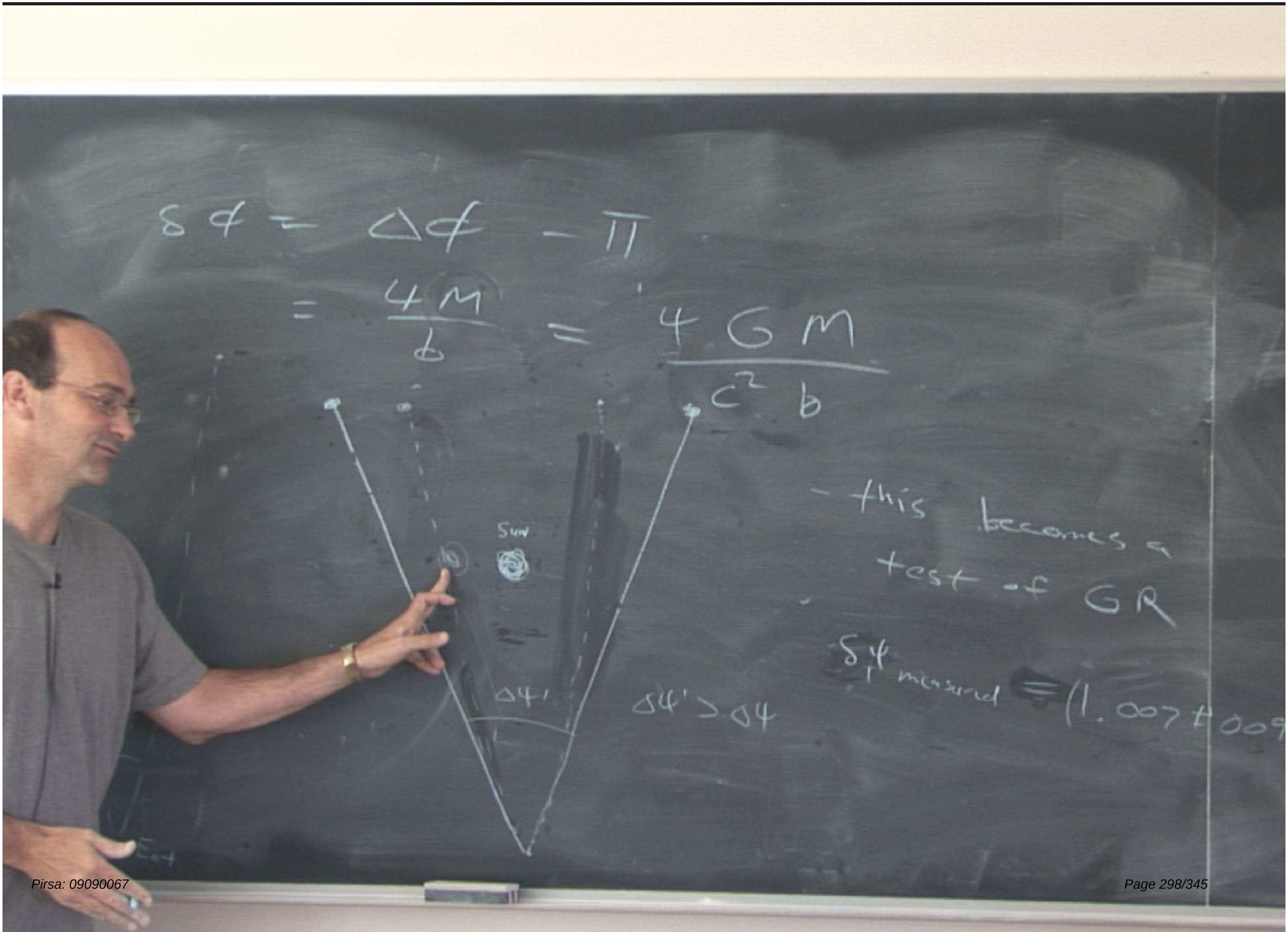

δφ = Δφ − π
= 4M/b = 4GM/…b
this becomes
Sun


δφ = Δφ − π
= 4M/b = 4GM/c² b
this becomes
test of
Δφ' > Δφ

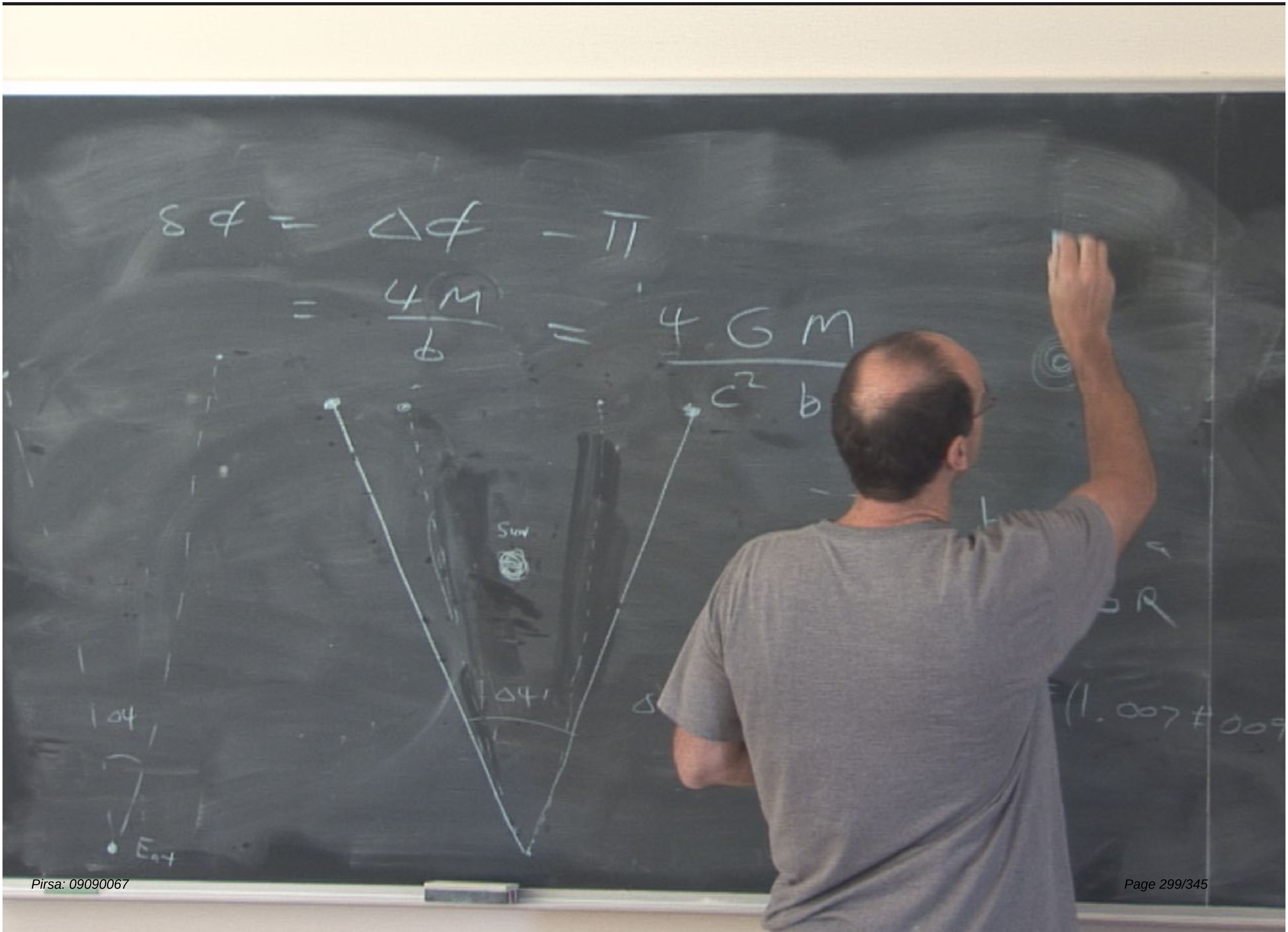

Δφ − π
4M/b = 4GM/c² b
W
Wmax = L/27M²
r=2M
Sun
Δψ'
δψ' > δψ
this becomes a test of GR
Δφ ≃ 2
δψ measured = (1.007 ± .009) δψ predicted
Δφ ≃

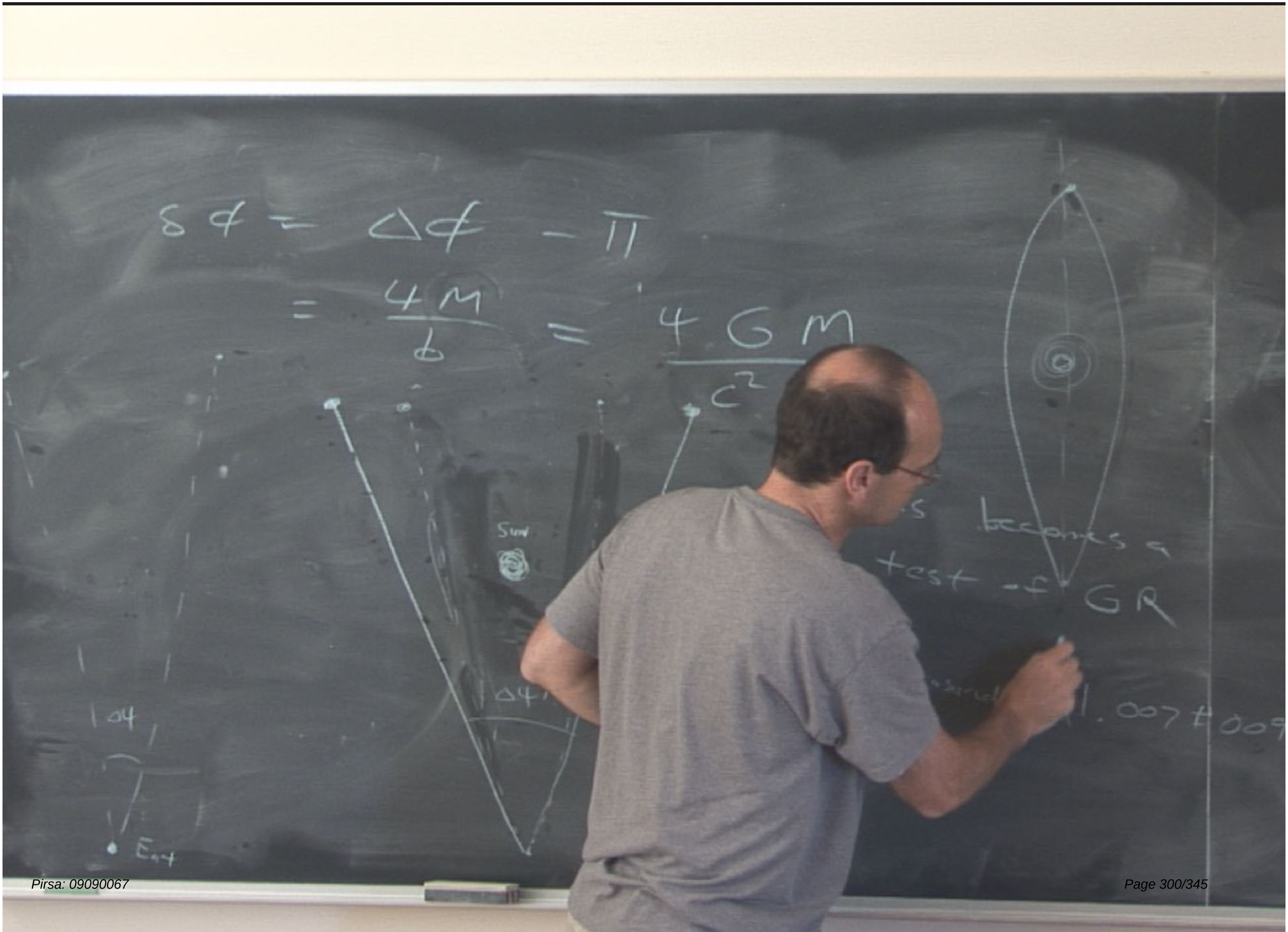

δφ = Δφ − π
= 4M/b = 4GM/c² b
Sun
Δφ'
Δφ' > Δφ
- this becomes a test of GR

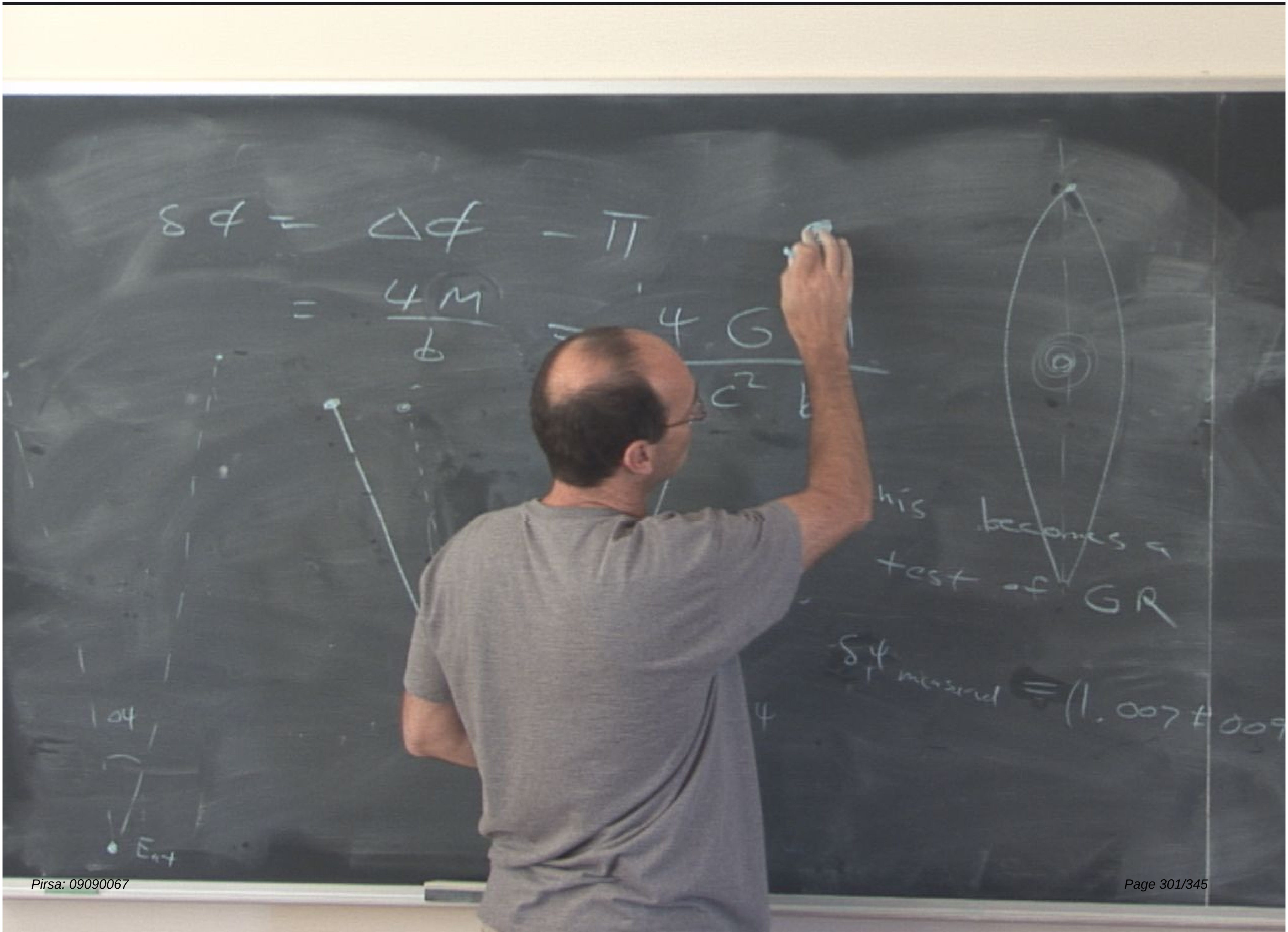

δφ = Δφ − π
= 4M / b = 4GM / c² b
Sun
Δφ'
Δφ' > Δφ
this becomes a
test of GR

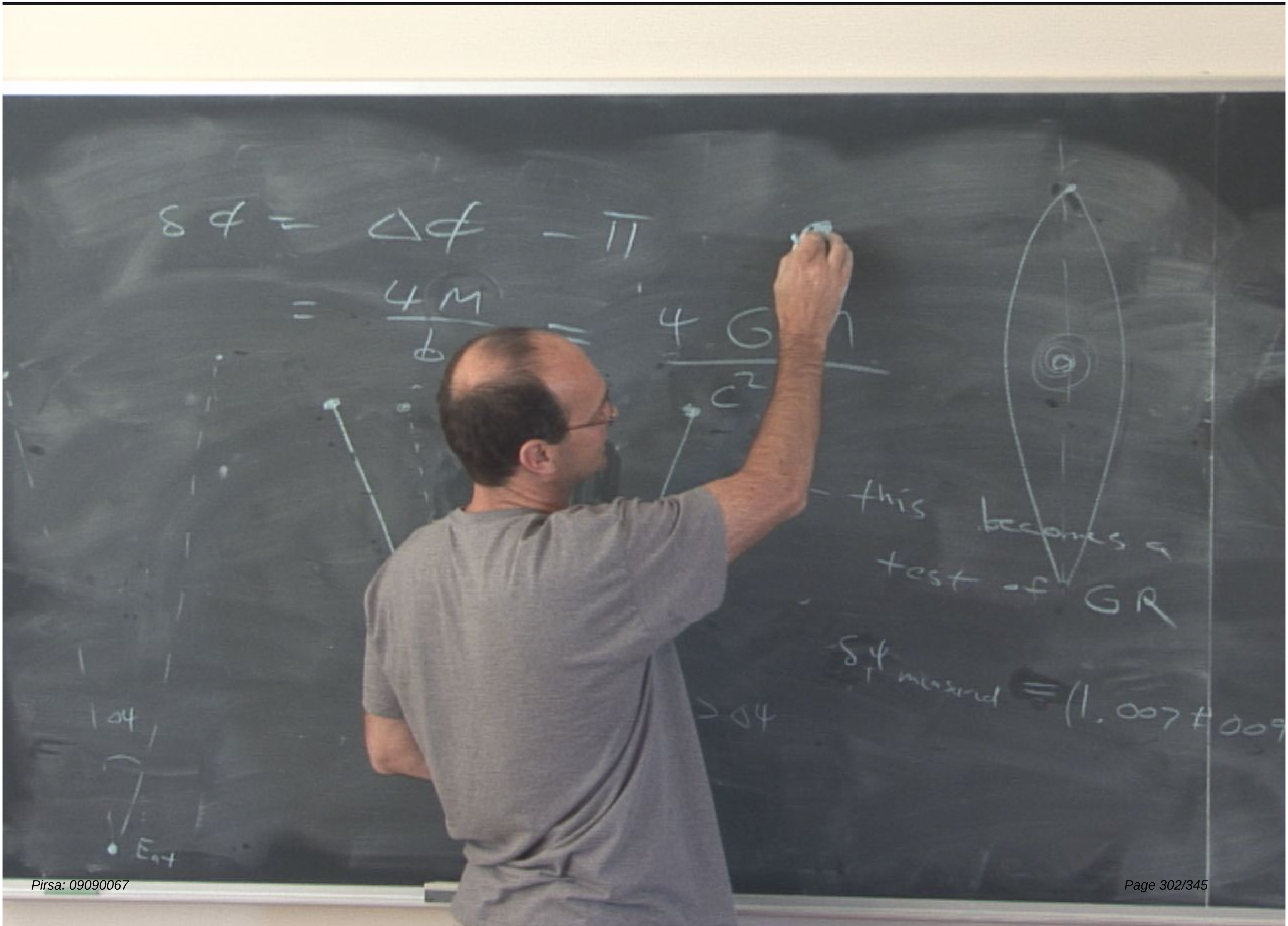


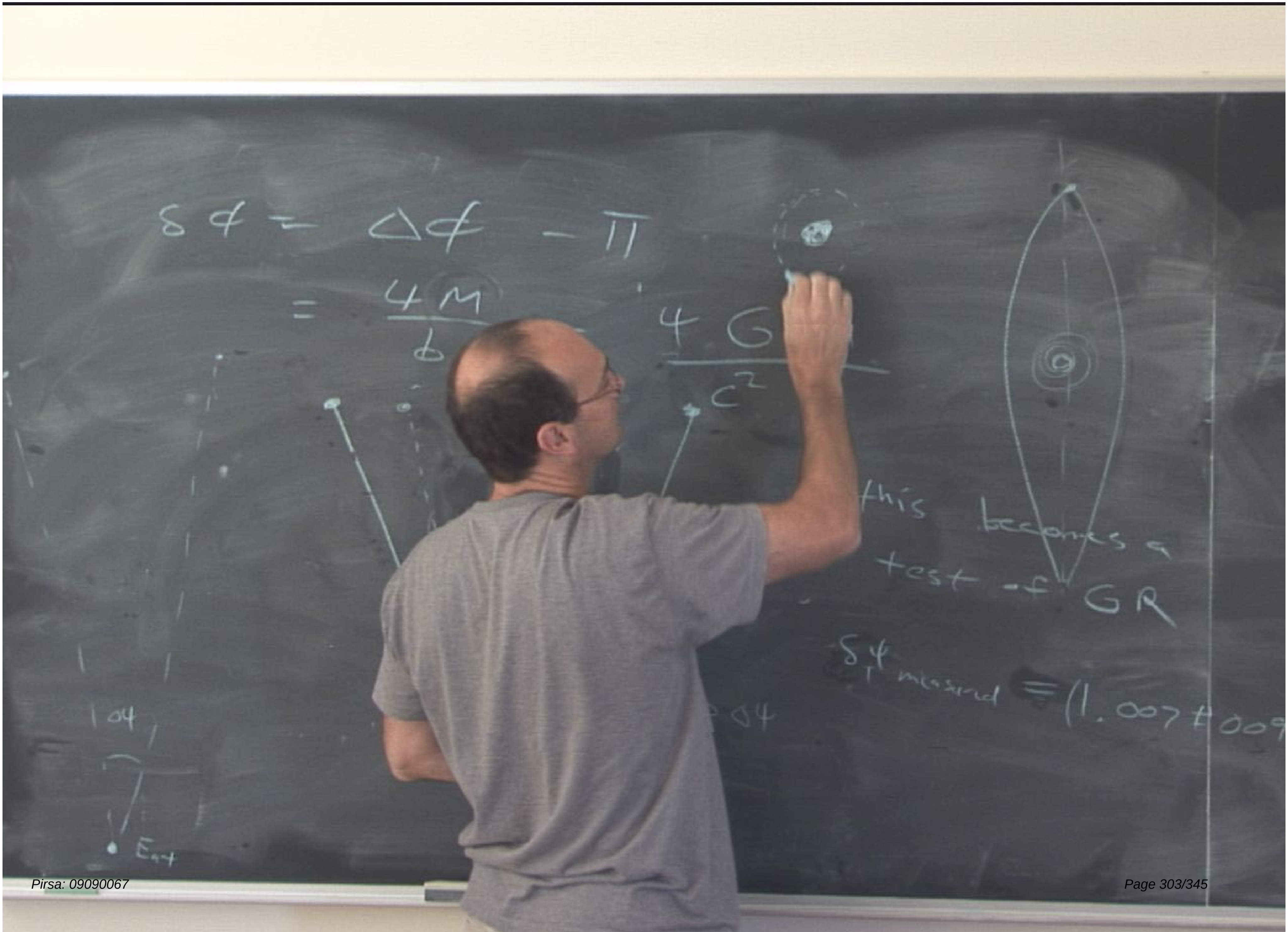
δφ = Δφ − π
= 4M/b = 4GM/c²b
Sun
Δφ'
Δφ' > Δφ
- this becomes a test of GR
δφ measured = (1.007 ± .009

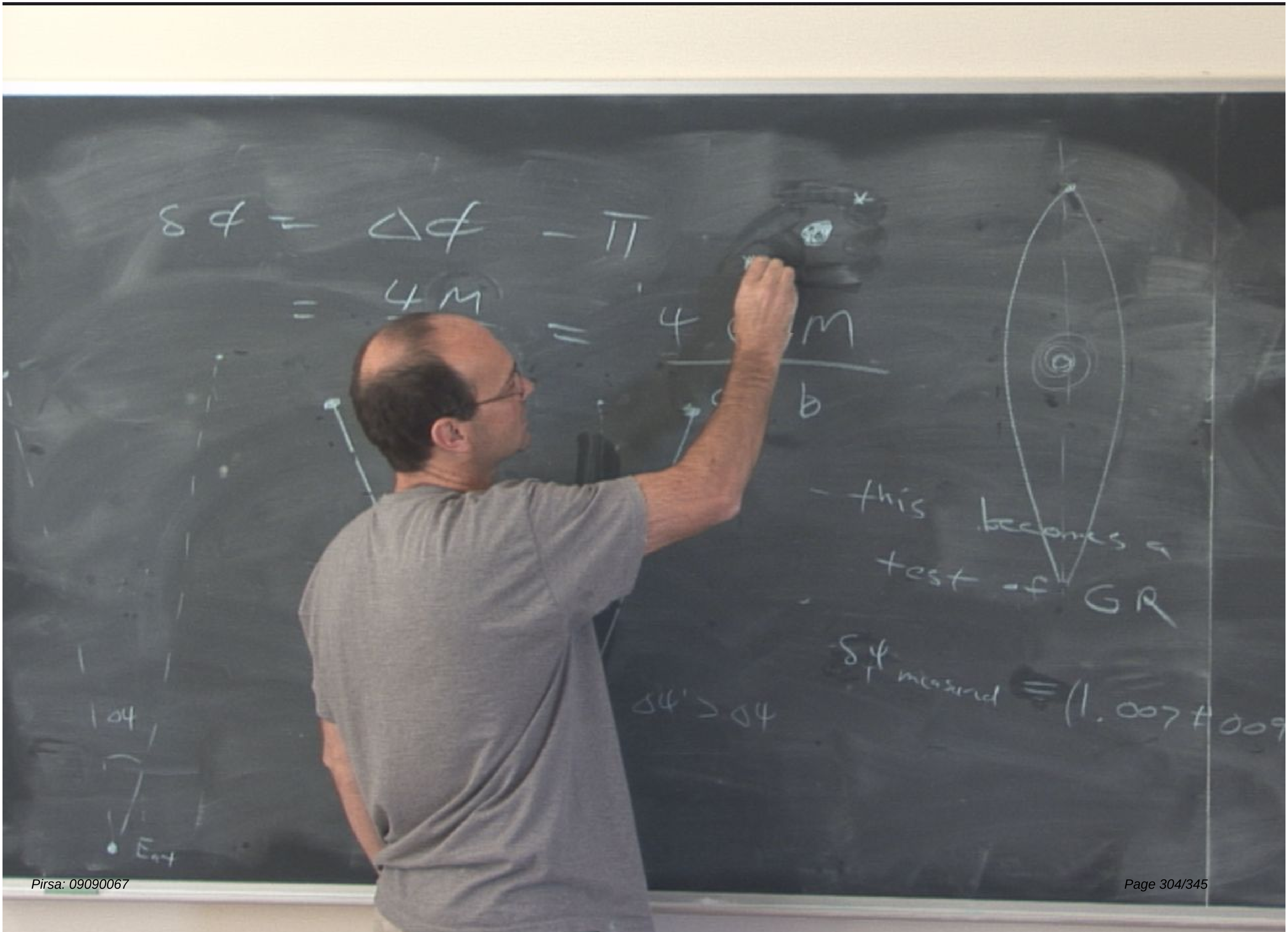

δφ = Δφ − π
= 4M/b = 4GM/c²b
Sun
Δφ

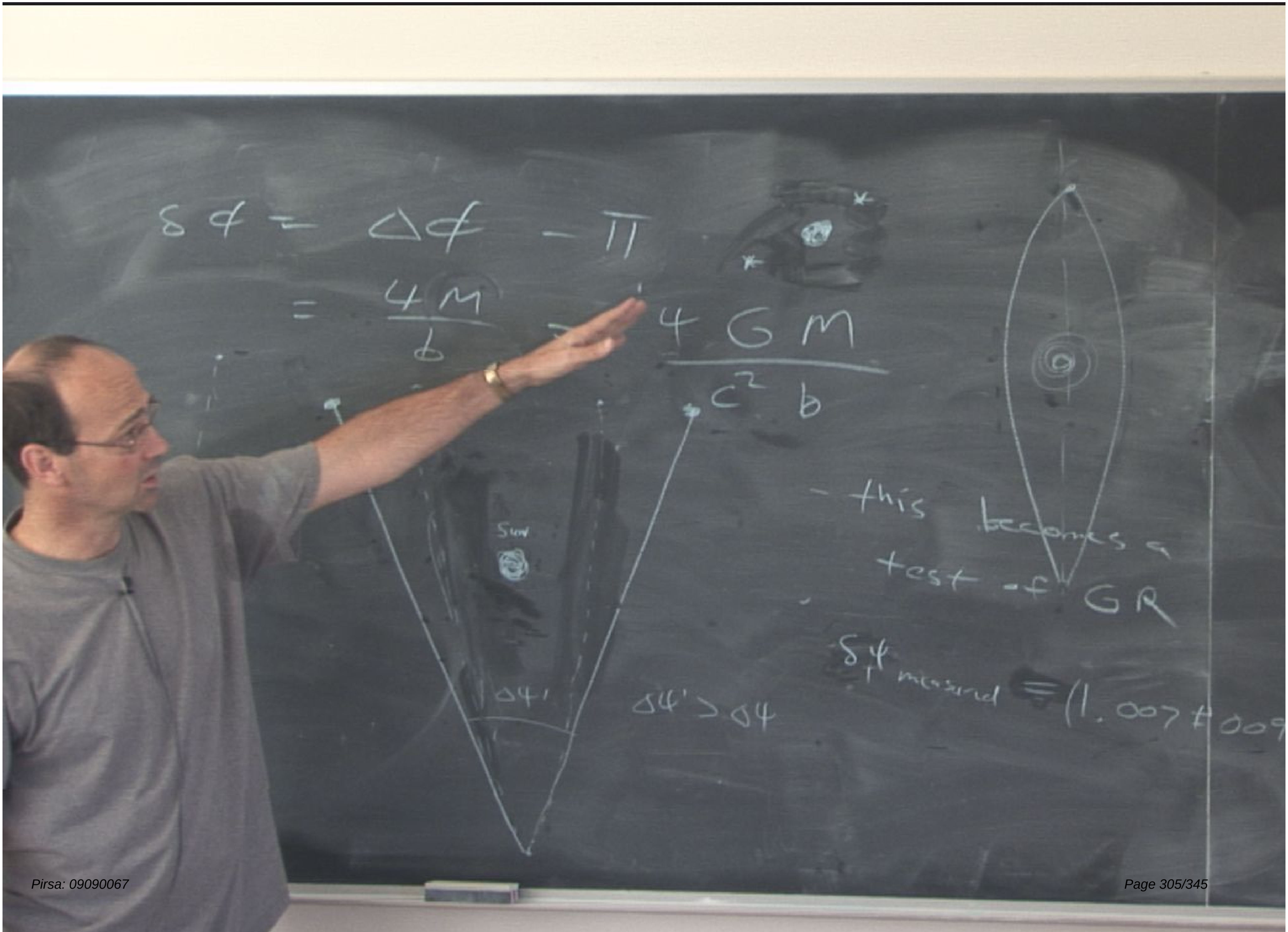

δφ = Δφ − π
= 4M/b = 4GM/c²
Sun
becomes a
test of GR
Earth

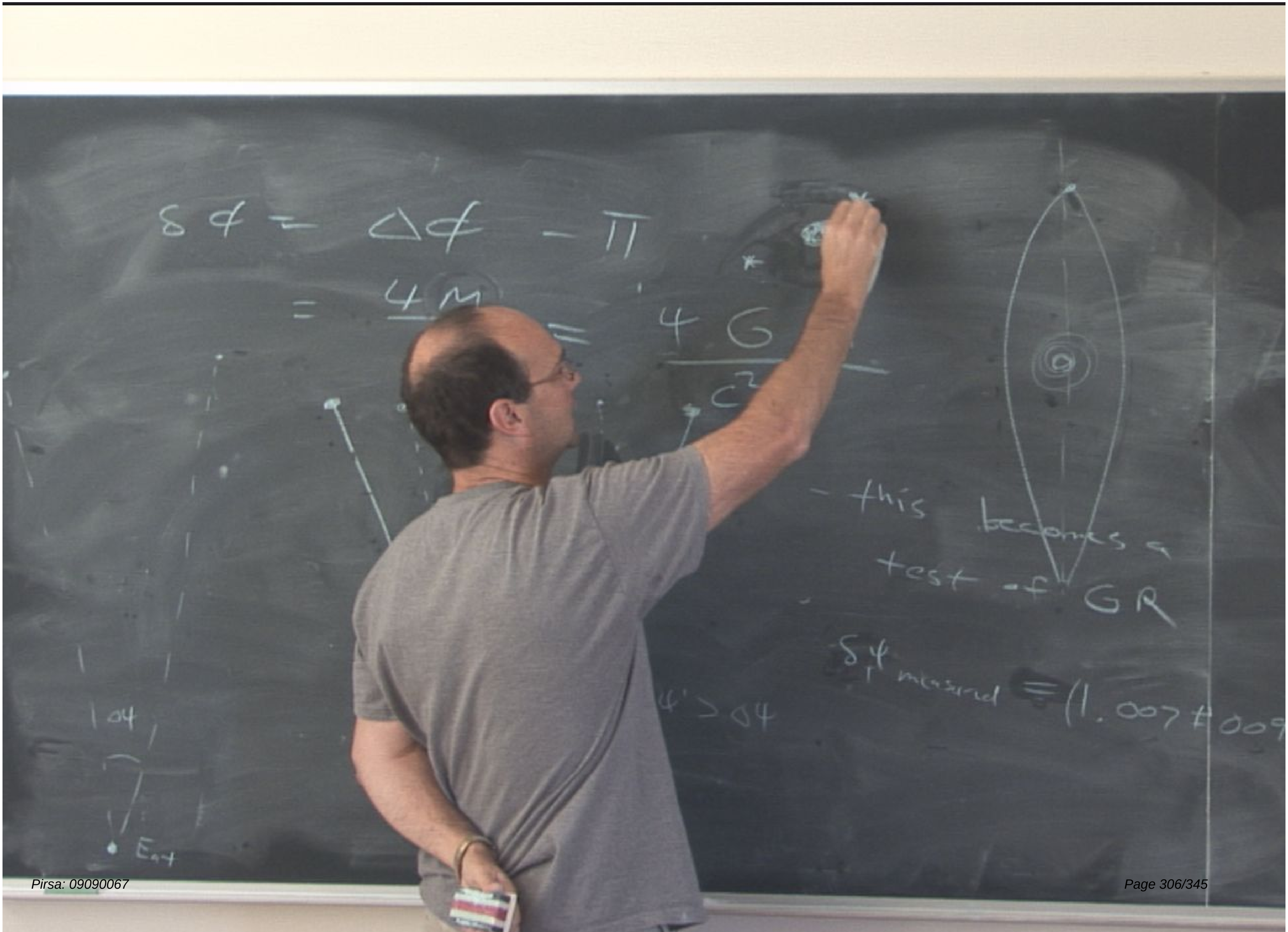

δφ = Δφ − π
= 4M/b = 4G / c² b
his becomes a
test of GR
δφ measured = (1.007 ± .009


δφ = Δφ − π
this becomes a
test of GR

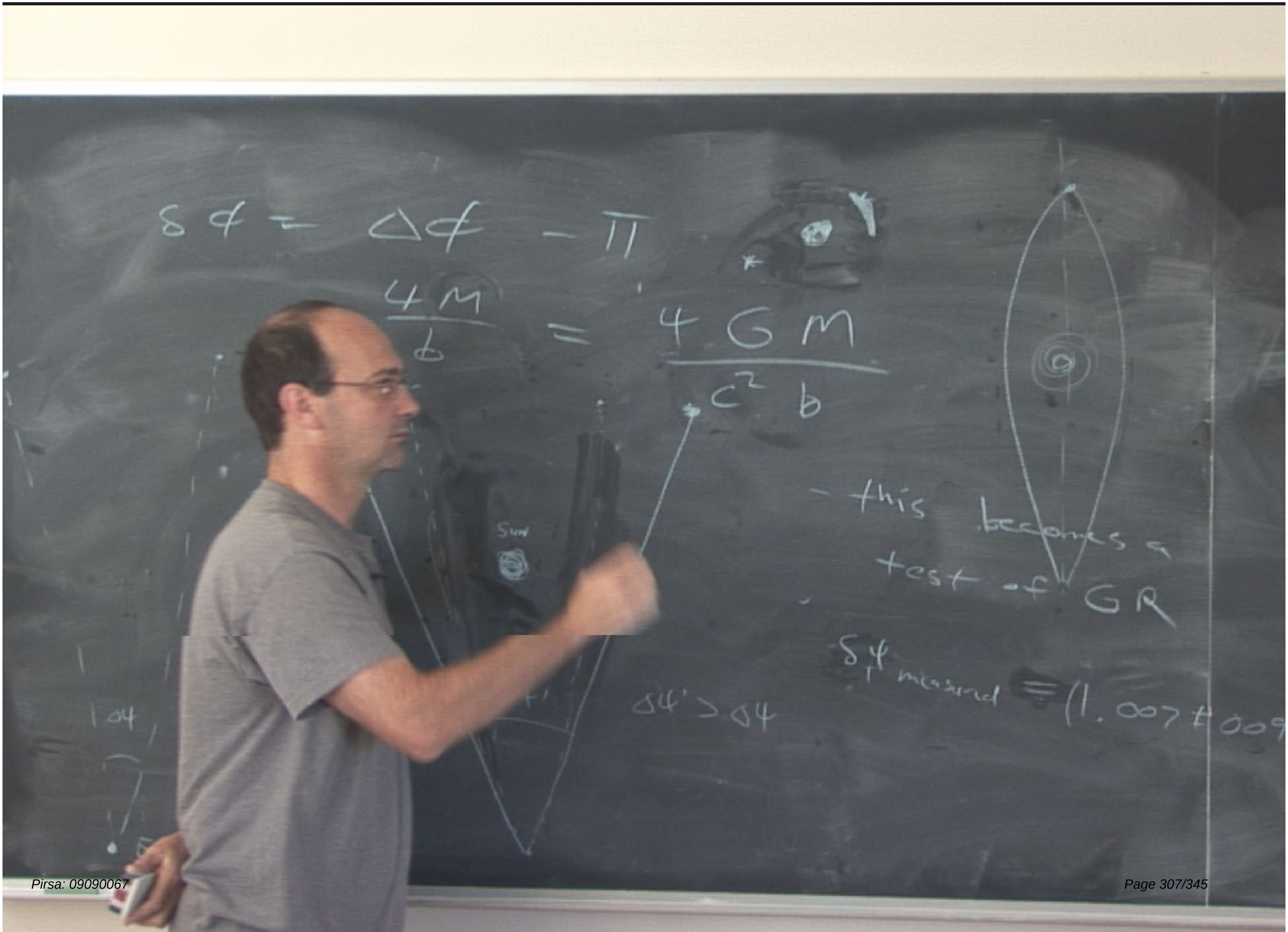

δφ = Δφ − π
this becomes a
test of GR

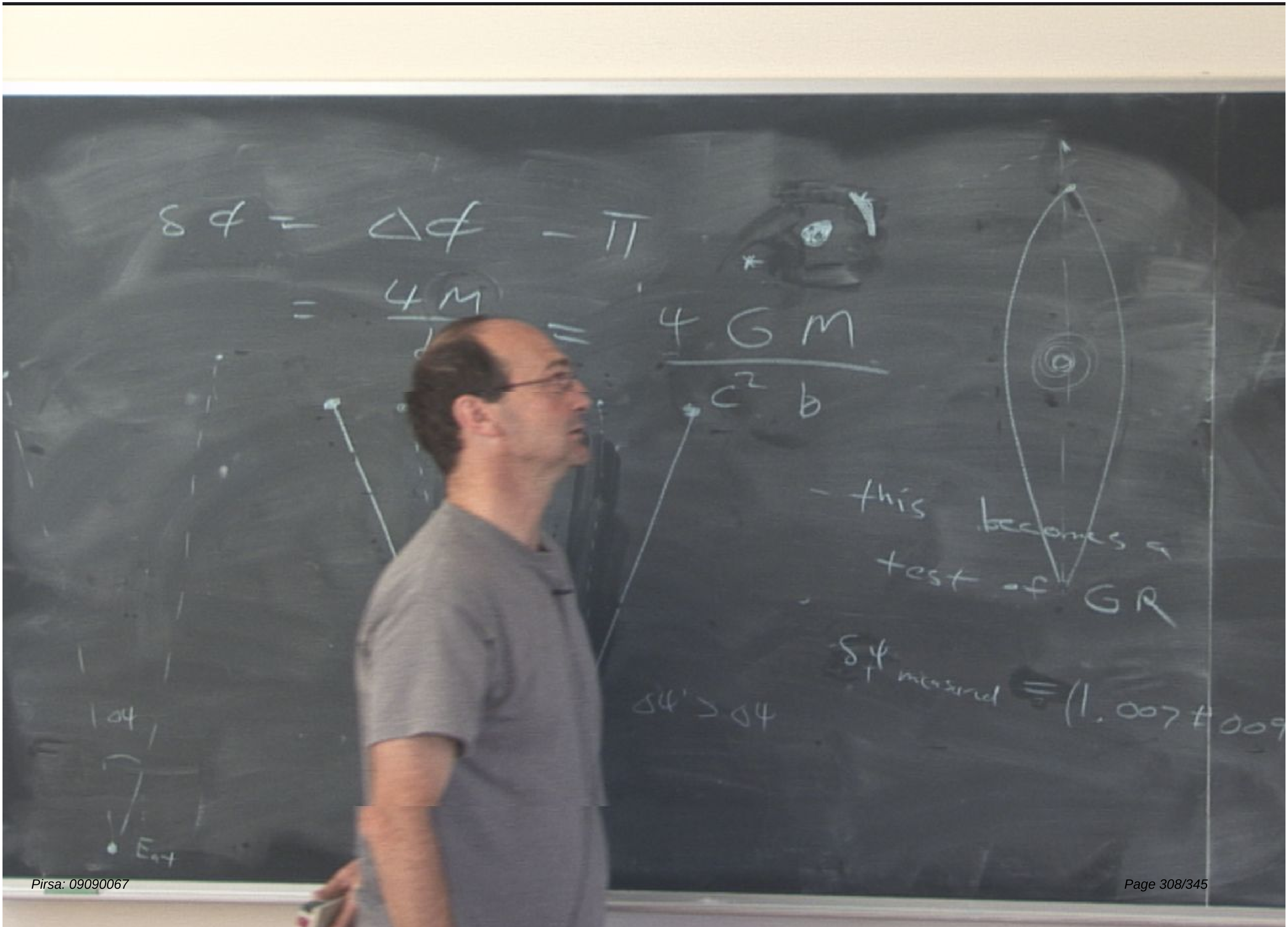

δφ = Δφ − π
this becomes a
test of GR
δφ measured = (1.007 ±.009

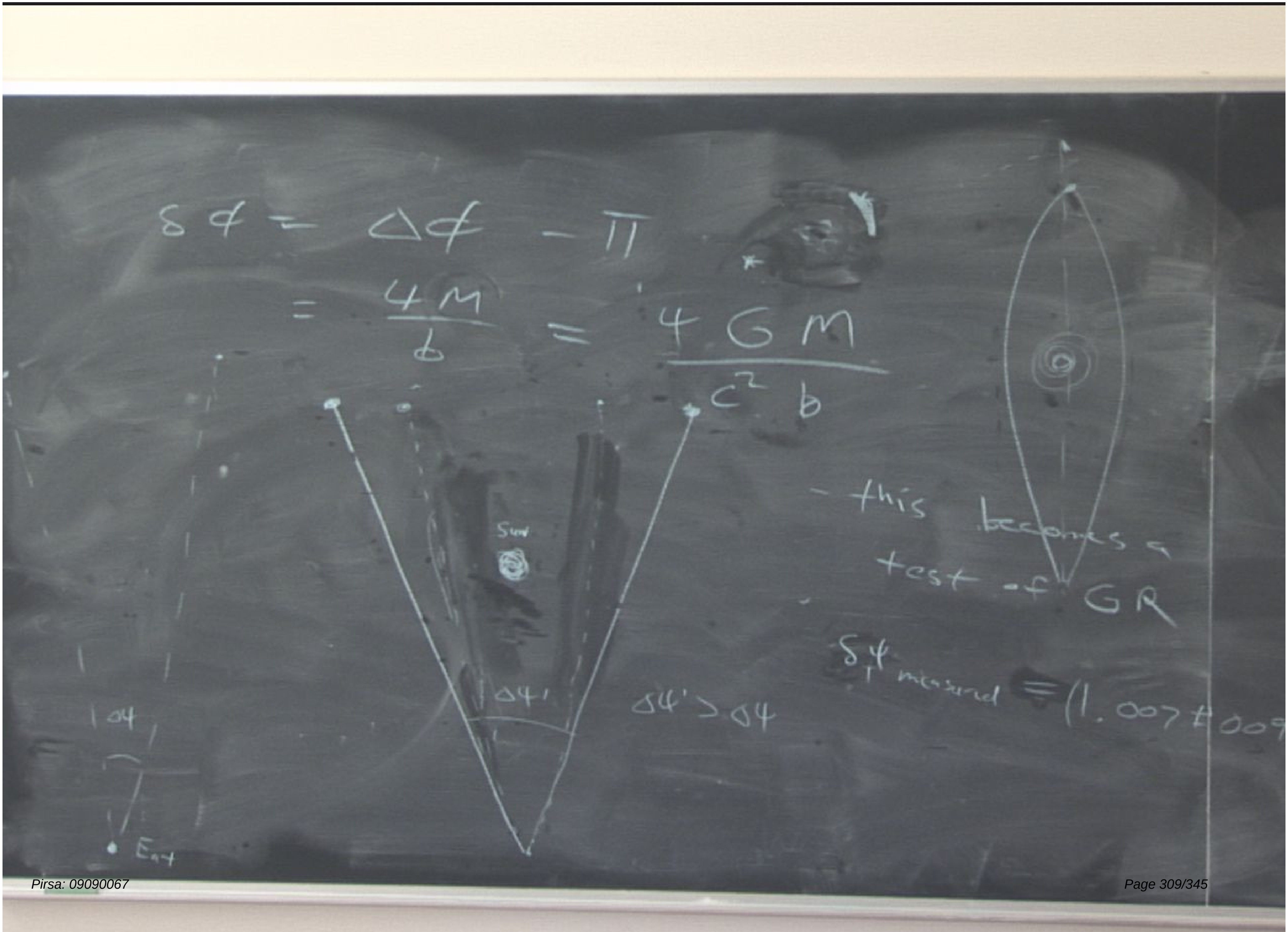

δφ = Δφ − π
= 4M/b → 4GM/c²b
Sun
Δφ'
Δφ' > Δφ
- this becomes a test of GR
δφ measured = (1.007 ± .009

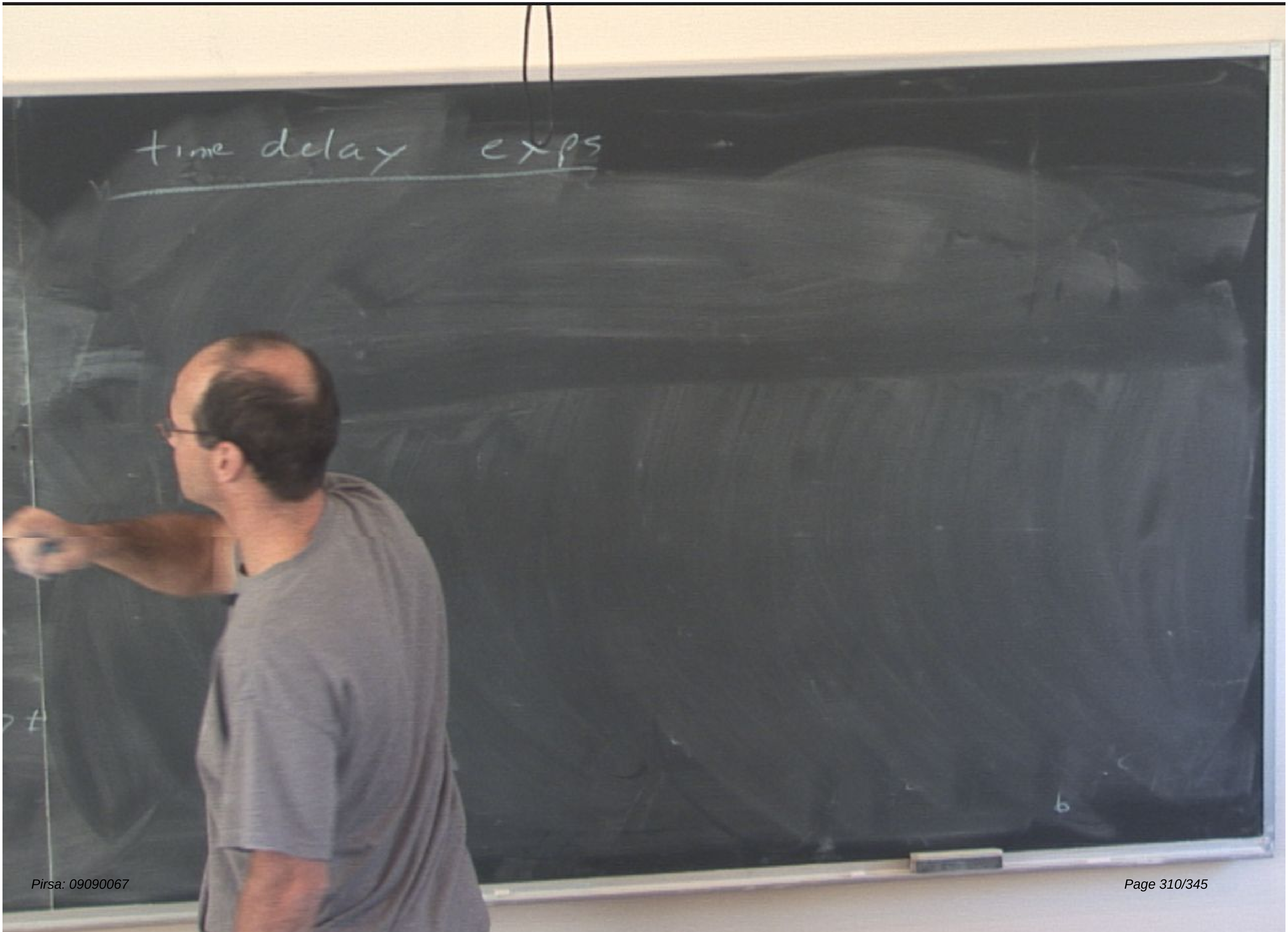


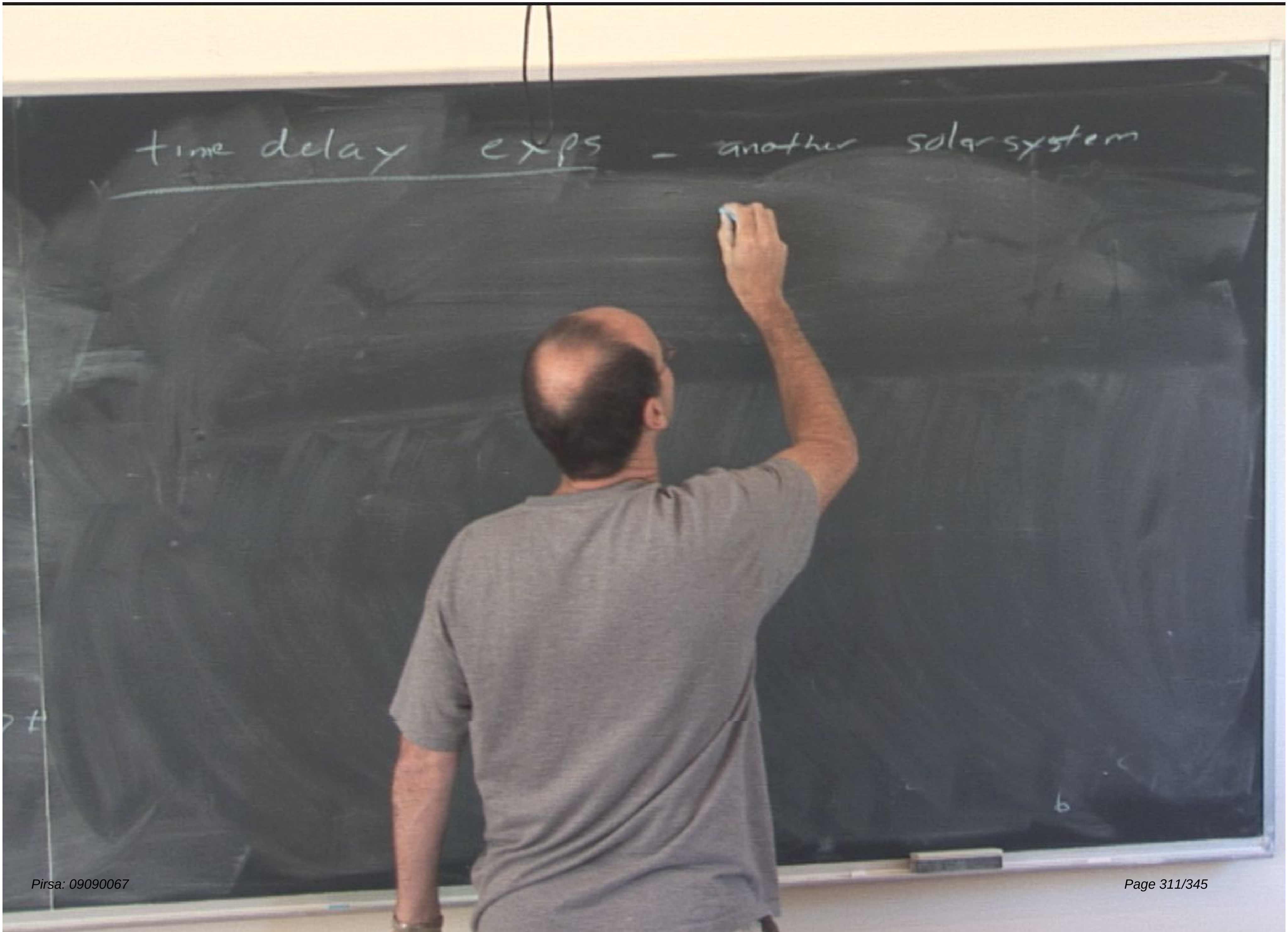
δφ = Δφ − π
= 4M
= 4G
c²
this becomes a
test of GR

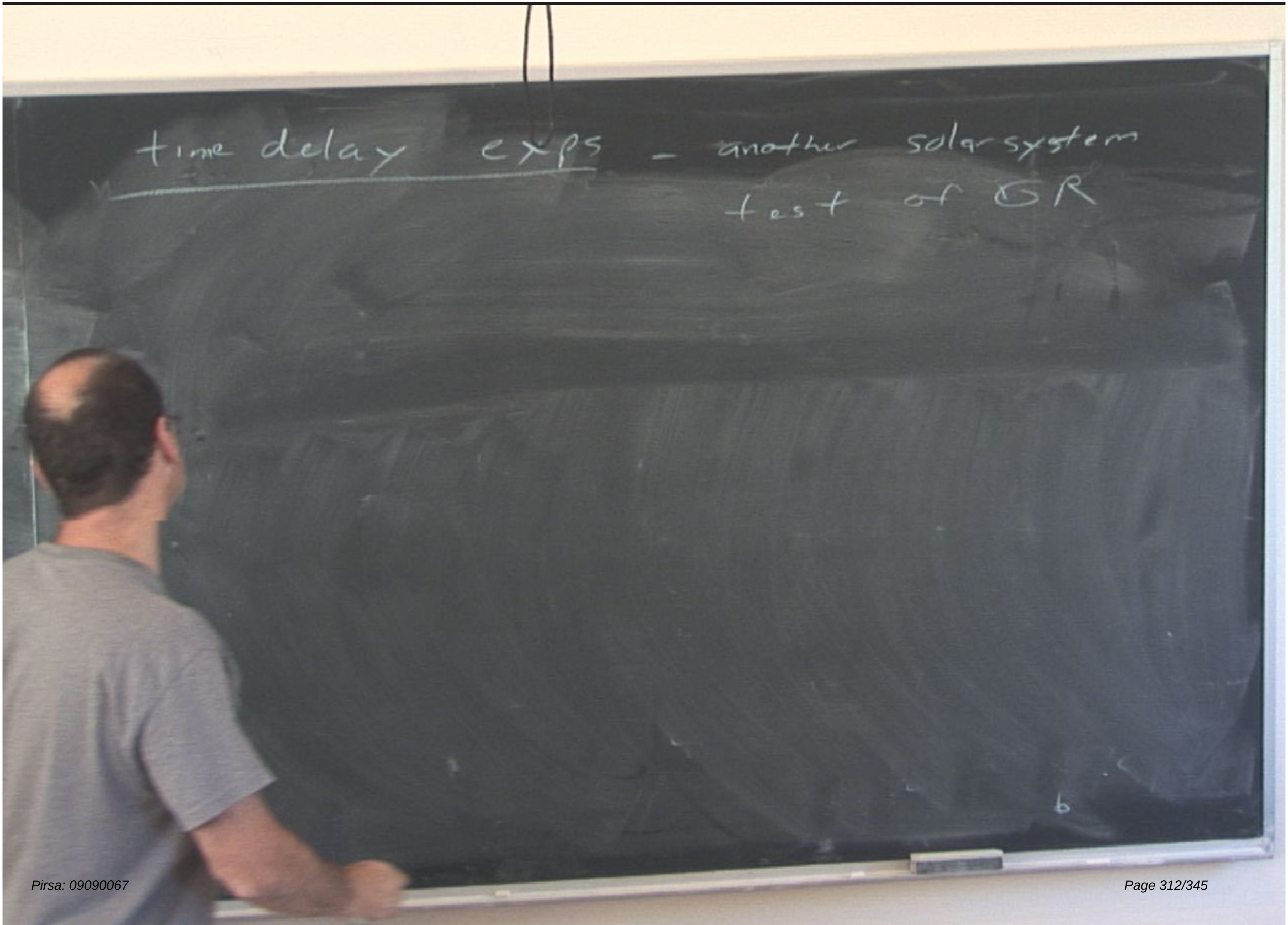

δφ = Δφ − π
4M/b = 4GM/c² b
Sun
this becomes a test of GR
δφ' > δφ
δφ measured = (1.007 ± .009

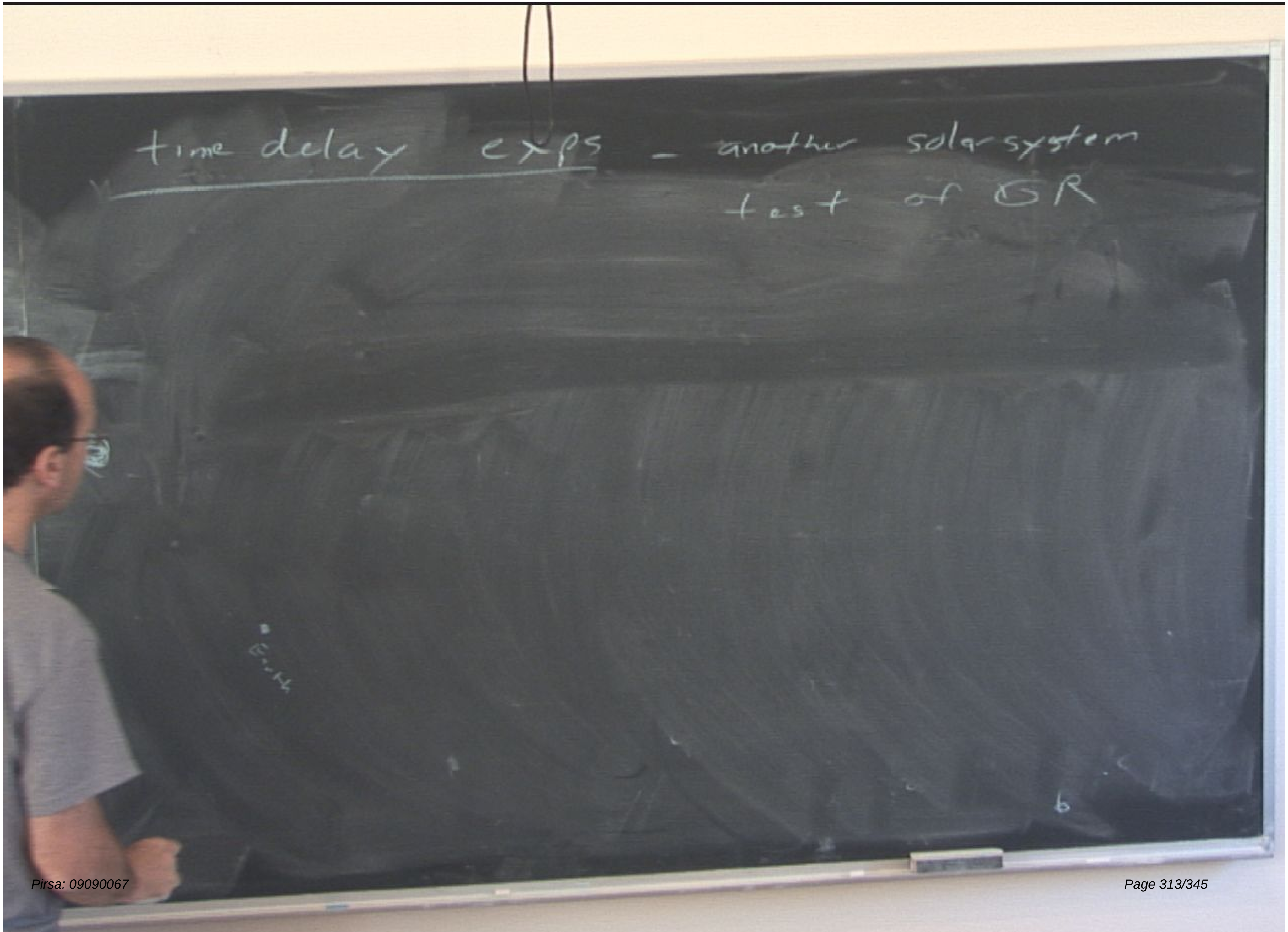

δφ = Δφ − π
= 4GM / c² b
this becomes a test of GR
δφ measured = (1.007 ±

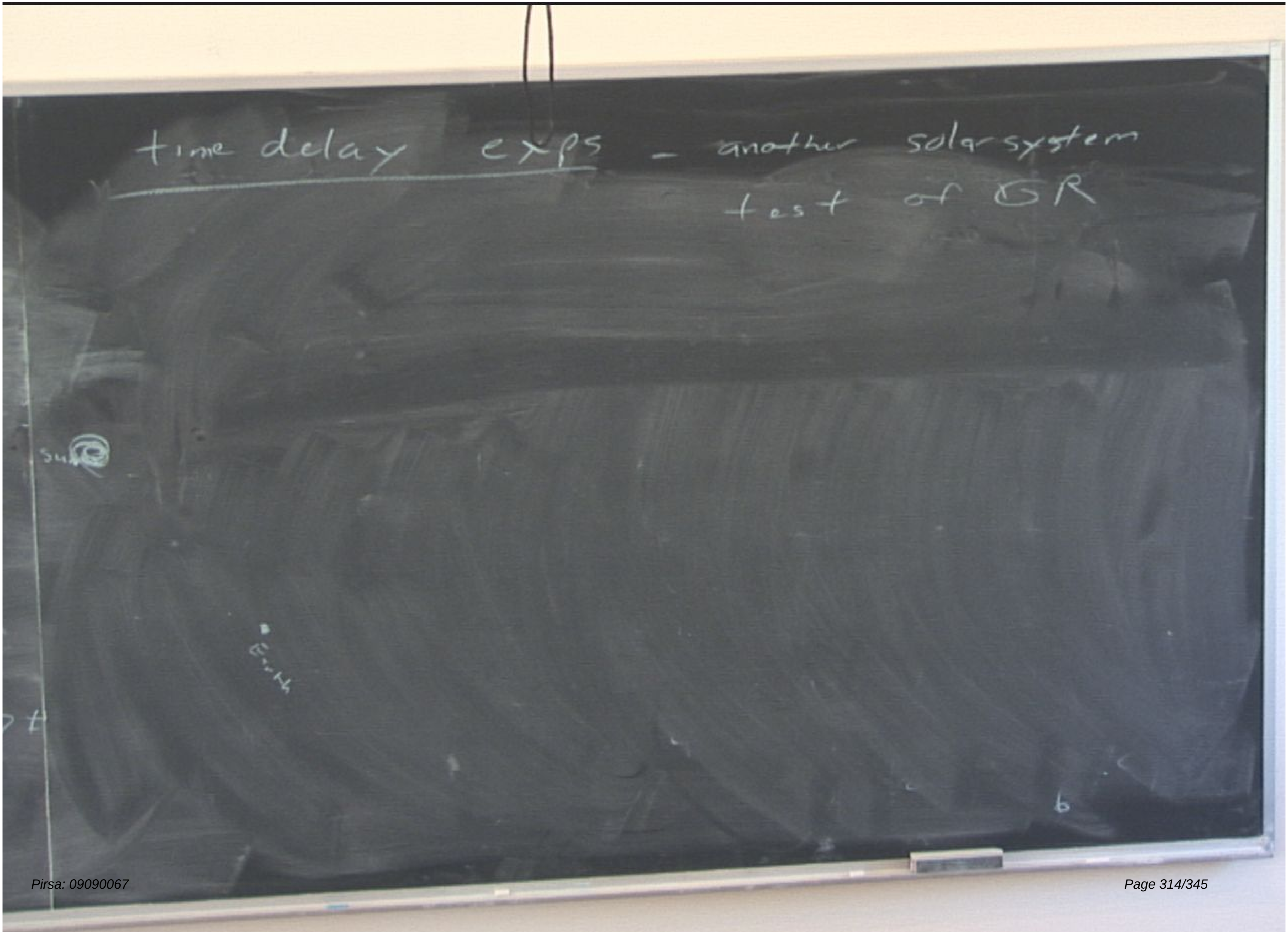

δφ = Δφ − π
= 4M/b = 4GM/c²b
Sun
Δφ'
Δφ' > Δφ
Δφ
Eart
- this becomes a test of GR
δφ measured = (1.007 ± .009

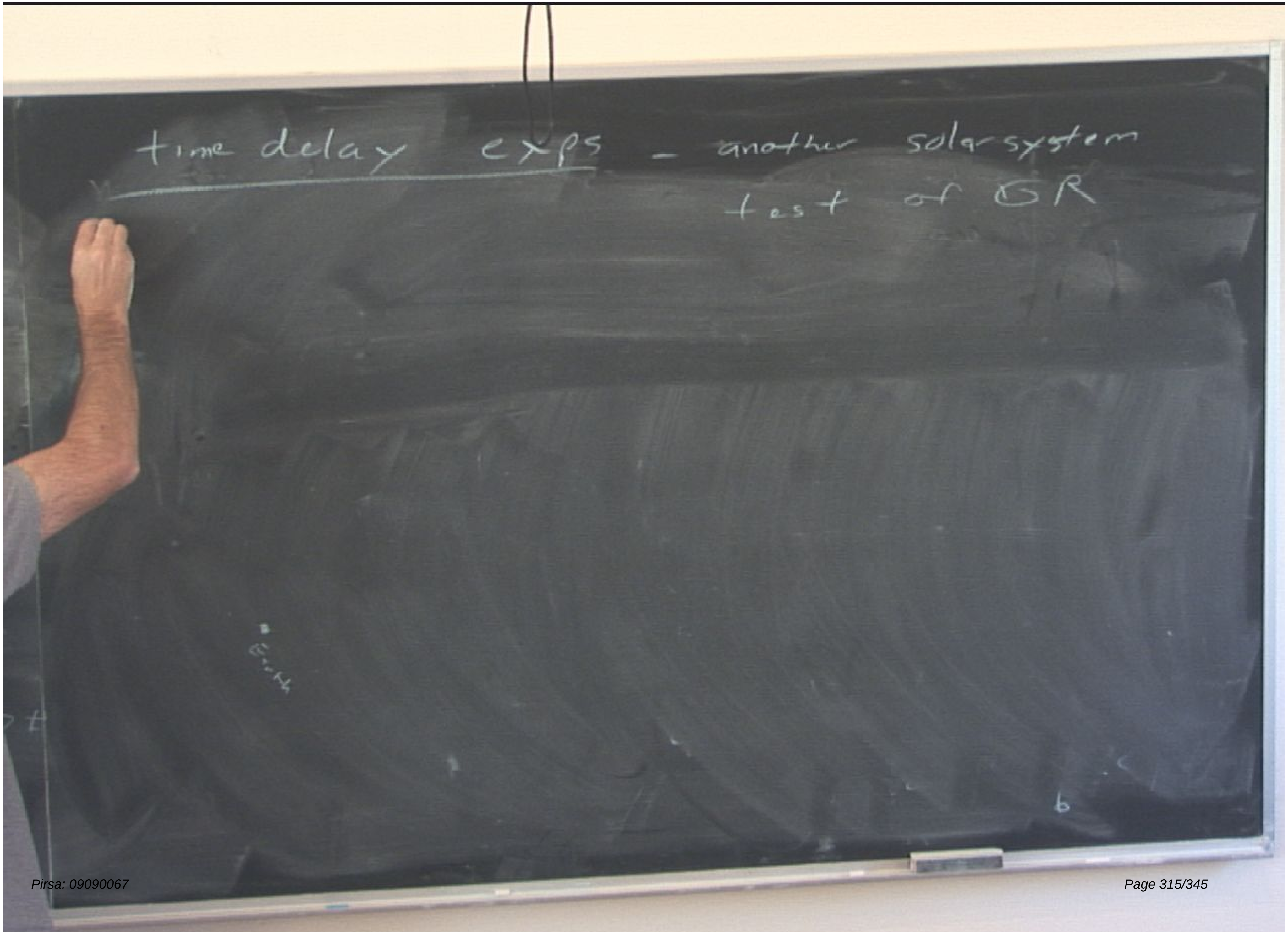

time delay exps

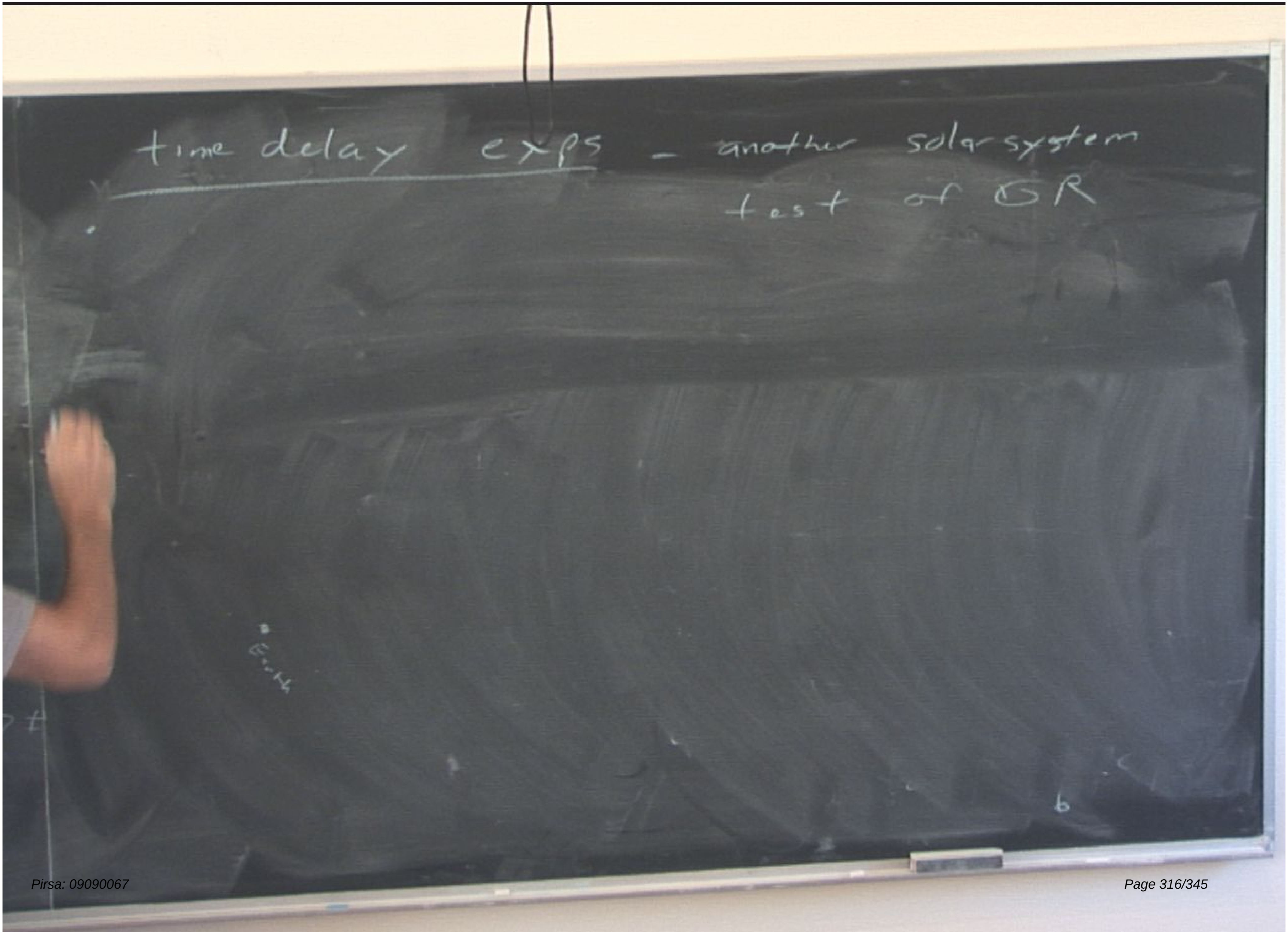

time delay exps - another solarsystem

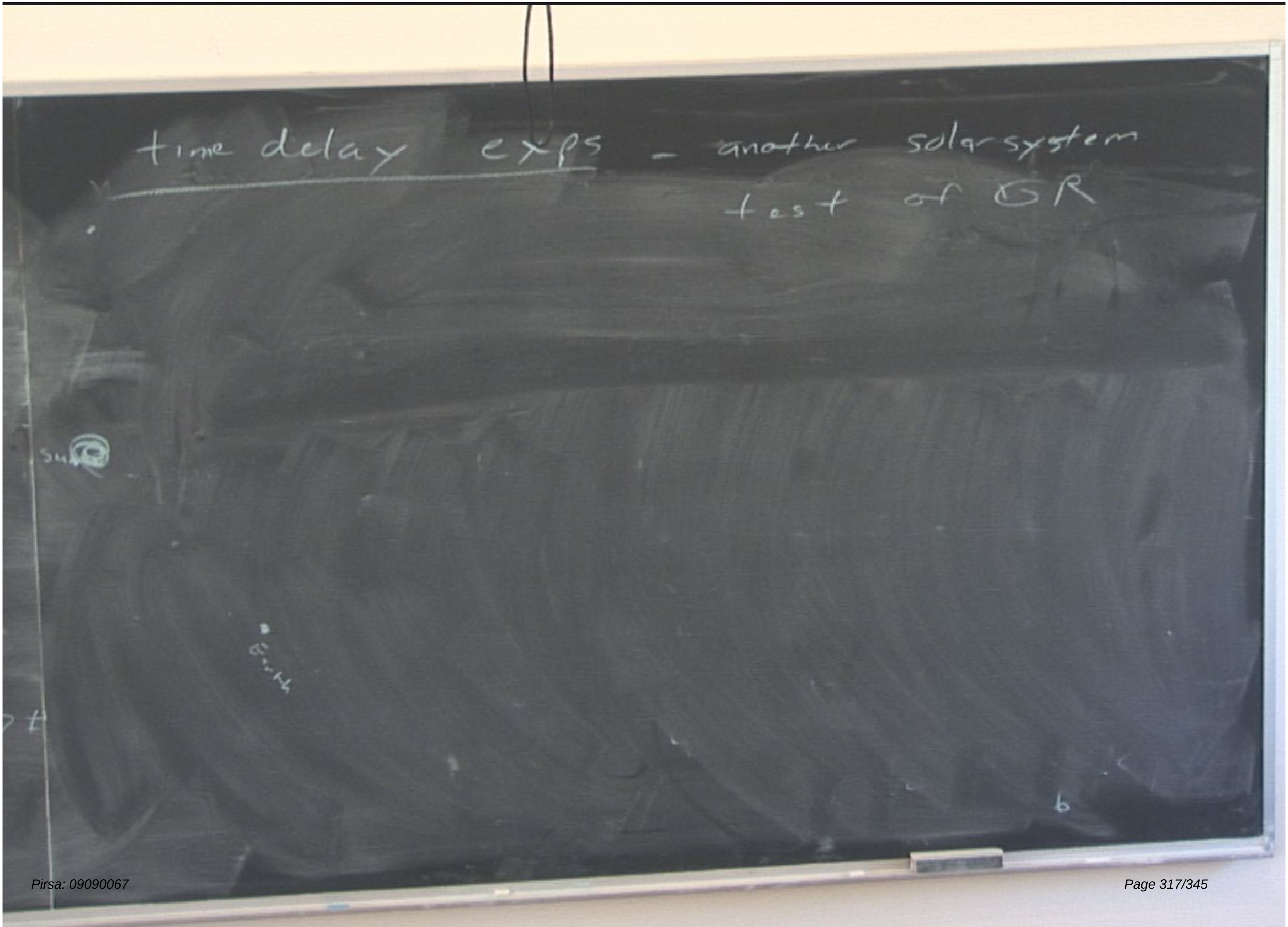

time delay exps - another solarsystem
test of GR

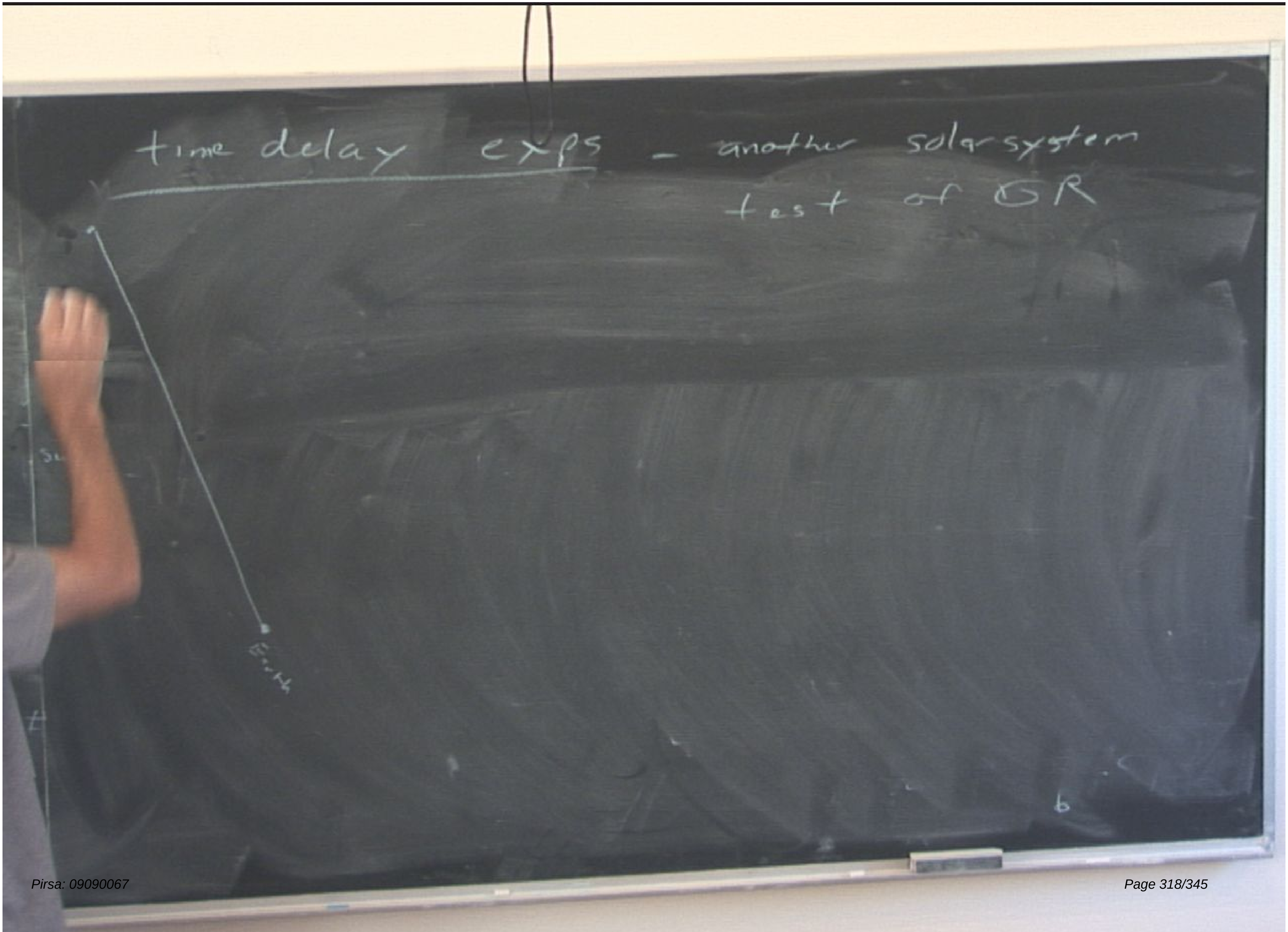

time delay exps – another solar system
test of GR

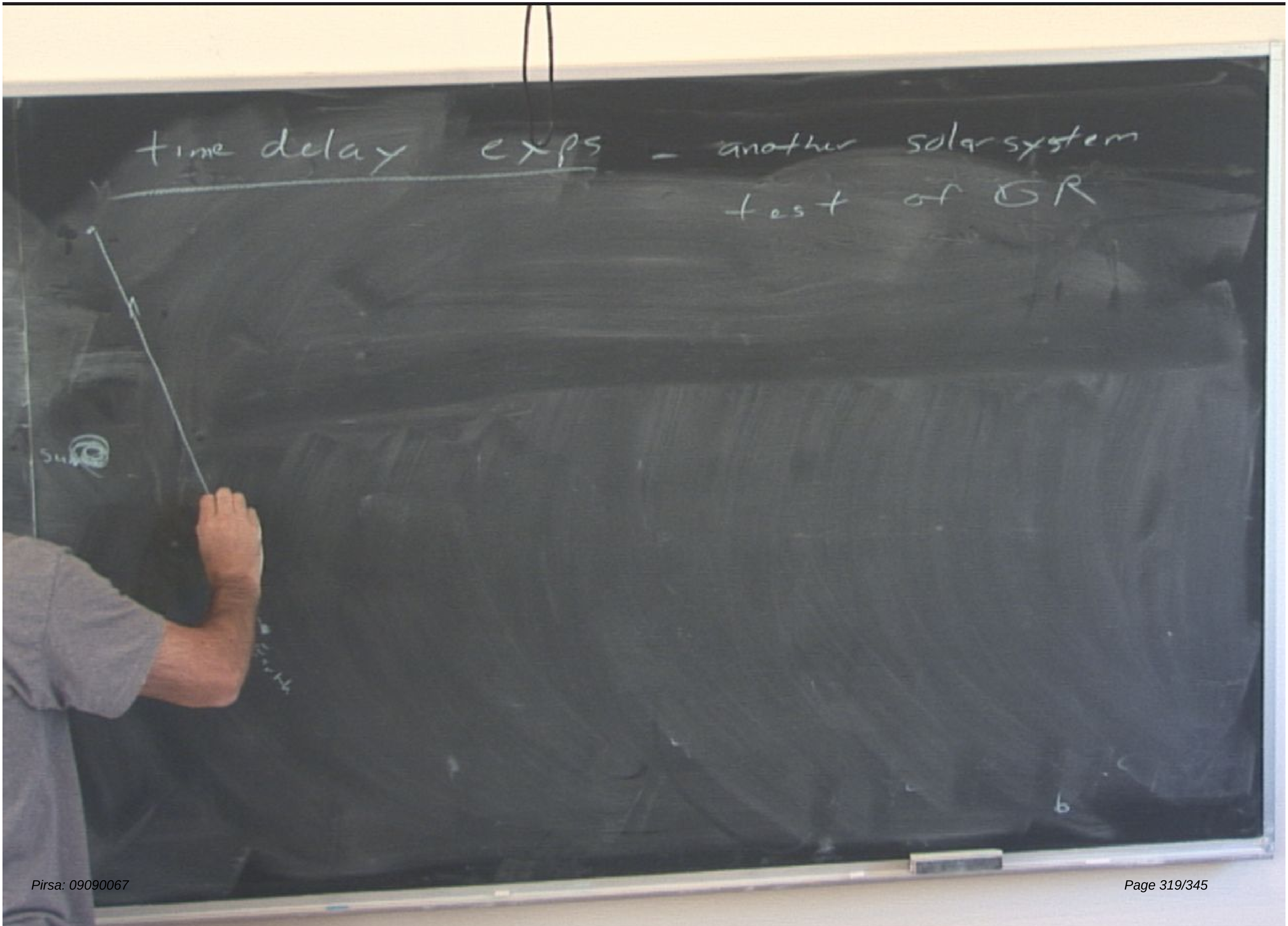

time delay exps - another solarsystem
test of GR
Sun

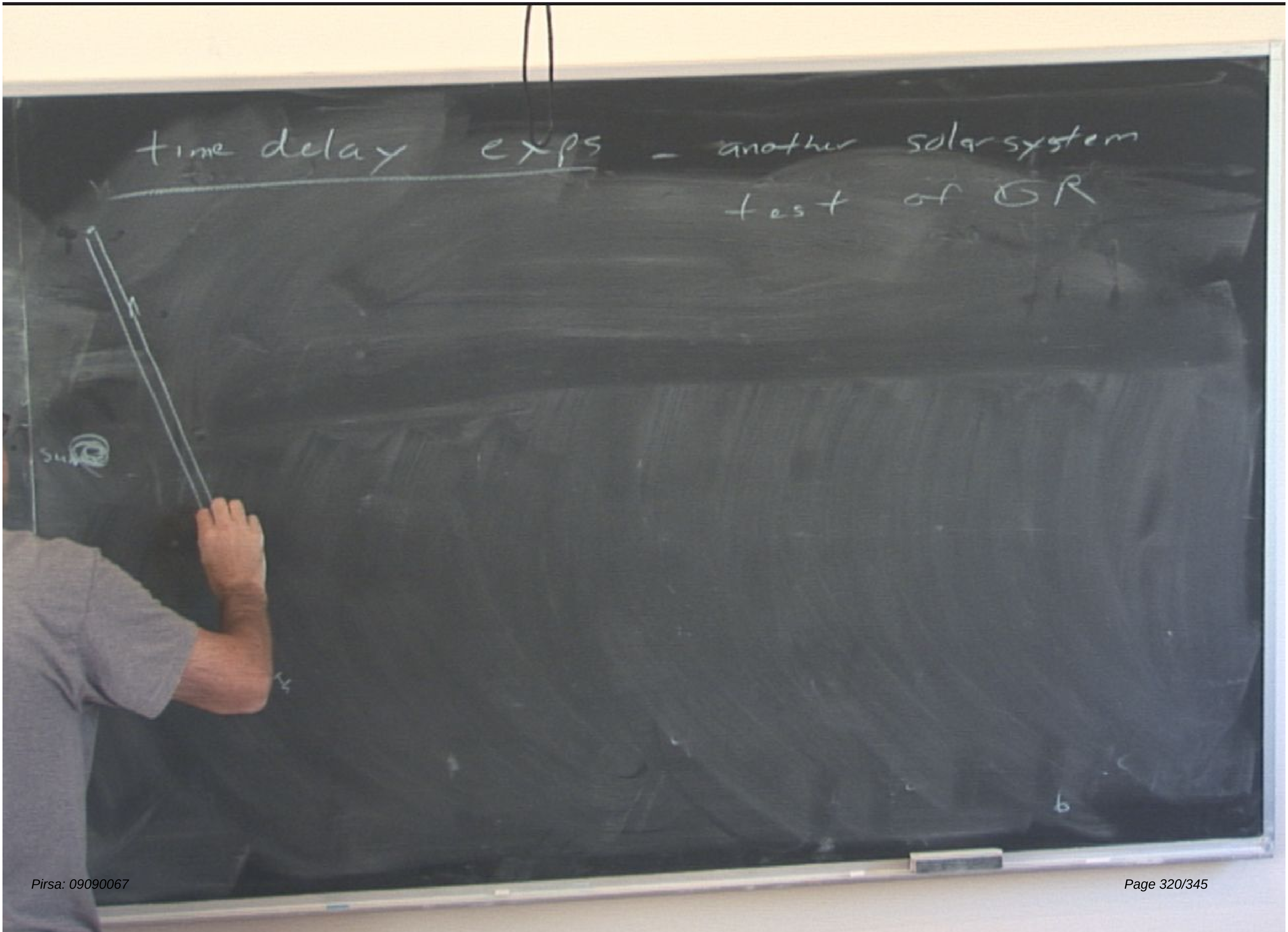

time delay exps - another solar system
test of GR

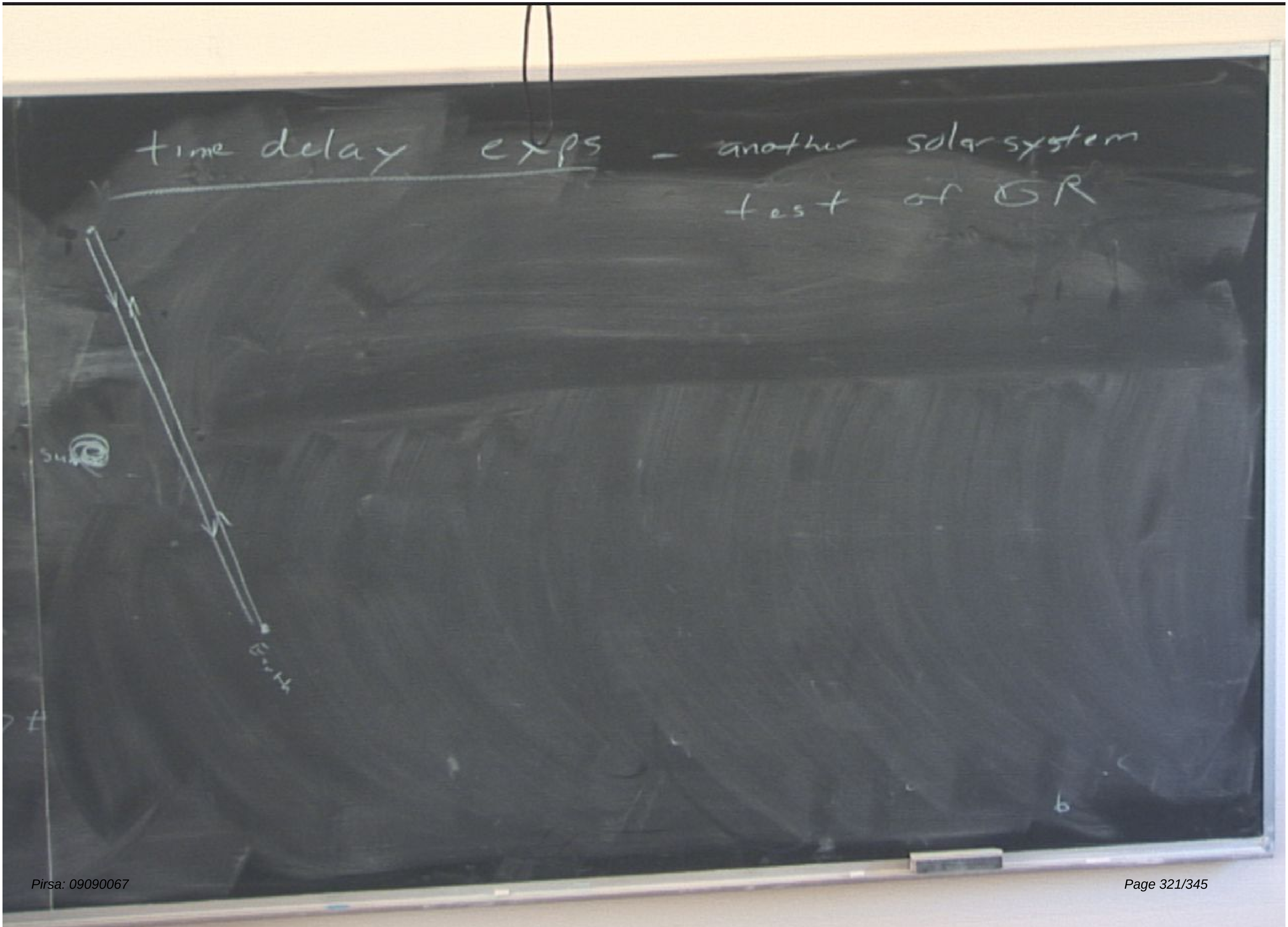

time delay exps — another solarsystem
test of GR

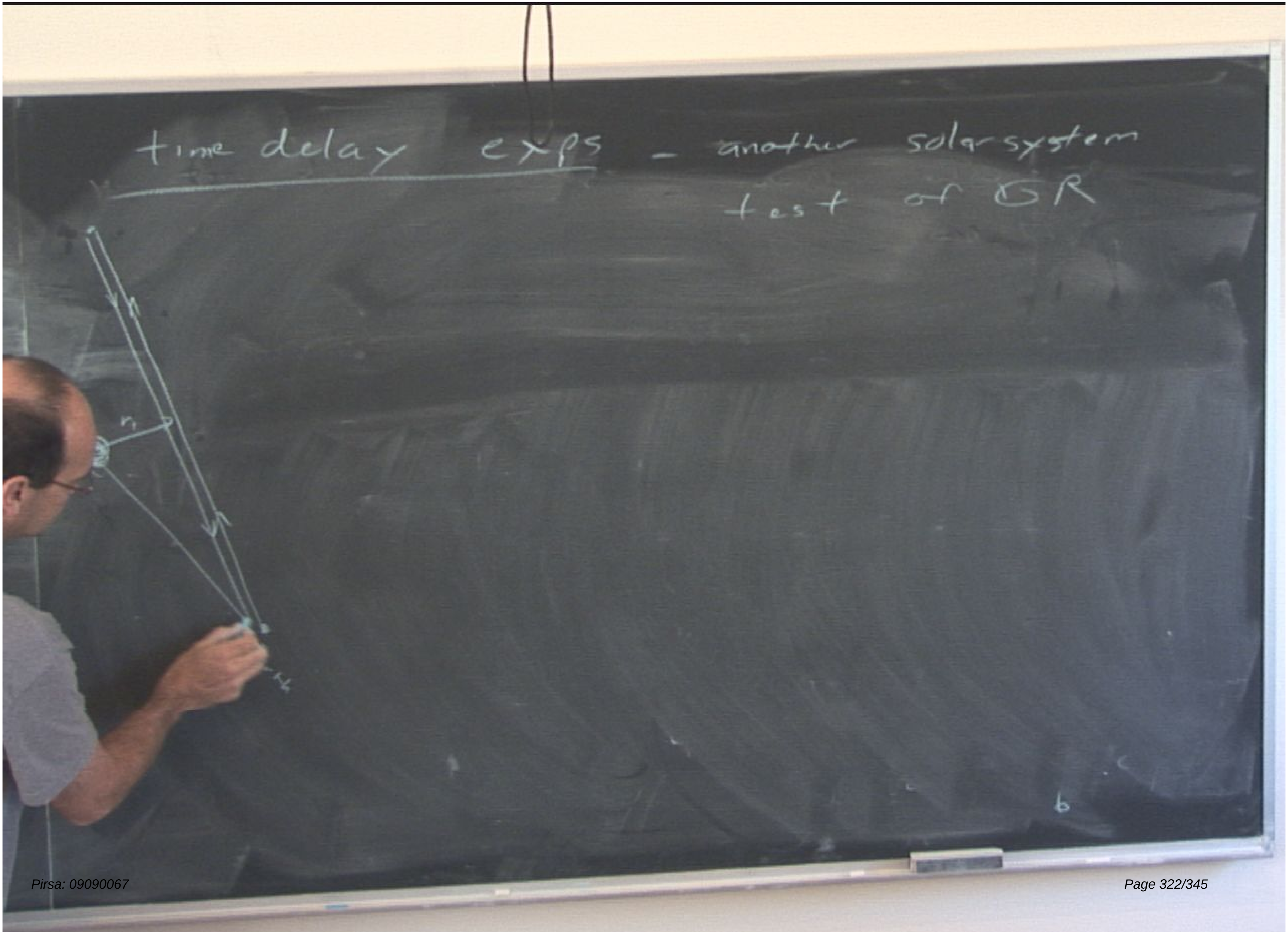

time delay exps - another solarsystem
test of GR
sun

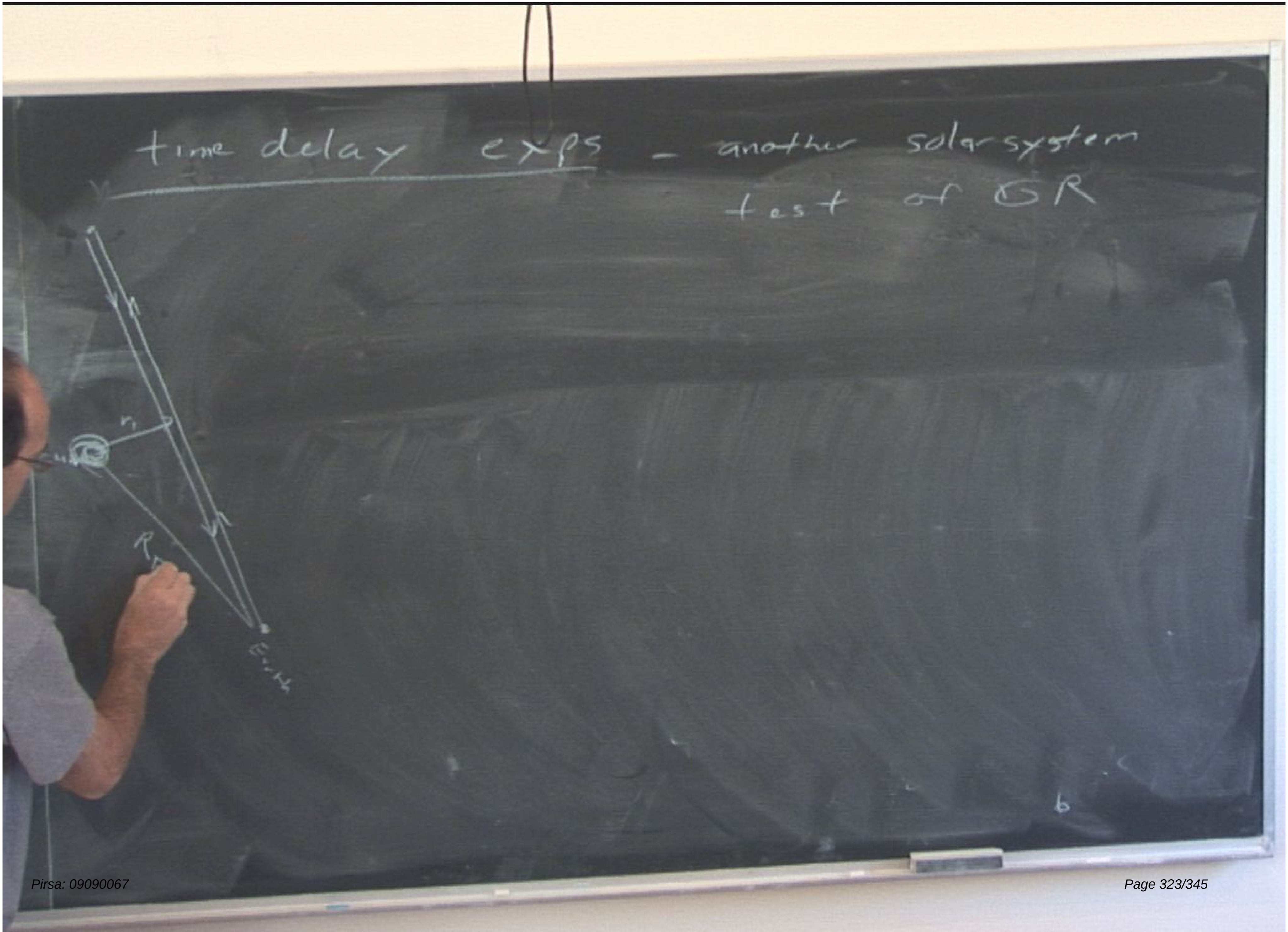

time delay exps - another solarsystem
test of GR
Earth


time delay exps - another solarsystem
test of GR

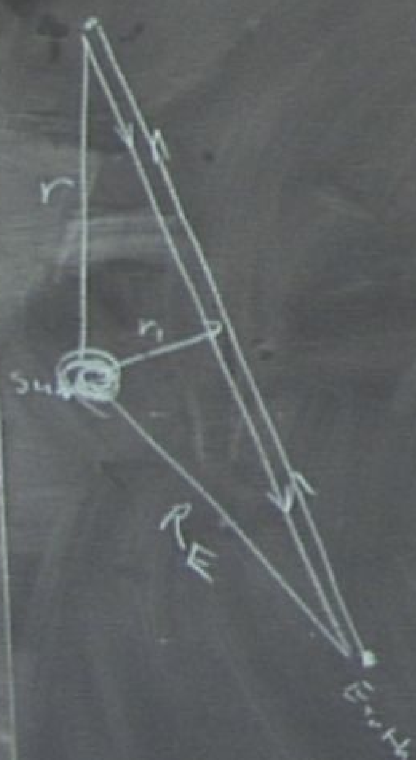

time delay exps - another solarsystem
test of GR
Sun

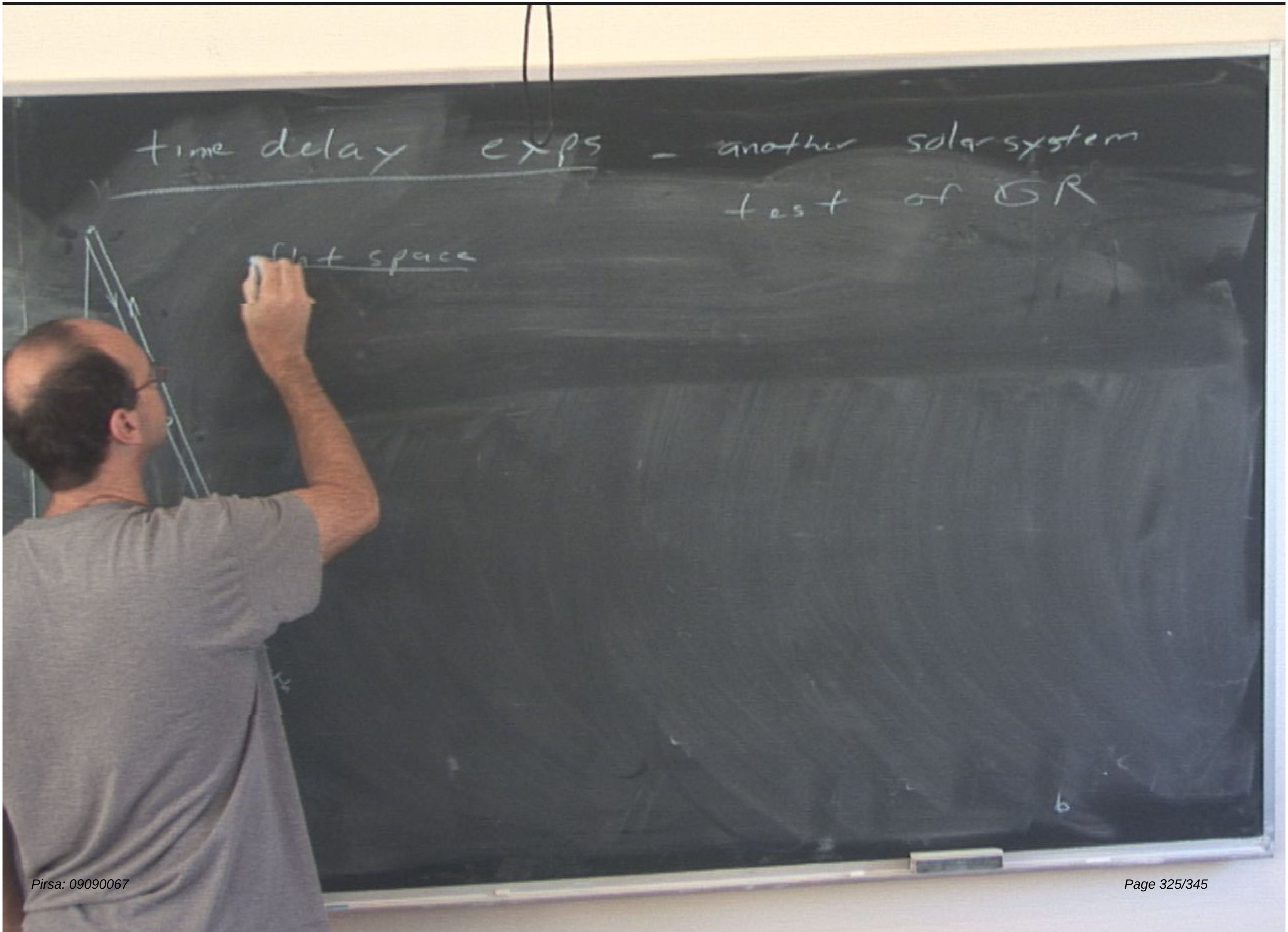

time delay exps - another solarsystem test of GR
Sun
Earth

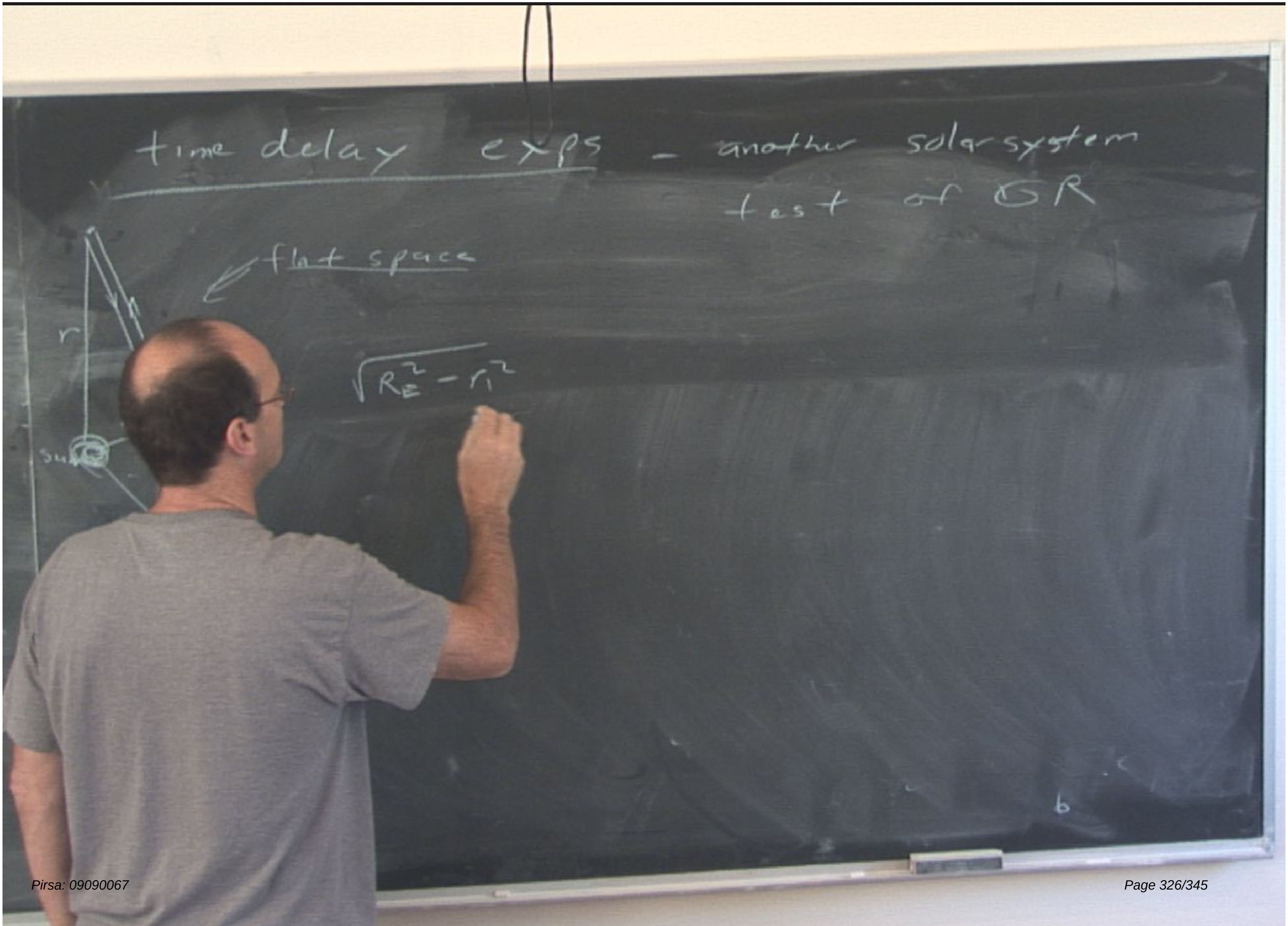

time delay exps - another solarsystem
test of GR

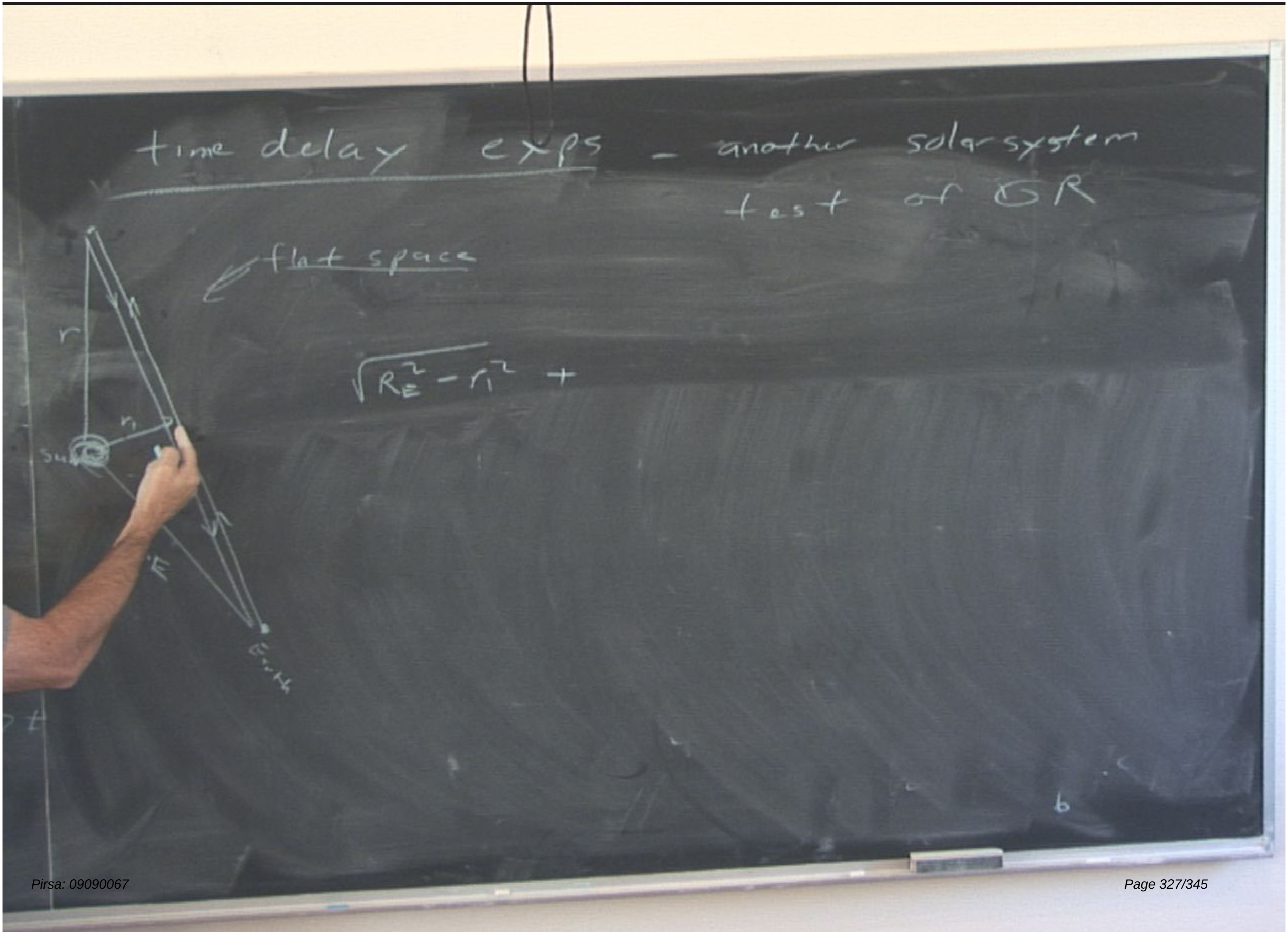

time delay exps - another solarsystem
test of GR
R

time delay exps – another solar system test of GR



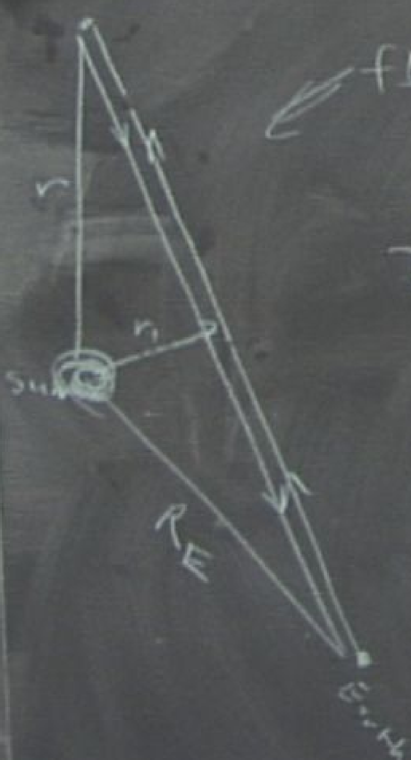

time delay exps - another solarsystem
test of GR
space


time delay exps - another solarsystem
test of GR
flat space
r
√(R_E² − r_1²)


time delay exps - another solar system
test of GR
flat space
r
r_1
Sun
R_E
Earth
$\sqrt{R_E^2 - r_1^2} +$

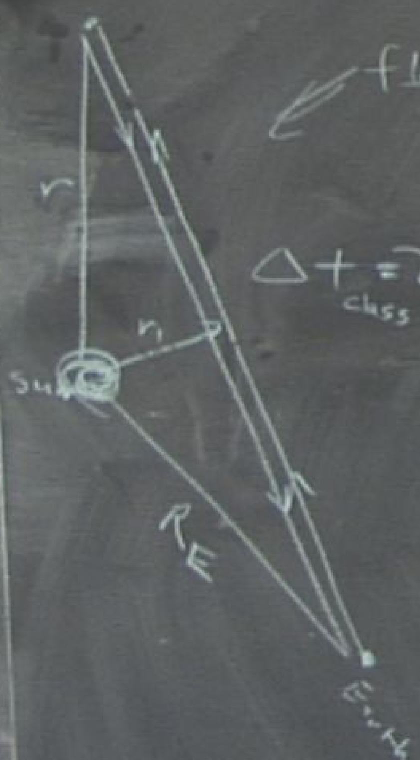
time delay exps – another solar system test of GR

flat space



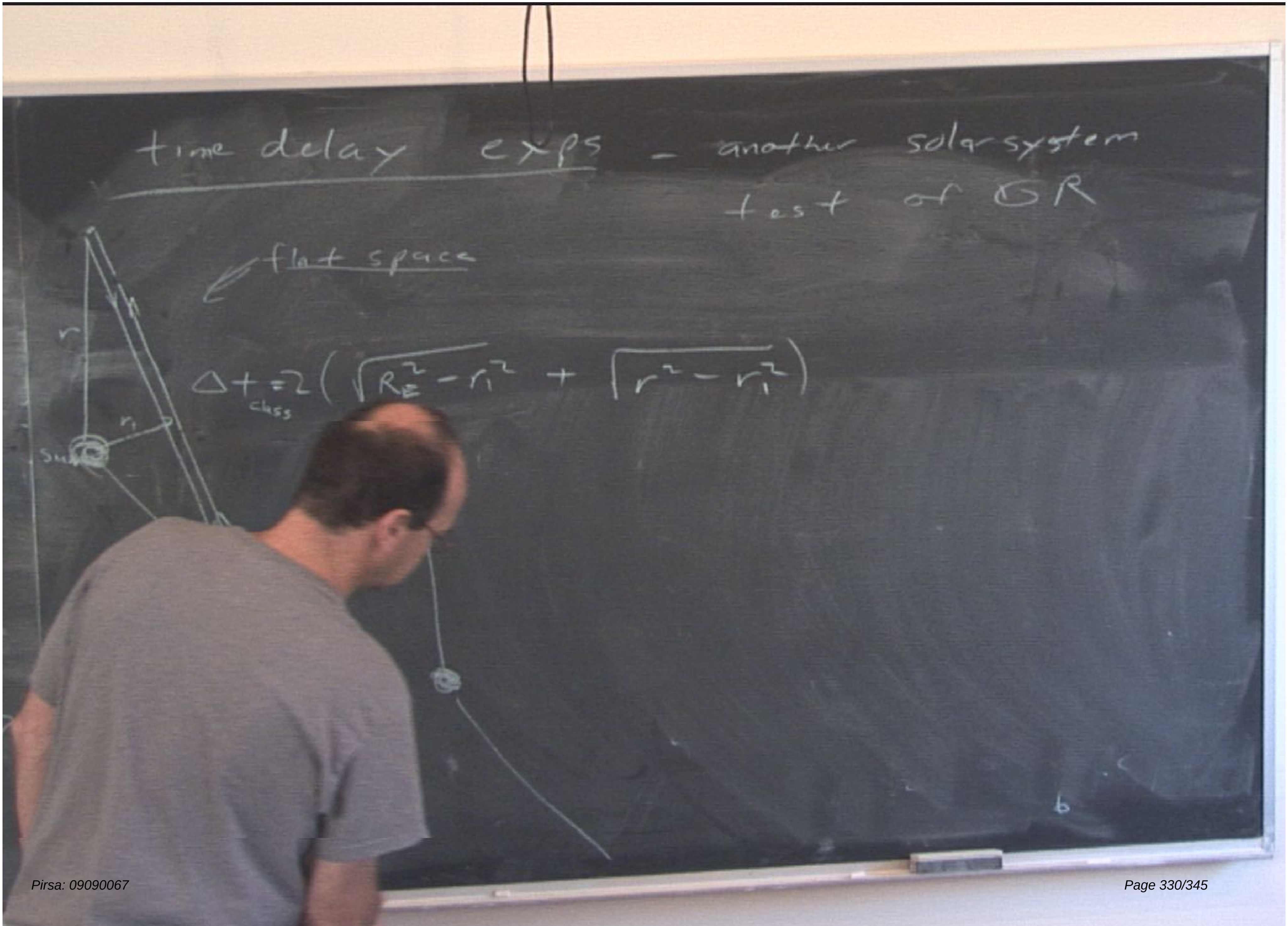
$$2\left(\sqrt{R_E^2 - r_1^2} + \sqrt{r^2 - r_1^2}\right)$$

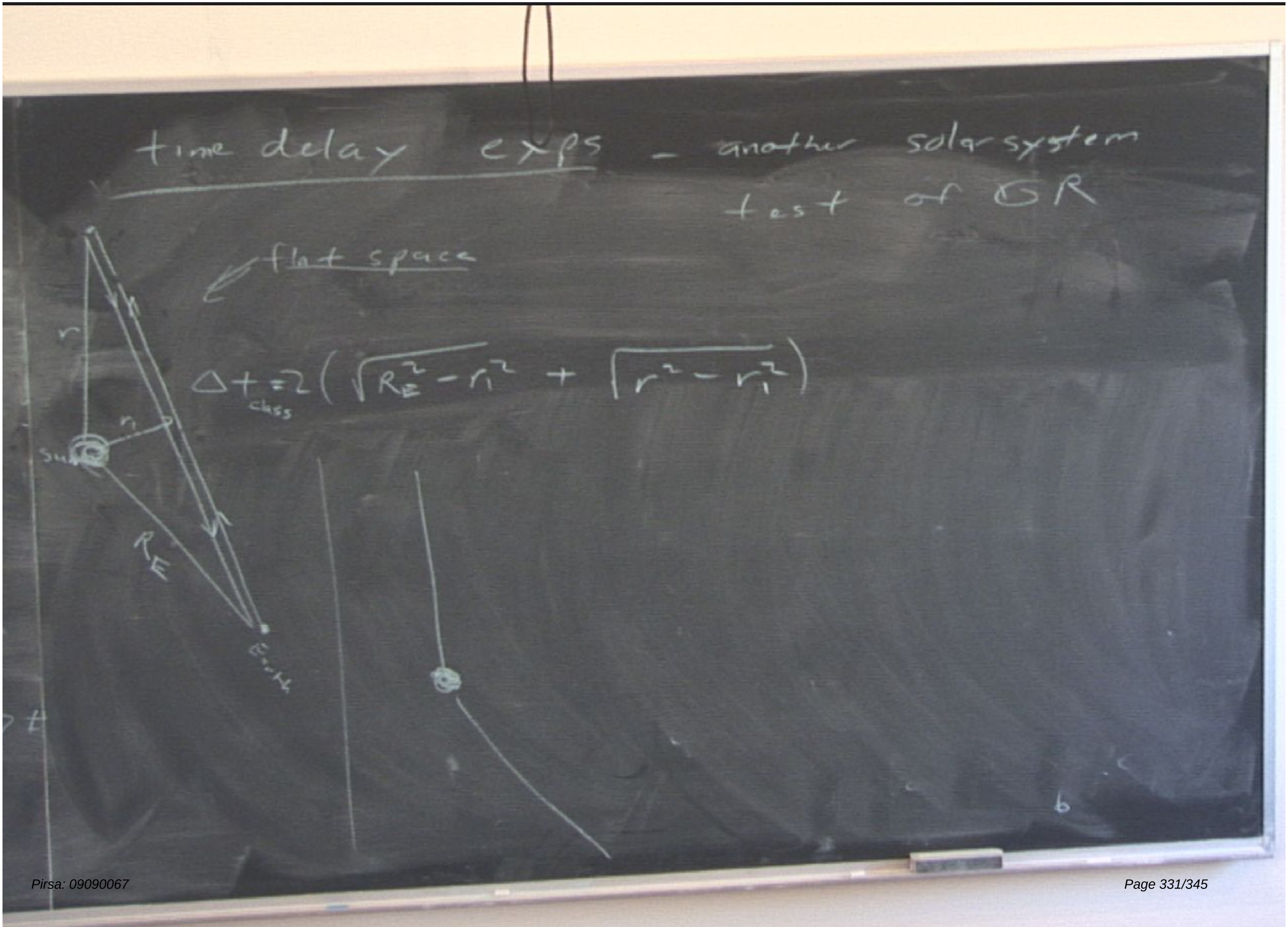
time delay exps – another solar system test of GR

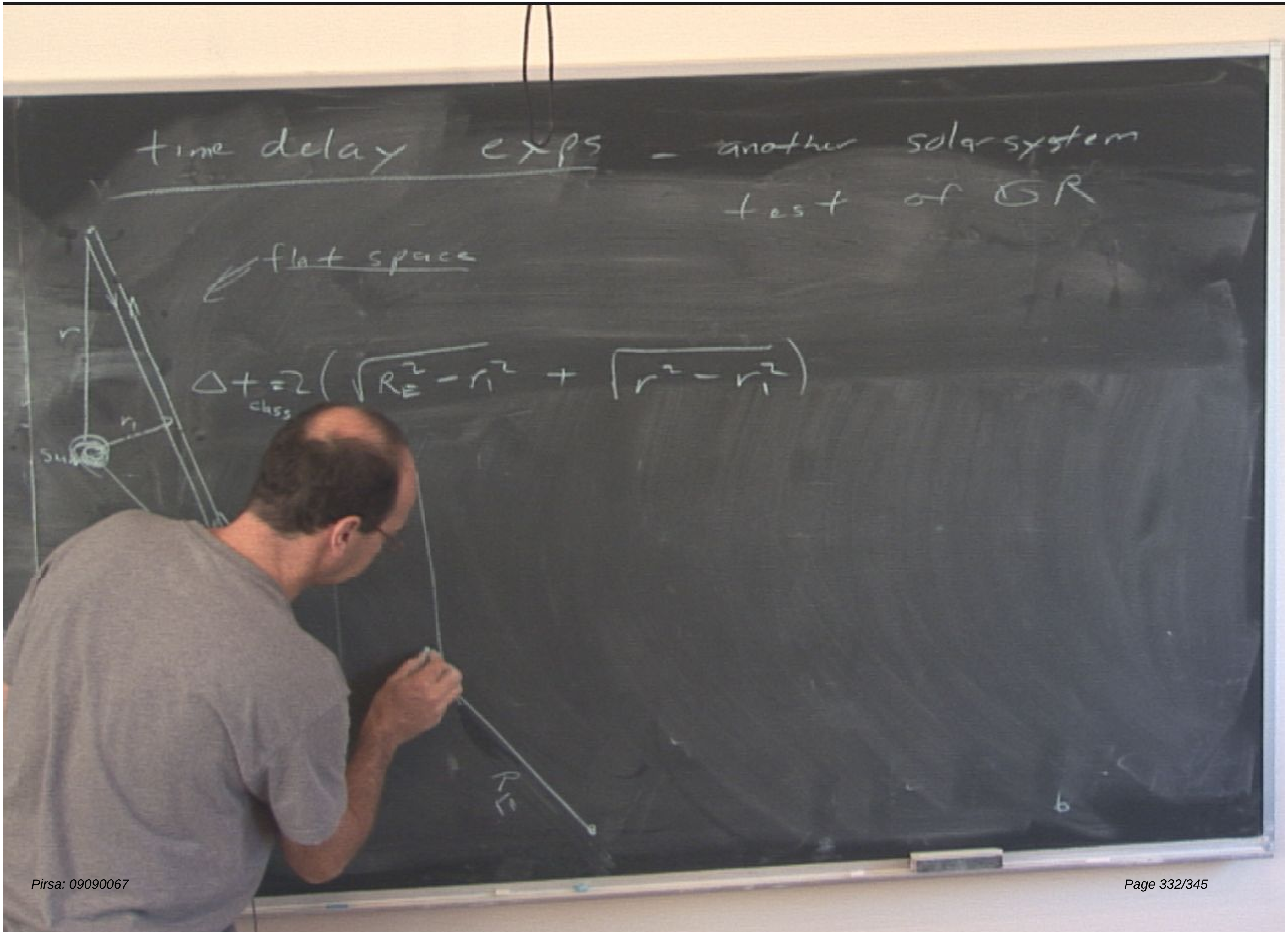


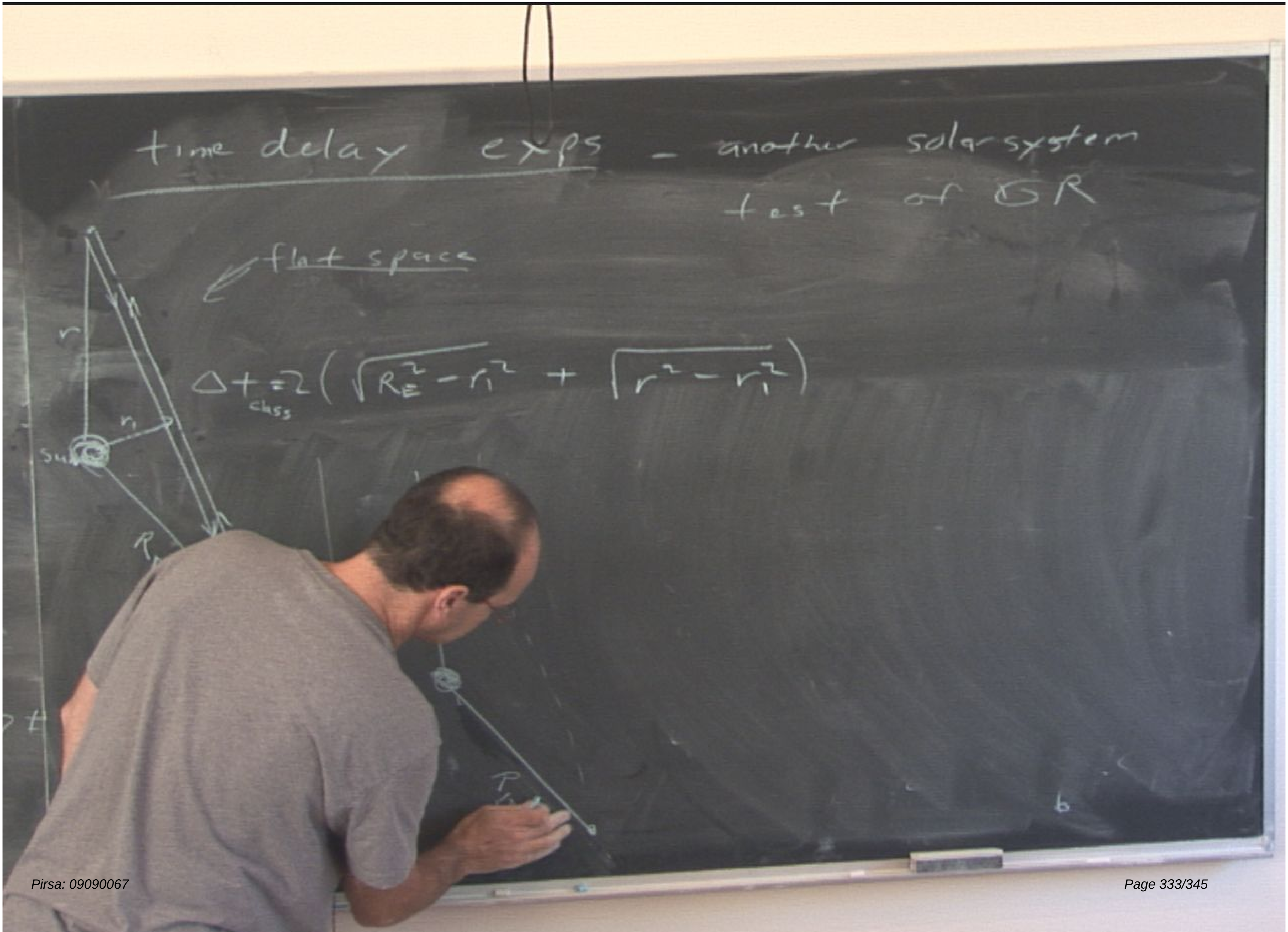
flat space

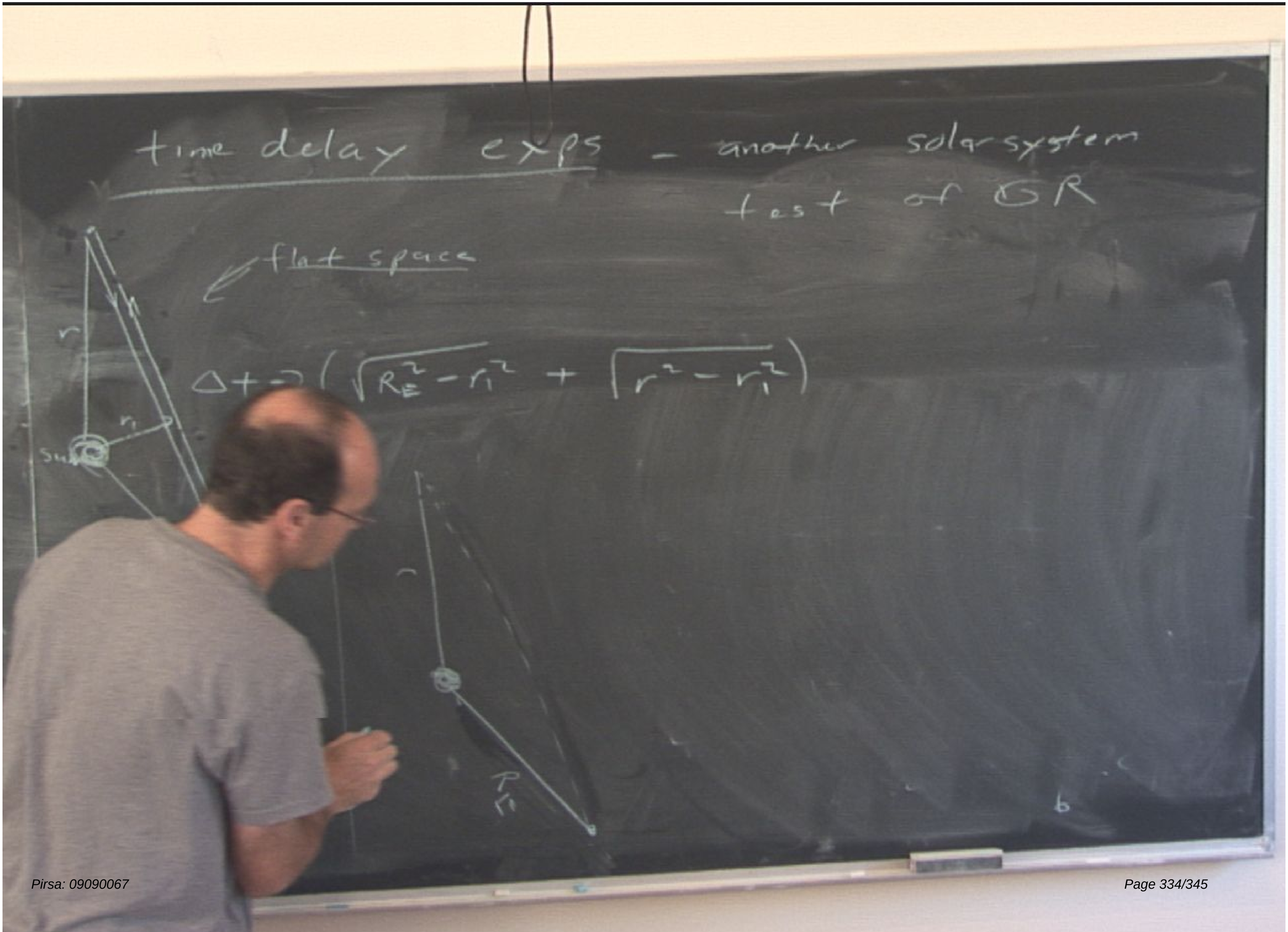
$$\Delta t_{class} = 2\left(\sqrt{R_E^2 - r_1^2} + \sqrt{r^2 - r_1^2}\right)$$

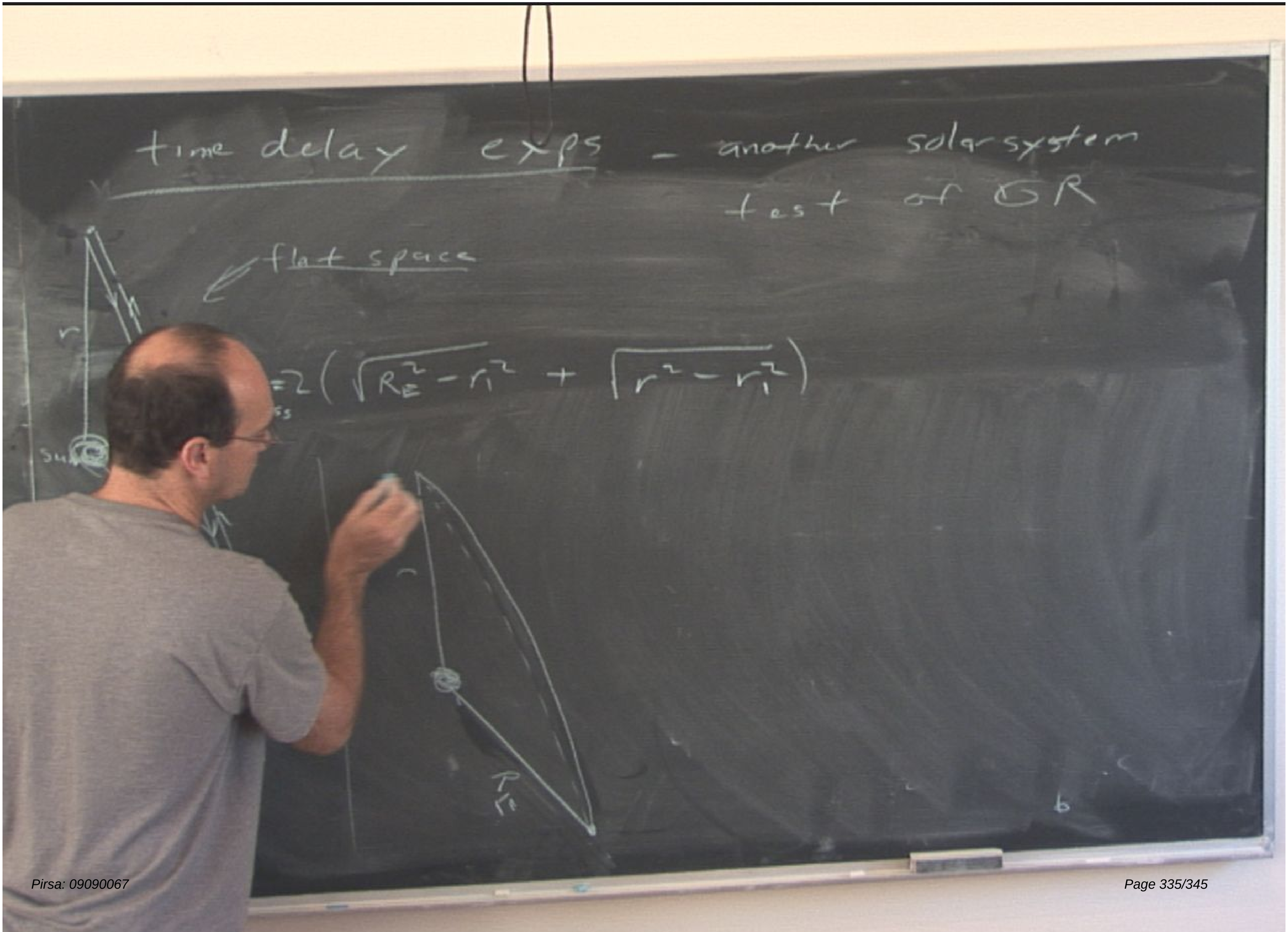

time delay exps – another solar system test of GR
flat space
r
r_1
Sun
$\Delta t_{class} = 2\left(\sqrt{R_E^2 - r_1^2} + \sqrt{r^2 - r_1^2}\right)$

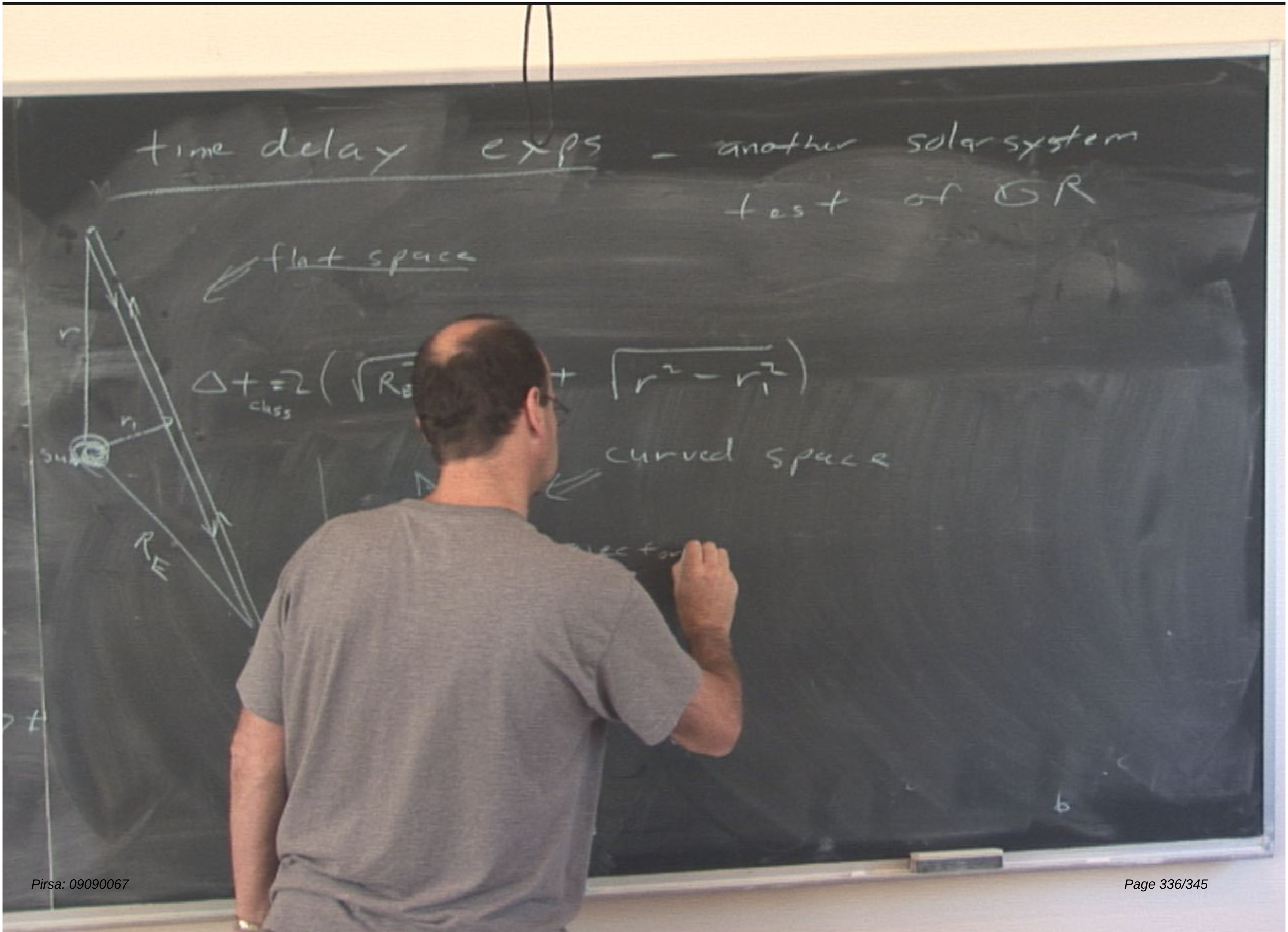

time delay exps - another solar system test of GR
flat space
$\Delta t_{class} = 2\left(\sqrt{R_E^2 - r_1^2} + \sqrt{r^2 - r_1^2}\right)$
r
r_1
Sun
R_E
Earth

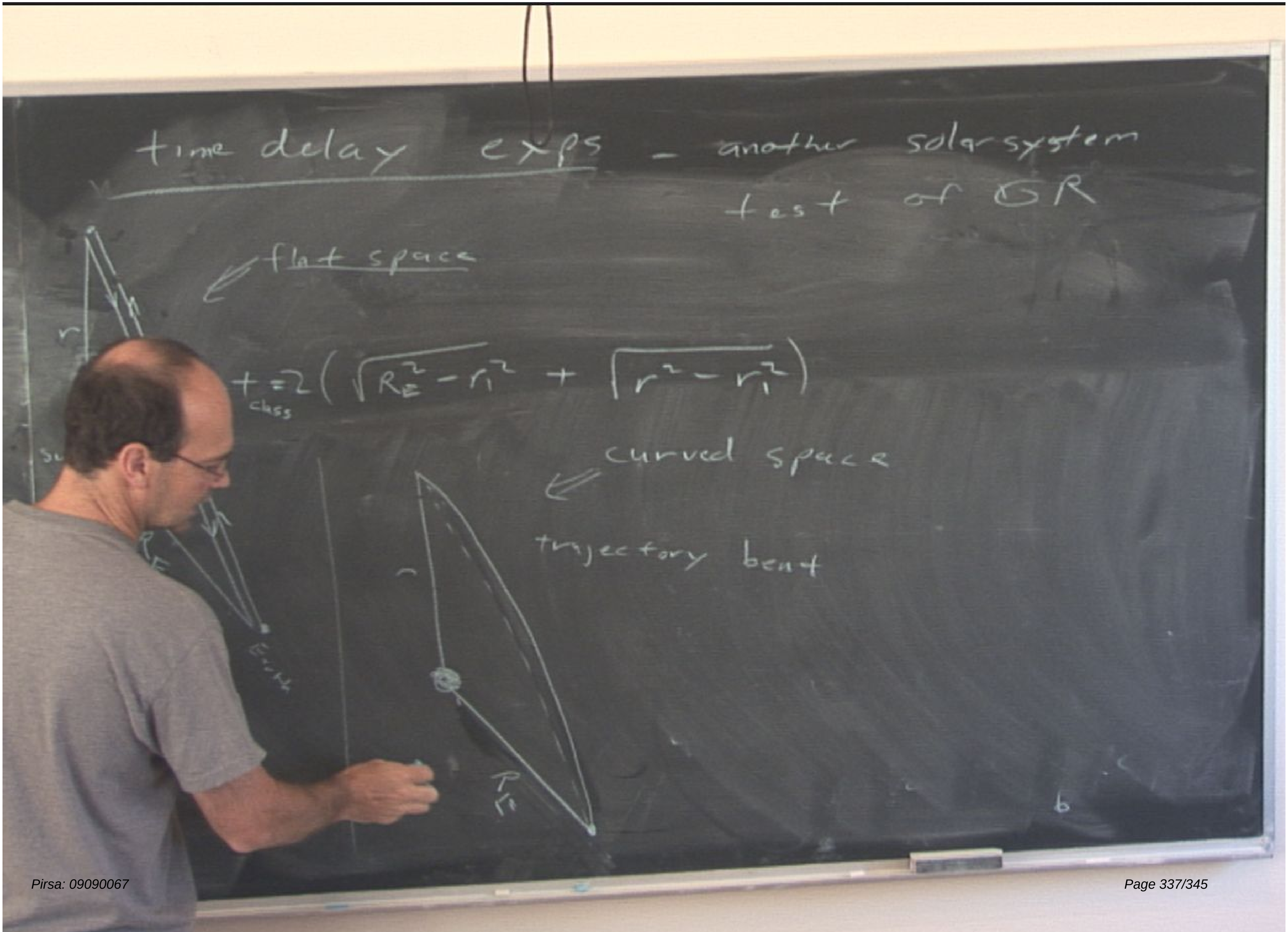

time delay exps - another solar system test of GR
flat space
Δt_class = 2(√(R_E² - r_1²) + √(r² - r_1²))

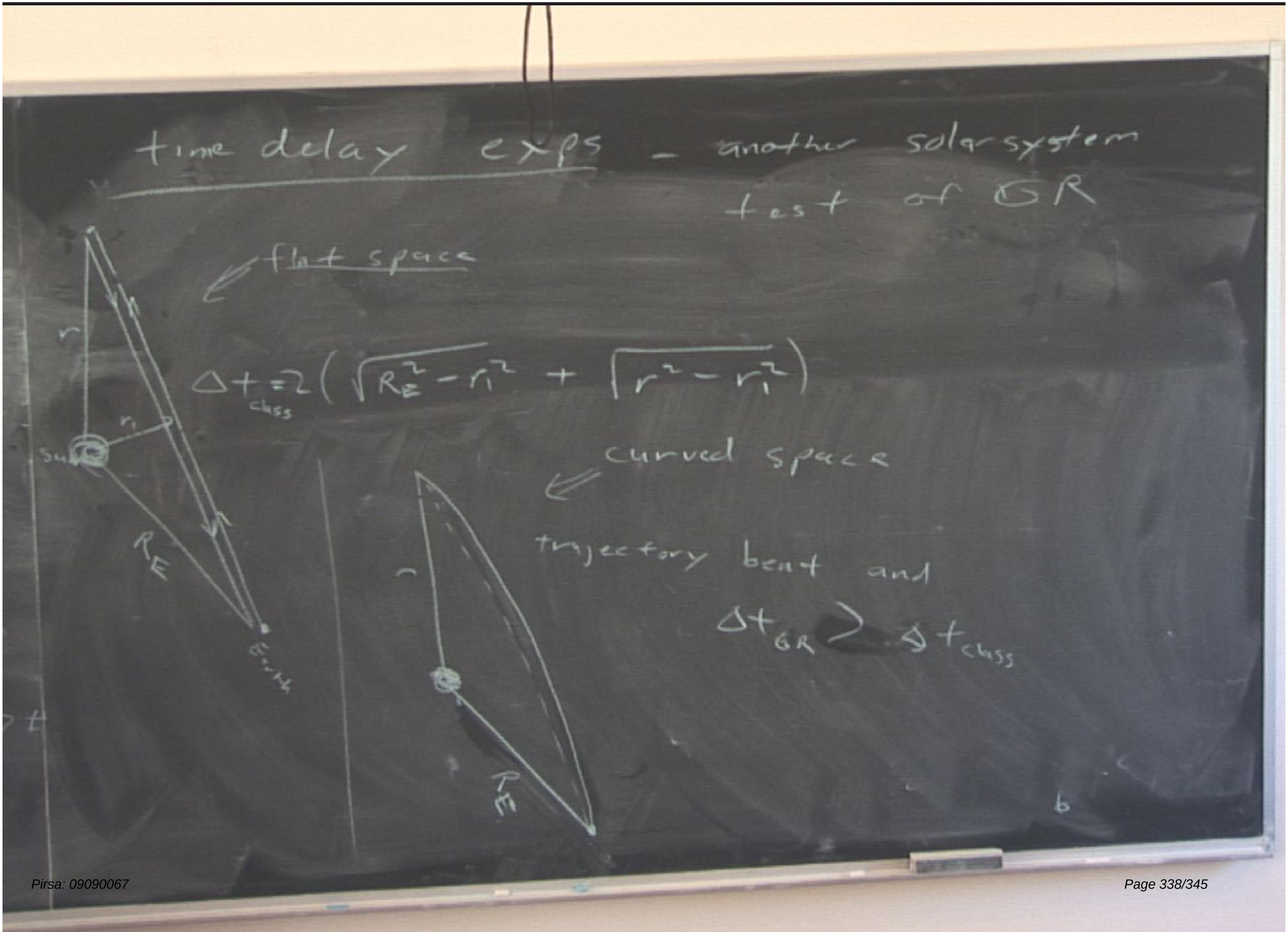

time delay exps - another solar system test of GR
flat space
Δt class = 2 (√(R_E² − r_1²) + √(r² − r_1²))
r
r_1
Sun
R

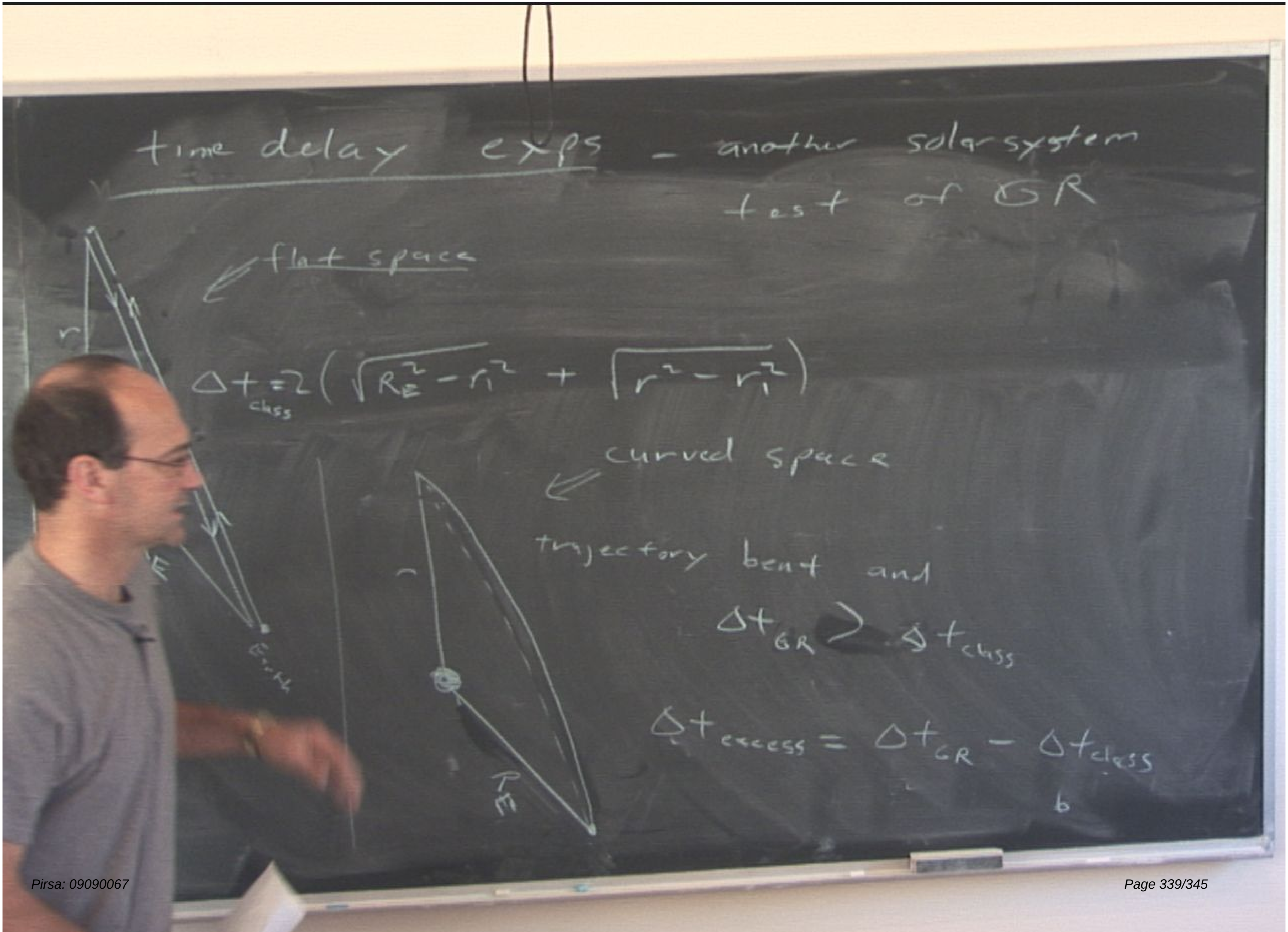

time delay exps - another solar system test of GR
flat space
Δt = (√(R_E^2 - r_1^2) + √(r^2 - r_1^2))
r
r1
Sun

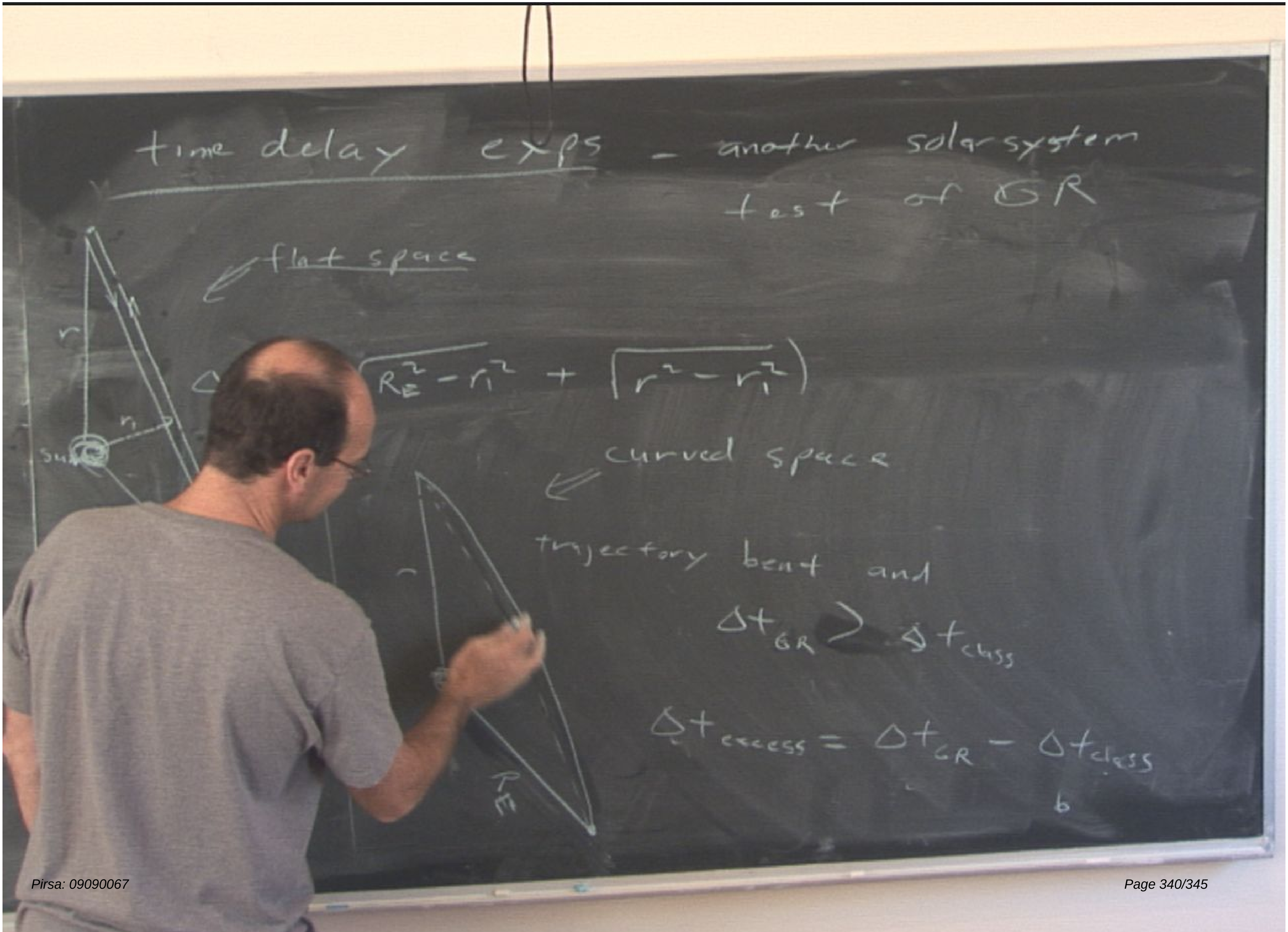

time delay exps - another solar system test of GR
flat space
r
sun
=2 (√(R_E² − r_1²) + √(r² − r_1²))

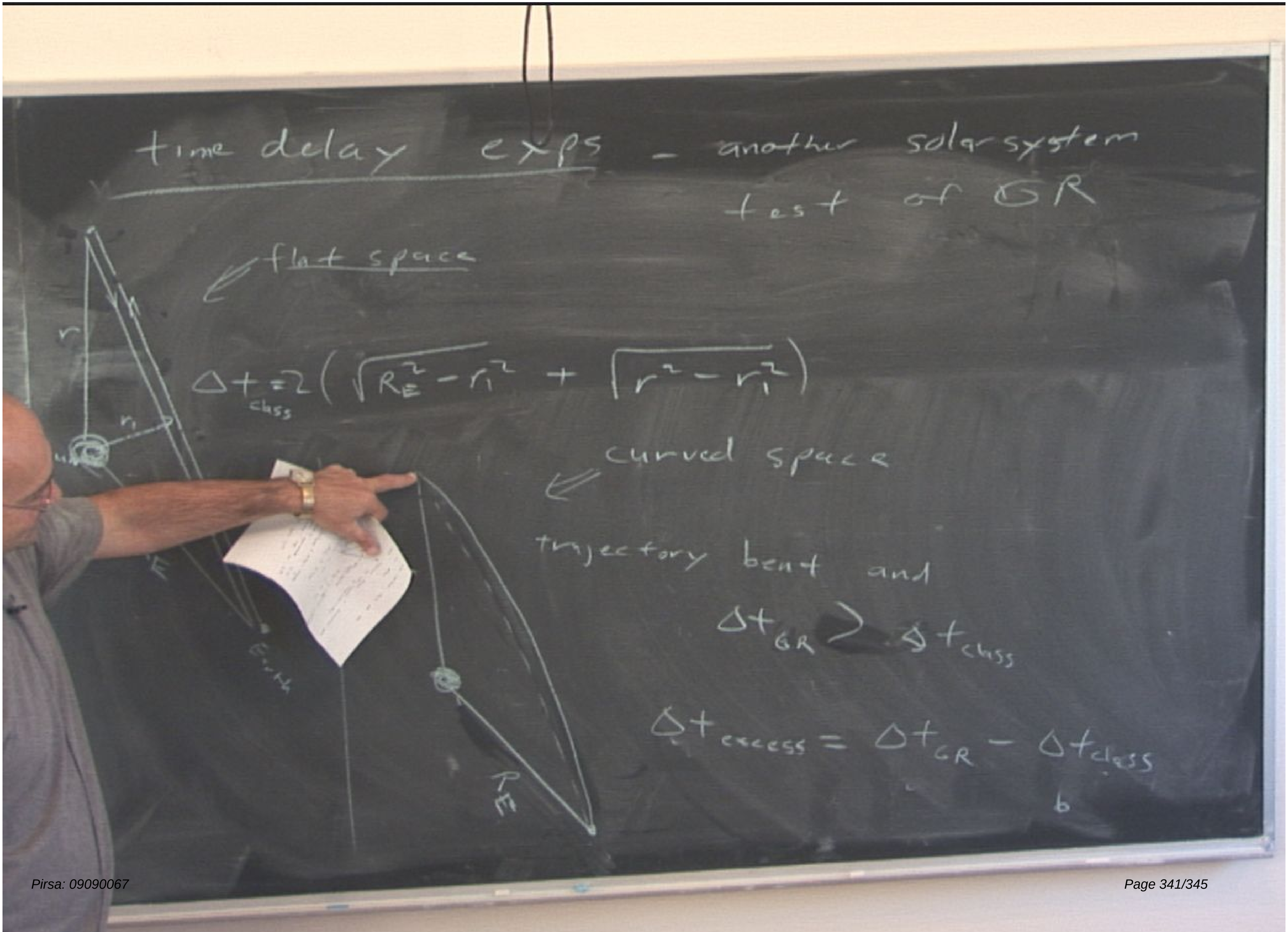

time delay exps - another solarsystem
test of GR
flat space
curved space
r
sun
R_E

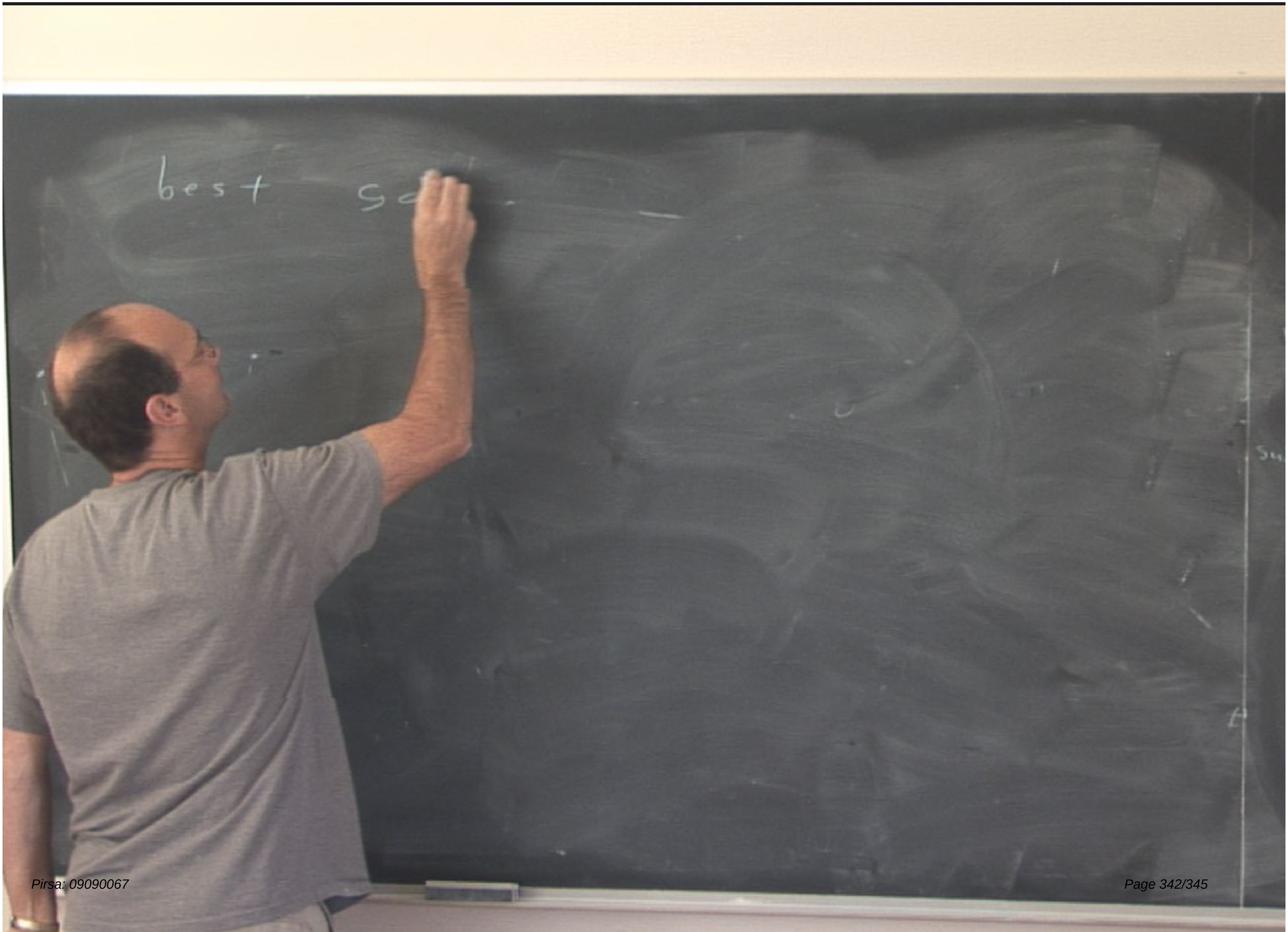

time delay exps - another solar system test of GR
flat space
$t_{class} = 2\left(\sqrt{R_E^2 - r_1^2} + \sqrt{r^2 - r_1^2}\right)$
curved space
trajectory bent

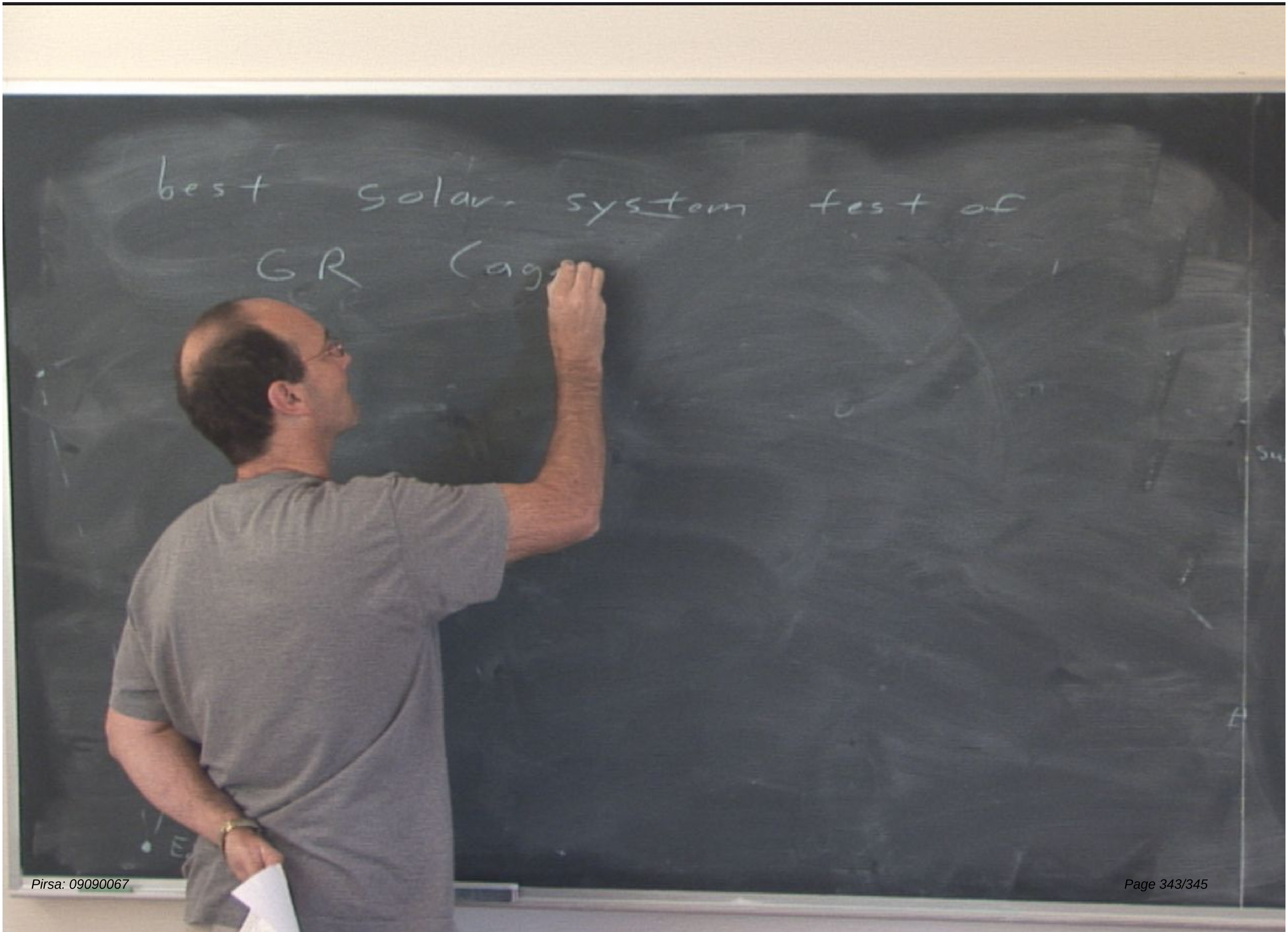

time delay exps - another solar system test of GR
flat space
curved space
trajectory bent and
$\Delta t_{GR} > \Delta t_{class}$
$\Delta t_{class} = 2\left(\sqrt{R_E^2 - r_1^2} + \sqrt{r^2 - r_1^2}\right)$

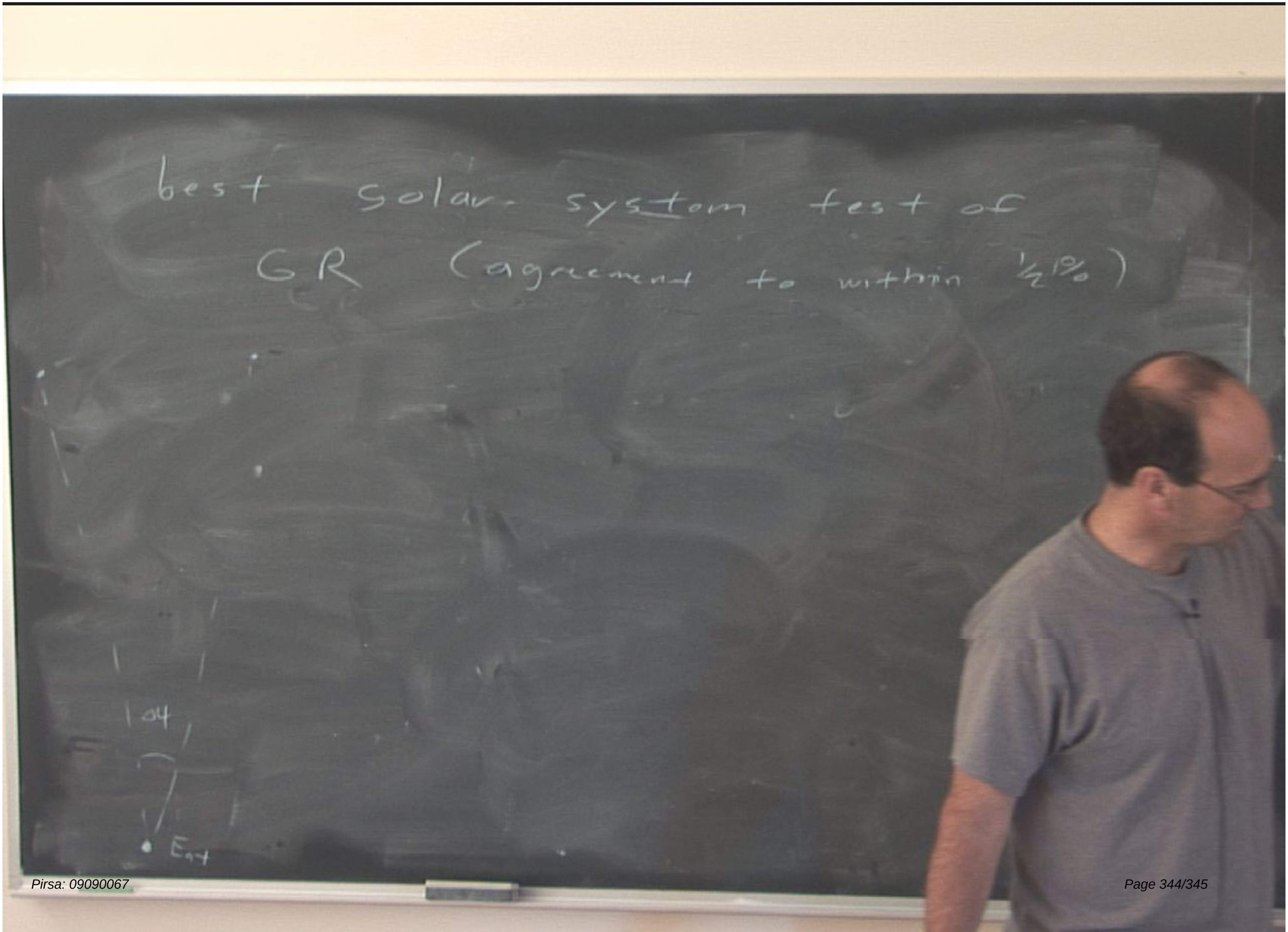

time delay exps – another solarsystem test of GR
flat space
Δt = 2(√(R_E² − r₁²) + √(r² − r₁²))
class
curved space
trajectory bent and
Δt_GR > Δt_class
Δt_excess = Δt_GR − Δt_class

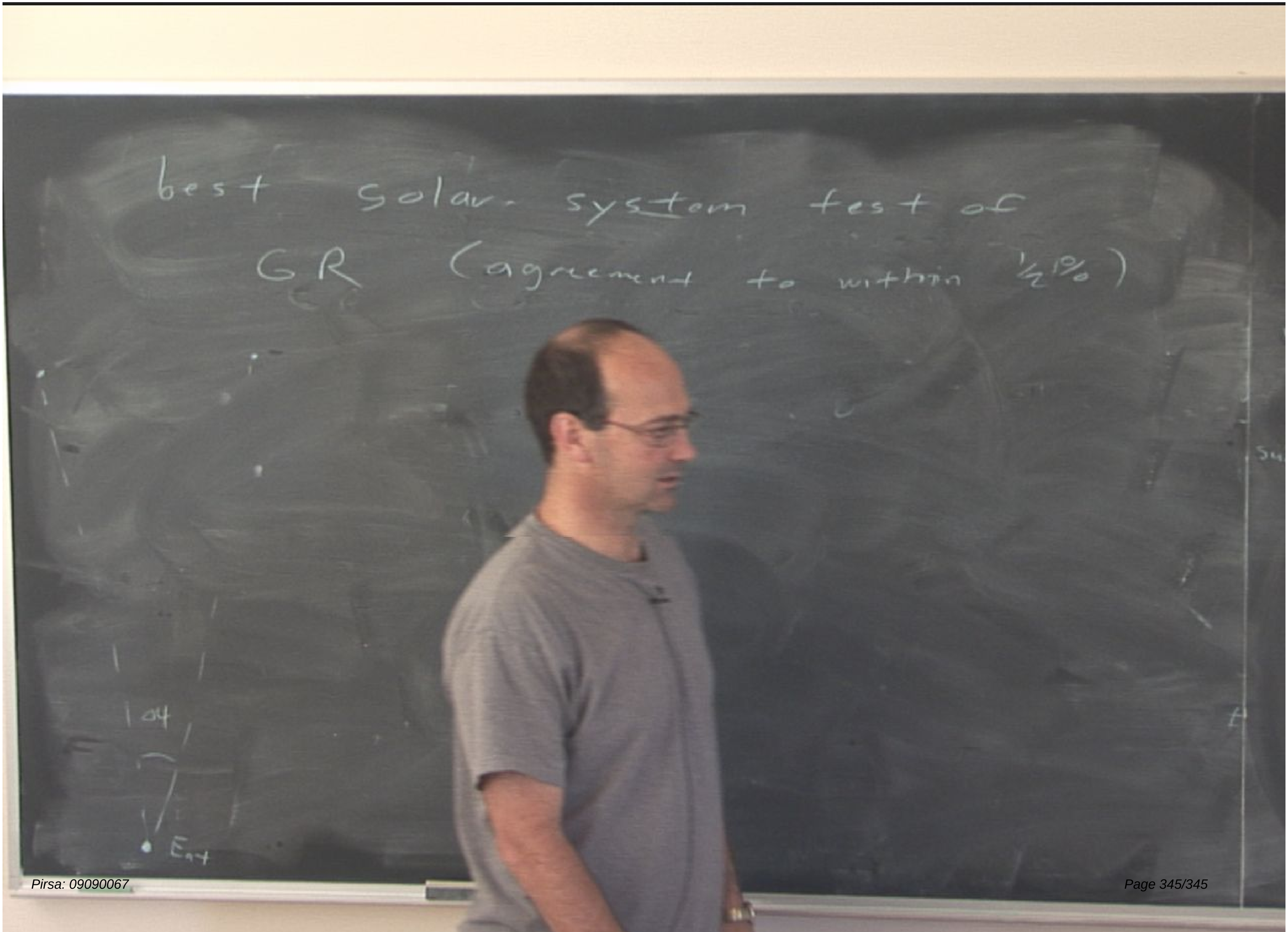

time delay exps - another solarsystem
test of GR
flat space
r
r1
Sun
curved space
trajectory bent and
Δt_GR > Δt_class
Δt_excess = Δt_GR − Δt_class
RE


time delay exps - another solar system
test of GR
flat space
$\Delta t_{class} = 2\left(\sqrt{R_E^2 - r_1^2} + \sqrt{r^2 - r_1^2}\right)$
curved space
trajectory bent and
$\Delta t_{GR} > \Delta t_{class}$
$\Delta t_{excess} = \Delta t_{GR} - \Delta t_{class}$


best


best solar system test of
GR (ag


best solar system test of
GR (agreement to within ½ 1%)



best solar system test of
GR (agreement to within 1/2 1%)