Title: Quantum Theory - Core (PHYS 605) - Lecture 10

Date: Sep 18, 2009 09:00 AM

URL: http://pirsa.org/09090051

Abstract:

Measure cross-sections

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spherically symmetric potential

spherically symmetric potential $V(r)$, $V(r) \to 0$ as $r \to \infty$

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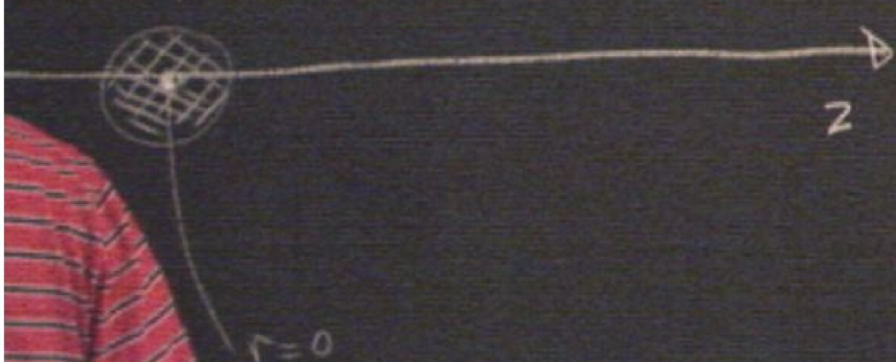
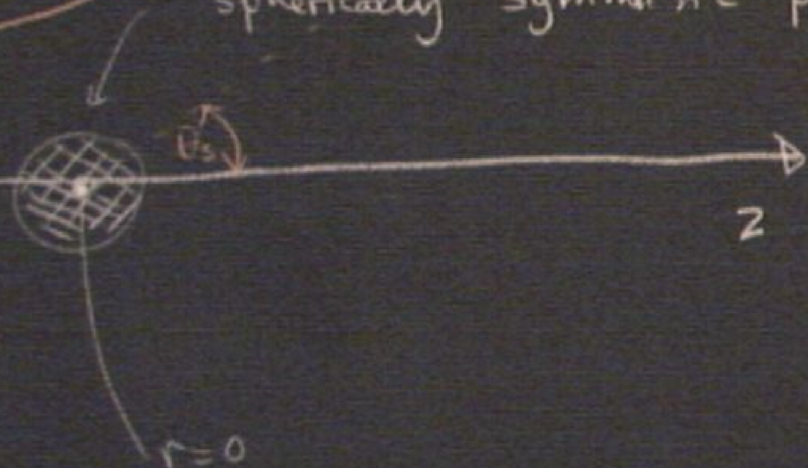


Thus at large distances, scattered particles are essentially free.

spherically symmetric potential $V(r)$, $V(r) \to 0$ as $r \to \infty$



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θ_s – scattering angle.

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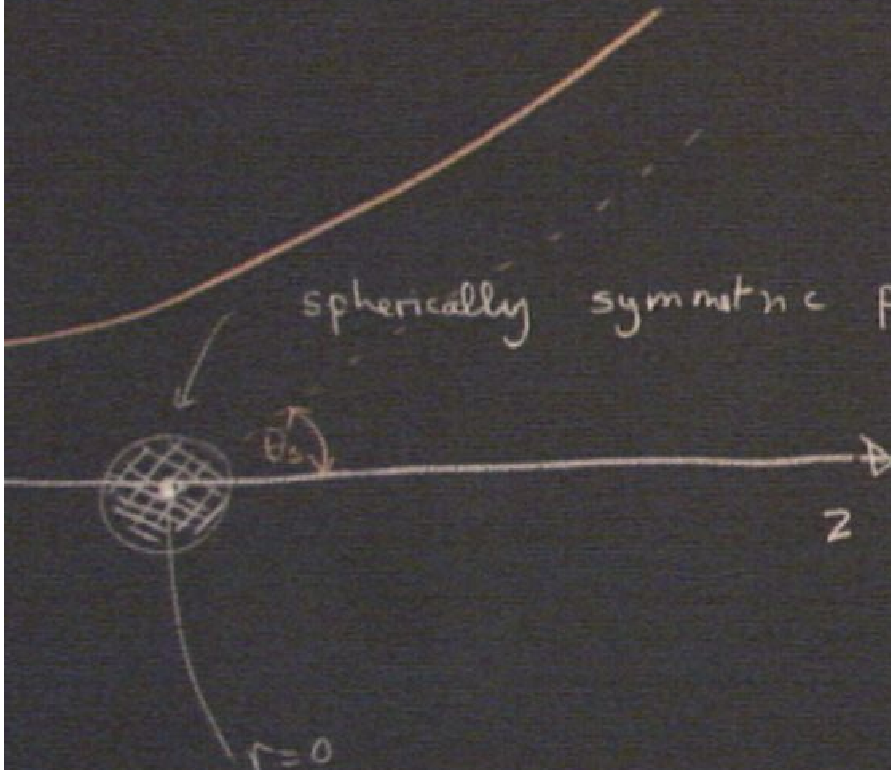

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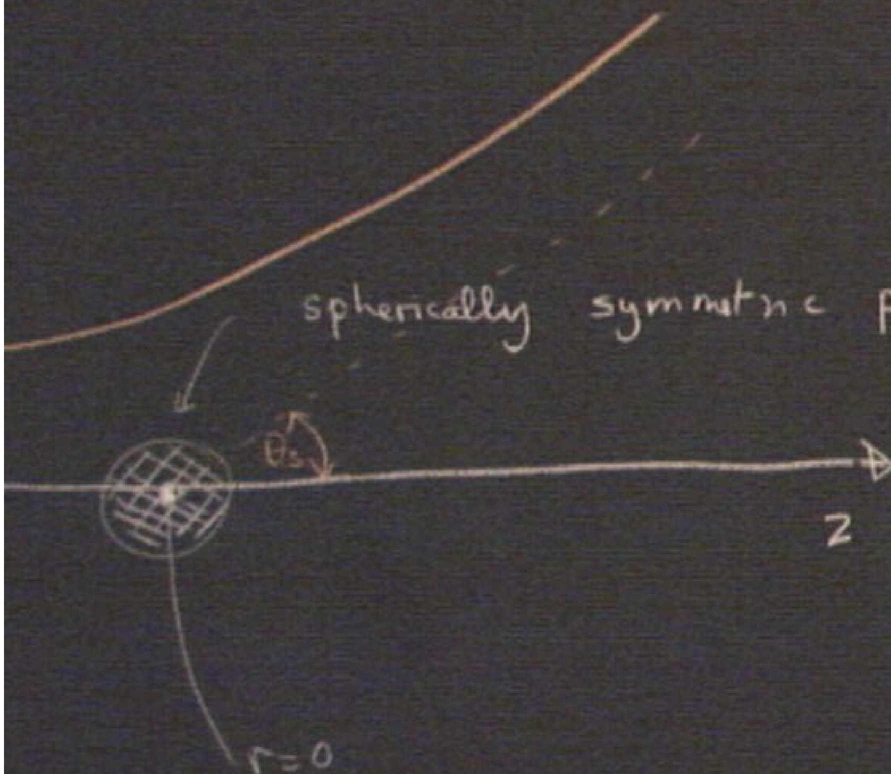

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$V(r)$ being spherically symmetric $\Rightarrow$ problem is axisymmetric about z-axis

$\longrightarrow$

z

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θ_s – scattering angle.

$V(r)$ being spherically symmetric $\Rightarrow$ problem is axisymmetric about z-axis

Independent of ϕ

$\longrightarrow$ z

x-z plane.

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

z

...ric potential $V(r)$, $V(r) \to 0$ as $r \to \infty$

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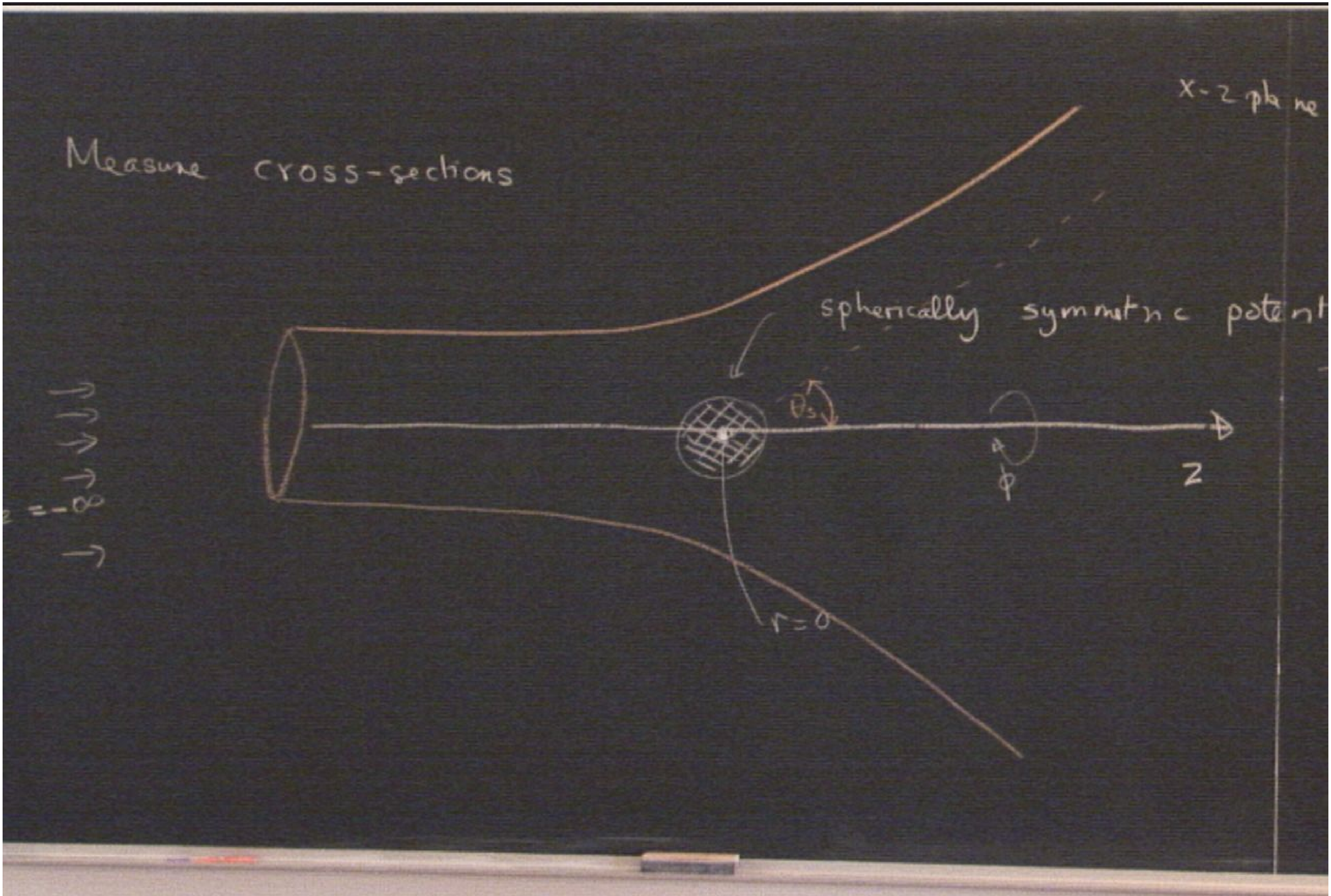
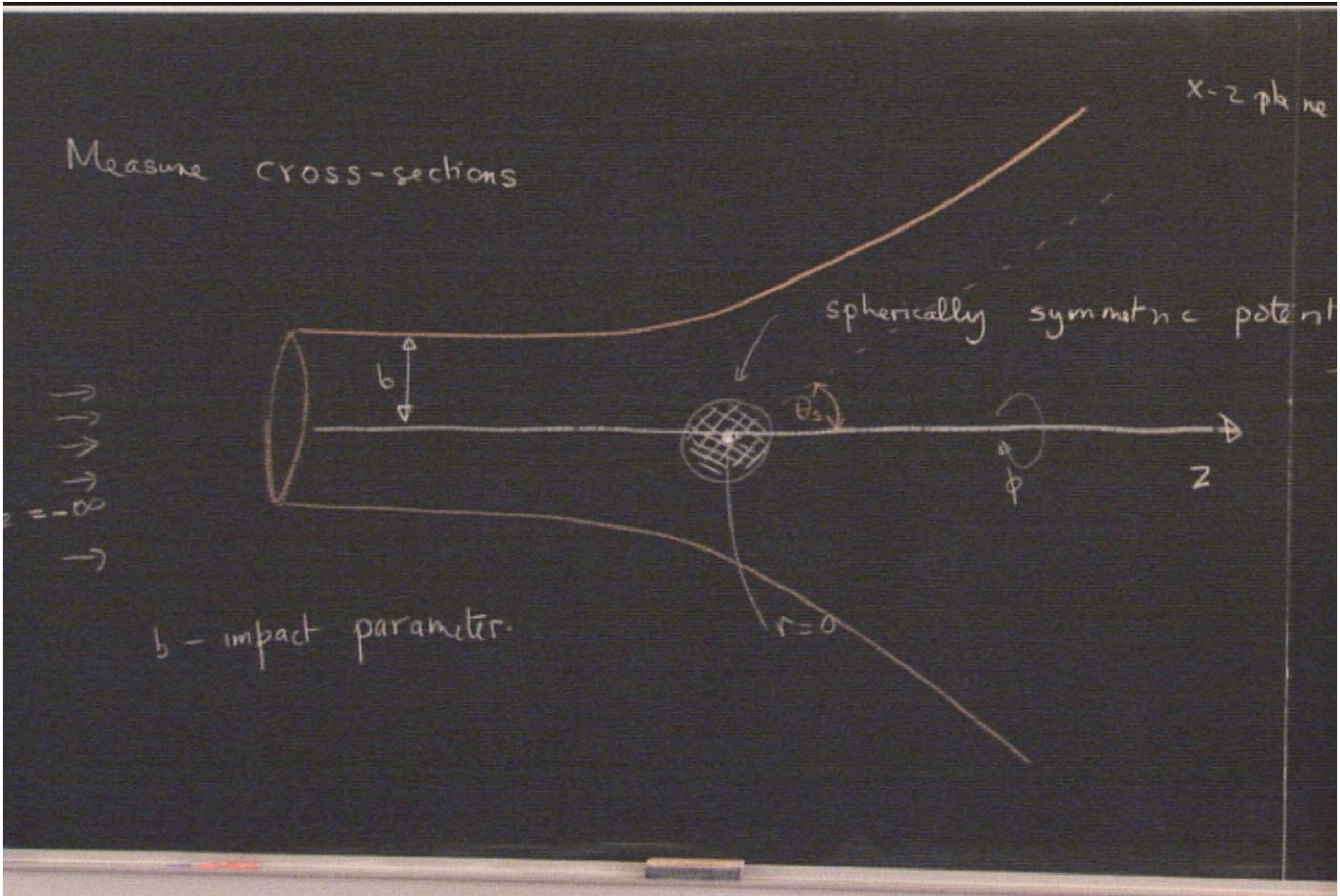
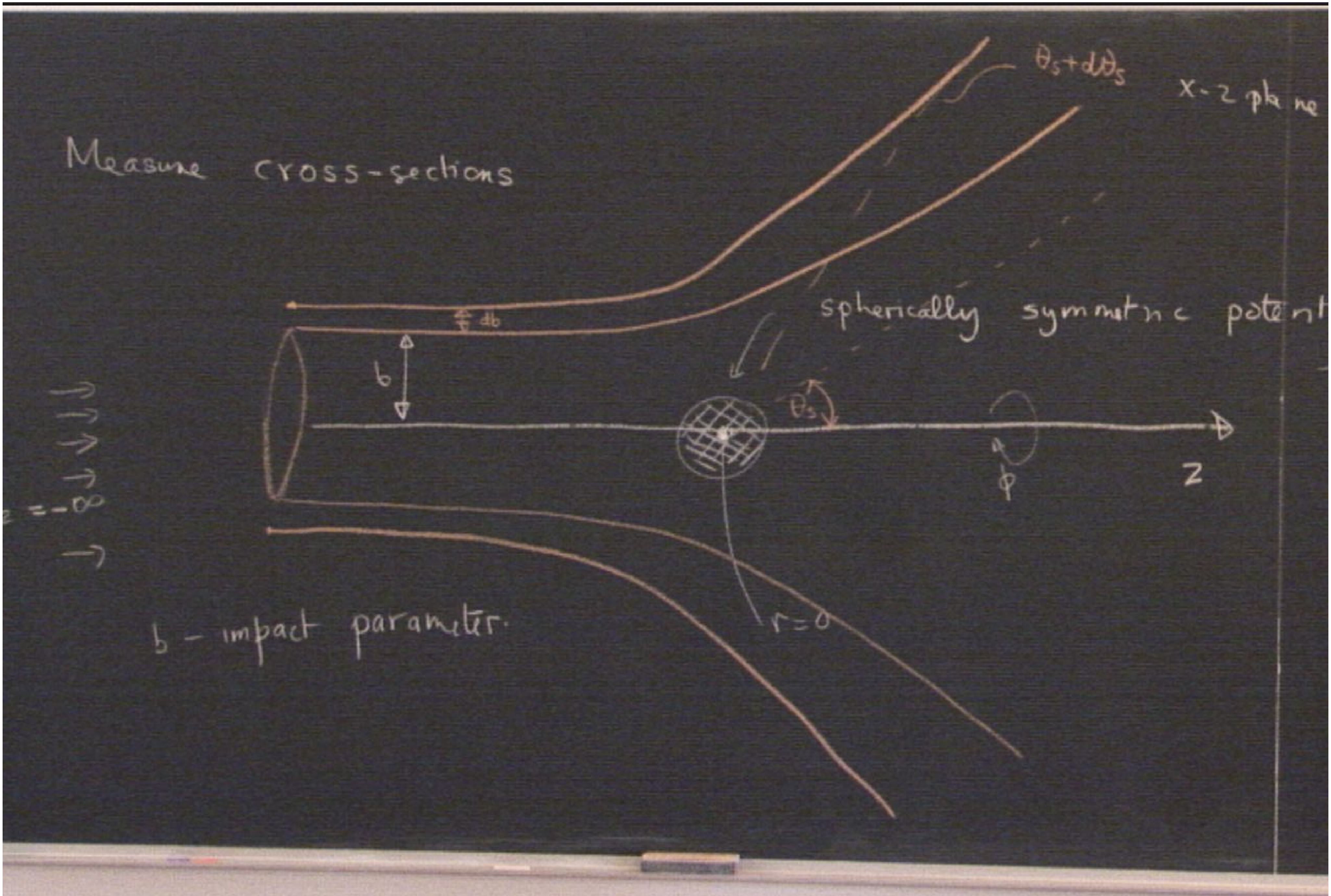
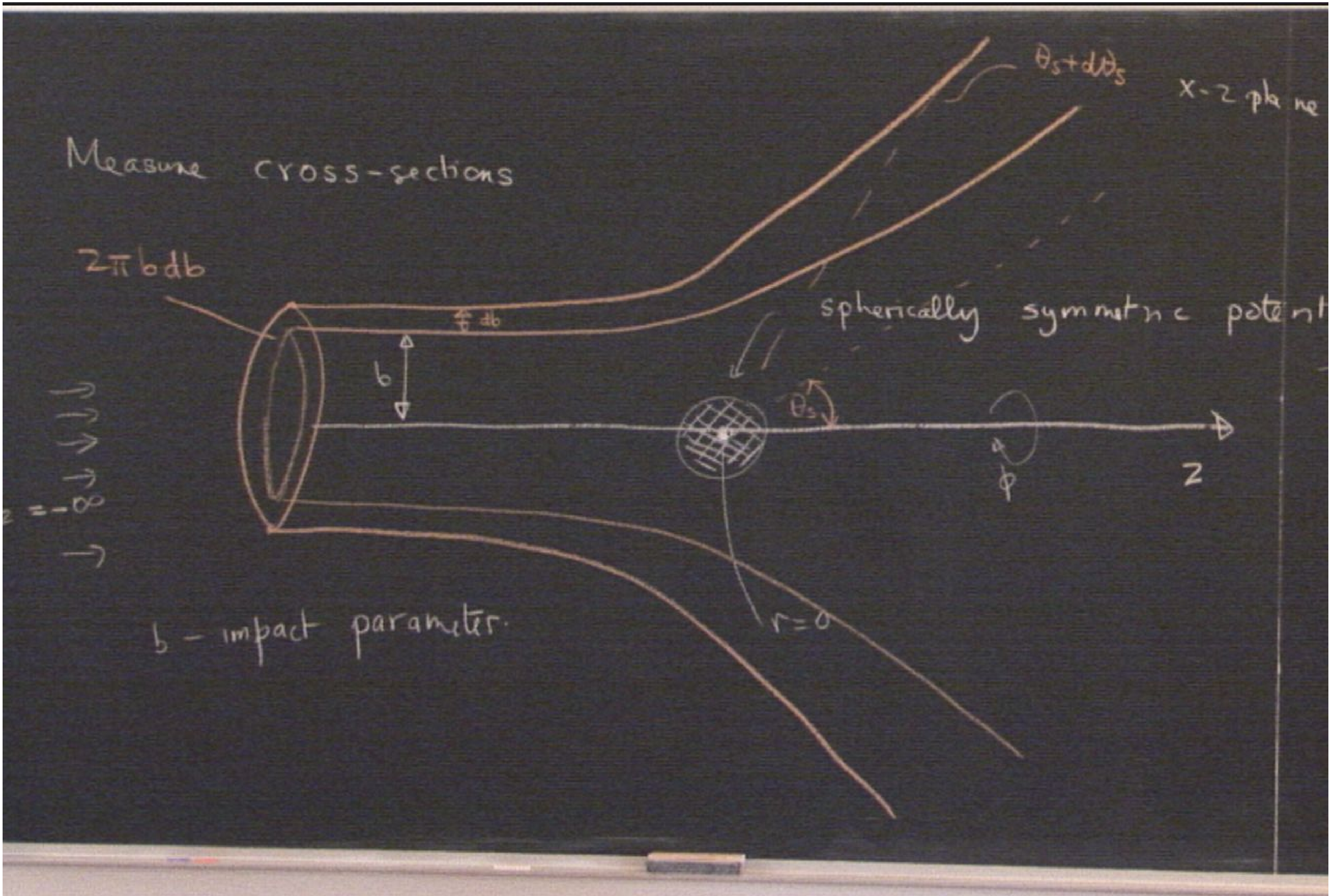


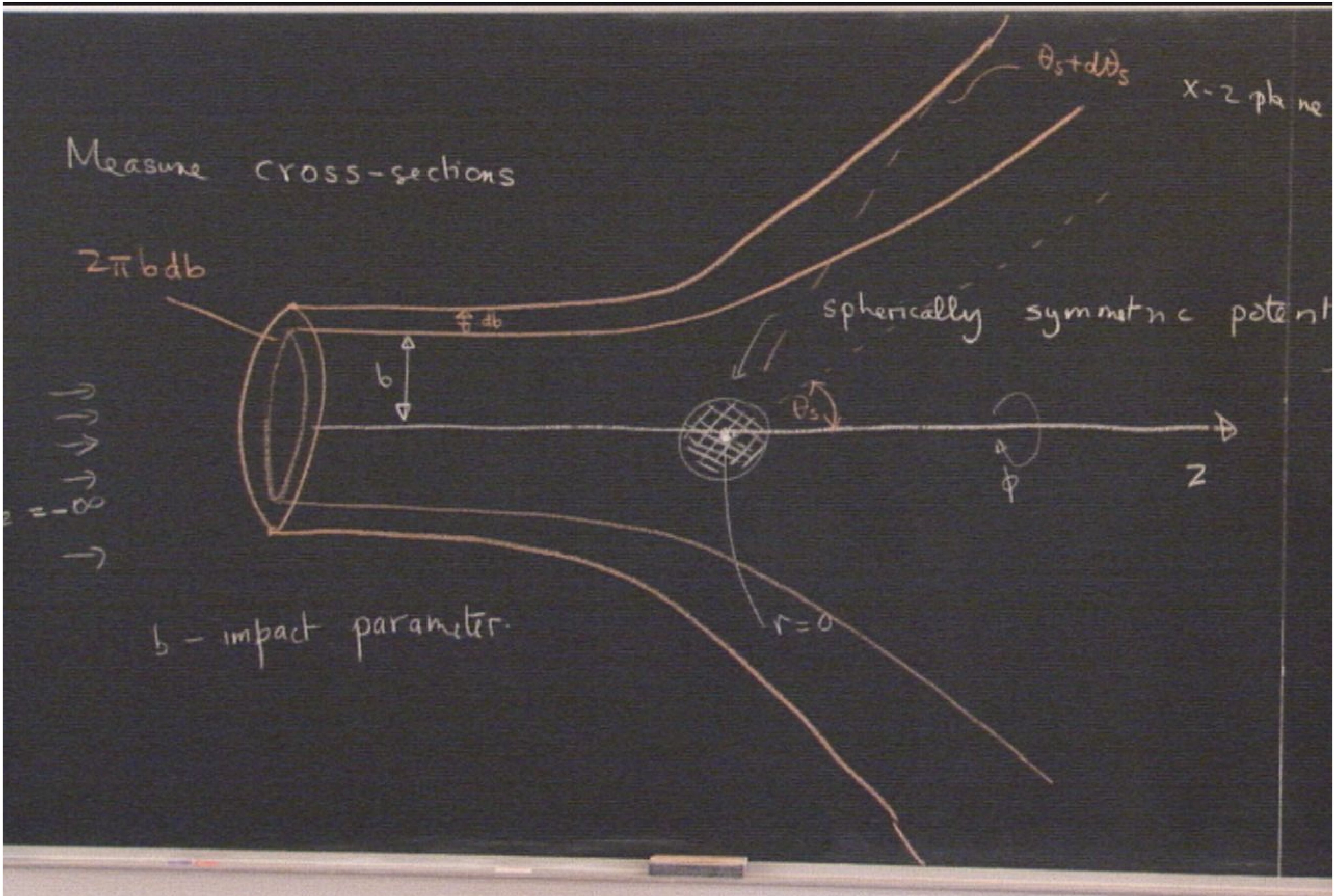
Measure cross-sections
x-z plane
spherically symmetric potent
θ_s
z
ϕ
$z = -\infty$
$r = 0$


x-z plane
Measure cross-sections
spherically symmetric potent
b
θ_s
ϕ
z
$z = -\infty$
b - impact parameter.
r=0



Measure cross-sections
$\theta_s + d\theta_s$
x-z plane
db
b
spherically symmetric potent
θ_s
z
ϕ
$z = -\infty$
b - impact parameter.
r=0


Measure cross-sections
2π b db
db
b
z = -∞
b - impact parameter.
θs+dθs
x-z plane
spherically symmetric potent
θs
φ
z
r=0


Measure cross-sections
2π b db
θs + dθs
x-z plane
db
b
spherically symmetric potent
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z
φ
z = -∞
b - impact parameter.
r=0

$\theta_s \longrightarrow \theta_s + d\theta_s$

Scan out near $r \to \infty$

a solid angle

$d\Omega = \sin\theta_s \, d\theta_s \, d\phi$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$V(r)$, $V(r) \longrightarrow 0$ as $r \to \infty$

at large distances, scattered particles are essentially free.

Independent

θ_s - scattering angle.

$V(r)$ being spherically symmetric $\Rightarrow$ problem is axisymmetric about z-axis

$\theta_s \rightarrow \theta_s + d\theta_s$

Scan out near $r \rightarrow \infty$

a solid angle

$d\Omega = \sin\theta_s \, d\theta_s \, d\phi$ Independent of ϕ

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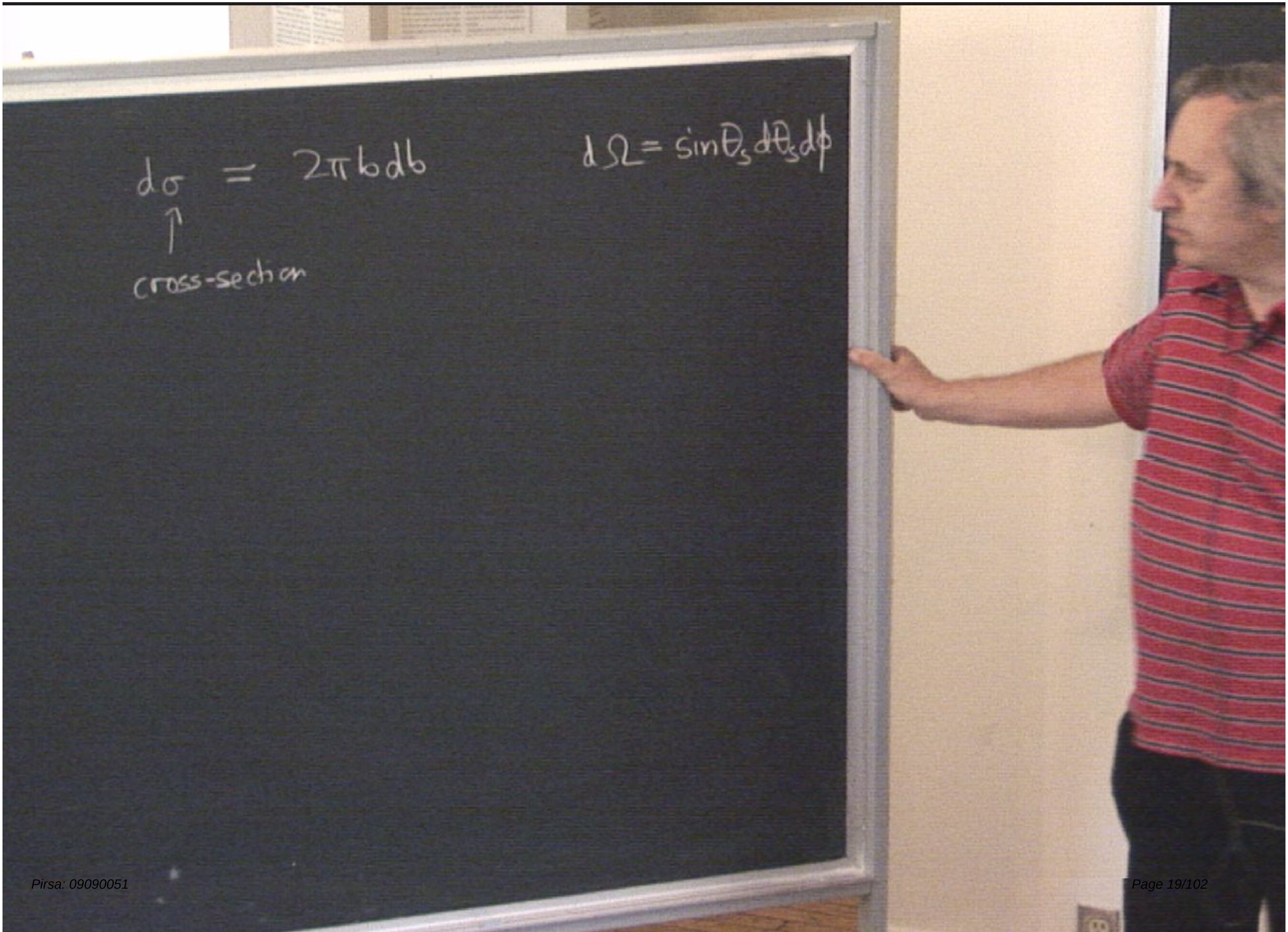

$d\sigma = 2\pi b\, db$
cross-section
$d\Omega = \sin\theta_s\, d\theta_s\, d\phi$

$$d\sigma = 2\pi b\, db$$

cross-section

$$d\Omega = \sin\theta_s \, d\theta_s \, d\phi$$

Differential scattering cross-section

$$\frac{d\sigma}{d\Omega} = \frac{2\pi b\, db}{\sin\theta_s \, d\theta_s \, d\phi}$$

$$d\sigma = 2\pi b \, db$$

$\uparrow$

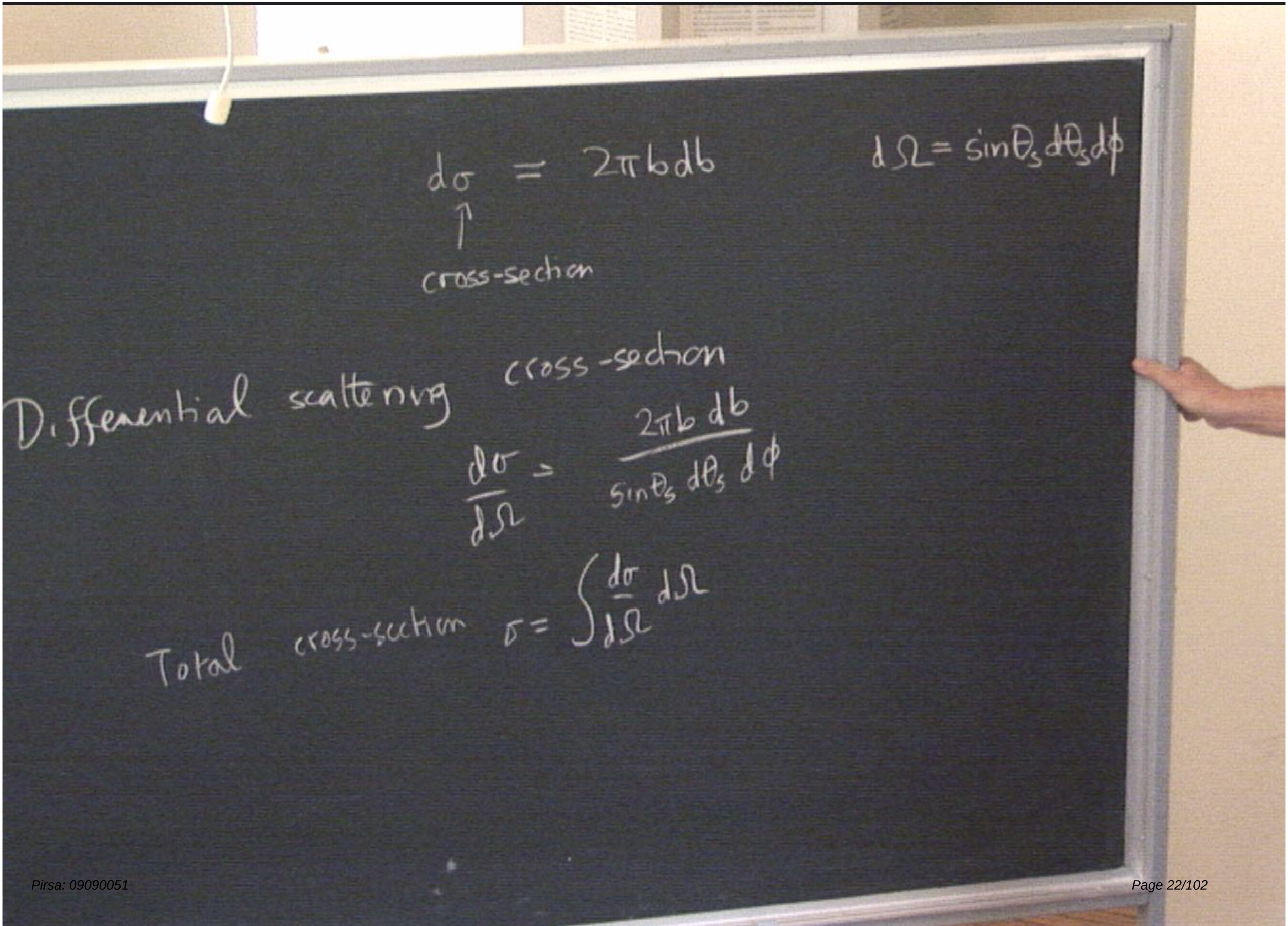
cross-section

$$d\Omega = \sin\theta_s \, d\theta_s \, d\phi$$

Differential scattering cross-section

$$\frac{d\sigma}{d\Omega} = \frac{2\pi b \, db}{\sin\theta_s \, d\theta_s \, d\phi}$$

Total cross-section


$d\sigma = 2\pi b\,db$
cross-section
$d\Omega = \sin\theta_s\, d\theta_s\, d\phi$
Differential scattering cross-section
$\frac{d\sigma}{d\Omega} = \frac{2\pi b\, db}{\sin\theta_s\, d\theta_s\, d\phi}$
Total cross-section $\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$

$$d\sigma = 2\pi b\, db \qquad d\Omega = \sin\theta_s\, d\theta_s\, d\phi$$

cross-section

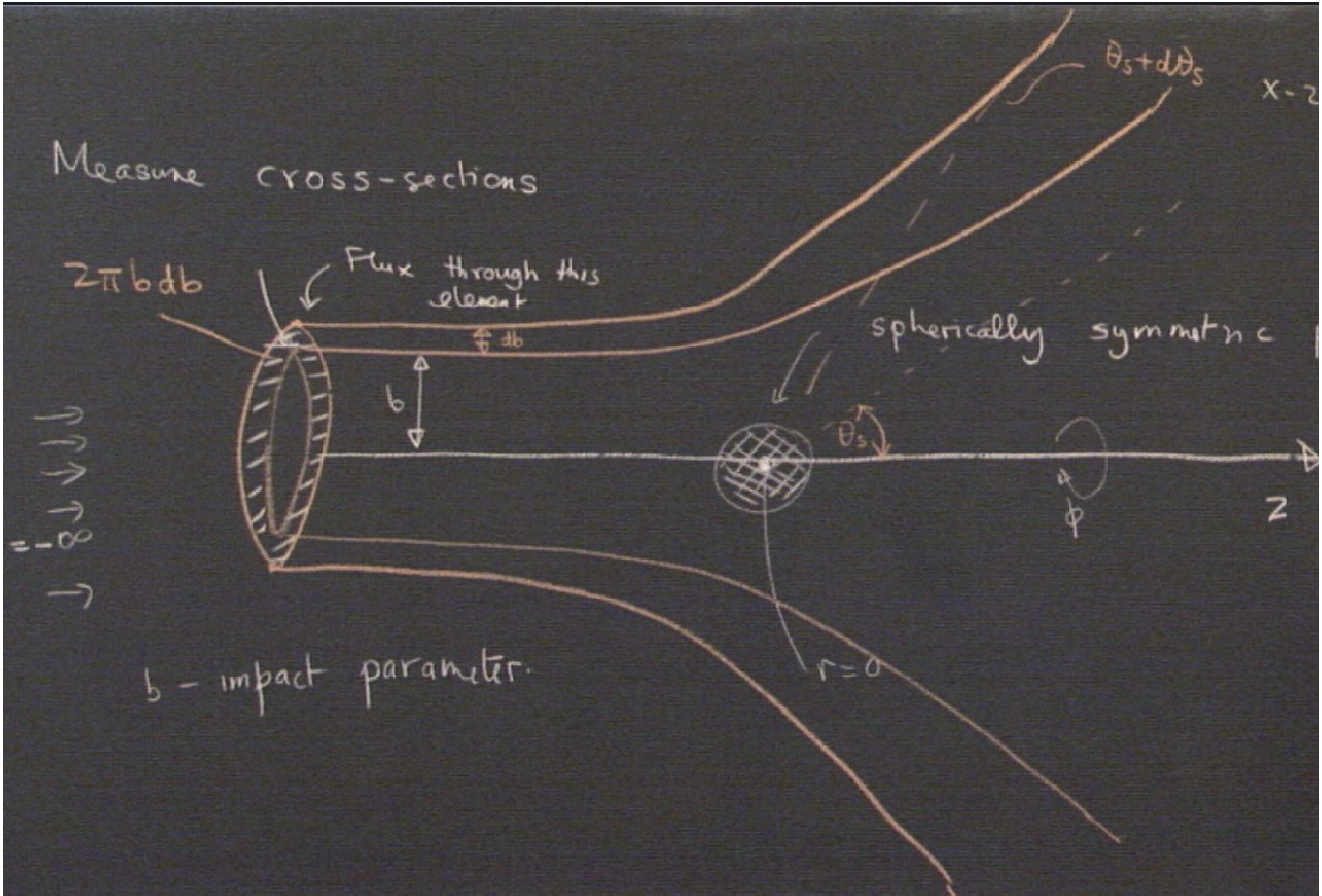
Differential scattering cross-section

$$\frac{d\sigma}{d\Omega} = \frac{2\pi b\, db}{\sin\theta_s\, d\theta_s\, d\phi}$$

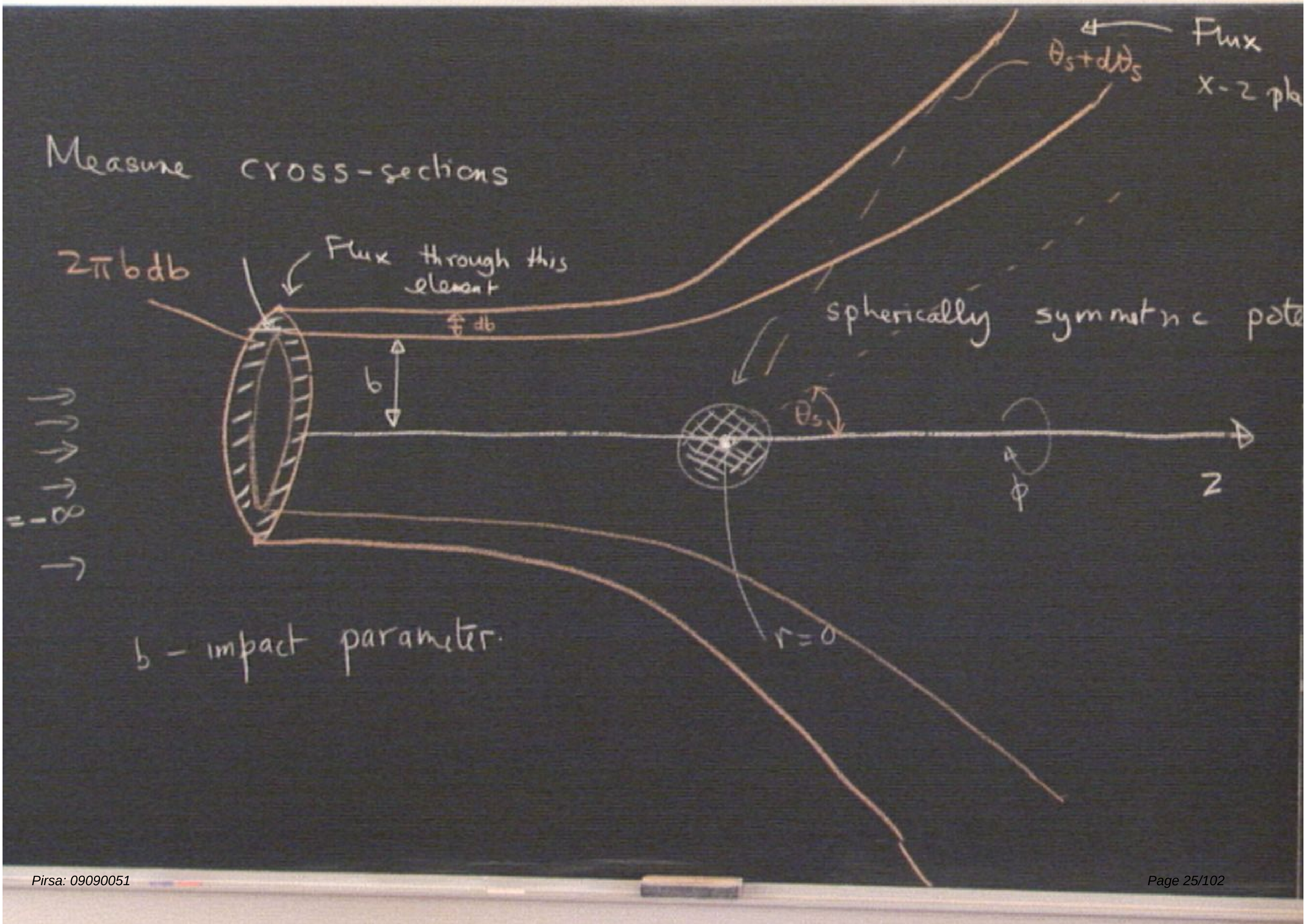
Total cross-section $\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega$ integrated over all solid angles

$0 < \theta_s < \pi$

$0 < \phi < 2\pi$



Measure cross-sections
$\theta_s + d\theta_s$
x-z
$2\pi b\,db$
Flux through this element
spherically symmetric
db
b
θ_s
ϕ
z
$=-\infty$
b - impact parameter.
r=0


Measure cross-sections
2πbdb
Flux through this element
db
b
spherically symmetric pote
Flux
X-Z pl
θs+dθs
θs
φ
Z
=-∞
r=0
b - impact parameter.

Flux

$\theta_s + d\theta_s$

x-z plane

$$\frac{d\sigma}{d\Omega} = \frac{\text{Flux through } d\Omega \ (\text{\# scattered particles})}{\text{Flux of incoming particles.}}$$

$\theta_s \longrightarrow \theta_s + d\theta_s$

Scan out near $r \to \infty$

a solid angle

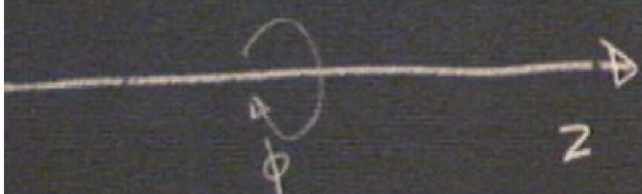
$$d\Omega = \sin\theta_s \, d\theta_s \, d\phi$$

$x =$

$y =$

z

…herically symmetric potential $V(r)$, $V(r) \to 0$ as $r \to \infty$



Thus at large distances, scattered particles are essentially free.

θ_s – scattering angle.

$V(r)$ being spherically symmetric

$d\sigma = 2\pi b\, db$

cross-section

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$$\frac{d\sigma}{d\Omega} = \frac{2\pi b\, db}{\sin\theta_s\, d\theta_s\, d\phi}$$

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$0 < \theta_s < \pi$

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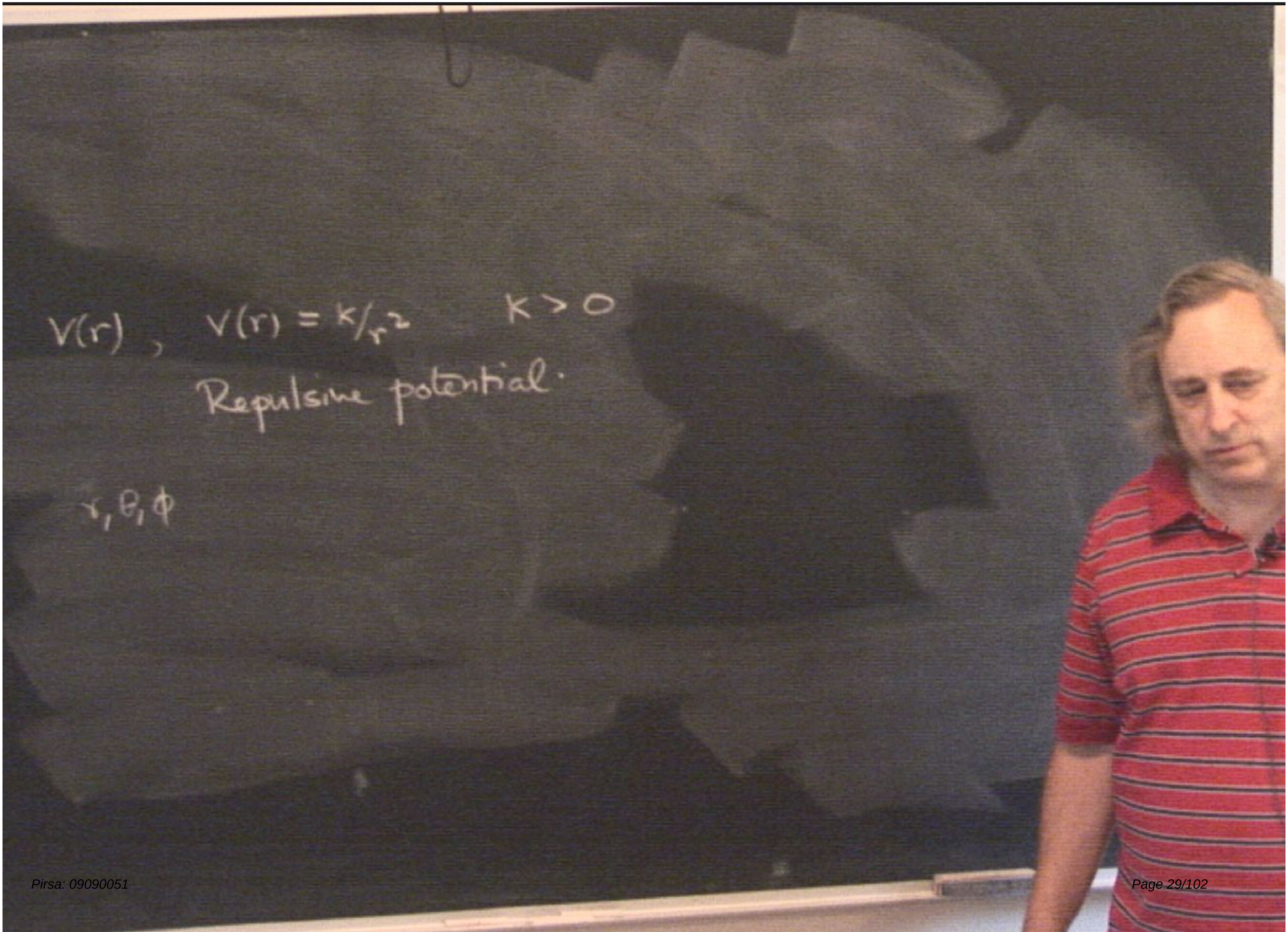
$V(r)$, $V(r) = k/r^2$ $\quad k > 0$

Repulsive potential.


V(r) , V(r) = k/r²   k > 0
Repulsive potential.
r, θ, φ

$V(r)$, $V(r) = k/r^2$ $k > 0$

Repulsive potential.

Start from the Lagrangian

r, θ, ϕ

$V(r)$, $V(r) = k/r^2$ $\quad k > 0$

Repulsive potential.

r, θ, ϕ

Start from the Lagrangian

Restrict to the $x-z$ plane

$$L = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2 \right)$$

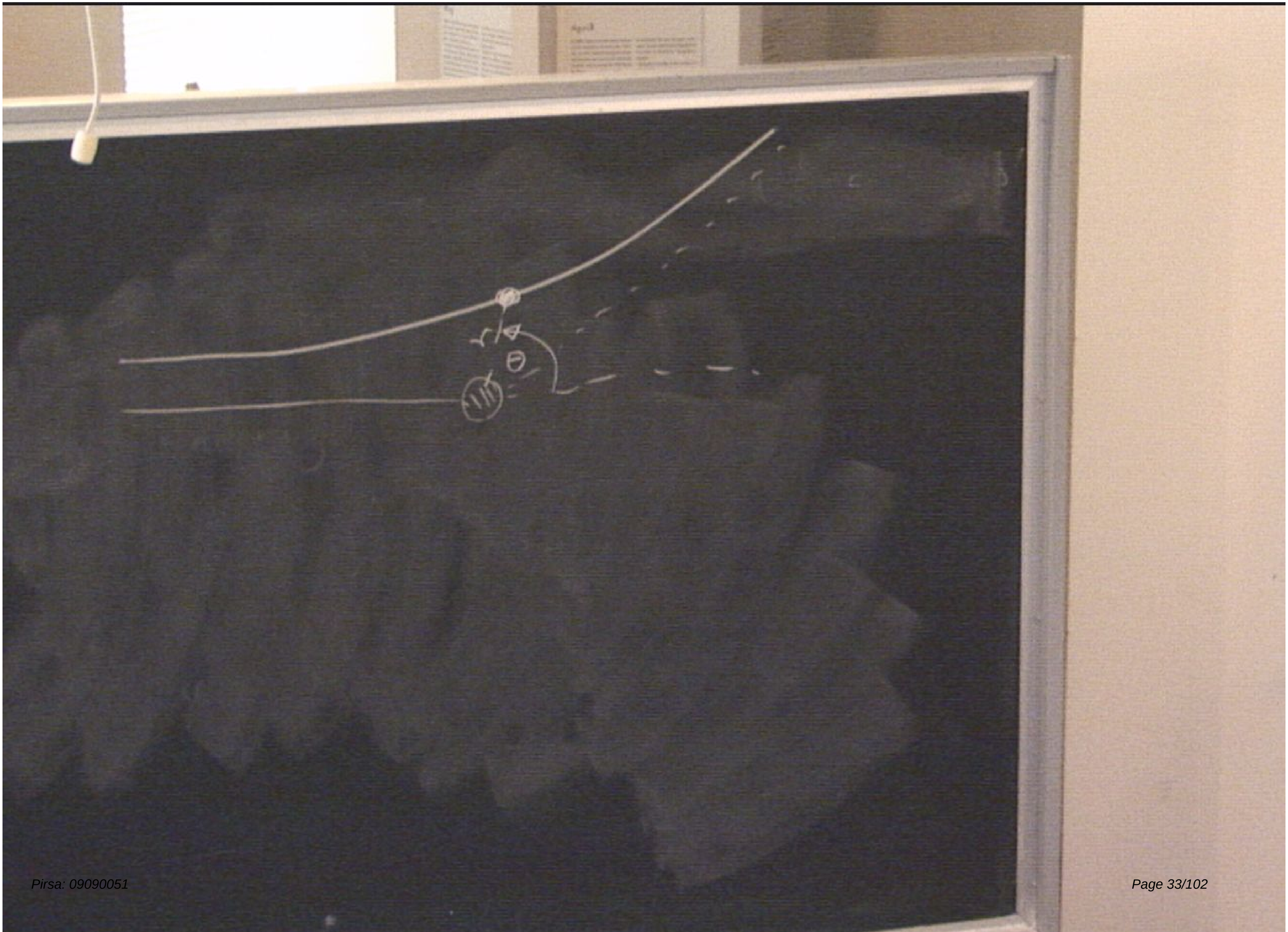
$V(r)$, $V(r) = K/r^2$ $K > 0$

Repulsive potential.

r, θ, ϕ Start from the Lagrangian

Restrict to the x - z plane

$$L = \underbrace{\tfrac{1}{2} m(\dot{r}^2 + r^2\dot{\theta}^2)}_{KE} - K/r^2$$



At $r \to \infty$, incoming particle has $\theta = \pi$

outgoing particle has $\theta = \theta_s$

Find an equation for the shape of the orbit

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θ ignorable co-ordinate $\qquad r^2\dot{\theta} = \text{const}$

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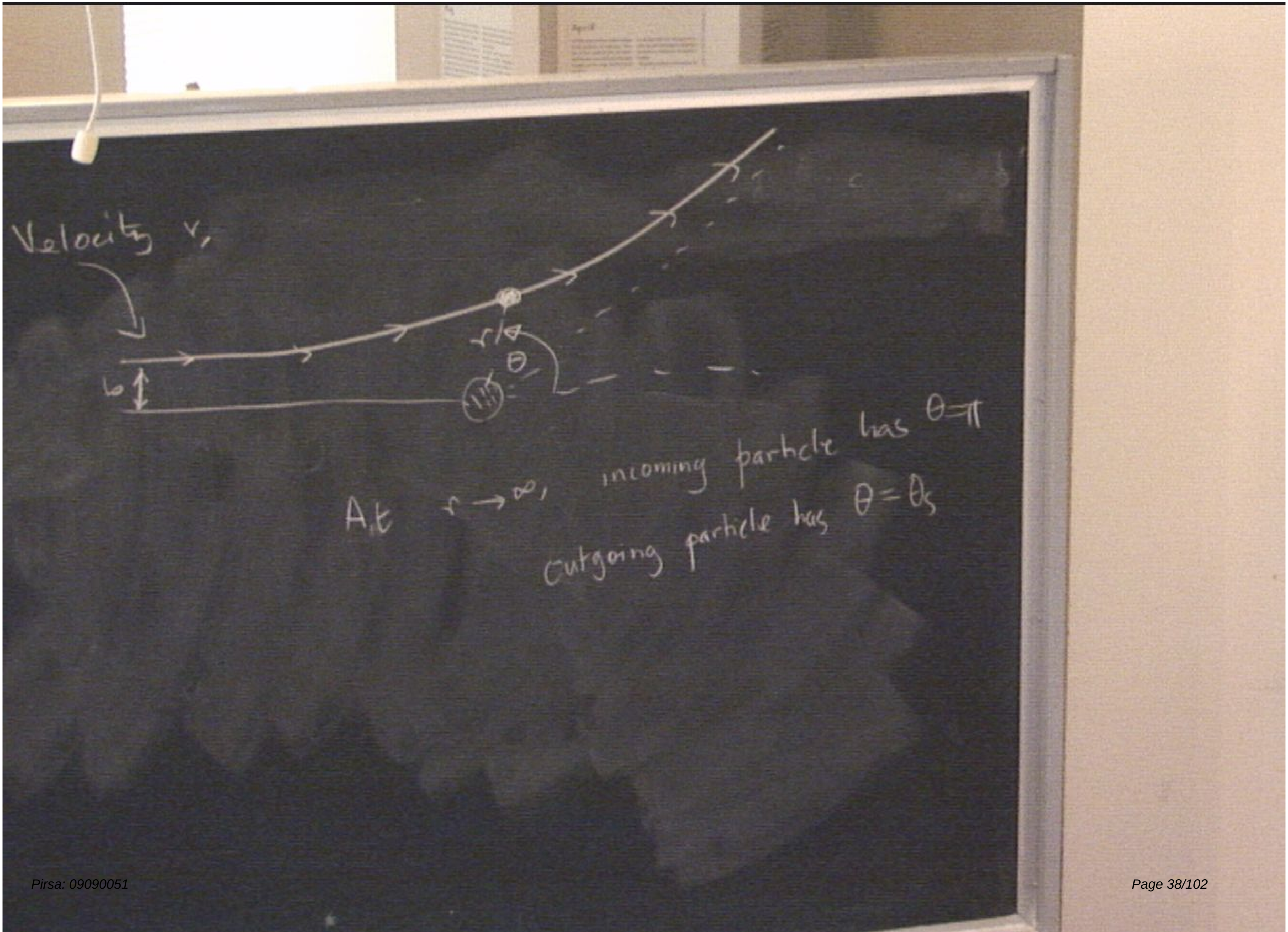
Angular momentum is conserved since V is spherically symmetric


Velocity v,
b
r
θ
At r → ∞, incoming particle has θ=π
outgoing particle has θ=θs

Find an equation for the shape of the orbit

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Find an equation for the shape of the orbit

θ ignorable co-ordinate $\qquad r^2\dot\theta = \text{const} = vb$

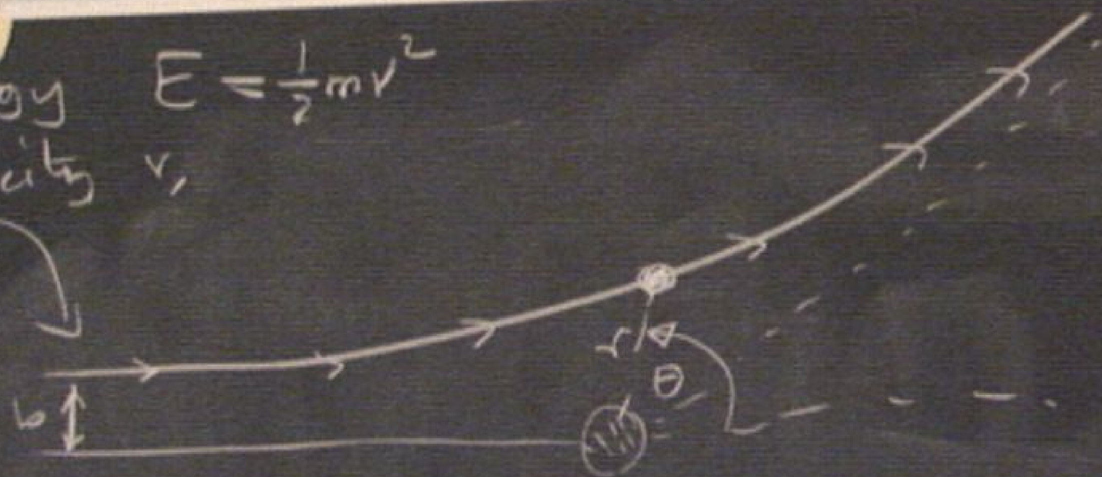
Angular momentum is conserved since V is spherically symmetric

$$\dot\theta = \sqrt{\frac{2E}{m}}\,\frac{b}{r^2}$$

Energy $E = \frac{1}{2} m v^2$

Velocity v,

Only out near ∞



At $r \to \infty$, incoming particle has $\theta = \pi$

outgoing particle has $\theta = \theta_s$

$$\frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \frac{2E}{m} \frac{b}{r^4} + \frac{K}{r^2} = \text{const}$$

$$L = \underbrace{\frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2 \right)}_{KE} - K/r^2$$

$$\frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\,\frac{2E}{m}\,\frac{b}{r^4} + \frac{K}{r^2} = \text{const} = E$$

Look at $r \to \infty$ $\qquad \frac{1}{2}m\dot{r}^2 = E$

$$L = \underbrace{\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)}_{KE} - K/r^2$$

$$\frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\frac{2E}{m}\frac{b}{r^4} + \frac{K}{r^2} = \text{const} = E$$

Look at $r \to \infty$ $\qquad \frac{1}{2}m\dot{r}^2 = E$

$$\dot{r}^2 = \frac{2E}{m} - \frac{1}{r^2}\left(\frac{2K}{m} + \frac{2Eb}{m}\right)$$

$$L = \underbrace{\frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right)}_{KE} - K/r^2$$

$$\frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\,\frac{2E}{m}\,\frac{b^2}{r^4} + \frac{K}{r^2} = \text{const} = E$$

Look at $r \to \infty$ $\quad \frac{1}{2}m\dot{r}^2 = E$

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$$\frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\frac{2E}{m}\frac{b^2}{r^4} + \frac{K}{r^2} = \text{const} = E$$

Look at $r \to \infty$ $\quad \frac{1}{2}m\dot{r}^2 = E$

$$\dot{r}^2 = \frac{2E}{m} - \frac{1}{r^2}\left(\frac{2K}{m} + \frac{2Eb^2}{m}\right) = \frac{2E}{m}\left(1 - \frac{\alpha}{r^2}\right)$$

$$L = \underbrace{\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)}_{KE} - K/r^2$$

$$\frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \frac{2E}{m} \frac{b^2}{r^4} + \frac{K}{r^2} = \text{const} = E$$

Look at $r \to \infty$ $\qquad \frac{1}{2} m \dot{r}^2 = E$

$$\dot{r}^2 = \frac{2E}{m} - \frac{1}{r^2}\left(\frac{2K}{m} + \frac{2Eb^2}{m}\right) = \frac{2E}{m}\left(1 - \frac{\alpha^2 b^2}{r^2}\right)$$

$$\alpha = \left(1 + \frac{K}{Eb^2}\right)^{1/2}$$

$$L = \underbrace{\frac{1}{2} m\left(\dot{r}^2 + r^2 \dot{\theta}^2\right)}_{KE} - K/r^2$$

$$\frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\,\frac{2E}{m}\,\frac{b^2}{r^4} + \frac{K}{r^2} = \text{const} = E$$

Look at $r \to \infty$ $\qquad \frac{1}{2}m\dot{r}^2 = E$

$$\dot{r}^2 = \frac{2E}{m} - \frac{1}{r^2}\left(\frac{2K}{m} + \frac{2Eb^2}{m}\right) = \frac{2E}{m}\left(1 - \frac{\alpha^2 b^2}{r^2}\right)$$

$$\alpha = \left(1 + \frac{K}{Eb^2}\right)^{1/2}$$

$$L = \underbrace{\frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right)}_{KE} - K/r^2$$

$\alpha = 1$ limit as

no scattering

$K = 0$

$\dot{r}$ starts out as < 0, turns around at $r = \alpha b$

then $\dot{r} > 0$

$\dot{r}$ starts out as < 0, turns around at $r = \alpha b$

then $\dot{r} > 0$

Want $r(\theta)$

$$\frac{dr}{d\theta} = \frac{\dot{r}}{\dot{\theta}} =$$

$\dot{r}$ starts out as < 0, turns around at $r = \alpha b$

then $\dot{r} > 0$

Want $r(\theta)$

$$\frac{dr}{d\theta} = \frac{\dot{r}}{\dot{\theta}} = \sqrt{\frac{2E}{m}} \left(1 - \frac{\alpha^2 b^2}{r^2}\right)^{1/2}$$

$\dot{r}$ starts out as < 0, turns around at $r = \alpha b$

then $\dot{r} > 0$

Want $r(\theta)$

$$\frac{dr}{d\theta} = \frac{\dot{r}}{\dot{\theta}} = \sqrt{\frac{2E}{m}} \left(1 - \frac{\alpha^2 b^2}{r^2}\right)^{1/2} \sqrt{\frac{m}{2E}}$$

$$\dot{\theta} = \sqrt{\frac{2E}{m}} \frac{b}{r^2}$$

starts out as < 0, turns around at $r = \alpha b$

then $\dot{r} > 0$

$r(\theta)$

$$\frac{dr}{d\theta} = \frac{\dot{r}}{\dot{\theta}} = \sqrt{\frac{2E}{m}} \left(1 - \frac{\alpha^2 b^2}{r^2}\right)^{1/2} \sqrt{\frac{m}{2E}} \ \frac{r^2}{b}$$

$$\dot{\theta} = \sqrt{\frac{2E}{m}} \frac{b}{r^2}$$

starts out as < 0, turns around at $r = \alpha b$

then $\dot{r} > 0$

$r(\theta)$

$$\frac{dr}{d\theta} = \frac{\dot{r}}{\dot{\theta}} = \cancel{\sqrt{\frac{2E}{m}}} \left(1 - \frac{\alpha^2 b^2}{r^2}\right)^{1/2} \cancel{\sqrt{\frac{m}{2E}}} \frac{r^2}{b}$$

$$\dot{\theta} = \sqrt{\frac{2E}{m}} \frac{b}{r^2}$$

starts out as < 0, turns around at $r = \alpha b$

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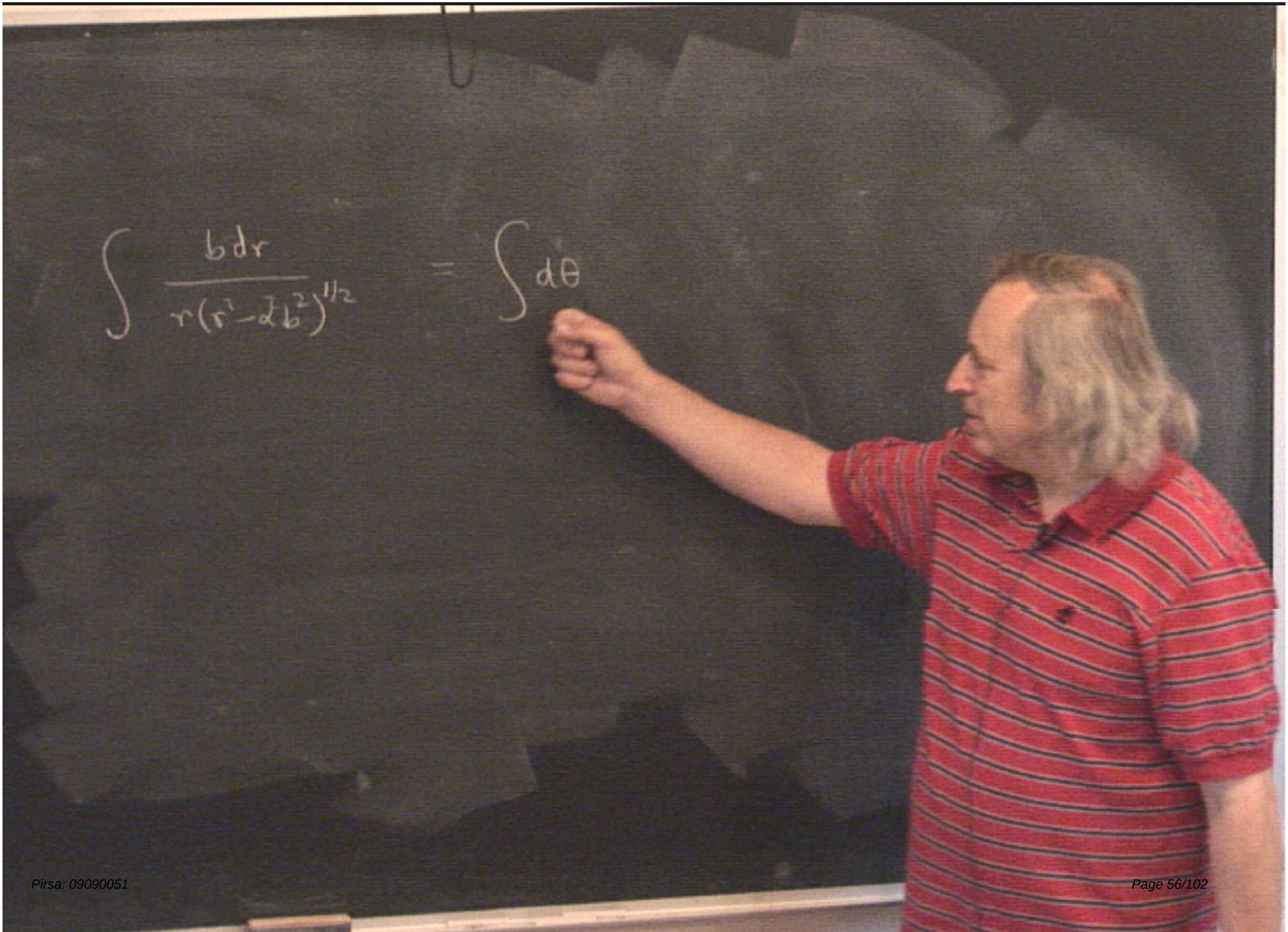
$r(\theta)$

$$\frac{dr}{d\theta} = \frac{\dot{r}}{\dot{\theta}} = \cancel{\sqrt{\frac{2E}{m}}} \left(1 - \frac{\alpha^2 b^2}{r^2}\right)^{1/2} \cancel{\sqrt{\frac{m}{2E}}} \; \frac{r^2}{b}$$

$$= \frac{r}{b}\left(r^2 - \alpha^2 b^2\right)^{1/2}$$

$$\dot{\theta} = \sqrt{\frac{2E}{m}} \frac{b}{r^2}$$

$\frac{1}{2}$


$\int \frac{b\,dr}{r(r^2-\alpha b^2)^{1/2}} = \int d\theta$

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$$\int \frac{b\,dr}{r(r^2-\alpha b^2)^{1/2}} = \int d\theta$$

$$r = \frac{\alpha b}{\sin x}$$

$\frac{1}{u}\Big)$

$$\int \frac{b\,dr}{r(r^2-\alpha b^2)^{1/2}} = \int d\theta$$

$$r = \frac{\alpha b}{\sin\chi} \qquad dr = \frac{\alpha b}{\sin^2\chi}\cos\chi \quad d\chi$$

$$\int \frac{\alpha b^2 \cos\chi\, d\chi}{\sin^2\chi \, \frac{\alpha b}{\sin\chi}\left(\frac{1}{\sin^2\chi}-1\right)^{1/2}}$$

$\frac{1}{u}\Big)$

$$\int \frac{b\,dr}{r(r^2-\alpha b^2)^{1/2}} = \int d\theta$$

$$r = \frac{\alpha b}{\sin x} \qquad dr = \frac{\alpha b}{\sin^2 x}\cos x \quad dx$$

$$\int \frac{\alpha b^2 \cos x\, dx}{\sin^2 x\, \frac{\alpha b}{\sin x}\left(\frac{1-\sin^2 x}{\sin^2 x}\right)^{1/2}}$$

$$\frac{1}{u}\Big)$$

$$\int \frac{b\,dr}{r(r^2-\alpha b^2)^{1/2}} = \int d\theta$$

$$r = \frac{\alpha b}{\sin\chi} \qquad dr = \frac{\alpha b}{\sin^2\chi}\cos\chi \; d\chi$$

$$\int \frac{\alpha b^2 \cancel{\cos\chi}\, d\chi}{\cancel{\sin^2\chi}\, \frac{\alpha^2 b^2}{\cancel{\sin^2\chi}} \left(\frac{1-\cancel{\sin^2\chi}}{\cancel{\sin^2\chi}}\right)^{1/2}} = \frac{\chi}{\alpha}$$

$\frac{1}{u}$)

$$\int \frac{b\,dr}{r(r^2-\alpha^2 b^2)^{1/2}} = \int d\theta$$

$$\frac{\chi}{\alpha} = \theta + \text{const}$$

$$r = \frac{\alpha b}{\sin\chi} \qquad dr = \frac{\alpha b}{\sin^2\chi}\cos\chi \quad d\chi$$

$$\int \frac{\alpha b^2 \cos\chi\, d\chi}{\cancel{\sin^2}\chi\,\frac{\alpha^2 b^2}{\cancel{\sin\chi}}\left(\frac{1-\cancel{\sin^2\chi}}{\cancel{\sin^2\chi}}\right)^{1/2}} = \frac{\chi}{\alpha}$$

$\frac{1}{u}$)

$$\frac{\alpha b}{r} = \sin x$$

$$x = \sin^{-1}\frac{\alpha b}{r} = \alpha\theta + \text{const}$$

$$\frac{\alpha b}{r} = \sin \chi$$

$$\chi = \sin^{-1} \frac{\alpha b}{r} = \alpha\theta + \text{const}$$

$r \to \infty \qquad \theta = \pi \qquad \sin^{-1} 0 = 0, \pi, \ldots$

so $\text{const} = -\alpha\pi$

$$\sin^{-1} \frac{\alpha b}{r} = \alpha(\theta - \pi)$$

$$\frac{db}{r} = \sin \chi$$

$$\chi = \sin^{-1} \frac{db}{r} = \alpha\theta + \text{const}$$

$r \to \infty \qquad \theta = \pi \qquad \sin^{-1} 0 = 0, \pi, \ldots$

so $\text{const} = -\alpha\pi$

$$\sin^{-1} \frac{db}{r} = \alpha(\theta - \pi)$$

Next zero is at π

$$\frac{\alpha b}{r} = \sin\chi$$

$$\chi = \sin^{-1}\frac{\alpha b}{r} = \alpha\theta + \text{const}$$

$r \to \infty \qquad \theta = \pi \qquad \sin^{-1} 0 = 0, \pi, \ldots$

so $\text{const} = -\alpha\pi$

$$\sin^{-1}\frac{\alpha b}{r} = \alpha(\theta - \pi)$$

$$\pi = \alpha(\theta_s - \pi)$$

Next zero is at π

$$\theta_s = \pi\left(1 - \frac{1}{\alpha}\right) \qquad \alpha = \left(1 + \frac{K}{Eb^2}\right)^{1/2}$$

$\frac{1}{u})$

x

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χ

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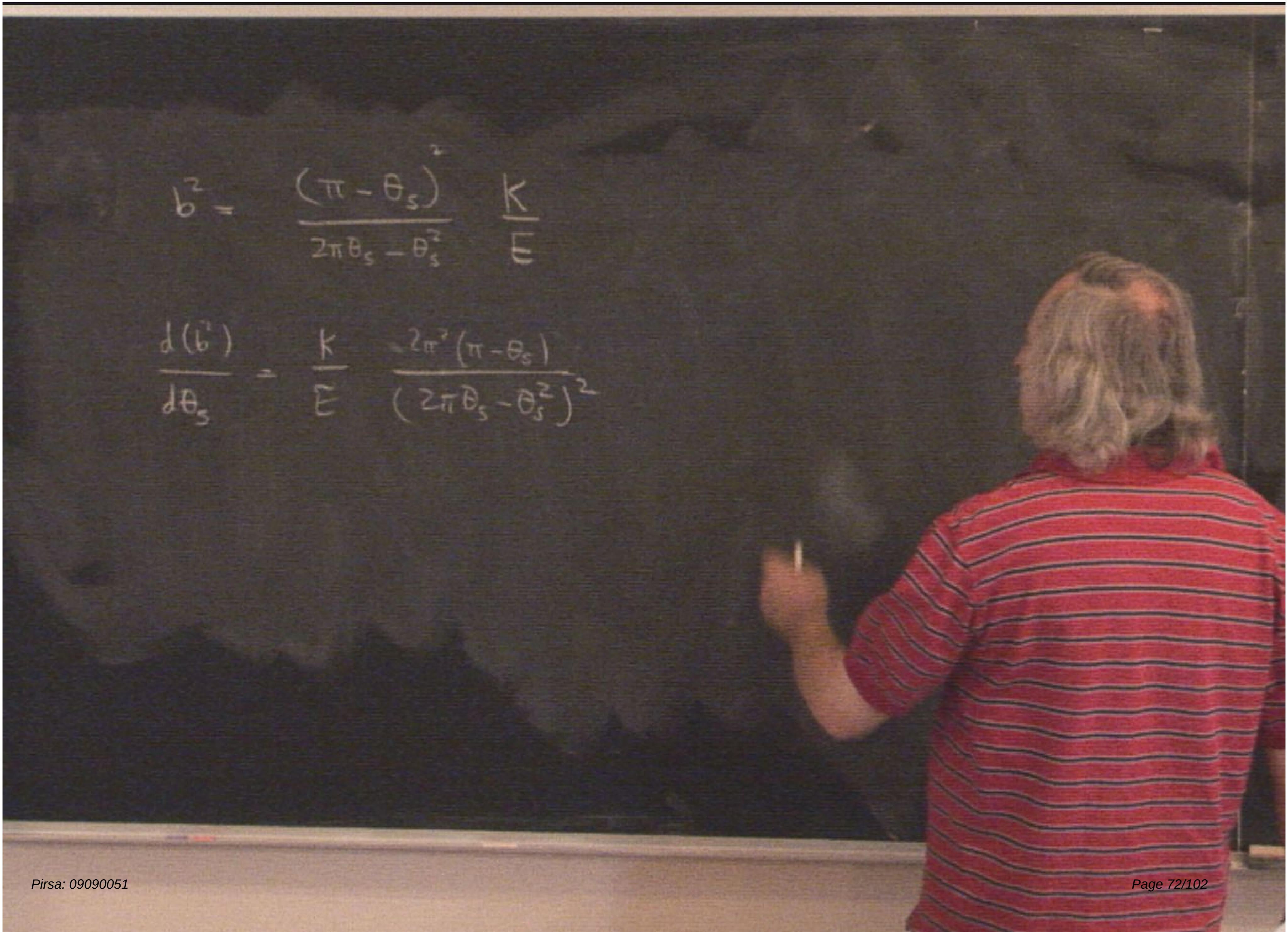
$$\frac{d\sigma}{d\Omega} = \frac{2\pi b\, db}{\sin\theta_s\, d\theta_s \cdot 2\pi} = \frac{1}{2\sin\theta_s} \frac{d(b^2)}{d\theta_s}$$

$\frac{1}{u}\Big)$

$$b^2 = \frac{(\pi - \theta_s)^2}{2\pi\theta_s - \theta_s^2} \frac{K}{E}$$



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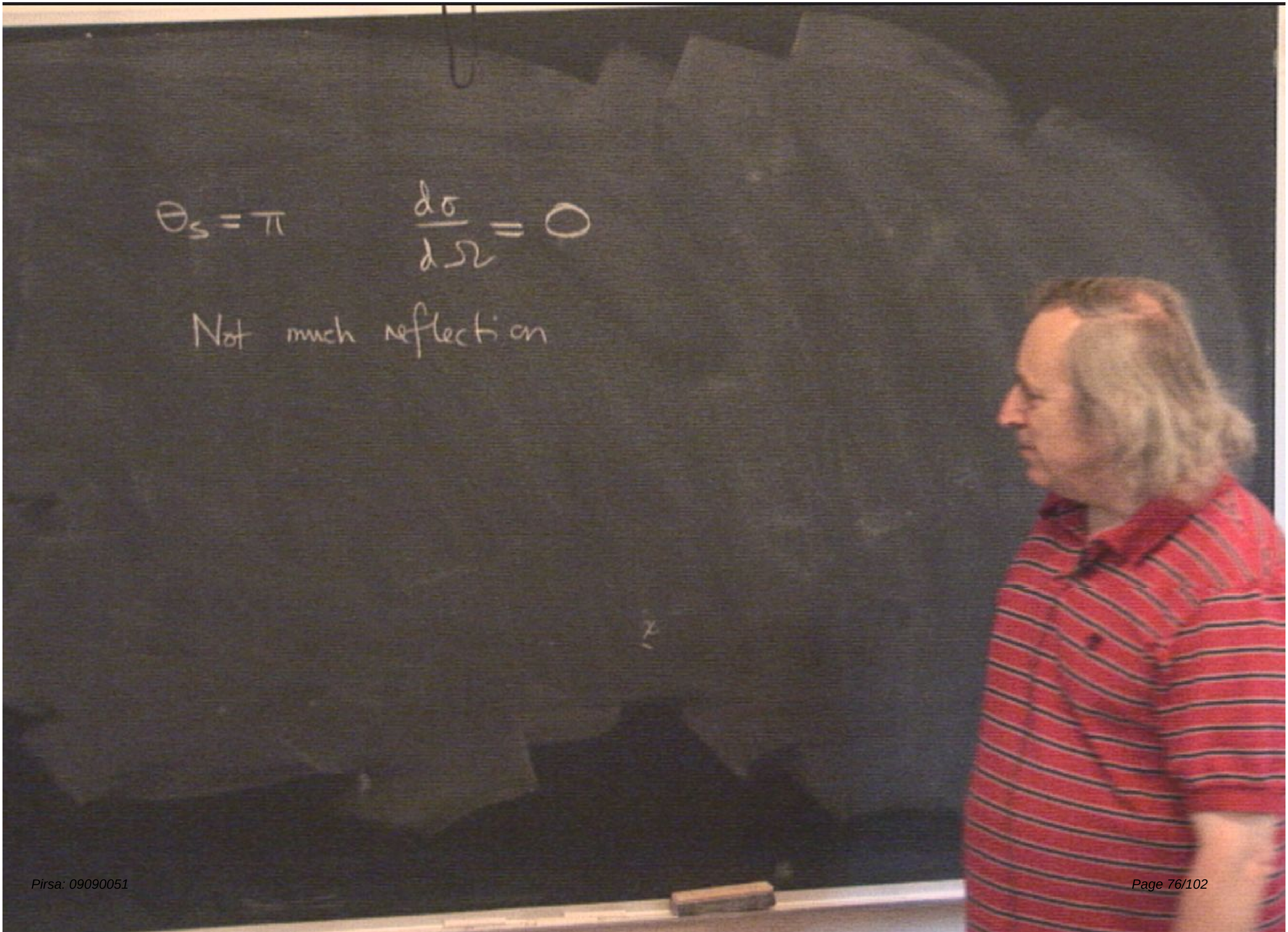
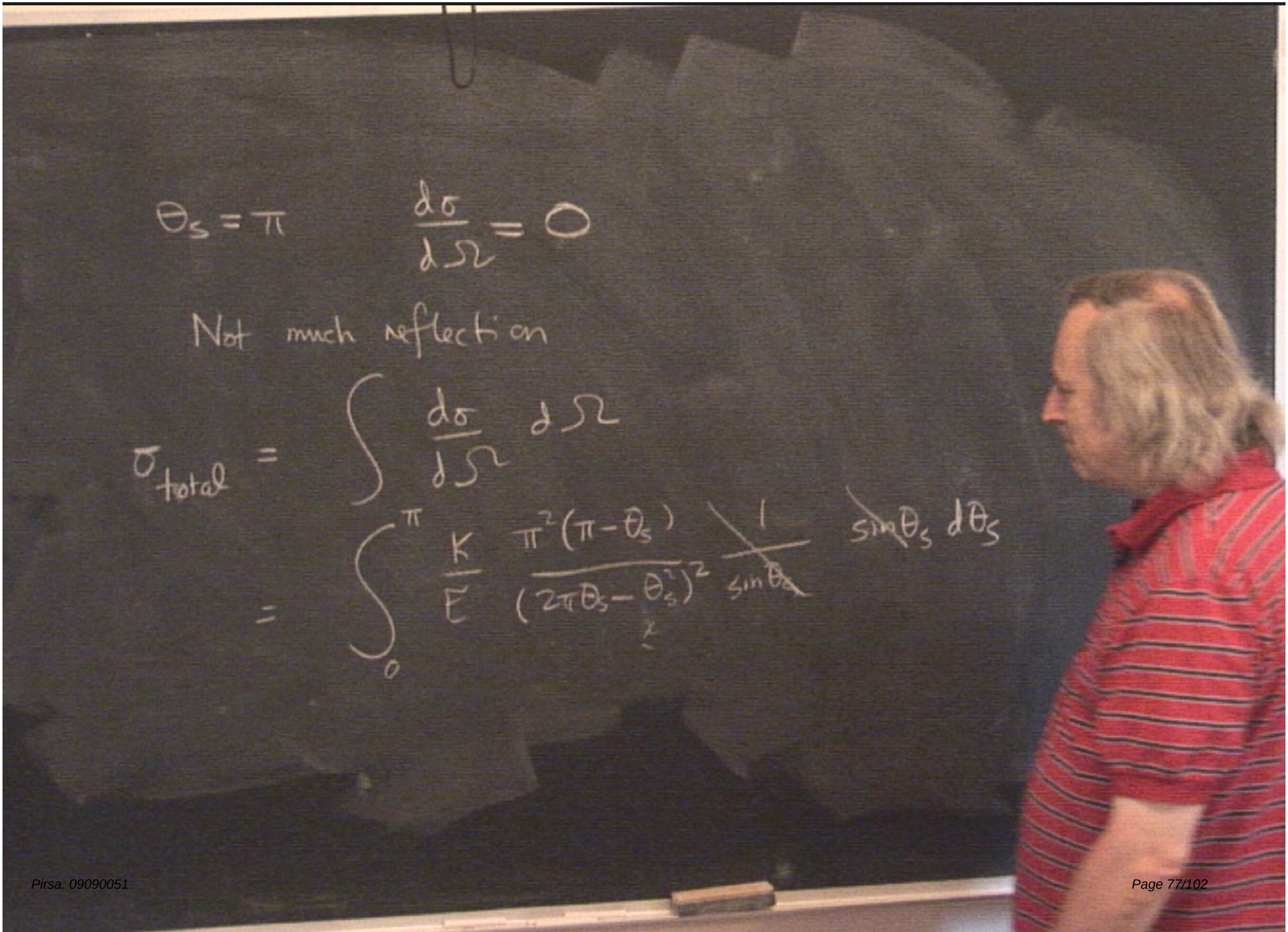
Decreasing function of E


$\theta_s = \pi$
$\frac{d\sigma}{d\Omega} = 0$
Not much reflection


$\theta_s = \pi \qquad \frac{d\sigma}{d\Omega} = 0$
Not much reflection
$\sigma_{total} = \int \frac{d\sigma}{d\Omega}\, d\Omega$
$= \int_0^\pi \frac{K}{E} \frac{\pi^2(\pi-\theta_s)}{(2\pi\theta_s - \theta_s^2)^2} \cancel{\frac{1}{\sin\theta_s}} \cancel{\sin\theta_s}\, d\theta_s$

$$\theta_s = \pi \qquad \frac{d\sigma}{d\Omega} = 0$$

Not much reflection

$$\sigma_{total} = \int \frac{d\sigma}{d\Omega}\, d\Omega$$

$$= \int_0^{\pi} \frac{K}{E}\, \frac{\pi^2(\pi-\theta_s)}{(2\pi\theta_s-\theta_s^2)^2}\, \cancel{\frac{1}{\sin\theta_s}}\, \cancel{\sin\theta_s}\, d\theta_s$$

$$= \frac{\pi^2 K}{E}\, \frac{1}{2}\, \frac{1}{(2\pi\theta_s-\theta_s^2)}\Bigg|_0^{\pi} = \infty$$

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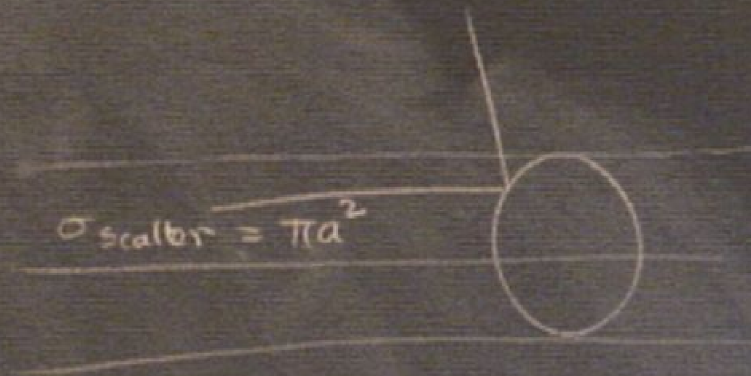
(Often happens)

$$= \frac{\pi^2 K}{E} \left(-\frac{1}{2}\right) \frac{1}{(2\pi\theta_s - \theta_s^2)} \Bigg|_0^\pi = \infty$$

Finite angles is OK

$$\theta_s = \pi \qquad \frac{d\sigma}{d\Omega} = 0$$

Not much reflection



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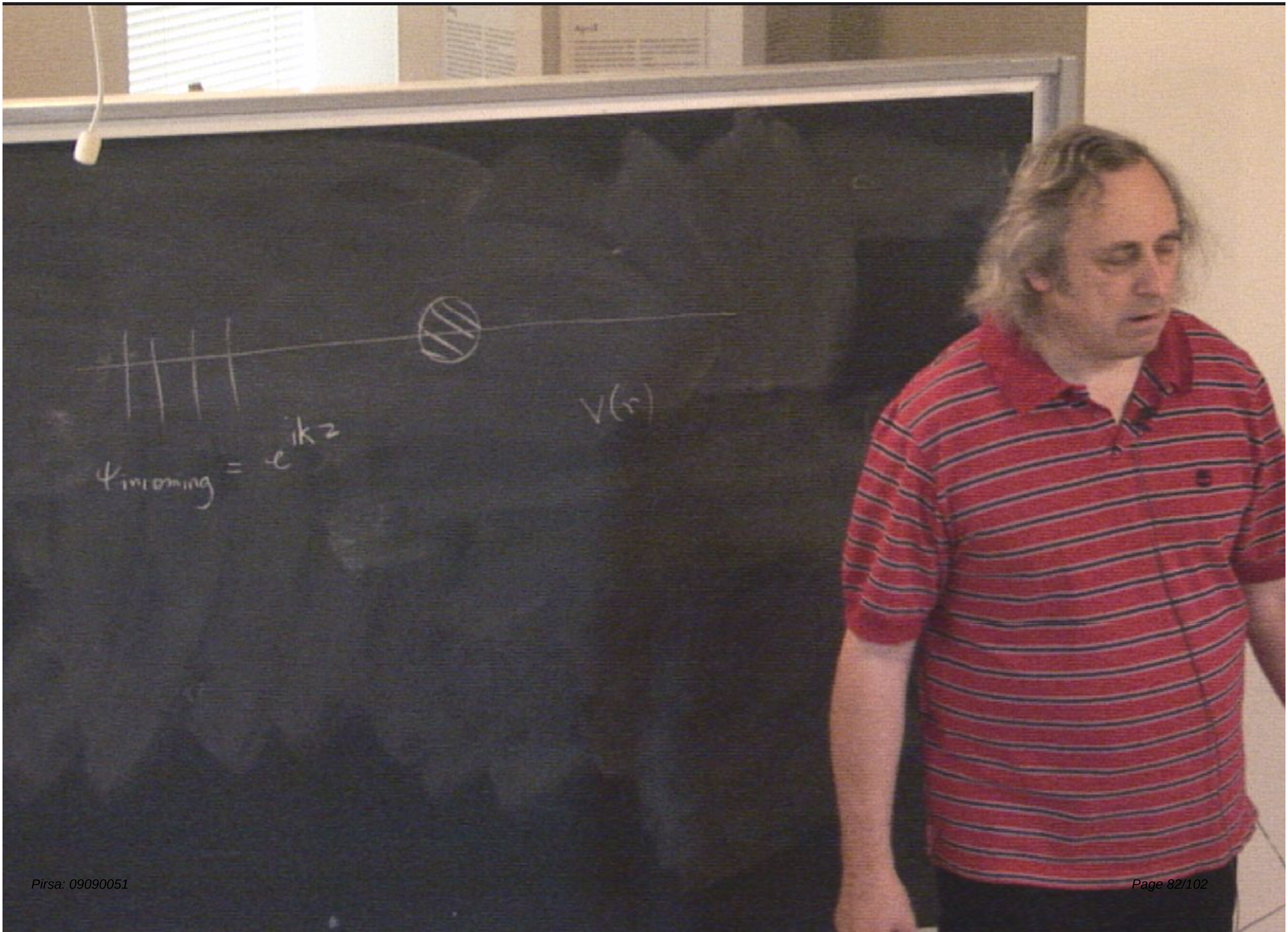
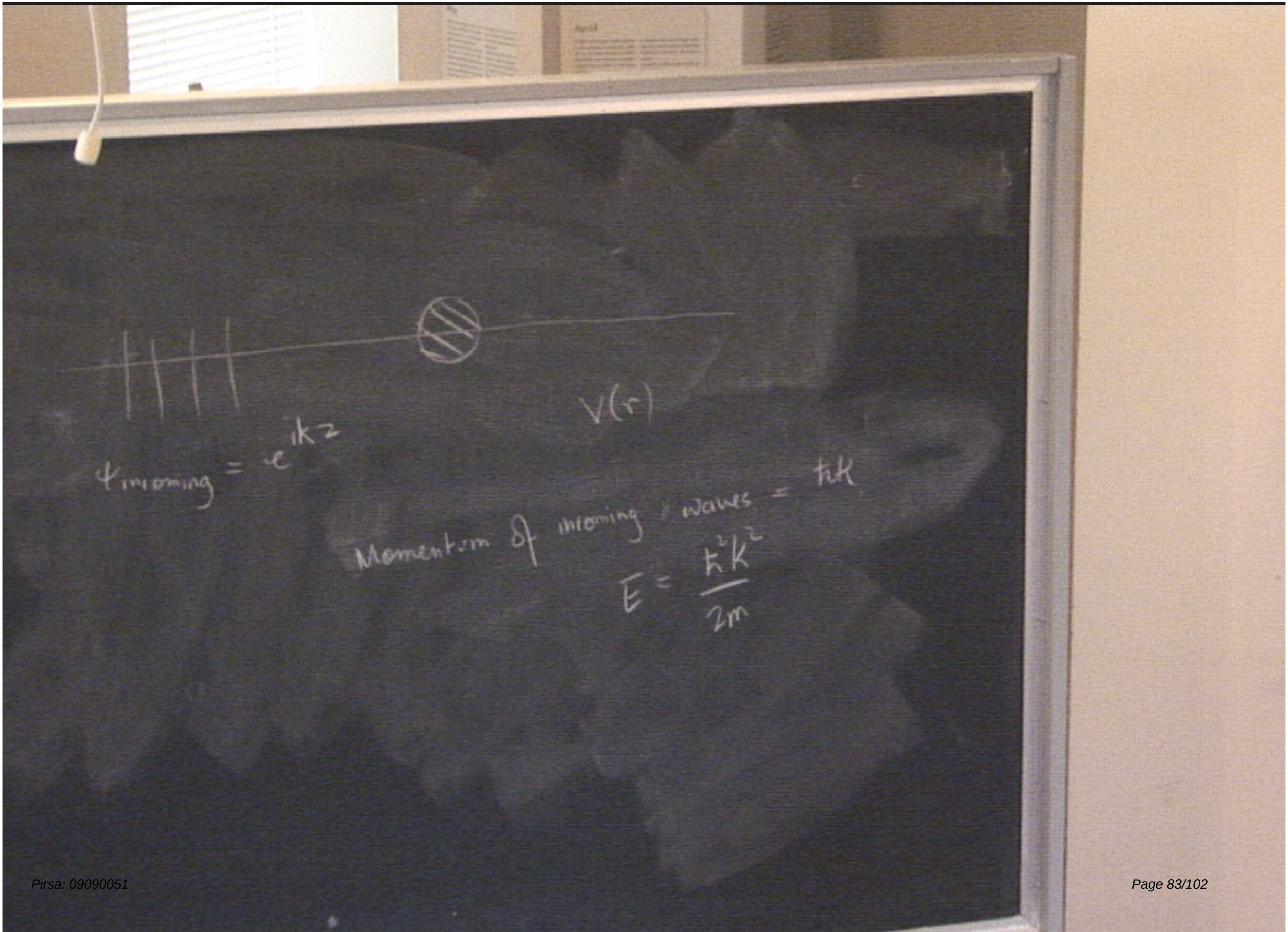
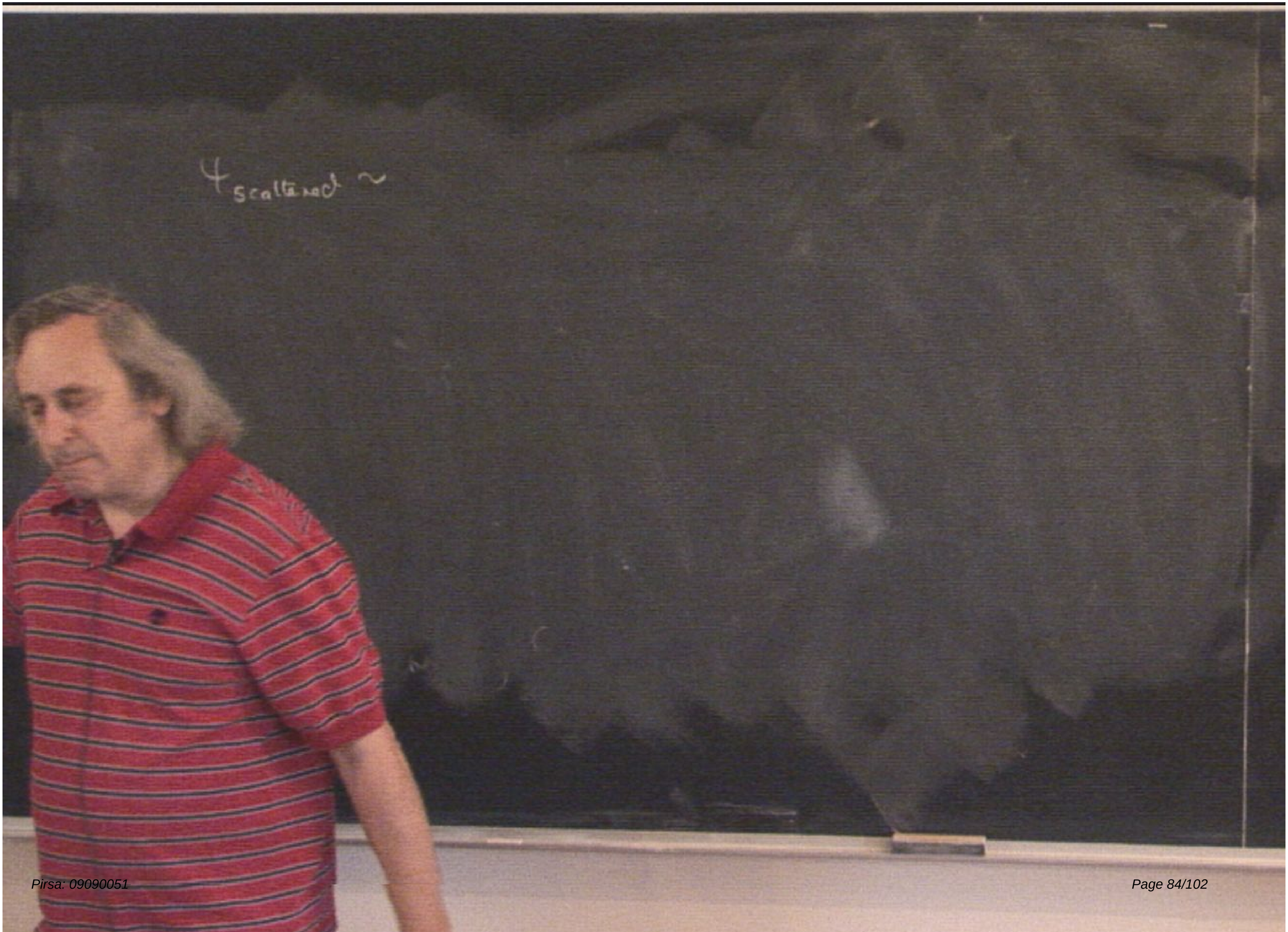
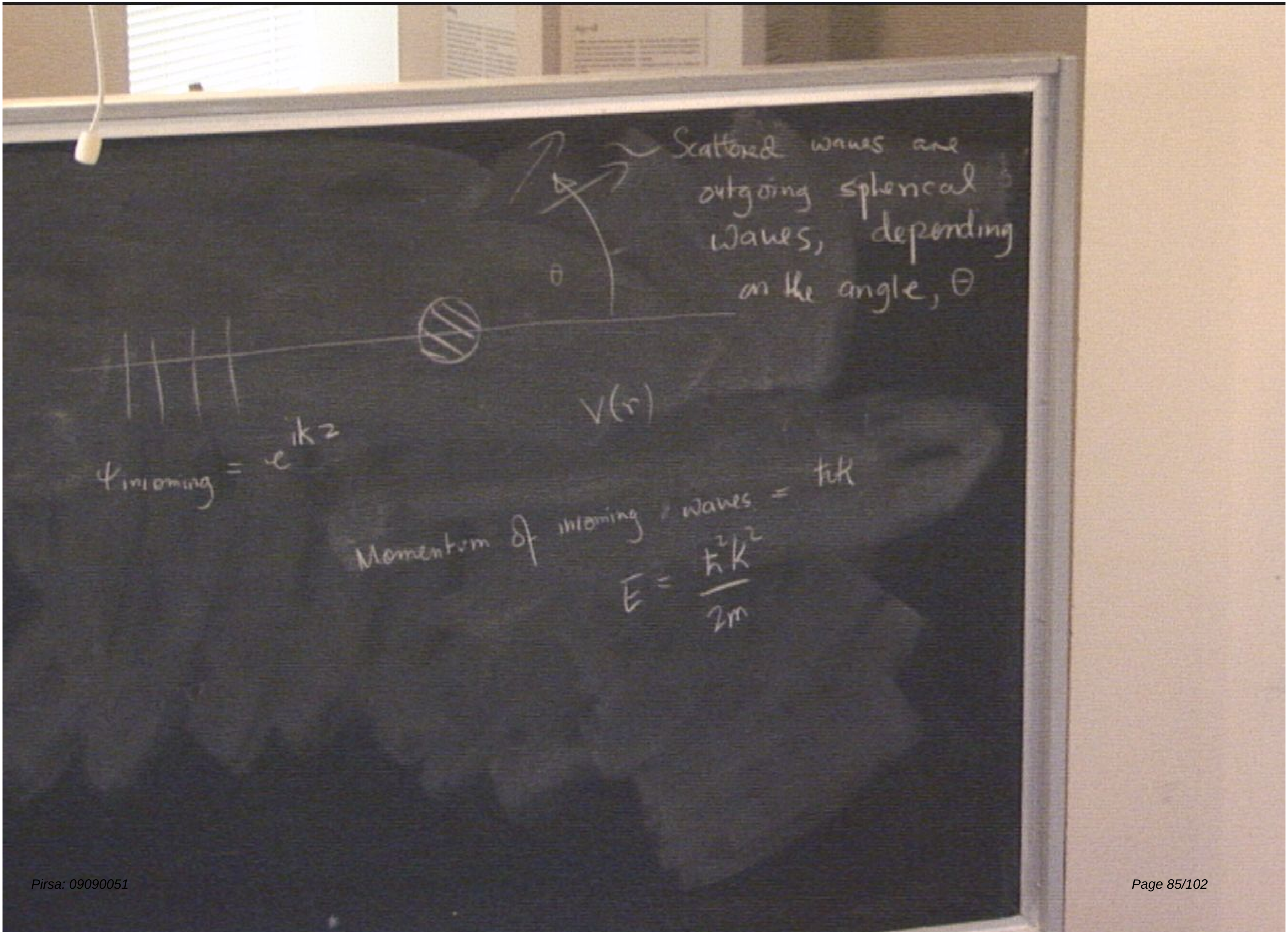

V(r)
ψincoming = e^ikz


V(r)
ψincoming = e^ikz
Momentum of incoming waves = ħk
E = ħ²k²/2m


ψ scattered ~


Scattered waves are outgoing spherical waves, depending on the angle, θ
θ
V(r)
ψ incoming = e^ikz
Momentum of incoming waves = ħk
E = ħ²k²/2m

$$\psi_{\text{scattered}} \sim \frac{e^{ikr}}{r} f(\theta) \quad \text{as } r \to \infty$$

$$\psi_{scattered} \sim \frac{e^{ikr}}{r} f(\theta) \qquad \text{as } r \to \infty$$

$$\psi_{outgoing} \quad \underbrace{e^{ikz}}_{\text{Not scattered}} + \frac{e^{ikr}}{r} f(\theta)$$

satisfies Schrödinger equation as $r \to \infty$

$\psi_{\text{scattered}} \sim \dfrac{e^{ikr}}{r} f(\theta)$ as $r \to \infty$

ψ_{outgoing} $\quad \underbrace{e^{ikz}}_{\text{Not scattered.}} + \dfrac{e^{ikr}}{r} f(\theta)$

satisfies Schrödinger equation as $r \to \infty$ $\forall f(\theta)$

OK - as long as $V(r)$ falls off fast enough.

Probability flux vector = $\frac{\hbar}{2im}\left(\psi^* \underline{\nabla} \psi - (\underline{\nabla}\psi)^* \psi \right)$

= No. of particles flow through
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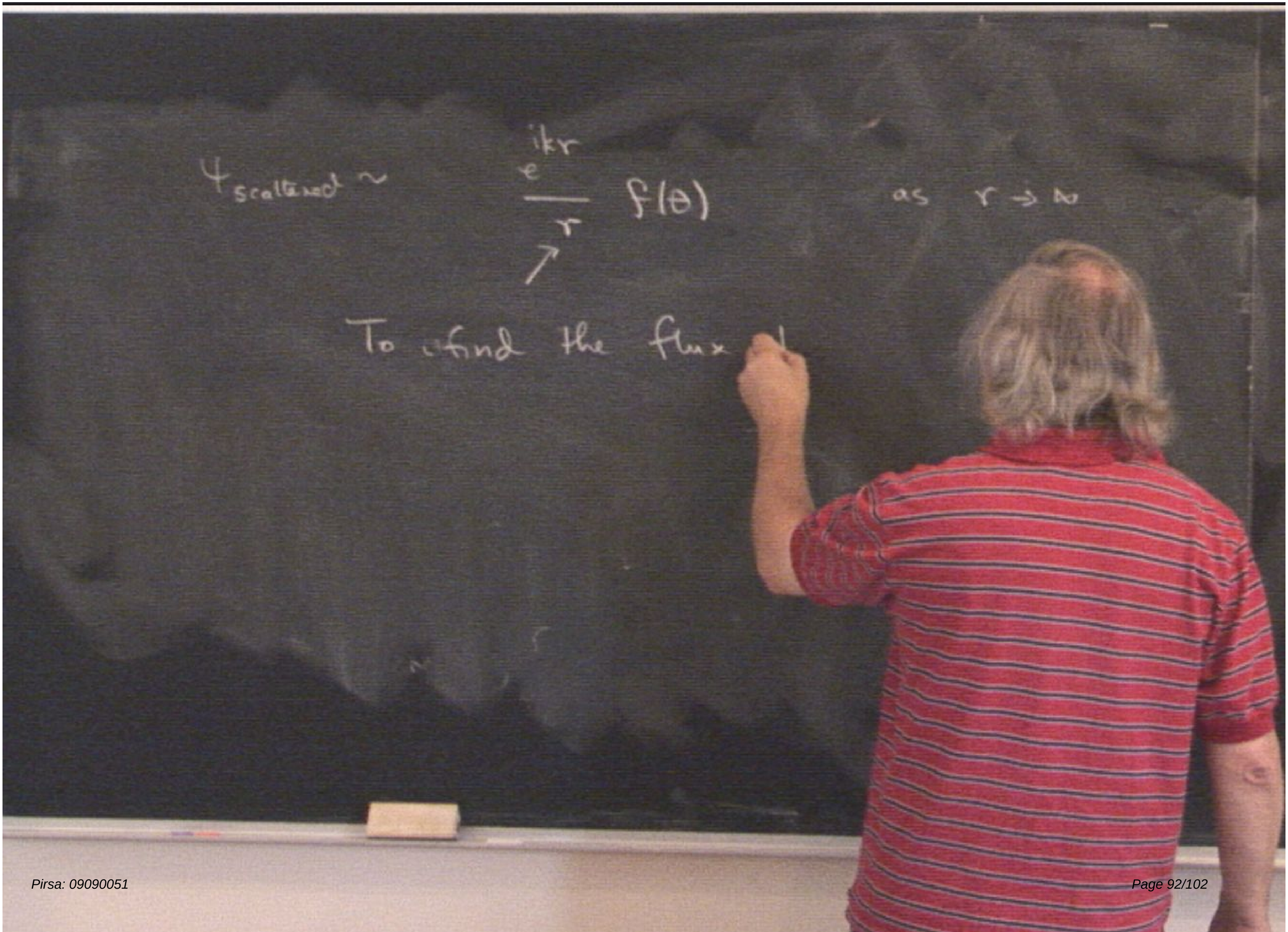
- For the incoming particles

$$\frac{\hbar}{2im}(2ik) = \frac{\hbar k}{m}$$ in positive z-dir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To find the flux

$\psi_{\text{scattered}} \sim \dfrac{e^{ikr}}{r} f(\theta)$ as $r \to \infty$

To find the flux at ∞, only interested in the $1/r^2$ part of the radial component of $\underline{J}$

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To find the flux at ∞, only interested in the $1/r^2$ part of the radial component of $\underline{J}$

$$\underline{J} = \frac{\hbar}{2im}\left[\psi^* \frac{\partial \psi}{\partial r} - \frac{\partial \psi^*}{\partial r}\psi\right] = \frac{\hbar}{2im}$$

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$\psi \sim \dfrac{e^{ikr}}{r} f(\theta)$ as $r \to \infty$

To find the flux at ∞, only interested in the $1/r^2$ part of the radial component of $\underline{J}$

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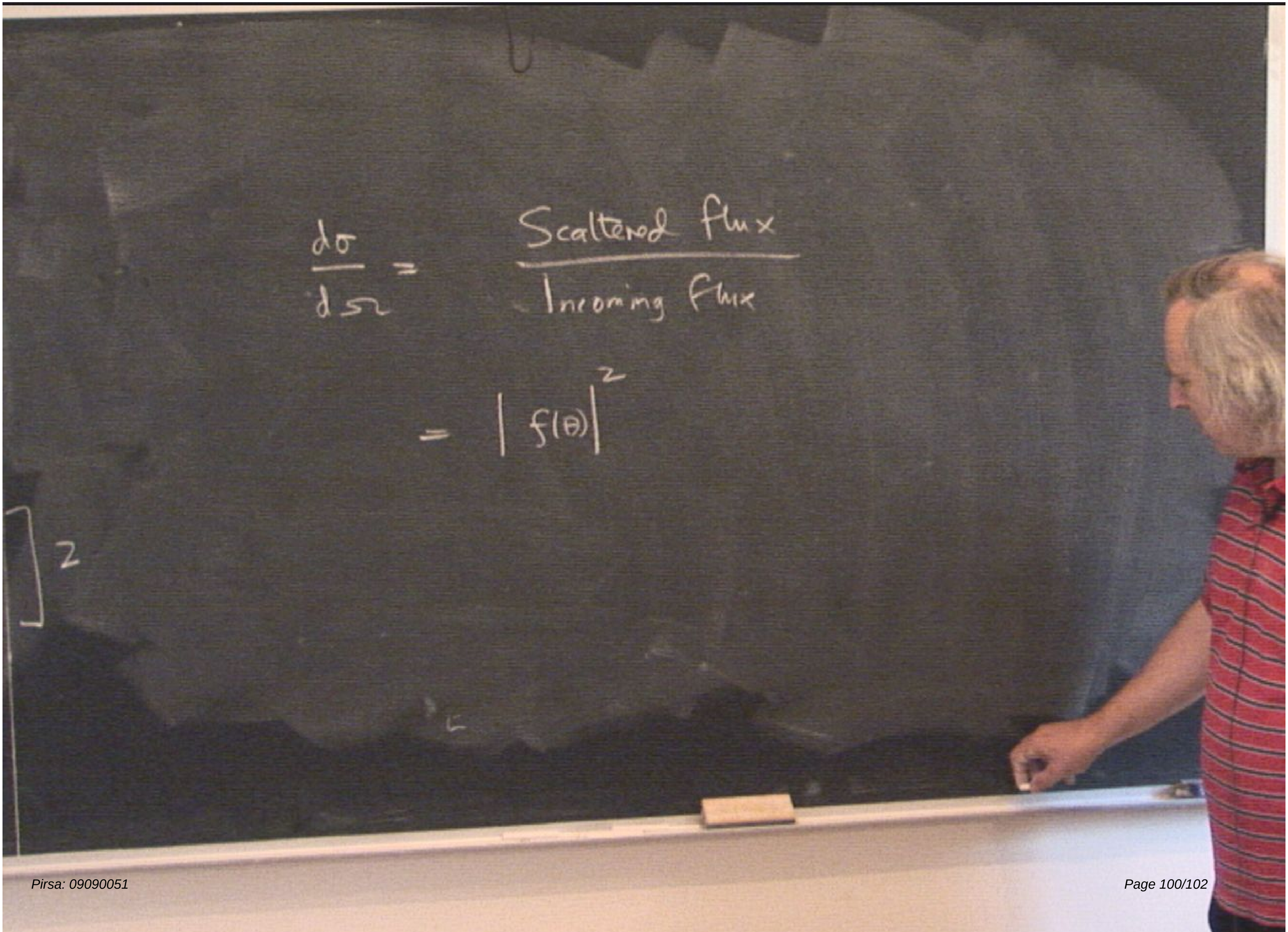
$$= \frac{\hbar k}{m} |f|^2$$

$$\frac{d\sigma}{d\Omega} = \frac{\text{Scattered flux}}{\text{Incoming flux}}$$

$=$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$= |f(\theta)|^2$
$]^2$

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Scattering amplitude.

$]^2$

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