Title: Aspects of SUSY breaking

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Abstract: We present some exact results and new approaches to SUSY breaking theories.

Aspects of SUSY Breaking

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ZK and Nathan Seiberg : arXiv:0907.2441

Motivations

Supersymmetry is important for particle physics, string theory and many other branches of physics and mathematics.

We do not yet have a good understanding of spontaneous SUSY breaking. This is important: If we want to make contact with experiment SUSY has to be broken.

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Breaking of global $\mathcal{N} = 1$ supersymmetry predicts the existence of a massless Weyl fermion G_α.

Studying it we hope to elucidate generalities of SUSY breaking and make contact with phenomenology. There are many open questions.

Open Questions

- What is the connection between the UV physics and the Goldstinos?

Models of SUSY breaking are sometimes incalculable and even worse, there is no superpotential description.

Can we say which UV operators are associated to the low energy Goldstino? Is there a useful superspace description at low energies?

How do we parameterize SUSY breaking in strongly coupled models?

Open Questions

- Is there any difference between D-term and F-term breaking?

How is D-term vs. F-term breaking reflected in the Goldstino Lagrangian?

Open Questions

- What are the interactions of the Goldstino particle with itself?

In the 1970's Akulov and Volkov proposed a Lagrangian:

$$\mathcal{L}_{A-V} \sim F^2 \det \left[\delta^\mu_\nu - \frac{i}{F^2} \partial_\nu G \sigma^\mu \bar{G} + \frac{i}{F^2} G \sigma^\mu \partial_\nu \bar{G} \right]$$

$$\sim F^2 + G\sigma^\mu \partial_\mu \bar{G} + \cdots$$

It was before the discovery of SUSY.

What is the role of this Lagrangian? Why is there a symmetry $G_\alpha \to e^{i\theta} G_\alpha$? How to describe corrections?

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- What are the interactions of the Goldstino particle with light matter particles?

If SUSY is broken in field theory, the Goldstino is out there.

What are its leading interactions with the visible sector? gluons? matter fermions? Higgs fields?

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Outline

We present the tools with which we address these issues.

- Models that Break SUSY
- Broken Symmetries
- The Supercurrent Multiplet
- Connecting the UV and the IR
- Goldstinos and some Lagrangians
- Matter Fields
- Summary

Basic Examples of SUSY Breaking

Consider a single chiral field, Φ

$$\Phi = \phi + \theta\psi_\phi + \theta^2 F$$

with a free Lagrangian

$$\int d^4\theta |\Phi|^2 + \int d^2\theta f\Phi + c.c.$$

The vacuum energy is $|f|^2 \neq 0$ and therefore SUSY is broken.

Since the theory is free the spectrum is supersymmetric, $m_\phi = m_{\psi_\phi} = 0$.

Basic Examples of SUSY Breaking

Now, think of:

$$\int d^4\theta \left(|\Phi|^2 - \frac{1}{M^2}|\Phi|^4 \right) + \int d^2\theta f \Phi + c.c.$$

The scalar potential is

$$V = 1 + \frac{4|f|^2}{M^2}|\phi|^2 + \ldots$$

There is nice vacuum at $\phi = 0$ with vacuum energy $|f|^2$.
The spectrum is $m_\phi = 2|f|/M$ and $m_{\psi_\phi} = 0$.

The fermion ψ_ϕ is massless because it is the Goldstino.

More Examples of SUSY Breaking

- Renormalizable models of SUSY breaking. E.g. all the O'Raifeartaigh-like models and their recent reincarnation.
- Calculable dynamical models. For example, 3-2 model (ADS), deformed quantum moduli space (ITIY), massive SQCD (ISS).
- Incalculable models with strong (but indirect) evidence of SUSY breaking, e.g. $SU(5)$ and $SO(10)$ (ADS).

What are the general principles common to all these examples?

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Symmetry Breaking

For a conserved charge Q we can associate a conserved current

$$\partial^\mu j_\mu = 0 \,.$$

Even if the symmetry is spontaneously broken, the operator equation $\partial^\mu j_\mu = 0$ holds but $Q = \int d^3x\, j_0$ diverges in the IR.

In spite of this,

$$[Q, \mathcal{O}] \,,$$

where $\mathcal{O}$ is any local operator is well defined and local.

Symmetry Breaking

We conclude that even if a symmetry is spontaneously broken,

All the operators sit in representations of the symmetry.

The Supercurrent Multiplet

$\mathcal{N} = 1$ supersymmetric theories have a conserved supercurrent,

$$\partial^\mu S_{\mu\alpha} = 0 \ .$$

We can study the multiplet of the supercurrent, i.e. calculate $\{Q, S_{\mu\alpha}\}$, $\{Q^\dagger, S_{\mu\alpha}\}$ etc.

In this way we find the conserved energy momentum tensor $T_{\mu\nu}$, some R-current j_μ^R (which may not be conserved) and an operator which we will call x.

They can all be written explicitly in a given microscopic theory.

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A nice way to package this multiplet was given by Ferrara and Zumino. They used a real multiplet $\mathcal{J}_\mu$. Its components

$$\mathcal{J}_\mu = j_\mu + \theta^\alpha \left(S_{\mu\alpha} + \frac{1}{3}(\sigma_\mu \bar{\sigma}^\rho S_\rho)_\alpha \right) + c.c.$$

$$+(\theta\sigma^\nu\bar{\theta}) \left(2T_{\nu\mu} - \frac{2}{3}\eta_{\nu\mu}T - \frac{1}{2}\epsilon_{\nu\mu\rho\sigma}\partial^\rho j^\sigma \right) + \frac{i}{2}\theta^2\partial_\mu x^\dagger - \frac{i}{2}\bar{\theta}^2\partial_\mu x \cdots .$$

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The information encoded in the superfield is equivalent to the current algebra

$$\{Q^\dagger_{\dot\beta}, S_{\mu\alpha}\} = \sigma^\nu_{\alpha\dot\beta}\left(2T_{\mu\nu} + i\eta_{\nu\mu}\partial j - i\partial_\nu j_\mu - \frac{1}{4}\epsilon_{\nu\mu\rho\sigma}\partial^{[\rho}j^{\sigma]}\right)$$

$$\{Q_\beta, S_{\mu\alpha}\} = 2i(\sigma_{\mu\nu})_{\alpha\beta}\partial^\nu x^\dagger$$

This holds even if SUSY is broken. We see that the mysterious x is a well defined operator in the theory. It can be obtained by varying the supercurrent.

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In the superfield language, the conservation equations follow from

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Relation to Goldstino

We need to understand X better. The supercurrent $S_{\mu\alpha}$ has two different Lorentz representations $(1, 1/2)$, $(0, 1/2)$. If supersymmetry is broken the $(0, 1/2)$ component is the Goldstino. (And $(1, 1/2)$ decouples.)

Therefore, at very long distances, ψ becomes the Goldstino

$$\psi \sim \sigma^{\mu}_{\alpha\dot{\alpha}} \bar{S}^{\dot{\alpha}}_{\mu} \sim G_{\alpha} .$$

The chiral superfield X must flow at low energies to a chiral superfield which contains the Goldstino!

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This superfield X is expected to be nonlinear at low energies. What could x flow to?

We know,

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In addition, the F component of X is just the vacuum energy T^μ_μ.

There is not much choice:

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Relation to Goldstino

SUSY fixes all the coefficients and we get:

$$X \to X_{NL} = \frac{G^2}{2F} + \sqrt{2}\theta G + \theta^2 F \ .$$

Note the equation

$$X_{NL}^2 = 0$$

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So far we have seen that

- The Goldstino always sits in a chiral superfield (regardless of D-terms), which can be defined in the UV. This makes perfect sense even in incalculable examples. We can therefore calculate in all theories

$$\langle x(\mathbf{r_1})x(\mathbf{r_2})x(\mathbf{r_3})...\rangle \sim \langle G^2(\mathbf{r_1})G^2(\mathbf{r_2})G^2(\mathbf{r_3})...\rangle$$

- X generalizes the SUSY-breaking “spurion." Therefore, a chiral spurion superfield exists not only in weakly coupled examples!

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$$\int d^4\theta |X_{NL}|^2 + \int d^2\theta f X_{NL} + c.c.$$

This gives the Akulov-Volkov theory! There is an accidental R-symmetry. Previous work from different perspectives:

Akulov,Volkov,Rocek,Lindstrom,Wess,Samuel,Clark,Love...

Corrections are controlled by scaling under which $S(X_{NL}) = -1$, $S(d\theta) = 1/2$. So, all the A-V terms have $S = 0$. E.g. we can write $\int d^4\theta |\partial X_{NL}|^2$ which has $S = 2$.

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Corrections are controlled by scaling under which $S(X_{NL}) = -1$, $S(d\theta) = 1/2$. So, all the A-V terms have $S = 0$. E.g. we can write $\int d^4\theta |\partial X_{NL}|^2$ which has $S = 2$.

Lagrangians

In components the meaning of this scaling is that the Goldstino is assigned dimension $-1/2$. The terms following from

$$\int d^4\theta |X_{NL}|^2 + \int d^2\theta f X_{NL} + c.c.$$

are of the form

$$|f|^2 + \bar{G}\bar{\sigma}\partial G + \bar{G}^2\partial^2(G^2) + ...$$

and we see that they all indeed have $S = 0$. Can prove that $\int d^4\theta |X_{NL}|^2$ is unique at scaling zero. The accidental R-symmetry can be broken at higher orders.

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Let us see how all these ideas work in the simplest nontrivial example.

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Reminder:

$$\mathcal{L} = \int d^4\theta \left(\bar{\Phi}\Phi - \frac{1}{M^2}(\bar{\Phi}\Phi)^2 \right) + \int d^2\theta f \Phi + c.c. \, .$$

$\phi = 0$ is a good vacuum. The vacuum energy is $|f|^2$ and the spectrum is

$$m_\phi^2 = |f|^2/M^2 \, , \qquad m_{\psi_\phi} = 0 \, .$$

ψ_ϕ is the Goldstino. Our description at low energies should include only ψ_ϕ as a physical field. We should integrate out ϕ.

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At zero-momentum the Lagrangian is

$$\mathcal{L}_{p=0} = -\frac{1}{M^2}(2\phi F_\phi - \psi_\phi^2)(2\phi^\dagger F_\phi^\dagger - \bar{\psi}_\phi^2) \ .$$

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The derivation and low energy results are independent of M. They are completely general.

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Matter Fields

With similar technics and ideas we can solve the problem of including matter fields. In addition to $X_{NL}^2 = 0$ we have

- Matter fermions (e.g. electrons) $X_{NL} Q_{NL} = 0$
- Nonlinear Wess-Zumino Gauge $XV = 0$
- Gauge fields (e.g. photon) $X_{NL} W_{\alpha NL} = 0$
- Higgs fields $X_{NL} \bar{\mathcal{H}} = chiral$
- Goldstone bosons (e.g. axions) $X_{NL} \left(\mathcal{A} - \bar{\mathcal{A}} \right) = 0$

From this we can calculate all the interesting couplings of matter fields to Goldstinos.

$$\int d^4\theta\, |X_{NL}|^2 + \int d^2\theta\, f X_{NL} + c.c.$$

$$\mathcal{L} \supset \frac{1}{f^2} G^2 \Box \bar{G}^2$$

$$\int d^4\theta \left(|X_{NL}|^2 + |Q_{NL}|^2\right) + \int d^2\theta\, f X_{NL}$$

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$$+ |\partial Q_{NL}|^2$$

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Conclusions

- Supersymmetry and superspace are useful even when SUSY is broken.
- We can follow the supercurrent multiplet $\mathcal{J}_{\alpha\dot{\alpha}}$ and the associated X along the flow.
- X flows to the Goldstino multiplet and satisfies $X^2 = 0$ at long distances.
- We can efficiently find the interesting interactions of the Goldstino and other particles.
- The deep low-energy theory is universal.

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