Title: Quantum Theory - Core (PHYS 605) - Lecture 15

Date: Sep 25, 2009 09:00 AM

URL: http://pirsa.org/09090058

Abstract:

Dirac Equation:

Charge q

Mass m

$$\left[\gamma^a\left(\partial_a + iqA_a\right) + m\right]\psi = 0$$

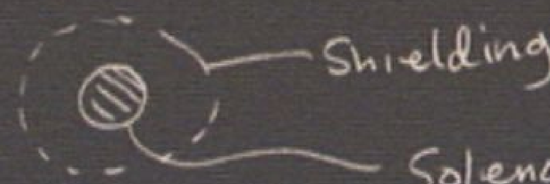

aronov – Bohm

Aharonov – Bohm

Solenoid
Huge B-field

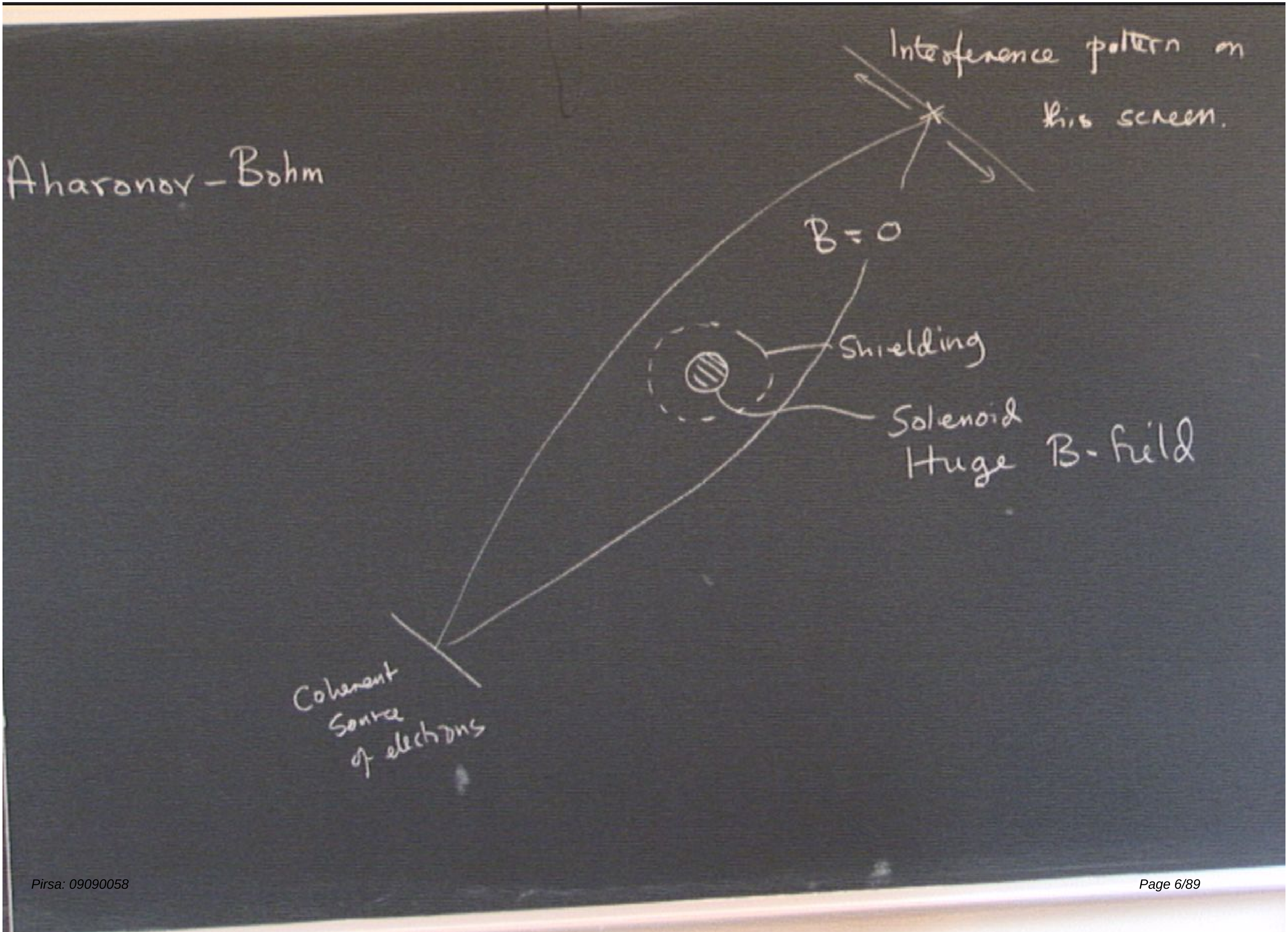
Aharonov – Bohm

$B = 0$



Huge B-field

Coherent source of electrons


Aharonov - Bohm
Interference pattern on this screen.
B = 0
Shielding
Solenoid
Huge B-field
Coherent source of electrons

Aharonov–Bohm

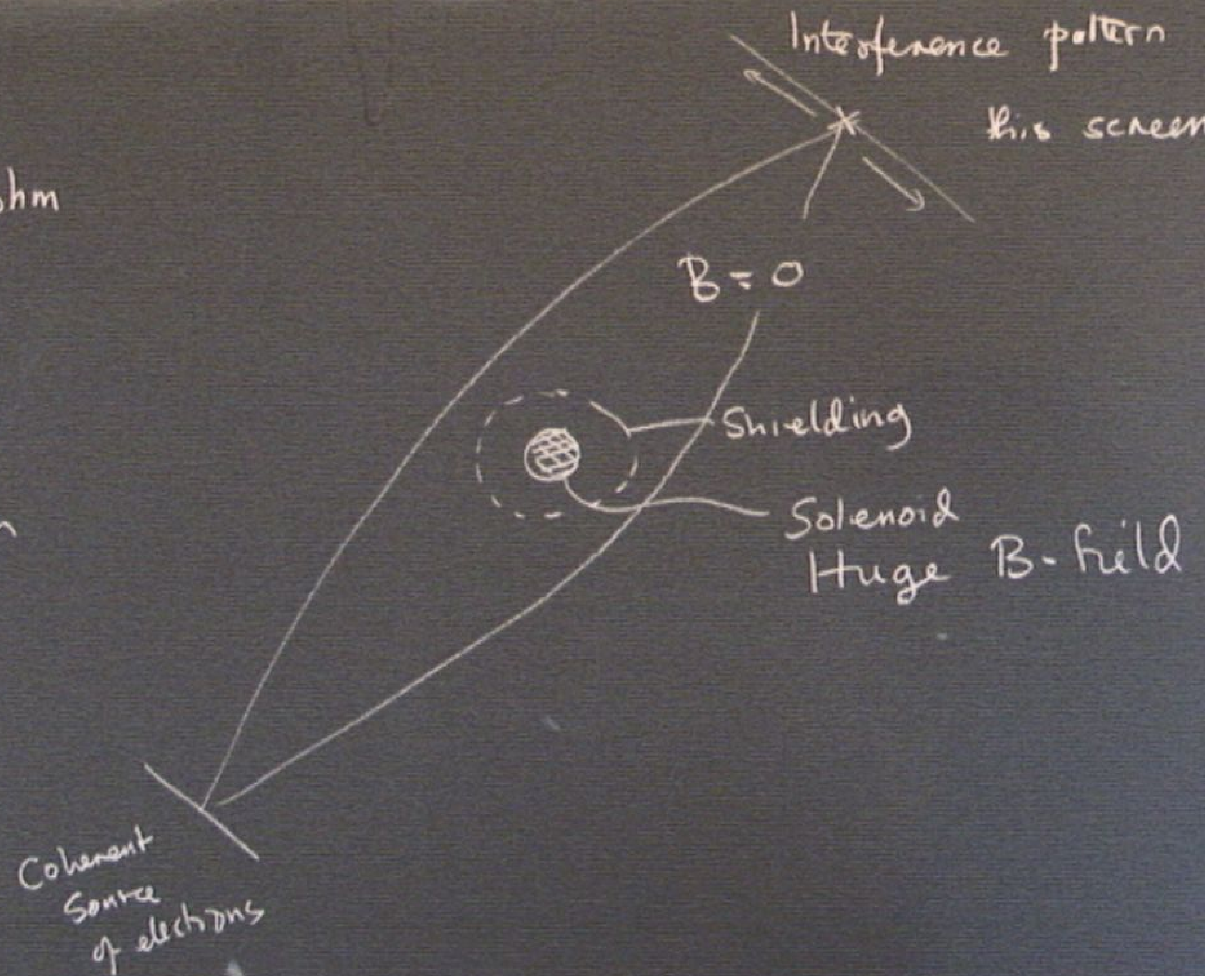
$\underline{B} = \text{curl}\,\underline{A}$

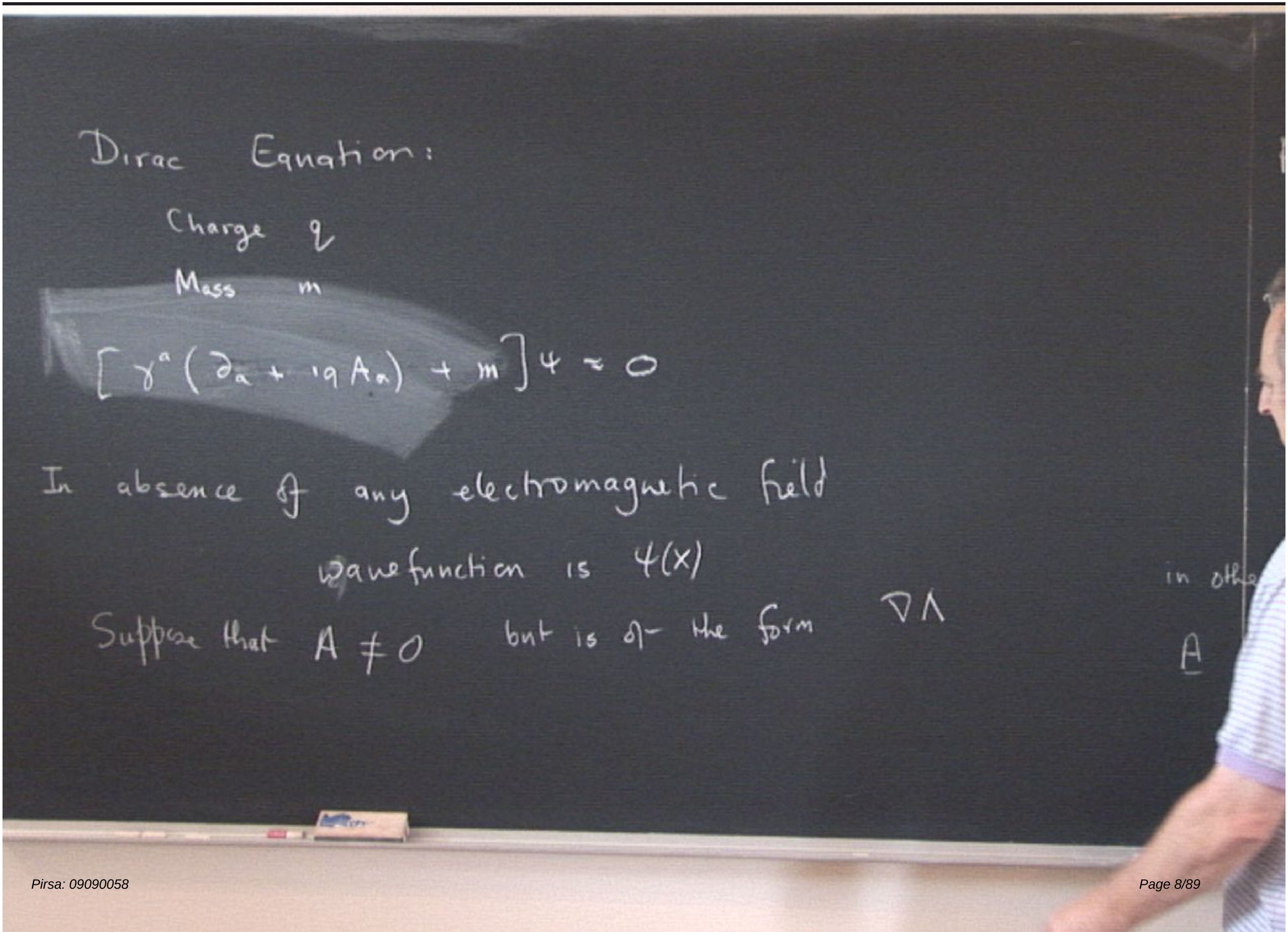
If $\underline{B} = 0$ then

$\underline{A} = \text{grad}\,\Lambda$

[I]n other words

$\underline{A}$ is pure gauge.




Dirac Equation:
Charge q
Mass m
$[\gamma^a(\partial_a + iqA_a) + m]\psi = 0$
In absence of any electromagnetic field
wavefunction is $\psi(x)$
Suppose that $A \neq 0$ but is of the form $\nabla\Lambda$
in othe
$\underline{A}$

Dirac Equation:

Charge q

Mass m

$$\left[\gamma^a\left(\partial_a + iqA_a\right) + m\right]\psi = 0$$

In absence of any electromagnetic field

wavefunction is $\psi(x)$

in other

Suppose that $A \neq 0$ but is of the form $\nabla\Lambda$

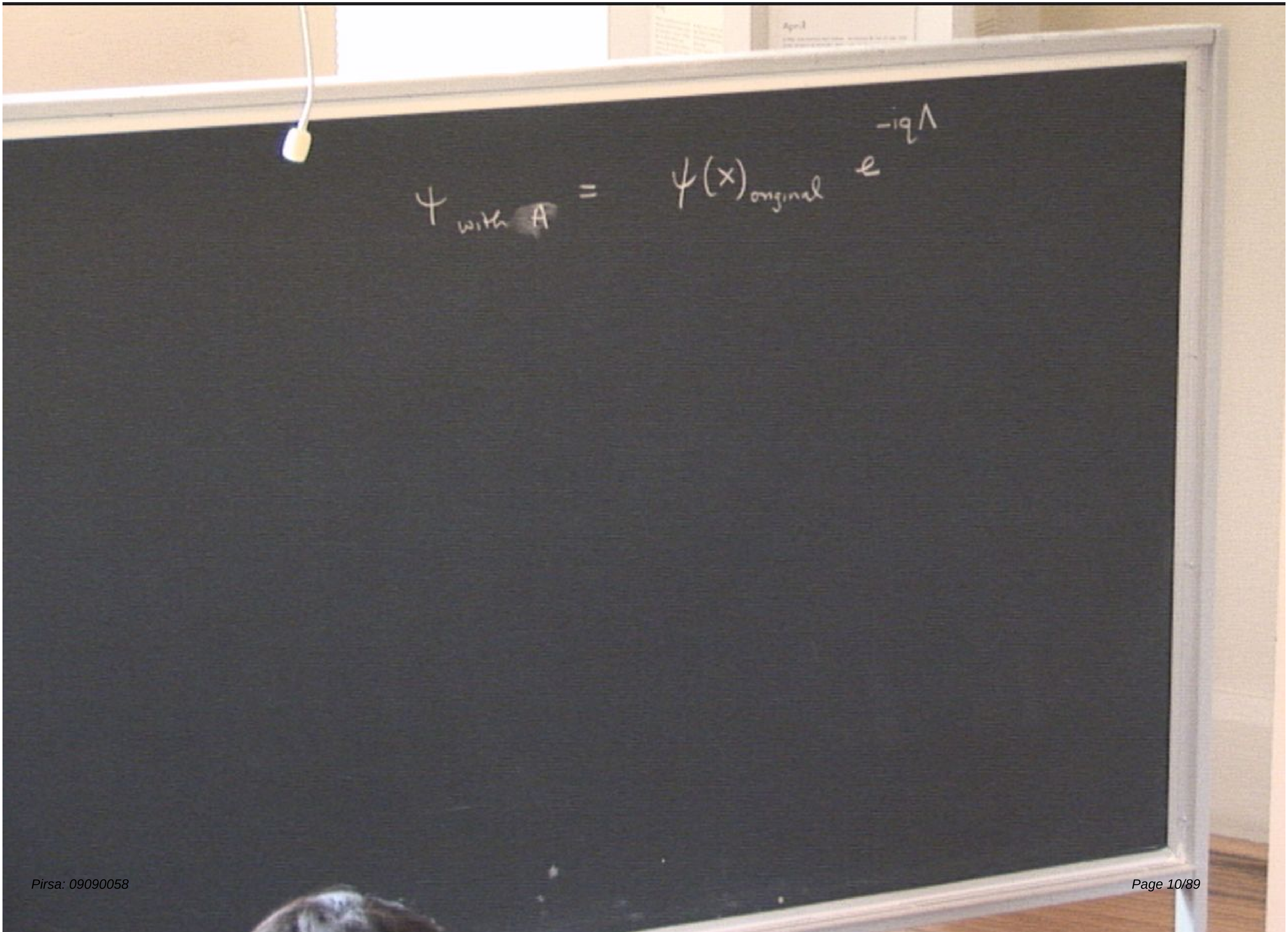
$\underline{A}$ is

$$A_i = \partial_i \Lambda$$

$$A_0 = 0$$

B

If


$\psi_{\text{with } A} = \psi(x)_{\text{original}}\, e^{-iq\Lambda}$

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Difference is phase between

AXB and AYB

$$\psi_{\text{with } A} = \psi(x)_{\text{original}}\, e^{-iq\Lambda}$$

Difference is phase between

AXB and AYB

$$X \int_A^B \nabla\Lambda \cdot d\ell$$

Phase change relative to original

$$\psi_{\text{with } A} = \psi(x)_{\text{original}} \, e^{-iq\Lambda}$$

Difference is phase between

AXB and AYB

Phase change relative to original via X $\rightarrow \displaystyle {}_{X}\!\int_{A}^{B} \nabla\Lambda \cdot d\mathbf{l}$

via Y $\displaystyle {}_{Y}\!\int_{A}^{B} \nabla\Lambda \cdot d\mathbf{l}$

$$\psi_{\text{with } A} = \psi(x)_{\text{original}}\, e^{-iq\Lambda}$$

Difference is phase between

AXB and AYB

Phase change relative to original via X $\rightarrow$

$$\int_{A\,X}^{B} \nabla\Lambda \cdot d\mathbf{l} = \Lambda(B) - \Lambda(A)$$

via Y

via Y

$$\int_{A\,Y}^{B} \nabla\Lambda \cdot d\mathbf{l} = \Lambda(B) - \Lambda(A)$$

via Y

Difference between the two

$$\oint \underline{\nabla}\Lambda \cdot d\underline{l} = \int_{AXB} \underline{\nabla}\Lambda \cdot d\underline{l} - \int_{AYB} \underline{\nabla}\Lambda \cdot d\underline{l}$$

$$= \int_{AXBYA} \nabla\Lambda \cdot dl$$

In

in other

$\underline{A}$ is

Difference between the two paths

$$\oint \underline{\nabla}\Lambda \cdot d\underline{\ell} = \int_{AXB} \underline{\nabla}\Lambda \cdot d\underline{\ell} - \int_{AYB} \underline{\nabla}\Lambda \cdot d\underline{\ell}$$

$$= \int_{AXBYA} \nabla\Lambda \cdot d\ell$$

In

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$\underline{A}$ is

Difference between the two paths

$$\oint \underline{\nabla}\Lambda \cdot d\underline{l} = \int_{AXB} \underline{\nabla}\Lambda \cdot d\underline{l} - \int_{AYB} \underline{\nabla}\Lambda \cdot d\underline{l}$$

$$= \int_{AXBYA} \nabla\Lambda \cdot d\underline{l}$$

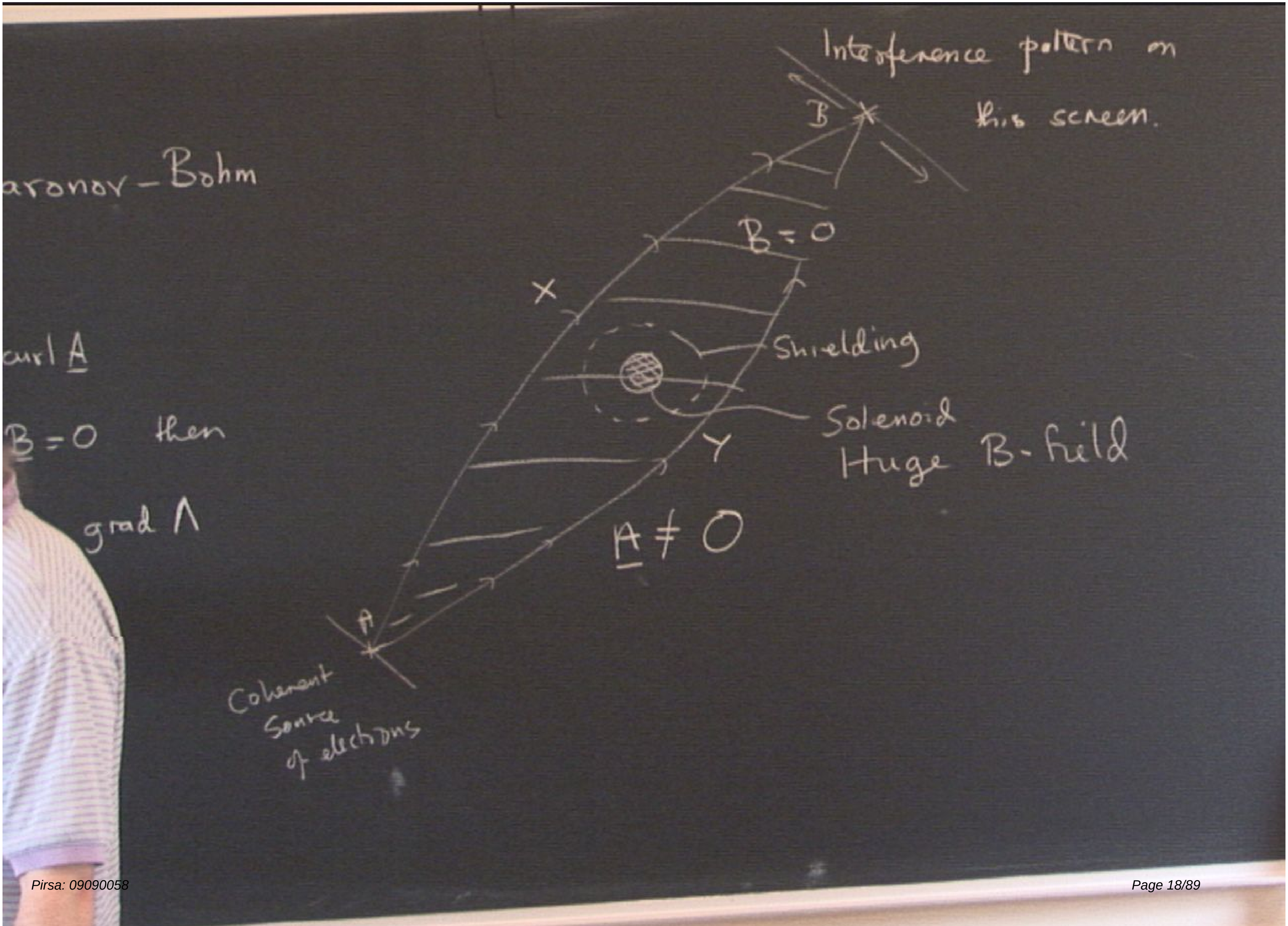
$$= \int \underline{A} \cdot d\underline{l}$$

$$= \int \underline{B} \cdot d\underline{S}$$

In

in other

$\underline{A}$ is



Interference pattern on
this screen.
B
aronov-Bohm
B = 0
X
curl A
Shielding
B = 0 then
Solenoid
Huge B-field
Y
grad Λ
A ≠ 0
A
Coherent
Source
of electrons

$$\ldots \cdot d\underline{l} - \int_{AYB} \underline{\nabla}\Lambda \cdot d\underline{l}$$

$$\ldots \Lambda \cdot d\underline{l}$$

$\ldots d\underline{l}$ Stokes theorem.

$\ldots d\underline{S}$

Aharonov – Bohm

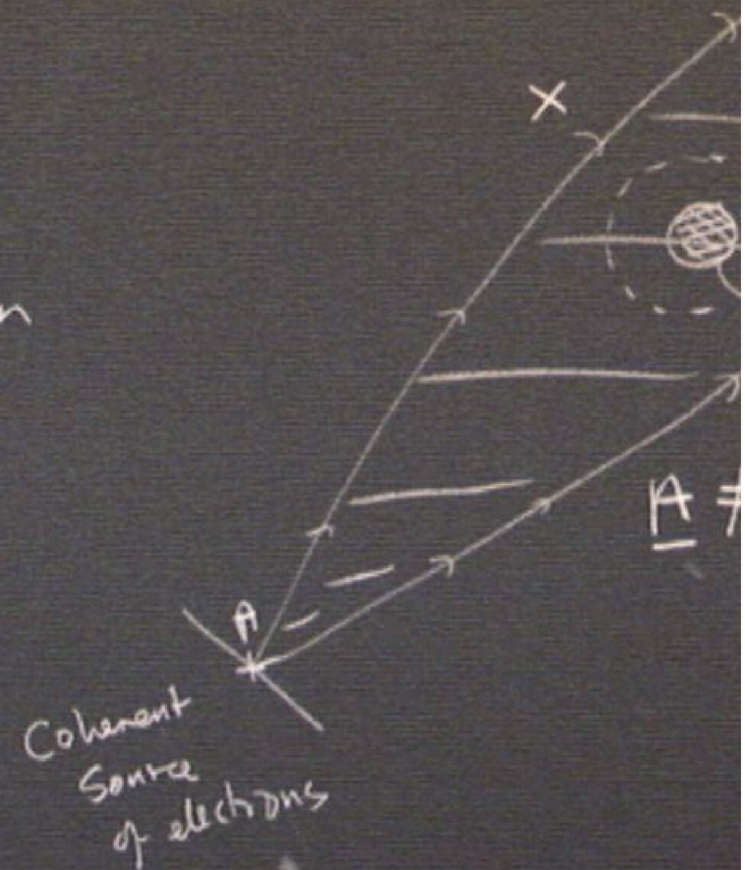
$\underline{B} = \mathrm{curl}\,\underline{A}$

If $\underline{B} = 0$ then

$\underline{A} = \mathrm{grad}\,\Lambda$

in other words

$\underline{A}$ is 'pure gauge'



...fference is

...$]_A$ along AxB

$$\psi_{\text{with } A} = \psi(x)_{\text{original}}\; e^{-iq\Lambda}$$

Difference is phase between

AXB and AYB

$$X\int_A^B \nabla\Lambda \cdot d\ell = \Lambda(B) - \Lambda(A)$$

via Y

Phase change relative to original

via X

$$\text{via } Y \quad Y\int_A^B \nabla\Lambda \cdot d\ell = \Lambda(B) - \Lambda(A)$$

via Y

$$\psi_{\text{with } A} = \psi(x)_{\text{original}}\, e^{-iq\Lambda}$$

hysical diffence is phases

$$q\int_{AXB} \nabla\Lambda \cdot d\ell = q[\Lambda(B) - \Lambda(A)]$$

via X

$$q\int_{AYB} \nabla\Lambda \cdot d\ell = q[\Lambda(B) - \Lambda(A)]$$

via Y

Diffenence is phase between AXB and AYB

Phase change relative to original

via X → $X\int_A^B \nabla\Lambda \cdot d\ell$

via Y $Y\int_A^B \nabla\Lambda \cdot d\ell = \Lambda(B)$

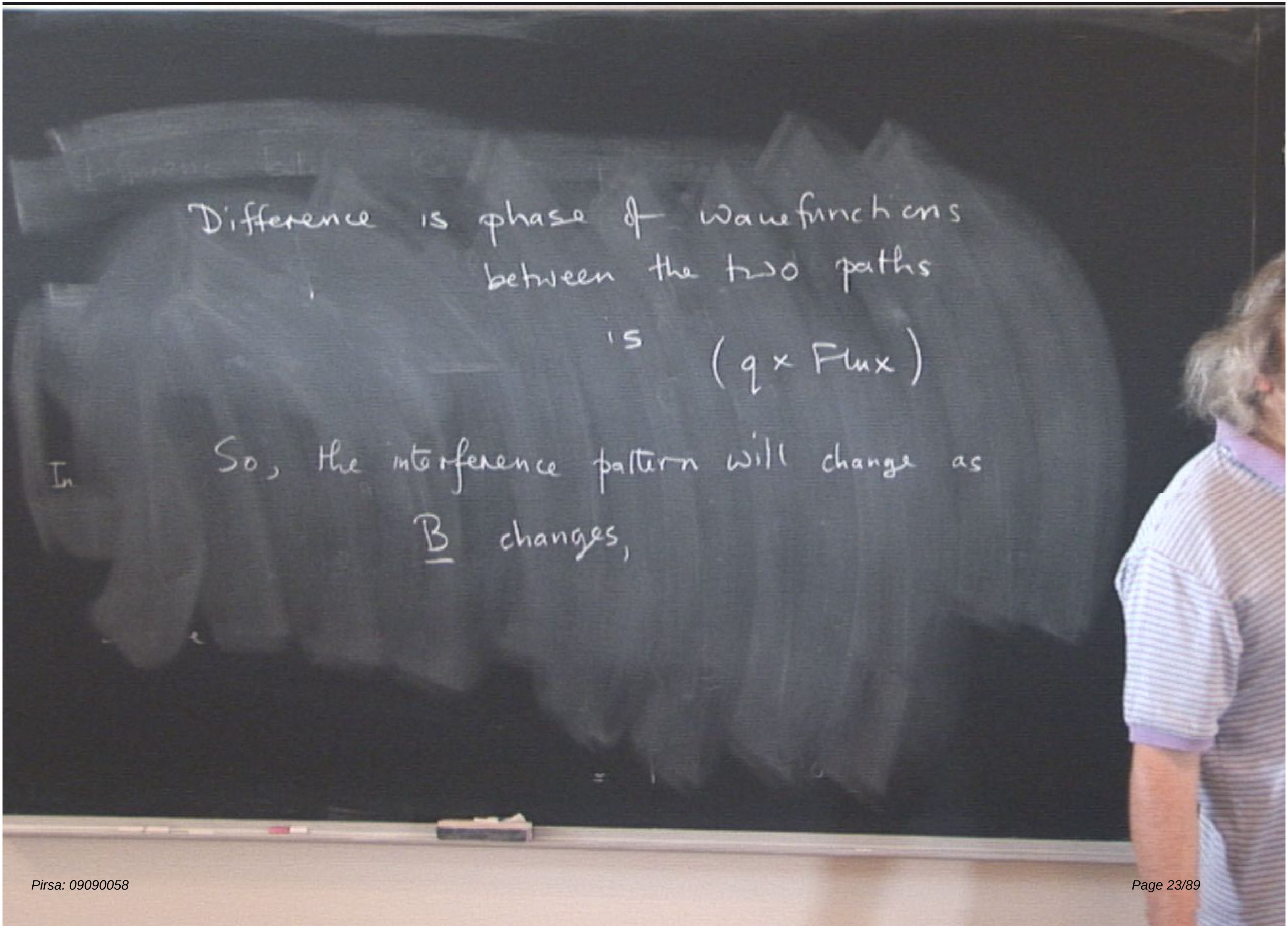
Difference is phase of wavefunctions between the two paths

is $(q \times \text{Flux})$

In

in other

$\underline{A}$ is


Difference is phase of wavefunctions
between the two paths
is (q × Flux)
So, the interference pattern will change as
B changes,

Difference is phase of wavefunctions between the two paths

is $(q \times \text{Flux})$

So, the interference pattern will change as $\underline{B}$ changes, even though the magnetic field experienced by the particles vanishes.

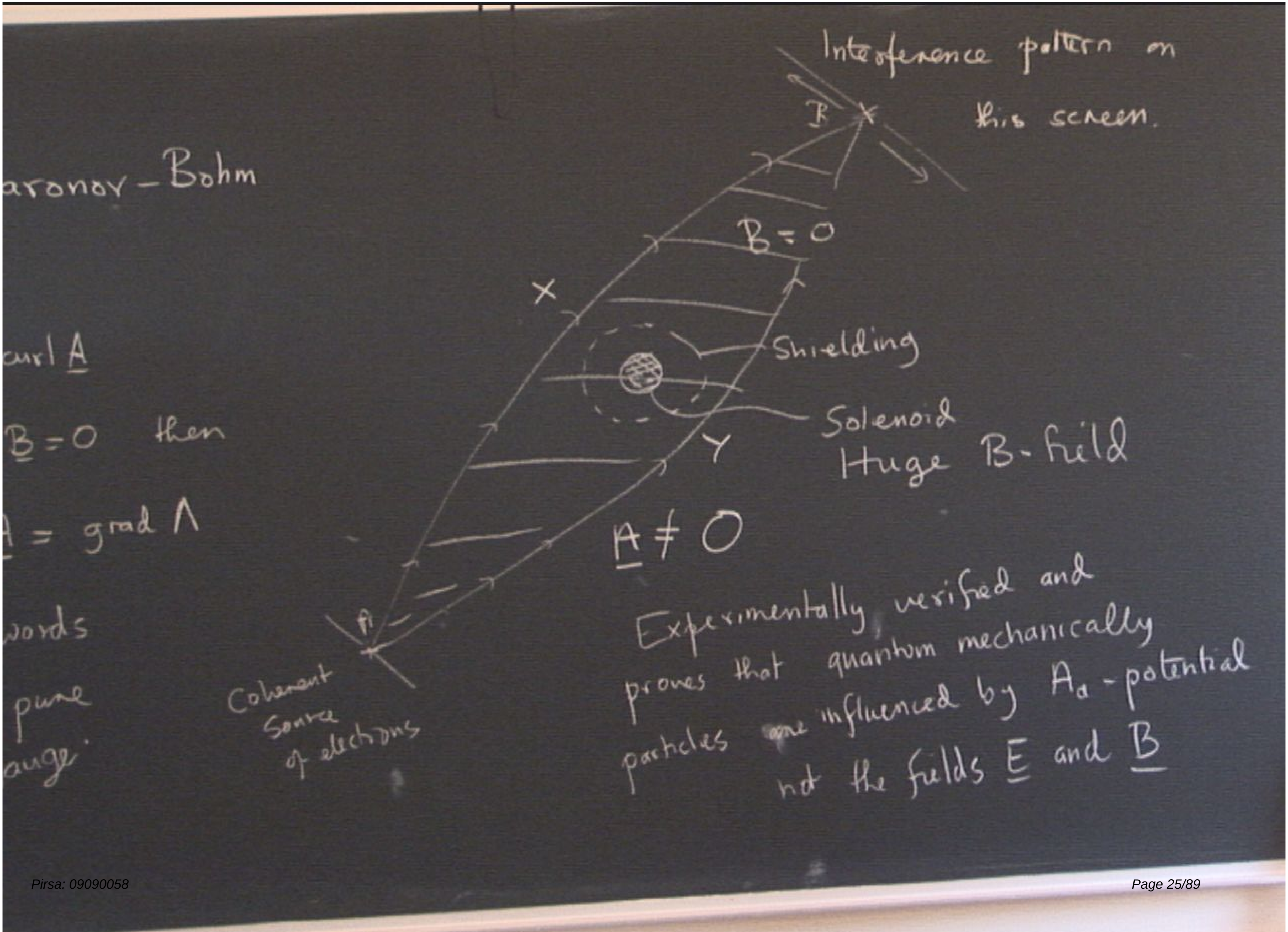
In

$\underline{B}$

If

in other

$\underline{A}$ is


aronov-Bohm
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Interference pattern on this screen.
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A ≠ 0
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Coherent source of electrons
Experimentally verified and proves that quantum mechanically particles are influenced by A_α-potential not the fields E and B

$$\psi_{\text{with } A} = \psi(x)_{\text{original}} \, e^{-iq\Lambda}$$

ffence is

s

$$d\ell = q\big[\Lambda(B) - \Lambda(A)\big]_{\text{via } X}$$

Difference is phase between

AXB and AYB

$$\int_{X,\,A}^{B} \nabla\Lambda \cdot d\underline{\ell} = \Lambda(B) - \Lambda(A)_{\text{via } Y}$$

Phase change relative to original via X

$$d\ell = q\big[\Lambda(B) - \Lambda(A)\big]_{\text{via } Y}$$

$$\text{via } Y \quad Y\int_A^B \nabla\Lambda \cdot d\underline{\ell} = \Lambda(B) - \Lambda(A)_{\text{via } Y}$$

$$\oint \underline{A} \cdot d\underline{\ell} = \oint \nabla\Lambda \cdot d\underline{\ell} = \Lambda(2\pi) - \Lambda(0)$$

$$= \int \underline{B} \cdot d\underline{S}$$

aronov – Bohm

curl $\underline{A}$

$\underline{B} = 0$ then

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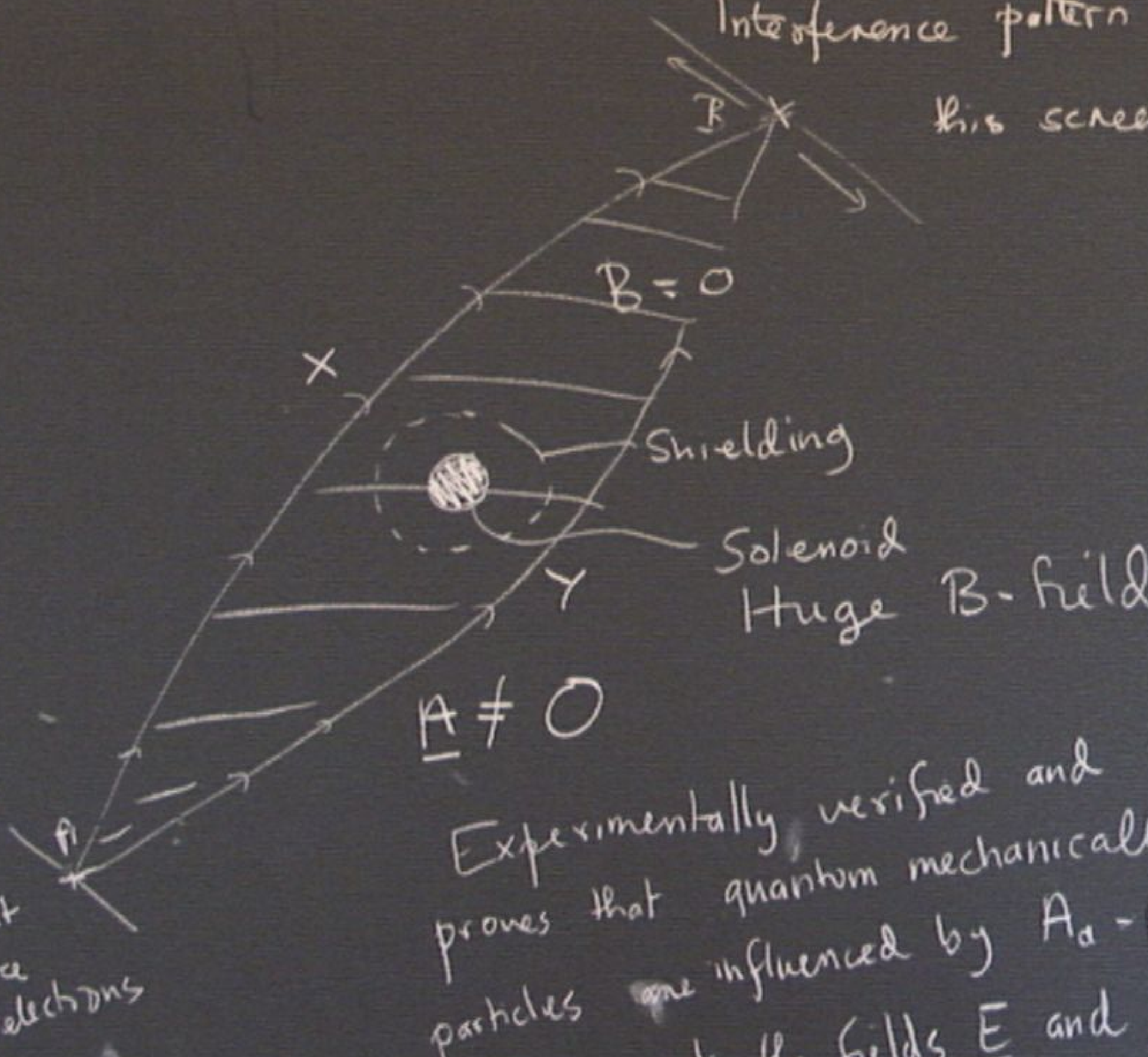
pure

auge'



$\underline{A} \neq 0$

Experimentally verified and
proves that quantum mechanically
particles are influenced by A_α - potential
not the fields $\underline{E}$ and $\underline{B}$

$$\psi_{\text{with } A} = \psi(x)_{\text{original}}\, e^{-iq\Lambda}$$

...fference is
...s

$$\ldots d\ell = q\big[\Lambda(B) - \Lambda(A)\big]_{\text{via } X}$$

$$\ldots d\ell = q\big[\Lambda(B) - \Lambda(A)\big]_{\text{via } Y}$$

Difference is phase between

AXB and AYB

Phase change relative to original via X

$$\int_{A\,(X)}^{B} \nabla\Lambda \cdot d\ell = \Lambda(B) - \Lambda(A) \quad \text{via } X$$

via Y

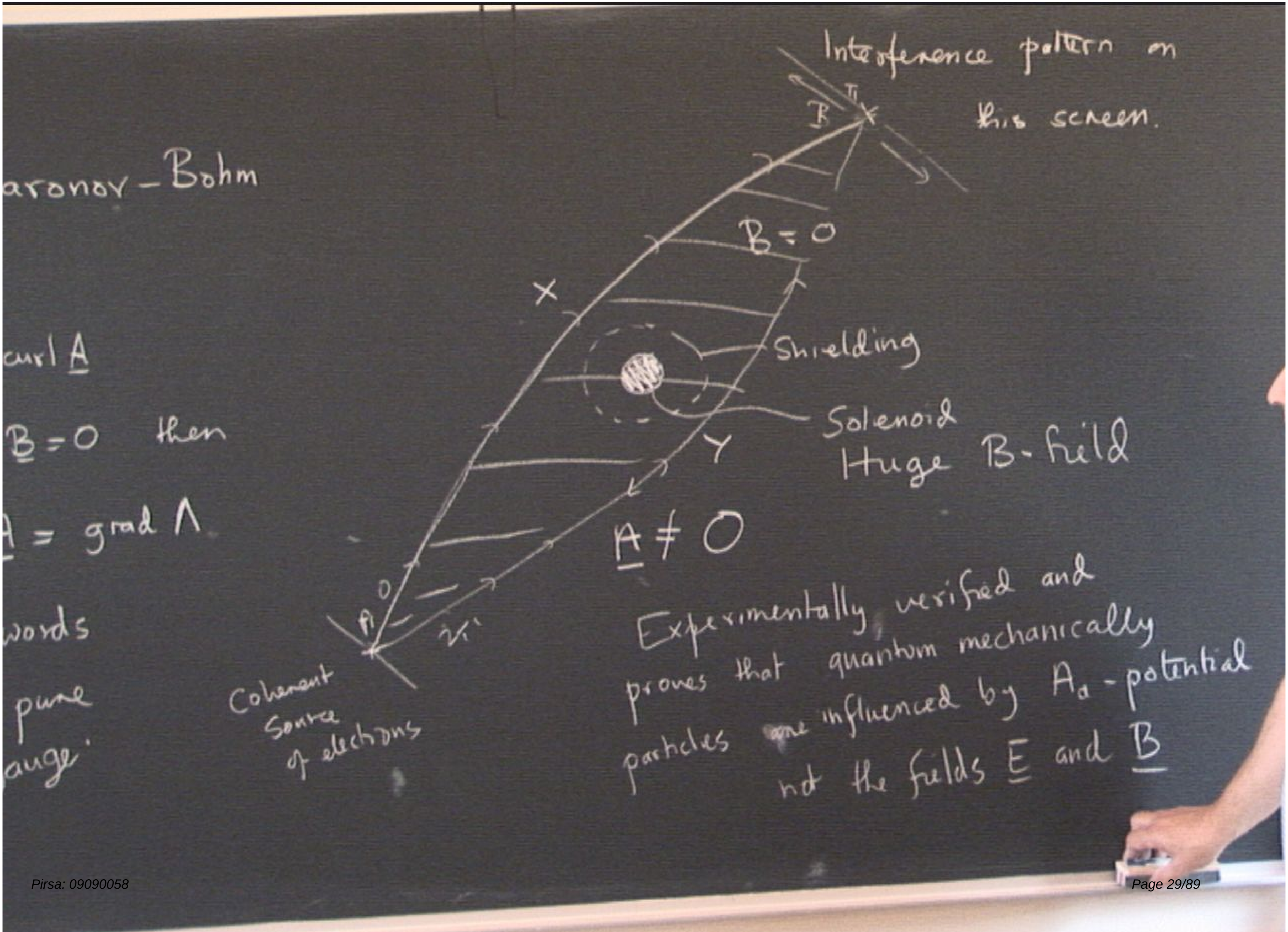
$$\int_{A\,(Y)}^{B} \nabla\Lambda \cdot d\ell = \Lambda(B) - \Lambda(A) \quad \text{via } Y$$

$$\oint \underline{A} \cdot d\ell = \oint \nabla\Lambda \cdot \underline{d\ell} = \Lambda(2\pi) - \Lambda(0)$$

$$\downarrow$$

$$\Lambda(\pi)_{AXB} - \Lambda(\pi)_{AYB}$$

$$= \int \underline{B} \cdot \underline{dS}$$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aronov – Bohm

curl $\underline{A}$

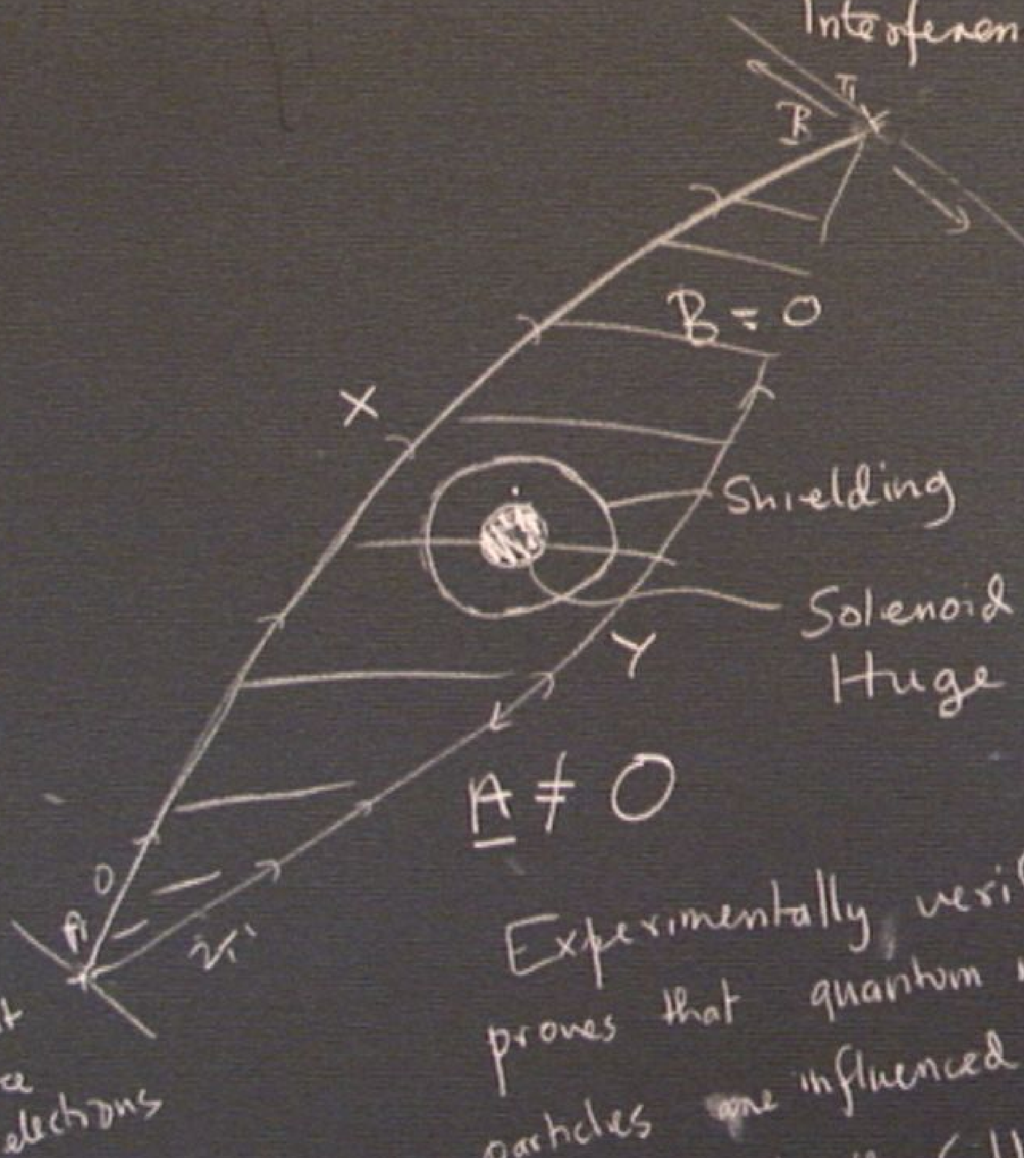
$\underline{B} = 0$ then

$\underline{A}$ = grad Λ

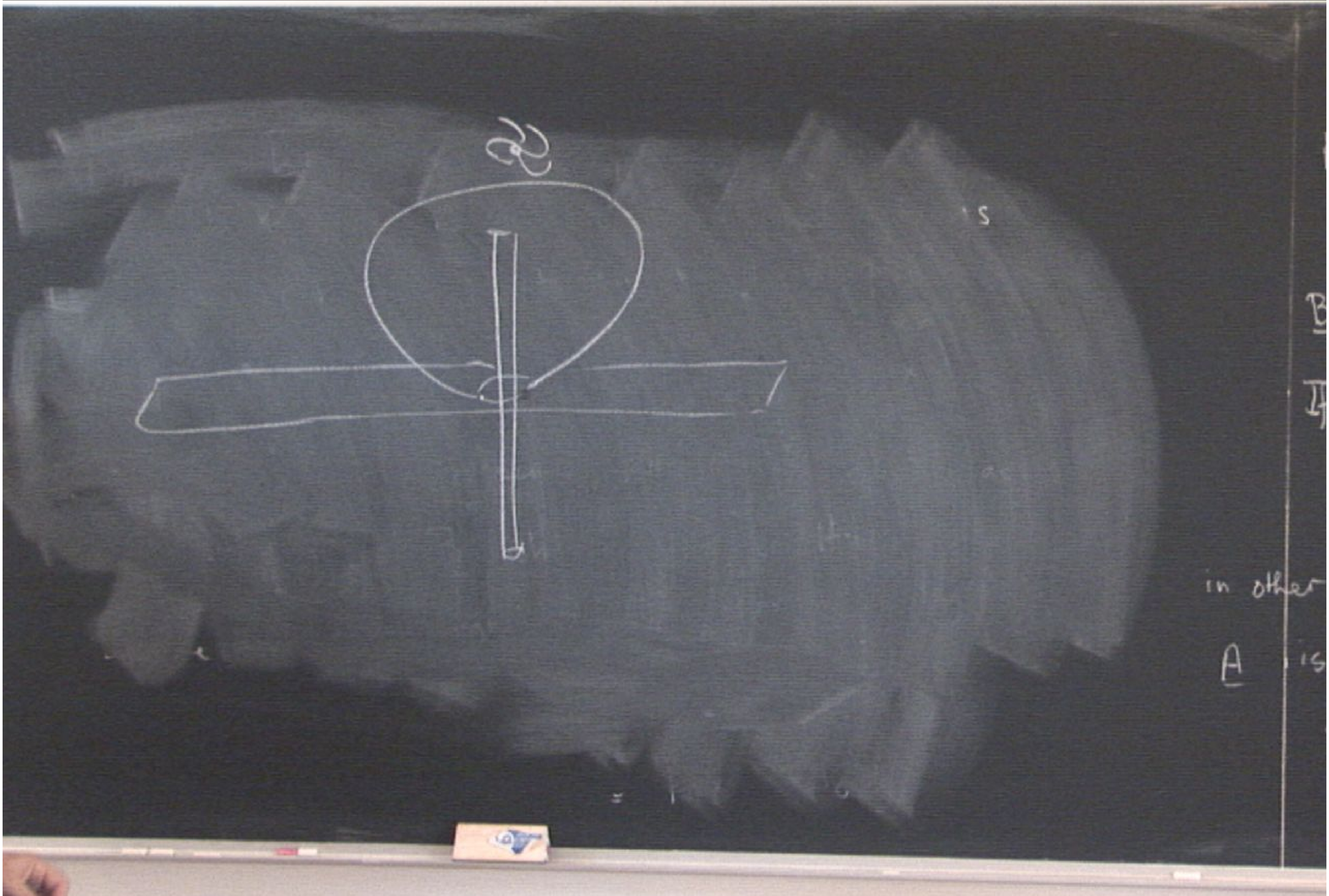
words

pure

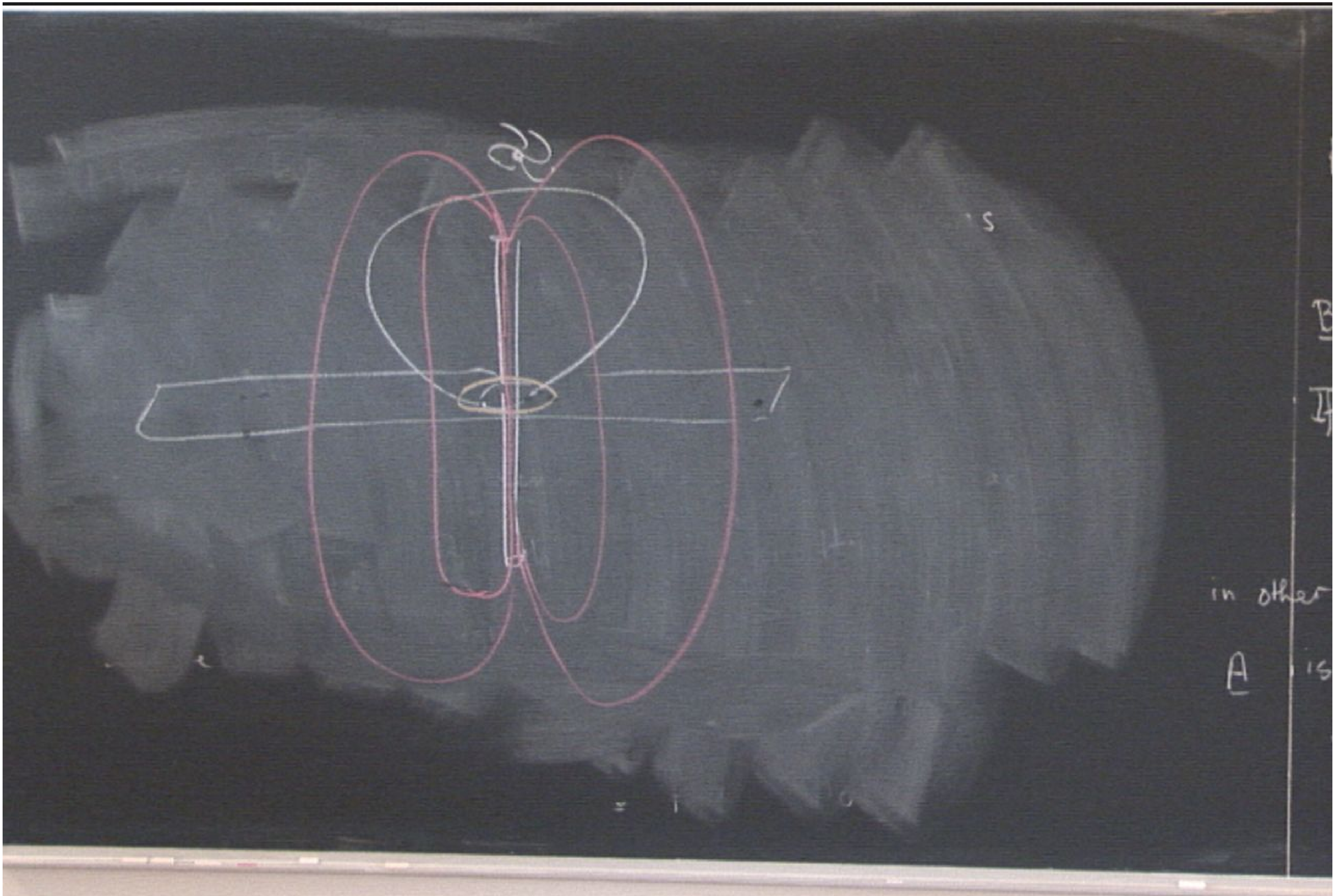
auge.

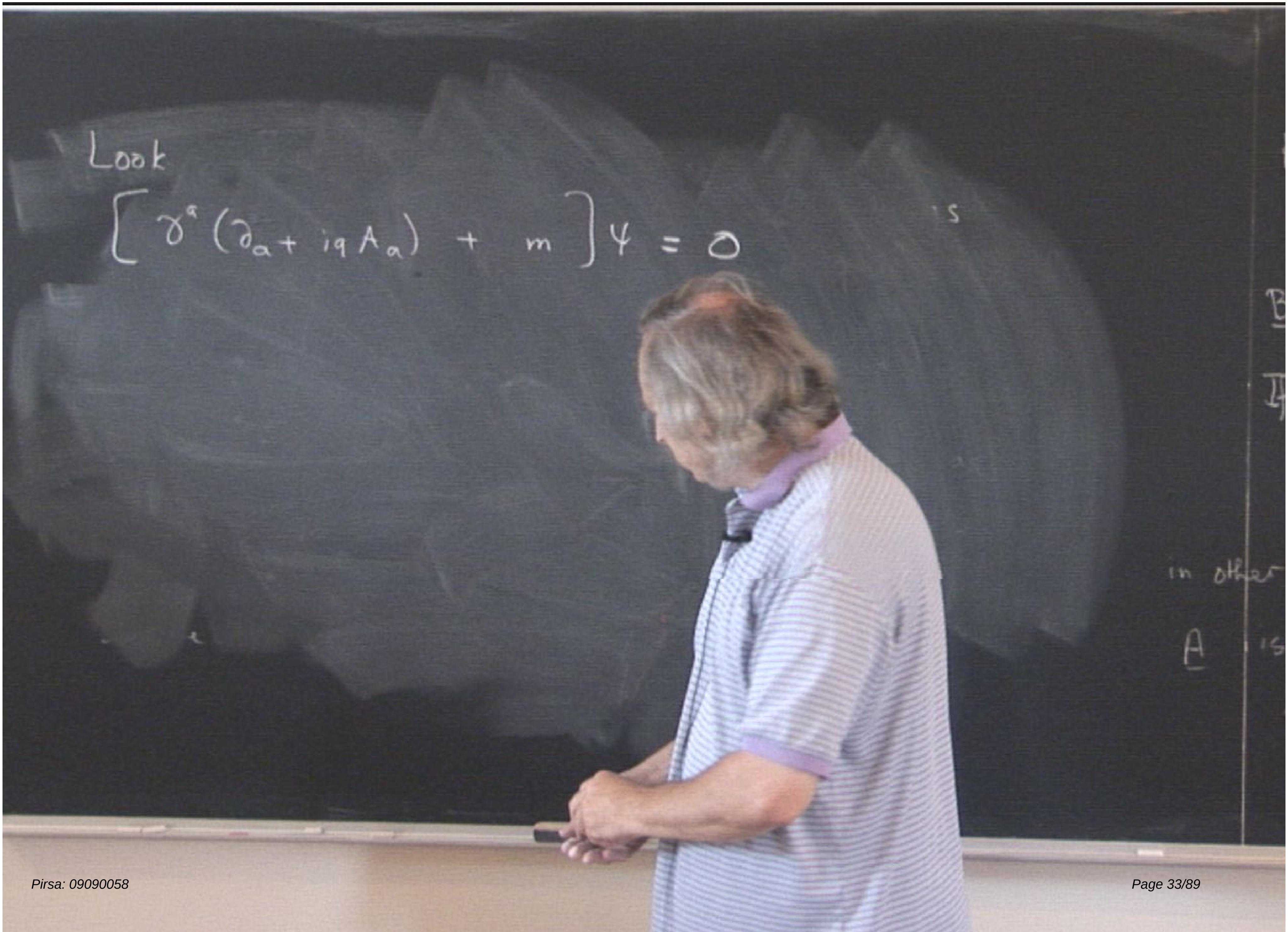


Experimentally verified and proves that quantum mechanically particles are influenced by A_α - potential not the fields $\underline{E}$ and $\underline{B}$



in other
A is




Look
$\left[\gamma^a(\partial_a + iqA_a) + m\right]\psi = 0$
in other
$\underline{A}$ is

$$\gamma^0 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}$$

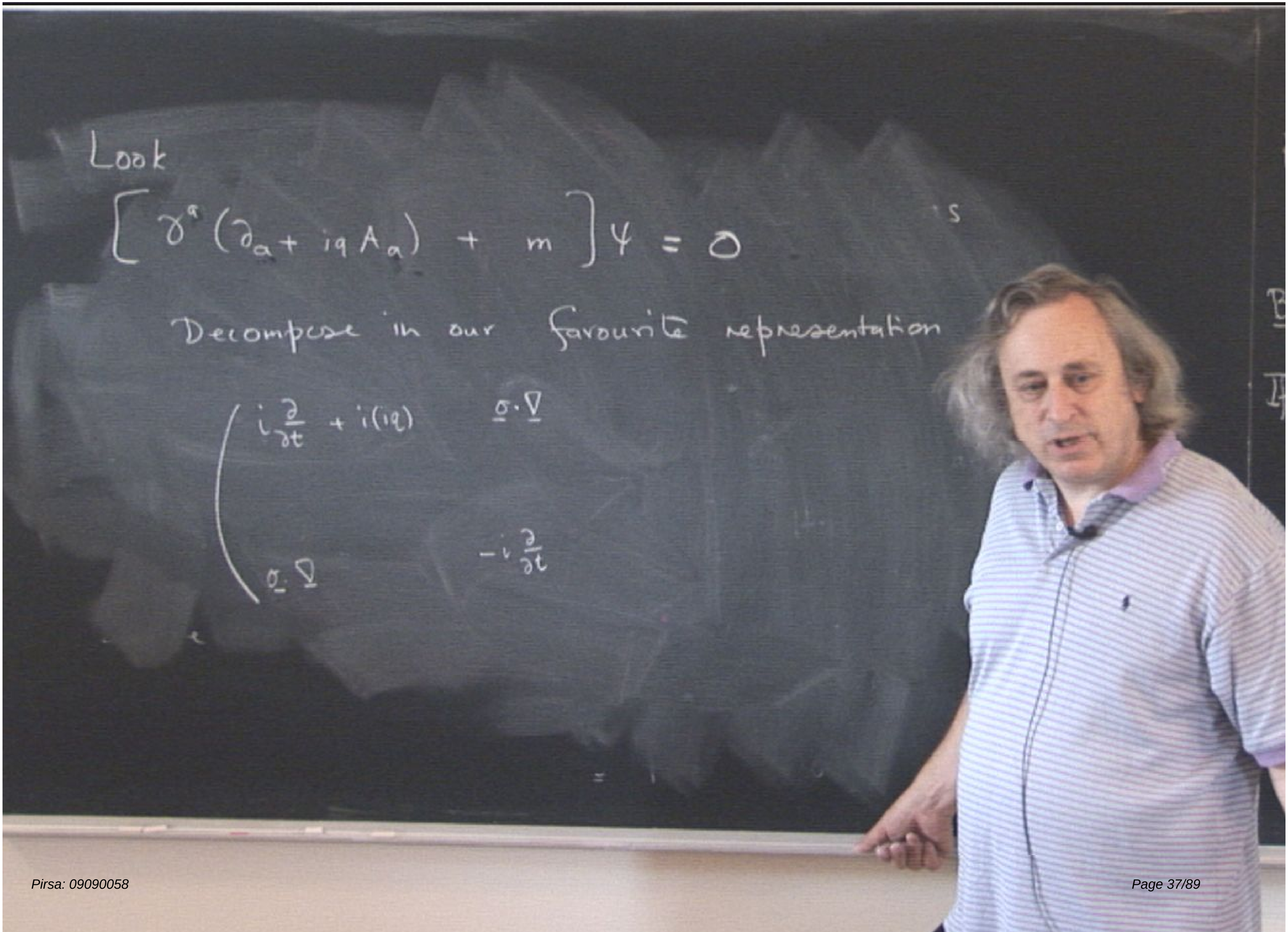
$$\gamma^0 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

$$A^a = \left(\phi, \underline{A} \right)$$

$$\gamma^0 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}$$

$$A^a = (\phi, \underline{A})$$

$$\psi = \begin{pmatrix} \chi \\ \Phi \end{pmatrix}$$


Look
$\left[\gamma^a(\partial_a + iqA_a) + m\right]\psi = 0$
Decompose in our favourite representation
$i\frac{\partial}{\partial t} + i(iq)$
$\underline{\sigma}\cdot\underline{\nabla}$
$\underline{\sigma}\cdot\underline{\nabla}$
$-i\frac{\partial}{\partial t}$

Look

$$\left[\gamma^a (\partial_a + iqA_a) + m \right] \psi = 0$$

Decompose in our favourite representation

$$\left(\begin{array}{cc} i\frac{\partial}{\partial t} + i(iq)(-\phi) & \underline{\sigma}\cdot\underline{\nabla} \\ & \\ \underline{\sigma}\cdot\underline{\nabla} & -i\frac{\partial}{\partial t} \end{array} \right.$$

in other

$\underline{A}$ is

$$\gamma^{0} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \qquad \gamma^{i} = \begin{pmatrix} 0 & \sigma^{i} \\ \sigma^{i} & 0 \end{pmatrix}$$

$$A^{a} = \left(\phi, \underline{A} \right) \qquad A_{a} = \left(-\phi, \underline{A} \right)$$

$$\psi = \begin{pmatrix} \chi \\ \Phi \end{pmatrix}$$

Look

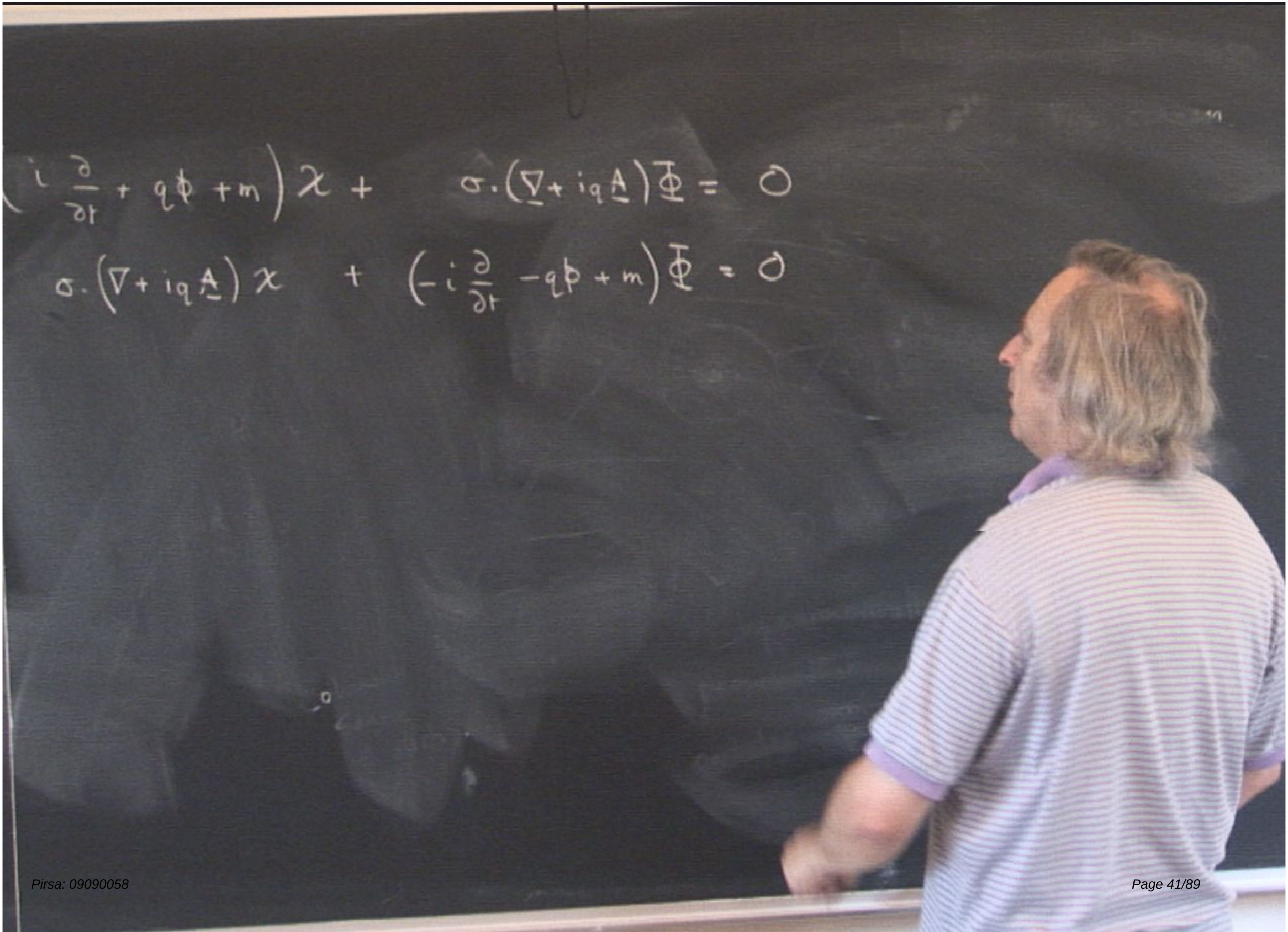
$$\left[\gamma^a (\partial_a + iqA_a) + m \right] \psi = 0$$

Decompose in our favourite representation

$$\begin{pmatrix} i\frac{\partial}{\partial t} + q\phi + m & \underline{\sigma}\cdot\underline{\nabla} + iq\,\underline{\sigma}\cdot\underline{A} \\ \underline{\sigma}\cdot\underline{\nabla} + iq\,\underline{\sigma}\cdot\underline{A} & -i\frac{\partial}{\partial t} - q\phi + m \end{pmatrix} \begin{pmatrix} \chi \\ \Phi \end{pmatrix} = 0$$

in other

$\underline{A}$ is



$\left(i\frac{\partial}{\partial t} + q\phi + m\right)\chi + \sigma\cdot(\nabla + iq\underline{A})\Phi = 0$
$\sigma\cdot(\nabla + iq\underline{A})\chi + \left(-i\frac{\partial}{\partial t} - q\phi + m\right)\Phi = 0$

$$\left(i\frac{\partial}{\partial t} + q\phi + m\right)\chi + \sigma\cdot\left(\underline{\nabla} + iq\underline{A}\right)\Phi = 0$$

$$\sigma\cdot\left(\nabla + iq\underline{A}\right)\chi + \left(-i\frac{\partial}{\partial t} - q\phi + m\right)\Phi = 0$$

$$\left(i\frac{\partial}{\partial t} + q\phi + m\right)\chi + \sigma\cdot\left(\underline{\nabla} + iq\underline{A}\right)\Phi = 0$$

$$\sigma\cdot\left(\nabla + iq\underline{A}\right)\chi + \left(-i\frac{\partial}{\partial t} - q\phi + m\right)\Phi = 0$$

ϕ, A small

Make a non-relativistic approximation,

$$\frac{E-m}{m} \ll 1$$

$$\left(i\frac{\partial}{\partial t} + q\phi + m\right)\chi + \sigma\cdot\left(\underline{\nabla} + iq\underline{A}\right)\Phi = 0$$

$$\sigma\cdot\left(\nabla + iq\underline{A}\right)\chi + \left(-i\frac{\partial}{\partial t} - q\phi + m\right)\Phi = 0$$

ϕ, A small

Make a non-relativistic approximation,

$$\frac{E-m}{m} \ll 1$$

$$i\frac{\partial}{\partial t} = E$$

$$\left(i\frac{\partial}{\partial t} + q\phi + m\right)\chi + \sigma\cdot(\underline{\nabla} + iq\underline{A})\Phi = 0$$

$$\sigma\cdot(\nabla + iq\underline{A})\chi + \left(-i\frac{\partial}{\partial t} - q\phi + m\right)\Phi = 0$$

ϕ, A small

Make a non-relativistic approximation,

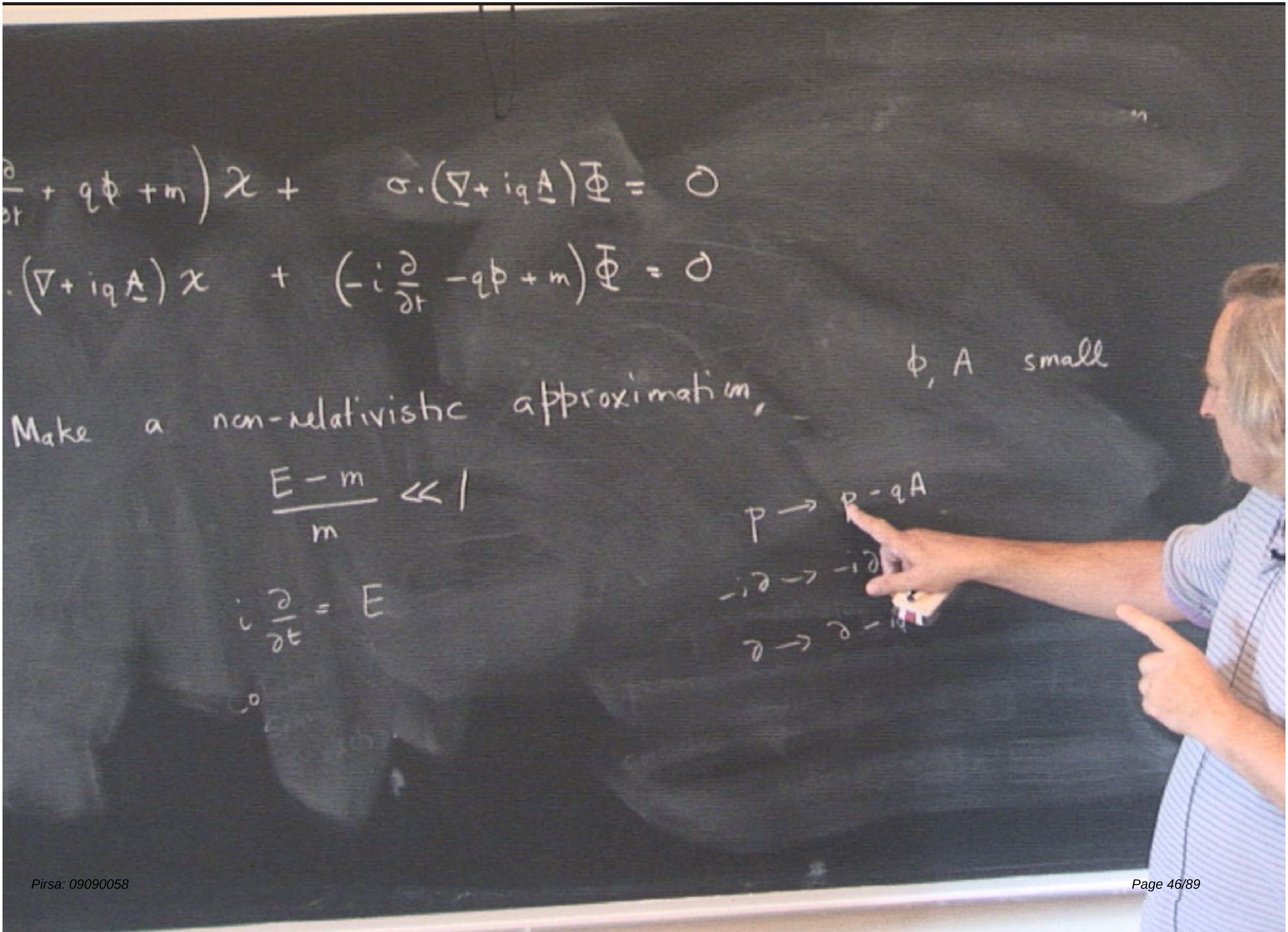
$$\frac{E-m}{m} \ll 1$$

$$p \to p - qA$$

$$-i\partial \to -i\partial - qA$$

$$i\frac{\partial}{\partial t} = E$$

$$\partial \to \partial - iqA$$


$\frac{\partial}{\partial t} + q\phi + m) \chi + \sigma.(\nabla + iqA)\Phi = 0$
$.(\nabla + iqA)\chi + (-i\frac{\partial}{\partial t} - q\phi + m)\Phi = 0$
ϕ, A small
Make a non-relativistic approximation,
$\frac{E-m}{m} \ll 1$
$p \to p - qA$
$i\frac{\partial}{\partial t} = E$

$$\ldots\frac{\partial}{\partial t} + q\phi + m\Big)\chi + \sigma\cdot(\underline{\nabla} + iq\underline{A})\Phi = 0$$

$$\ldots\cdot(\nabla + iq\underline{A})\chi + \left(-i\frac{\partial}{\partial t} - q\phi + m\right)\Phi = 0$$

ϕ, A small

Make a non-relativistic approximation,

$$\frac{E-m}{m} \ll 1$$

$$i\frac{\partial}{\partial t} = E$$

$$p \to p - qA$$

$$-i\partial \to -i\partial - qA$$

$$\partial \to \partial - iqA$$

$$\left(i\frac{\partial}{\partial t} - q\phi + m\right)\chi + \sigma\cdot(\underline{\nabla} - iq\underline{A})\Phi = 0$$

$$\sigma\cdot(\nabla - iq\underline{A})\chi + \left(-i\frac{\partial}{\partial t} + q\phi + m\right)\Phi = 0$$

ϕ, A s

Make a non-relativistic approximation,

$$\frac{E-m}{m} \ll 1$$

$$p \to p - qA$$

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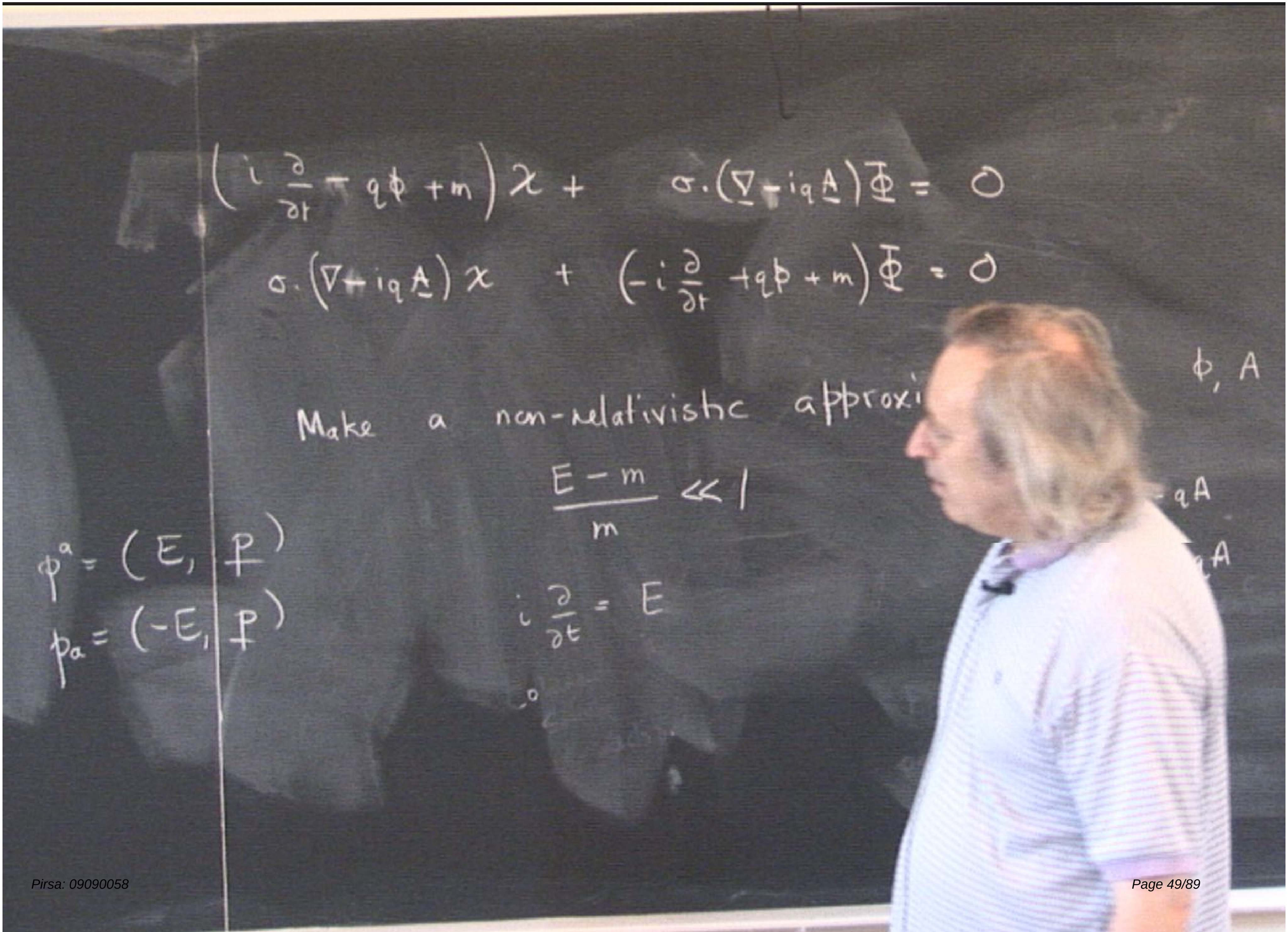
$$i\frac{\partial}{\partial t} = E$$


$\left(i\frac{\partial}{\partial t} - q\phi + m\right)\chi + \sigma\cdot(\underline{\nabla} - iq\underline{A})\underline{\Phi} = 0$
$\sigma\cdot(\nabla + iq\underline{A})\chi + \left(-i\frac{\partial}{\partial t} + q\phi + m\right)\underline{\Phi} = 0$
ϕ, A
Make a non-relativistic approxi
$\frac{E-m}{m} \ll 1$
$p^a = (E, \underline{p})$
$p_a = (-E, \underline{p})$
$i\frac{\partial}{\partial t} = E$

$$\left(i\frac{\partial}{\partial t} - q\phi + m\right)\chi + \sigma\cdot(\underline{\nabla} - iq\underline{A})\underline{\Phi} = 0$$

$$\sigma\cdot(\nabla + iq\underline{A})\chi + \left(-i\frac{\partial}{\partial t} + q\phi + m\right)\underline{\Phi} = 0$$

ϕ, A

Make a non-relativistic approximation,

$$\frac{E-m}{m} \ll 1$$

$p \to p - qA$

$-i\partial \to -i\partial - qA$

$\partial \to \partial - iqA$

$p^a = (E, \underline{p})$

$p_a = (-E, \underline{p})$

$p_a \to -i\partial_a$

$$i\frac{\partial}{\partial t} = E$$

$$\left(i\frac{\partial}{\partial t} - q\phi + m\right)\chi + \sigma\cdot(\underline{\nabla} - iq\underline{A})\,\underline{\Phi} = 0$$

$$\sigma\cdot(\nabla + iq\underline{A})\,\chi + \left(-i\frac{\partial}{\partial t} + q\phi + m\right)\underline{\Phi} = 0$$

ϕ, A

Make a non-relativistic approximation,

$$\frac{E-m}{m} \ll 1$$

$p \rightarrow p - qA$

$-i\partial \rightarrow -i\partial - qA$

$\partial \rightarrow \partial - iqA$

$p^a = (E, \underline{p})$

$p_a = (-E, \underline{p})$

$$i\frac{\partial}{\partial t} = E$$

$p_a \rightarrow -i\partial_a$

$-E \rightarrow -i\frac{\partial}{\partial t}$

$$\left(i\frac{\partial}{\partial t} - q\phi + m \right)\chi + \sigma\cdot(\underline{\nabla} - iq\underline{A})\Phi = 0$$

$$\sigma\cdot(\nabla + iq\underline{A})\chi + \left(-i\frac{\partial}{\partial t} + q\phi + m \right)\Phi = 0$$

ϕ, A

Make a non-relativistic approximation,

$$\frac{E-m}{m} \ll 1$$

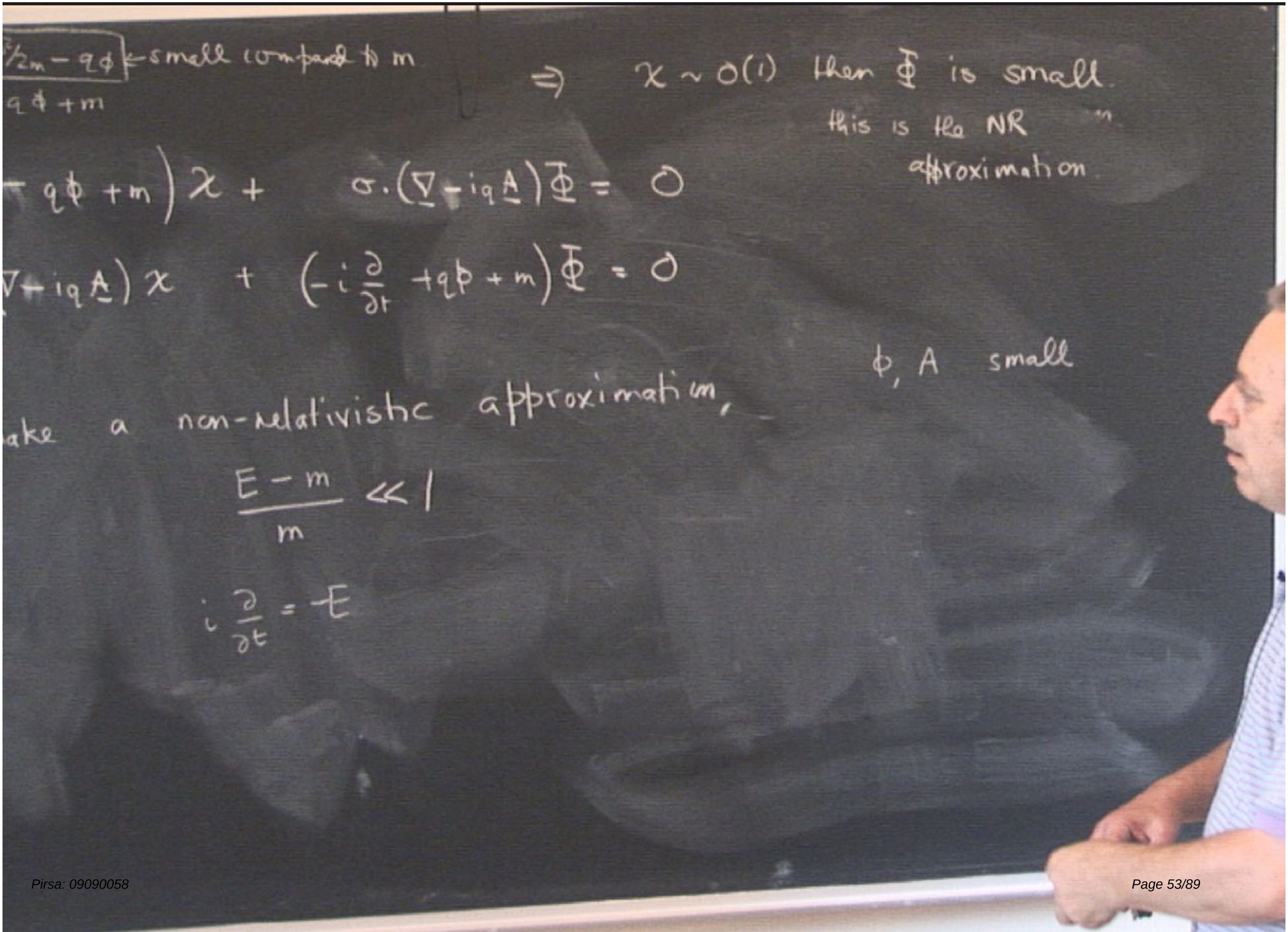
$\phi^a = (E, \underline{p})$

$p_a = (-E, \underline{p})$

$i\frac{\partial}{\partial t} = -E$

$p_a \to -i\partial_a$

$-E \to -i\frac{\partial}{\partial t}$


/2m − qφ ← small compared to m
qφ + m
⇒ χ ~ O(1) then Φ is small
this is the NR approximation
− qφ + m) χ + σ.(∇ − iqA) Φ = 0
∇ − iqA) χ + (−i ∂/∂t + qφ + m) Φ = 0
φ, A small
ake a non-relativistic approximation,
(E − m)/m ≪ 1
i ∂/∂t = −E

$\boxed{{}^2/_{2m} - q\phi}$ ← small compared to m

$q\phi + m$

$\Rightarrow \quad \chi \sim O(1)$ then Φ is small

this is the NR "

approximation.

$$-q\phi + m\Big)\chi + \quad \sigma\cdot(\underline{\nabla} - iq\underline{A})\Phi = 0$$

$$\nabla - iq\underline{A})\chi \quad + \quad \left(-i\frac{\partial}{\partial t} + q\phi + m\right)\Phi = 0$$

ϕ, A small

ake a non-relativistic approximation,

$$\frac{E-m}{m} \ll 1$$

$$i\frac{\partial}{\partial t} = E$$

$\boxed{p^2/2m - q\phi}$ ← small compared to m

$E - q\phi + m$

$\Rightarrow \quad \chi \sim O(1)$ then Φ is small.

this is the NR "approximation"

$$\left(\frac{\partial}{\partial t} - q\phi + m\right)\chi + \sigma\cdot(\nabla - iq\underline{A})\Phi = 0$$

$$\sigma\cdot(\nabla + iq\underline{A})\chi + \left(-i\frac{\partial}{\partial t} + q\phi + m\right)\Phi = 0$$

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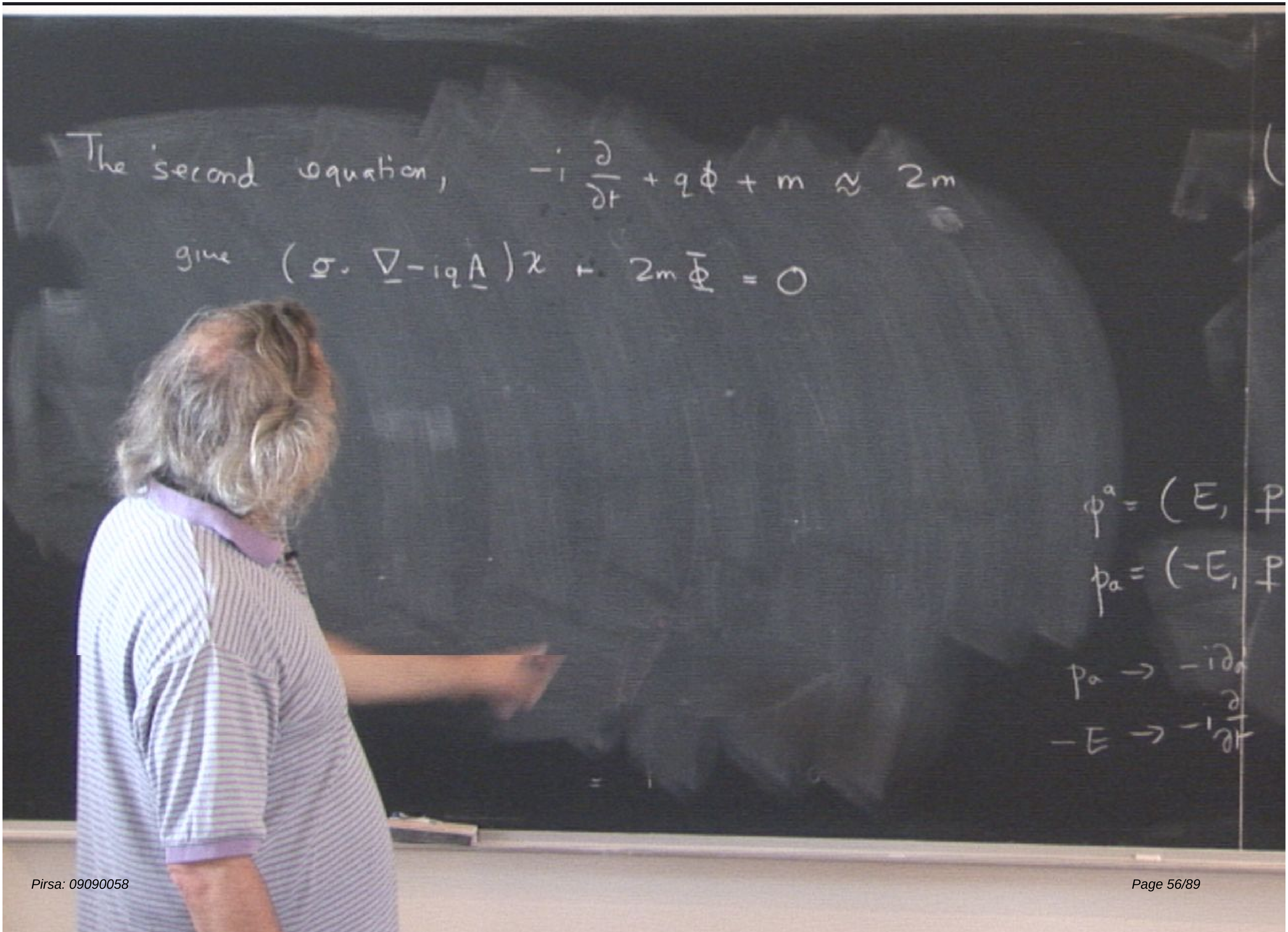
Make a non-relativistic approximation,

$$\frac{E-m}{m} \ll 1$$

$$i\frac{\partial}{\partial t} = -E$$


The second equation, $-i\frac{\partial}{\partial t} + q\Phi + m \approx 2m$
give $(\underline{\sigma}\cdot\underline{\nabla} - iq\underline{A})\chi + 2m\Phi = 0$
$p^a = (E, \underline{p}$
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substitute into the first equation

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$$\left(i\frac{\partial}{\partial t} - q\phi + m\right)\chi - \frac{1}{2m}\sigma_i\sigma_j\left(\partial_i - iqA_i\right)\left(\partial_j - iqA_j\right)\chi = 0$$

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$$-iqA_j\partial_i\chi$$

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$$- iqA_j\partial_i\chi - q^2A_iA_j\chi\Big) =$$

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$$+\frac{1}{2m}i\varepsilon_{ijk}\sigma_k\frac{1}{2}iq\left(\partial_iA_j-\partial_jA_i\right)\chi=0$$

$$\left(i\frac{\partial}{\partial t} - iq\phi + m\right)\chi - \frac{1}{2m}\left(\nabla^2\chi - 2iq\,\underline{A}\,\underline{\nabla}\chi - iq\,(\mathrm{div}\,\underline{A})\right.$$

$$\left(i\frac{\partial}{\partial t}+m - iq\phi\right)\chi-\frac{1}{2m}\left(\nabla^2\chi-2iq\,\underline{A}\,\underline{\nabla}\chi-iq\,(\mathrm{div}\,\underline{A})\chi-q^2A^2\chi\right)$$

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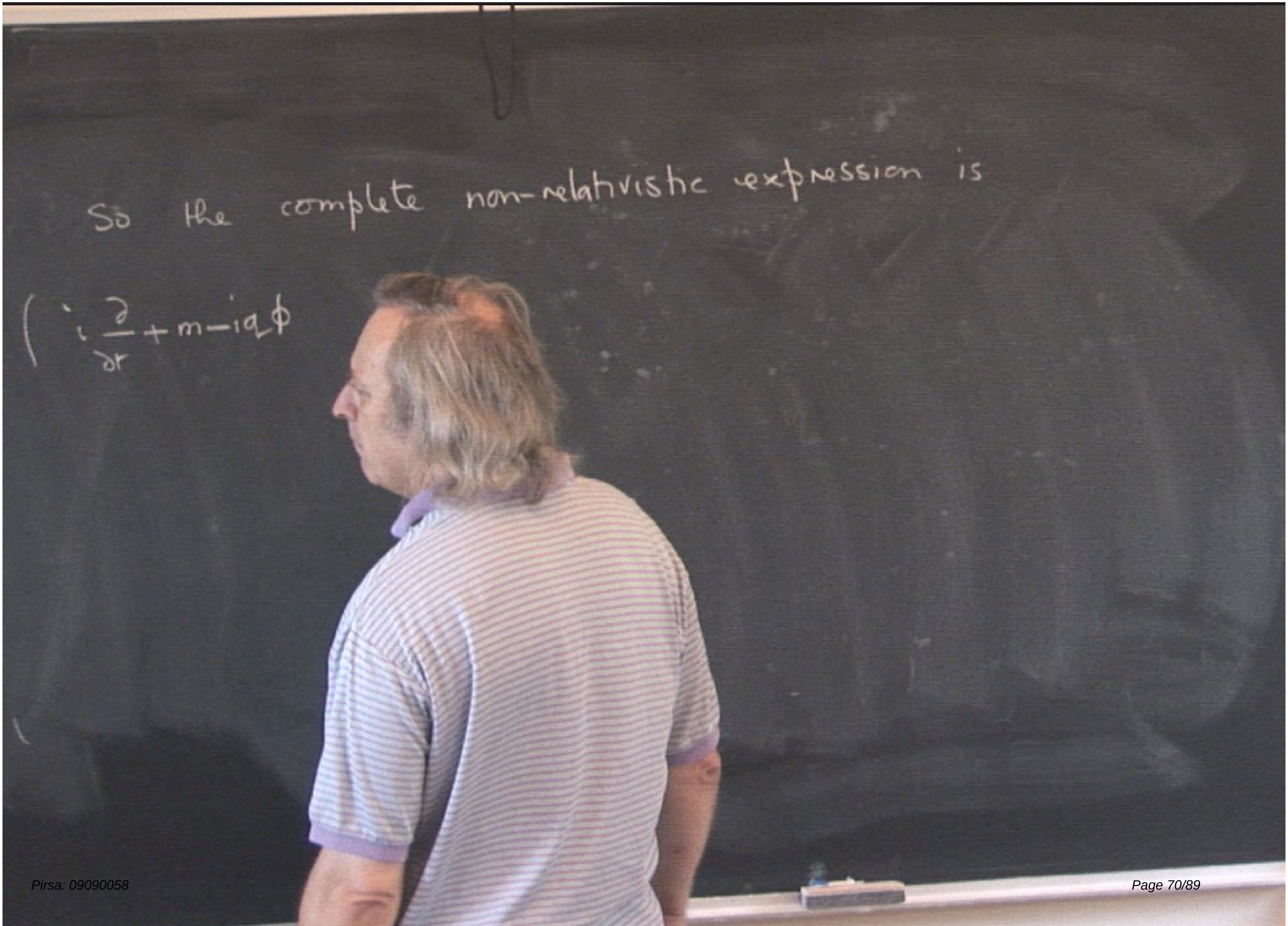
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$$\left(\underbrace{i\frac{\partial}{\partial t} + m}_{\text{KE}} - \underbrace{q\phi}_{\text{PE due to a electric potential } \phi}\right)\chi - \frac{1}{2m}\left(\overbrace{\nabla^2\chi}^{\text{KE operator}} - \overbrace{2iq\,A\cdot\nabla\chi}^{\text{Lorentz force law}} - q^2A^2\right) - \frac{q}{2m}\,\underline{\sigma}\cdot\underline{B}\,\chi = 0$$

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KE operator

Lorentz force law

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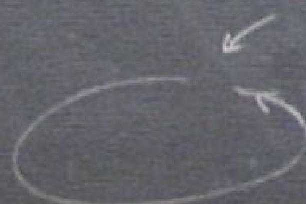

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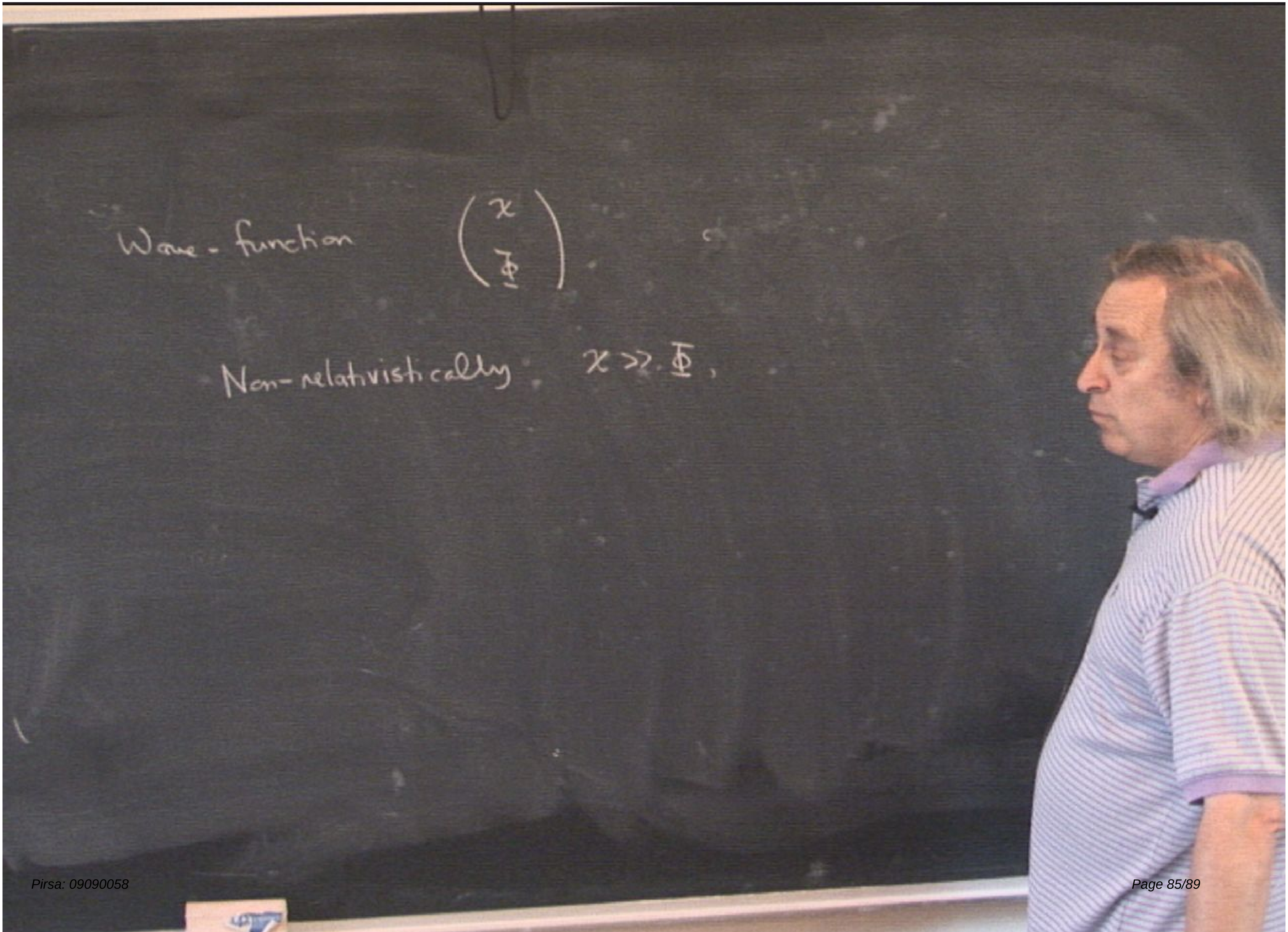
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Describes positrons (i.e. antimatter)