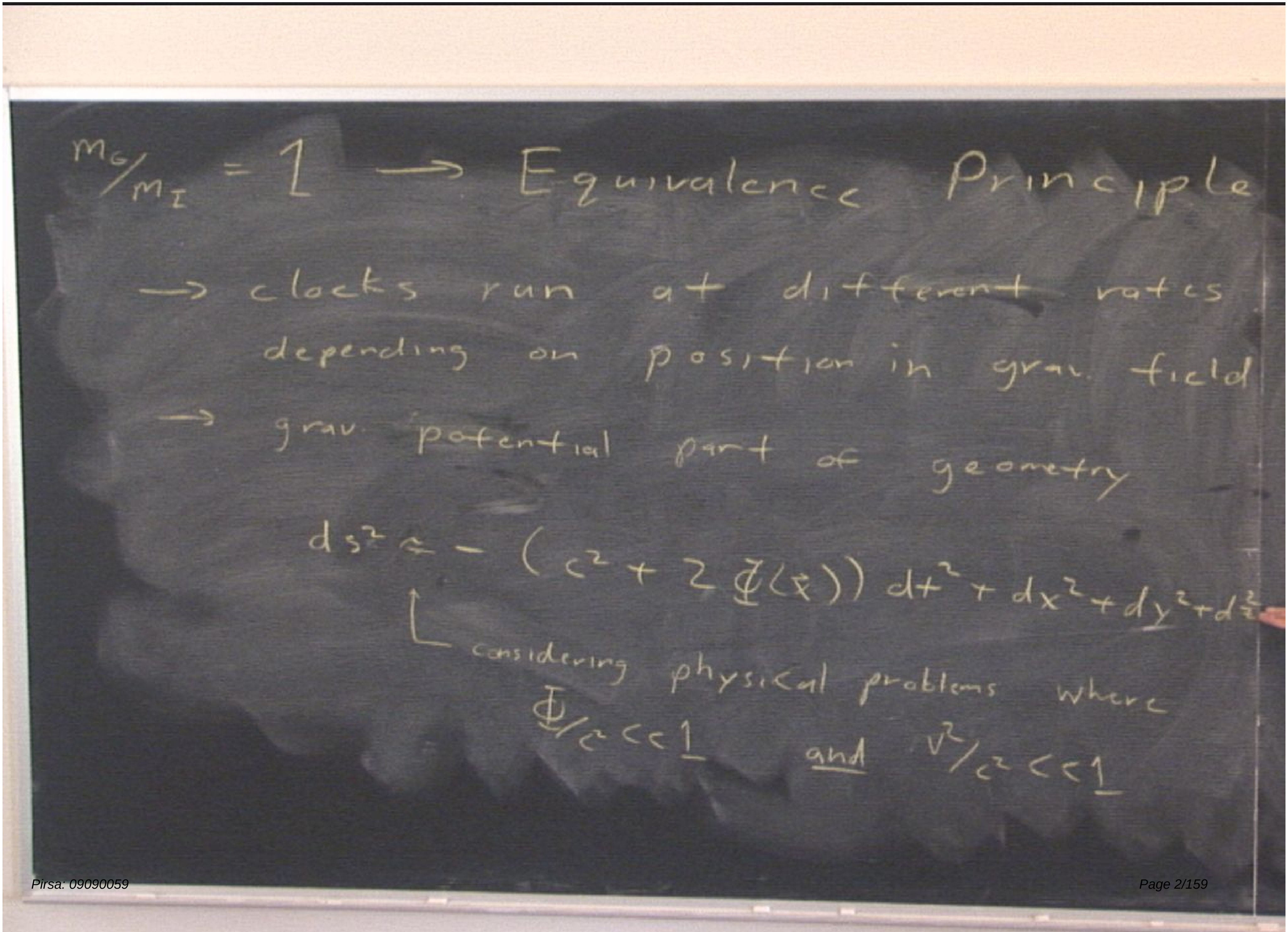
Title: PSI - Relativity (PHYS 604) - 4

Date: Sep 08, 2009 10:30 AM

URL: http://pirsa.org/09090059

Abstract:



$m_G/m_I = 1 \longrightarrow$ Equivalence Principle

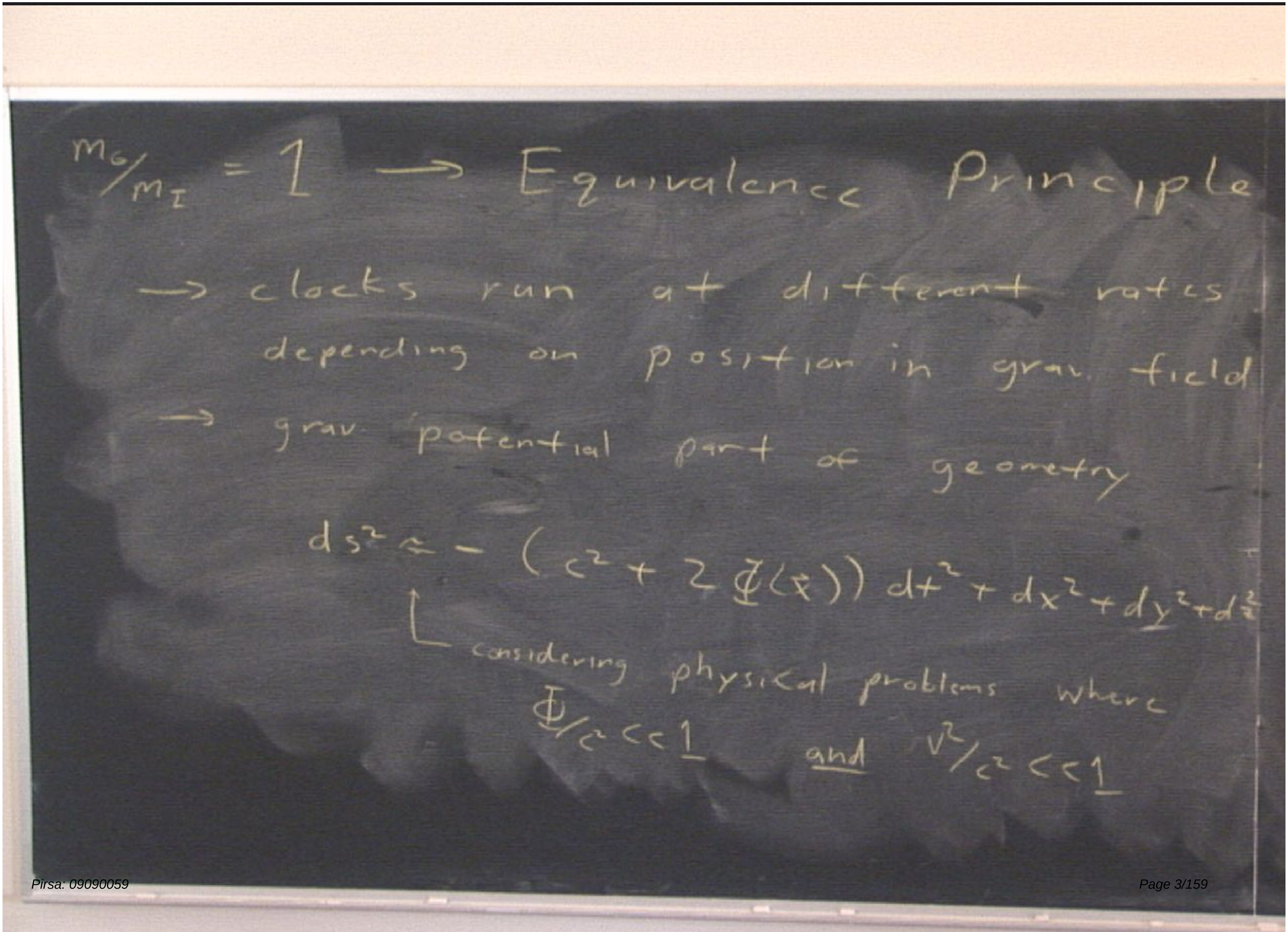
→ clocks run at different rates depending on position in grav. field

→ grav. potential part of geometry

$$ds^2 \simeq -\left(c^2 + 2\Phi(\vec{x})\right)dt^2 + dx^2 + dy^2 + dz^2$$

↳ considering physical problems where

$\Phi/c^2 \ll \underline{1}$ **and** $v^2/c^2 \ll \underline{1}$



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$$ds^2 \simeq -(c^2 + 2\Phi(\vec{x}))\,dt^2 + dx^2 + dy^2 + dz^2$$

└ considering physical problems where $\Phi/c^2 \ll 1$ and $v^2/c^2 \ll 1$

- we can test this conclusion with exp.

"gravitational redshift"

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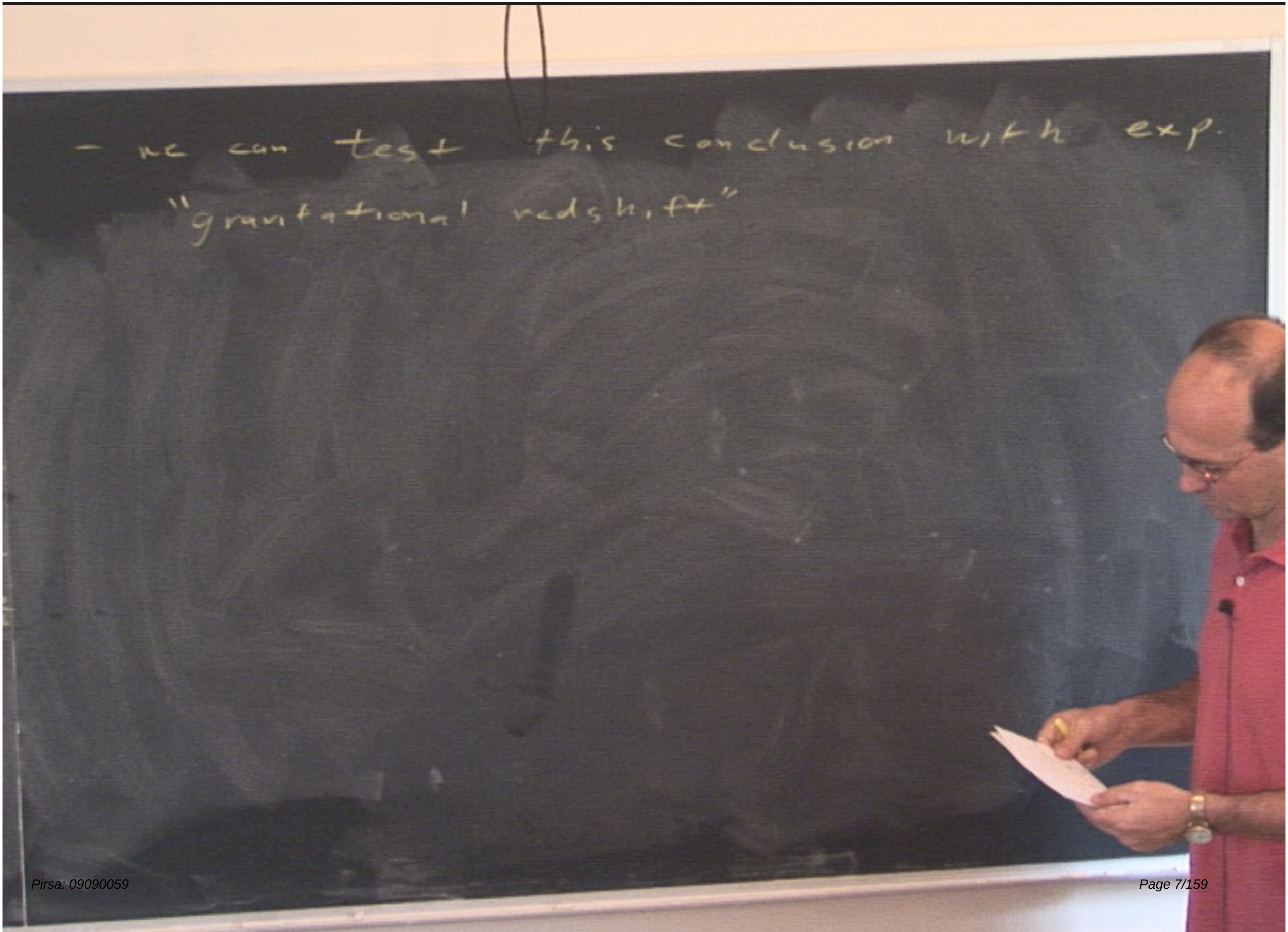

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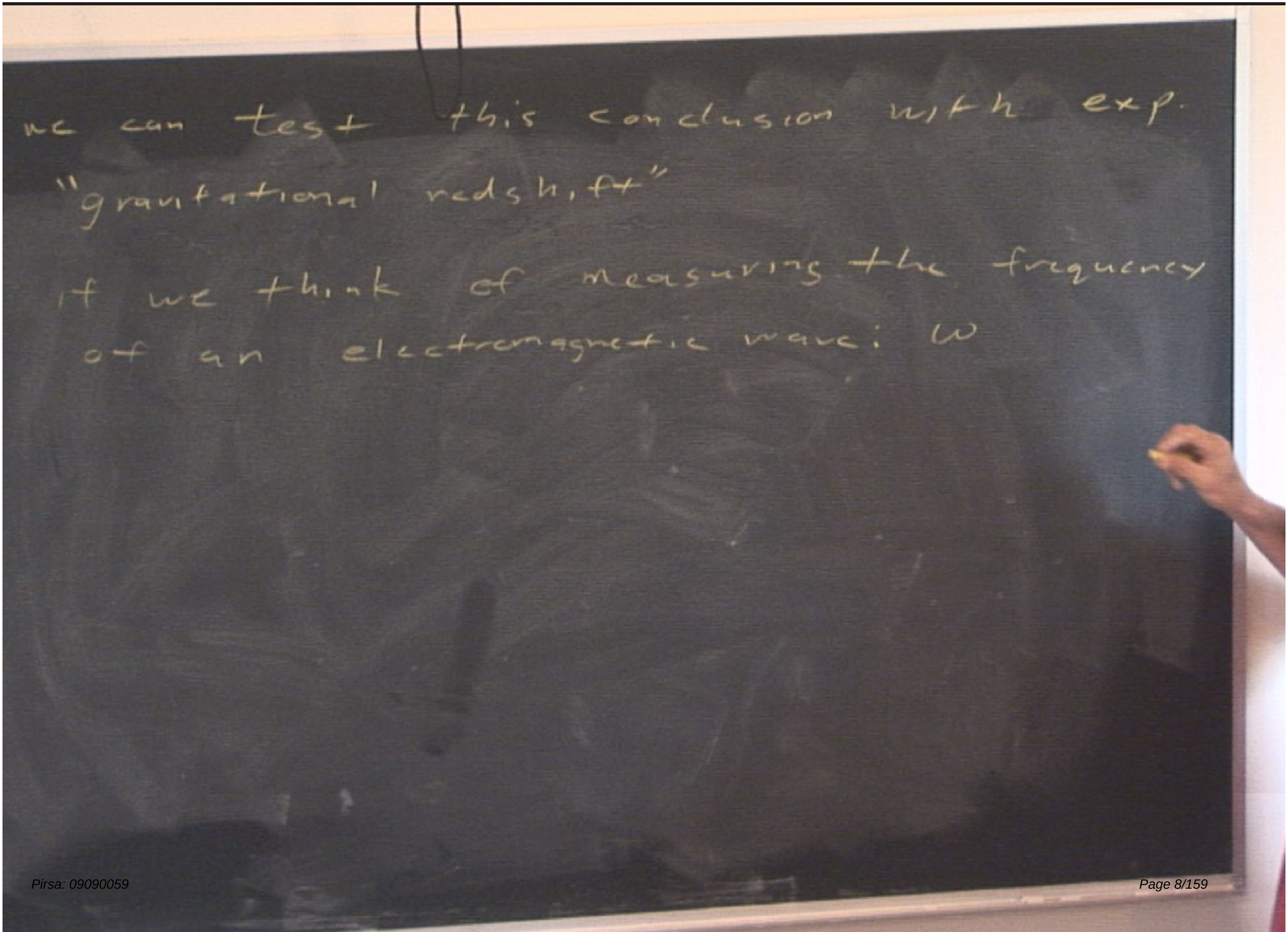

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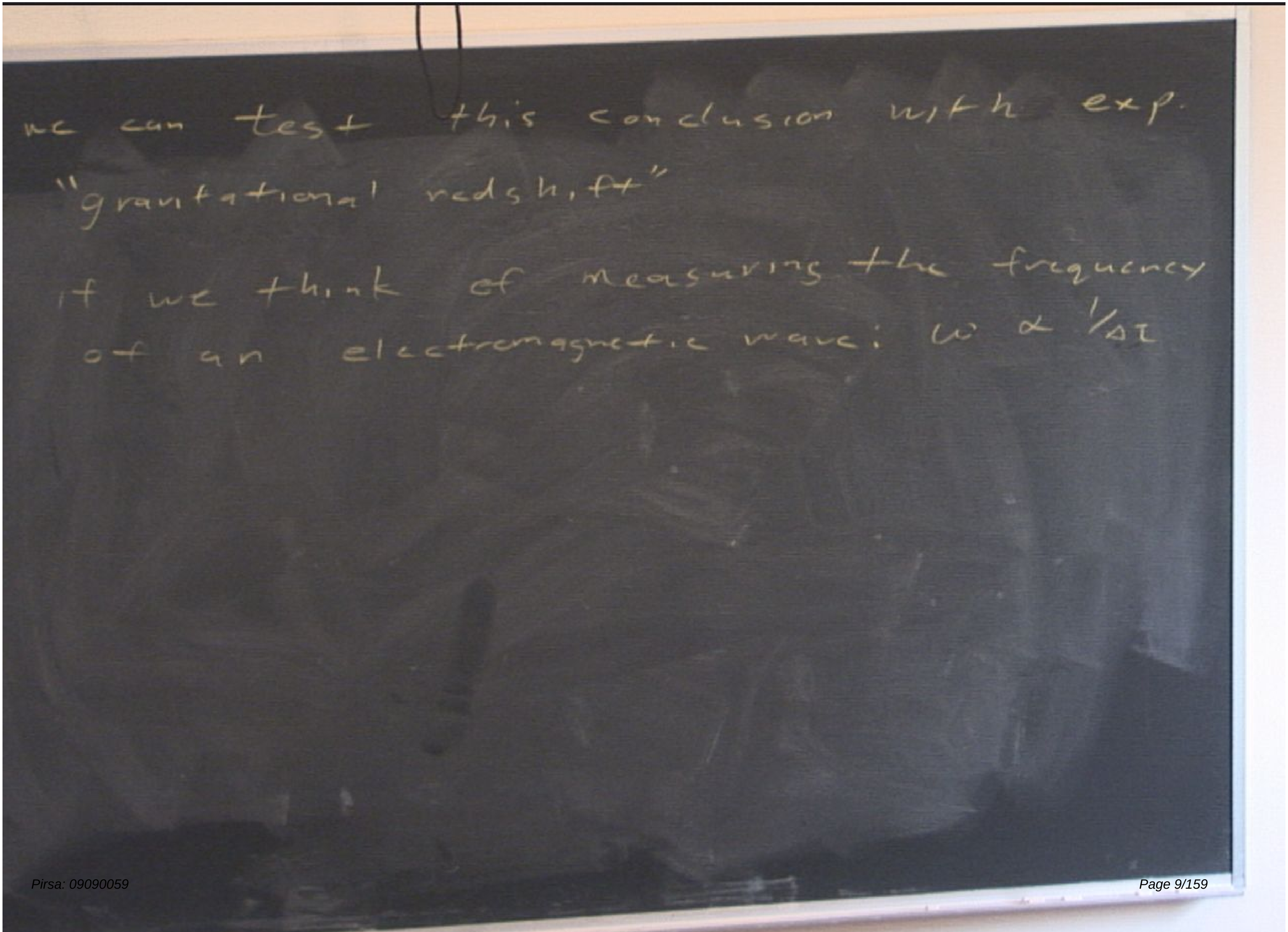

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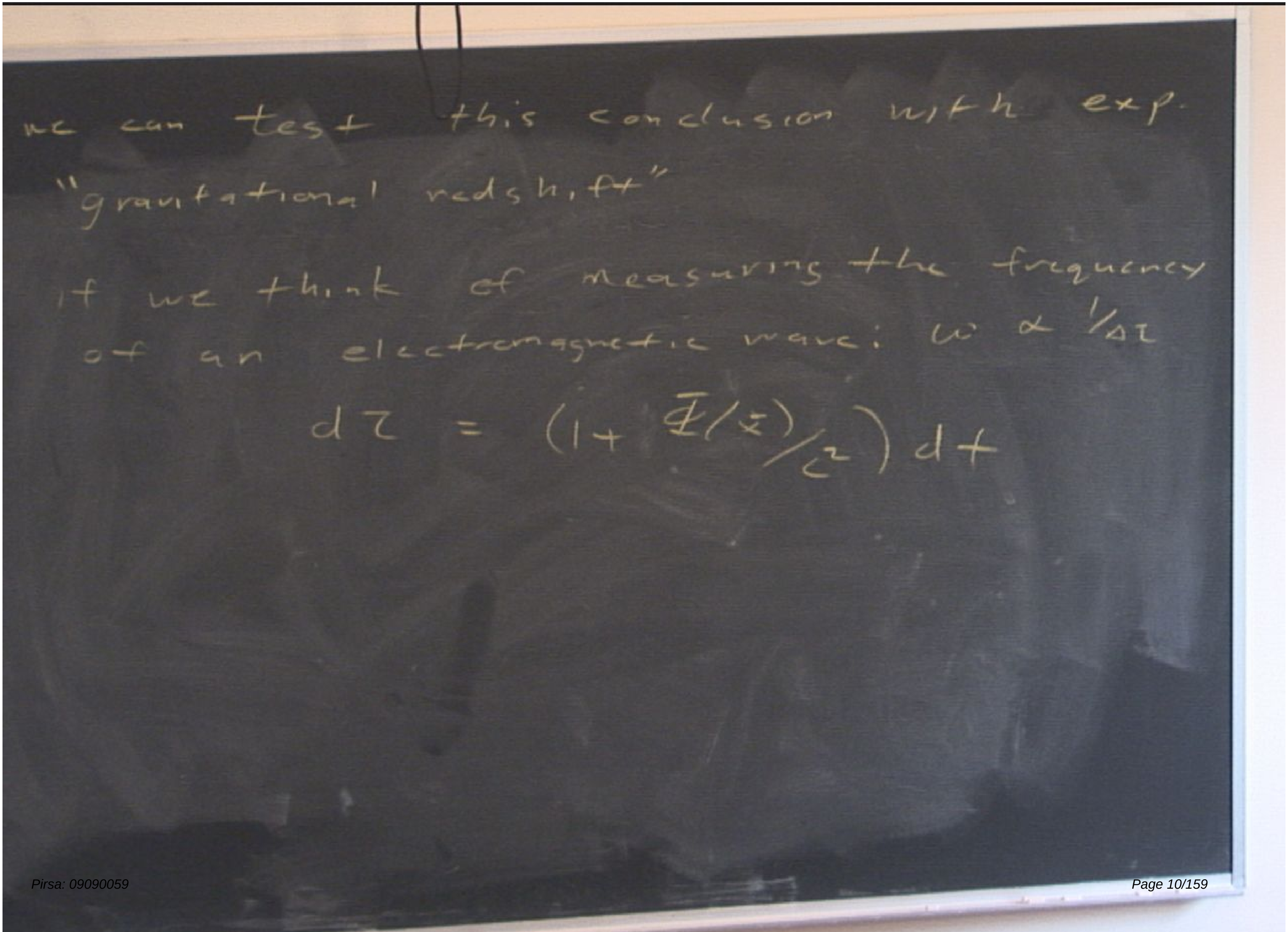

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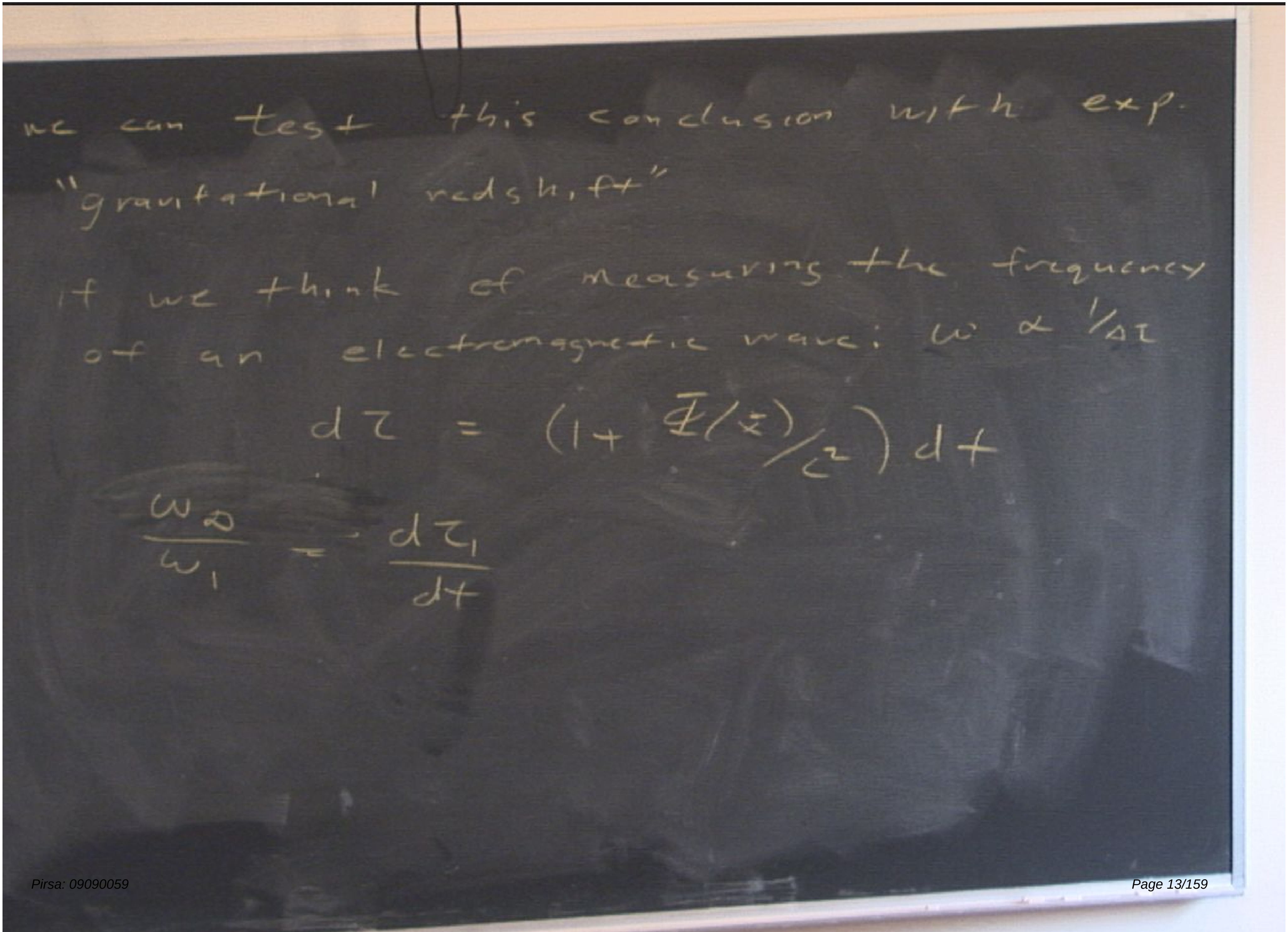

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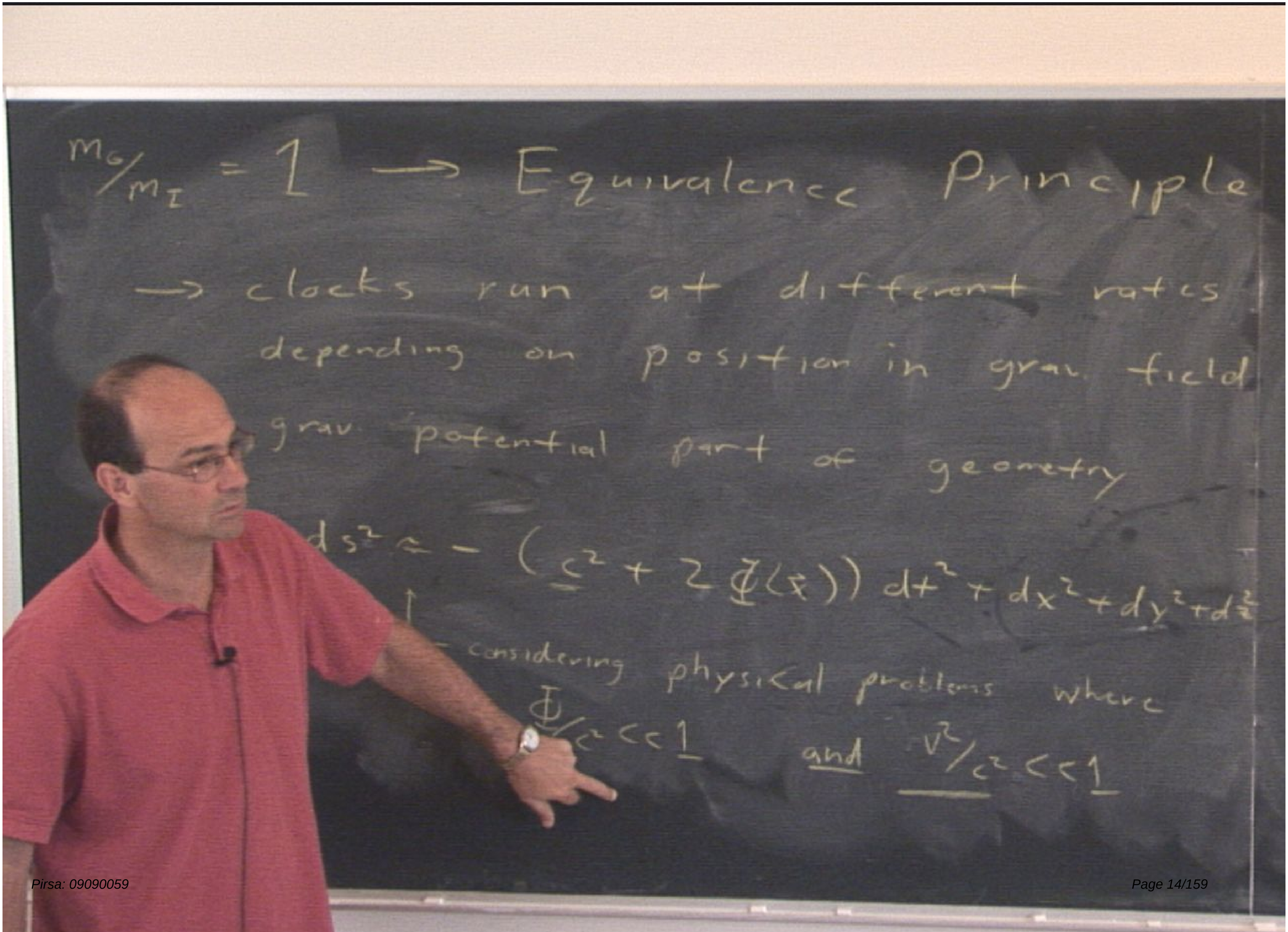

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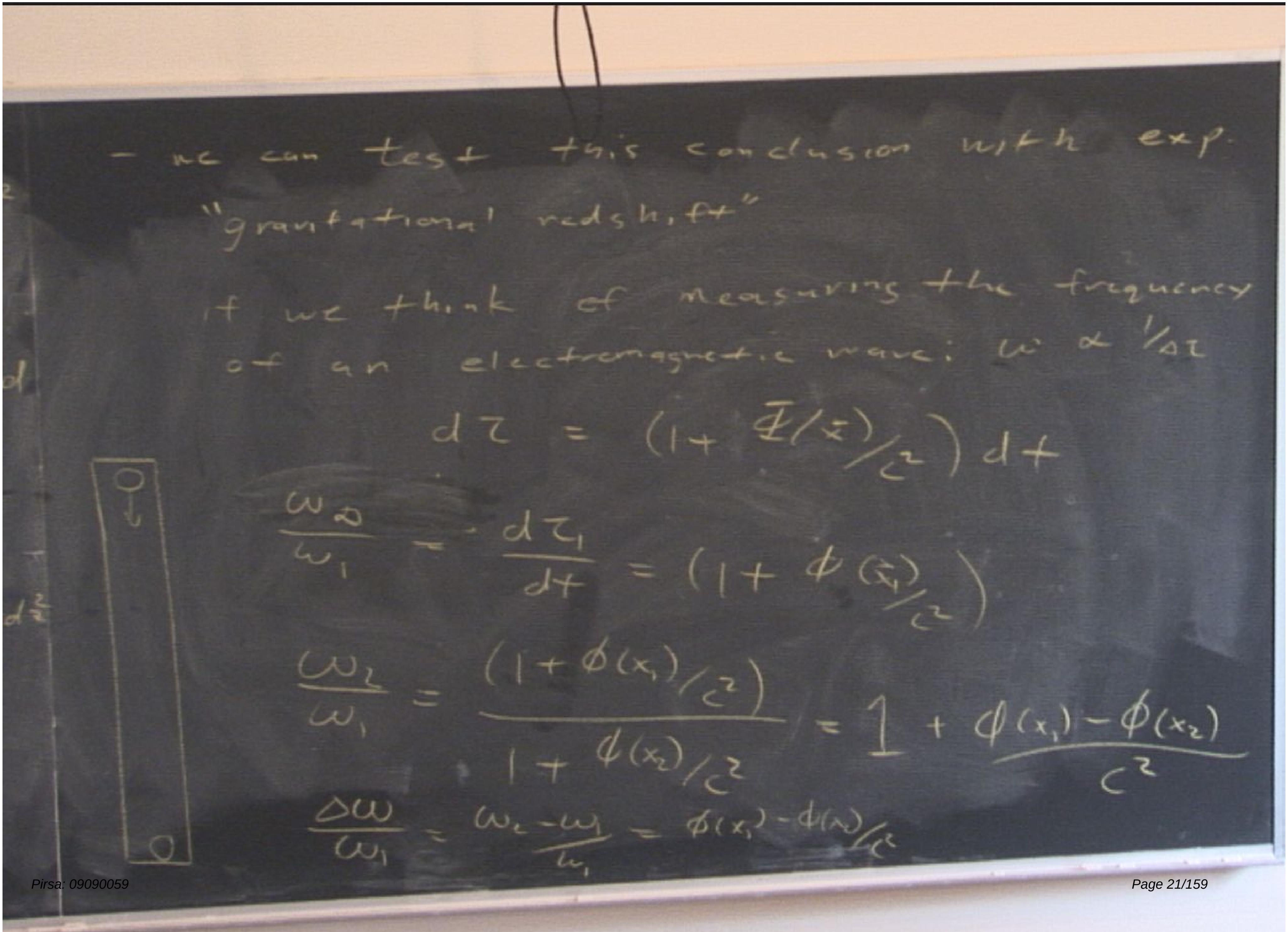

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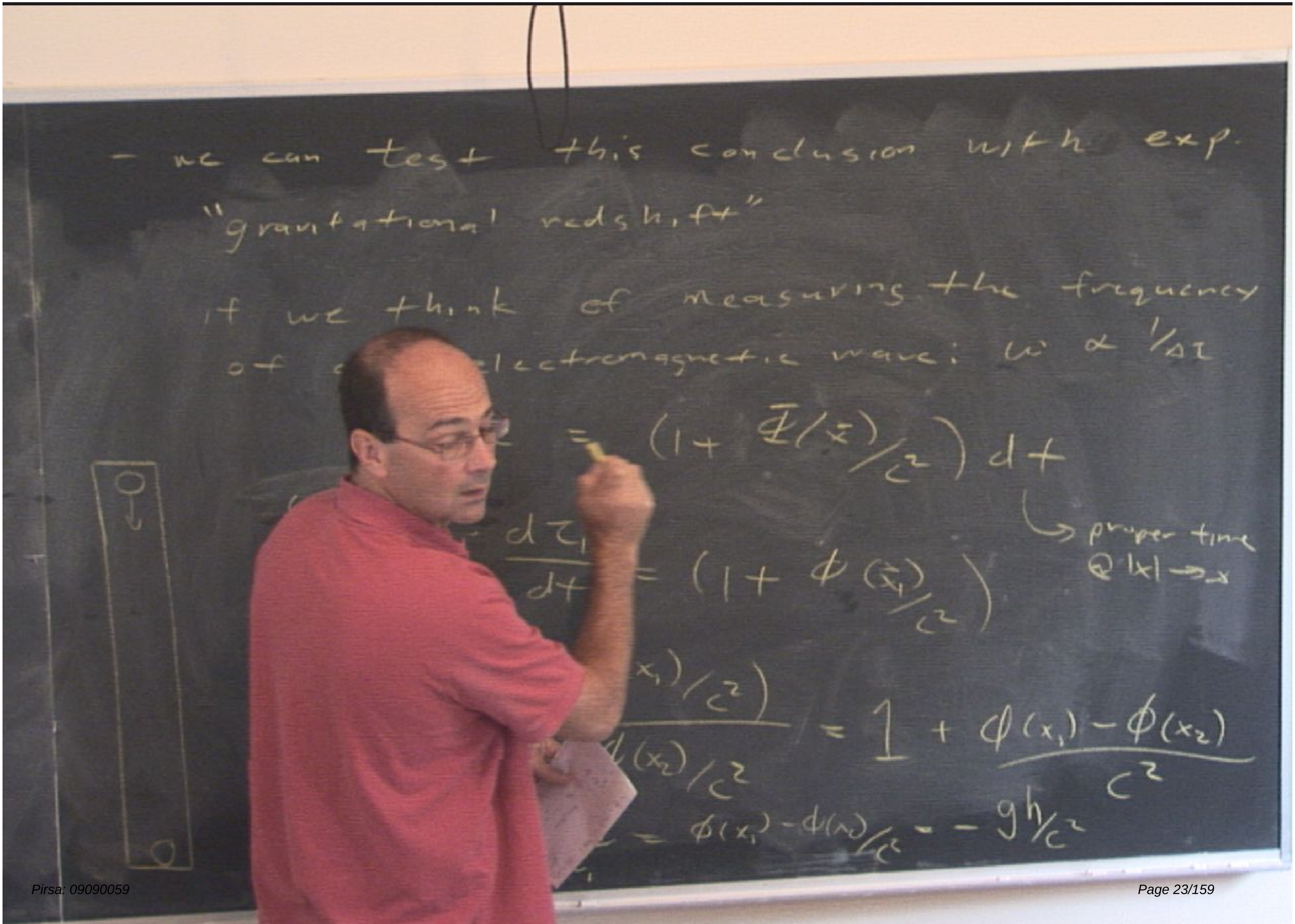

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↳ proper time @ $|x| \rightarrow \infty$

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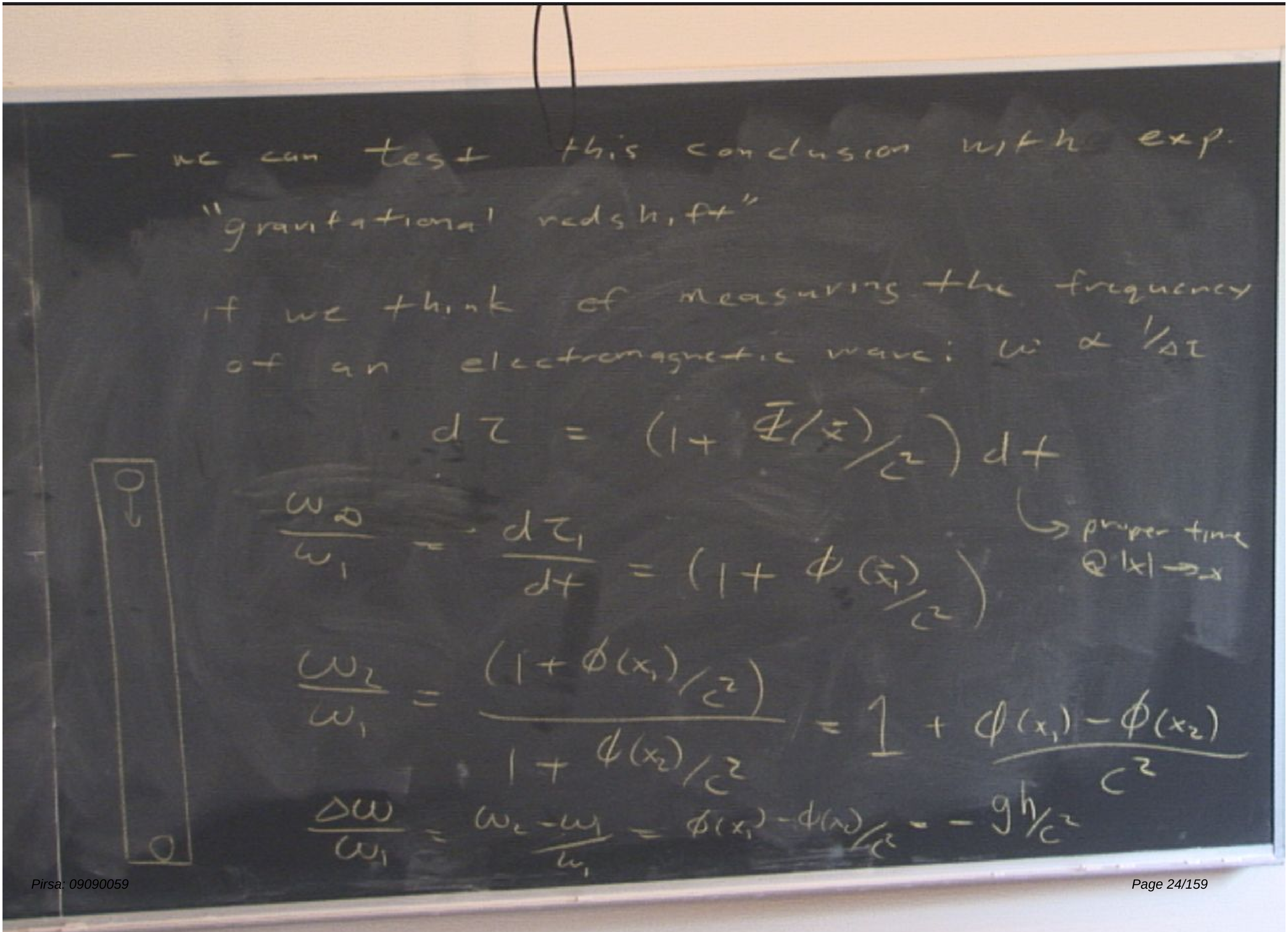

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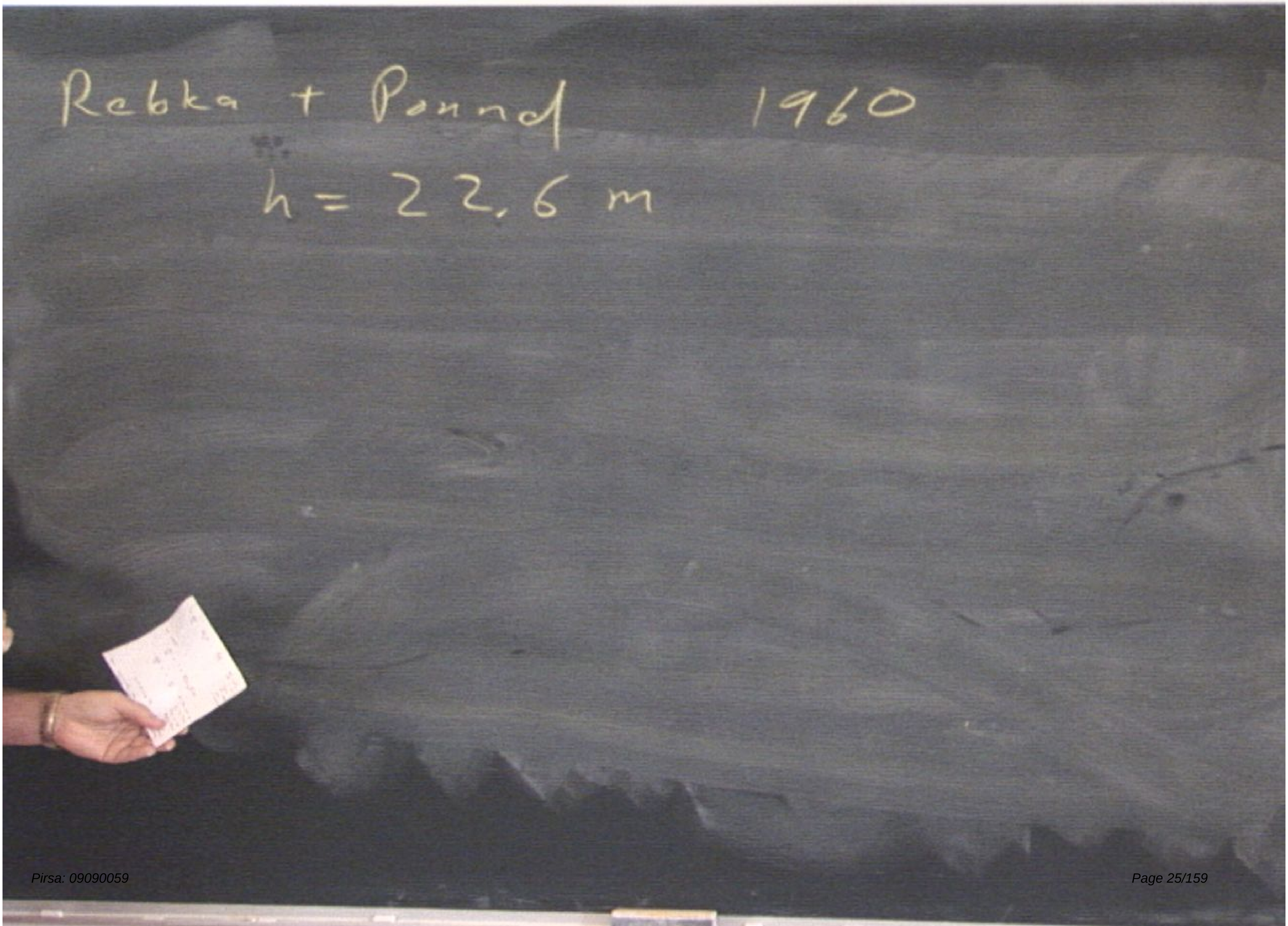

Rebka + Pound 1960

$h = 22.6$ m

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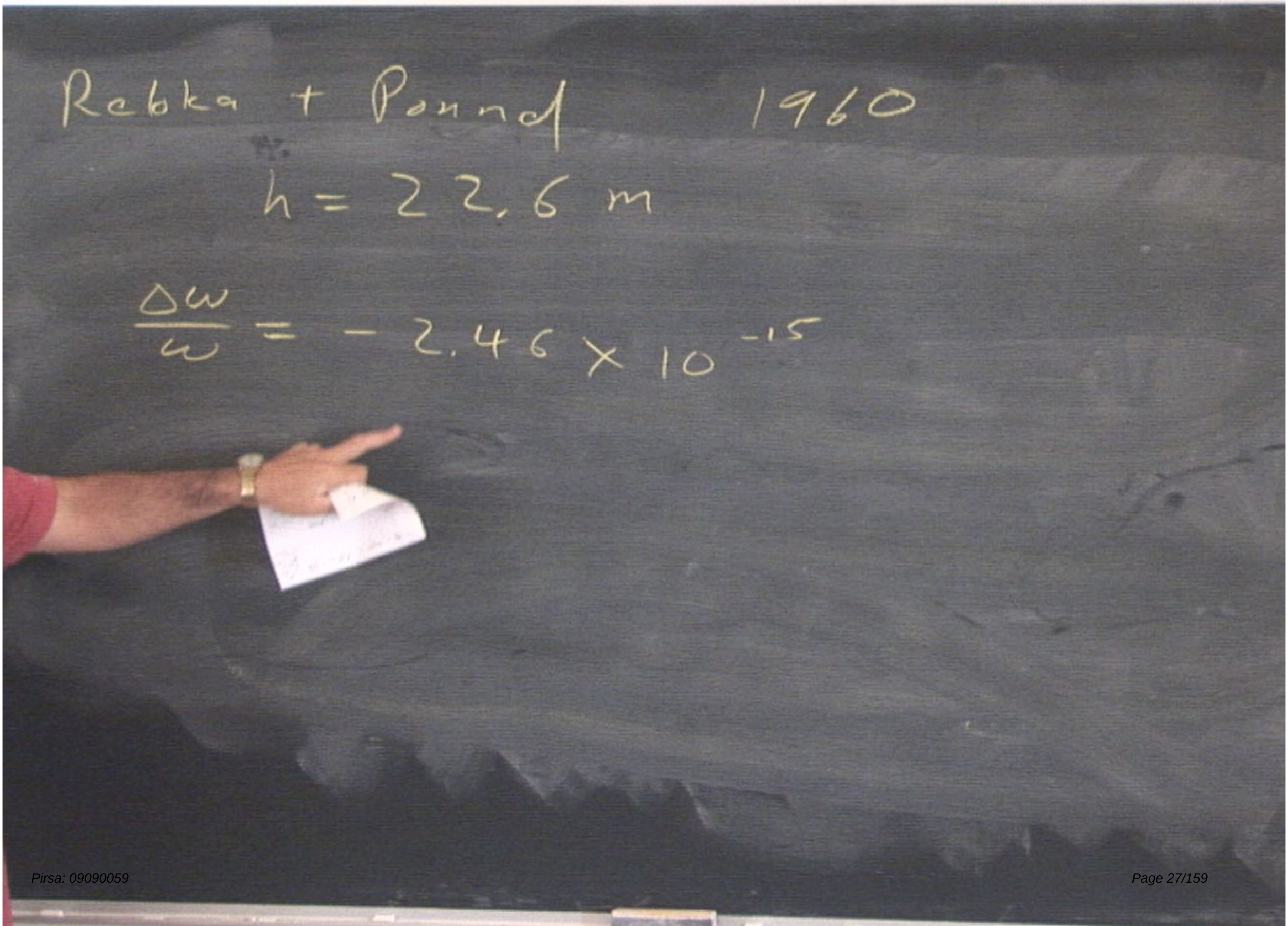

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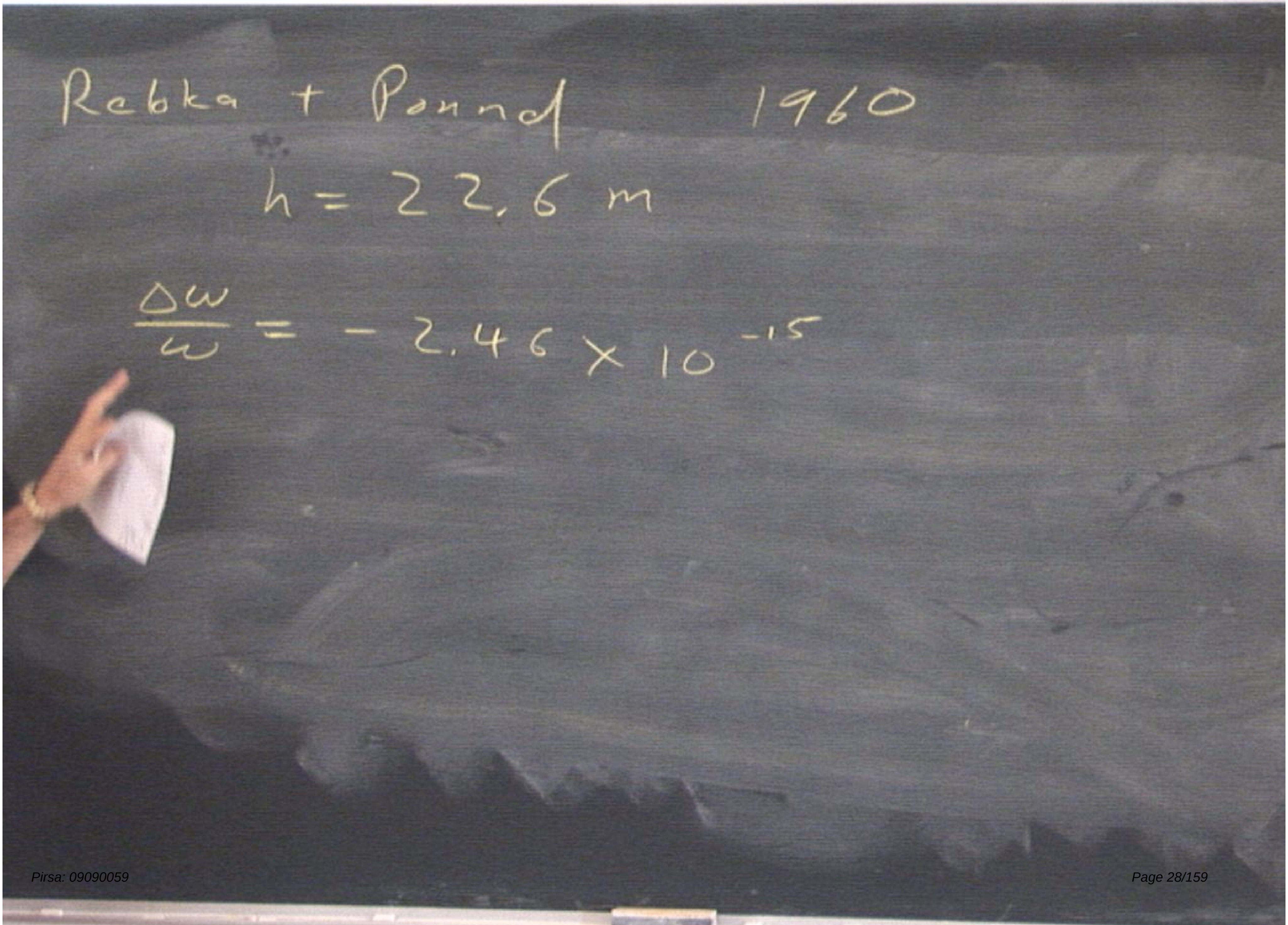

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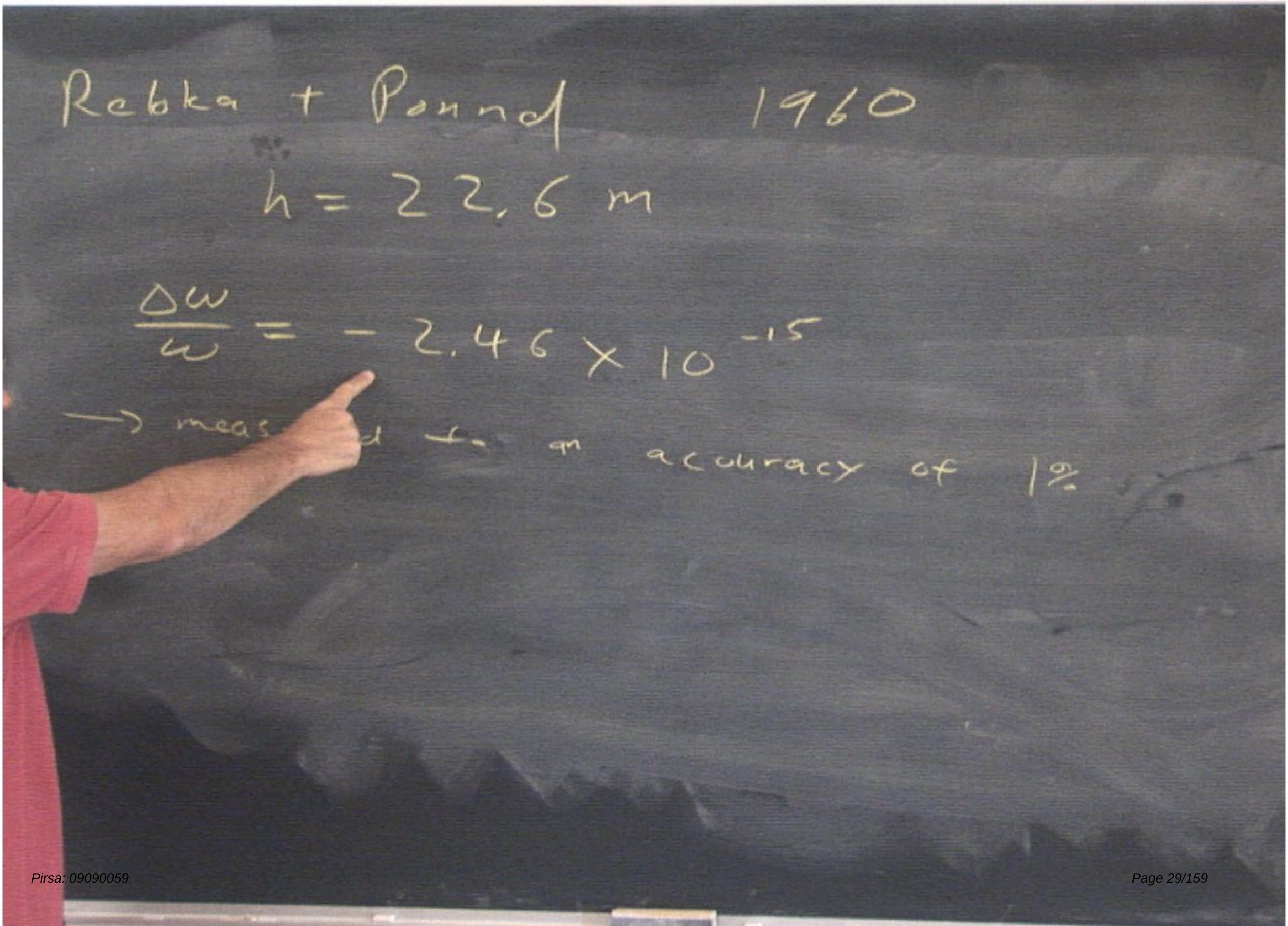

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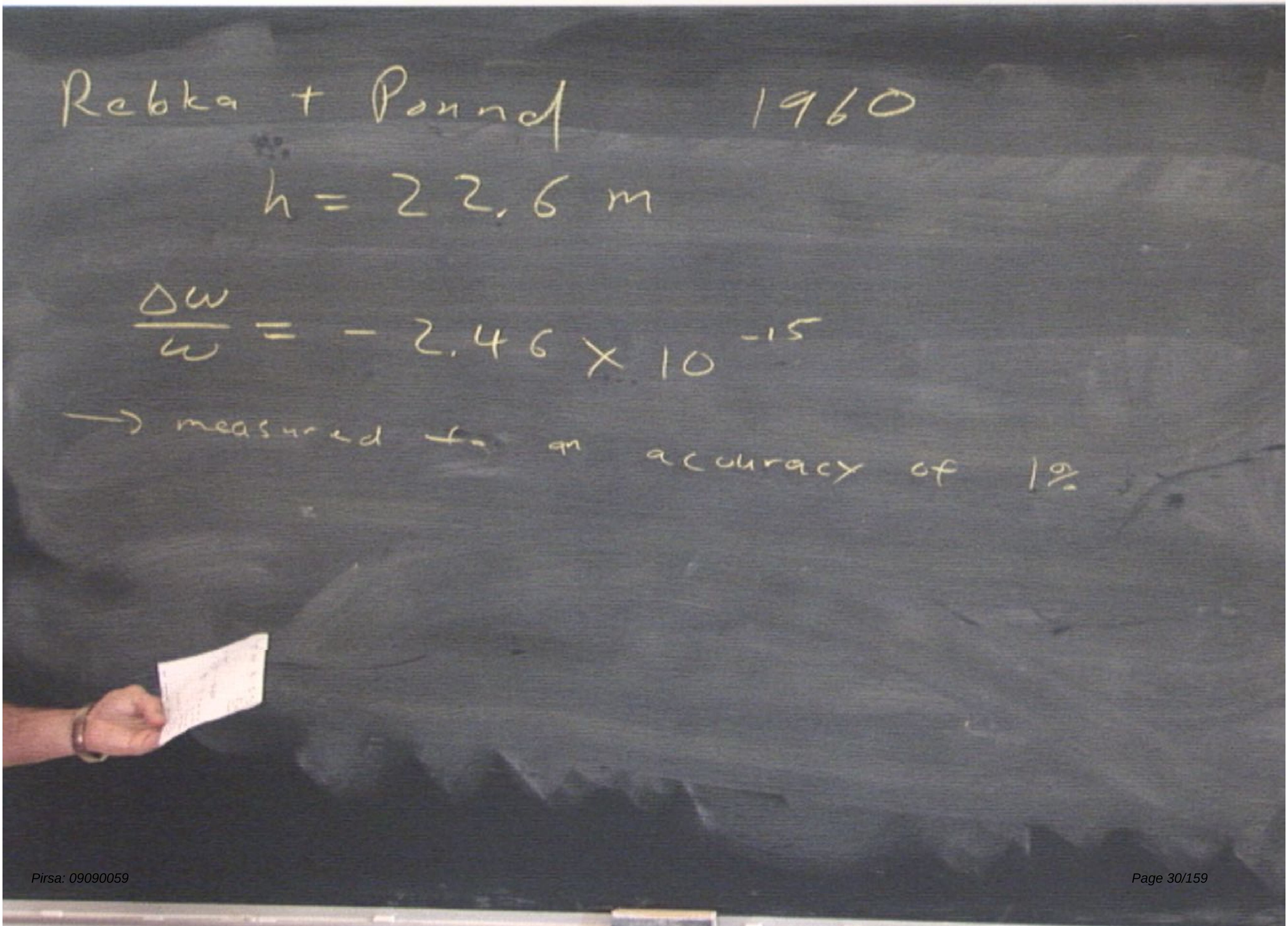

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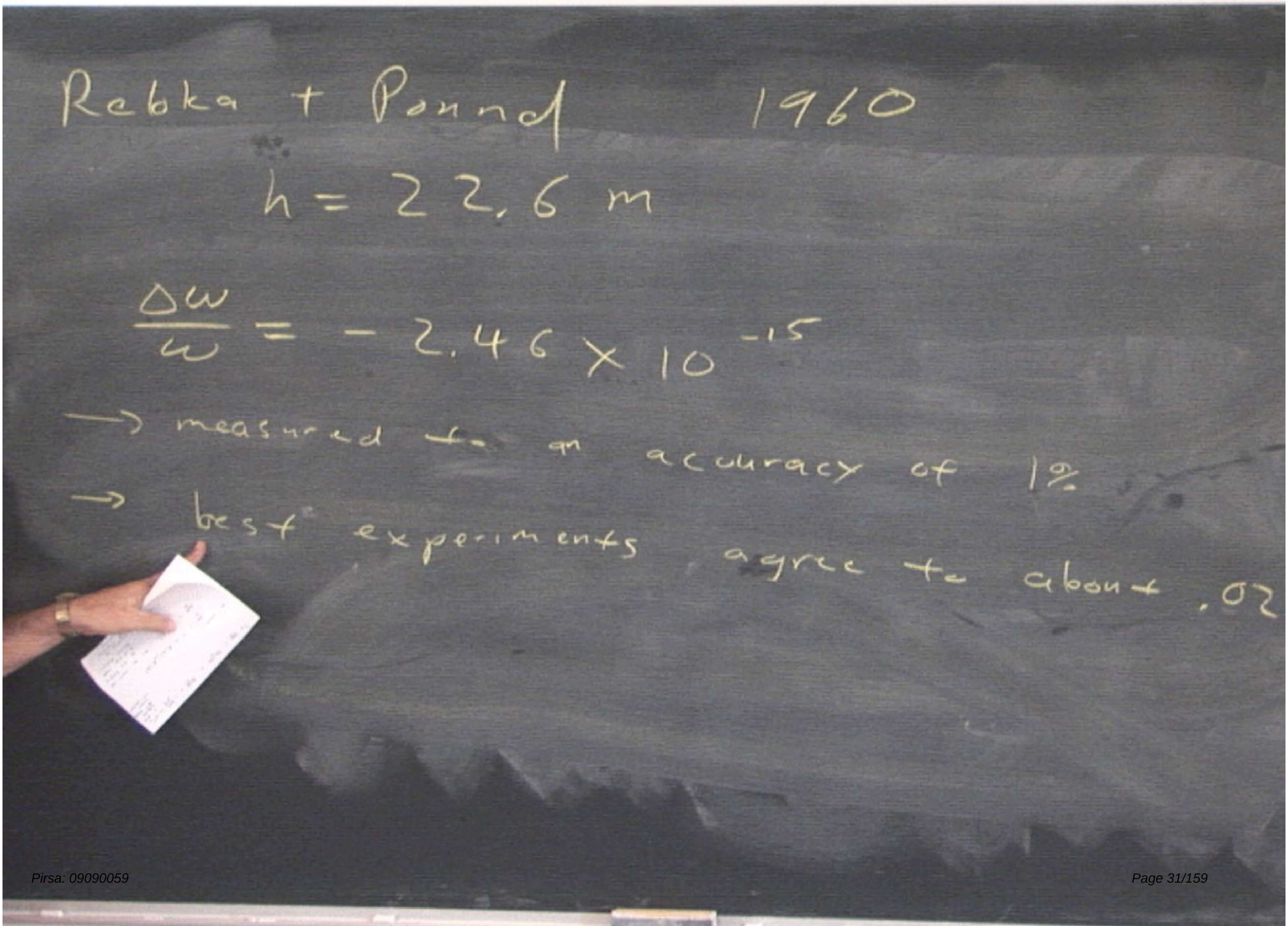

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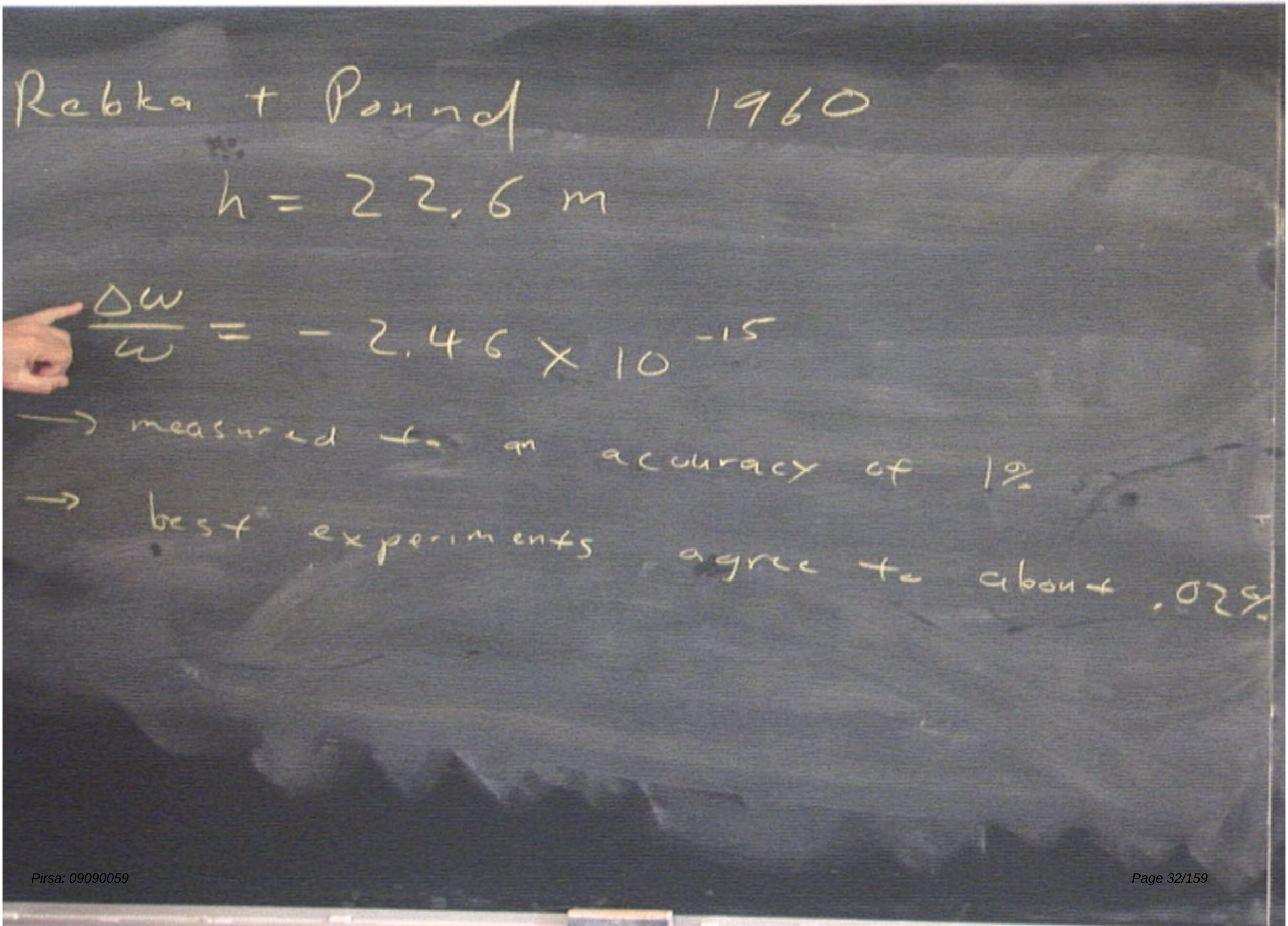

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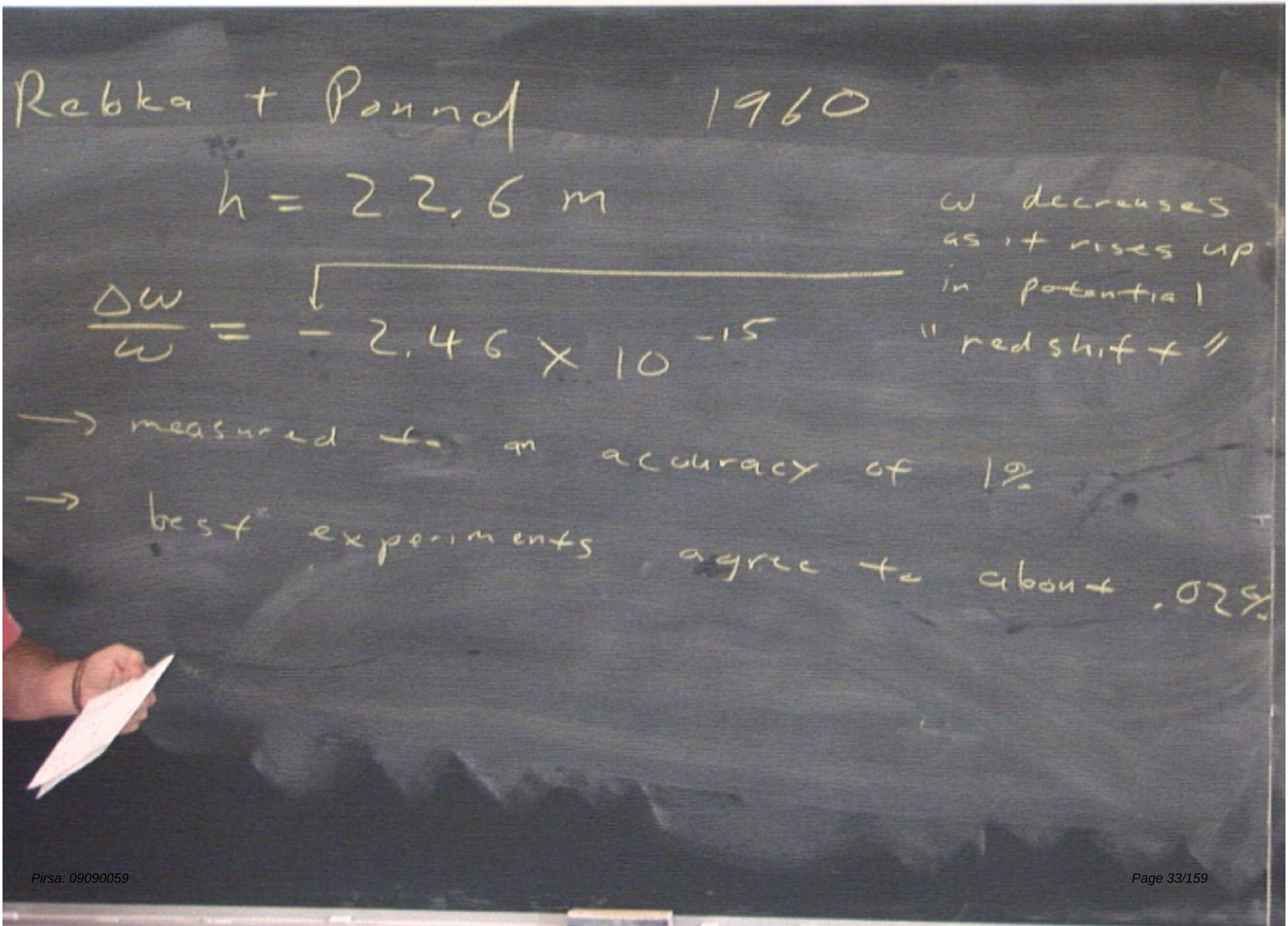

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ω decreases as it rises up in potential

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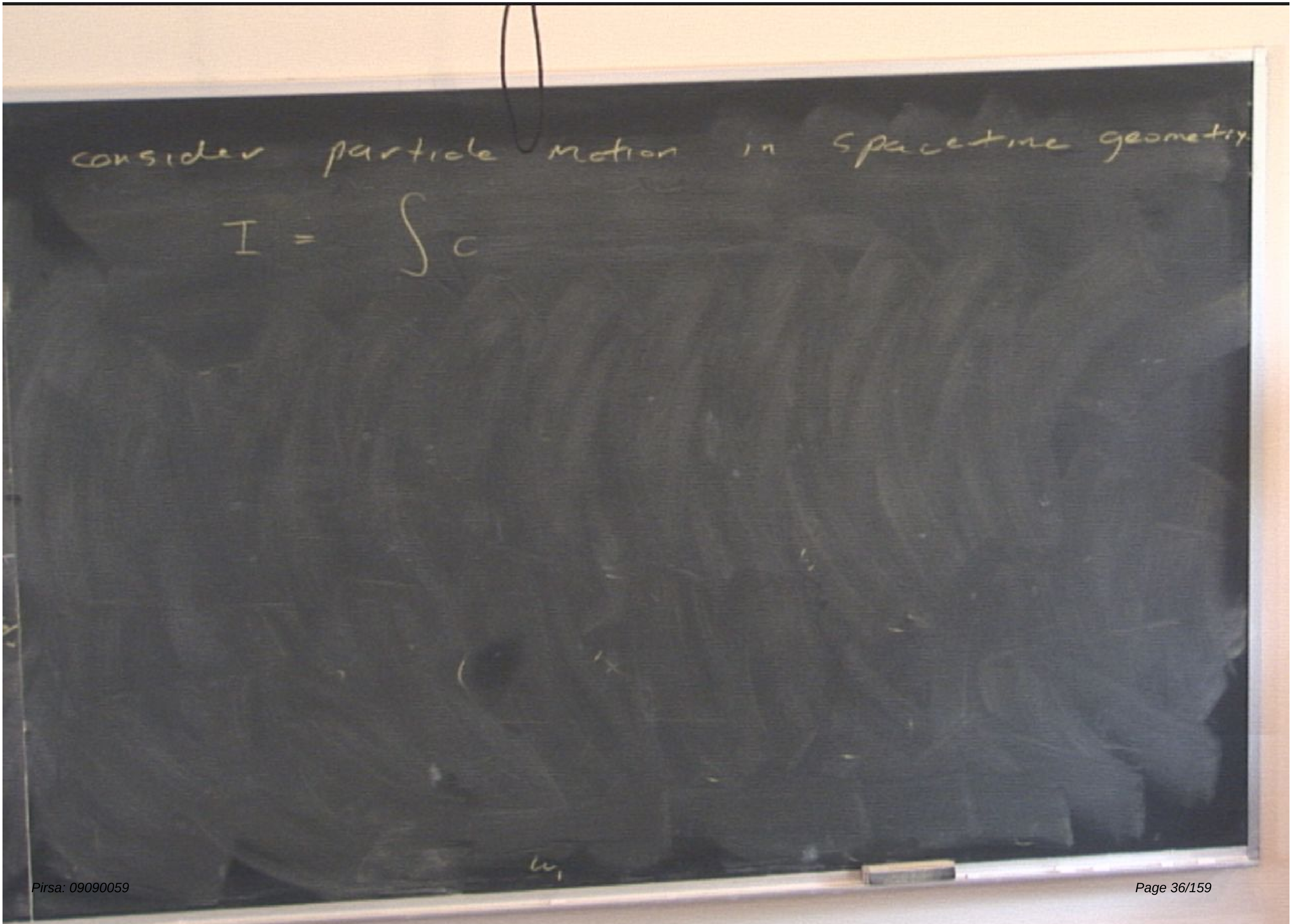

consider particle motion in spacetime geometry

$$I = \int_C$$

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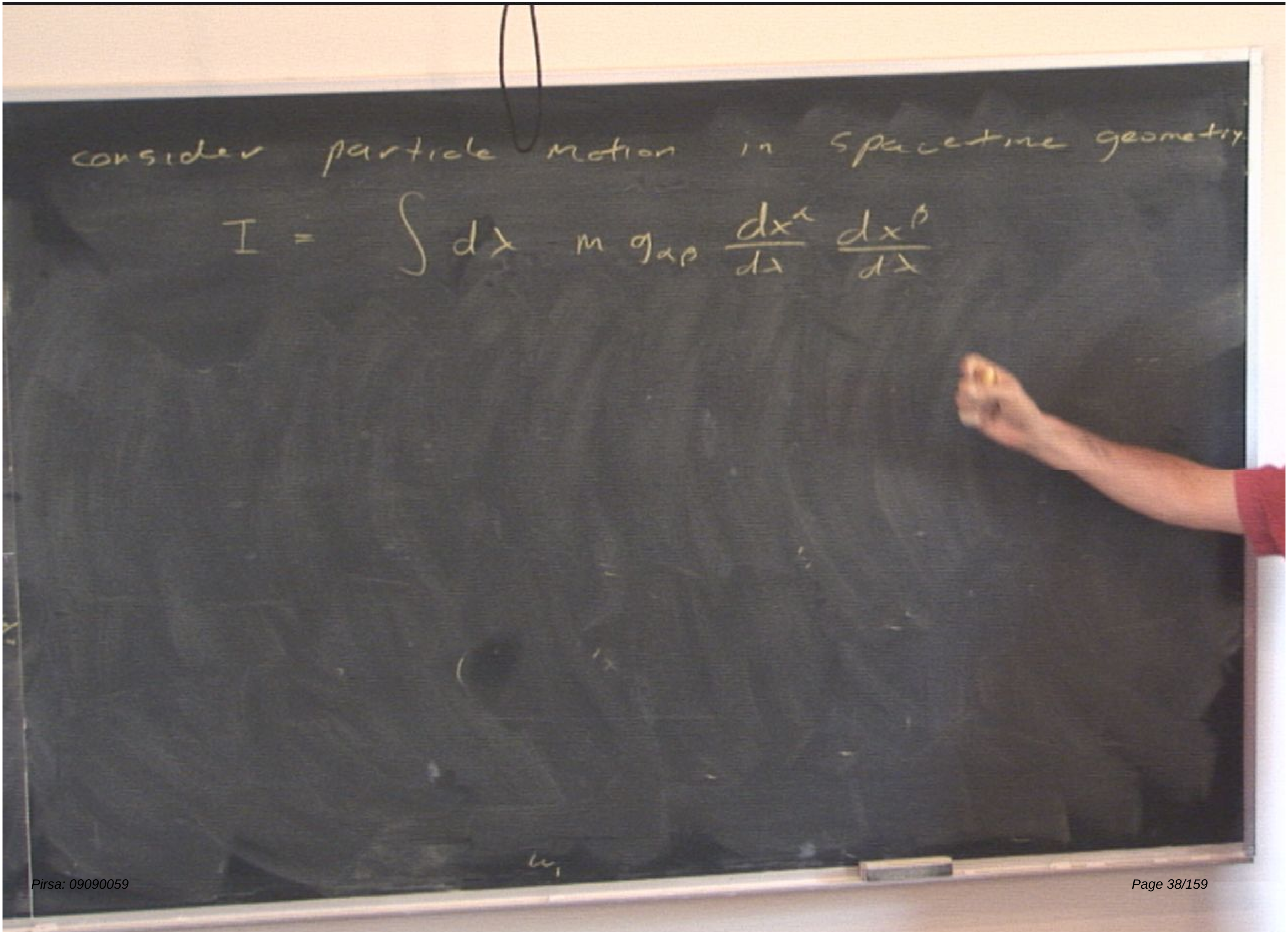
$$I = \int d\lambda$$



consider particle motion in spacetime geometry.

$$I = \int d\lambda \; m \, g_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda}$$

consider particle motion in spacetime geometry.

$$I = \int d\lambda \; m \, \underset{\substack{\uparrow \\ \text{replace } \eta_{\alpha\beta} \text{ by new ``metric''}}}{g_{\alpha\beta}} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda}$$

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replace $\eta_{\alpha\beta}$ by new "metric"

$$= \int d\lambda \; m \left[-\left(c^2 + 2\Phi(\underline{x})\right)\left(\frac{dt}{d\lambda}\right)^2 + \left(\frac{dx^i}{d\lambda}\right)^2 \right]$$

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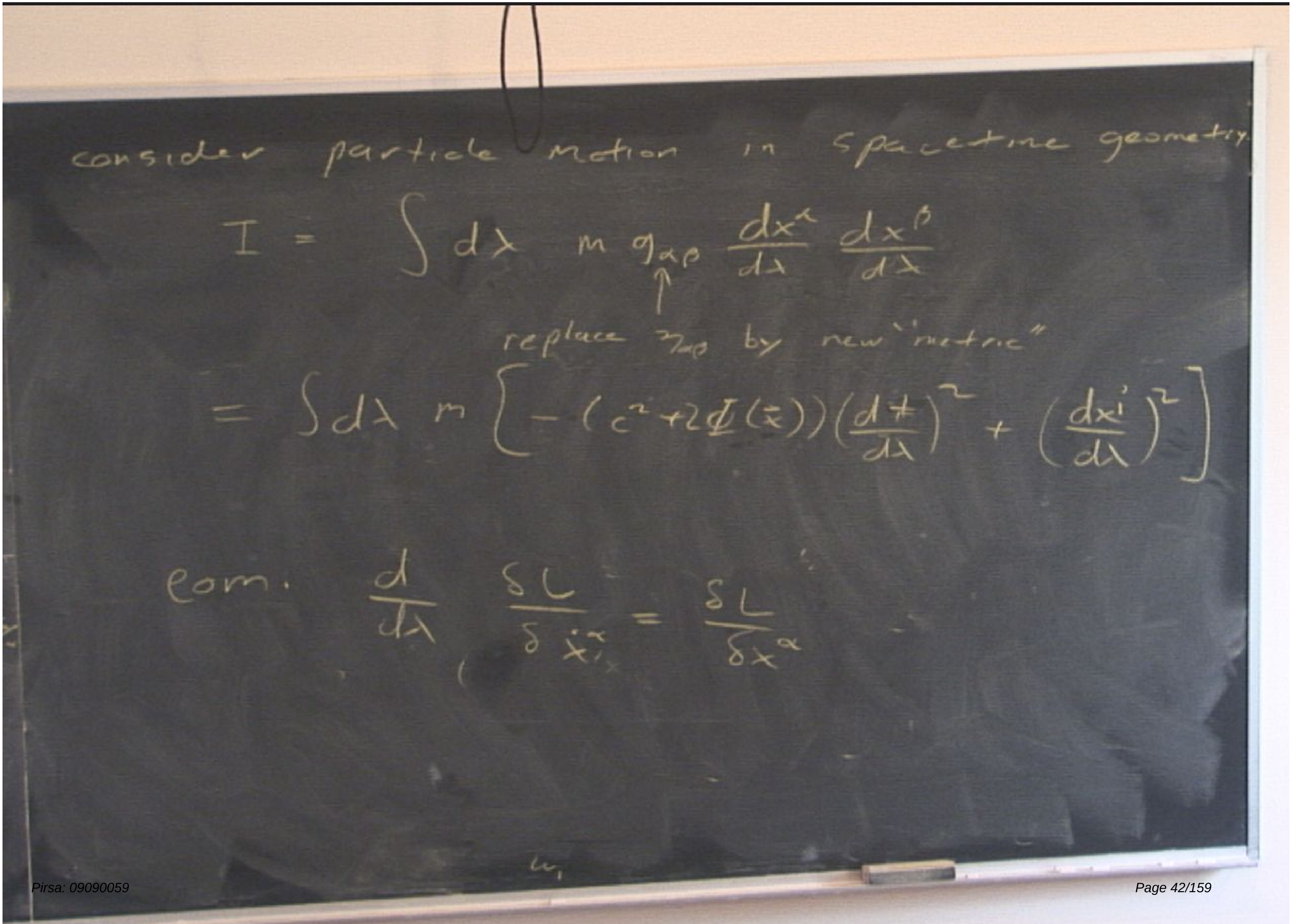

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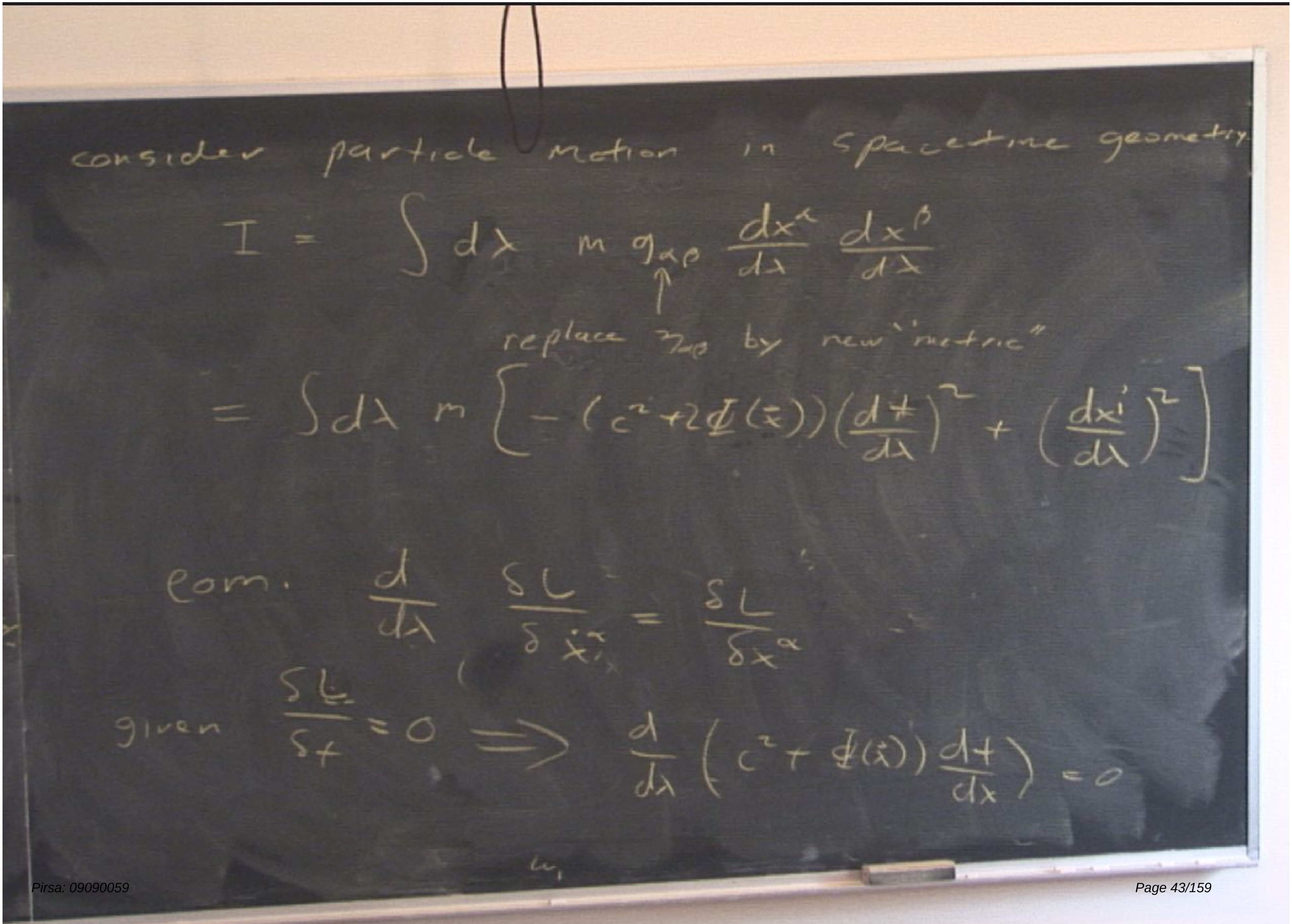

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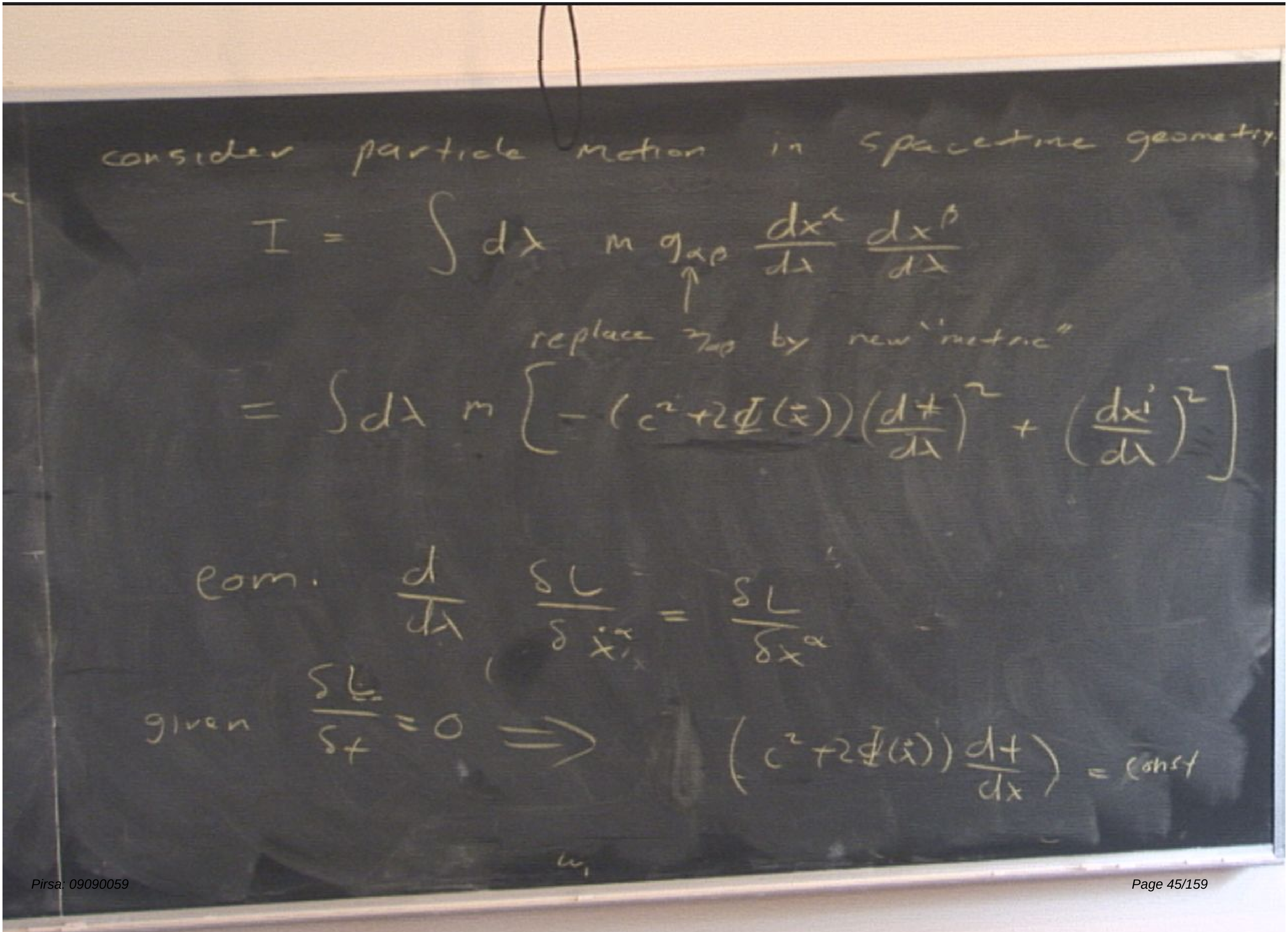

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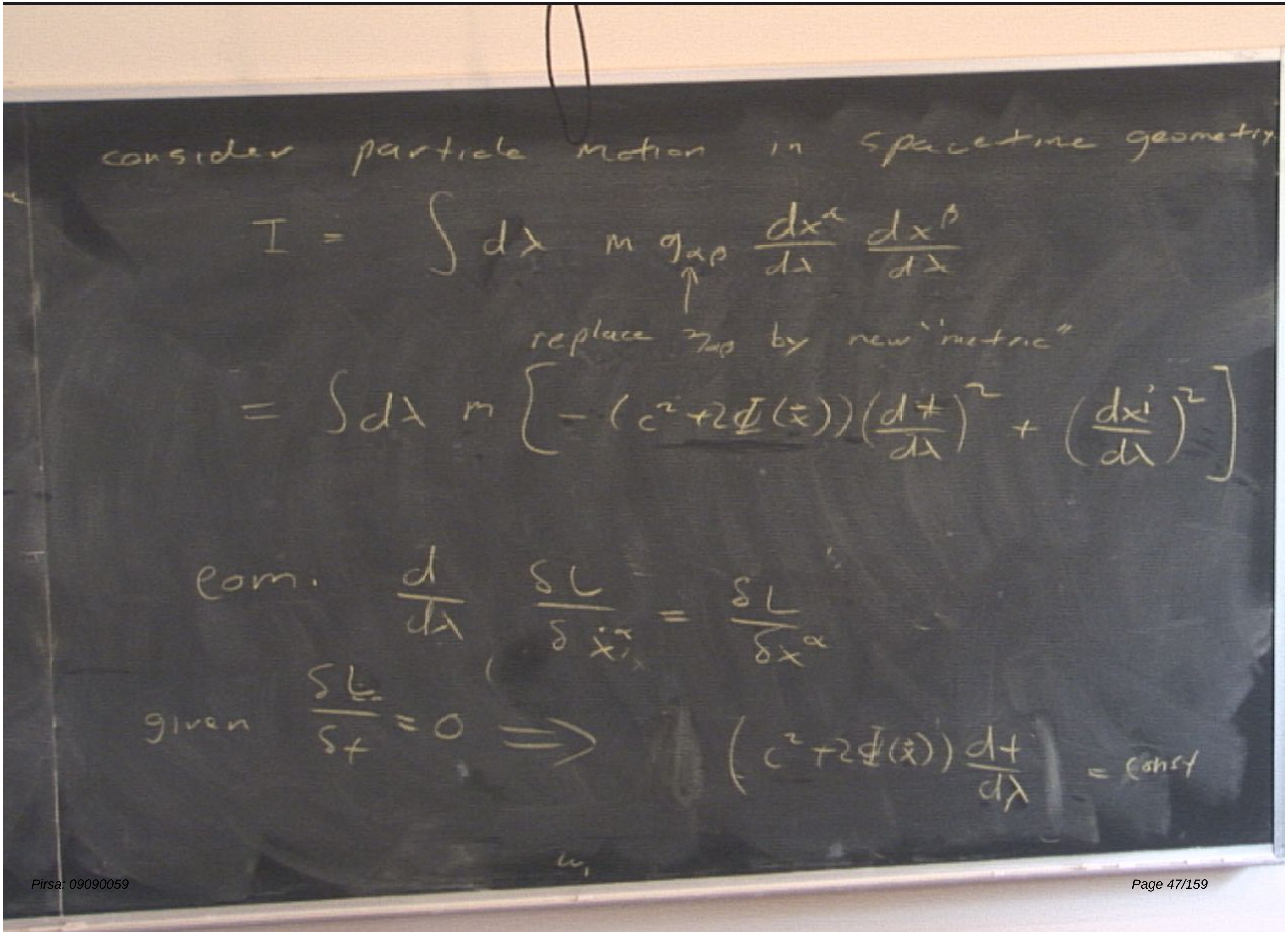

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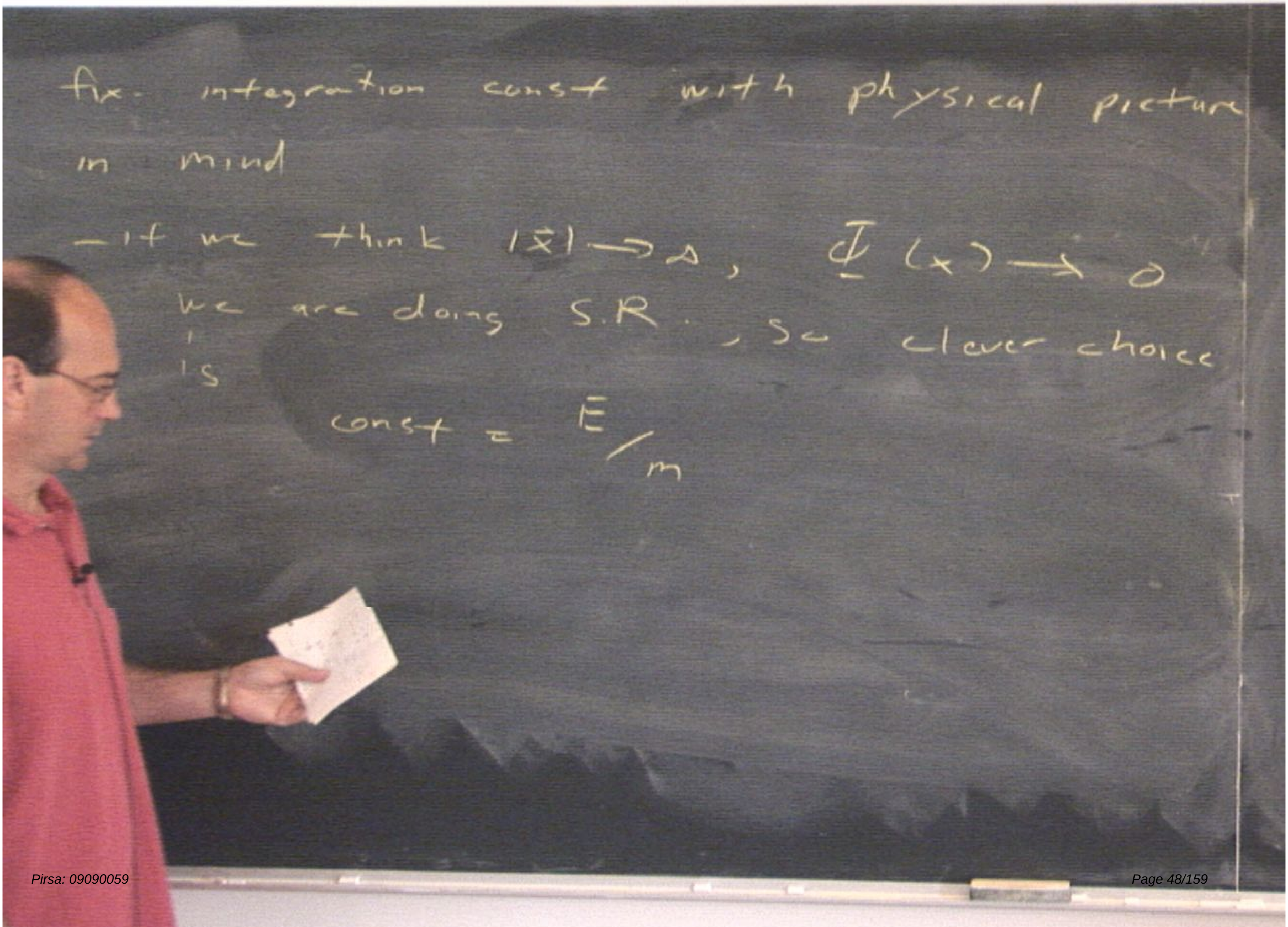

fix. integration const with physical picture in mind

– if we think $|\vec{x}| \Longrightarrow \infty$, $\Phi(x) \longrightarrow 0$

we are doing S.R. , so clever choice is

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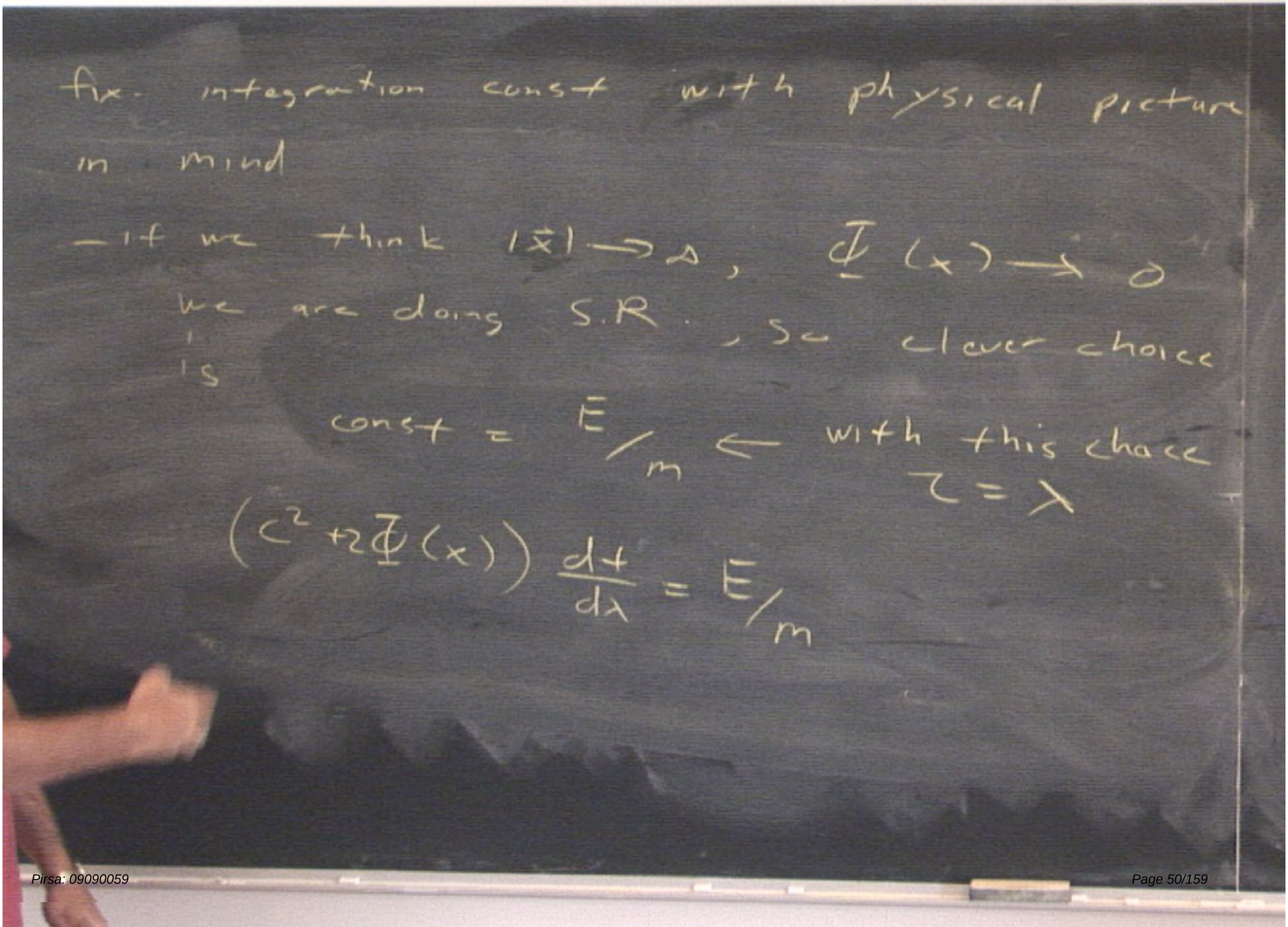

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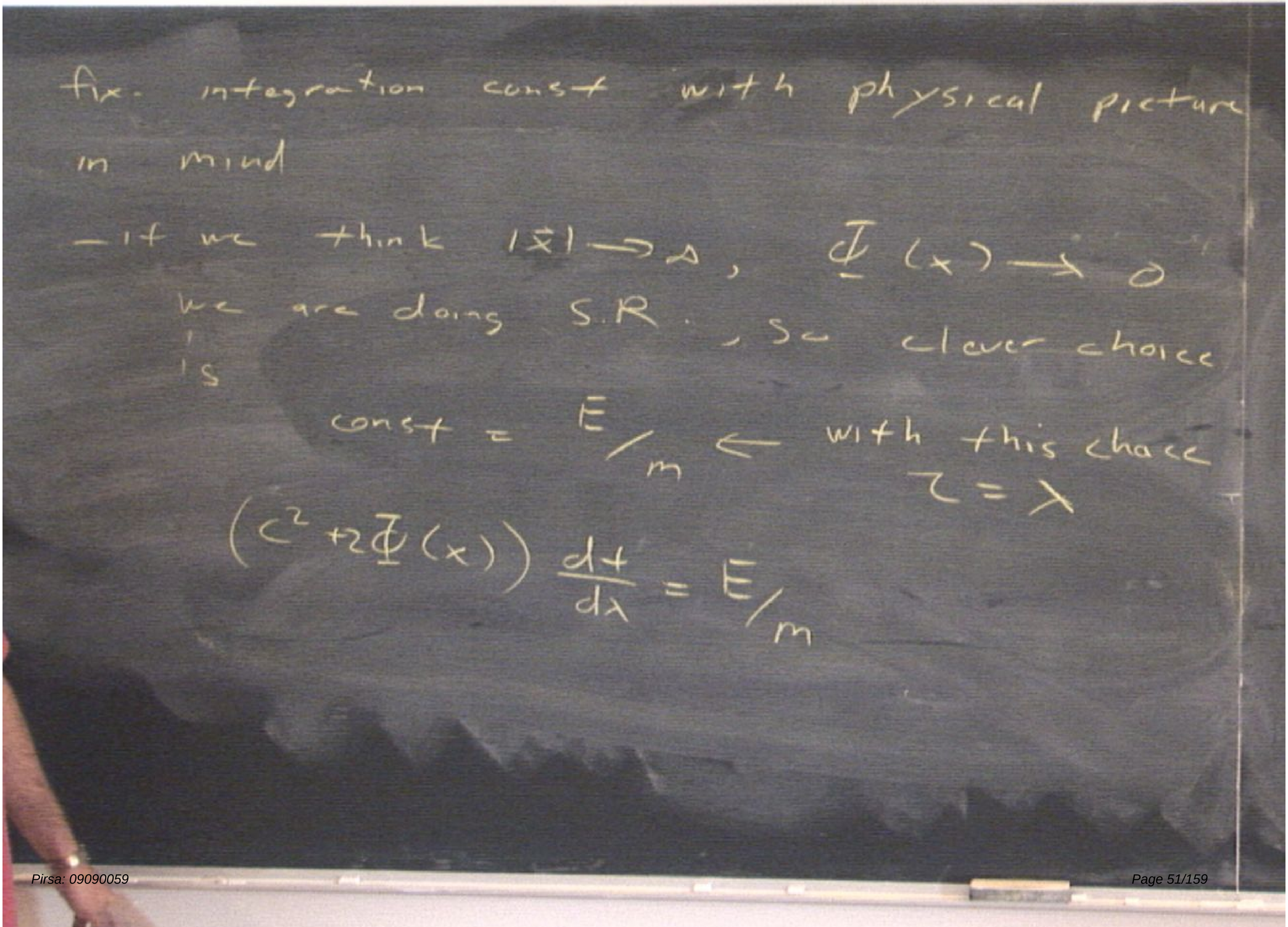
$$\text{const} = E/m \Leftarrow \text{with this choice } \tau = \lambda$$

$$\left(c^2 + 2\Phi(x)\right)\frac{dt}{d\lambda} = E/m$$



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$$\Longrightarrow \frac{dt}{d\tau} \simeq \frac{E}{mc^2}\left(1 - 2\phi/c^2\right)$$

fix integration const with physical picture in mind

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SR

$$\frac{dt}{d\tau} = E/mc^2 \longleftarrow \quad \text{const} = E/m \longleftarrow \text{with this choice}$$

$$\tau = \lambda$$

$$(c^2 + 2\Phi(x)) \frac{dt}{d\tau} = E/m$$

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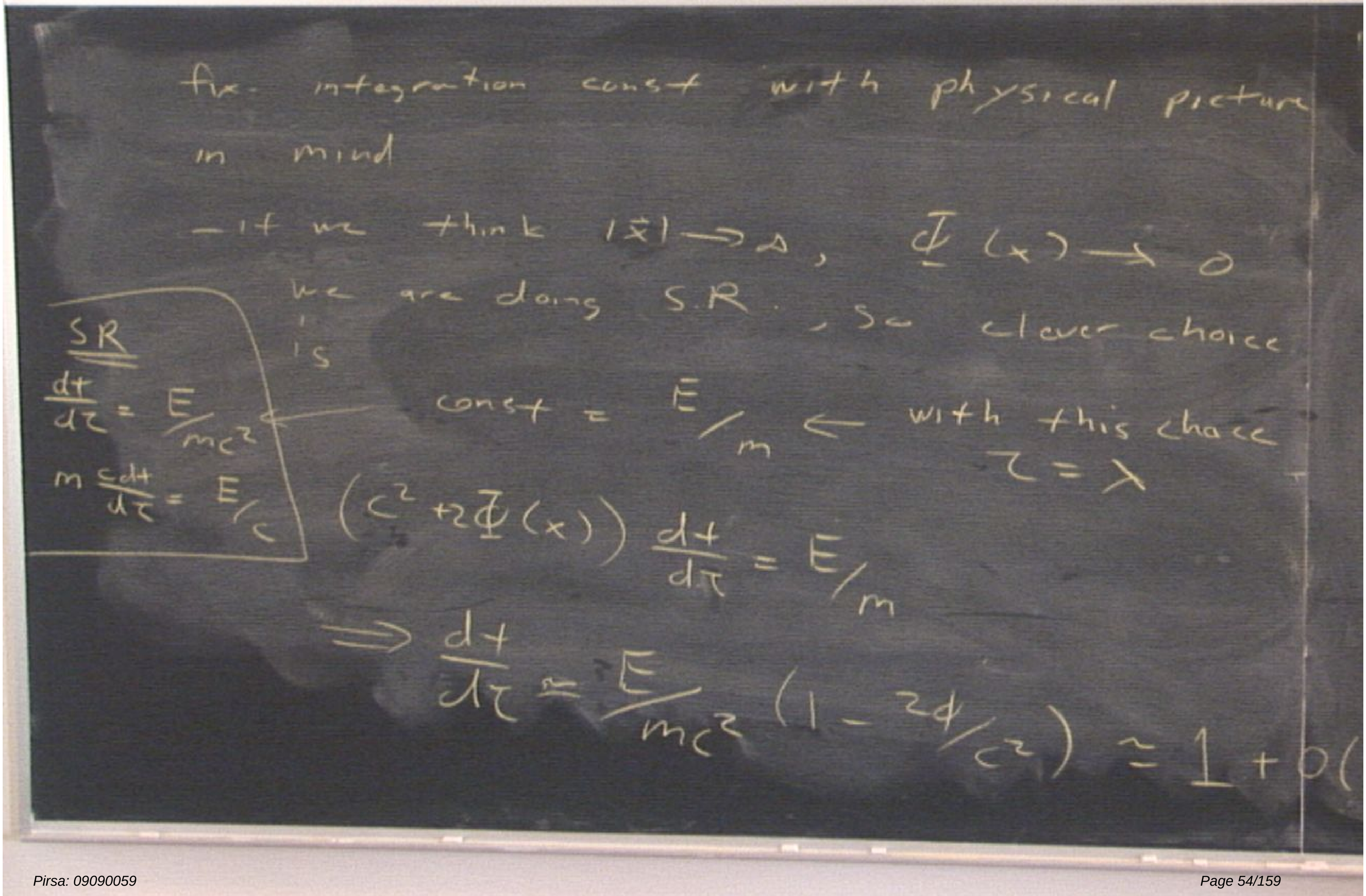

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we are doing S.R., so clever choice is

const $= E/m$ ← with this choice $\tau = \lambda$

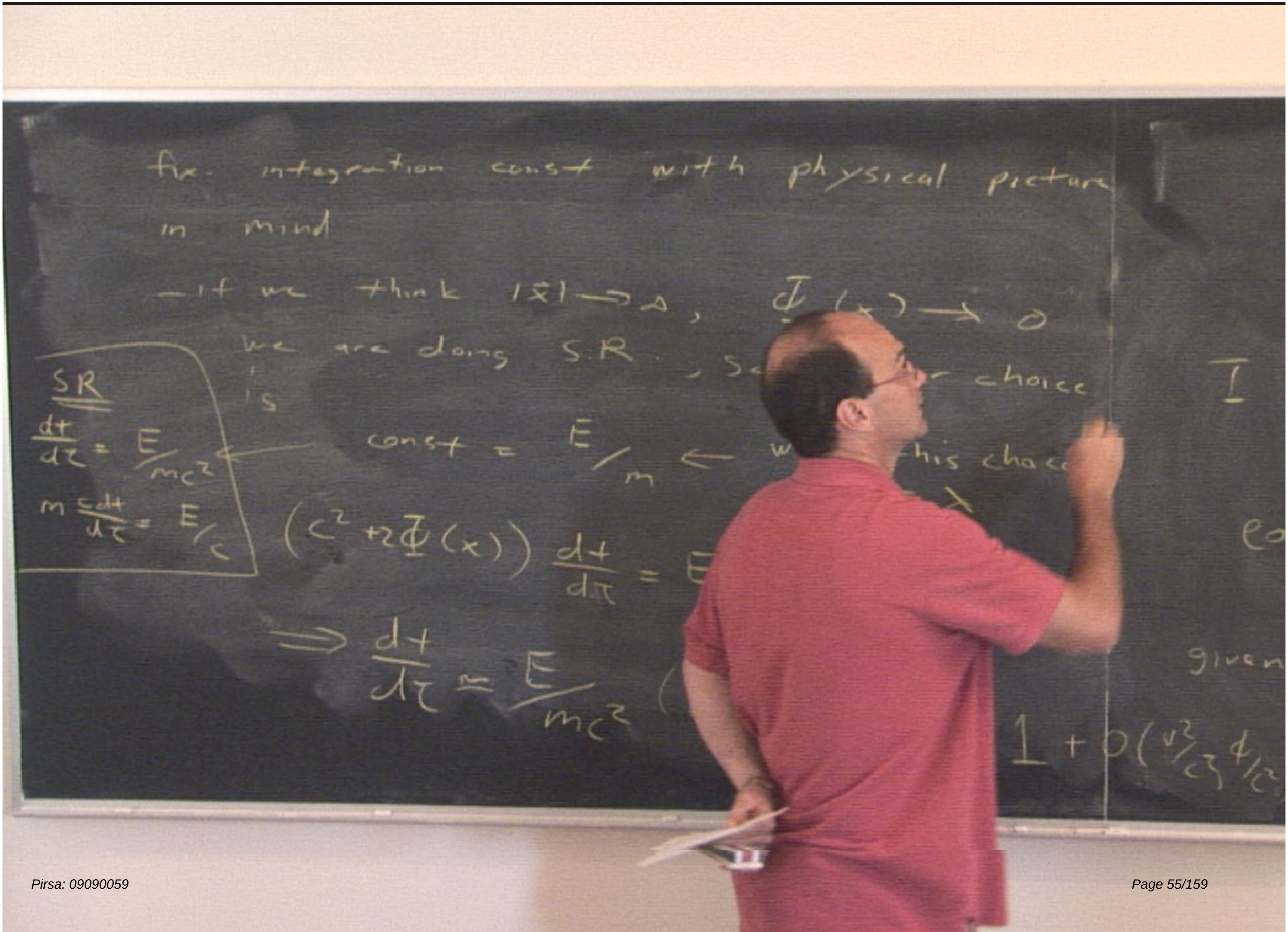
SR

$$\frac{dt}{d\tau} = E/mc^2$$

$$m\frac{c\,dt}{d\tau} = E/c$$

$$(c^2 + 2\Phi(x))\frac{dt}{d\tau} = E/m$$

$$\Longrightarrow \frac{dt}{d\tau} \simeq \frac{E}{mc^2}\left(1 - 2\phi/c^2\right) \simeq 1 + O($$



fix integration const with physical picture in mind

– if we think $|\vec{x}| \to \infty$, $\Phi(x) \to 0$

we are doing S.R., so [illegible] choice

is

const $= E/_m$ ← w[illegible] this choice

SR

$\frac{dt}{d\tau} = \frac{E}{mc^2}$

$m\frac{c\,dt}{d\tau} = E/_c$

$(c^2 + 2\Phi(x))\frac{dt}{d\tau} = E$

$\Longrightarrow \frac{dt}{d\tau} \simeq \frac{E}{mc^2} ($

$1 + O(v^2/_{c^2}, \phi/_{c^2}$

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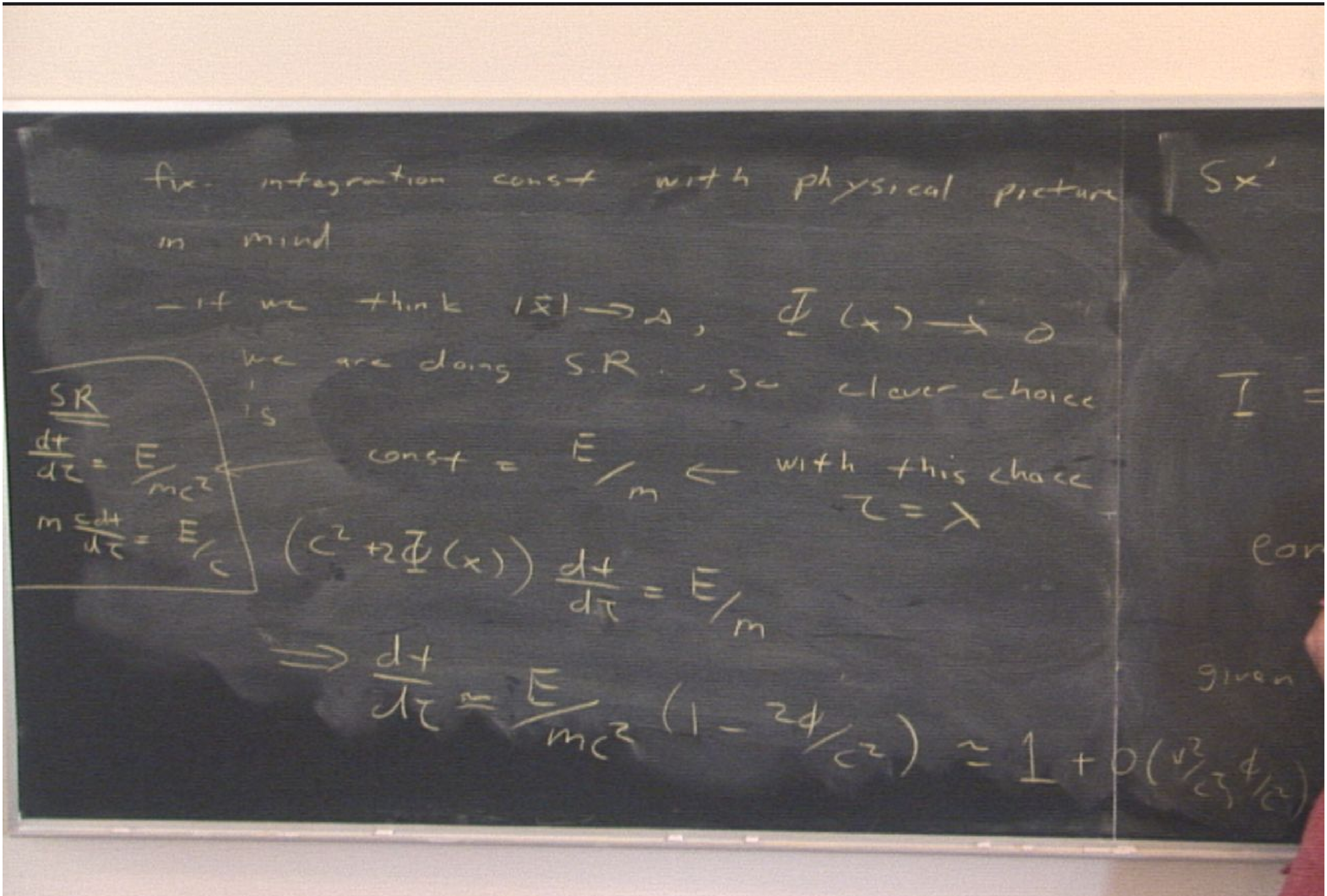

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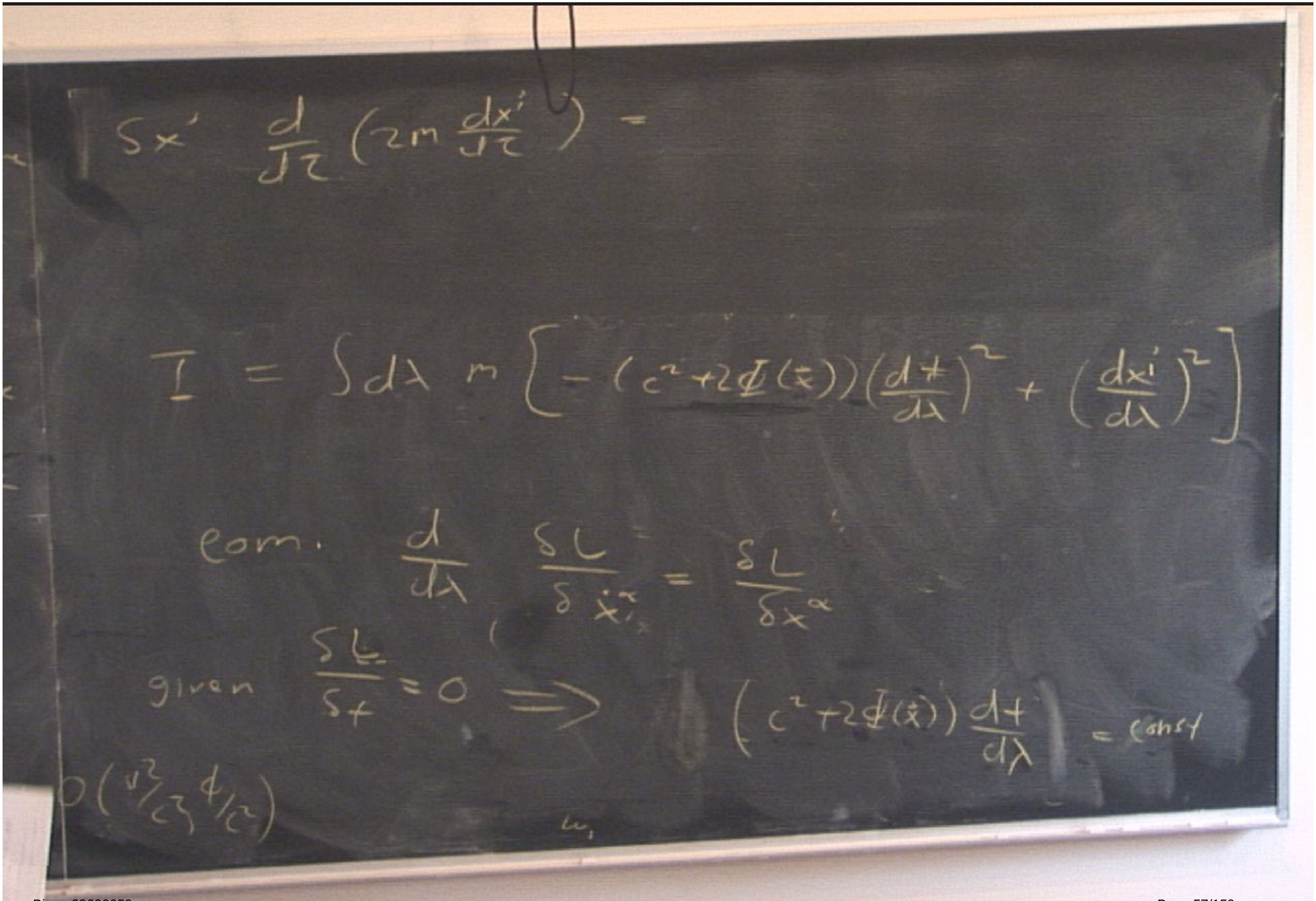
$$\Rightarrow \frac{dt}{d\tau} \simeq \frac{E}{mc^2}\left(1 - 2\phi/c^2\right) \simeq 1 + O\left(v^2/c^2, \phi/c^2\right)$$

$\delta x'$

$\Phi =$

cor

given



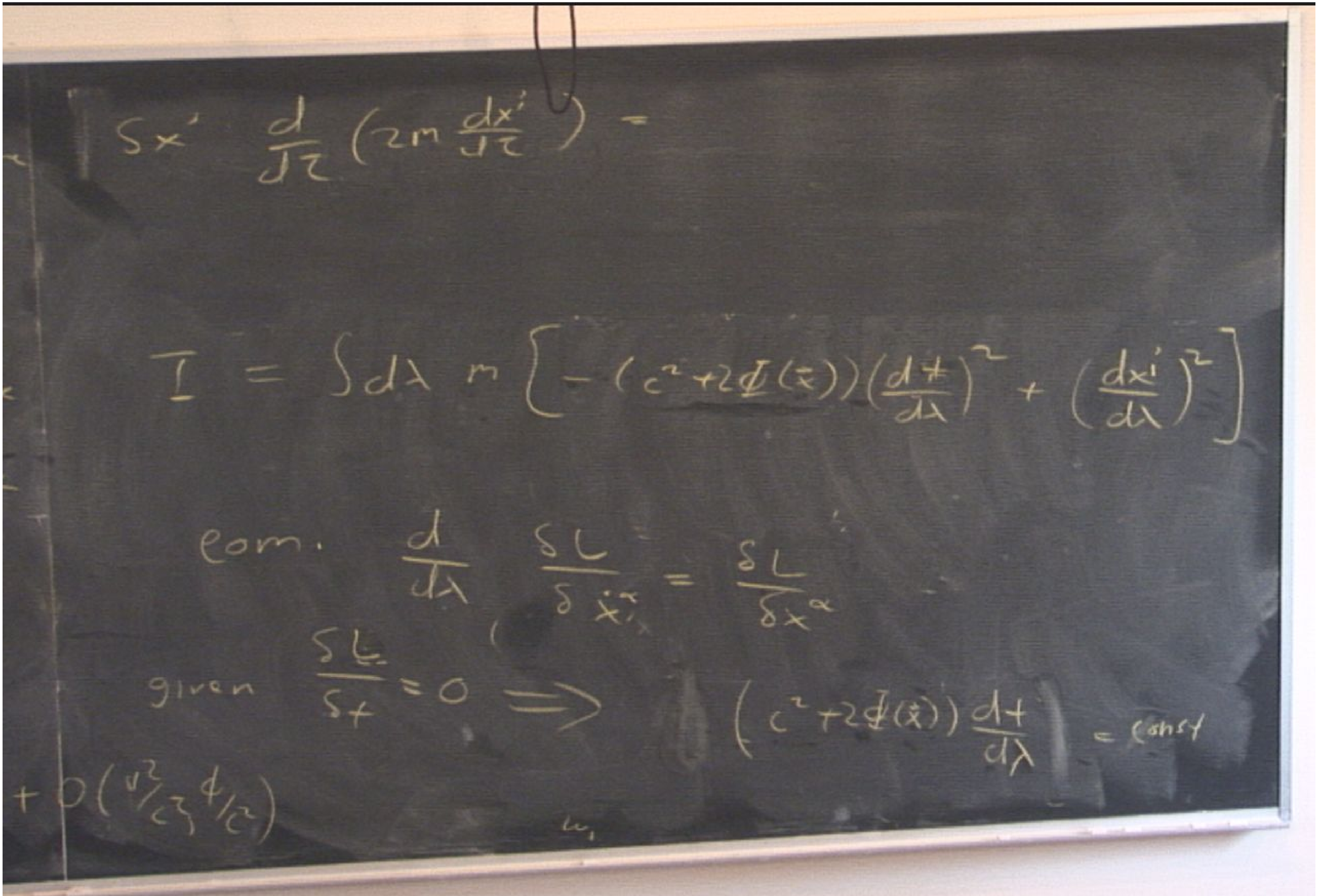
$\delta x^i \quad \frac{d}{d\tau}\left(2m\frac{dx^i}{d\tau}\right) =$

$$I = \int d\lambda \, m\left[-\left(c^2+2\Phi(\vec{x})\right)\left(\frac{dt}{d\lambda}\right)^2 + \left(\frac{dx^i}{d\lambda}\right)^2\right]$$

eom. $\frac{d}{d\lambda}\frac{\delta L}{\delta \dot{x}^\alpha} = \frac{\delta L}{\delta x^\alpha}$

given $\frac{\delta L}{\delta t} = 0 \Longrightarrow \left(c^2+2\Phi(\vec{x})\right)\frac{dt}{d\lambda} = \text{const}$

$O\left(v^2/c^2, \phi/c^2\right)$



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$+\, O\left(v^2/c^2, \Phi/c^2\right)$

$$\delta x^i \quad \frac{d}{d\tau}\left(2m\frac{dx^i}{d\tau}\right) = m(-)\,2\,\frac{\partial \Phi}{\partial x^i}\left(\frac{dt}{d\tau}\right)^2$$

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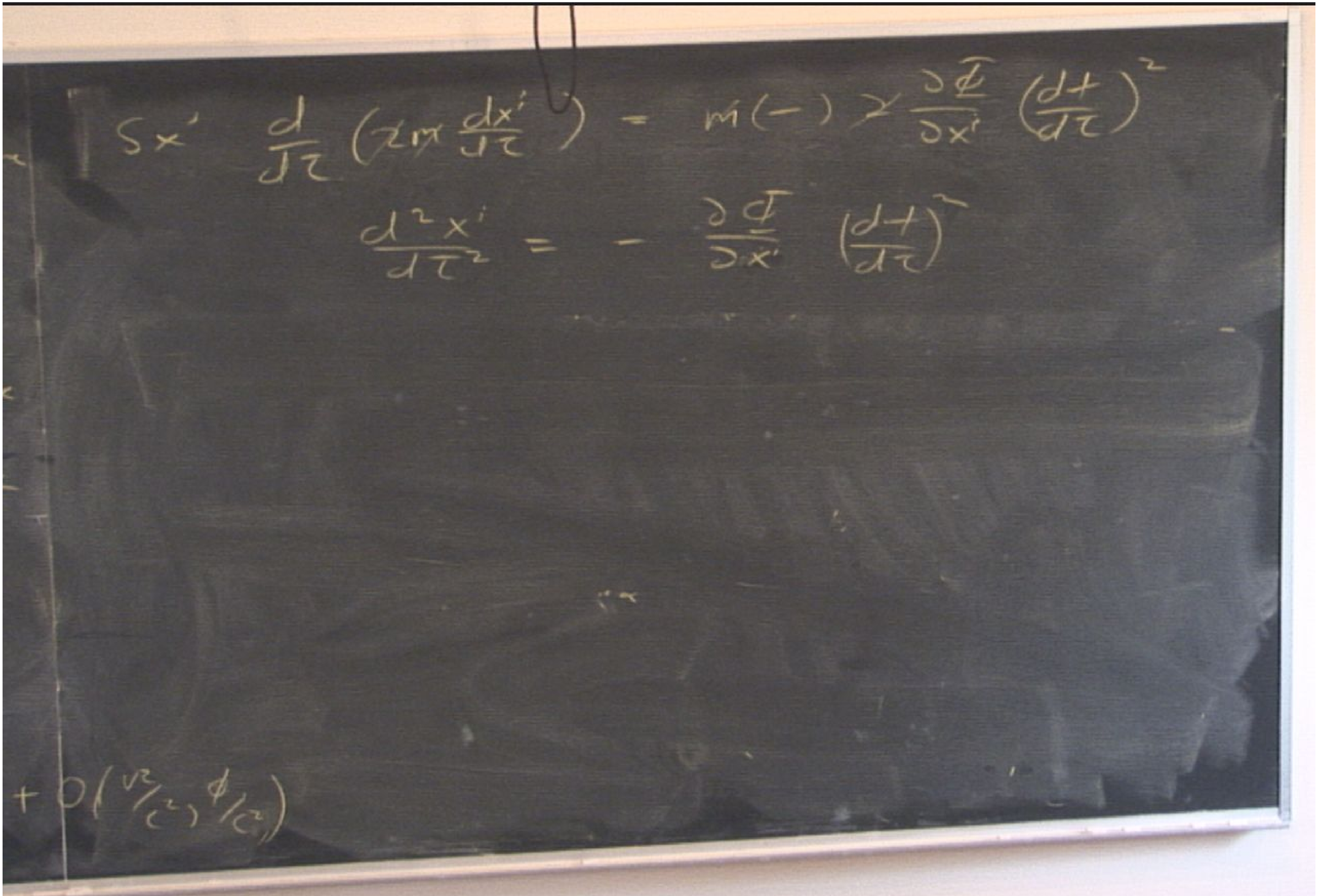

$$\delta x^i \quad \frac{d}{d\tau}\left(\cancel{2}\, m \frac{dx^i}{d\tau}\right) = m(-)\,\cancel{2}\,\frac{\partial \Phi}{\partial x^i}\left(\frac{dt}{d\tau}\right)^2$$

$$\frac{d^2 x^i}{d\tau^2} = -\frac{\partial \Phi}{\partial x^i}\left(\frac{dt}{d\tau}\right)^2$$

$$+\,O\left(v^2/c^2,\ \phi/c^2\right)$$

$$\delta x^i \quad \frac{d}{d\tau}\left(\gamma m \frac{dx^i}{d\tau}\right) = m(-)\,\gamma \frac{\partial \Phi}{\partial x^i}\left(\frac{dt}{d\tau}\right)^2$$

$$\frac{d^2 x^i}{d\tau^2} = -\frac{\partial \Phi}{\partial x^i}\left(\frac{dt}{d\tau}\right)^2$$

$$\frac{d^2 x^i}{dt^2} = -\frac{\partial \Phi}{\partial x^i}$$

$$+\,O\left(v^2/c^2,\ \phi/c^2\right)$$

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$$\bar{I} = \int d\lambda\, m\, g_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}$$

$+ O(v^2/c^2, \phi/c^2)$

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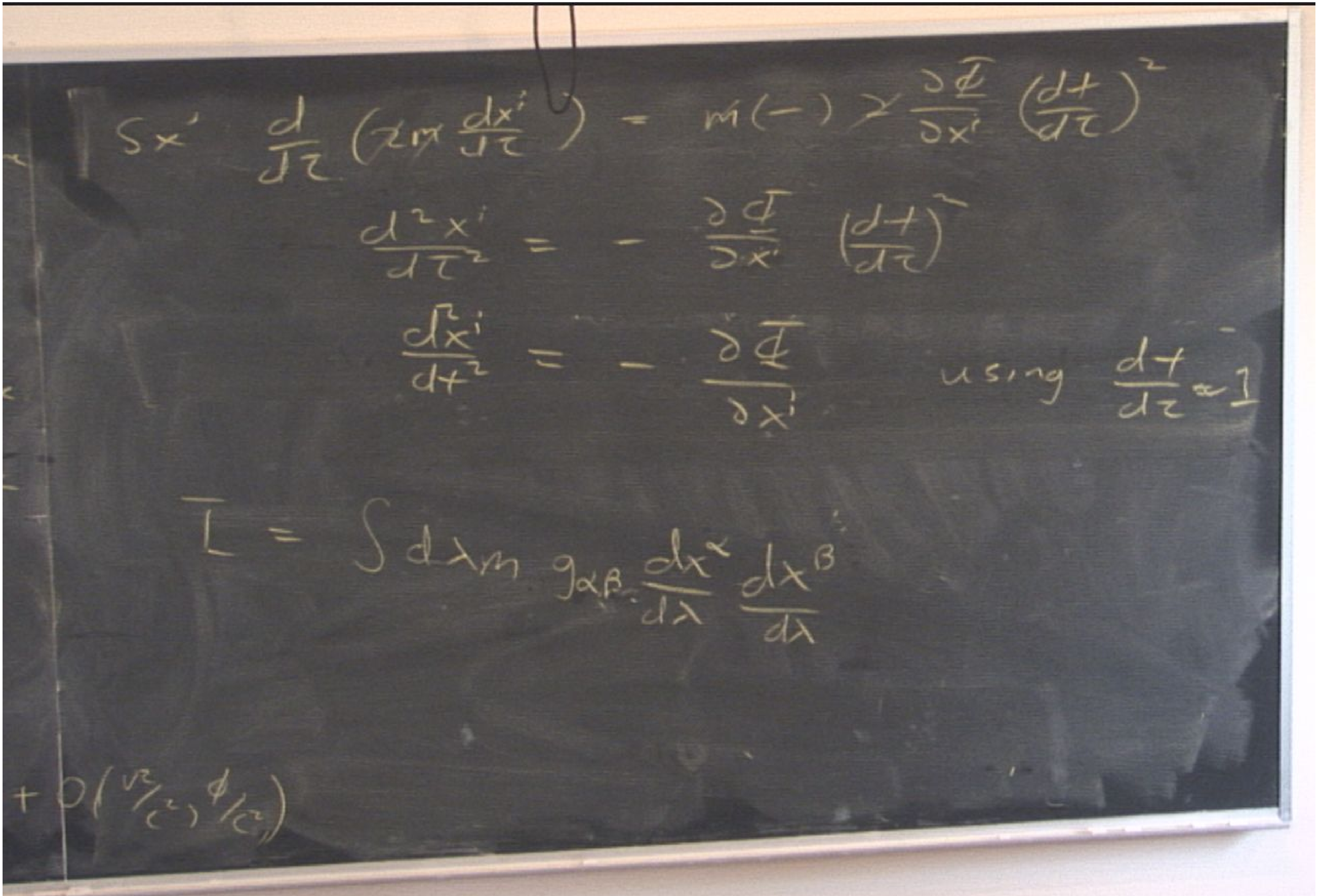

$$\delta x^i \quad \frac{d}{d\tau}\left(\cancel{2}\cancel{m}\frac{dx^i}{d\tau}\right) = \cancel{m}(-)\cancel{2}\frac{\partial \Phi}{\partial x^i}\left(\frac{dt}{d\tau}\right)^2$$

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$$+ O(v^2/c^2, \phi/c^2)$$

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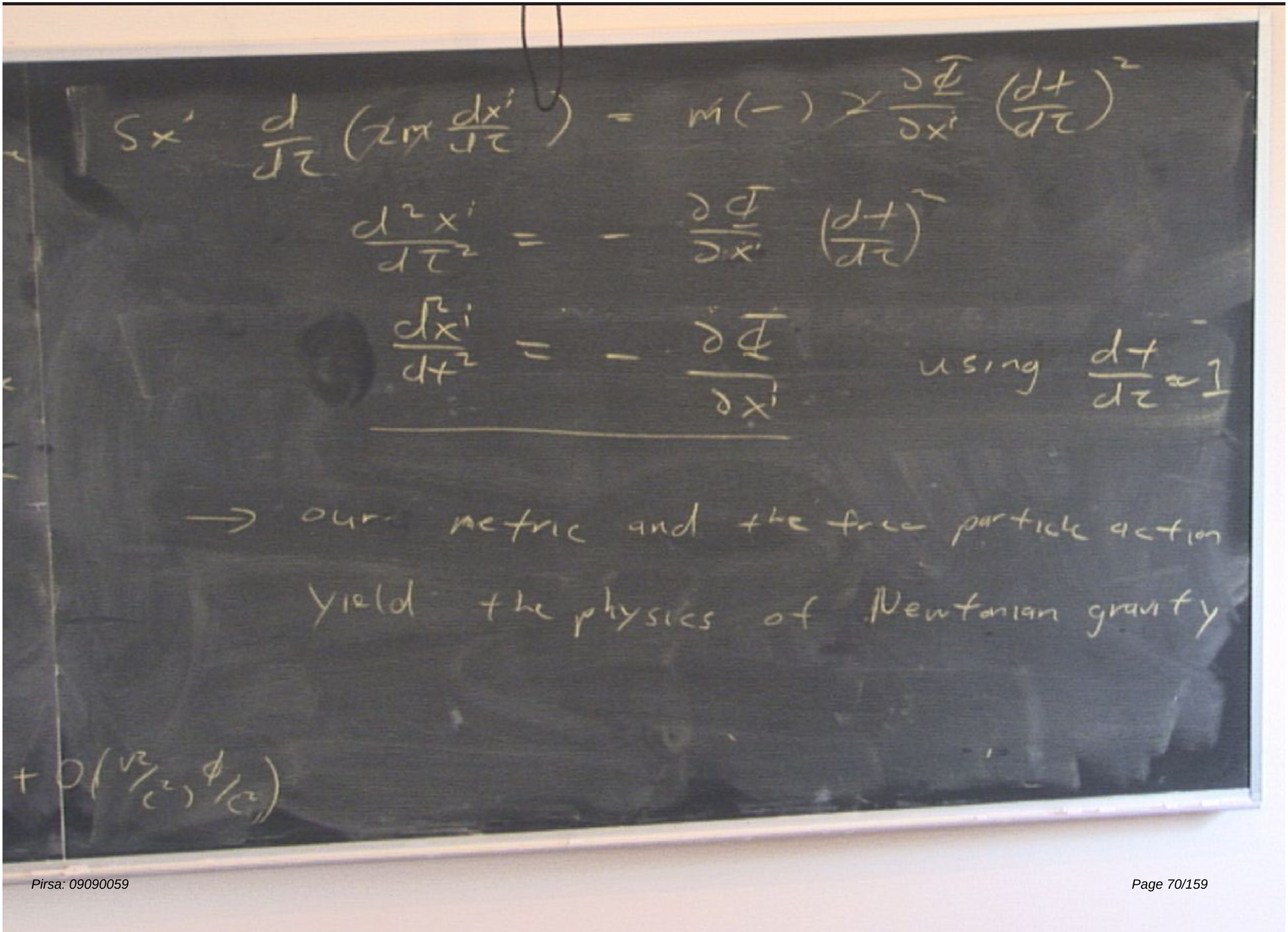
→ our metric and the free particle action yield the physics of Newtonian gravity

$+ O(v^2/c^2, \phi/c^2)$



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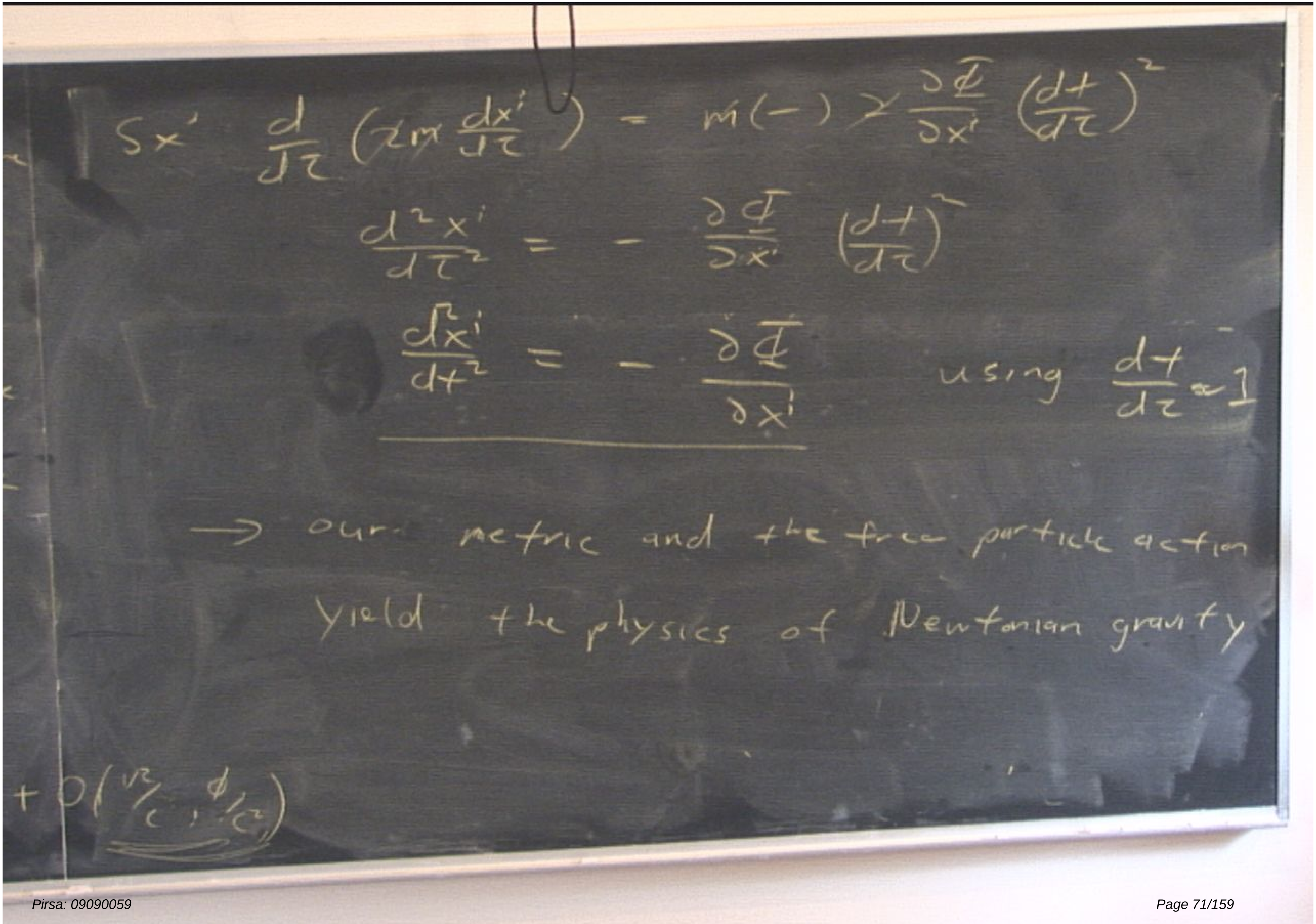

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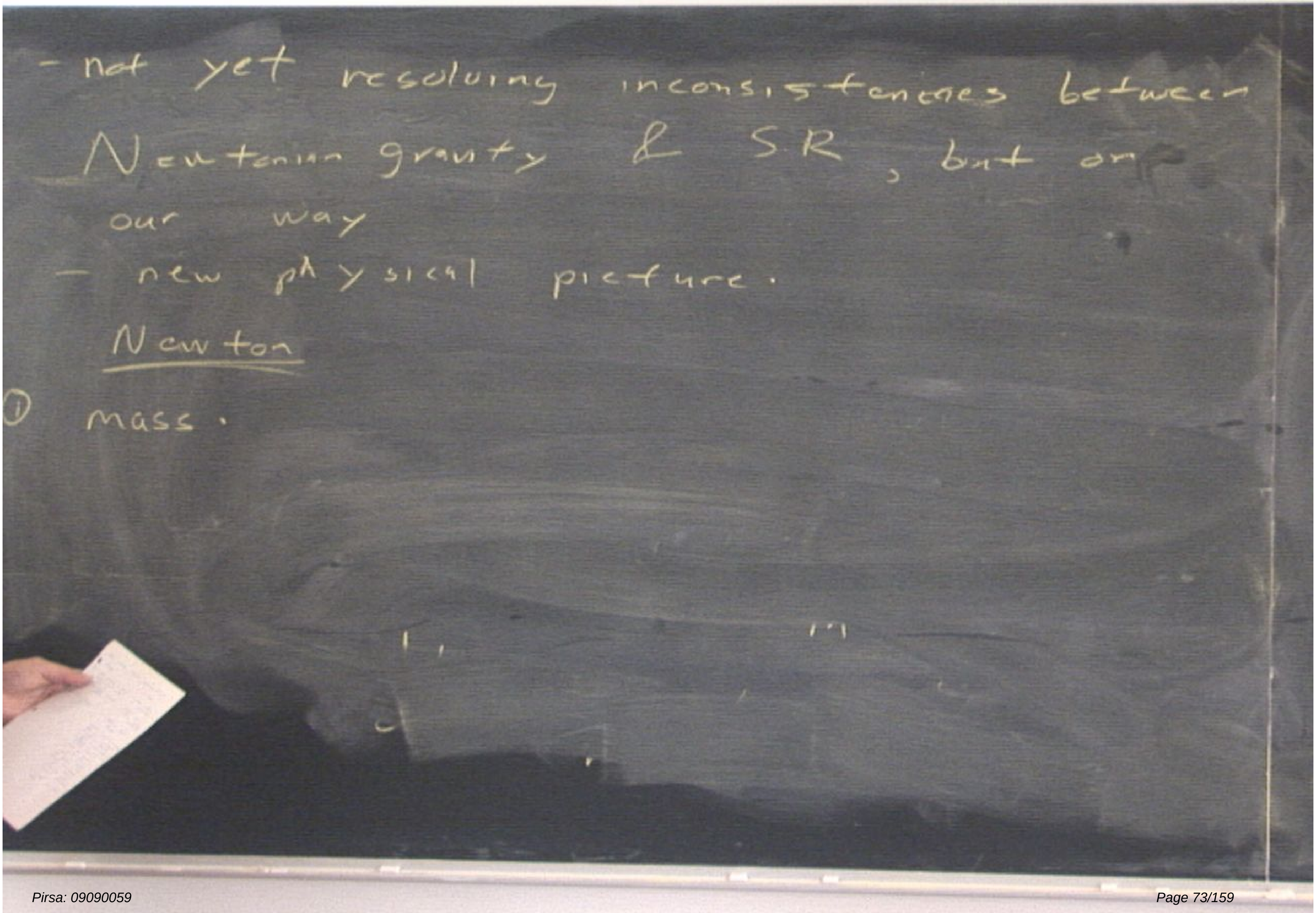
→ our metric and the free particle action yield the physics of Newtonian gravity

$$+\,\underline{\underline{O\left(v^2/c,\ \phi/c^2\right)}}$$

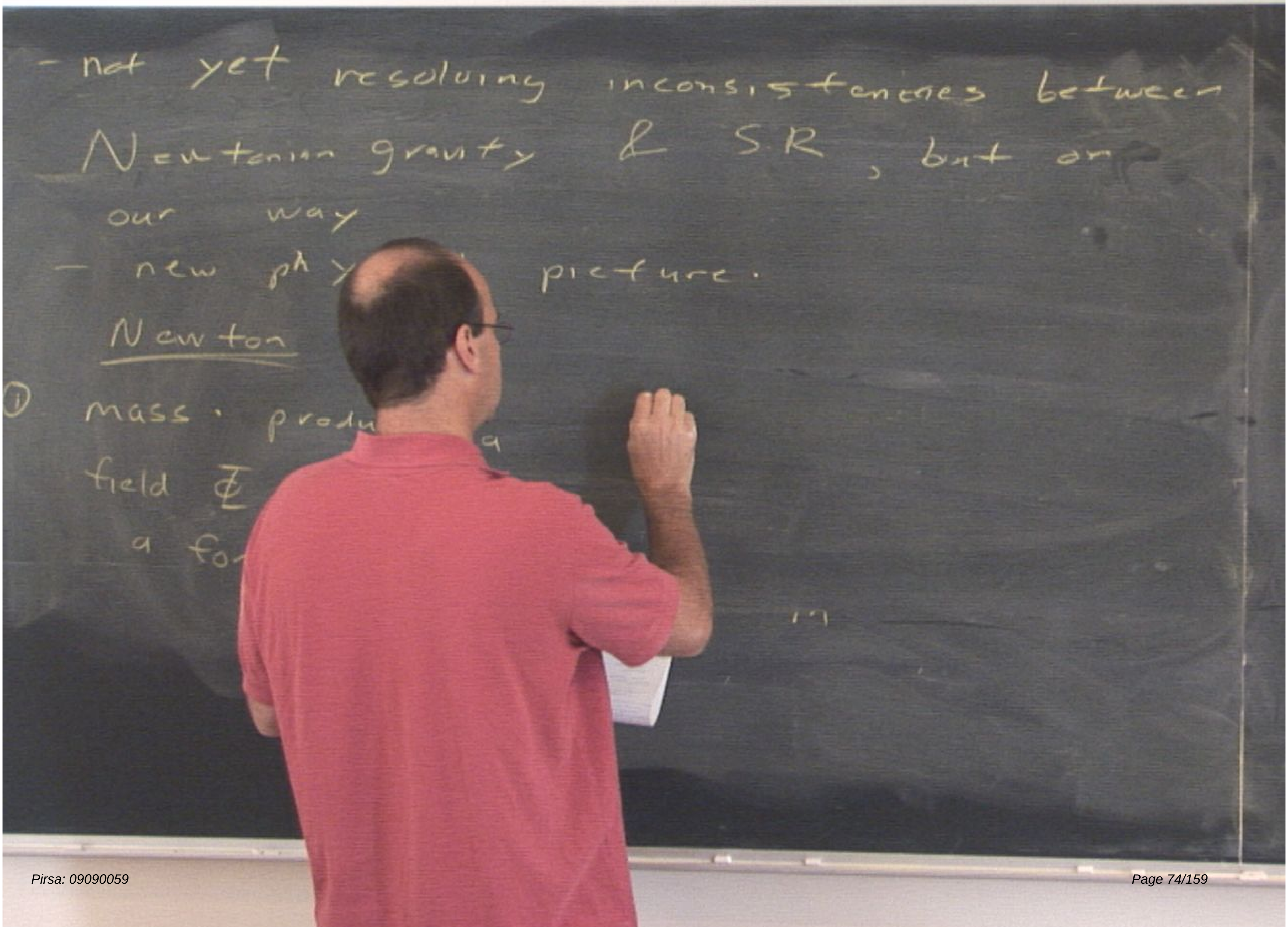
- not yet resolving inconsistencies between Newtonian gravity & SR, but on our way
- new physical picture.



- not yet resolving inconsistencies between Newtonian gravity & S.R, but on our way
- new physical picture.

Newton

① mass.



- not yet resolving inconsistencies between Newtonian gravity & S.R, but on our way
- new ph y picture.

Newton

① mass. produ a

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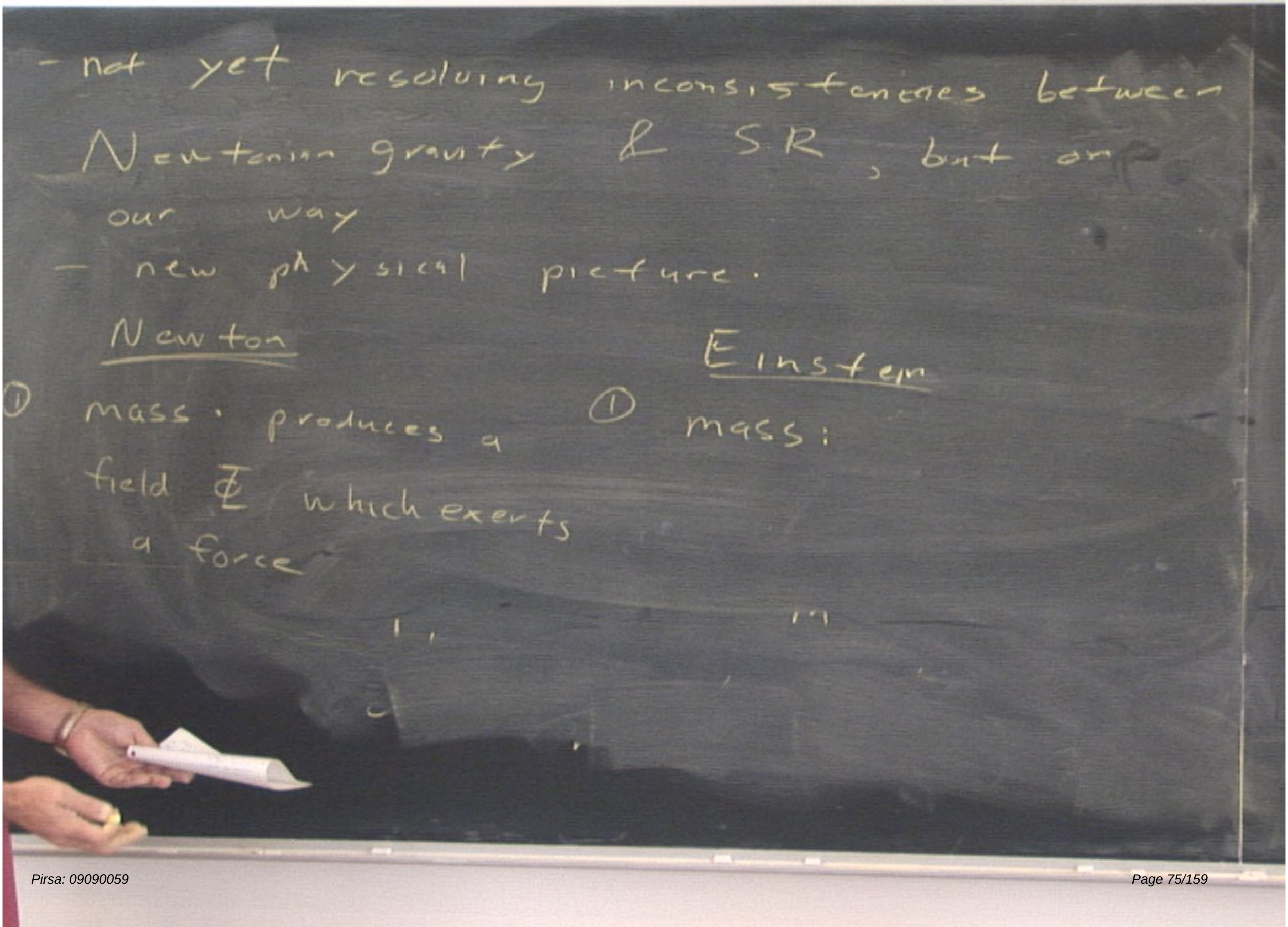


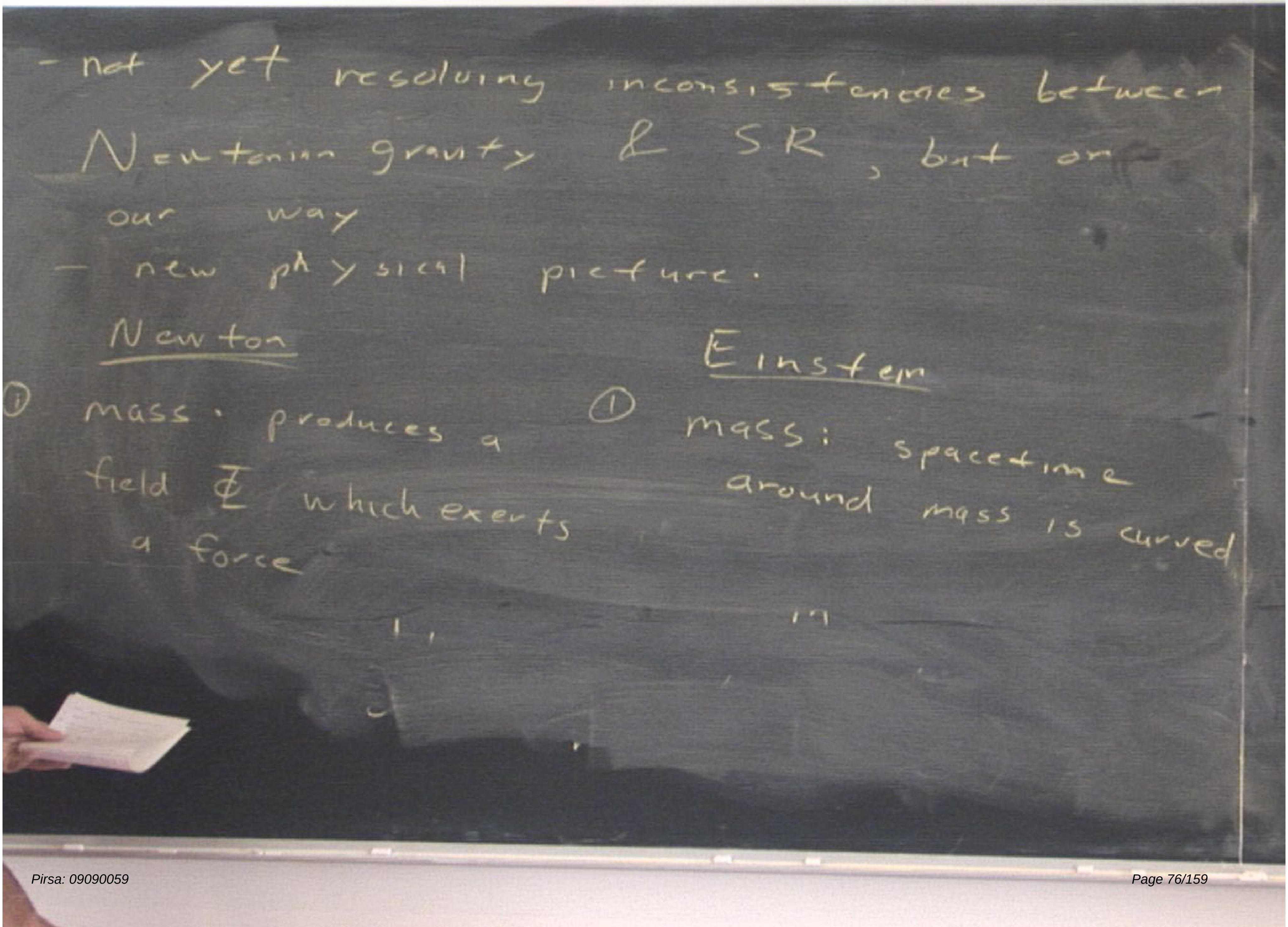
- not yet resolving inconsistencies between Newtonian gravity & S.R, but on our way
- new physical picture.

Newton

① mass: produces a field Φ which exerts a force

Einstein

① mass:



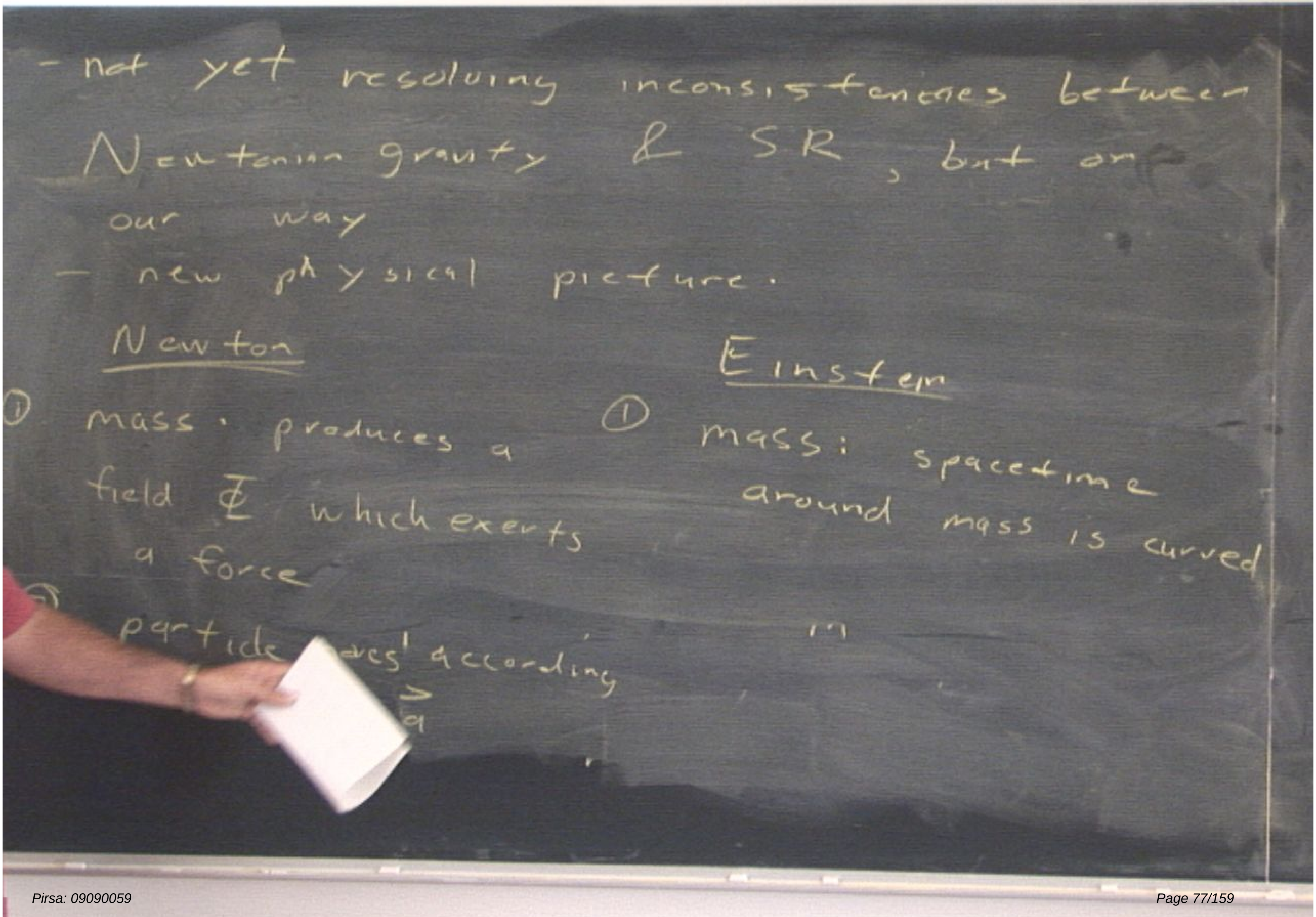
- not yet resolving inconsistencies between Newtonian gravity & S.R, but on our way
- new physical picture.

Newton

① mass: produces a field Φ which exerts a force

Einstein

① mass: spacetime around mass is curved



- not yet resolving inconsistencies between Newtonian gravity & SR, but on our way
- new physical picture.

Newton

① mass: produces a field Φ which exerts a force

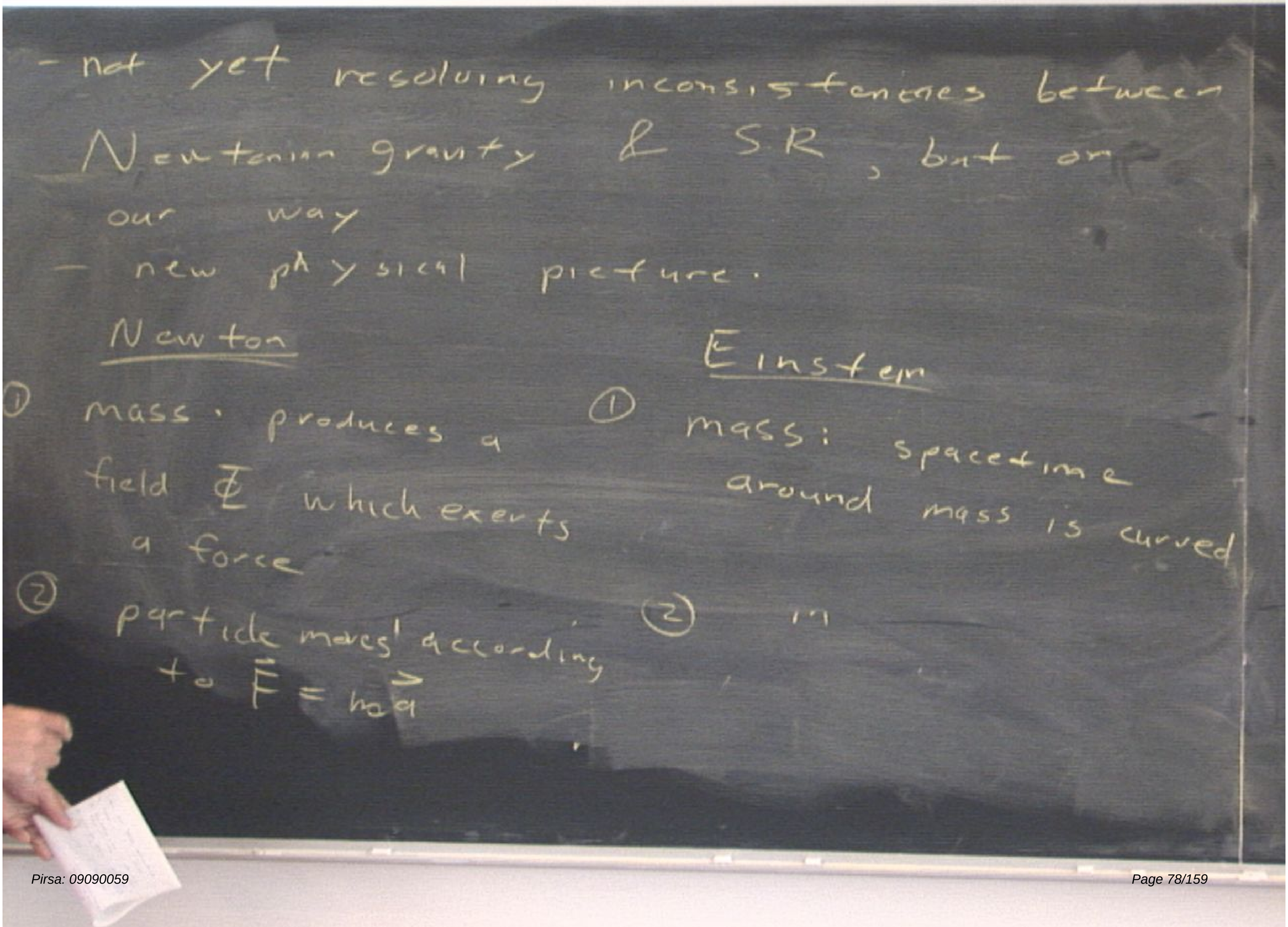
② particle [illegible]ves' according

$\vec{a}$

Einstein

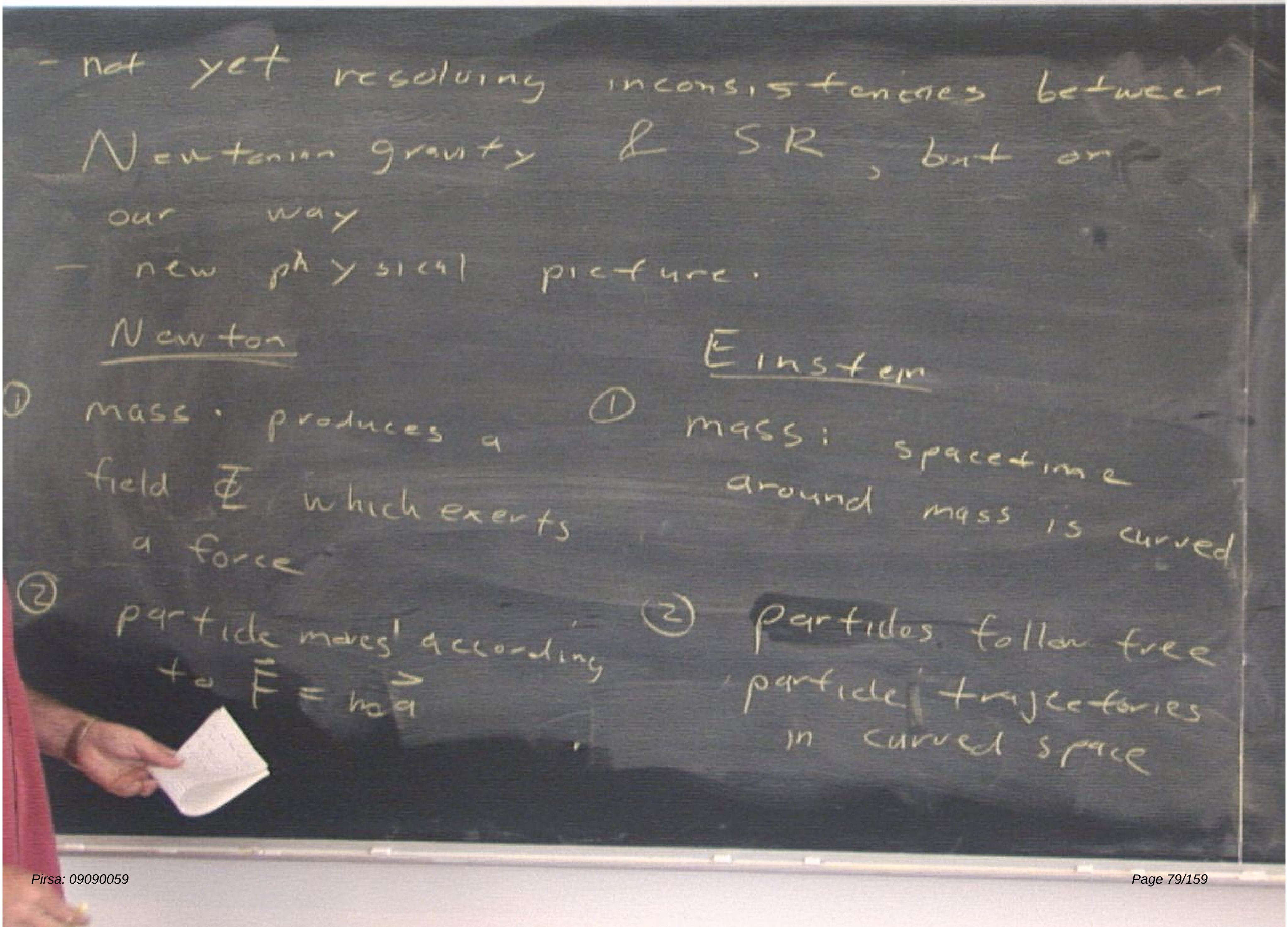
① mass: spacetime around mass is curved

m



- not yet resolving inconsistencies between Newtonian gravity & SR, but on our way
- new physical picture.

Newton	Einstein
① mass: produces a field Φ which exerts a force	① mass: spacetime around mass is curved
② particle moves according to $\vec{F} = m\vec{a}$	② in



- not yet resolving inconsistencies between Newtonian gravity & SR, but on our way
- new physical picture.

Newton

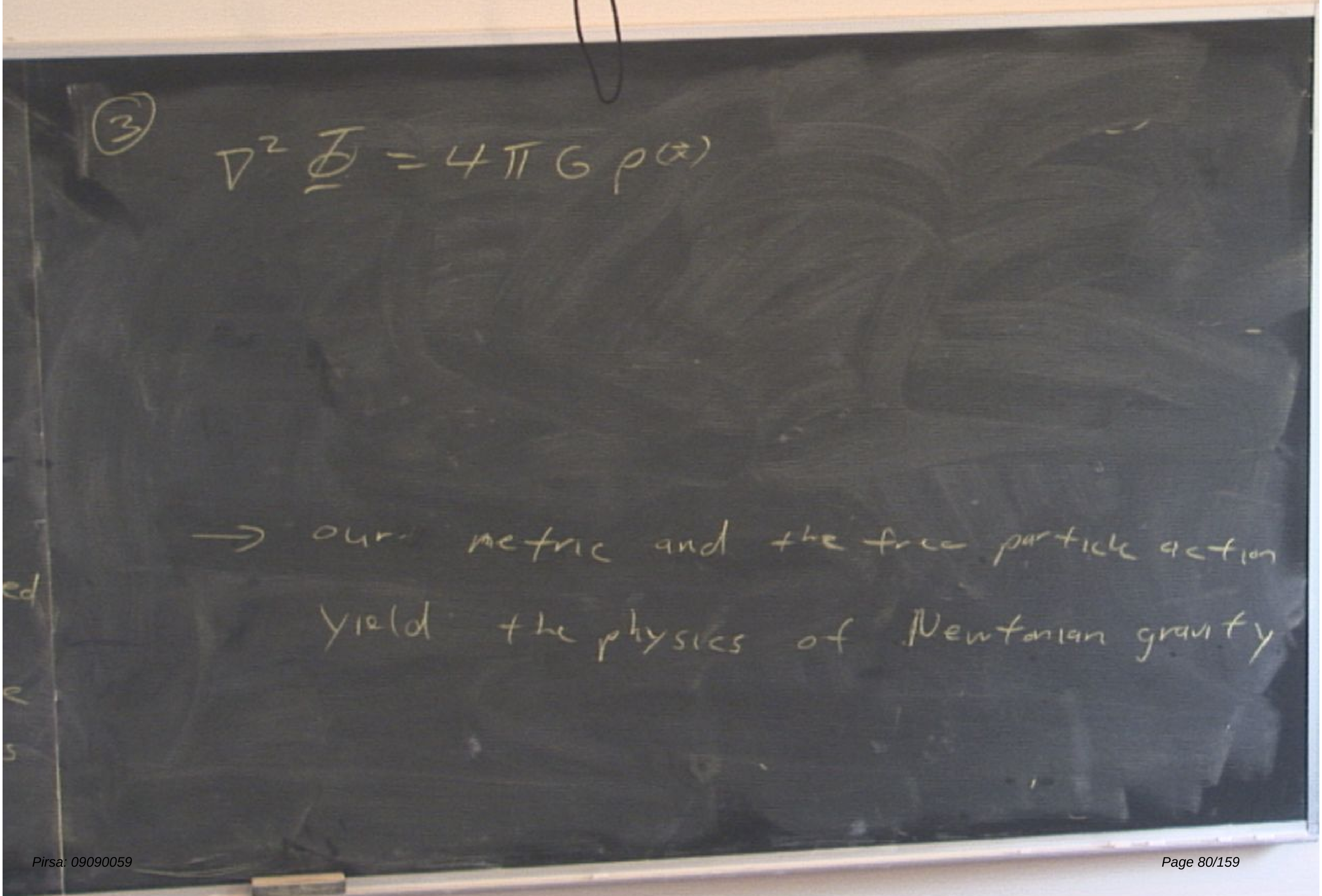
① mass: produces a field Φ which exerts a force

② particle moves according to $\vec{F} = m\vec{a}$

Einstein

① mass: spacetime around mass is curved

② particles follow free particle trajectories in curved space



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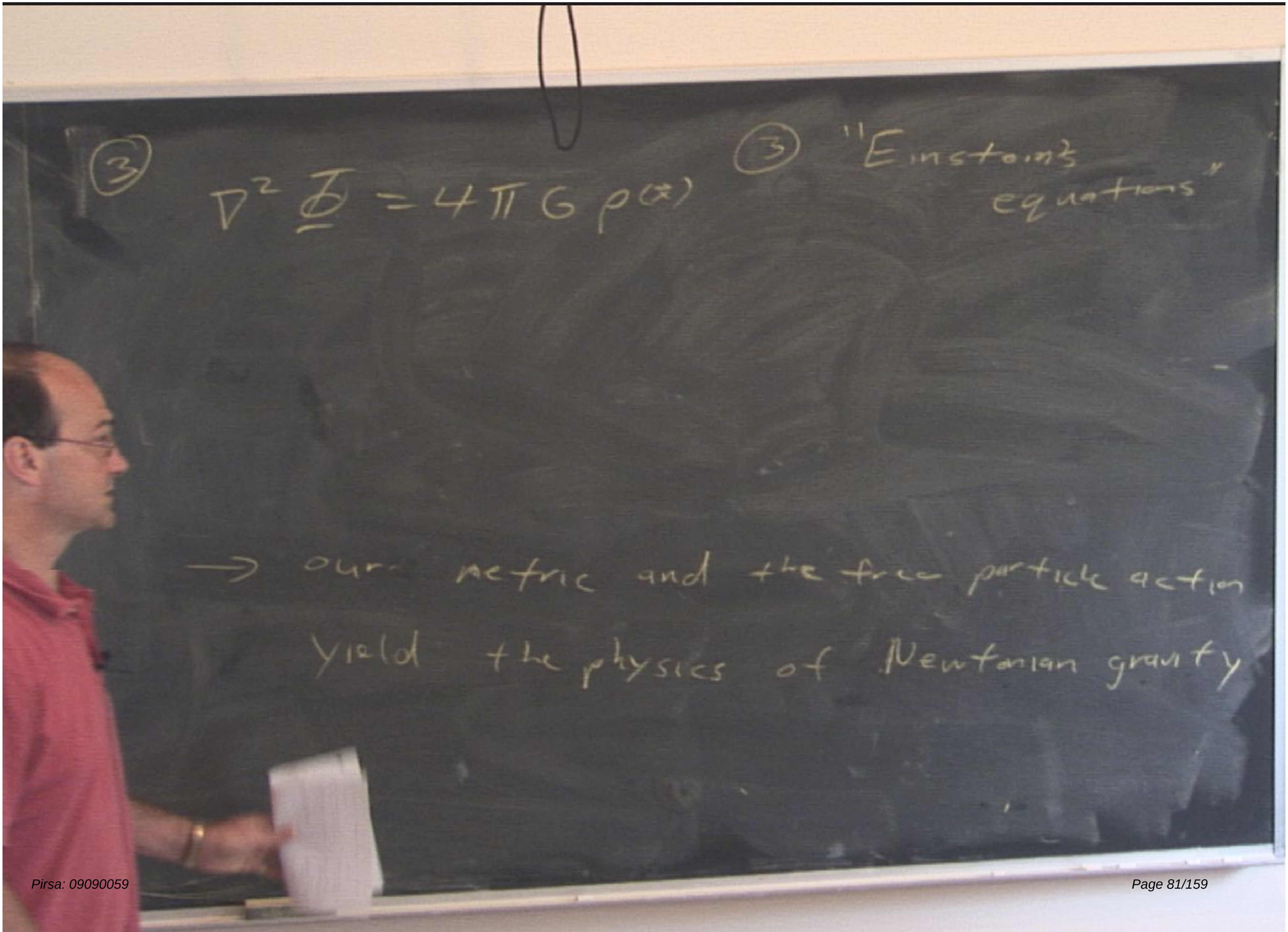
③ $\nabla^2 \underline{\Phi} = 4\pi G \rho(\vec{x})$

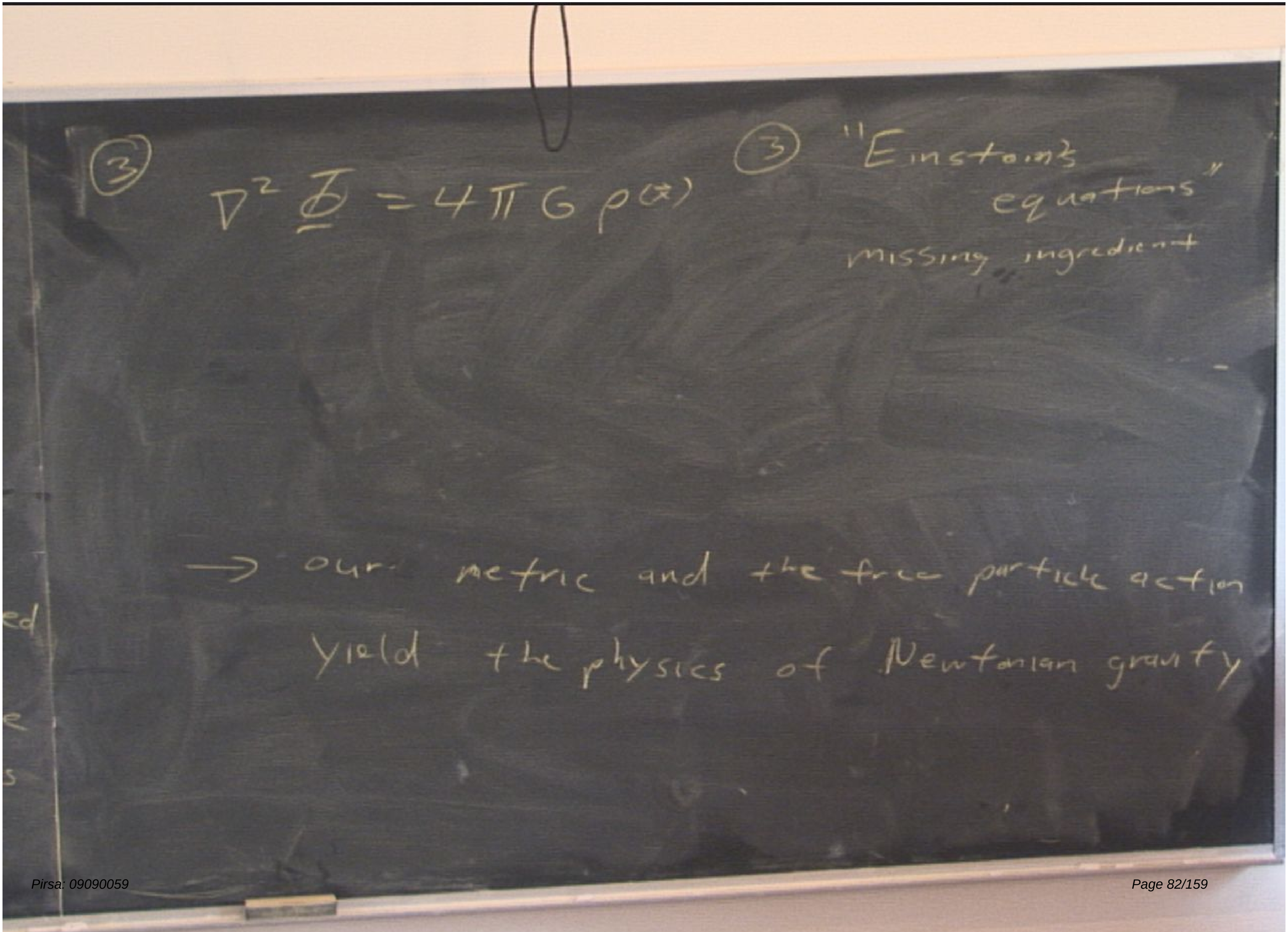
→ our metric and the free particle action yield the physics of Newtonian gravity



③ $\nabla^2 \underline{\Phi} = 4\pi G \rho(\vec{x})$

③ "Einstein's equations"

→ our metric and the free particle action yield the physics of Newtonian gravity



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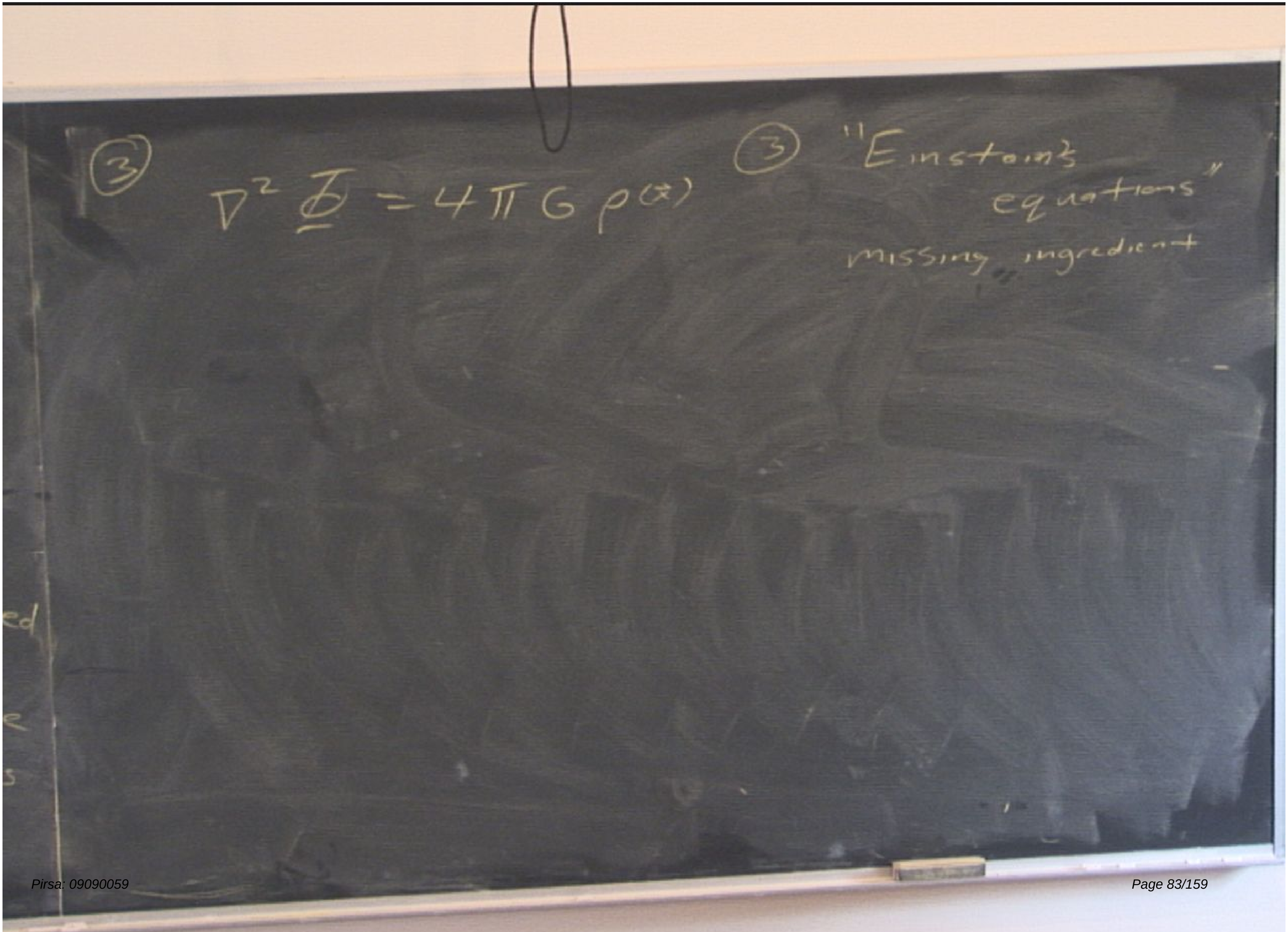
③ "Einstein's equations"

missing ingredient

→ our metric and the free particle action yield the physics of Newtonian gravity



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③ "Einstein's equations"

missing ingredient

ed

e

s

③

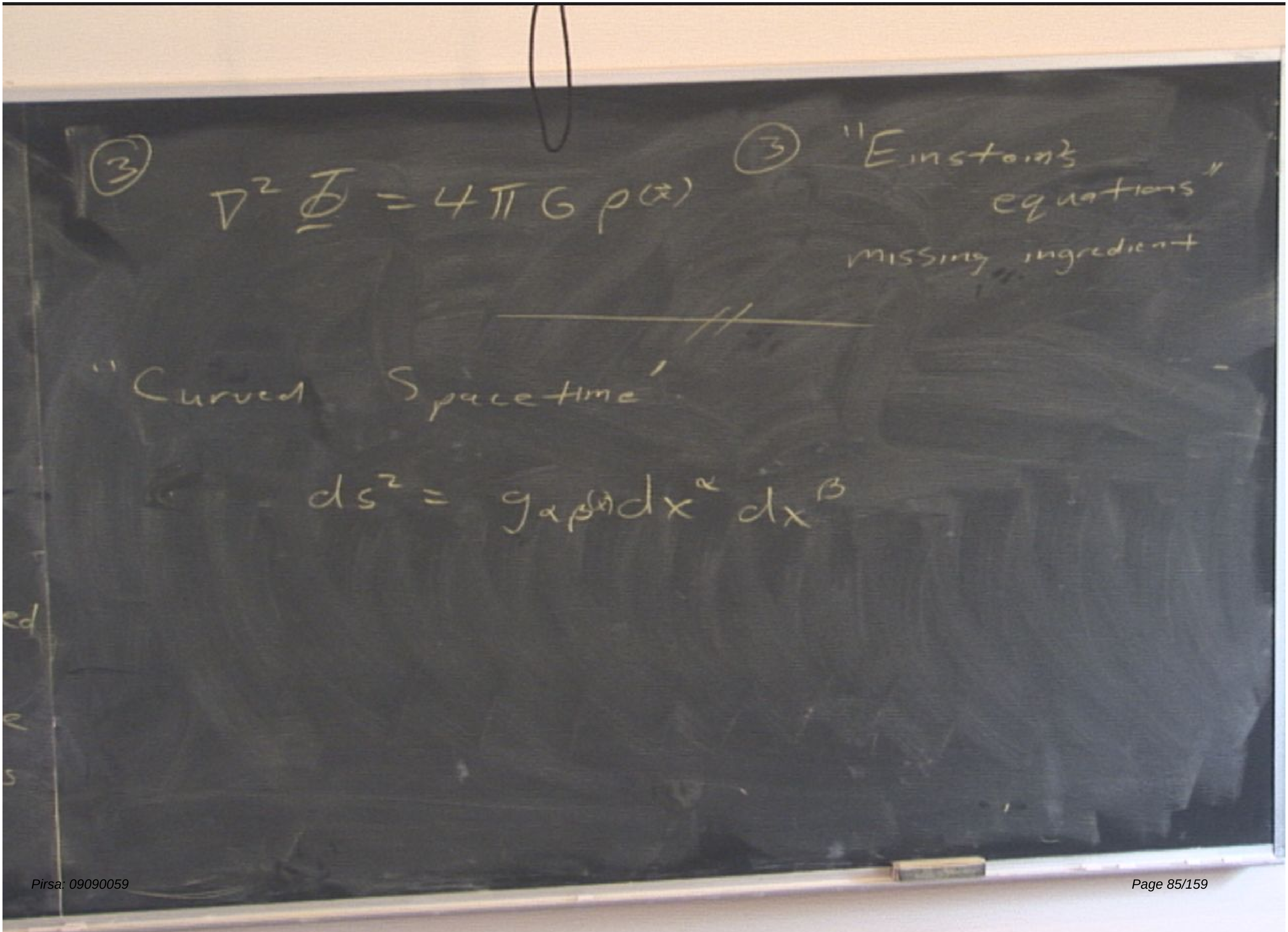
$$\nabla^2 \underline{\Phi} = 4\pi G \rho(\vec{x})$$

③ "Einstein's equations"

missing ingredient

"Curved Spacetime"

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$



③ $\nabla^2 \underline{\Phi} = 4\pi G \rho(\vec{x})$

③ "Einstein's equations"

missing ingredient

"Curved Spacetime".

$$ds^2 = g_{\alpha\beta}(x)\, dx^\alpha\, dx^\beta$$

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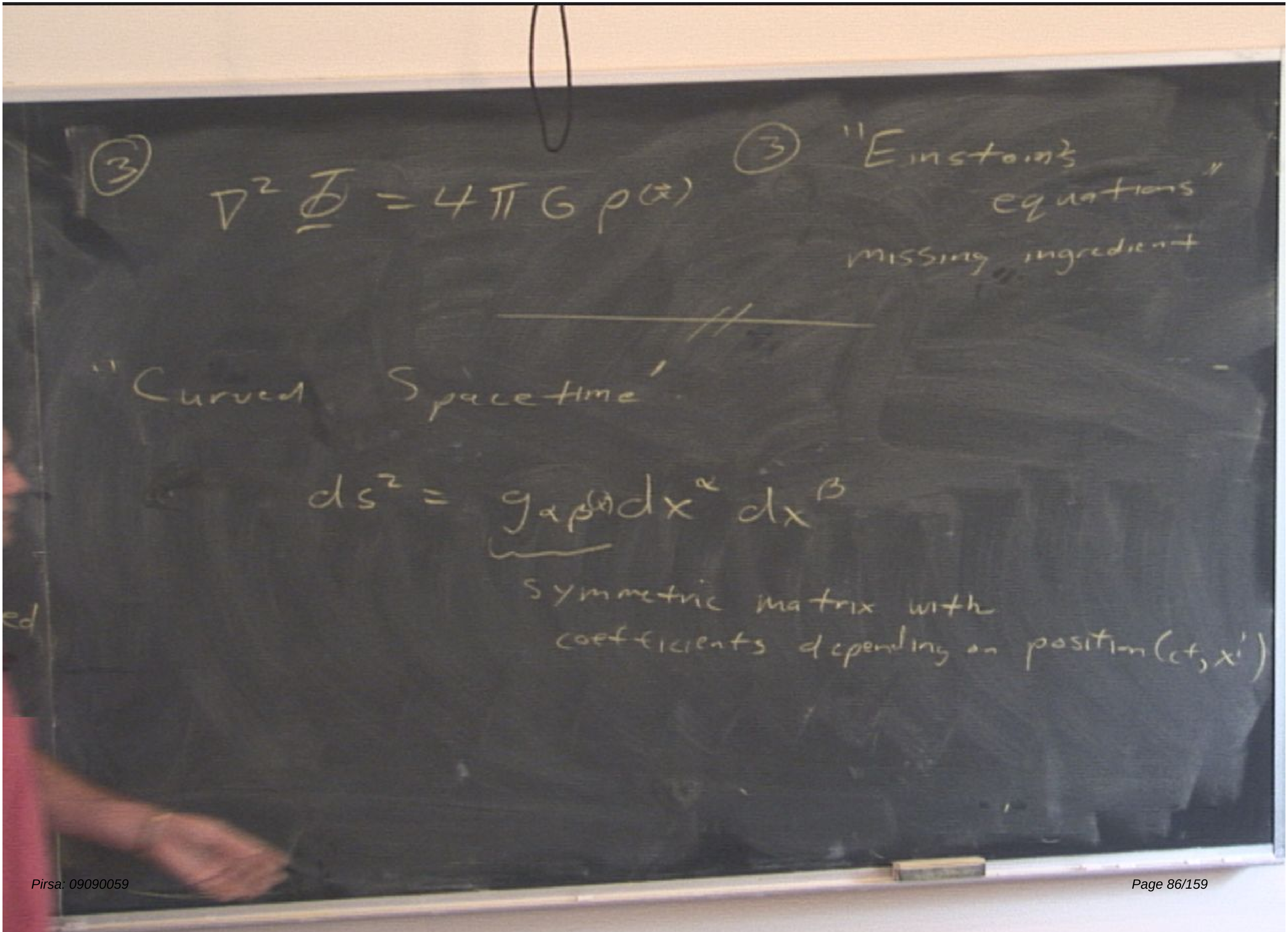


③ $\nabla^2 \underline{\Phi} = 4\pi G \rho(\vec{x})$

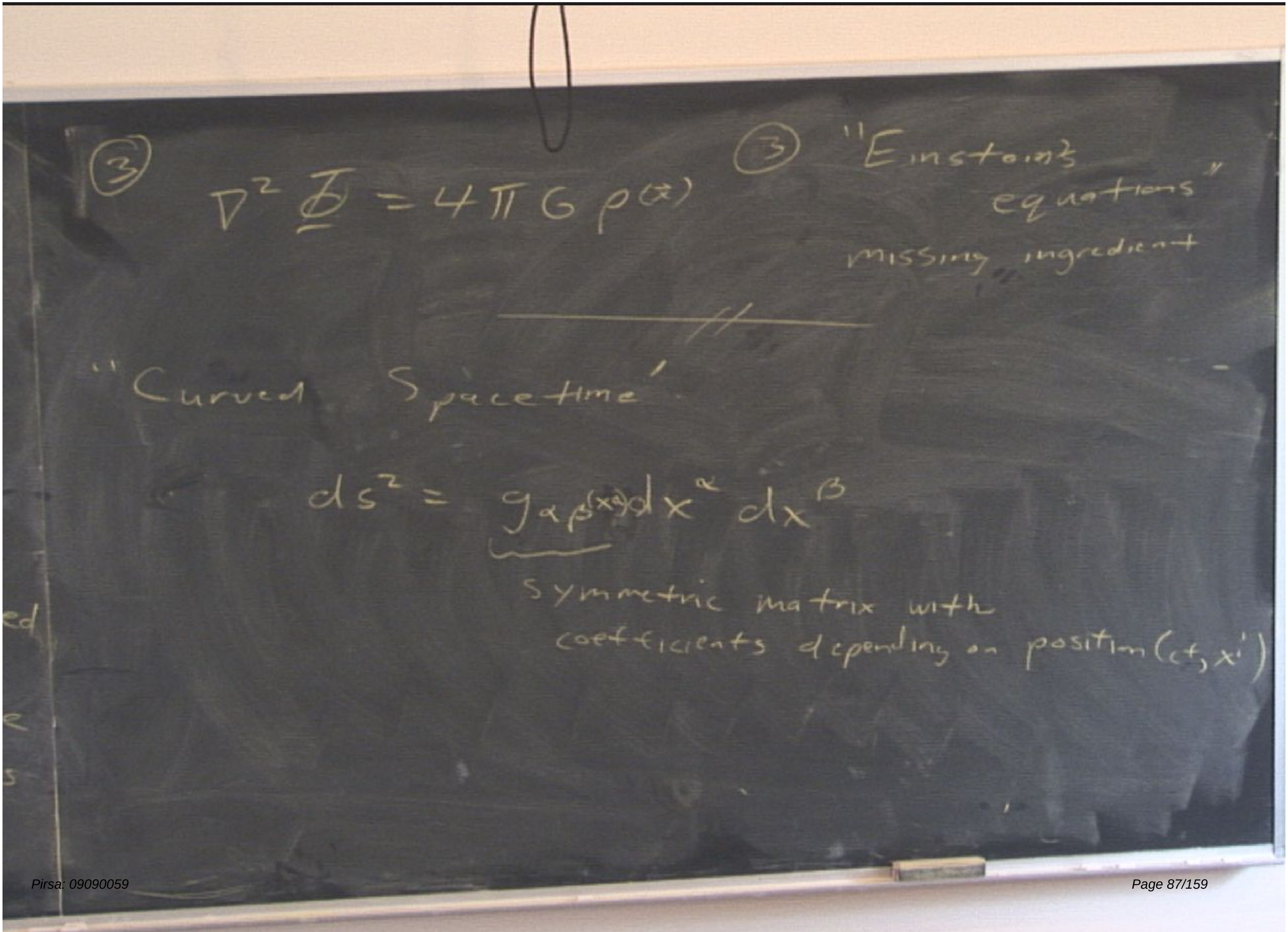
③ "Einstein's equations"

missing ingredient

"Curved Spacetime"

$$ds^2 = \underbrace{g_{\alpha\beta}(x)} dx^\alpha dx^\beta$$

symmetric matrix with coefficients depending on position (ct, x^i)



③ $$\nabla^2 \underline{\Phi} = 4\pi G \rho(\vec{x})$$

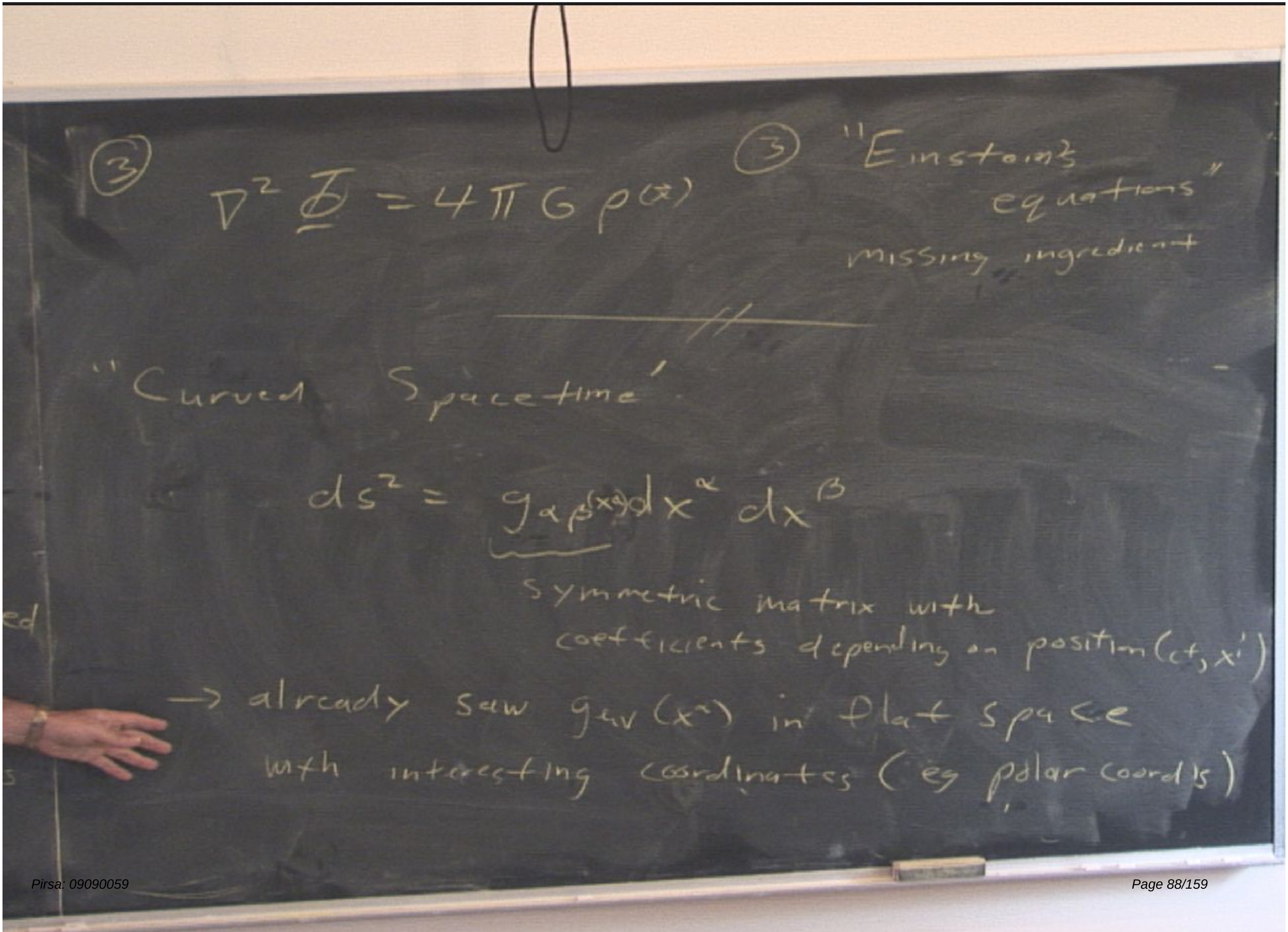
③ "Einstein's equations"

missing ingredient

"Curved Spacetime"

$$ds^2 = \underbrace{g_{\alpha\beta}(x^\sigma)}\, dx^\alpha dx^\beta$$

symmetric matrix with coefficients depending on position (ct, x^i)



③ $\nabla^2 \underline{\Phi} = 4\pi G \rho(\vec{x})$

③ "Einstein's equations"

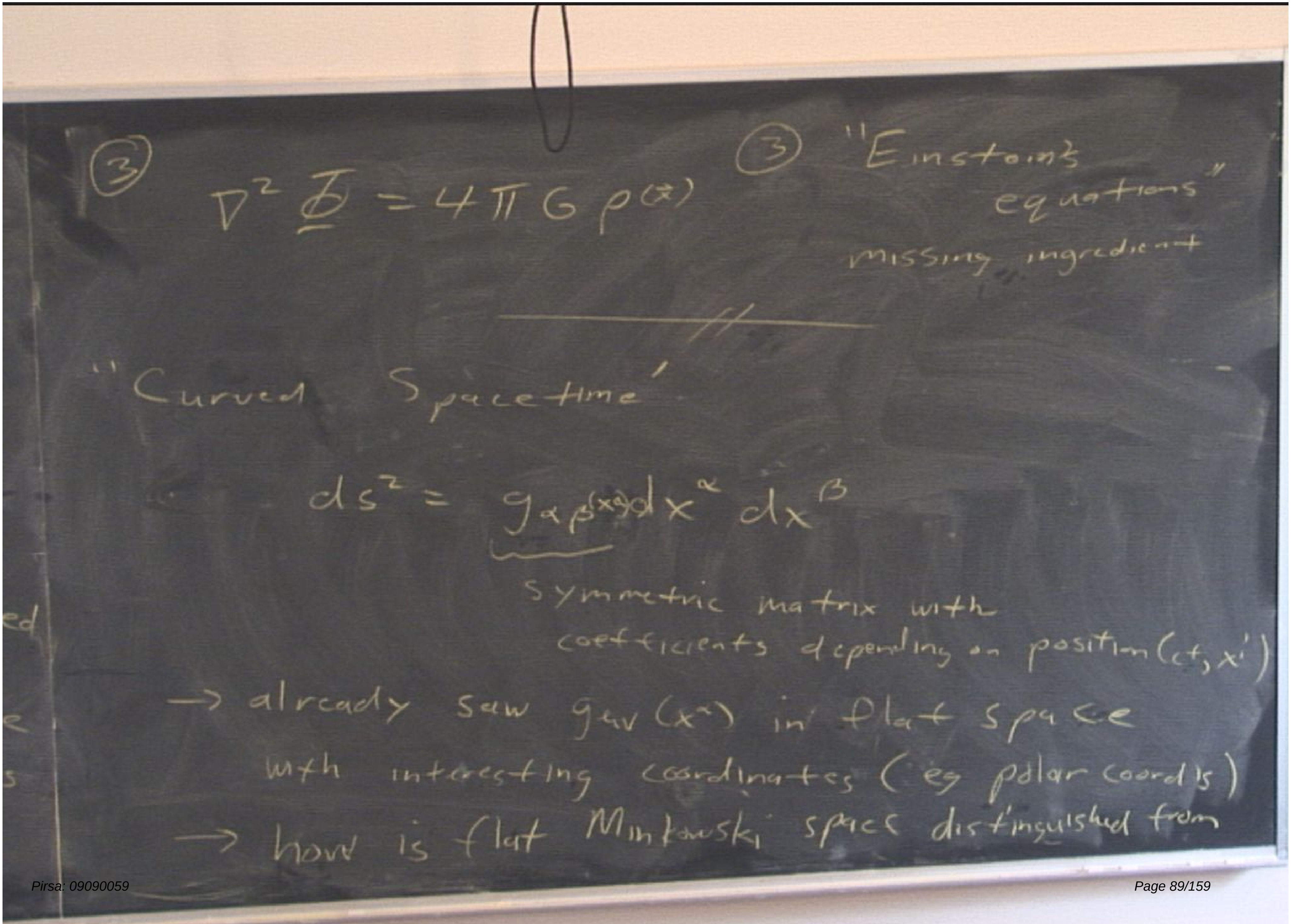
missing ingredient

"Curved Spacetime"

$$ds^2 = \underline{g_{\alpha\beta}(x^\gamma)}\, dx^\alpha\, dx^\beta$$

symmetric matrix with coefficients depending on position (ct, x^i)

→ already saw $g_{\mu\nu}(x^\alpha)$ in flat space with interesting coordinates (eg polar coord's)



③ $\nabla^2 \underline{\Phi} = 4\pi G \rho(\vec{x})$

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missing ingredient

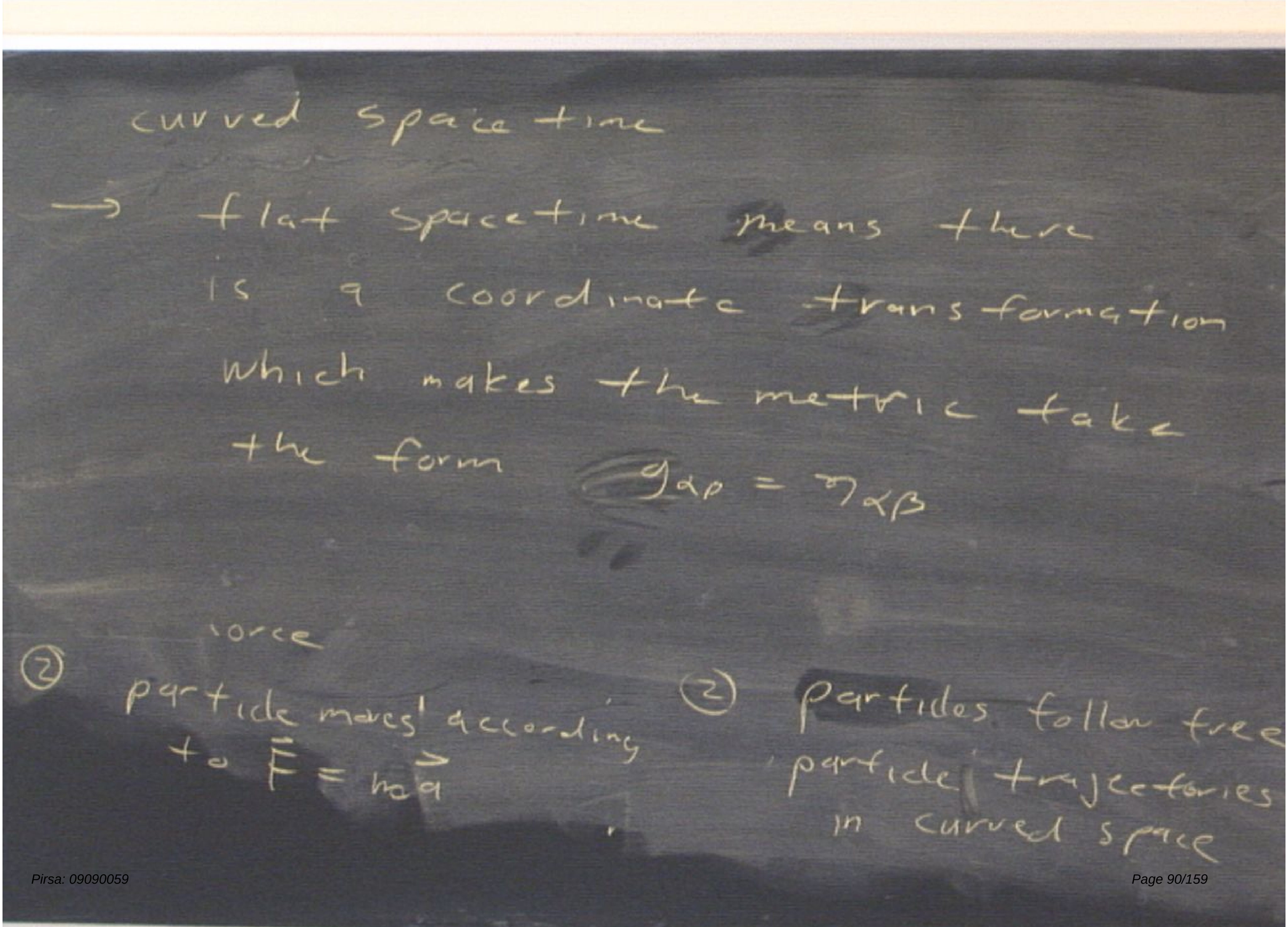
"Curved Spacetime"

$$ds^2 = \underbrace{g_{\alpha\beta}(x^\gamma)}\, dx^\alpha\, dx^\beta$$

symmetric matrix with coefficients depending on position (ct, x^i)

→ already saw $g_{\mu\nu}(x^\alpha)$ in flat space with interesting coordinates (eg polar coord's)

→ how is flat Minkowski space distinguished from



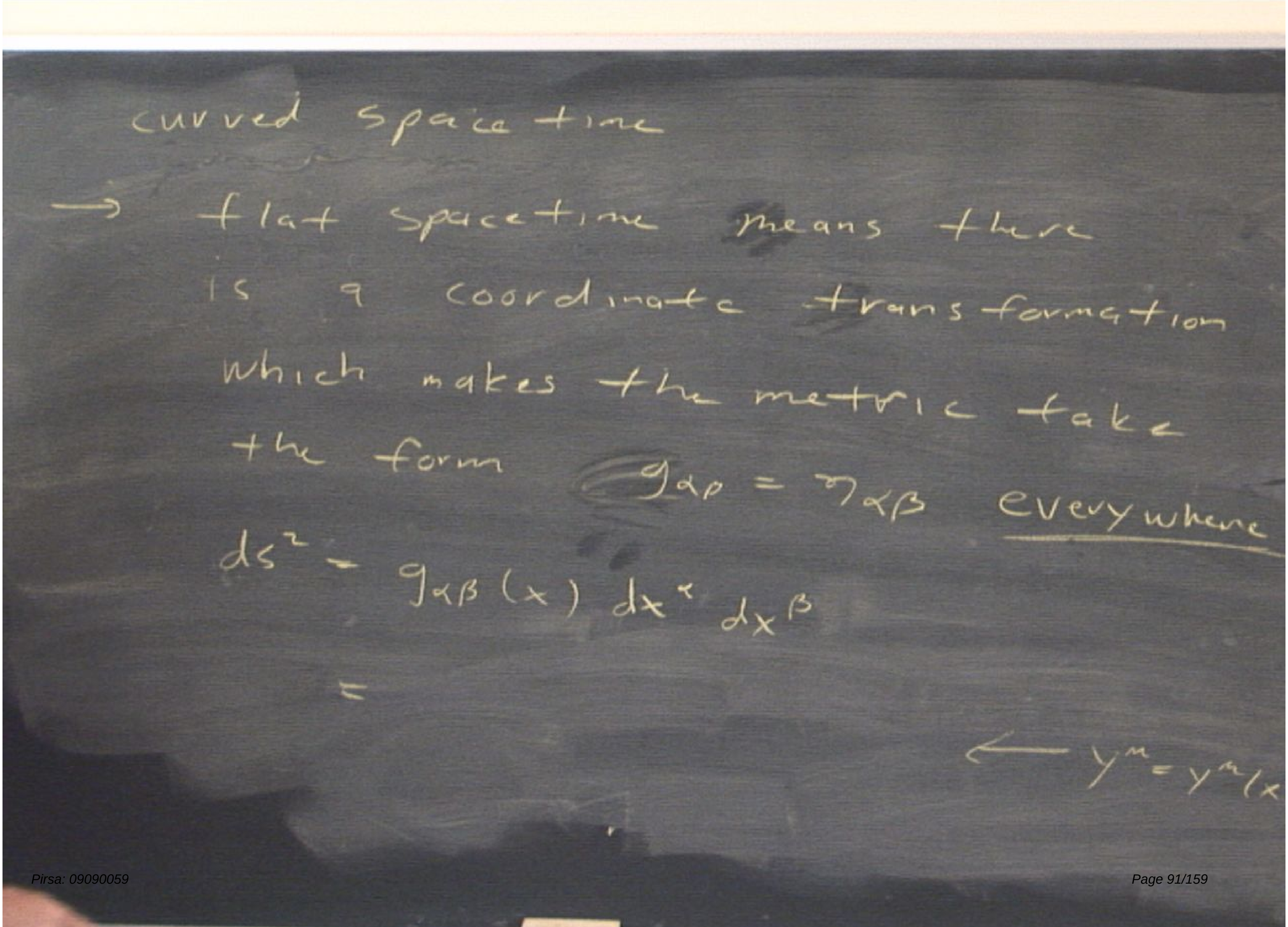
curved spacetime

→ flat spacetime means there is a coordinate transformation which makes the metric take the form $g_{\alpha\beta} = \eta_{\alpha\beta}$

orce

② particle moves according to $\vec{F} = m\vec{a}$

② particles follow free particle trajectories in curved space



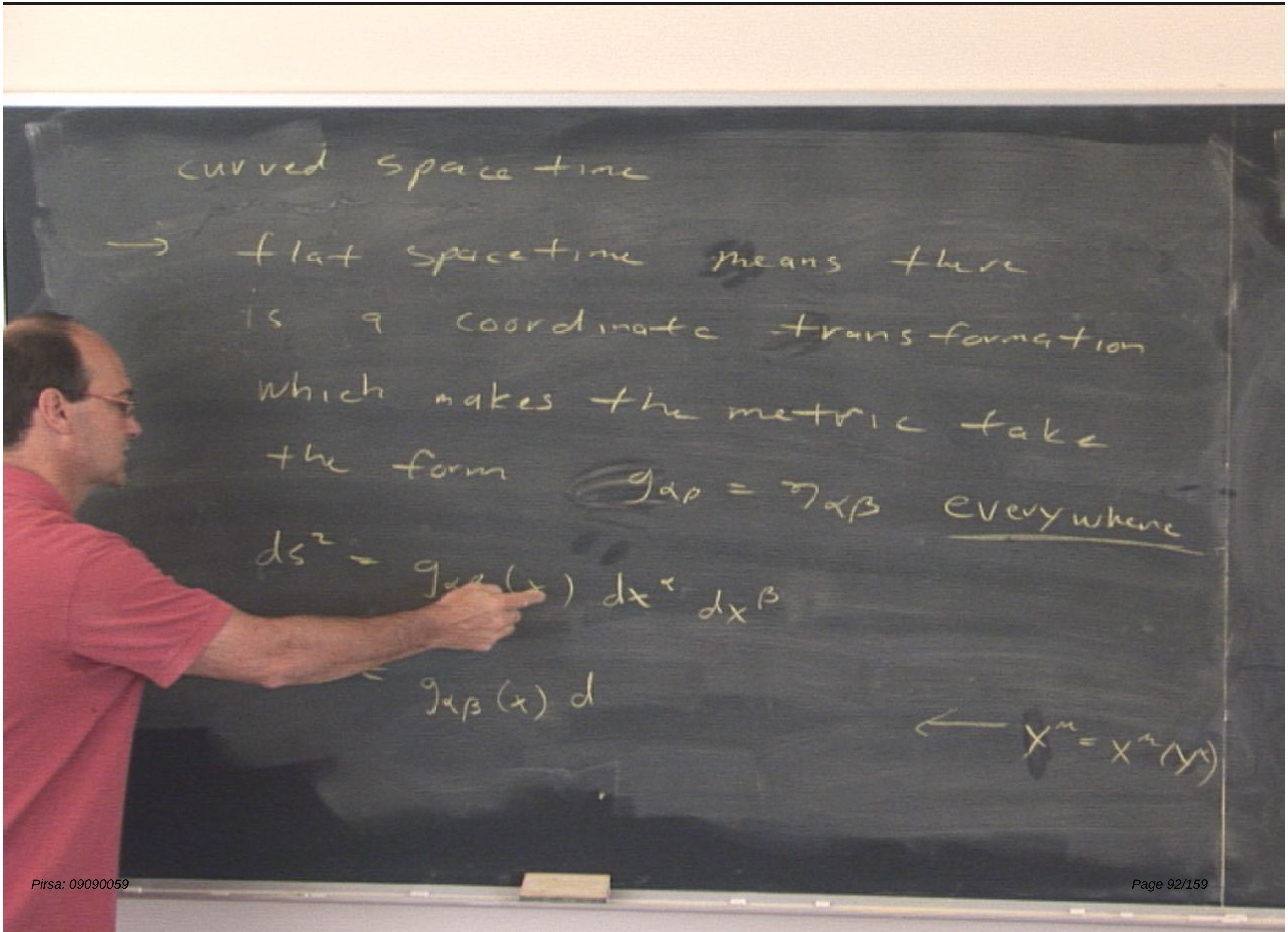
curved spacetime

→ flat spacetime means there is a coordinate transformation which makes the metric take the form $g_{\alpha\beta} = \eta_{\alpha\beta}$ $\underline{\text{everywhere}}$

$$ds^2 = g_{\alpha\beta}(x)\, dx^\alpha\, dx^\beta$$

$$=$$

$\longleftarrow y^\mu = y^\mu(x$



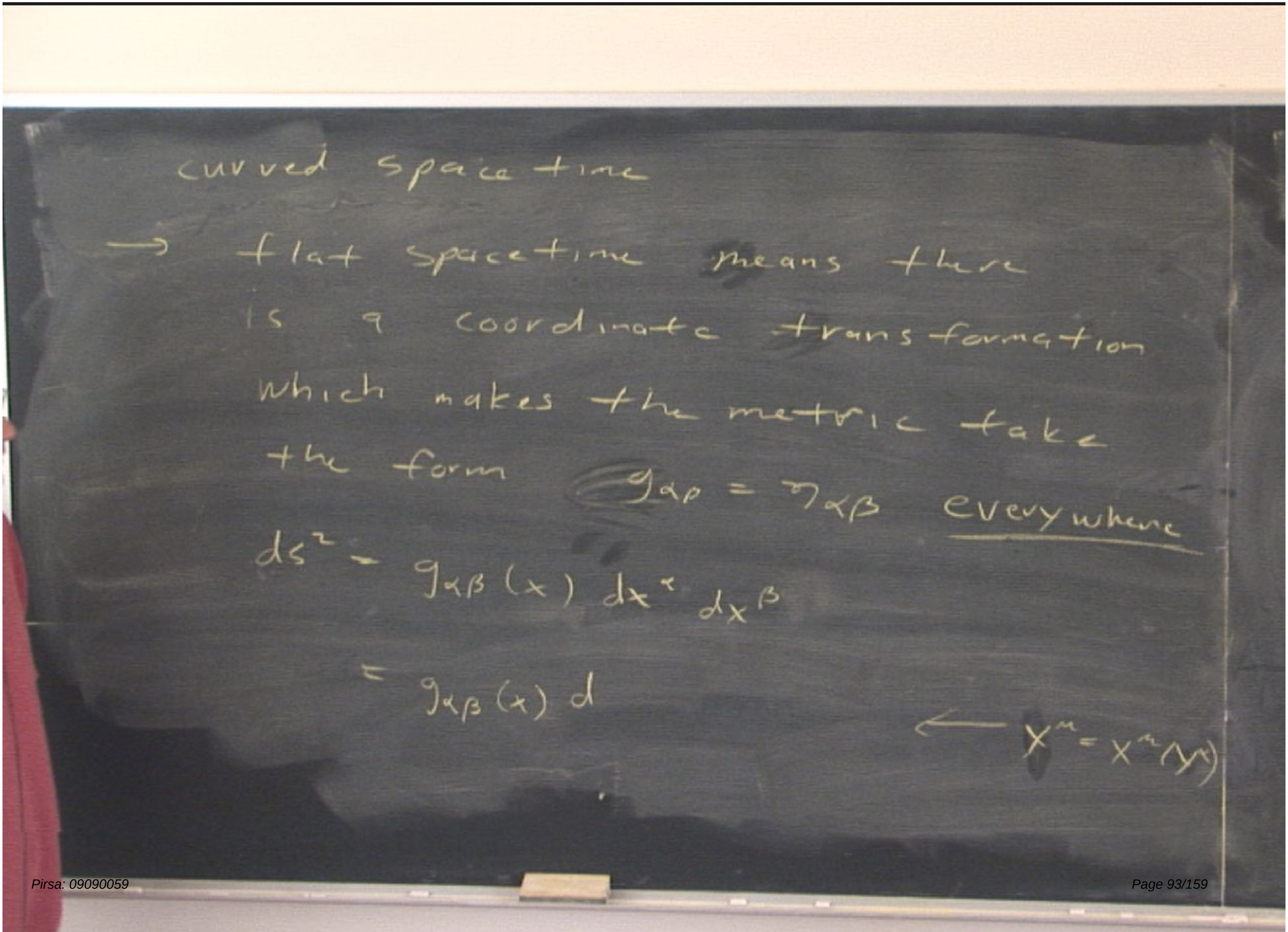
curved space time

→ flat spacetime means there is a coordinate transformation which makes the metric take the form $g_{\alpha\beta} = \eta_{\alpha\beta}$ <u>everywhere</u>

$$ds^2 = g_{\alpha\beta}(x)\, dx^\alpha\, dx^\beta$$

$$g_{\alpha\beta}(x)\, d$$

$\longleftarrow X^\mu = X^\mu(yx)$



curved spacetime

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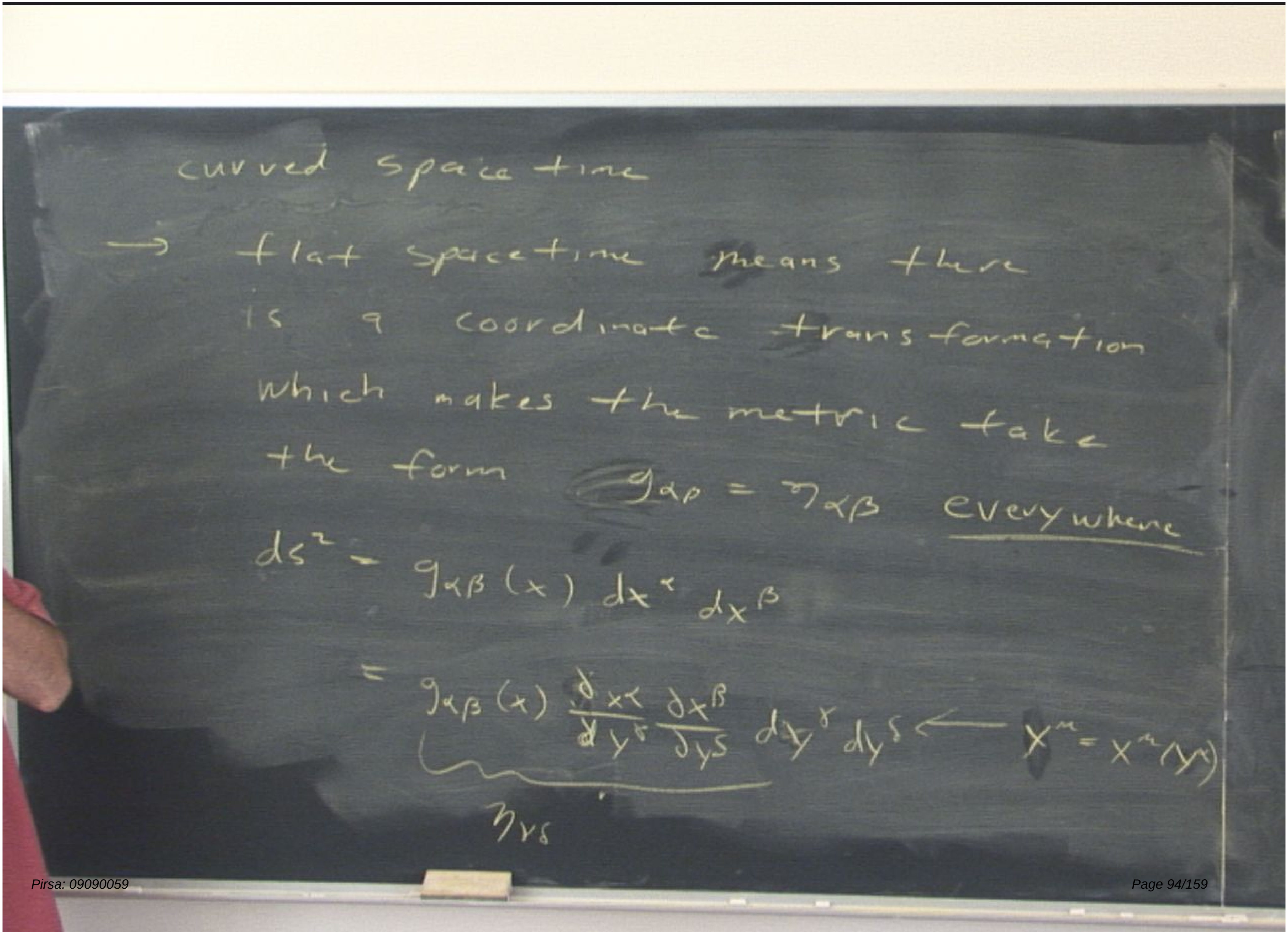
$ds^2 = g_{\alpha\beta}(x)\, dx^\alpha \, dx^\beta$

$= g_{\alpha\beta}(x)\, d$

$\longleftarrow X^\mu = X^\mu(y^\alpha)$



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$$ds^2 = g_{\alpha\beta}(x)\, dx^\alpha\, dx^\beta$$

$$= \underbrace{g_{\alpha\beta}(x) \frac{\partial x^\alpha}{\partial y^\gamma} \frac{\partial x^\beta}{\partial y^\delta}}_{\eta_{\gamma\delta}} dy^\gamma\, dy^\delta \longleftarrow x^\mu = x^\mu(y^\alpha)$$

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(if we can't do such a transformation

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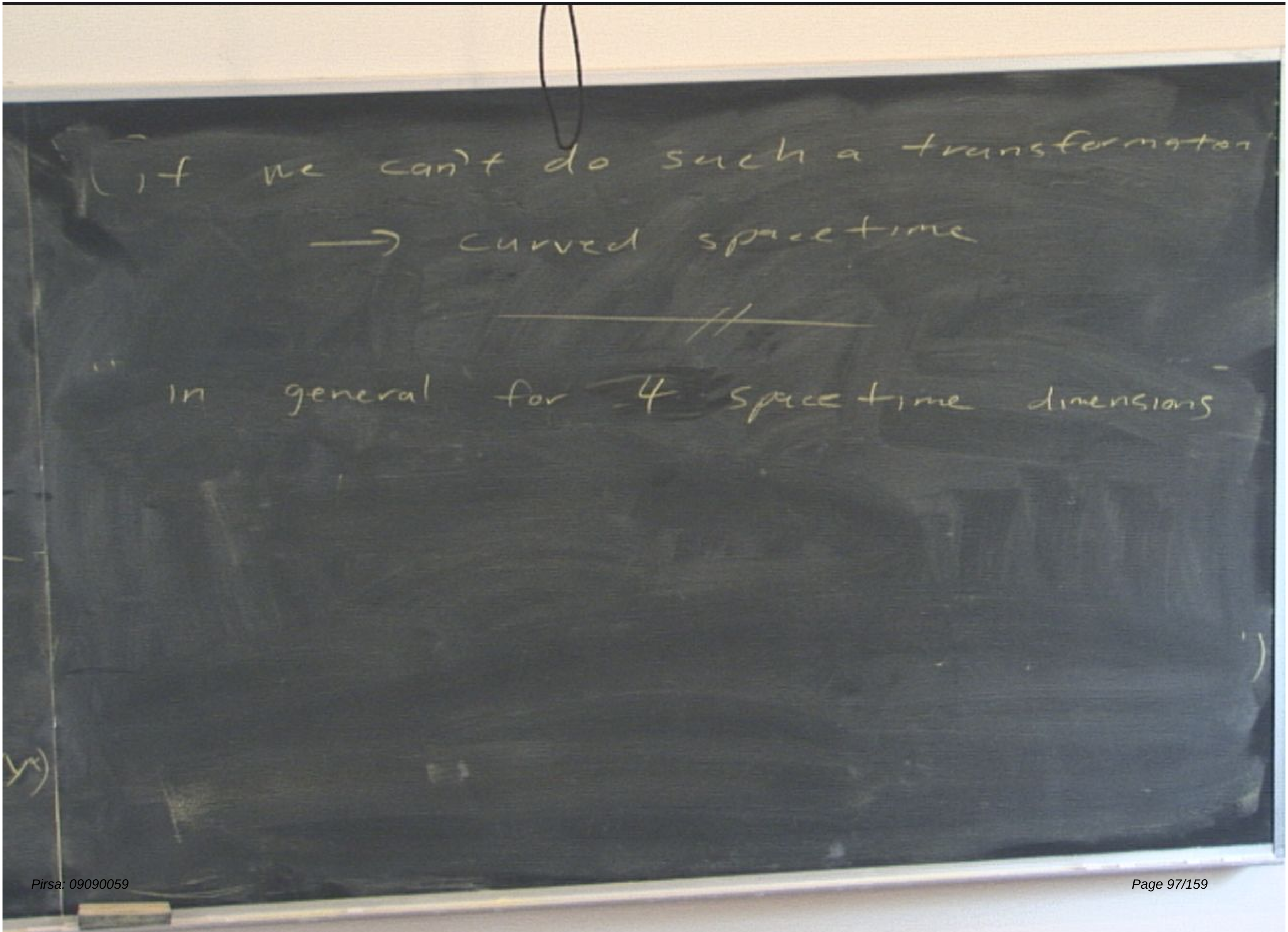

(if we can't do such a transformation

$\longrightarrow$ curved spacetime

" in general for 4 spacetime dimensions "

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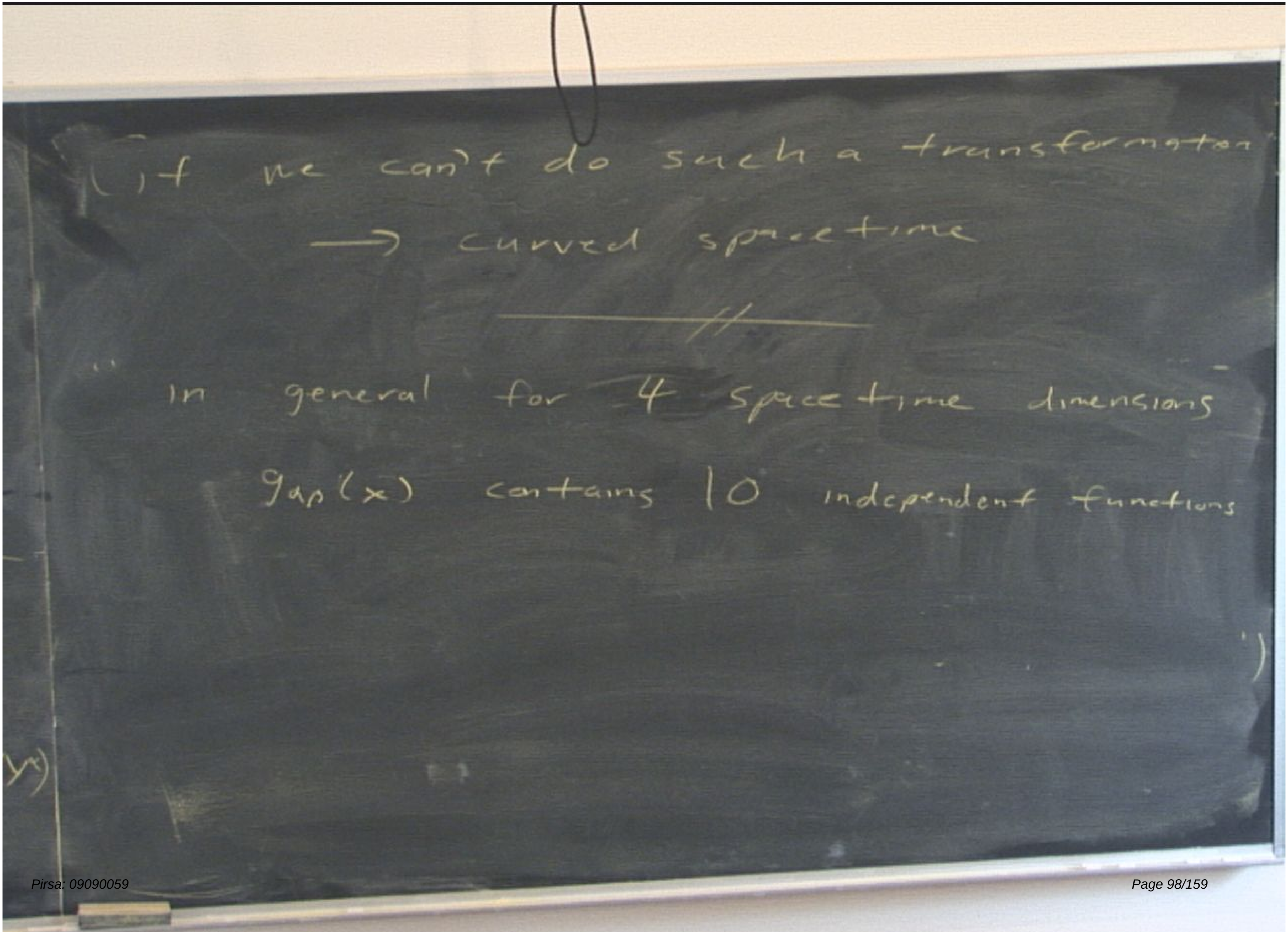
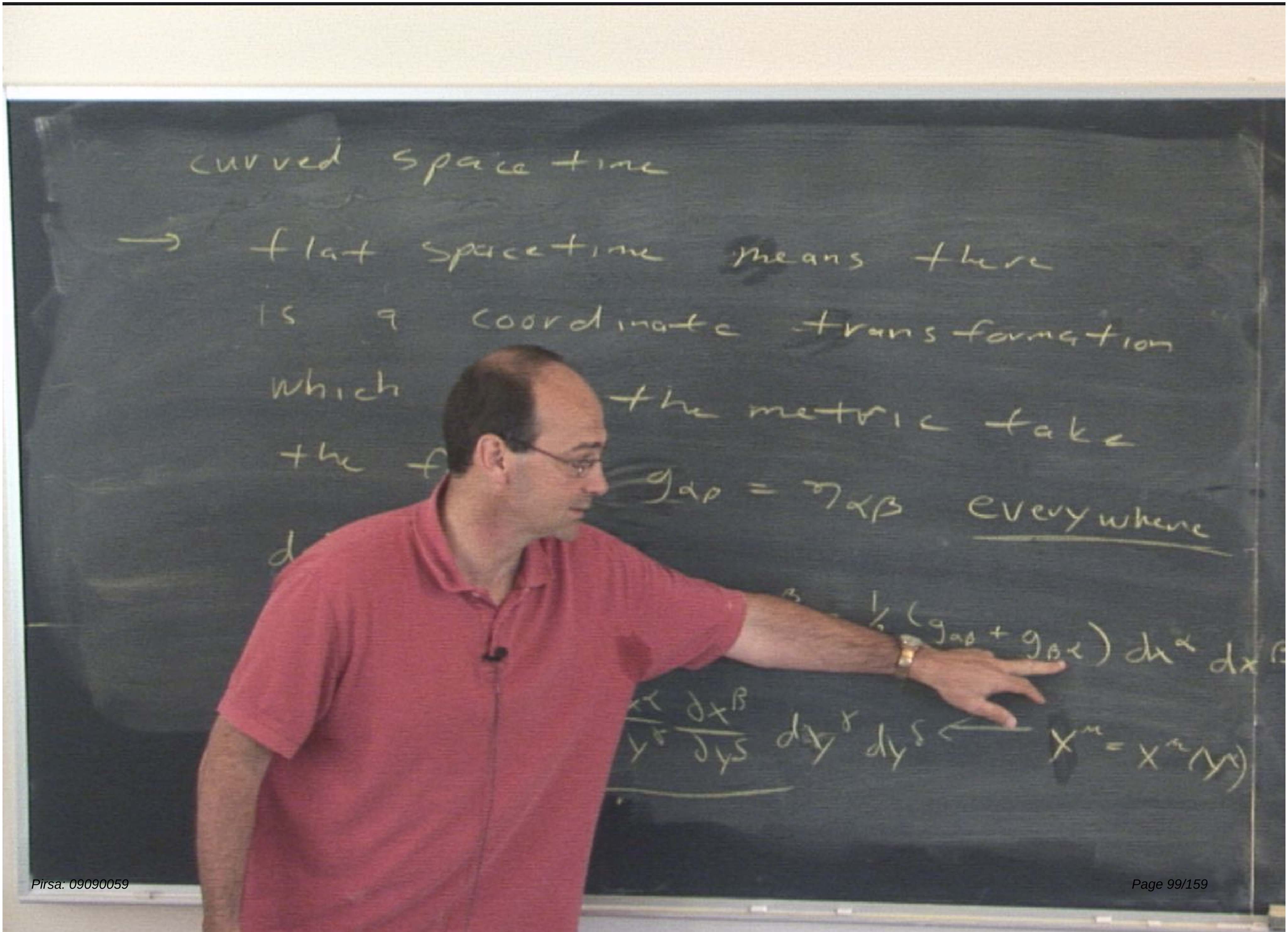


if we can't do such a transformation

→ curved spacetime

in general for 4 spacetime dimensions

$g_{\alpha\beta}(x)$ contains 10 independent functions



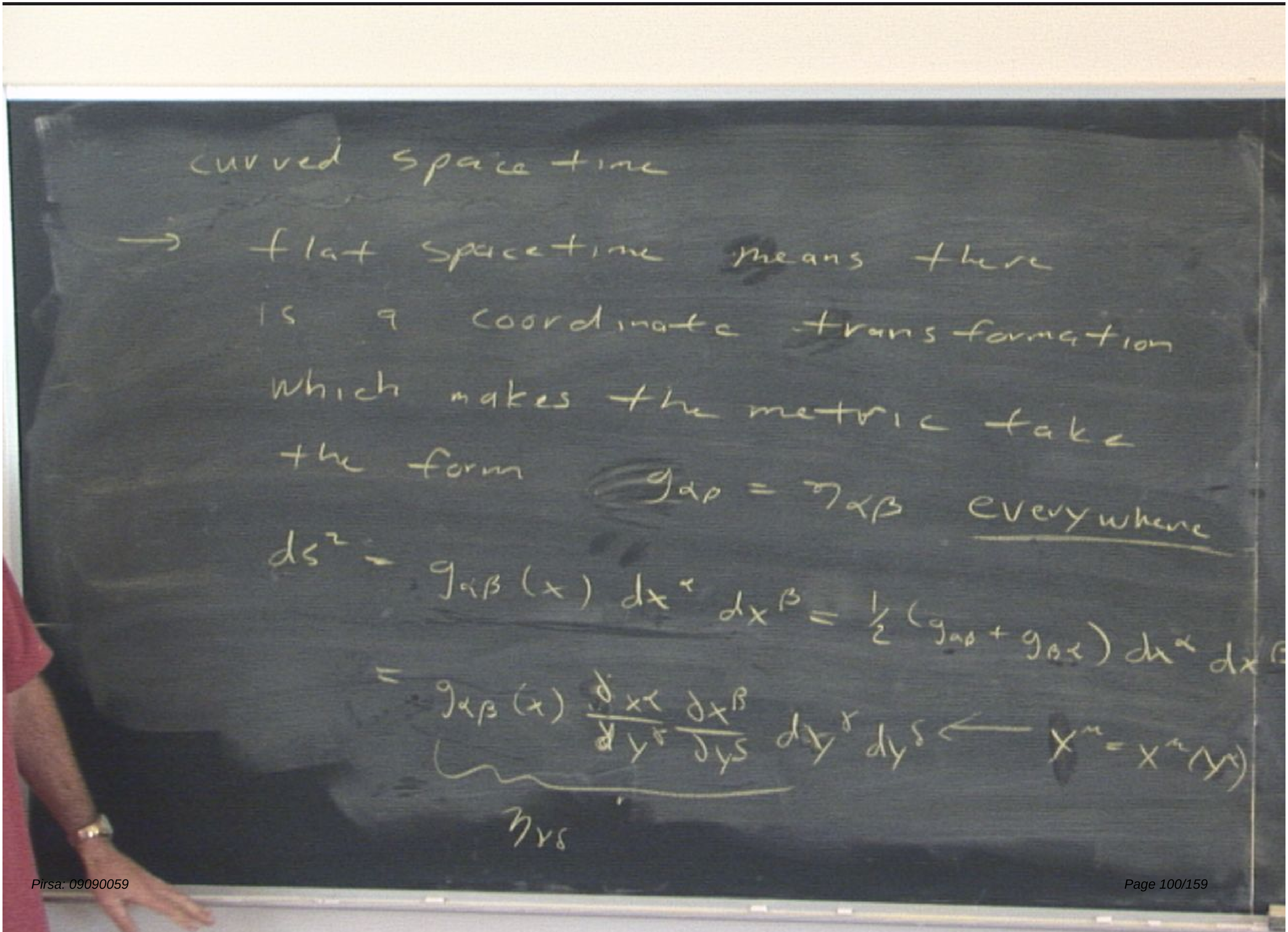
curved spacetime

→ flat spacetime means there is a coordinate transformation which the metric take the f $g_{\alpha\beta} = \eta_{\alpha\beta}$ everywhere

d

$\beta = \frac{1}{2}(g_{\alpha\beta} + g_{\beta\alpha})\, dx^{\alpha}\, dx^{\beta}$

$\frac{\partial x^{\alpha}}{\partial y^{\gamma}} \frac{\partial x^{\beta}}{\partial y^{\delta}}\, dy^{\gamma}\, dy^{\delta} \leftarrow x^{\mu} = x^{\mu}(y^{\nu})$



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$$ds^2 = \underline{g_{\alpha\beta}(x)\, dx^\alpha\, dx^\beta} = \frac{1}{2}\left(g_{\alpha\beta} + g_{\beta\alpha}\right) dx^\alpha\, dx^\beta$$

$$= \underbrace{g_{\alpha\beta}(x)\, \frac{\partial x^\alpha}{\partial y^\gamma} \frac{\partial x^\beta}{\partial y^\delta}}_{\eta_{\gamma\delta}}\, dy^\gamma\, dy^\delta \longleftarrow x^\mu = x^\mu(y)$$

(if we can't do such a transformation

$\longrightarrow$ curved spacetime

In general for 4 spacetime dimensions

$g_{\alpha\beta}(x)$ contains 10 independent functions

since symmetric matrix

Go

dx^{β}

$y^{\lambda})$

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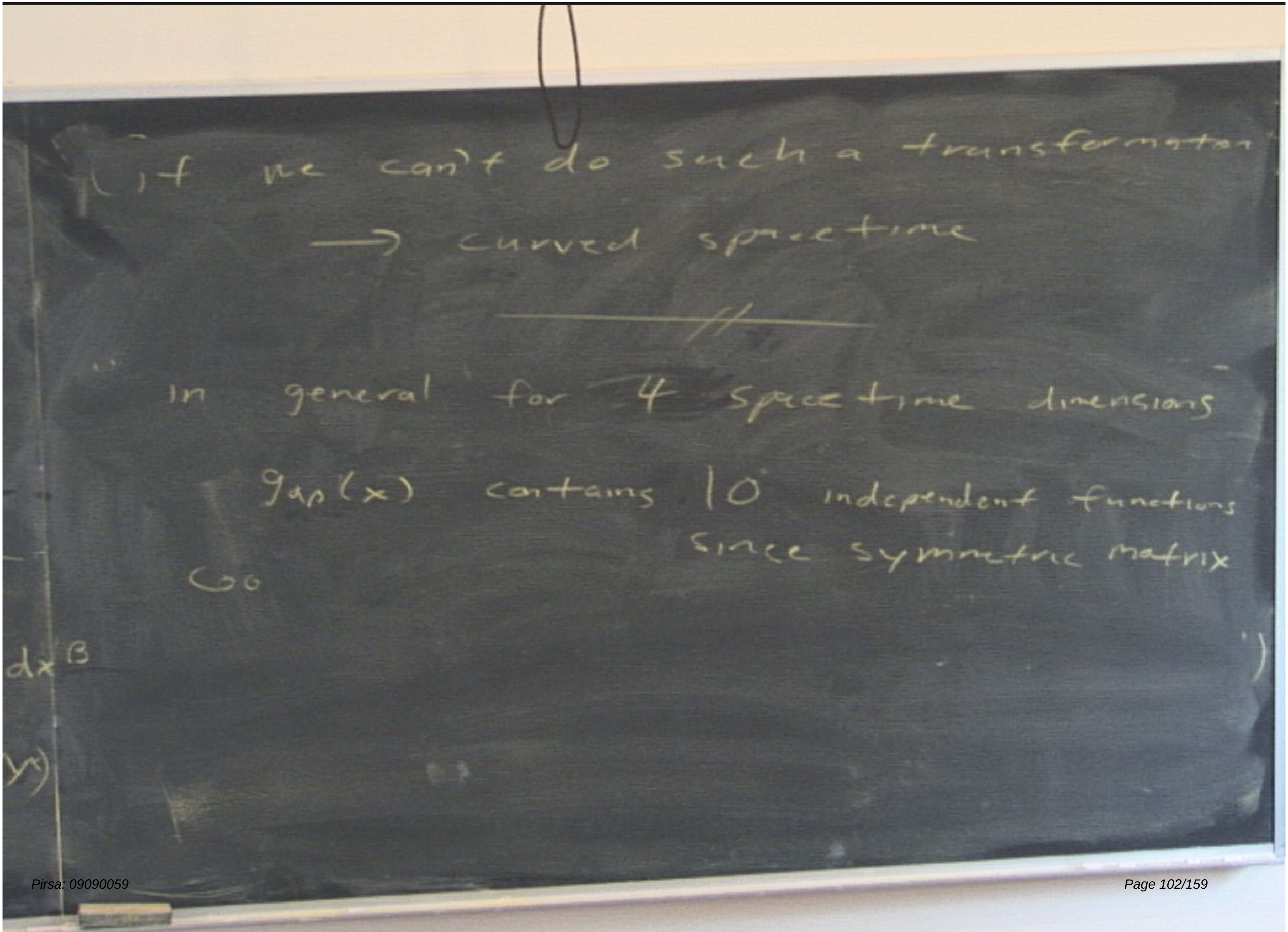

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Coordinate transformations

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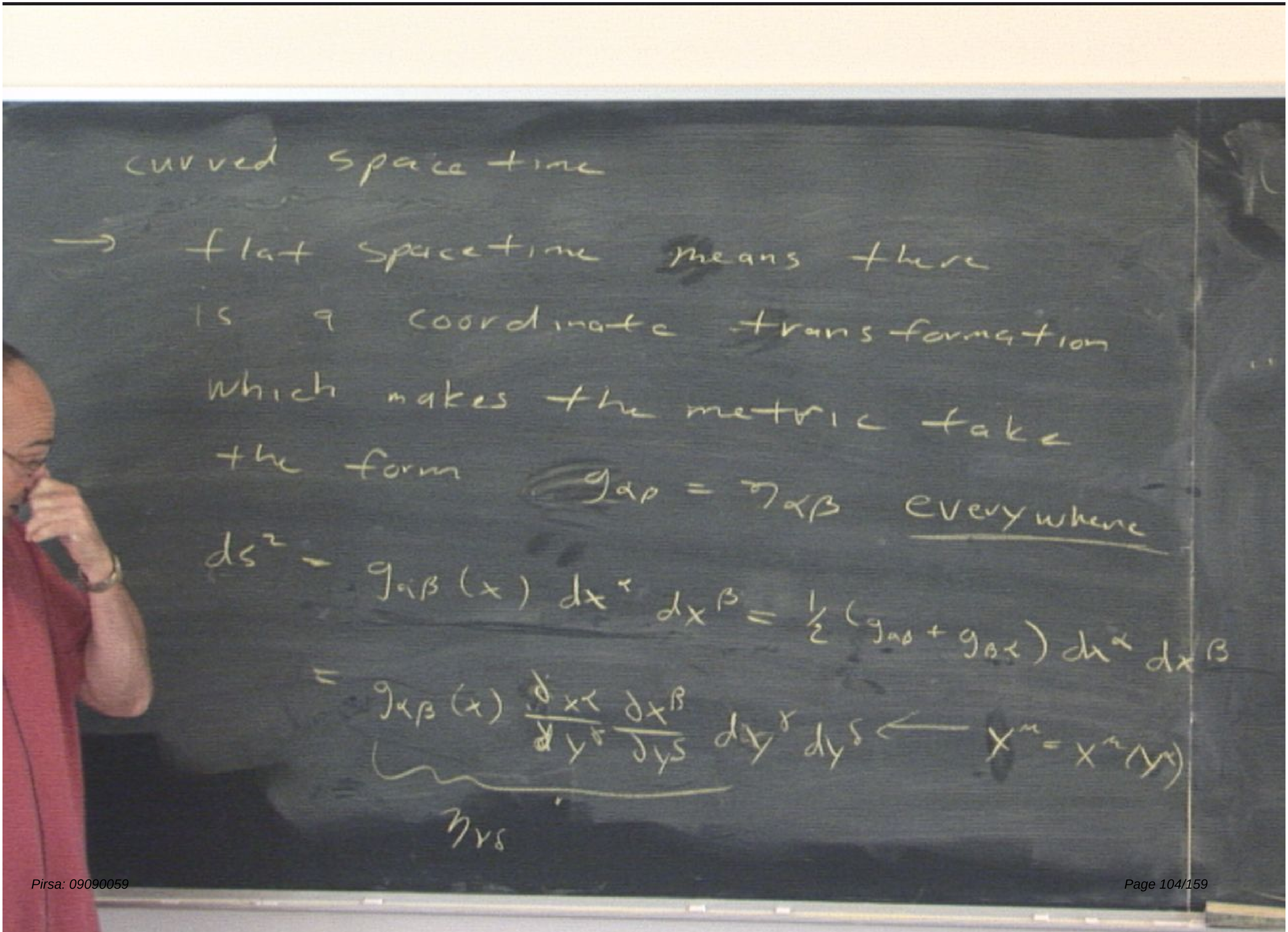

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coordinate transformation

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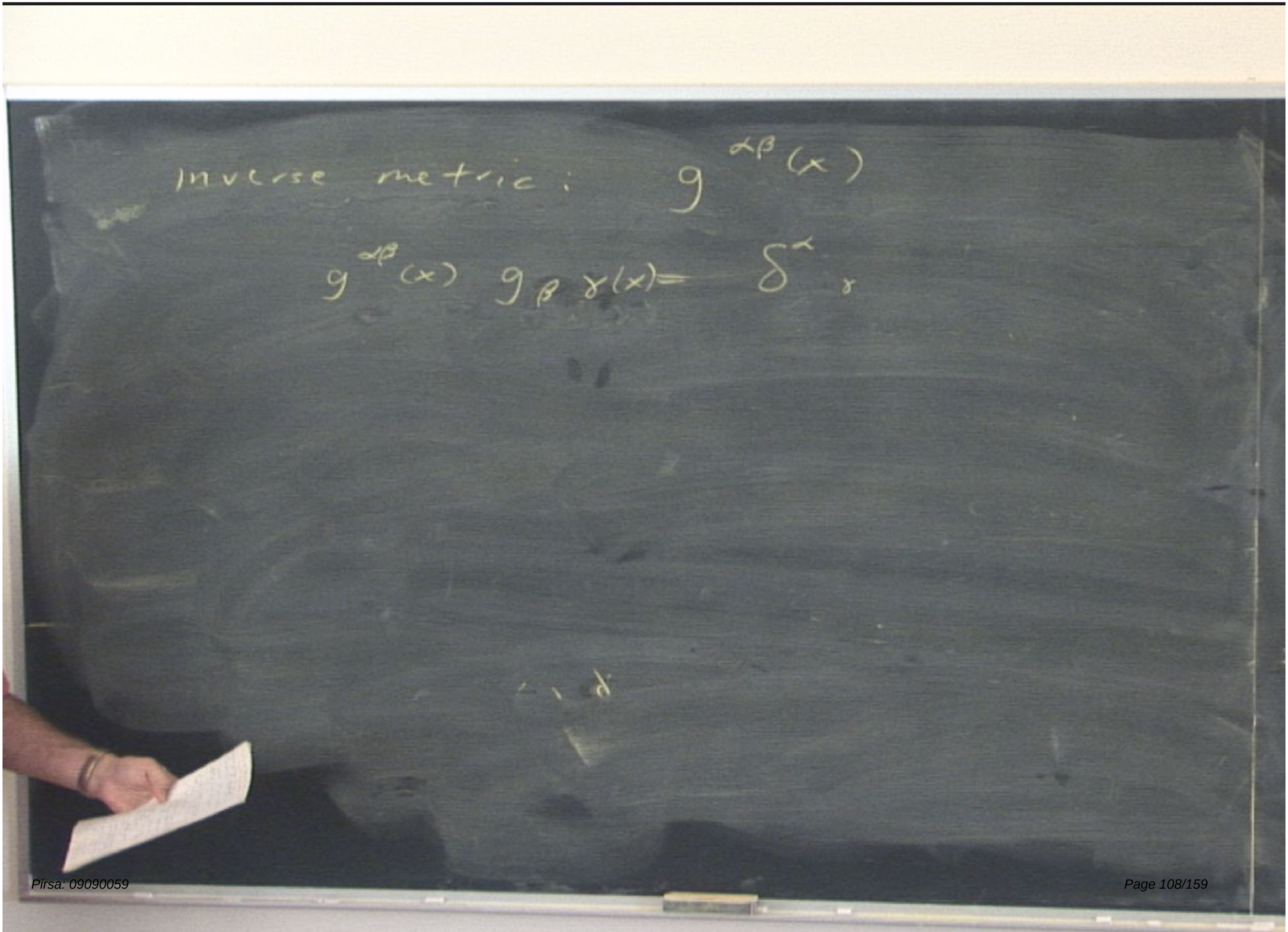
$y^\alpha)$

→ describes the same geometry
with "new labels"



inverse metric: $g^{\alpha\beta}(x)$

$$g^{\alpha\beta}(x)\, g_{\beta\gamma}(x) = \delta^{\alpha}{}_{\gamma}$$

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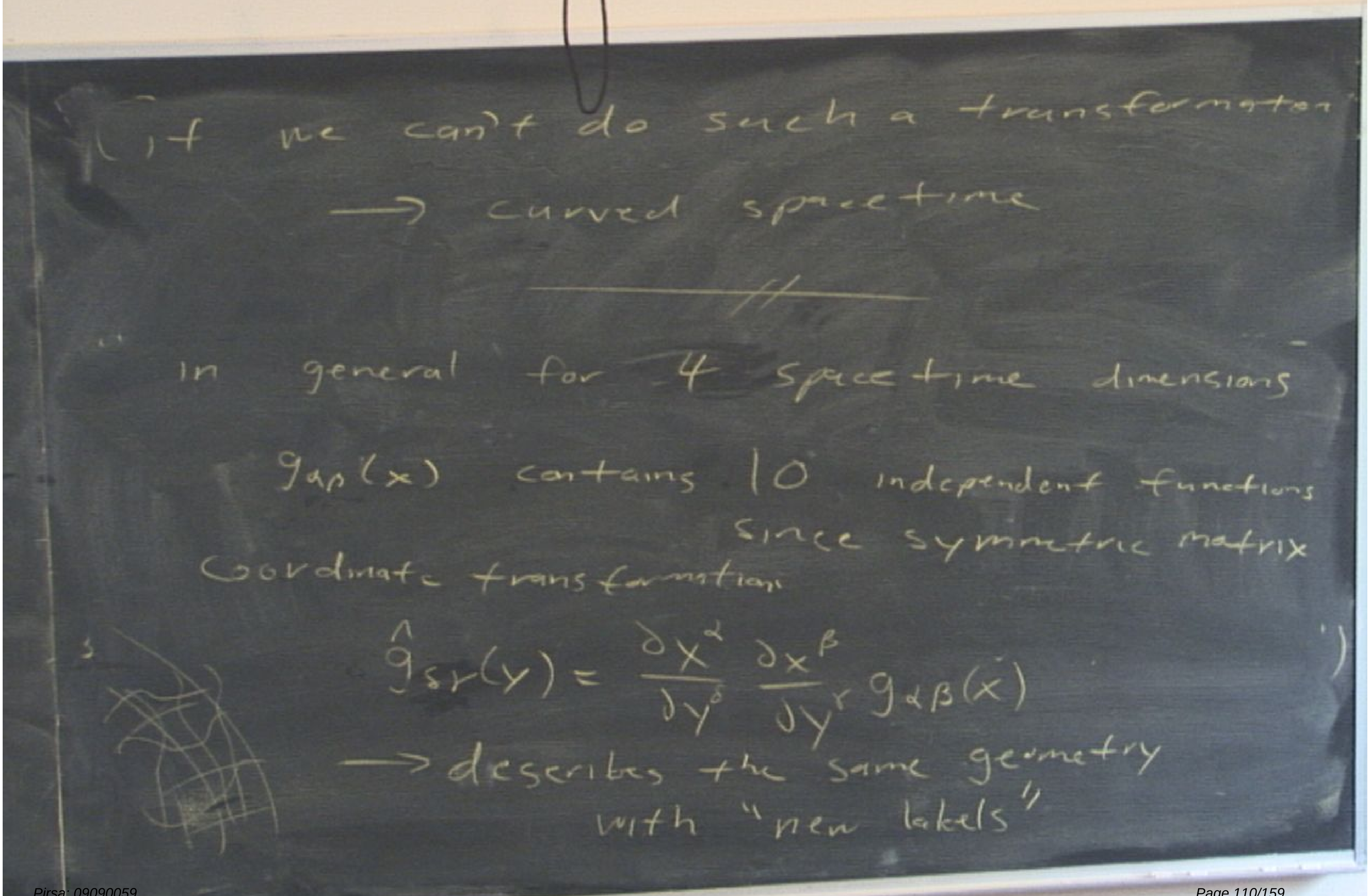

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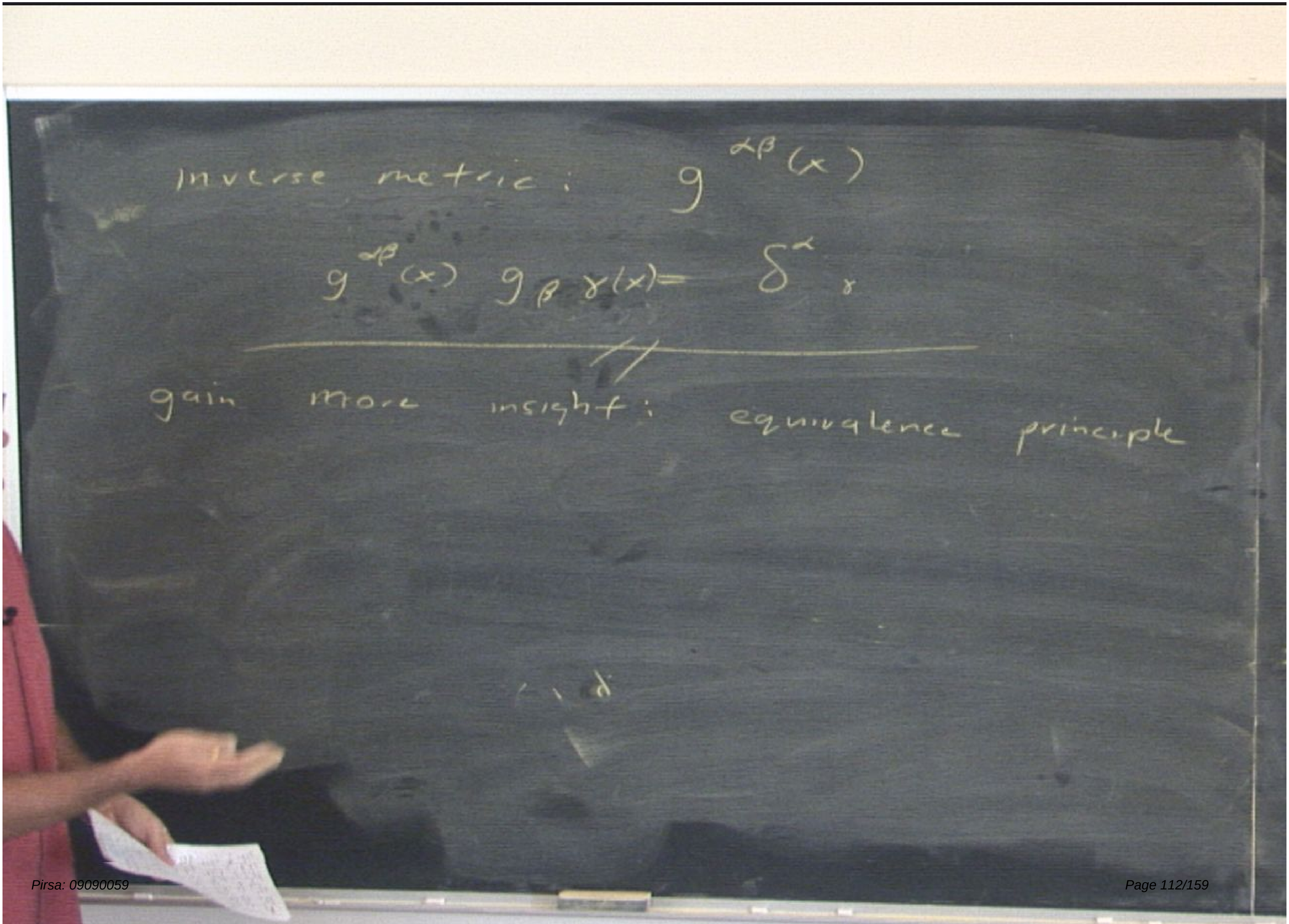
gain more insight:



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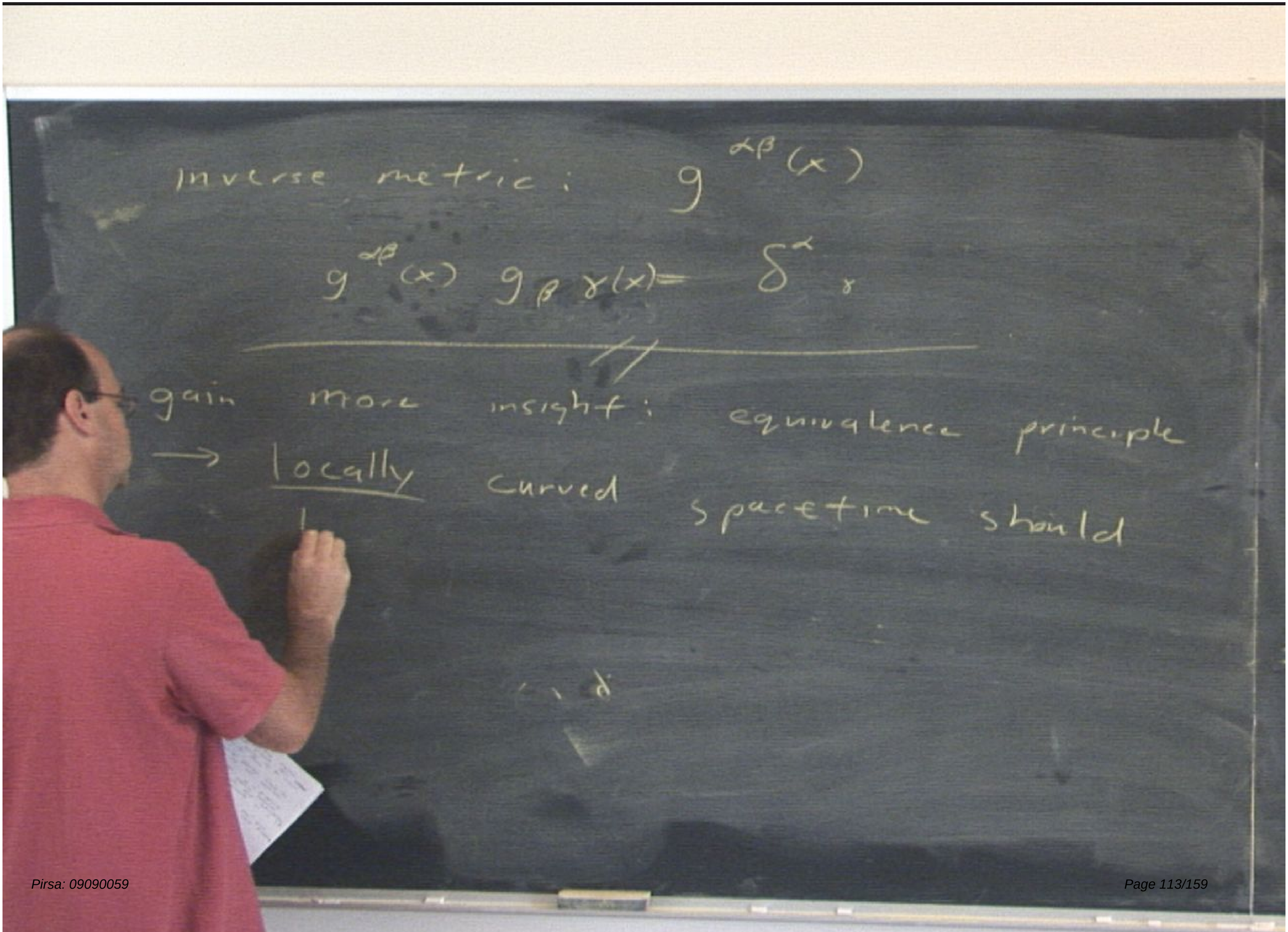
gain more insight: equivalence principle



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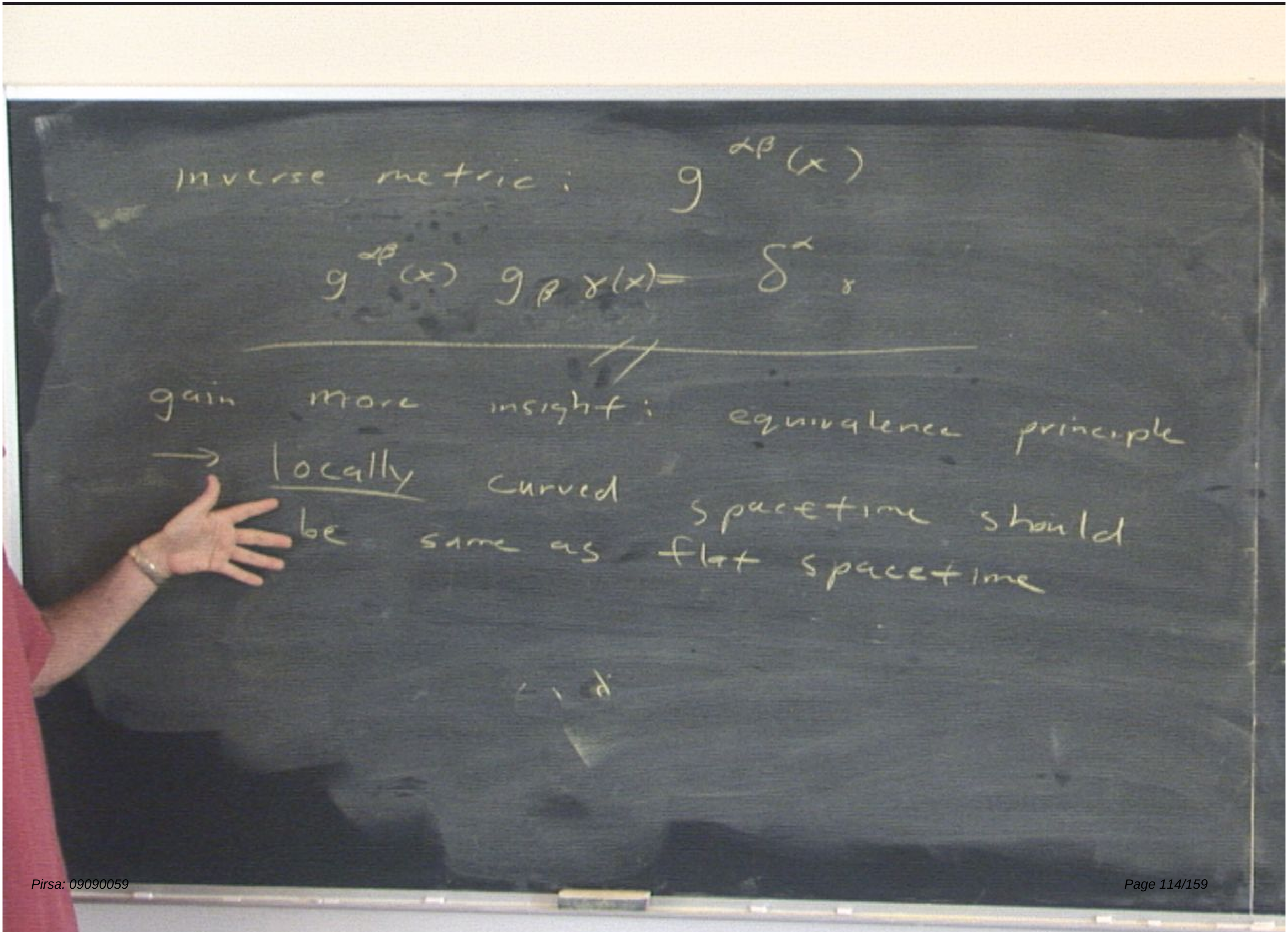
gain more insight: equivalence principle

→ locally curved spacetime should



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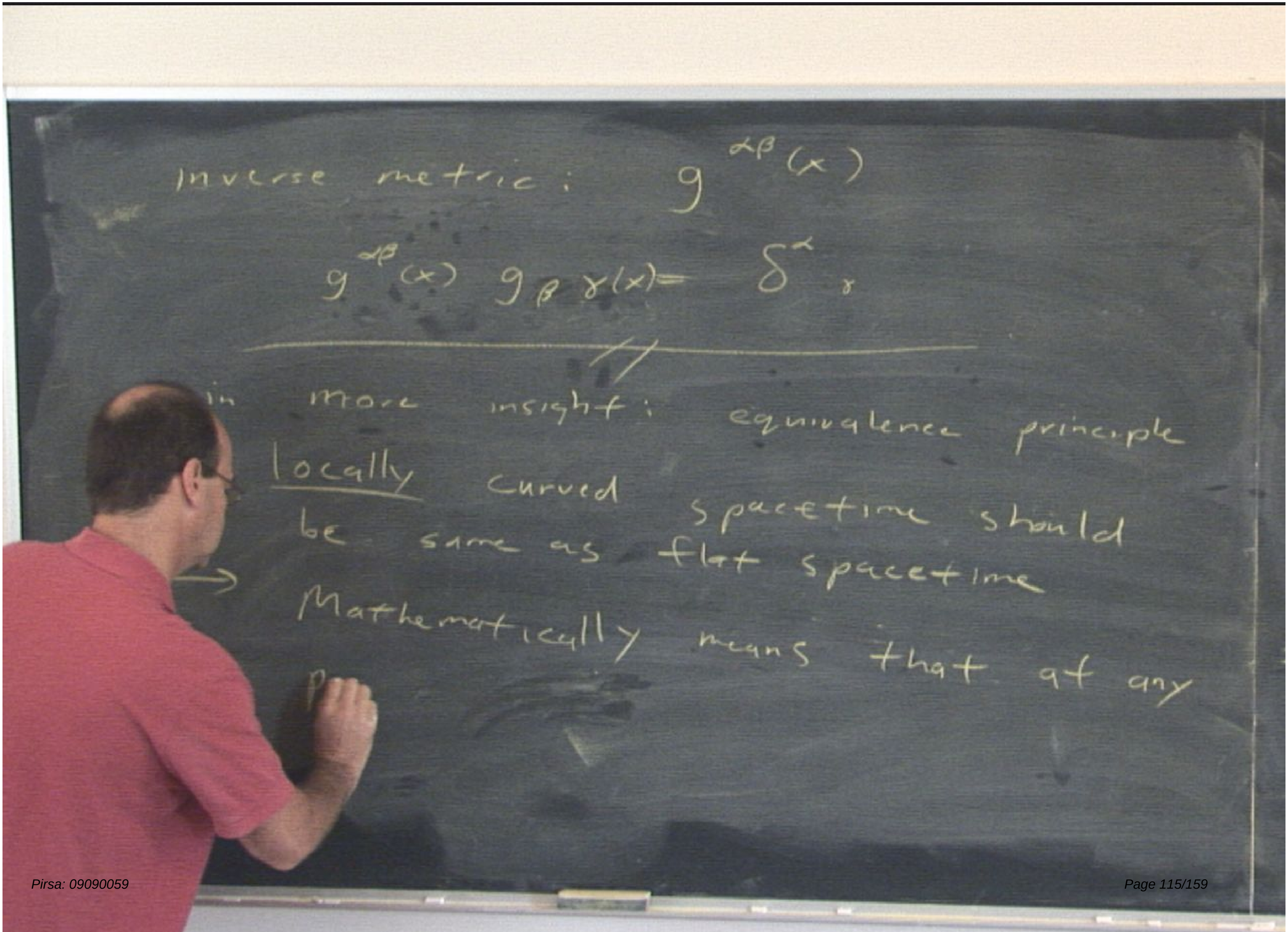
gain more insight: equivalence principle

→ <u>locally</u> curved spacetime should be same as flat spacetime



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in more insight: equivalence principle

locally curved spacetime should be same as flat spacetime

→ Mathematically means that at any

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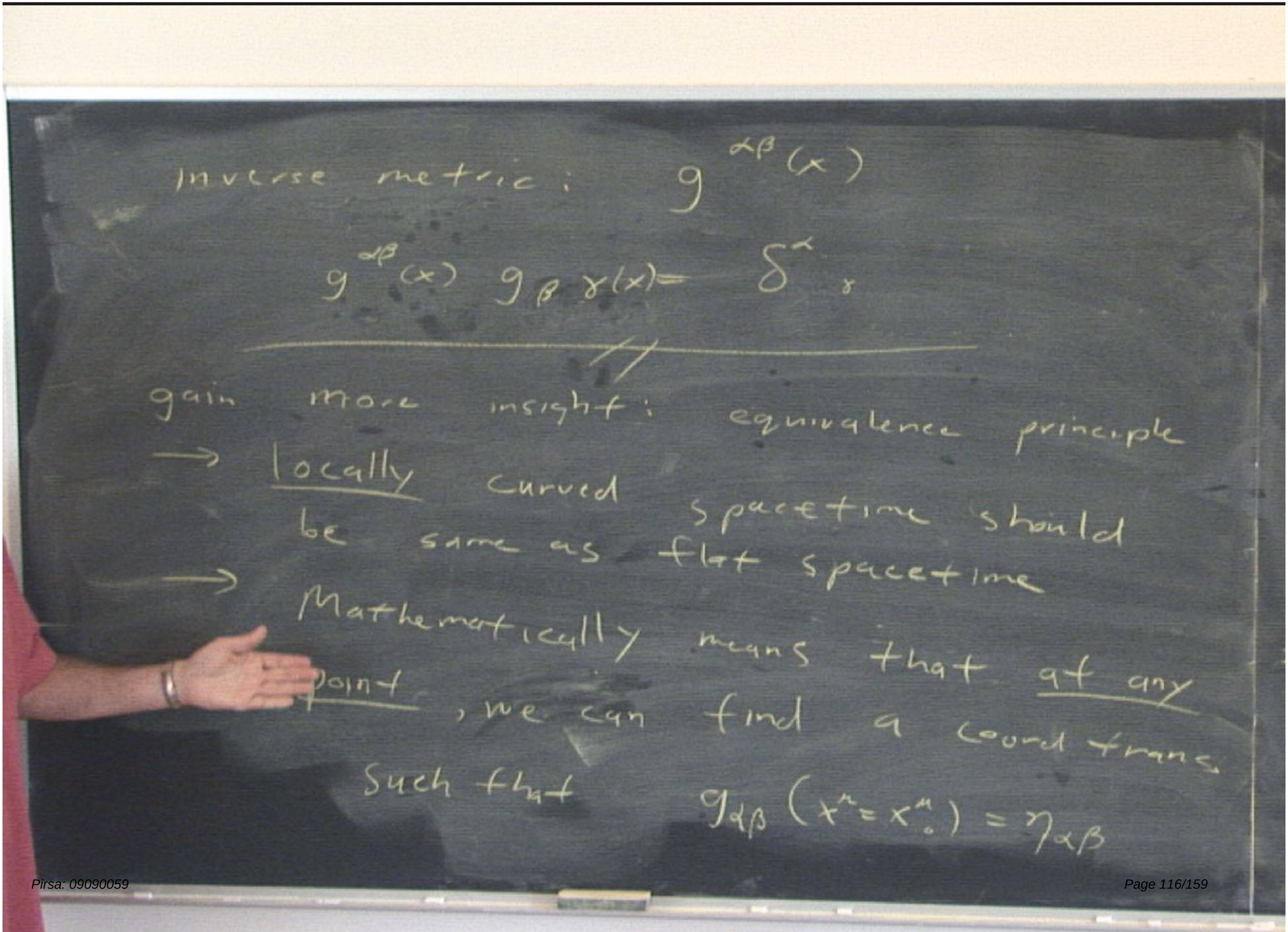

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gain more insight: equivalence principle

→ locally curved spacetime should be same as flat spacetime

→ Mathematically means that at any point, we can find a coord trans such that $g_{\alpha\beta}(x^\mu = x^\mu_0) = \eta_{\alpha\beta}$

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Coordinate transformation

$$\hat{g}_{\delta\gamma}(y) = \frac{\partial x^\alpha}{\partial y^\delta}\frac{\partial x^\beta}{\partial y^\gamma} g_{\alpha\beta}(x)$$

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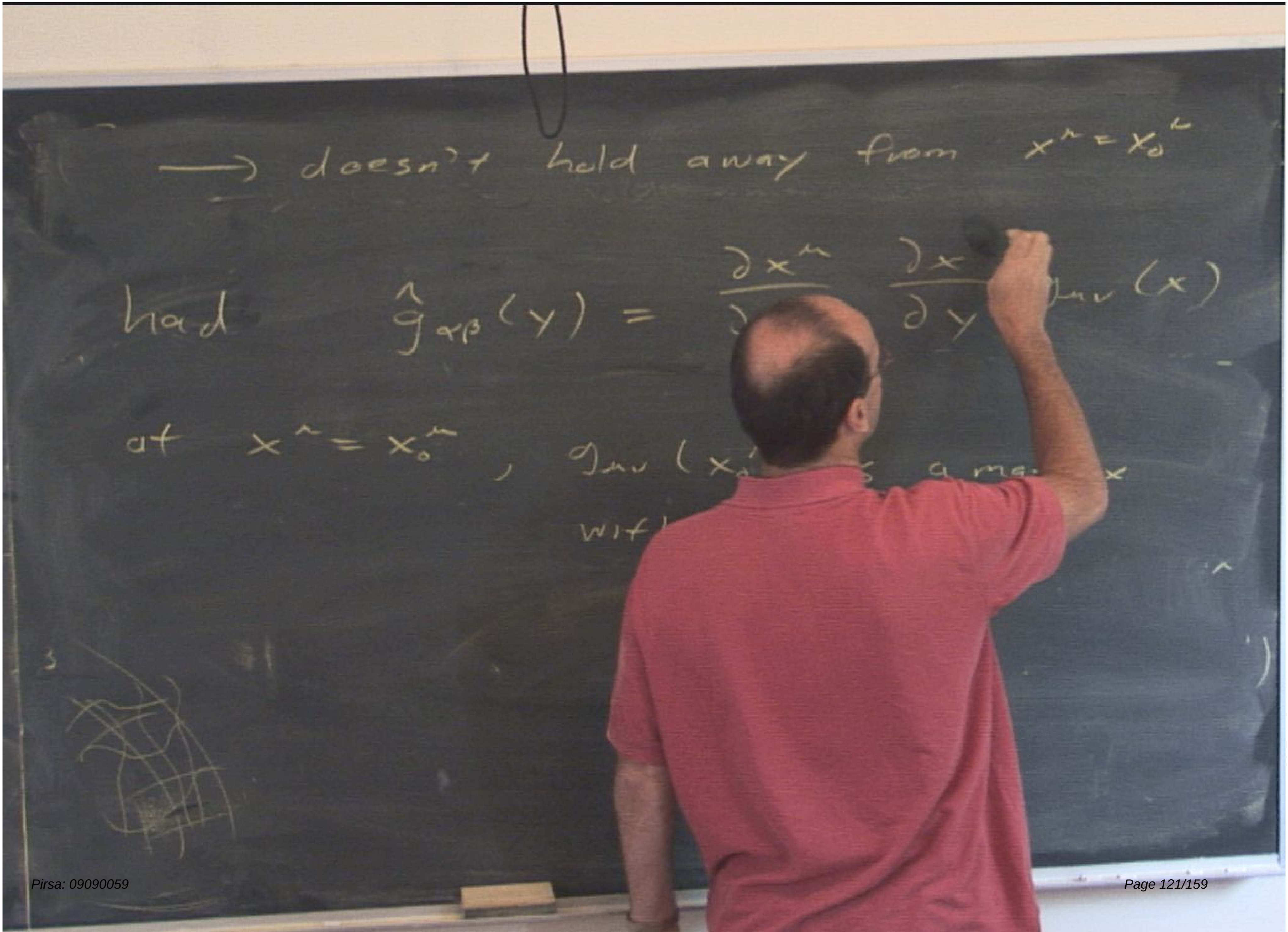

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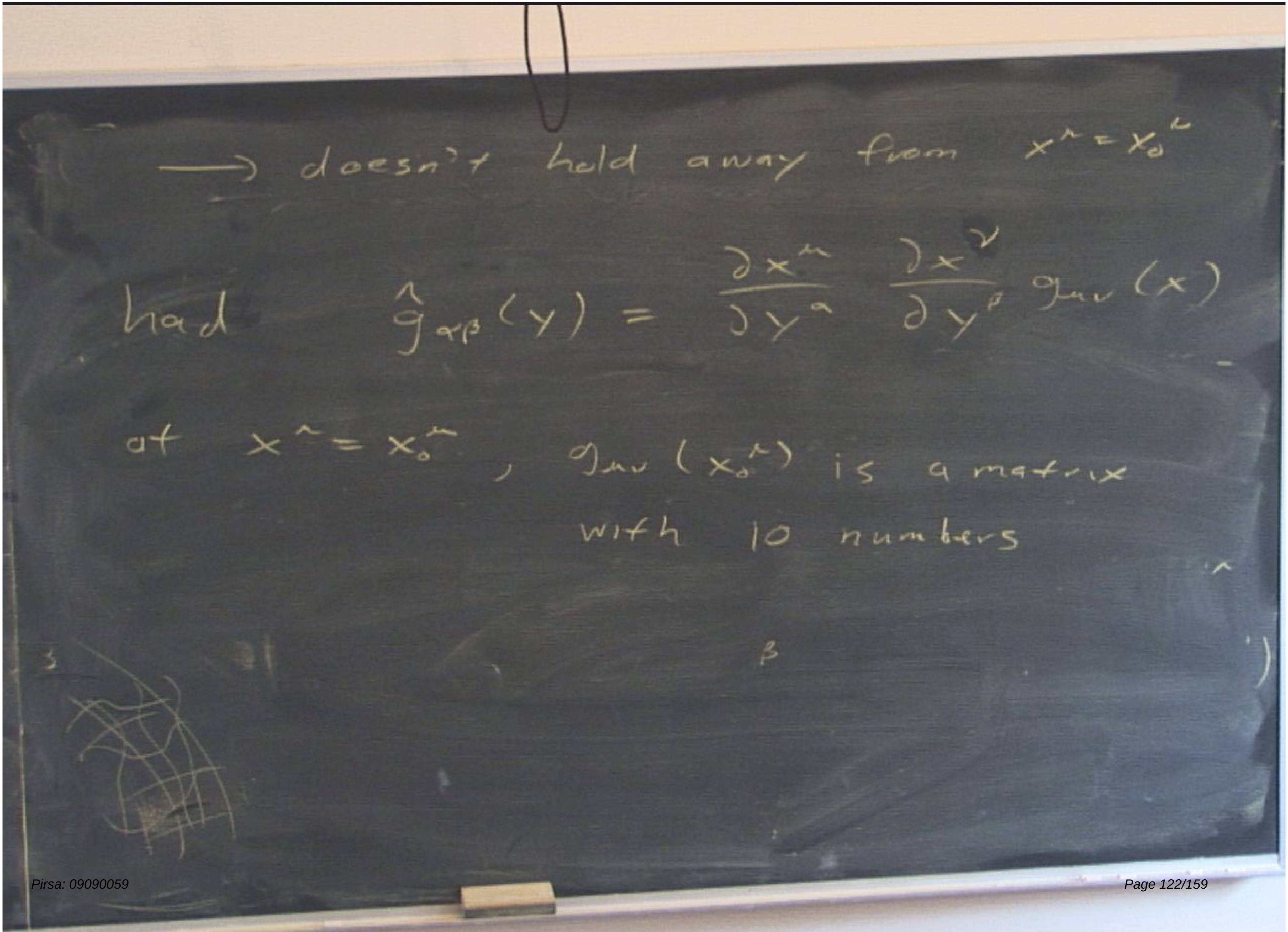

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similarly $\left.\frac{\partial x^\mu}{\partial y^\alpha}\right|_{x=x^0}$ is a matrix with 16 numbers

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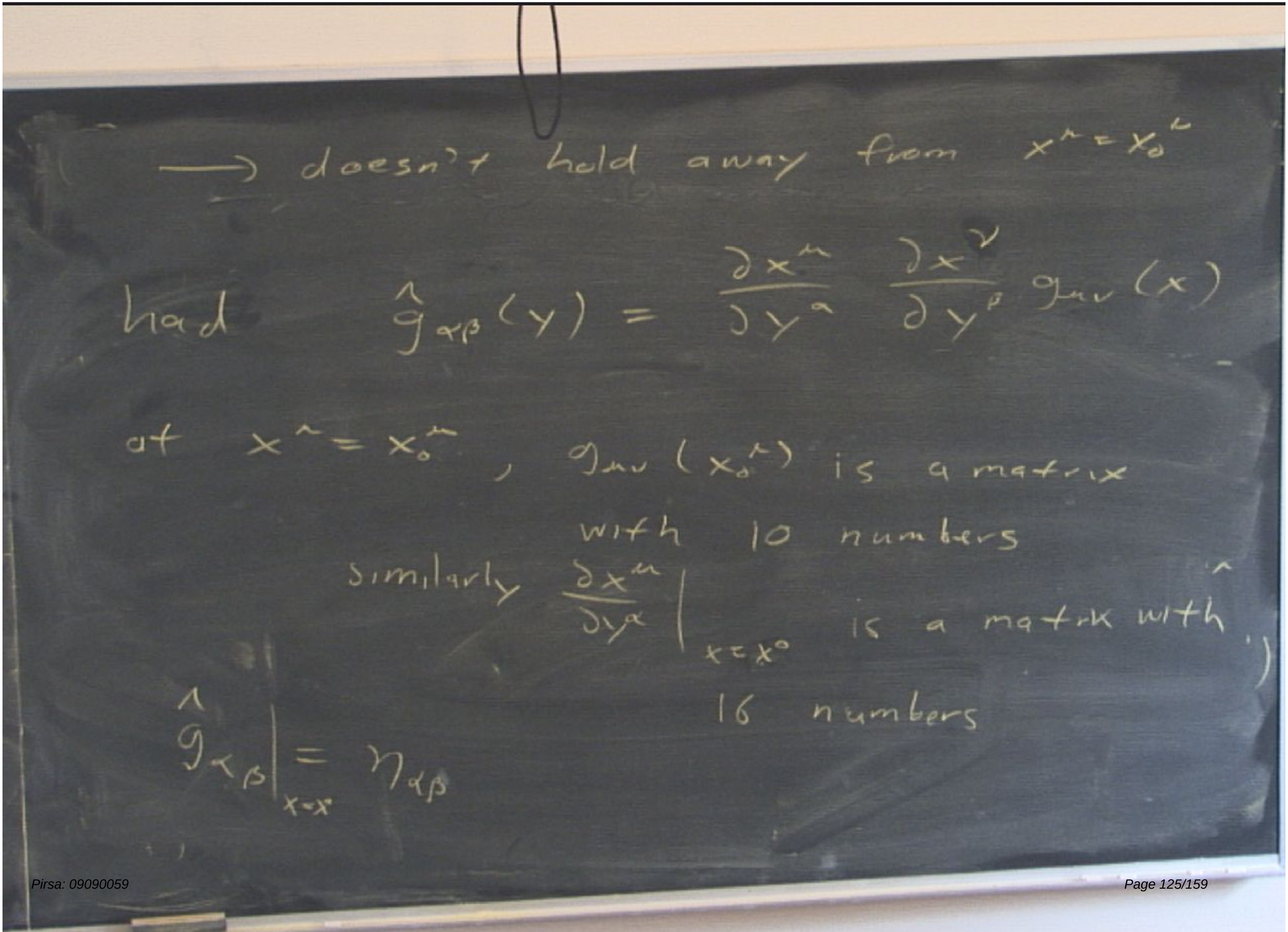

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$\hat{g}_{\alpha\beta}\Big|_{x=x^0} = \eta_{\alpha\beta}$ ← 10 eq's

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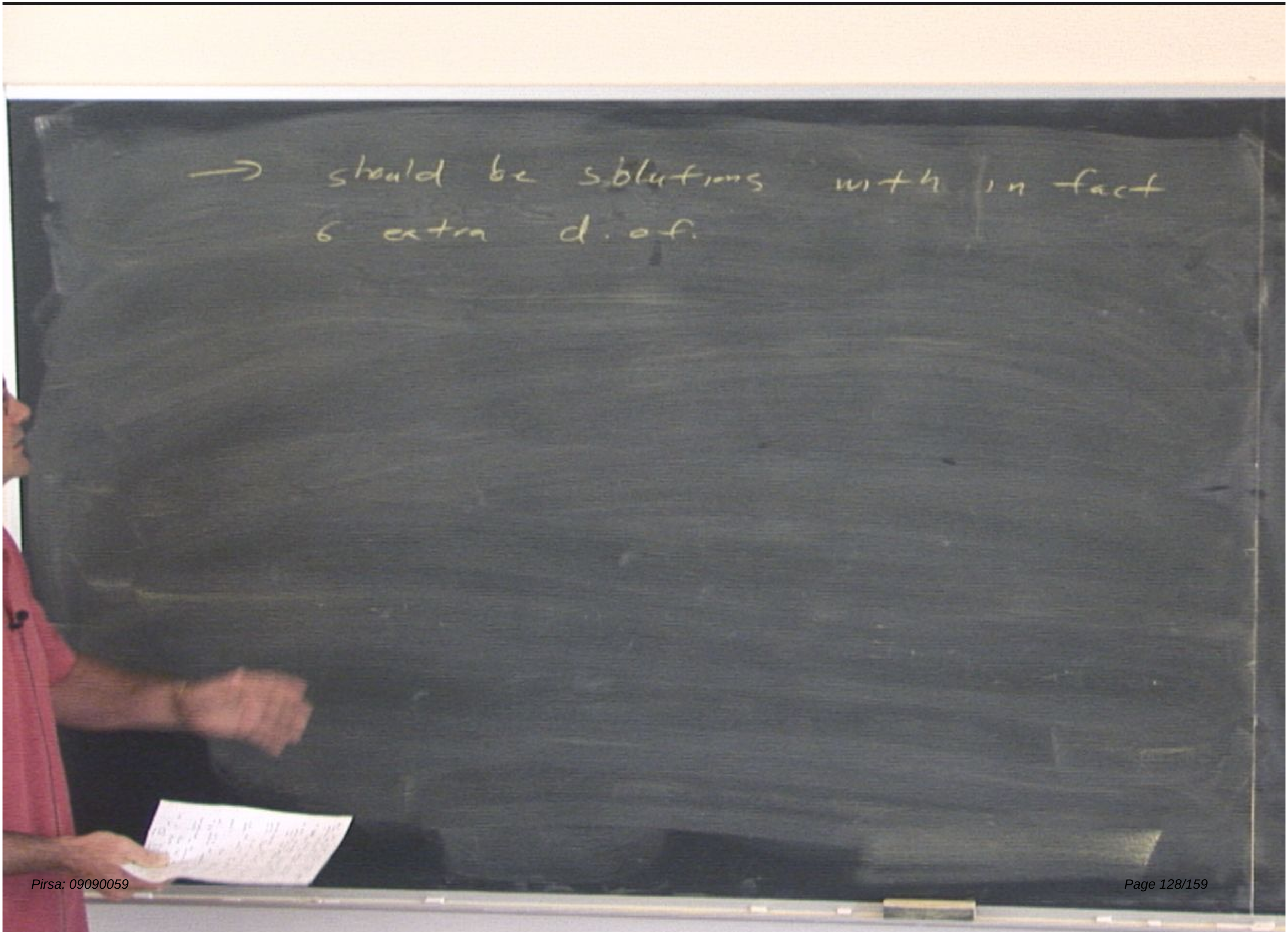
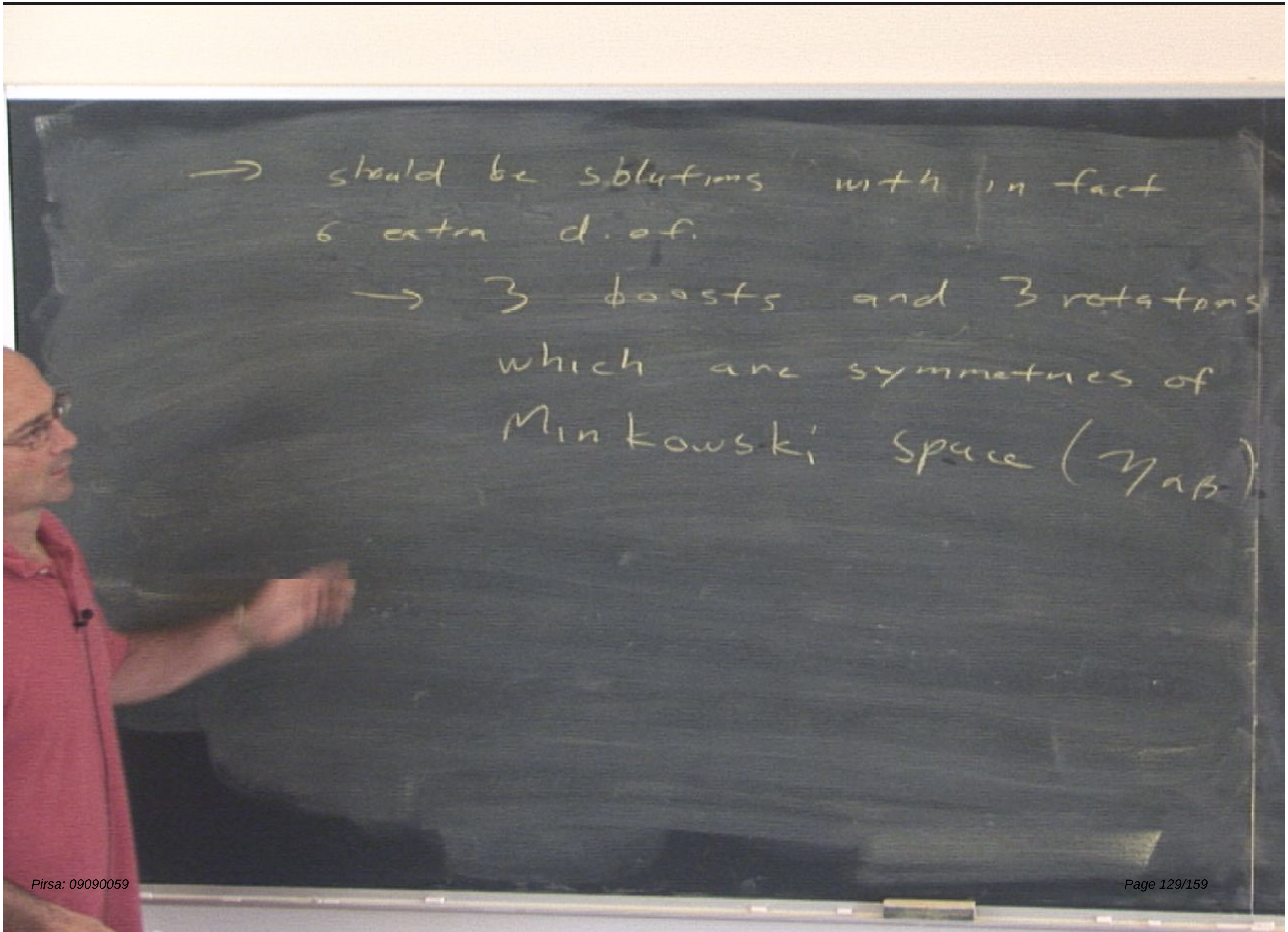
$\left.\hat{g}_{\alpha\beta}\right|_{x=x^0} = \eta_{\alpha\beta}$ ← 10 eq's in 16 unknowns



→ should be solutions with | in fact
6 extra d.o.f.



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→ 3 boosts and 3 rotations
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$(g^{\alpha\beta} g_{\beta\mu} = \delta^\alpha{}_\mu)$

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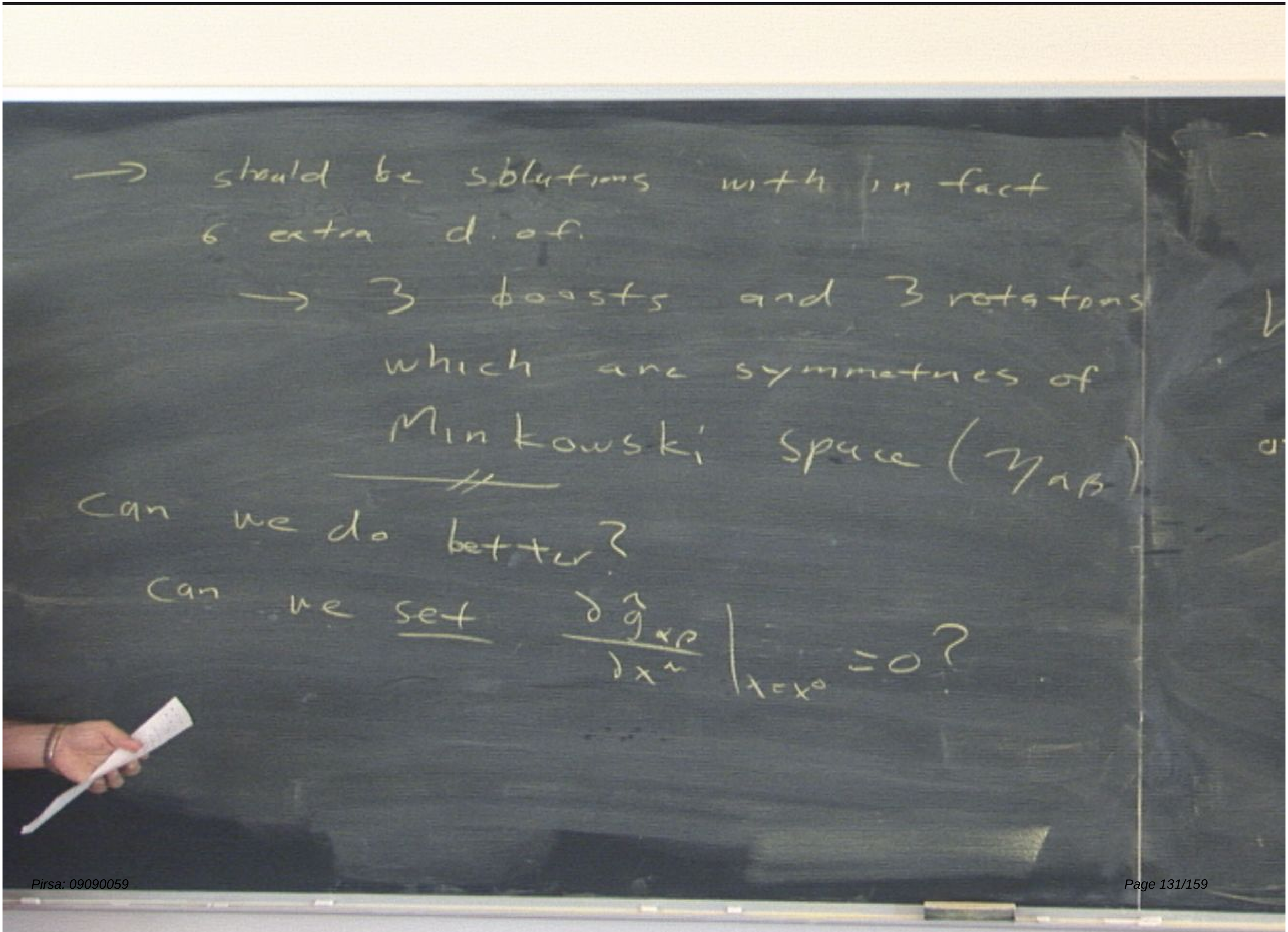
$$\left.\hat{g}_{\alpha\beta}\right|_{x=x^0} = \eta_{\alpha\beta} \quad \leftarrow 10 \text{ eq's in } 16 \text{ unknowns}$$



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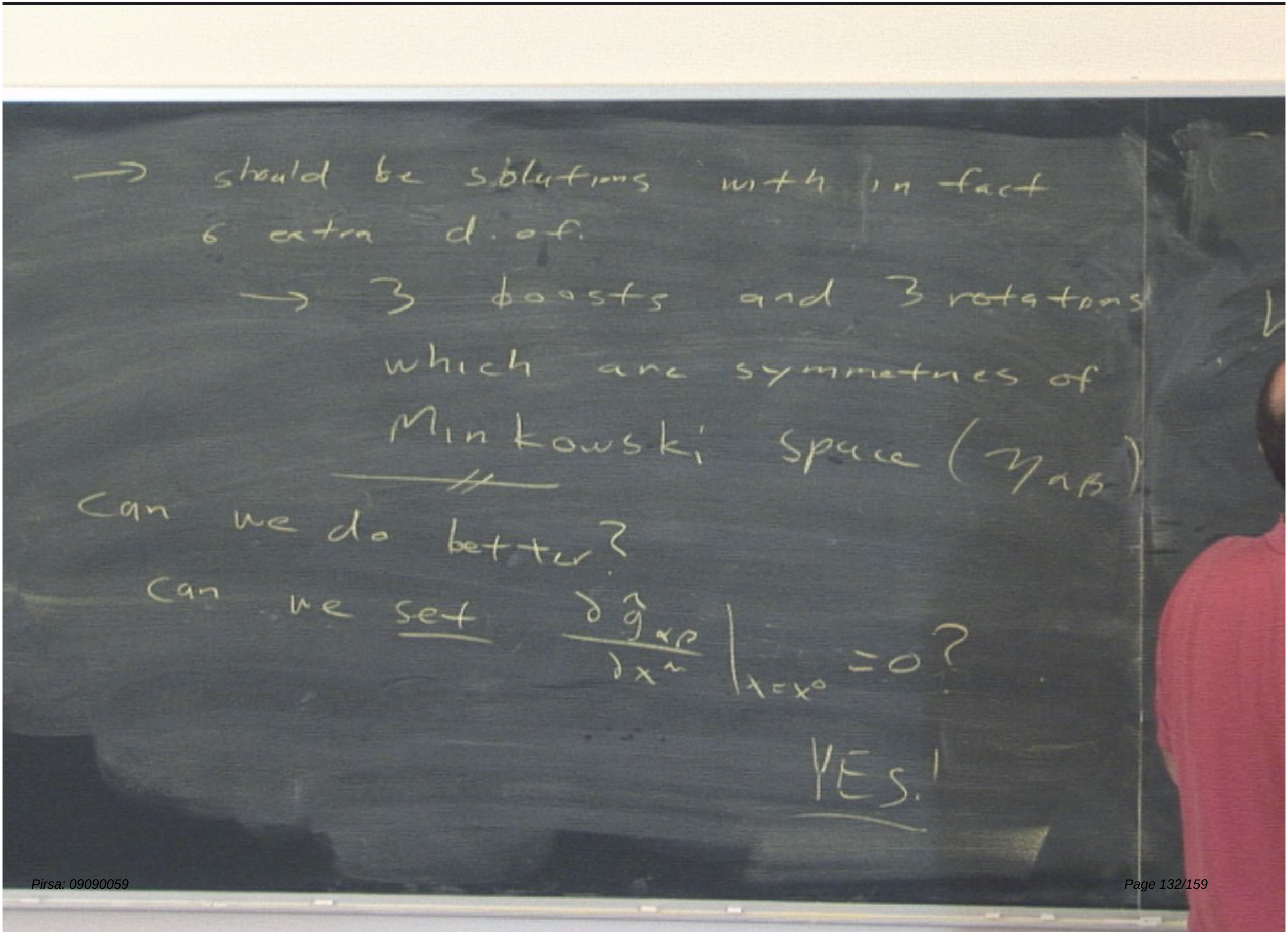
Can we do better?

Can we set $\frac{\partial \hat{g}_{\alpha\beta}}{\partial x^{\mu}}\Big|_{x=x^0} = 0$?



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YES!

→ should be solutions with in fact 6 extra d.o.f.

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Can we do better?

Can we set $\frac{\partial \hat{g}_{\alpha\beta}}{\partial y^{\mu}}\Big|_{x=x^0} = 0$?

YES!

$$\frac{\partial}{\partial y^\gamma}\hat{g}_{\alpha\beta}(y) = \frac{\partial x^\lambda}{\partial y^\gamma}\frac{\partial}{\partial x^\lambda}\left[\boxed{\frac{\partial x^\mu}{\partial y^\alpha}}\frac{\partial x^\nu}{\partial y^\beta}\, g_{\mu\nu}(x)\right]$$

at $x^\mu = x_0^\mu$, $g_{\mu\nu}(x_0^\mu)$ is a matrix with 10 numbers

Similarly $\left.\frac{\partial x^\mu}{\partial y^\alpha}\right|_{x=x^0}$ is a matrix with 16 numbers

$\left.\hat{g}_{\alpha\beta}\right|_{x=x^0} = \eta_{\alpha\beta}$ ← 10 eq's in 16 unknowns

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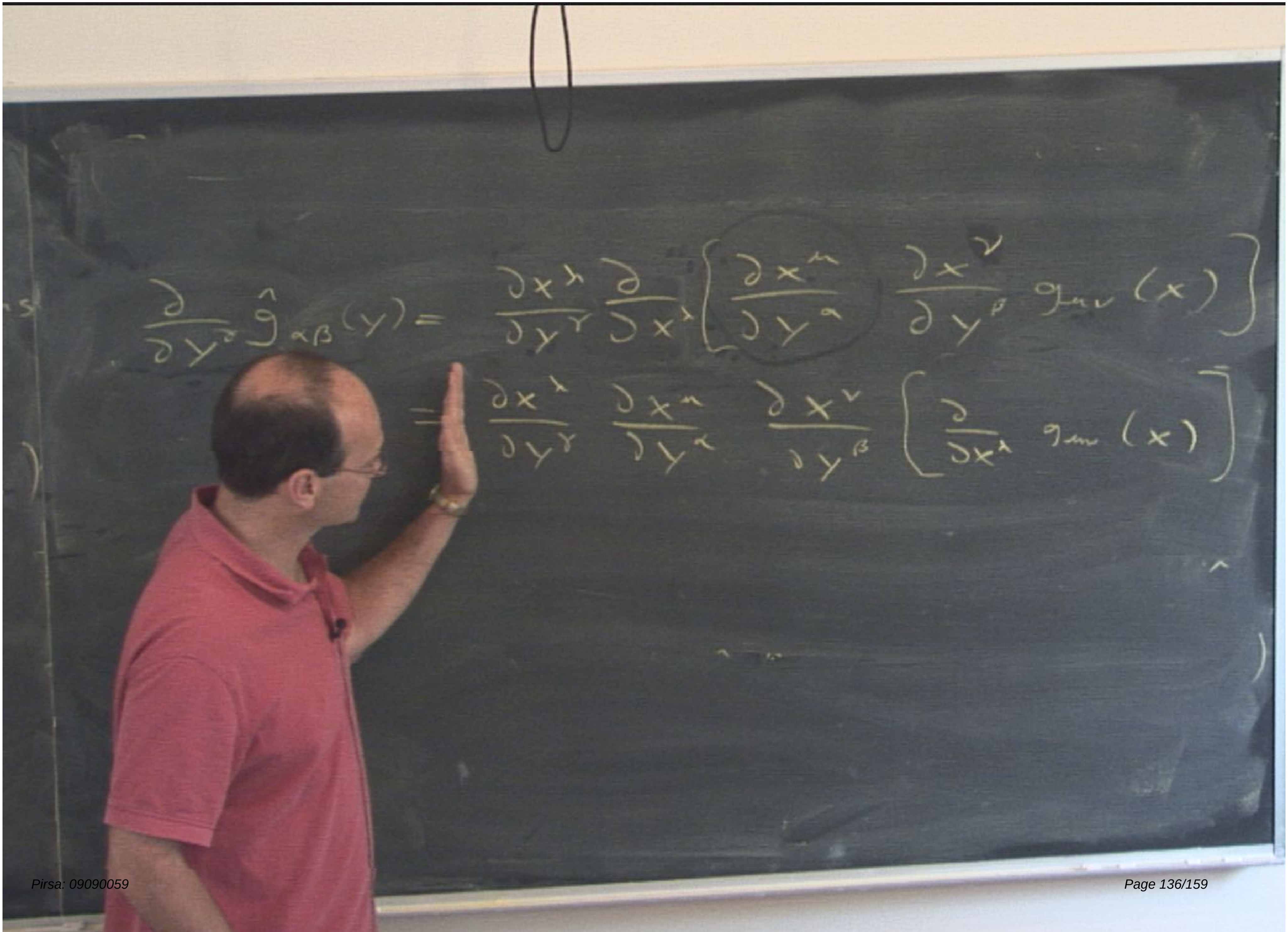

$$\frac{\partial}{\partial y^\gamma}\hat{g}_{\alpha\beta}(y) = \frac{\partial x^\lambda}{\partial y^\gamma}\frac{\partial}{\partial x^\lambda}\left[\frac{\partial x^\mu}{\partial y^\alpha}\frac{\partial x^\nu}{\partial y^\beta}\, g_{\mu\nu}(x)\right]$$

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$$= \frac{\partial x^\lambda}{\partial y^\gamma}\frac{\partial x^\mu}{\partial y^\alpha}\frac{\partial x^\nu}{\partial y^\beta}\left[\frac{\partial}{\partial x^\lambda} g_{\mu\nu}(x)\right]$$

$$+ \frac{\partial^2 x^\mu}{\partial y^\gamma \partial y^\alpha}\frac{\partial x^\nu}{\partial y^\beta} g_{\mu\nu}(x)$$

$$+ \frac{\partial x^\mu}{\partial y^\alpha}\frac{\partial^2 x^\nu}{\partial y^\gamma \partial y^\beta} g_{\mu\nu}(x)$$

$$\frac{\partial}{\partial y^\gamma}\hat{g}_{\alpha\beta}(y) = \frac{\partial x^\lambda}{\partial y^\gamma}\frac{\partial}{\partial x^\lambda}\left\{\frac{\partial x^\mu}{\partial y^\alpha}\frac{\partial x^\nu}{\partial y^\beta} g_{\mu\nu}(x)\right\}$$

$$= \frac{\partial x^\lambda}{\partial y^\gamma}\frac{\partial x^\mu}{\partial y^\alpha}\frac{\partial x^\nu}{\partial y^\beta}\left[\frac{\partial}{\partial x^\lambda} g_{\mu\nu}(x)\right]$$

<u>NOT</u> tensor transformation rule →

$$+ \frac{\partial^2 x^\mu}{\partial y^\gamma \partial y^\alpha}\frac{\partial x^\nu}{\partial y^\beta} g_{\mu\nu}(x)$$

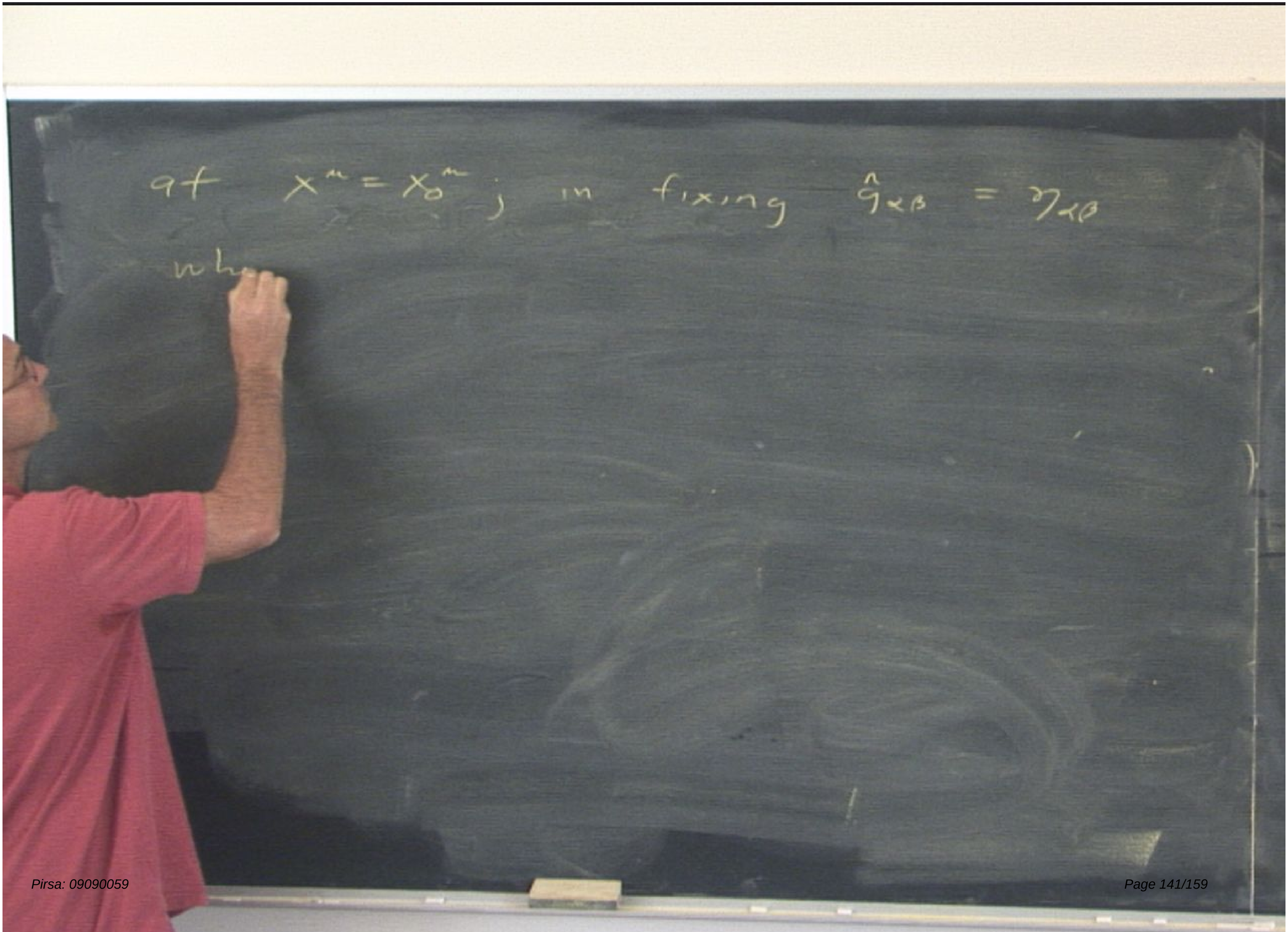
$$+ \frac{\partial x^\mu}{\partial y^\alpha}\frac{\partial^2 x^\nu}{\partial y^\gamma \partial y^\beta} g_{\mu\nu}(x)$$

$$\frac{\partial}{\partial y^\gamma}\hat{g}_{\alpha\beta}(y) = \frac{\partial x^\lambda}{\partial y^\gamma}\frac{\partial}{\partial x^\lambda}\left\{\boxed{\frac{\partial x^\mu}{\partial y^\alpha}}\frac{\partial x^\nu}{\partial y^\beta}\, g_{\mu\nu}(x)\right\}$$

$$= \frac{\partial x^\lambda}{\partial y^\gamma}\frac{\partial x^\mu}{\partial y^\alpha}\frac{\partial x^\nu}{\partial y^\beta}\left[\frac{\partial}{\partial x^\lambda}\, g_{\mu\nu}(x)\right]$$

<u>NOT</u> tensor transformation rule →

$$\left\{\begin{array}{l} + \dfrac{\partial^2 x^\mu}{\partial y^\gamma \partial y^\alpha}\dfrac{\partial x^\nu}{\partial y^\beta}\, g_{\mu\nu}(x) \\[2ex] + \dfrac{\partial x^\mu}{\partial y^\alpha}\dfrac{\partial^2 x^\nu}{\partial y^\gamma \partial y^\beta}\, g_{\mu\nu}(x) \end{array}\right.$$



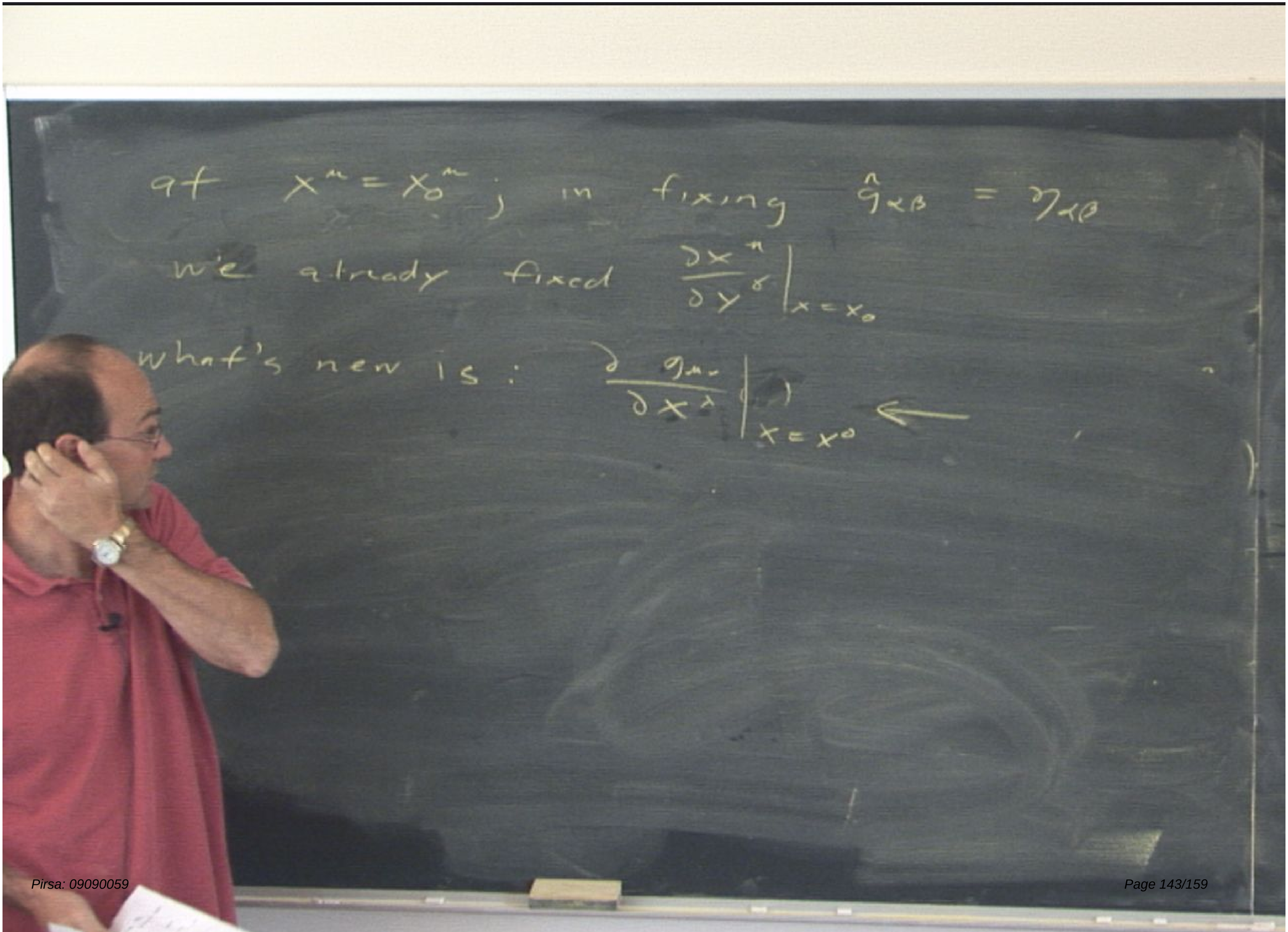
at $x^\mu = x_0^\mu$; in fixing $\hat{g}_{\alpha\beta} = \eta_{\alpha\beta}$

wh

at $x^\mu = x_0^\mu$; in fixing $\hat{g}_{\alpha\beta} = \eta_{\alpha\beta}$

w'e already fixed $\left.\frac{\partial x^\mu}{\partial y^\delta}\right|_{x=x_0}$

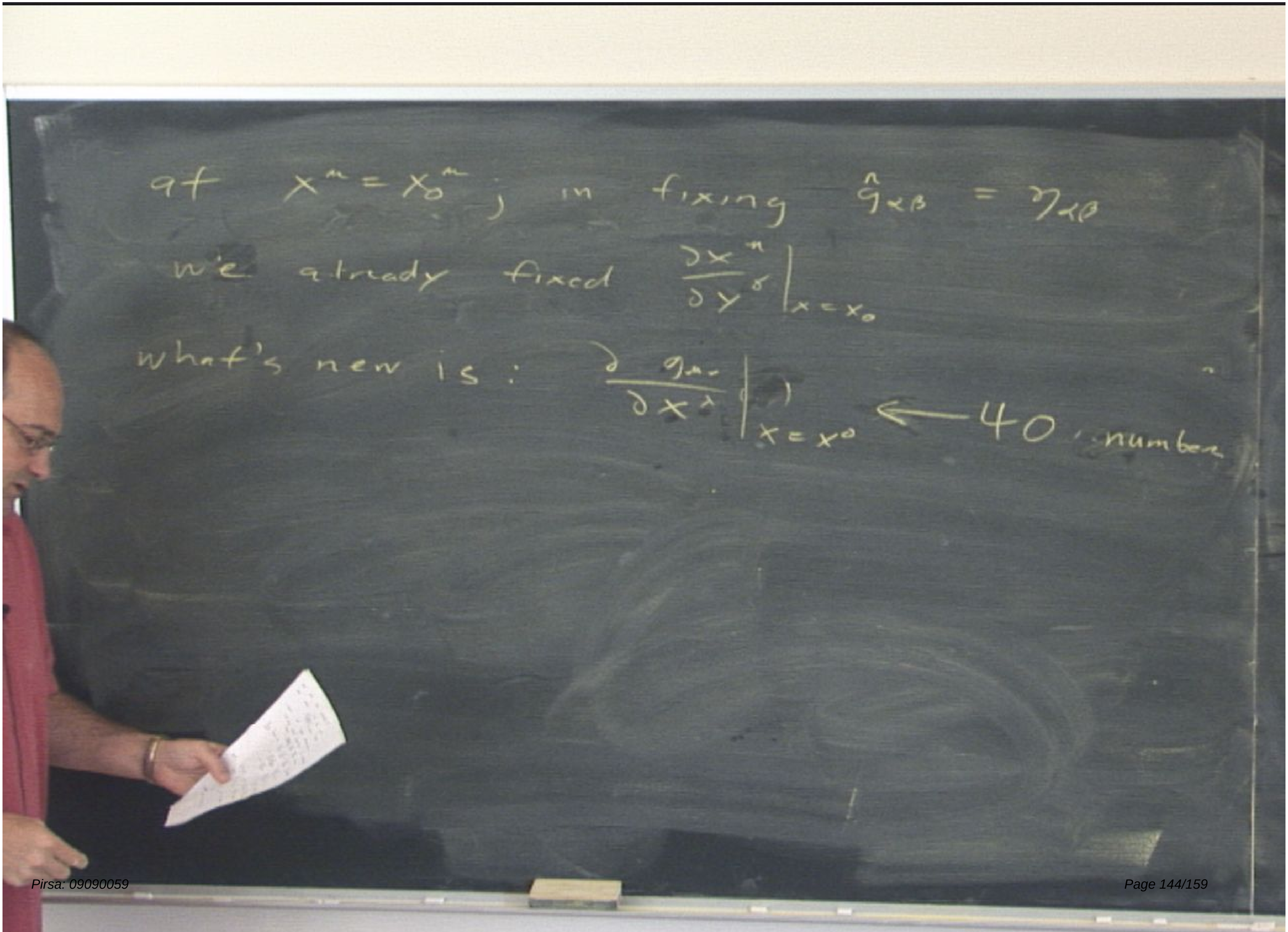
what's new is :



at $x^\mu = x_0^\mu$; in fixing $\hat{g}_{\alpha\beta} = \eta_{\alpha\beta}$

we already fixed $\left.\frac{\partial x^\mu}{\partial y^\sigma}\right|_{x=x_0}$

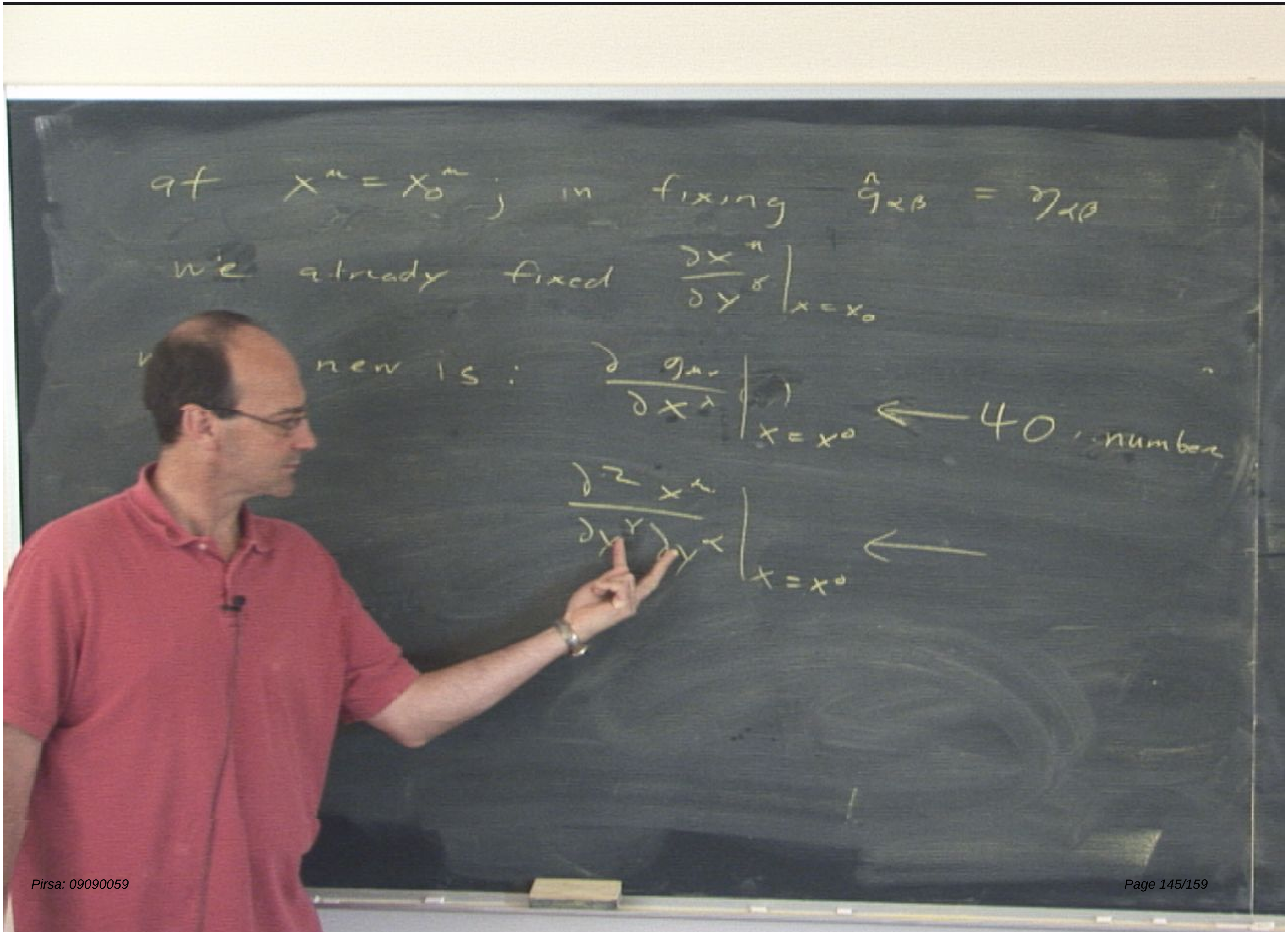
what's new is: $\left.\frac{\partial g_{\mu\nu}}{\partial x^\lambda}\right|_{x=x^0} \longleftarrow$



at $x^\mu = x_0^\mu$, in fixing $\hat{g}_{\alpha\beta} = \eta_{\alpha\beta}$

we already fixed $\left.\frac{\partial x^\mu}{\partial y^\sigma}\right|_{x=x_0}$

what's new is: $\left.\frac{\partial g_{\mu\nu}}{\partial x^\lambda}\right|_{x=x^0}$ ← 40 numbers

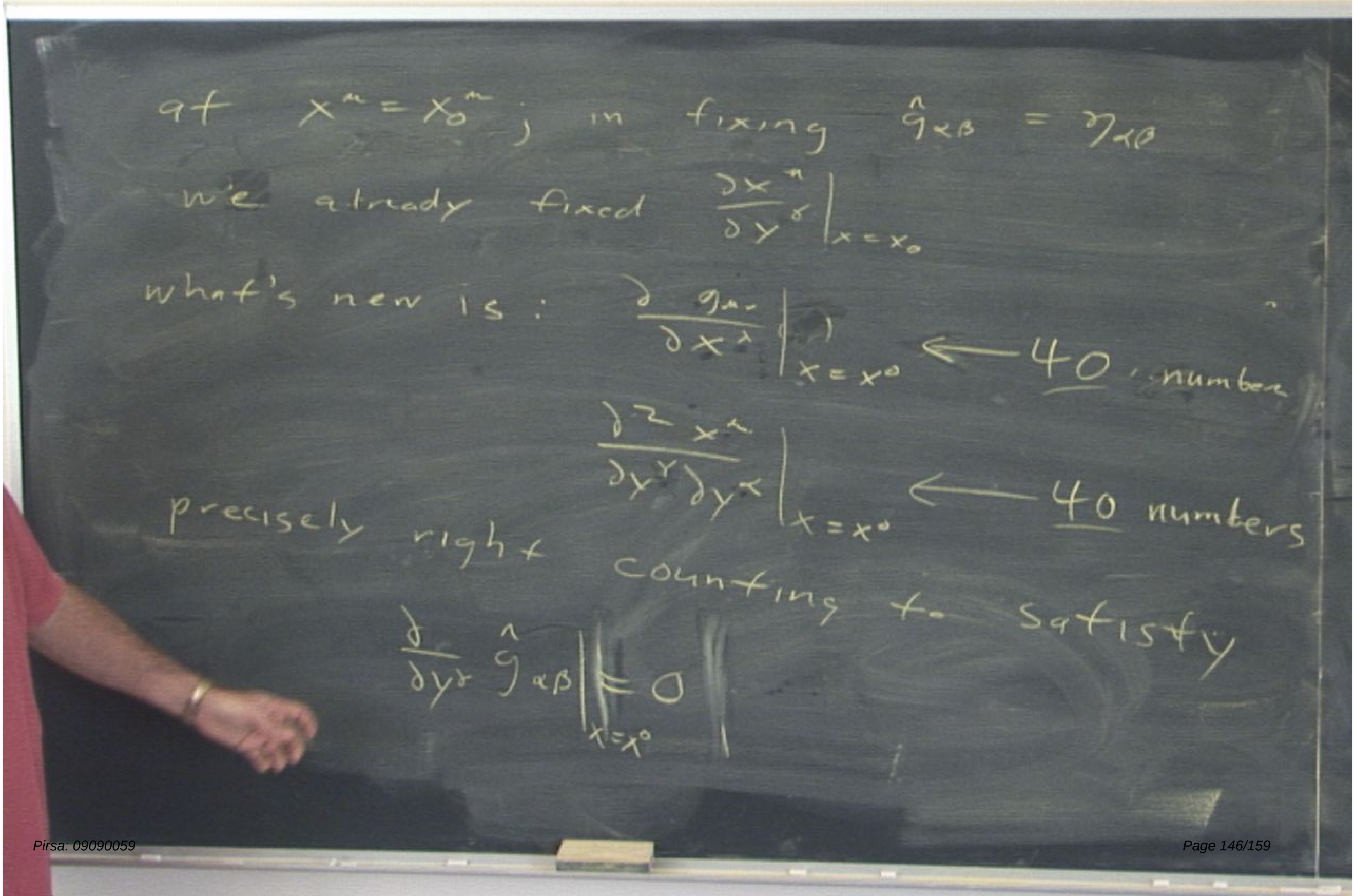


at $x^\mu = x_0^\mu$; in fixing $\hat{g}_{\alpha\beta} = \eta_{\alpha\beta}$

we already fixed $\left.\frac{\partial x^\mu}{\partial y^\sigma}\right|_{x=x_0}$

w new is: $\left.\frac{\partial g_{\mu\nu}}{\partial x^\lambda}\right|_{x=x^0} \longleftarrow$ 40 numbers

$\left.\frac{\partial^2 x^\mu}{\partial y^\gamma \partial y^\kappa}\right|_{x=x^0} \longleftarrow$



at $x^\mu = x_0^\mu$; in fixing $\hat{g}_{\alpha\beta} = \eta_{\alpha\beta}$

we already fixed $\left.\frac{\partial x^\mu}{\partial y^\delta}\right|_{x=x_0}$

what's new is: $\left.\frac{\partial g_{\mu\nu}}{\partial x^\lambda}\right|_{x=x^0} \longleftarrow \underline{40}$ numbers

$\left.\frac{\partial^2 x^\mu}{\partial y^\gamma \partial y^\alpha}\right|_{x=x^0} \longleftarrow \underline{40}$ numbers

precisely right counting to satisfy

$$\left.\frac{\partial}{\partial y^\delta} \hat{g}_{\alpha\beta}\right|_{x=x^0} = 0$$

at $x^\mu = x_0^\mu$; in fixing $\hat{g}_{\alpha\beta} = \eta_{\alpha\beta}$

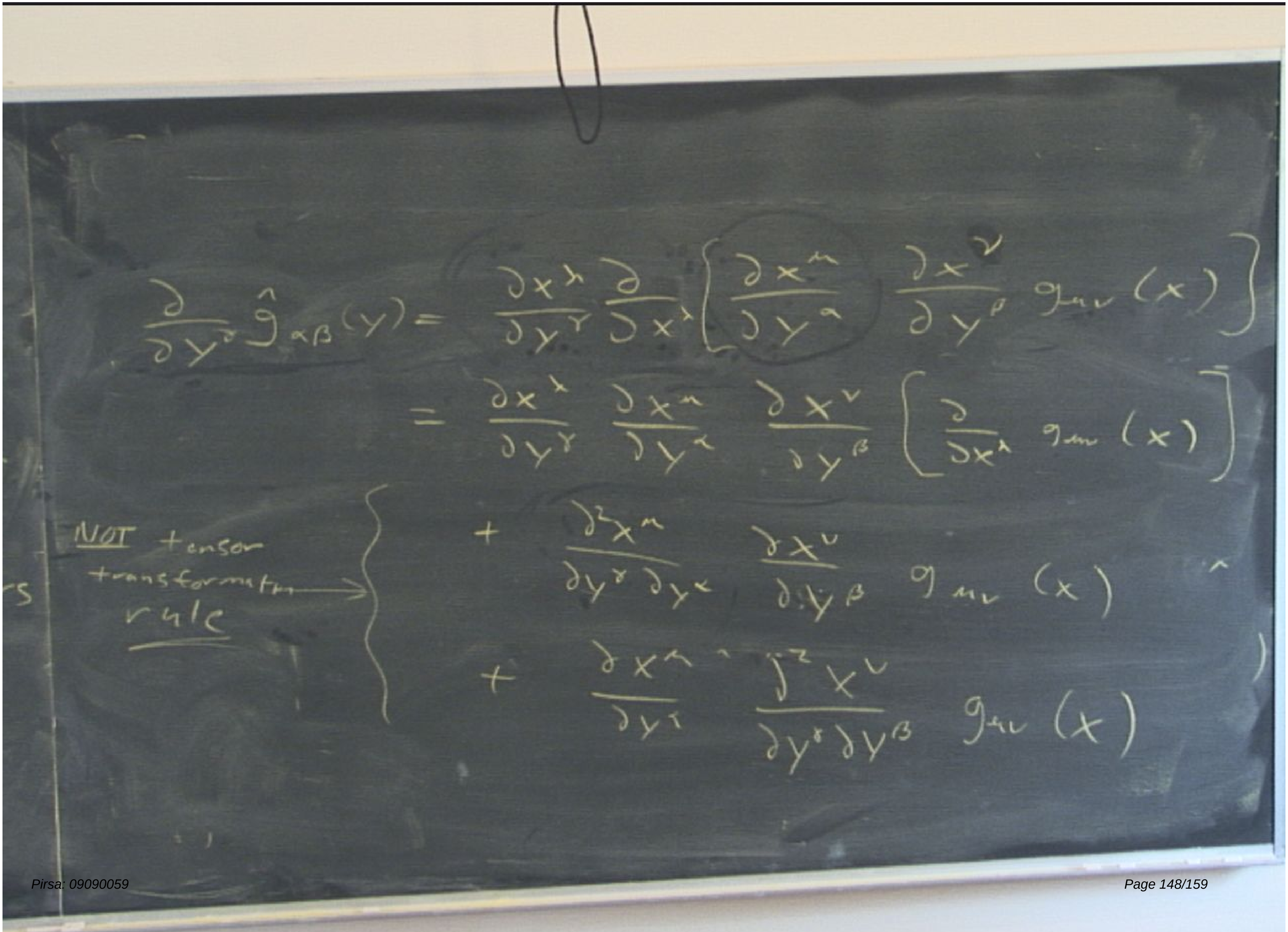
we already fixed $\left.\frac{\partial x^\mu}{\partial y^\gamma}\right|_{x=x_0}$

what's new is: $\left.\frac{\partial g_{\mu\nu}}{\partial x^\lambda}\right|_{x=x^0} \longleftarrow \underline{40}$ numbers

$$\left.\frac{\partial^2 x^\mu}{\partial y^\gamma \partial y^\alpha}\right|_{x=x^0} \longleftarrow \underline{40} \text{ numbers}$$

precisely right counting to satisfy

$$\left.\frac{\partial}{\partial y^\gamma}\hat{g}_{\alpha\beta}\right|_{x=x^0} = 0$$



$$\frac{\partial}{\partial y^\gamma}\hat{g}_{\alpha\beta}(y) = \frac{\partial x^\lambda}{\partial y^\gamma}\frac{\partial}{\partial x^\lambda}\left[\frac{\partial x^\mu}{\partial y^\alpha}\frac{\partial x^\nu}{\partial y^\beta}\, g_{\mu\nu}(x)\right]$$

$$= \frac{\partial x^\lambda}{\partial y^\gamma}\frac{\partial x^\mu}{\partial y^\alpha}\frac{\partial x^\nu}{\partial y^\beta}\left[\frac{\partial}{\partial x^\lambda}\, g_{\mu\nu}(x)\right]$$

rs

<u>NOT</u> tensor transformation rule →

$$\left\{\begin{aligned} &+ \frac{\partial^2 x^\mu}{\partial y^\gamma\,\partial y^\alpha}\frac{\partial x^\nu}{\partial y^\beta}\, g_{\mu\nu}(x) \\ &+ \frac{\partial x^\mu}{\partial y^\alpha}\frac{\partial^2 x^\nu}{\partial y^\gamma\,\partial y^\beta}\, g_{\mu\nu}(x) \end{aligned}\right.$$

at $x^\mu = x_0^\mu$; in fixing $\hat{g}_{\alpha\beta} = \eta_{\alpha\beta}$

we already fixed $\left.\frac{\partial x^\mu}{\partial y^\delta}\right|_{x=x_0}$

what's new is: $\left.\frac{\partial g_{\mu\nu}}{\partial x^\lambda}\right|_{x=x^0}$ ← $\underline{40}$ numbers

$\left.\frac{\partial^2 x^\mu}{\partial y^\gamma \partial y^\alpha}\right|_{x=x^0}$ ← $\underline{40}$ numbers

precisely right counting to satisfy

$\left.\frac{\partial}{\partial y^\delta}\hat{g}_{\alpha\beta}\right|_{x=x^0} = 0$ ← can also eliminate at $x^\mu = x_0^\mu$, in fixing $\hat{g}_{\alpha\beta} = \eta_{\alpha\beta}$

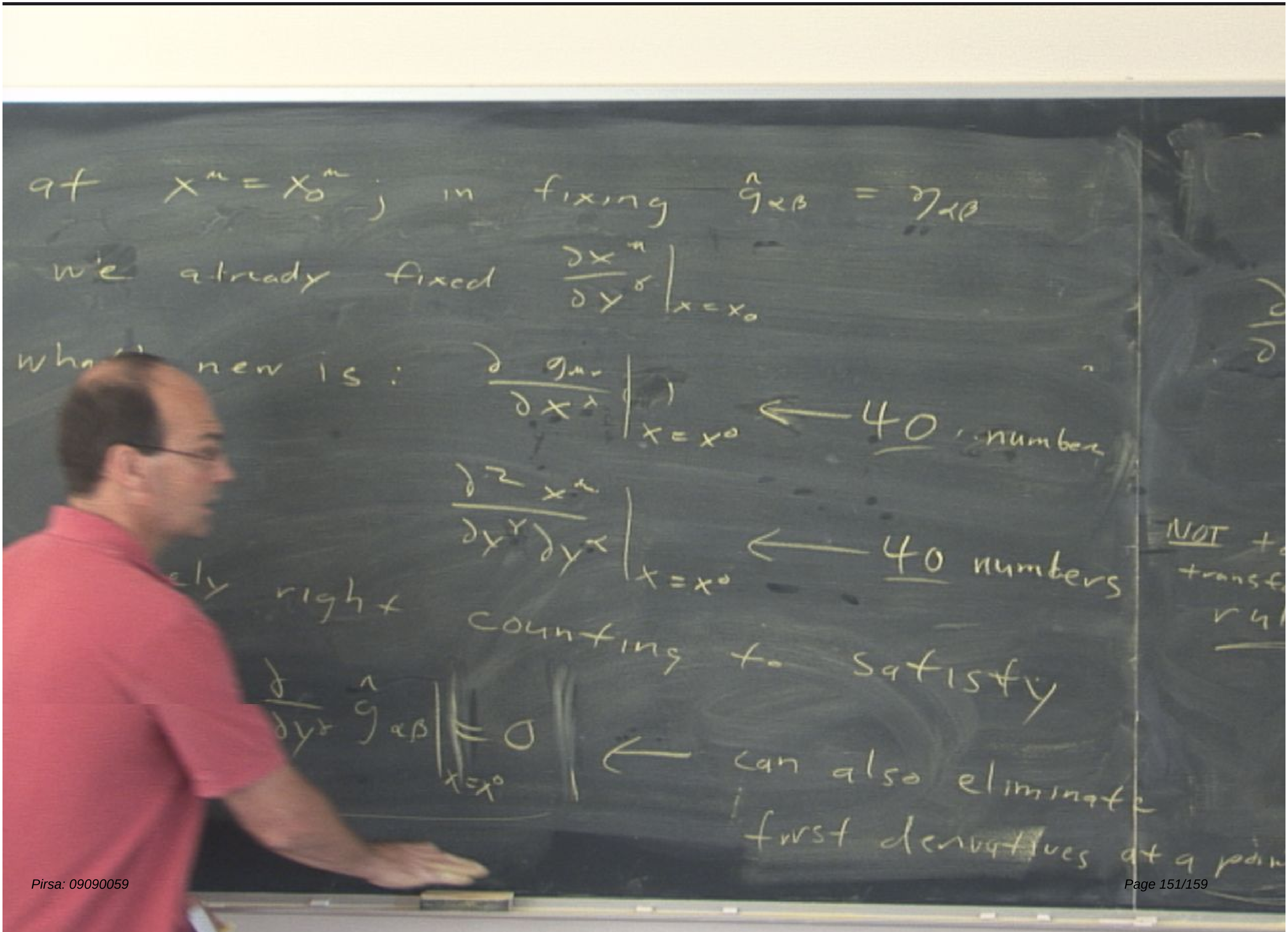
we already fixed $\left.\frac{\partial x^\mu}{\partial y^\delta}\right|_{x=x_0}$

what's new is: $\left.\frac{\partial g_{\mu\nu}}{\partial x^\lambda}\right|_{x=x^0}$ ← $\underline{40}$ number

$\left.\frac{\partial^2 x^\mu}{\partial y^\gamma \partial y^\alpha}\right|_{x=x^0}$ ← $\underline{40}$ numbers

precisely right counting to satisfy

$\left.\frac{\partial}{\partial y^\delta}\hat{g}_{\alpha\beta}\right|_{x=x^0} = 0$ ← can also eliminate first derivatives



at $x^\mu = x_0^\mu$; in fixing $\hat{g}_{\alpha\beta} = \eta_{\alpha\beta}$

we've already fixed $\left.\frac{\partial x^\mu}{\partial y^\gamma}\right|_{x=x_0}$

what's new is: $\left.\frac{\partial g_{\mu\nu}}{\partial x^\lambda}\right|_{x=x^0} \longleftarrow$ 40 numbers

$\left.\frac{\partial^2 x^\mu}{\partial y^\gamma \partial y^\alpha}\right|_{x=x^0} \longleftarrow$ 40 numbers

ely right counting to satisfy

$\left.\frac{\partial}{\partial y^\gamma}\hat{g}_{\alpha\beta}\right|_{x=x^0} = 0 \longleftarrow$ can also eliminate first derivatives at a poin

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at $x^\mu = x_0^\mu$; in fixing $\hat{g}_{\alpha\beta} = \eta_{\alpha\beta}$

we already fixed $\left.\frac{\partial x^\mu}{\partial y^\gamma}\right|_{x=x_0}$

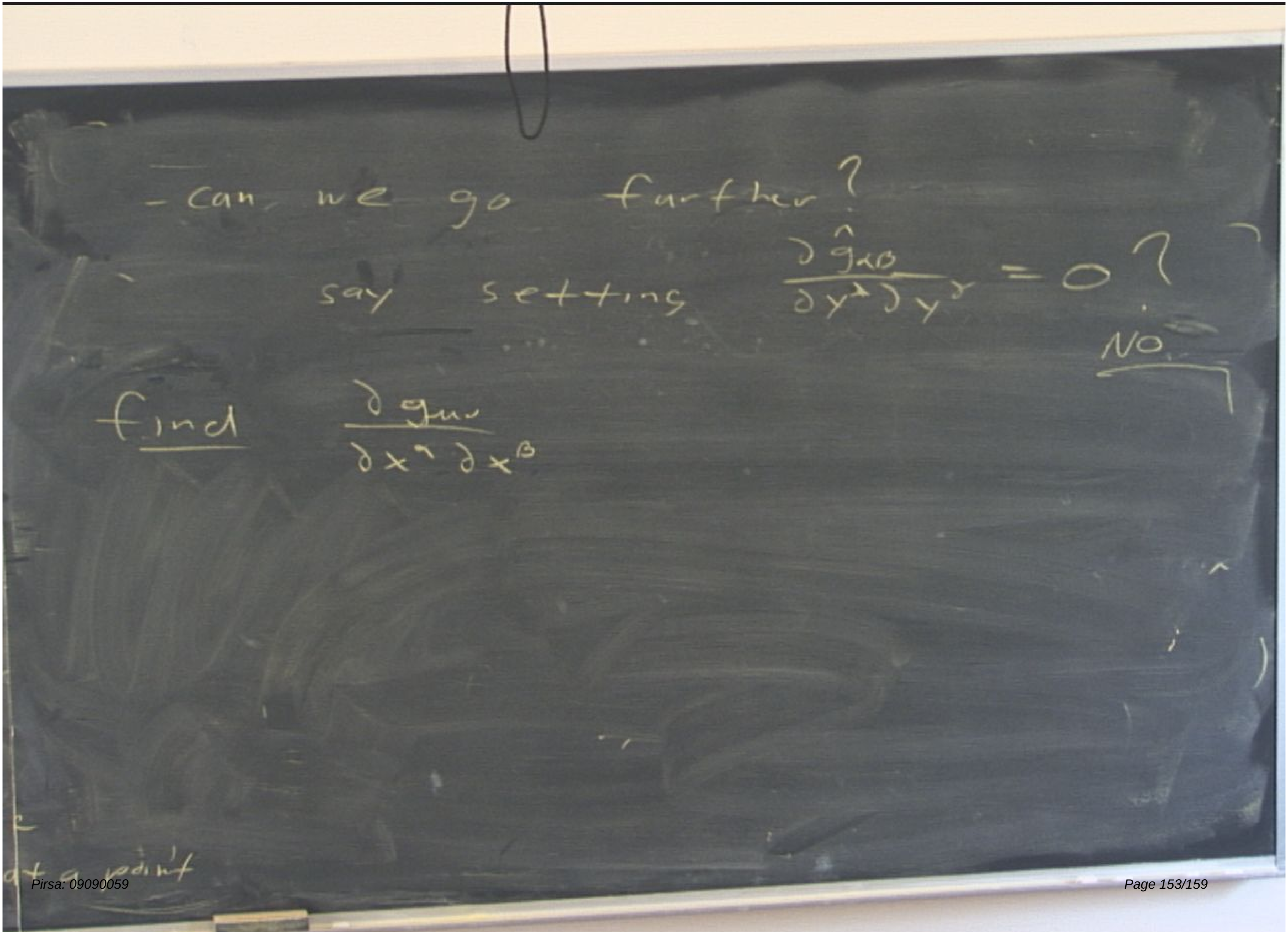
what's new is: $\left.\frac{\partial g_{\mu\nu}}{\partial x^\lambda}\right|_{x=x^0} \longleftarrow \underline{40}$ numbers

$\left.\frac{\partial^2 x^\mu}{\partial y^\gamma \partial y^\alpha}\right|_{x=x^0} \longleftarrow \underline{40}$ numbers

precisely right counting to satisfy

$\left.\frac{\partial}{\partial y^\gamma}\hat{g}_{\alpha\beta}\right|_{x=x^0} = 0 \longleftarrow$ can also eliminate first derivatives at a point

<u>NOT</u> tensor
transforma
<u>rule</u>



- can we go further?

say setting $\frac{\partial \hat{g}_{\alpha\beta}}{\partial y^{\lambda} \partial y^{\gamma}} = 0$?

NO

find $\frac{\partial g_{\mu\nu}}{\partial x^{\alpha} \partial x^{\beta}}$

at a point

- can we go further?

say setting $\frac{\partial \hat{g}_{\alpha\beta}}{\partial y^\lambda \partial y^\gamma} = 0$?

NO

<u>find</u> $\frac{\partial g_{\mu\nu}}{\partial x^\alpha \partial x^\beta} \leftarrow$ 100 new numbers

c

at a point

- can we go further?

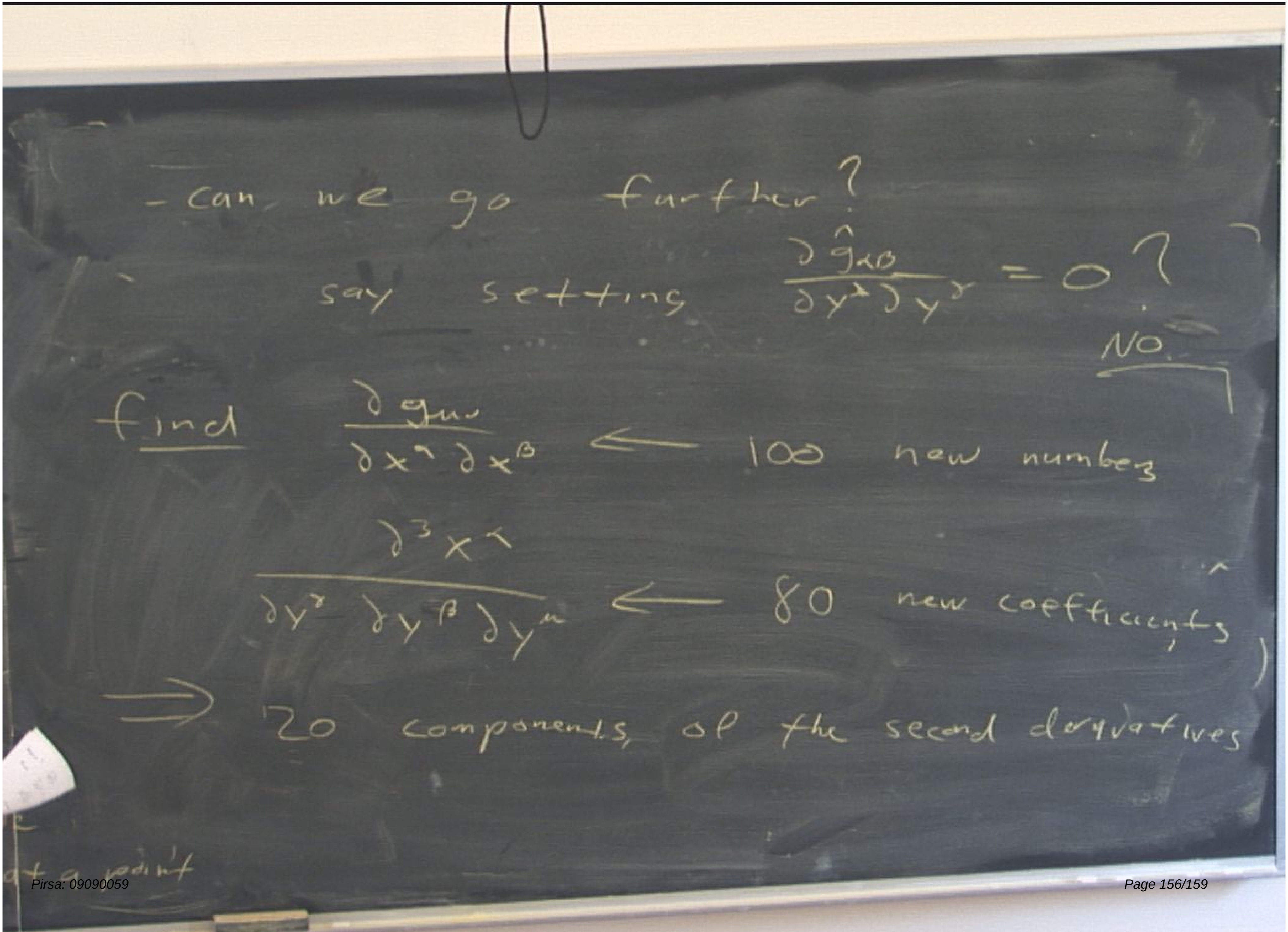
say setting $\frac{\partial \hat{g}_{\alpha\beta}}{\partial y^{\lambda} \partial y^{\gamma}} = 0$?

NO

find $\frac{\partial g_{\mu\nu}}{\partial x^{\alpha} \partial x^{\beta}}$ ⟵ 100 new numbers

$$\frac{\partial^3 x^{\alpha}}{\partial y^{\gamma}\, \partial y^{\beta}\, \partial y^{\mu}} \longleftarrow 80$$

at a point



- can we go further?

say setting $\frac{\partial \hat{g}_{\alpha\beta}}{\partial y^\lambda \partial y^\gamma} = 0$?

...

NO.

find $\frac{\partial g_{\mu\nu}}{\partial x^\alpha \partial x^\beta}$ ⟸ 100 new numbers

$\frac{\partial^3 x^\alpha}{\partial y^\gamma \partial y^\beta \partial y^\mu}$ ⟸ 80 new coefficients

$\Longrightarrow$ 20 components of the second derivatives

at a point

- can we go further?

say setting $\frac{\partial \hat{g}_{\alpha\beta}}{\partial y^\lambda \partial y^\gamma} = 0$?

NO

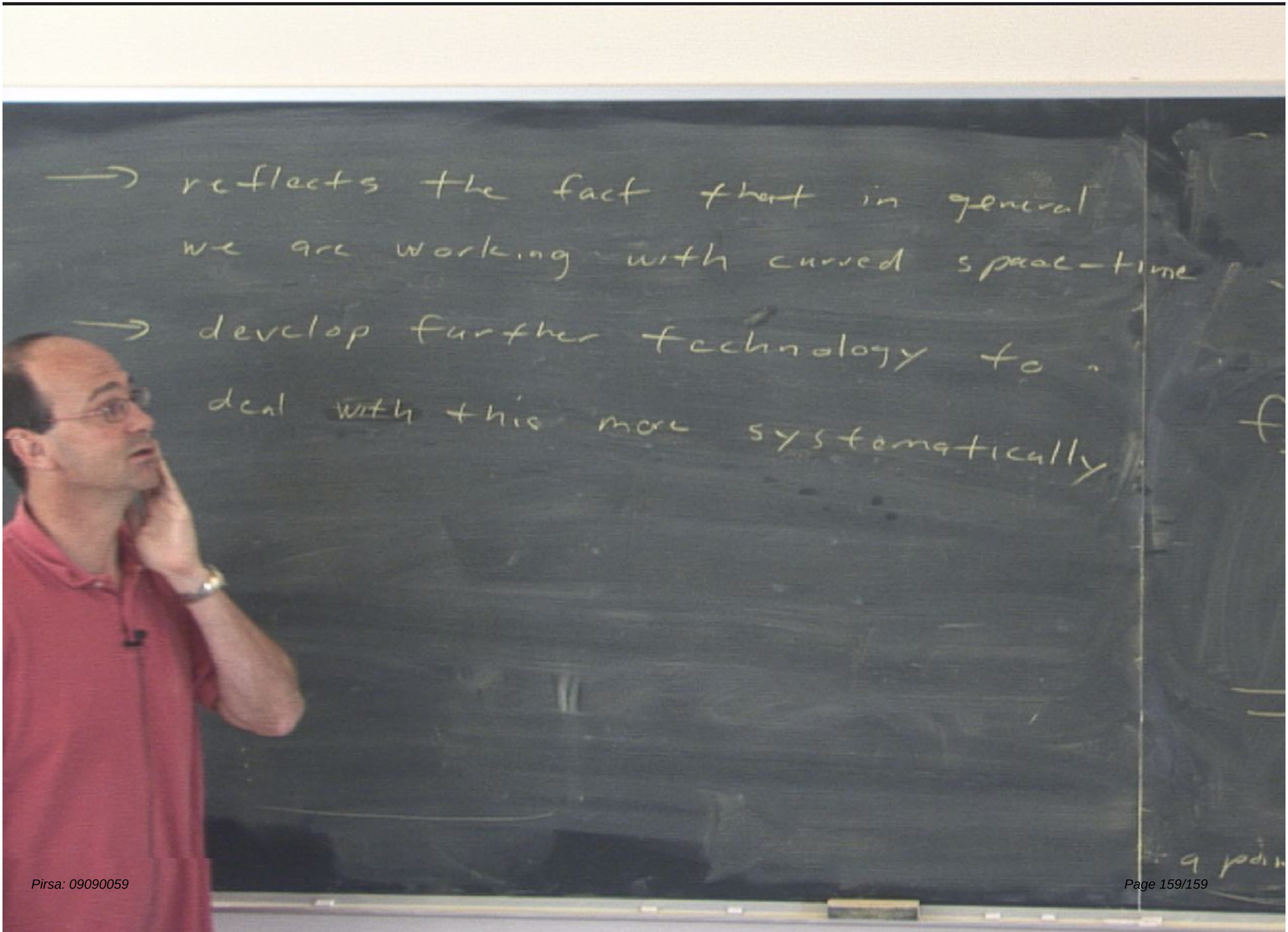
find $\frac{\partial g_{\mu\nu}}{\partial x^\alpha \partial x^\beta} \leftarrow$ 100 new numbers

$\frac{\partial^3 x^\alpha}{\partial y^\gamma \partial y^\beta \partial y^\mu} \leftarrow$ 80 new coefficients

$\Rightarrow$ 20 components of the second derivatives can NOT be set to zero in general

at a point

→ reflects the fact that in general
we are working with curved space-time



→ reflects the fact that in general we are working with curved space-time

→ develop further technology to deal with this more systematically