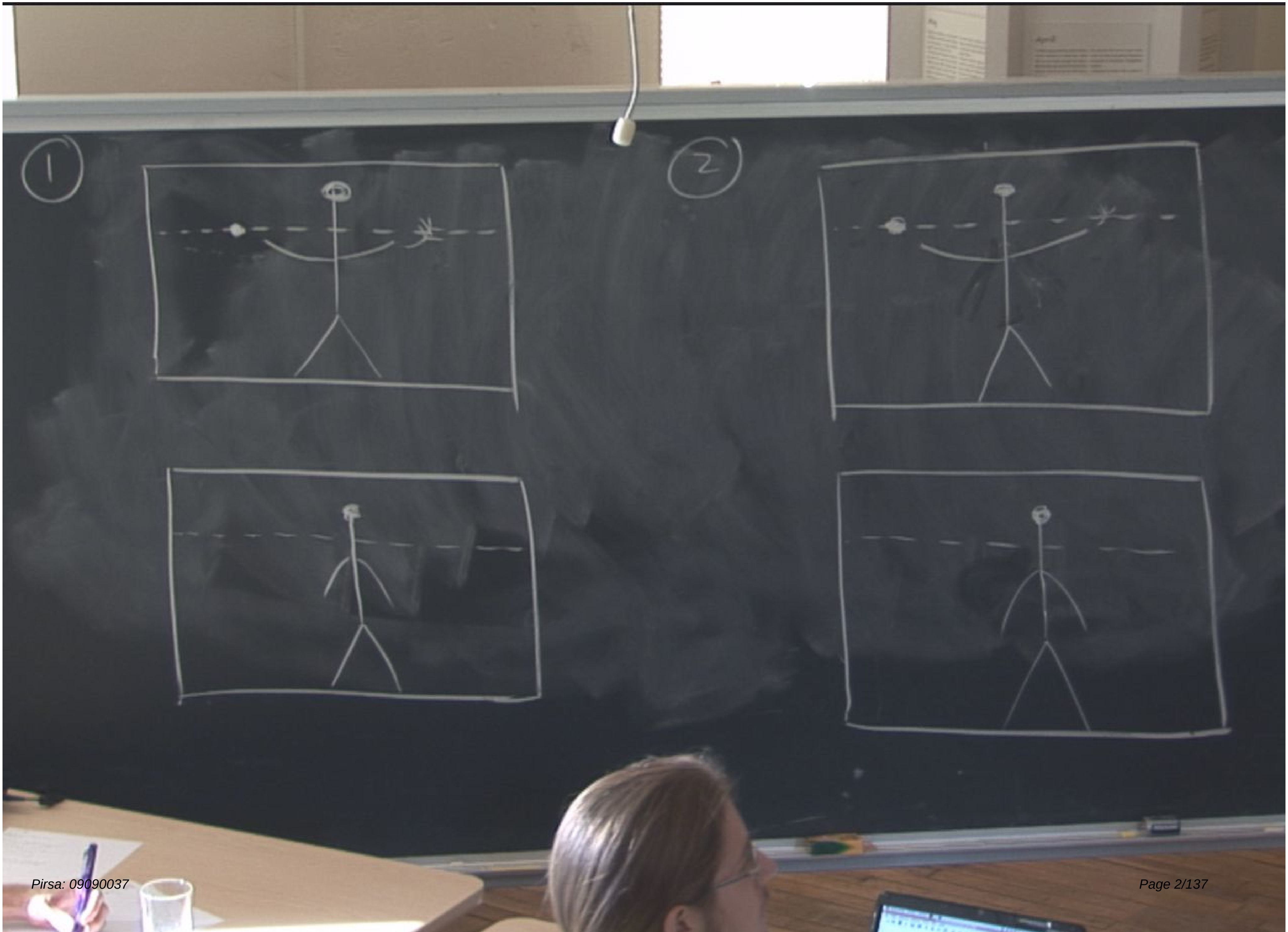
Title: Relativity - Core (PHYS 604) - Lecture 3

Date: Sep 07, 2009 09:00 AM

URL: http://pirsa.org/09090037

Abstract:



1
2

Newtonian gravity

$$\vec{F}_{12} = - \frac{G m_1 m_2}{|\vec{r}_1 - \vec{r}_2|^2} \hat{\rho}_{12}$$

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- inconsistent with S.R

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Newtonian gravity

$$\vec{F}_{12} = - \frac{G m_1 m_2}{|\vec{r}_1 - \vec{r}_2|^2} \hat{\rho}_{12}$$

- inconsistent with S.R.
  - $\vec{F}(t)$ depends on $\vec{r}_1(t)$ and $\vec{r}_2(t)$ at the same instant of time — but different inertial observers will disagree "same time"

- one key idea in Einstein's construction of

key idea in Einstein's construction
G.R. was the equality of
avitational and inertial mass

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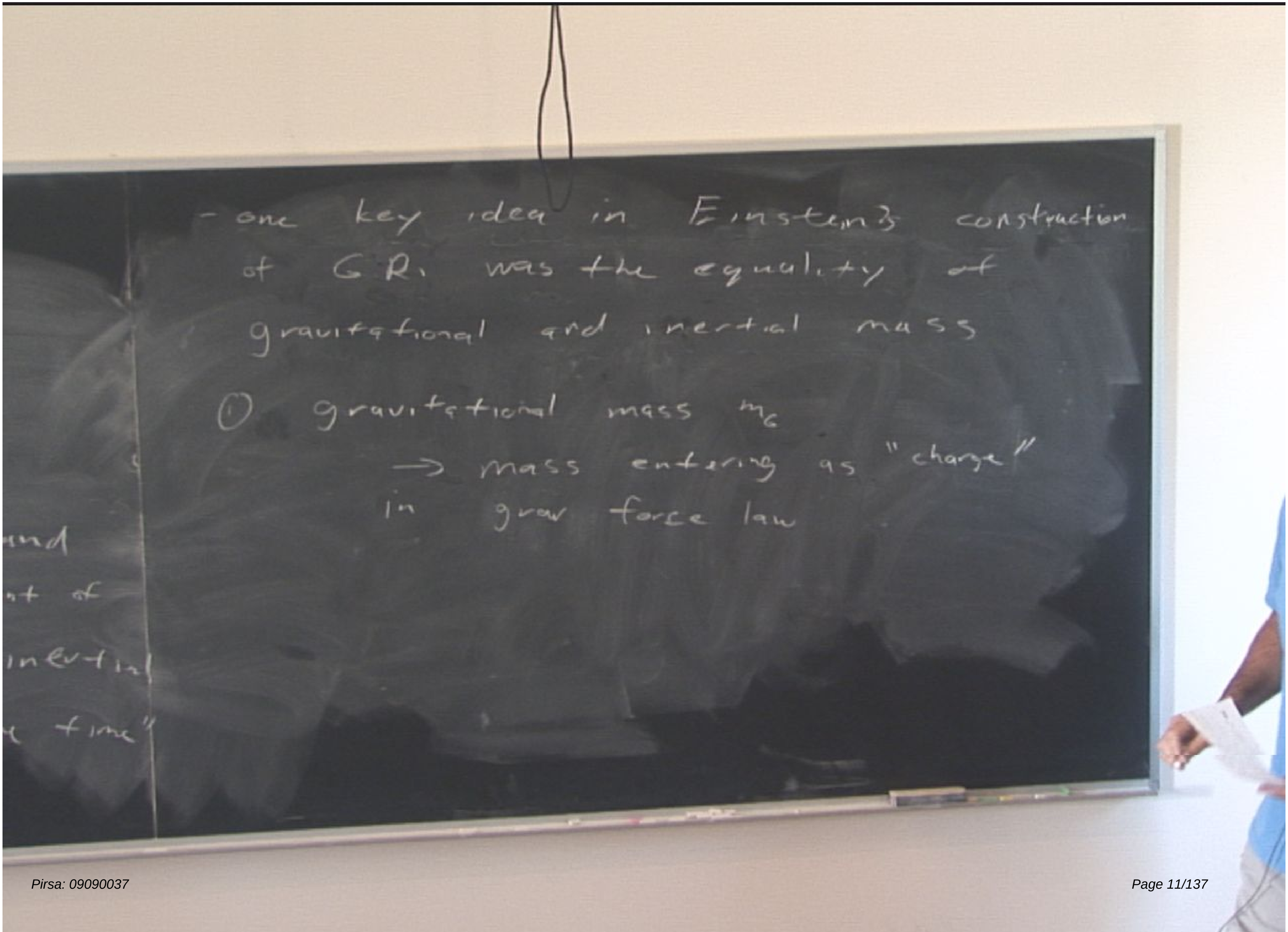
inertial

time"

- one key idea in Einstein's construction of G.R. was the equality of gravitational and inertial mass

(1) gravitational mass m_G


- one key idea in Einstein's construction
of G.R. was the equality of
gravitational and inertial mass
① gravitational mass m_G
→ mass entering as "charge"
in grav force law
and
nt of
inertial
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② inertial mass, m_I

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(1) gravitational mass m_G

$\rightarrow$ mass entering as "charge" in grav force law

(2) inertial mass, m_I

$\rightarrow$

- one key idea in Einstein's construction of G.R. was the equality of gravitational and inertial mass

① gravitational mass m_G

→ mass entering as "charge" in grav force law

② inertial mass, m_I

→ Mass that enters in the Newton's 2nd law: $\vec{F} = m_I \vec{a}$

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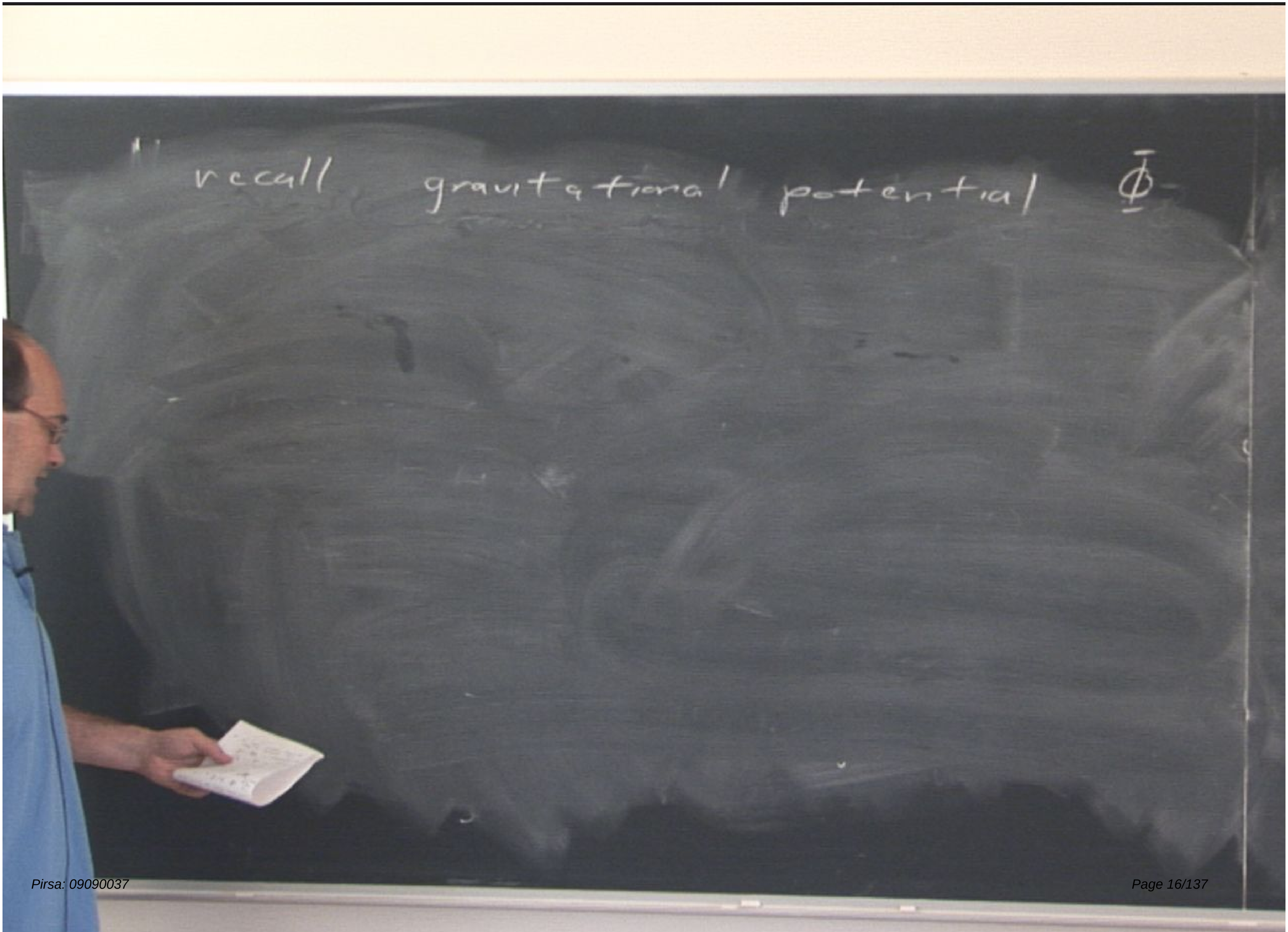
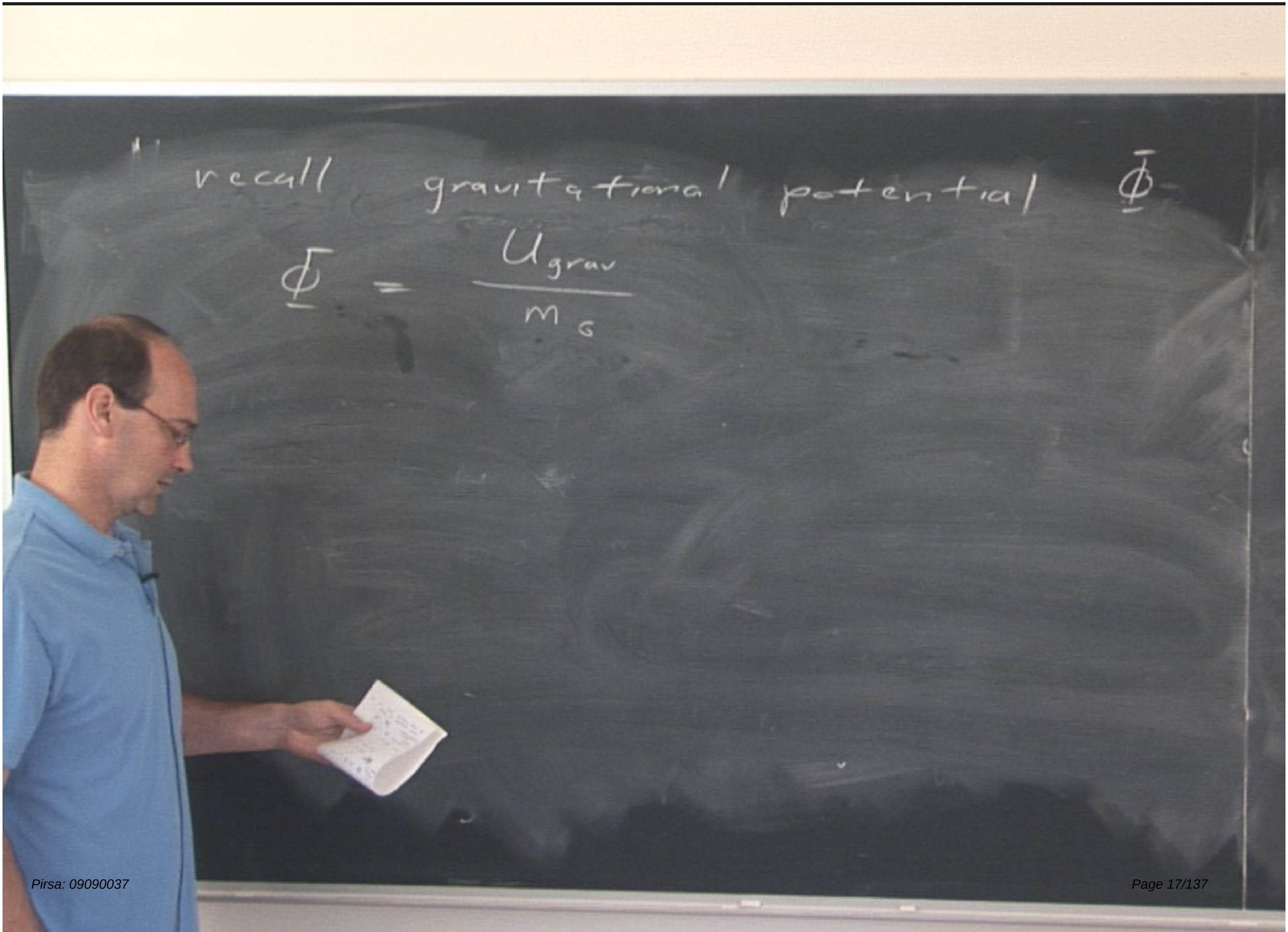

recall gravitational potential Φ


"recall gravitational potential Φ
$\Phi = \frac{U_{grav}}{m_G}$

recall gravitational potential Φ

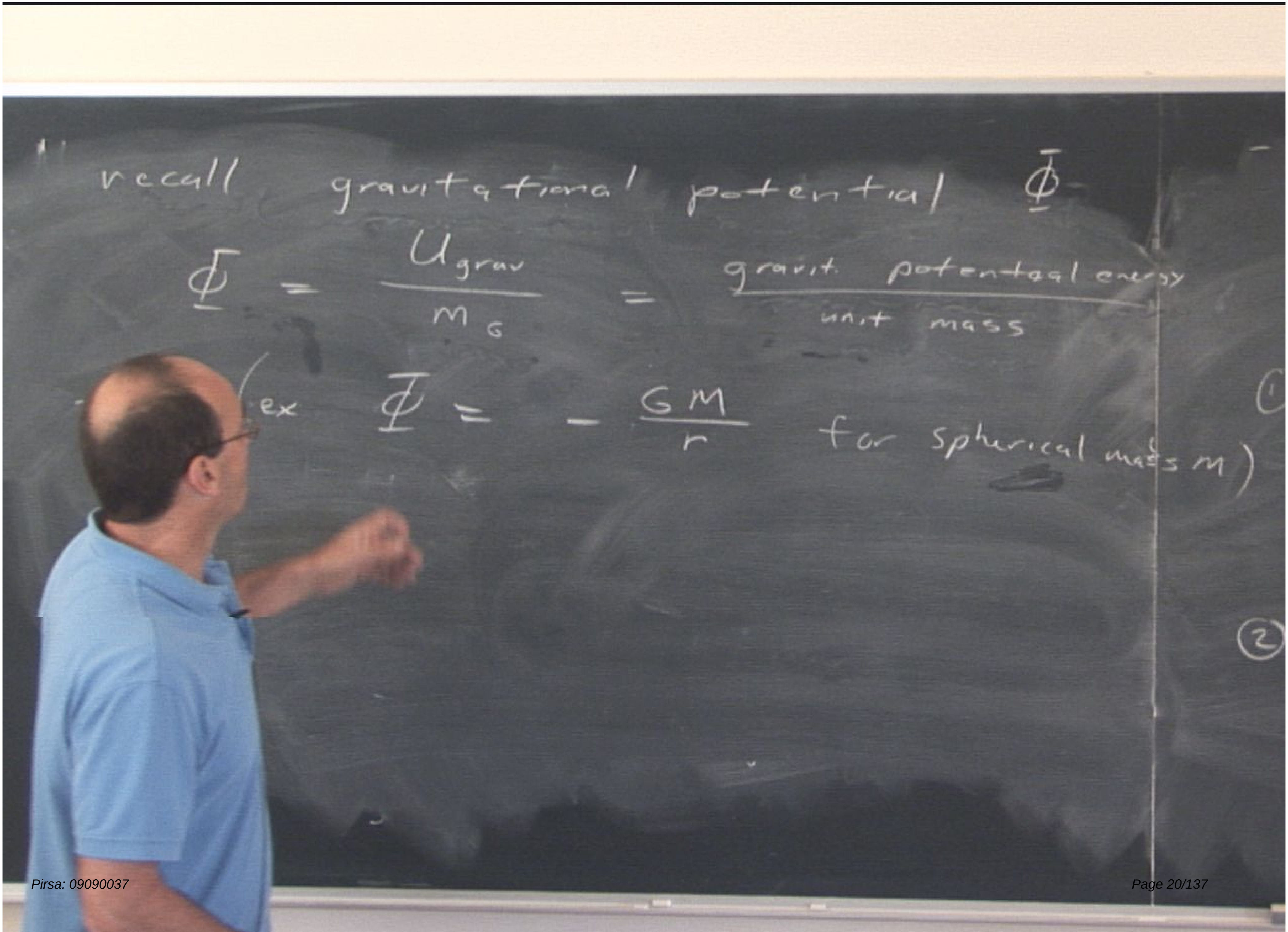
$$\Phi = \frac{U_{grav}}{m_G} = \frac{\text{gravit. potential energy}}{\text{unit mass}}$$

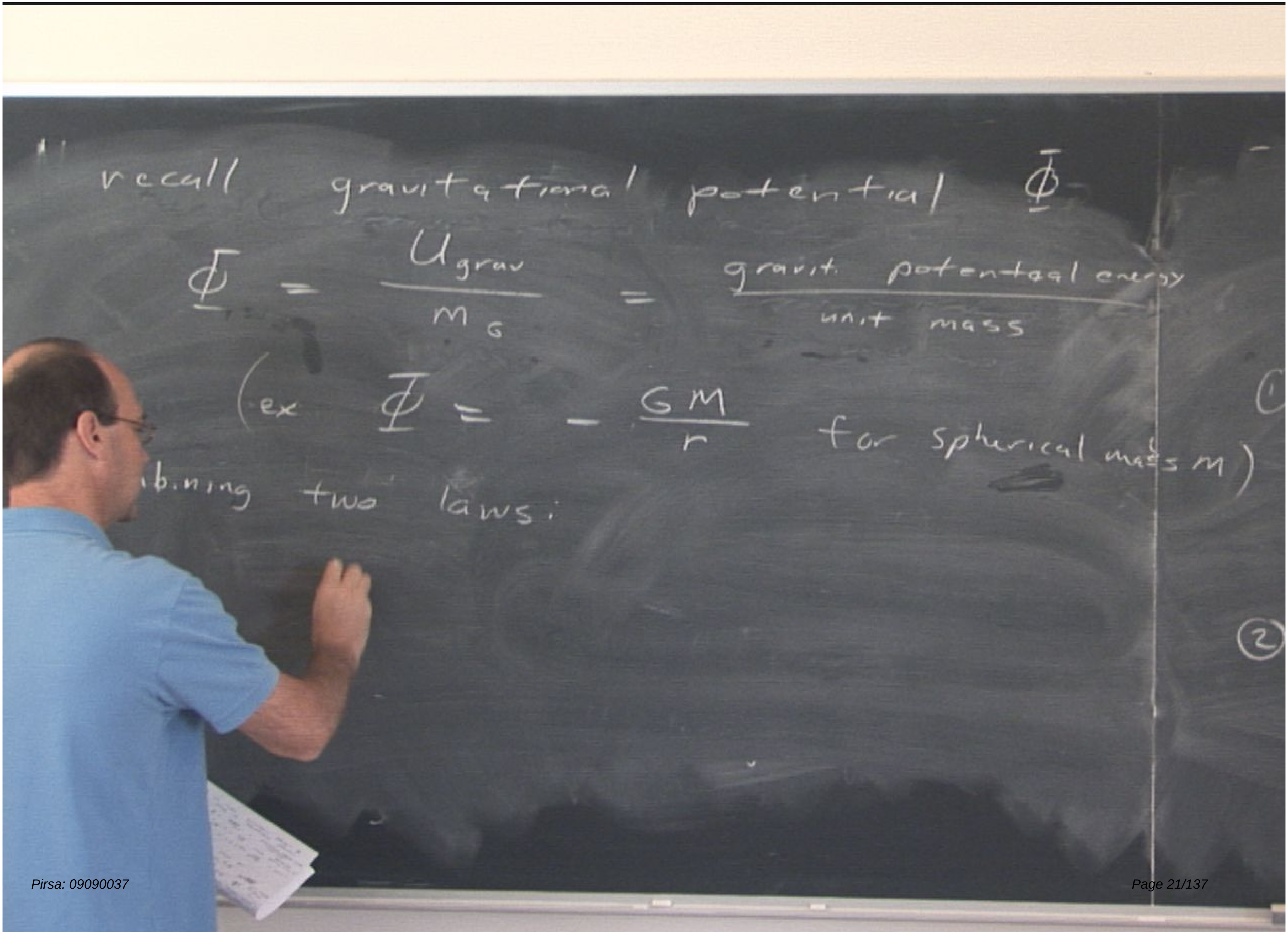
(ex $\Phi = -$

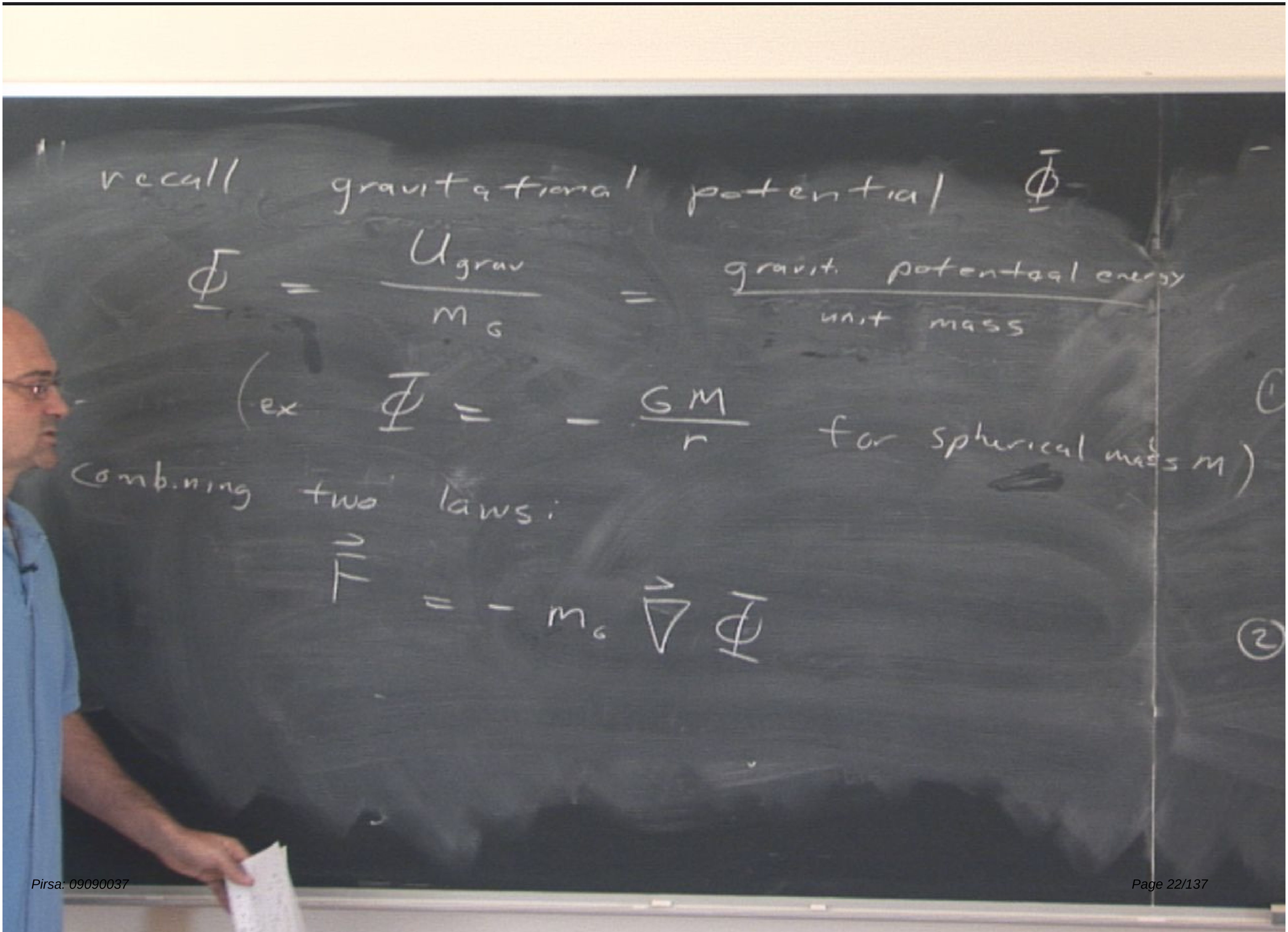
recall gravitational potential Φ

$$\Phi = \frac{U_{grav}}{m_G} = \frac{\text{gravit. potential energy}}{\text{unit mass}}$$

(ex $\Phi = -\frac{GM}{r}$


recall gravitational potential Φ
Φ = U_grav / m_G = gravit. potential energy / unit mass
(ex Φ = − GM / r for spherical mass M)
1
2


recall gravitational potential Φ
$\Phi = \frac{U_{grav}}{m_G} = \frac{\text{gravit. potential energy}}{\text{unit mass}}$
(ex $\Phi = -\frac{GM}{r}$ for spherical mass M)
bining two laws:
①
②


recall gravitational potential Φ
$\Phi = \frac{U_{grav}}{m_G} = \frac{\text{gravit. potential energy}}{\text{unit mass}}$
(ex $\Phi = -\frac{GM}{r}$ for spherical mass M)
combining two laws:
$\vec{F} = -m_G \vec{\nabla} \Phi$
①
②

recall gravitational potential Φ

$$\Phi = \frac{U_{grav}}{m_G} = \frac{\text{gravit. potential energy}}{\text{unit mass}}$$

(ex $\Phi = -\frac{GM}{r}$ for spherical mass M) ①

combining two laws:

$$\vec{F} = -m_G \vec{\nabla} \Phi = m_I \vec{a}$$ ②

recall gravitational potential Φ

$$\Phi = \frac{U_{grav}}{m_G} = \frac{\text{gravit. potential energy}}{\text{unit mass}}$$

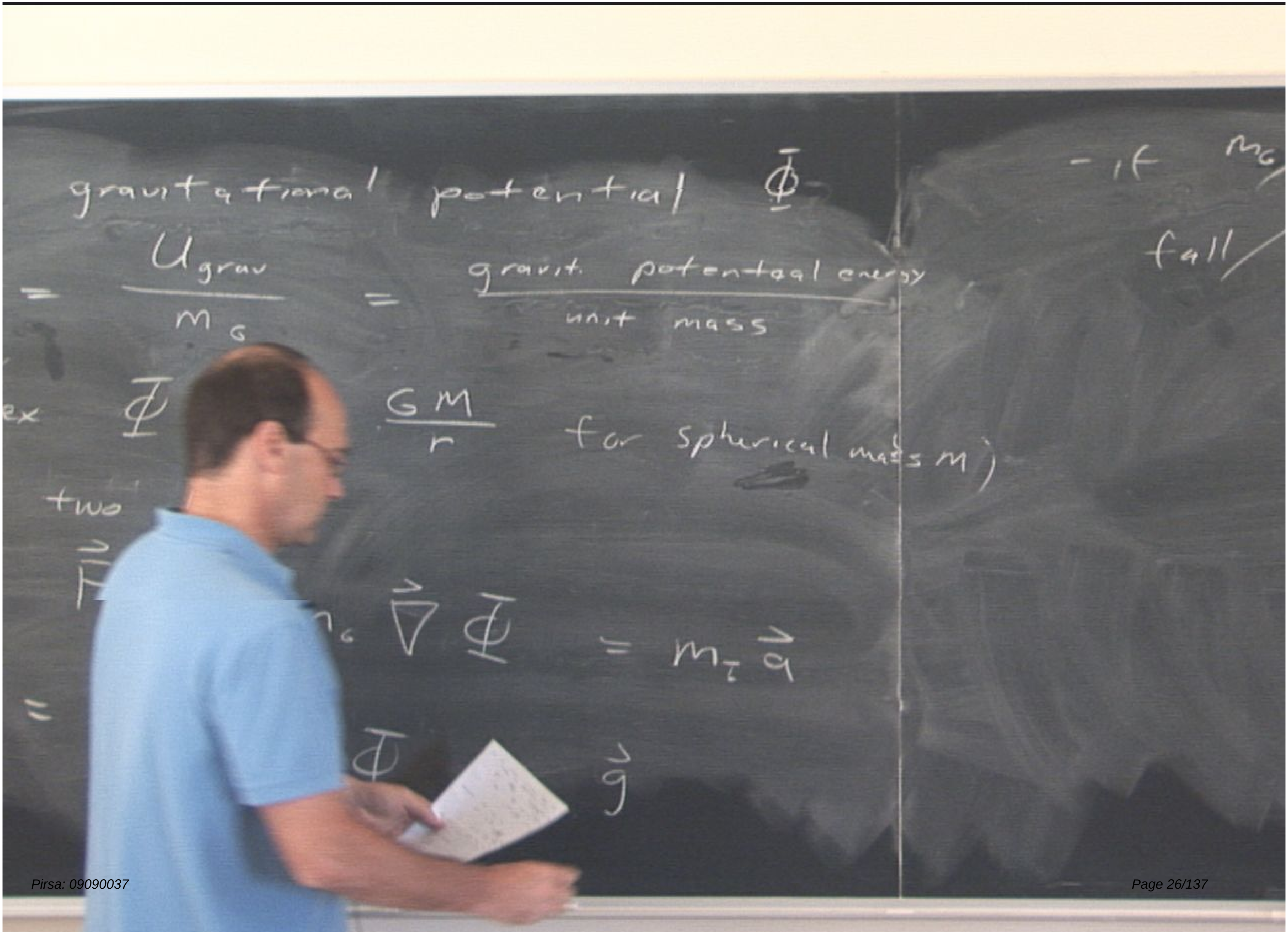
(ex $\Phi = -\frac{GM}{r}$ for spherical mass M) ①

combining two laws:

$$\vec{F} = -m_G \vec{\nabla} \Phi = m_I \vec{a}$$ ②

$$\vec{a} = -\frac{m_G}{m_I} \vec{\nabla} \Phi .$$

- if $m_G/m_I = 1$, then all bodies fall/accelerate in the same way


gravitational potential Φ
= U grav / m G = gravit. potential energy / unit mass
ex Φ
GM / r
for spherical mass M)
two
= m I a
g
- if M G
fall

recall gravitational potential Φ

$$\Phi = \frac{U_{grav}}{m_G} = \frac{\text{gravit. potential energy}}{\text{unit mass}}$$

$\left(\text{ex } \Phi = -\frac{GM}{r} \text{ for spherical mass } M\right)$

combining two laws:

$$\vec{F} = -m_G \vec{\nabla} \Phi = m_I \vec{a}$$

$$\vec{a} = -\frac{m_G}{m_I} \vec{\nabla} \Phi \Rightarrow \vec{g} \leftarrow$$

local acceler- due to gravity with $m_G / m_I = 1$

- if $m_G/m_I = 1$, then all bodies fall/accelerate in the same way

- modern exp's verify $m_G/m_I = 1$ to few parts in 10^{13}

recall gravitational potential Φ

$$\Phi = \frac{U_{grav}}{m_G} = \frac{\text{gravit. potential energy}}{\text{unit mass}}$$

$$\left(\text{ex} \quad \Phi = -\frac{GM}{r} \right. \quad \text{for spherical mass}$$

Combining two laws:

$$\vec{F} = -m_G \vec{\nabla} \Phi = m_I \vec{a}$$

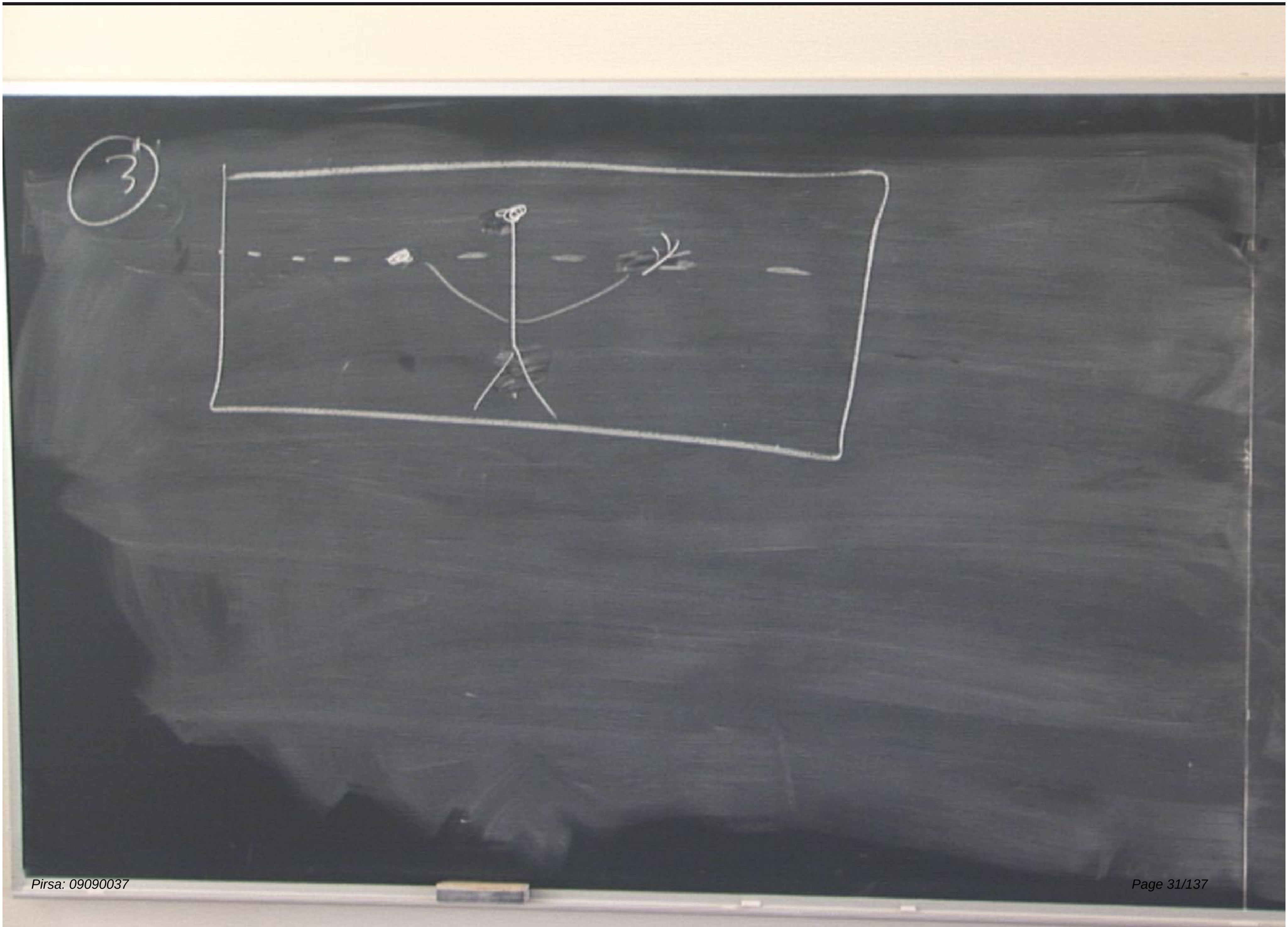
$$\vec{a} = -\frac{m_G}{m_I} \vec{\nabla} \Phi \Rightarrow \vec{g} \Leftarrow$$

local acc…
due t…
with m…

- if $m_G/m_I = 1$, then all bodies fall/accelerate in the same way

- modern exp's verify $m_G/m_I = 1$ to few parts in 10^{13}

(read Hartle Chap 6)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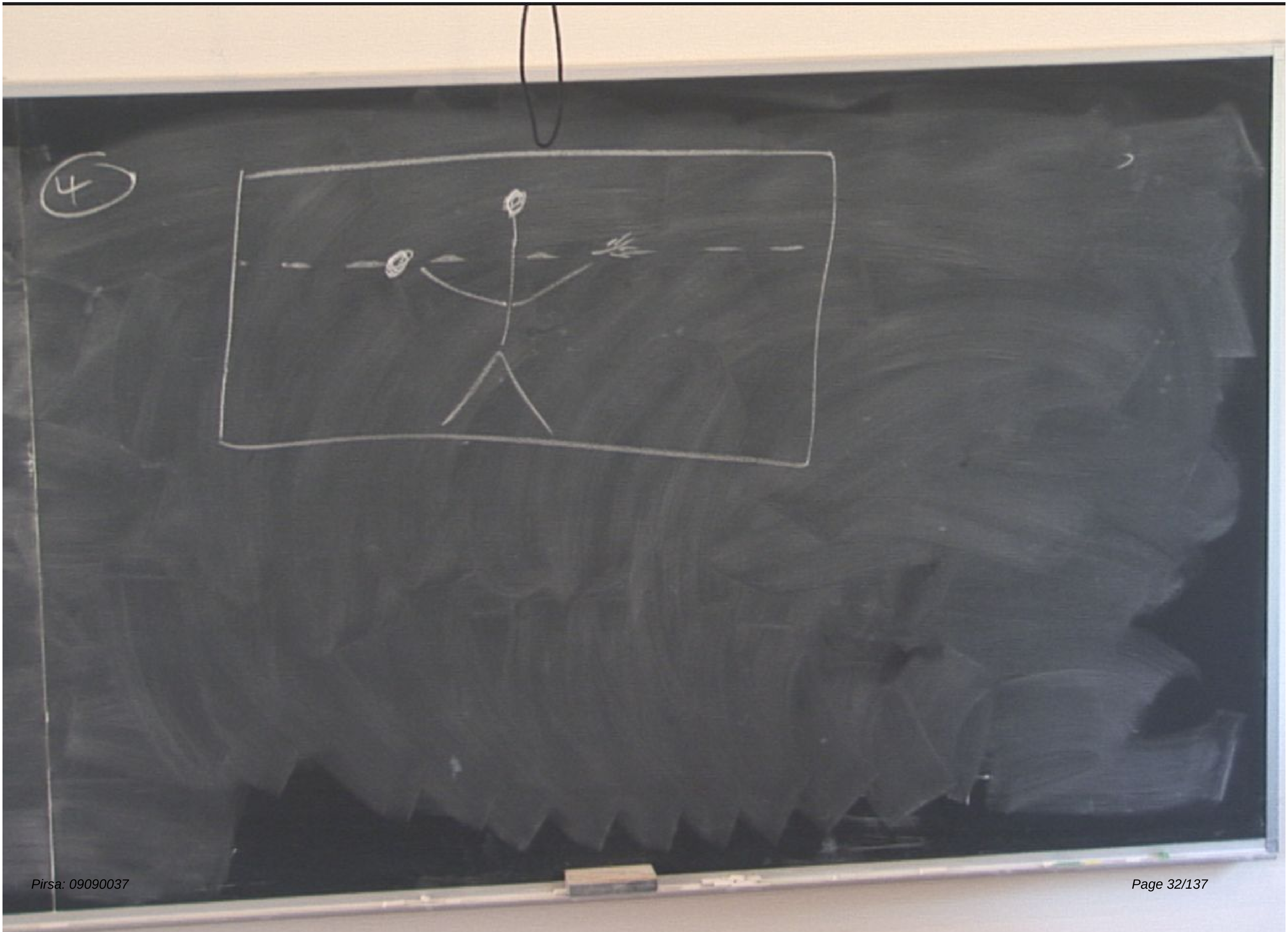
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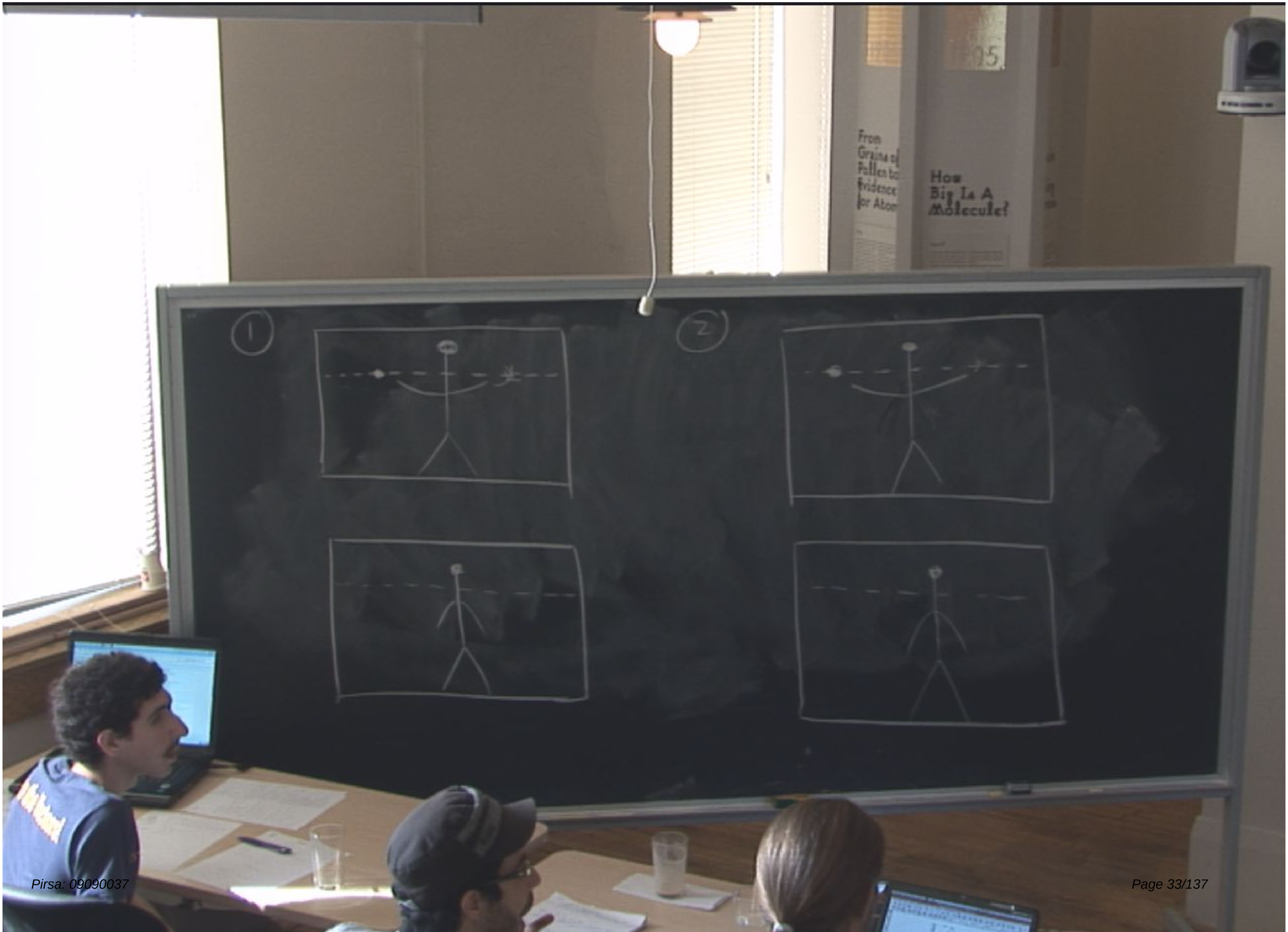
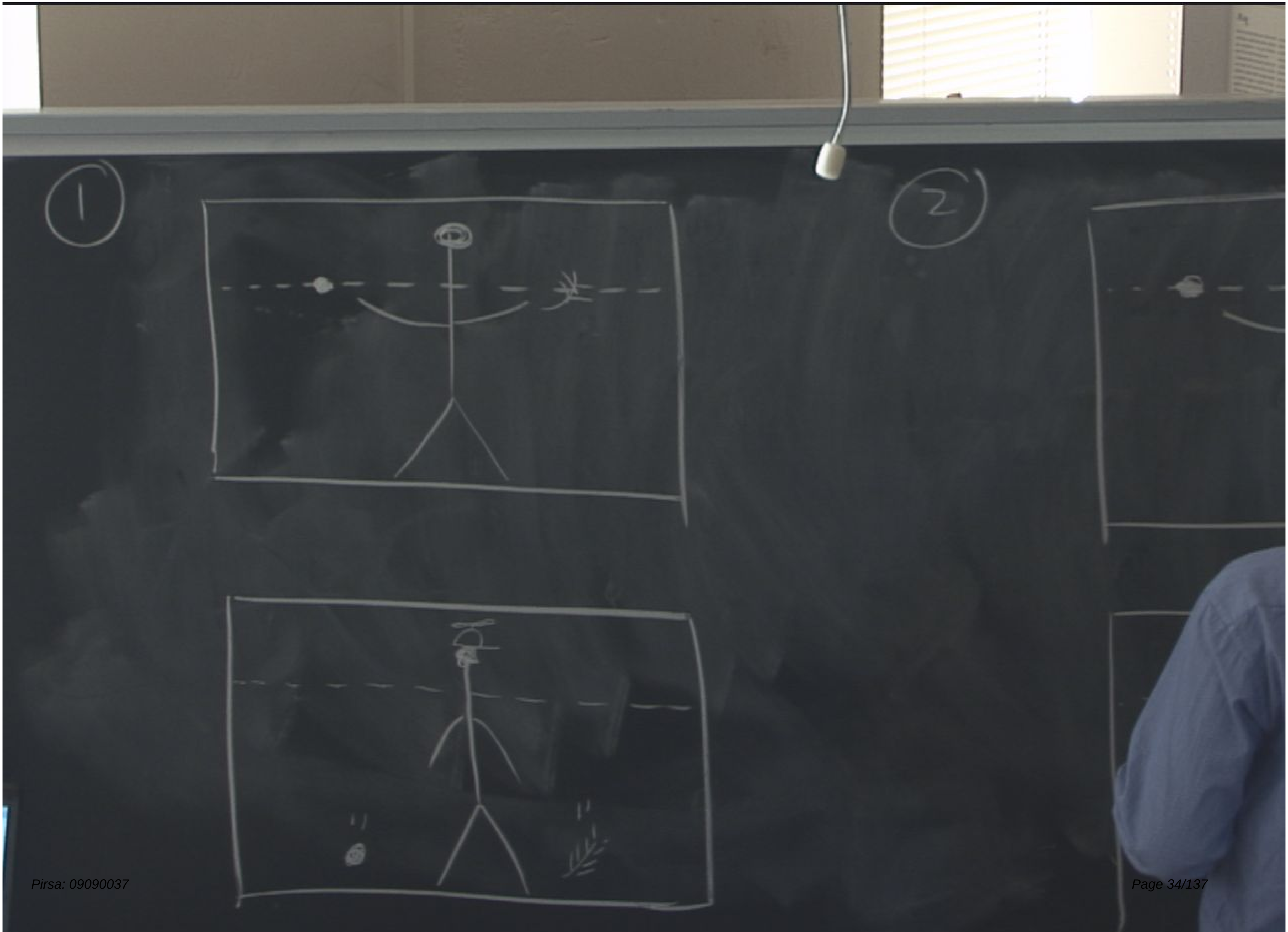
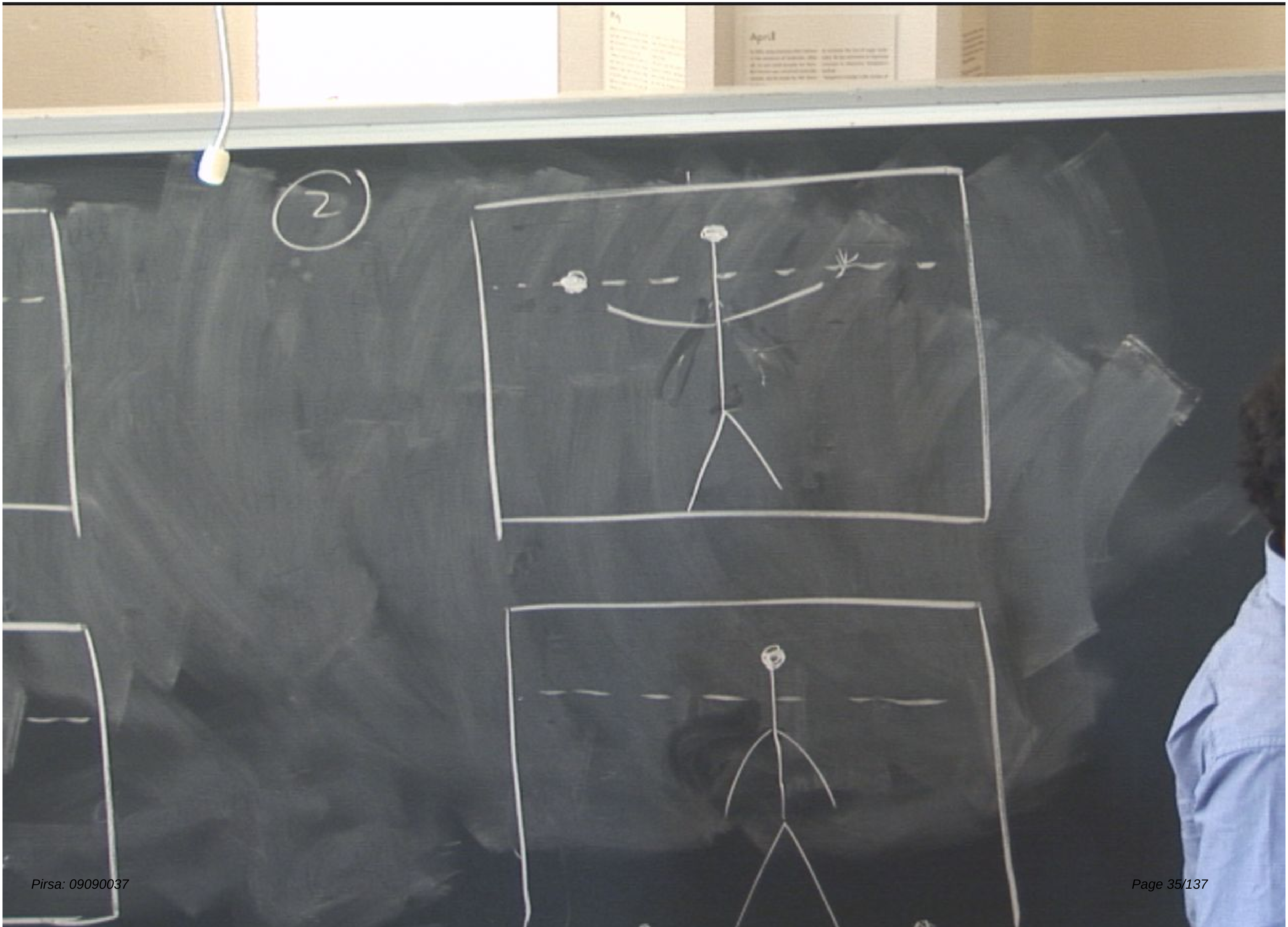

From
Grains of
Pollen to
Evidence
for Atom
How
Big Is A
Molecule?
1
2


1
2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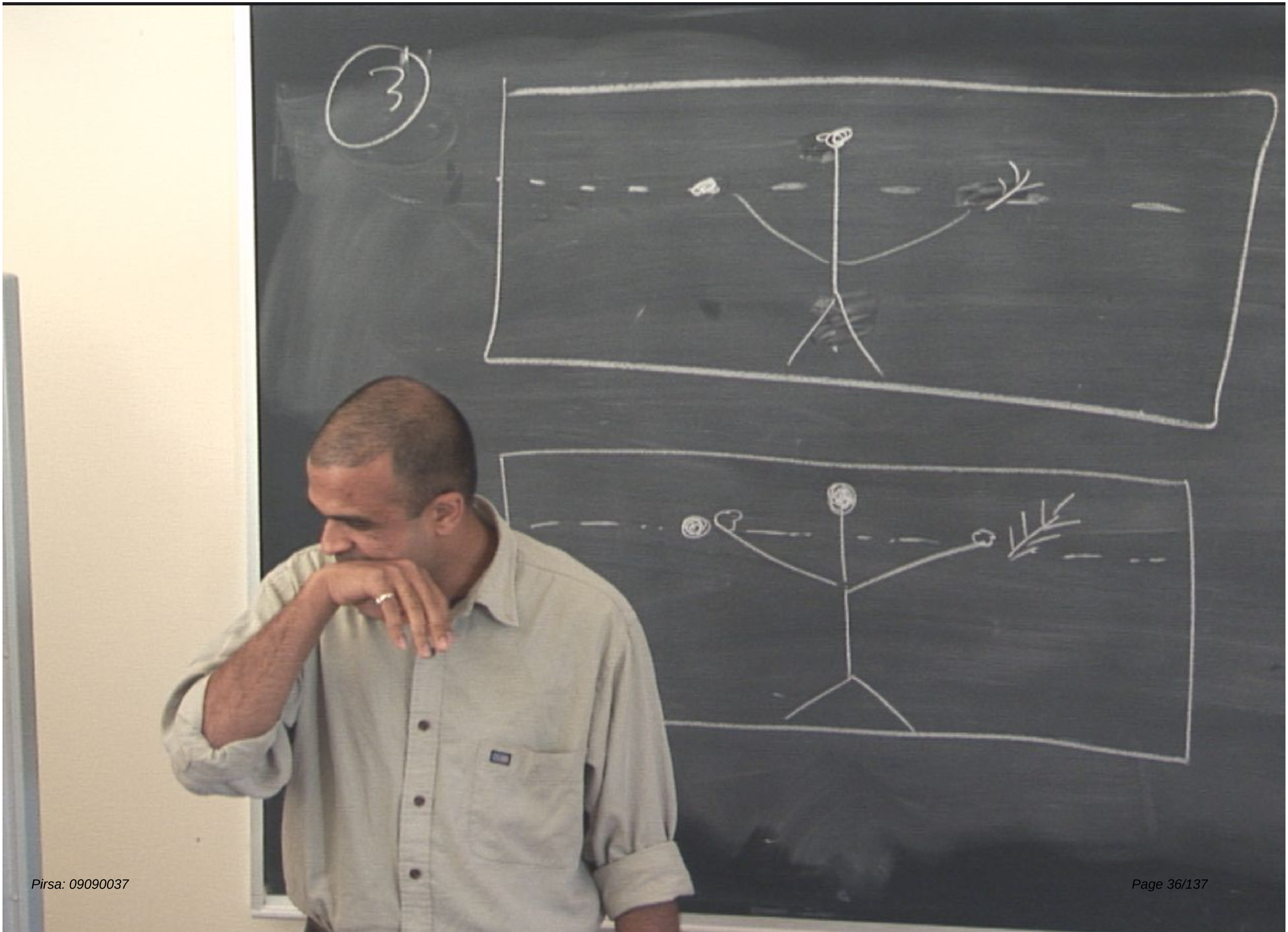
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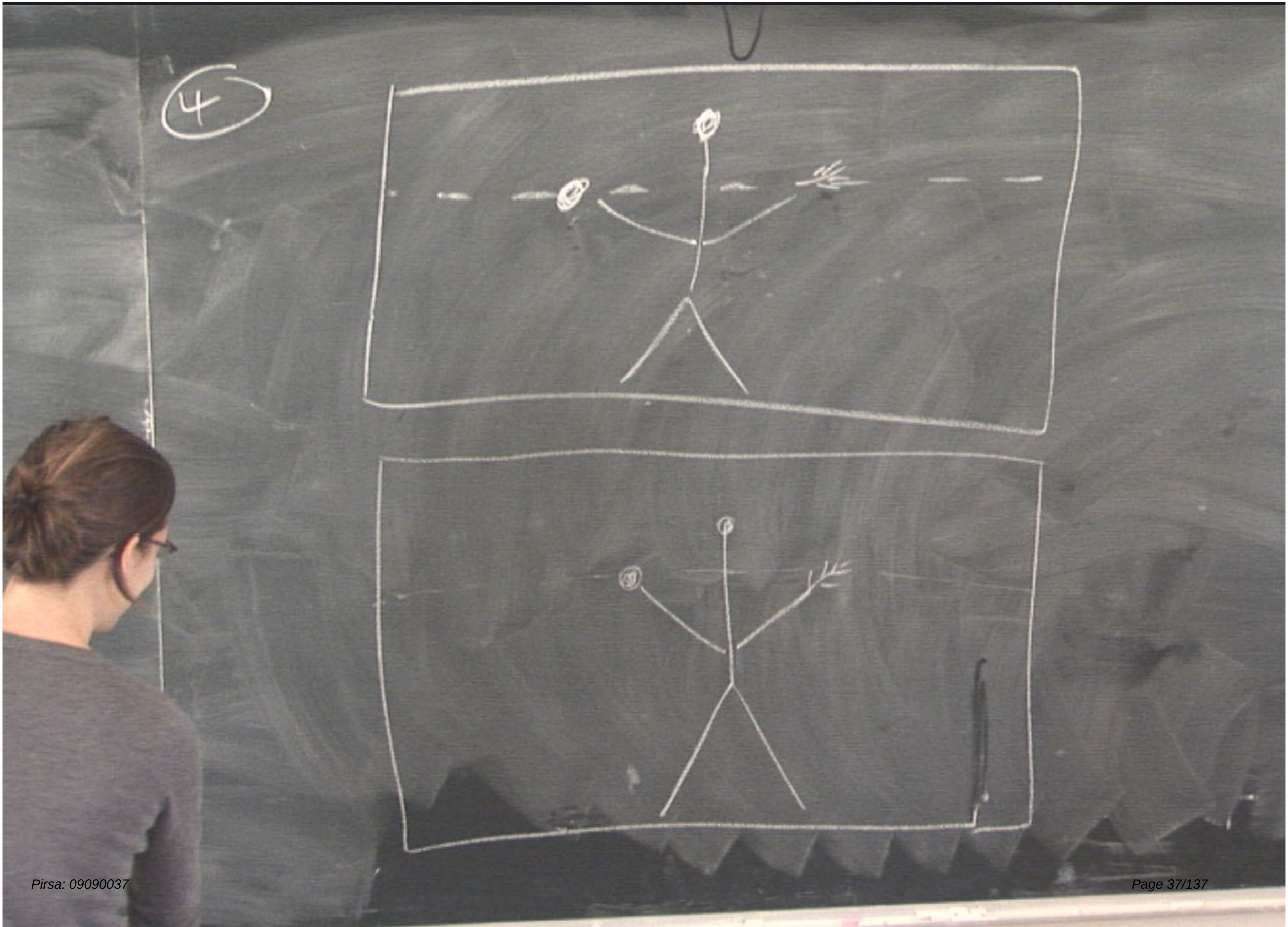
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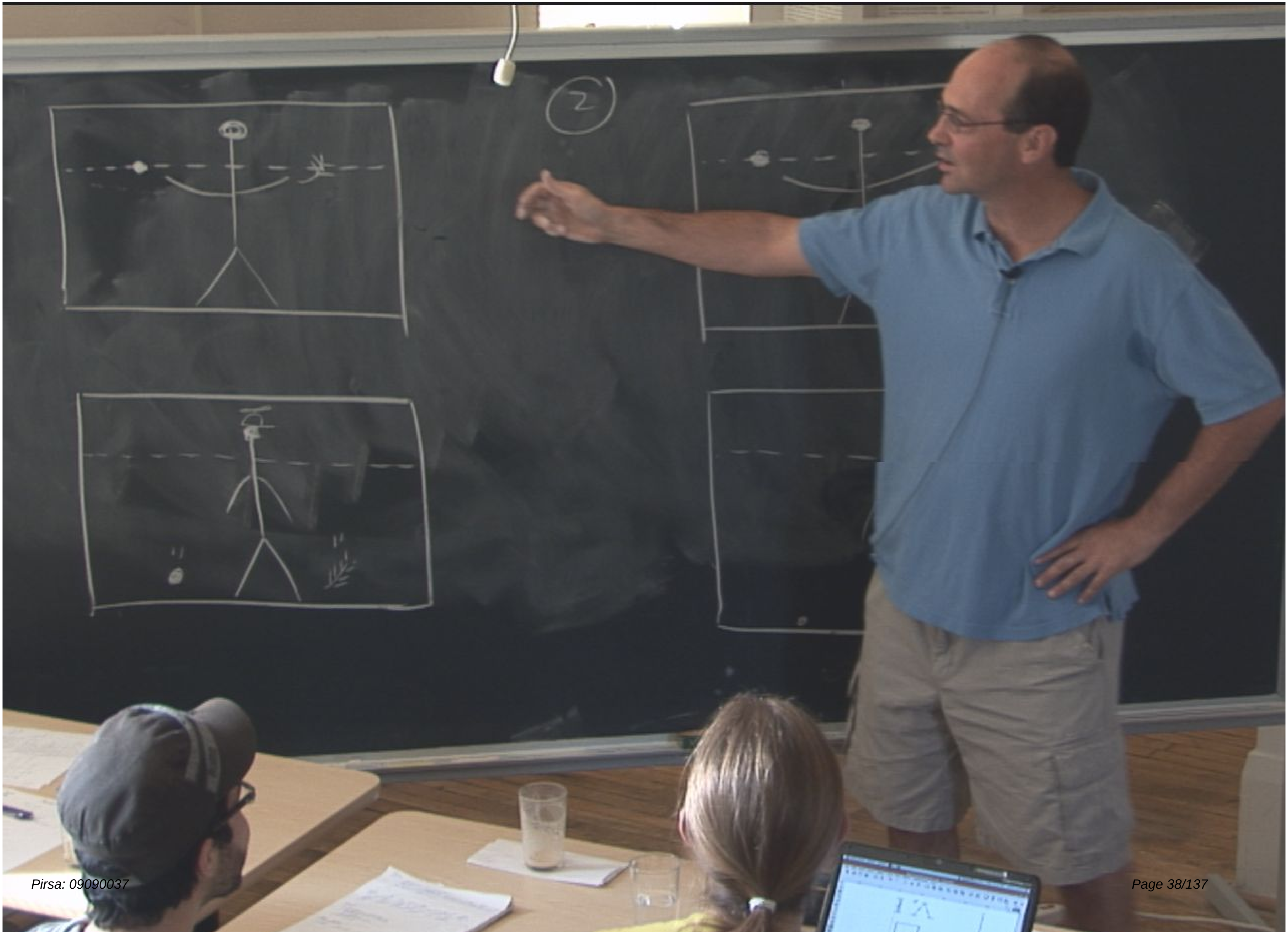
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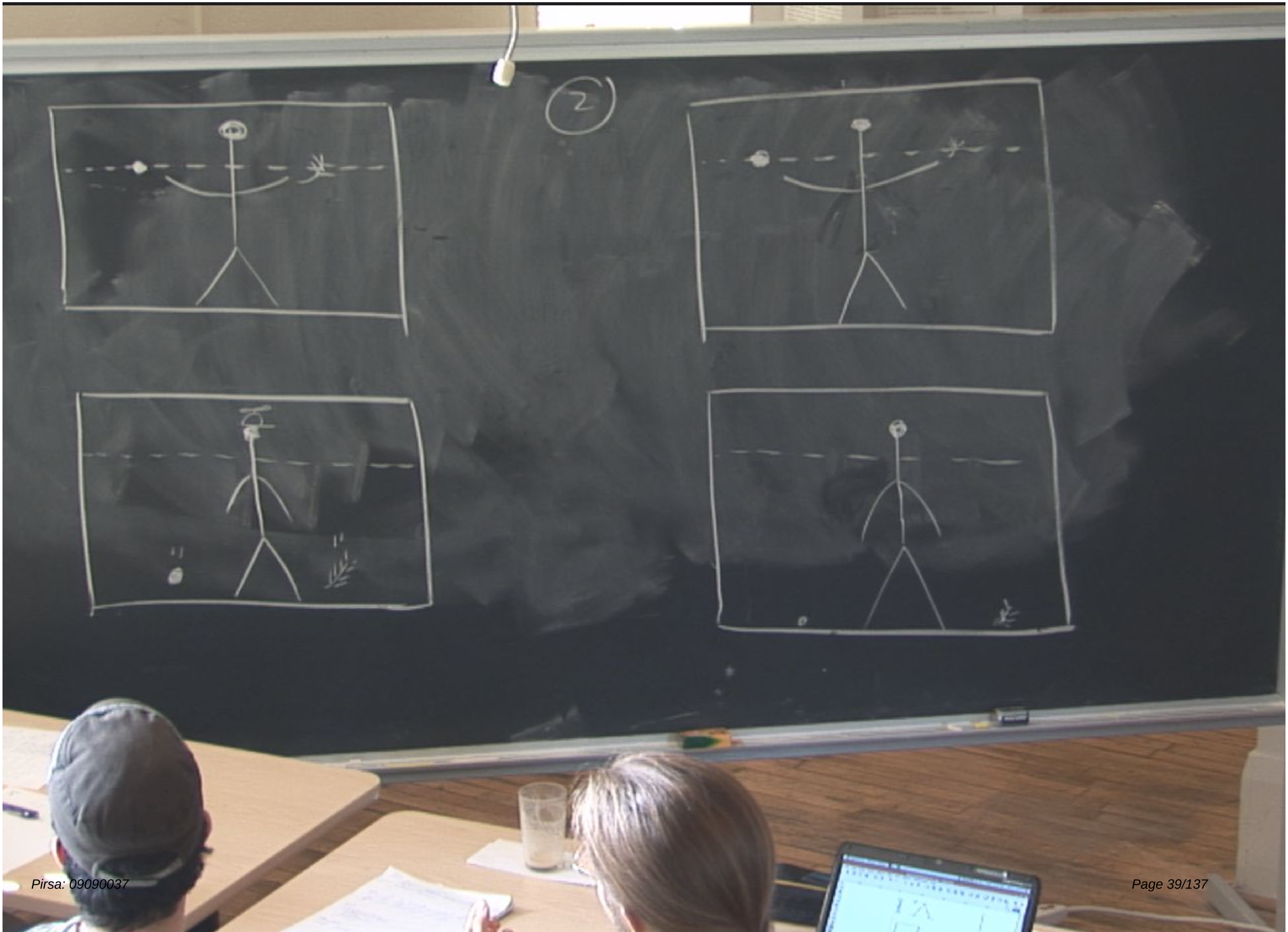
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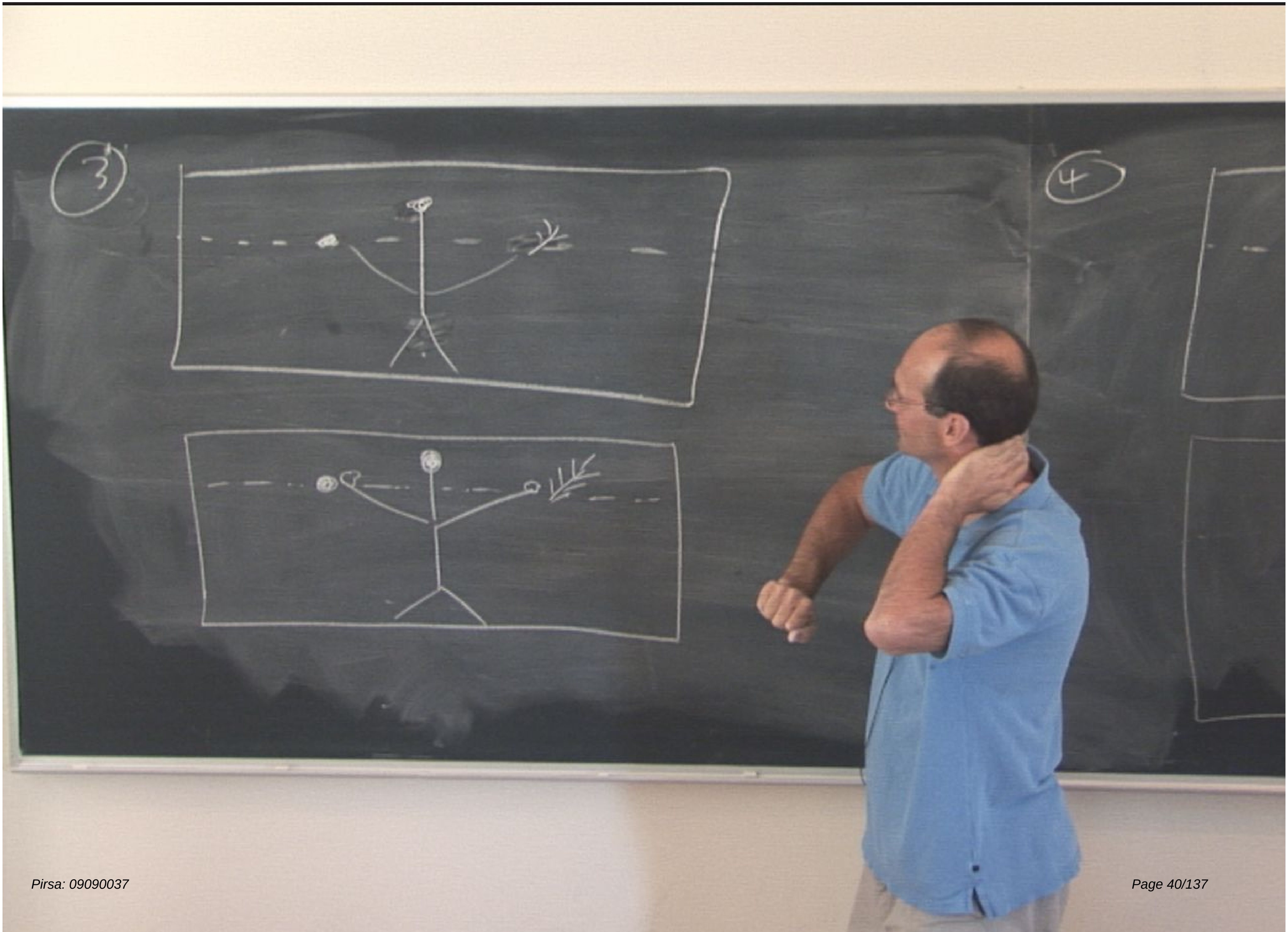
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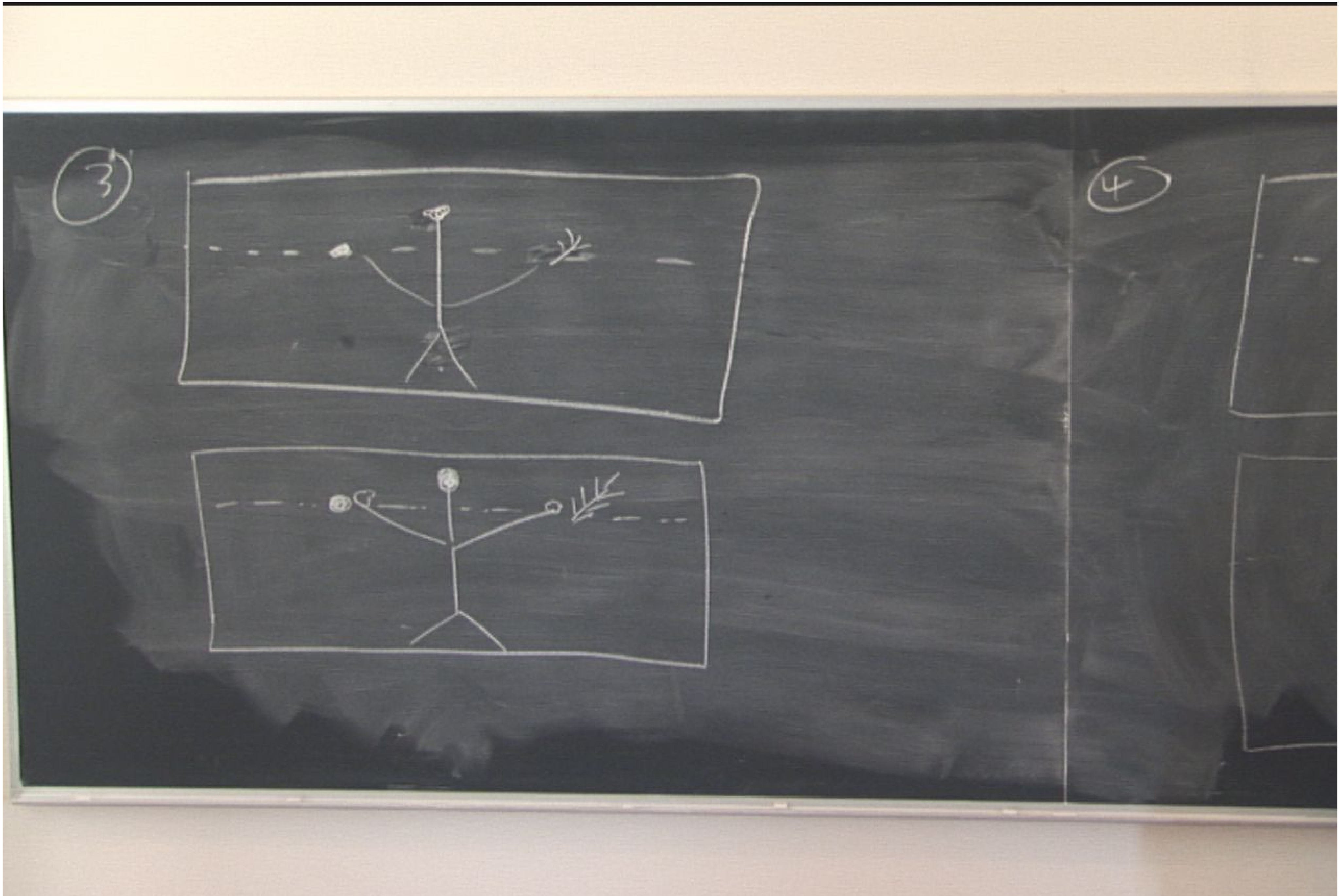
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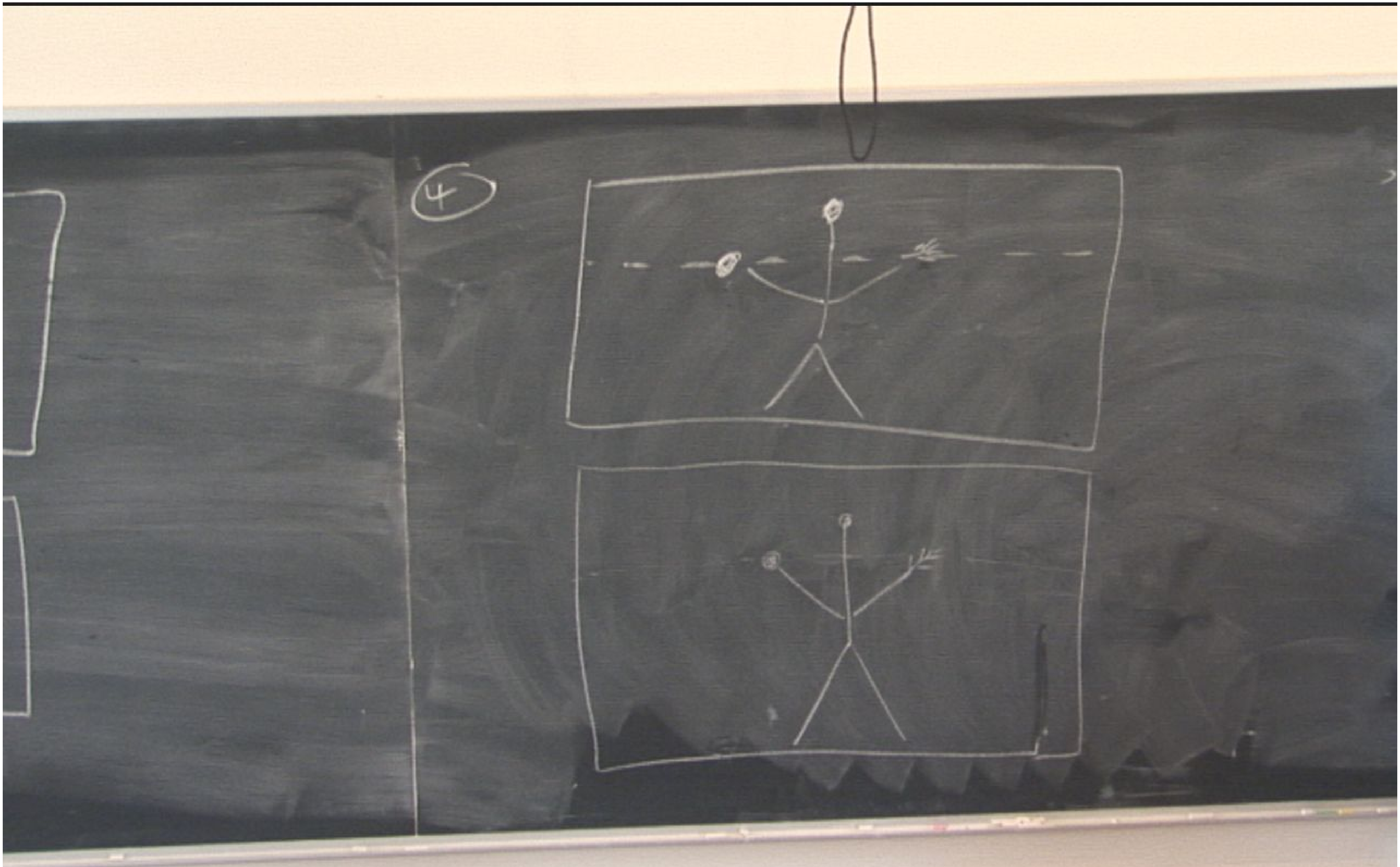
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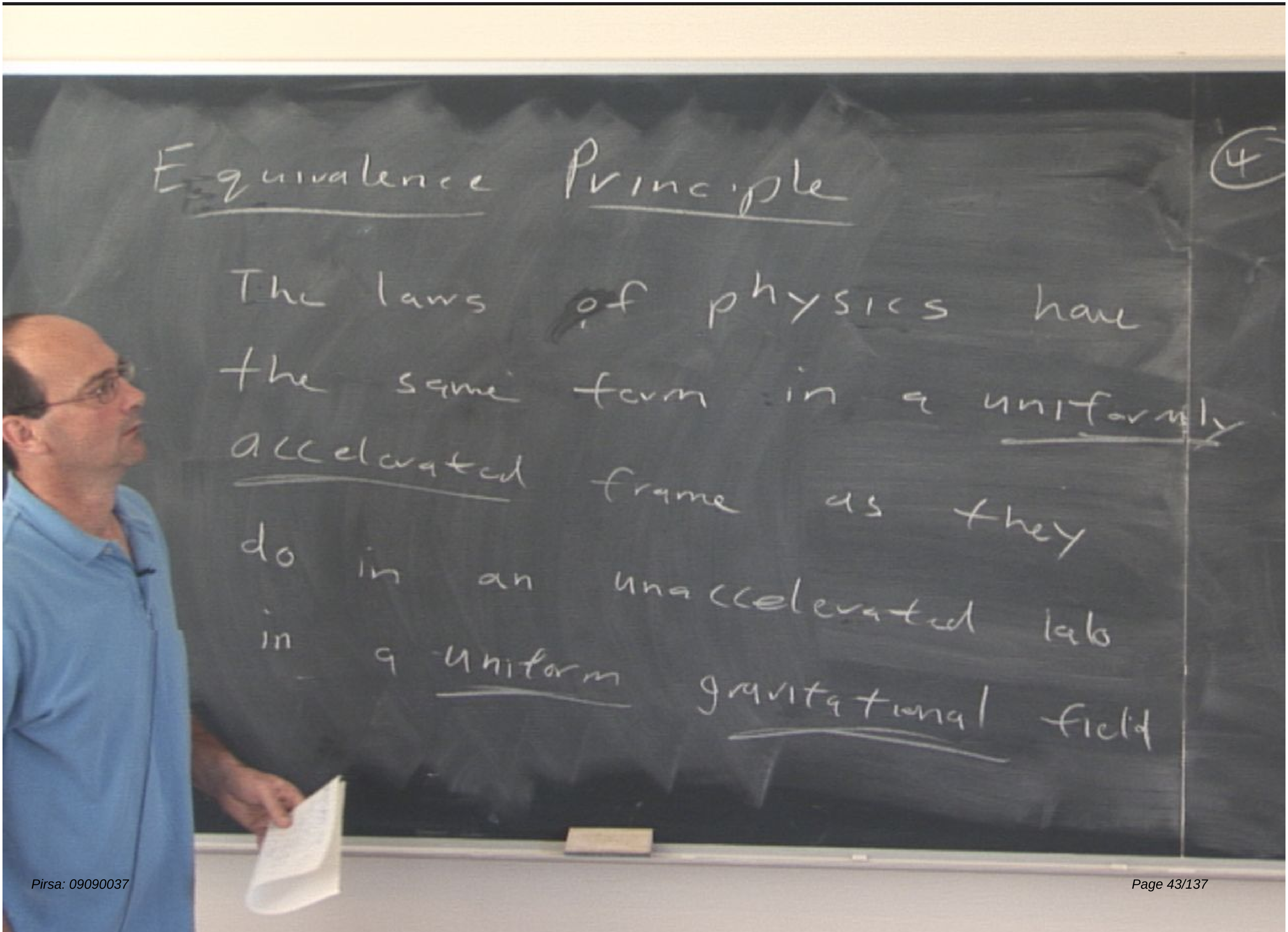
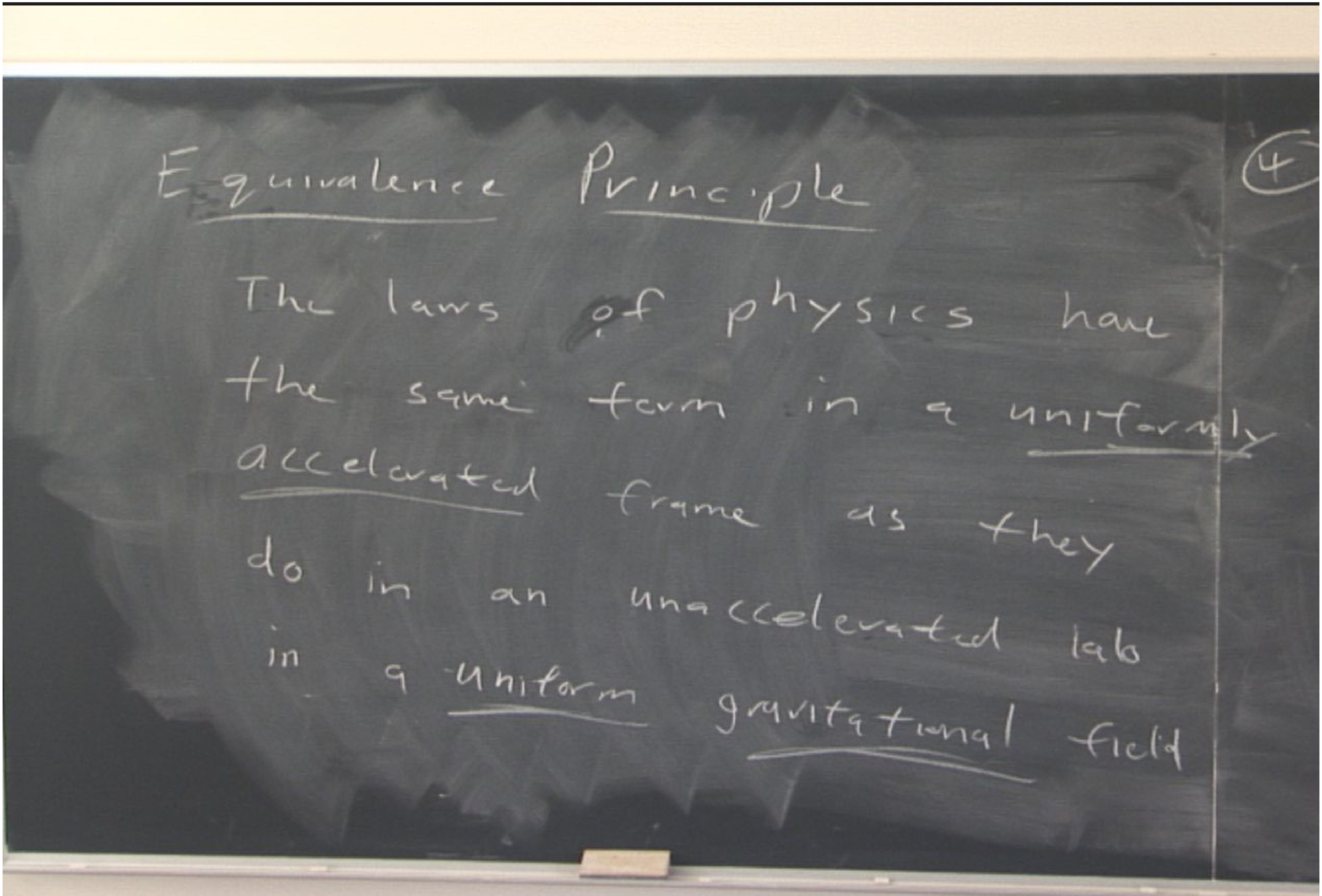

Equivalence Principle
4
The laws of physics have
the same form in a uniformly
accelerated frame as they
do in an unaccelerated lab
in a uniform gravitational field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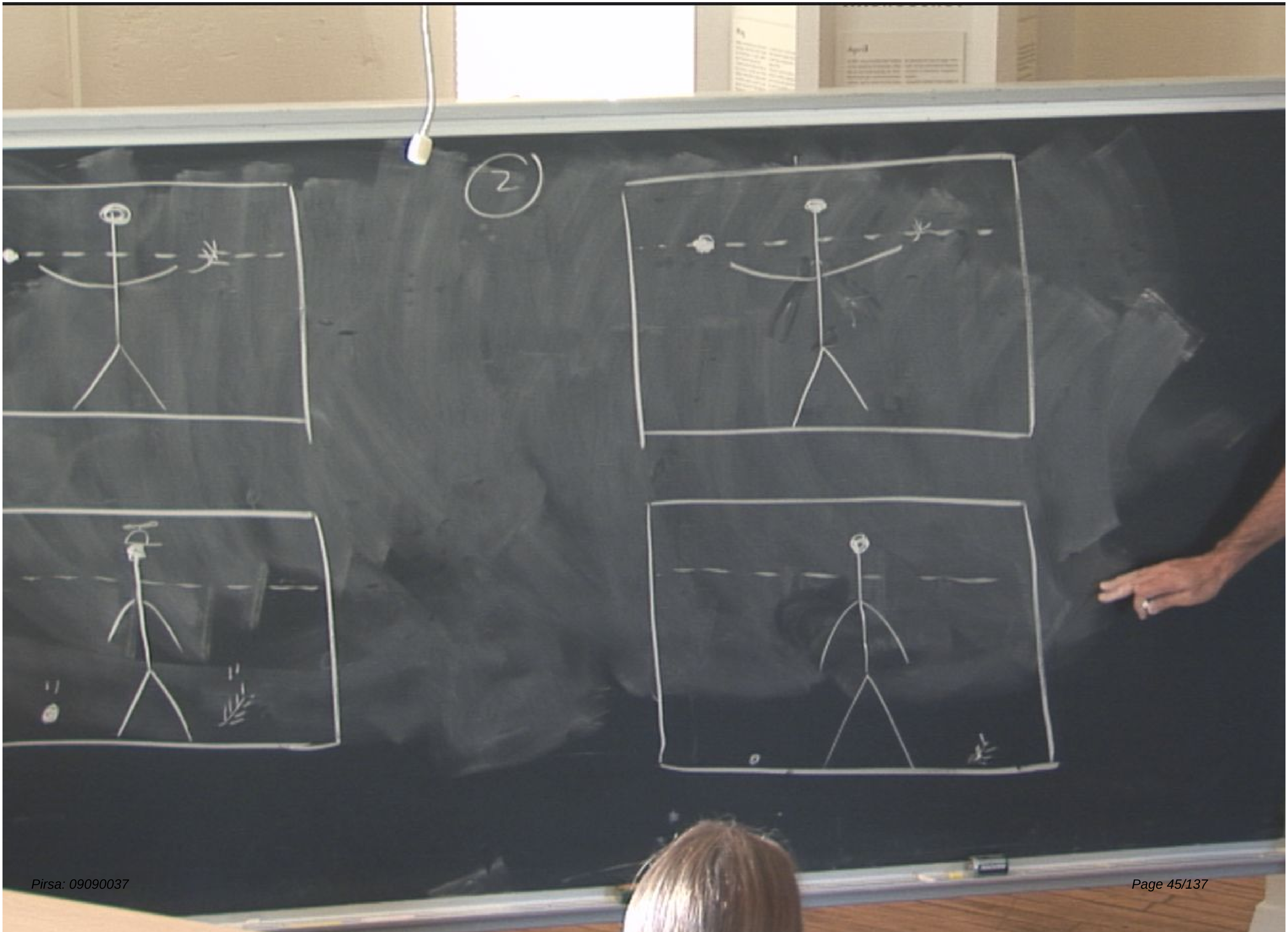
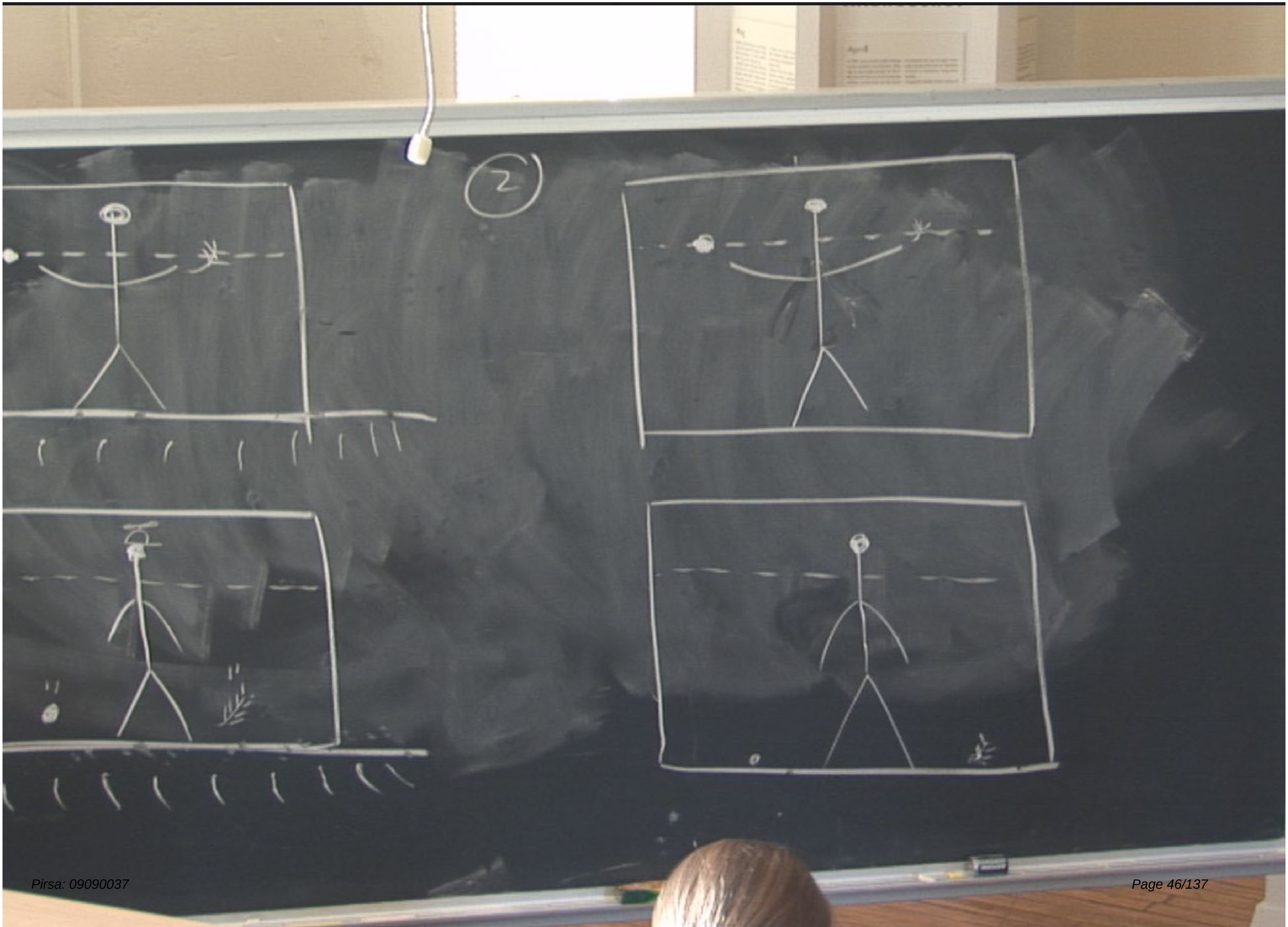
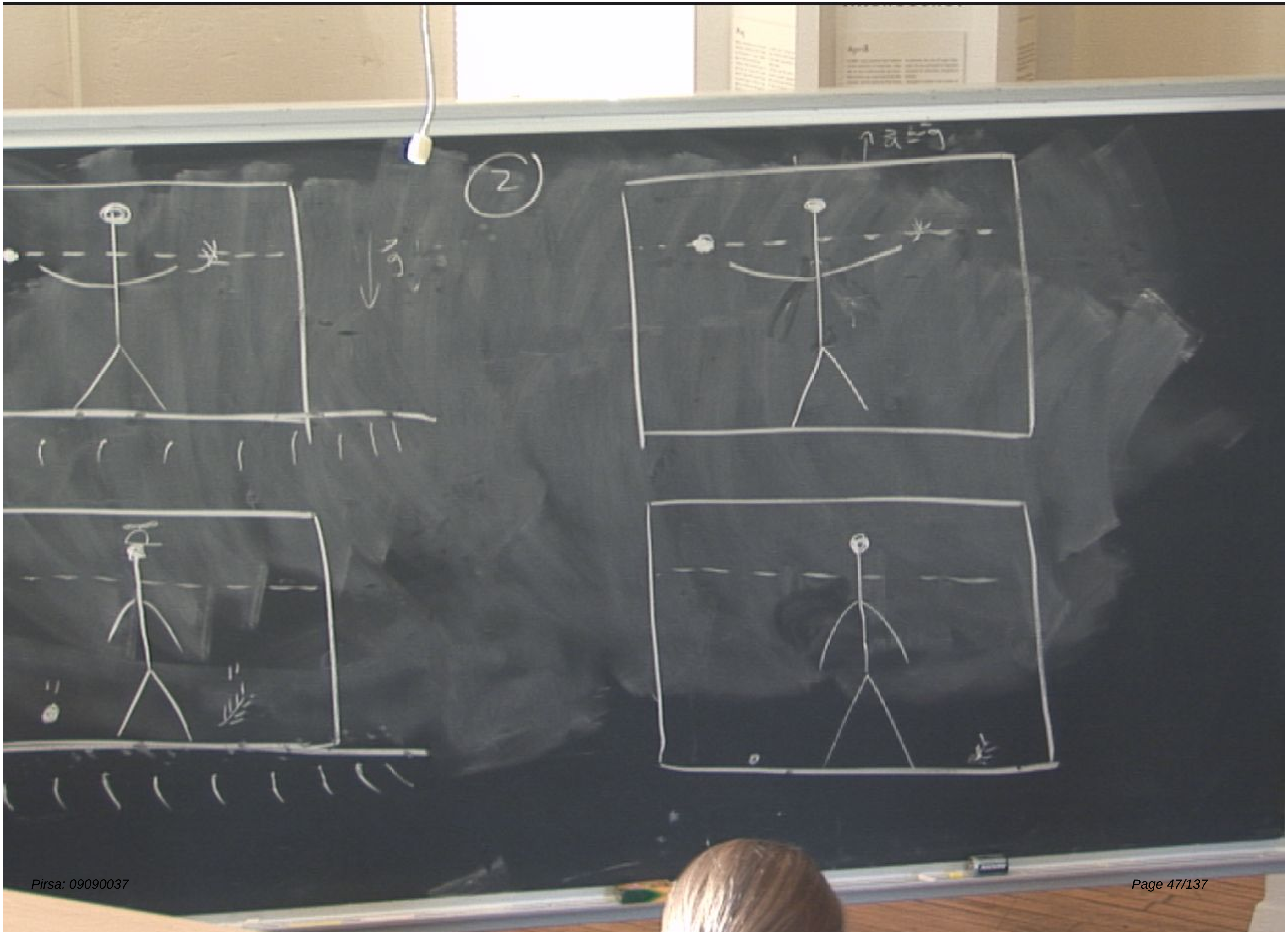

2





Consequences

1

Consequences

① locally always possible to find a frame where gravitational effects vanish → free falling frame

Consequences

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Consequences

① locally always possible to find a frame where gravitational effects vanish $\longrightarrow$ free falling frame

"gravitational field has only a relative existence"

(2) light falls in a gravitational field

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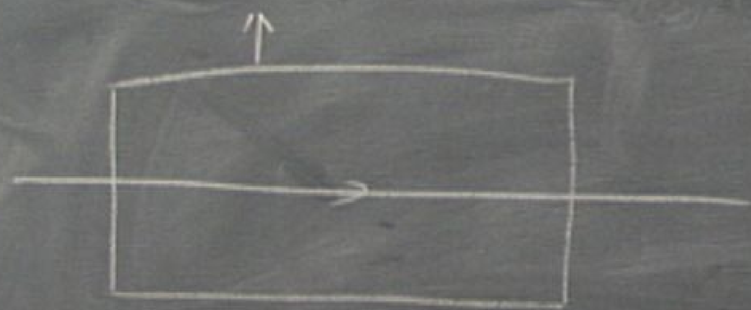
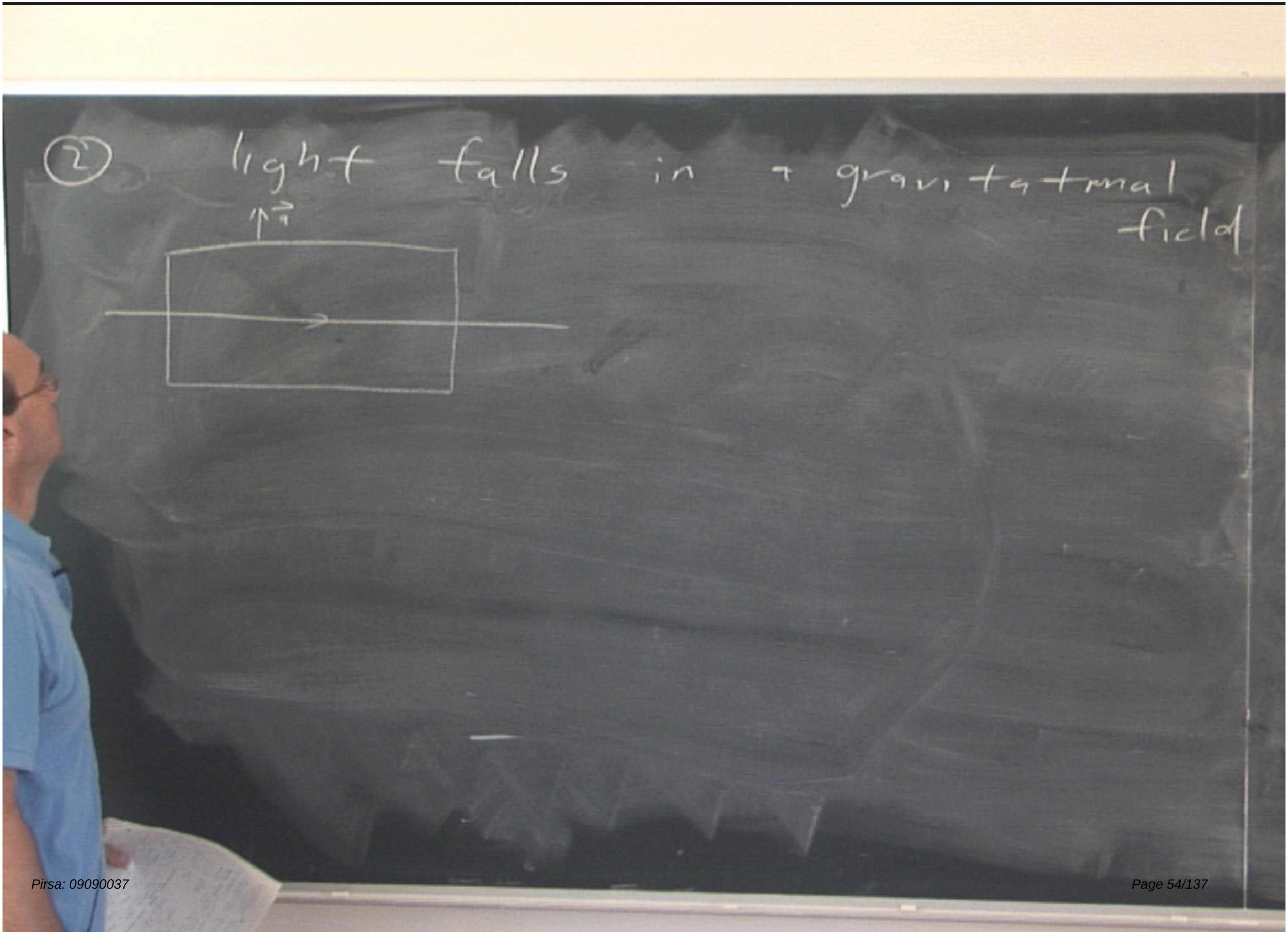
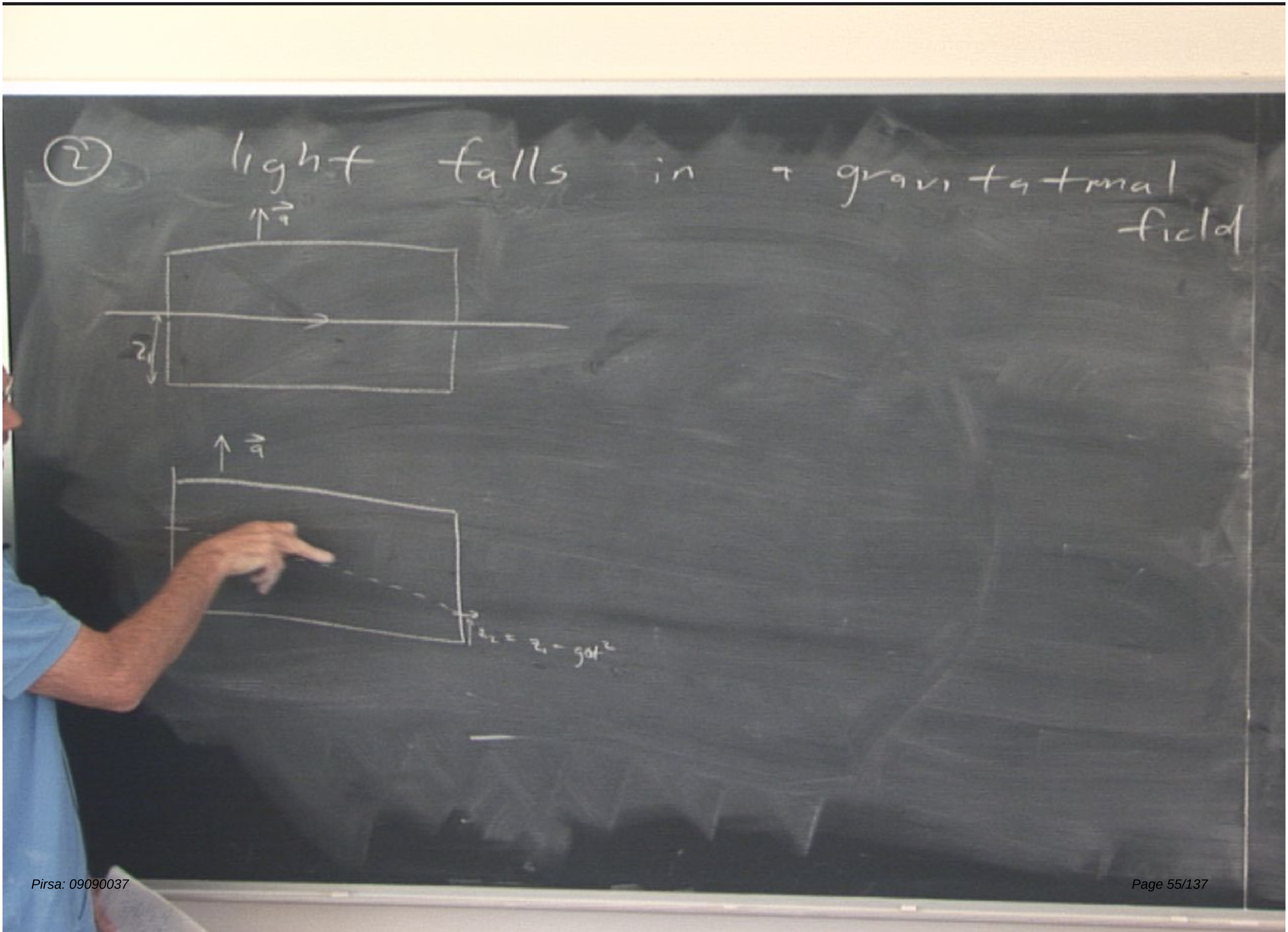
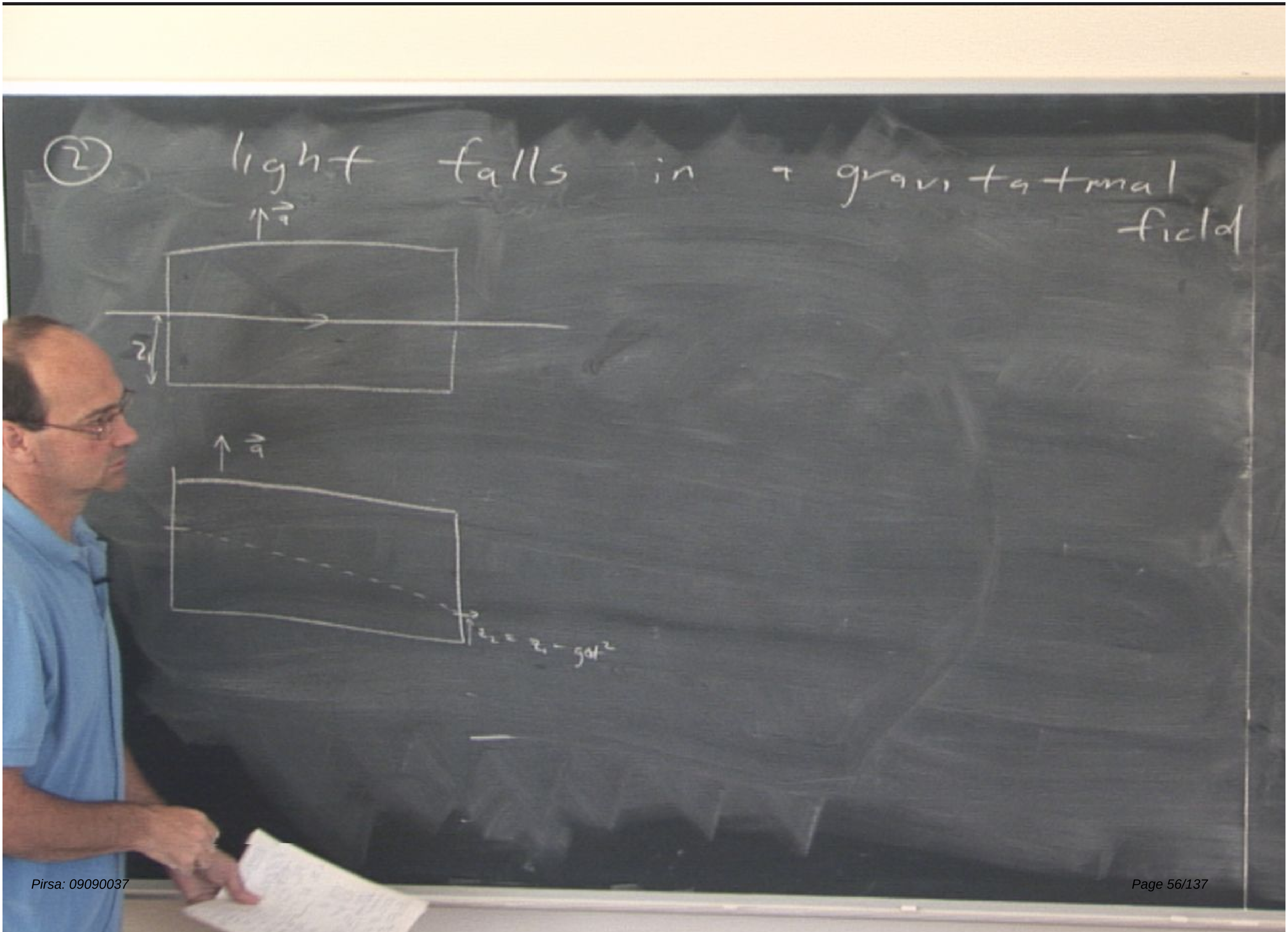
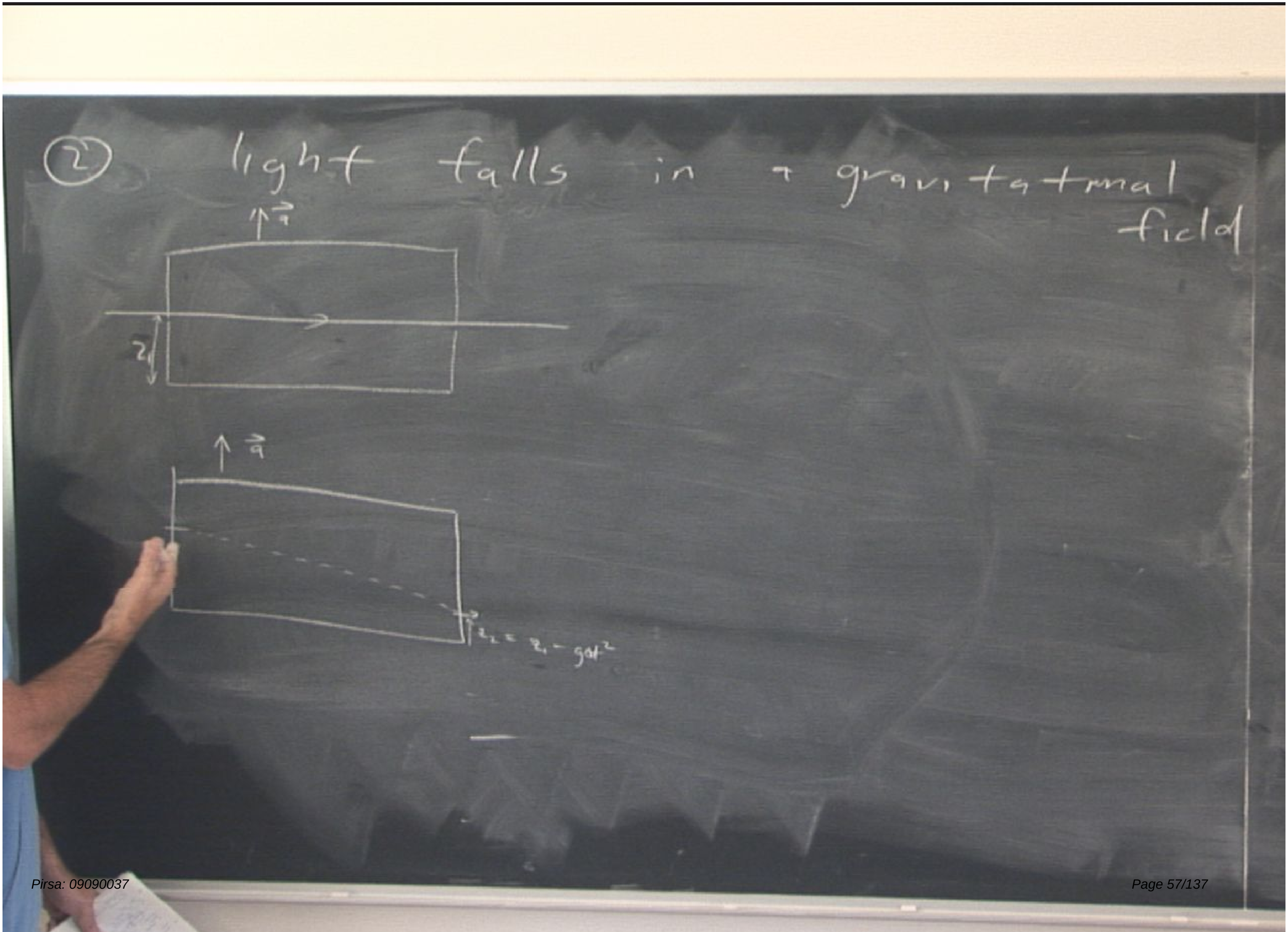



② light falls in a gravitational field

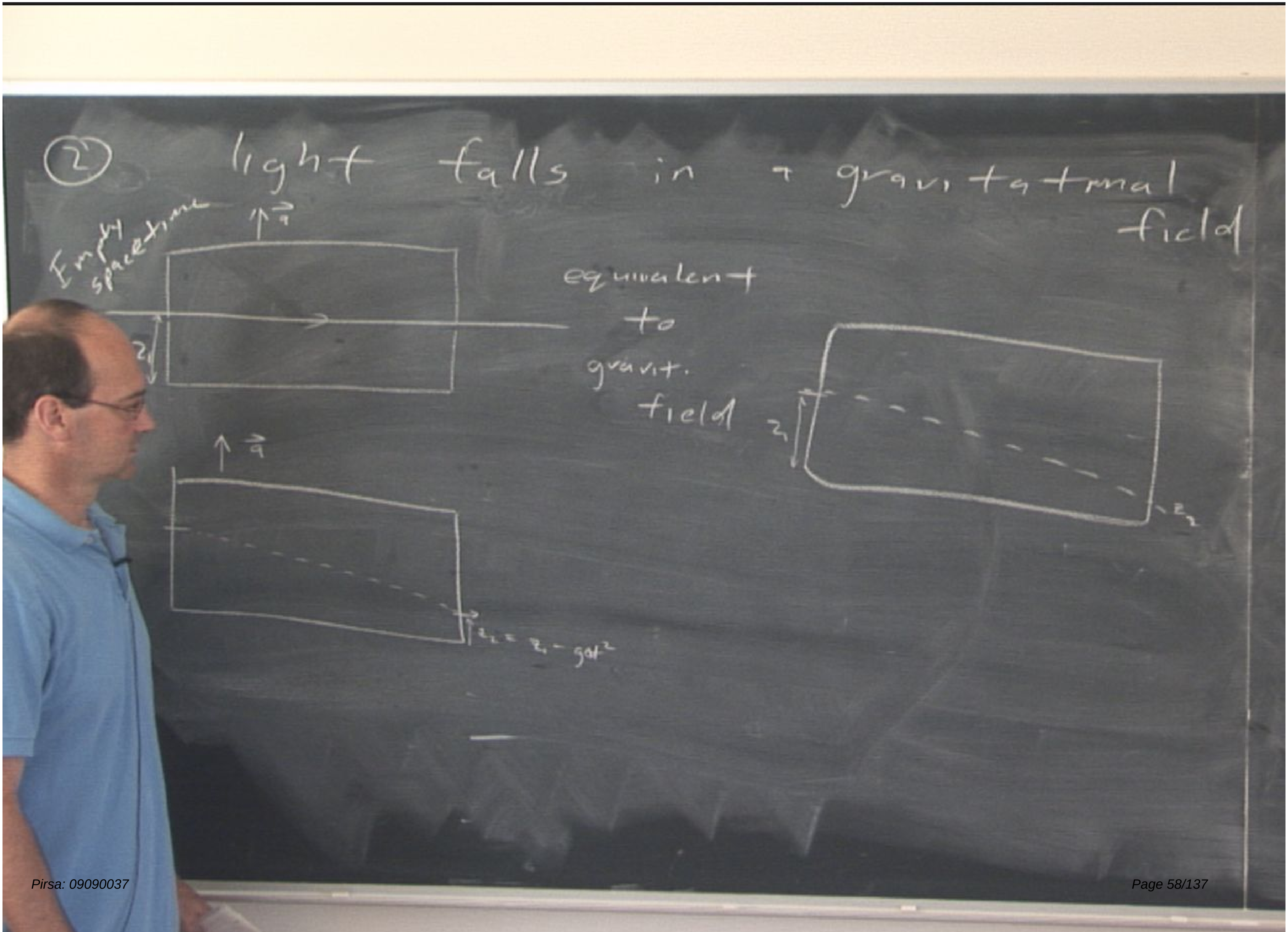


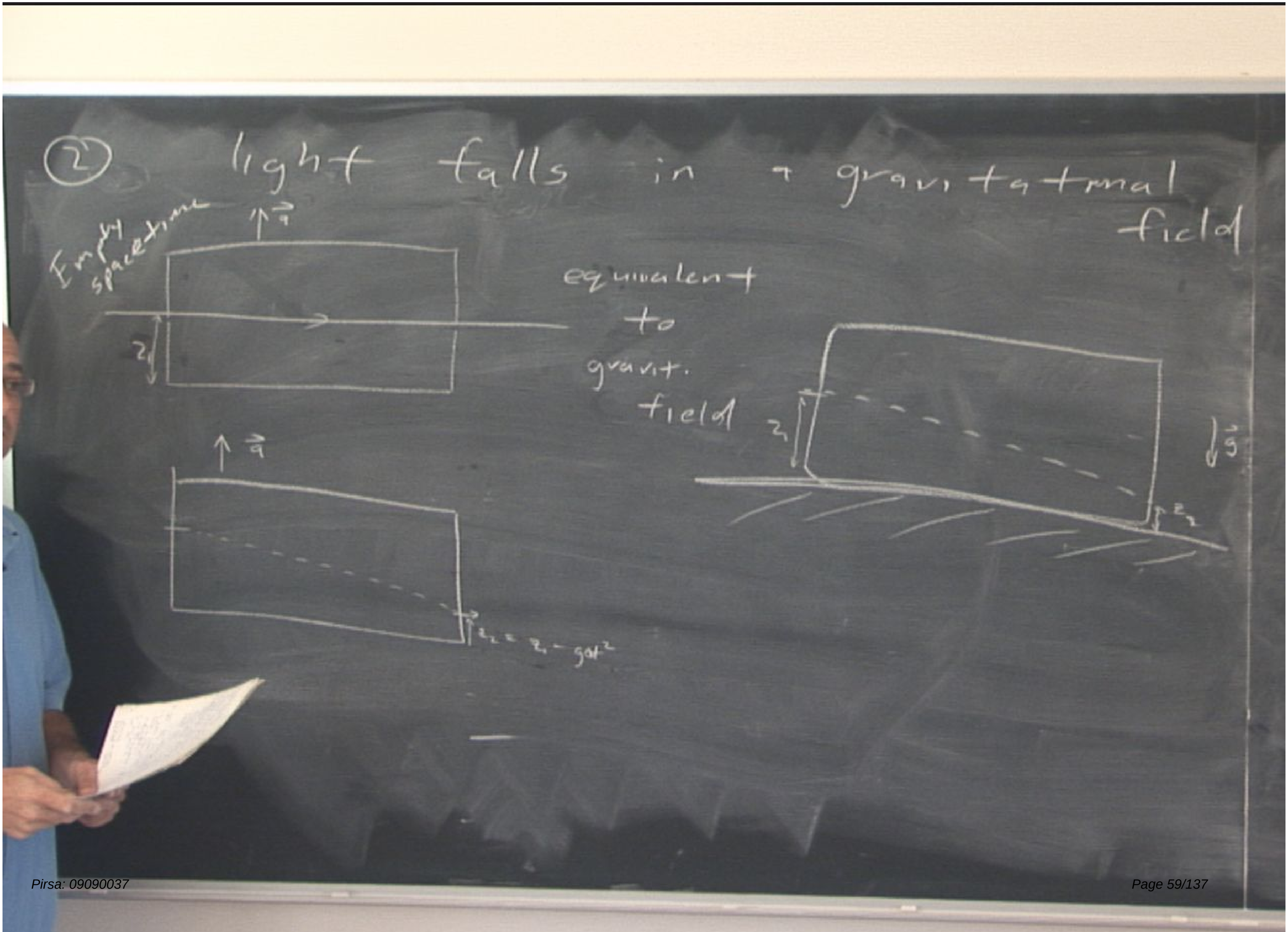
② light falls in a gravitational field
$\uparrow \vec{g}$
z_1
$\uparrow \vec{g}$
$z_2 = z_1 - g\,dt^2$


2 light falls in a gravitational field
$\vec{g}$
z_1
$\vec{g}$
$z_2 = z_1 - g\Delta t^2$



2
light falls in a gravitational field
z1
z2 = z1 - gat2


② light falls in a gravitatmal field
Empty spacetime
$\vec{a}$
z_1
equivalent to gravit. field
$\vec{a}$
$z_2 = z_1 - g\delta t^2$
z_1
z_2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z_1
equivalent to gravit. field
$\vec{a}$
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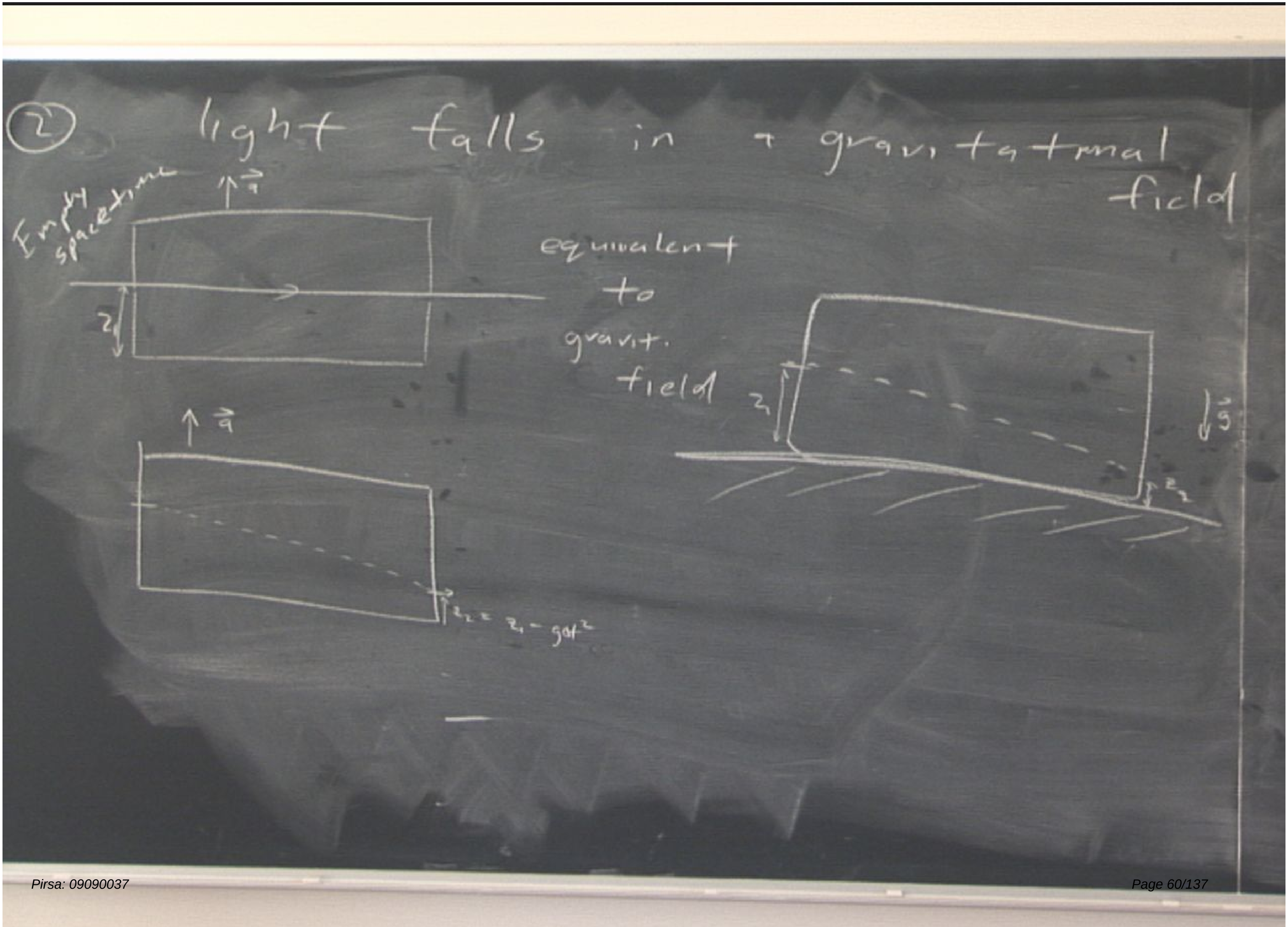
z_1
$\vec{g}$
z_2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equivalent to gravit. field
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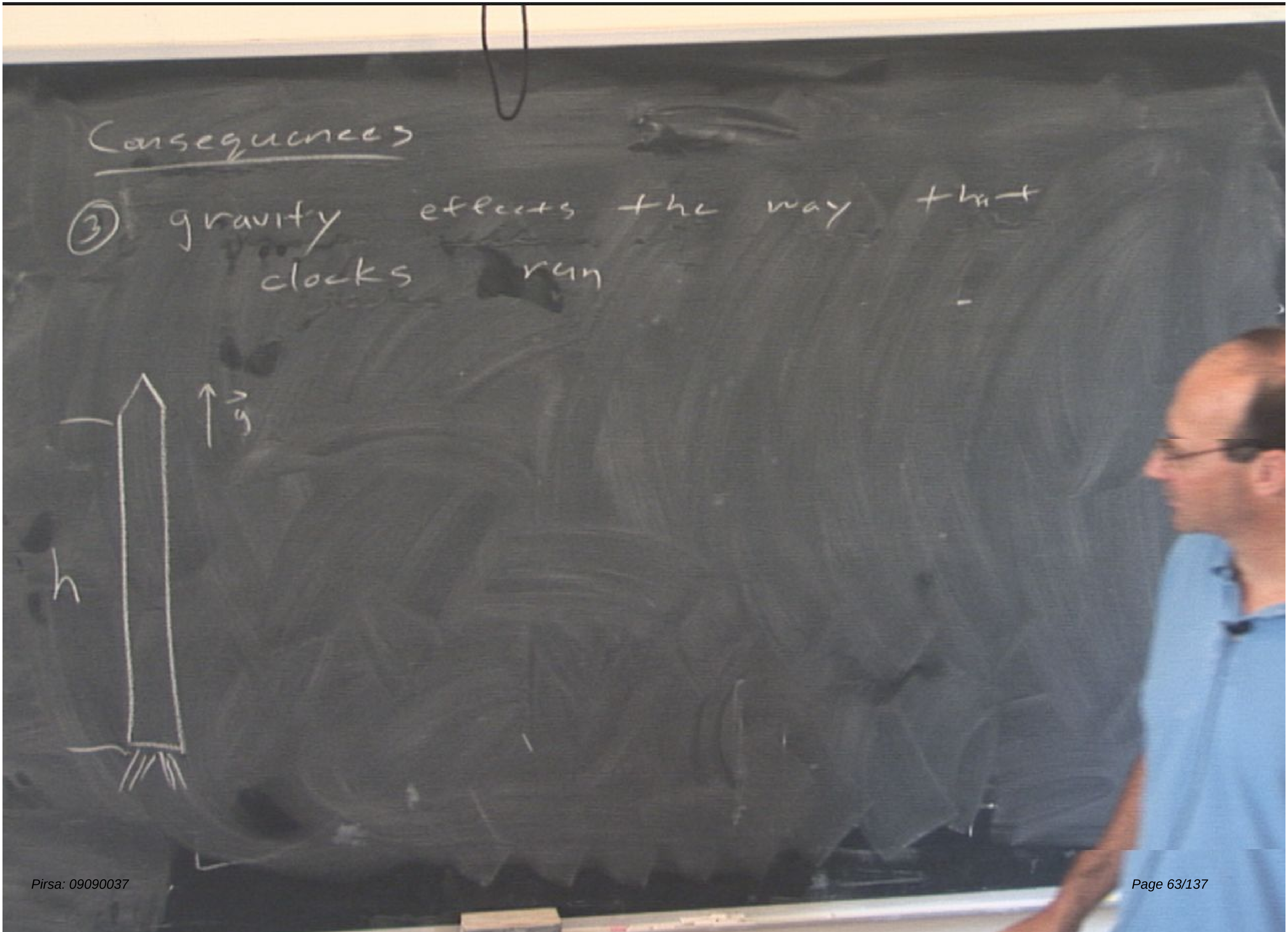
z_1
$\downarrow \vec{g}$
z_2

Consequences

③ gravity effects the way that clocks run

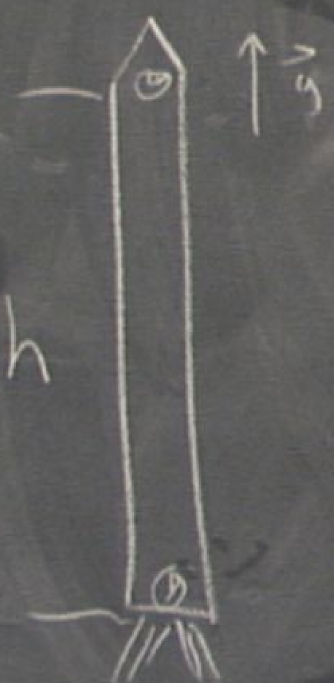
Consequences

(3) gravity effects the way that clocks run


Consequences
③ gravity effects the way that
clocks run
$\vec{g}$
h

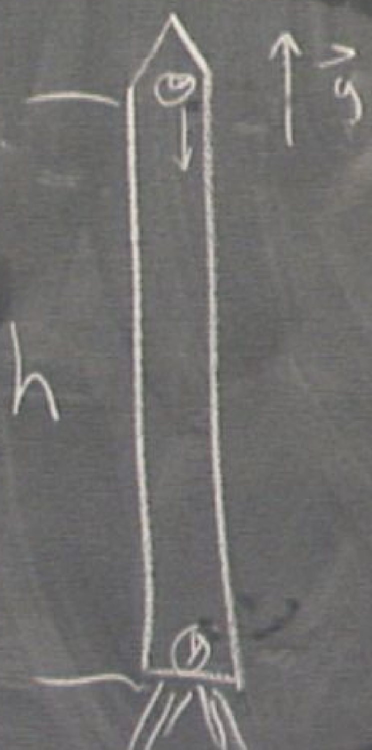
Consequences

3 gravity effects the way that clocks run



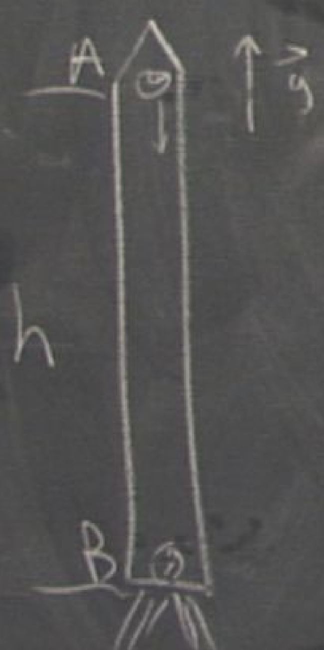
Consequences

③ gravity effects the way that clocks run



Consequences

(3) gravity effects the way that clocks run

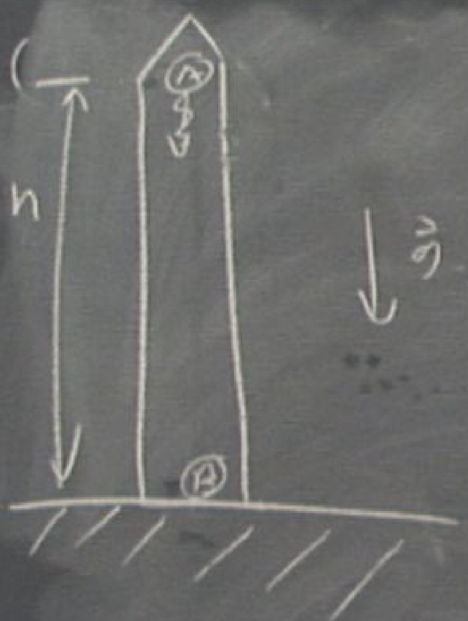


- if A sends pulses down at a regular rate Δt_A, then because B accelerates towards A, the pulses arrive with different time separation Δt_B

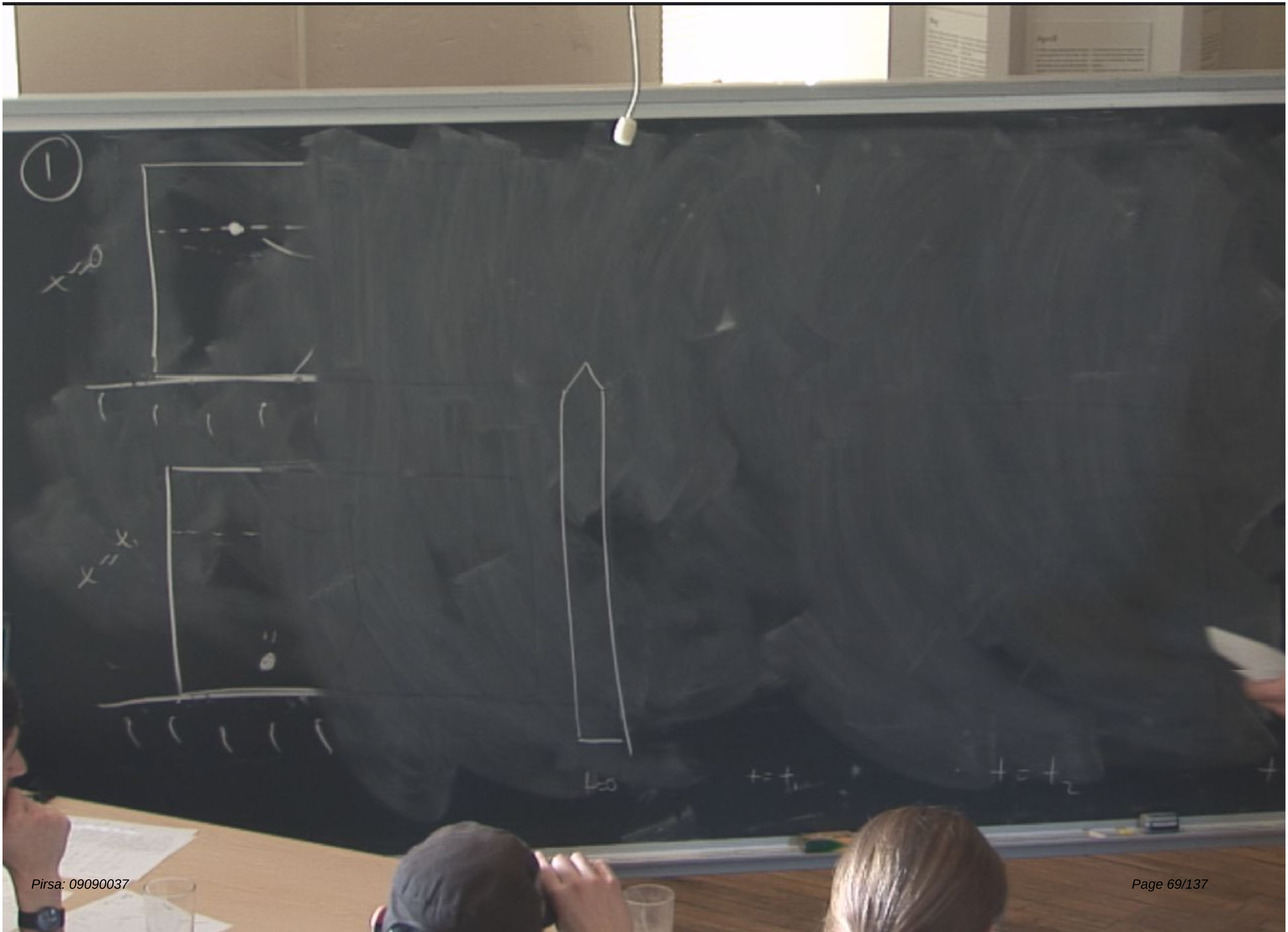
Consequences

(3) gravity effects the way that clocks run

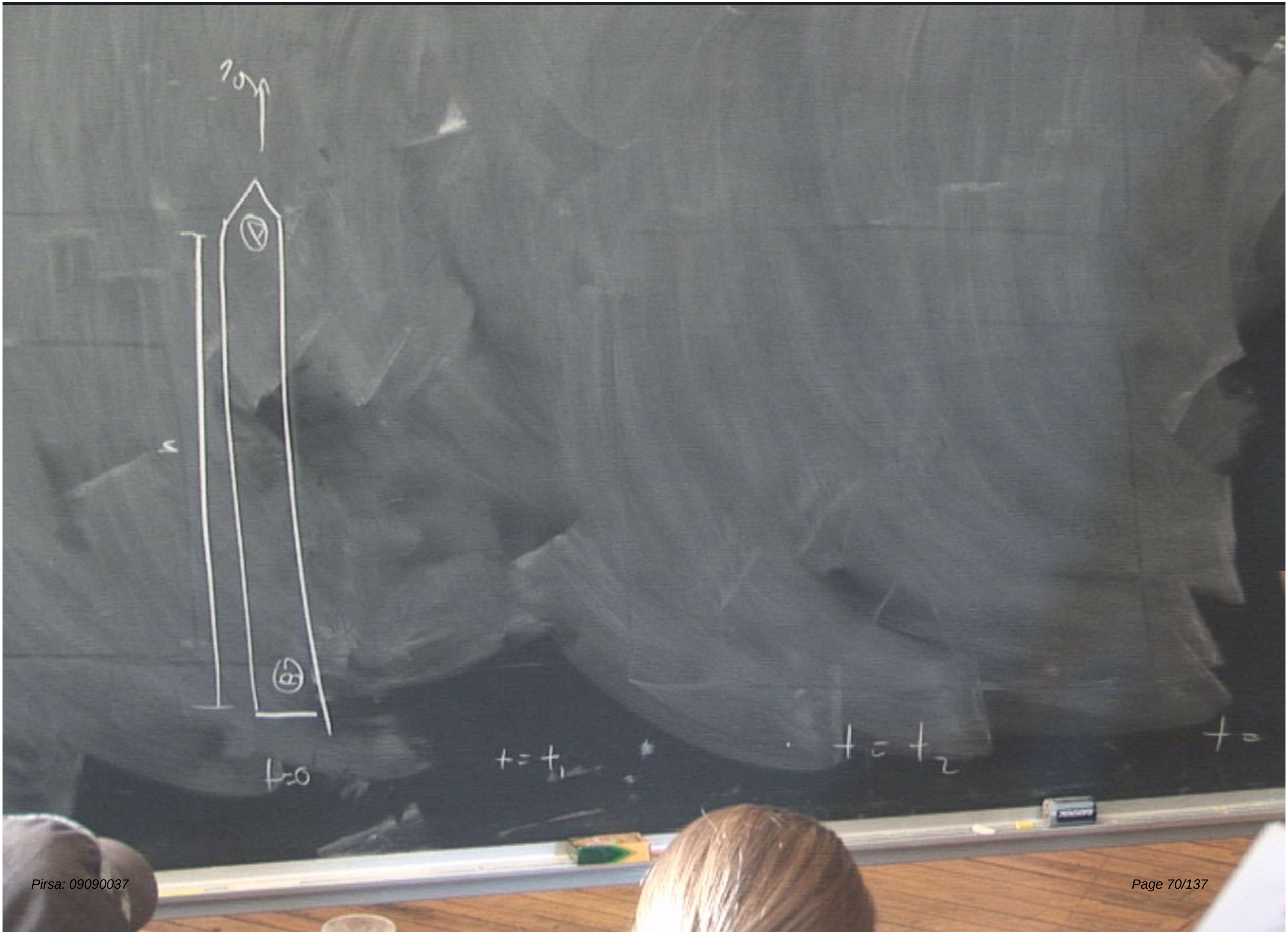
- if A sends pulses down at a regular rate Δt_A, then because B accelerates towards A, the pulses arrive with different time separation Δt_B

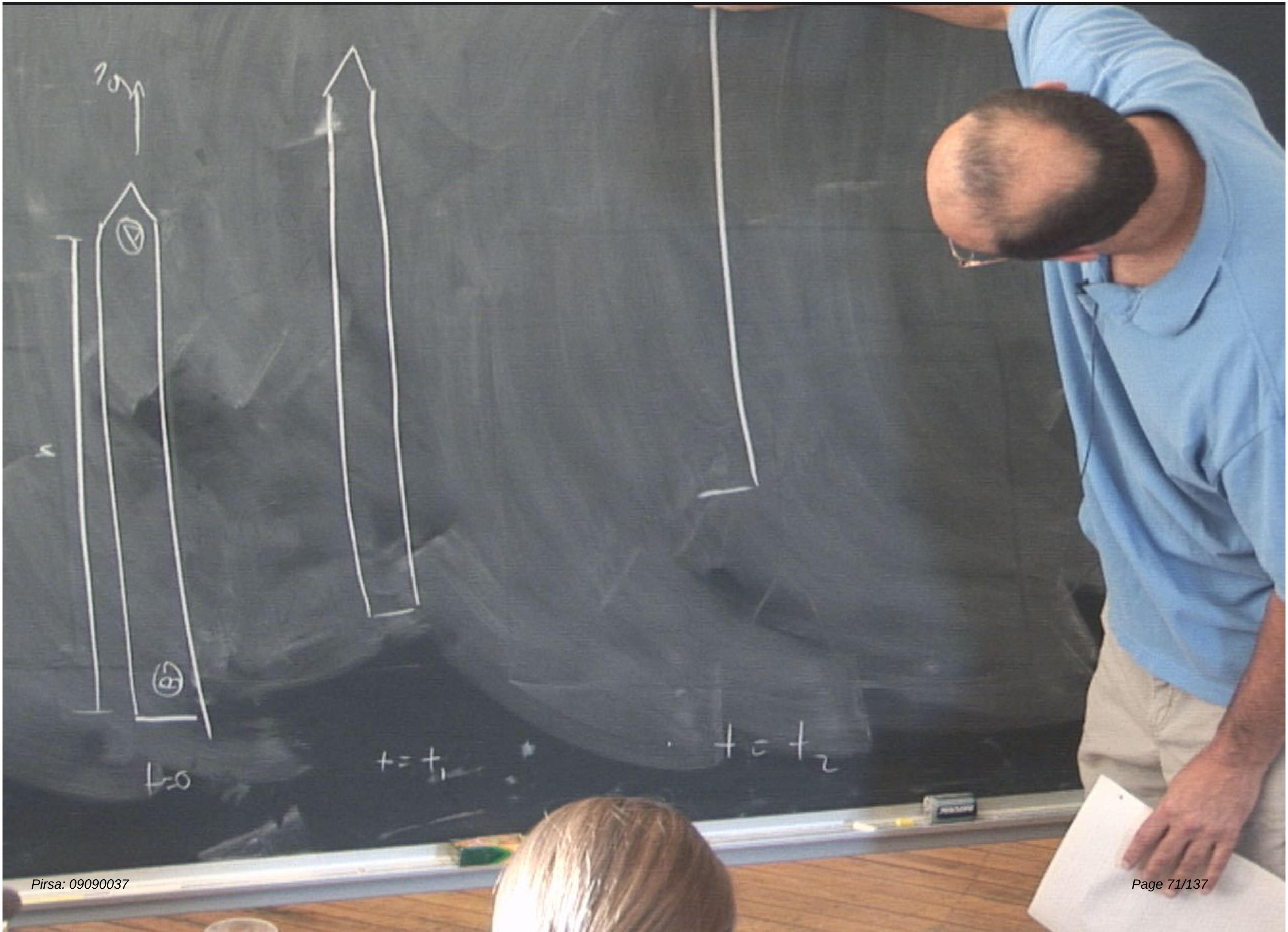


equivalence principle says we see same physics in a uniform grav. field $\longrightarrow$ interprete the result as indicating that clocks at different places in grav field run at different rates



x = 0
x = x₁
t=0
t = t₁
t = t₂


t=0
t = t1
t = t2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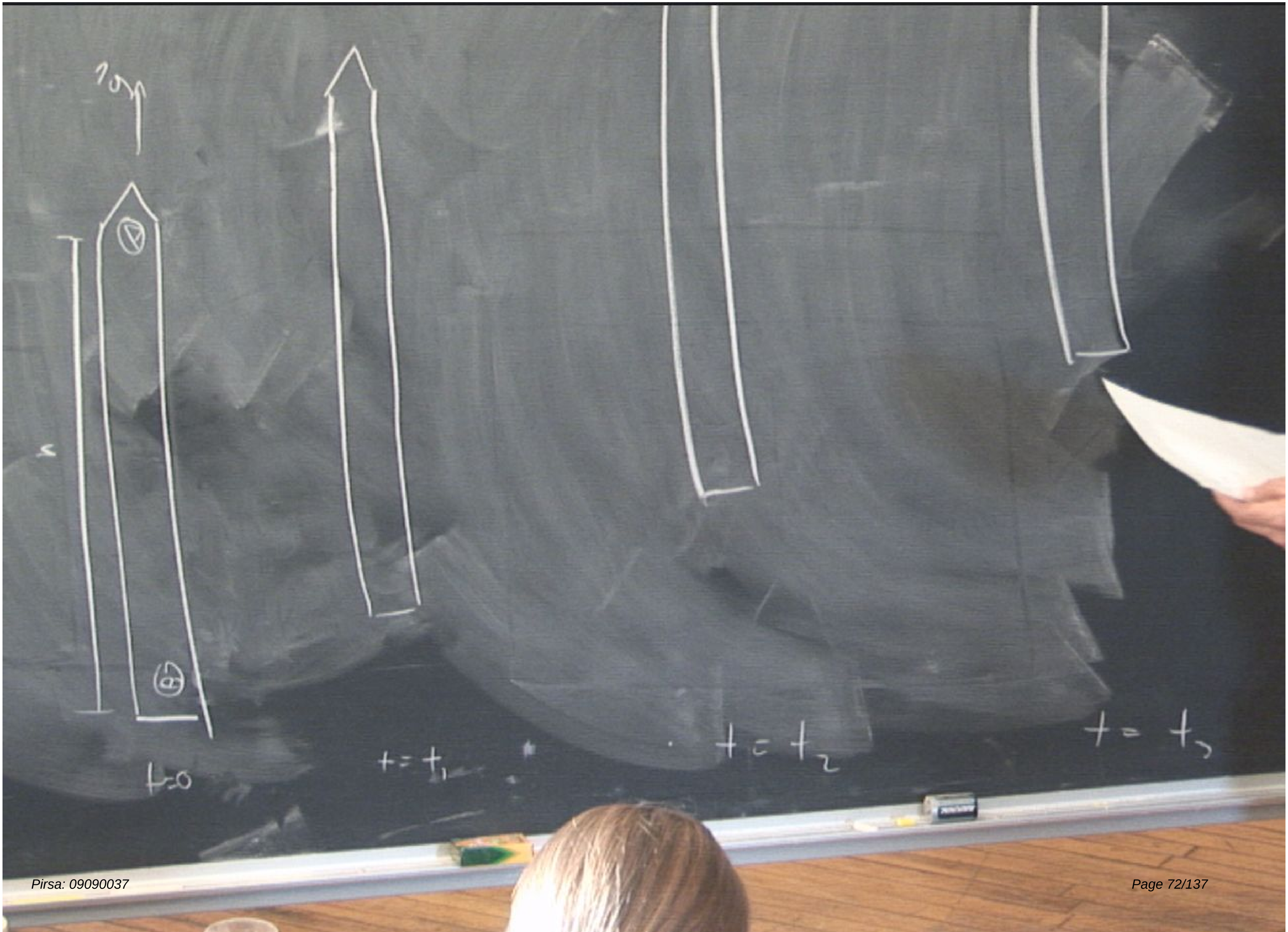
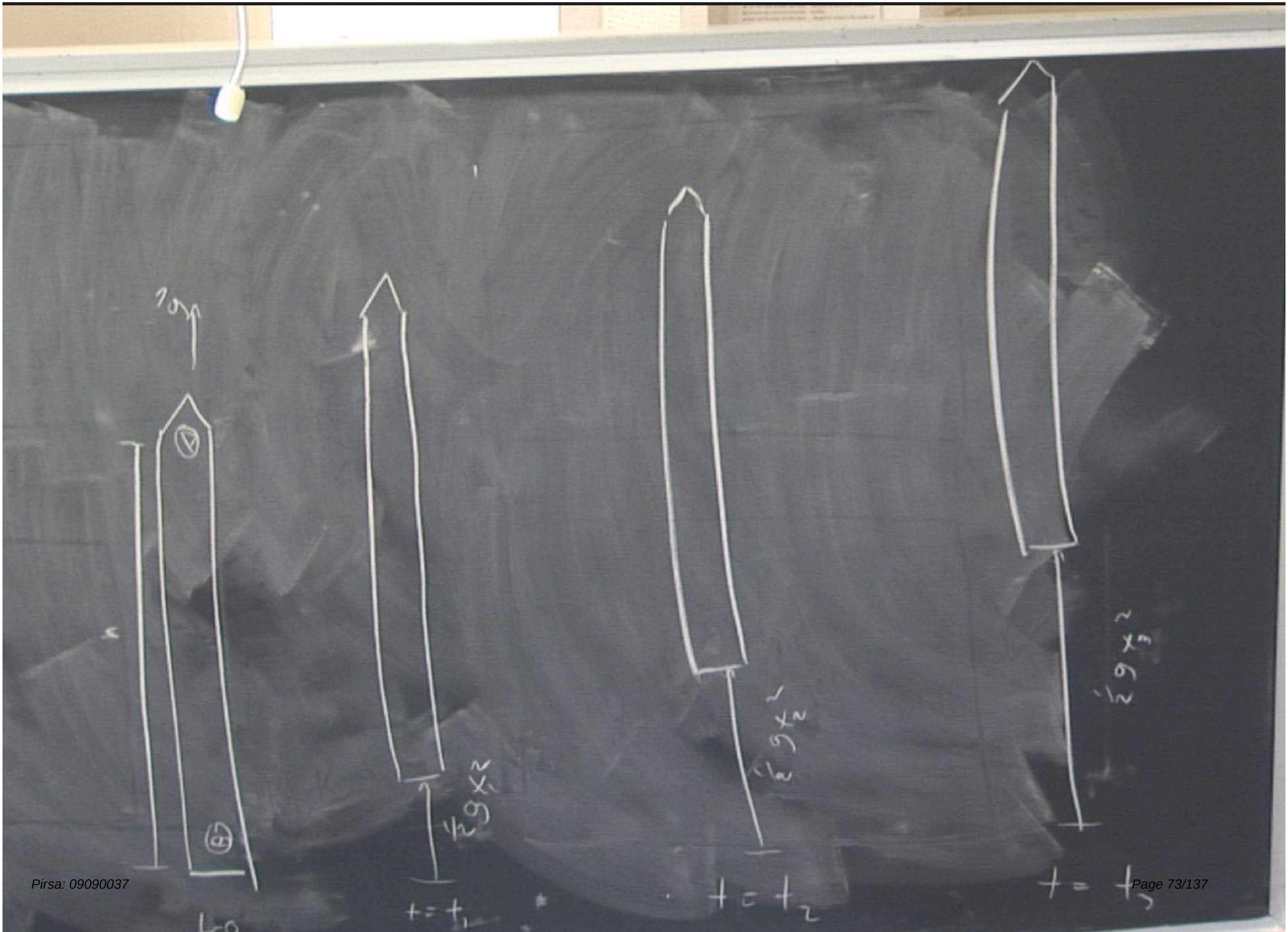
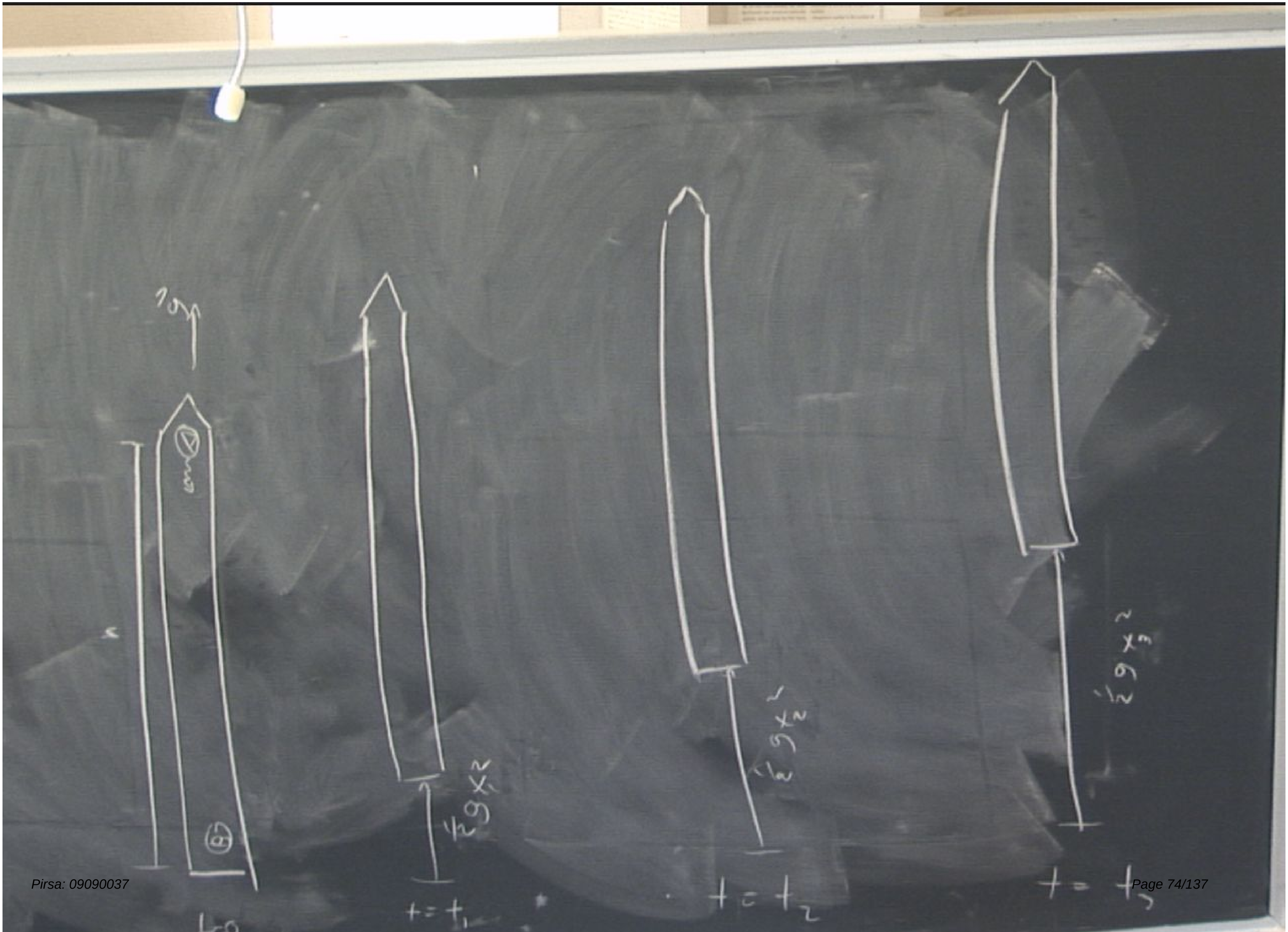
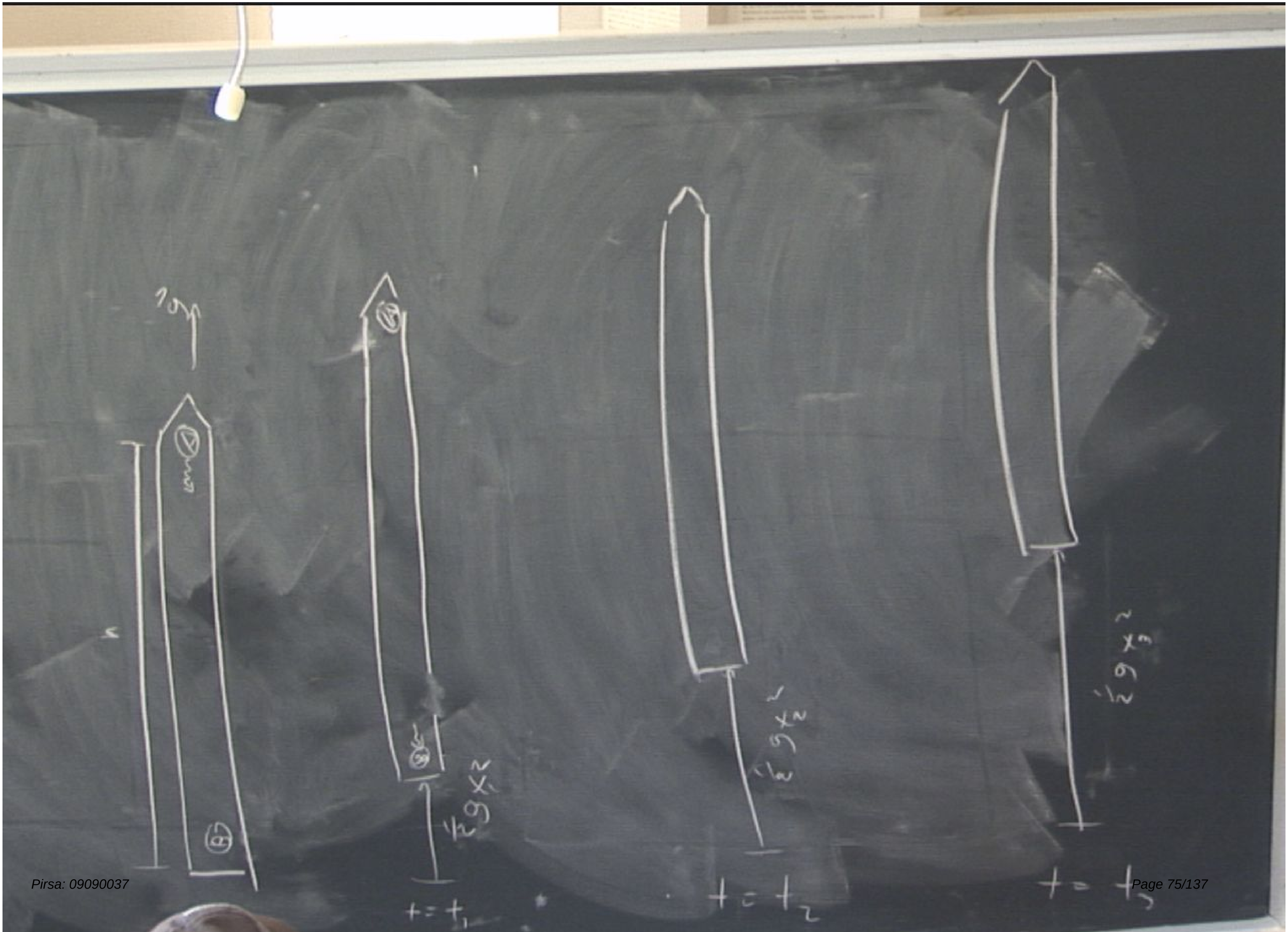
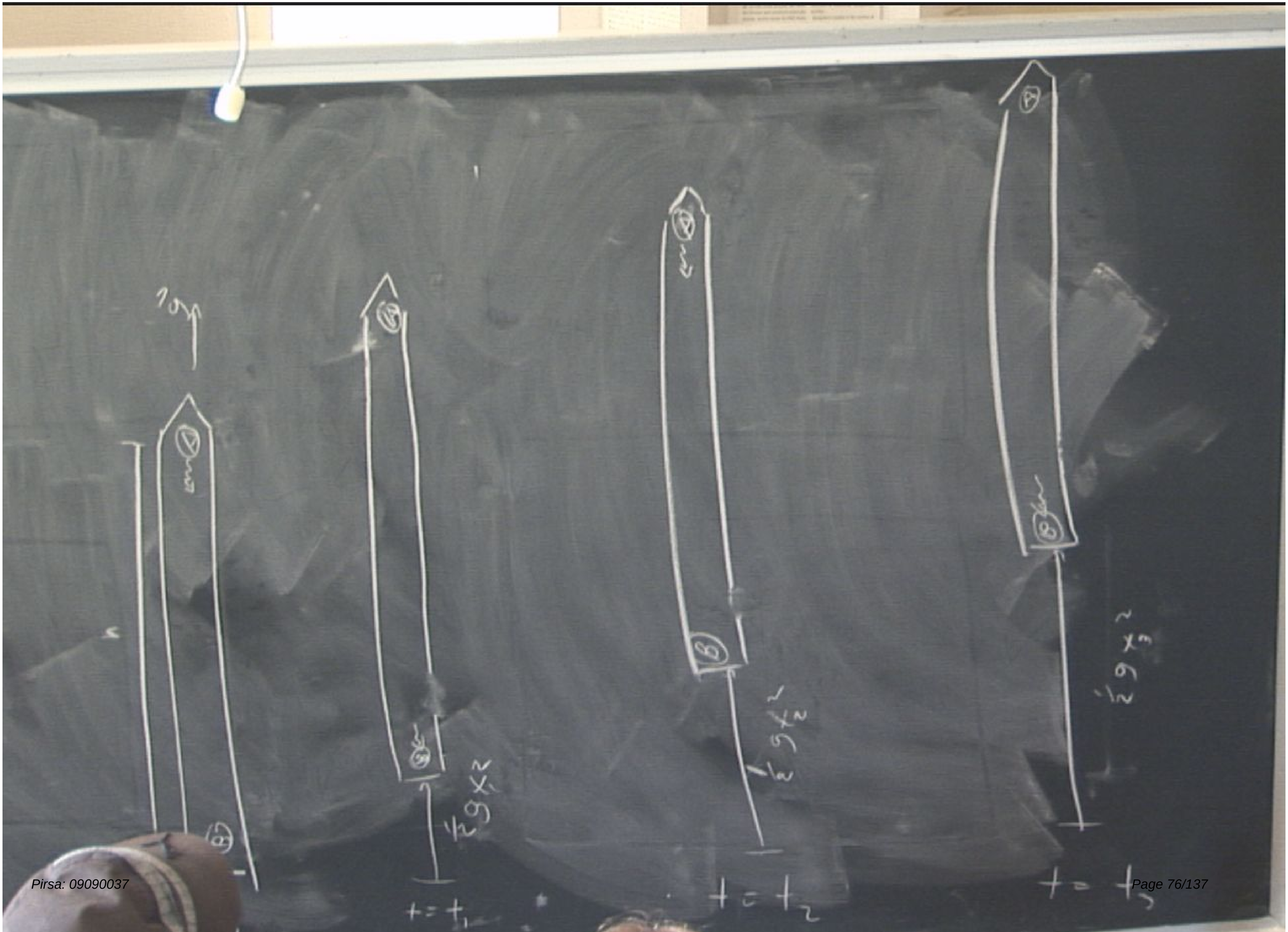

t=0
t = t1
t = t2
t = t3

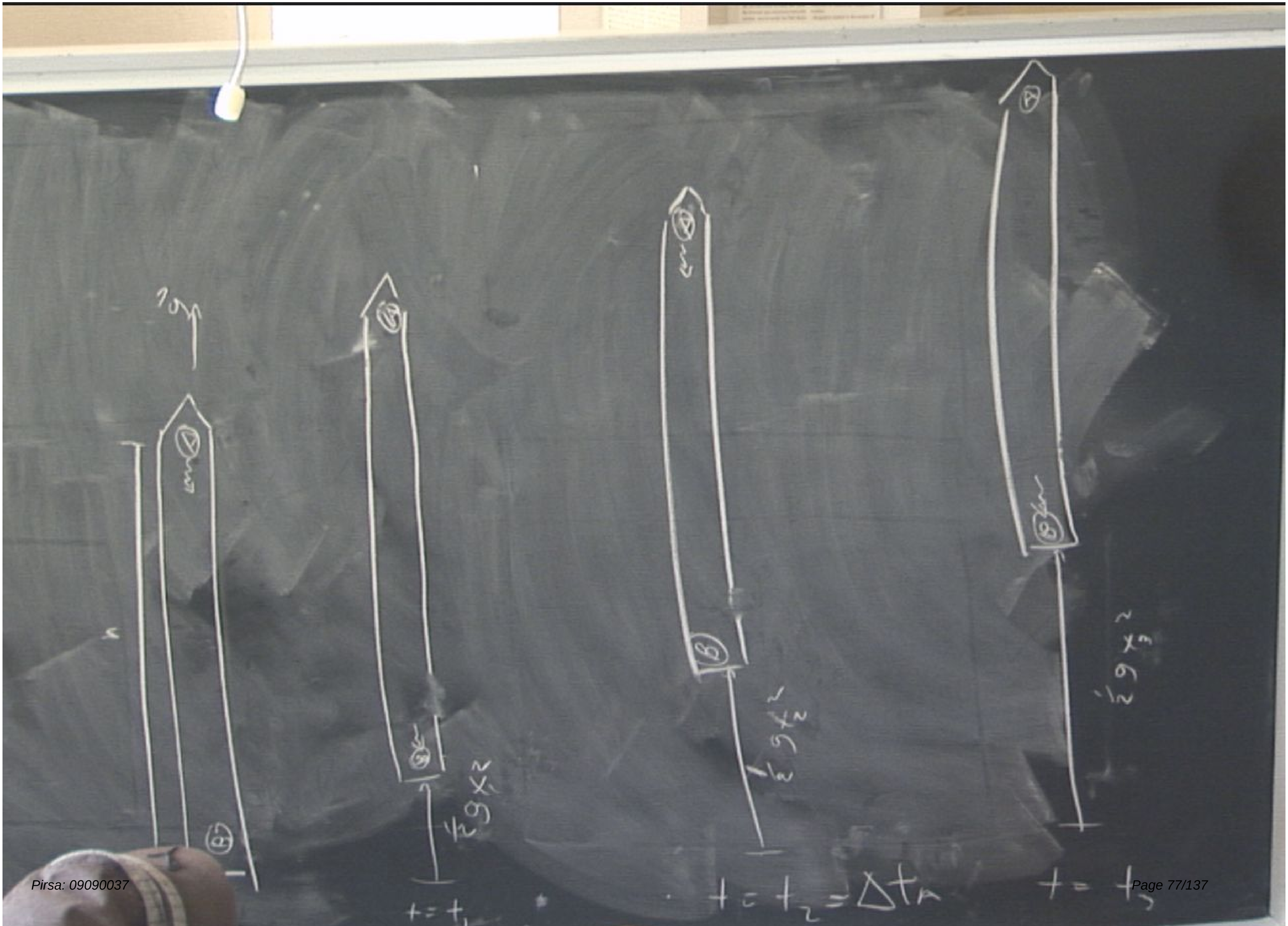

t = t1
t = t2
t = t3

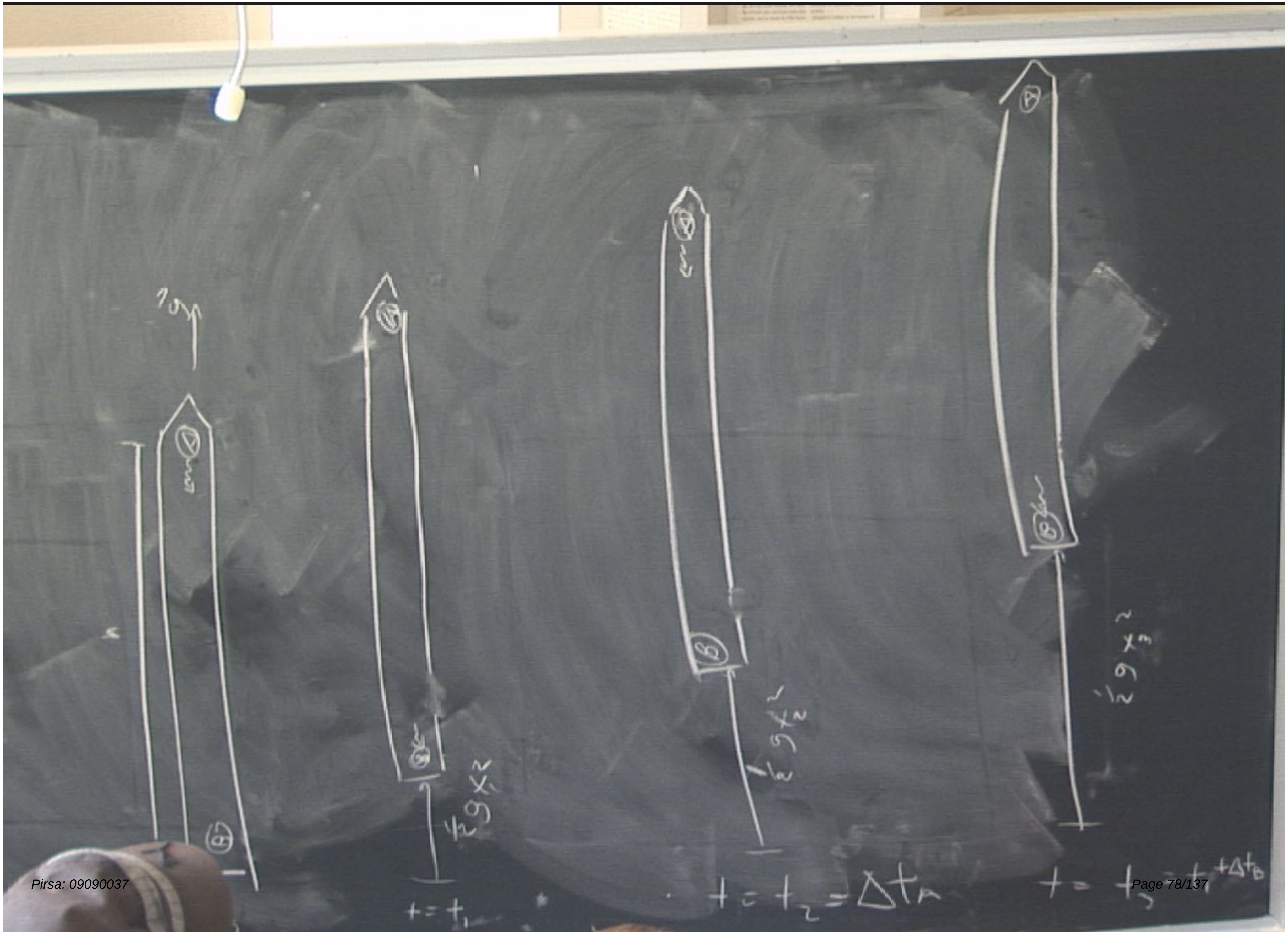

10m
$t = t_1$
$\frac{1}{2} g x_1^2$
$t = t_2$
$\frac{1}{2} g x_2^2$
$t = t_3$
$\frac{1}{2} g x_3^2$



$\frac{1}{2} g x_1^2$
$t = t_1$
$\frac{1}{2} g x_2^2$
$t = t_2$
$\frac{1}{2} g x_3^2$
$t = t_3$


½ g x₁²
½ g x₂²
½ g x₃²
t = t₁
t = t₂
t = t₃


$\frac{1}{2} g \tilde{x}_1^2$
$\frac{1}{2} g \tilde{x}_2^2$
$\frac{1}{2} g \tilde{x}_3^2$
$t = t_1$
$t = t_2 = \Delta t_A$
$t = t_3$


½ g x̃²
t = t₁
t = t₂ = Δt_A

assumptions

① $(v/c)^2 \ll 1$

assumptions

① $(v/c)^2 \ll 1$ ← relativistic effects (time dilation or length contraction) can be ignored

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<u>assumptions</u>

① $(v/c)^2 \ll 1$ ← relativistic effects (time dilation or length contraction) can be ignored

② $gh/c^2 \ll 1$

assumptions

① $(v/c)^2 \ll 1$ ← relativistic effects (time dilation or length contraction) can be ignored

② $gh/c^2 \ll 1$ ← gravitational potential energy $\ll$ relativistic rest energy

assumptions

① $(v/c)^2 \ll 1$ ← relativistic effects (time dilation or length contraction) can be ignored

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trajectories:

assumptions

① $(v/c)^2 \ll 1$ ← relativistic effects (time dilation or length contraction) can be ignored

② $gh/c^2 \ll 1$ ← gravitational potential energy $\ll$ relativistic rest energy

trajectories:

$$x_B = \tfrac{1}{2} g t^2$$

$$x_A = h + \tfrac{1}{2} g t^2$$

assumptions

① $(v/c)^2 \ll 1$ ← relativistic effects (time dilation or length contraction) can be ignored

② $gh/c^2 \ll 1$ ← gravitational potential energy $\ll$ relativistic rest energy

trajectories:

$$x_B = \tfrac{1}{2} g t^2$$

$$x_A = h + \tfrac{1}{2} g t^2$$

first photon

$c\,(t_1 - t_0) = h -$

first photon

$$c\,(t_1 - \cancelto{0}{t_0}) = h - \tfrac{1}{2} g t_1^2$$

first pulse

$$c\,(t_1 - \cancelto{0}{t_0}) = h - \tfrac{1}{2} g t_1^2$$

2nd pulse

first pulse

$$c\,(t_1 - t_0) = h - \tfrac{1}{2} g t_1^2$$

2nd pulse

$$c\,(t_3 - t_2) = h + \tfrac{1}{2} g t_2^2 -$$

first pulse

$$c\,(t_1 - \cancel{t_0}^{\,0}) = h - \tfrac{1}{2} g t_1^2$$

2nd pulse

$$c\,(t_3 - t_2) = h + \tfrac{1}{2} g t_2^2 - \tfrac{1}{2} g t_3^2$$

first pulse

$$c\,(t_1 - \overset{0}{\cancel{t_0}}) = h - \tfrac{1}{2} g t_1^2$$

2nd pulse

$$c\,(t_3 - t_2) = h + \tfrac{1}{2} g t_2^2 - \tfrac{1}{2} g t_3^2$$

$$c\,(t_1 + \Delta t_B - \Delta t_A) = h - \tfrac{1}{2} g \,(t_1^2 + 2 t_1 \Delta t_B + \Delta t_B^2$$

first pulse

$$c\,(t_1 - \cancelto{0}{t_0}) = h - \tfrac{1}{2} g t_1^2$$

2nd pulse

$$c\,(t_3 - t_2) = h + \tfrac{1}{2} g t_2^2 - \tfrac{1}{2} g t_3^2$$

$$c\,(t_1 + \Delta t_B - \Delta t_A) = h - \tfrac{1}{2} g \left(t_1^2 + 2 t_1 \Delta t_B + \Delta t_B^2 - \Delta t_A^2\right)$$

first pulse

① $c(t_1 - t_0) = h - \frac{1}{2} g t_1^2$

2nd pulse

$c(t_3 - t_2) = h + \frac{1}{2} g t_2^2 - \frac{1}{2} g t_3^2$

② $c(t_1 + \Delta t_B - \Delta t_A) = h - \frac{1}{2} g (t_1^2 + 2 t_1 \Delta t_B + \Delta t_B^2 - \Delta t_A^2)$

② − ① $\longrightarrow$ $c(\Delta t_B - \Delta t_A) =$

first pulse

① $c\,(t_1 - \cancel{t_0}^{\,0}) = h - \tfrac{1}{2} g t_1^2$

2nd pulse

$c\,(t_3 - t_2) = h + \tfrac{1}{2} g t_2^2 - \tfrac{1}{2} g t_3^2$

② $c\,(t_1 + \Delta t_B - \Delta t_A) = h - \tfrac{1}{2} g\,(t_1^2 + 2 t_1 \Delta t_B + \Delta t_B^2 - \Delta t_A^2)$

② − ① $\longrightarrow$ $c\,(\Delta t_B - \Delta t_A) = -g t_1 \Delta t_B$

first pulse

① $c\,(t_1 - \cancel{t_0}^{\,0}) = h - \frac{1}{2} g t_1^2$

2nd pulse

$c\,(t_3 - t_2) = h + \frac{1}{2} g t_2^2 - \frac{1}{2} g t_3^2$

② $c\,(t_1 + \Delta t_B - \Delta t_A) = h - \frac{1}{2} g\,(t_1^2 + 2 t_1 \Delta t_B + \Delta t_B^2 - \Delta t_A^2)$

② − ① $\longrightarrow$ $c\,(\Delta t_B - \Delta t_A) = -g t_1 \Delta t_B - \frac{1}{2} g\,(\Delta t_B^2 - \Delta t_A^2)$

first pulse

$$① \quad c\,(t_1 - \cancelto{0}{t_0}) = h - \tfrac{1}{2} g t_1^2$$

2nd pulse

$$c\,(t_3 - t_2) = h + \tfrac{1}{2} g t_2^2 - \tfrac{1}{2} g t_3^2$$

$$② \quad c\,(t_1 + \Delta t_B - \Delta t_A) = h - \tfrac{1}{2} g \left(t_1^2 + 2 t_1 \Delta t_B + \Delta t_B^2 - \Delta t_A^2\right)$$

$$② - ① \longrightarrow \quad c\,(\Delta t_B - \Delta t_A) = - g t_1 \Delta t_B - \tfrac{1}{2} g (\Delta t_B^2 - \Delta t_A^2)$$

first pulse

$$① \quad c\,(t_1 - \cancel{t_0}^{\,0}) = h - \tfrac{1}{2} g t_1^2$$

2nd pulse

$$c\,(t_3 - t_2) = h + \tfrac{1}{2} g t_2^2 - \tfrac{1}{2} g t_3^2$$

$$② \quad c\,(t_1 + \Delta t_B - \Delta t_A) = h - \tfrac{1}{2} g\,(t_1^2 + 2 t_1 \Delta t_B + \Delta t_B^2 - \Delta t_A^2)$$

$$② - ① \longrightarrow \quad c\,(\Delta t_B - \Delta t_A) = -g t_1 \Delta t_B - \tfrac{1}{2} g\,(\Delta t_B^2 - \Delta t_A^2)$$

$$\sim g\,\Delta t_B^2$$

assumptions

① $(v/c)^2 \ll 1$ ← relativistic effects (time dilation or length contraction) can be ignored

② $gh/c^2 \ll 1$ ← gravitational potential energy $\ll$ relativistic rest energy

trajectories: $x_B = \frac{1}{2} g t^2$

$x_A = h + \frac{1}{2} g t^2$

tion

red

first pulse

(1) $c\,(t_1 - \cancel{t_0}) = h - \frac{1}{2} g t_1^2$

2nd pulse

$c\,(t_3 - t_2) = h + \frac{1}{2} g t_2^2 - \frac{1}{2} g t_3^2$

(2) $c\,(t_1 + \Delta t_B - \Delta t_A) = h - \frac{1}{2} g\,(t_1^2 + 2 t_1 \Delta t_B + \Delta t_B^2 - \Delta t_A^2)$

(2) $-$ (1) $\longrightarrow$ $c\,(\Delta t_B - \Delta t_A) = -g t_1 \Delta t_B - \frac{1}{2} g\,(\Delta t_B^2 - \Delta t_A^2)$

$\sim g\,\Delta t_B^2$

$\sim O(g^2)$

tion

red

first pulse

① $c\,(t_1 - \cancel{t_0}^{\,0}) = h - \tfrac{1}{2} g t_1^2$

2nd pulse

$$c\,(t_3 - t_2) = h + \tfrac{1}{2} g t_2^2 - \tfrac{1}{2} g t_3^2$$

② $c(t_1 + \Delta t_B - \Delta t_A) = h - \tfrac{1}{2} g \,(t_1^2 + 2 t_1 \Delta t_B + \Delta t_B^2 - \Delta t_A^2)$

② − ① $\longrightarrow$ $c(\Delta t_B - \Delta t_A) = -g t_1 \Delta t_B \; \underbrace{- \tfrac{1}{2} g (\Delta t_B^2 - \Delta t_A^2)}_{\sim g \Delta t_B^2}$

t_1

$\sim O(g^2)$ drop

first pulse

① $c(t_1 - t_0) = h - \frac{1}{2} g t_1^2$

2nd pulse

$c(t_3 - t_2) = h + \frac{1}{2} g t_2^2 - \frac{1}{2} g t_3^2$

② $c(t_1 + \Delta t_B - \Delta t_A) = h - \frac{1}{2} g (t_1^2 + 2 t_1 \Delta t_B + \Delta t_B^2 - \Delta t_A^2)$

② − ① $\longrightarrow$ $c(\Delta t_B - \Delta t_A) = -g t_1 \Delta t_B - \frac{1}{2} g (\Delta t_B^2 - \Delta t_A^2)$

$t_1 \sim h/c$

$\sim g \Delta t_B^2$

$\sim O(g^2)$ drop

tion

red

first pulse

$$① \quad c\,(t_1 - \cancelto{0}{t_0}) = h - \tfrac{1}{2} g t_1^2$$

2nd pulse

$$c\,(t_3 - t_2) = h + \tfrac{1}{2} g t_2^2 - \tfrac{1}{2} g t_3^2$$

$$② \quad c\,(t_1 + \Delta t_B - \Delta t_A) = h - \tfrac{1}{2} g\,(t_1^2 + 2 t_1 \Delta t_B + \Delta t_B^2 - \Delta t_A^2)$$

$$② - ① \longrightarrow \quad c\,(\Delta t_B - \Delta t_A) = -g t_1 \Delta t_B - \tfrac{1}{2} g\,(\Delta t_B^2 - \Delta t_A^2)$$

$$t_1 \sim h/c - \frac{1}{2c} g\,(h/c)^2$$

$$\sim g\,\Delta t_B^2$$

$\sim O(g^2)$ drop

first pulse

① $c\,(t_1 - \cancel{t_0}^{\,0}) = h - \frac{1}{2} g t_1^2$

2nd pulse

$c\,(t_3 - t_2) = h + \frac{1}{2} g t_2^2 - \frac{1}{2} g t_3^2$

② $c\,(t_1 + \Delta t_B - \Delta t_A) = h - \frac{1}{2} g\,(t_1^2 + 2 t_1 \Delta t$

$+ \Delta t_B^2 -$

② $-$ ① $\longrightarrow$ $c\,(\Delta t_B - \Delta t_A) = -g t_1 \Delta t_B \; - \ldots \Delta t_B - \Delta t_A^2)$

$t_1 \sim h/c - \frac{1}{2c} g (h/c)^2$

$\sim h/c \, (1 - \frac{1}{2} g h/c^2 + \ldots)$

$\sim g \Delta t_B^2$

$\sim O(g^2)$ drop

tion

red

first pulse

① $c\,(t_1 - \cancel{t_0}^{\,0}) = h - \tfrac{1}{2} g t_1^2$

2nd pulse

$c\,(t_3 - t_2) = h + \tfrac{1}{2} g t_2^2 - \tfrac{1}{2} g t_3^2$

② $c\,(t_1 + \Delta t_B - \Delta t_A) = h - \tfrac{1}{2} g\,(t_1^2 + 2 t_1 \Delta t_B + \Delta t_B^2 - \Delta t_A^2)$

② − ① $\longrightarrow$ $c\,(\Delta t_B - \Delta t_A) = -g t_1 \Delta t_B - \tfrac{1}{2} g\,(\Delta t_B^2 - \Delta t_A^2)$

$t_1 \sim h/c - \frac{1}{2c} g \left(h/c\right)^2 + \cdots$

$\sim h/c\,\left(1 - \tfrac{1}{2}\, g h/c^2 + \cdots\right)$

$\sim g\,\Delta t_B^2$

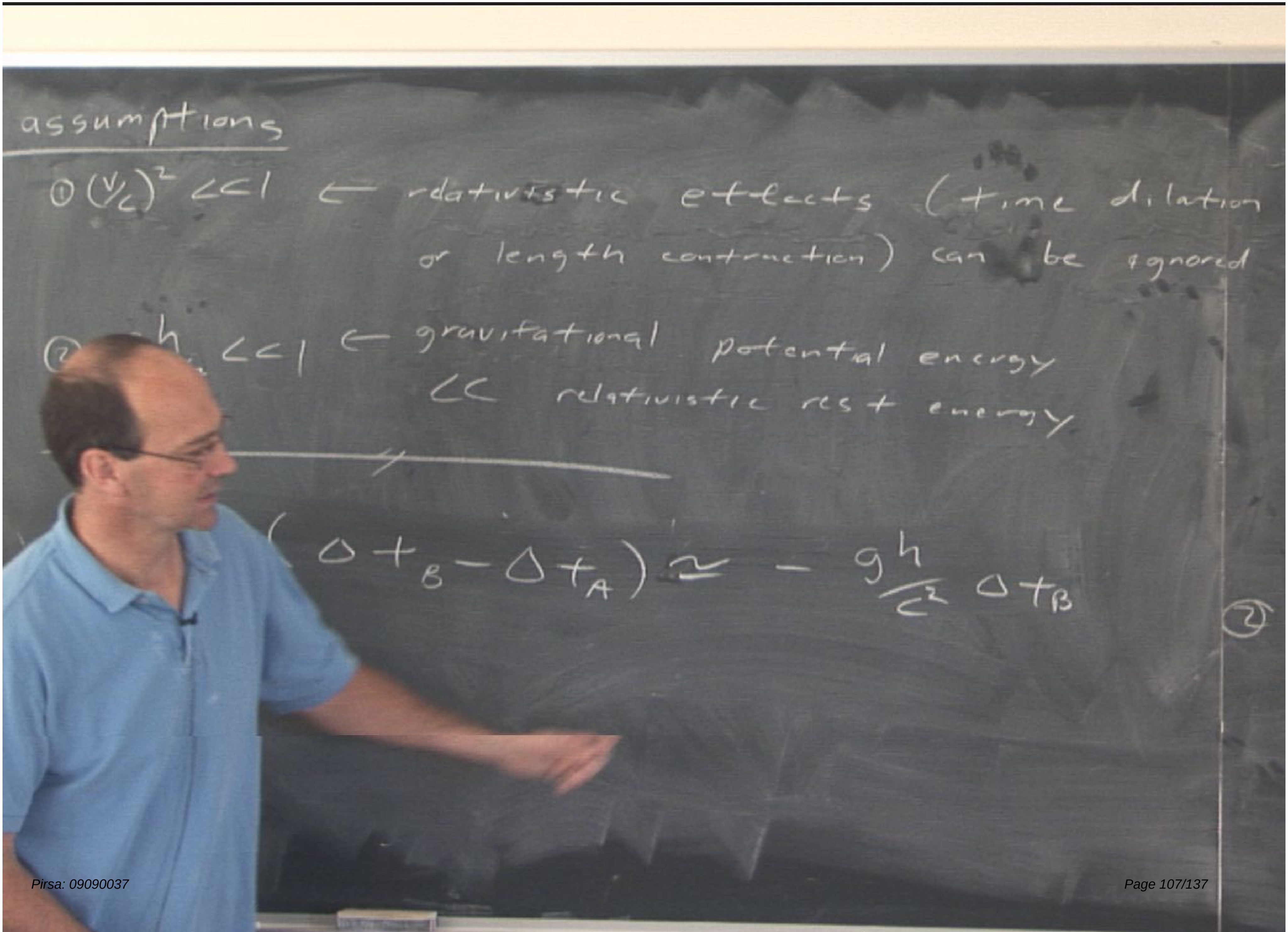
$\sim O(g^2)$ drop

assumptions

① $(v/c)^2 \ll 1$ ← relativistic effects (time dilation or length contraction) can be ignored

$gh/c^2 \ll 1$ ← gravitational potential energy $\ll$ relativistic rest energy

$$(\Delta t_B - \Delta t_A) \simeq -\frac{gh}{c}\,\Delta t_B \qquad ②$$


assumptions
① $(v/c)^2 \ll 1$ ← relativistic effects (time dilation
or length contraction) can be ignored
② $gh/c^2 \ll 1$ ← gravitational potential energy
$\ll$ relativistic rest energy
$(\Delta t_B - \Delta t_A) \simeq - \frac{gh}{c^2} \Delta t_B$
②

assumptions

① $(v/c)^2 \ll 1$ ← relativistic effects (time dilation or length contraction) can be ignored

② $gh/c^2 \ll 1$ ← gravitational potential energy $\ll$ relativistic rest energy

$$(\Delta t_B - \Delta t_A) \simeq - \frac{gh}{c^2} \Delta t_B$$ ②

assumptions

① $(v/c)^2 \ll 1$ ← relativistic effects (time dilation or length contraction) can be ignored

② $gh/c^2 \ll 1$ ← gravitational potential energy $\ll$ relativistic rest energy

$$(\Delta t_B - \Delta t_A) \simeq -\frac{gh}{c^2}\,\Delta t_B$$

②

assumptions

① $(v/c)^2 \ll 1$ ← relativistic effects (time dilation or length contraction) can be ignored

② $gh/c^2 \ll 1$ ← gravitational potential energy $\ll$ relativistic rest energy

$$\Rightarrow (\Delta t_B - \Delta t_A) \simeq - \frac{gh}{c^2} \Delta t_B \qquad ②$$

$$\Delta t_B \left(1 + gh/c^2\right) \simeq \Delta t_A$$

assumptions

① $(v/c)^2 \ll 1$ ← relativistic effects (time dilation or length contraction) can be ignored

② $gh/c^2 \ll 1$ ← gravitational potential energy $\ll$ relativistic rest energy

$$\Rightarrow (\Delta t_B - \Delta t_A) \simeq - \frac{gh}{c^2} \Delta t_B$$

$$\Delta t_B \left(1 + gh/c^2\right) \simeq \Delta t_A$$

$$\Delta t_B < \Delta t_A$$

invoking "equivalence principle"

find same effect in rocket

- sitting at rest in grav. field,

invoking "equivalence principle"

find same effect in rocket

sitting at rest in grav. field

invoking "equivalence principle":

find same effect in rocket

sitting at rest in grav. field

→ clock lower in the grav. field runs slow

$$\Rightarrow \quad \Delta t_A \Big|_{\substack{\text{at height} \\ z=h}} \simeq \Delta t_B \Big|_{\substack{\text{at height} \\ z=0}} \left(1 + gh/c^2\right)$$

invoking "equivalence principle"

find same effect in rocket

sitting at rest in grav. field

→ clock lower in the grav. field runs slow

$$\Rightarrow \quad \Delta t_A\Big|_{\substack{\text{at height}\\ z=h}} \simeq \Delta t_B\Big|_{\substack{\text{at height}\\ z=0}} \left(1+\frac{gh}{c^2}\right)$$

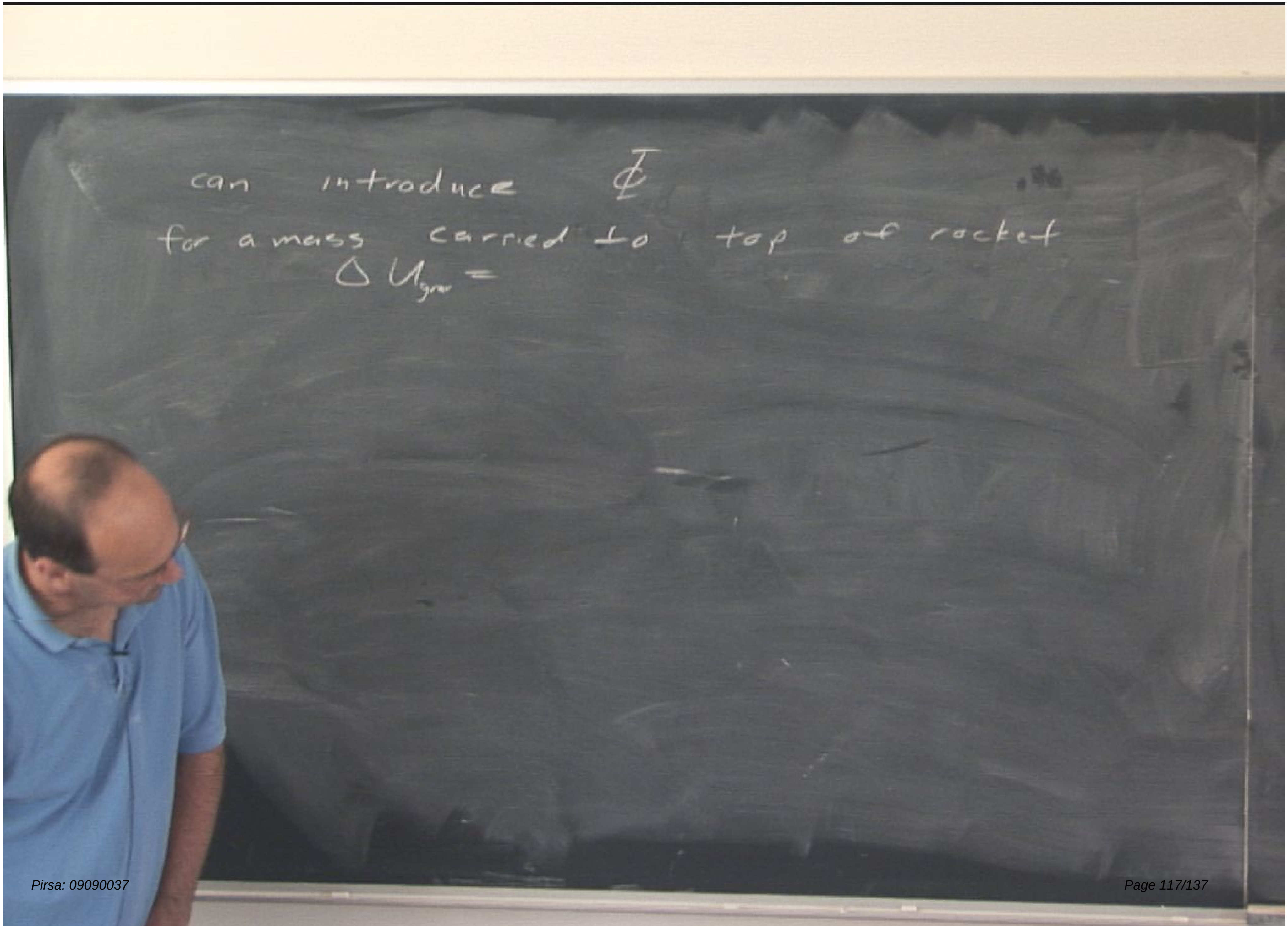
invoking "equivalence principle"

find same effect in rocket

sitting at rest in grav. field

$\longrightarrow$ clock lower in the grav. field runs slow

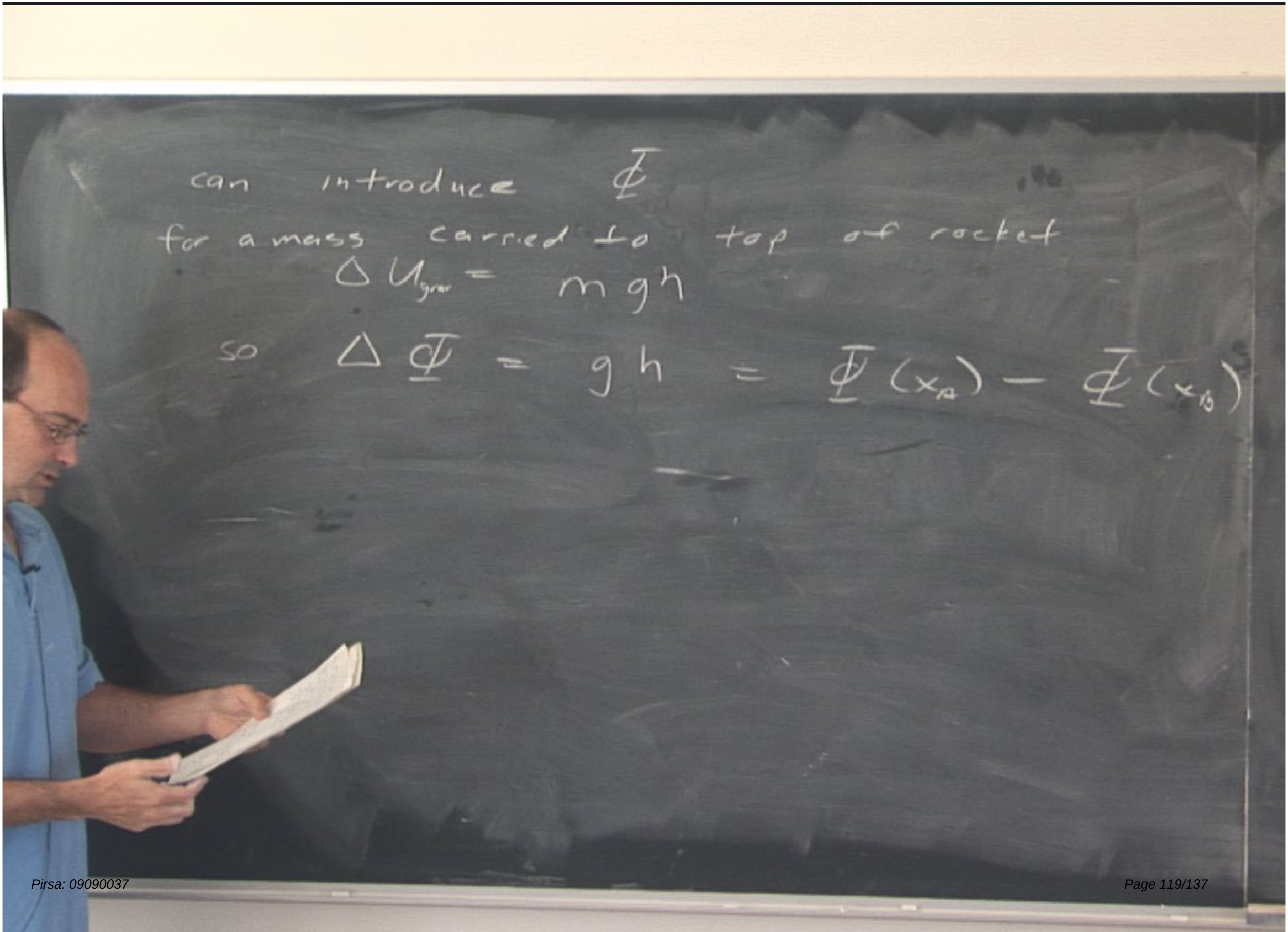
$$\Rightarrow \quad \Delta t_A \Big|_{\substack{\text{at height} \\ z=h}} \simeq \Delta t_B \Big|_{\substack{\text{at height} \\ z=0}} \left(1 + gh/c^2\right)$$

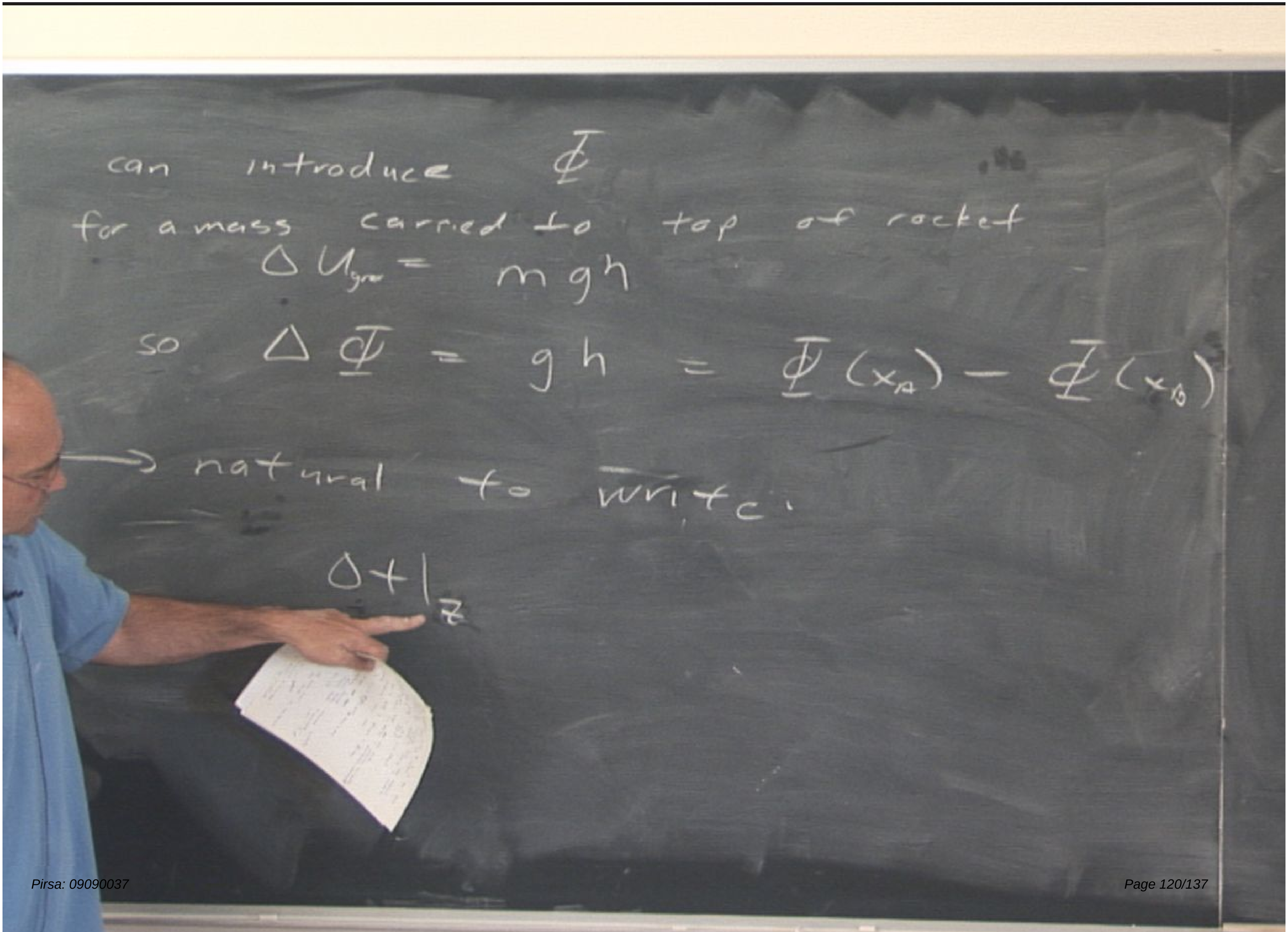

can introduce Φ
for a mass carried to top of rocket
$\Delta U_{grav} =$

can introduce Φ

for a mass carried to top of rocket

$\Delta U_{grav} =$


can introduce Φ
for a mass carried to top of rocket
$\Delta U_{grav} = mgh$
so $\Delta \Phi = gh = \Phi(x_A) - \Phi(x_B)$


can introduce Φ
for a mass carried to top of rocket
$\Delta U_{grav} = mgh$
so $\Delta \Phi = gh = \Phi(x_A) - \Phi(x_B)$
$\longrightarrow$ natural to write:
$\Delta t|_Z$

can introduce Φ

for a mass carried to top of rocket

$$\Delta U_{grav} = mgh$$

so $\Delta \Phi = gh = \Phi(x_A) - \Phi(x_B)$

$\longrightarrow$ natural to write:

$$\Delta t\big|_z = \left(1 + \frac{\Phi(z)}{c^2}\right) \Delta t\big|_{z \text{ where } \Phi = 0}$$

can introduce Φ

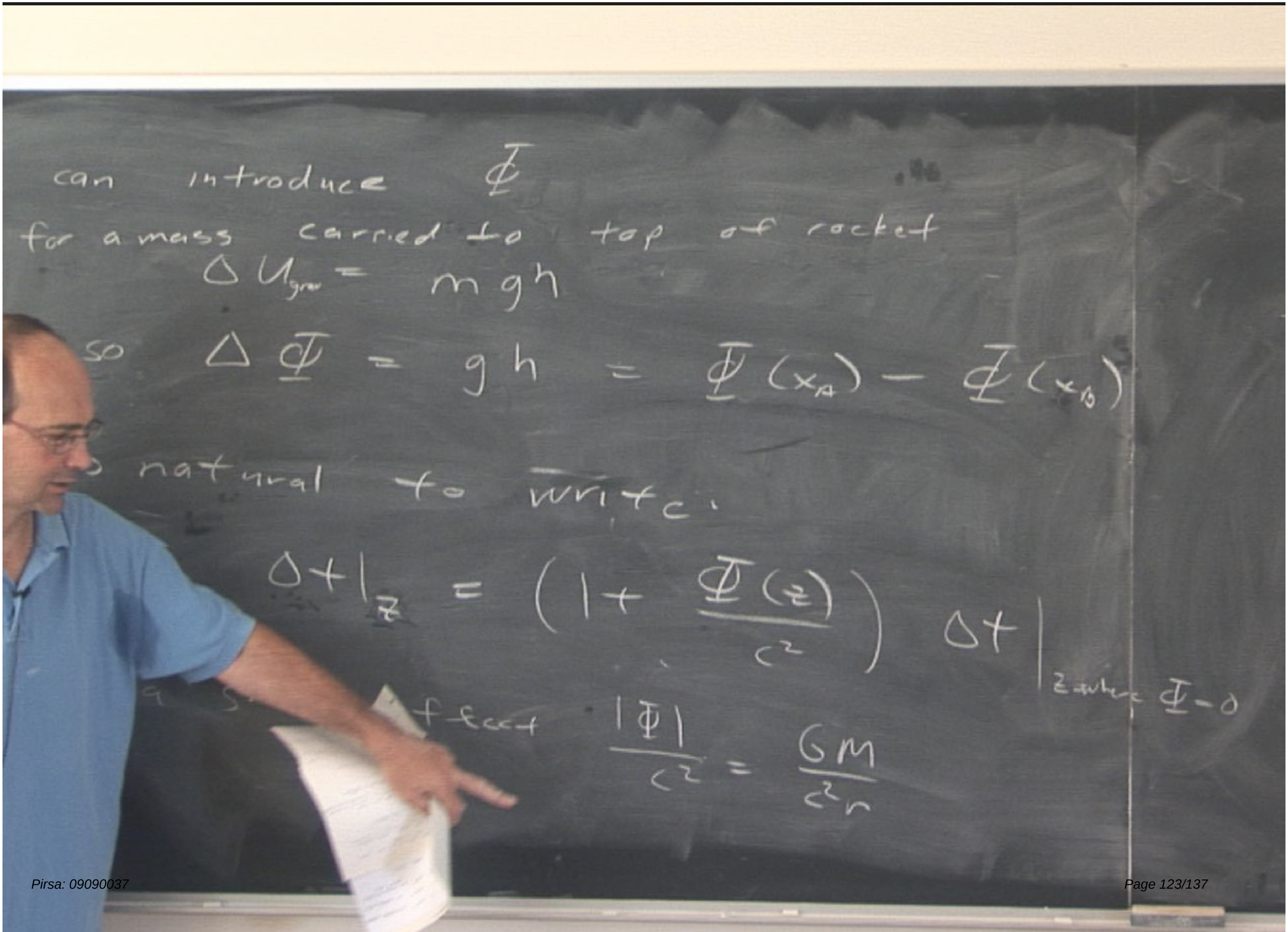
for a mass carried to top of rocket

$$\Delta U_{grav} = mgh$$

so $\Delta \Phi = gh = \Phi(x_A) - \Phi(x_B)$

$\longrightarrow$ natural to write:

$$\Delta t\big|_z = \left(1 + \frac{\Phi(z)}{c^2}\right) \Delta t\big|_{z \text{ where } \Phi = 0}$$


can introduce Φ
for a mass carried to top of rocket
$\Delta U_{grav} = mgh$
so $\Delta \Phi = gh = \Phi(x_A) - \Phi(x_B)$
natural to write:
$\Delta t|_z = \left(1 + \frac{\Phi(z)}{c^2}\right) \Delta t|_{z \text{ where } \Phi = 0}$
effect $\frac{|\Phi|}{c^2} = \frac{GM}{c^2 r}$

can introduce Φ

for a mass carried to top of rocket

$$\Delta U_{grav} = mgh$$

so $$\Delta \Phi = gh = \Phi(x_A) - \Phi(x_B)$$

$\longrightarrow$ natural to write:

$$\Delta t\big|_z = \left(1 + \frac{\Phi(z)}{c^2}\right) \Delta t\big|_{z \text{ where } \Phi = 0}$$

$\longrightarrow$ a small effect $\dfrac{|\Phi|}{c^2} = \dfrac{GM}{c^2 r}$

can introduce Φ

for a mass carried to top of rocket

$$\Delta U_{grav} = m g h$$

so $\Delta \Phi = g h = \Phi(x_A) - \Phi(x_B)$

$\longrightarrow$ natural to write:

$$\Delta t \big|_{z} = \left(1 + \frac{\Phi(z)}{c^2}\right) \Delta t \Big|_{z \text{ where } \Phi = 0}$$

a small effect $\dfrac{|\Phi|}{c^2} = \dfrac{GM}{c^2 r} \simeq 10^{-9}$

can introduce Φ

for a mass carried to top of rocket

$$\Delta U_{grav} = mgh$$

so $$\Delta \Phi = gh = \Phi(x_A) - \Phi(x_B)$$

→ natural to write:

$$\Delta t\big|_z = \left(1 + \frac{\Phi(z)}{c^2}\right) \Delta t\big|_{z \text{ where } \Phi = 0}$$

→ a small effect $\frac{|\Phi|}{c^2} = \frac{GM}{c^2 r} \approx 10^{-9}$ on surface of earth

$\approx 10^{-6}$ on surface of sun

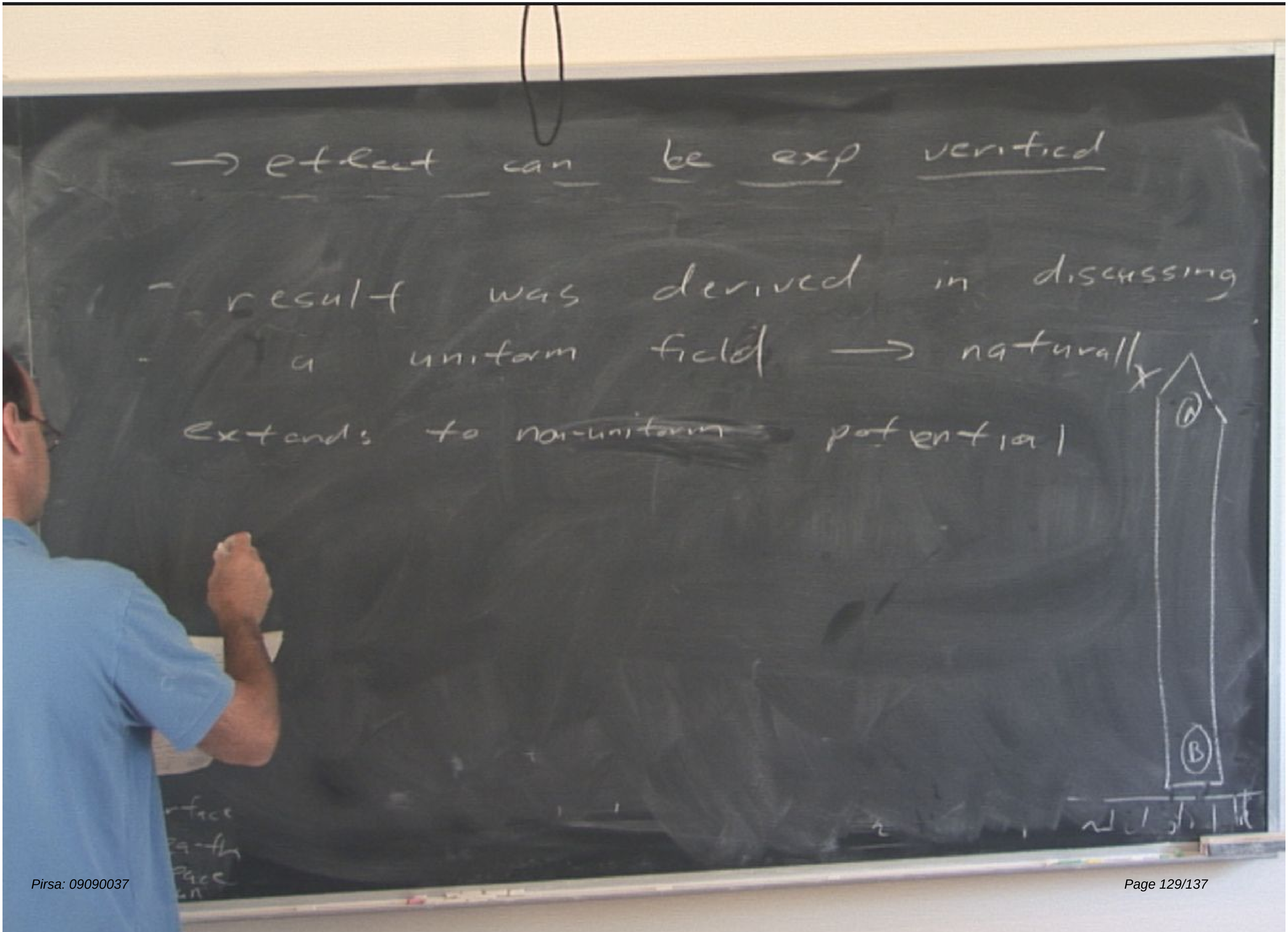

where $\Phi = 0$
on surface of earth
on surface of sun

→ effect can be exp verified

where $\Phi = 0$

on surface of earth

on surface of sun


→ effect can be exp verified
- result was derived in discussing
a uniform field → naturally
extends to non-uniform potential
@
B

→ effect can be exp verified

- result was derived in discussing a uniform field → naturally extends to ~~non-uniform~~ potential

$$\Delta\tau = \left(1 + \frac{\Phi(\vec{x})}{c^2}\right)\Delta t$$

on surface of earth

on surface of sun

→ effect can be exp verified

– result was derived in discussing a uniform field → naturally

extends to non-uniform potential

$$\Delta\tau = \left(1 + \frac{\Phi(\vec{x})}{c^2}\right)\Delta t$$

local proper time of $\vec{x}$

where $\Phi = 0$

on surface of earth

on surface of sun

→ effect can be exp verified

- result was derived in discussing a uniform field → naturally

extends to ~~non-uniform~~ potential

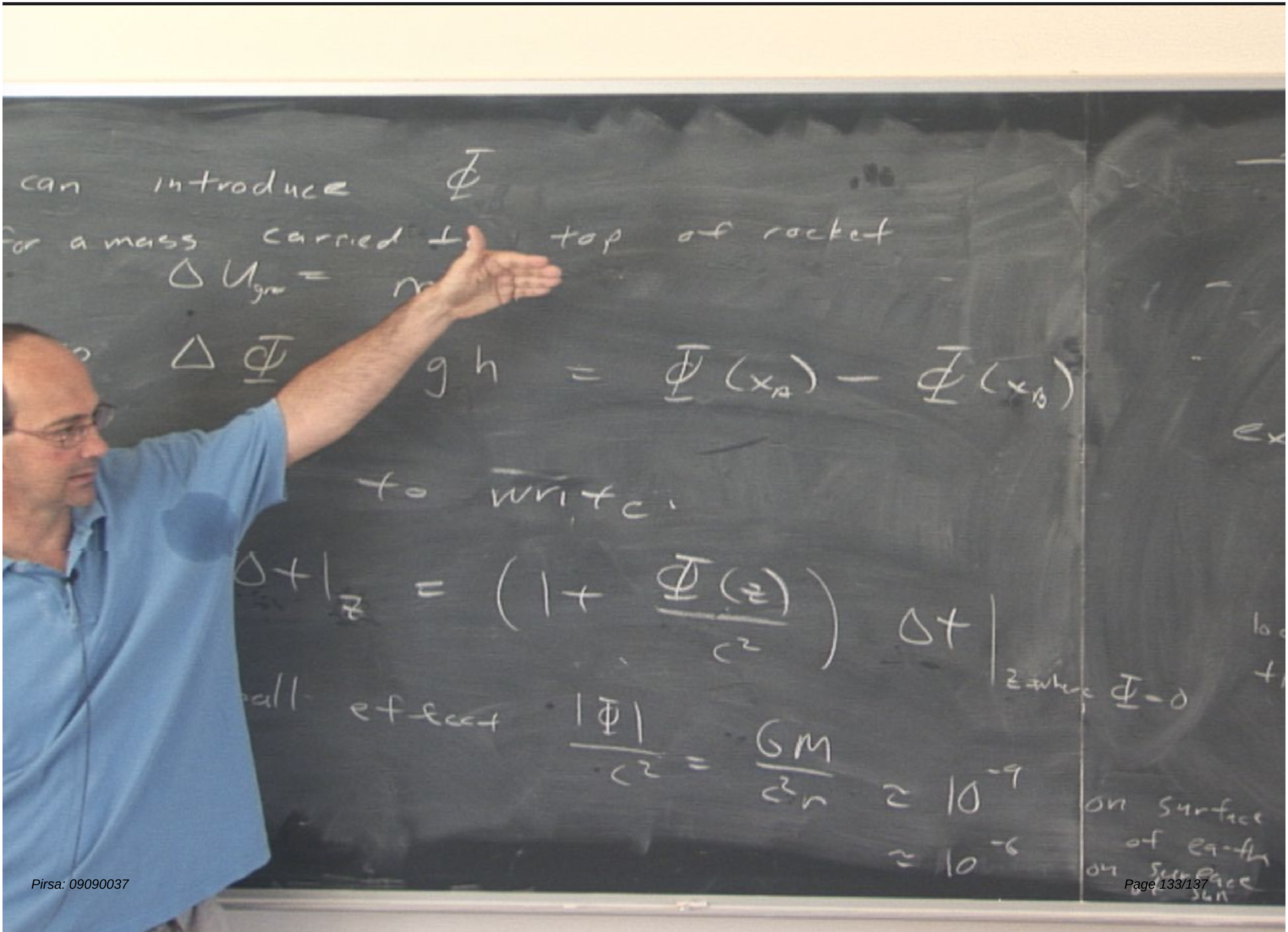
$$\Delta\tau = \left(1 + \frac{\Phi(\vec{x})}{c^2}\right)\Delta t$$

local proper time of $\vec{x}$

where $\Phi = 0$

on surface of earth

on surface of sun


can introduce Φ
for a mass carried to top of rocket
ΔU_grav = m
ΔΦ = gh = Φ(x_A) − Φ(x_B)
to write:
Δt|_z = (1 + Φ(z)/c²) Δt|_z where Φ=0
all effect |Φ|/c² = GM/c²r ≃ 10^-9
on surface of earth
≃ 10^-6
on surface of sun

can introduce Φ

for a mass carried to top of rocket

$$\Delta U_{grav} = mgh$$

so $$\Delta \Phi = gh = \Phi(x_A) - \Phi(x_B)$$

$\Rightarrow$ natural to write:

$$\Delta t|_z = \left(1 + \frac{\Phi(z)}{c^2}\right) \Delta t \Big|_{z \text{ where } \Phi = 0}$$

$\Rightarrow$ a small effect $\frac{|\Phi|}{c^2} = \frac{GM}{c^2 r} \simeq 10^{-9}$ on sur[face] of e[arth]

$\simeq 10^{-6}$ on surf[ace] ...

→ effect can be exp verified

- result was derived in discussing a uniform field → naturally extends to non-uniform potential

$$\Delta\tau = \left(1 + \frac{\Phi(\vec{x})}{c^2}\right)\Delta t$$

local proper time at $\vec{x}$

proper time measured at infinity - assume $\Phi \to 0$ @ $|\vec{x}| \to \infty$

surface
earth

→ effect can be exp verified

- result was derived in discussing a uniform field → naturally

extends to ~~non-uniform~~ potential

$$\Delta \tau = \left(1 + \frac{\Phi(\vec{x})}{c^2}\right) \Delta t$$

local proper time at $\vec{x}$

$\Phi < 0$

proper time measured at infinity - assume $\Phi \to 0$ @ $|\vec{x}| \to \infty$

→ effect can be exp verified

– result was derived in discussing a uniform field → naturally extends to non-uniform potential

$$\Delta\tau = \left(1 + \frac{\Phi(\vec{x})}{c^2}\right)\Delta t$$

local proper time at $\vec{x}$

$\Phi < 0$

proper time measured at infinity – assume $\Phi \to 0$ @ $|\vec{x}| \to \infty$

surface earth