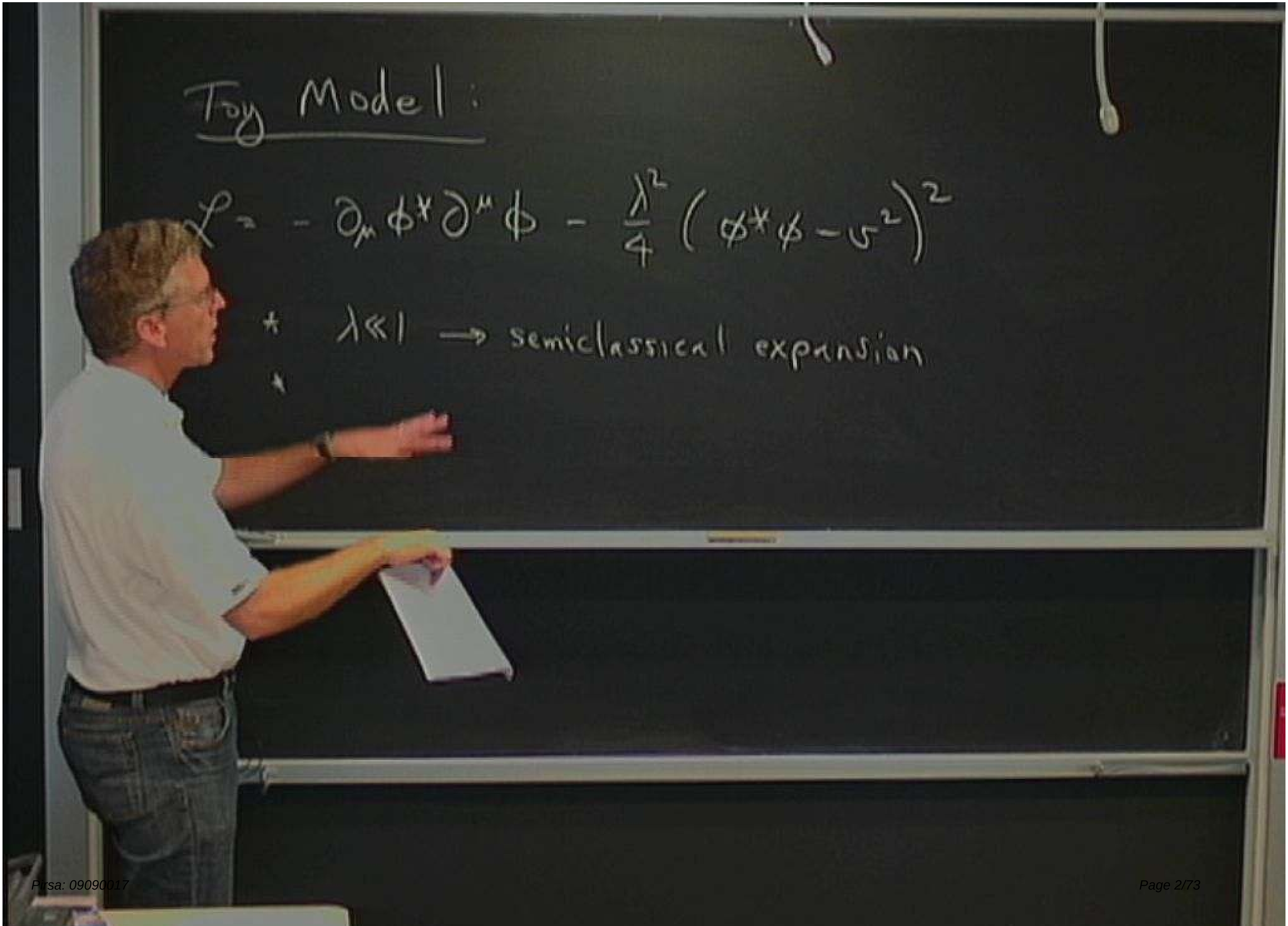
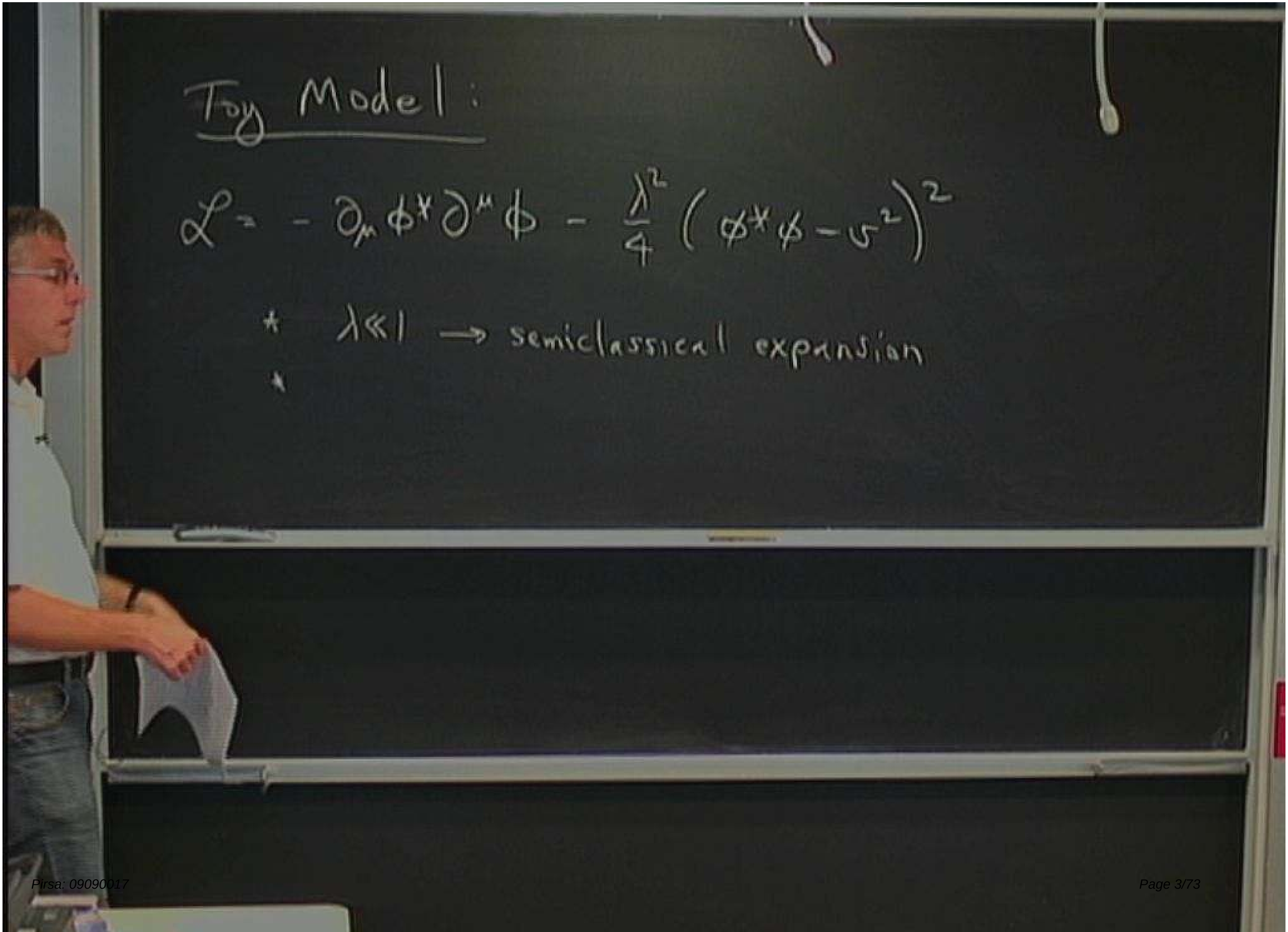
Title: Introduction to Effective Field Theory - Lecture 2A

Date: Sep 23, 2009 10:00 AM

URL: http://pirsa.org/09090017

Abstract:


Toy Model:
$\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - \frac{\lambda^2}{4}(\phi^*\phi - v^2)^2$
* $\lambda \ll 1 \longrightarrow$ semiclassical expansion
*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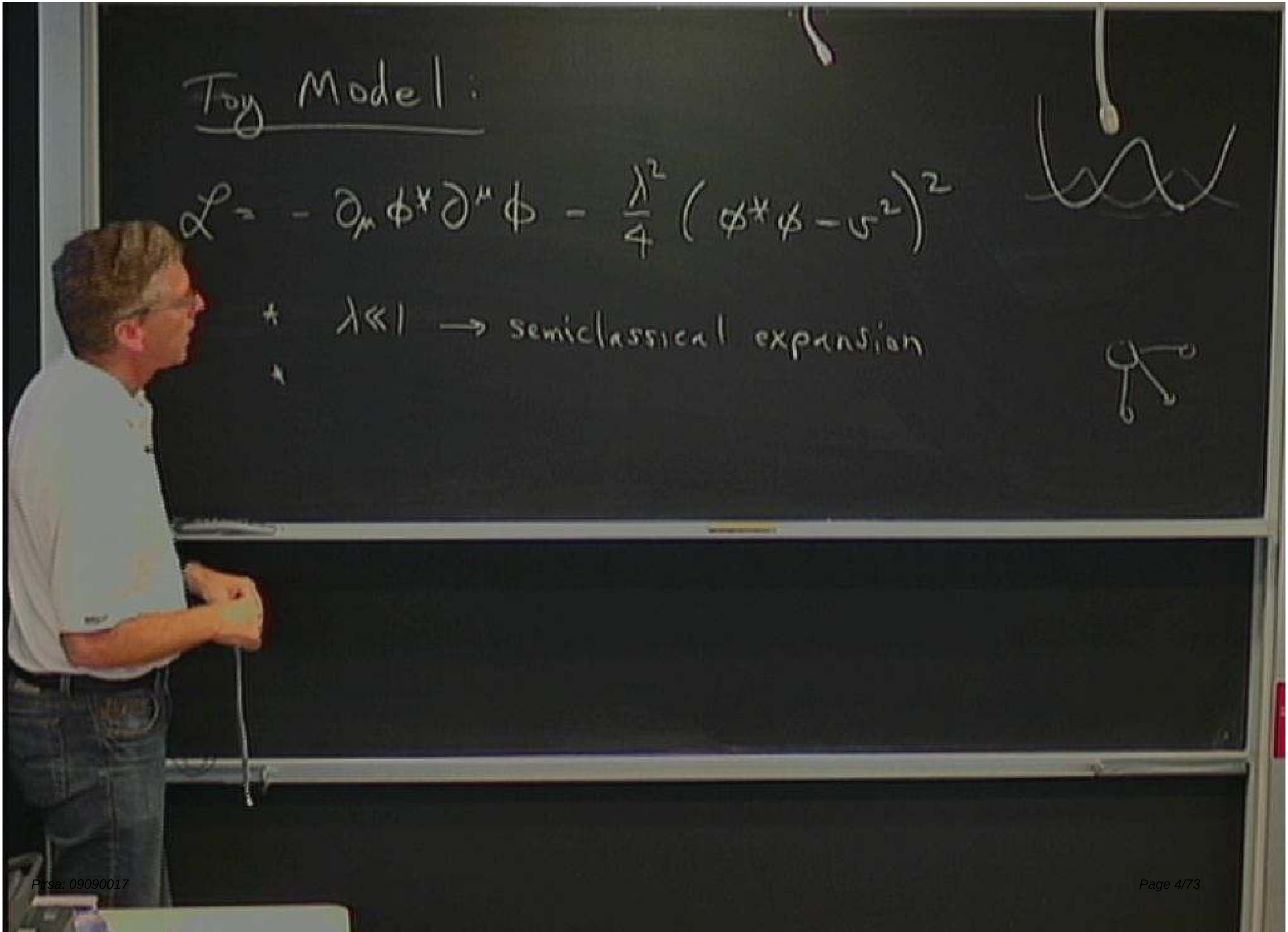
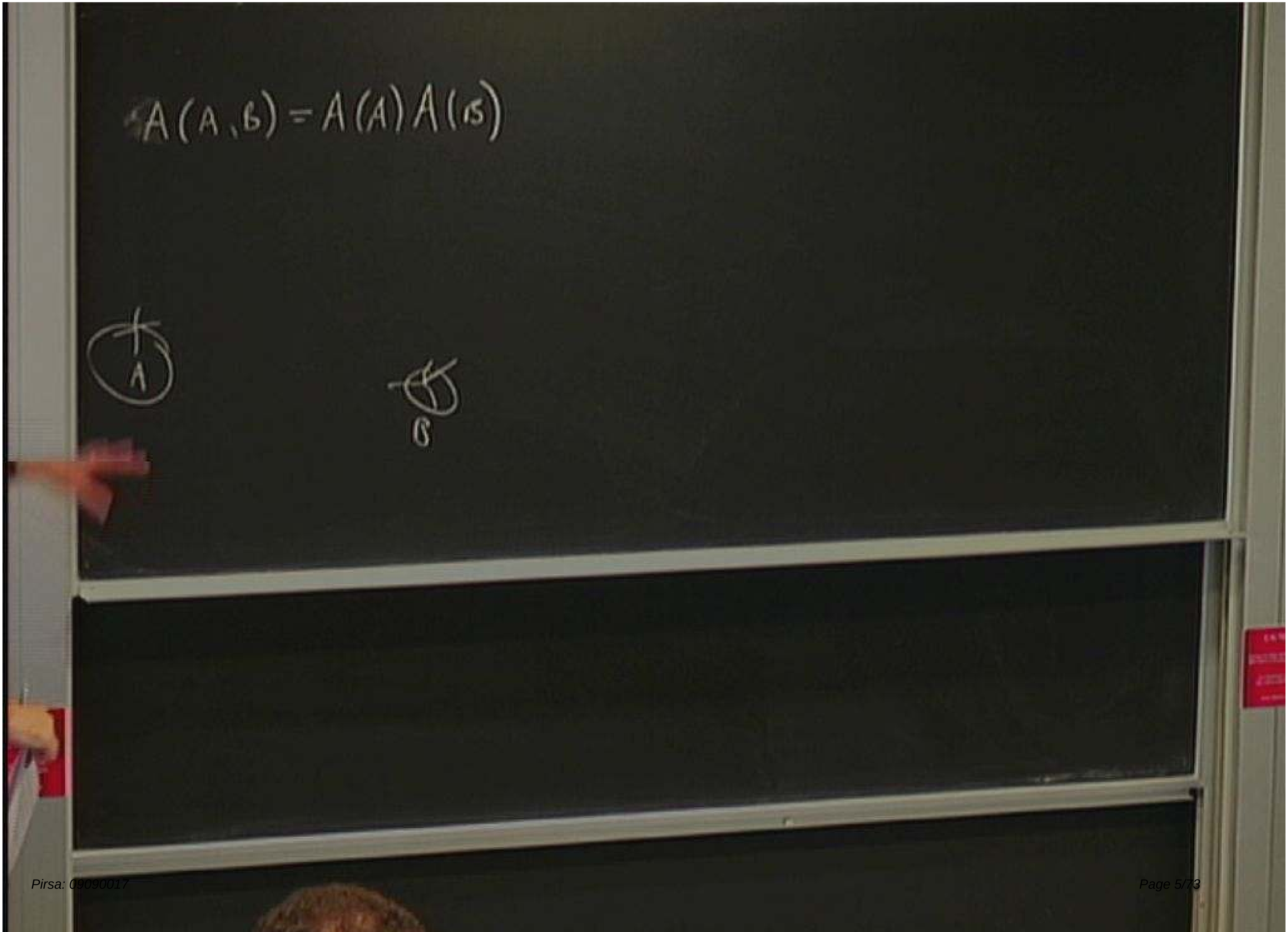
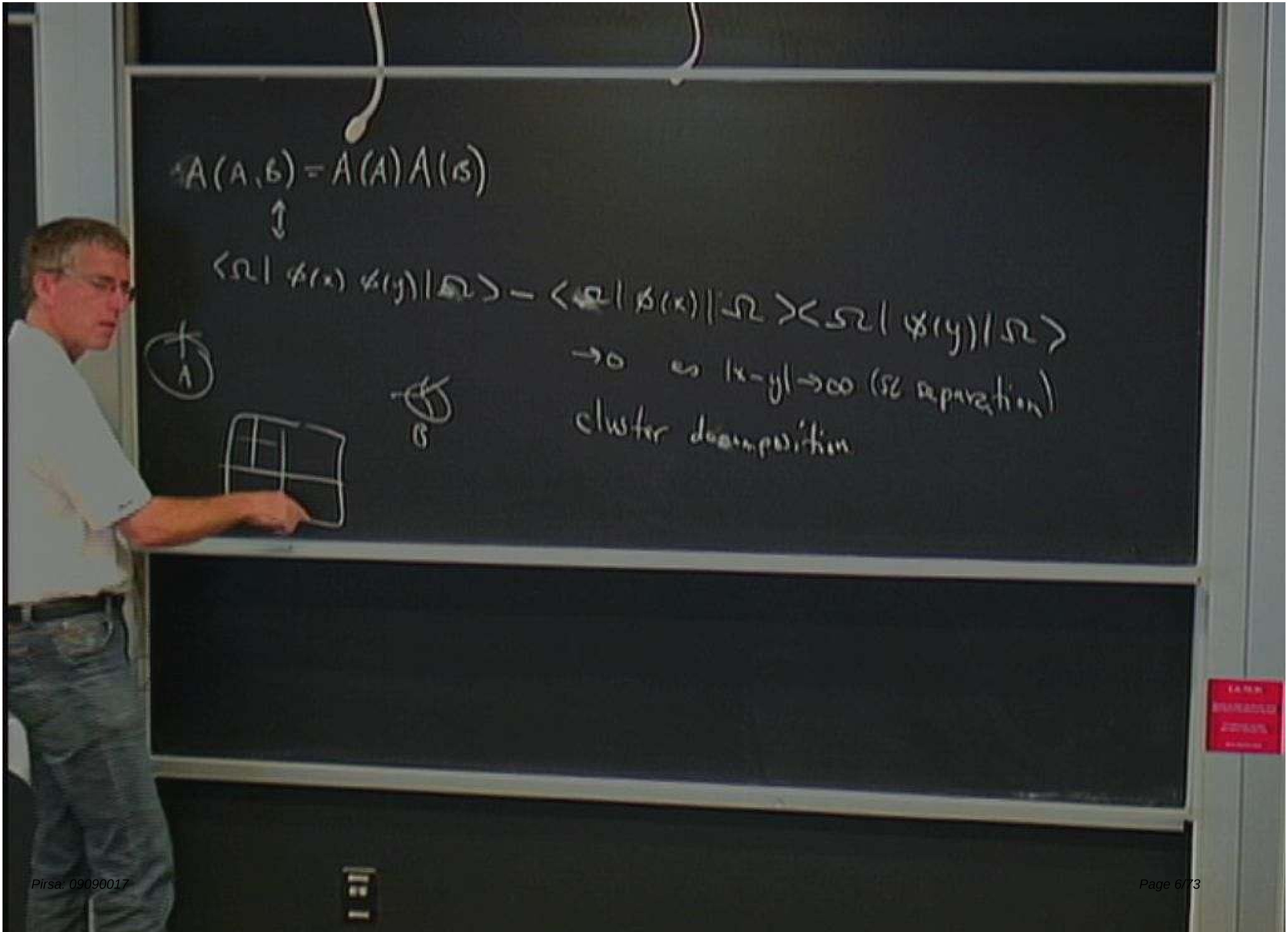

Toy Model:
$\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - \frac{\lambda^2}{4}(\phi^* \phi - v^2)^2$
* $\lambda \ll 1$ → semiclassical expansion
*


A(A,B) = A(A) A(B)
A
B


A(A,B) = A(A)A(B)
⟨Ω| φ(x) φ(y)|Ω⟩ − ⟨Ω| φ(x)|Ω⟩⟨Ω| φ(y)|Ω⟩
→0 as |x−y|→∞ (sl separation)
cluster decomposition
A
B

$A(A,B) = A(A)\,A(B)$

$\Updownarrow$

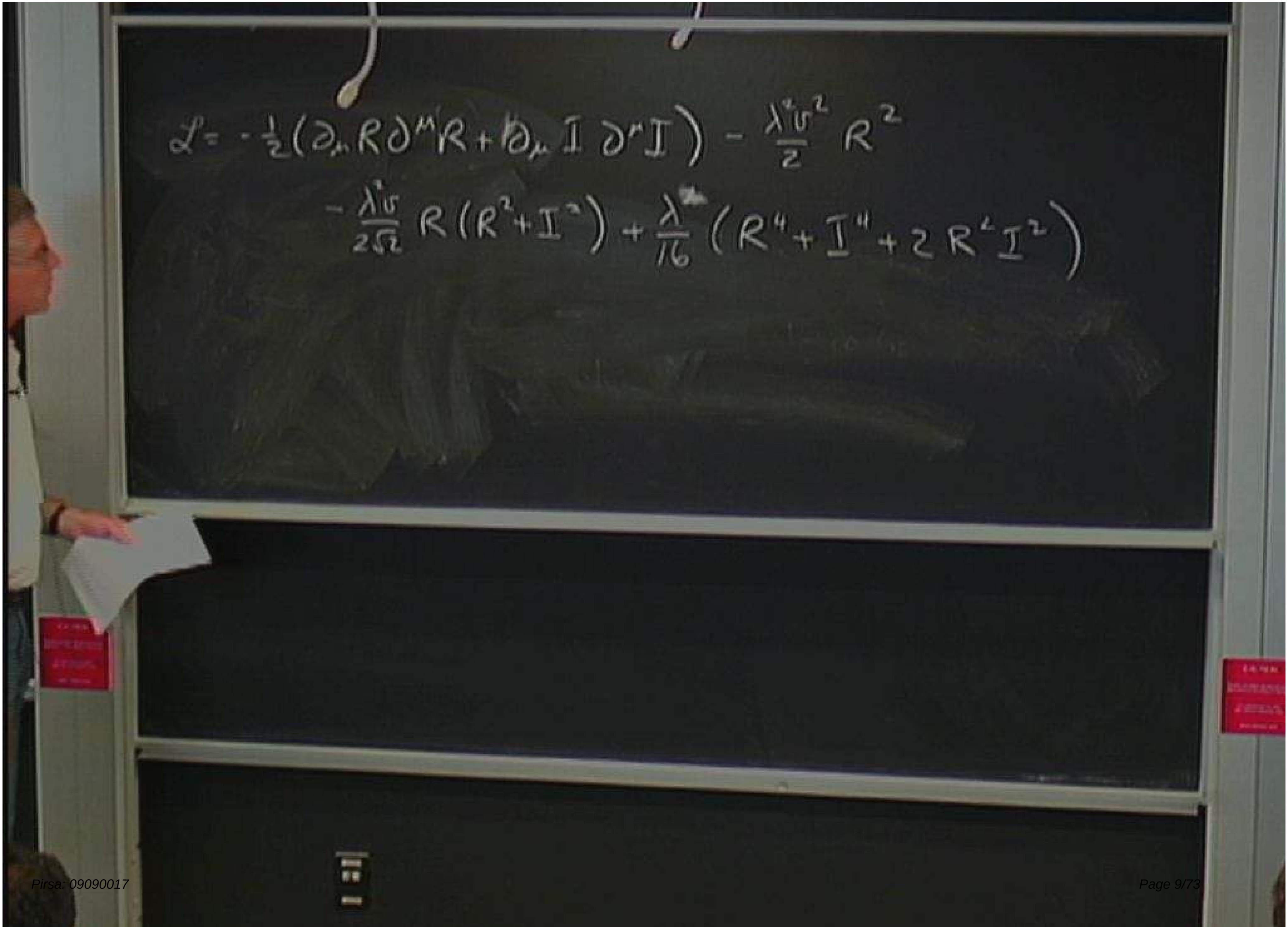
$\langle\Omega|\phi(x)\phi(y)|\Omega\rangle - \langle\Omega|\phi(x)|\Omega\rangle\langle\Omega|\phi(y)|\Omega\rangle$

A $\quad e^{-iHt}$ $\quad$ B

$\to 0$ as $|x-y| \to \infty$ (sl separation)

cluster decomposition

$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu R\, \partial^\mu R + \partial_\mu I\, \partial^\mu I\right) - \lambda$$


$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu R \partial^\mu R + \partial_\mu I \partial^\mu I\right) - \frac{\lambda^2 v^2}{2} R^2$
$- \frac{\lambda^2 v}{2\sqrt{2}} R\left(R^2 + I^2\right) + \frac{\lambda^2}{16}\left(R^4 + I^4 + 2R^2 I^2\right)$

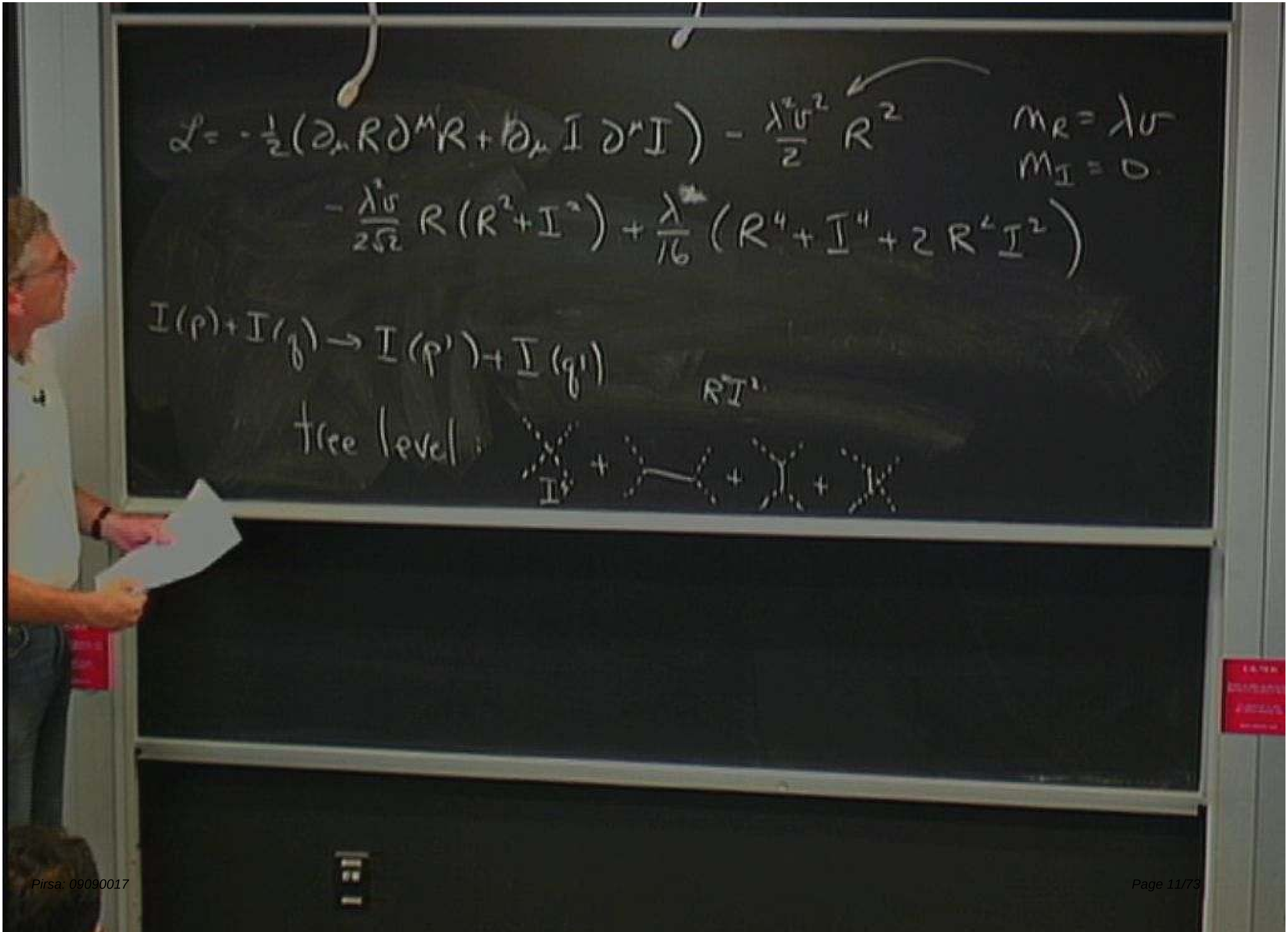
$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu R\, \partial^\mu R + \partial_\mu I\, \partial^\mu I\right) - \frac{\lambda^2 v^2}{2} R^2$$

$$- \frac{\lambda^2 v}{2\sqrt{2}} R\left(R^2 + I^2\right) + \frac{\lambda^2}{16}\left(R^4 + I^4 + 2R^2 I^2\right)$$

$m_R = \lambda v$

$m_I = 0$

$$I(p) + I(q) \to I(p') + I(q')$$


$\mathcal{L} = -\frac{1}{2}(\partial_\mu R \partial^\mu R + \partial_\mu I \partial^\mu I) - \frac{\lambda^2 v^2}{2} R^2$
$m_R = \lambda v$
$m_I = 0$
$-\frac{\lambda^2 v}{2\sqrt{2}} R(R^2 + I^2) + \frac{\lambda^2}{16}(R^4 + I^4 + 2R^2 I^2)$
$I(p) + I(q) \to I(p') + I(q')$
RI^2
tree level :
I^4

$$S(p, q \to p', q') = -(2\pi i)^4 \delta^4(p + q - p' - q') \mathcal{A}$$

$$\mathcal{A} =$$

$$S(p,q \to p',q') = -(2\pi i)^4 \delta^4(p+q-p'-q')\,\mathcal{A}$$

$$\mathcal{A} = -\frac{\lambda^2}{16}\cdot 4\cdot 3\cdot 2$$

$$S(p,q\to p',q') = -(2\pi i)^4\,\delta^4(p+q-p'-q')$$

$$A = -\frac{\lambda^2}{16}\cdot 4\cdot 3\cdot 2 + (-)^2\,\frac{1}{2!}$$

$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu R\,\partial^\mu R + \partial_\mu I\,\partial^\mu I\right) - \frac{\lambda^2 v^2}{2} R^2$$

$m_R = \lambda v$

$m_I = 0$

$$- \frac{\lambda^2 v}{2\sqrt{2}} R\left(R^2 + I^2\right) - \frac{\lambda}{16}\left(R^4 + I^4 + 2R^2 I^2\right)$$

$$I(p) + I(q) \to I(p') + I(q')$$

RI^2

$e^{iS_{int}} = 1 + iS_{int} + \frac{i^2}{2} S_{int}^2$

tree level:

$$S(p,q \to p',q') = -(2\pi i)^4 \delta^4(p+q-p'-q') \, A$$

$$A = -\frac{\lambda^2}{16} \cdot 4 \cdot 3 \cdot 2 + (-)^2 \frac{1}{2!} \, 4 \cdot 2 \left[\frac{1}{(p+q)^2+m_K^2} + \frac{1}{(p-p')^2+m_K^2} + \frac{1}{(p-q')^2+m_K^2} \right]$$

$$S(p,q\to p',q') = -(2\pi i)^4\,\delta^4(p+q-p'-q')\,\mathcal{A}$$

$$\mathcal{A} = -\frac{\lambda^2}{16}\cdot 4\cdot 3\cdot 2 + (-)^2\frac{1}{2!}\,4\cdot 2\left[\frac{1}{(p+q)^2+m_R^2}+\frac{1}{(p-p')^2+m_R^2}+\frac{1}{(p-q')^2+m_R^2}\right]\left(\frac{\lambda^2 v}{2\sqrt{2}}\right)^2$$

$$= -\frac{3\lambda^2}{2} + \frac{\lambda^4 v^2}{2}\left[\frac{1}{(p+q)^2+m_R^2}+\dots\right]$$

$$\phi = v + \frac{1}{\sqrt{2}}(R + iI)$$

Expand A in powers of $\frac{\text{Energy}}{m_R}$

$$\frac{1}{k^2 + m_R^2} = \frac{1}{m_R^2}\left[1 - \frac{k^2}{m_R^2} + \frac{k^4}{m_R^4} + \cdots\right]$$

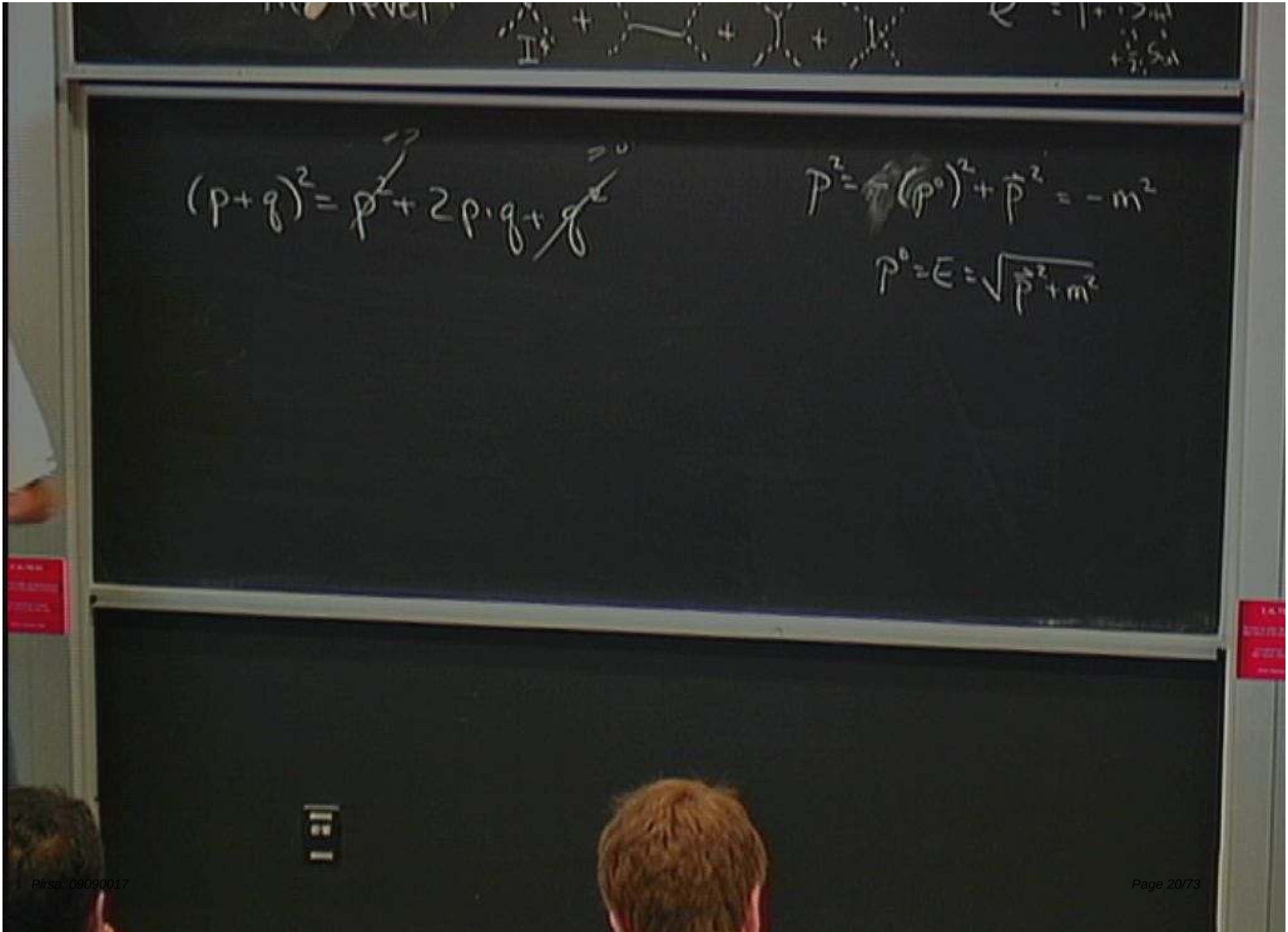
$$A = \lambda^2\left[-\frac{3}{2}\right.$$

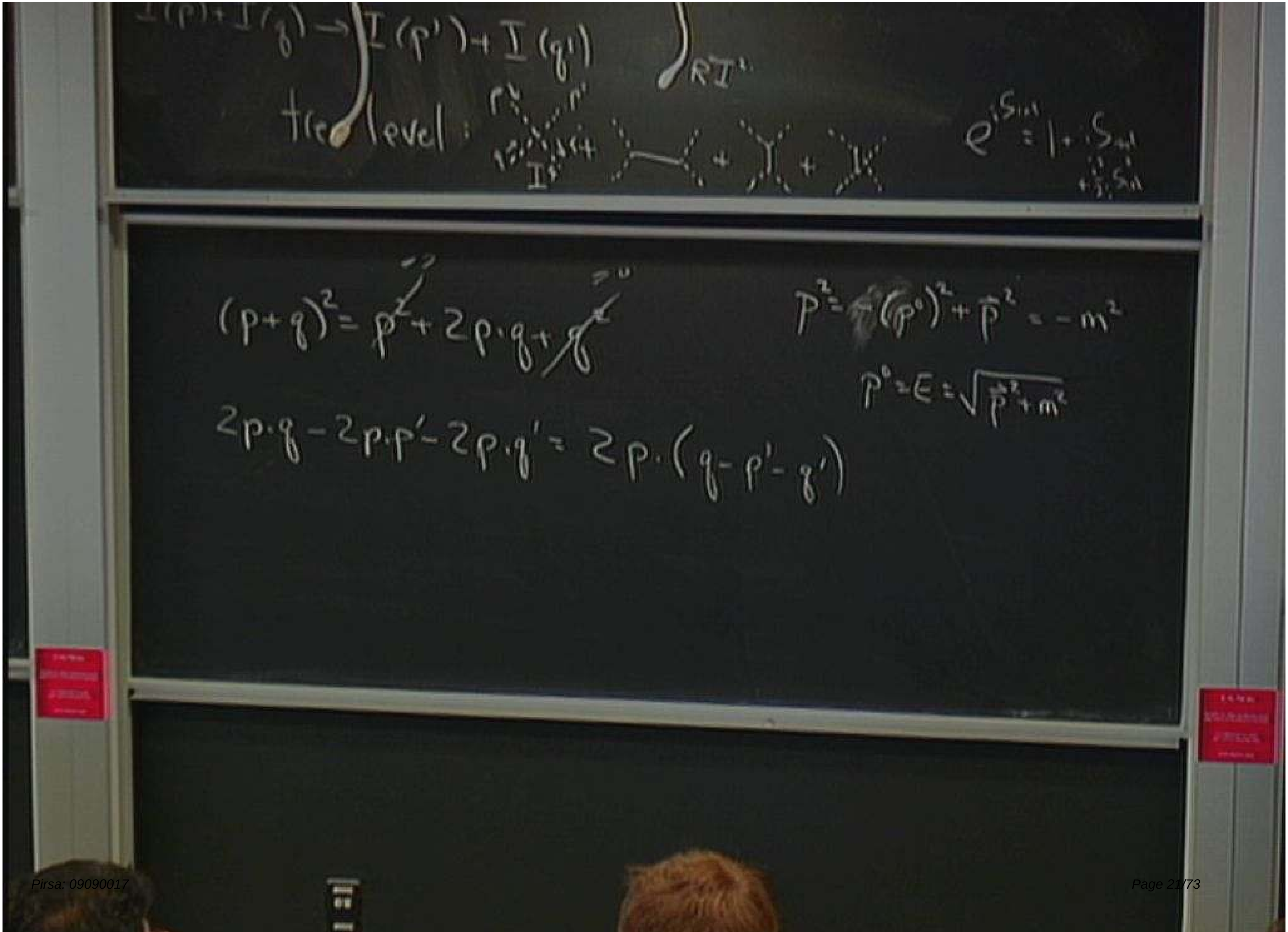
$$\varphi = v + \frac{1}{\sqrt{2}}(R + iI)$$

Expand A in powers of $\frac{\text{Energy}}{m_R}$

$$\frac{1}{k^2 + m_R^2} = \frac{1}{m_R^2}\left[1 - \frac{k^2}{m_R^2} + \frac{k^4}{m_R^4} + \cdots\right]$$

$$A = \lambda^2\left[\left(-\frac{3}{2} + \frac{\lambda^2 v^2}{2}\cdot\frac{3}{m_R^2}\right) + \frac{\lambda^2 v^2}{2m_R^4}\left[\underbrace{(p+q)^2 + (p-p')^2 + (p-q')^2}\right]\right]$$


$(p+q)^2 = \underbrace{p^2}_{=-m^2} + 2p\cdot q + \underbrace{\cancel{q^2}}_{=0}$
$p^2 = -(p^0)^2 + \vec{p}^{\,2} = -m^2$
$p^0 = E = \sqrt{\vec{p}^{\,2} + m^2}$


tree level :
(p+q)^2 = p^2 + 2p·q + q^2
p^2 = -(p^0)^2 + p^2 = -m^2
p^0 = E = √(p^2 + m^2)
2p·q - 2p·p' - 2p·q' = 2p·(q - p' - q')

tree level: 

$$e^{iS_{int}} = 1 + iS_{int} + \frac{1}{2}(iS_{int})^2$$

$$(p+q)^2 = \overset{=0}{\cancel{p^2}} + 2p\cdot q + \overset{=0}{\cancel{q^2}}$$

$$p^2 = -(p^0)^2 + \vec{p}^{\,2} = -m^2$$

on shell $\longrightarrow$ $p^0 = E = \sqrt{\vec{p}^{\,2} + m^2}$

$$2p\cdot q - 2p\cdot p' - 2p\cdot q' = 2p\cdot(q - p' - q') = -2p^2 = 0$$

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Expand A in powers of $\frac{\text{Energy}}{m_R}$

$$\frac{1}{k^2+m_R^2} = \frac{1}{m_R^2}\left[1 - \frac{k^2}{m_R^2} + \frac{k^4}{m_R^4} + \dots\right]$$

$$A = \lambda^2\left[\left(-\frac{3}{2} + \frac{\lambda^2 v^2}{2}\cdot\frac{3}{m_R^2}\right) - \frac{\lambda^2 v^2}{2m_R^4}\cancel{\left[\underbrace{(p+q)^2+(p-p')^2+(p-q')^2}_{2p\cdot q - 2p\cdot p' - 2p\cdot q'}\right]}^{=0} + \frac{\lambda^2 v^2}{2m_R^6}\left[(2p\cdot q)^2 + (2p\cdot p')^2 + (2p\cdot q')^2\right] + \dots\right.$$

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Expand A in powers of $\frac{\underline{\text{Energy}}}{m_R}$

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tree level:  $e^{iS_{int}} = 1 + iS_{int} + \frac{i^2}{2} S_{int}^2 \ldots$

$$(p+q)^2 = \cancel{p^2} + 2p\cdot q + \cancel{q^2}$$

$$p^2 = -(p^0)^2 + \vec{p}^{\,2} = -m^2$$

on shell $\longrightarrow$ $p^0 = E = \sqrt{\vec{p}^{\,2} + m^2}$

$$2p\cdot q - 2p\cdot p' - 2p\cdot q' = 2p\cdot(q - p' - q') = -2p^2 = 0$$

$$s = -(p+q)^2 \qquad t = (p-p')^2 \qquad u = (p-q')^2 \qquad s+t+u=0$$

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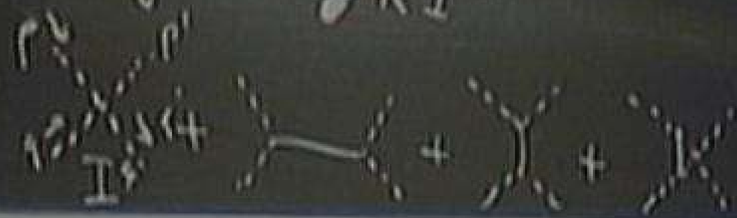
using $p^2 = q^2 = \ldots = 0$

$$A = \frac{\lambda^2 v^2}{2m_R^4}\left(s^2 + t^2 + u^2\right) + O\left(\frac{1}{m_R^8}\right)$$

tree level: 

$e^{iS_{int}} = 1 + iS_{int} + \frac{i^2}{2}S_{int}^2$

$(p+q)^2 = \underbrace{\cancel{p^2}}_{=0} + 2p\cdot q + \underbrace{\cancel{q^2}}_{=0}$ $\qquad p^2 = -(p^0)^2 + \vec{p}^{\,2} = -m^2$

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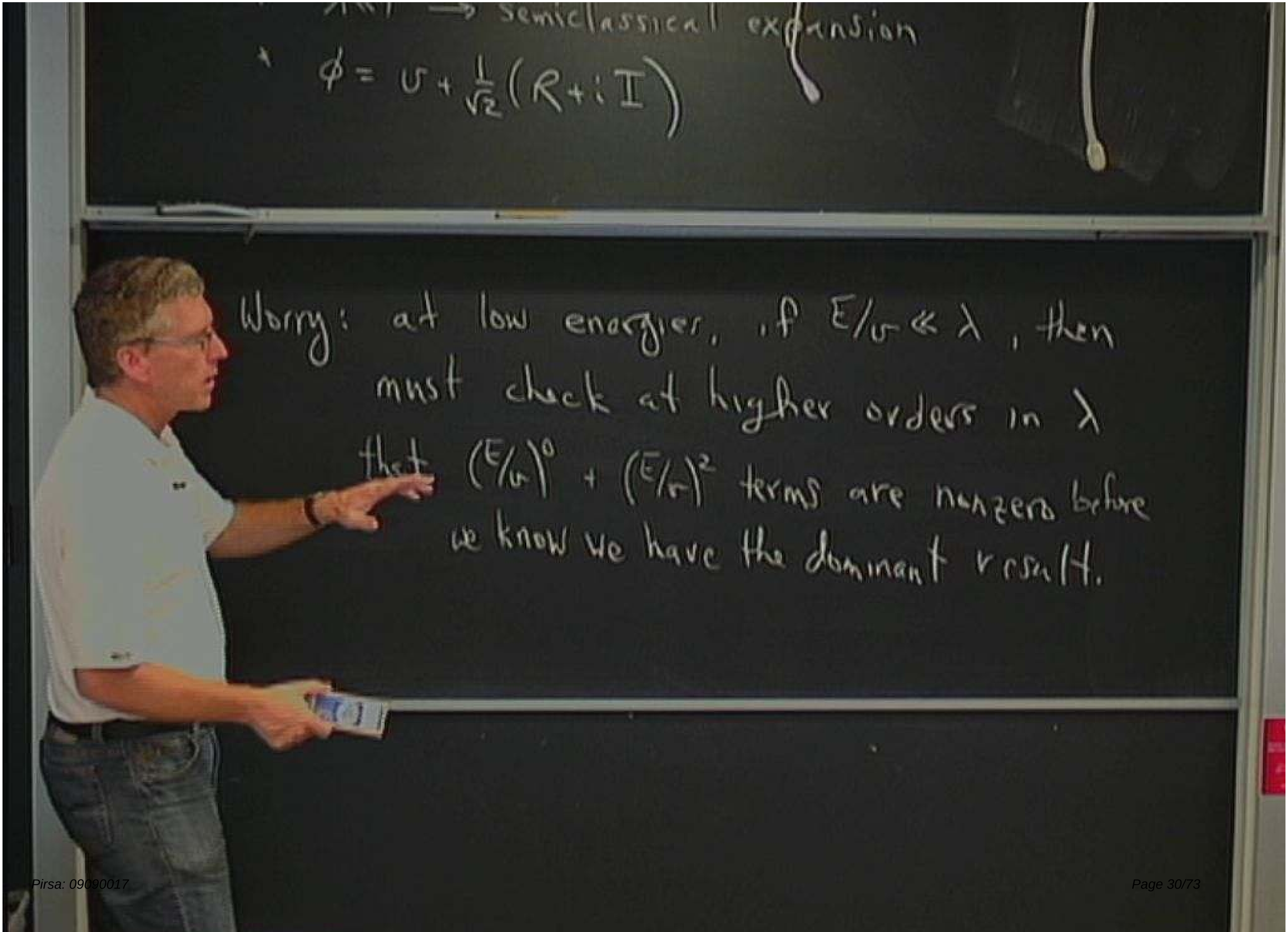
$$A = \frac{\lambda^4 v^2}{2m_R^6}\left(s^2+t^2+u^2\right) + O\left(\frac{1}{m_R^8}\right) \simeq \frac{1}{2\lambda^2 v^4}\left(s^2+t^2+u^2\right) + \dots$$

First two terms vanish, seemingly by accident, in the energy expansion

$$\mathcal{A} = \frac{1}{2\lambda^4}\left(\frac{s^2+t^2+u^2}{v^4}\right) + \lambda^0\left[a_1 + b_1\left(\frac{E}{v}\right)^2 + c_1\left(\frac{E}{v}\right)^4 + \dots\right]$$

$$+ \lambda^2\left[a_2 + b_2\left(\frac{E}{v}\right)^2 + \dots\right] + \dots$$


semiclassical expansion
$\phi = v + \frac{1}{\sqrt{2}}(R + i I)$
Worry: at low energies, if $E/v \ll \lambda$, then
must check at higher orders in λ
that $(E/v)^0$ + $(E/v)^2$ terms are nonzero before
we know we have the dominant result.

$\lambda \ll 1 \Rightarrow$ semiclassical expansion

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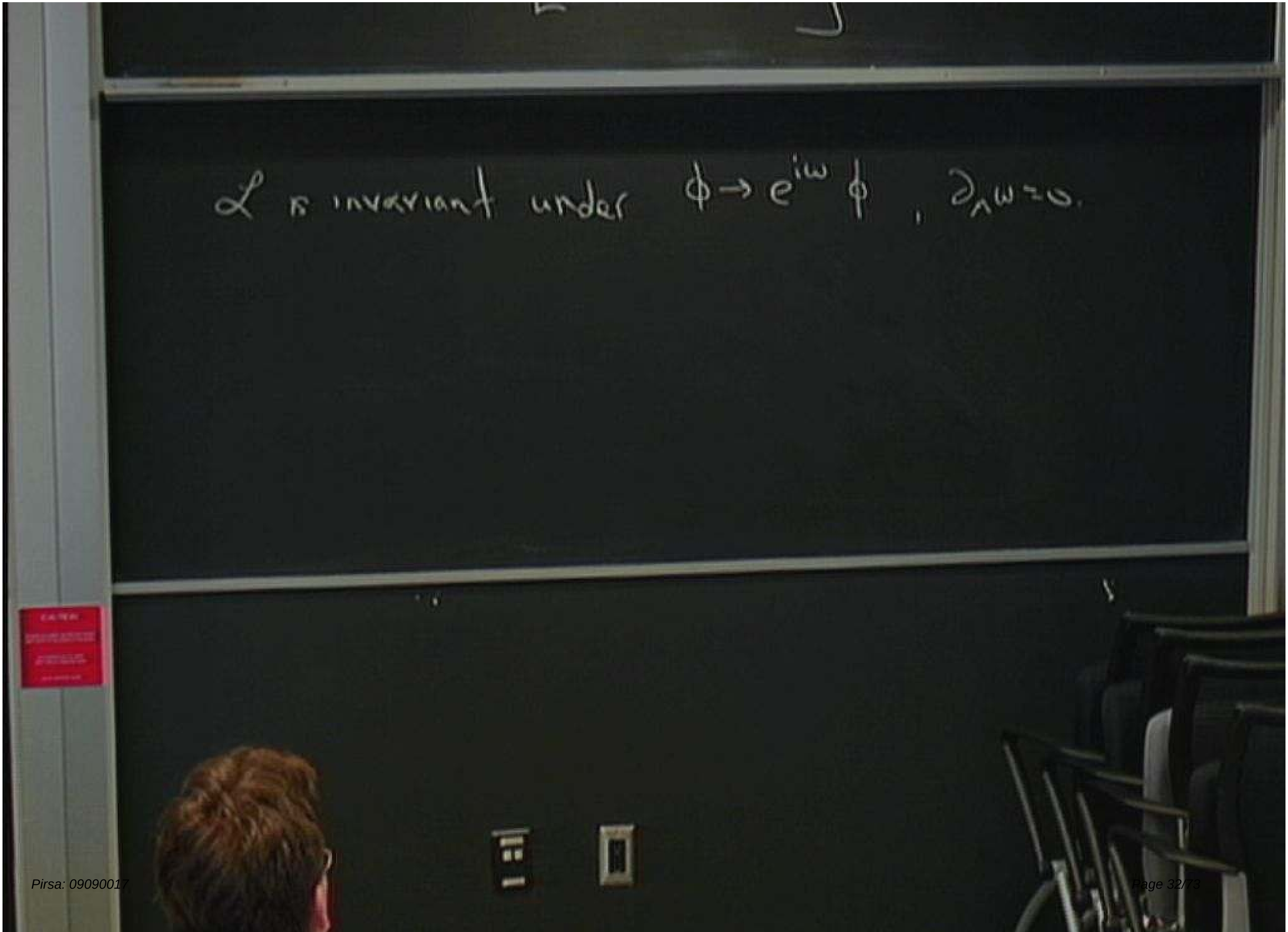
Worry: at low energies, if $E/v \ll \lambda$, then must check at higher orders in λ that $(E/v)^0$ + $(E/v)^2$ terms are nonzero before we know we have the dominant result.

Claim: $A \simeq \left(\frac{E}{v}\right)^4$ to all orders in λ when $E \ll v$.


$\mathcal{L}$ is invariant under $\phi \to e^{i\omega}\phi$, $\partial_\lambda \omega = 0$.

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$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \qquad \left(\frac{\phi_1}{\sqrt{2}} = v + \frac{R}{\sqrt{2}} \right) \qquad \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \to \begin{pmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

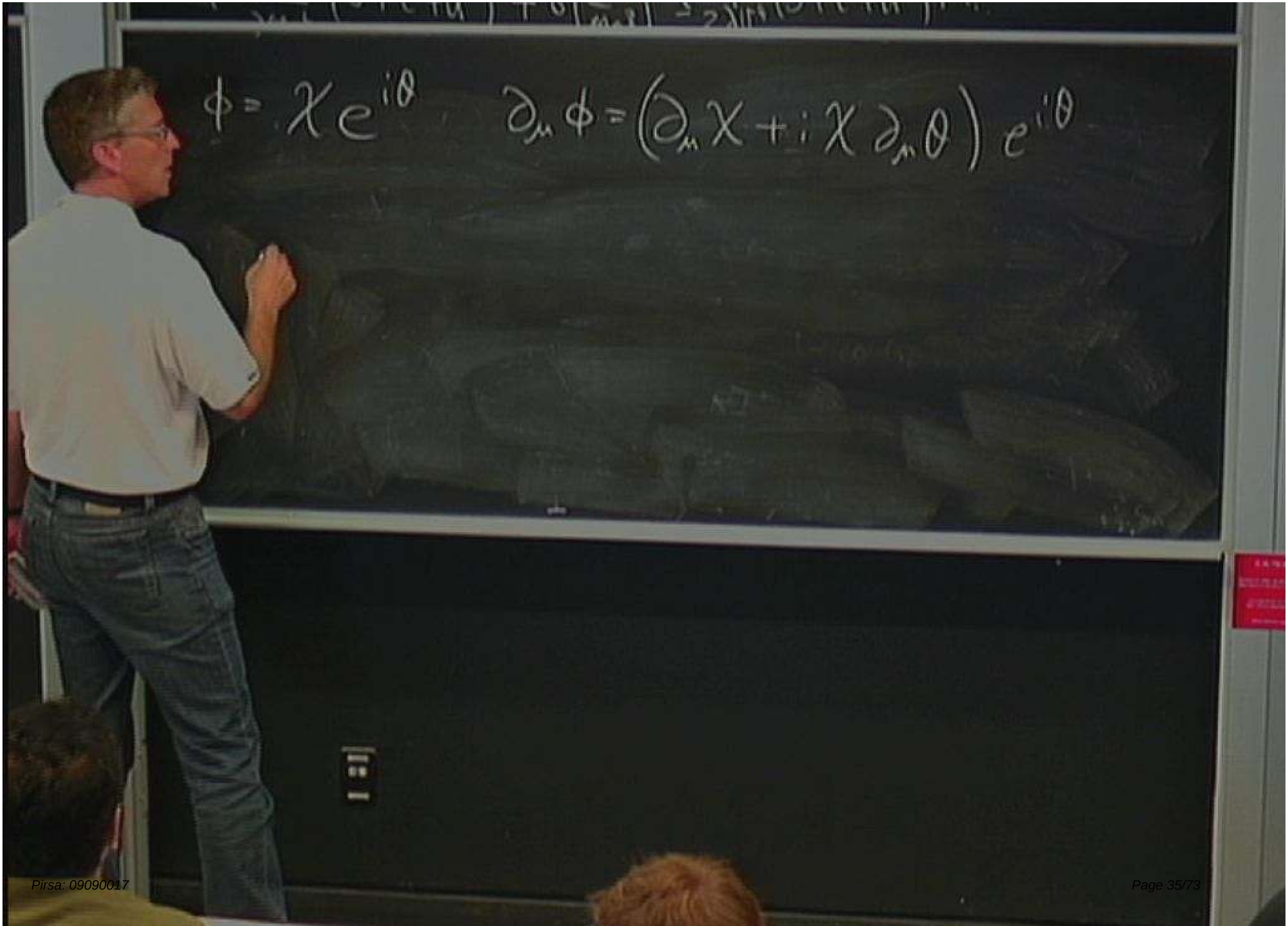
$$\phi_2 = I$$

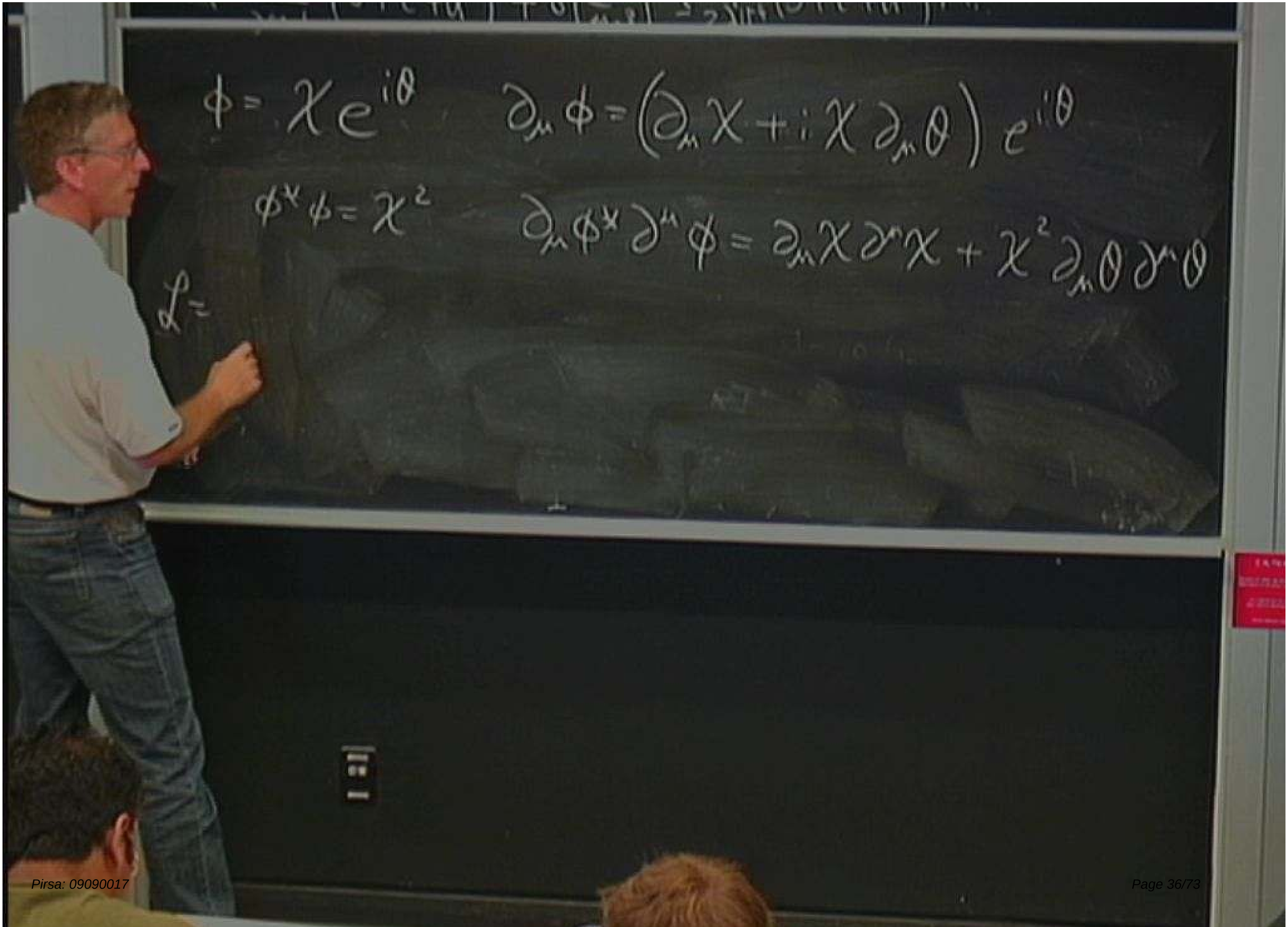
$$\mathcal{A} = \lambda^2\left[\left(-\frac{3}{2} + \frac{\lambda^2 v^2}{2}\cdot\frac{3}{m_R^2}\right) - \frac{\lambda^2 v^2}{2m_R^4}\left[\underbrace{(p+q)^2 + (p-q')^2 + (p-q')^2}_{2p\cdot q - 2p\cdot p' - 2p\cdot q'}\right] + \frac{\lambda^2 v^2}{2m_R^2}\left[(2p\cdot q)^2 + (2p\cdot p')^2 + (2p\cdot q')^2\right] + \dots\right] = 0$$

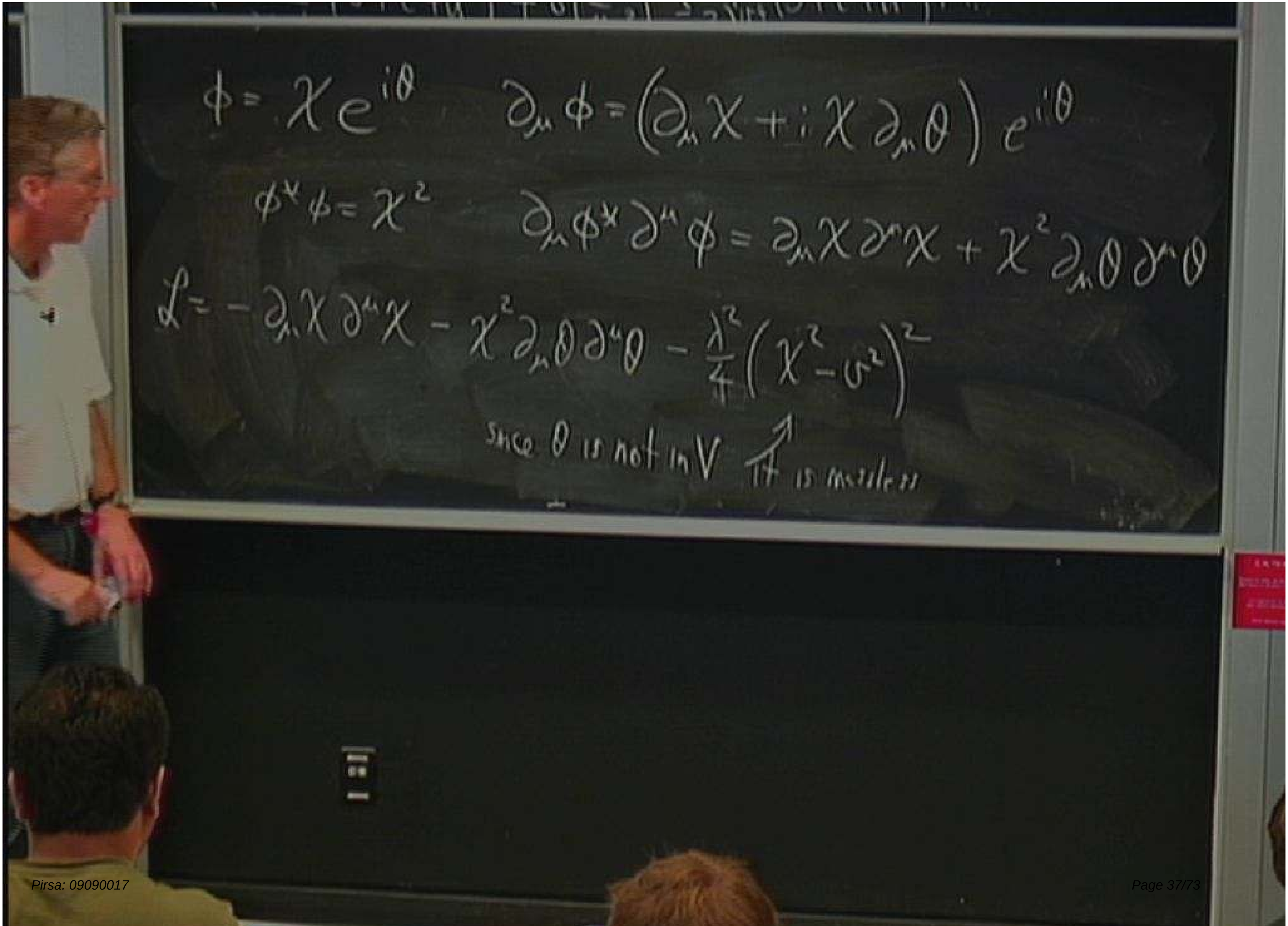
Toy Model:

$$\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - \frac{\lambda^2}{4}\left(\phi^*\phi - v^2\right)^2$$

- $\lambda \ll 1 \longrightarrow$ semiclassical expansion
- $\phi = v + \frac{1}{\sqrt{2}}(R + iI)$


$\phi = \chi e^{i\theta}$
$\partial_\mu \phi = (\partial_\mu \chi + i \chi \partial_\mu \theta) e^{i\theta}$


$\phi = \chi e^{i\theta}$
$\partial_\mu \phi = (\partial_\mu \chi + i \chi \partial_\mu \theta) e^{i\theta}$
$\phi^* \phi = \chi^2$
$\partial_\mu \phi^* \partial^\mu \phi = \partial_\mu \chi \partial^\mu \chi + \chi^2 \partial_\mu \theta \partial^\mu \theta$
$\mathcal{L} =$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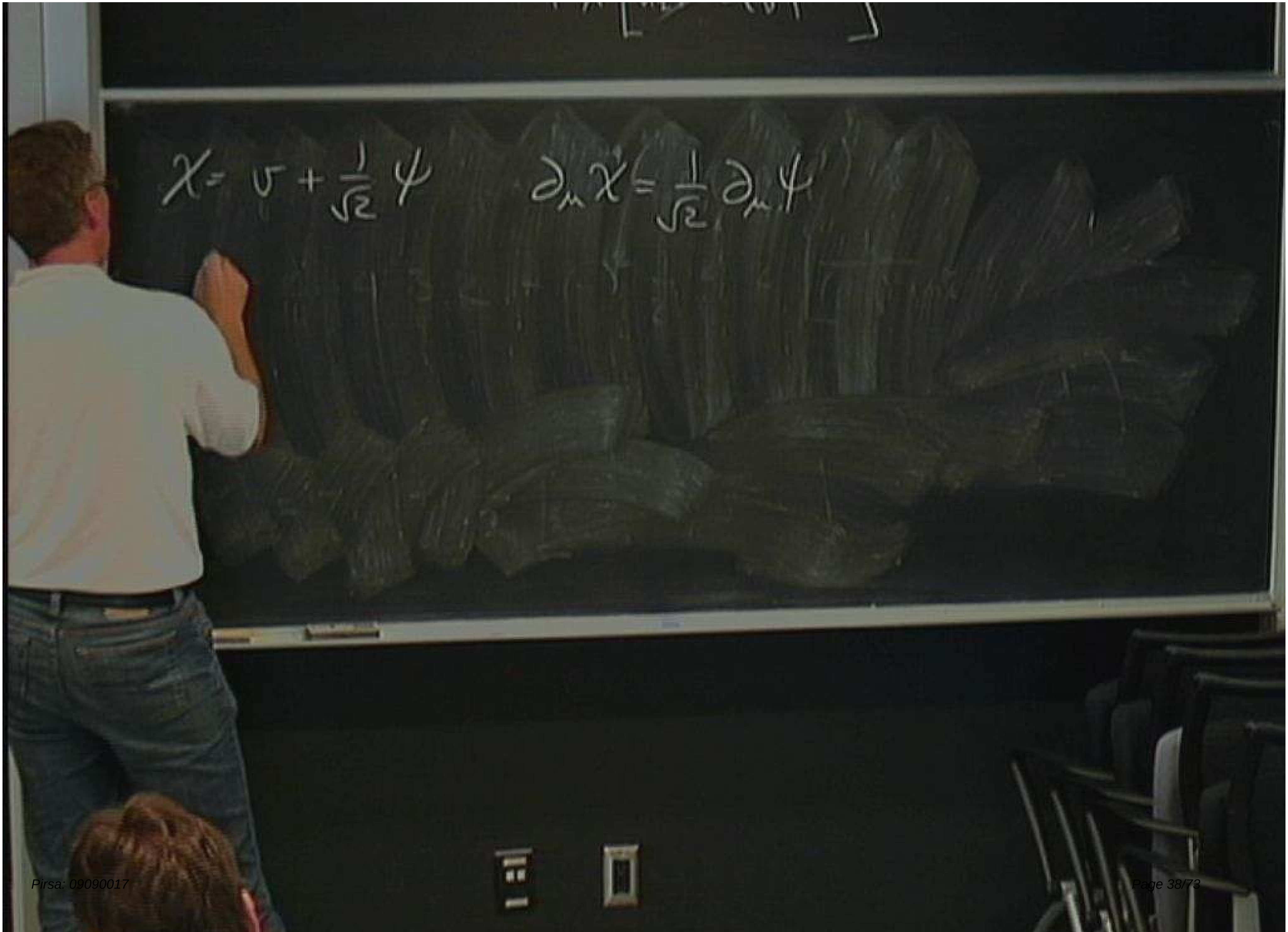
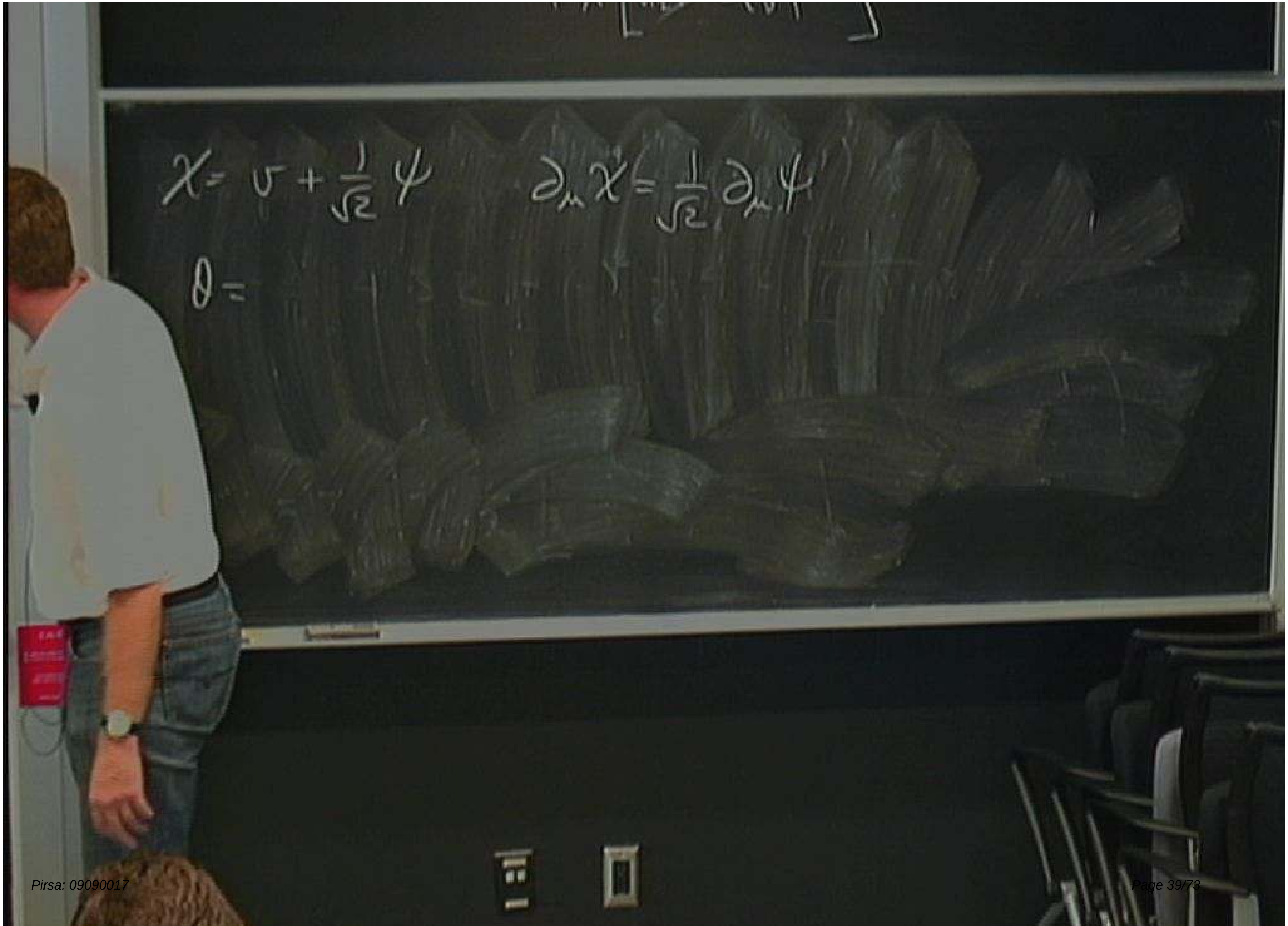
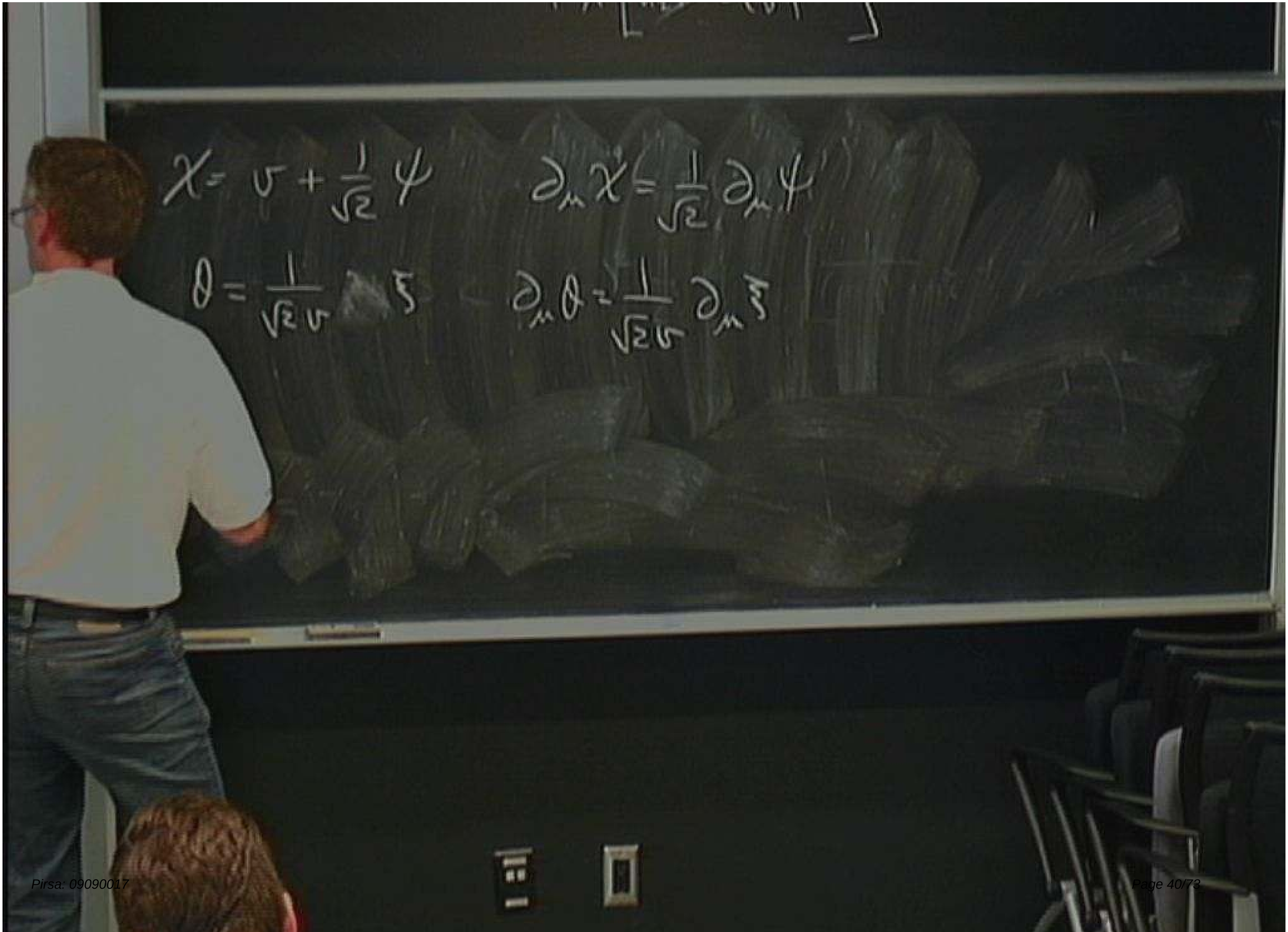
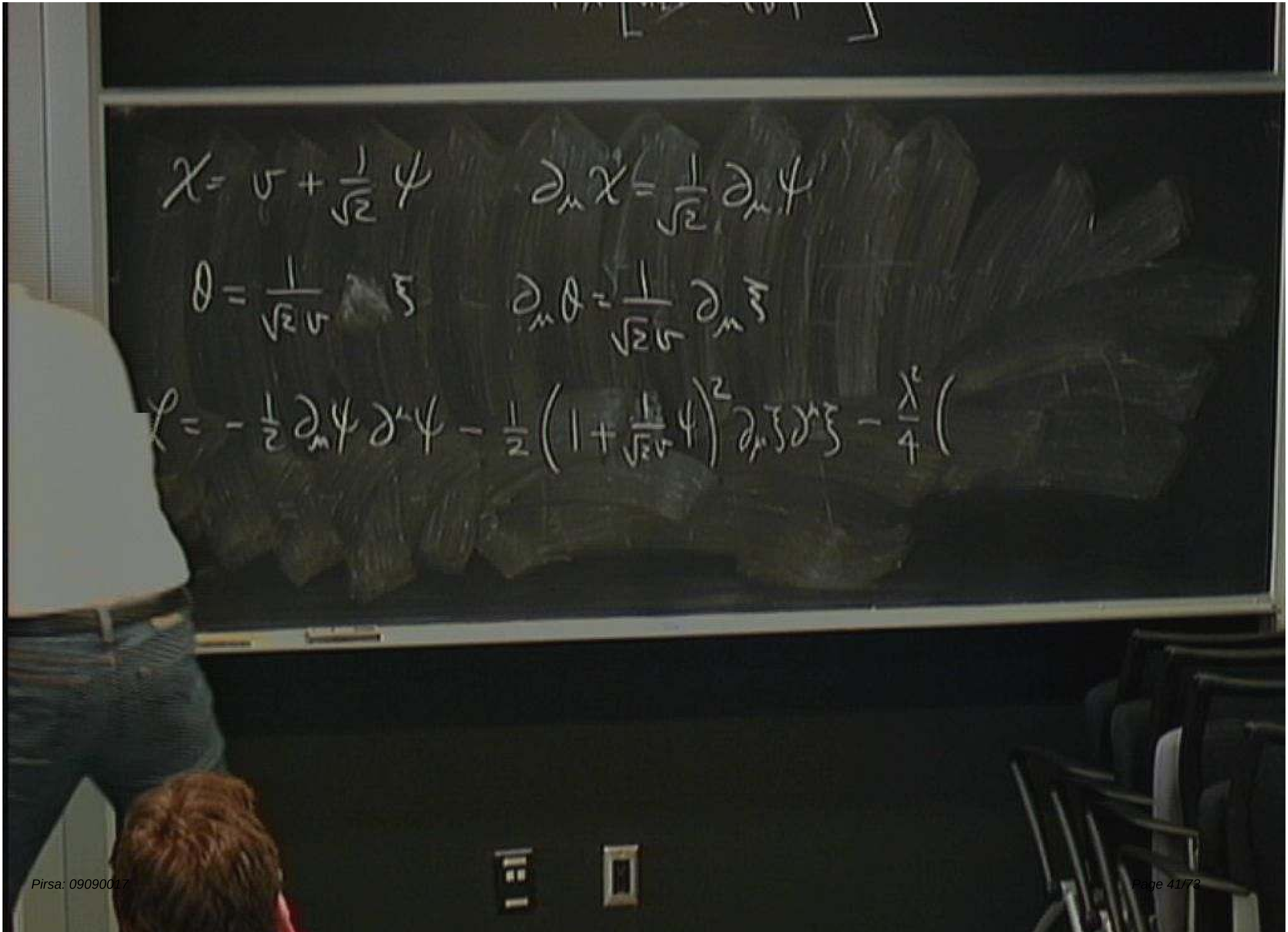
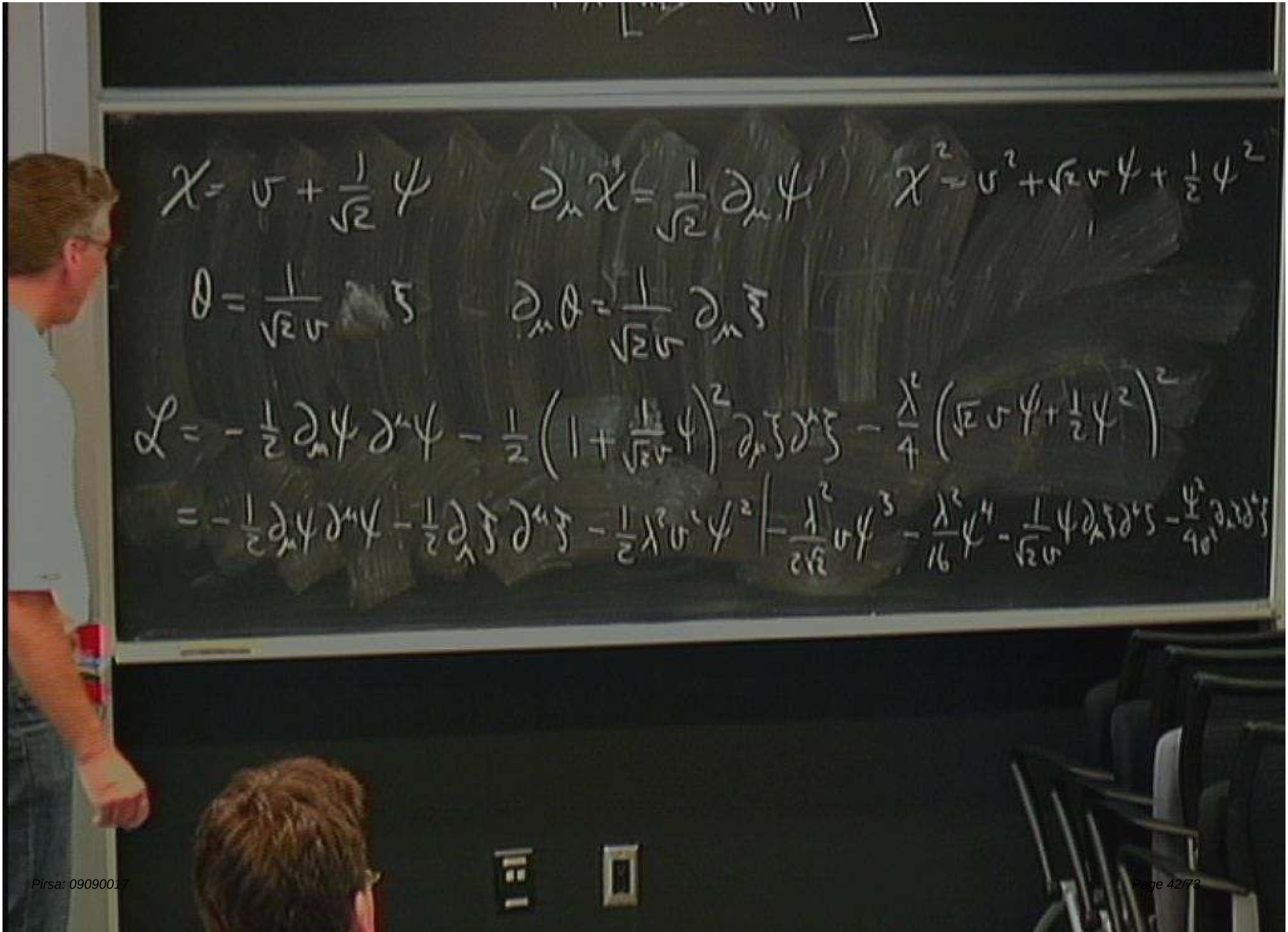
$\phi^* \phi = \chi^2$
$\partial_\mu \phi^* \partial^\mu \phi = \partial_\mu \chi \partial^\mu \chi + \chi^2 \partial_\mu \theta \partial^\mu \theta$
$\mathcal{L} = -\partial_\mu \chi \partial^\mu \chi - \chi^2 \partial_\mu \theta \partial^\mu \theta - \frac{\lambda^2}{4}(\chi^2 - v^2)^2$
since θ is not in V
it is massless


$\chi = v + \frac{1}{\sqrt{2}}\psi$
$\partial_\mu \chi = \frac{1}{\sqrt{2}}\partial_\mu \psi$

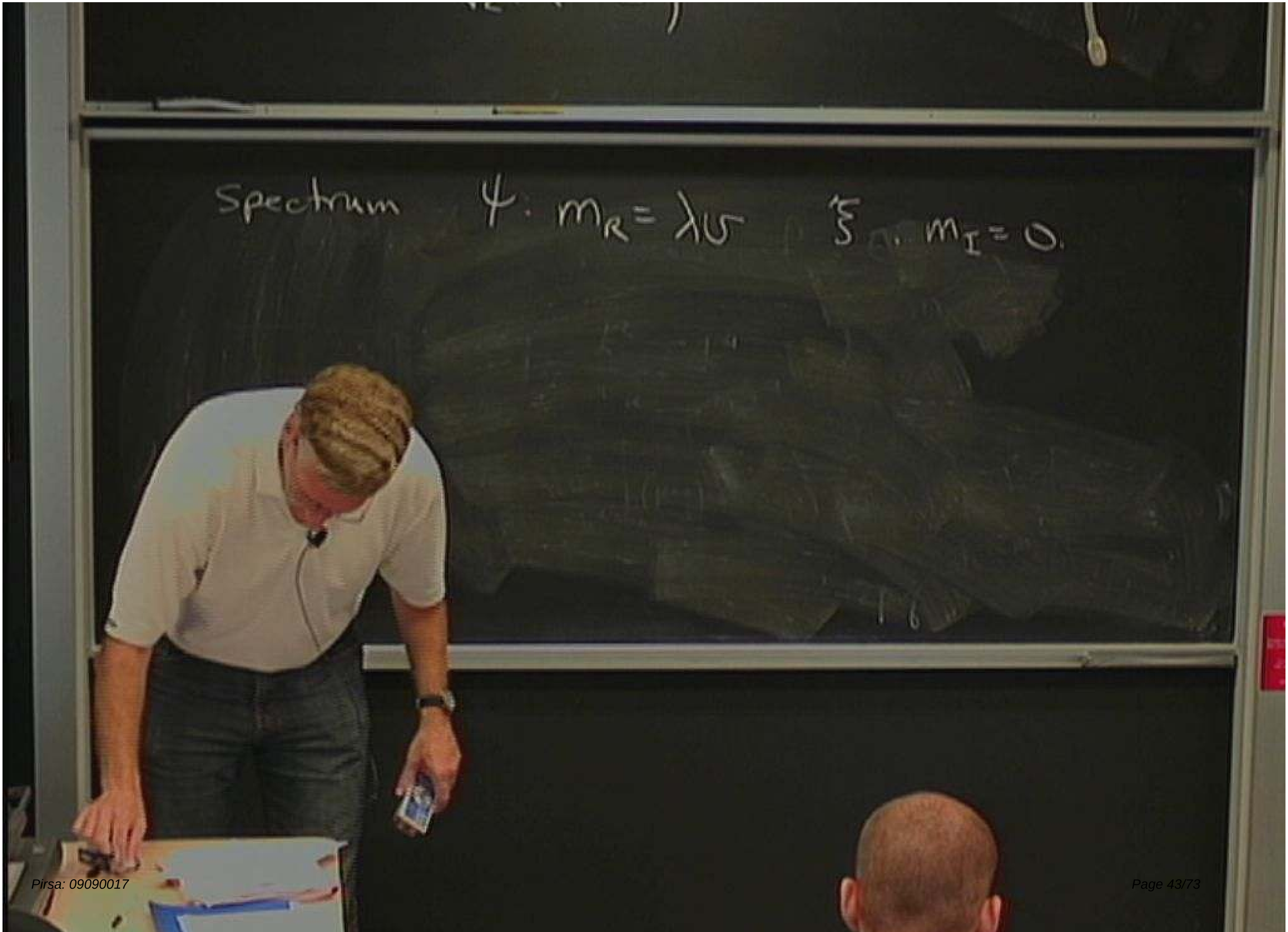

$\chi = v + \frac{1}{\sqrt{2}}\psi$
$\partial_\mu \chi = \frac{1}{\sqrt{2}}\partial_\mu \psi$
$\theta =$


$\chi = v + \frac{1}{\sqrt{2}} \psi \qquad \partial_\mu \chi = \frac{1}{\sqrt{2}} \partial_\mu \psi$
$\theta = \frac{1}{\sqrt{2} v} \xi \qquad \partial_\mu \theta = \frac{1}{\sqrt{2} v} \partial_\mu \xi$


$\chi = v + \frac{1}{\sqrt{2}}\psi$ $\partial_\mu \chi = \frac{1}{\sqrt{2}}\partial_\mu \psi$
$\theta = \frac{1}{\sqrt{2}v}\xi$ $\partial_\mu \theta = \frac{1}{\sqrt{2}v}\partial_\mu \xi$
$\mathcal{L} = -\frac{1}{2}\partial_\mu\psi\,\partial^\mu\psi - \frac{1}{2}\left(1+\frac{1}{\sqrt{2}v}\psi\right)^2\partial_\mu\xi\,\partial^\mu\xi - \frac{\lambda^2}{4}($



$\chi = v + \frac{1}{\sqrt{2}}\psi \qquad \partial_\mu \chi = \frac{1}{\sqrt{2}}\partial_\mu \psi \qquad \chi^2 = v^2 + \sqrt{2}v\psi + \frac{1}{2}\psi^2$
$\theta = \frac{1}{\sqrt{2}v}\xi \qquad \partial_\mu \theta = \frac{1}{\sqrt{2}v}\partial_\mu \xi$
$\mathcal{L} = -\frac{1}{2}\partial_\mu\psi\,\partial^\mu\psi - \frac{1}{2}\left(1+\frac{1}{\sqrt{2}v}\psi\right)^2 \partial_\mu\xi\,\partial^\mu\xi - \frac{\lambda^2}{4}\left(\sqrt{2}v\psi + \frac{1}{2}\psi^2\right)^2$
$= -\frac{1}{2}\partial_\mu\psi\,\partial^\mu\psi - \frac{1}{2}\partial_\lambda\xi\,\partial^\lambda\xi - \frac{1}{2}\lambda^2 v^2\psi^2 - \frac{\lambda^2}{2\sqrt{2}}v\psi^3 - \frac{\lambda^2}{16}\psi^4 - \frac{1}{\sqrt{2}v}\psi\,\partial_\lambda\xi\,\partial^\lambda\xi - \frac{\psi^2}{4v^2}\partial_\lambda\xi\,\partial^\lambda\xi$

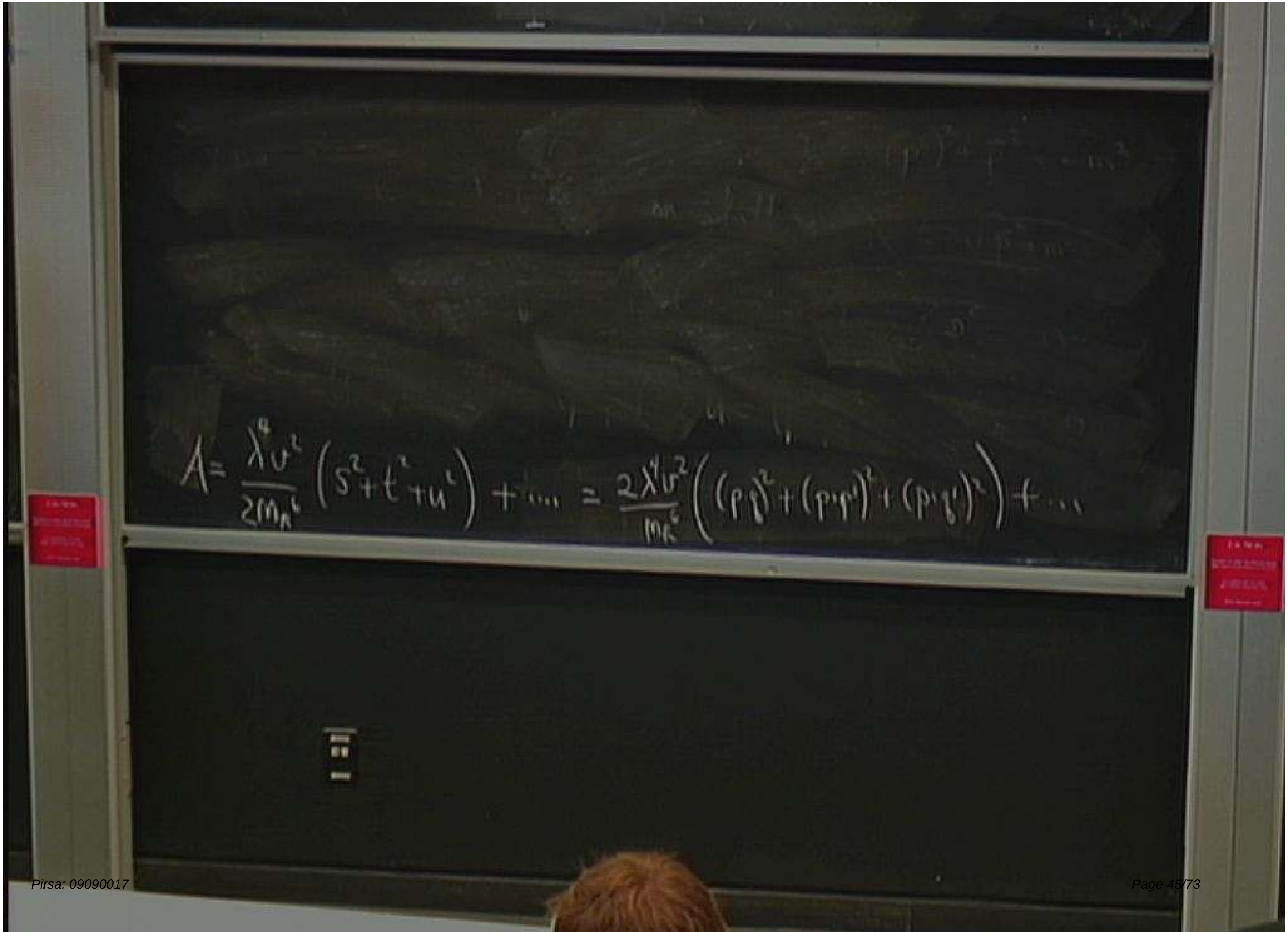

Spectrum ψ: m_R = λv   ξ: m_I = 0.

Spectrum ψ: $m_R = \lambda v$ $\quad \xi$: $m_I = 0$.

 ψ^4  ψ^3  $\psi \partial_\mu \xi \partial^\mu \xi$  $\psi^2 \partial_\mu \xi \partial^\mu \xi$

$\xi(p) + \xi(q) \to \xi(p') + \xi(q')$

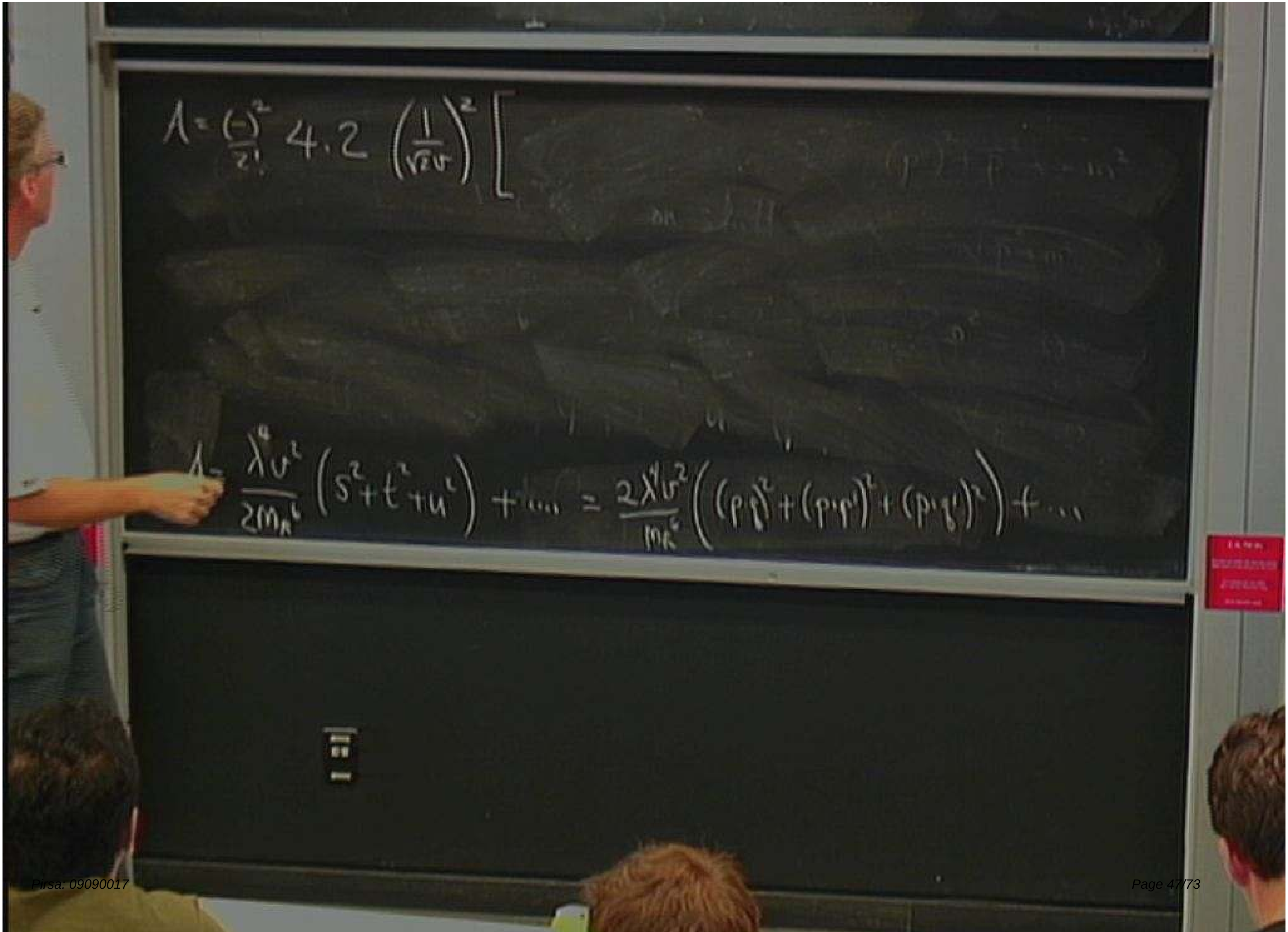
Tree graphs:  $+$  $+$ ...



$A = \frac{\lambda^4 v^2}{2 m_h^6}\left(s^2+t^2+u^2\right) + \cdots = \frac{2\lambda^4 v^2}{m_h^6}\left((p\cdot q)^2+(p\cdot p')^2+(p\cdot q')^2\right) + \cdots$

$$A = \frac{(-)^2}{2!}\, 4 \cdot 2$$

$$A = \frac{\lambda^4 v^2}{2 m_R^6}\left(s^2 + t^2 + u^2\right) + \cdots = \frac{2\lambda^4 v^2}{m_R^6}\left((p\cdot q)^2 + (p\cdot p')^2 + (p\cdot q')^2\right) + \ldots$$


$A = \frac{(-)^2}{2!} 4.2 \left(\frac{1}{\sqrt{2}v}\right)^2 [$
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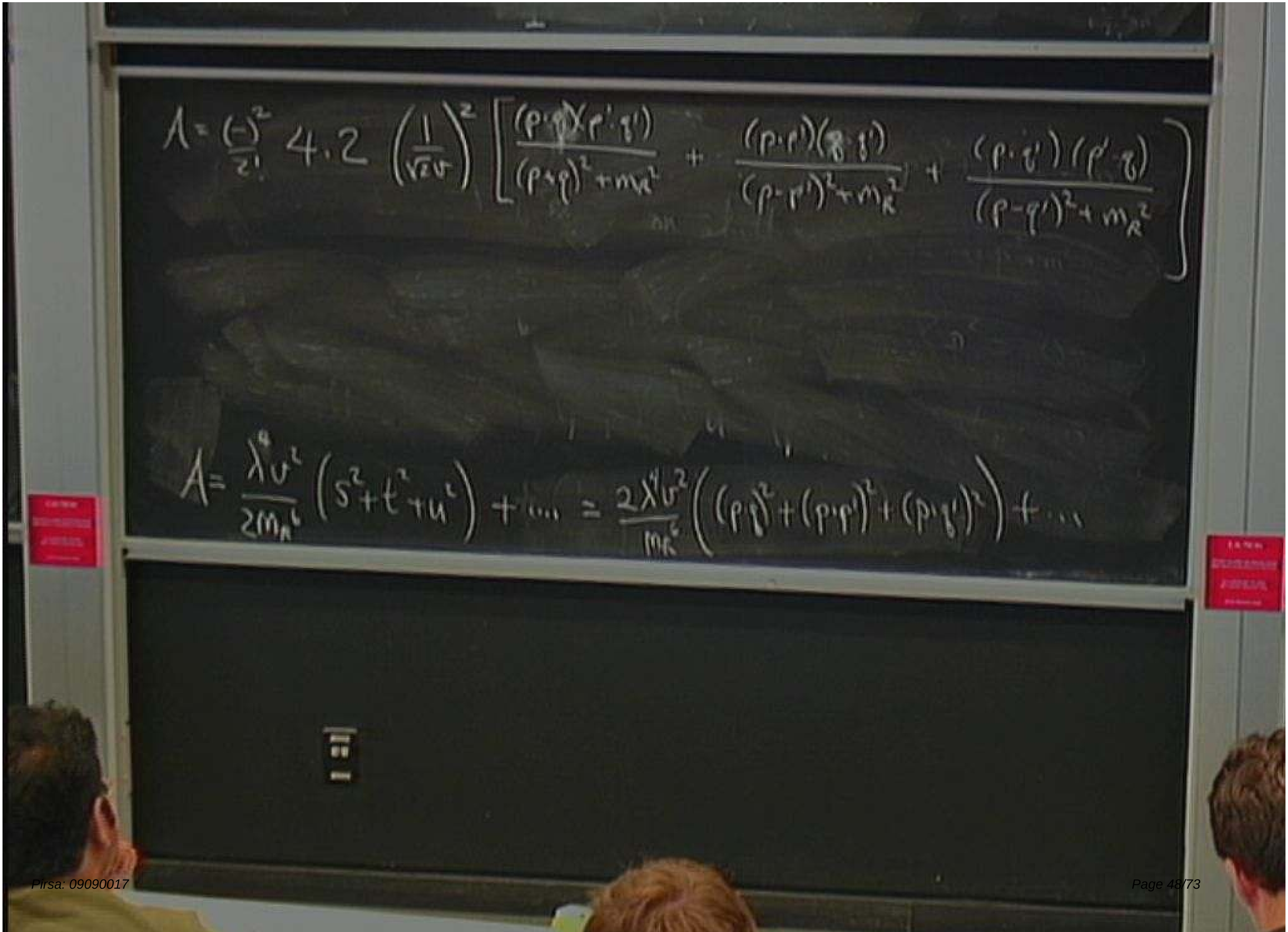
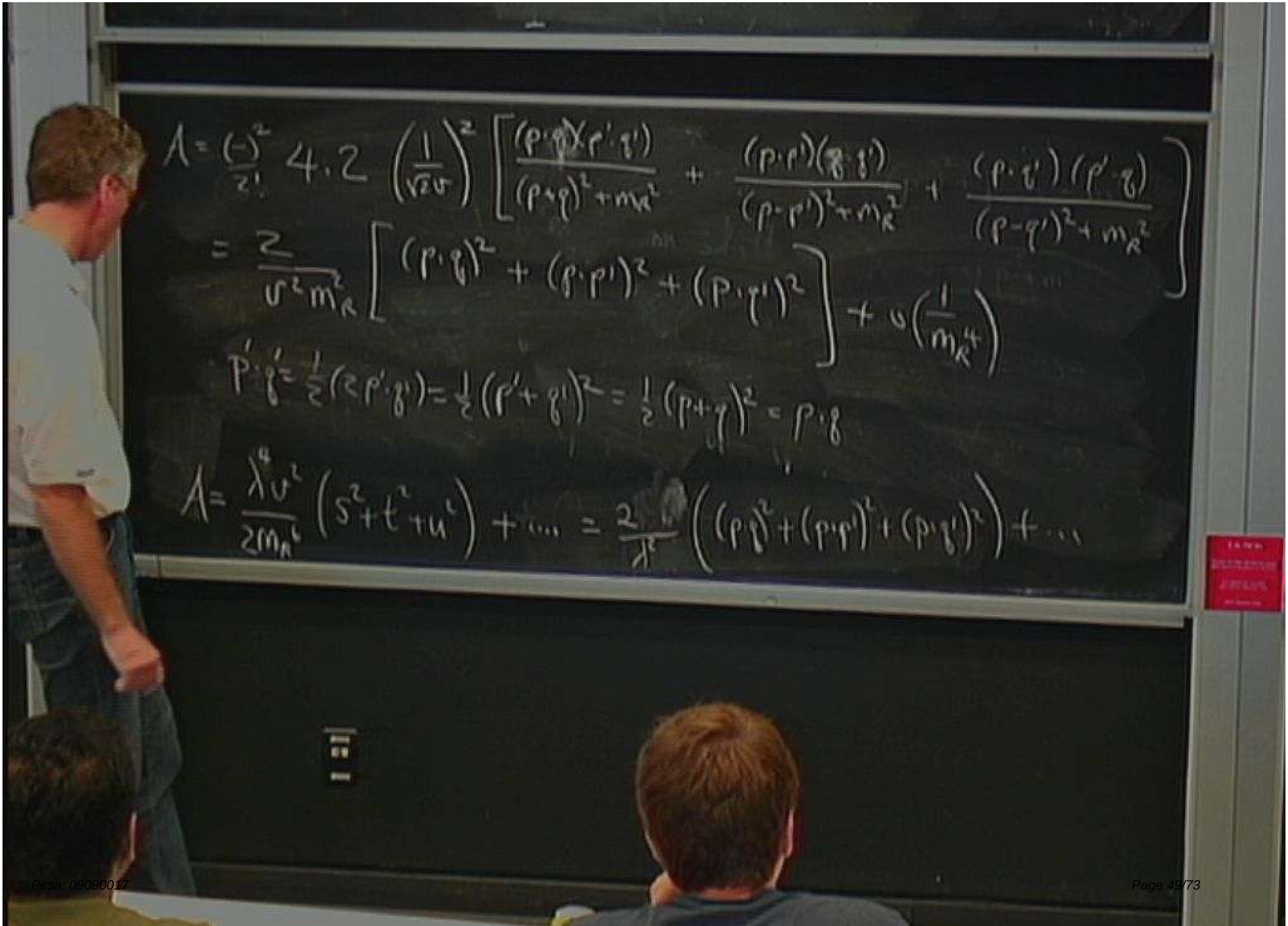


$$A = \frac{(-)^2}{2!}\, 4\cdot 2 \left(\frac{1}{\sqrt{2}v}\right)^2 \left[\frac{(p\cdot q)(p'\cdot q')}{(p+q)^2+m_R^2} + \frac{(p\cdot p')(q\cdot q')}{(p-p')^2+m_R^2} + \frac{(p\cdot q')(p'\cdot q)}{(p-q')^2+m_R^2} \right]$$
$$A = \frac{\lambda^4 v^2}{2m_R^6}\left(s^2+t^2+u^2\right) + \dots = \frac{2\lambda^4 v^2}{m_R^6}\left((p\cdot q)^2+(p\cdot p')^2+(p\cdot q')^2\right) + \dots$$



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$= \frac{2}{v^2 m_R^2} \left[(p\cdot q)^2 + (p\cdot p')^2 + (p\cdot q')^2 \right] + O\left(\frac{1}{m_R^4}\right)$
$p'\cdot q' = \frac{1}{2}(2p'\cdot q') = \frac{1}{2}((p'+q')^2) = \frac{1}{2}(p+q)^2 = p\cdot q$
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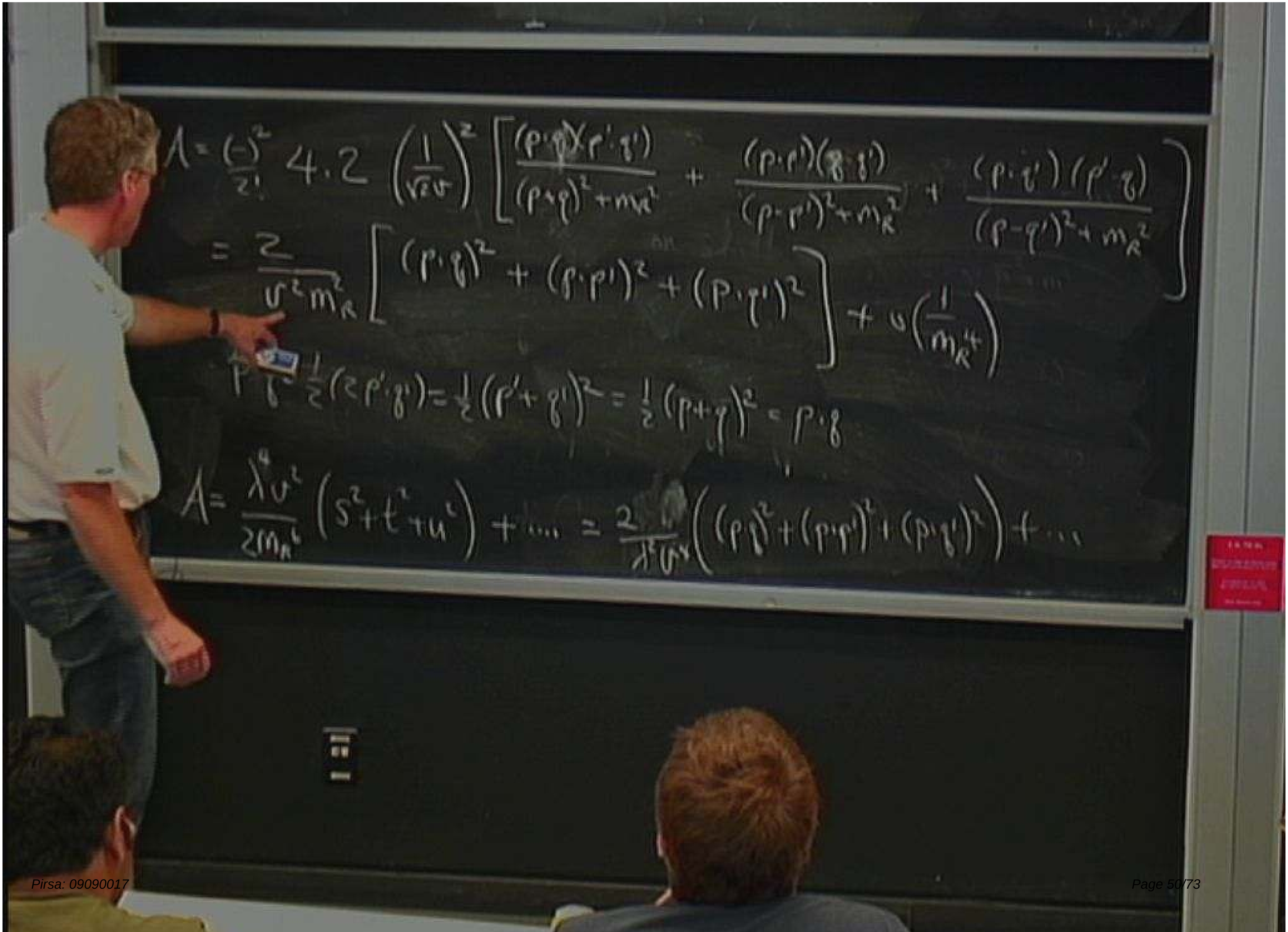
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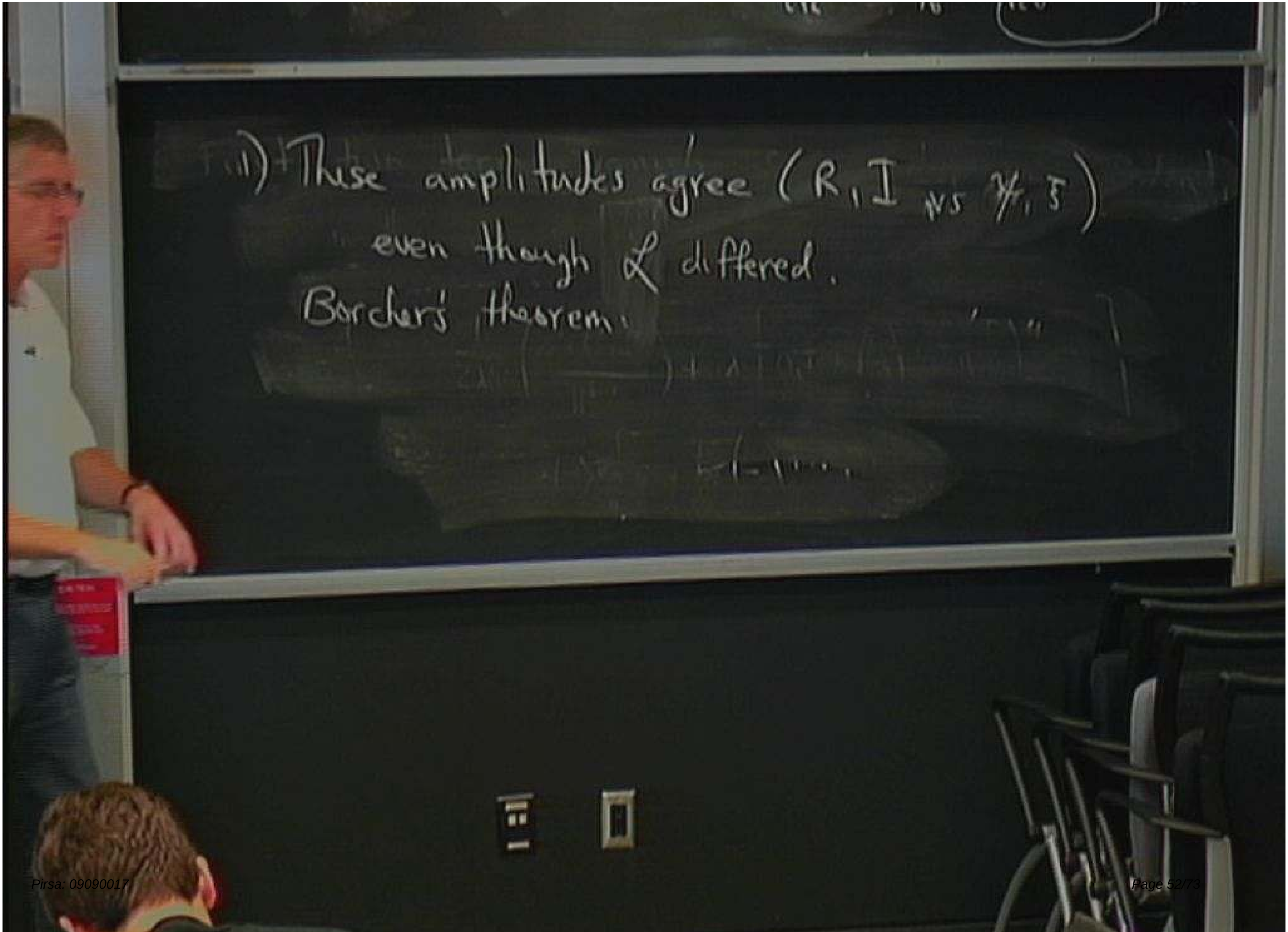
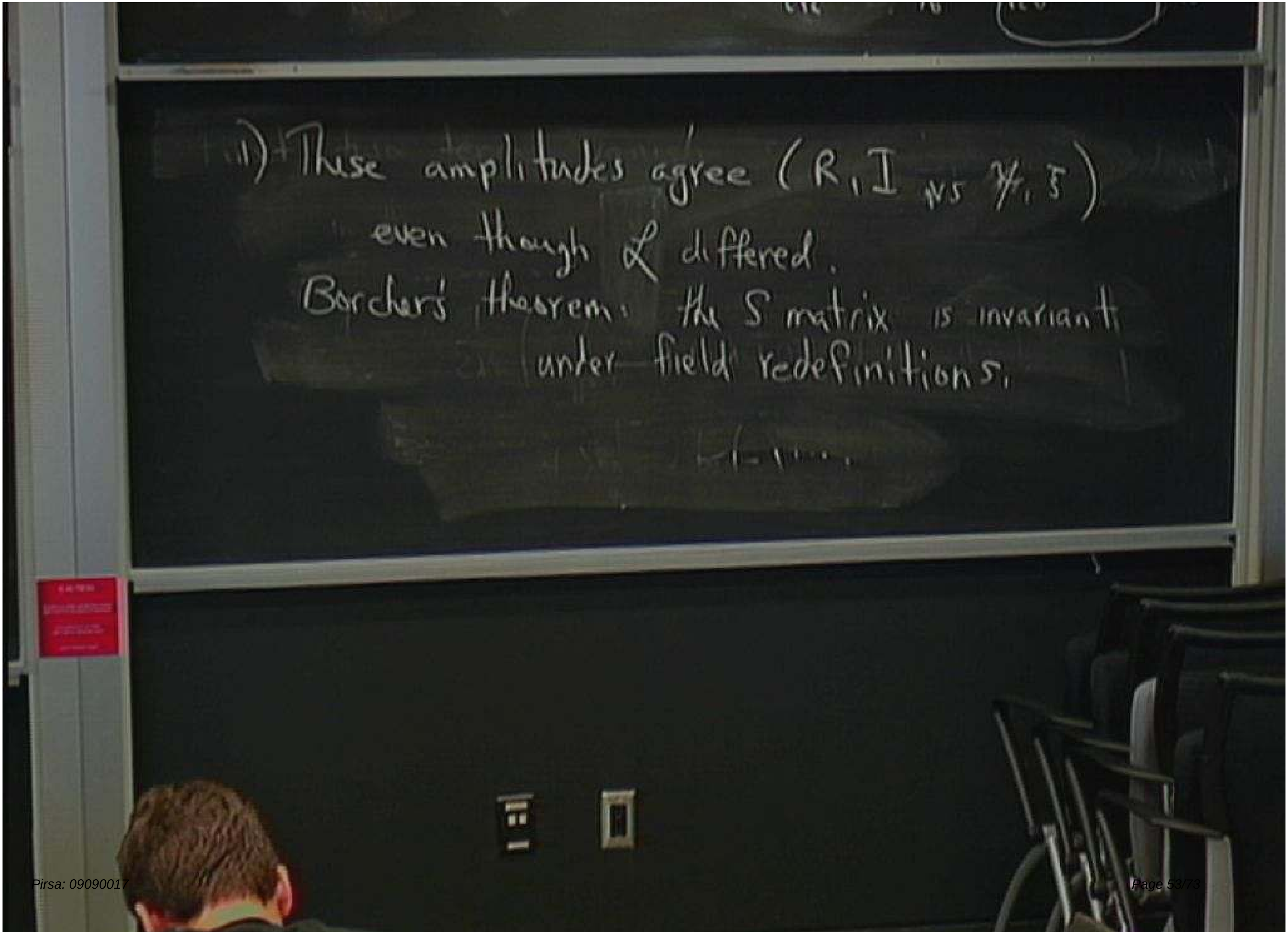

1) These amplitudes agree (R, I vs ψ, ξ)
even though ℒ differed.
Borcher's theorem: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Borcher's theorem: the S matrix is invariant
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even though $\mathcal{L}$ differed.

Borcher's theorem: the S matrix is invariant under field redefinitions.

ψ, ξ are convenient for seeing ϵ dependence of A.

$$= -\frac{1}{2}\partial_\mu\psi\,\partial^\mu\psi - \frac{1}{2}\partial_\mu\xi\,\partial^\mu\xi - \frac{1}{2}\lambda^2 v^2\psi^2 - \frac{\lambda^2}{2\sqrt{2}}v\psi^3 - \frac{\lambda^2}{16}\psi^4 - \frac{1}{\sqrt{2}v}\psi\,\partial_\mu\xi\,\partial^\mu\xi + \frac{\psi^2}{4v^2}\partial_\mu\xi\,\partial^\mu\xi$$

(R, I vs ψ, ξ)

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Borcher's theorem: the S matrix is invariant under field redefinitions.

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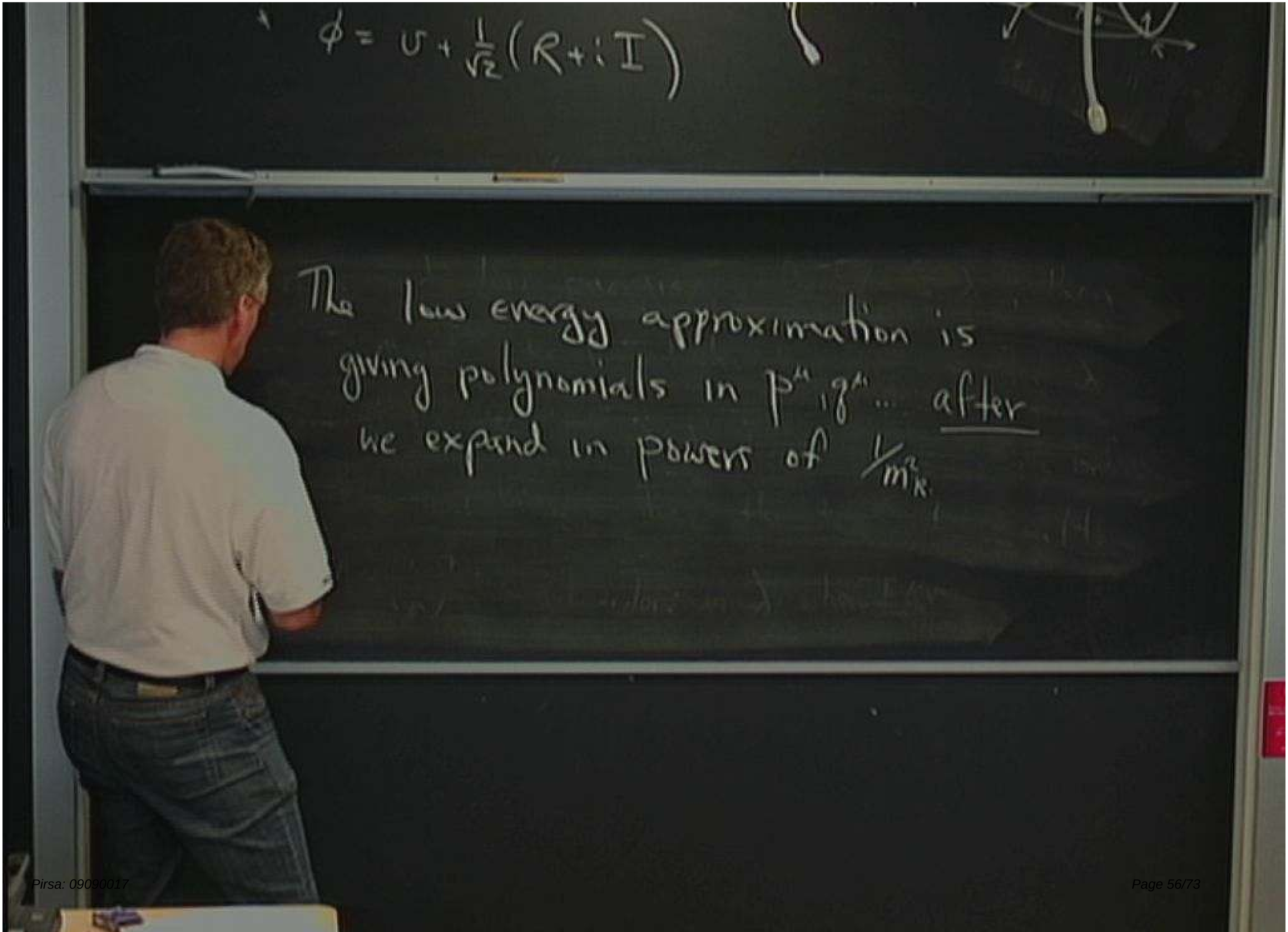
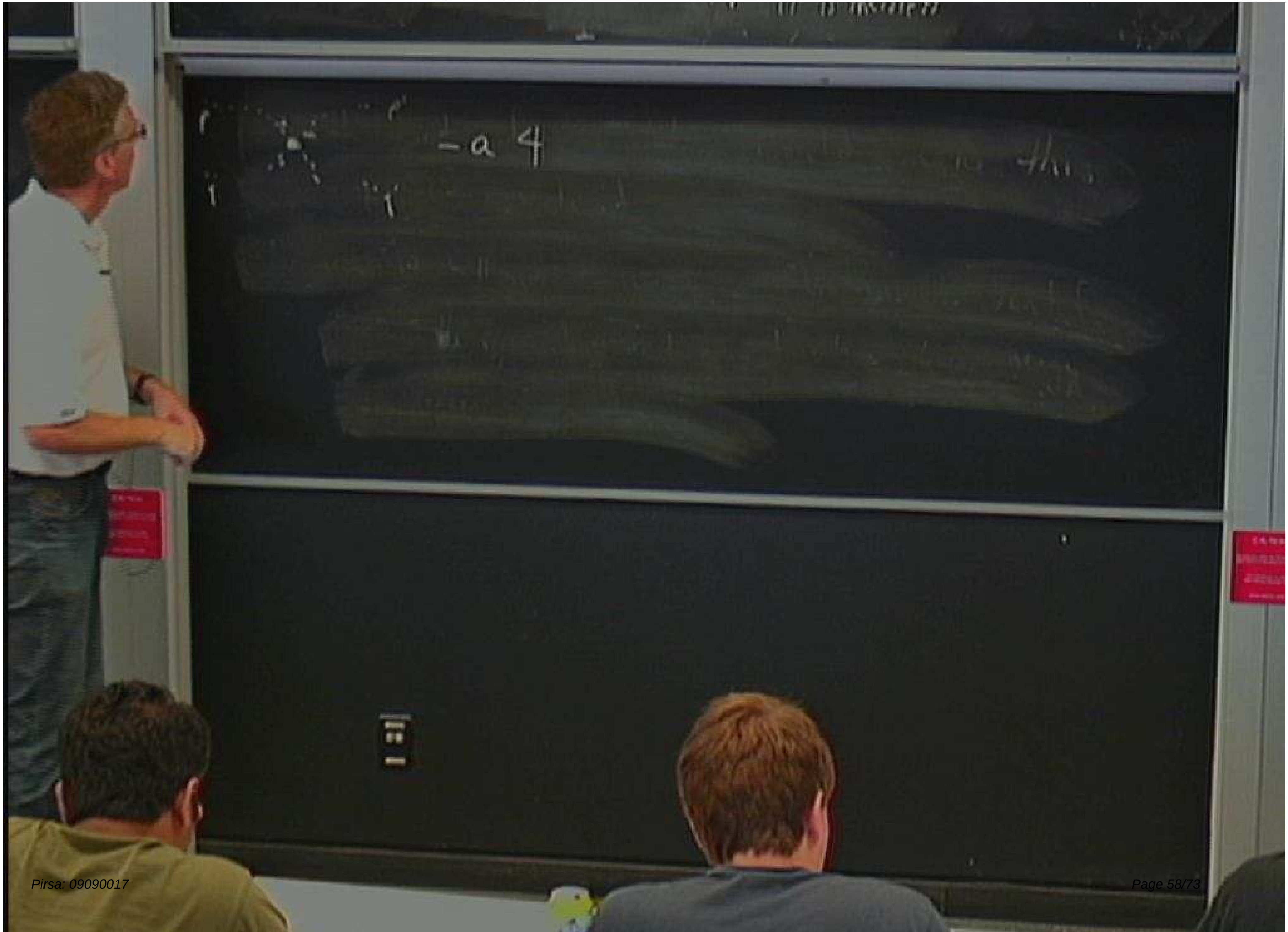
R, I " " " renormalizability.

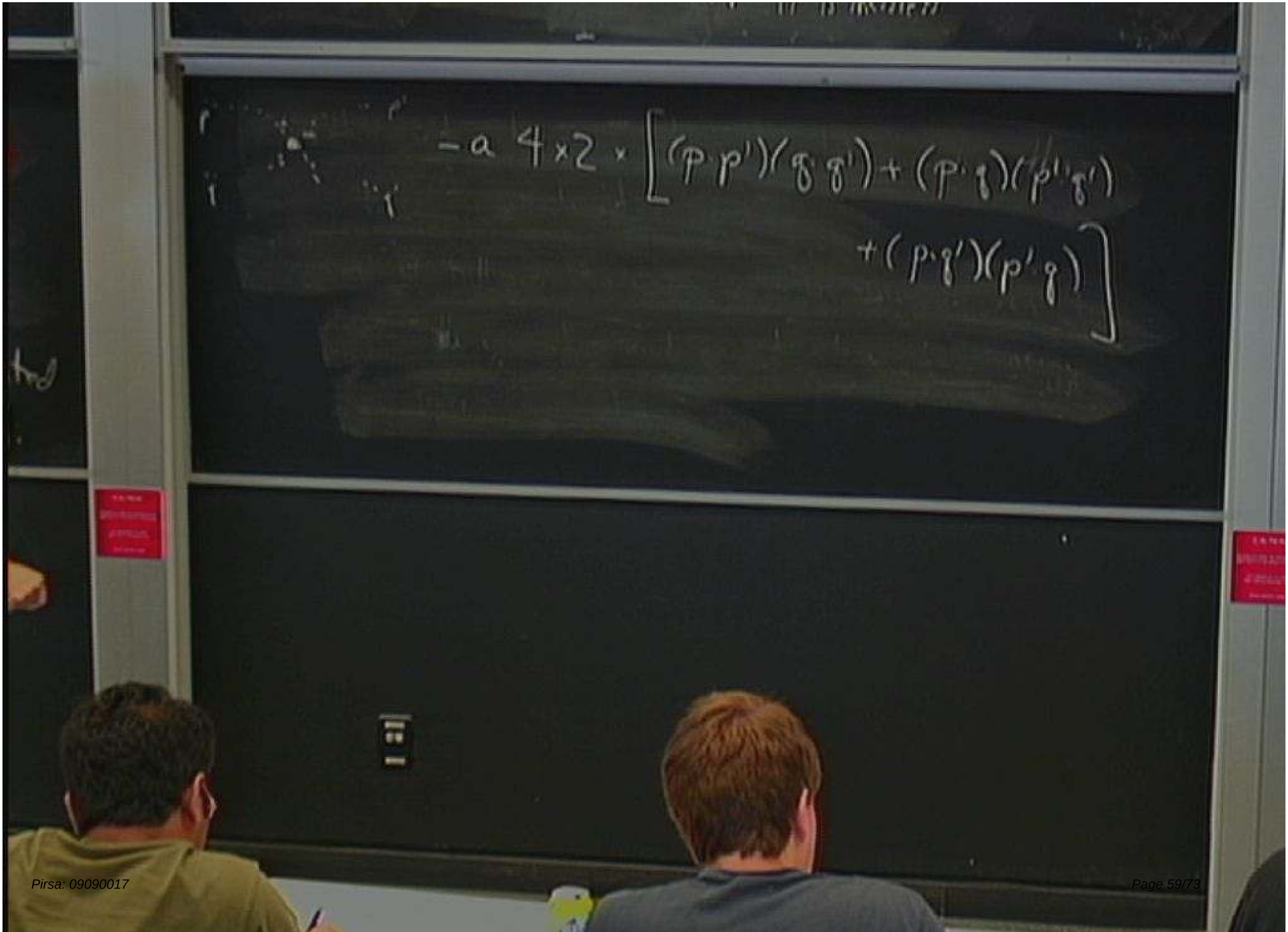

$\phi = v + \frac{1}{\sqrt{2}}(R + iI)$
The low energy approximation is giving polynomials in p^μ, q^μ... after we expand in powers of $1/m_R^2$

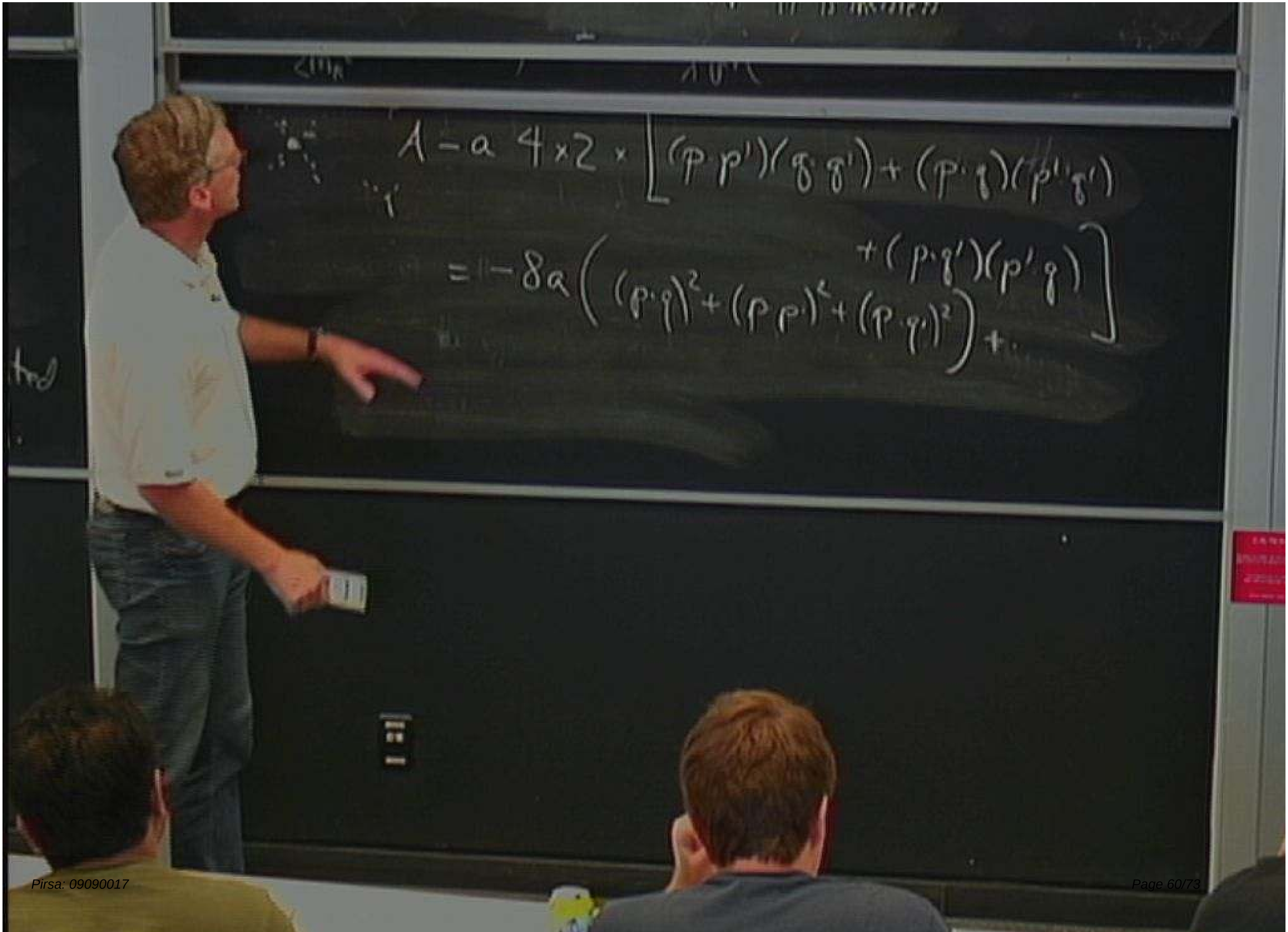
$$\phi = v + \frac{1}{\sqrt{2}}(R + i\,I)$$

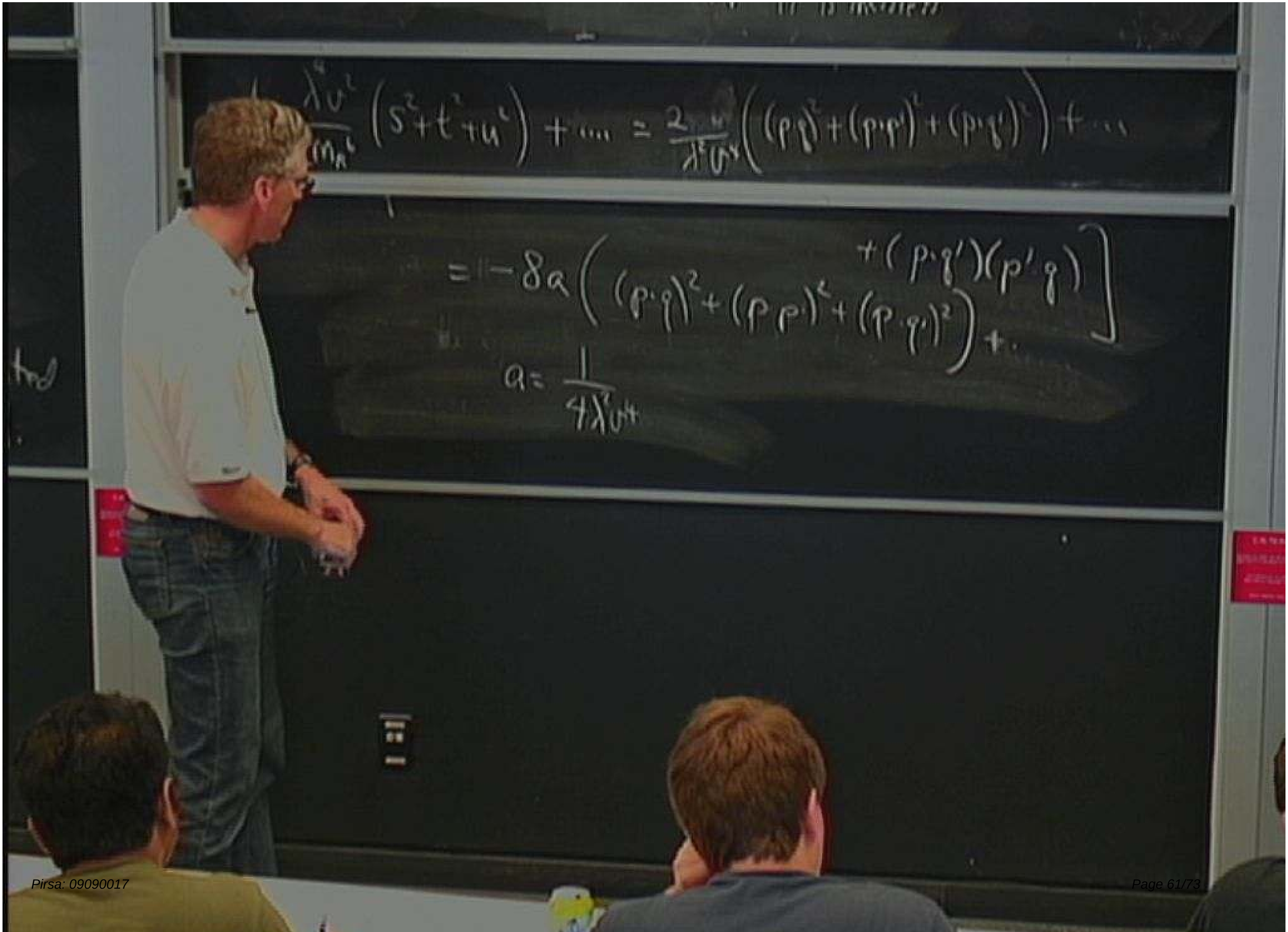
2) The low energy approximation is giving polynomials in $p^\mu, q^\mu \ldots$ after we expand in powers of $1/m_R^2$

Notice: our A would equally well have been computed from $\mathcal{L} = -\frac{1}{2}\partial_\mu\xi\,\partial^\mu\xi - a(\partial_\mu\xi\,\partial^\mu\xi)^4$

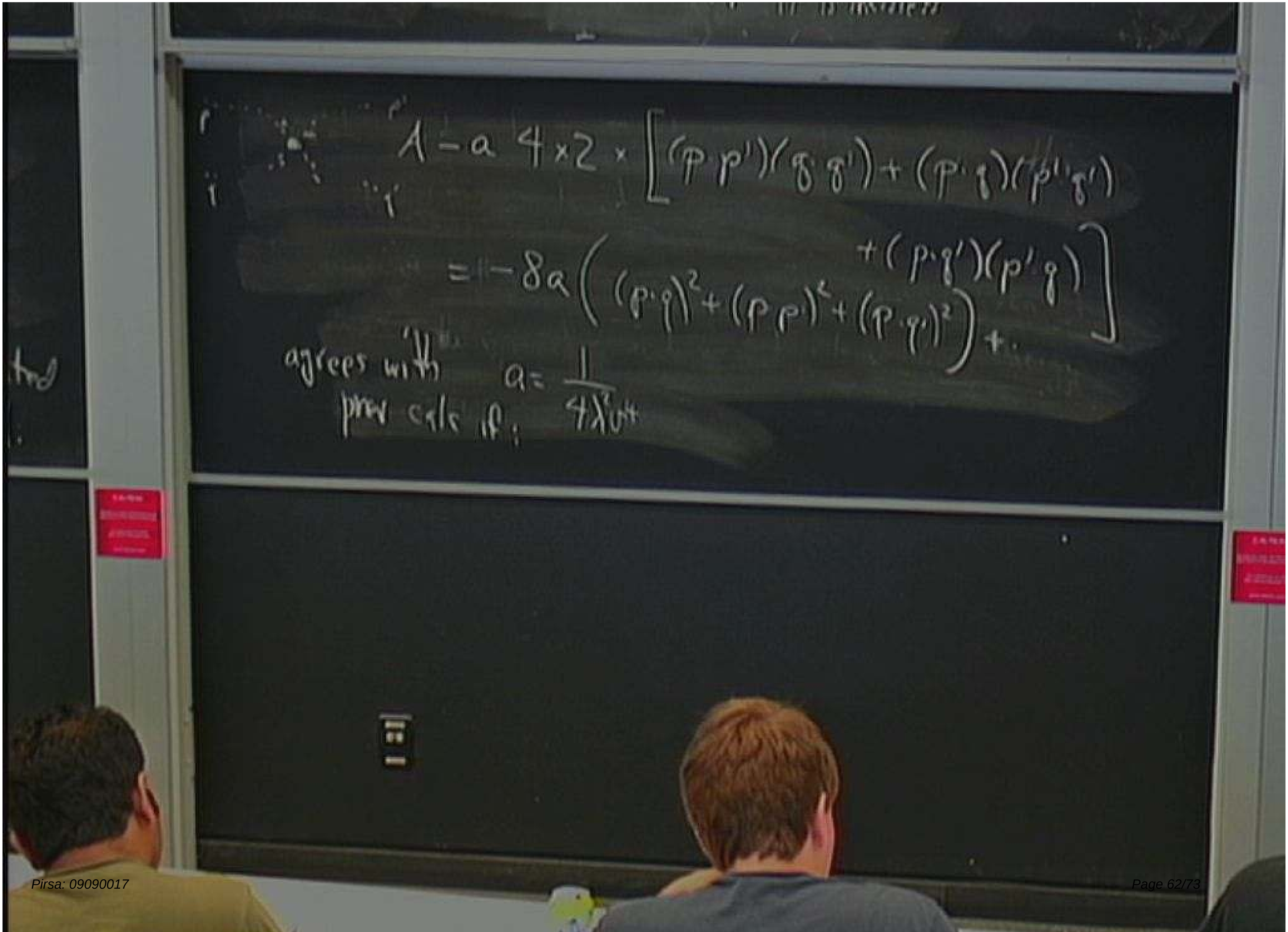

-a 4


$-a\ 4\times 2\times\left[(p\cdot p')(q\cdot q')+(p\cdot q)(p'\cdot q')+(p\cdot q')(p'\cdot q)\right]$


A ~ a 4×2 × [(p·p')(q·q') + (p·q)(p'·q') + (p·q')(p'·q)]
= -8a((p·q)^2 + (p·p')^2 + (p·q')^2) + ...



$\frac{\lambda^4 v^2}{m_h^4}\left(s^2+t^2+u^2\right)+\ldots = \frac{2}{\lambda^2 v^4}\left((p\cdot q)^2+(p\cdot p')^2+(p\cdot q')^2\right)+\ldots$
$= -8a\left((p\cdot q)^2+(p\cdot p')^2+(p\cdot q')^2\right)+\ldots + (p\cdot q')(p'\cdot q)$
$a = \frac{1}{4\lambda^2 v^4}$


$\lambda \sim a\, 4\times 2 \times \Big[(p\cdot p')(q\cdot q') + (p\cdot q)(p'\cdot q') + (p\cdot q')(p'\cdot q)\Big]$
$= -8a\big((p\cdot q)^2 + (p\cdot p')^2 + (p\cdot q')^2\big) + \ldots$
agrees with prev calc if: $a = \frac{1}{4\lambda^2 v^4}$

It is because A is a polynomial in E
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$$G(x,y) = -i\int \frac{d^4p}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2+m_R^2} = \frac{-i}{m_R^2}\int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)}\left(1 - \frac{p^2}{m_R^2} + \frac{p^4}{m_R^4} + \cdots\right)$$

$\xi(p) + \xi(q) \to \xi(p') + \xi(q')$

Tree graphs:

$$\int \frac{d^4p}{(2\pi)^4}\, p^{2n}\, e^{ip(x-y)} = (-\Box)^n \int \frac{d^4p}{(2\pi)^4}\, e^{ip(x-y)}$$

$$= (-\Box)\,\delta^4(x-y)$$

$$G(x,y) = \frac{-i}{m_R^2}\left(1 + \frac{\Box}{m_R^2} + \frac{\Box^2}{m_R^2} + \dots\right)\delta^4(x-y)$$

$$G(x,y) = \frac{-1}{m_R^2}\left(1 + \frac{\Box}{m_R^2} + \frac{\Box^2}{m_R^4} + \dots\right)\delta^4(x-y)$$

2) The low energy approximation is effective giving polynomials in $p^\mu, q^\mu \dots$ after interact. we expand in powers of $1/m_R^2$

Notice: our A would equally well have been computed from $\mathcal{L} = -\frac{1}{2}\partial_\mu \xi \partial^\mu \xi - a(\partial_\mu \xi \partial^\mu \xi)^2$ at the level.

$$G(x,y) = \frac{-1}{m_R^2}\left(1 + \frac{\Box}{m_R^2} + \frac{\Box^2}{m_R^2} + \dots\right)\delta^4(x-y)$$

$\phi \to \phi e^{i\omega}$ $\phi = \chi e^{i\theta}$ $\theta \to \theta + \omega$ $\chi \to \chi$ $\xi \to \xi + \omega\sqrt{2}v$ $\psi \to \psi$

2) ... the low energy approximation is effective giving polynomials in p^μ, q^μ .. after interact. we expand in powers of $1/m_R^2$

... our A would equally well have been computed from $\mathcal{L} = -\frac{1}{2}\partial_\mu\xi\partial^\mu\xi - a(\partial_\mu\xi\partial^\mu\xi)^2$ at tree level.

$$A = \frac{\ldots}{2m^6}\left(s^2+t^2+u^2\right) + \cdots = \frac{2}{\ldots}\left((p\cdot q)^2 + (p\cdot p')^2 + (p\cdot q')^2\right) + \cdots$$

Another Toy Model (with which we will build up the general formulation)

$$A = \frac{\lambda \ldots}{2m^4}\left(s^2+t^2+u^2\right) + \ldots = \frac{2 \ldots}{\ldots}\left((p\cdot q)^2+(p\cdot q')^2+(p\cdot q')^2\right) + \ldots$$

Another Toy Model (with which we will build up the general formulation)

two spinless particles: h (heavy), ℓ (light)

$$\mathcal{L} = -\frac{1}{2}\partial_\mu h\,\partial^\mu h - \frac{1}{2}\partial_\mu \ell\,\partial^\mu \ell - \frac{1}{2}M^2 h^2 - \frac{1}{2}m^2 \ell^2 - \ldots$$

$$A = \frac{\lambda [illegible]}{2m^6}\left(s^2+t^2+u^2\right) + \cdots = \frac{2}{[illegible]}\left((p\cdot q)^2 + (p\cdot q')^2 + (p\cdot q')^2\right) + \cdots$$

Another Toy Model (with which we will build up the general formulation)

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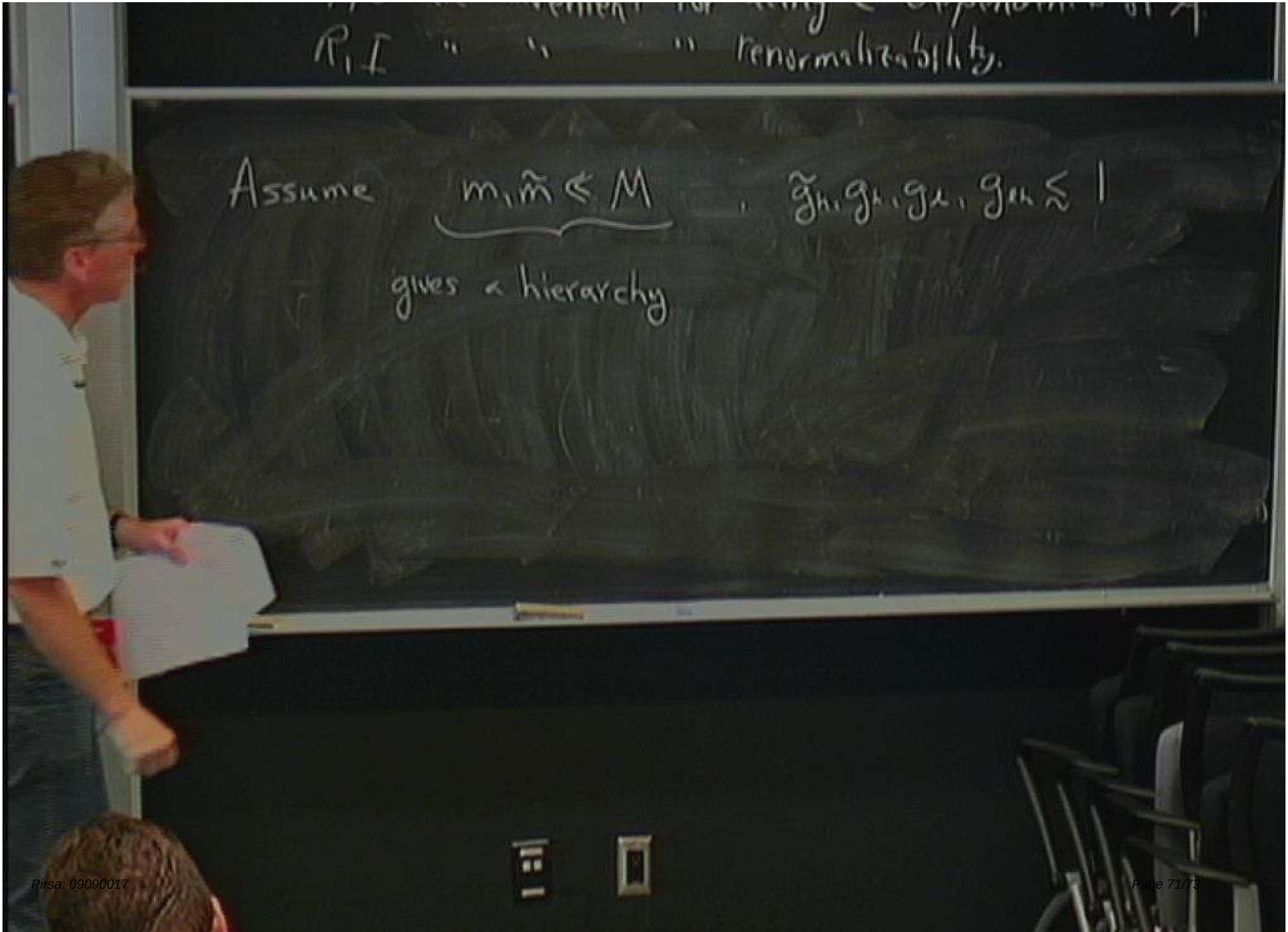
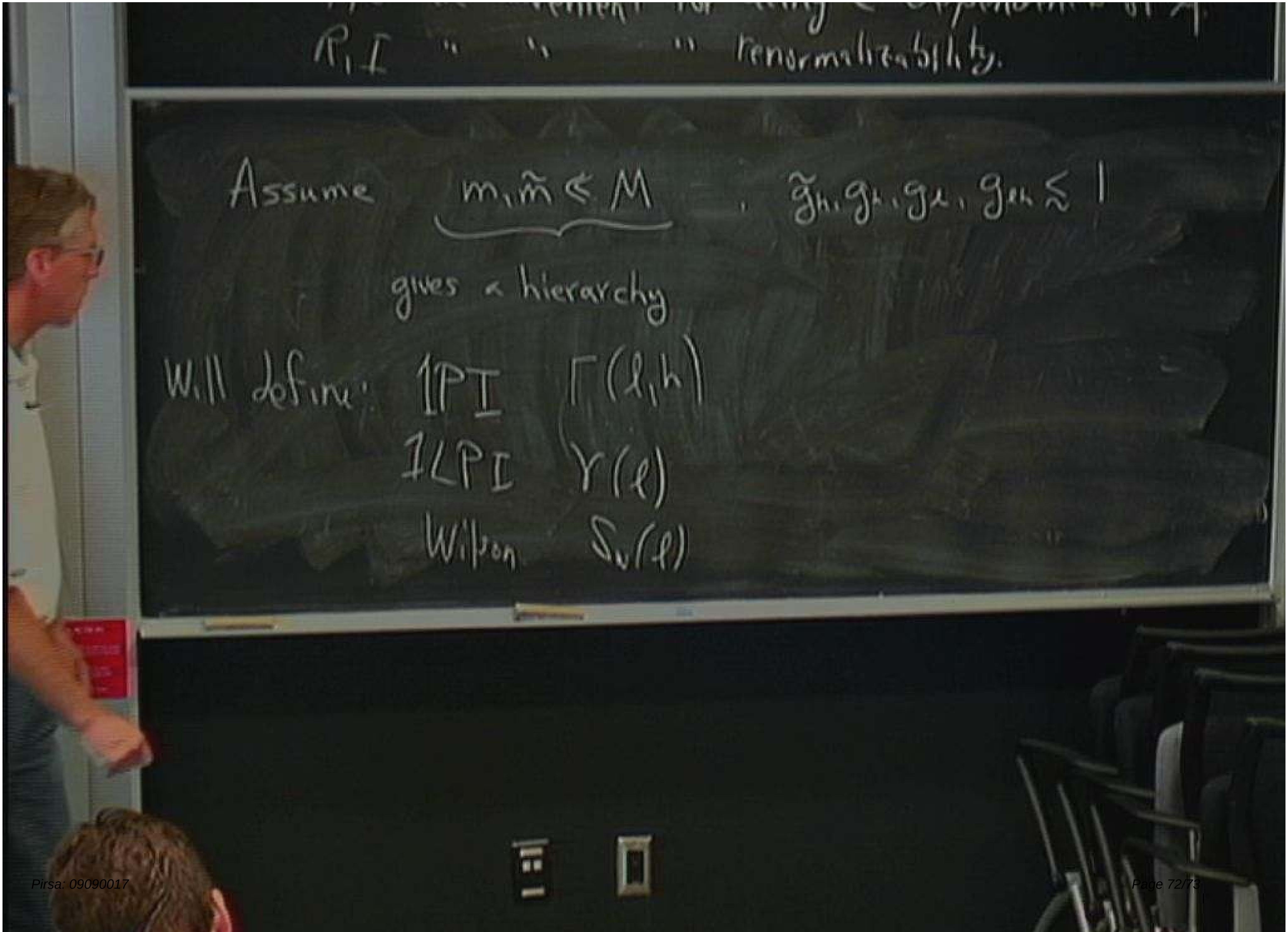

R, I " " " renormalizability.
Assume $\underbrace{m, \tilde{m} \ll M}_{\text{gives a hierarchy}}$, $\tilde{g}_h, g_h, g_L, g_{eh} \lesssim 1$


R, I " " " renormalizability.
Assume m, m̃ ≪ M ; g̃h, gh, gl, glh ≲ 1
gives a hierarchy
Will define: 1PI Γ(l,h)
1LPI γ(l)
Wilson Sw(l)

$) + \ldots$

we will

$\frac{\tilde{g}_h}{3!} M h^3$

$- \frac{g_h}{4} \ell^2 h^2$

... field redefinitions.

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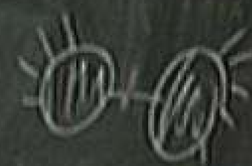
Assume $\underbrace{m, \tilde{m} \ll M}_{\text{gives a hierarchy}}$, $\tilde{g}_h, g_\ell, g_{h\ell}, g_{\ell h} \lesssim 1$



Will define: 1PI $\Gamma(\ell, h)$

1LPI $Y(\ell)$

Wilson $S_W(\ell)$