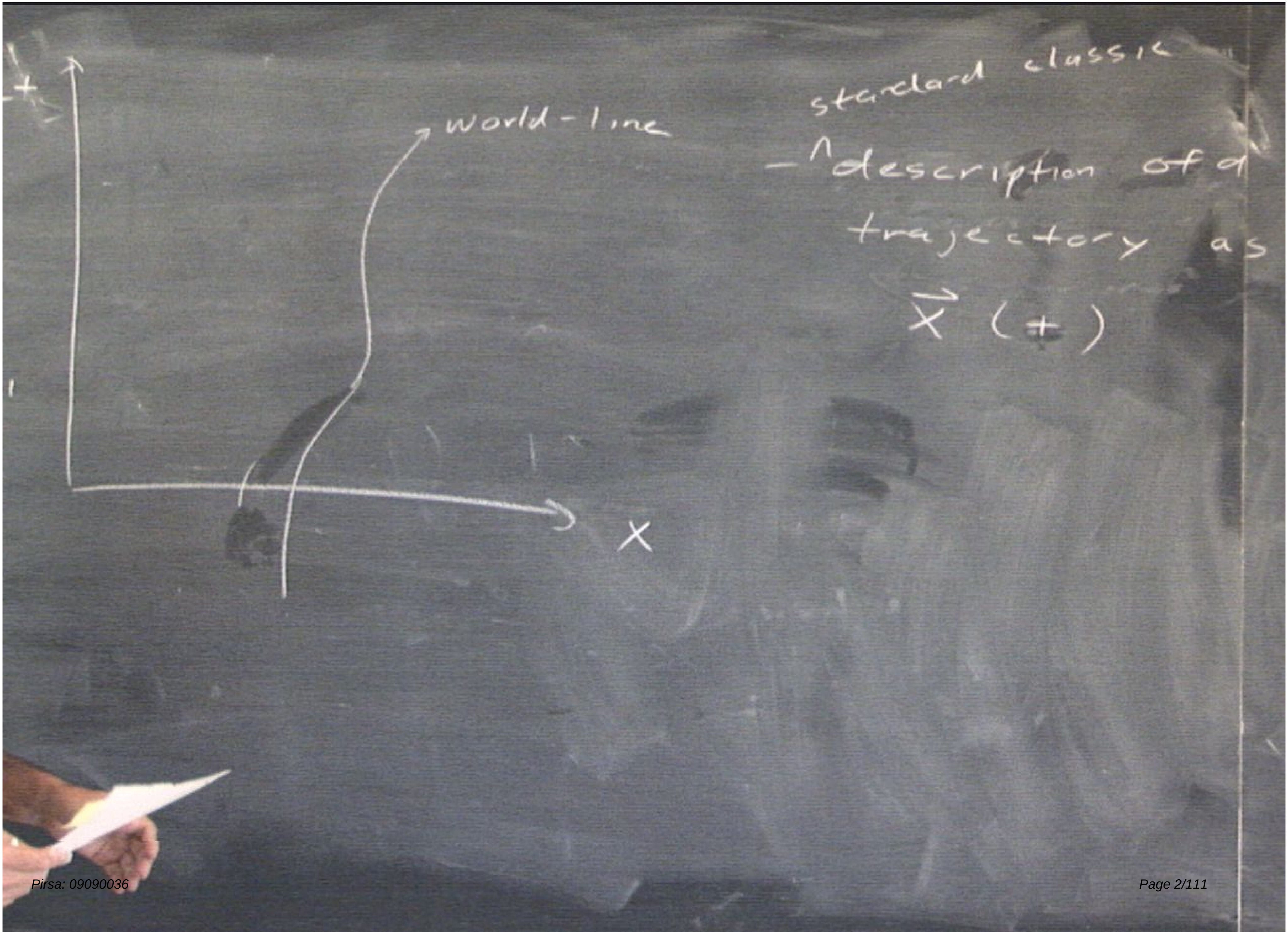
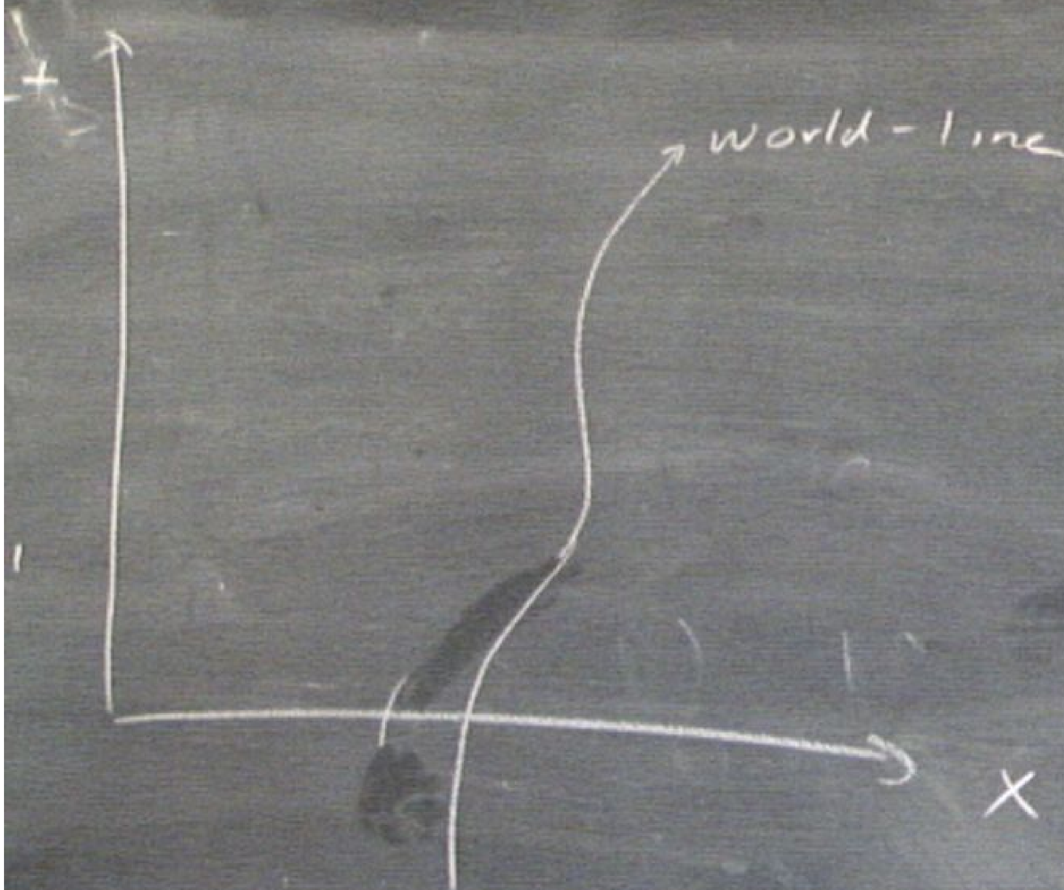
Title: PSI - Relativity (PHYS 604) - 2B

Date: Sep 04, 2009 11:00 AM

URL: http://pirsa.org/09090036

Abstract:

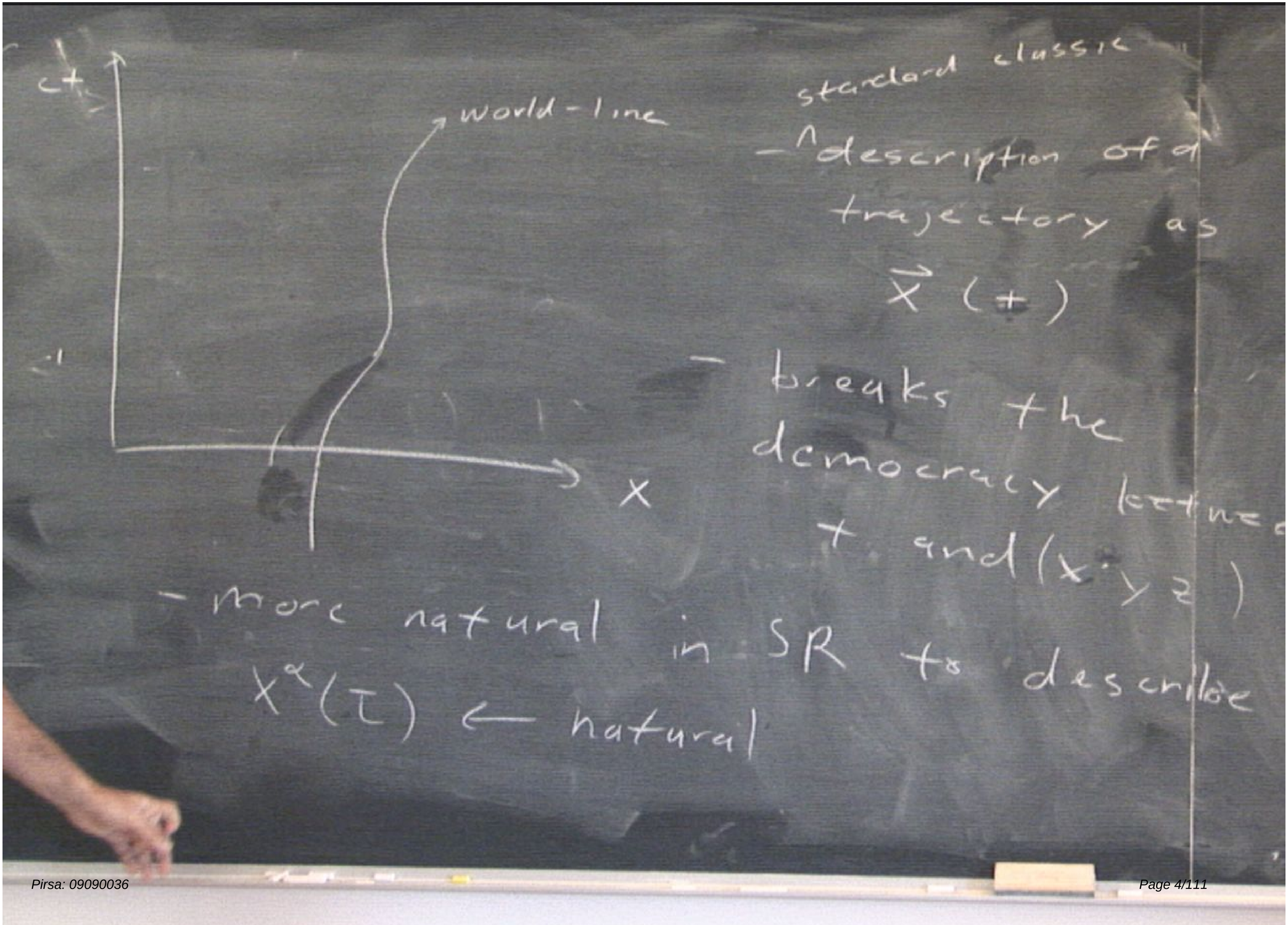

t
world-line
x
standard classic
description of a
trajectory as
$\vec{X}(t)$

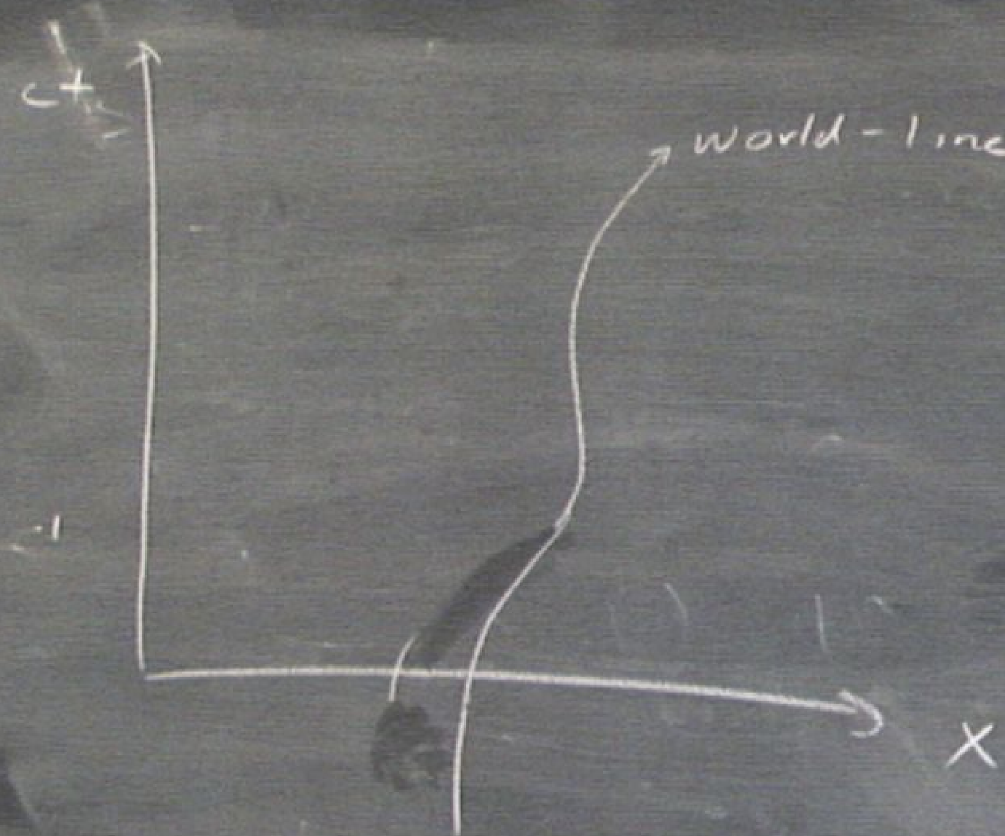


standard classic
- ^description of a
trajectory as
$\vec{x}(t)$

- breaks the
democracy betw
t and (x, y, z)

- more natural in SR to describe
x^α


ct
world-line
x
standard classic
description of a
trajectory as
$\vec{x}(t)$
breaks the
democracy between
t and (x y z)
more natural in SR to describe
$x^\alpha(\tau)$ ← natural



standard classic

— ^description of a

trajectory as

$\vec{x}(t)$

— breaks the democracy betwee

t and (x y z)

— more natural in SR to describe

$x^{\alpha}(\tau) \leftarrow$ natural choice for τ is

time measured by a clock fo

the world-line $\Longrightarrow$ proper time

en

following

the world-line $\Longrightarrow$ proper time

$$ds^2 = -c^2 \, d\tau^2$$

llowing

the world-line $\Longrightarrow$ proper time

$$ds^2 = -c^2\, d\tau^2 = -c\,dt^2 + dx^2$$

the world-line $\Longrightarrow$ <u>proper</u> <u>time</u>

$$ds^2 = - c^2 \, d\tau^2 = - c\,dt^2 + dx^2$$

for a particle with constant v

the world-line $\Longrightarrow$ proper time

$$ds^2 = -c^2\,d\tau^2 = -c\,dt^2 + dx^2$$

for a particle with constant v

$$c\,dt = \frac{c\,d\tau}{\sqrt{1 - v^2/c^2}}$$

the world-line $\Longrightarrow$ <u>proper time</u>

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for a particle with constant v

$$c\,dt = \frac{c\,d\tau}{\sqrt{1 - v^2/c^2}}$$

$$dx^i = \frac{v^i\,d\tau}{\sqrt{1 - v^2/c^2}}$$

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in general for infinitesimal displacements on more general world line would have $v(\tau)$

the world-line $\Longrightarrow$ <u>proper time</u>

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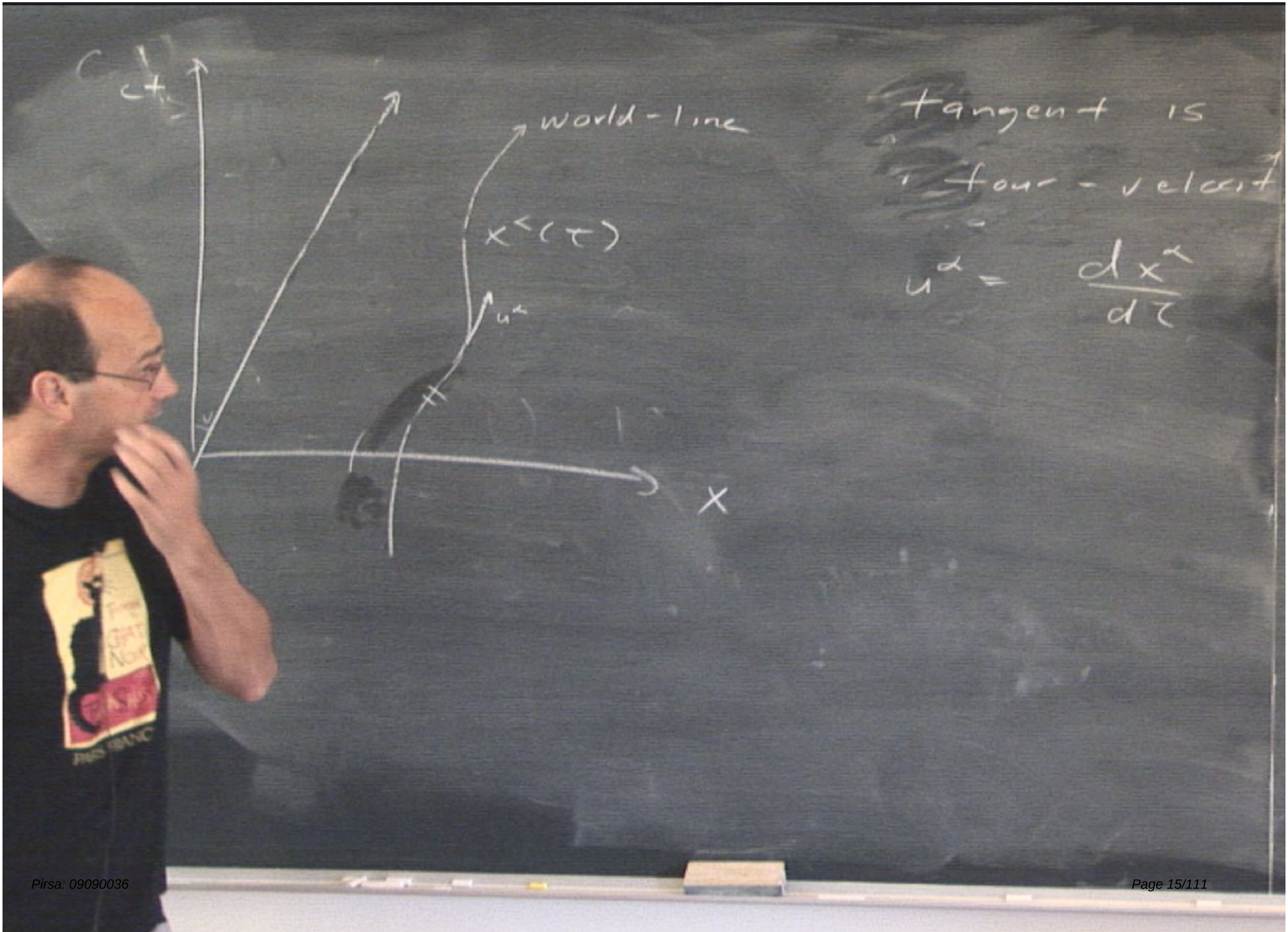
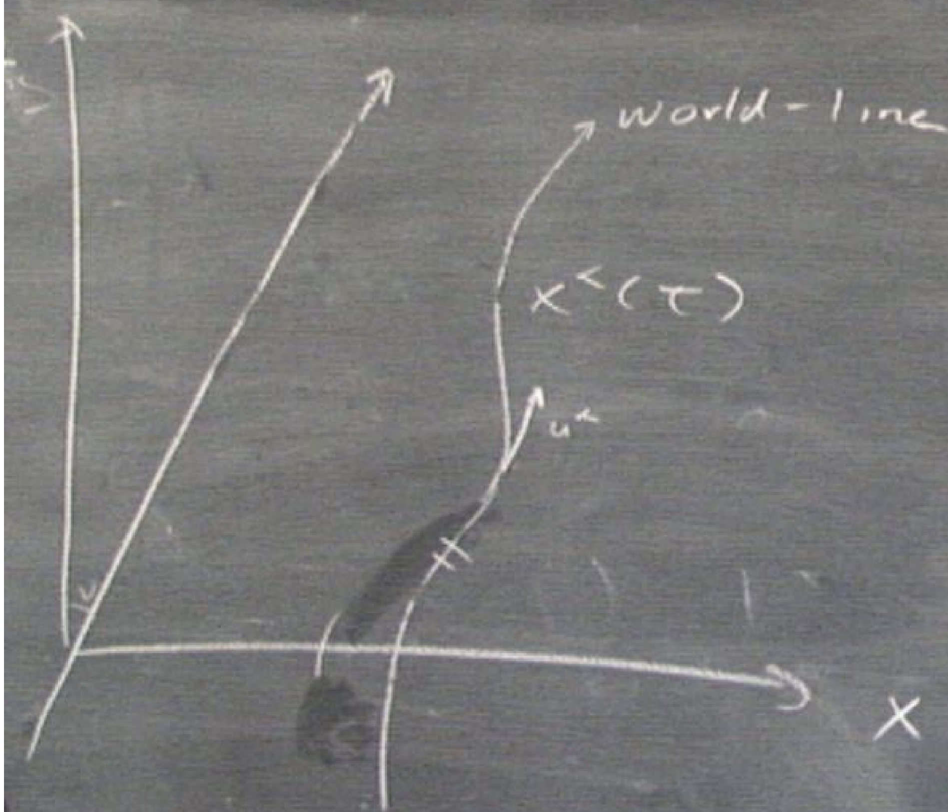
in general for infinitesimal displacements on more general world line would have $v(\tau)$

$\longrightarrow$ no acceleration effects


ct
world-line
$x^\alpha(\tau)$
u^α
x
tangent is
four-velocit
$u^\alpha = \frac{dx^\alpha}{d\tau}$

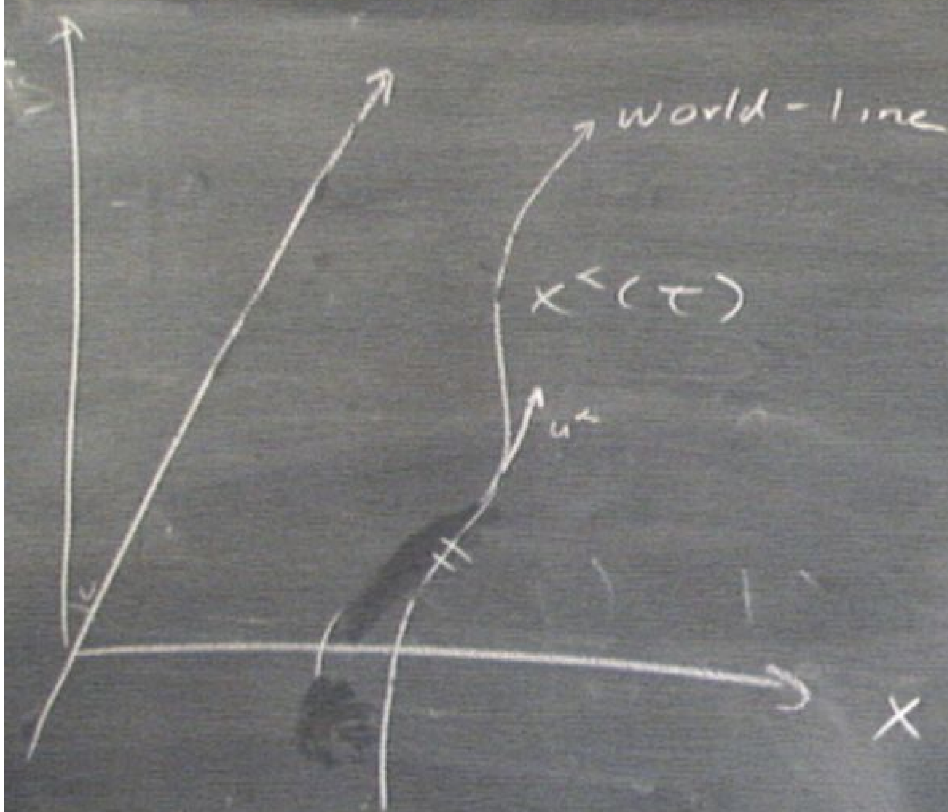


tangent is four-velocity:

$$u^\alpha = \frac{dx^\alpha}{d\tau}$$

$$u^0 = c\frac{dt}{d\tau} = \frac{c}{\sqrt{1-v^2/c^2}}$$

$$u^i = \frac{dx^i}{d\tau} = \frac{v^i}{\sqrt{1-v^2/c^2}}$$



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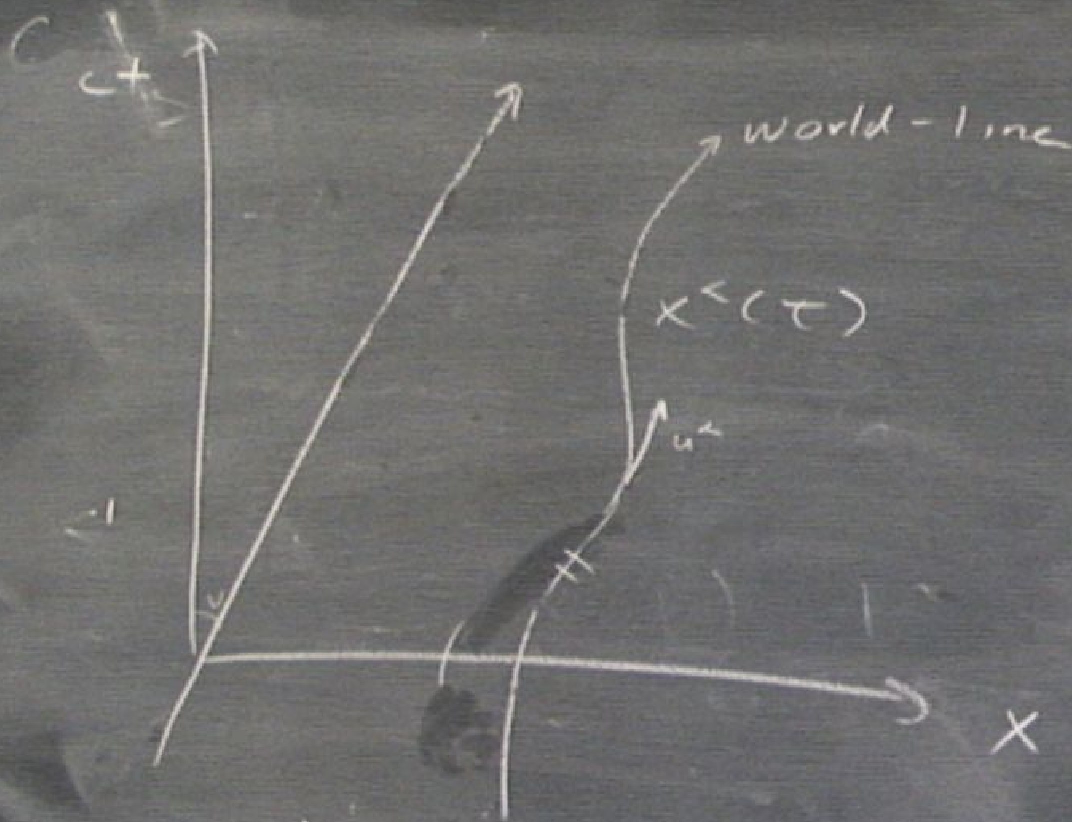
$$u^i = \frac{dx^i}{d\tau} = \frac{v^i}{\sqrt{1-v^2/c^2}}$$

$$c\,u^i/u^0 = v^i = \left(\frac{dx^i/d\tau}{dt/d\tau}\right)$$



tangent is

four-veloc

$$u^\alpha = \frac{dx^\alpha}{d\tau}$$

$$u^0 = c\,\frac{dt}{d\tau} =$$

$$u^i = \frac{dx^i}{d\tau} =$$

recover $\vec{v}$: $c\,{u^i}/{u^0} = \underline{v^i} = \left(\frac{dx^i/d\tau}{dt/d\tau}\right)$

inner product:

$$\eta_{\alpha\beta}\,u^\alpha u^\beta = -c^2 \leftarrow \underline{\text{norm}} \text{ is constan}$$

relativistic 4 - momentum

$$p^\alpha = m u^\alpha$$

relativistic 4 – momentum

$$p^\alpha = m\, u^\alpha$$

m ← rest mass

→ conserved in interactions

relativistic 4 - momentum

$$p^{\alpha} = m\, u^{\alpha}$$

rest mass

$\longrightarrow$ conserved in interactions

$$p^{i} = \frac{m v^{i}}{\sqrt{1 - v^2/c^2}}$$

relativistic 4 - momentum

$$p^\alpha = m\, u^\alpha$$

rest mass

$\longrightarrow$ conserved in interactions

$$p^i = \frac{m v^i}{\sqrt{1 - v^2/c^2}} = m v^i \left(1 + O\left(v^2/c^2\right)\right)$$

recover classical result for $v/c \ll 1$

relativistic 4 – momentum

$$p^\alpha = m\, u^\alpha$$

rest mass

$\longrightarrow$ conserved in interactions

$$p^i = \frac{m v^i}{\sqrt{1 - v^2/c^2}} = m v^i \left(1 + O\left(v^2/c^2 \right) \right)$$

recover classical result for $v/c \ll 1$

$$p^0 = \frac{m c}{\sqrt{1 - v^2/c^2}}$$

relativistic 4 - momentum

$$p^\alpha = m\, u^\alpha$$

rest mass

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$$p^i = \frac{m v^i}{\sqrt{1 - v^2/c^2}} = m v^i \left(1 + O\left(v^2/c^2\right)\right)$$

recover classical result for $v/c \ll 1$

$$p^0 = \frac{1}{c}\left(\frac{m c^2}{\sqrt{1 - v^2/c^2}}\right)$$

relativistic 4-momentum

$$p^\alpha = m\, u^\alpha$$

rest mass

$\longrightarrow$ conserved in interactions

$$p^i = \frac{m v^i}{\sqrt{1 - v^2/c^2}} = m v^i \left(1 + O\left(v^2/c^2\right)\right)$$

recover classical result for $v/c \ll 1$

$$p^0 = \frac{1}{c}\left(\frac{m c^2}{\sqrt{1 - v^2/c^2}}\right) = \frac{1}{c}\left(m c^2\right.$$

relativistic 4-momentum

$$p^\alpha = m\, u^\alpha$$

rest mass

$\longrightarrow$ conserved in interactions

$$p^i = \frac{m v^i}{\sqrt{1 - v^2/c^2}} = m v^i \left(1 + O\left(v^2/c^2\right)\right)$$

recover classical result for $v/c \ll 1$

$$p^0 = \frac{1}{c}\left(\frac{m c^2}{\sqrt{1 - v^2/c^2}}\right) = \frac{1}{c}\left(m c^2 + \frac{1}{2} m v^2 + O\left(v^4/c^2\right)\right)$$

relativistic 4 - momentum

$$p^\alpha = m\, u^\alpha$$

rest mass

$\longrightarrow$ conserved in interactions

$$p^i = \frac{m v^i}{\sqrt{1 - v^2/c^2}} = m v^i \left(1 + O\left(v^2/c^2\right)\right)$$

recover classical result for $v/c \ll 1$

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$$= E/c$$

relativistic 4 - momentum

$$p^\alpha = m\, u^\alpha$$

rest mass

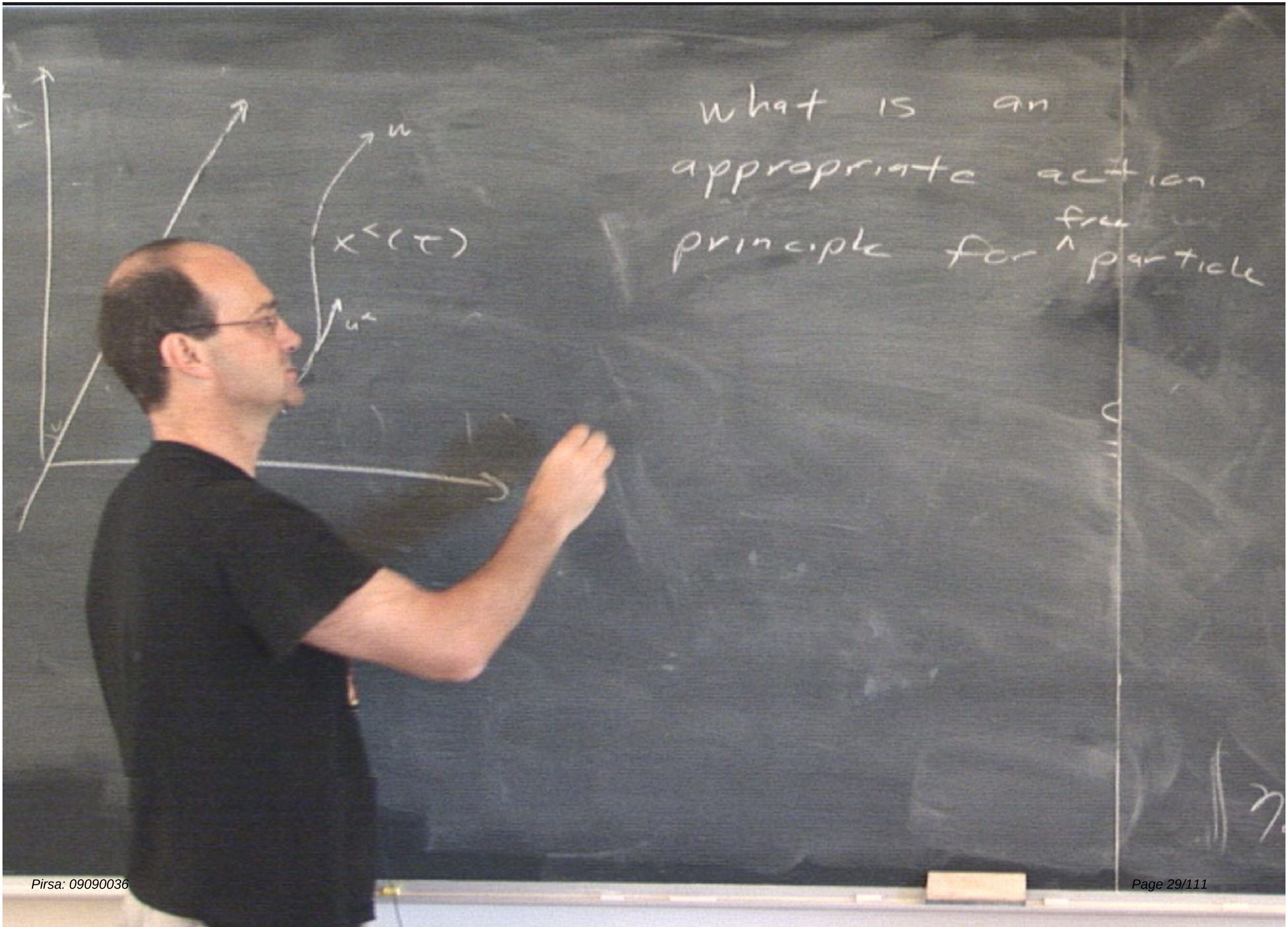
$\longrightarrow$ conserved in interactions

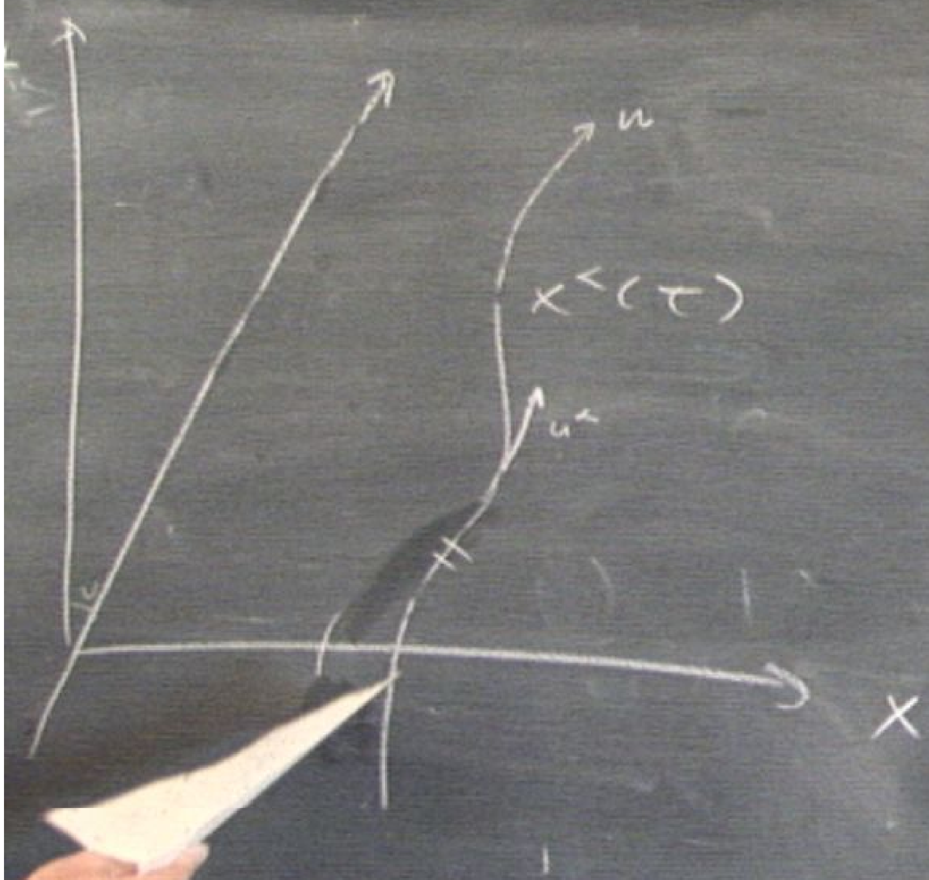
$$p^i = \frac{m v^i}{\sqrt{1 - v^2/c^2}} = m v^i \left(1 + O\left(\frac{v^2}{c^2}\right)\right)$$

recover classical result for $v/c \ll 1$

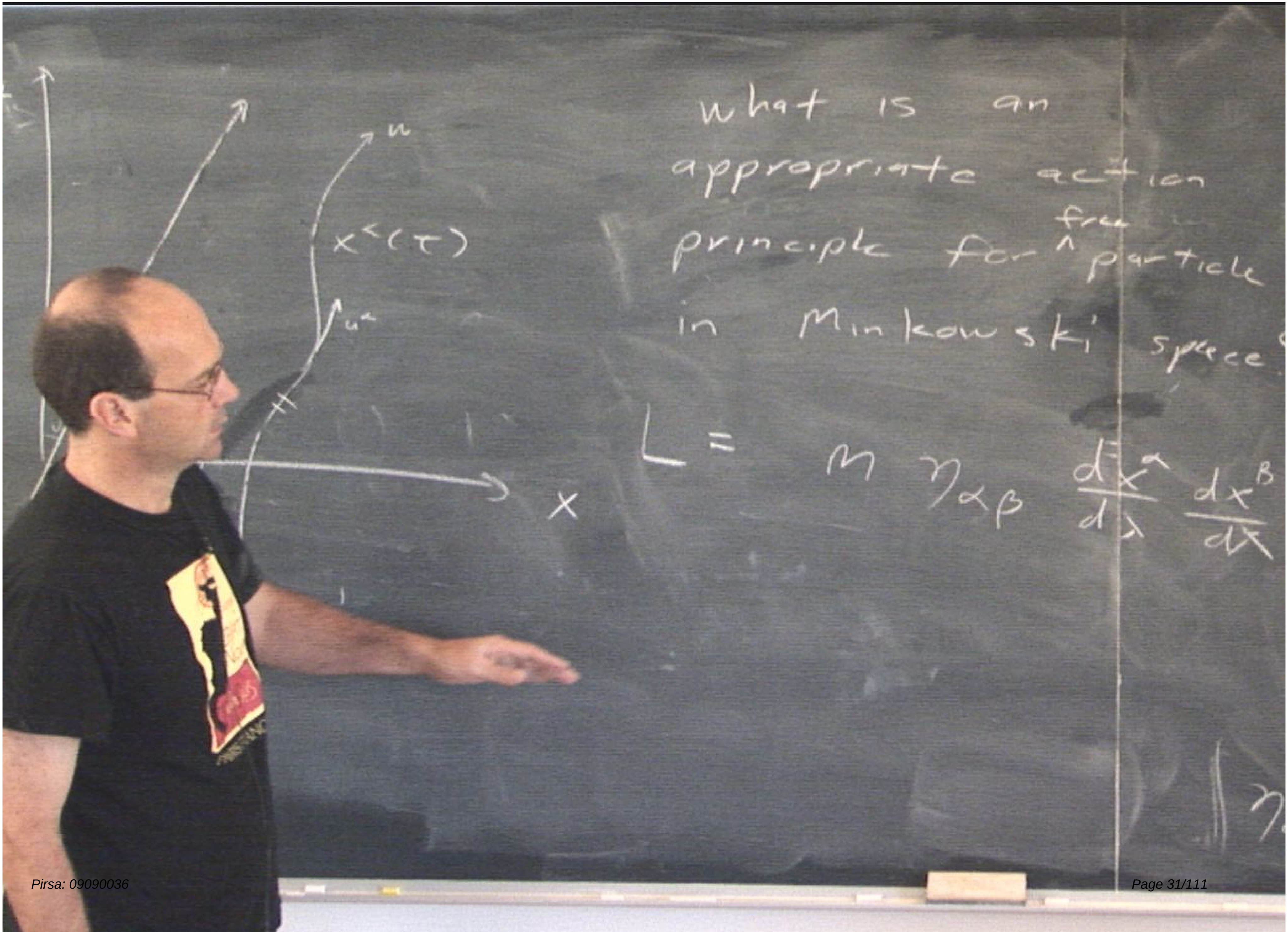
$$p^0 = \frac{1}{c}\left(\frac{m c^2}{\sqrt{1 - v^2/c^2}}\right) = \frac{1}{c}\left(m c^2 + \tfrac{1}{2} m v^2 + O\left(\frac{v^4}{c^2}\right)\right) = E/c$$

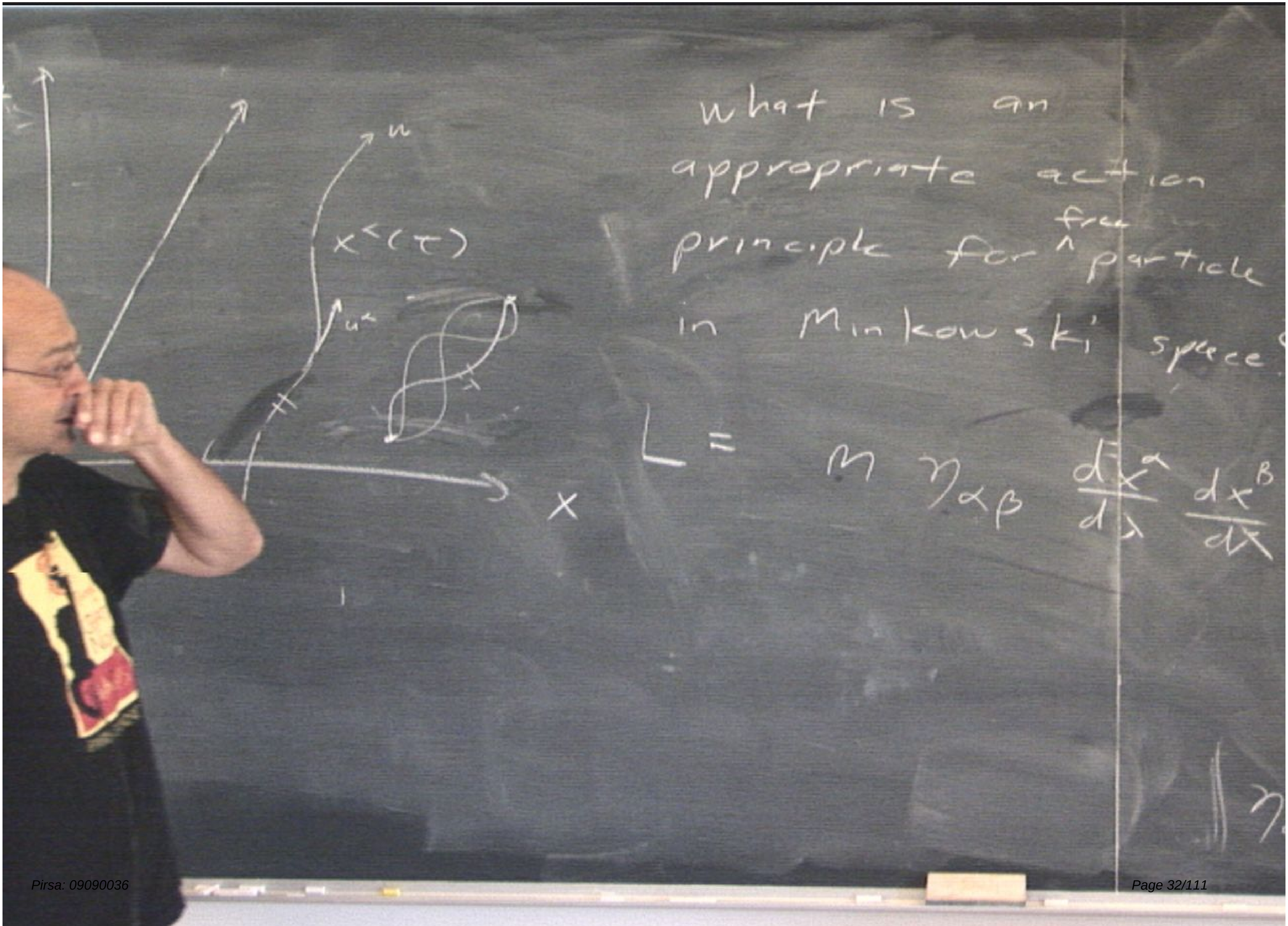
$$\eta_{\alpha\beta}\, p^\alpha p^\beta = -m^2 c^2$$


what is an
appropriate action
principle for free particle
x^a(τ)
u^a



what is an appropriate action principle for free particle in Minkowski space?


what is an appropriate action principle for free particle in Minkowski space
$x^\alpha(\tau)$
u
u^α
x
$L = m\,\eta_{\alpha\beta}\,\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}$


what is an
appropriate action
principle for free particle
in Minkowski space?
L = m η_αβ dx^α/dλ dx^β/dλ
x^α(τ)
u^α
x

x

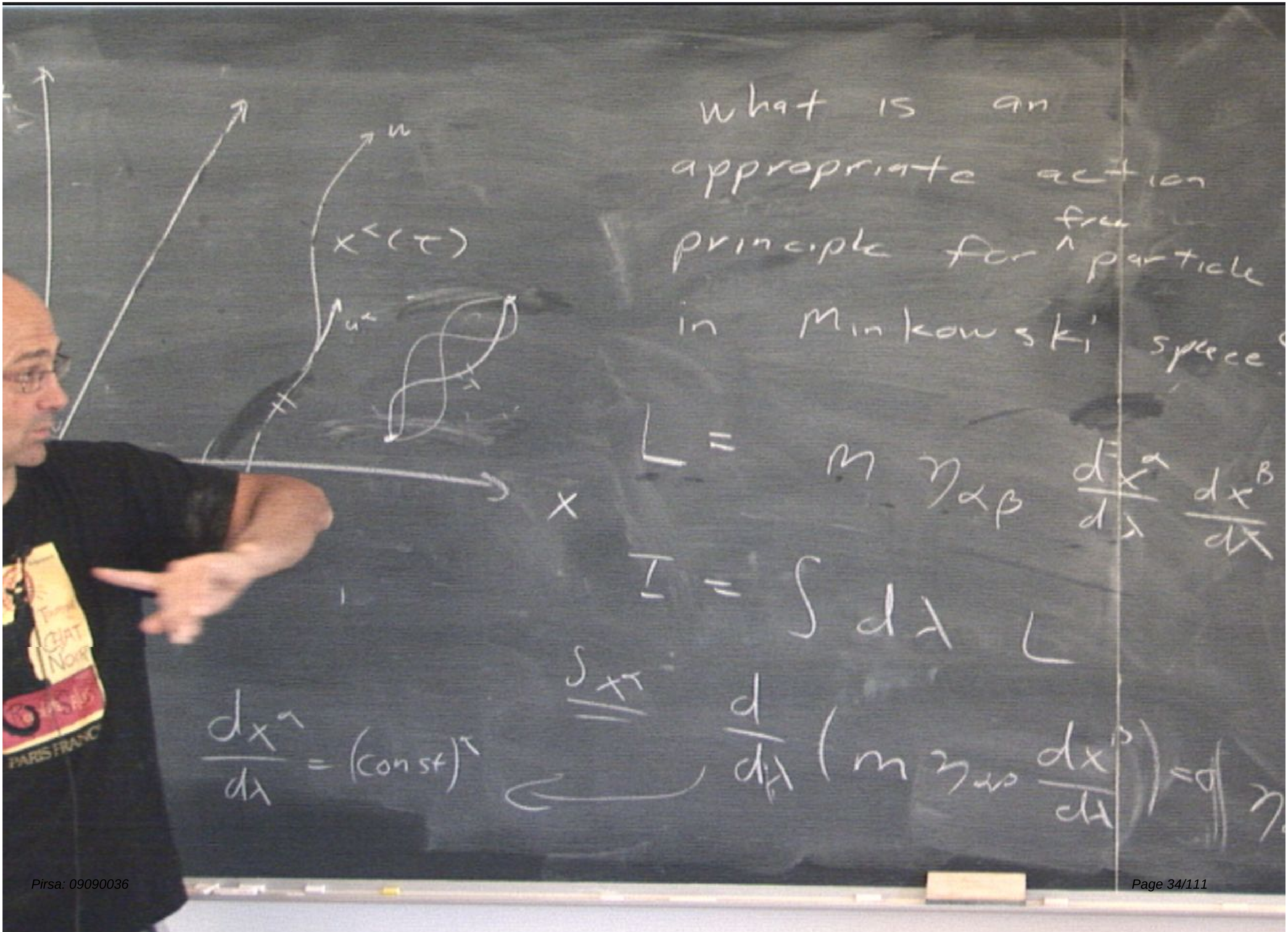
what is an appropriate action principle for free particle in Minkowski space

$$L = m\,\eta_{\alpha\beta}\,\frac{dx^\alpha}{d\lambda}\,\frac{dx^\beta}{d\lambda}$$

$$I = \int d\lambda\; L$$

$\underline{\delta x^\alpha}$

$$\frac{d}{d\lambda}\left(m\,\eta_{\alpha\beta}\,\frac{dx^\beta}{d\lambda}\right) = 0 \quad \eta$$


what is an
appropriate action
principle for free particle
in Minkowski space
$x^\alpha(\tau)$
u^α
x
$L = m\,\eta_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}$
$I = \int d\lambda\, L$
$\frac{dx^\alpha}{d\lambda} = (\mathrm{const})^\alpha$
$\frac{d}{d\lambda}\left(m\,\eta_{\alpha\beta}\frac{dx^\beta}{d\lambda}\right) = 0$

$$\boxed{\frac{dx^a}{d\tau} = (\mathrm{const})^a}$$

$$\eta_{\alpha\beta}\left(m\frac{dx^\alpha}{d\lambda}\right)\left(m\frac{dx^\beta}{d\lambda}\right) = \underline{\text{constant}}$$

scale $\lambda = \tau \implies -\underline{m^2 c^2}$

scale $\lambda = \tau \implies -m^2 c^2$

Observers + Observations

– observer carries a clock and rulers along world-line scale $\lambda = \tau \implies$

<u>Observers</u> + <u>Observations</u>

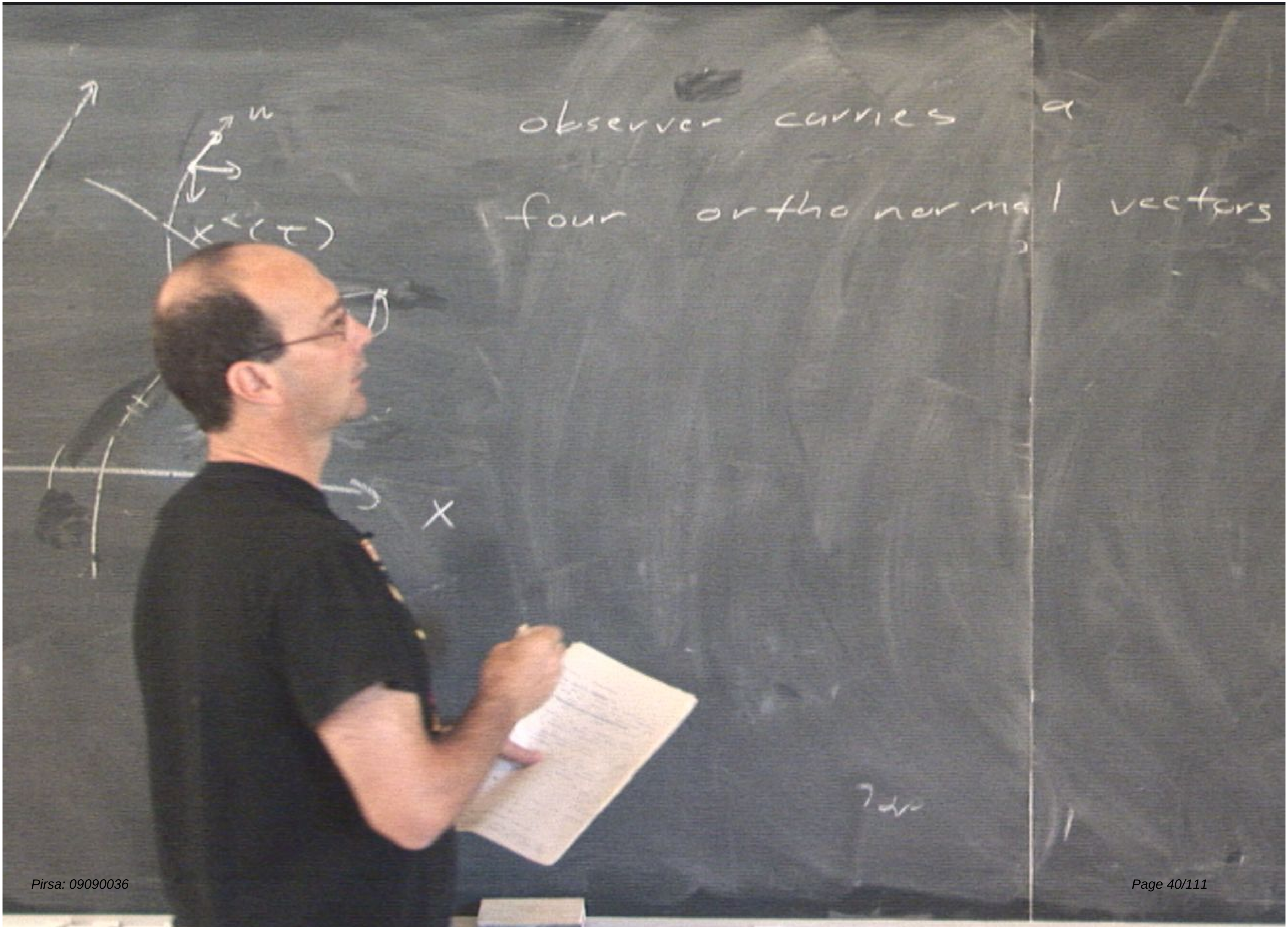
- observer carries a clock and set rulers along world-line
  ie, works in a local laboratory

scale $\lambda = \tau \Longrightarrow -m^2 c^2$

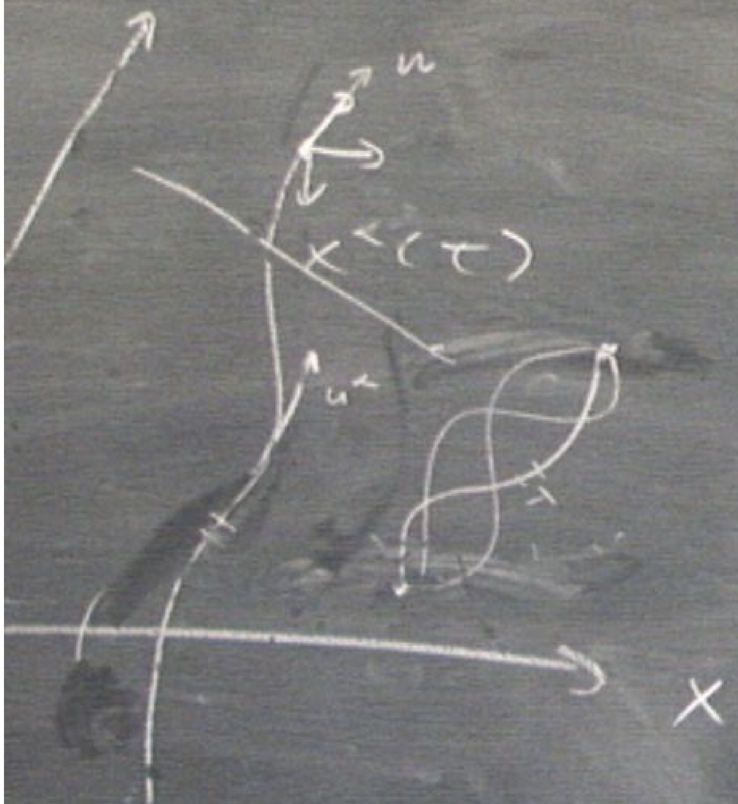
Observers + Observations

- observer carries a clock and set rulers along world-line

i.e. works in a local laboratory (idealizing that this lab infinitesimal)

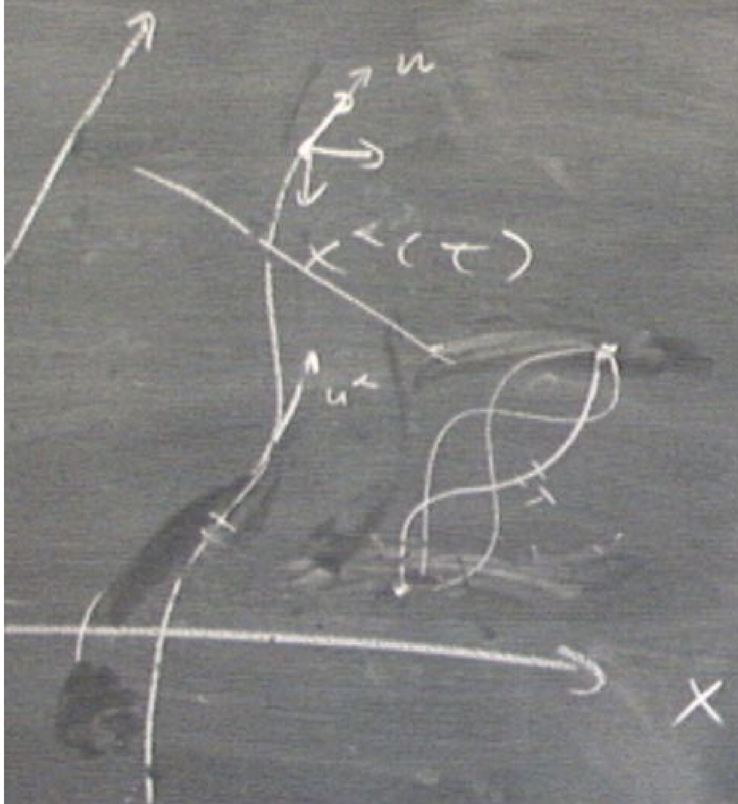


u
x^α(τ)
observer carries a
four orthonormal vectors
x

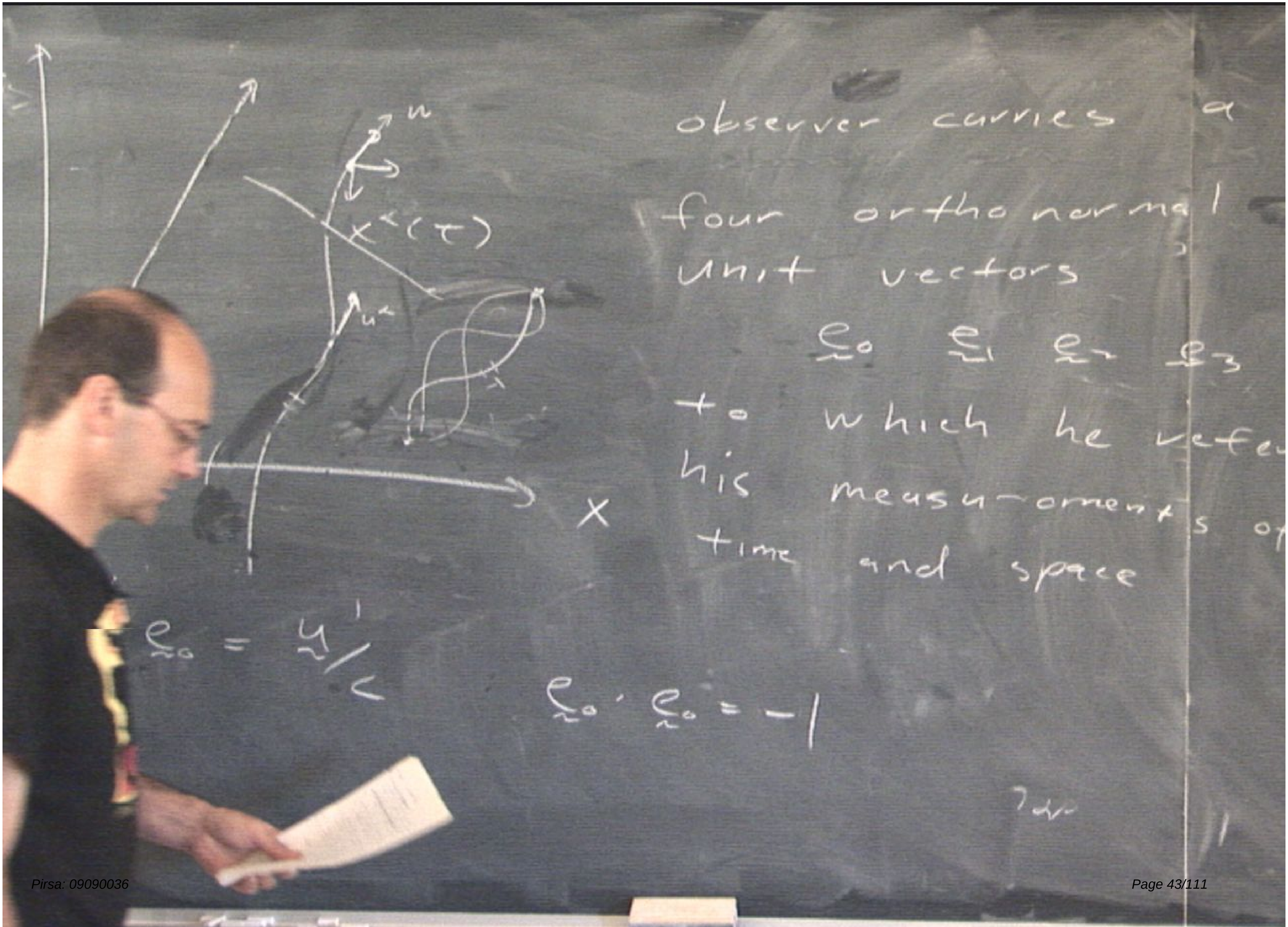


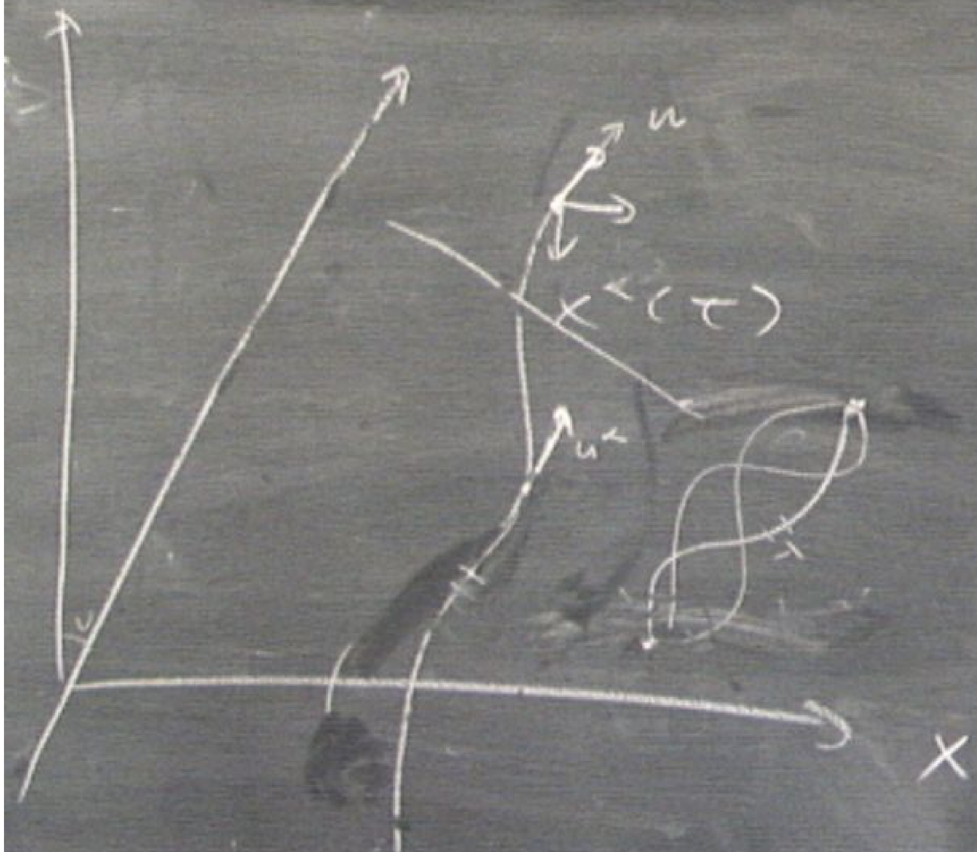
observer carries a
four orthonormal
unit vectors

$e_0 \quad e_1 \quad e_2 \quad e_3$



observer carries a
four orthonormal
unit vectors
e_0 e_1 e_2 e_3
to which he refers
his measurements of
time and space


observer carries a
four orthonormal
unit vectors
e_0 e_1 e_2 e_3
to which he refer
his measurements of
time and space
$x^\alpha(\tau)$
u^α
x
$e_0 = u/c$
$e_0 \cdot e_0 = -1$



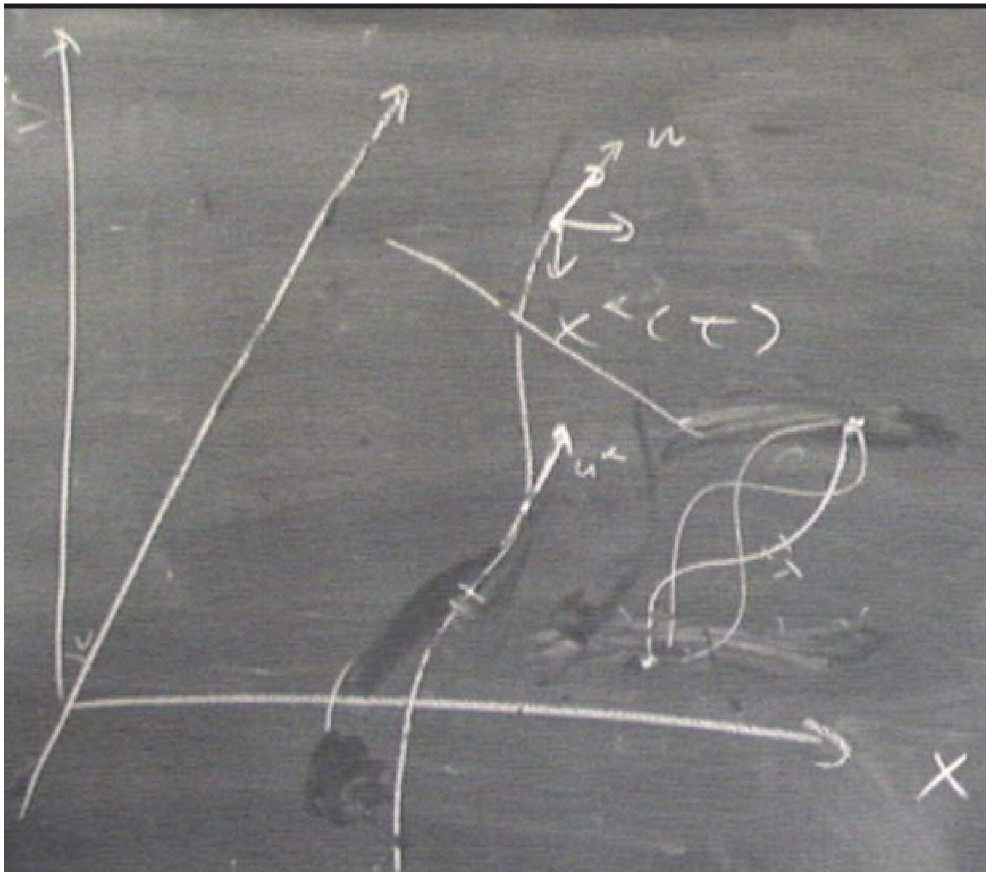
observer carries a
four orthonormal
unit vectors

$\underset{\sim}{e}_0$ $\underset{\sim}{e}_1$ $\underset{\sim}{e}_2$ $\underset{\sim}{e}_3$

to which he refe
his measurements of
time and space

$\underset{\sim}{e}_0 = \underset{\sim}{u}$

$\underset{\sim}{e}_0 \cdot \underset{\sim}{e}_0 = -1$ ← defines ha
his clock, ru



observer carries a
four orthonormal
unit vectors
$$\underset{\sim}{e}_0 \quad \underset{\sim}{e}_1 \quad \underset{\sim}{e}_2 \quad \underset{\sim}{e}_3$$
to which he refer[s]
his measurements of
time and space

$$\underset{\sim}{e}_0 = \underset{\sim}{u}/c$$

$$\underset{\sim}{e}_0 \cdot \underset{\sim}{e}_0 = -1 \longleftarrow$$ defines how his clock ru[ns]

for spatial vectors; $\underset{\sim}{e}_i \cdot \underset{\sim}{e}_j$

$$\underset{\sim}{e}_a \cdot \underset{\sim}{e}_b = \eta_{ab} = \eta_{\alpha\beta}\, e^{\alpha}{}_{a}\, e^{\beta}{}_{b}$$

$e^{\alpha}{}_a \leftarrow$ which vector

$$\underset{\sim}{e}_a \cdot \underset{\sim}{e}_b = \eta_{ab} = \eta_{\alpha\beta}\, e^{\alpha}{}_a \, e^{\beta}{}_b$$

$\alpha \leftarrow$ which component of the vector

$e_a \leftarrow$ which vector

$$\underset{\sim}{e}_a \cdot \underset{\sim}{e}_b = \eta_{ab} = \eta_{\alpha\beta}\, e^{\alpha}{}_{a}\, e^{\beta}{}_{b}$$

— any four vector along world-line can be expressed in terms of e_a

$\alpha \leftarrow$ which component of the vector

$e_a \leftarrow$ which vector

$$\underset{\sim}{e}_a \cdot \underset{\sim}{e}_b = \eta_{ab} = \eta_{\alpha\beta}\, e^\alpha{}_a\, e^\beta{}_b$$

— any four vector along world-line can be expressed in terms of $\underset{\sim}{e}_a$

$$\text{eg } \underset{\sim}{p} = p^a \underset{\sim}{e}_a$$

$\alpha \leftarrow$ which component of the vector

$e_a \leftarrow$ which vector

$$\underset{\sim}{e}_a \cdot \underset{\sim}{e}_b = \eta_{ab} = \eta_{\alpha\beta}\, e^{\alpha}{}_{a}\, e^{\beta}{}_{b}$$

— any four vector along world-line can be expressed in terms of $\underset{\sim}{e}_a$

$$\text{eg } \underset{\sim}{p} = p^a \underset{\sim}{e}_a$$ $\leftarrow$ implicit sum $a = 0, 1, 2, 3$

$\alpha \leftarrow$ which component of the vector

$e_a \leftarrow$ which vector

$$\underset{\sim}{e}_a \cdot \underset{\sim}{e}_b = \eta_{ab} = \eta_{\alpha\beta}\, e^{\alpha}{}_{a}\, e^{\beta}{}_{b}$$

— any four vector along world-line can be expressed in terms of $\underset{\sim}{e}_a$

eg $\underset{\sim}{p} = p^a \underset{\sim}{e}_a \leftarrow$ implicit sum $a = 0, 1, 2, 3$

measurements are expressed in terms of p^a

$\alpha \leftarrow$ which component of the vector

$e_a \leftarrow$ which vector

$$\underset{\sim}{e}_a \cdot \underset{\sim}{e}_b = \eta_{ab} = \eta_{\alpha\beta}\, e^{\alpha}{}_{a}\, e^{\beta}{}_{b}$$

— any four vector along world-line can be expressed in terms of $\underset{\sim}{e}_a$

$$\text{eg } \underset{\sim}{p} = p^a \underset{\sim}{e}_a \leftarrow \text{implicit sum } a = 0,1,2,3$$

measurements are expressed in terms of $\underline{p^a}$

g recall $\underset{\sim}{p} = (E/c, \vec{p})$

$\alpha \leftarrow$ which component of the vector

$e_a \leftarrow$ which vector

$$\underset{\sim}{e}_a \cdot \underset{\sim}{e}_b = \eta_{ab} = \eta_{\alpha\beta}\, e^{\alpha}{}_a\, e^{\beta}{}_b$$

– any four vector along world-line can be expressed in terms of $\underset{\sim}{e}_a$

eg $\underset{\sim}{p} = p^a \underset{\sim}{e}_a \leftarrow$ implicit sum $a = 0, 1, 2, 3$

measurements are expressed in terms of $\underline{p}^a$

eg recall $\underset{\sim}{p} = (E/c, \vec{p})$

observer measures $E = c\,p^0 = -c\,\underset{\sim}{p} \cdot \underset{\sim}{e}_0$

$\alpha \leftarrow$ which component of the vector

$e_a \leftarrow$ which vector

$$\underset{\sim}{e}_a \cdot \underset{\sim}{e}_b = \eta_{ab} = \eta_{\alpha\beta}\, e^{\alpha}{}_{a}\, e^{\beta}{}_{b}$$

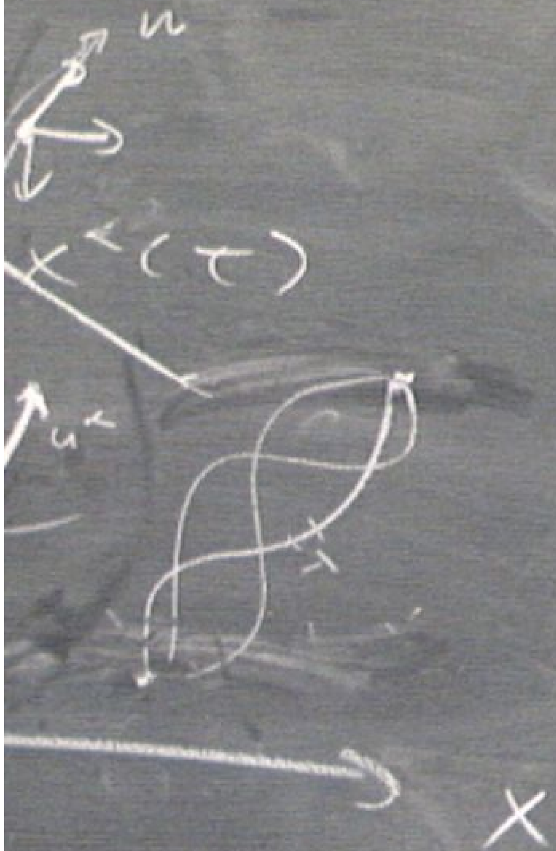
– any four vector along world-line can be expressed in terms of $\underset{\sim}{e}_a$

$$\text{eg } \underset{\sim}{p} = p^a \underset{\sim}{e}_a \leftarrow \text{implicit sum } a = 0,1,2,3$$

measurements are expressed in terms of $\underline{p}^a$

eg recall $\underset{\sim}{p} = (E/c, \vec{p})$

observer measures $E = c p^0 = -c\, \underset{\sim}{p} \cdot \underset{\sim}{e}_0$



$$E = -c\, \underset{\sim}{P} \cdot \underset{\sim}{e}_0$$

projection of $\underset{\sim}{P}$ on time basis vector $\underset{\sim}{e}_0$

$$\underset{\sim}{e}_0 \cdot \underset{\sim}{e}_0 = -1 \quad \longleftarrow \text{defines how}$$

$$E = -c\, \underset{\sim}{p} \cdot \underset{\sim}{e}_0$$

projection of $\underset{\sim}{p}$

on time basis vector $\underset{\sim}{e}_0$

$$E = -\underset{\sim}{p} \cdot \underset{\sim}{u}$$

x

accelerating observer

consider trajectory

$t(\lambda)$

e_0

s vector e_0

u s

f

es then

clock run

$\underset{\sim}{e}_0$

$\underset{\sim}{p}$

s vector $\underset{\sim}{e}_0$

$\underset{\sim}{u}$ s

f

times then

s clock run

accelerating observer

consider trajectory

$$t(\lambda) = \frac{1}{a} \sinh \lambda$$

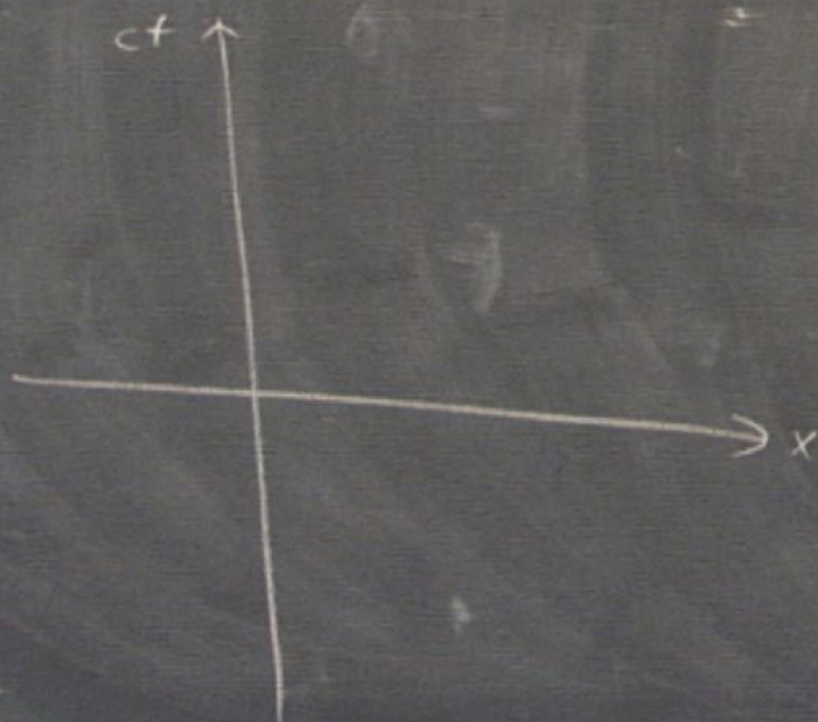
$$x(\lambda) = \frac{1}{a} \cosh \lambda$$

accelerating observer

consider trajectory

$$t(\lambda) = \frac{1}{a} \sinh \lambda$$

$$x(\lambda) = \frac{1}{a} \cosh \lambda$$



$\underset{\sim}{e}_0$

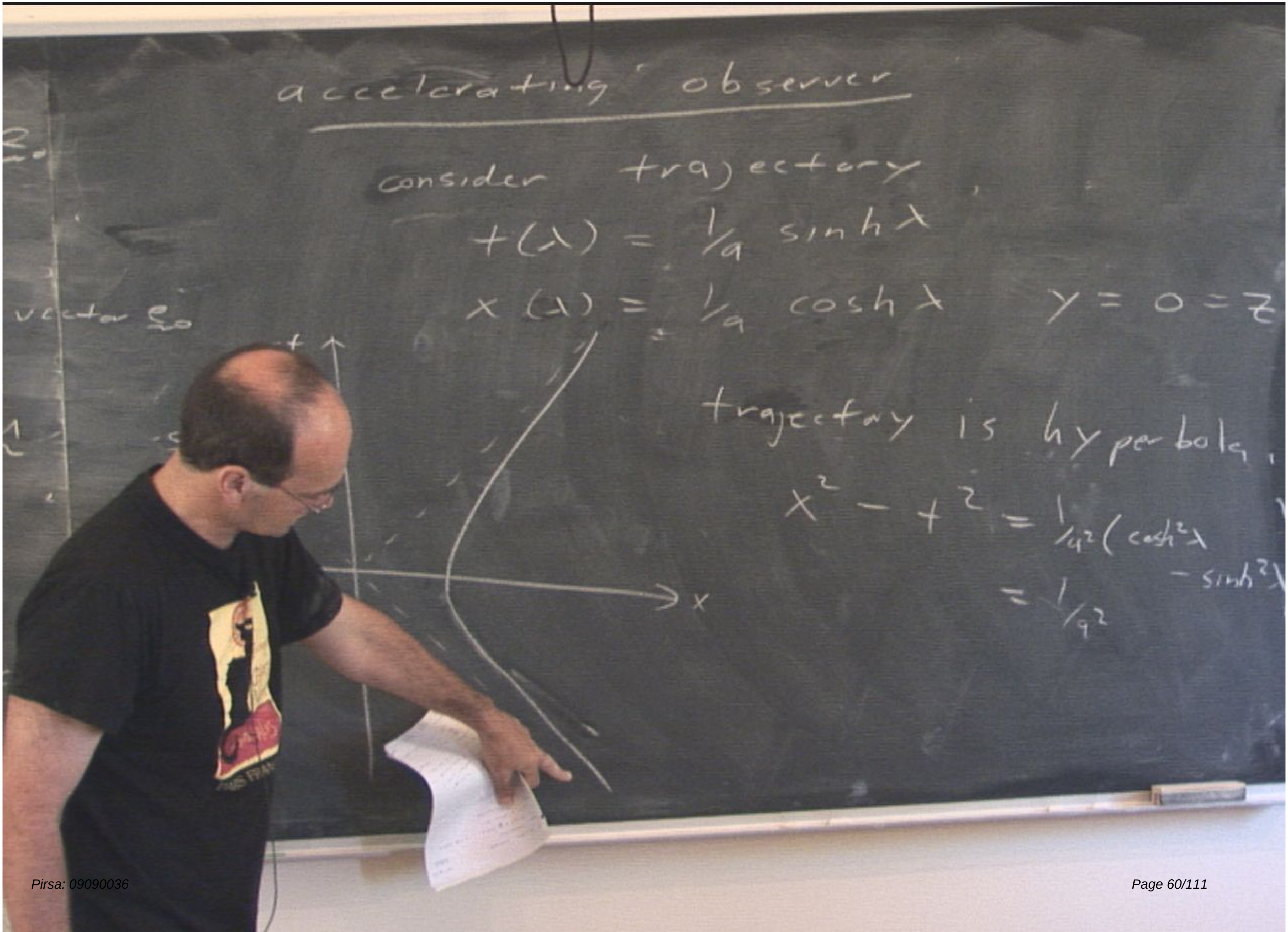
s vector $\underset{\sim}{e}_0$

$\underset{\sim}{u}$ s

t

fines [illegible]

s clock run


accelerating observer
consider trajectory
$t(\lambda) = 1/a \sinh\lambda$
$x(\lambda) = 1/a \cosh\lambda$
$y = 0 = z$
vector e_0
t
x
trajectory is hyperbola
$x^2 - t^2 = 1/a^2(\cosh^2\lambda - \sinh^2\lambda)$
$= 1/a^2$

$\underset{\sim}{e}_0$

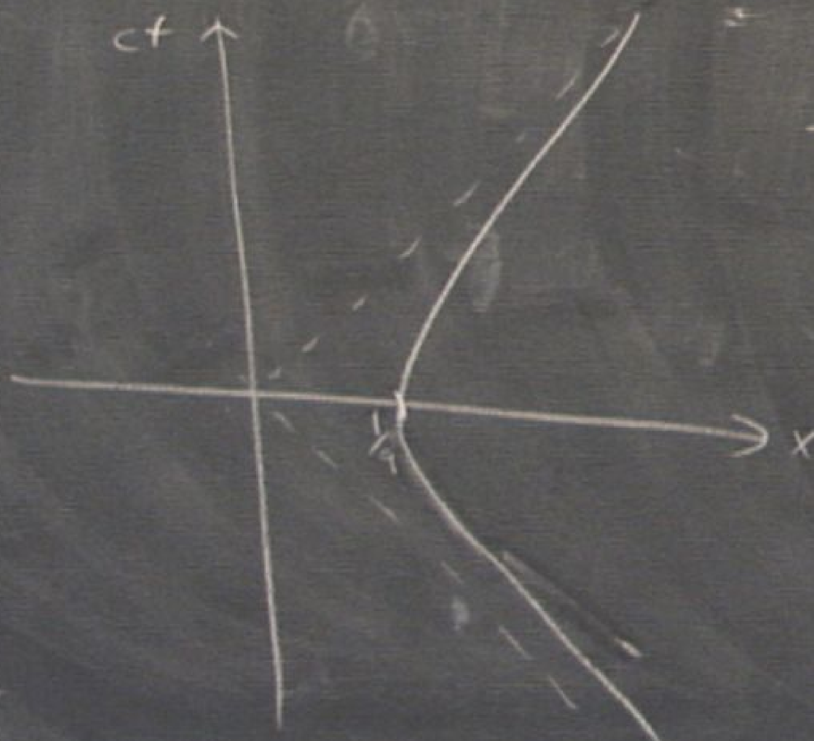
vector $\underset{\sim}{e}_0$

accelerating observer

consider trajectory

$$t(\lambda) = \frac{1}{a}\sinh\lambda$$

$$x(\lambda) = \frac{1}{a}\cosh\lambda \qquad y = 0 = z$$



trajectory is hyperbola,

$$x^2 - t^2 = \frac{1}{a^2}\left(\cosh^2\lambda - \sinh^2\lambda\right) = \frac{1}{a^2}$$

times than

clock run

$\underset{\sim}{e}_0$

vector $\underset{\sim}{e}_0$

$\underset{\sim}{\Lambda}$ ·s

t

times than

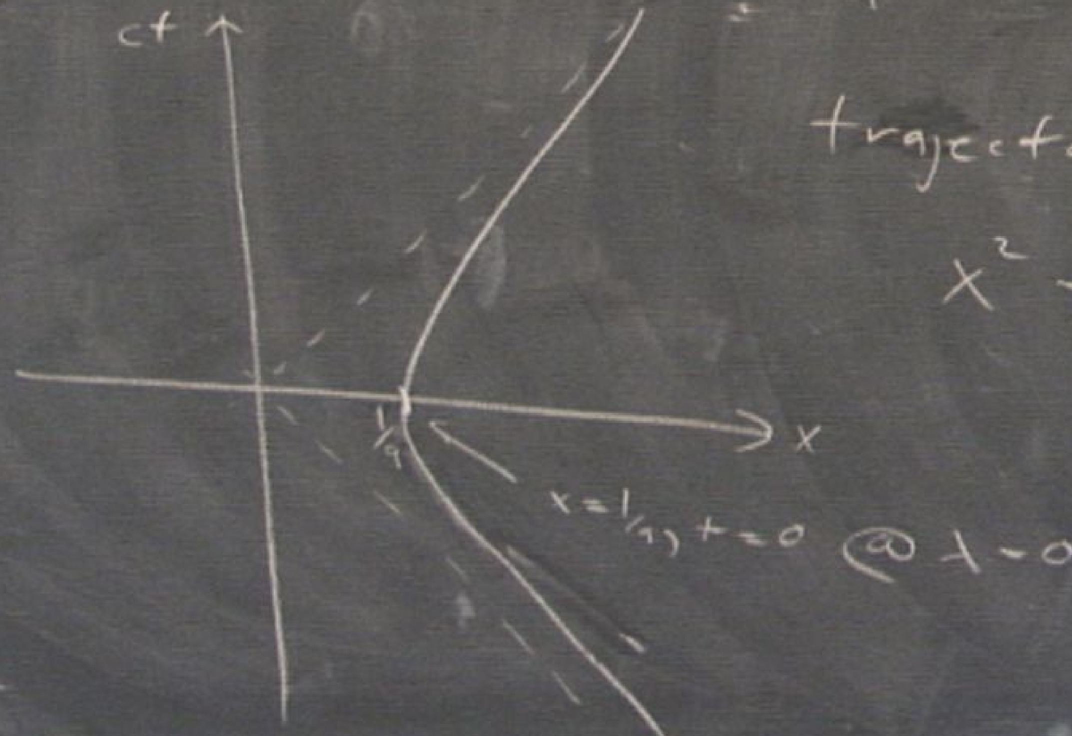
clock run

accelerating observer

consider trajectory

$$t(\lambda) = \tfrac{1}{a} \sinh\lambda$$

$$x(\lambda) = \tfrac{1}{a} \cosh\lambda \qquad y = 0 = z$$



trajectory is hyperbola

$$x^2 - t^2 = \tfrac{1}{a^2}(\cosh^2\lambda - \sinh^2\lambda)$$

$$= \tfrac{1}{a^2}$$

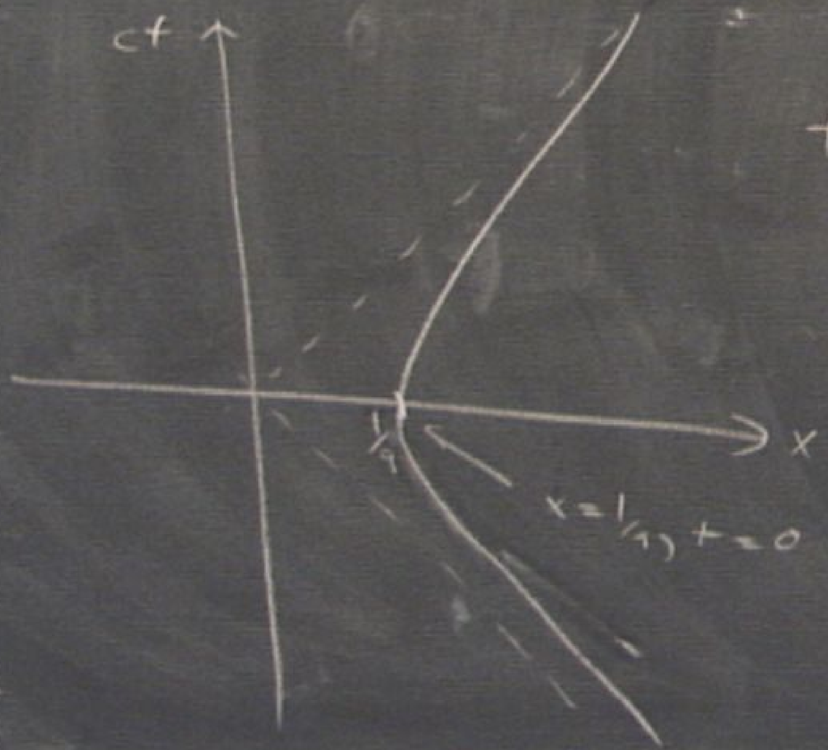
accelerating observer

consider trajectory

$$t(\lambda) = \frac{1}{a} \sinh \lambda$$

$$x(\lambda) = \frac{1}{a} \cosh \lambda \qquad y = 0 = z$$

vector e_0



$x = 1/a,\ t = 0$ @ $\lambda = 0$

trajectory is hyperbola,

$$x^2 - t^2 = \frac{1}{a^2}(\cosh^2\lambda - \sinh^2\lambda) = \frac{1}{a^2}$$

← curved trajectory hence accelerating

s

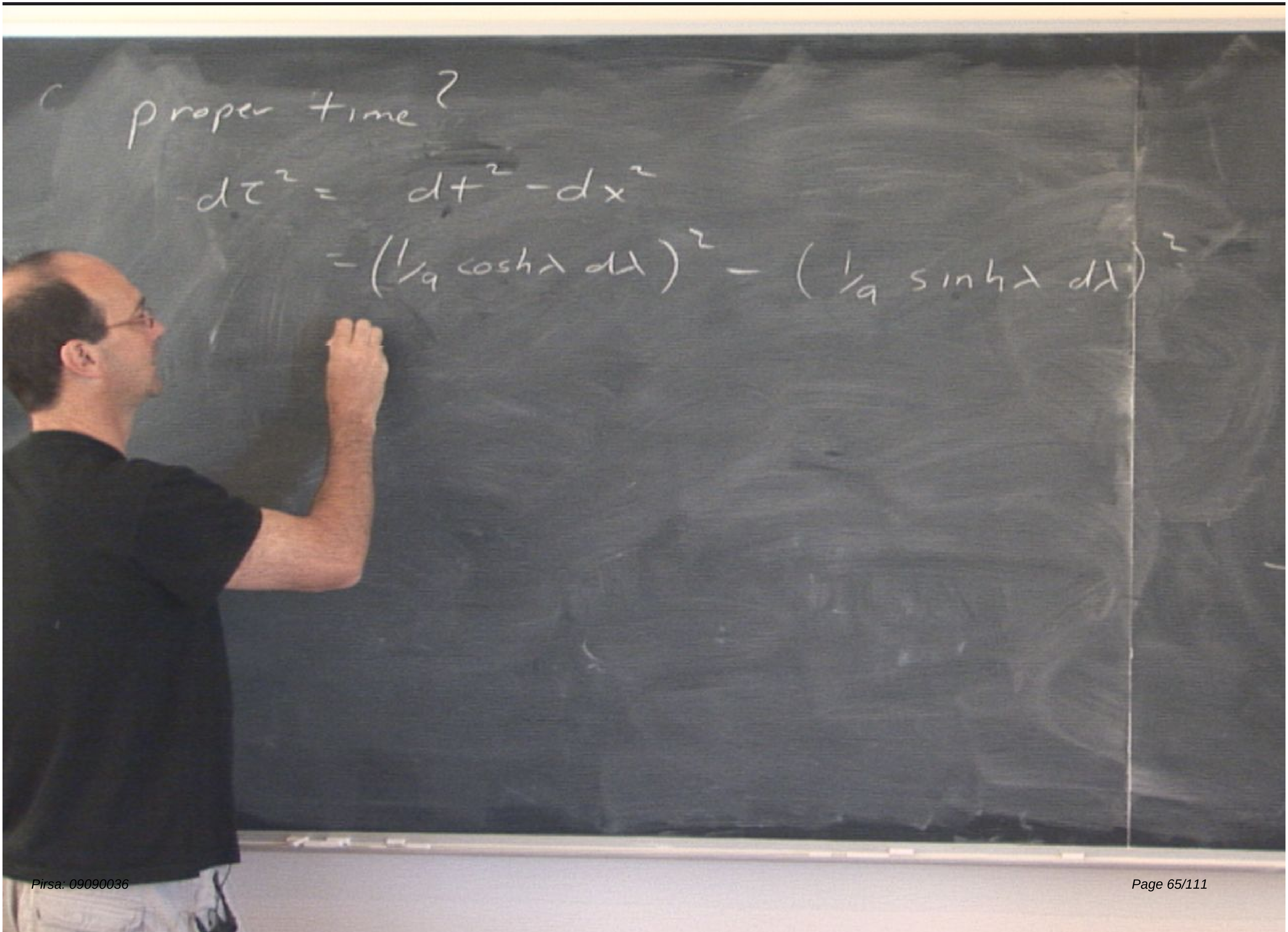
t

times han

clock run

proper time?

$$d\tau^2 = dt^2 - dx^2$$


proper time?
$d\tau^2 = dt^2 - dx^2$
$= (\frac{1}{a}\cosh\lambda\, d\lambda)^2 - (\frac{1}{a}\sinh\lambda\, d\lambda)^2$

proper time?

$$-d\tau^2 = dt^2 - dx^2$$

$$= \left(\tfrac{1}{a}\cosh\lambda \, d\lambda\right)^2 - \left(\tfrac{1}{a}\sinh\lambda \, d\lambda\right)^2$$

$$= \tfrac{1}{a^2}\, d\lambda^2$$

$$d\tau = \pm \tfrac{1}{a}\, d\lambda$$

proper time?

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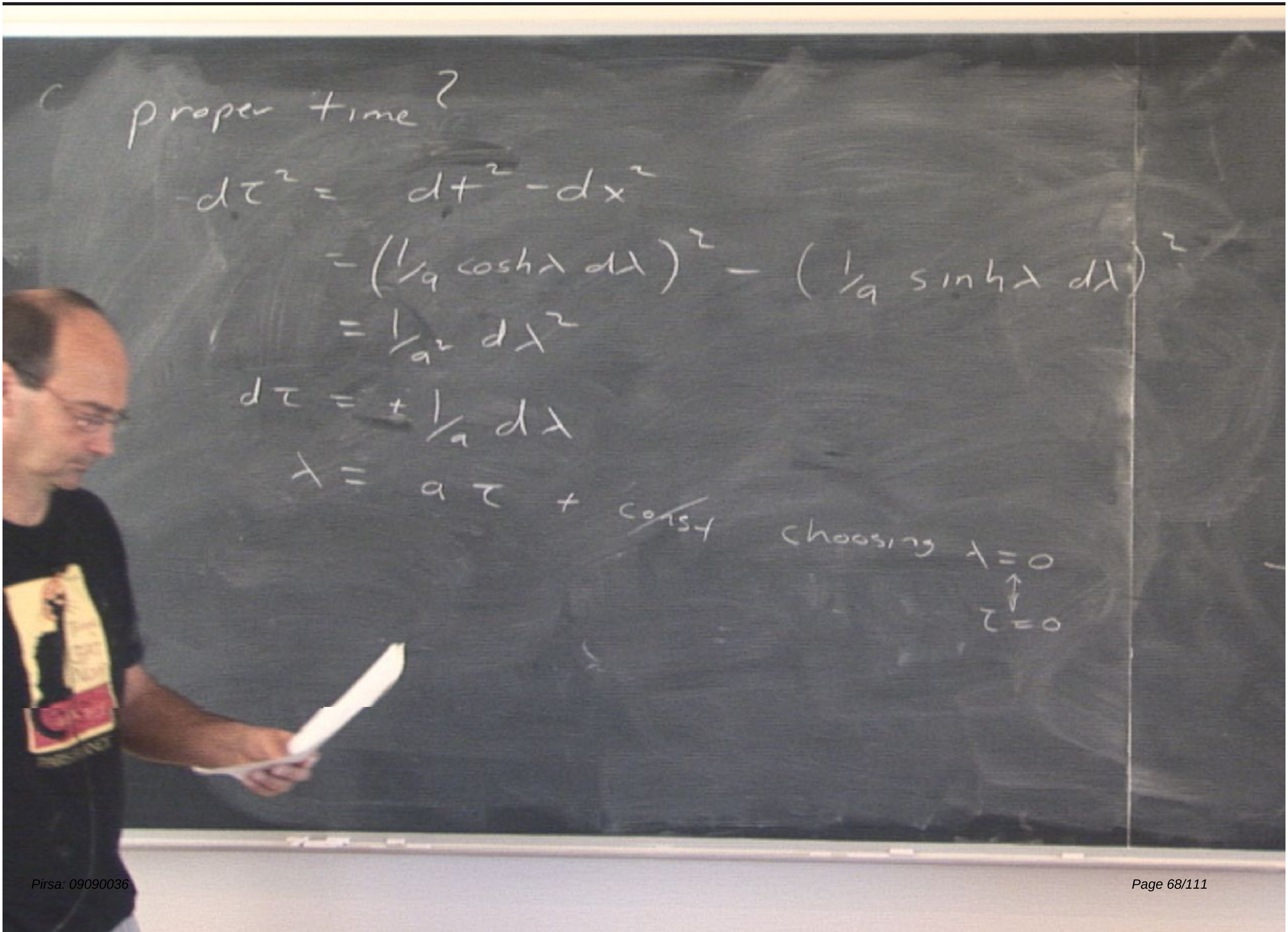
$$d\tau = \pm \tfrac{1}{a}\, d\lambda$$

$$\lambda = a\tau + \cancel{\text{const}}$$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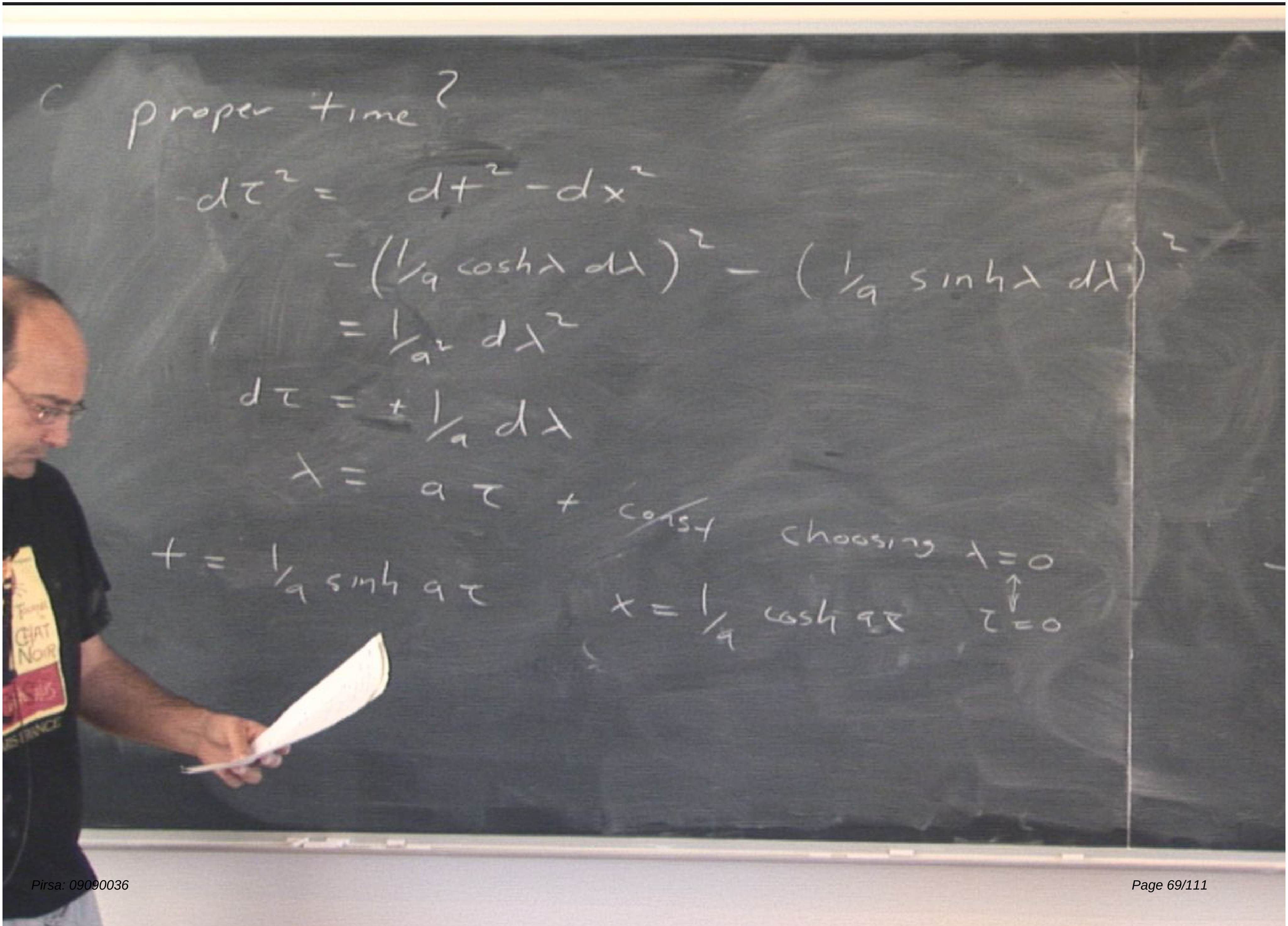
choosing $\lambda = 0$
$\updownarrow$
$\tau = 0$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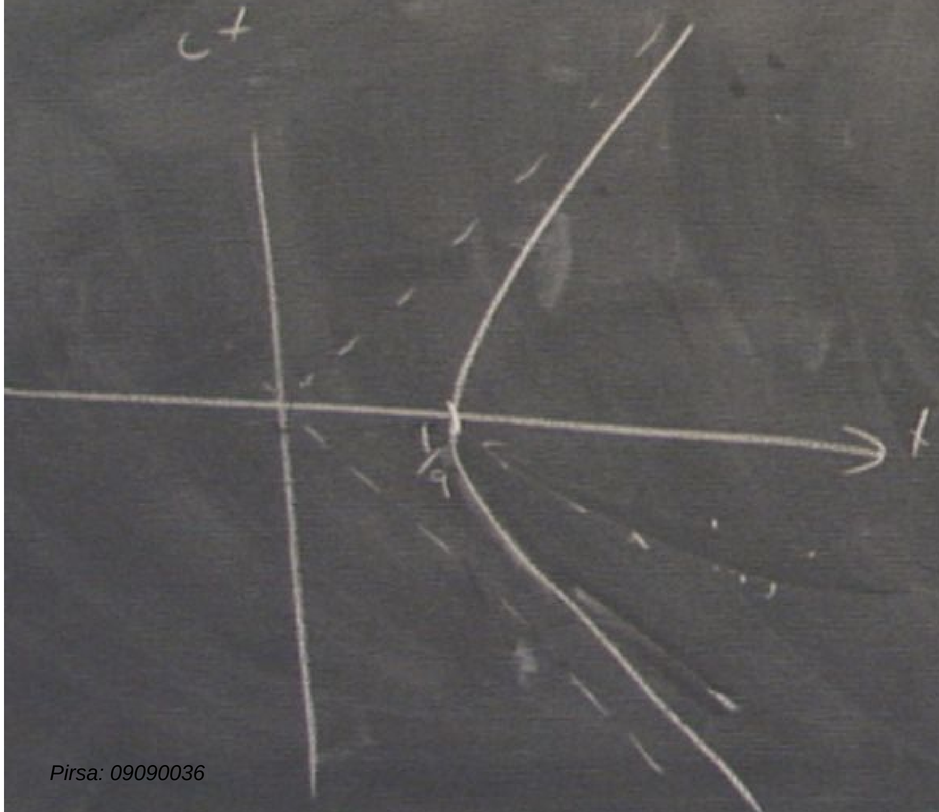
choosing $\lambda = 0 \leftrightarrow \tau = 0$
$t = \frac{1}{a}\sinh a\tau$
$x = \frac{1}{a}\cosh a\tau$

accelerating observer

four velocity

$$u^\alpha = \frac{dx^\alpha}{d\tau}$$

$$= ($$



← curved trajectory hence accelerating

"accelerating" observer

four velocity

$$u^\alpha = \frac{dx^\alpha}{d\tau}$$

$$= (\cosh a\tau,\ \sinh a\tau,\ 0,\ 0)$$

← curved trajectory
hence accelerating

accelerating observer

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three velocity

$$\frac{dx}{dt}$$

← curved trajectory
hence accelerating

"accelerating" observer

four velocity

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three velocity

$$\frac{dx}{dt} = \frac{u^x}{u^0} = \frac{dx/d\tau}{dt/d\tau}$$

$$=$$

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$$= \tanh a\tau$$

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$$= \tanh a\tau$$

$$= 0 \text{ at } \tau = 0$$

$$\Rightarrow \qquad \tau \to \pm\infty$$

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$= 0$ at $\tau = 0$

$\longrightarrow \pm 1$ at $\tau \longrightarrow$

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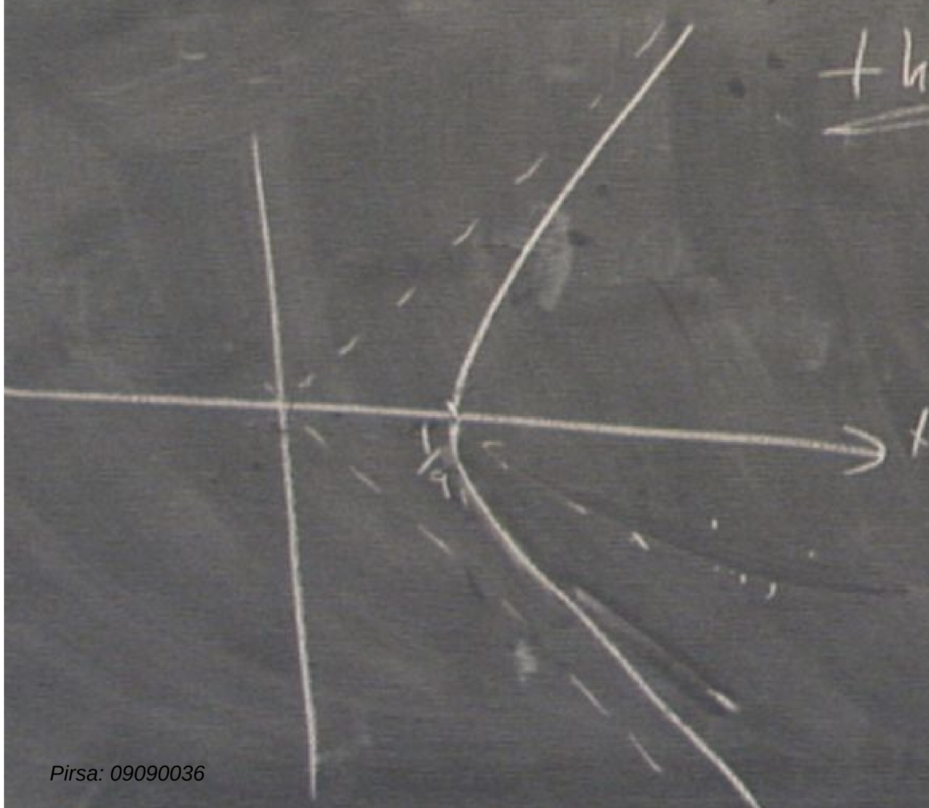

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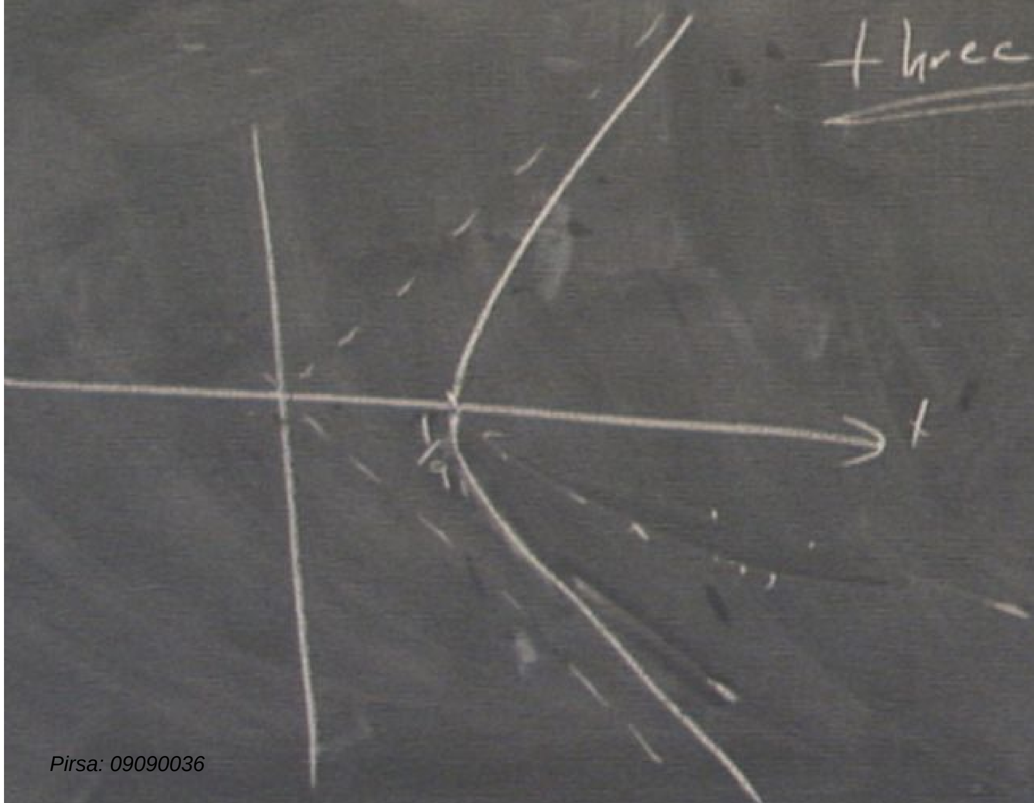

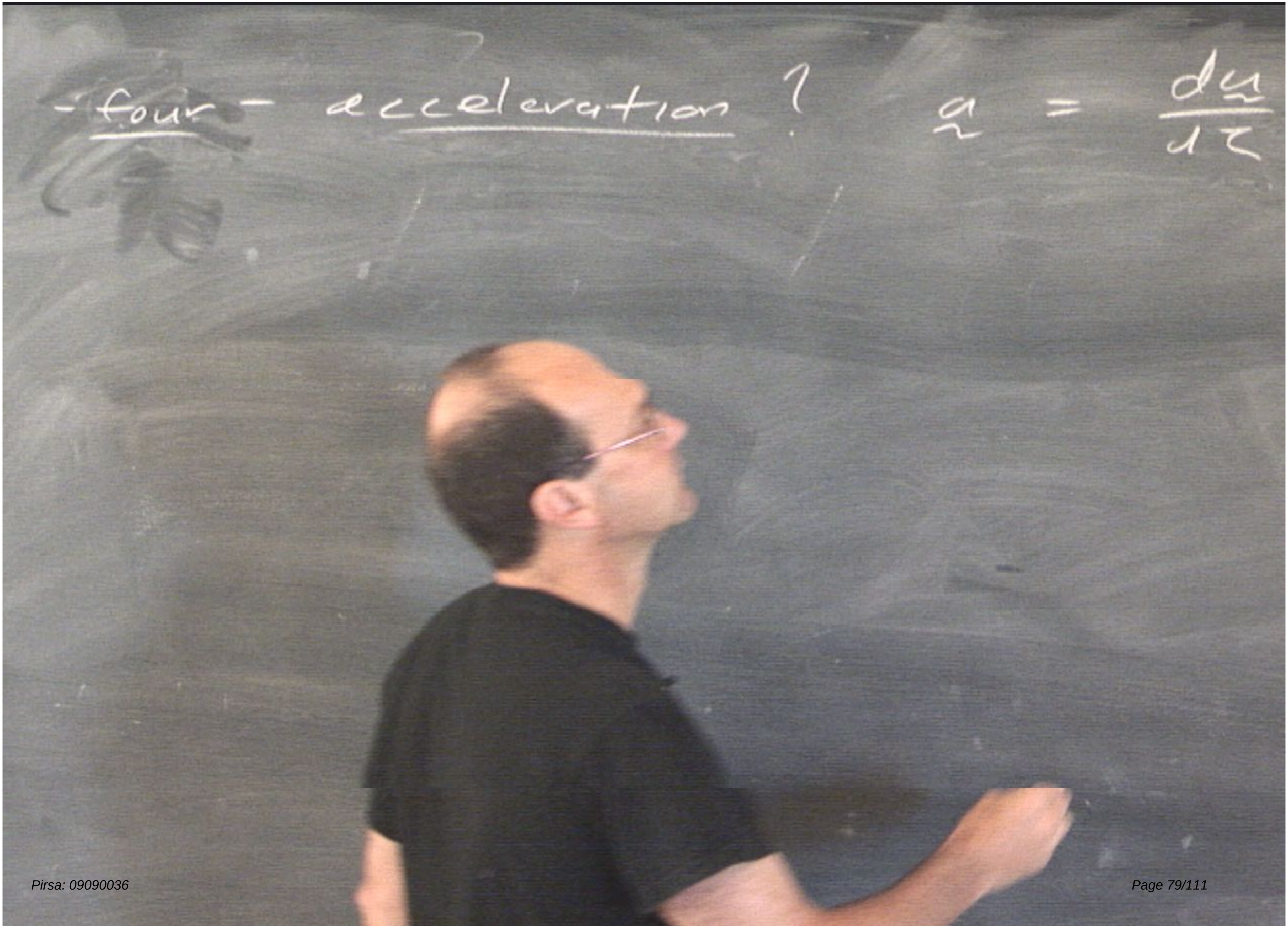
four - acceleration ?
a = du / dτ

acceleration ? $\underset{\sim}{a} = \frac{d\underset{\sim}{u}}{d\tau} = \frac{d^2\underset{\sim}{x}}{d\tau^2}$

four-acceleration ? $\underset{\sim}{a} = \frac{d\underset{\sim}{u}}{d\tau} = \frac{d^2\underset{\sim}{x}}{d\tau^2}$

naturally appears relativistic version of the second law:

$$m \frac{d\underset{\sim}{u}}{d\tau}$$

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$$\underset{\sim}{a} \cdot \underset{\sim}{a} =$$

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$\longrightarrow$ relativistic notation

$$\underset{\sim}{a} = (a \sinh a\tau, a \cosh a\tau, 0, 0)$$

$$\underset{\sim}{a} \cdot \underset{\sim}{a} = +a^2 \longleftarrow \underline{\text{constant}}$$

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accelerating observer

$\left(\quad \underset{\sim}{u} \cdot \underset{\sim}{u} = -1 \right.$

accelerating observer

$$\underset{\sim}{u} \cdot \underset{\sim}{u} = -1$$

$$\frac{d}{d\tau}(\underset{\sim}{u} \cdot \underset{\sim}{u}) = 0$$

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accelerating observer

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$$\frac{d}{d\tau}\left(\underset{\sim}{u} \cdot \underset{\sim}{u}\right) = 0$$

$$\underset{\sim}{u} \cdot \underset{\sim}{a} = 0 \longleftarrow \underline{\text{general}}$$

accelerating observer

$$\left[\begin{array}{l} \underset{\sim}{u} \cdot \underset{\sim}{u} = -1 \\ \frac{d}{d\tau}(\underset{\sim}{u} \cdot \underset{\sim}{u}) = 0 \\ \underset{\sim}{u} \cdot \underset{\sim}{a} = 0 \end{array}\right.$$

$\longleftarrow$ general

$$\underset{\sim}{u} \cdot \underset{\sim}{a} = (\cosh a\tau)(a \sinh a\tau) + (\sinh a\tau)$$

accelerating observer

$$\underset{\sim}{u} \cdot \underset{\sim}{u} = -1$$

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$$\underset{\sim}{u} \cdot \underset{\sim}{a} = 0 \quad \longleftarrow \text{general}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{a} = -(\cosh a\tau)(a \sinh a\tau) + (\sinh a\tau)(a \cosh a\tau)$$

accelerating observer

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$$\frac{d}{d\tau}(\underset{\sim}{u} \cdot \underset{\sim}{u}) = 0$$

$$\underset{\sim}{u} \cdot \underset{\sim}{a} = 0 \quad \leftarrow \text{general}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{a} = -(\cosh a\tau)(a \sinh a\tau) + (\sinh a\tau)(a \cosh a\tau)$$

$$= 0$$

Orthonormal basis?

$$\underset{\sim}{e}_0 = \underset{\sim}{u}$$

Orthonormal basis?

$$\underset{\sim}{e}_0 = \underset{\sim}{u} = (\cosh a\tau, \sinh a\tau, 0, 0)$$

$y, z.$ $$\underset{\sim}{e}_2 = (0, 0, 1, 0)$$

$$\underset{\sim}{e}_3 = (0, 0, 0, 1)$$

Orthonormal basis ?

time $\underset{\sim}{e}_0 = \underset{\sim}{u} = (\cosh a\tau, \sinh a\tau, 0, 0)$

y, z. $\underset{\sim}{e}_2 = (0, 0, 1, 0)$

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x direction $\underset{\sim}{e}_1 = 1/a \, \underset{\sim}{a}$

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$\Uparrow$

$\underset{\sim}{e}_1 \cdot \underset{\sim}{e}_0 = 0$

$\underset{\sim}{e}_1 \cdot \underset{\sim}{e}_1 = 1$

Orthonormal basis ?

time $\underset{\sim}{e}_0 = \underset{\sim}{u} = (\cosh a\tau, \sinh a\tau, 0, 0)$

y, z. $\underset{\sim}{e}_2 = (0, 0, 1, 0)$

$\underset{\sim}{e}_3 = (0, 0, 0, 1)$

x direction $\underset{\sim}{e}_1 = \frac{1}{a}\,\underset{\sim}{a} = (\sinh a\tau, \cosh a\tau, 0, 0)$

$\Uparrow$

$\underset{\sim}{e}_1 \cdot \underset{\sim}{e}_0 = 0$

$\underset{\sim}{e}_1 \cdot \underset{\sim}{e}_1 = 1$

measurement imagine there is a source of photons at $x = 0$

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source of photons at $x = 0$

constantly emits with (angular)

frequency ω_*

measurement imagine there is a source of photons at $x = 0$ constantly emits with (angular) frequency ω_*

what frequency does accelerating observer measure?

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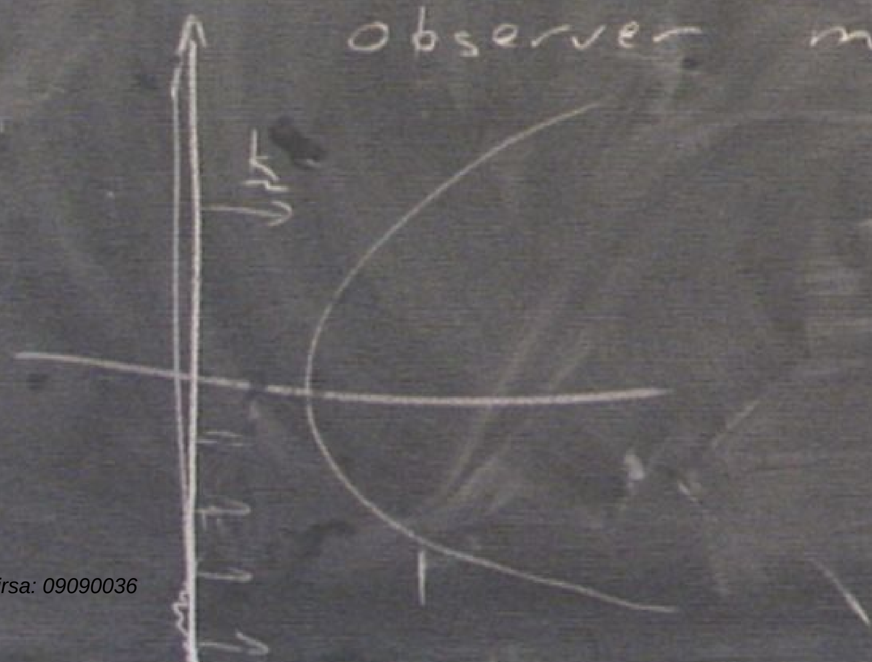

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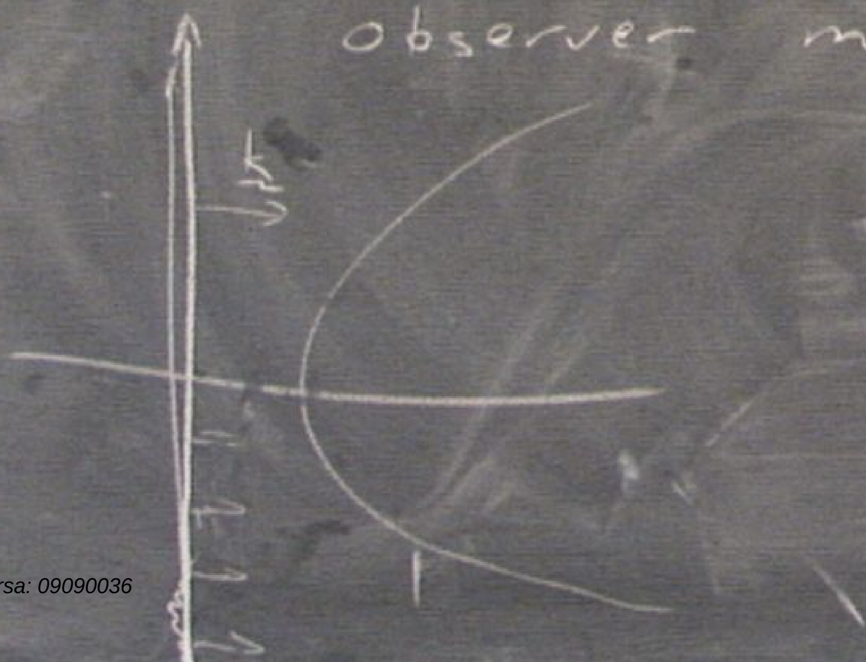

$\omega_0 = -k \cdot e_0$

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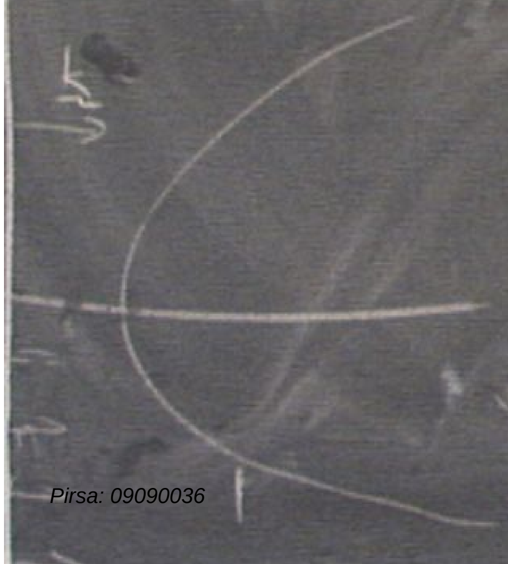

$$\omega_0 = -\underset{\sim}{k} \cdot \underset{\sim}{e}_0 = -\underset{\sim}{k} \cdot \underset{\sim}{u}$$

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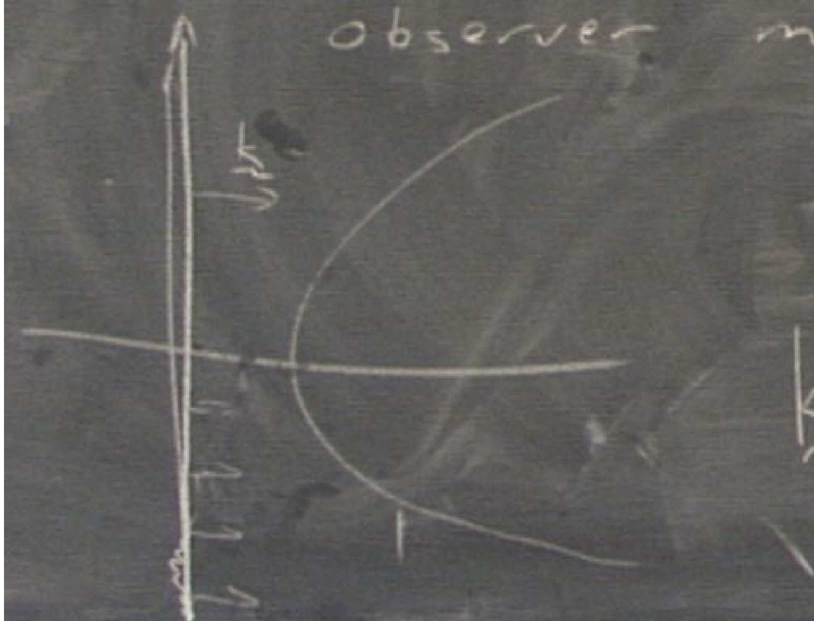

$$\omega_0 = -\underset{\sim}{k}\cdot \underset{\sim}{e}_0 = -\underset{\sim}{k}\cdot \underset{\sim}{u}$$

$$\underset{\sim}{k} = (\omega_*, \underset{\overset{\parallel}{\omega_*}}{k_y}, 0, 0) \leftarrow \boxed{\underset{\sim}{k}\cdot\underset{\sim}{k} = 0}$$

orthonormal basis?

time $\underset{\sim}{e}_0 = \underset{\sim}{u} = (\cosh a\tau, \sinh a\tau, 0, 0)$

$\omega_0 = -\underset{\sim}{k} \cdot \underset{\sim}{u}$

$= -[-\omega_* \cdot \cosh a\tau + \omega_* \sinh a\tau$

$= \omega_* (\cosh a\tau - \sinh a\tau)$

$=$

orthonormal basis?

time $\underset{\sim}{e}_0 = \underset{\sim}{u} = (\cosh a\tau, \sinh a\tau, 0, 0)$

$$\begin{aligned}
\omega_0 &= -\underset{\sim}{k} \cdot \underset{\sim}{u} \\
&= -\left[-\omega_* \cdot \cosh a\tau + \omega_* \sinh a\tau\right] \\
&= \omega_* (\cosh a\tau - \sinh a\tau) \\
&= \omega_* e^{-a\tau}
\end{aligned}$$

orthonormal basis?

time $\underset{\sim}{e}_0 = \underset{\sim}{u} = (\cosh a\tau, \sinh a\tau, 0, 0)$

$$\omega_0 = -\underset{\sim}{k}\cdot\underset{\sim}{u}$$
$$= -\left[-\omega_* \cosh a\tau + \omega_* \sinh a\tau\right]$$
$$= \omega_* (\cosh a\tau - \sinh a\tau)$$
$$= \omega_* e^{-a\tau} \rightarrow \omega_0 = \omega_* \text{ @ } \tau = 0$$

orthonormal basis ?

$\underset{\sim}{e}_0 = \underset{\sim}{u} = (\cosh a\tau, \sinh a\tau, 0, 0)$

$\omega_0 = - \underset{\sim}{k} \cdot \underset{\sim}{u}$

$= - \left(-\omega_* \cdot \cosh a\tau + \omega_* \sinh a\tau \right)$

$= \omega_* (\cosh a\tau - \sinh a\tau)$

$= \omega_* e^{-a\tau}$

$\longrightarrow \omega_0 = \omega_*$ @ $\tau = 0$

$\omega_0 \longrightarrow 0$ @ $\tau \to \infty$

ω_0

ormal basis ?

$= \underset{\sim}{u} = (\cosh a\tau, \sinh a\tau, 0, 0)$

$= - \underset{\sim}{k} \cdot \underset{\sim}{u}$

$= - \left(- \omega_* \cdot \cosh a\tau + \omega_* \sinh a\tau \right)$

$= \omega_* (\cosh a\tau - \sinh a\tau)$

$= \omega_* e^{-a\tau} \rightarrow \omega_0 = \omega_*$ @ $\tau = 0$

$\omega_0 \rightarrow 0$ @ $\tau \rightarrow \infty$

$\omega_0 \rightarrow +\infty$ @ $\tau \rightarrow -\infty$

measu

spur

cons

freq

what

$\underset{\sim}{k}$