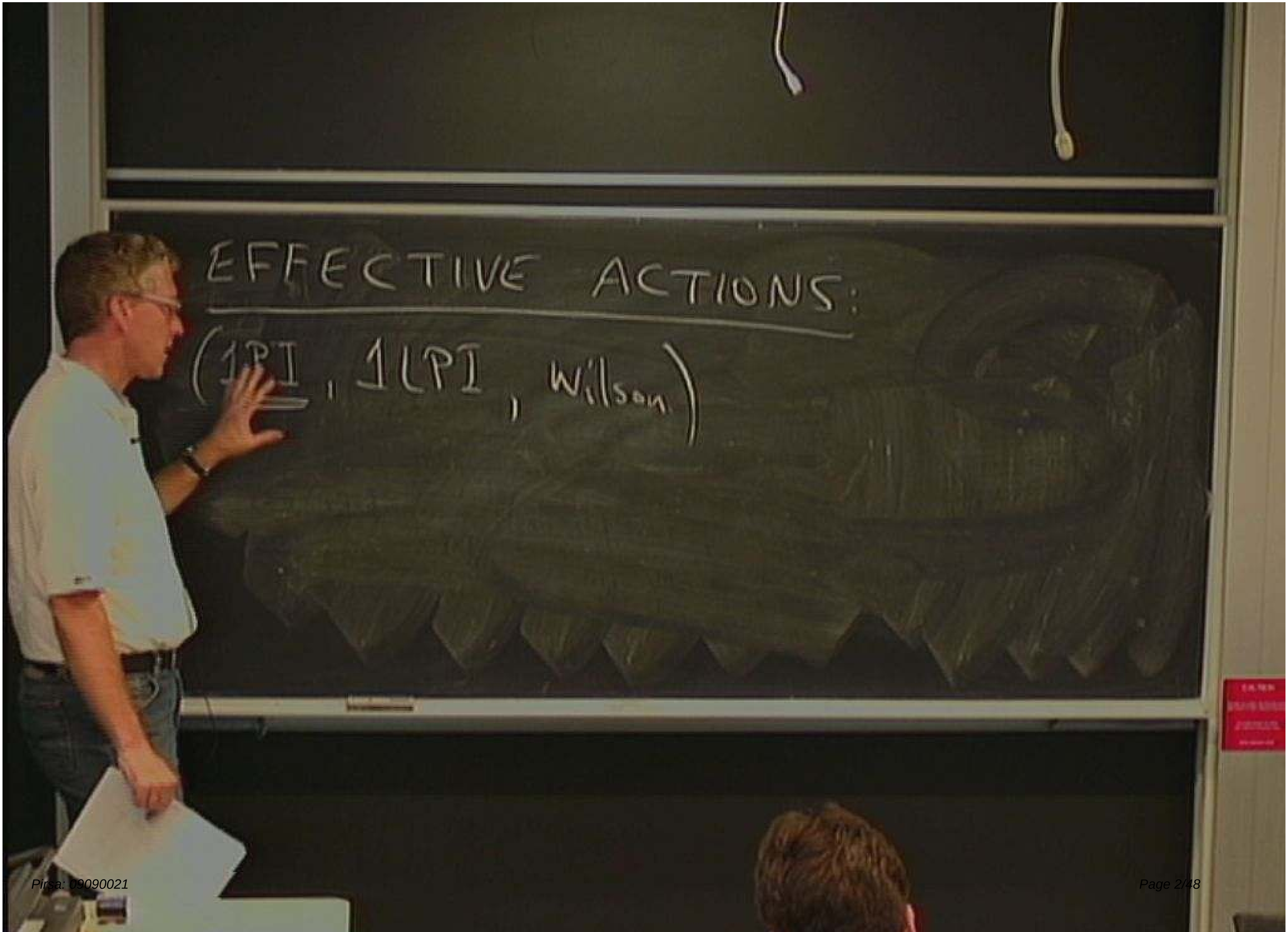
Title: Introduction to Effective Field Theory - Lecture 2B

Date: Sep 23, 2009 03:30 PM

URL: http://pirsa.org/09090021

Abstract:


EFFECTIVE ACTIONS:
(1PI, 1LPI, Wilson)

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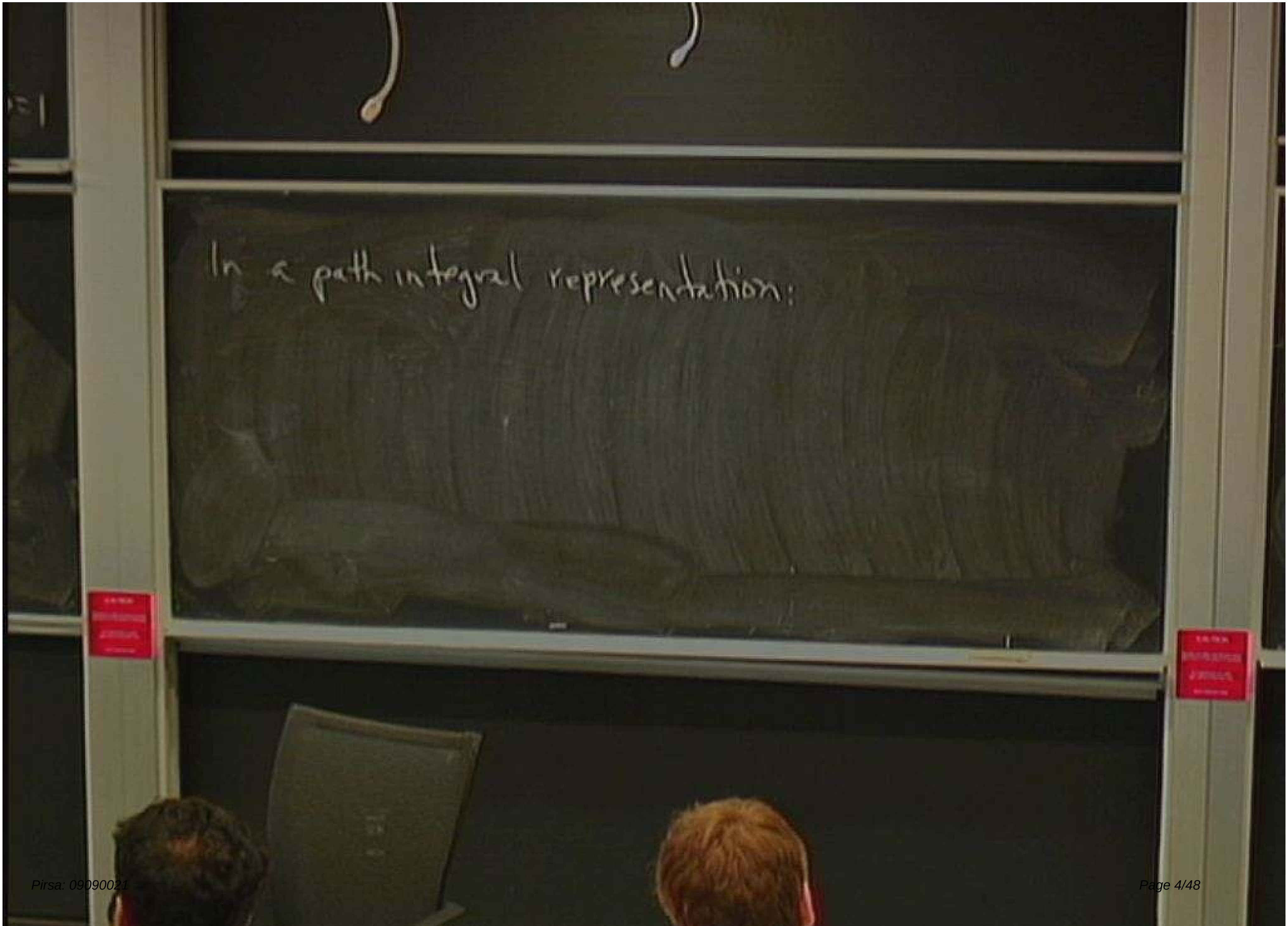
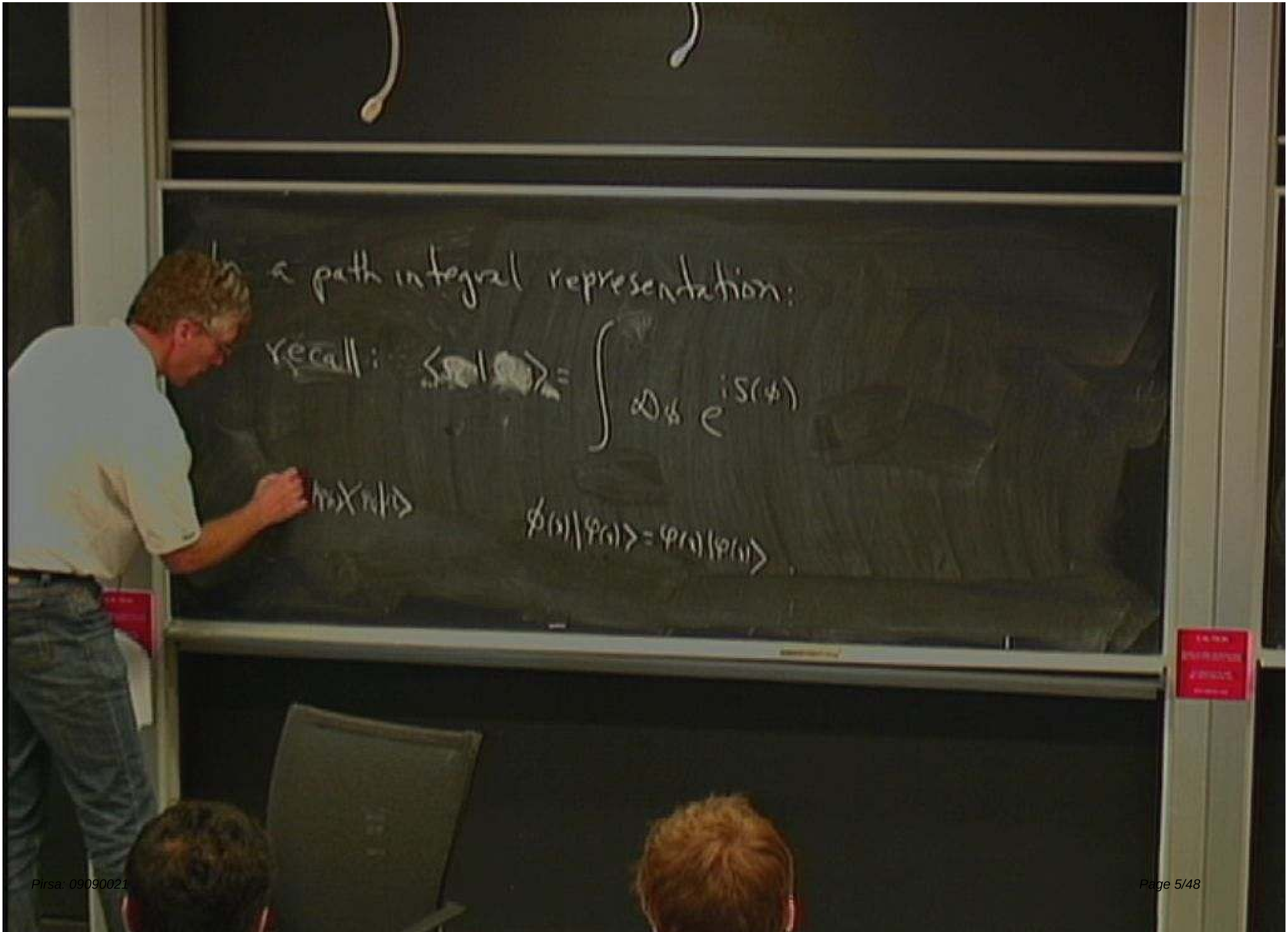
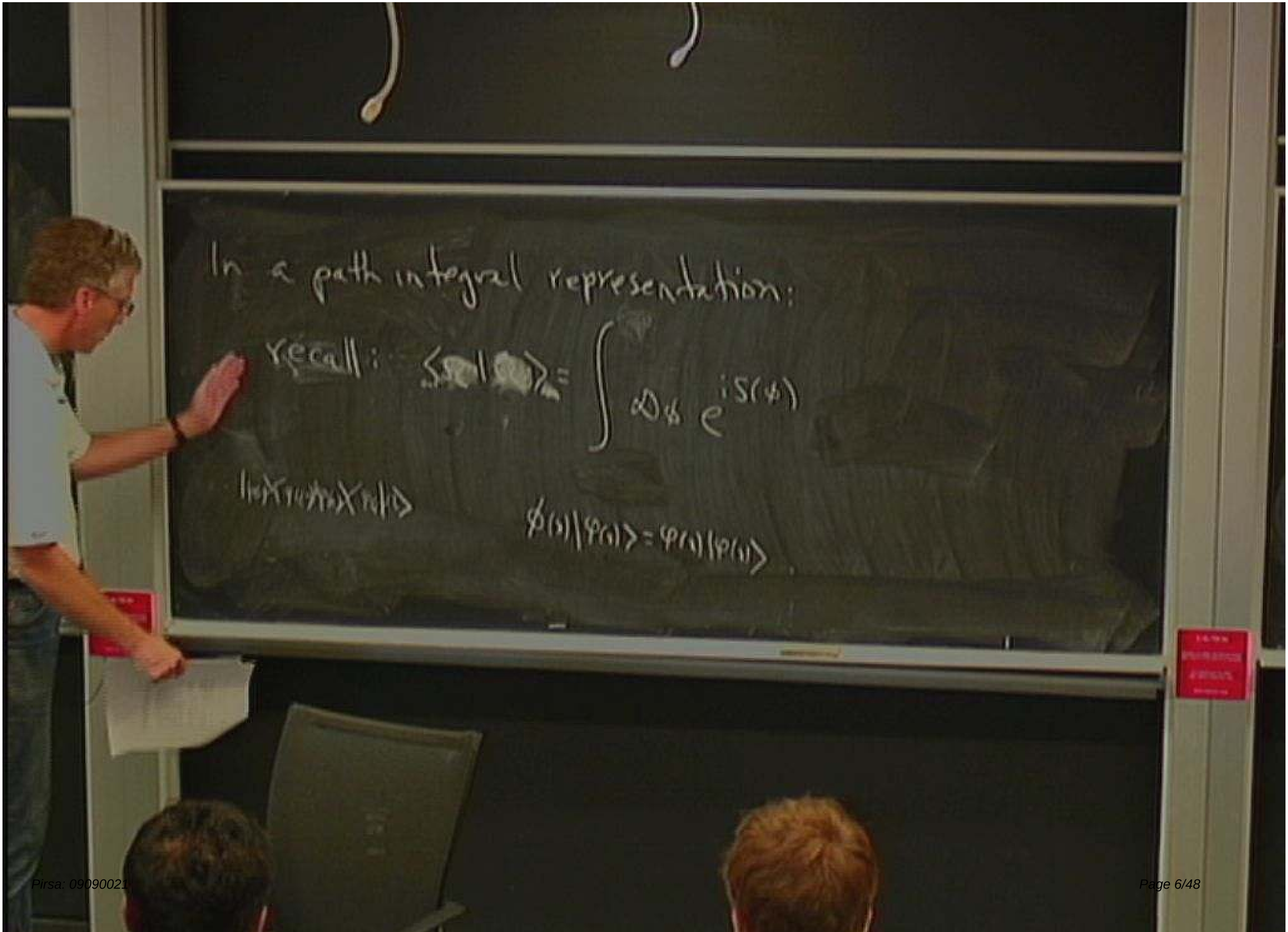
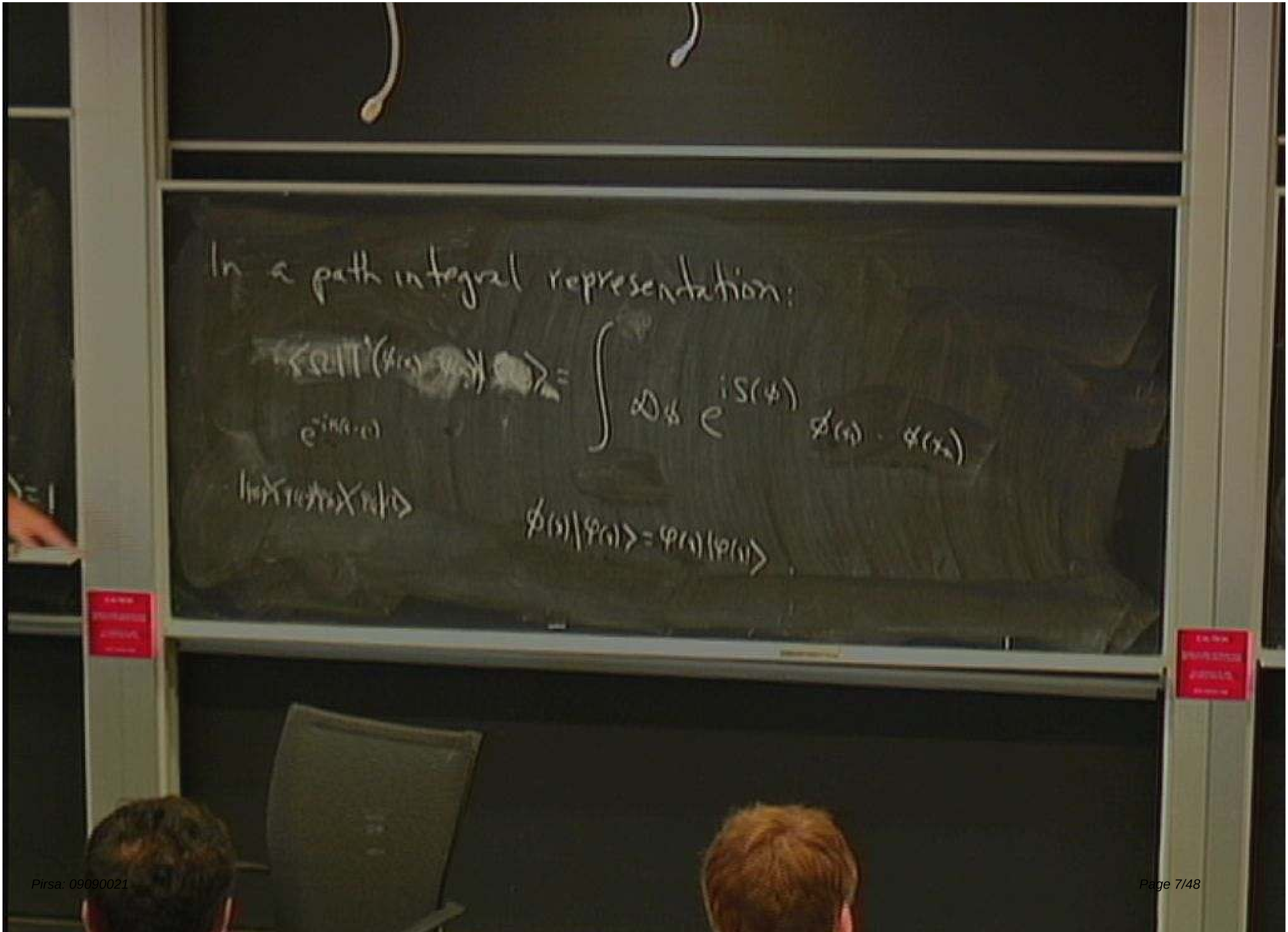
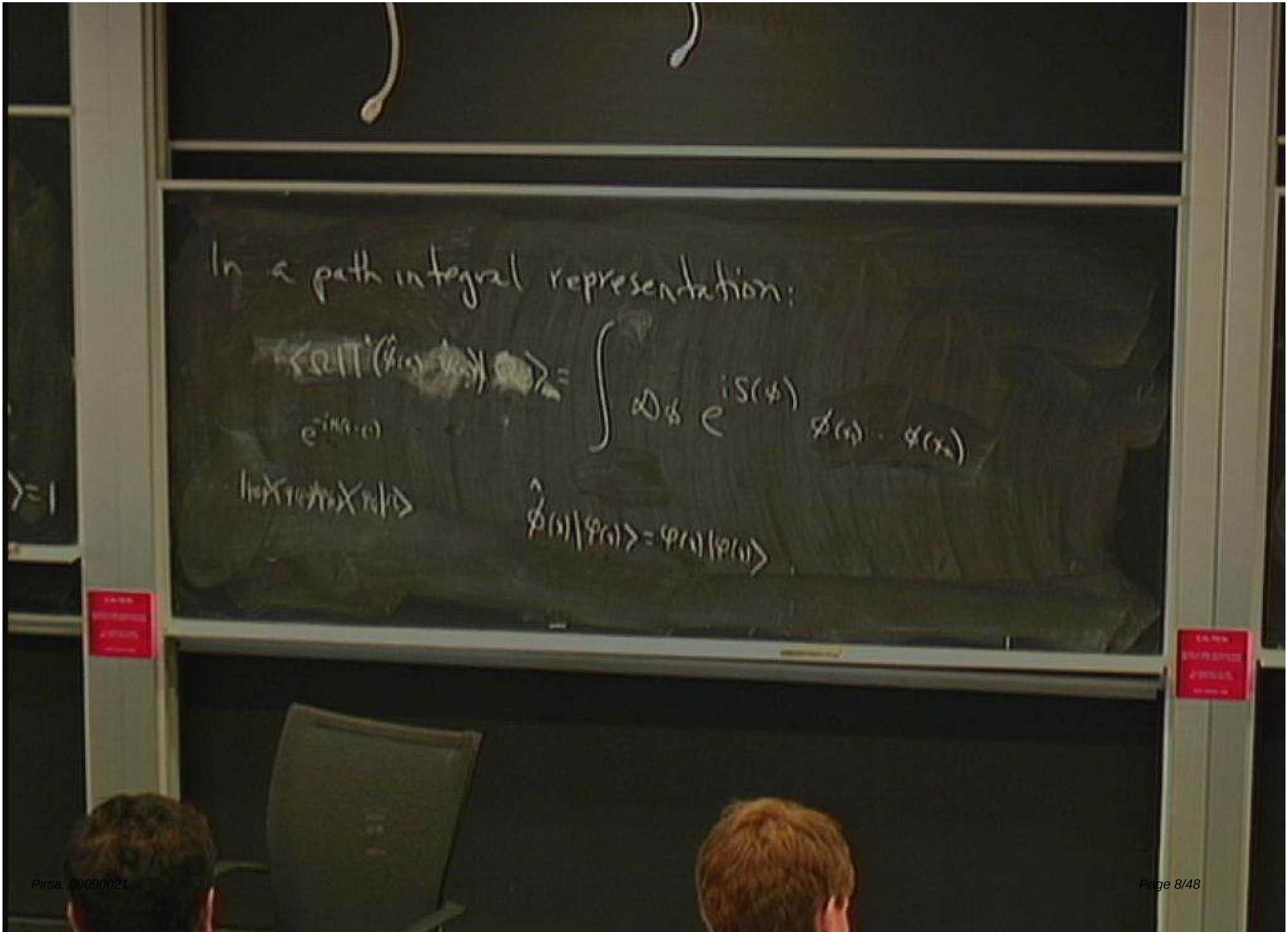
Consider the generating functional for correlation functions: $\langle \Omega | T(\phi(x_1) \cdots \phi(x_n)) | \Omega \rangle \qquad \langle \Omega | \Omega \rangle = 1$

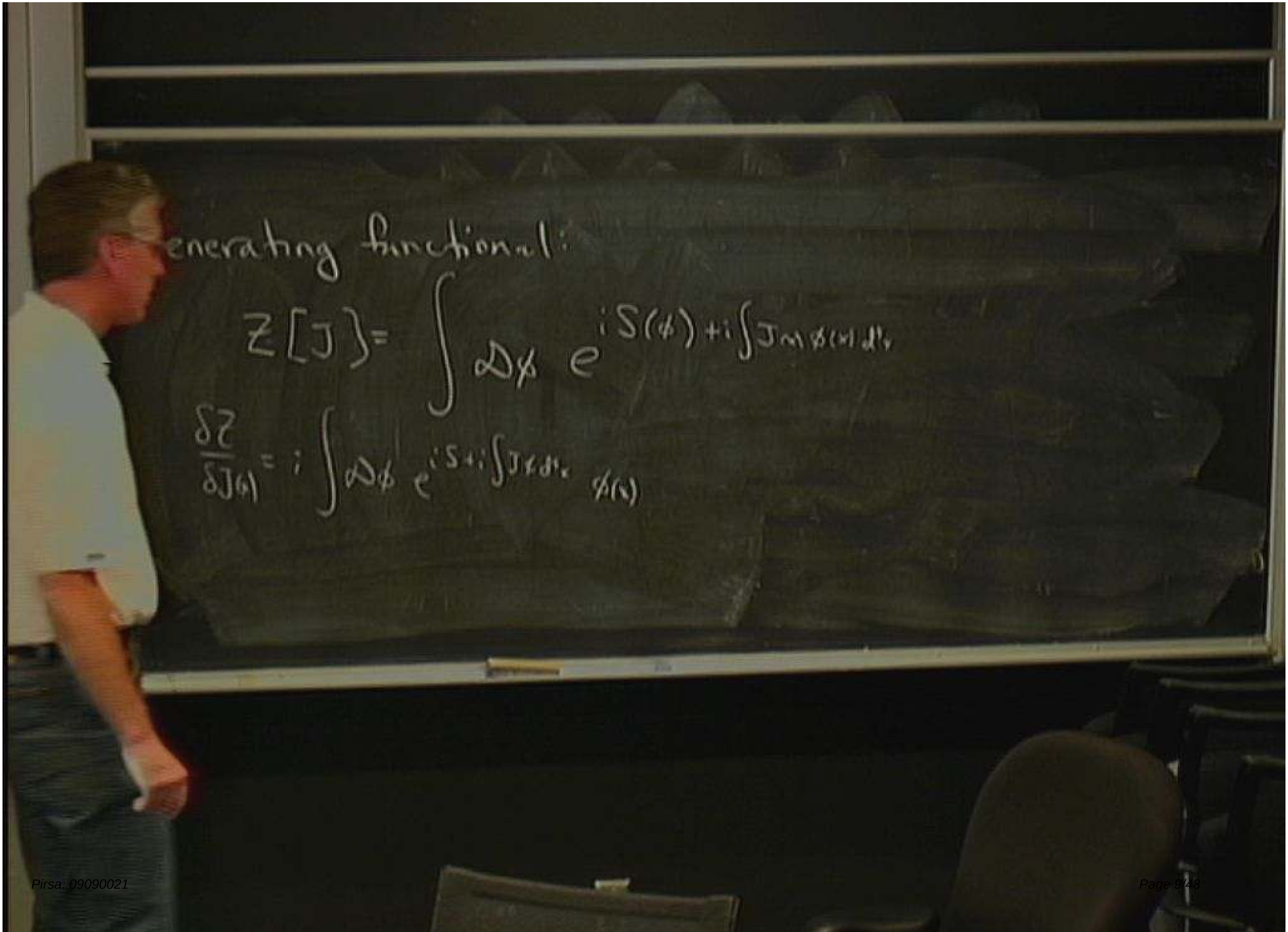

In a path integral representation:

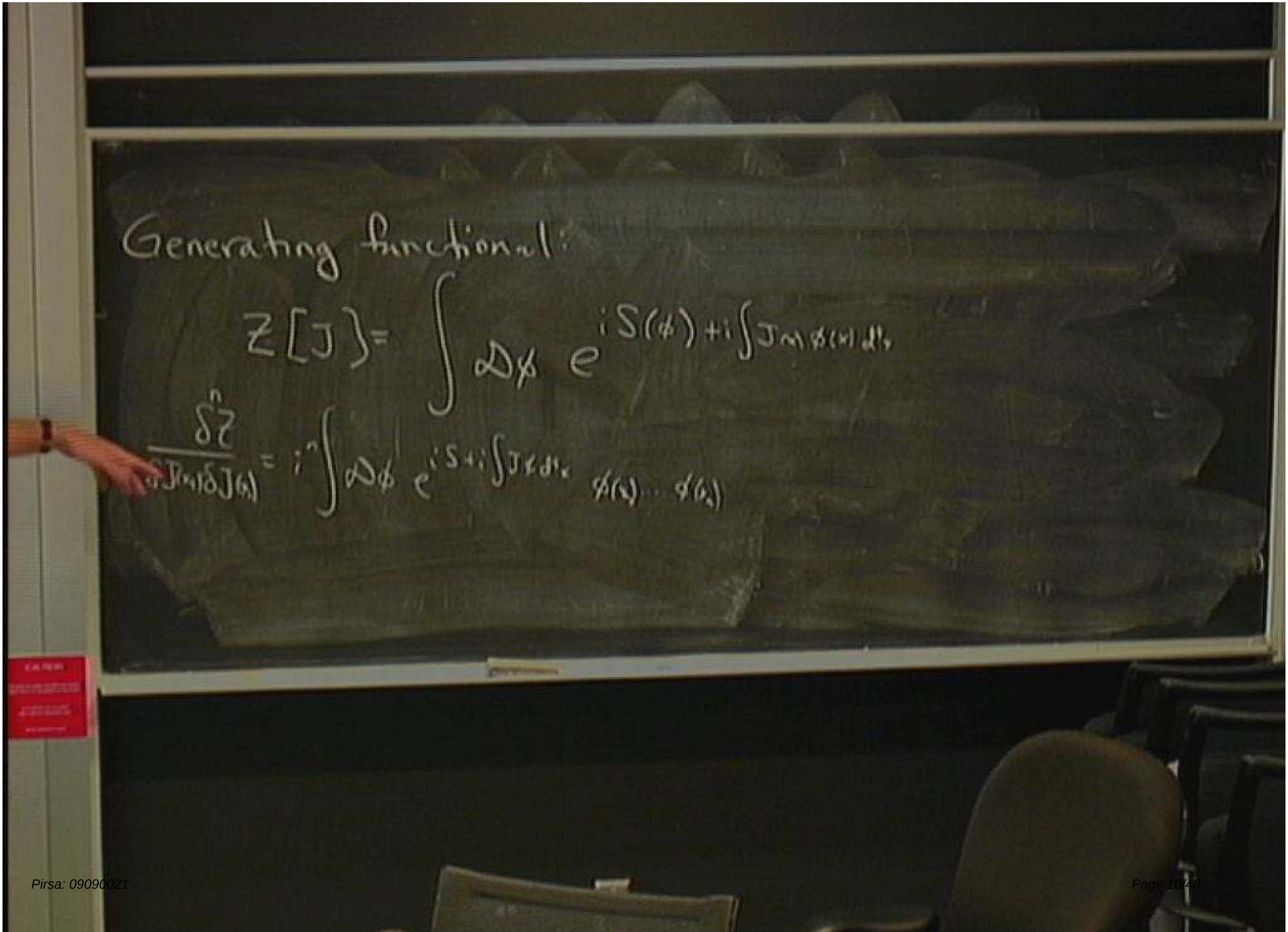

a path integral representation:
recall:

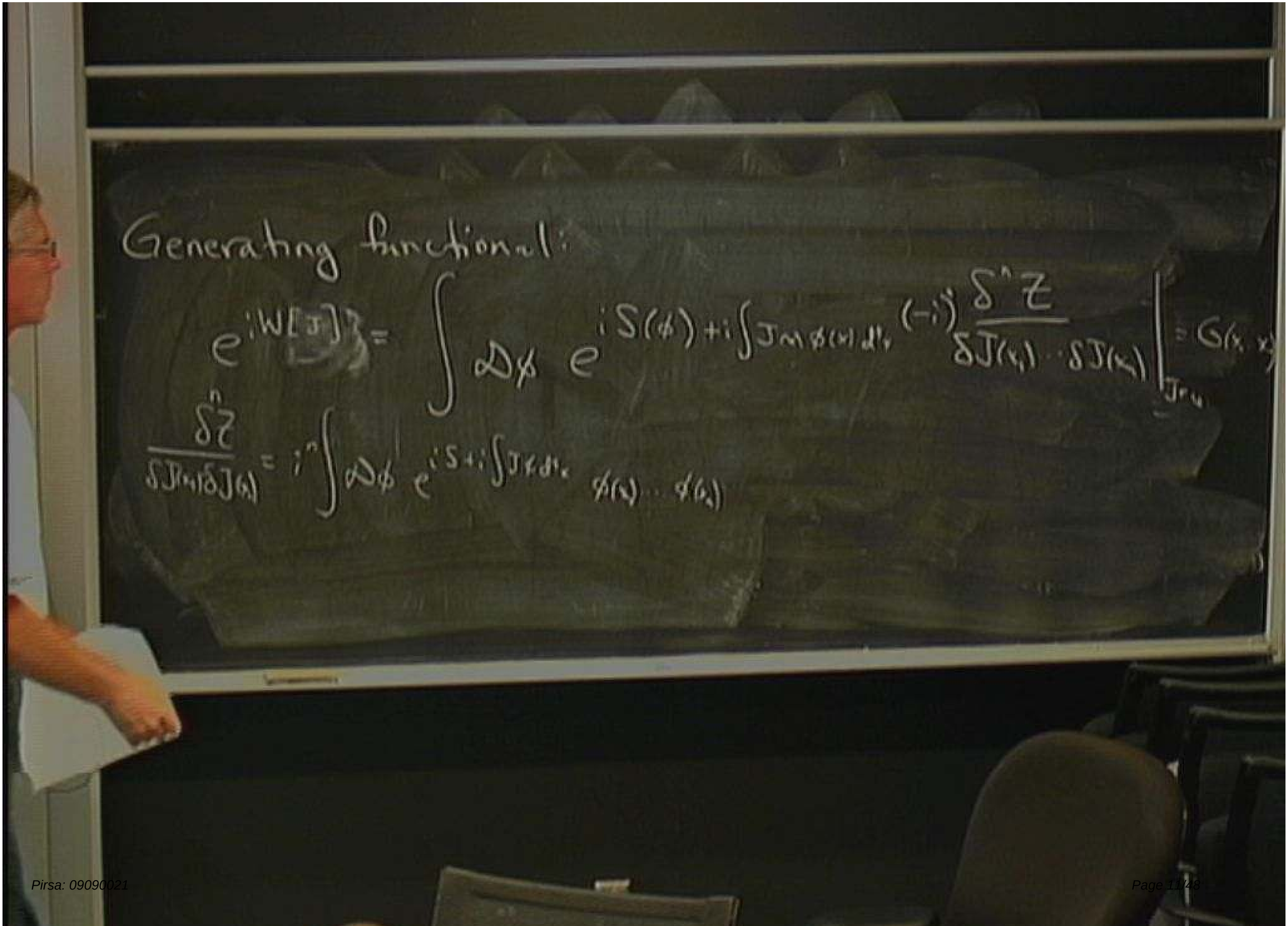

In a path integral representation:
recall:
$\int \mathcal{D}\phi \, e^{iS(\phi)}$
$\phi(x)|\varphi(x)\rangle = \varphi(x)|\varphi(x)\rangle$


In a path integral representation:
$= \int \mathcal{D}\phi \, e^{iS(\phi)} \phi(x_1) \cdots \phi(x_n)$
$\phi(x)|\varphi(x)\rangle = \varphi(x)|\varphi(x)\rangle$


In a path integral representation:
$= \int \mathcal{D}\phi \, e^{iS(\phi)} \phi(x_1) \cdots \phi(x_n)$
$\hat{\phi}(x)|\varphi(x)\rangle = \varphi(x)|\varphi(x)\rangle$


enerating functional:
$Z[J] = \int \mathcal{D}\phi \, e^{iS(\phi) + i\int J(x)\phi(x)\, d^4x}$
$\frac{\delta Z}{\delta J(x)} = i \int \mathcal{D}\phi \, e^{iS + i\int J\phi\, d^4x} \, \phi(x)$


Generating functional:
$Z[J] = \int \mathcal{D}\phi \, e^{iS(\phi) + i\int J(x)\phi(x)\,d^4x}$
$\frac{\delta^n Z}{\delta J(x_1)\cdots\delta J(x_n)} = i^n \int \mathcal{D}\phi \, e^{iS + i\int J\phi\,d^4x} \, \phi(x_1)\cdots\phi(x_n)$


Generating functional:
$e^{iW[J]} = \int \mathcal{D}\phi \, e^{iS(\phi) + i\int J(x)\phi(x)\,d^4x}$
$(-i)^n \frac{\delta^n Z}{\delta J(x_1)\cdots\delta J(x_n)}\Big|_{J=0} = G(x_1 \ldots x_n)$
$\frac{\delta^n Z}{\delta J(x_1)\cdots\delta J(x_n)} = i^n \int \mathcal{D}\phi \, e^{iS + i\int J\phi\,d^4x} \, \phi(x_1)\cdots\phi(x_n)$

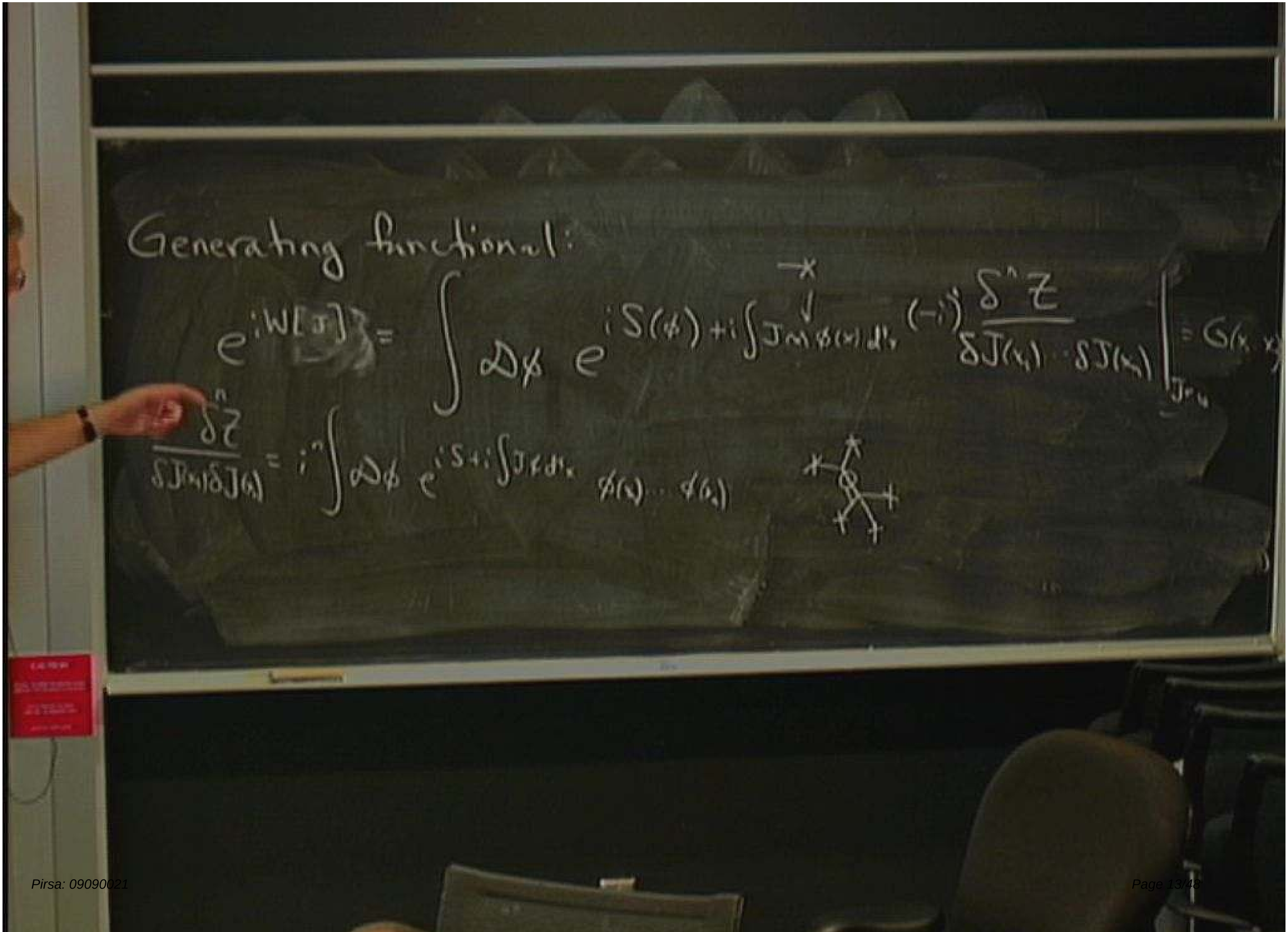
In a path integral representation: $\int_{-\infty}^{\infty} dx\, e^{-\frac{1}{2}\lambda x^2} = \sqrt{\frac{2\pi}{\lambda}}\, e^{i\pi/4}$

$$\langle\Omega|T(\hat\phi(x_1)\cdots\hat\phi(x_n))|\Omega\rangle = \int \mathcal{D}\phi\, e^{iS(\phi)}\, \phi(x_1)\cdots\phi(x_n) = G(x_1\ldots x_n)$$

$e^{-iH(t-t')}$ [illegible]

[illegible]

$$\hat\phi(x)|\varphi(x)\rangle = \varphi(x)|\varphi(x)\rangle$$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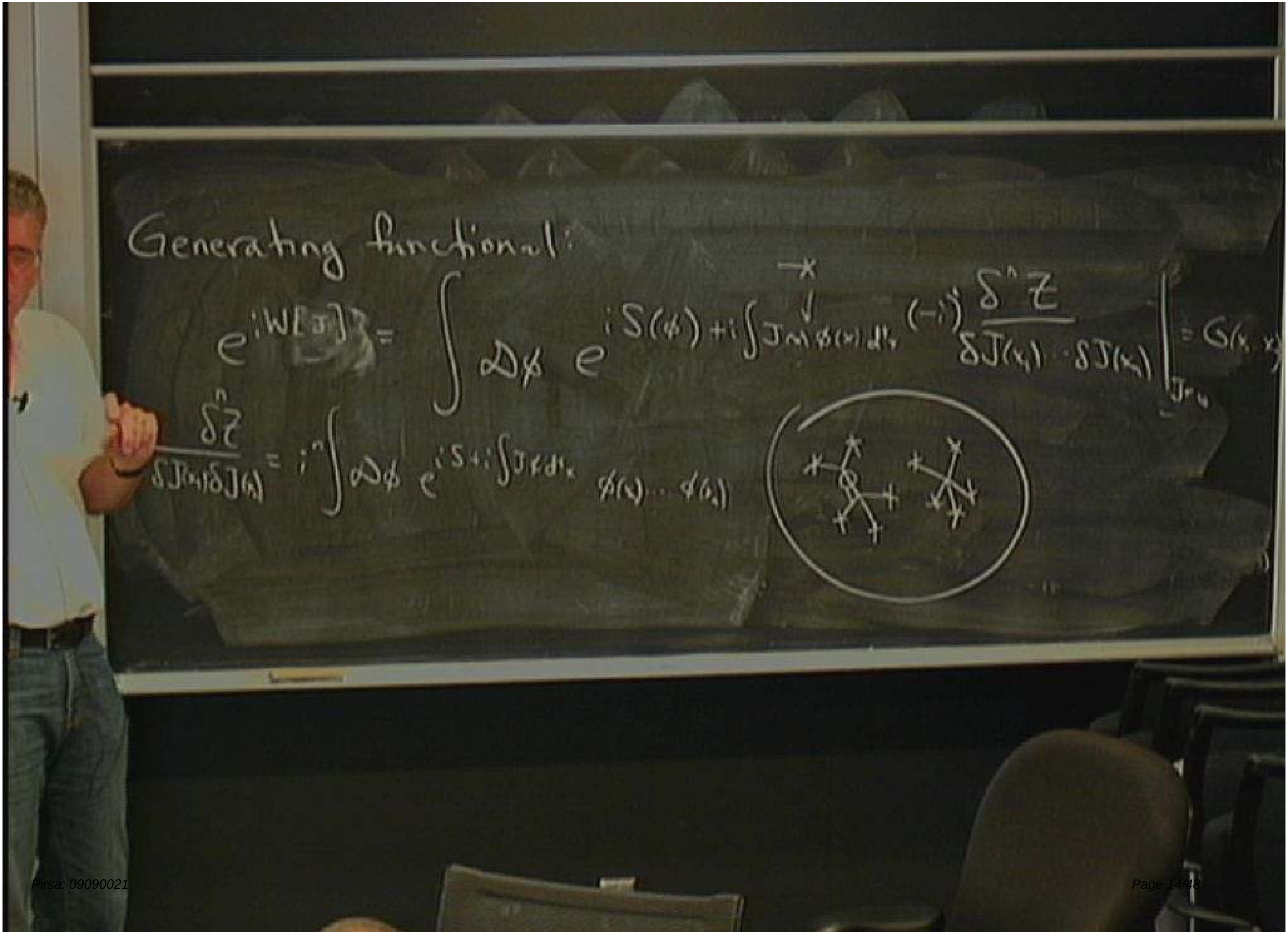
Generating functional:
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$\frac{\delta^n Z}{\delta J(x_1)\cdots\delta J(x_n)} = i^n \int \mathcal{D}\phi \; e^{iS + i\int J\phi\, d^4x} \; \phi(x_1)\cdots\phi(x_n)$

$$\langle \Omega | T(\phi(x_1) \ldots\ldots \phi(x_n)) | \Omega \rangle \qquad \langle \Omega | \Omega \rangle = 1$$

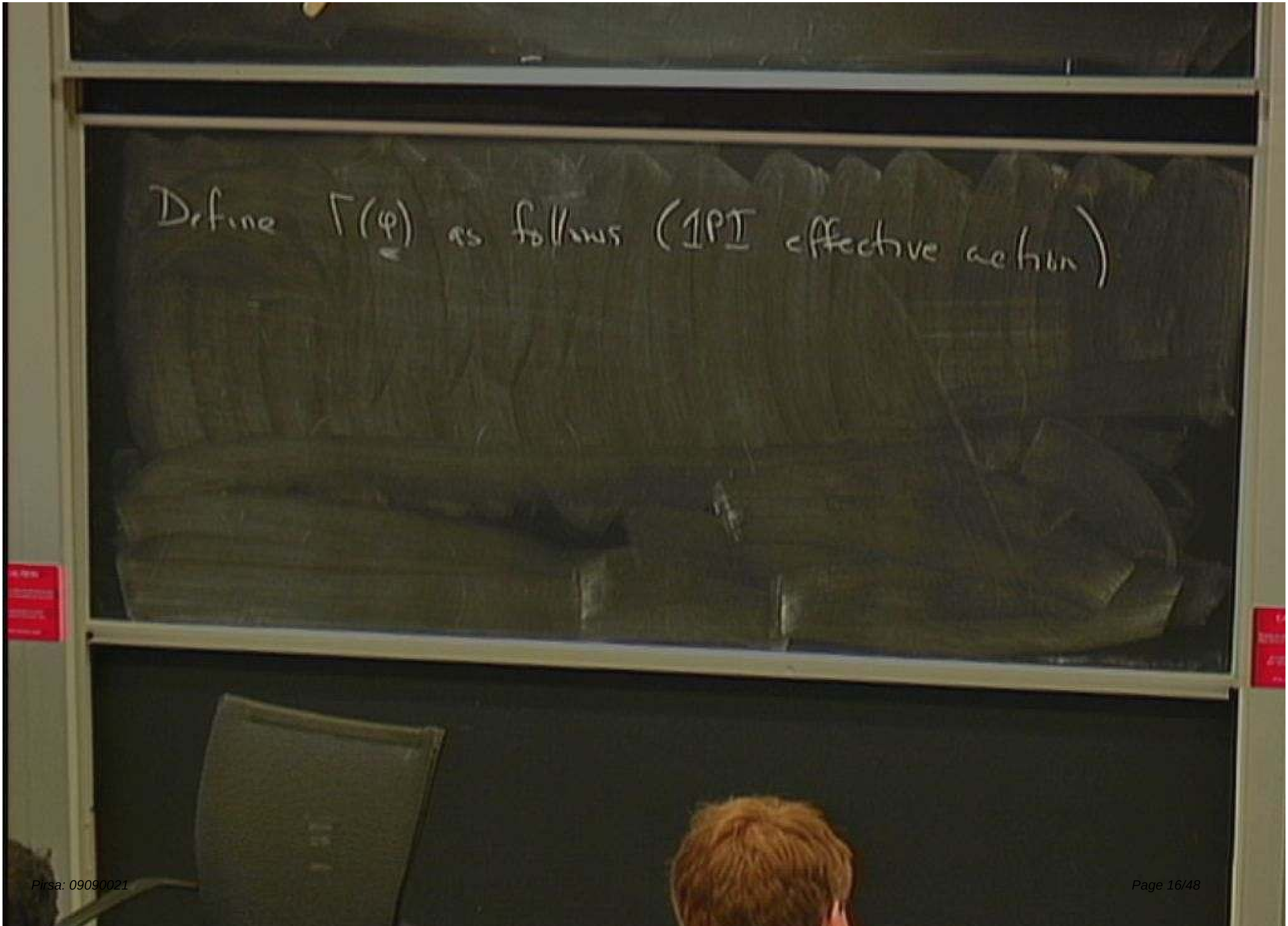
in perturbation theory $S(\phi) = S_{quad} + S_{int}$

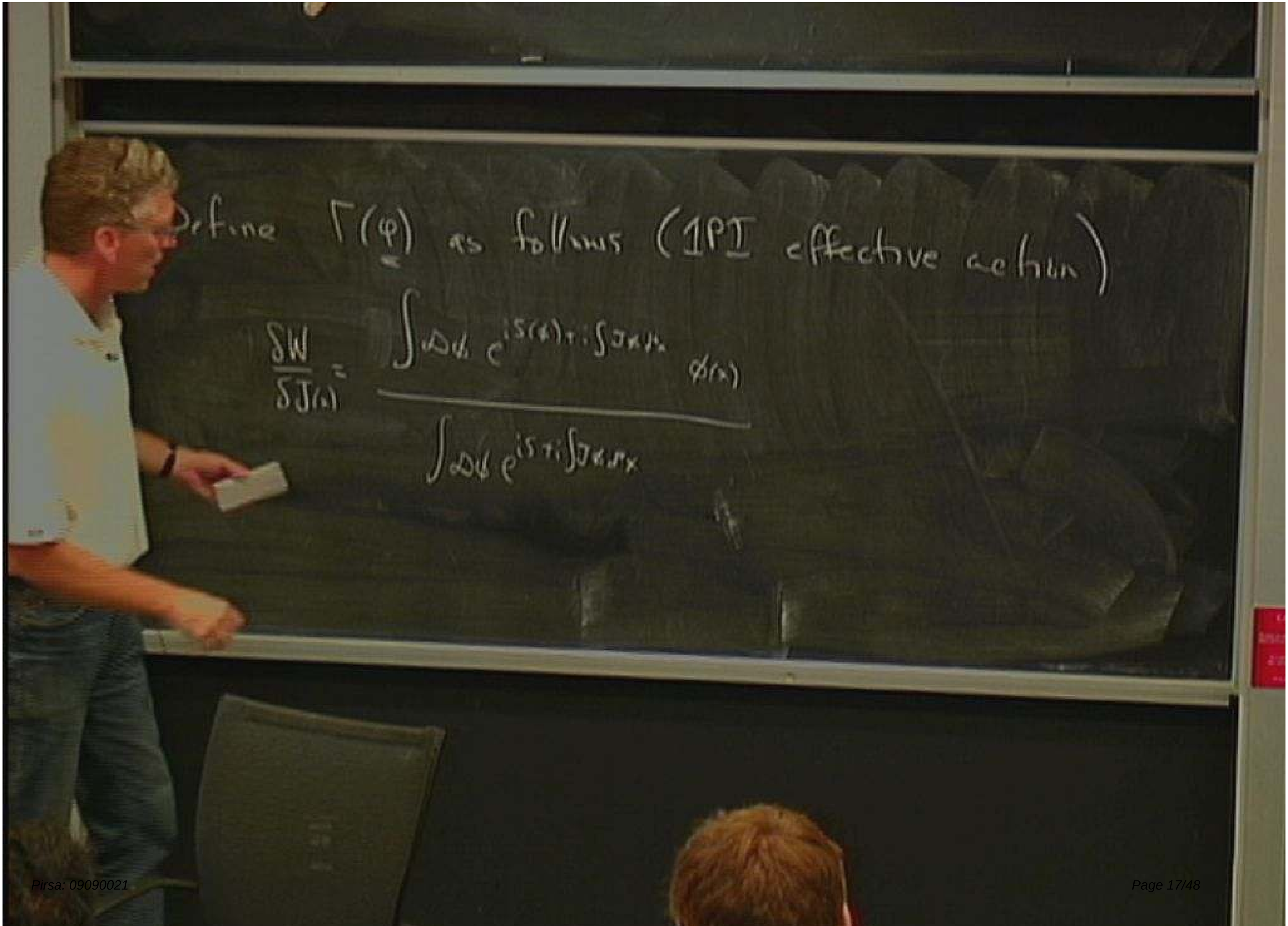
$$S_{quad} = \int d^4x \, \phi \Delta \phi$$

and $e^{iS_{int}} = 1 + iS_{int} + \frac{i^2}{2!} S_{int}^2 + \ldots$

gives $Z[J]$ as a sum of Feynman graphs where $\text{—} \rightarrow \Delta^{-1}$

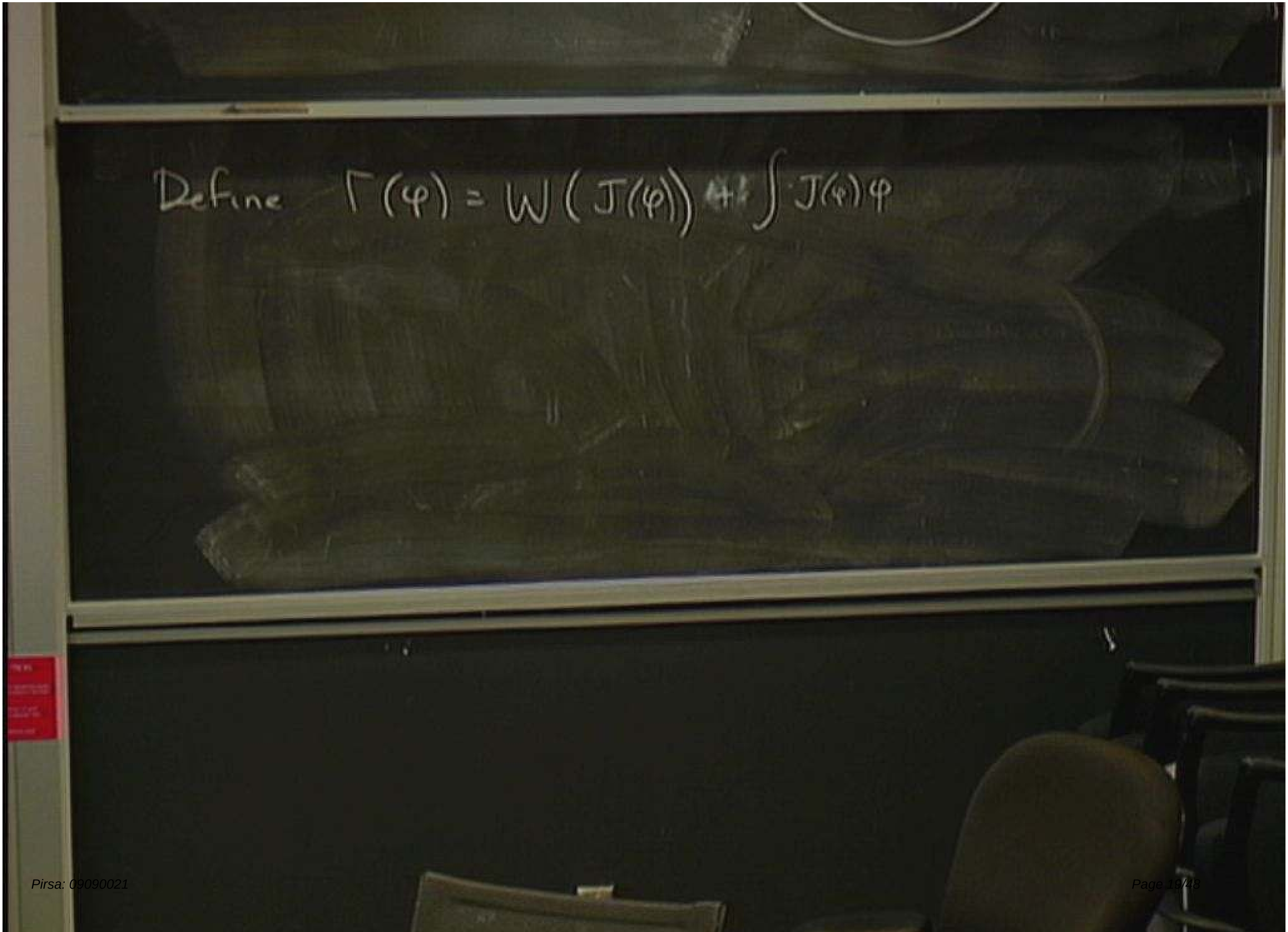
in usual way; and $W(J) = -i \ln Z$ = sum of connected graphs


Define $\Gamma(\varphi)$ as follows (1PI effective action)


Define Γ(φ) as follows (1PI effective action)
$\frac{\delta W}{\delta J(x)} = \frac{\int \mathcal{D}\phi \, e^{iS(\phi)+i\int J\phi\,dx}\,\phi(x)}{\int \mathcal{D}\phi \, e^{iS+i\int J\phi\,dx}}$

Define $\Gamma(\varphi)$ as follows (1PI effective action)

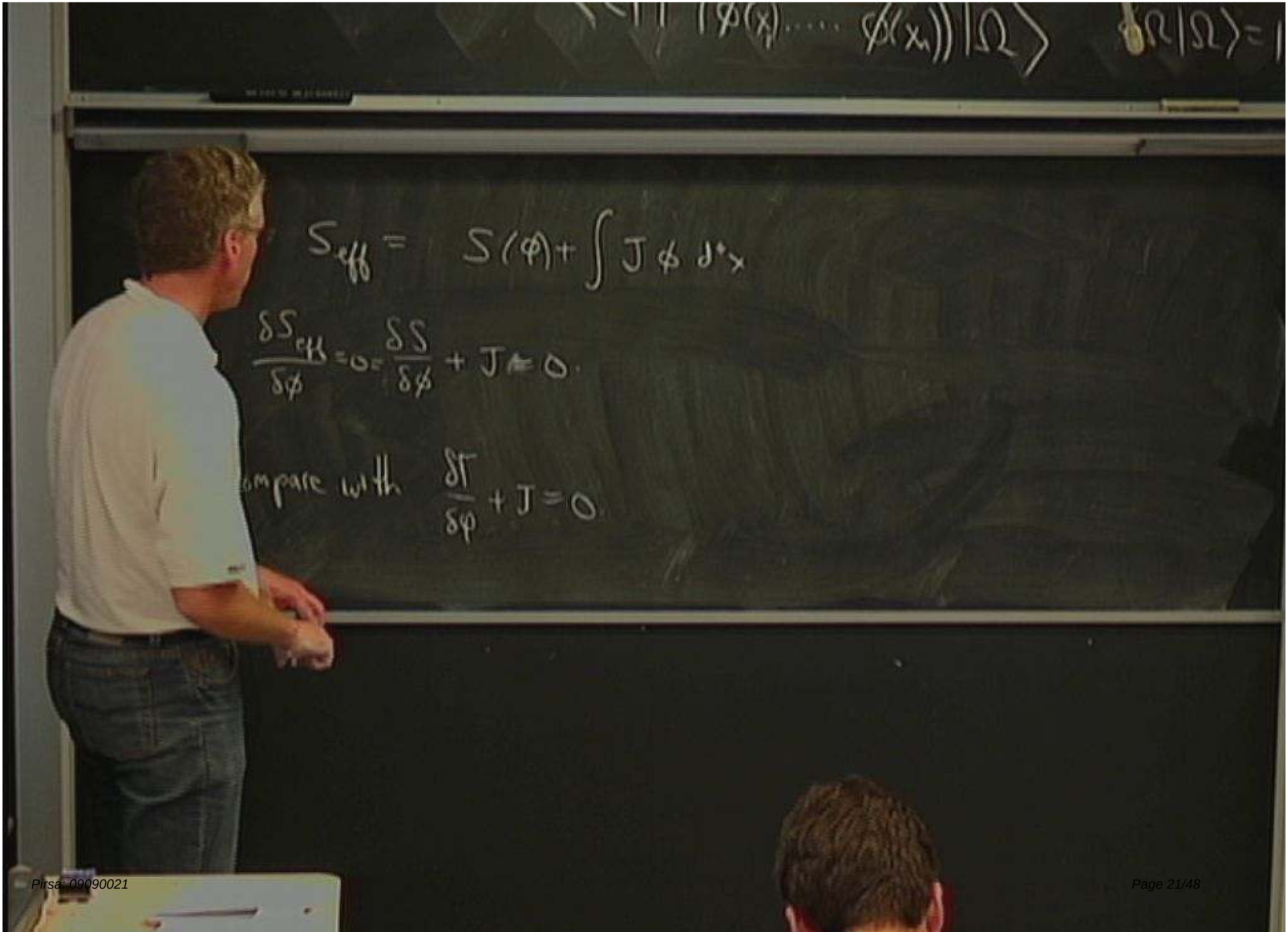
$$\frac{\delta W}{\delta J(x)} = \frac{\int \mathcal{D}\phi \, e^{iS(\phi) + i\int J\phi \, d^4x} \, \phi(x)}{\int \mathcal{D}\phi \, e^{iS + i\int J\phi \, d^4x}} = \varphi = \frac{\langle \Omega | \phi(x) | \Omega \rangle}{\langle \Omega | \Omega \rangle}$$


Define $\Gamma(\varphi) = W(J(\varphi)) + \int J(\varphi)\varphi$

Define $\Gamma(\varphi) = W(J(\varphi)) - \int J(\varphi)\varphi$

Legendre transform of $W(J)$

$$\frac{\delta \Gamma}{\delta \varphi(x)} = \int d^4y \, \frac{\delta W}{\delta J(y)} \frac{\delta J(y)}{\delta \varphi(x)} - \int d^4y \, \frac{\delta J(y)}{\delta \varphi(x)} \varphi(y) - J(x)$$



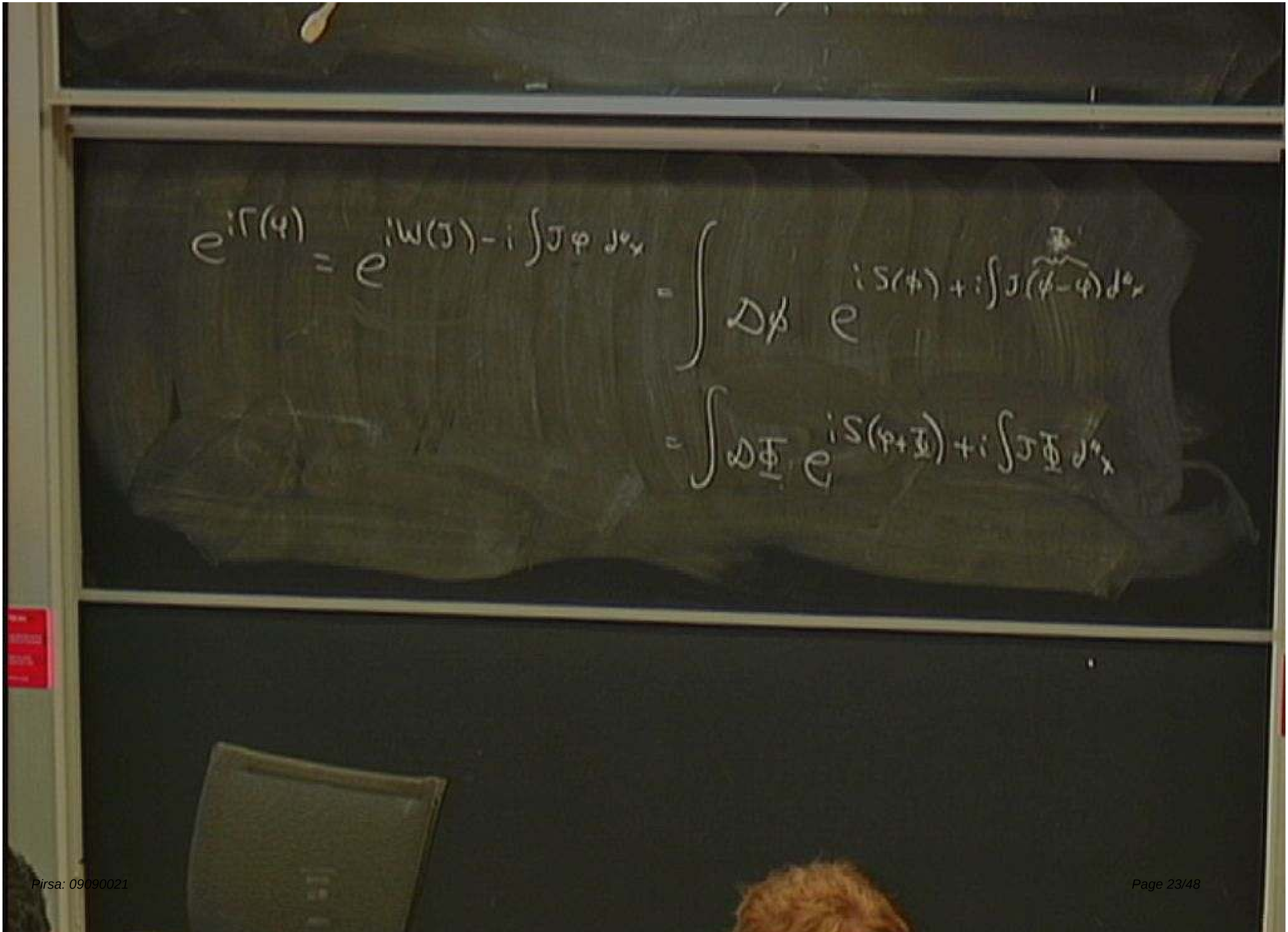
$S_{eff} = S(\phi) + \int J \phi \, d^4x$
$\frac{\delta S_{eff}}{\delta \phi} = 0 = \frac{\delta S}{\delta \phi} + J = 0$
ompare with $\frac{\delta \Gamma}{\delta \phi} + J = 0$

$$\langle \Omega | T\, \phi(x_1) \ldots \phi(x_n) | \Omega \rangle \qquad \langle \Omega | \Omega \rangle = 1$$

$$S_{eff} = S(\phi) + \int J \phi \, d^4x$$

$$\frac{\delta S_{eff}}{\delta \phi} = 0 = \frac{\delta S}{\delta \phi} + J = 0.$$

Compare with $\dfrac{\delta \Gamma}{\delta \varphi} + J = 0.$ when $J = 0$ $\dfrac{\delta \Gamma}{\delta \varphi} = 0$ determines $\varphi(J=0)$


$e^{i\Gamma(\varphi)} = e^{iW(J) - i\int J\varphi\, d^4x}$
$= \int \mathcal{D}\phi\; e^{iS(\phi) + i\int J\overbrace{(\phi-\varphi)}^{\Phi} d^4x}$
$= \int \mathcal{D}\Phi\; e^{iS(\varphi+\Phi) + i\int J\Phi\, d^4x}$

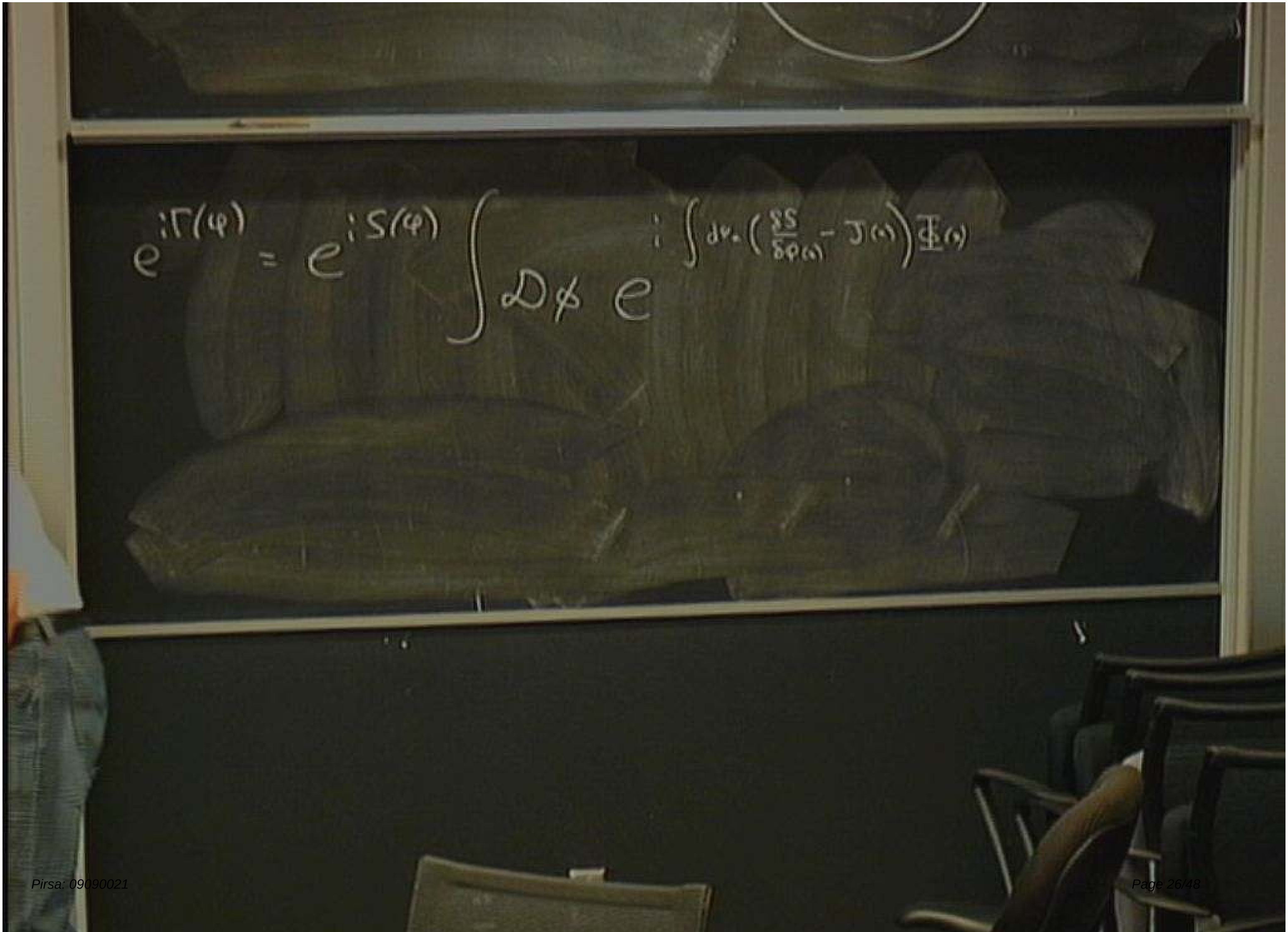
$$e^{i\Gamma(\varphi)} = e^{iW(J) - i\int J\varphi\, d^4x} = \int \mathcal{D}\phi\; e^{iS(\phi) + i\int J\overbrace{(\phi-\varphi)}^{\Phi} d^4x}$$

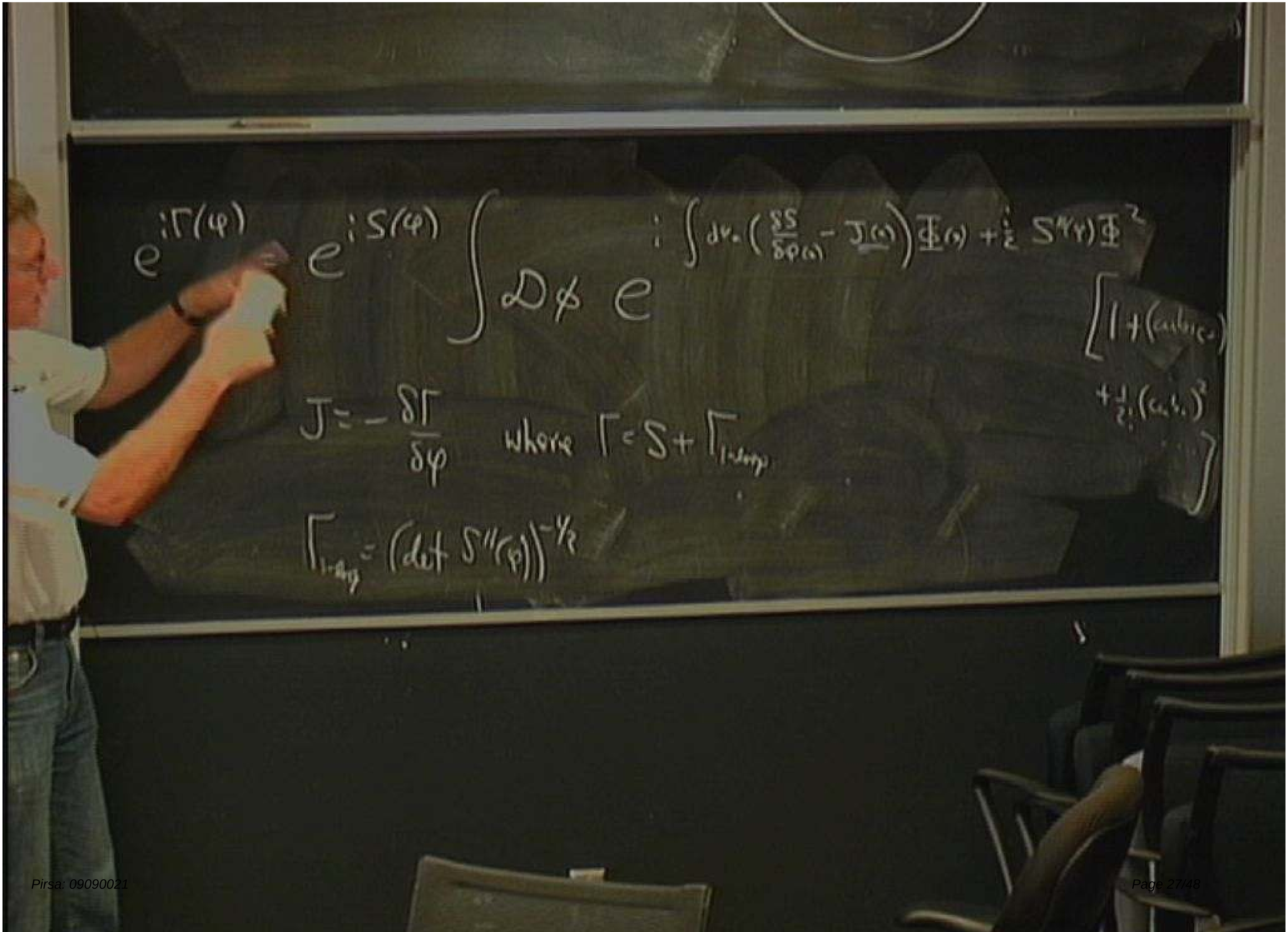
$$\int d^4y\, \frac{\delta S}{\delta\varphi(y)}\, \Phi(y)$$

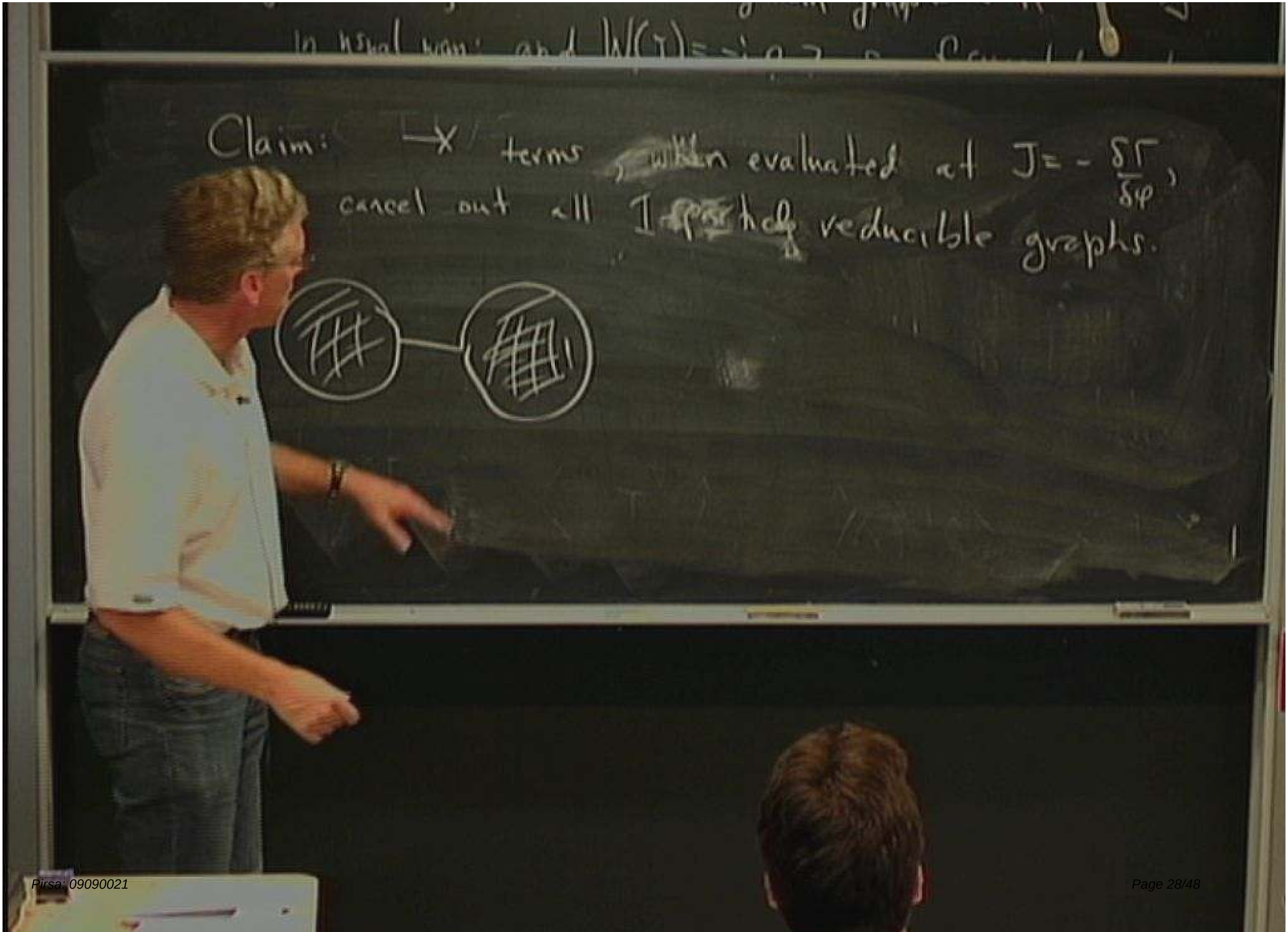
$$= \int \mathcal{D}\Phi\; e^{iS(\varphi+\Phi) + i\int J\Phi\, d^4x}$$

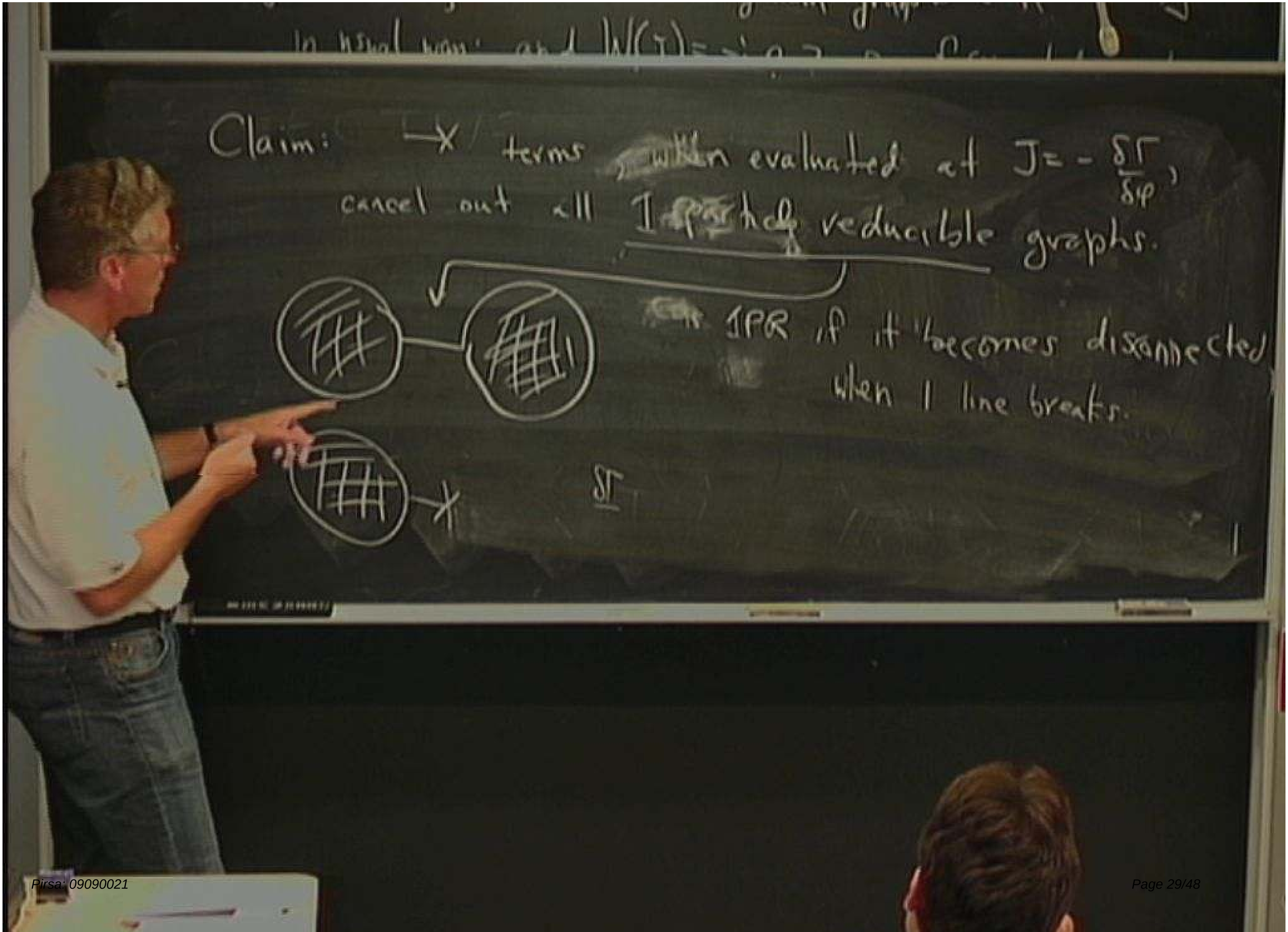
$$S(\varphi+\Phi) = S(\varphi) + S'(\varphi)\Phi + \tfrac{1}{2} S''(\varphi)\Phi^2 + \text{cubic} + \text{higher}.$$

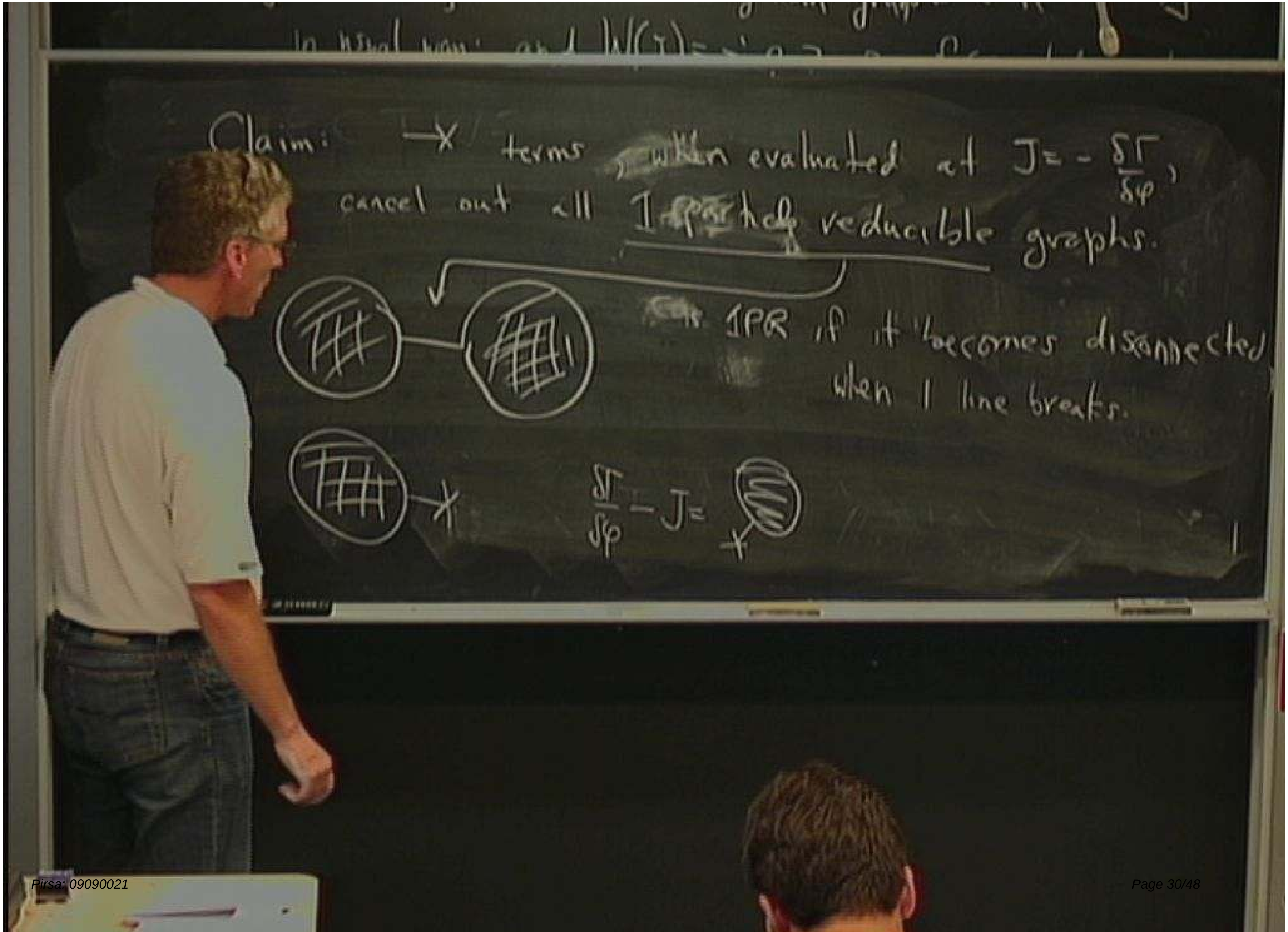

$e^{i\Gamma(\varphi)} = e^{iS(\varphi)} \int \mathcal{D}\phi$

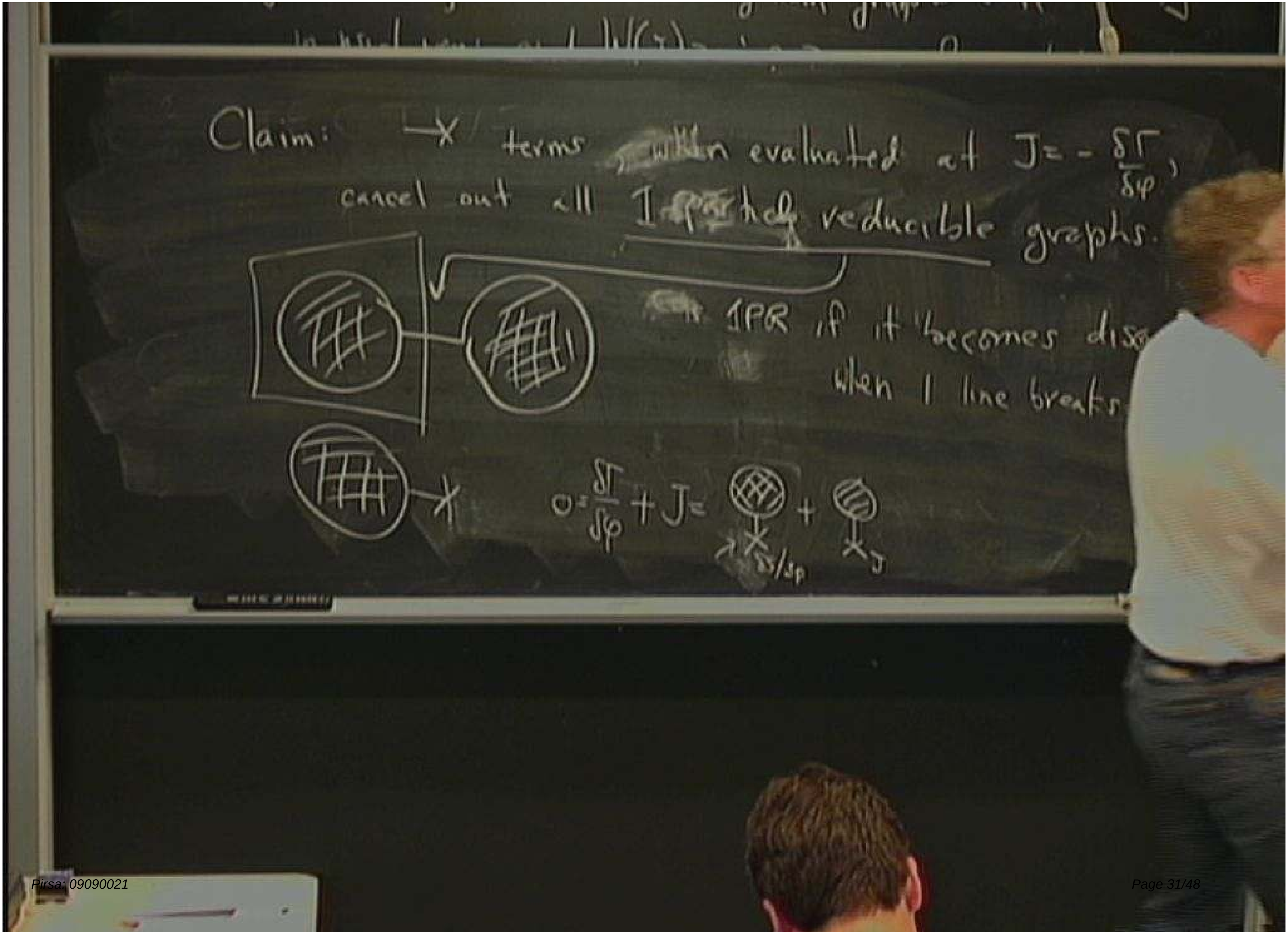

$$e^{i\Gamma(\varphi)} = e^{iS(\varphi)} \int \mathcal{D}\phi \; e^{i\int dv_x \left(\frac{\delta S}{\delta\varphi(x)} - J(x)\right)\Phi(x)}$$


e^{iΓ(φ)} = e^{iS(φ)} ∫ Dφ e^{i ∫dx (δS/δφ(x) − J(x)) Φ(x) + i/2 S''(φ) Φ²}
[1 + (cubic) + 1/2! (cubic)²]
J = − δΓ/δφ where Γ = S + Γ_{1-loop}
Γ_{1-loop} = (det S''(φ))^{-1/2}

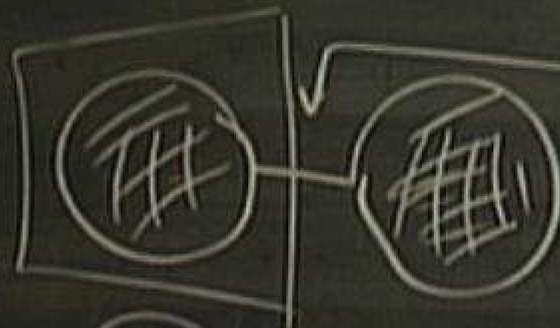

Claim: terms, when evaluated at J = - δΓ/δφ,
cancel out all 1 particle reducible graphs.


Claim: terms when evaluated at J = - δΓ/δφ,
cancel out all 1 particle reducible graphs.
1PR if it becomes disconnected
when 1 line breaks.
δΓ

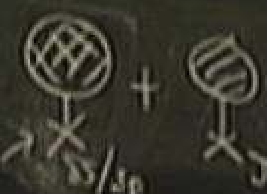

Claim: →✗ terms when evaluated at J = - δΓ/δφ,
cancel out all 1 particle reducible graphs.
1PR if it becomes disconnected
when 1 line breaks.
δΓ/δφ - J =


Claim: —x terms, when evaluated at J= - δΓ/δφ,
cancel out all 1 particle reducible graphs.
1PR if it becomes disc[onnected]
when 1 line breaks
0 = δΓ/δφ + J =
δ/δφ
J

Claim: $\to\!\times$ terms, when evaluated at $J = -\frac{\delta\Gamma}{\delta\varphi}$, cancel out all 1 particle reducible graphs.



1PR if it becomes disconnected when 1 line breaks.

$0 = \frac{\delta\Gamma}{\delta\varphi} + J =$ 

... φ as a function of J. Assume this can be inverted

Upshot:

$$\Gamma = S + \Gamma_{\text{1-loop}} + (\text{sum of 2 loop + higher 1PI graphs}$$

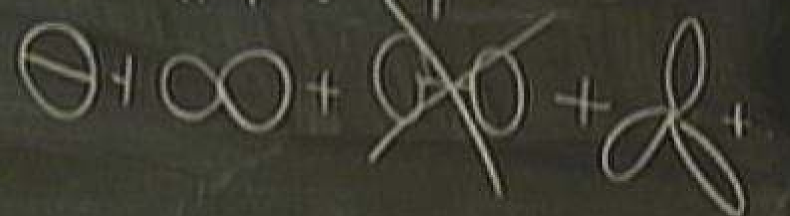
$$\Gamma_{\text{1-loop}} \downarrow (\det S'')^{-1/2}$$

... φ as a function of J. Assume this can be inverted

Upshot:

$$\Gamma(\varphi) = S(\varphi) + \Gamma_{1\text{-loop}} + (\text{sum of 2 loop + higher 1PI graphs with no external lines})$$

$$\Gamma_{1\text{-loop}} \downarrow (\det S'')^{-1/2}$$

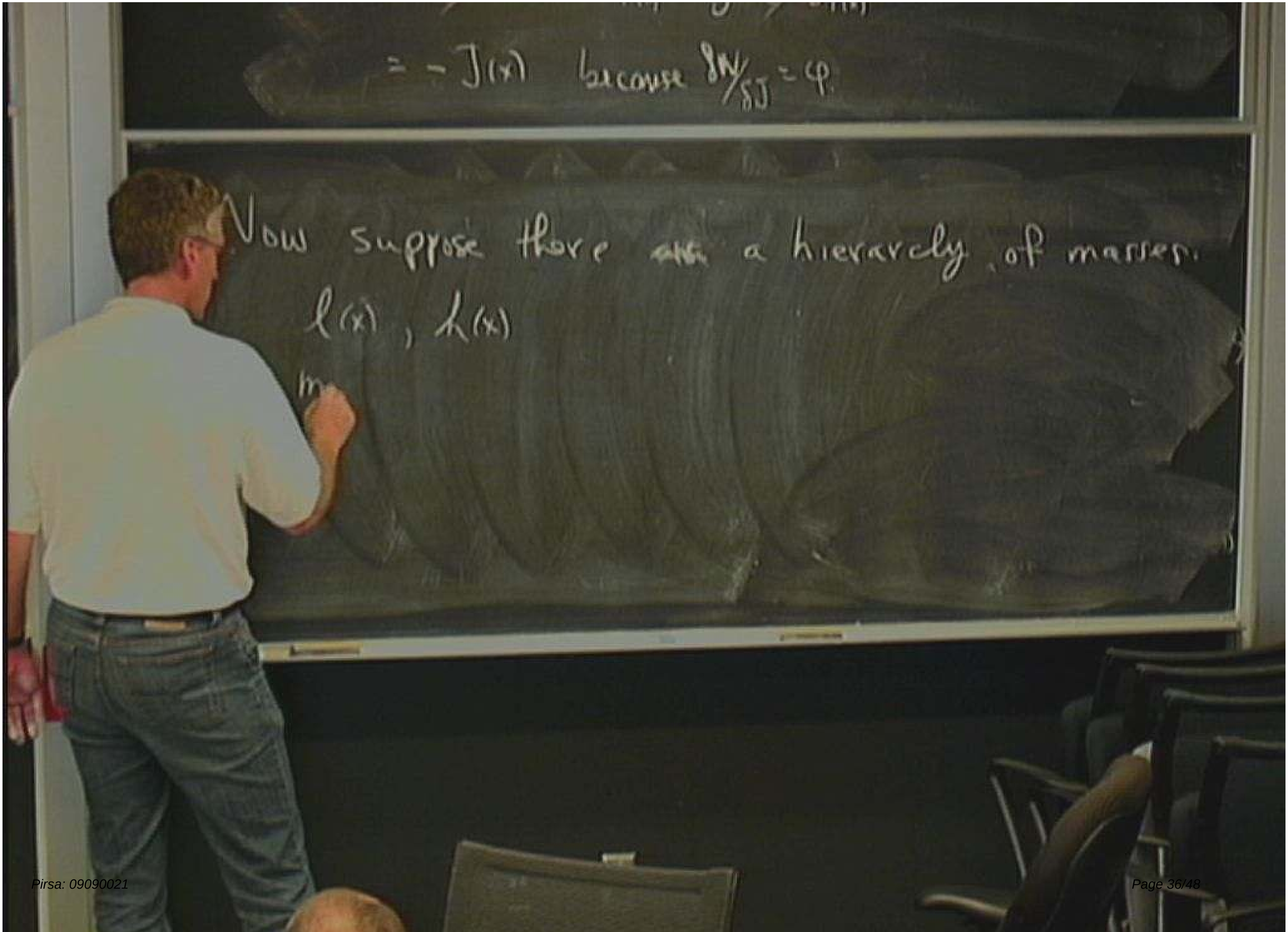


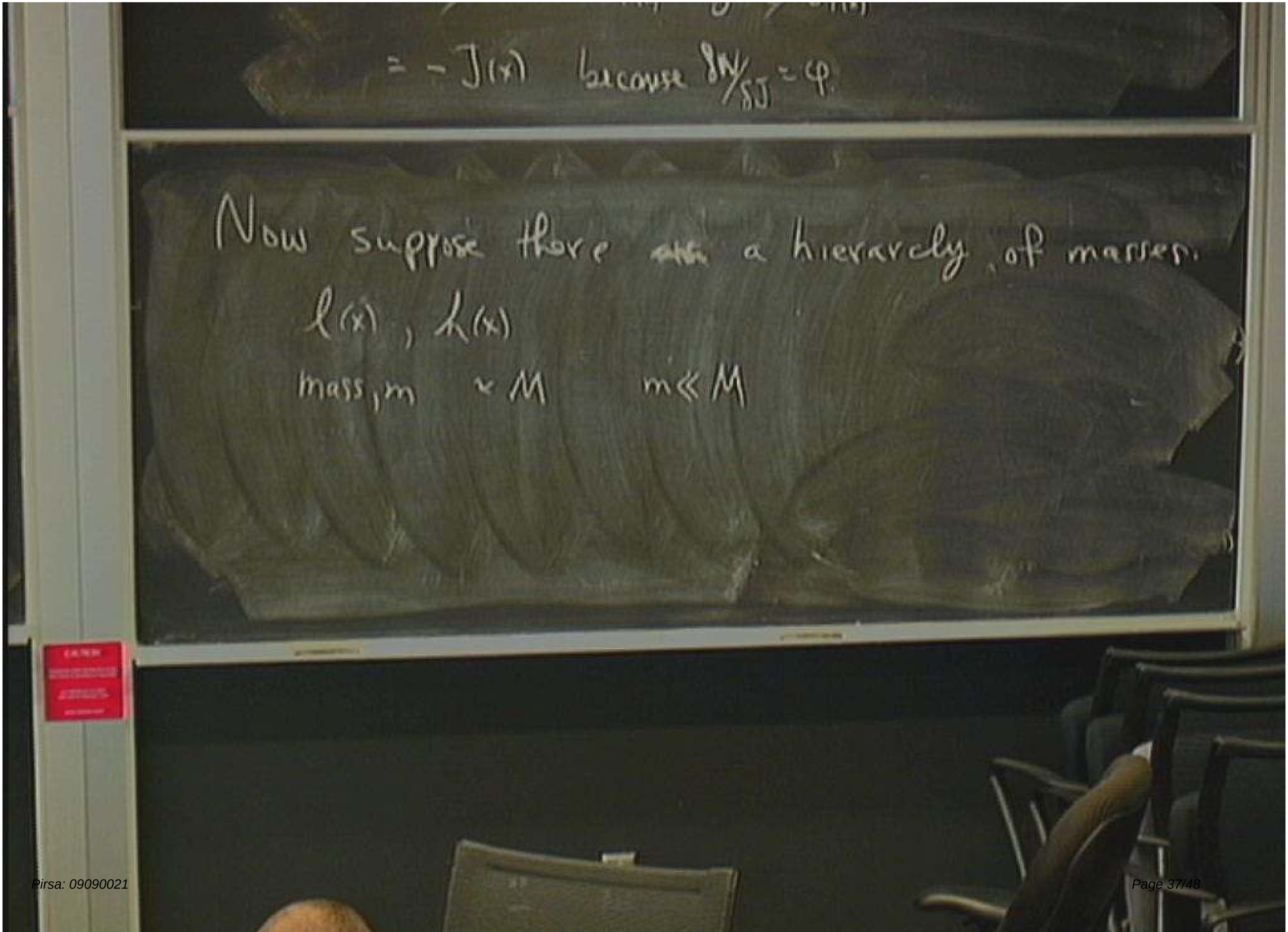
... Φ as a function of J. Assume this can be inverted

Upshot: 

$\Gamma(\phi) = S(\phi) + \Gamma_{1\text{-loop}} + $ (sum of 2 loop + higher 1PI graphs with no external lines)

$\Gamma_{1\text{-loop}} \Downarrow (\det S'')^{-1/2}$


= − J(x) because δW/δJ = φ
Now suppose there a hierarchy of masses.
l(x), h(x)
m


= − J(x) because δW/δJ = φ.
Now suppose there a hierarchy of masses.
l(x), h(x)
mass, m ~ M m≪M

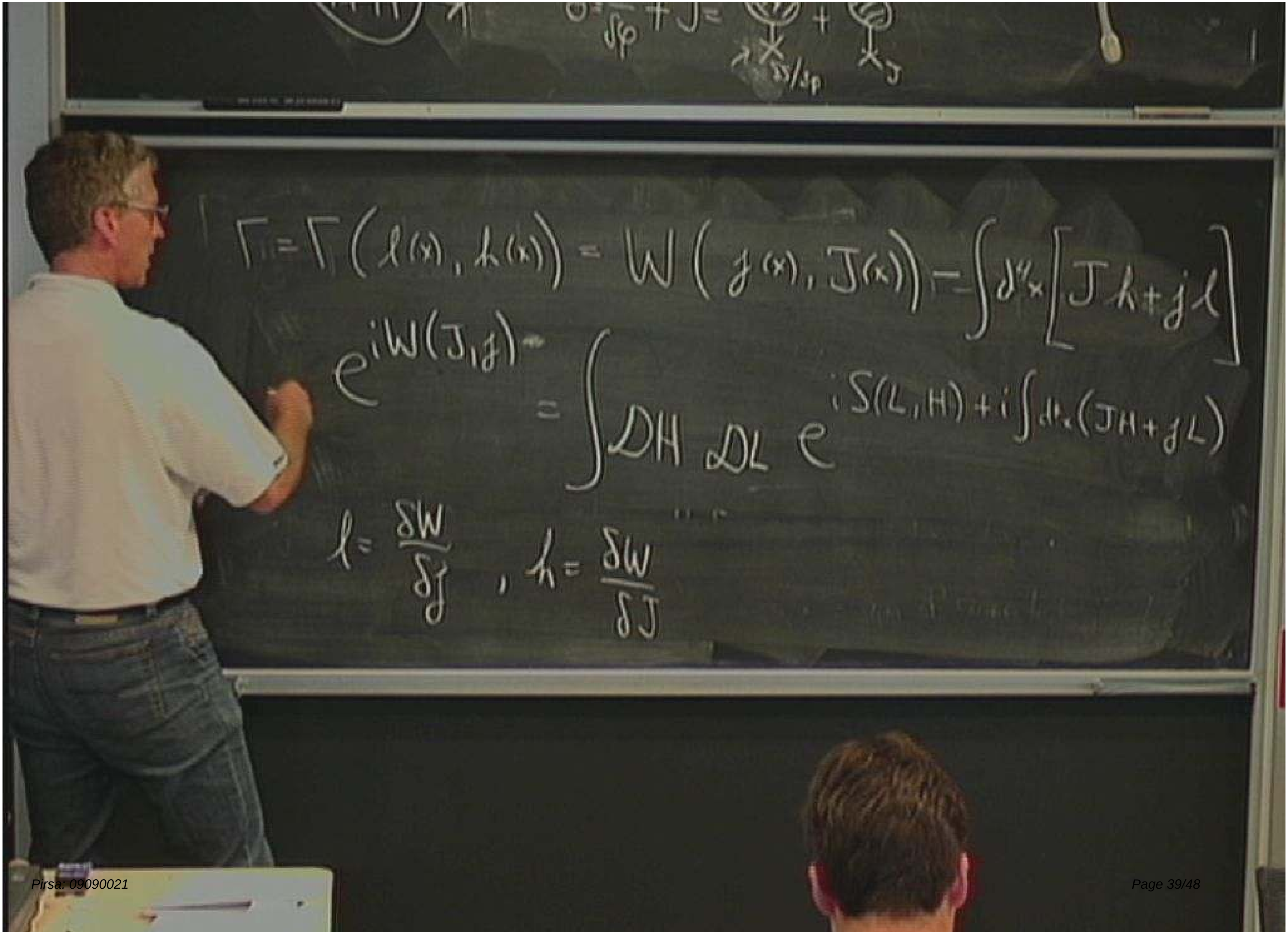
$= -J(x)$ because $\delta W / \delta J = \varphi$.

Now suppose there ~~are~~ a hierarchy of masses.

$\ell(x)$, $h(x)$

mass, $m \sim M$ $\quad m \ll M$

If your interest is only in energies $E \ll M$, or distances $\lambda \gg 1/M$, then you don't need $\Gamma(\varphi(x))$ for all $\varphi(x)$

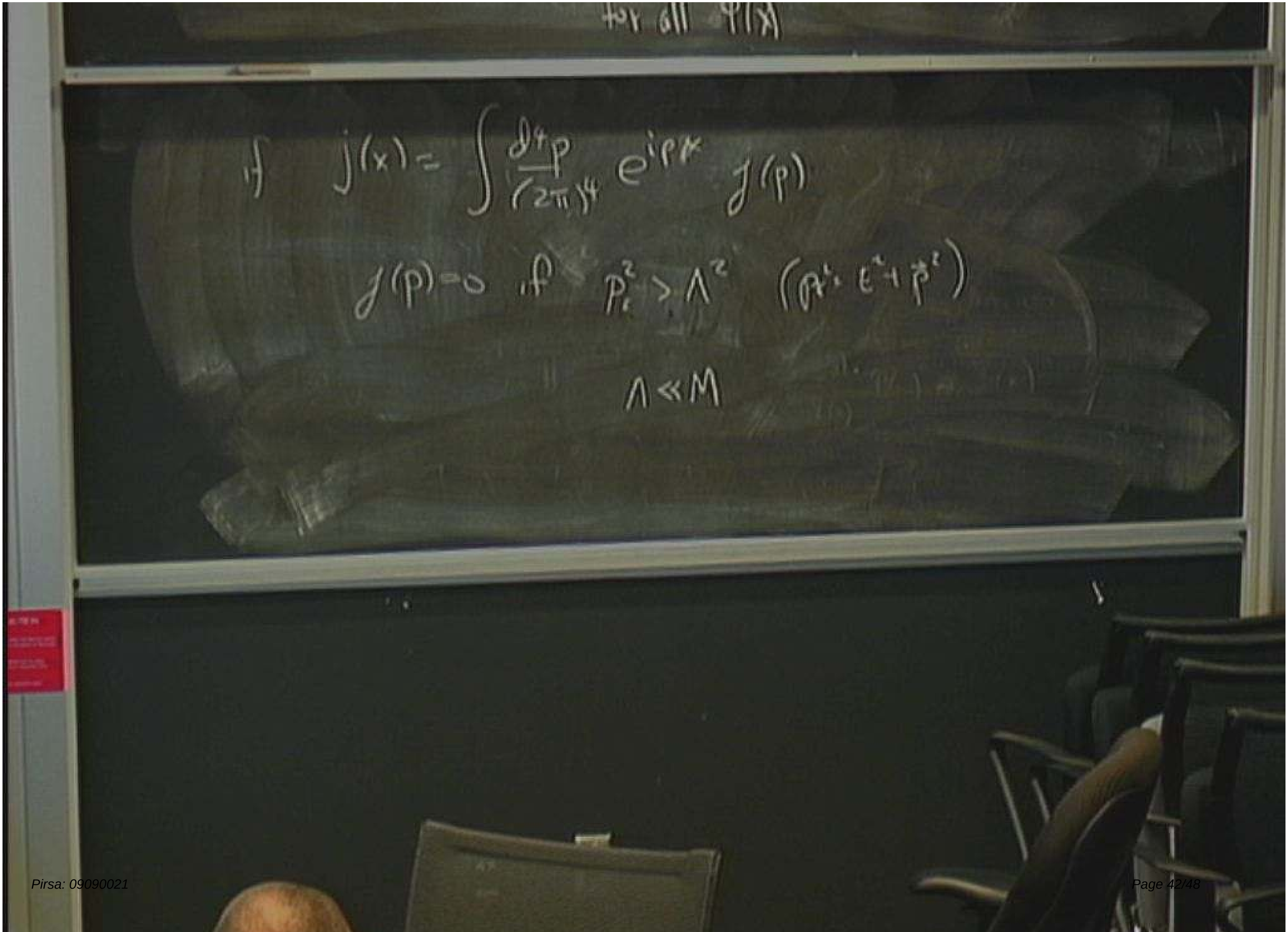

$\Gamma = \Gamma(l(x), h(x)) = W(j(x), J(x)) - \int d^4x \left[J h + j l \right]$
$e^{iW(J,j)} = \int \mathcal{D}H \, \mathcal{D}L \, e^{iS(L,H) + i\int d^4x (JH + jL)}$
$l = \frac{\delta W}{\delta j} \, , \; h = \frac{\delta W}{\delta J}$

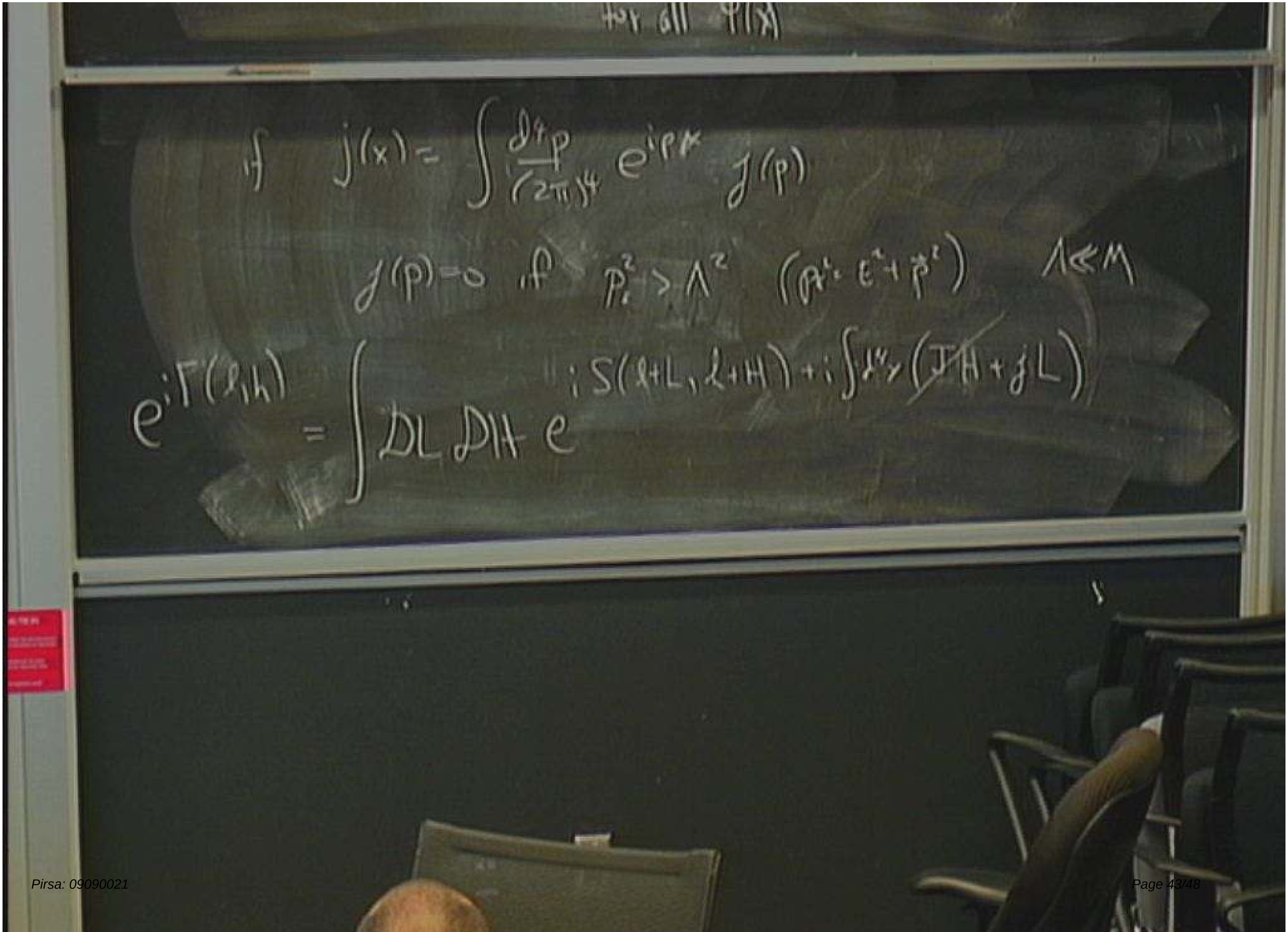
$\Gamma = S(\ell, h) + \Gamma_{1\text{-loop}}$ + sum of vacuum 2-loop & higher graphs that are 1PI.

$\Gamma = S(l,h) + \Gamma_{1\text{-loop}} +$ sum of vacuum 2-loop & higher graphs that are 1PI.

To specialize only to correlations of light fields should take $J=0$.

To " " " " " " of long-wavelength parts of $L(x)$ should require $j(x)$ to only involve long wavelengths.


for all $\varphi(x)$
if $j(x) = \int \frac{d^4p}{(2\pi)^4} e^{ipx} \, j(p)$
$j(p) = 0$ if $p_E^2 > \Lambda^2$ $(p_E^2 = t^2 + \vec{p}^2)$
$\Lambda \ll M$


for all φ(x)
if j(x) = ∫ d⁴p/(2π)⁴ e^{ipx} j(p)
j(p) = 0 if p_E² > Λ² (p_E² = E² + p⃗²) Λ ≪ M
e^{iΓ(l,h)} = ∫ DL DH e^{iS(l+L, l+H) + i∫d⁴x (JH + jL)}

cancelled if $J = -\delta\Gamma/\delta\ell$  if $J = -\frac{\delta\Gamma}{\delta\ell}$

$\Gamma = S(\ell, h) + \Gamma_{1\text{-loop}} +$ Sum of vacuum 2-loop & higher graphs that are 1PI.

To specialize only to correlations of light fields should take $J = 0$.

To " " " " " of long-wavelength parts of $L(x)$ should require $J(x)$ to only involve long wavelengths.

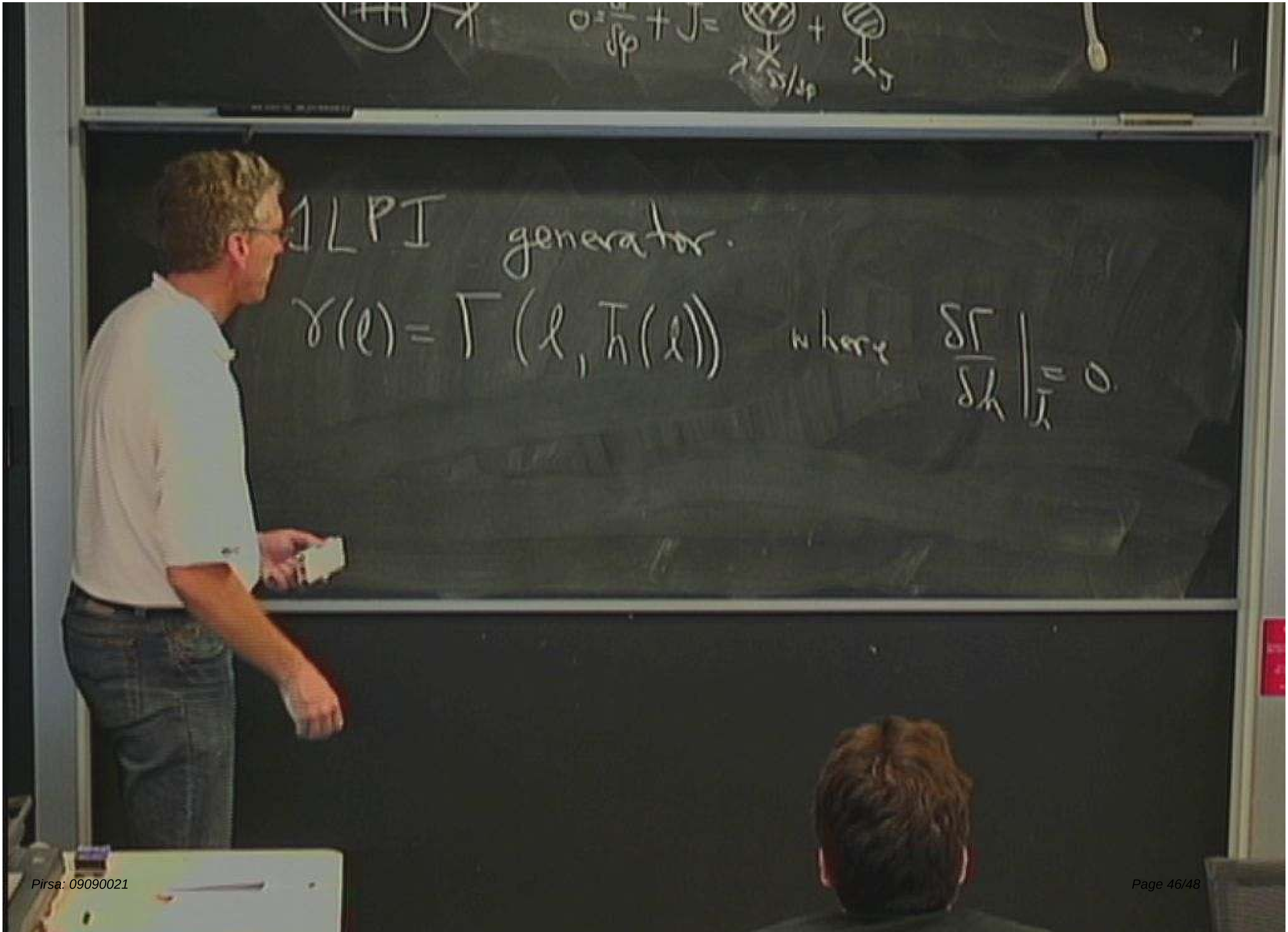
for all $\varphi(x)$

if $\quad j(x) = \int \frac{d^4p}{(2\pi)^4}\, e^{ipx}\, j(p)$

$j(p) = 0$ if $p_E^2 > \Lambda^2 \quad (p_E^2 = E^2 + \vec{p}^{\,2}) \qquad \Lambda \ll M$

$$e^{i\Gamma(\ell,\bar{h})} = \int \mathcal{D}L\,\mathcal{D}H\; e^{iS(\ell+L,\,\ell+H) + i\int d^4y\,(\cancel{J}H + jL)}$$

$\bar{h} = \frac{\delta W}{\delta J}\Big|_{J=0}$ satisfies $\frac{\delta\Gamma}{\delta h}\Big|_{\bar{h}} = 0$


1LPI generator.
$\gamma(\ell) = \Gamma(\ell, \bar{h}(\ell))$ where $\frac{\delta\Gamma}{\delta h}\Big|_{\bar{h}} = 0$

1LPI generator.

$$\gamma(\ell) = \Gamma(\ell, \bar{h}(\ell)) \quad \text{where} \quad \left.\frac{\delta\Gamma}{\delta h}\right|_{\bar{h}} = 0.$$

$$= S(\ell, \bar{h}(\ell)) + \Gamma_{1\text{-loop}} + \text{sum of 1LPI vacuum graphs}$$

$$0 = \frac{\delta}{\delta\varphi} + J = \ldots$$

1LPI generator.

$$\longrightarrow \gamma(\ell) = \Gamma(\ell, \bar{h}(\ell)) \quad \text{where} \quad \left.\frac{\delta\Gamma}{\delta h}\right|_{\bar{h}} = 0.$$

all info of LE theory $< \Lambda$

$$= S(\ell, \bar{h}(\ell)) + \Gamma_{1\text{-loop}} + \text{sum of 1LPI vacuum graphs}$$