

Title: Introduction to Effective Field Theory - Lecture 2B

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URL: <http://pirsa.org/09090021>

Abstract:

EFFECTIVE ACTIONS:

(IPI, ILPI, Wilson)

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(1PI, 1LPI, Wilson)

Consider the generating functional for correlation functions: $\langle \mathcal{R} | T(\phi(x_1) \dots \phi(x_n)) | \Omega \rangle$ $\langle \mathcal{R} | \Omega \rangle = 1$

In a path integral representation:

← path integral representation:

recall: $\langle \psi | \psi \rangle = \int \mathcal{D}\phi e^{iS(\phi)}$

$\langle \psi | \psi \rangle$

$\langle \psi | \psi \rangle = \langle \psi | \psi \rangle$

In a path integral representation:

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$\langle \psi | \psi \rangle = \langle \psi | \psi \rangle$

In a path integral representation:

$$\langle \psi_1 | \psi_2 \rangle = \int \mathcal{D}\phi e^{iS(\phi)} \phi(x_1) \phi(x_2)$$

$$\langle \psi_1 | \psi_2 \rangle$$

$$\phi(x_1) \phi(x_2) = \phi(x_1) \phi(x_2)$$

In a path integral representation:

$$\langle \psi(x_1) | \psi(x_2) \rangle = \int \mathcal{D}\phi e^{iS(\phi)} \phi(x_1) \cdot \phi(x_2)$$

$$\langle \psi(x_1) | \psi(x_2) \rangle$$

$$\hat{\phi}(x_1) | \phi(x_1) \rangle = \phi(x_1) | \phi(x_1) \rangle$$

generating functional:

$$Z[J] = \int \mathcal{D}\phi e^{iS(\phi) + i \int J(x)\phi(x) dx}$$

$$\frac{\delta Z}{\delta J(x)} = i \int \mathcal{D}\phi e^{iS + i \int J\phi dx} \phi(x)$$

Generating functional:

$$Z[J] = \int \mathcal{D}\phi e^{iS(\phi) + i\int J(x)\phi(x)dx}$$

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Generating functional:

$$e^{iW[J]} = \int \mathcal{D}\phi \, e^{iS(\phi) + i\int J(x)\phi(x)dx} \left. \frac{\delta^n Z}{\delta J(x_1) \cdots \delta J(x_n)} \right|_{J=0} = G(x_1, \dots, x_n)$$

$$\frac{\delta^n Z}{\delta J(x_1) \cdots \delta J(x_n)} = i^n \int \mathcal{D}\phi \, e^{iS + i\int J\phi dx} \phi(x_1) \cdots \phi(x_n)$$

In a path integral representation:

$$\int_{-\infty}^{\infty} dx e^{-i\lambda x^2} = \sqrt{\frac{2\pi}{\lambda}} e^{i\pi/4}$$

$$\langle \Omega | T \left(\prod_{i=1}^n \phi(x_i) \right) | \Omega \rangle = \int \mathcal{D}\phi e^{iS(\phi)} \phi(x_1) \dots \phi(x_n) = G(x_1, \dots, x_n)$$

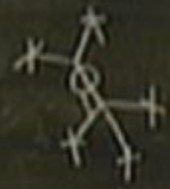
$$\langle \Omega | \phi(x) | \Omega \rangle = 0$$

$$\hat{\phi}(x) | \phi(x) \rangle = \phi(x) | \phi(x) \rangle$$

Generating functional:

$$e^{iW[J]} = \int \mathcal{D}\phi \ e^{iS(\phi) + i \int J(x) \phi(x) dx} \quad (-i)^n \frac{\delta^n Z}{\delta J(x_1) \cdots \delta J(x_n)} \Big|_{J=0} = G(x_1, \dots, x_n)$$

$$\frac{\delta^n Z}{\delta J(x_1) \delta J(x_2)} = i^n \int \mathcal{D}\phi \ e^{iS + i \int J \phi dx} \ \phi(x_1) \cdot \phi(x_2)$$



Generating functional:

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$$\langle \Omega | (\phi(x_1) \dots \phi(x_n)) | \Omega \rangle \quad \langle \Omega | \Omega \rangle = 1$$

in perturbation theory $S(\phi) = S_{\text{quad}} + S_{\text{int}}$

$$S_{\text{quad}} = \int d^d x \phi \Delta \phi$$

$$\text{and } e^{iS_{\text{int}}} = 1 + iS_{\text{int}} + \frac{i^2}{2!} S_{\text{int}}^2 + \dots$$

gives $Z[J]$ as a sum of Feynman graphs where $\text{---} \rightarrow \Delta^{-1}$
 in usual way; and $W[J] = -i \ln Z = \text{sum of connected graphs}$

Define $\Gamma(\varphi)$ as follows (1PI effective action)

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$$\frac{\delta W}{\delta J(x)} = \frac{\int \mathcal{D}\phi e^{iS(\phi) + i\int \mathcal{J}\phi} \phi(x)}{\int \mathcal{D}\phi e^{iS(\phi) + i\int \mathcal{J}\phi}}$$

Define $\Gamma(\varphi)$ as follows (IPT effective action)

$$\frac{\delta W}{\delta J(x)} = \frac{\int \mathcal{D}\phi e^{iS(\phi) + i\int J\phi} \phi(x)}{\int \mathcal{D}\phi e^{iS(\phi) + i\int J\phi}} = \varphi = \frac{\langle \Omega | \phi(x) | \Omega \rangle}{\langle \Omega | \Omega \rangle}$$

Define $\Gamma(\varphi) = W(J(\varphi)) + \int J(\varphi)\varphi$

Define $\Gamma(\varphi) = W(J(\varphi)) + \int J(\varphi)\varphi$

Legendre transform of $W(J)$

$$\frac{\delta \Gamma}{\delta \varphi(x)} = \int d^4 y \frac{\delta W}{\delta J(y)} \frac{\delta J(y)}{\delta \varphi(x)} + \int d^4 y \frac{\delta J(y)}{\delta \varphi(x)} \varphi(y) + J(x)$$

$$|\varphi(x_1), \dots, \varphi(x_n)\rangle |\Omega\rangle \quad \langle \Omega | \Omega \rangle = 1$$

$$S_{\text{eff}} = S(\varphi) + \int J \phi \, d^4x$$

$$\frac{\delta S_{\text{eff}}}{\delta \phi} = 0 = \frac{\delta S}{\delta \phi} + J \approx 0.$$

compare with $\frac{\delta \Gamma}{\delta \phi} + J = 0$

$$|\varphi(x_1) \dots \varphi(x_n)\rangle |\Omega\rangle \quad \langle \Omega | \Omega \rangle = 1$$

$$S_{\text{eff}} = S(\varphi) + \int J \phi J^* x$$

$$\frac{\delta S_{\text{eff}}}{\delta \varphi} = 0 = \frac{\delta S}{\delta \varphi} + J \approx 0.$$

Compare with $\frac{\delta \Gamma}{\delta \varphi} + J = 0$ when $J=0$ $\frac{\delta \Gamma}{\delta \varphi} = 0$ determines $\varphi(J=0)$

$$\begin{aligned}
 e^{i\Gamma(\varphi)} &= e^{iW(J) - i \int J\varphi d^4x} = \int \mathcal{D}\varphi e^{iS(\varphi) + i \int J(\varphi - \varphi_0) d^4x} \\
 &= \int \mathcal{D}\Phi e^{iS(\varphi + \varphi_0) + i \int J\Phi d^4x}
 \end{aligned}$$

$$e^{i\Gamma(\varphi)} = e^{iW(J) - i \int J\varphi d^4x} = \int \mathcal{D}\phi e^{iS(\phi) + i \int J(\phi - \varphi) d^4x}$$

$$= \int \mathcal{D}\Phi e^{iS(\varphi + \Phi) + i \int J\Phi d^4x}$$

$\int d^4y \frac{\delta S}{\delta \varphi(y)} \Phi(y)$

$$S(\varphi + \Phi) = S(\varphi) + S'(\varphi)\Phi + \frac{1}{2} S''(\varphi)\Phi^2 + \text{cubic} + \text{higher.}$$

$$e^{i\Gamma(\varphi)} = e^{iS(\varphi)} \int \mathcal{D}\phi$$

$$e^{i\Gamma(\varphi)} = e^{iS(\varphi)} \int \mathcal{D}\phi e^{i \int d^4x \left(\frac{\delta S}{\delta \phi(x)} - J(x) \right) \Phi(x)}$$

$$e^{i\Gamma(\varphi)} = e^{iS(\varphi)} \int \mathcal{D}\phi e^{i \int d^4x \left(\frac{\delta S}{\delta \phi(x)} - \underline{J}(x) \right) \Phi(x) + \frac{i}{2} S''(\varphi) \Phi^2}$$

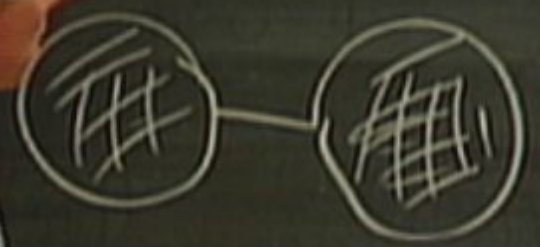
$$J = -\frac{\delta \Gamma}{\delta \varphi} \quad \text{where } \Gamma = S + \Gamma_{1-loop}$$

$$\Gamma_{1-loop} = (\det S''(\varphi))^{-1/2}$$

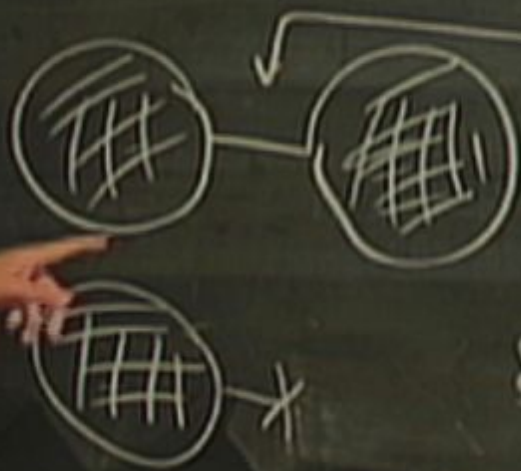
$$\left[1 + (\text{cubic}) + \frac{1}{2} (\text{cubic})^2 \right]$$

In usual way: and $W(\Gamma) = \dots$

Claim: \rightarrow terms ~~will~~ be evaluated at $J = -\frac{5\Gamma}{8\mu}$,
cancel out all ~~irreducible~~ reducible graphs.



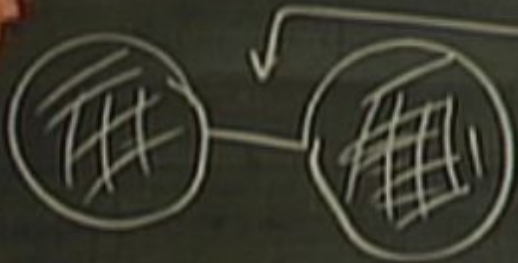
Claim: \times terms, when evaluated at $J = -\frac{\delta\Gamma}{\delta\varphi}$, cancel out all 1-prong reducible graphs.



IPR if it becomes disconnected when 1 line breaks.

ST

Claim: \times terms when evaluated at $J = -\frac{\delta\Gamma}{\delta\varphi}$,
cancel out all 1-prong reducible graphs.



is IPR if it becomes disconnected when 1 line breaks.



$$\frac{\delta\Gamma}{\delta\varphi} - J = \times$$

Claim: \times terms ~~will~~ be evaluated at $J = -\frac{\delta\Gamma}{\delta\varphi}$,
 cancel out all 1PR reducible graphs.



1PR if it becomes disconnected when 1 line breaks



$$0 = \frac{\delta\Gamma}{\delta\varphi} + J = \begin{array}{c} \text{[Grid Circle]} \\ \times \\ \delta\varphi \end{array} + \begin{array}{c} \text{[Grid Circle]} \\ \times \\ J \end{array}$$

Claim: \times terms when evaluated at $J = -\frac{\delta\Gamma}{\delta\varphi}$,
cancel out all 1-partly reducible graphs.



1PR if it becomes disconnected
when 1 line breaks.



$$0 = \frac{\delta\Gamma}{\delta\varphi} + J = \text{[diagram of a circle with diagonal hatching and a cross through it, labeled } \frac{\delta\Gamma}{\delta\varphi}] + \text{[diagram of a circle with diagonal hatching and a cross through it, labeled } J]$$

Ψ as a function of J . Assume this can be inverted

Upshot:

$$\Gamma = S + \Gamma_{1\text{-loop}} + (\text{sum of 2 loop + higher IPI graphs})$$

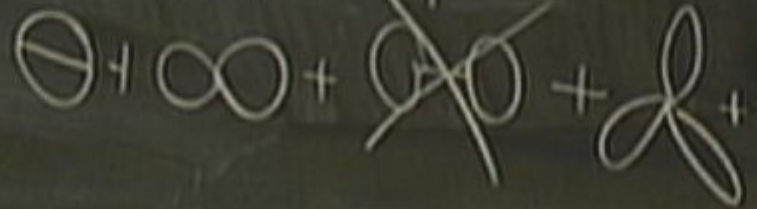
\downarrow

$$(\det S^{-1})^{1/2}$$


Ψ as a function of J Assume this can be inverted

Upshot:

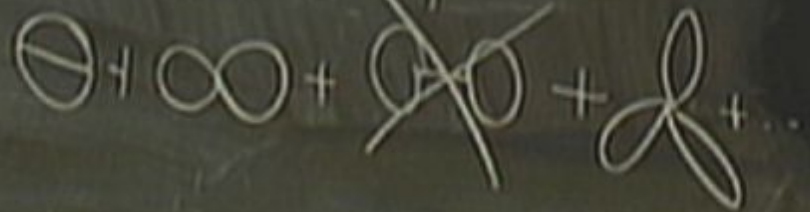
$$\Gamma_{(q)} = S_{\text{tree}} + \underbrace{\Gamma_{1\text{-loop}}}_{(\det S')^{-1/2}} + (\text{sum of 2 loop + higher JPI graphs with no external lines})$$



Let φ be a function of J . Assume this can be inverted

Upshot: 

$$\Gamma(\varphi) = S(\varphi) + \left[\begin{array}{l} \text{1-loop} \\ \downarrow \\ (\det S(\varphi))^{-1/2} \end{array} \right] + (\text{sum of 2-loop + higher JPI graphs with no external lines})$$



$$= -J(x) \text{ because } \frac{\partial W}{\partial J} = \varphi.$$

Now suppose there ~~are~~ a hierarchy of masses.

$l(x), k(x)$

m

$$= -J(x) \text{ because } \frac{\delta W}{\delta J} = \varphi.$$

Now suppose there is a hierarchy of masses.

$$l(x), h(x)$$

$$\text{mass, } m \sim M \quad m \ll M$$

$$= -J(x) \quad \text{because } \frac{\delta W}{\delta J} = \varphi.$$

Now suppose there are a hierarchy of masses.

$$l(x), k(x)$$

$$\text{mass, } m \sim M \quad m \ll M$$

If your interest is only in energies $E \ll M$, or distances
 $\lambda \gg 1/m$, then you don't need $\Gamma(\varphi(x))$
for all $\varphi(x)$

$$\Gamma = \Gamma(l(x), h(x)) = W(j(x), J(x)) - \int d^4x [Jh + jl]$$

$$e^{iW(J, j)} = \int \mathcal{D}H \mathcal{D}L e^{iS(L, H) + i \int d^4x (JH + jL)}$$

$$l = \frac{\delta W}{\delta j}, \quad h = \frac{\delta W}{\delta J}$$

$\Gamma = S(l, h) + \Gamma_{1\text{-loop}} + \text{sum of vacuum } 2\text{-loop } +$
higher graphs that are
1PI.

$$F = S(l, h) + \Gamma_{1\text{-loop}} + \text{sum of vacuum } 2\text{-loop \& higher graphs that are } \mathbb{1PI}.$$

To specialize only to correlations of light fields
should take $J=0$.

To " " " " of long-wavelength parts of $L(x)$
should require $f(x)$ to only involve long wavelengths,

for all $\psi(x)$

$$\text{if } j(x) = \int \frac{d^4 p}{(2\pi)^4} e^{i p x} j(p)$$

$$j(p) = 0 \text{ if } p_c^2 > \Lambda^2 \quad (p_c^2 = E^2 + \vec{p}^2)$$

$$\Lambda \ll M$$

for all $\psi(x)$

$$\text{if } j(x) = \int \frac{d^4 p}{(2\pi)^4} e^{i p x} j(p)$$

$$j(p) = 0 \text{ if } p^2 > \Lambda^2 \quad (p^2 = E^2 - \vec{p}^2) \quad \Lambda \ll M$$

$$e^{i \Gamma(L, H)} = \int \mathcal{D}L \mathcal{D}H e^{i S(L+H, L+H) + i \int d^4 x (JH + jL)}$$

$$F = S(p, h) + \int_{1\text{-loop}}^{\text{cancelled if } J = -\frac{5F}{5L}} + \text{Sum of vacuum } 2\text{-loop } J = -\frac{5F}{5L} \text{ if higher graphs that are } \mathbb{ZP}\mathbb{I}.$$

To specialize only to correlations of light fields should take $J=0$.

To " " " " of long-wavelength parts of $L(x)$ should require $f(x)$ to only involve long wavelength.

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$$j(p) = 0 \text{ if } p^2 > \Lambda^2 \quad (p^2 = E^2 + \vec{p}^2) \quad \Lambda \ll M$$

$$e^{i\Gamma(\ell, \bar{h})} = \int \mathcal{D}L \mathcal{D}H e^{i S(\ell+L, \ell+H) + i \int \ell^{\mu\nu} (J_{\mu\nu} H + j L)}$$

$$\bar{h} = \left. \frac{\delta W}{\delta J} \right|_{J=0} \text{ satisfies } \left. \frac{\delta \Gamma}{\delta \bar{h}} \right|_{\bar{h}} = 0$$

ΔLPI generator.

$$\gamma(\ell) = \Gamma(\ell, \bar{h}(\ell)) \quad \text{where} \quad \left. \frac{\delta \Gamma}{\delta \bar{h}} \right|_{\bar{h}} = 0.$$

1LPI generator.

$$\gamma(\ell) = \Gamma(\ell, \bar{h}(\ell)) \quad \text{where} \quad \left. \frac{\delta \Gamma}{\delta h} \right|_{\bar{h}} = 0.$$
$$= S(\ell, \bar{h}(\ell)) + \Gamma_{1\text{-loop}} + \text{sum of 1LPI vacuum graphs}$$

1LPI generator.

$\rightarrow \gamma(\ell) = \Gamma(\ell, \bar{h}(\ell))$ where $\left. \frac{\delta \Gamma}{\delta h} \right|_{\bar{h}} = 0$.

all info of LE theory $\triangleleft \Lambda$ = $S(\ell, \bar{h}(\ell)) + \Gamma_{1\text{-loop}} + \text{sum of 1LPI vacuum graphs}$