Title: PSI - Relativity (PHYS 604) - 2A

Date: Sep 04, 2009 09:00 AM

URL: http://pirsa.org/09090035

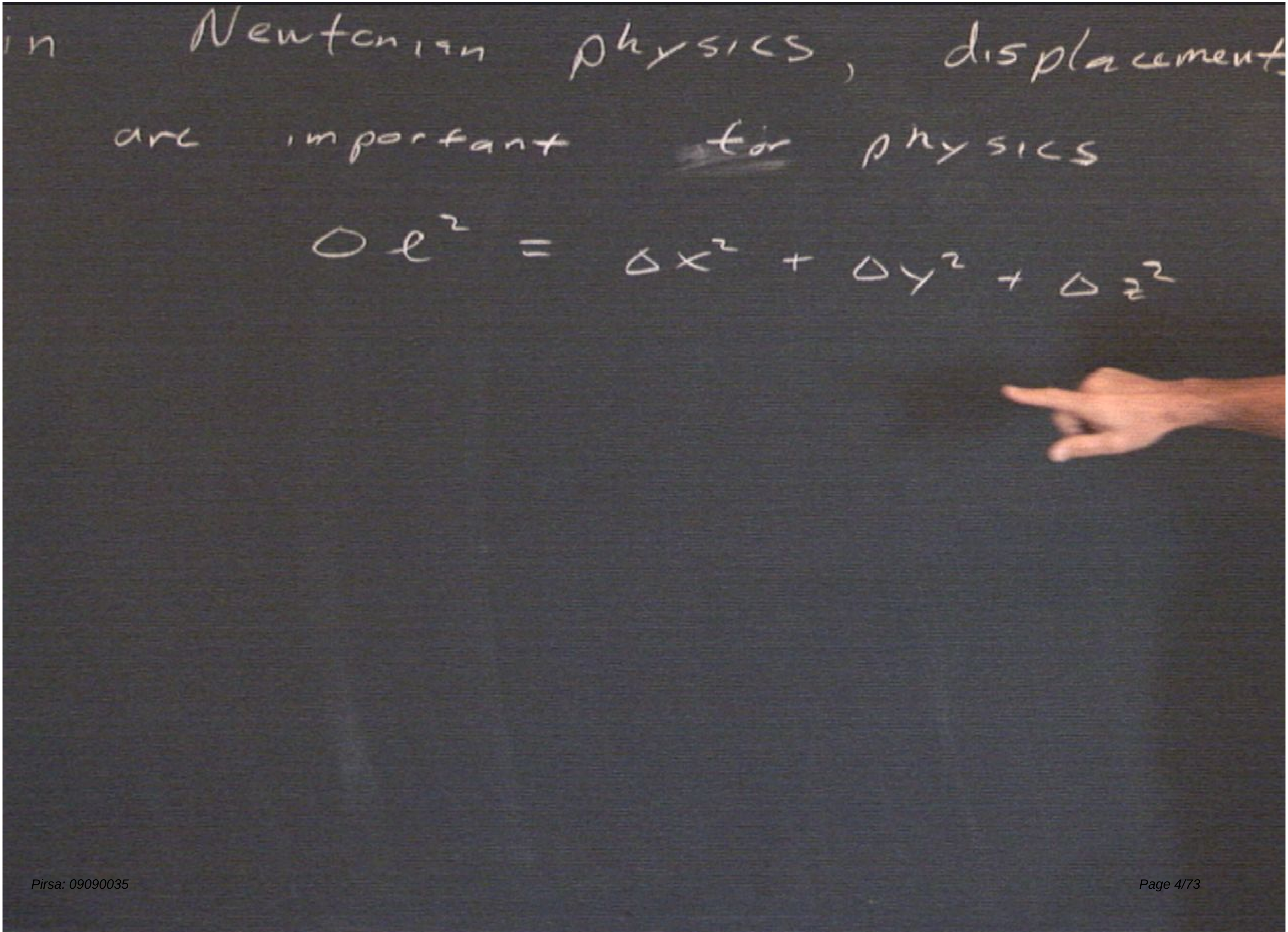
Abstract:

nstein's postulates

) Principle of relativity

) c fixed for all inertial frames

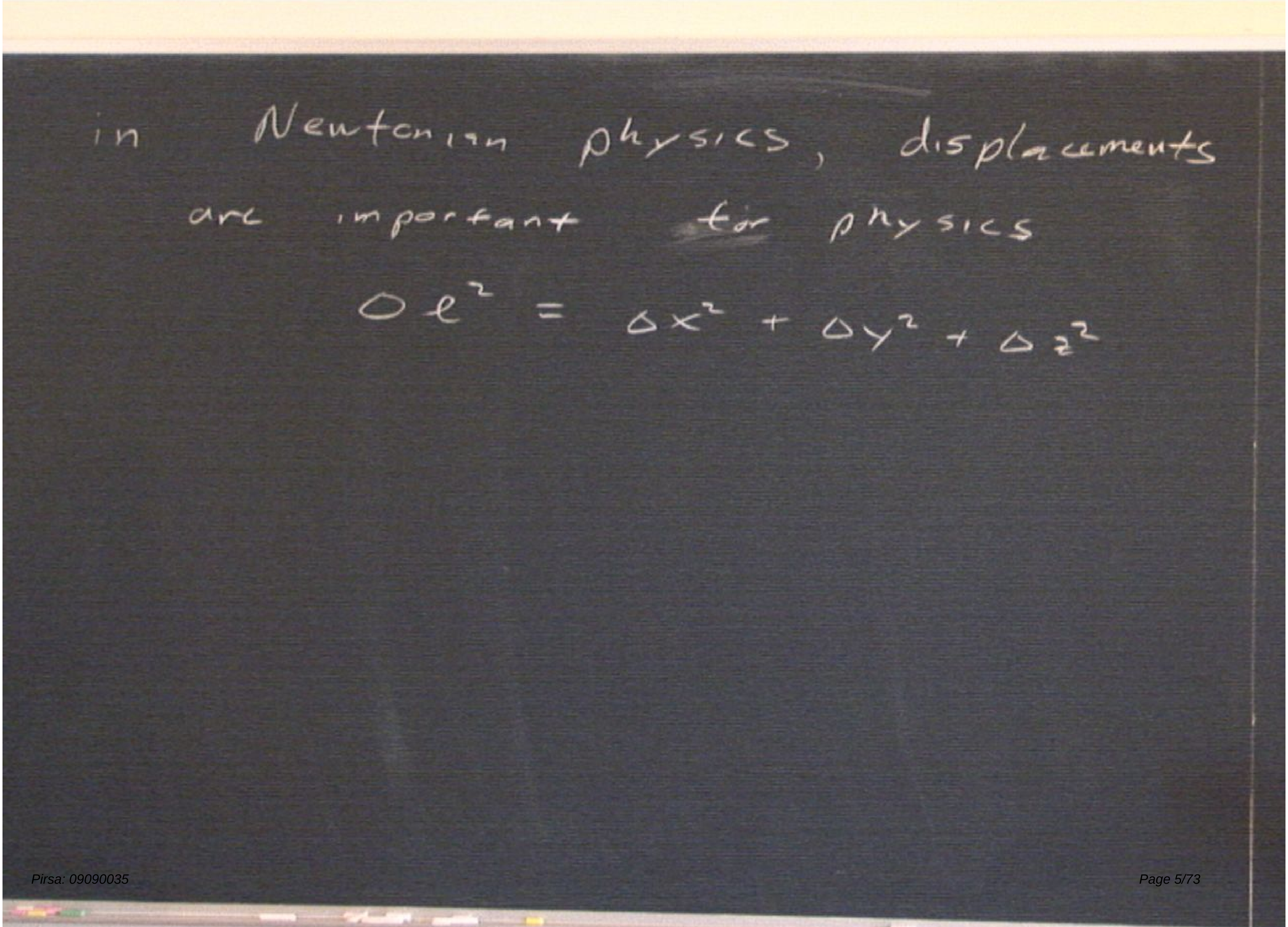
in Newtonian physics



in Newtonian physics, displacement

are important for physics

$$\Delta \ell^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$



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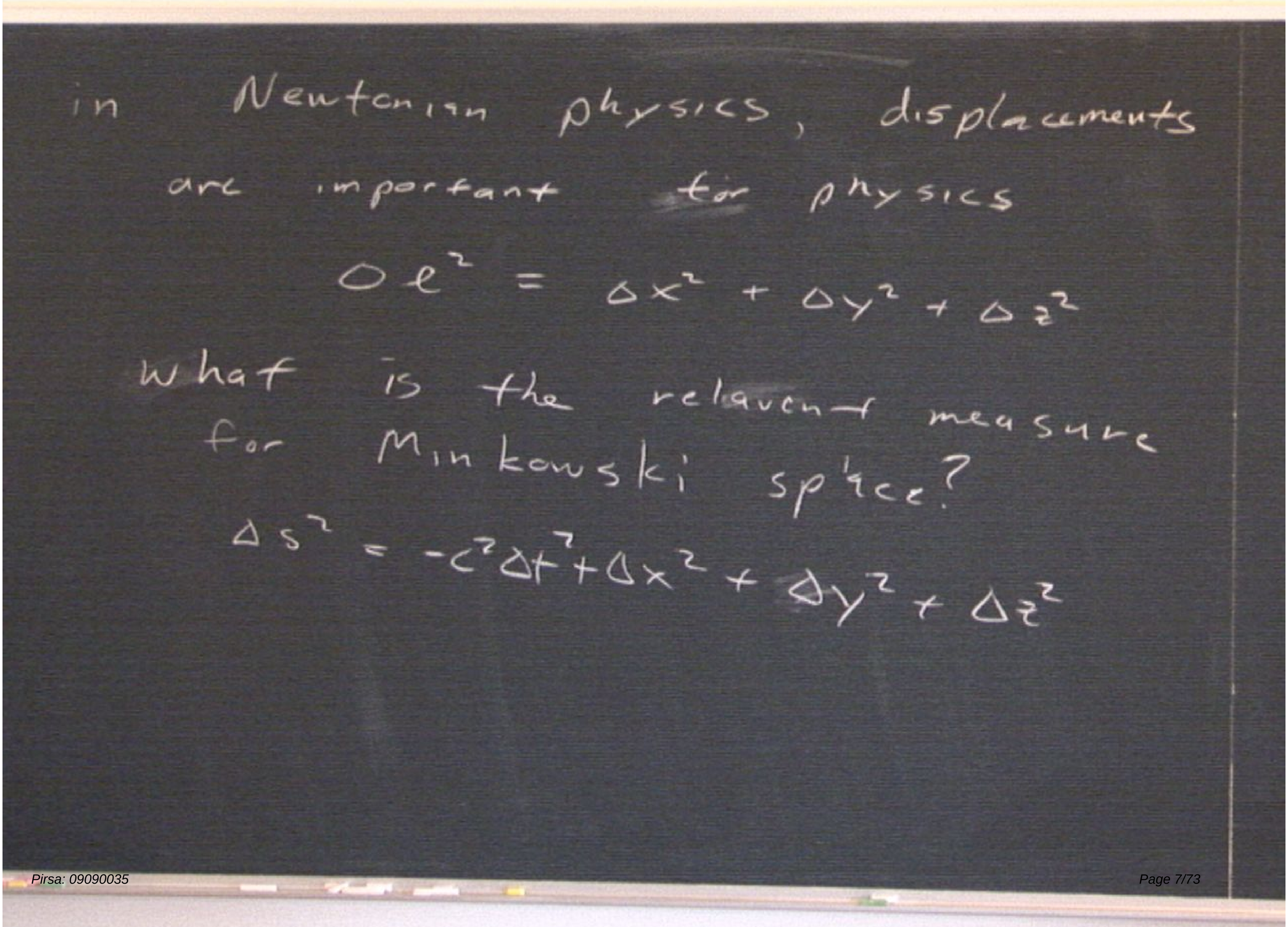
what is the relavent measure for Minkowski space?



in Newtonian physics, displacements are important for physics

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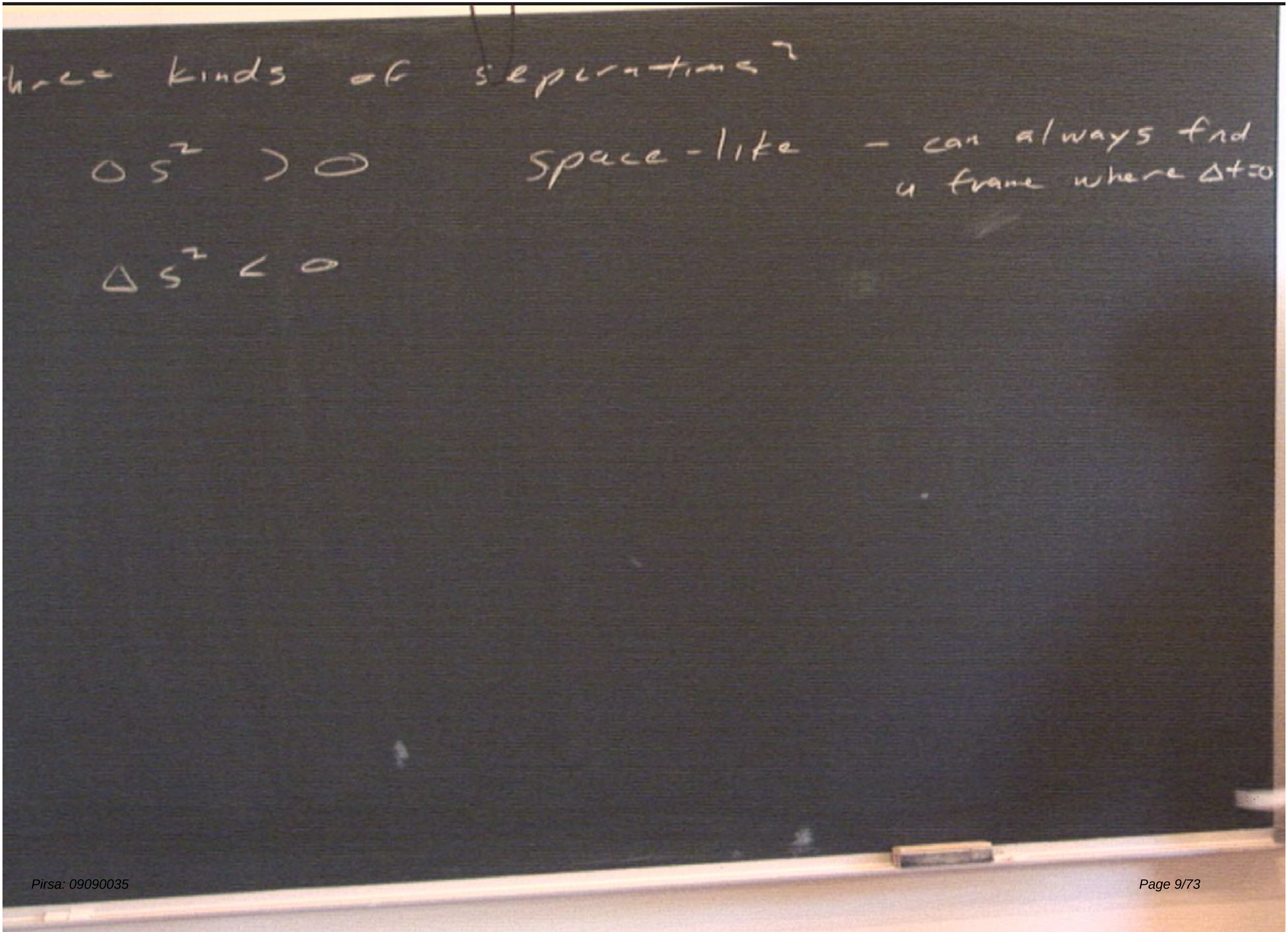
what is the relavent measure for Minkowski space?

$$\Delta s^2 = -c^2 \Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$

three kinds of seperations?

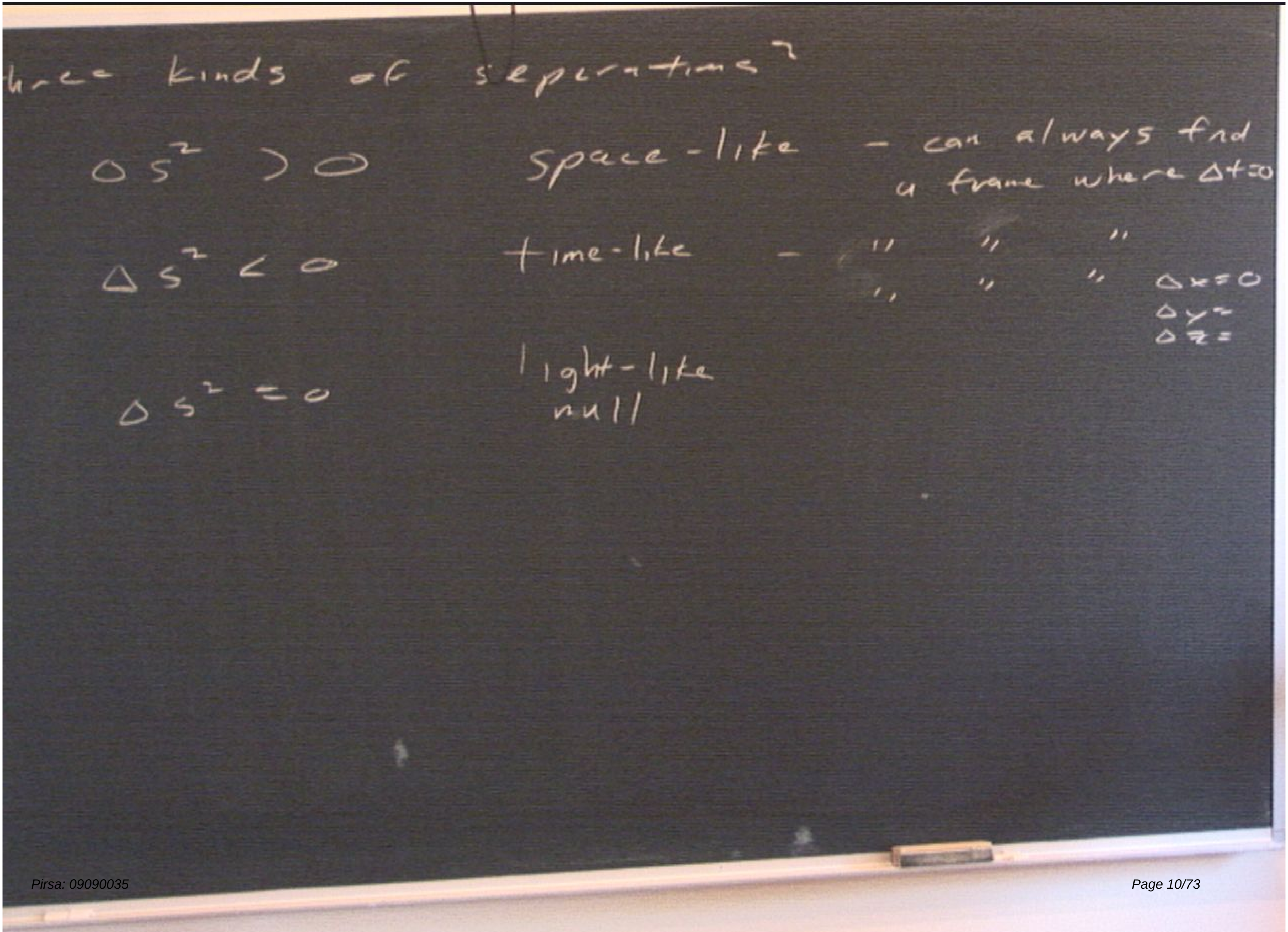
$\Delta s^2 > 0$ space-like



hree kinds of separations?

$\Delta s^2 > 0$ space-like - can always find a frame where $\Delta t = 0$

$\Delta s^2 < 0$



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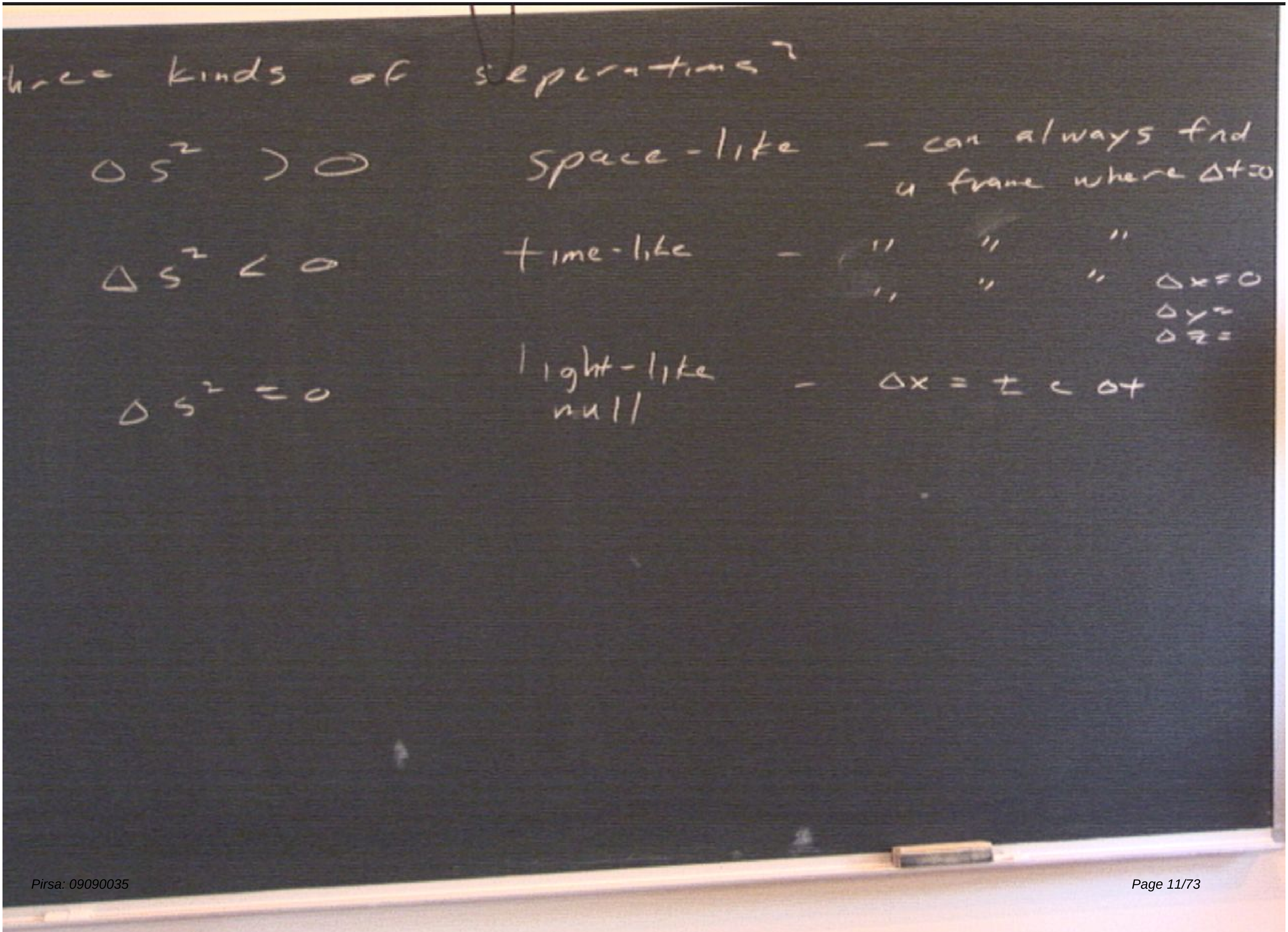
" " " $\Delta y =$
$\Delta z =$

$\Delta s^2 = 0$ light-like
null



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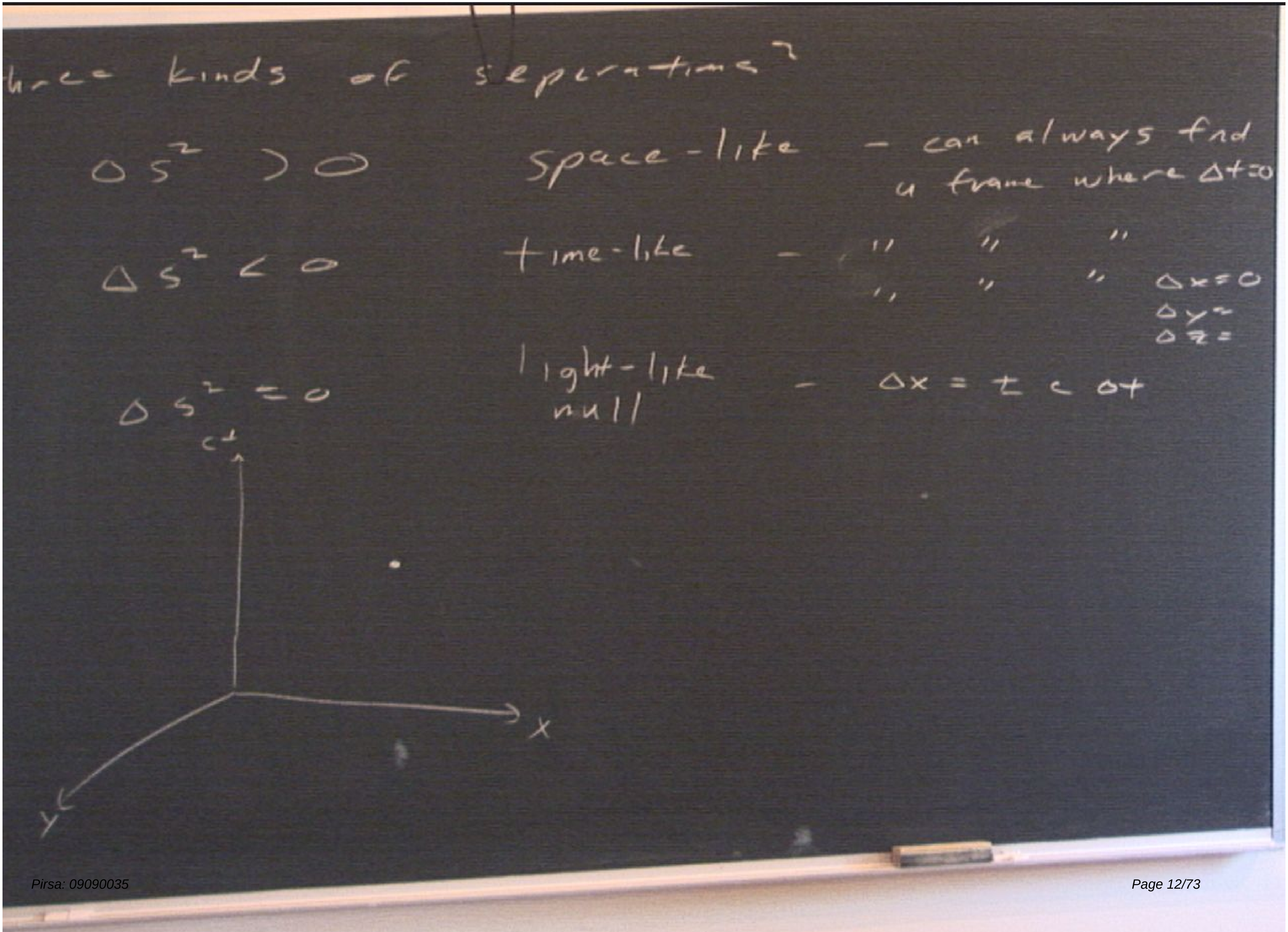

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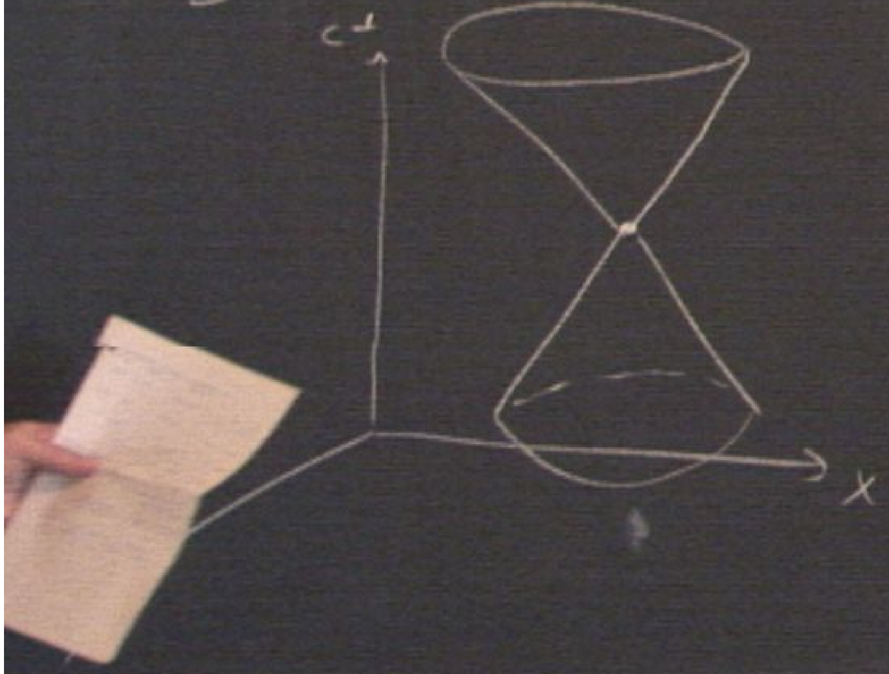

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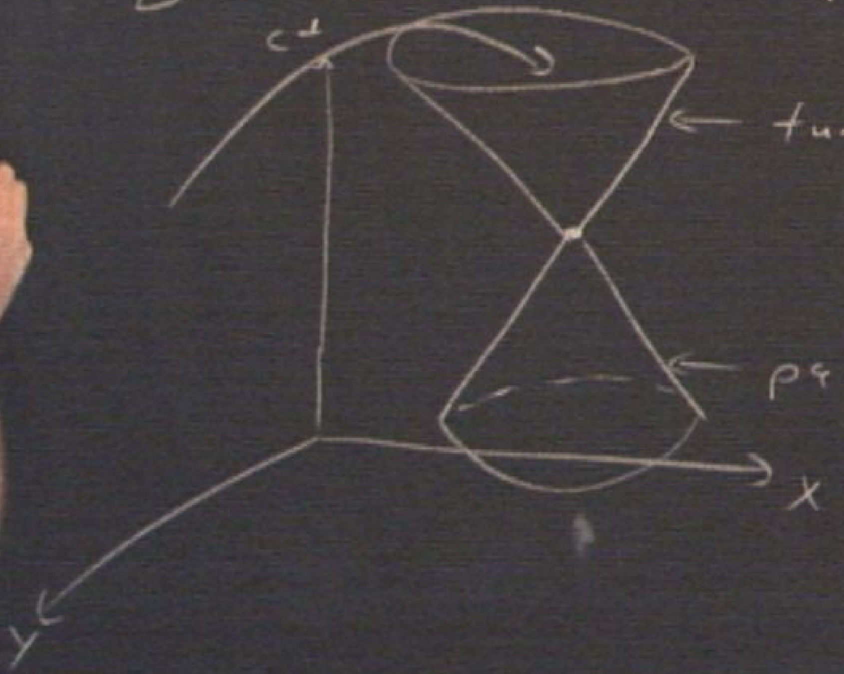

← future light cone

← past light cone

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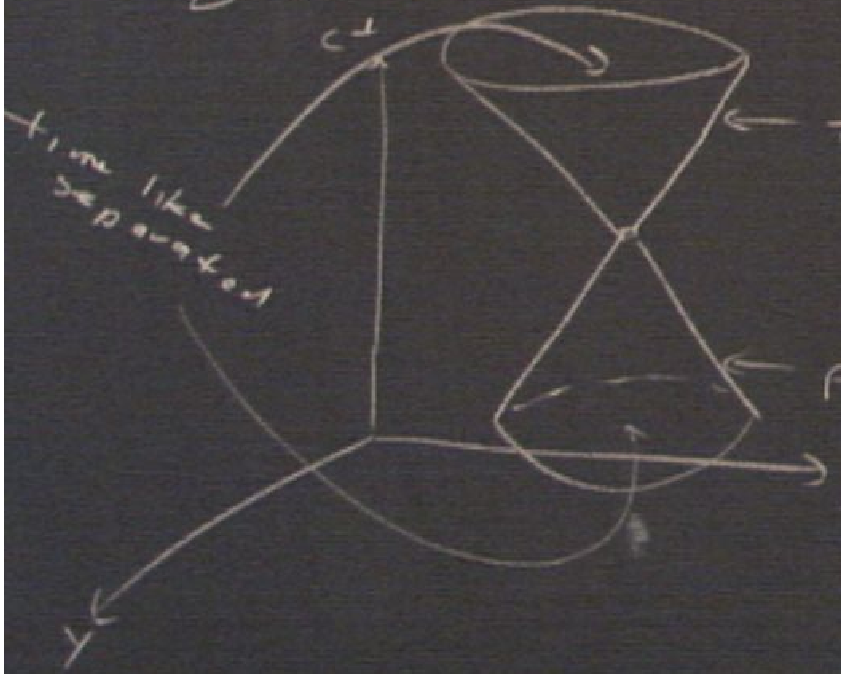

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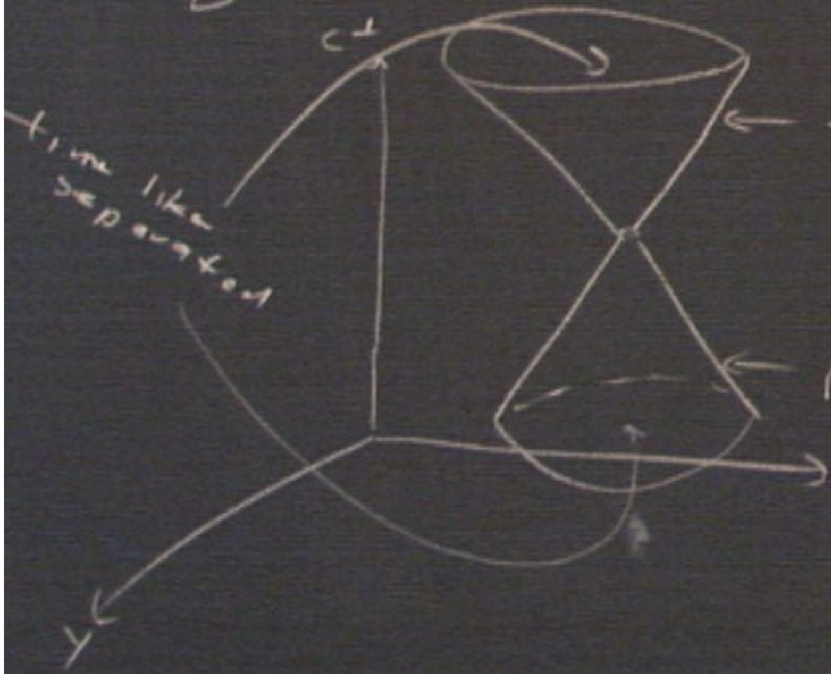
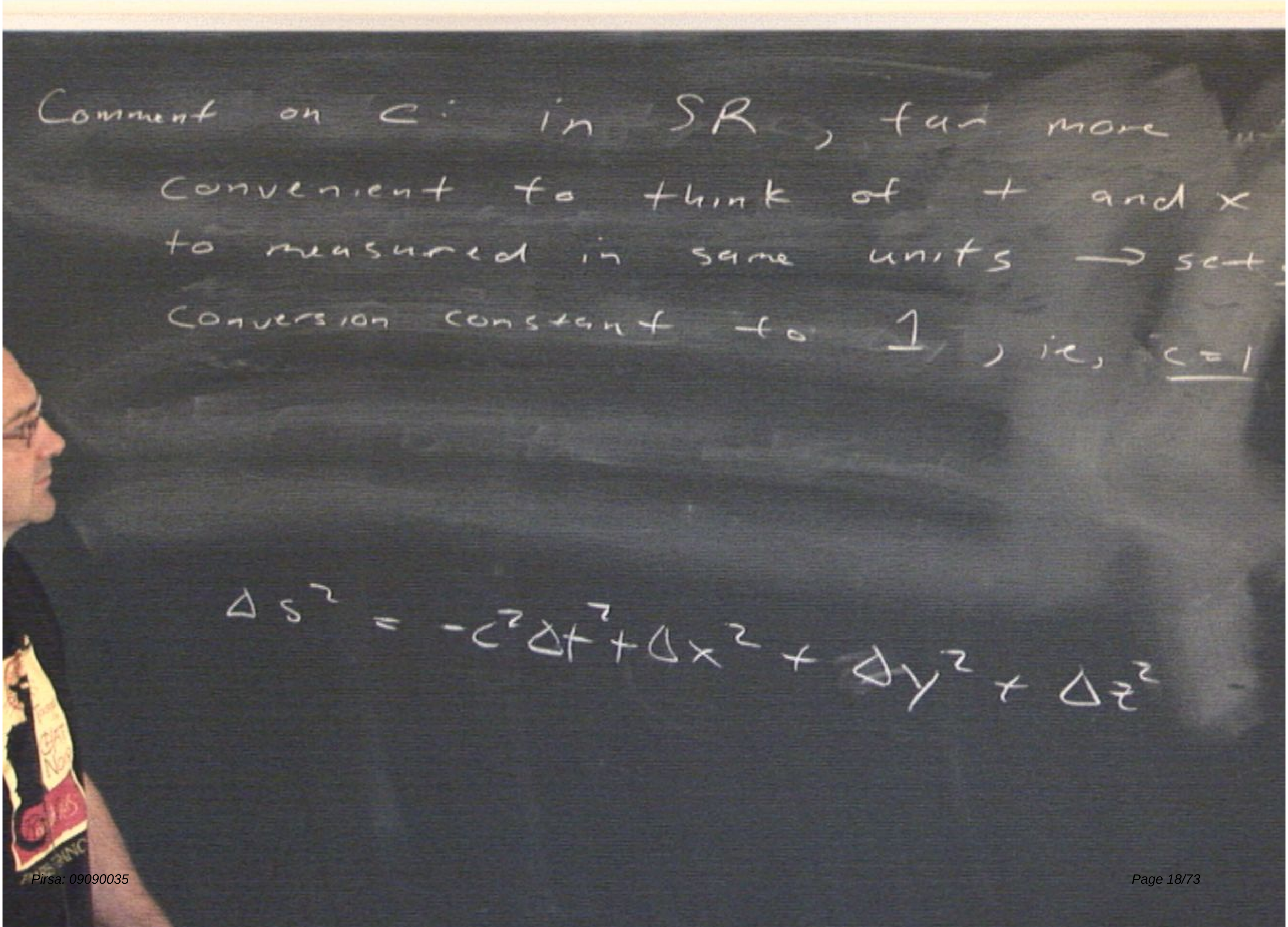


← future light cone

outside is spacelike separated

← past light cone

$$\Delta s^2 = -c^2 \Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$



Comment on c: in SR, far more
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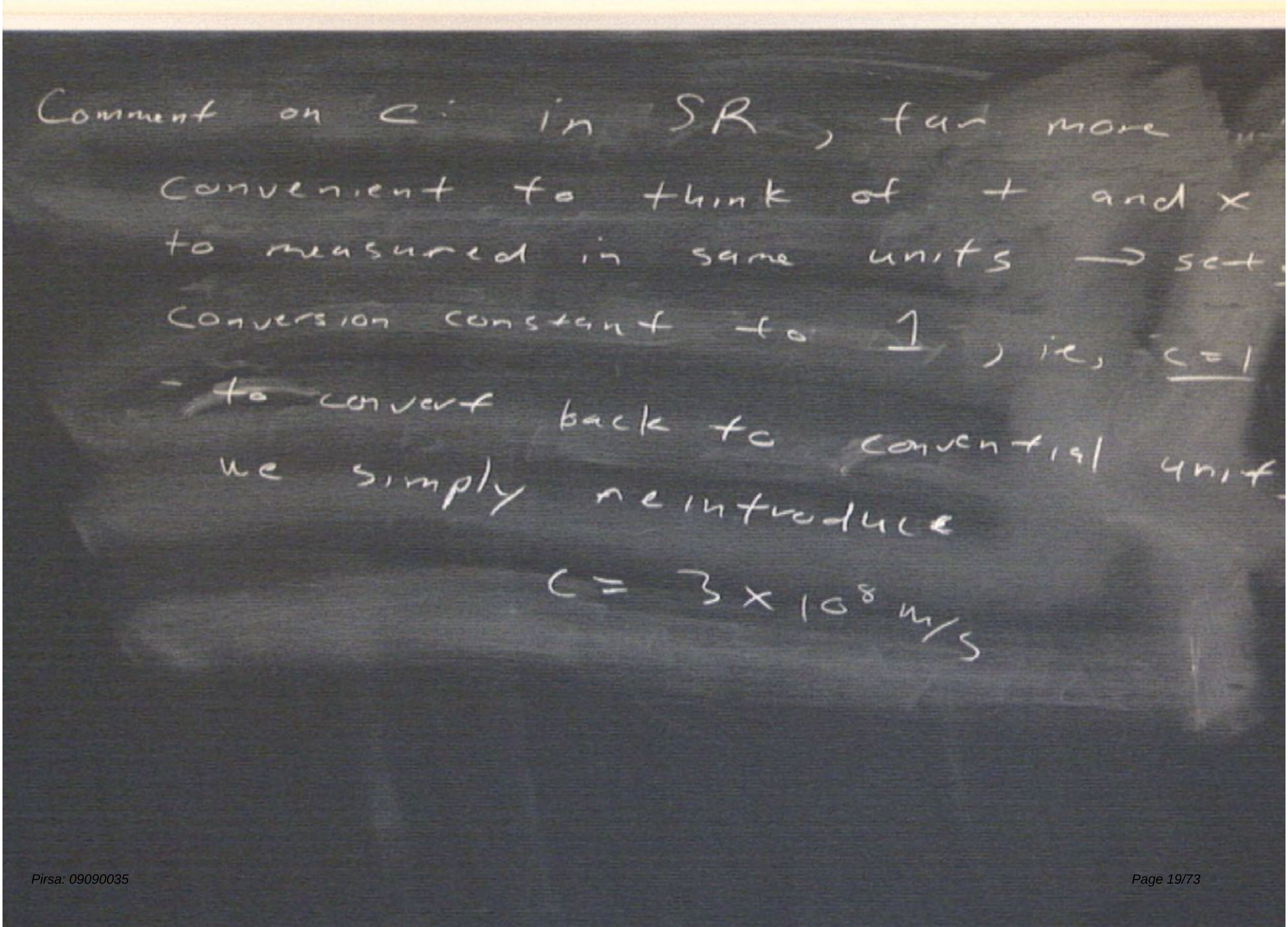

Comment on c: in SR, far more convenient to think of t and x to measured in same units → set conversion constant to $\underline{1}$, ie, $\underline{c=1}$

- to convert back to conventional unit we simply reintroduce

$$c = 3 \times 10^8 \, m/s$$

4 - vector notation:

$$(x^\alpha) = (x^0, x^1, x^2, x^3)$$

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↖ bold - face

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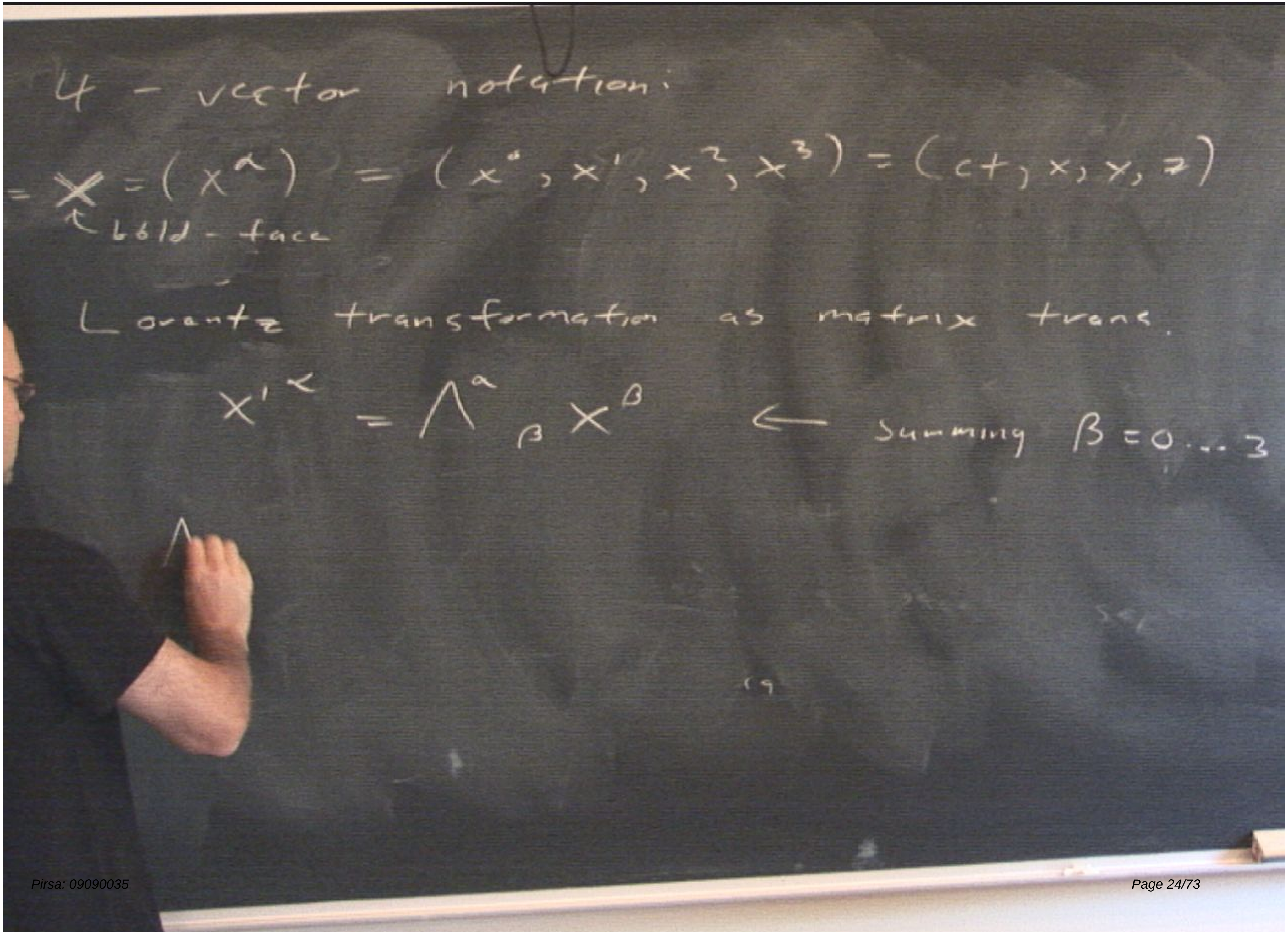
$$\underset{\sim}{x} = \underset{\nwarrow \text{bold-face}}{\mathbf{x}} = (x^{\alpha}) = (x^0, x^1, x^2, x^3) = (ct, x, y, z)$$



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Lorentz transformation as matrix trans.

$$x'^{\alpha} = \Lambda^{\alpha}{}_{\beta}\, x^{\beta} \quad \leftarrow \text{summing } \beta = 0 \ldots 3$$

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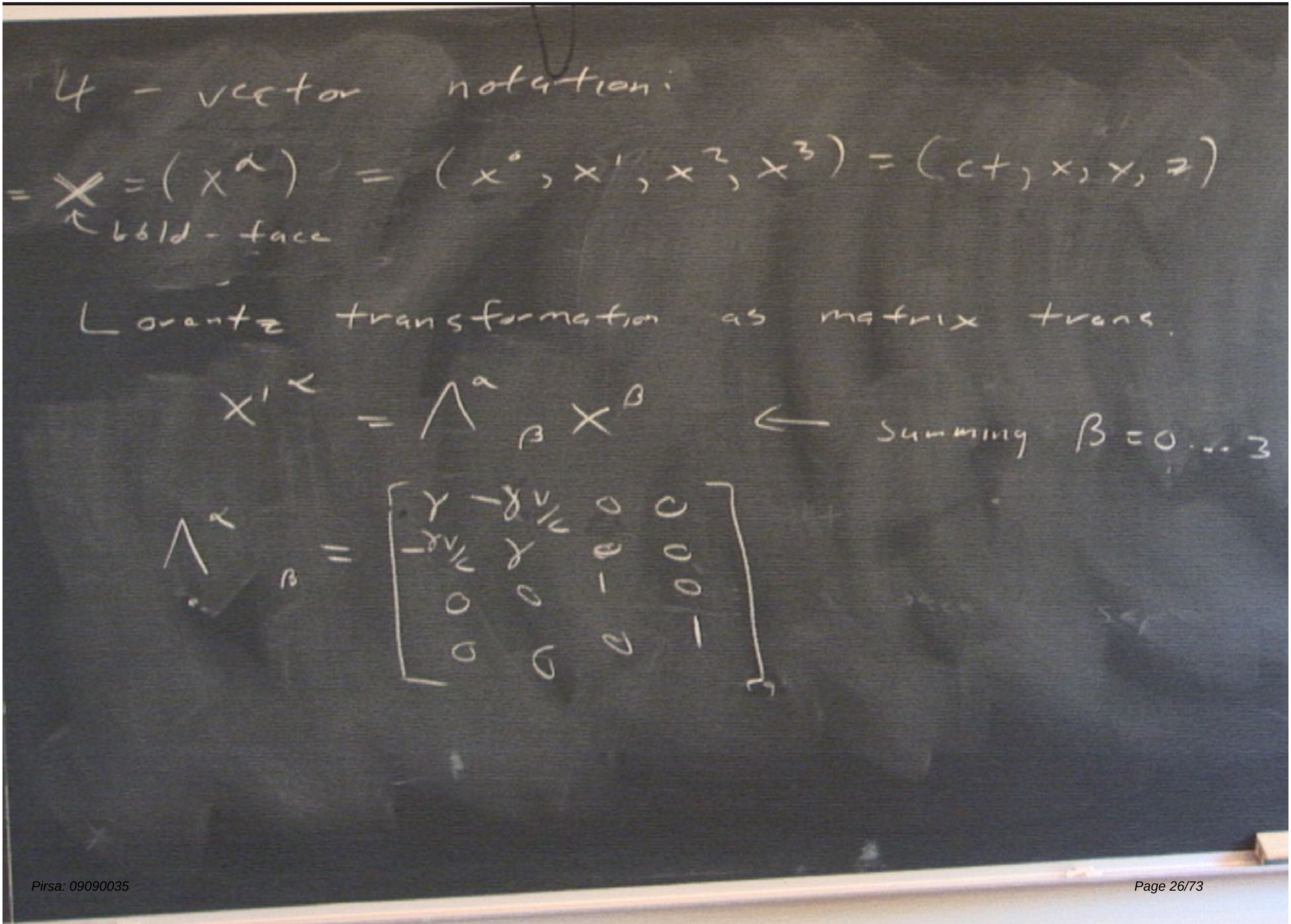

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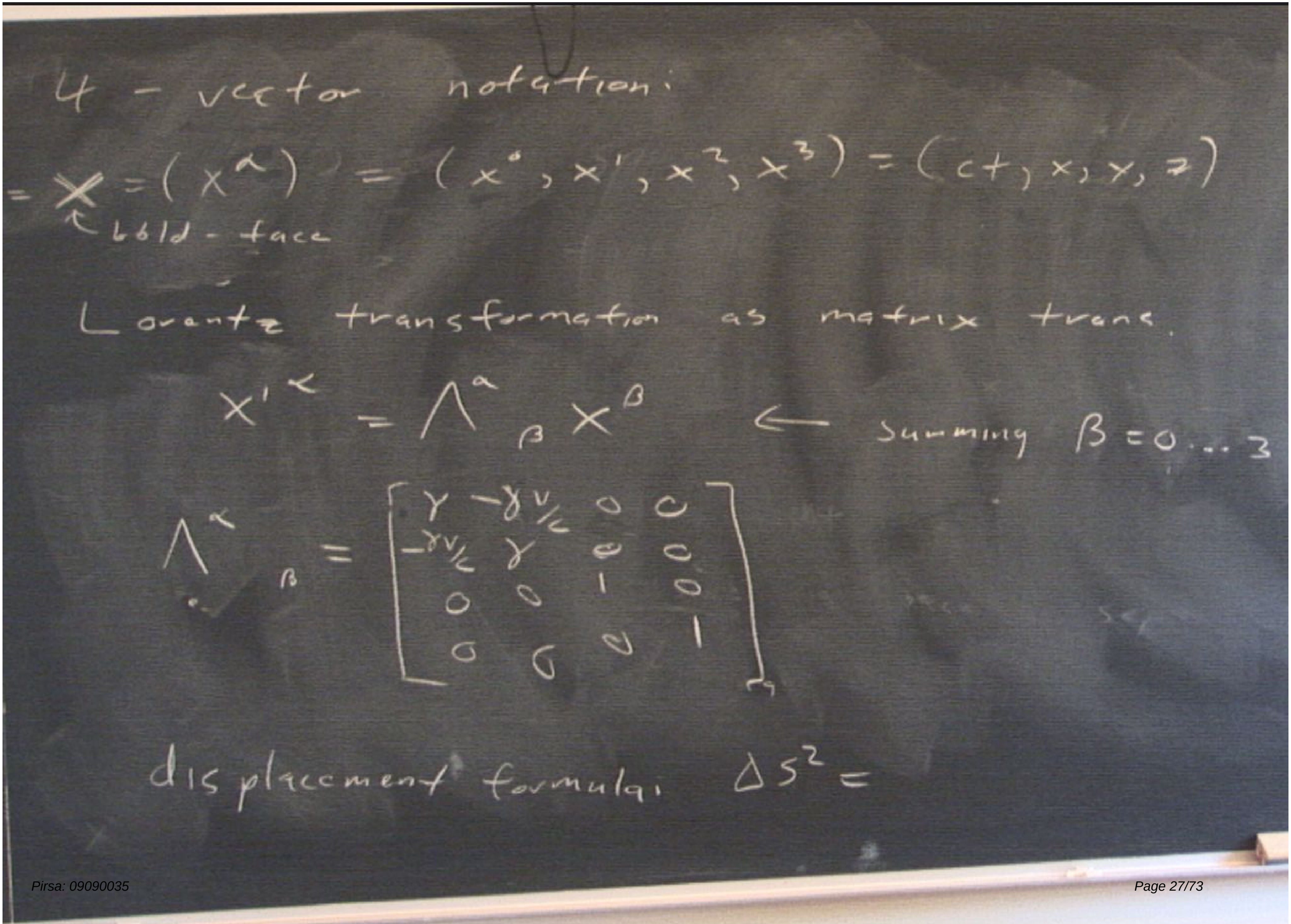

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displacement formula: $\Delta s^2 =$

α

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β

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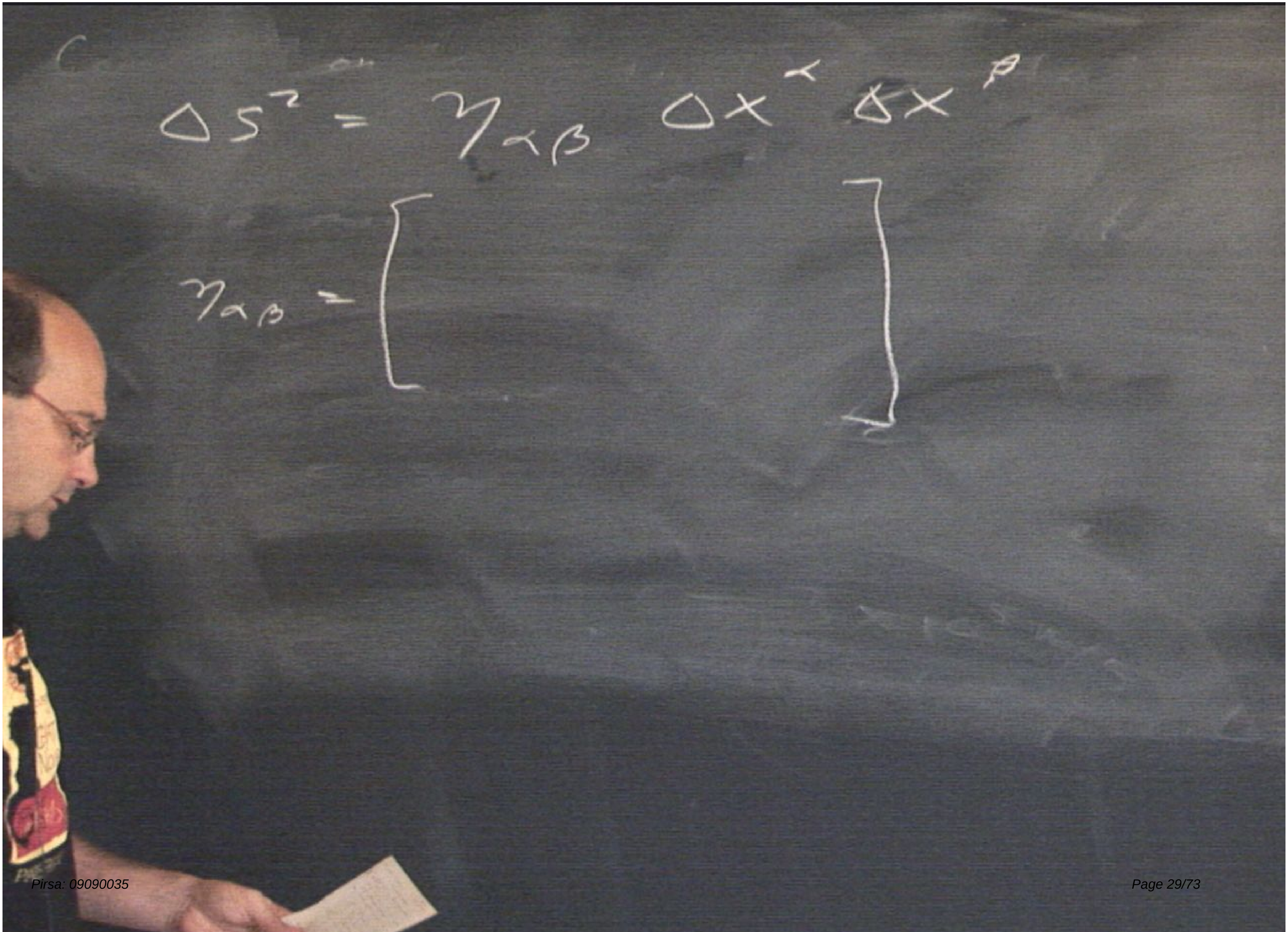
acement formula: $\Delta s^2 \neq \delta x^{\tau}$



$$\Delta s^2 = \eta_{\alpha\beta} \, \Delta x^\alpha \, \Delta x^\beta$$

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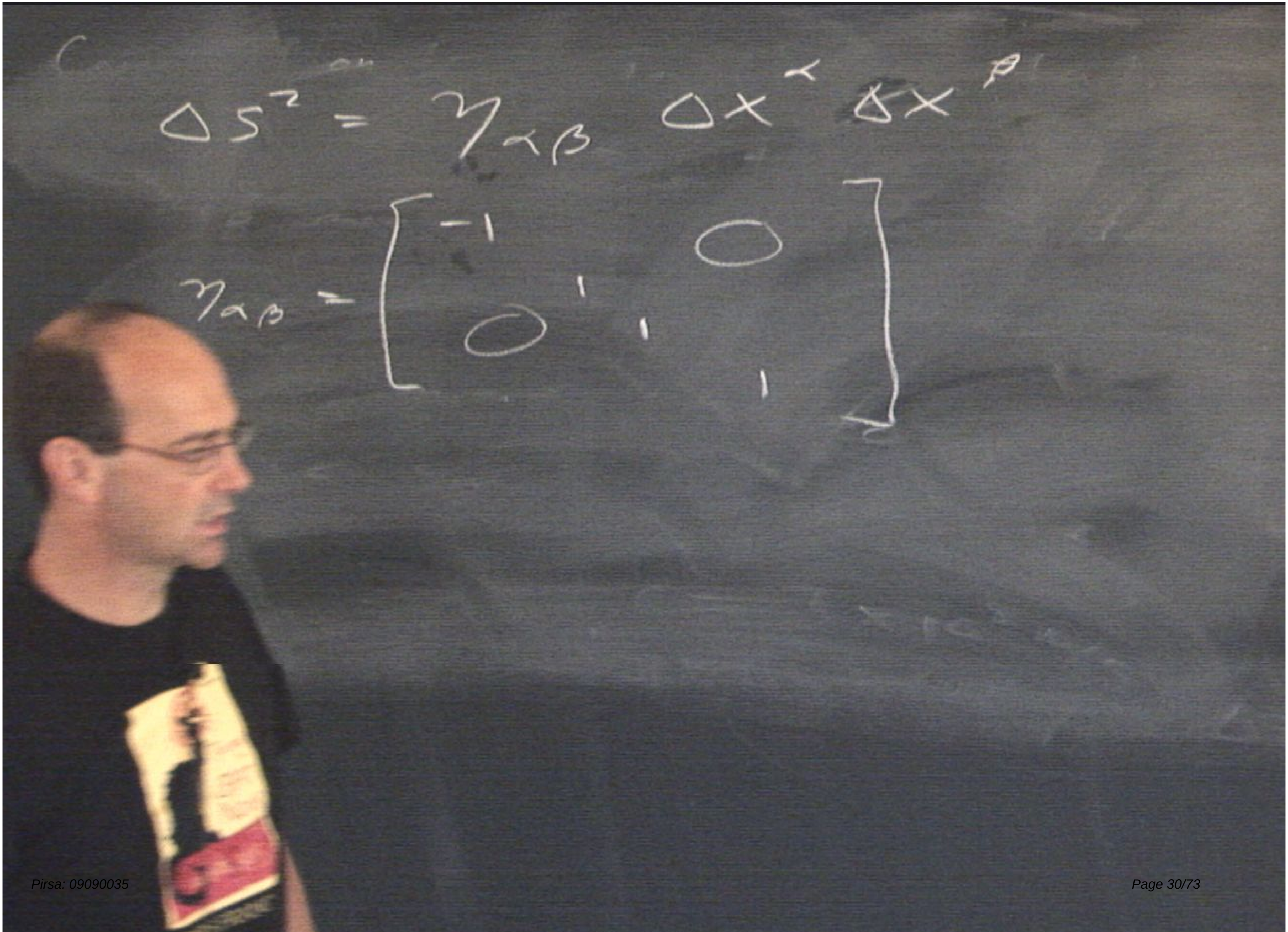

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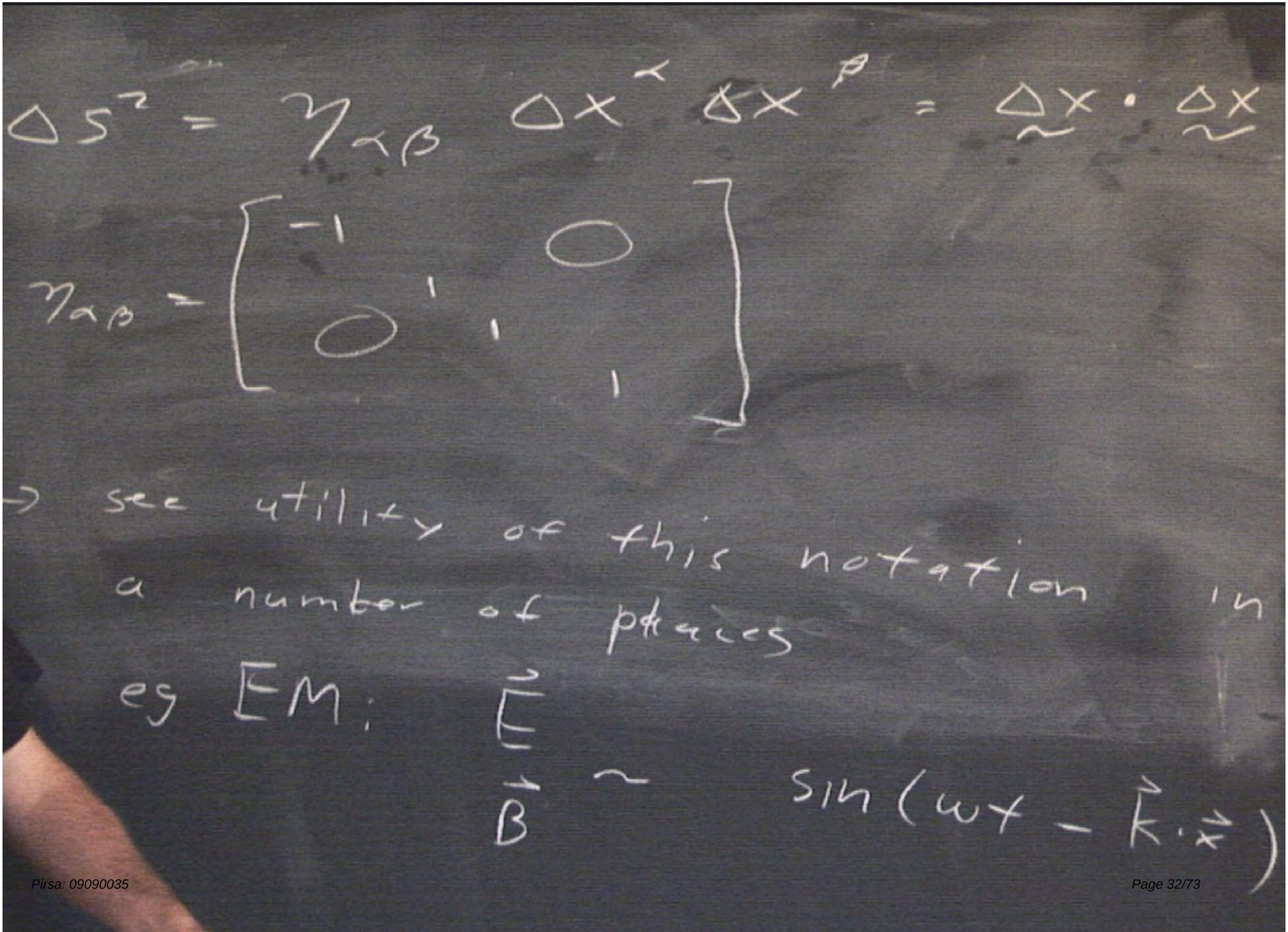

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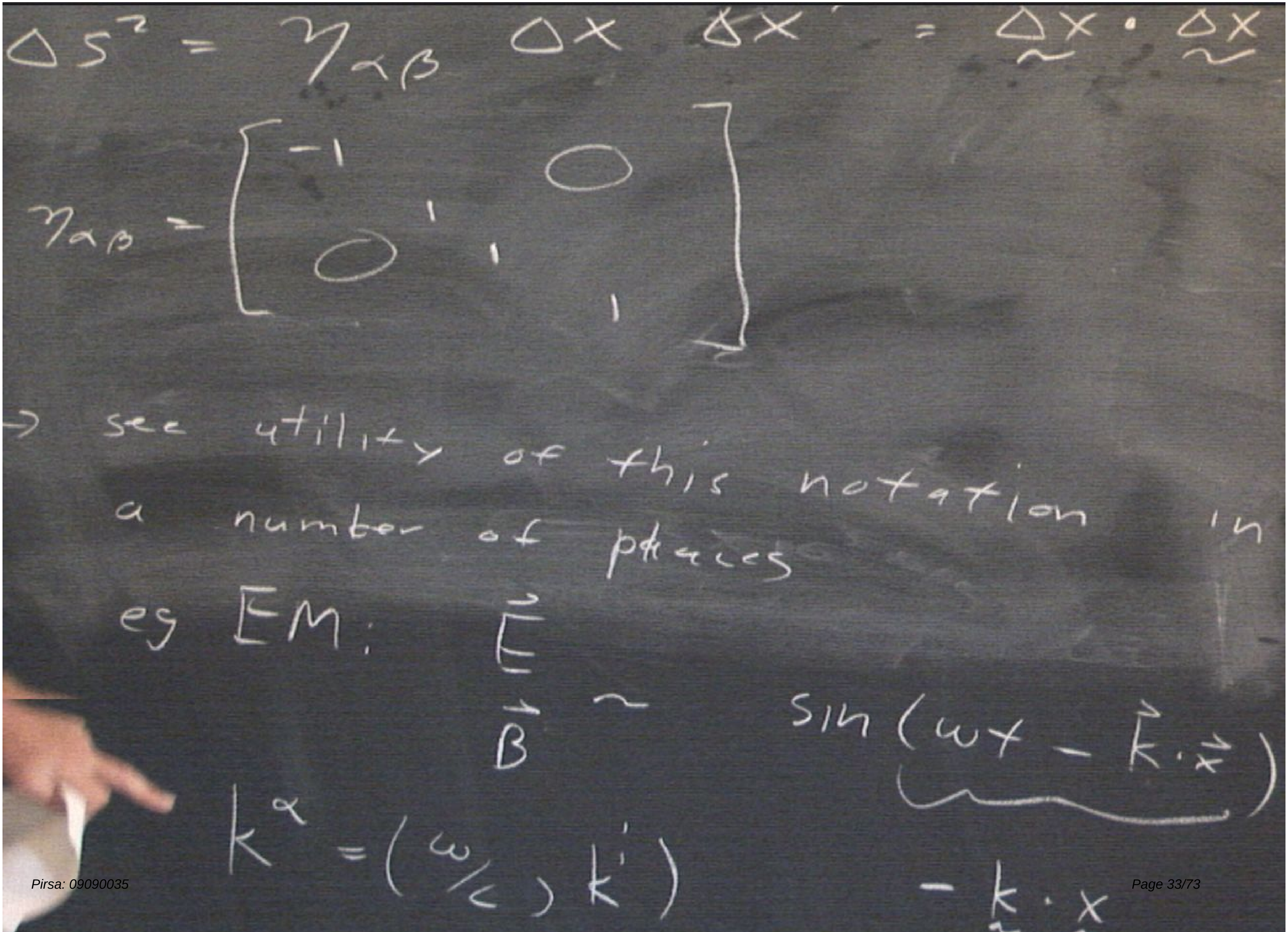
→ see utility of this notation in a number of places

eg EM: $\vec{E}$

$\vec{B} \sim \sin(\omega t - \vec{k} \cdot \vec{x})$



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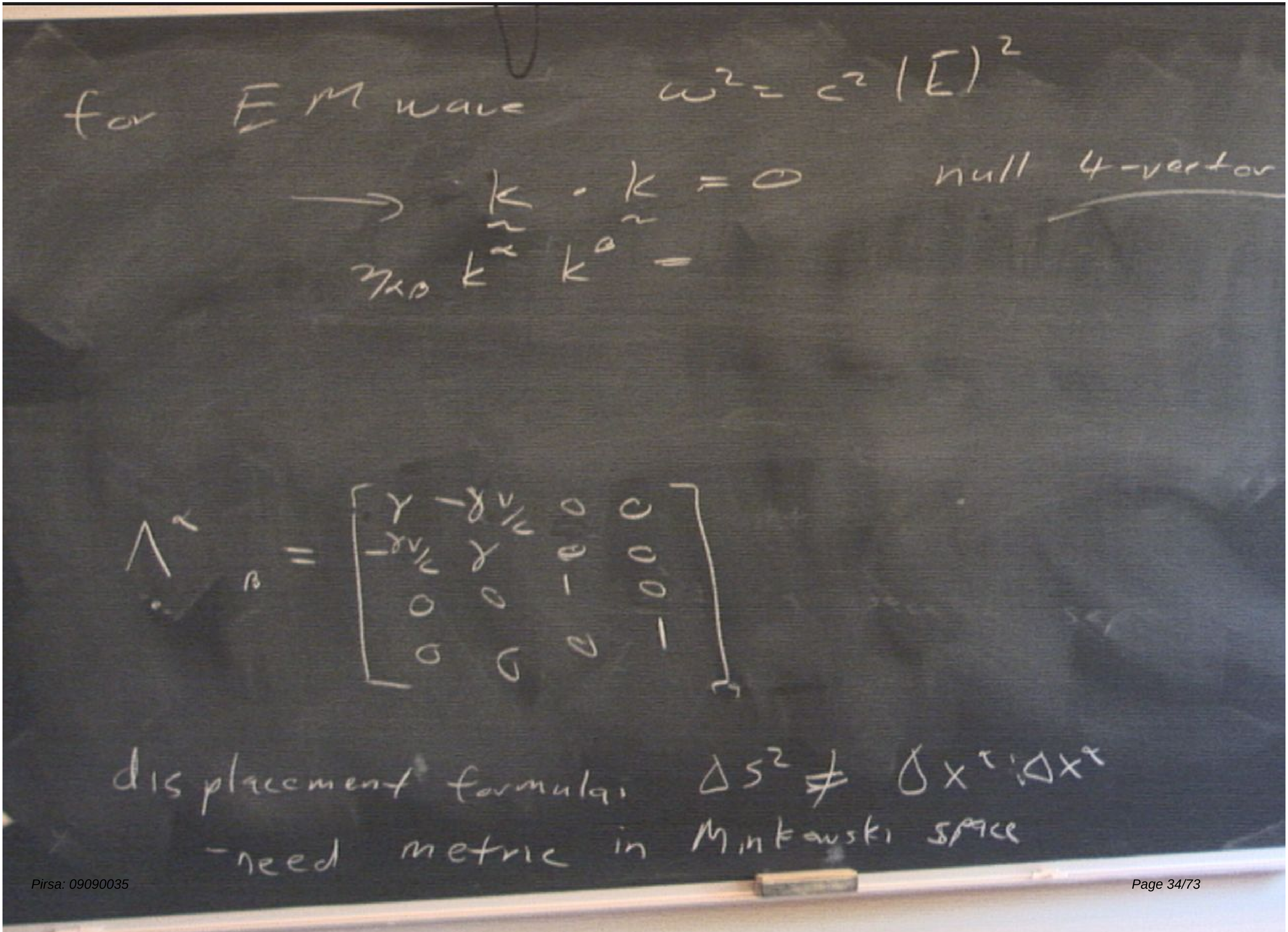
$$k^{\alpha} = (\omega/c, k^{i}) \qquad -\underset{\sim}{k}\cdot x$$



for EM wave $\omega^2 = c^2 (\underline{k})^2$

$\longrightarrow \underset{\sim}{k} \cdot \underset{\sim}{k} = 0$ null 4-vector

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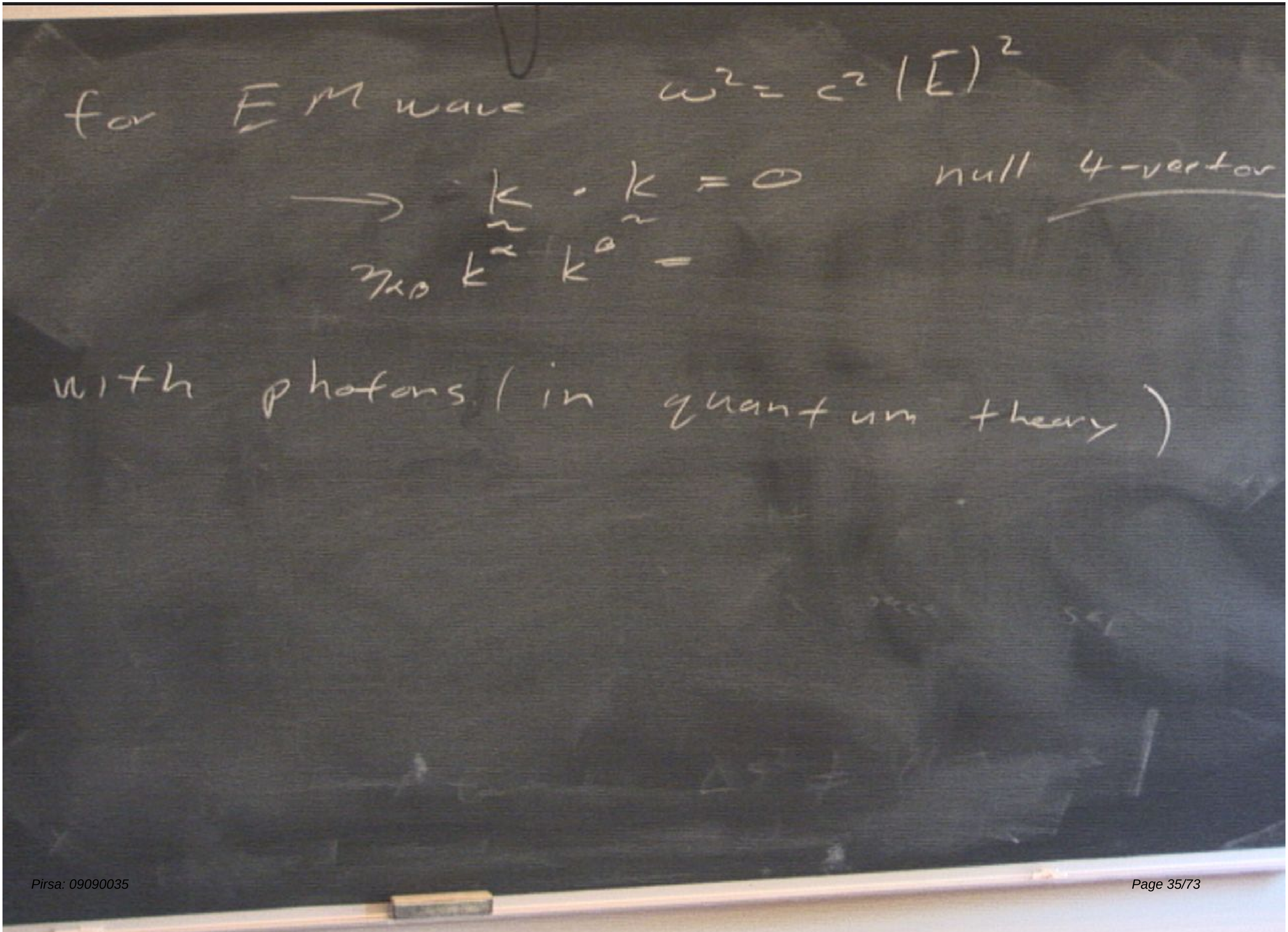
displacement formula: $\Delta s^2 \neq \Delta x^\alpha \Delta x^\alpha$

- need metric in Minkowski space



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with photons (in quantum theory)

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$E = \hbar \omega$ $\vec{p} = \hbar \underline{k}$

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$$\eta_{\alpha\beta}\, k^\alpha\, k^\beta =$$

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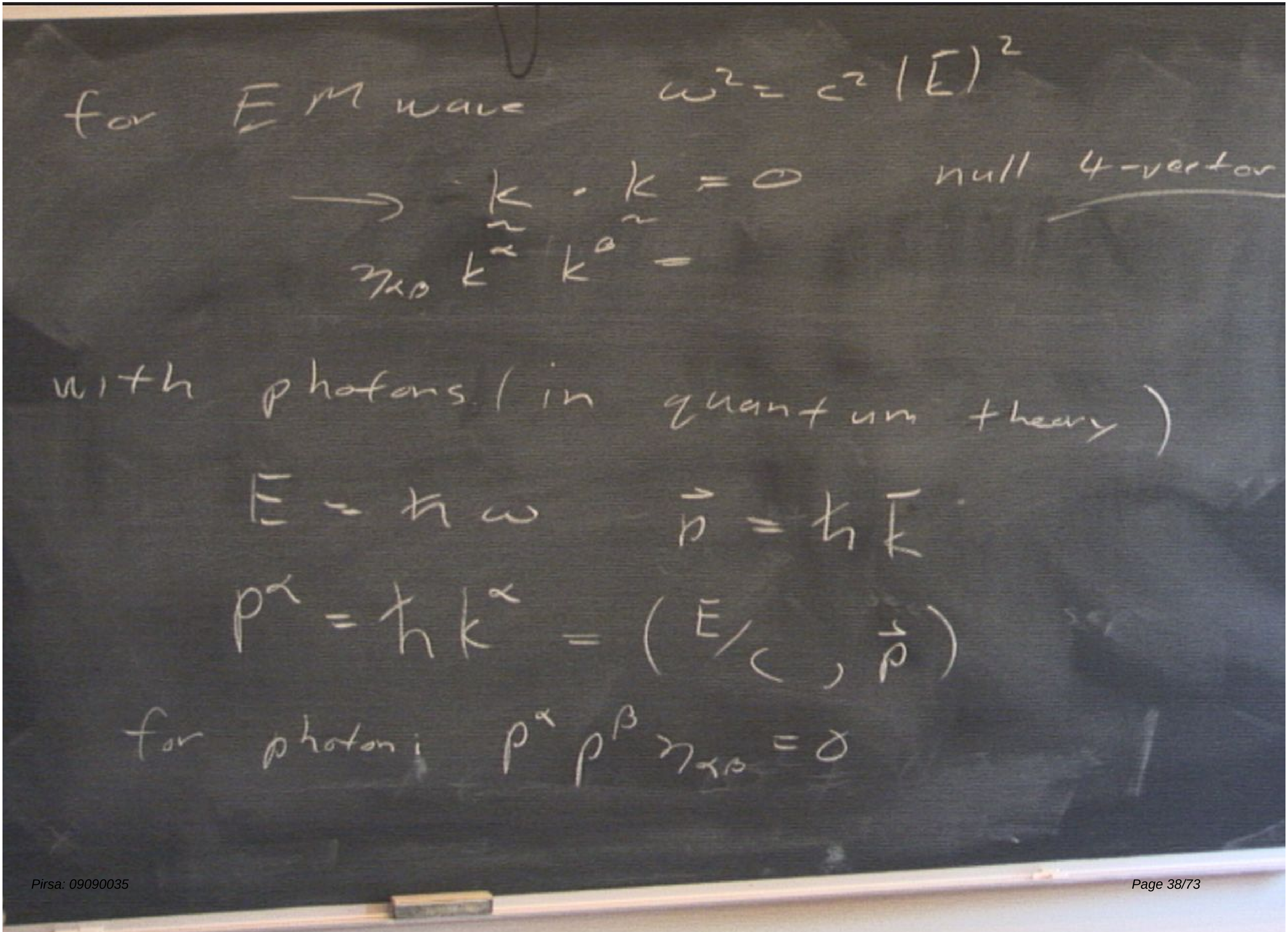

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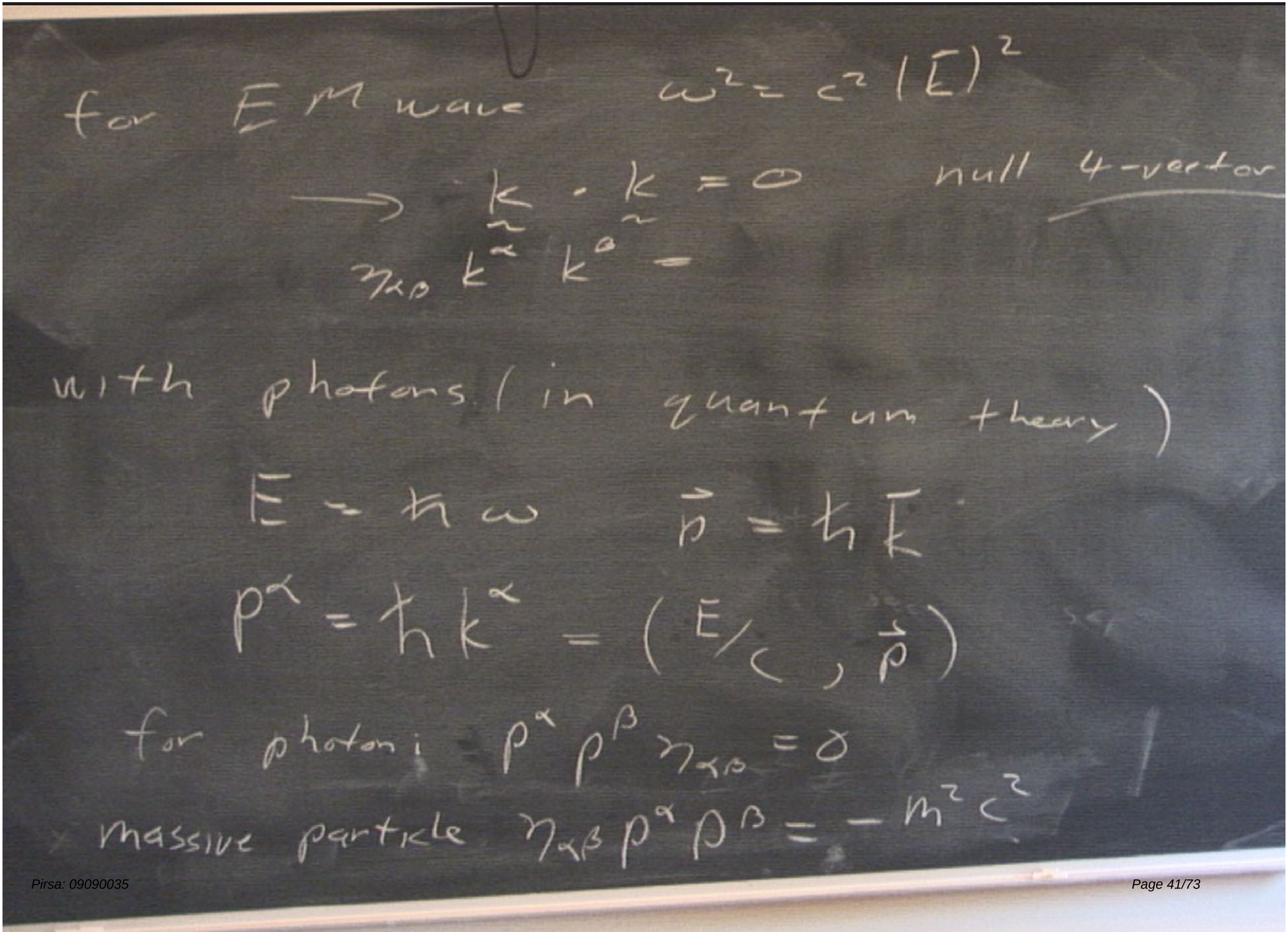

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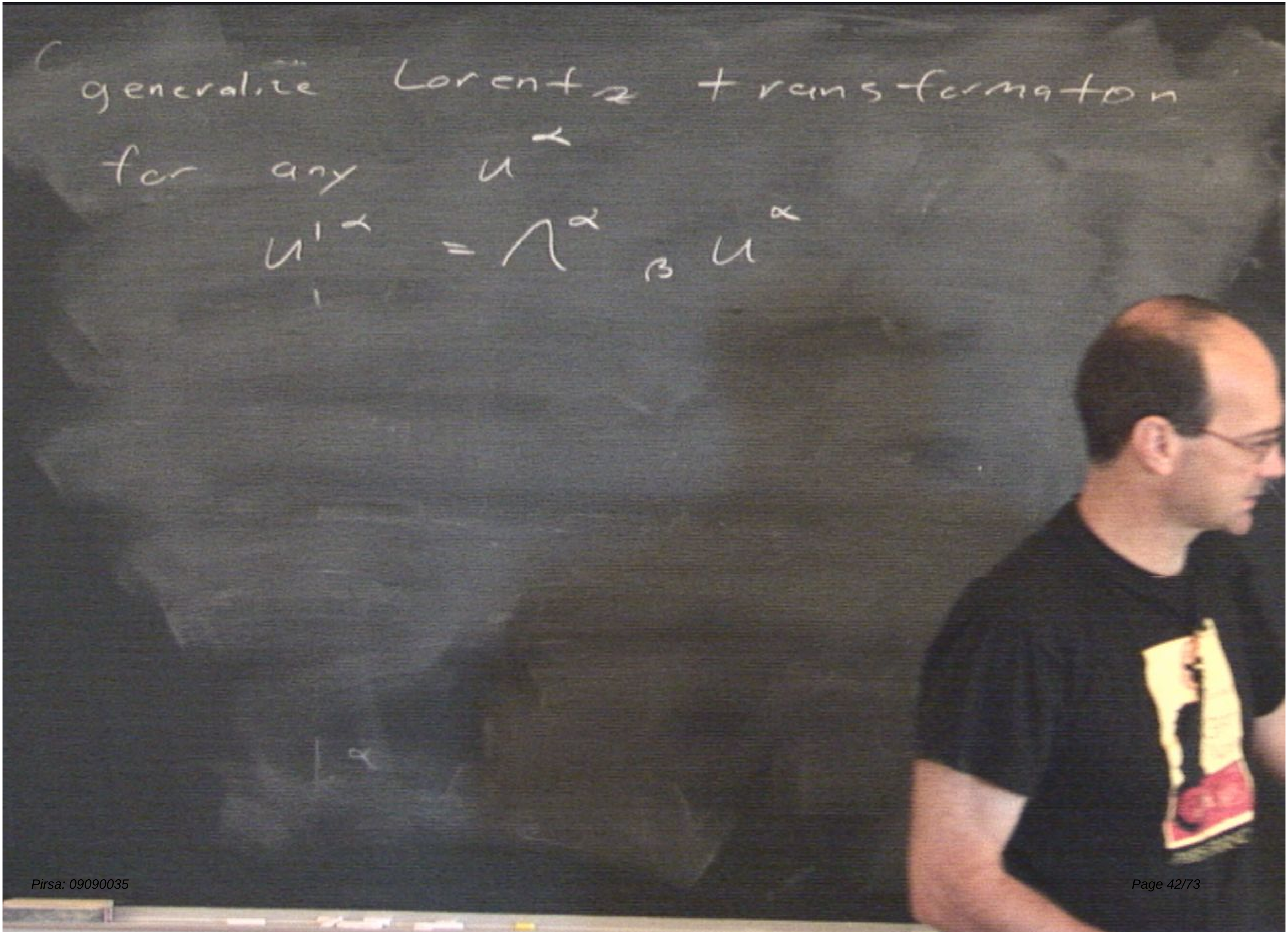
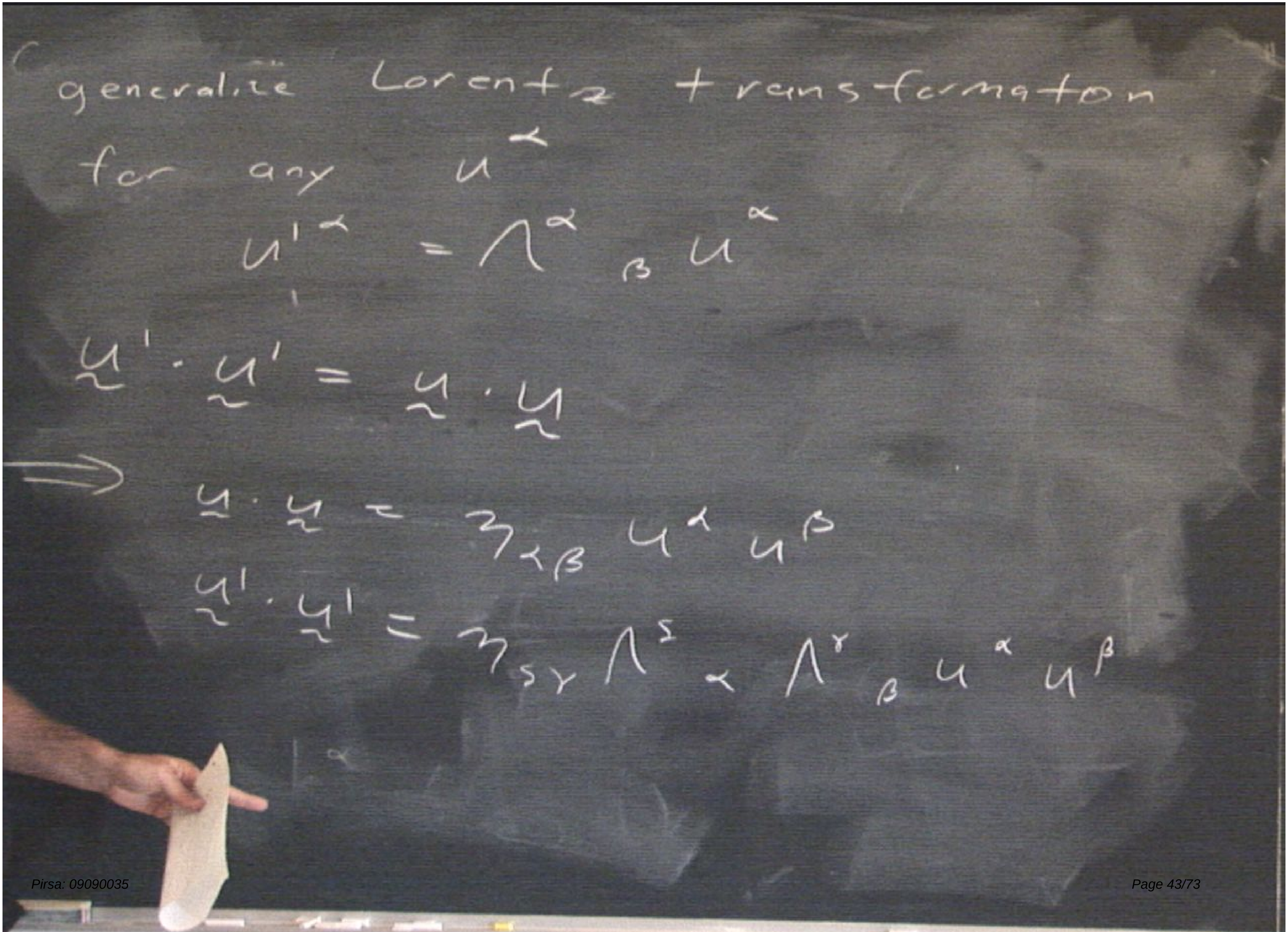


generalize Lorentz transformation

for any u^α

$$u'^\alpha = \Lambda^\alpha{}_\beta \, u^\alpha$$



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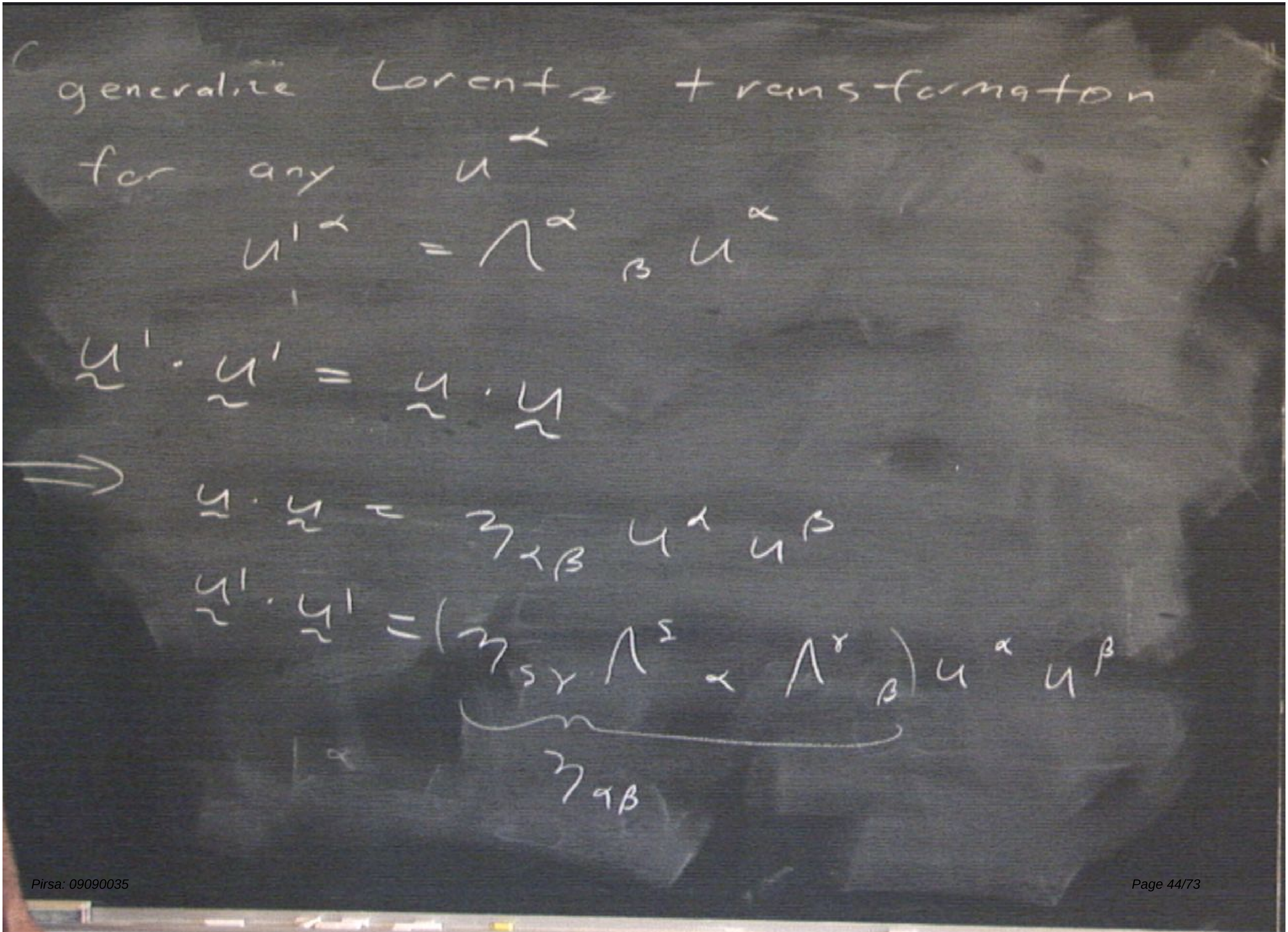
$$\Longrightarrow \quad \underset{\sim}{u} \cdot \underset{\sim}{u} = \eta_{\alpha\beta} \, u^\alpha u^\beta$$

$$\underset{\sim}{u}' \cdot \underset{\sim}{u}' = \eta_{\delta\gamma} \Lambda^\delta{}_\alpha \Lambda^\gamma{}_\beta \, u^\alpha u^\beta$$



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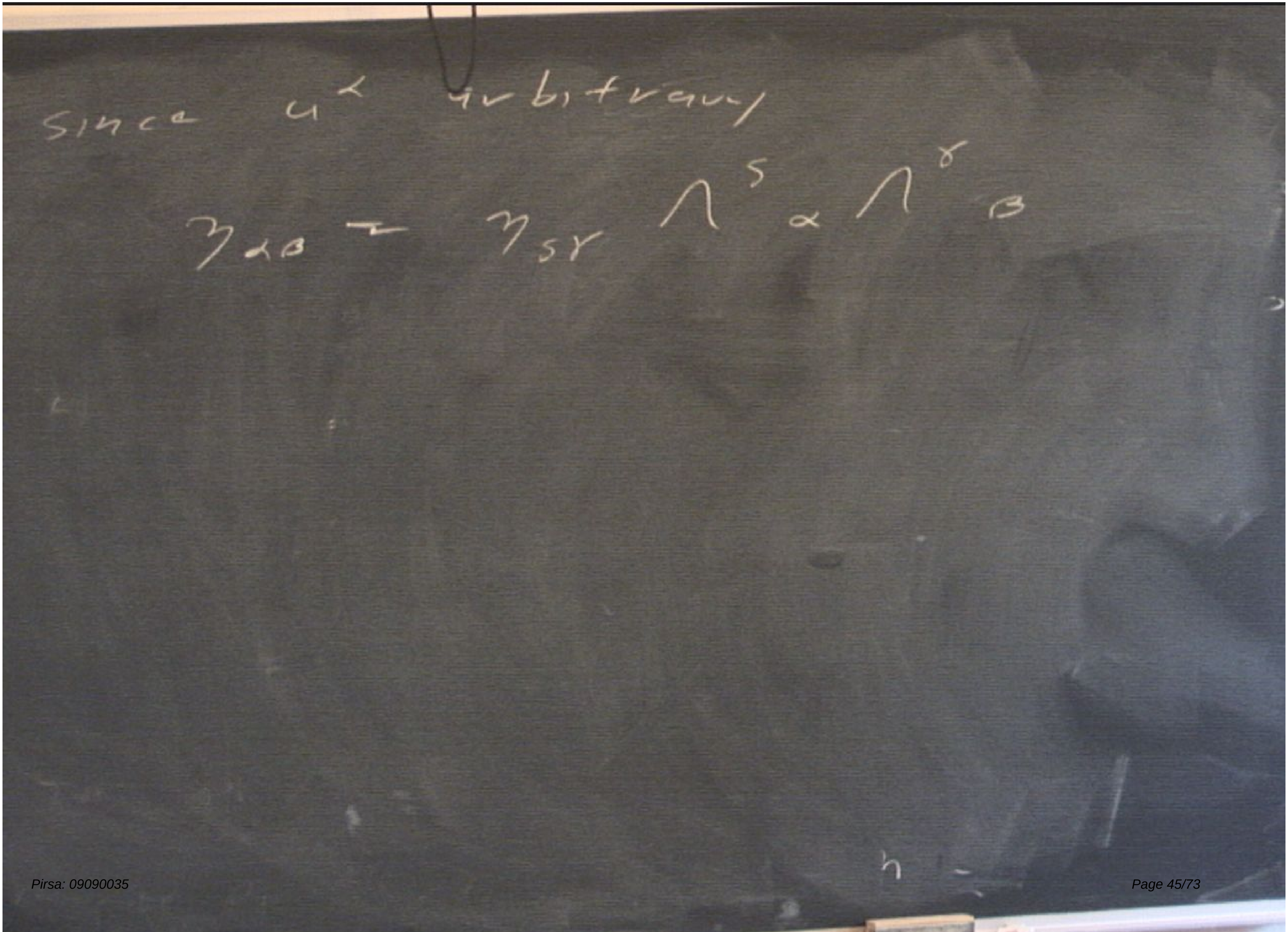
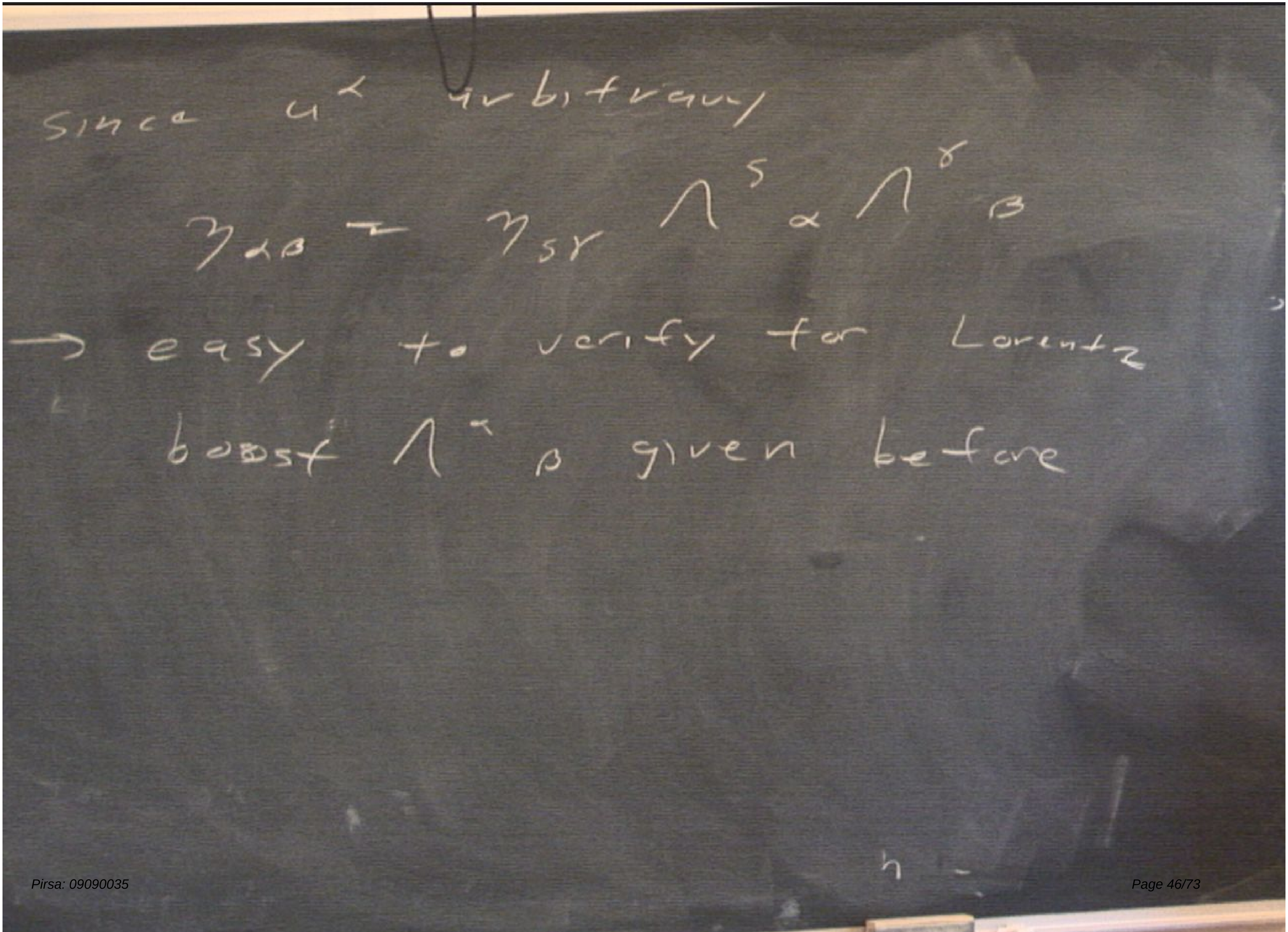


since u^α arbitrary

$$\eta_{\alpha\beta} = \eta_{\delta\gamma} \Lambda^\delta{}_\alpha \Lambda^\gamma{}_\beta$$



since u^α arbitrary

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→ easy to verify for Lorentz boost $\Lambda^{\alpha}{}_{\beta}$ given before since u^α arbitrary

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Summation convention (refined)

since u^α arbitrary

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→ easy to verify for Lorentz boost $\Lambda^\alpha{}_\beta$ given before

---

Summation convention (refined)

repeated index is summed over 0 1 2 3

$_{\alpha\beta}$ $_{\delta\gamma}$

→ easy to verify for Lorentz boost $\Lambda^{\alpha}{}_{\beta}$ given before

---

Summation convention (refined)

- repeated index is summed over 0 1 2 3
- in each pair one index is raised and one index is lowered

$_{\alpha\beta}$ $\quad$ $_{\delta\gamma}$

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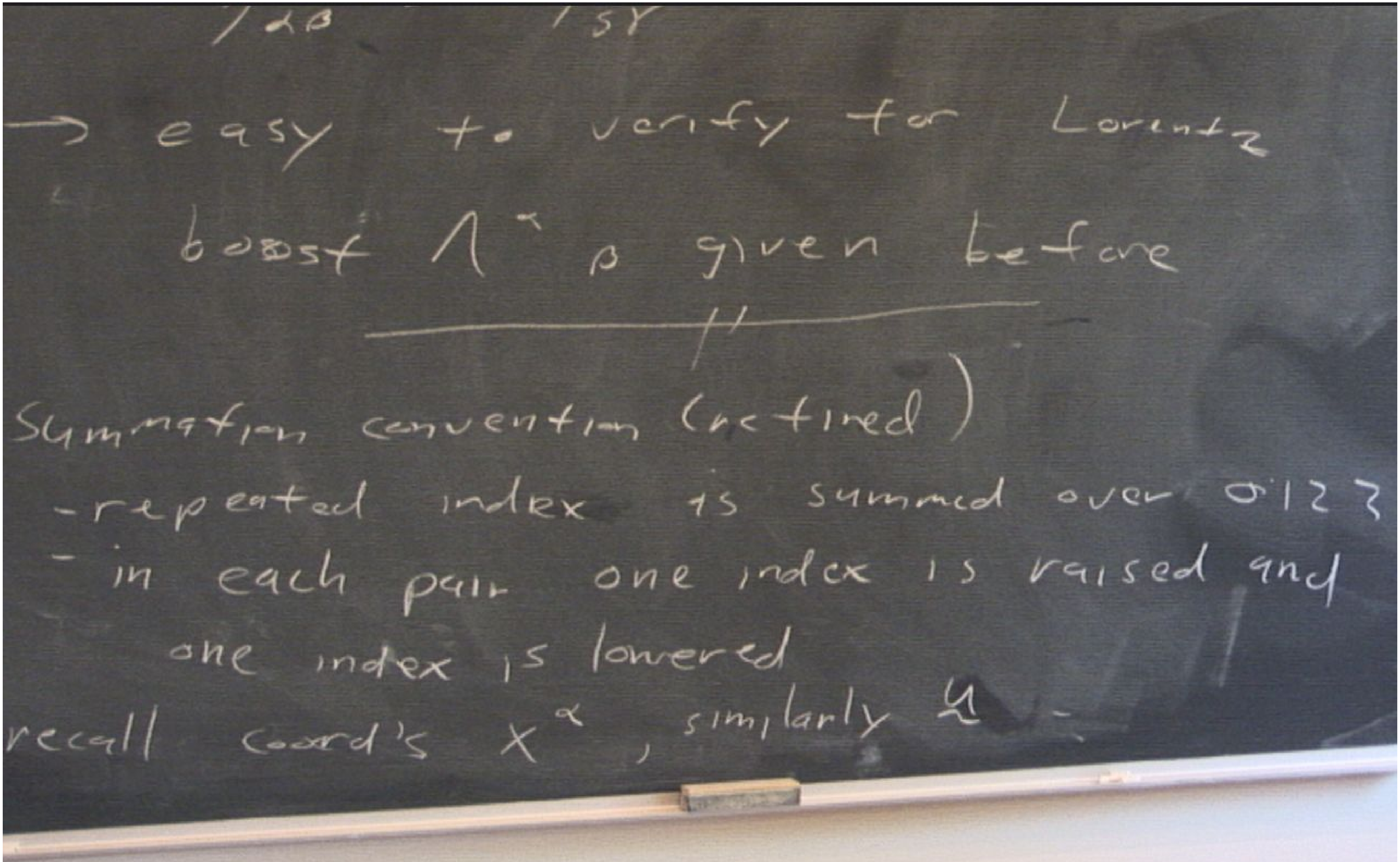
recall coord's x^{α} $\quad$ h



$/_{\alpha\beta}$ $/_{\delta\gamma}$

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recall coord's x^{α}, similarly $\underset{\sim}{u}$ -

$\eta_{\alpha\beta}$ $\eta_{\delta\gamma}$

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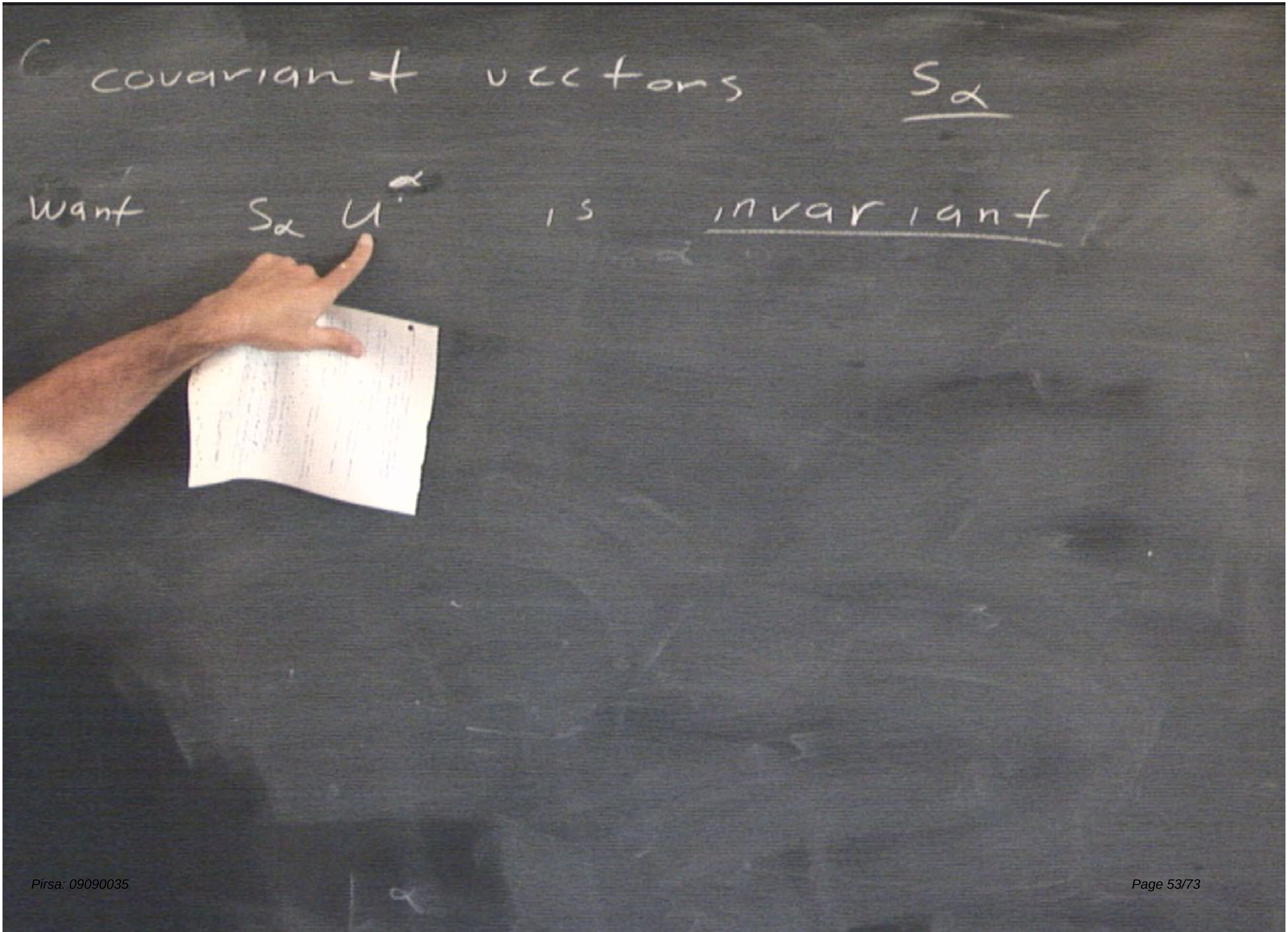
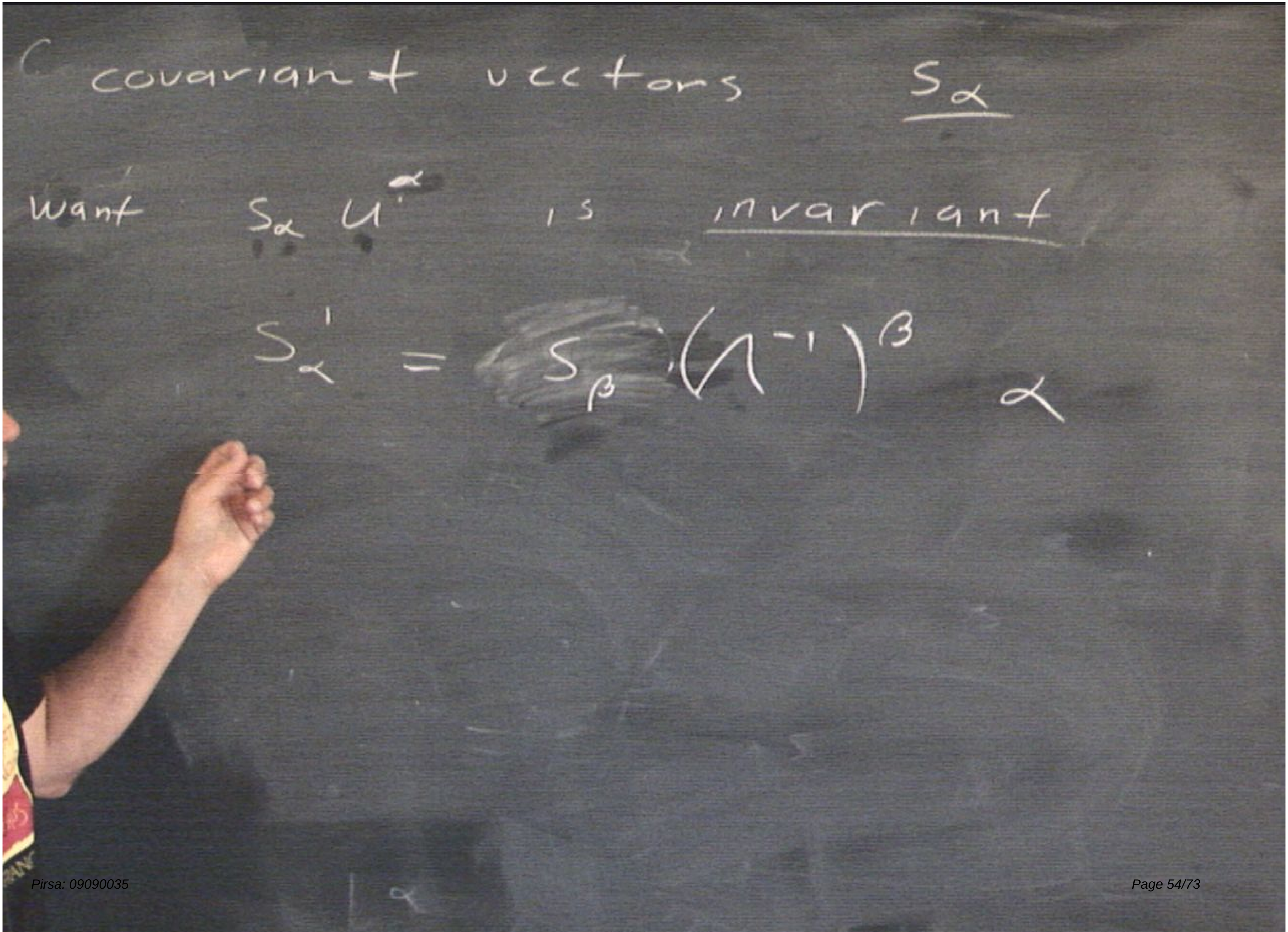
recall coord's x^{α}, similarly $\underset{\sim}{u}$ implicitly u^{α}



covariant vectors $\underline{S_\alpha}$

want $S_\alpha U^\alpha$ is <u>invariant</u>



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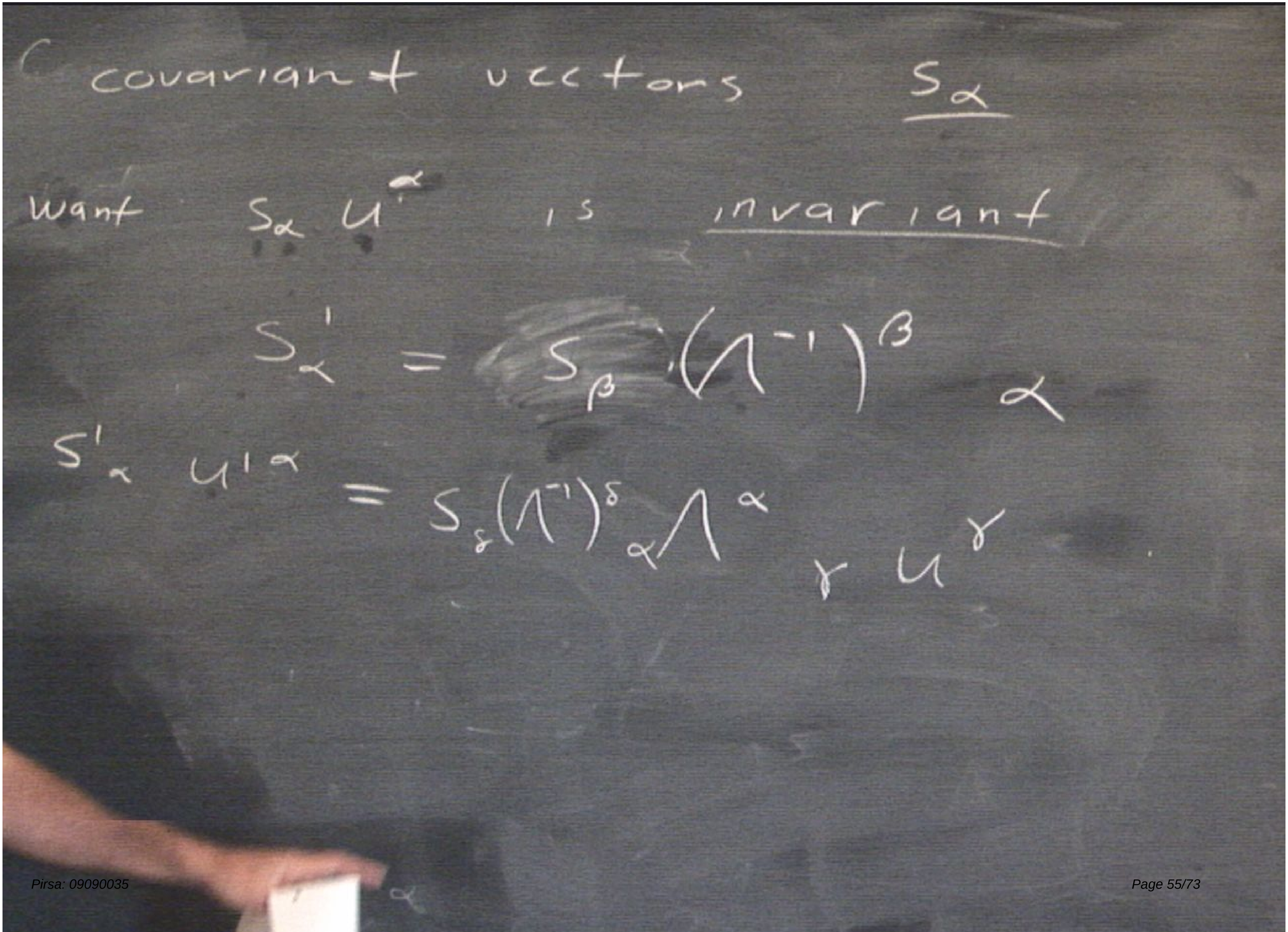
$$S'_\alpha = S_\beta (\Lambda^{-1})^\beta{}_\alpha$$



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$$S'_\alpha = S_\beta (\Lambda^{-1})^\beta{}_\alpha$$

$$S'_\alpha u'^\alpha = S_\delta (\Lambda^{-1})^\delta{}_\alpha \Lambda^\alpha{}_\gamma u^\gamma .$$

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$$S'_\alpha u'^\alpha = \underbrace{S_\delta (\Lambda^{-1})^\delta{}_\alpha \Lambda^\alpha{}_\gamma}_{\delta^\delta{}_\gamma} u^\gamma$$

$$= S_\gamma u^\gamma$$

covariant vectors $\underline{S_\alpha}$

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$$S'_\alpha = S_\beta \underline{(\Lambda^{-1})}^\beta{}_\alpha$$

$$S'_\alpha u'^\alpha = S_\delta \underbrace{(\Lambda^{-1})^\delta{}_\alpha \Lambda^\alpha{}_\gamma}_{\delta^\delta{}_\gamma} u^\gamma .$$

$$= S_\gamma u^\gamma$$

<u>heck</u>: $(\Lambda^{-1})^\beta{}_\alpha = \begin{bmatrix} \gamma & \frac{v}{c}\gamma & \\ \frac{v}{c}\gamma & \gamma & 0 \end{bmatrix}$

is <u>invariant</u>

$$S_\beta \underline{(\Lambda^{-1})}^\beta{}_\alpha$$

$$S_\delta \underbrace{(\Lambda^{-1})^\delta{}_\alpha \Lambda^\alpha{}_\gamma}_{\delta^\delta{}_\gamma} u^\gamma$$

$$S_\gamma u^\gamma$$

$$= \begin{bmatrix} \gamma & \frac{v}{c}\gamma & & \\ \frac{v}{c}\gamma & \gamma & & 0 \\ & & 1 & \\ & 0 & & 1 \end{bmatrix} = \Lambda_\alpha{}^\beta$$

$$= \eta_{\alpha\delta} \Lambda^\delta{}_\gamma \eta^{\gamma\beta}$$

inverse of η

$$(\eta^{-1})^{\alpha\beta}\,\eta_{\beta\gamma} = \delta^{\alpha}{}_{\gamma}$$

inverse of η

$$(\eta^{-1})^{\alpha\beta}\,\eta_{\beta\gamma} = \delta^{\alpha}{}_{\gamma}$$

$$\shortparallel$$

$$\eta^{\alpha\beta} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$(\eta^{-1})^{\alpha\beta}\,\eta_{\beta\gamma} = \delta^{\alpha}{}_{\gamma}$$

$$\parallel$$

$$\eta^{\alpha\beta} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$k^{\alpha}\,\eta_{\alpha\beta}\,x^{\alpha}$$

inverse

$$(\eta^{-1})^{\alpha\beta}\,\eta_{\beta\gamma} = \delta^{\alpha}{}_{\gamma}$$

$$\equiv$$

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$$\equiv$$

$$k_{\beta}$$

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$$(k^{\alpha}\,\boxed{\eta_{\alpha\beta}})\,x^{\beta} = k_{\beta}\,x^{\beta}$$

$$\parallel$$

$$k_{\beta}$$

$x\beta$

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$$\equiv$$

$$k_{\beta}$$

← we can use $\eta_{\alpha\beta}$ to lo

$\eta^{\alpha\beta}$ to r

$x\beta$

inverse η

$$(\eta^{-1})^{\alpha\beta}\,\eta_{\beta\gamma} = \delta^{\alpha}{}_{\gamma}$$

$$\|$$

$$\eta^{\alpha\beta} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$(k^{\alpha}\,\eta_{\alpha\beta})\,x^{\beta} = k_{\beta}\,x^{\beta}$$

$$\|$$

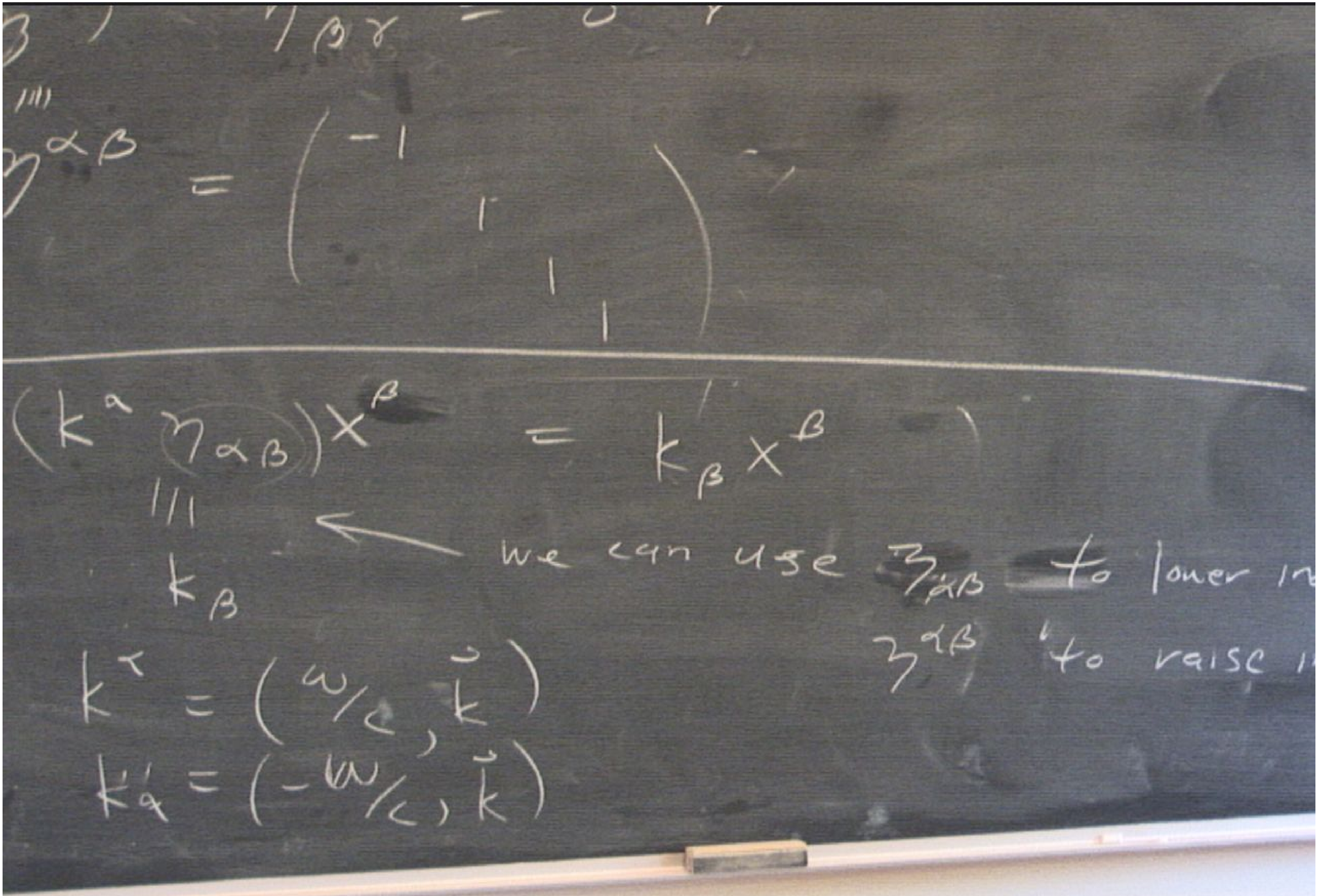
k_{β} ← we can use $\eta_{\alpha\beta}$ to lo

$\eta^{\alpha\beta}$ to re

$$k^{\alpha} = (\omega/c, \vec{k})$$

$$\|$$

β



$\beta) \quad \eta_{\beta\gamma} = \delta^{\gamma}$

$$\eta^{\alpha\beta} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$(k^{\alpha} \eta_{\alpha\beta}) x^{\beta} = k_{\beta} x^{\beta}$$

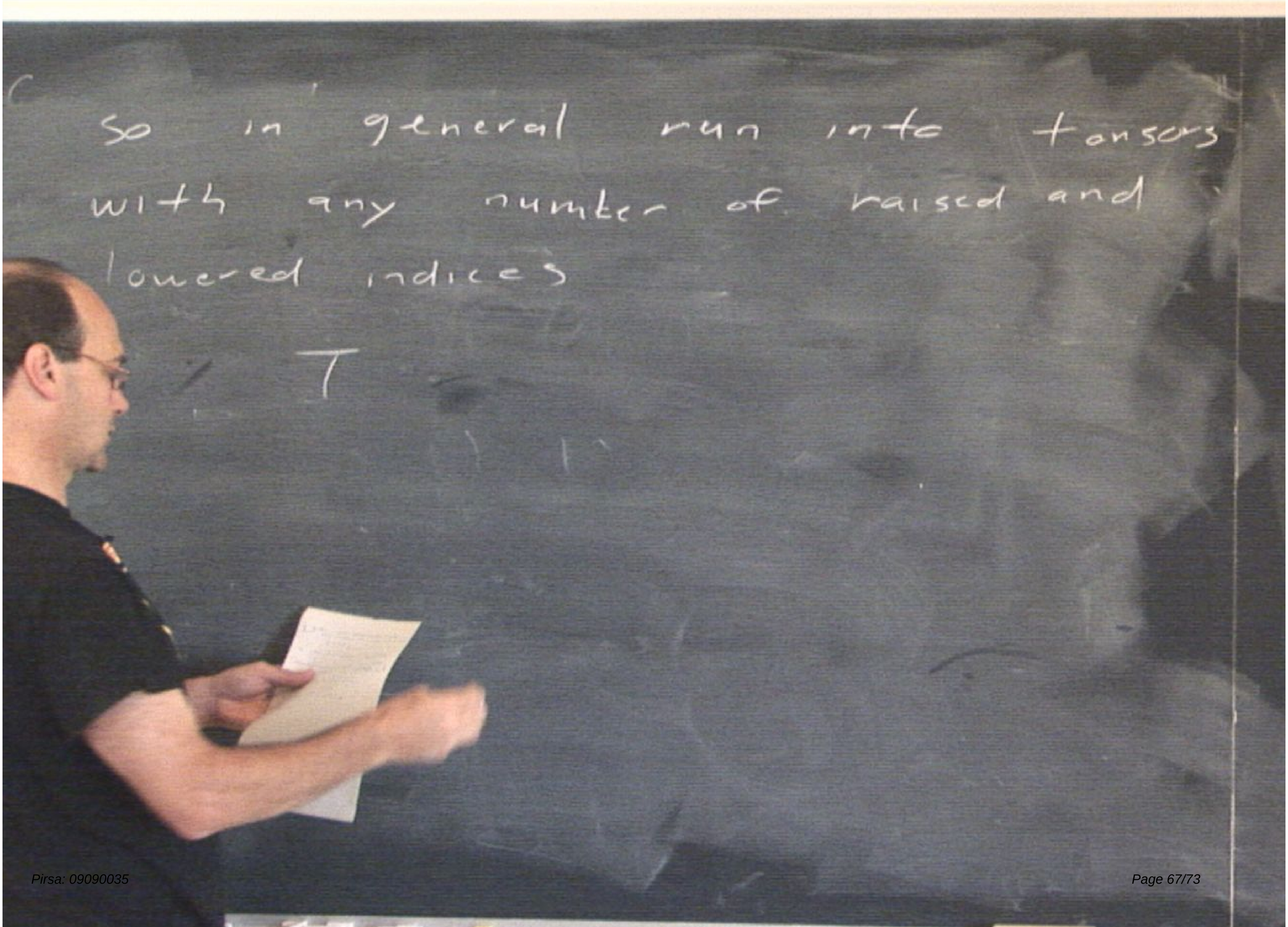
$$\underset{k_\beta}{\overset{|||}{}}$$

we can use $\eta_{\alpha\beta}$ to lower in

$\eta^{\alpha\beta}$ to raise i

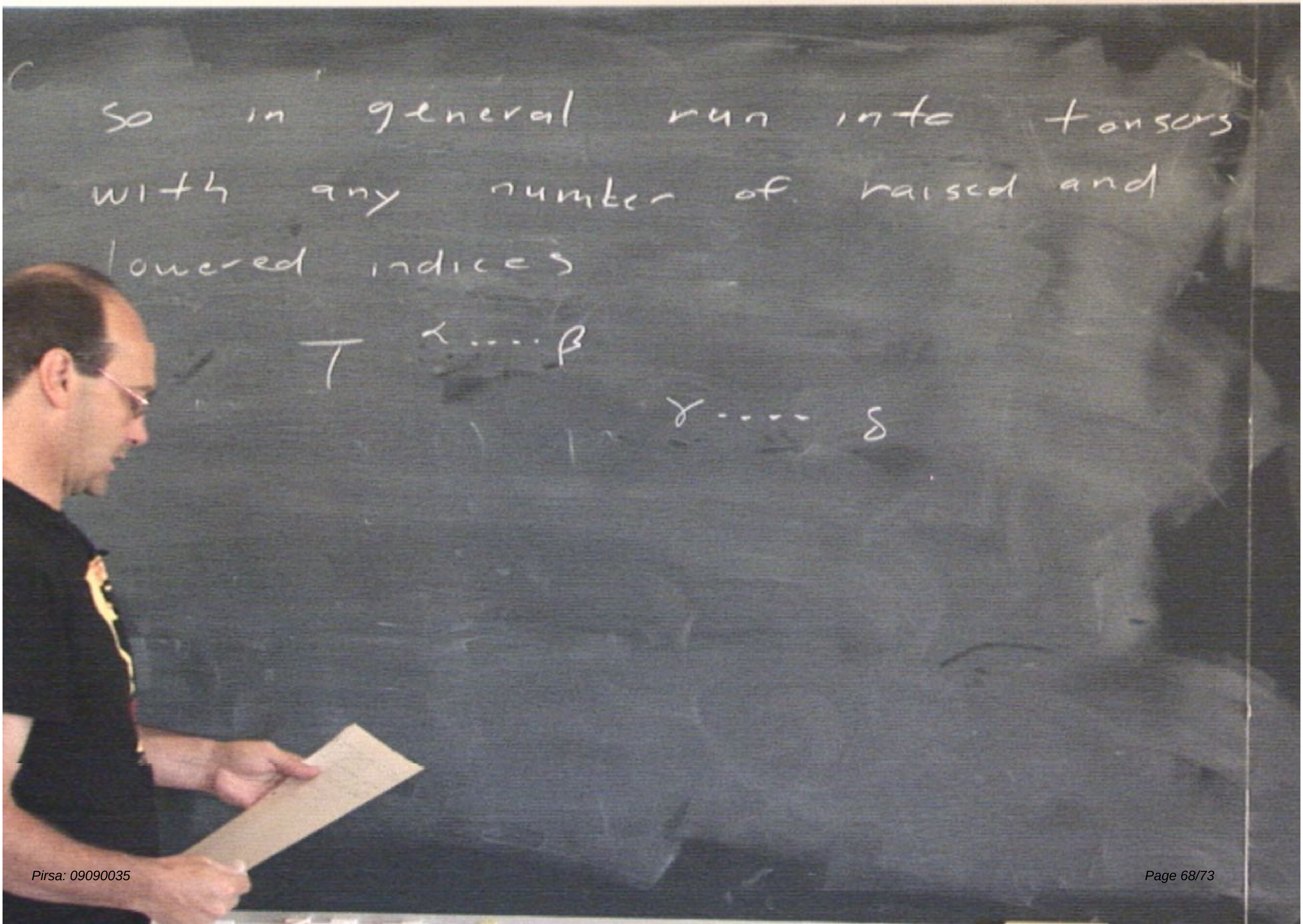
$$k^{\tau} = (\omega/c, \vec{k})$$

$$k_{\alpha} = (-\omega/c, \vec{k})$$



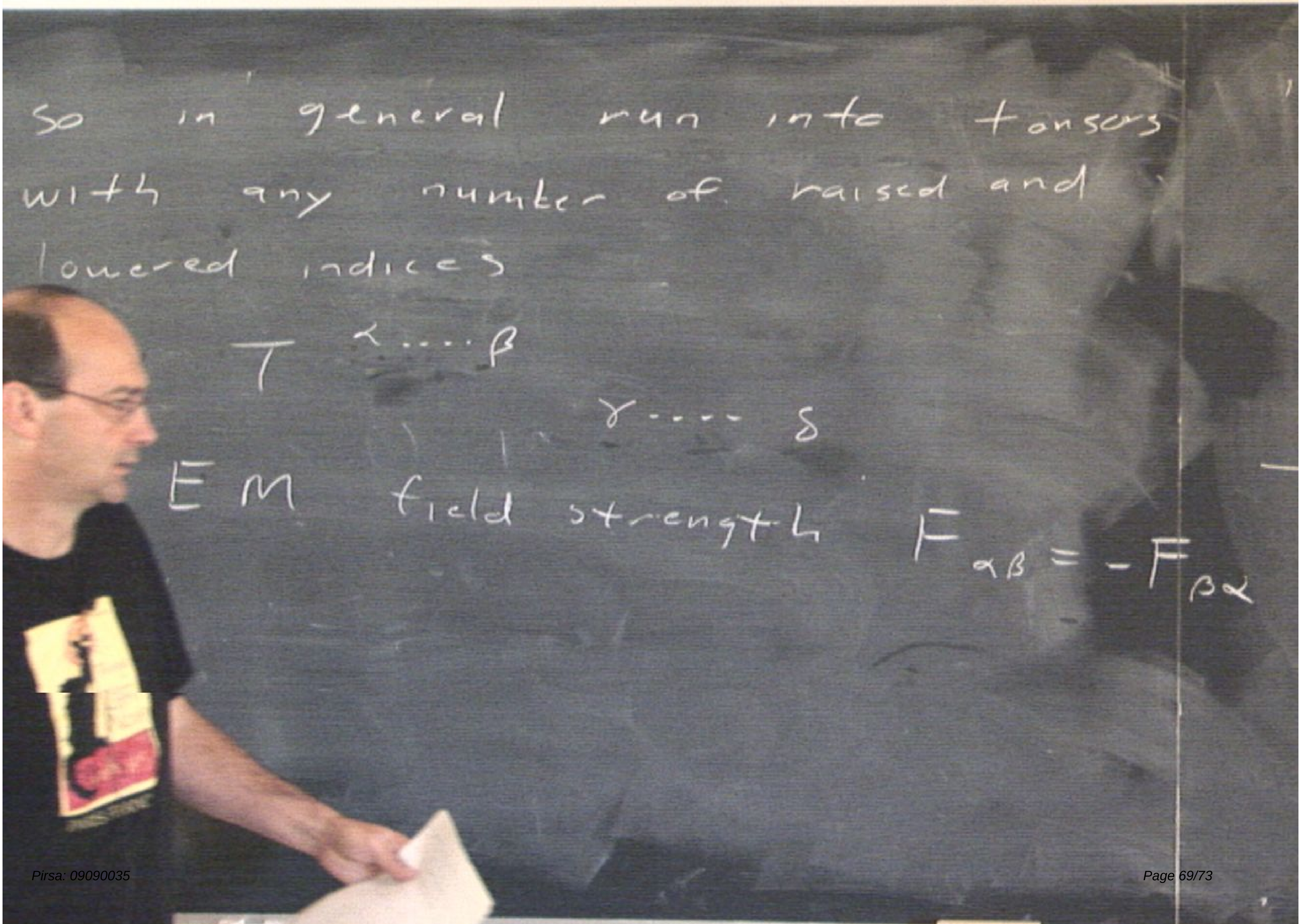
So in general run into tensors with any number of raised and lowered indices

T



So in general run into tensors with any number of raised and lowered indices

$$T^{\alpha \ldots\ldots \beta}{}_{\gamma \ldots\ldots \delta}$$



so in general run into tensors with any number of raised and lowered indices

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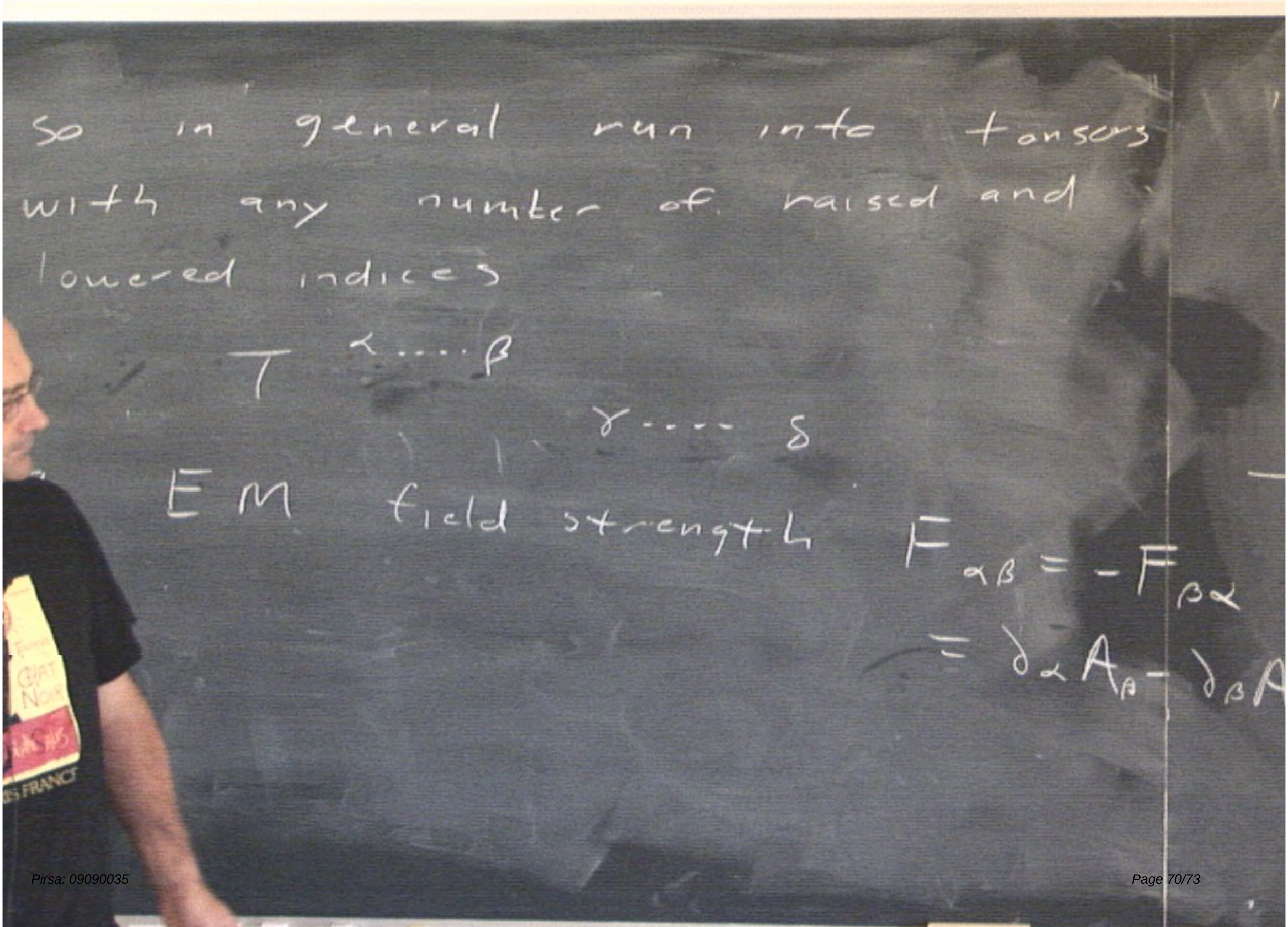
EM field strength $F_{\alpha\beta} = -F_{\beta\alpha}$



So in general run into tensors with any number of raised and lowered indices

$$T^{\alpha\ldots\beta}{}_{\gamma\ldots\delta}$$

EM field strength $F_{\alpha\beta} = -F_{\beta\alpha}$

$$= \partial_\alpha A_\beta - \partial_\beta A$$

$- \quad \delta$

$g + L$

$$F_{\alpha\beta} = -F_{\beta\alpha}$$

$$= \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

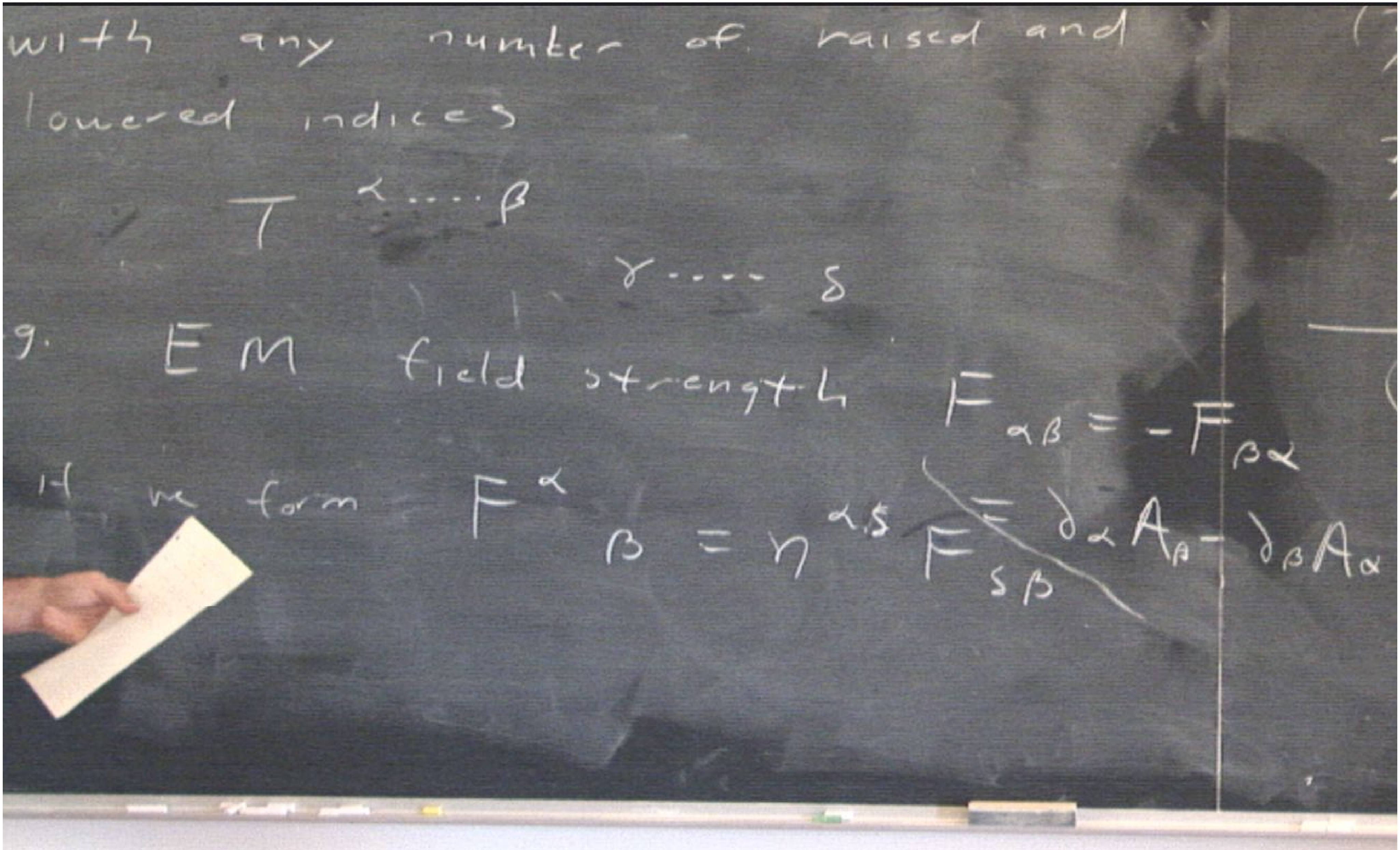
$(k^\alpha$

$||$

k

k^τ

k'_α



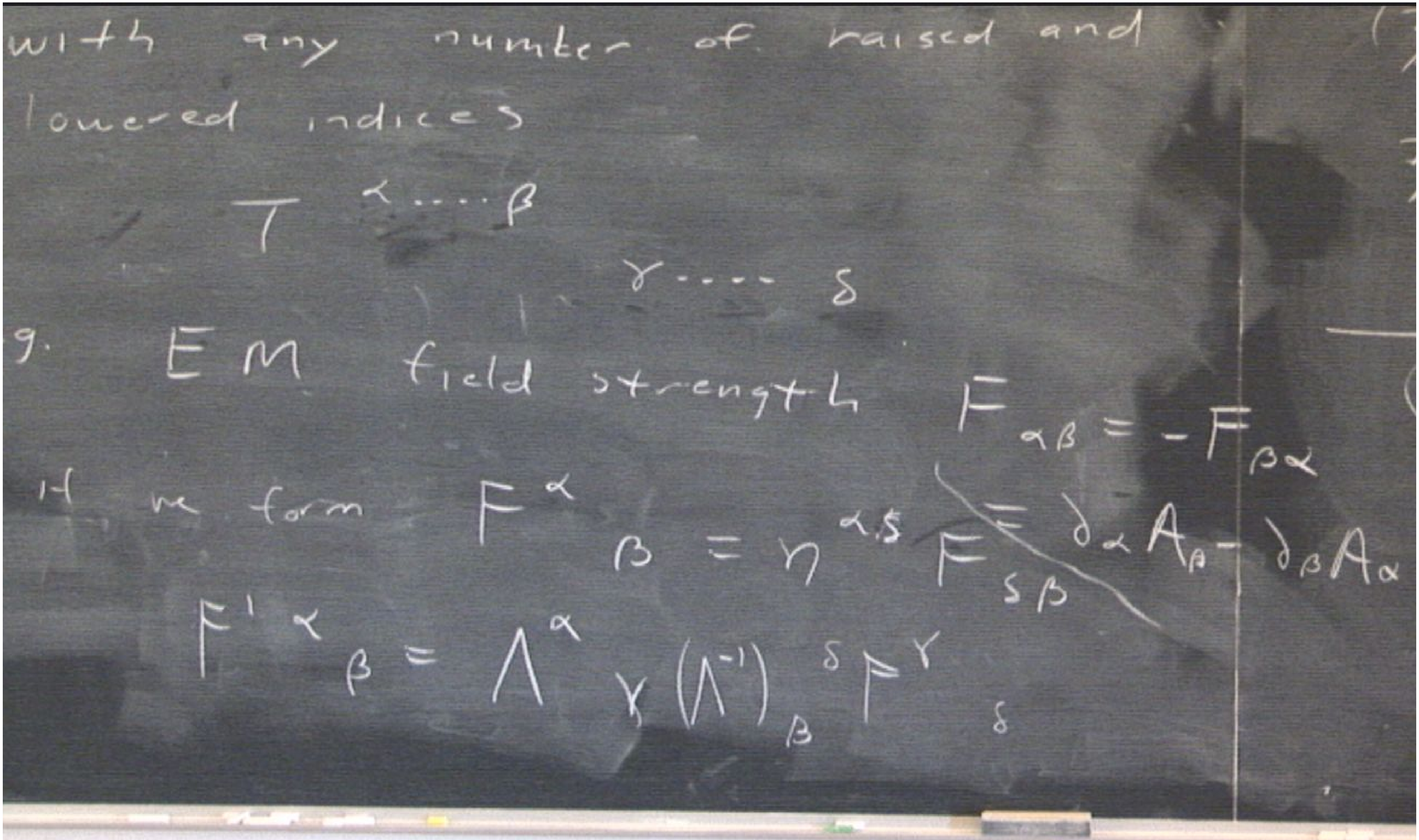
with any number of raised and
lowered indices

$$T^{\alpha\cdots\cdots\beta}{}_{\gamma\cdots\cdots\delta}$$

g. EM field strength $F_{\alpha\beta} = -F_{\beta\alpha}$

$= \partial_\alpha A_\beta - \partial_\beta A_\alpha$

If we form $F^{\alpha}{}_{\beta} = \eta^{\alpha\delta} F_{\delta\beta}$



with any number of raised and lowered indices

$$T^{\alpha\cdots\cdots\beta}{}_{\gamma\cdots\cdots\delta}$$

g. EM field strength $F_{\alpha\beta} = -F_{\beta\alpha}$ $= \partial_\alpha A_\beta - \partial_\beta A_\alpha$

H we form $F^{\alpha}{}_{\beta} = \eta^{\alpha\delta} F_{\delta\beta}$

$$F'^{\alpha}{}_{\beta} = \Lambda^{\alpha}{}_{\gamma} (\Lambda^{-1})_{\beta}{}^{\delta} F^{\gamma}{}_{\delta}$$