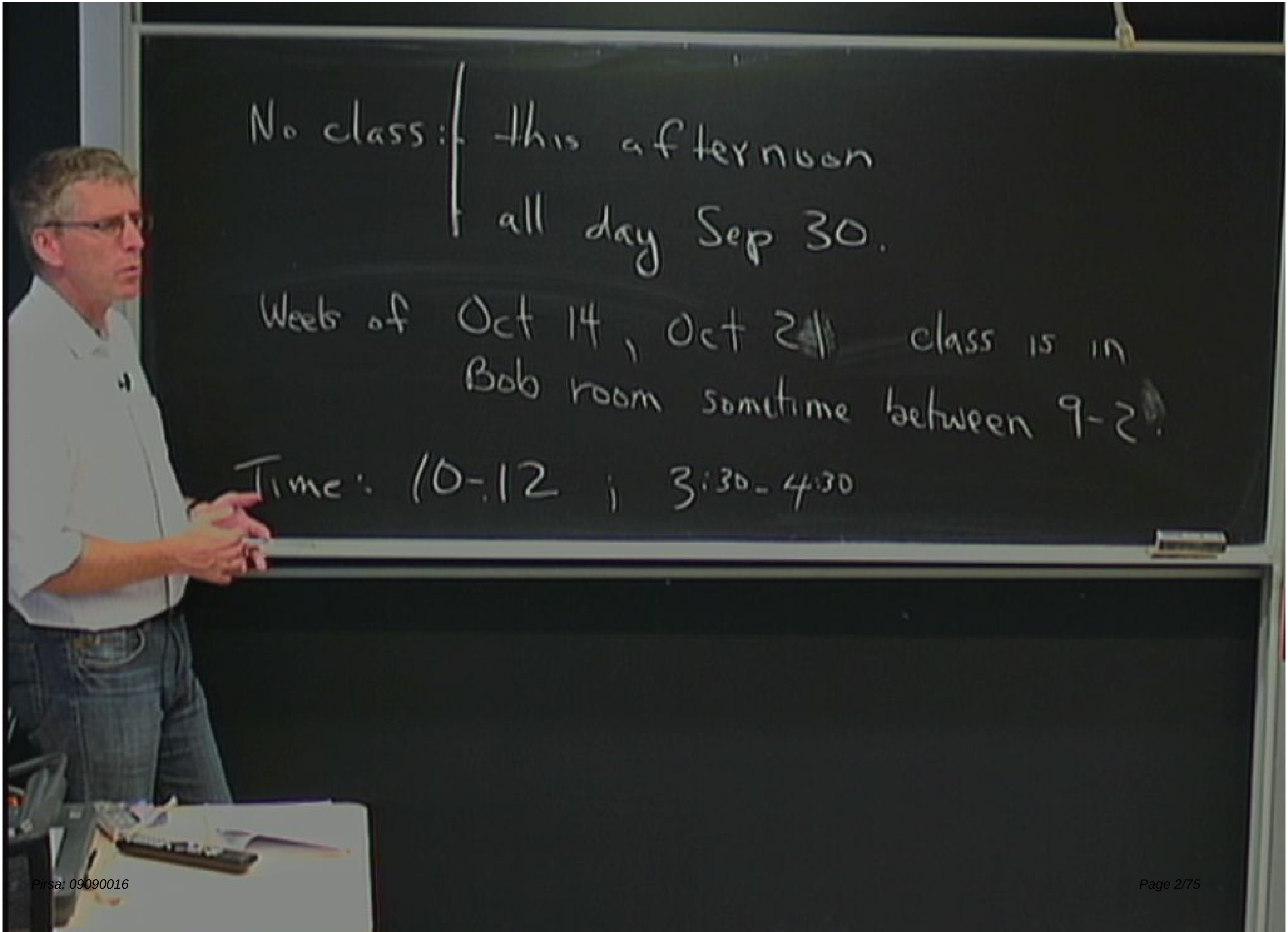
Title: Introduction to Effective Field Theory - Lecture 1A

Date: Sep 16, 2009 10:00 AM

URL: http://pirsa.org/09090016

Abstract:


No class: this afternoon
all day Sep 30.
Weeks of Oct 14, Oct 21 class is in
Bob room sometime between 9-2.
Time: 10-12 ; 3:30-4:30

EFFECTIVE FIELD THEORY METHODS.

Motivation: What is the point of EFT techniques?

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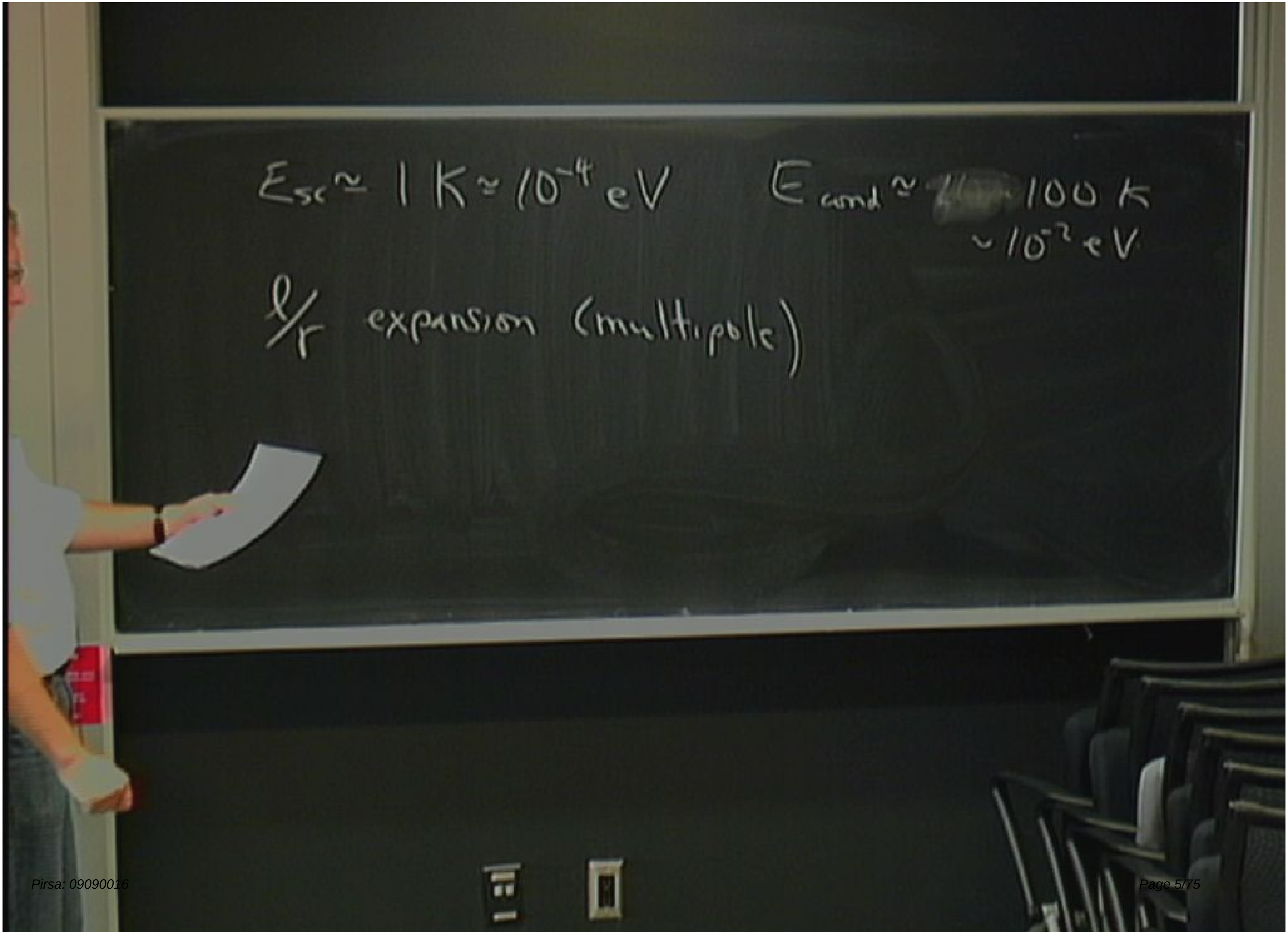
a) Physical systems often come with a variety of scales: $m_e, m_p \ll M_W, M_Z$

$$E_0 \simeq \frac{1}{2}\alpha^2 m_e \ll m_e \longleftarrow (\hbar = c = 1 = k_B)$$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ℓ/r expansion (multipole)

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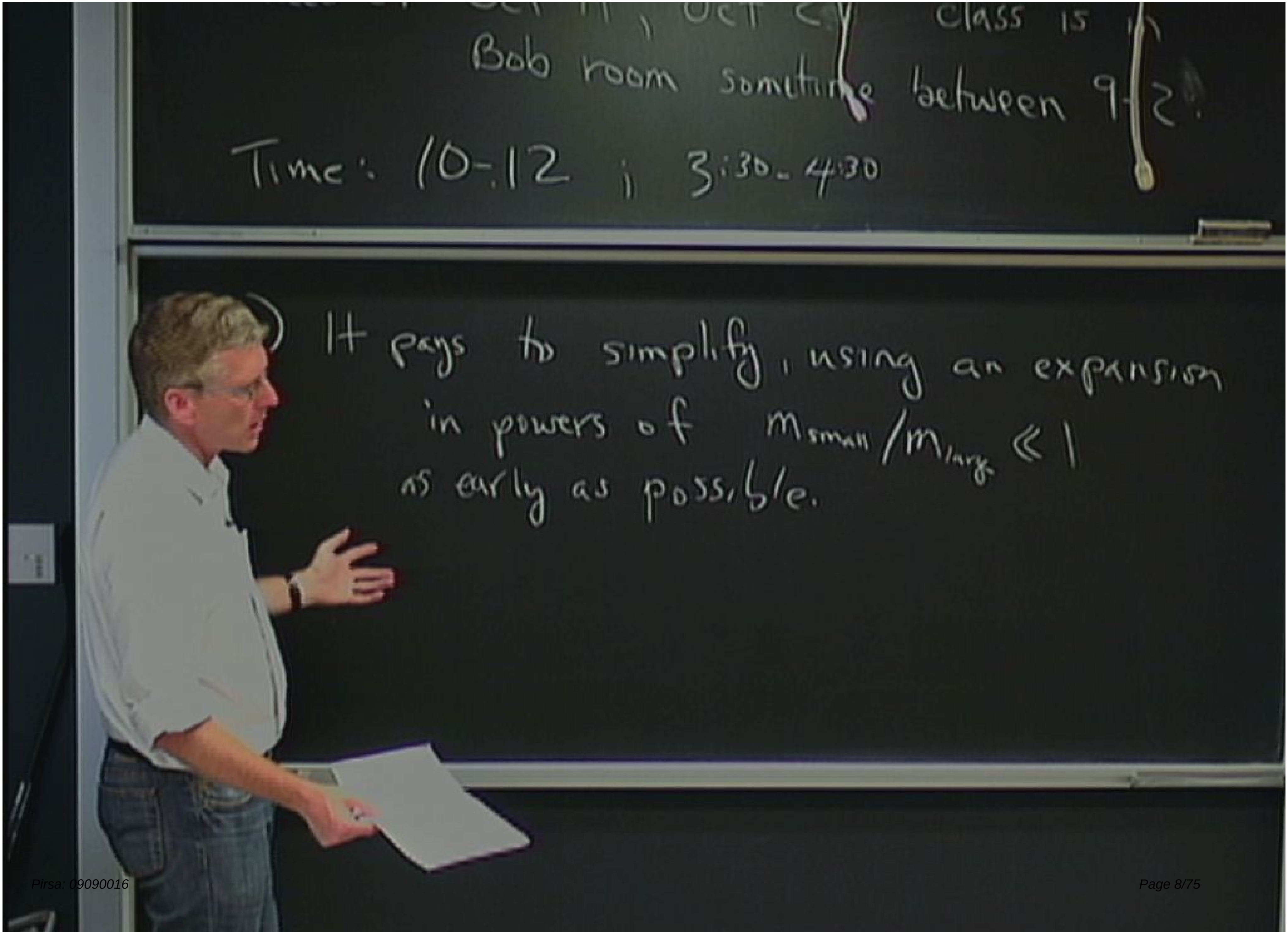
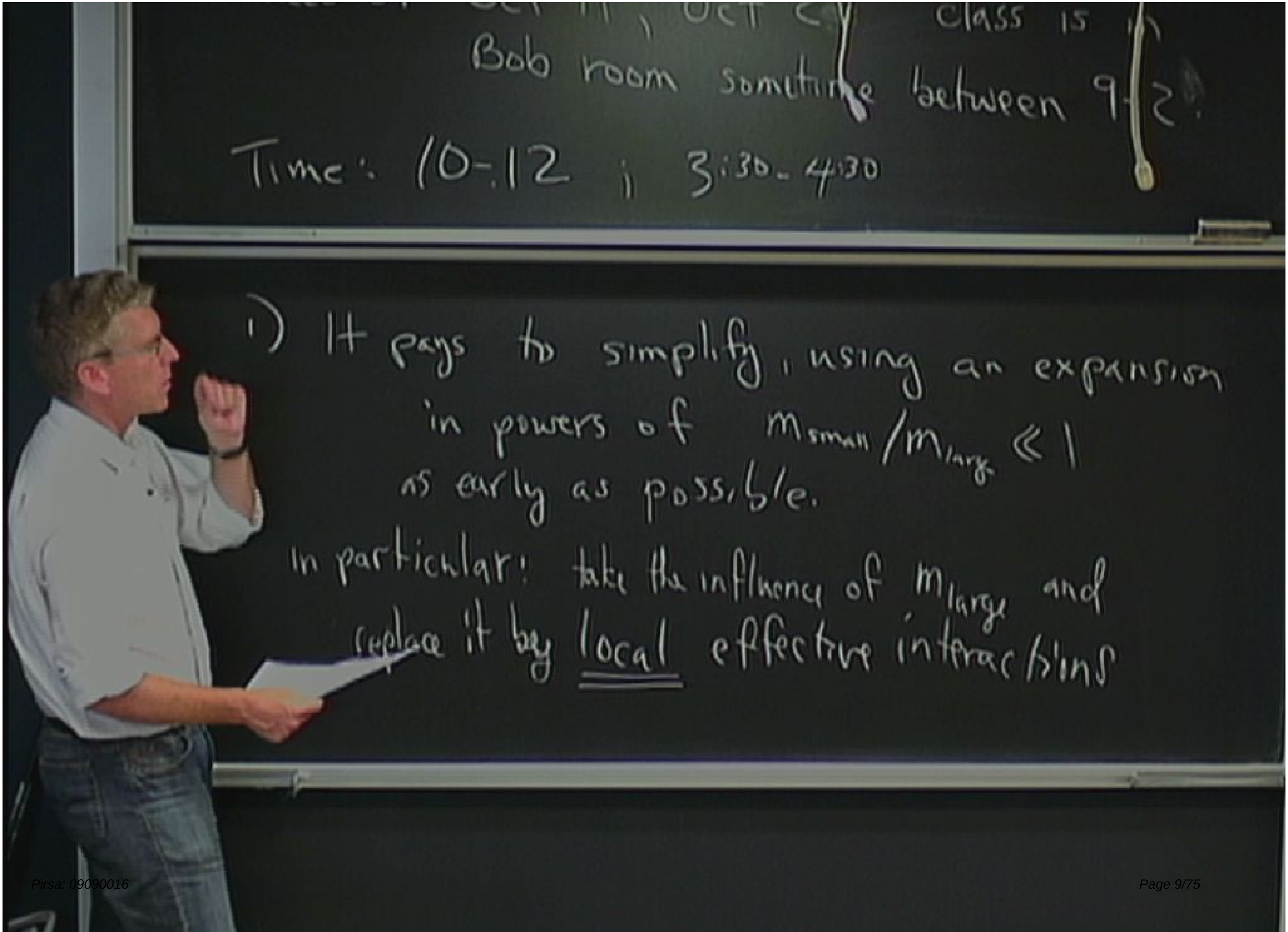
everything $\ll M_p$.


class is
Bob room sometime between 9-2
Time: 10-12 ; 3:30-4:30
It pays to simplify, using an expansion
in powers of $m_{small}/m_{large} \ll 1$
as early as possible.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Oct 17, Oct 24 class is in
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$m_p \ll M_W, M_Z$

$E_b \simeq \frac{1}{2}\alpha^2 m_e \ll m_e \leftarrow (\hbar = c = 1 = k_B)$

2) Experimental fact: most details of high energy part of a problem are irrelevant to low part: "decoupling".

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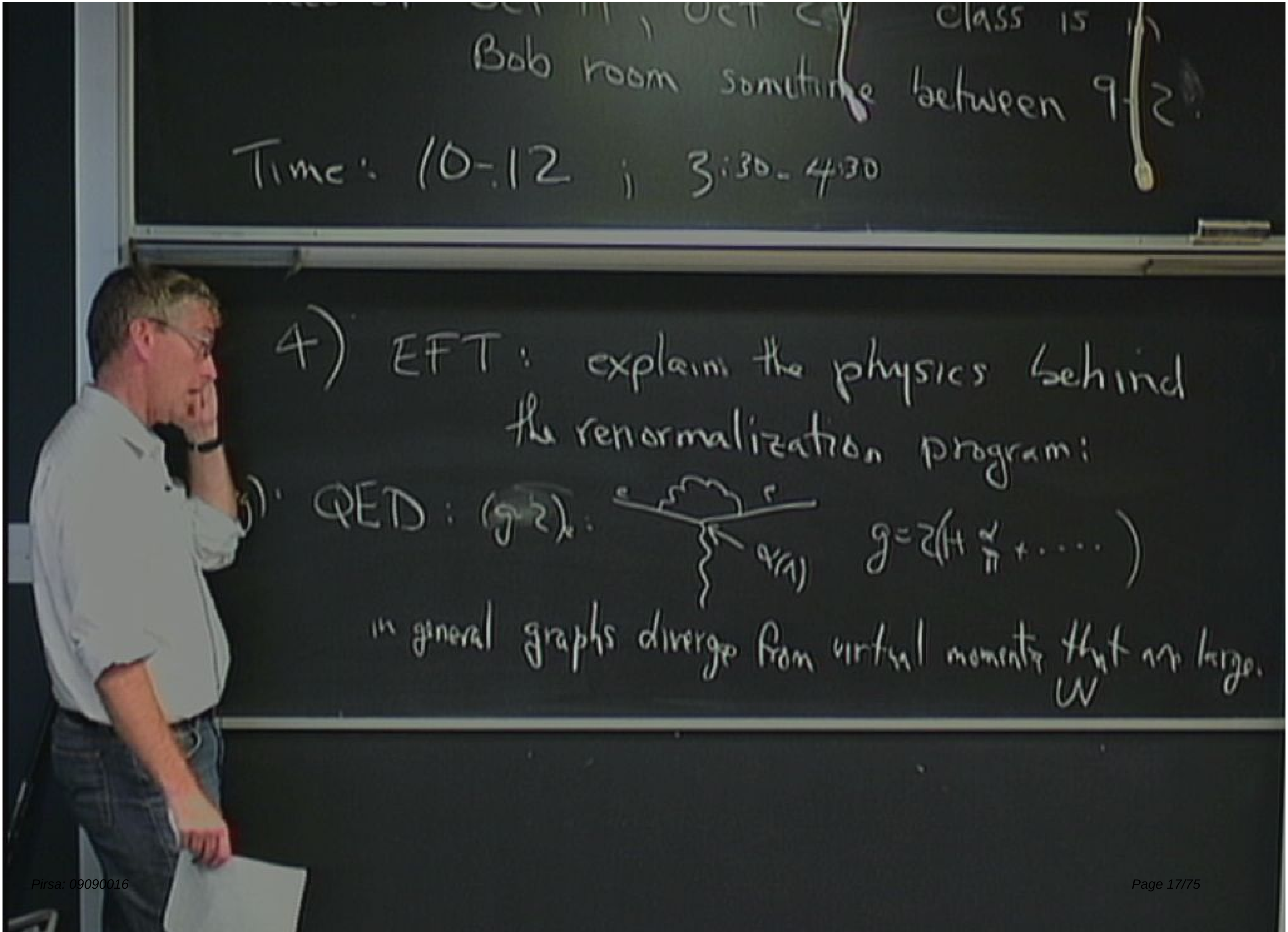
2) Which low energy processes are most sensitive (least sensitive) to HE?

everything $\ll M_p$.

3) It can happen that some predictions depend only on symmetries + the low energy expansion, even if this is not obvious in the full theory. $\rightarrow$ very robust

eg) Josephson effect, QHE, BCS


class is
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Time: 10-12 ; 3:30-4:30
4) EFT: explain the physics behind the renormalization program:
QED: (g-2)e:
q(Λ)
g=2(1+ α/π + ...)
in general graphs diverge from virtual momenta that are large.

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(and to quantify when quantum effects are (+ are not) important.)

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$g = 2\left(1 + \frac{\alpha}{\pi} + \cdots\right)$

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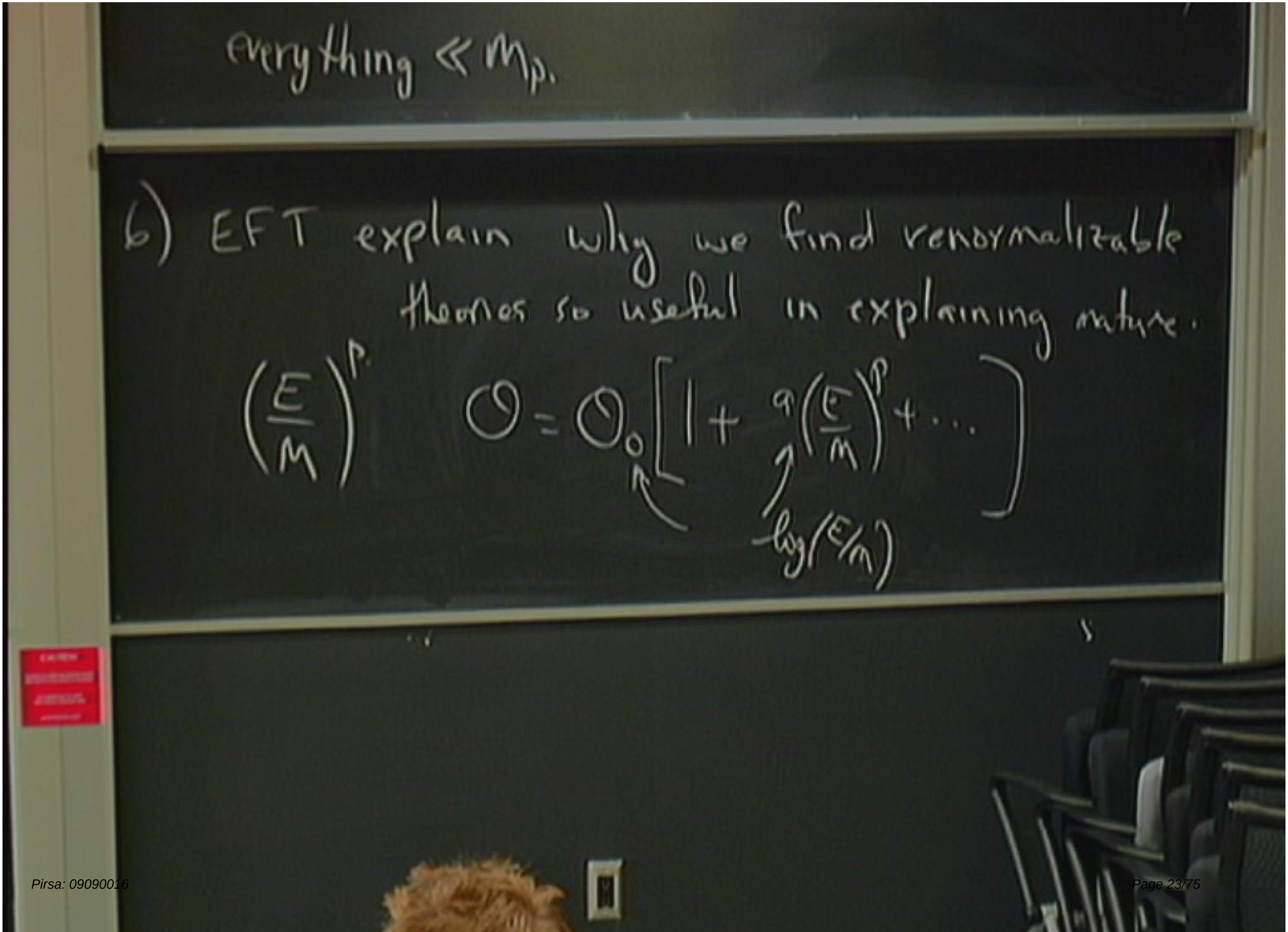
$$\left(\frac{E}{M}\right)^p \qquad \mathcal{O} = \mathcal{O}_0\left[1 + a\left(\frac{E}{M}\right)^p + \dots\right]$$

$\log(E/m)$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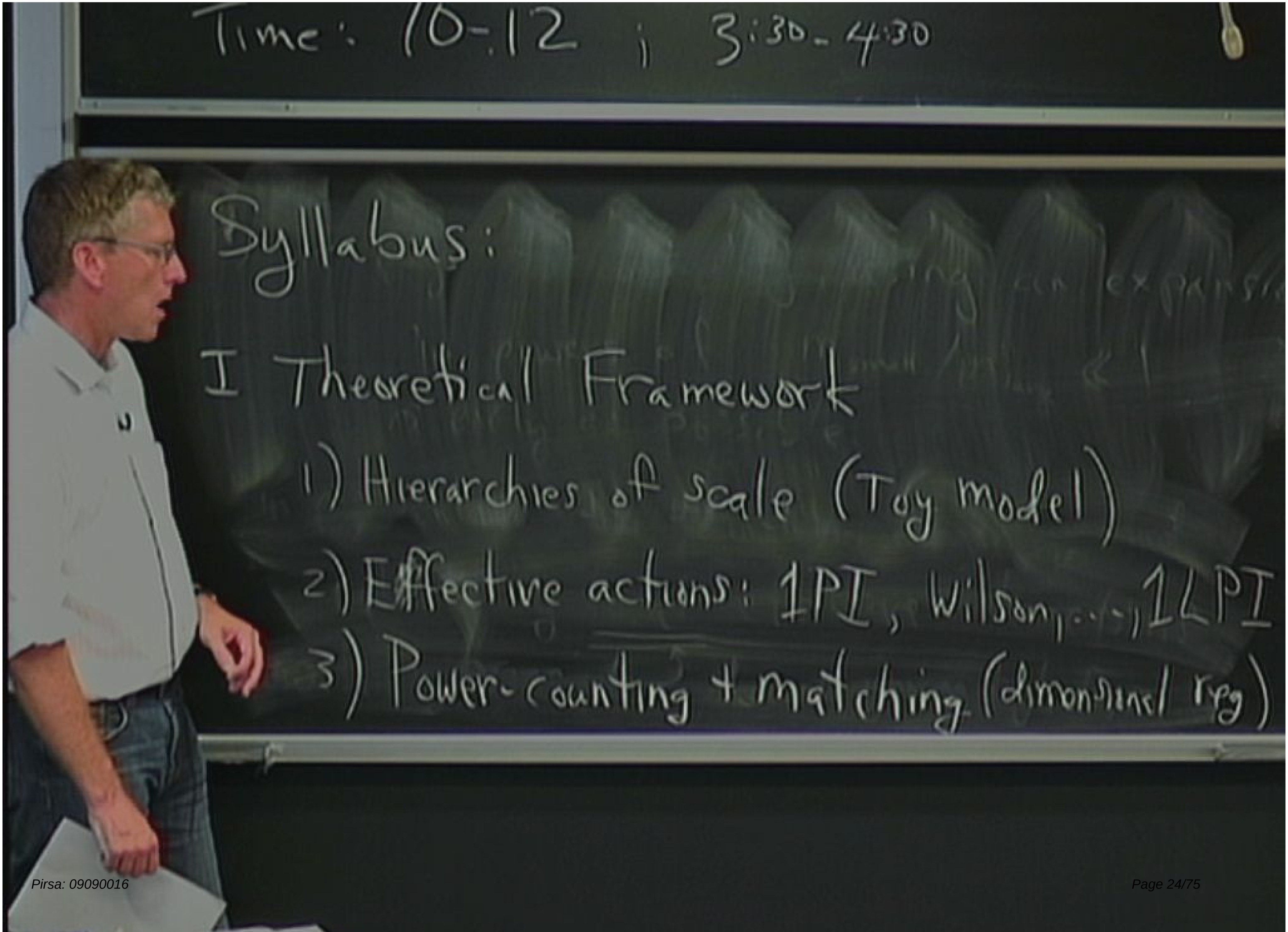
Time: 10-12 ; 3:30-4:30
Syllabus:
I Theoretical Framework
1) Hierarchies of scale (Toy model)
2) Effective actions: 1PI, Wilson, ..., 1LPI
3) Power-counting + matching (dimensional reg)

Time: 10-12 ; 3:30-4:30

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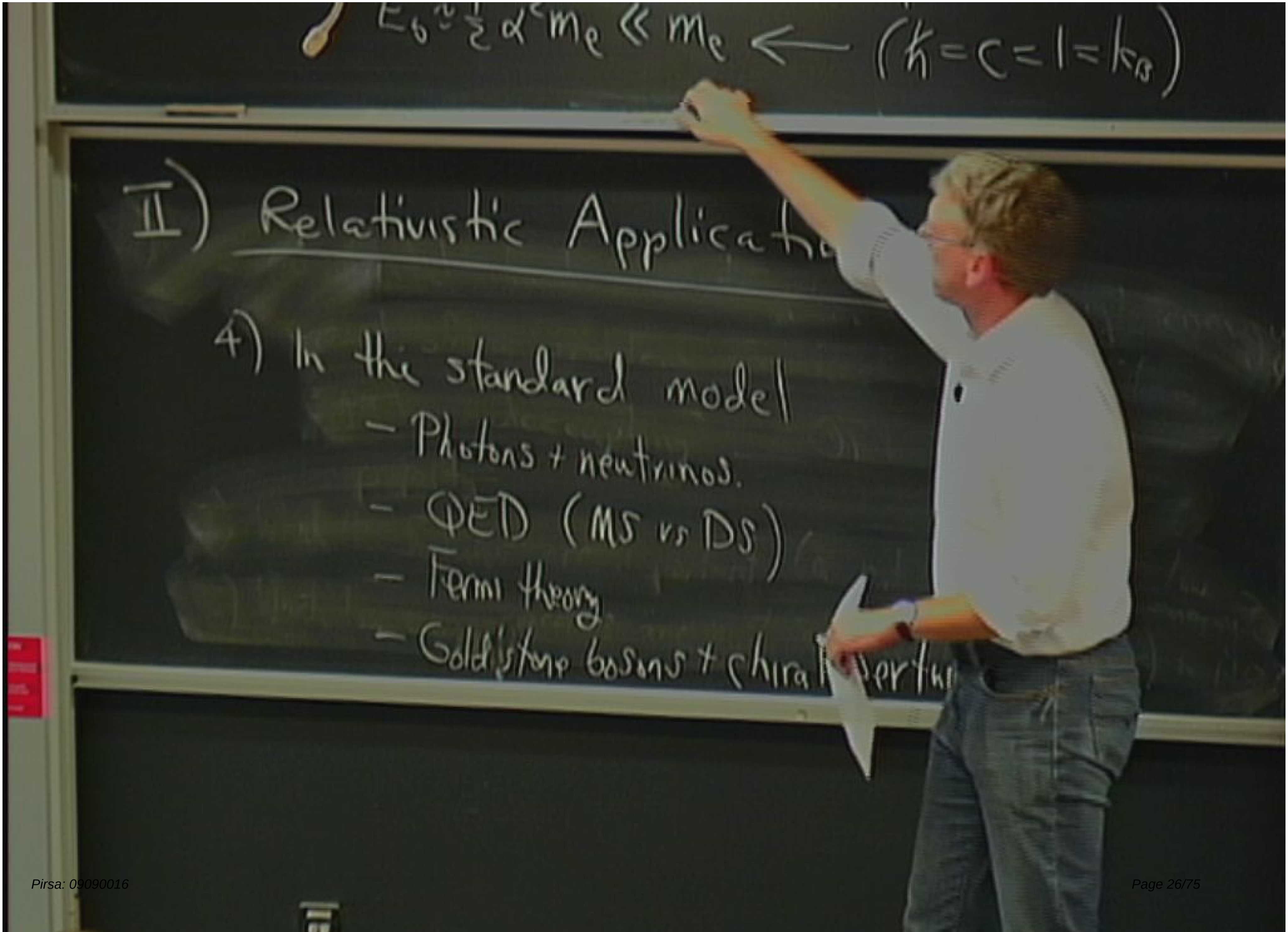
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II) Relativistic Applica
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– Photons + neutrinos.
– QED (MS vs DS)
– Fermi theory
– Goldstone bosons + chira

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II) Relativistic Applications:

4) In the standard model

- Photons + neutrinos.
- QED (MS vs DS)
- Fermi theory
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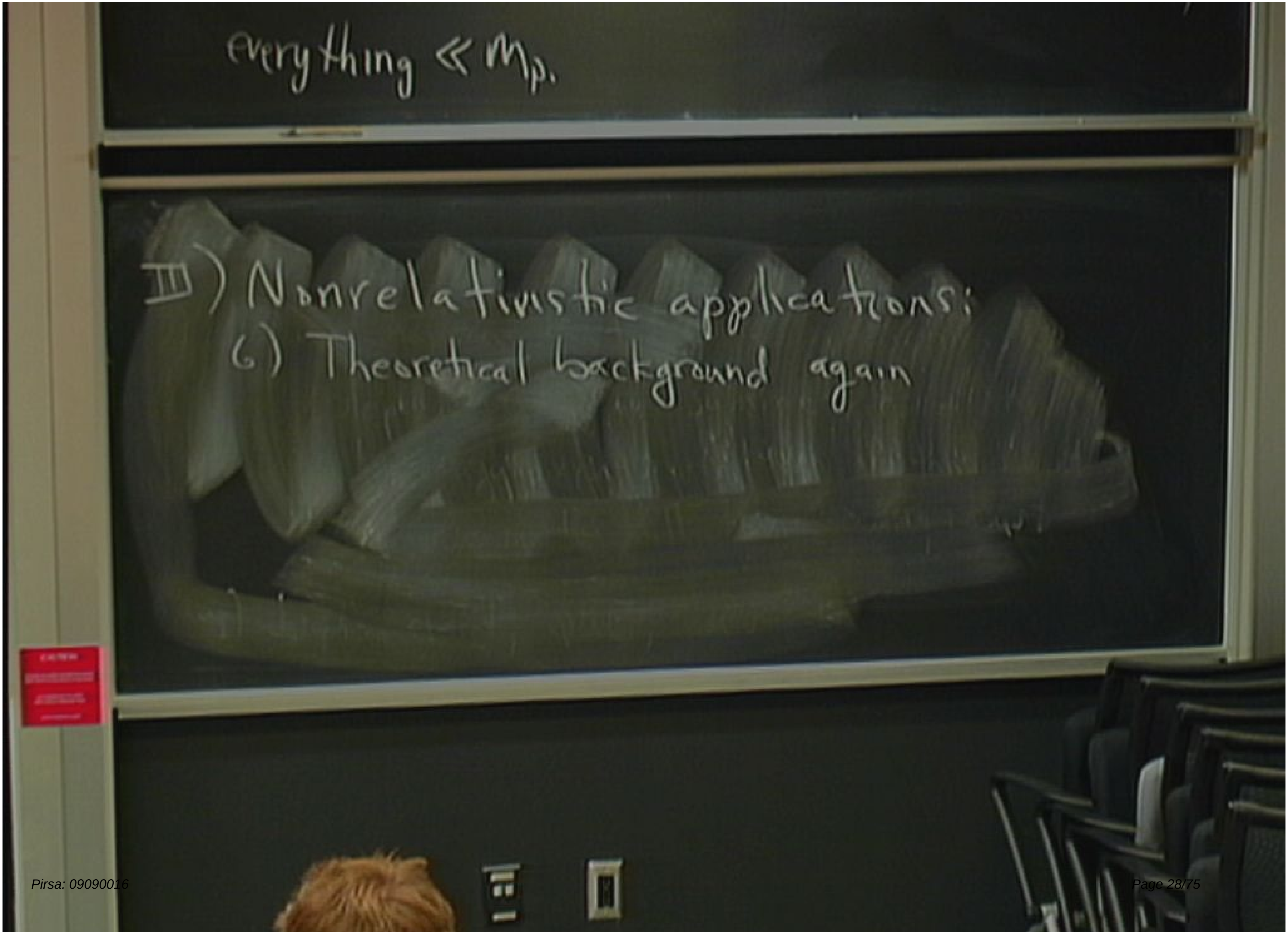
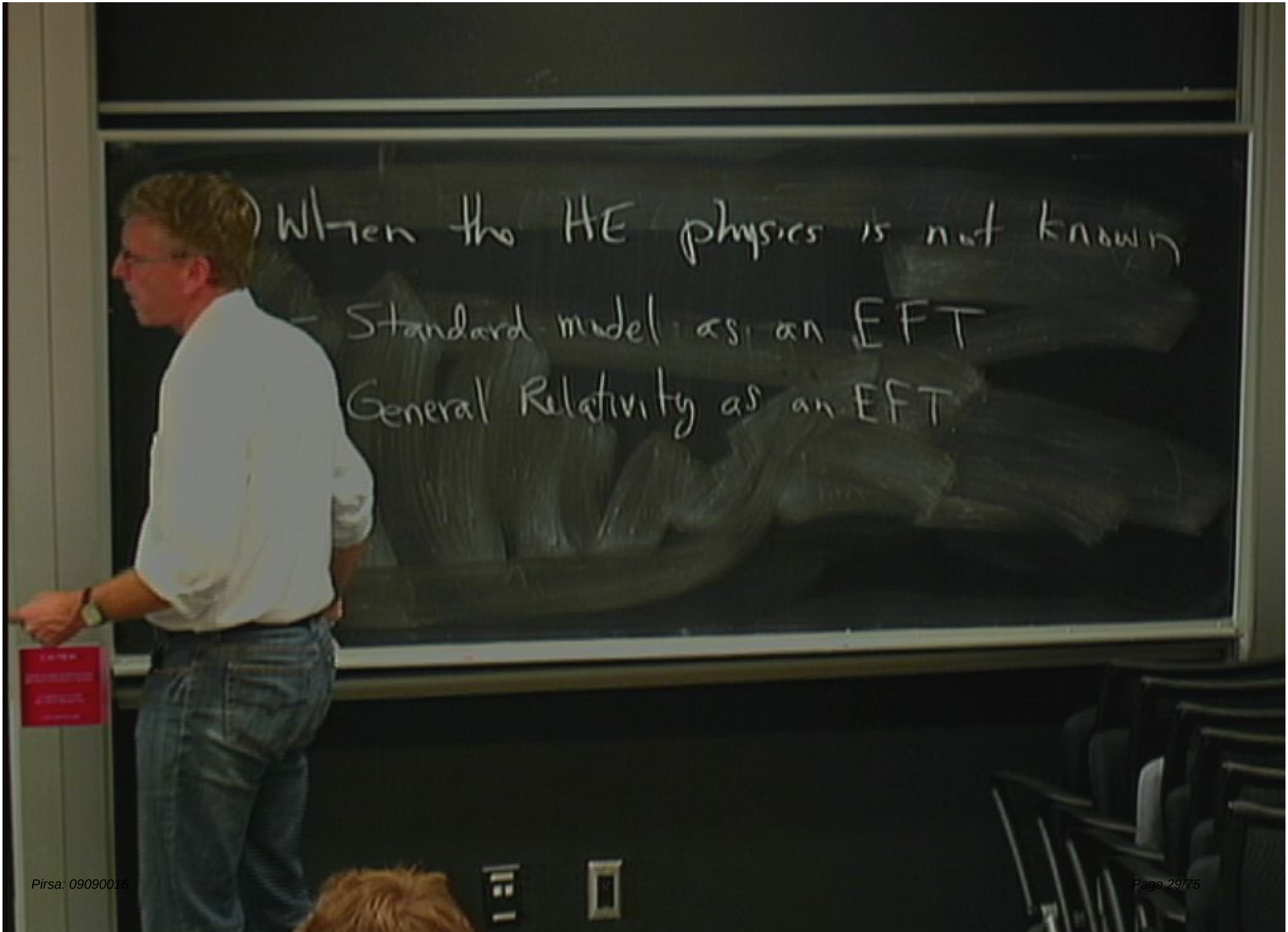
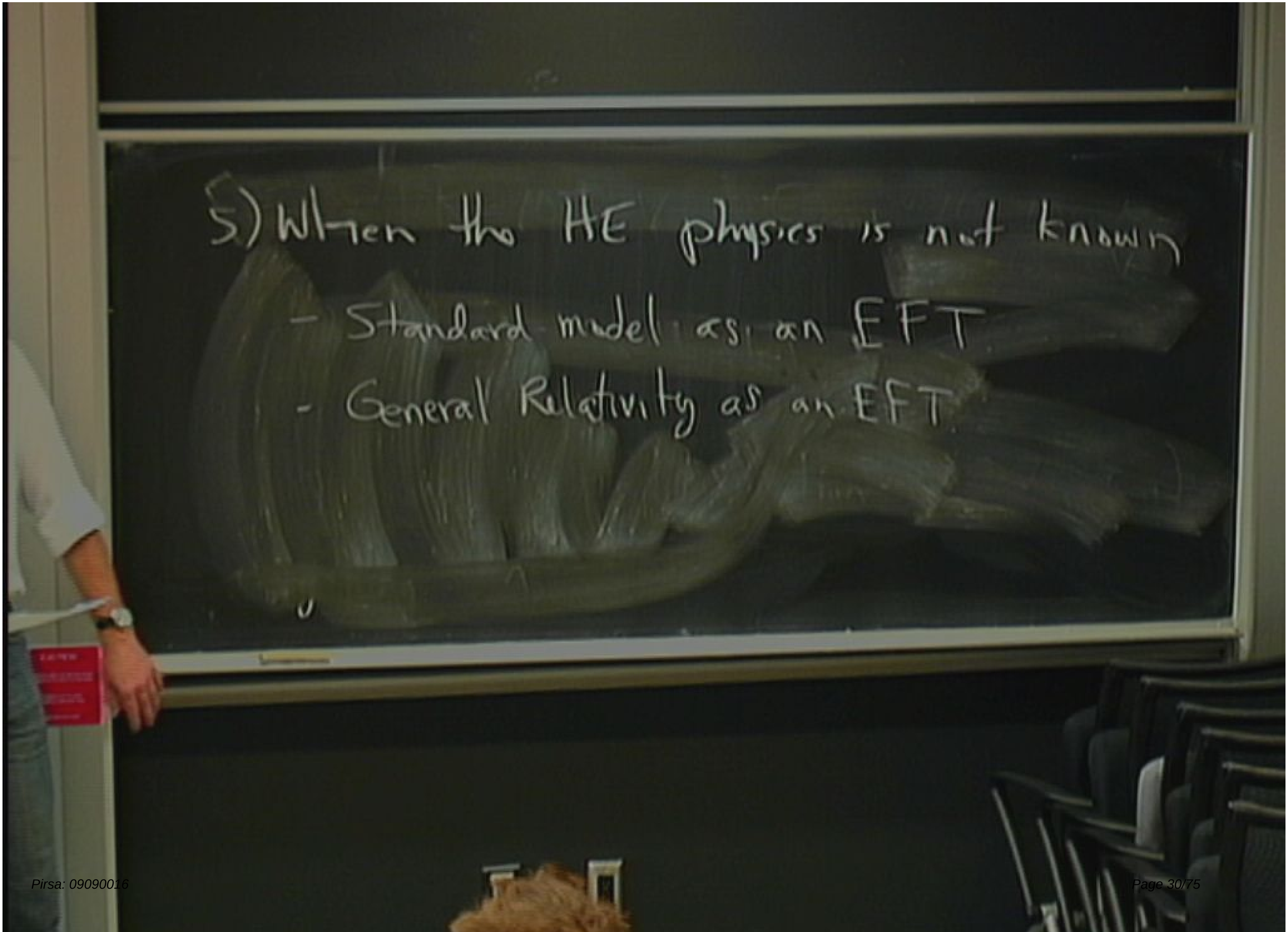

everything $\ll M_p$.
III) Nonrelativistic applications:
6) Theoretical background again


When the HE physics is not known
– Standard model as an EFT
General Relativity as an EFT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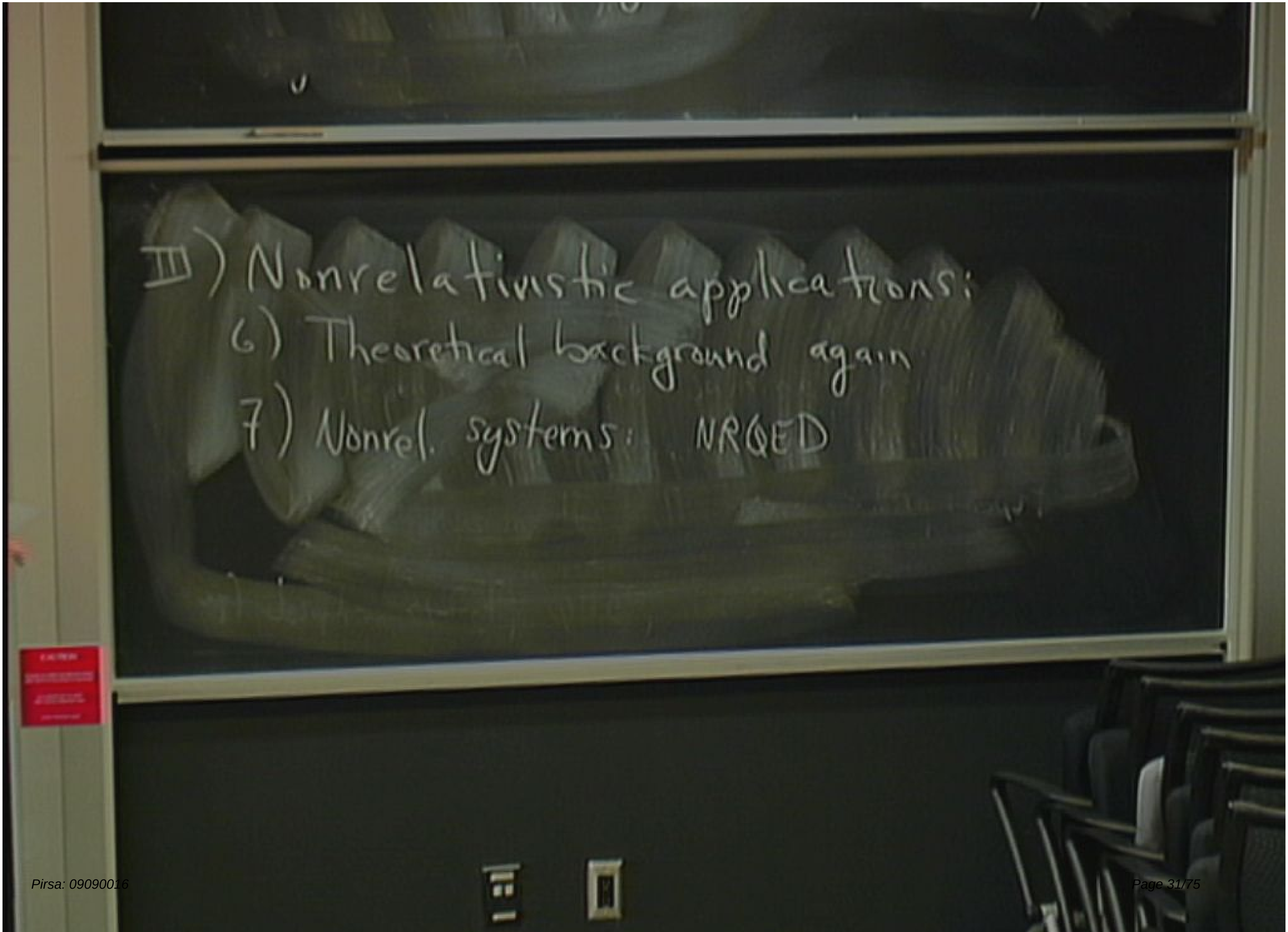
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- Positronium + unstable particles

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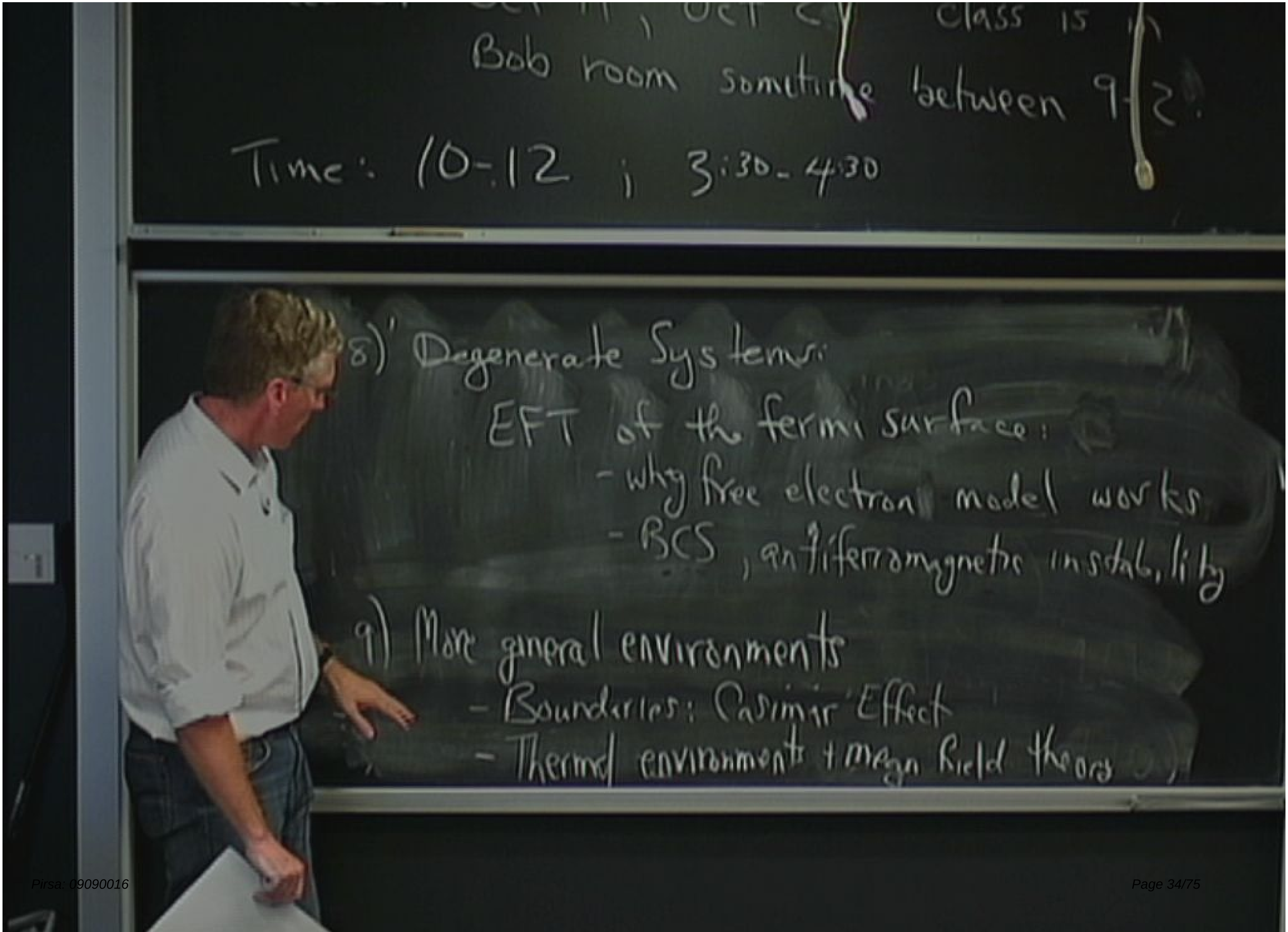
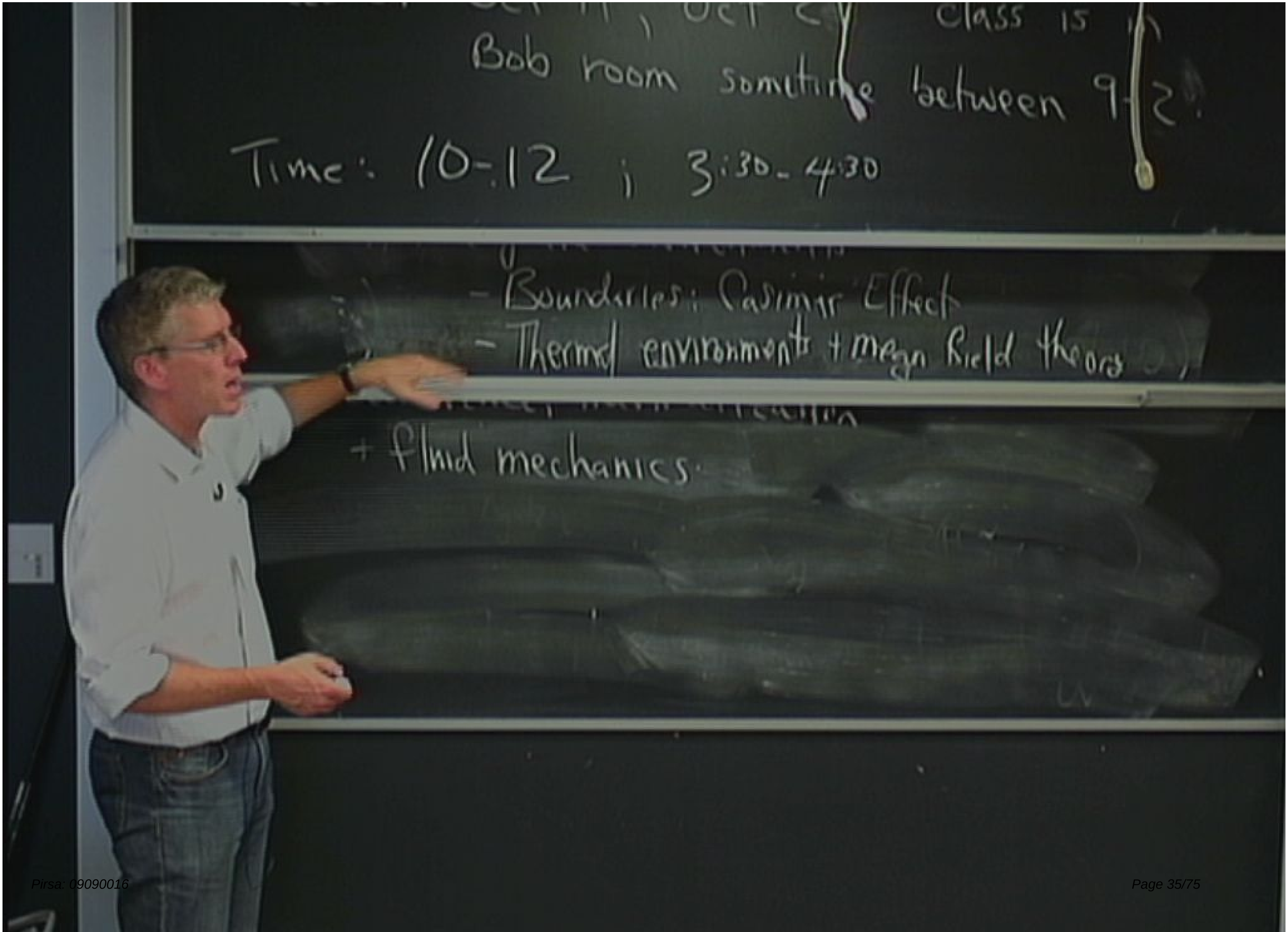
7) Nonrel. systems: - NRQED

- Positronium + unstable particles
- light scattering from molecules
- ferromagnets + antiferromagnets


class is
Bob room sometime between 9-2?
Time: 10-12 ; 3:30-4:30
8)' Degenerate Systems:
EFT of the fermi surface:
- why free electron model works
- BCS, antiferromagnetic instability
9) More general environments
- Boundaries: Casimir Effect
- Thermal environments + mean field theory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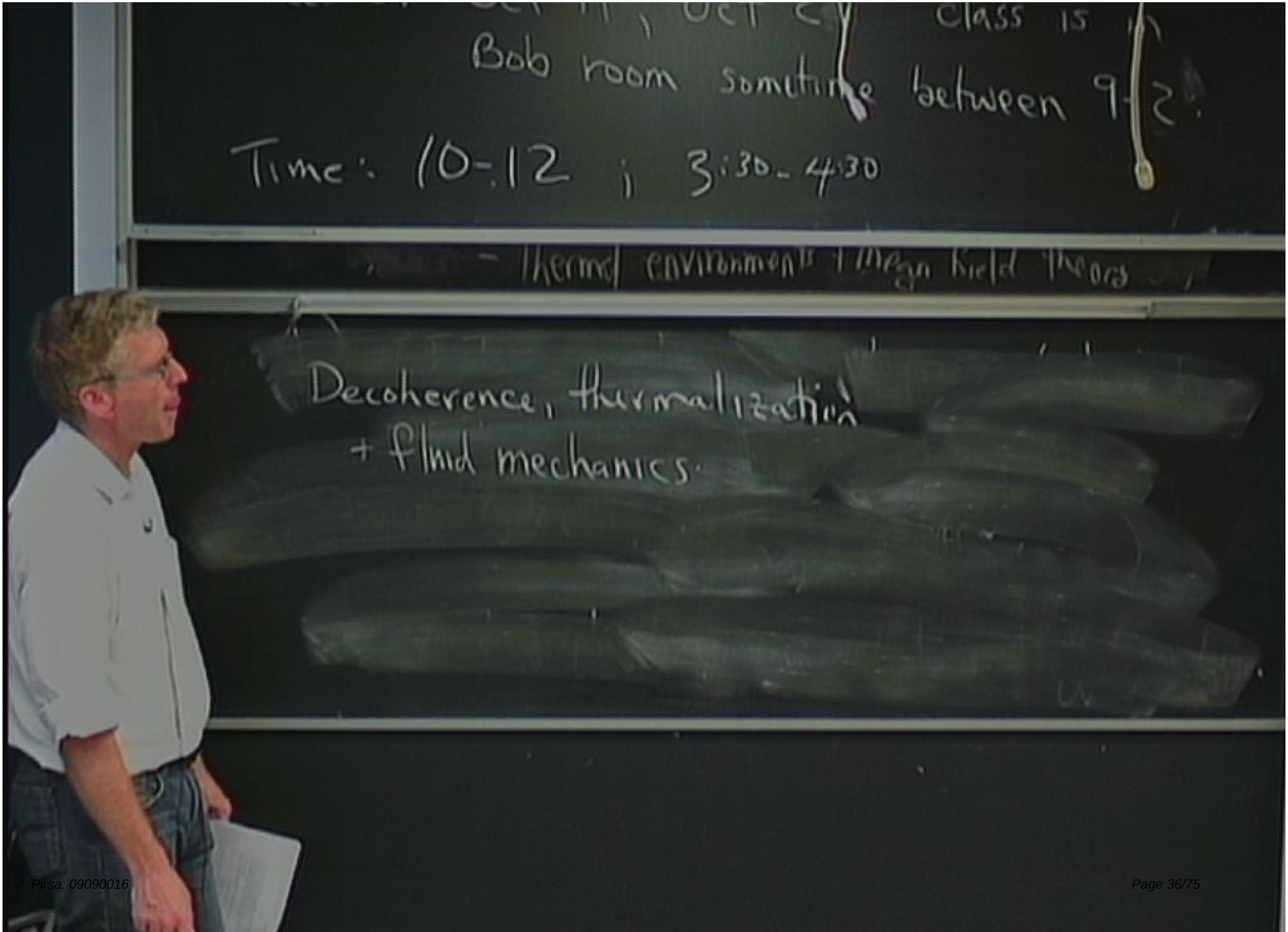
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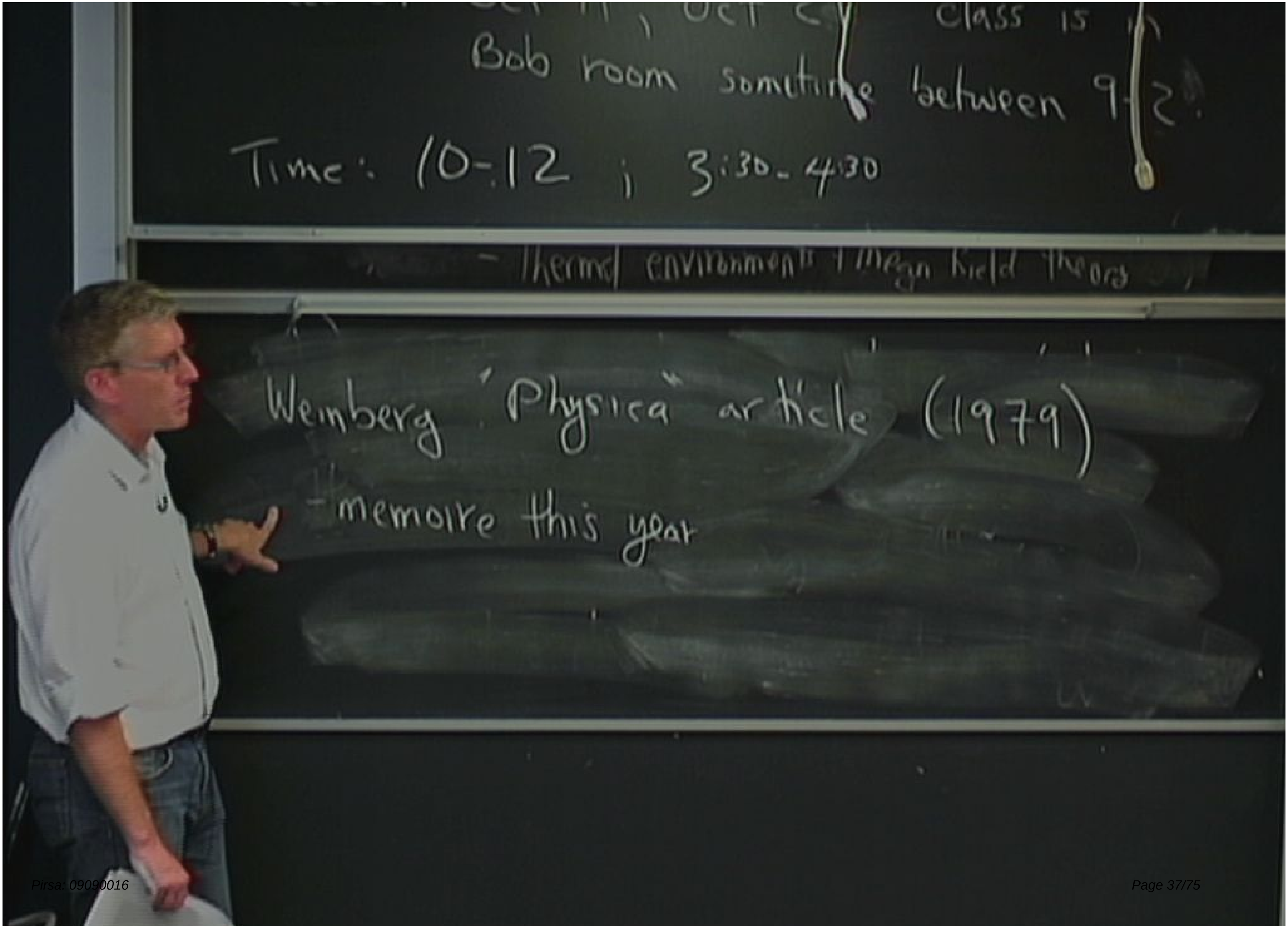
Thermal environments + mean field theory
Decoherence, thermalization
+ fluid mechanics.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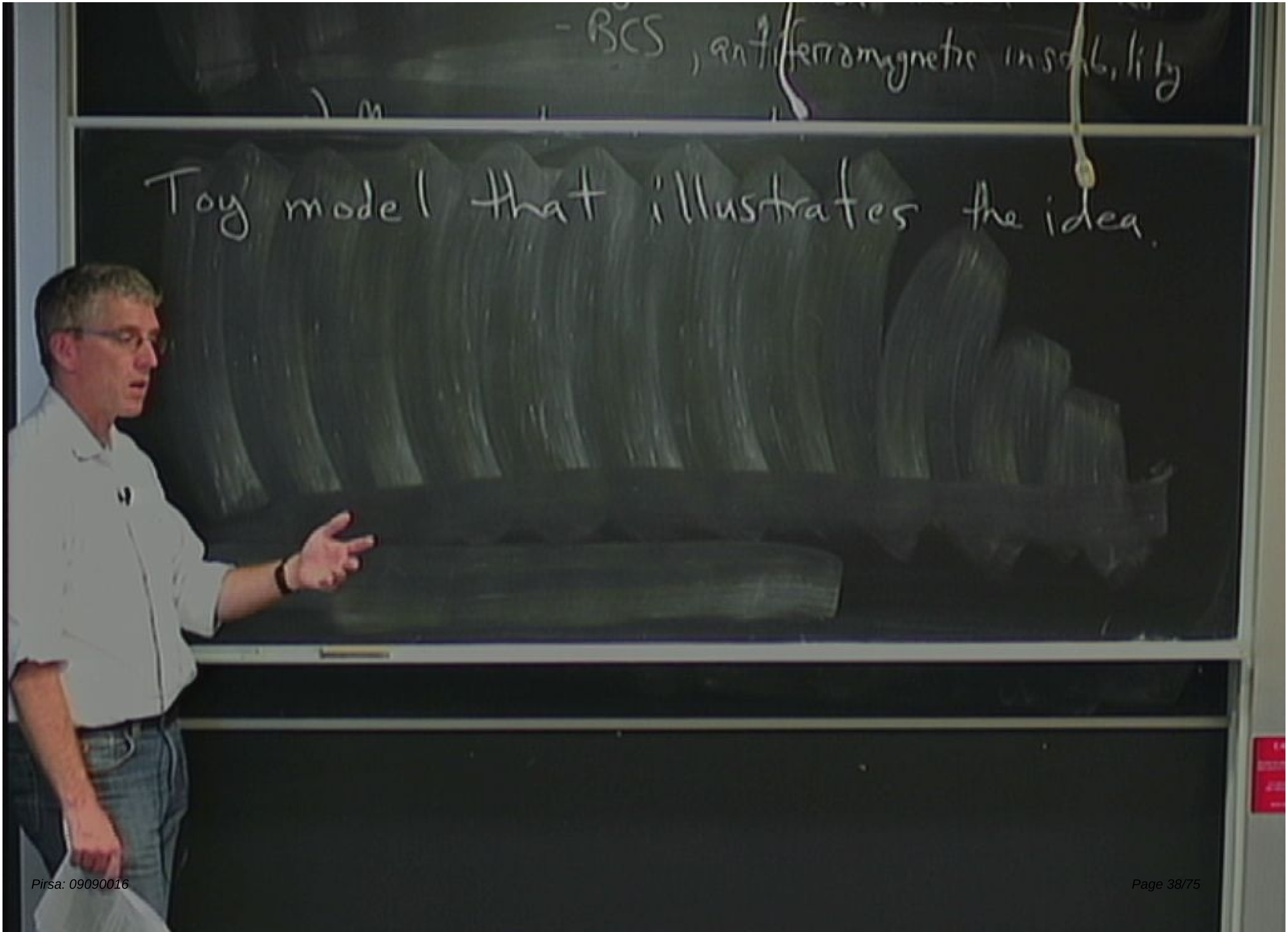
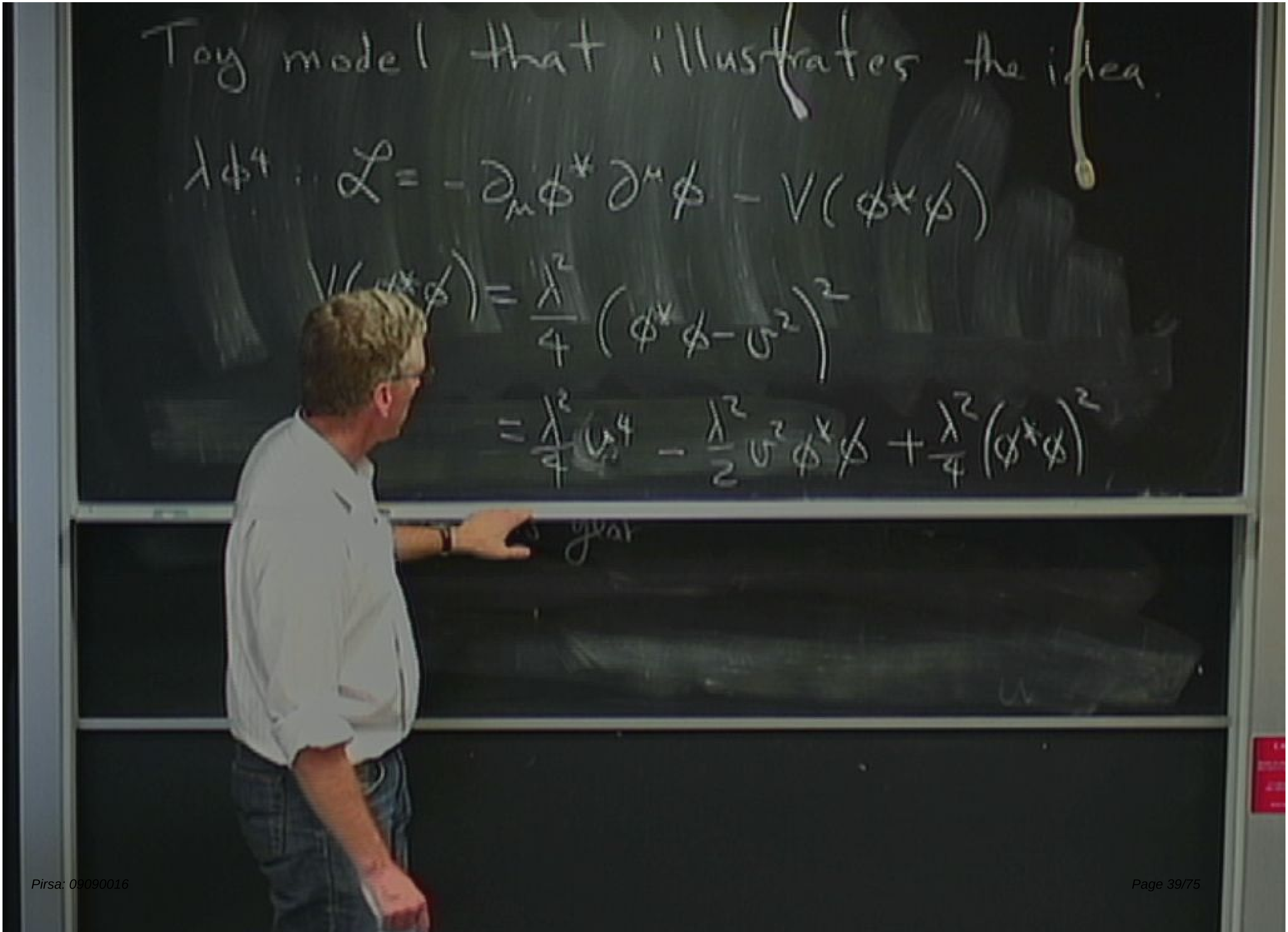
Weinberg "Physica" article (1979)
memoire this year


- BCS, antiferromagnetic instability
Toy model that illustrates the idea.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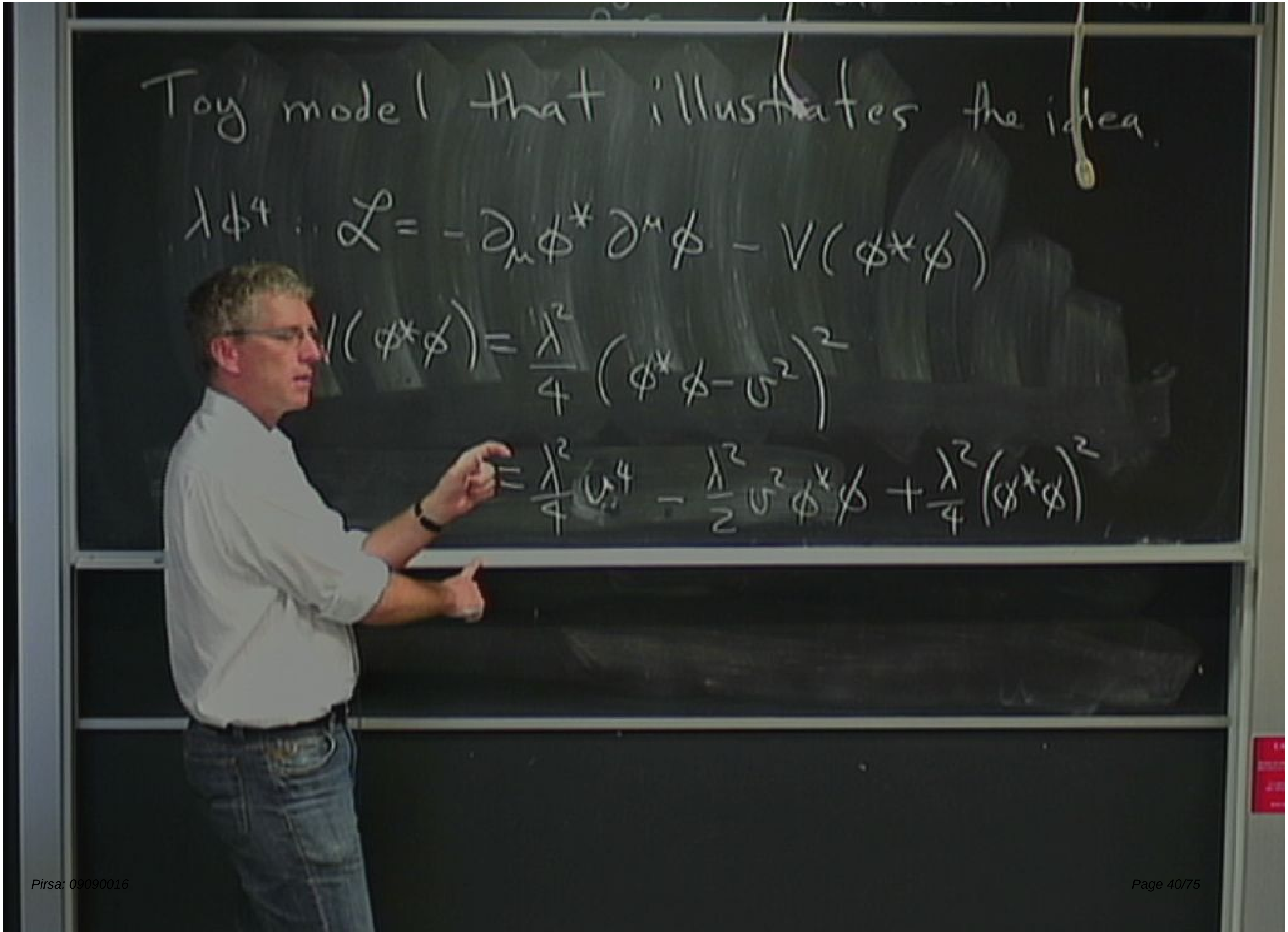
$\lambda\phi^4$: $\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - V(\phi^*\phi)$
$V(\phi^*\phi) = \frac{\lambda^2}{4}(\phi^*\phi - v^2)^2$
$= \frac{\lambda^2}{4} v^4 - \frac{\lambda^2}{2} v^2 \phi^*\phi + \frac{\lambda^2}{4}(\phi^*\phi)^2$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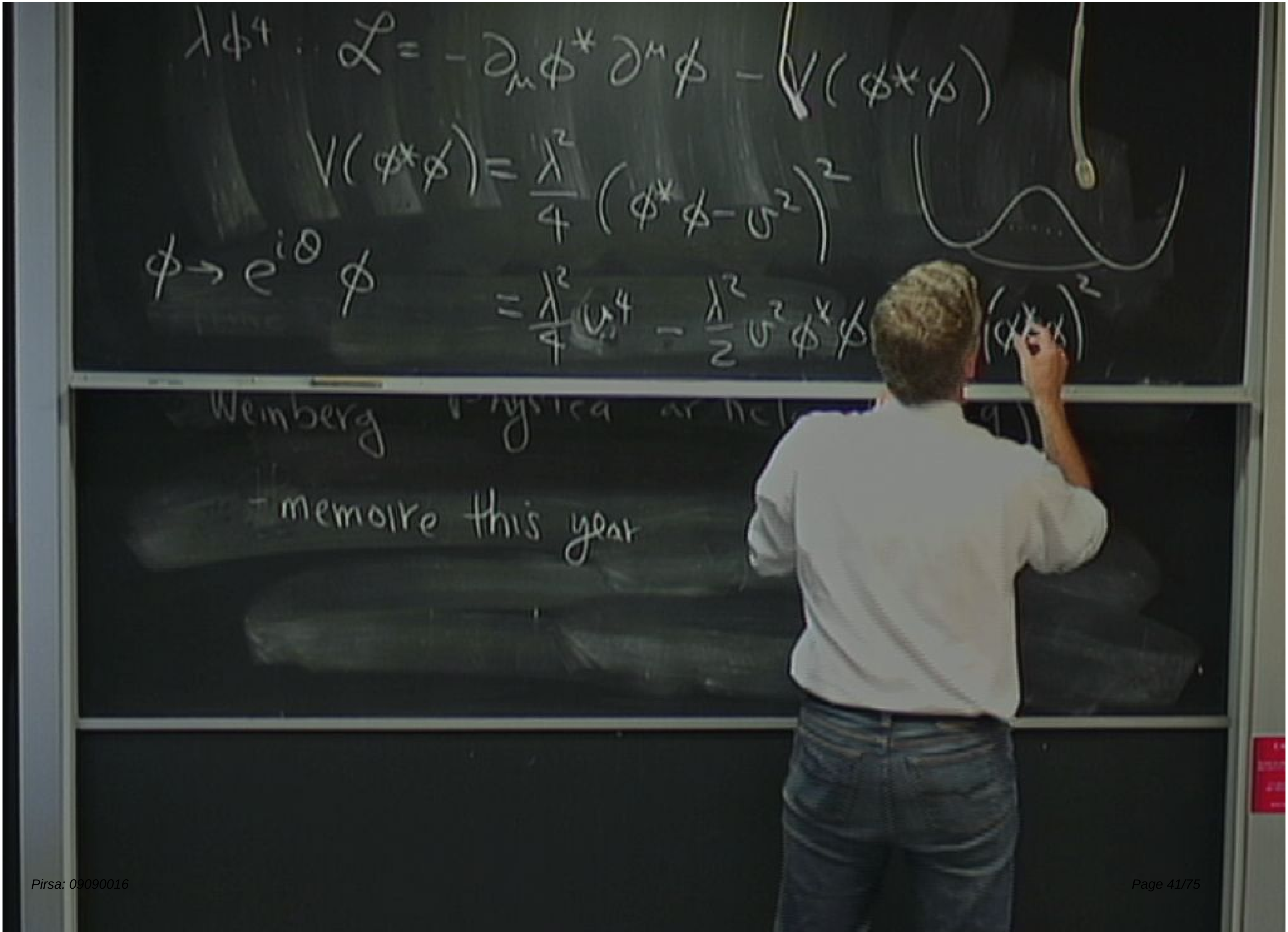
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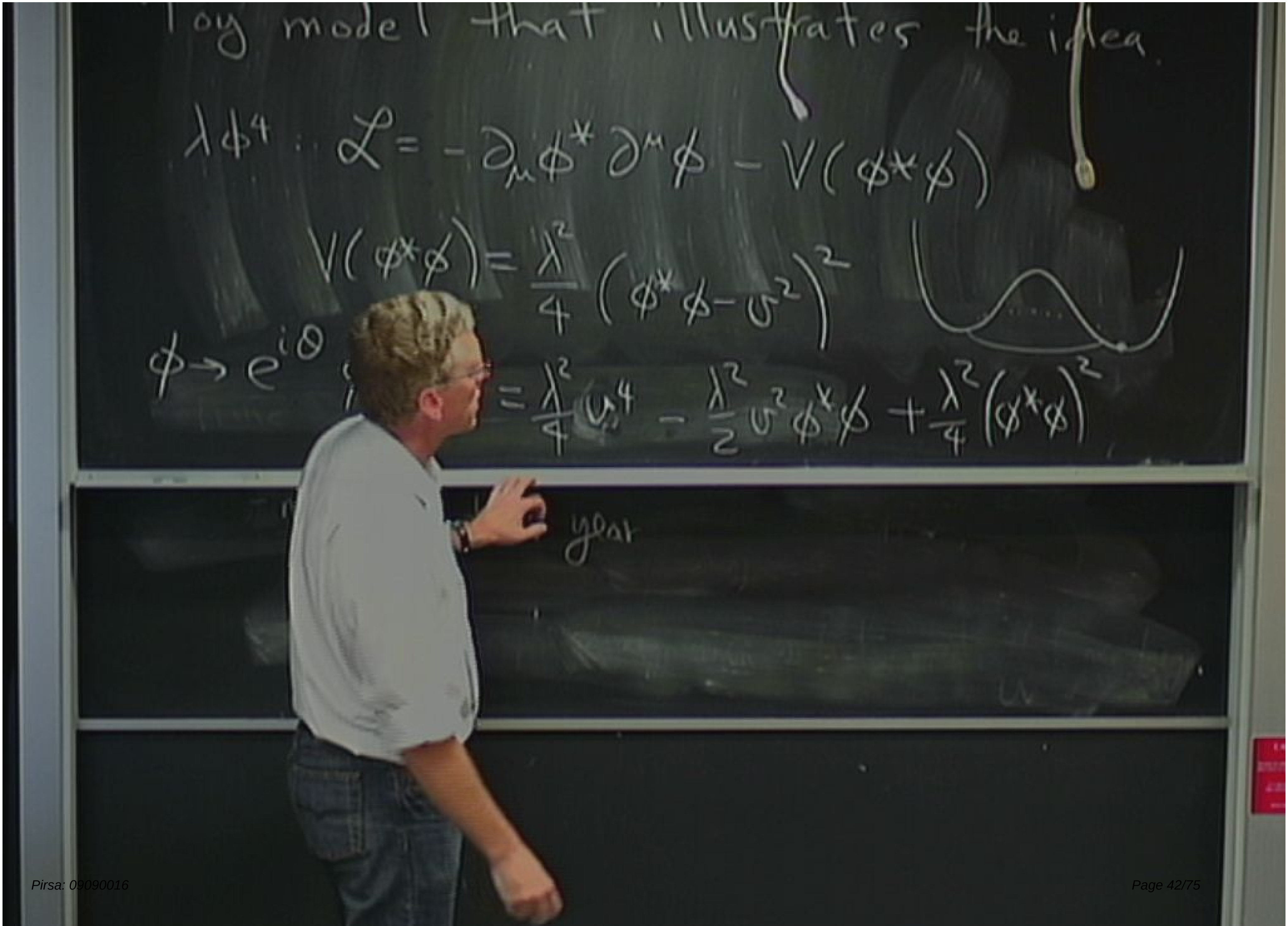
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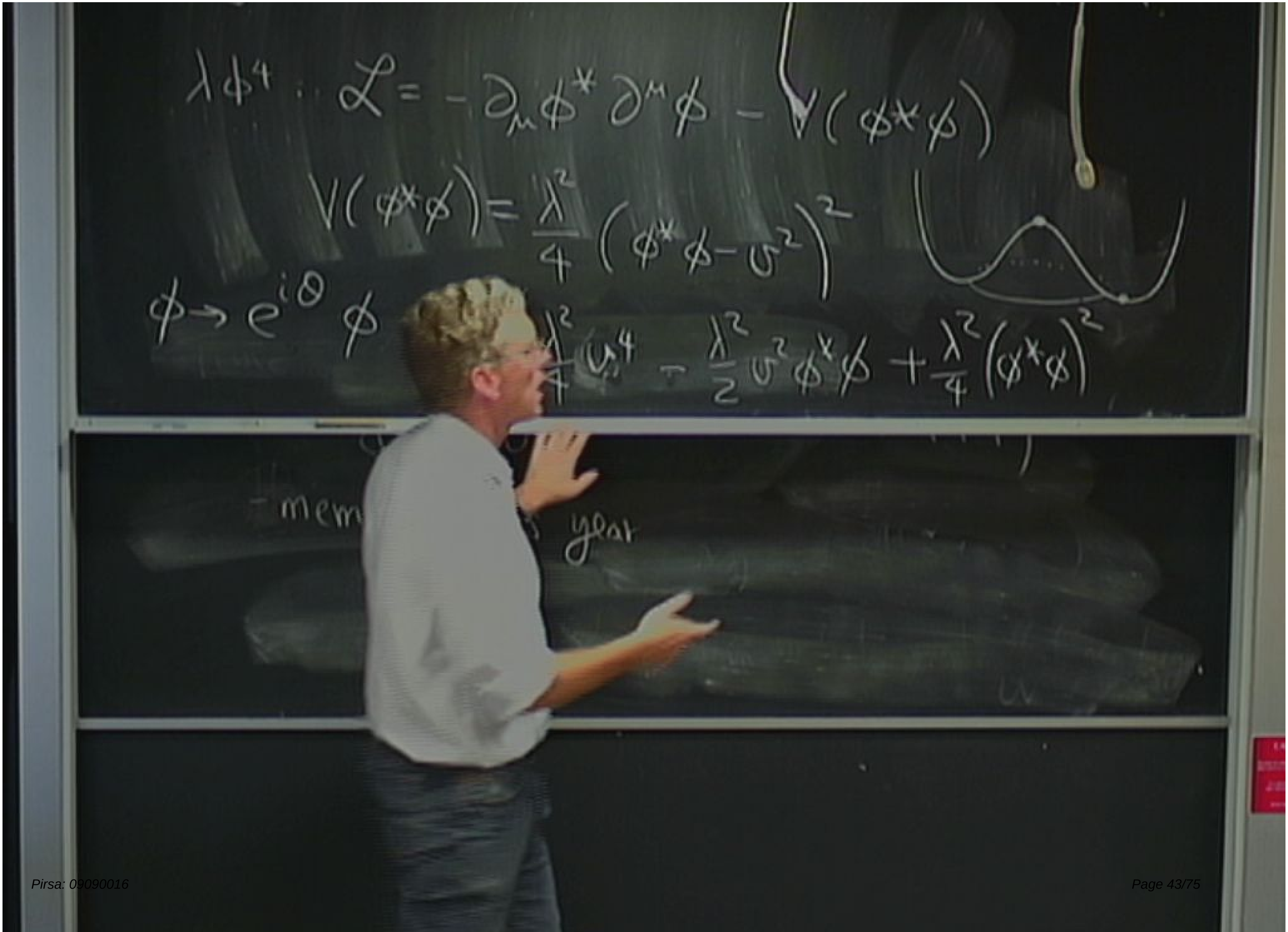
year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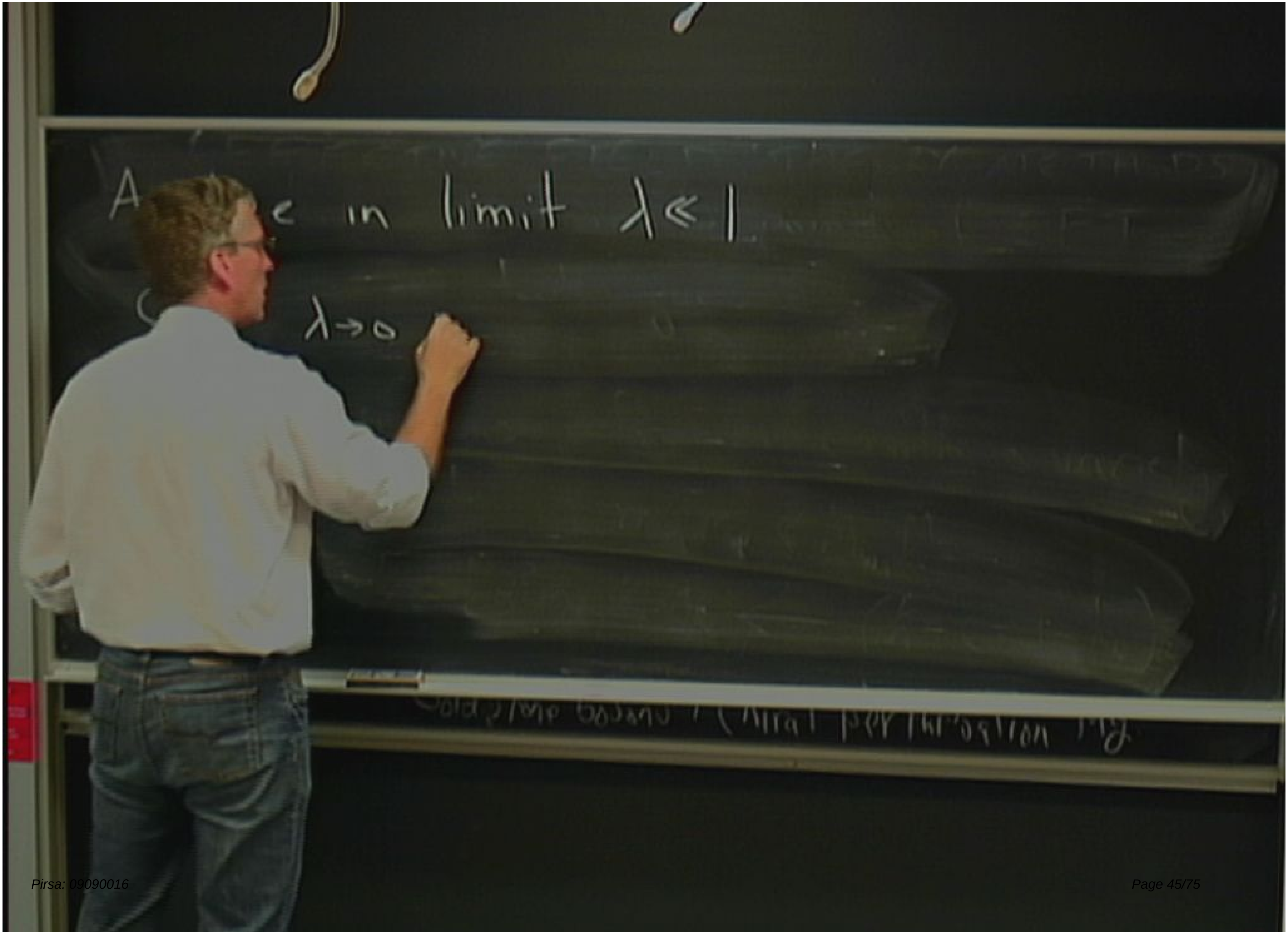
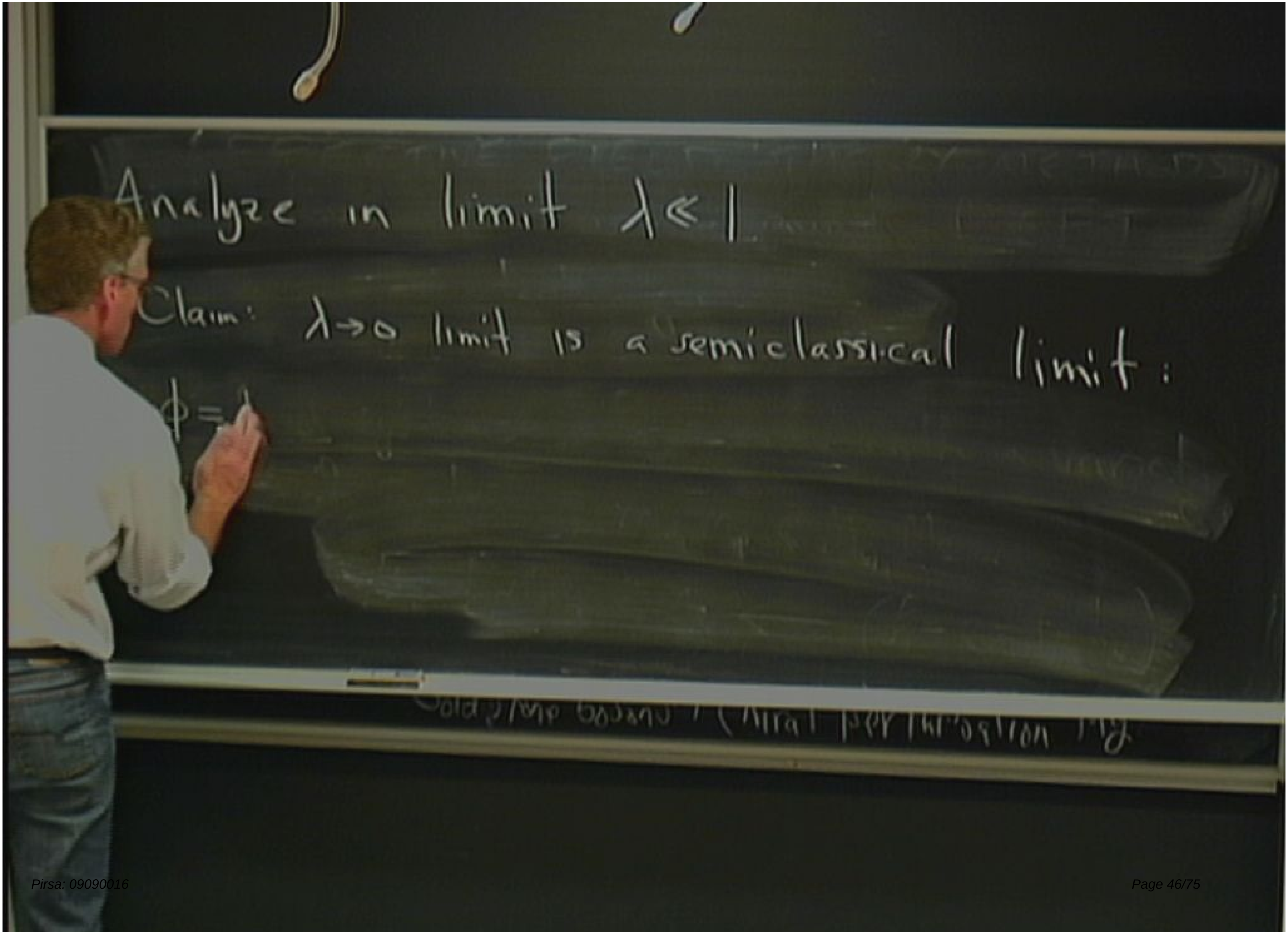

in limit λ ≪ 1
λ → 0


Analyze in limit $\lambda \ll 1$
Claim: $\lambda \to 0$ limit is a semiclassical limit:
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$\phi = \frac{1}{\lambda}\varphi \qquad v = \frac{\mu}{\lambda}$ hold φ, μ fixed:

$$\mathcal{L} = \frac{1}{\lambda^2}\left[-\partial_\mu \varphi \partial^\mu \varphi - \frac{1}{4}\left(\varphi^* \varphi - \mu^2\right)^2\right]$$

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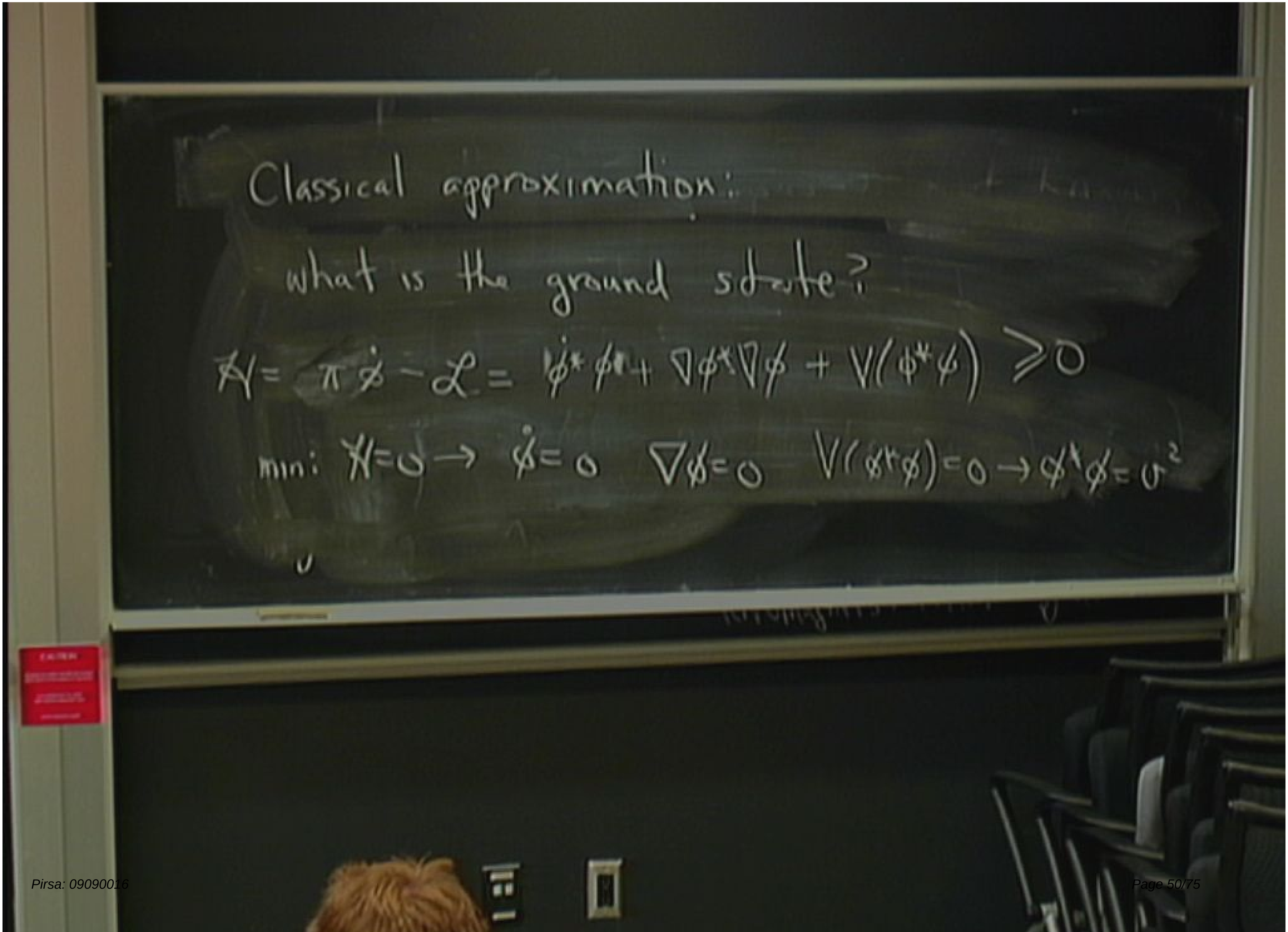
Classical approximation:

what is the ground state?

$$\mathcal{H} = \pi\dot{\phi} - \mathcal{L} = \dot{\phi}^*\dot{\phi} + \nabla\phi^*\nabla\phi + V(\phi^*\phi) \geq 0$$

$$\text{min: } \mathcal{H} = 0 \rightarrow \dot{\phi} = 0 \quad \nabla\phi = 0 \quad V(\phi^*\phi) = 0 \rightarrow \phi^*\phi = v^2$$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min: $\mathcal{H}=0 \to \dot{\phi}=0 \quad \nabla\phi=0 \quad V(\phi^*\phi)=0 \to \phi^*\phi = v^2$

Analyze in limit $\lambda \ll 1$

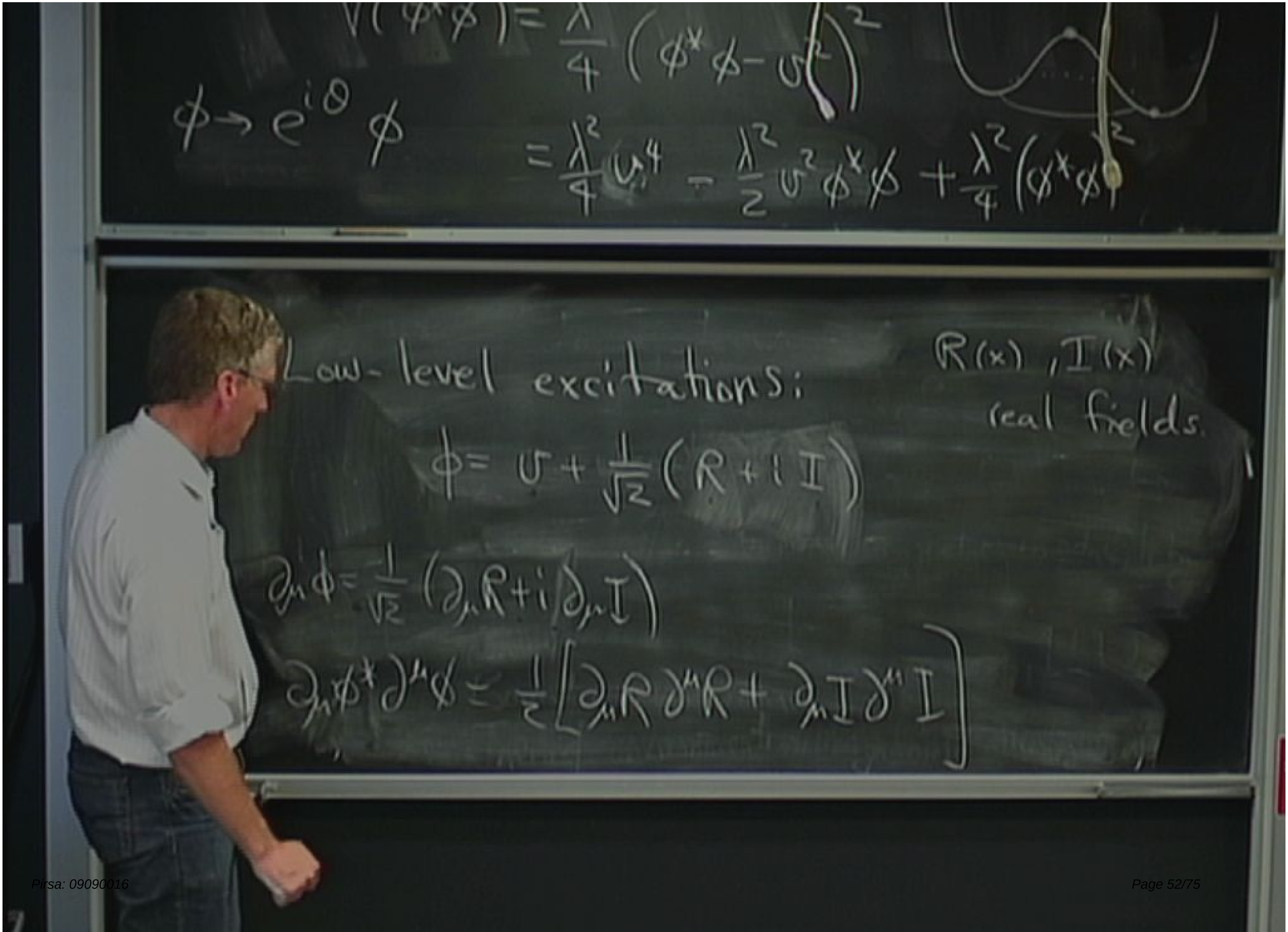
$$\eta_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\partial_\mu = \partial / \partial x^\mu \qquad x^\mu = \{t, \vec{x}\}$$

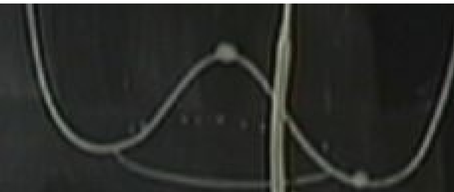
Claim: $\lambda \to 0$ limit is a semiclassical limit: $\overset{\uparrow}{0}$ $\overset{\uparrow}{1,2,3}$

$$\phi = \frac{1}{\lambda}\varphi \qquad v = \frac{\mu}{\lambda} \quad (\text{i.e. hold } \varphi, \mu \text{ fixed})$$

$$\mathcal{L} = \frac{1}{\lambda^2}\left[-\partial_\mu \varphi^* \partial^\mu \varphi - \frac{1}{4}\left(\varphi^*\varphi - \mu^2\right)^2\right] \qquad e^{iS/\hbar} = e^{i\int d^4x\, \mathcal{L}}$$


$V(\phi^*\phi) = \frac{\lambda}{4}(\phi^*\phi - v^2)^2$
$\phi \to e^{i\theta}\phi$
$= \frac{\lambda^2}{4}v^4 - \frac{\lambda^2}{2}v^2\phi^*\phi + \frac{\lambda^2}{4}(\phi^*\phi)^2$
Low-level excitations:
$R(x), I(x)$
real fields
$\phi = v + \frac{1}{\sqrt{2}}(R + iI)$
$\partial_\mu\phi = \frac{1}{\sqrt{2}}(\partial_\mu R + i\partial_\mu I)$
$\partial_\mu\phi^*\partial^\mu\phi = \frac{1}{2}\left[\partial_\mu R\partial^\mu R + \partial_\mu I\partial^\mu I\right]$

$$V(\phi^*\phi) = \frac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$



$$\phi \to e^{i\theta}\phi$$

$$= \frac{\lambda^2}{4}v^4 - \frac{\lambda^2}{2}v^2\phi^*\phi + \frac{\lambda^2}{4}(\phi^*\phi)^2$$

Low-level excitations: $R(x)$, $I(x)$ real fields

$$\phi = v + \frac{1}{\sqrt{2}}(R + iI)$$

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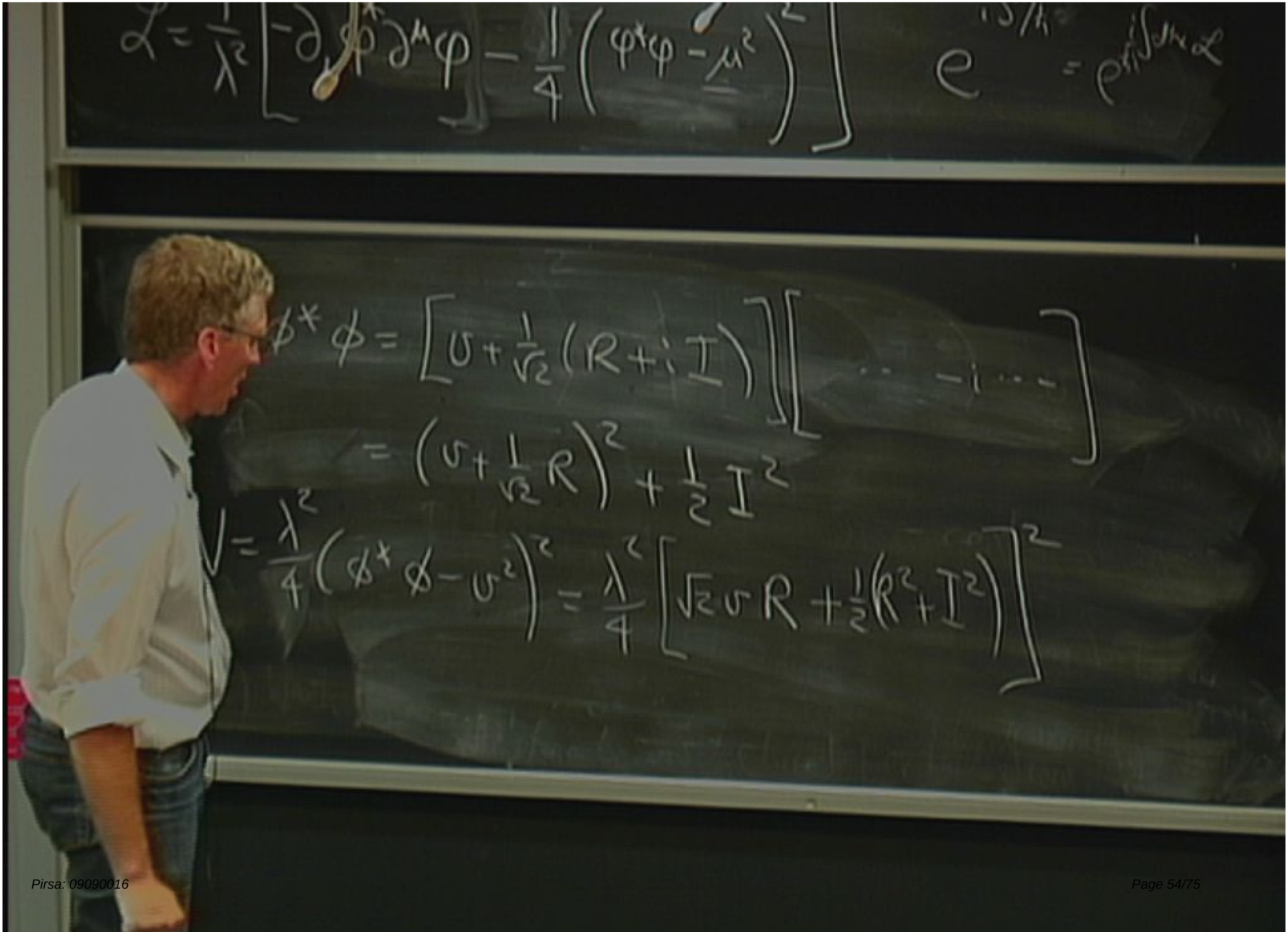
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$e^{iS/\hbar} = e^{i\int d^4x\, \mathcal{L}}$
$\phi^*\phi = \left[v + \frac{1}{\sqrt{2}}(R + iI)\right]\left[\ \cdots\ - i \cdots\ \right]$
$= \left(v + \frac{1}{\sqrt{2}} R\right)^2 + \frac{1}{2} I^2$
$V = \frac{\lambda^2}{4}\left(\phi^*\phi - v^2\right)^2 = \frac{\lambda^2}{4}\left[\sqrt{2}\, v R + \frac{1}{2}\left(R^2 + I^2\right)\right]^2$

$$\mathcal{L} = \frac{1}{\lambda^2}\left[-\partial_\mu\phi^*\partial^\mu\phi - \frac{1}{4}\left(\phi^*\phi - \mu^2\right)^2\right] \qquad e^{iS/\hbar} = e^{\frac{i}{\hbar}\int d^4x\,\mathcal{L}}$$

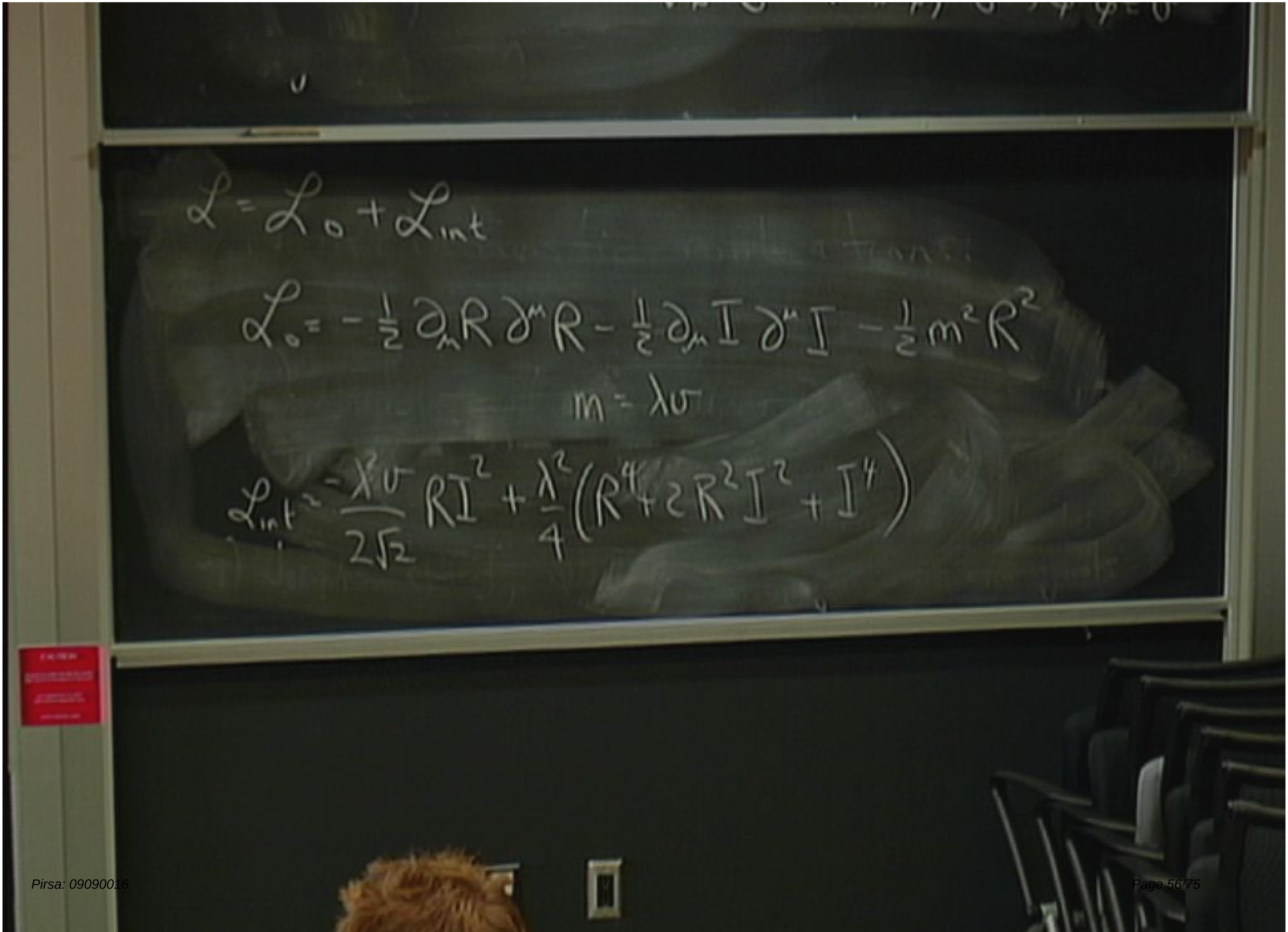
5) ... make

$$\phi^*\phi = \left[v + \frac{1}{\sqrt{2}}(R + iI)\right]\left[\cdots - i \cdots\right]$$

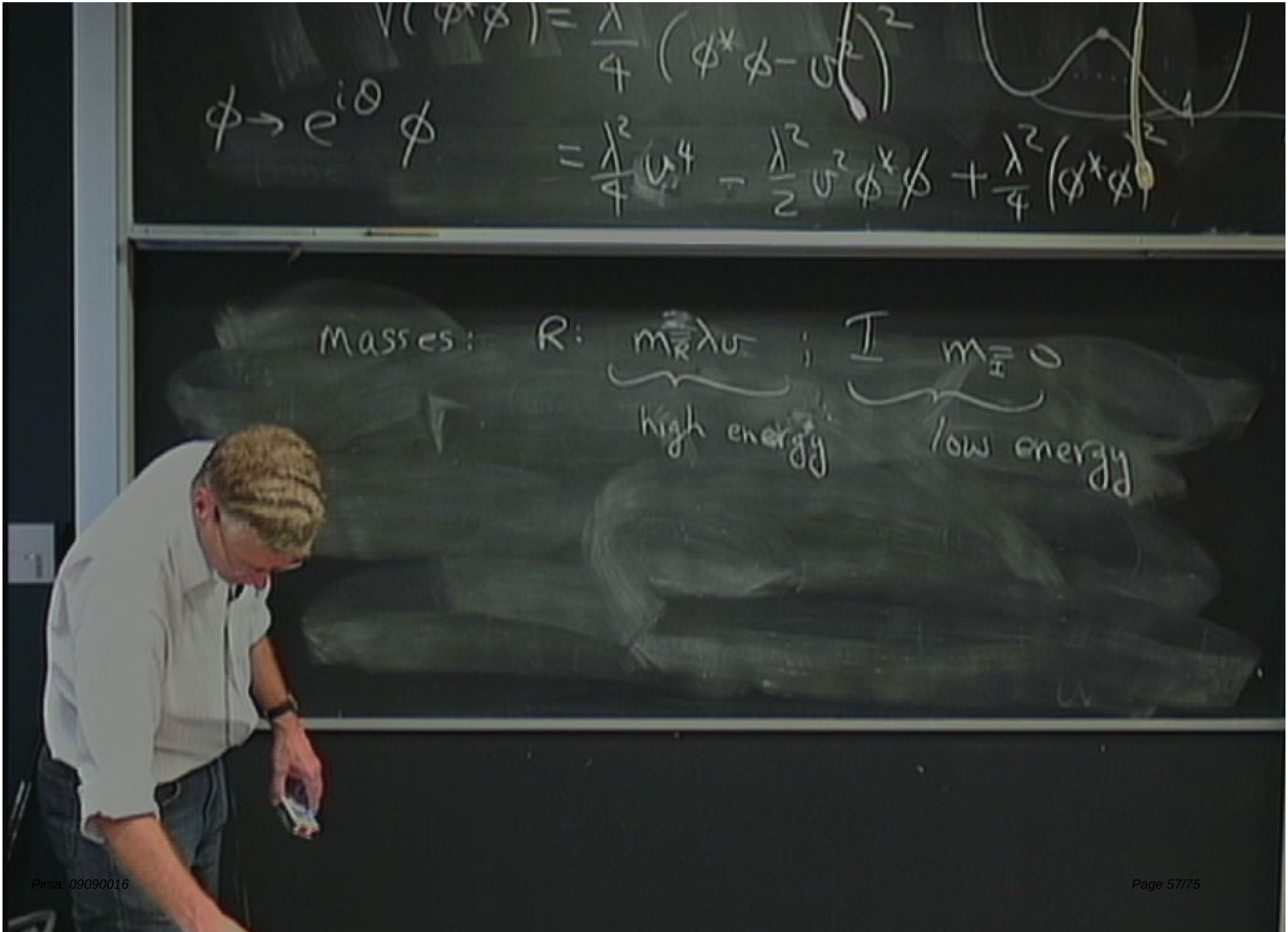
$$= \left(v + \frac{1}{\sqrt{2}}R\right)^2 + \frac{1}{2}I^2$$

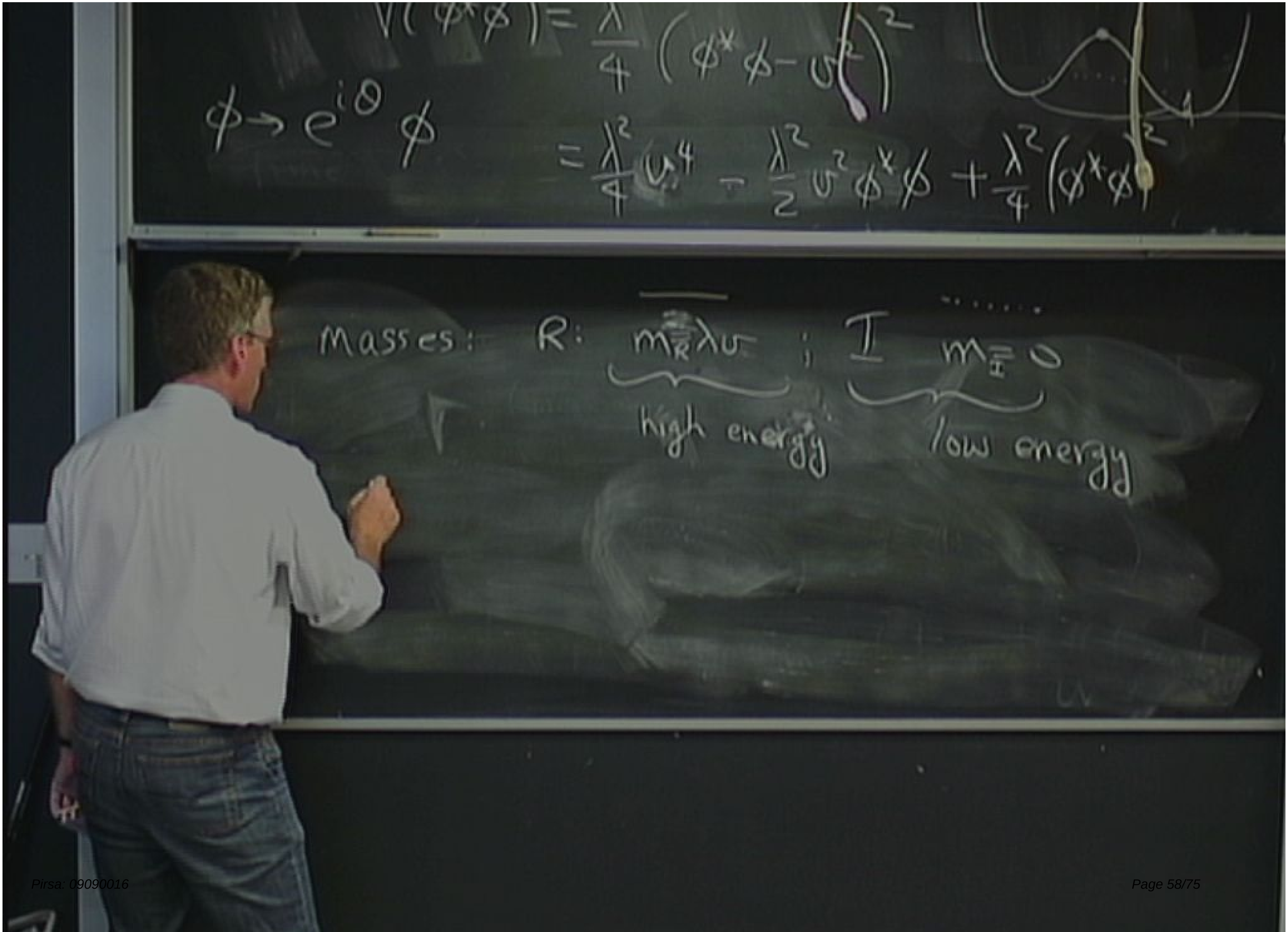
$$V = \frac{\lambda^2}{4}\left(\phi^*\phi - v^2\right)^2 = \frac{\lambda^2}{4}\left[\sqrt{2}vR + \frac{1}{2}\left(R^2 + I^2\right)\right]^2$$

$$= \frac{1}{2}\lambda^2 v^2 R^2 + \frac{\lambda^2 v}{2\sqrt{2}} R\left(R^2 + I^2\right) + \frac{\lambda^2}{4}\left(R^4 + I^4 + 2R^2I^2\right)$$



$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$
$\mathcal{L}_0 = -\frac{1}{2}\partial_\mu R\,\partial^\mu R - \frac{1}{2}\partial_\mu I\,\partial^\mu I - \frac{1}{2}m^2 R^2$
$m = \lambda v$
$\mathcal{L}_{int} = -\frac{\lambda^2 v}{2\sqrt{2}} R I^2 + \frac{\lambda^2}{4}\left(R^4 + 2R^2 I^2 + I^4\right)$


V(φ*φ) = λ/4 (φ*φ − v²)²
φ → e^{iθ} φ
= λ²/4 v⁴ − λ²/2 v² φ*φ + λ²/4 (φ*φ)²
Masses: R: m_R = λv ; I m_I = 0
high energy
low energy


$V(\phi^*\phi) = \frac{\lambda}{4}(\phi^*\phi - v^2)^2$
$\phi \to e^{i\theta}\phi$
$= \frac{\lambda^2}{4}v^4 - \frac{\lambda^2}{2}v^2\phi^*\phi + \frac{\lambda^2}{4}(\phi^*\phi)^2$
Masses: R: $\underbrace{m_R = \lambda v}_{\text{high energy}}$; I $\underbrace{m_I = 0}_{\text{low energy}}$

$$V(\phi^*\phi) = \frac{\lambda}{4}(\phi^*\phi - v^2)^2$$

$$\phi \to e^{i\theta}\phi$$

$$= \frac{\lambda^2}{4}v^4 - \frac{\lambda^2}{2}v^2\phi^*\phi + \frac{\lambda^2}{4}(\phi^*\phi)^2$$

Masses: R: $\underbrace{m_R \sim \lambda v}_{\text{high energy}}$; I $\underbrace{m_I = 0}_{\text{low energy}}$

R^3  RI^2  I^4 

R^4  R^2I^2 

$$V(\phi^*\phi) = \frac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$

$$\phi \to e^{i\theta}\phi$$

$$= \frac{\lambda^2}{4} v^4 - \frac{\lambda^2}{2} v^2 \phi^*\phi + \frac{\lambda^2}{4}\left(\phi^*\phi\right)^2$$

Masses: R: $\underbrace{m_R \approx \lambda v}_{\text{high energy}}$; I $\underbrace{m_I = 0}_{\text{low energy}}$

R^3 $\quad$ RI^2 $\quad$ I^4

R^4 $\quad$ R^2I^2

$$V(\phi^*\phi) = \frac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$ 

$$\phi \to e^{i\theta}\phi$$

$$= \frac{\lambda^2}{4}v^4 - \frac{\lambda^2}{2}v^2\phi^*\phi + \frac{\lambda^2}{4}\left(\phi^*\phi\right)^2$$

Masses: R: $\underbrace{m_R = \lambda v}_{\text{high energy}}$; I $\underbrace{m_I = 0}_{\text{low energy}}$

R^3  RI^2  I^4 

R^4  R^2I^2 

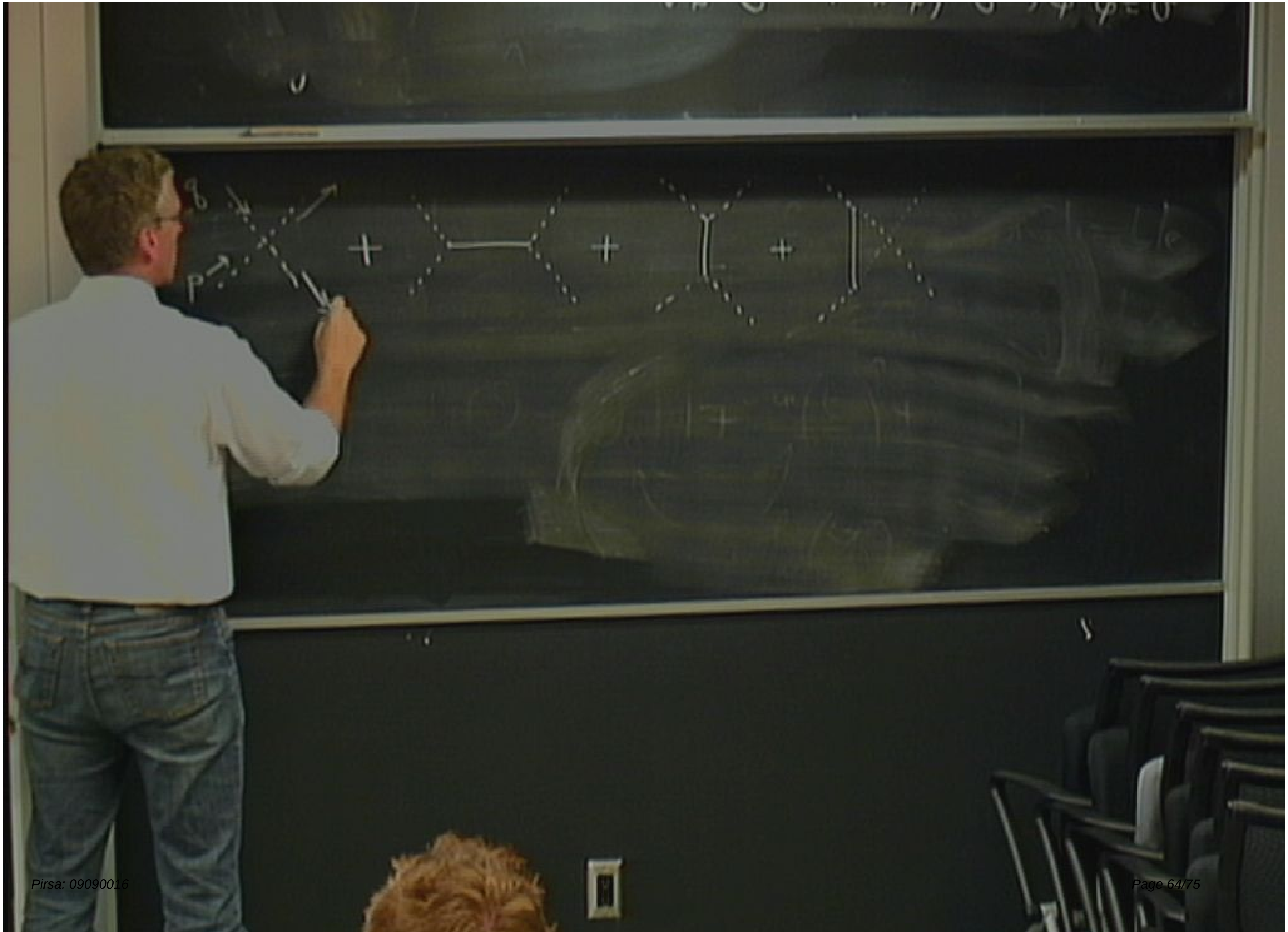
$$\mathcal{L} = \frac{1}{\lambda^2}\left[\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{4}\left(\phi\phi - \mu^2\right)\right] \qquad e$$

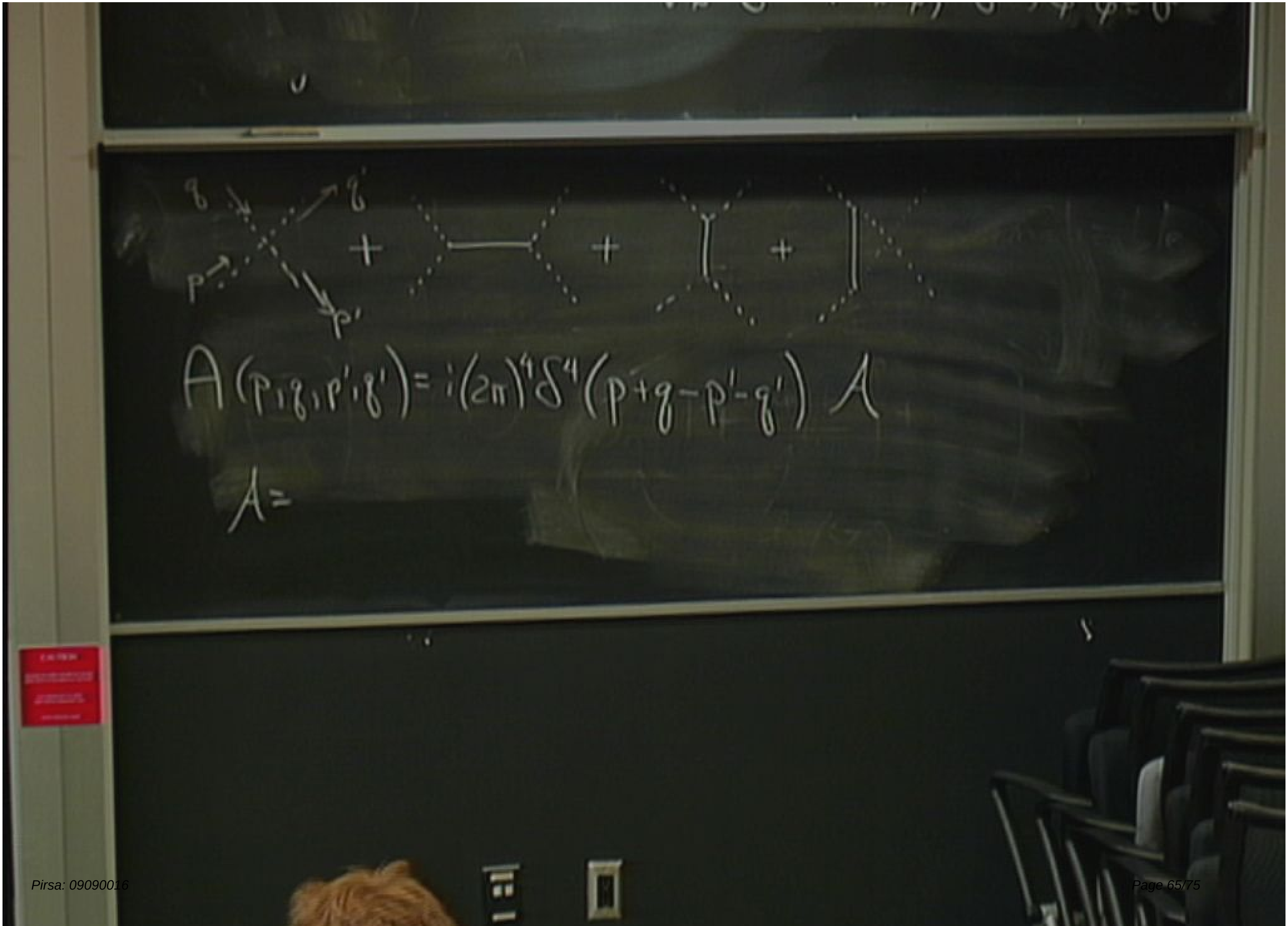
Calculate I scattering, in the limit where $E_{cm} \ll m_R$.

$$\mathcal{L} = \frac{1}{\lambda^2}\left[\partial_\mu \phi\, \partial^\mu \phi - \frac{1}{4}\left(\phi\phi - \mu^2\right)\right] \qquad e \ldots = e^{\ldots}$$

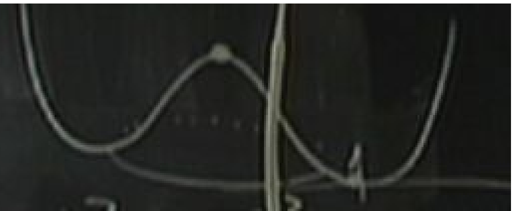
Calculate I scattering, in the limit where $E_{cm} \ll m_R$.

in loop expansion (semiclassical approximation)
(ie powers of λ)




q
q'
p
p'
+
+
+
$A(p,q,p',q') = i(2\pi)^4 \delta^4(p+q-p'-q')\ \mathcal{A}$
$\mathcal{A} =$

$$V(\phi^*\phi) = \frac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$



$$\phi \to e^{i\theta}\phi$$

$$= \frac{\lambda^2}{4}v^4 - \frac{\lambda^2}{2}v^2\phi^*\phi + \frac{\lambda^2}{4}\left(\phi^*\phi\right)^2$$

Masses: R: $\underbrace{m_R = \lambda v}_{\text{high energy}}$; I $\underbrace{m_I = 0}_{\text{low energy}}$

RI^2 

I^4 

$$V(\phi^*\phi) = \frac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$

$$\phi \to e^{i\theta}\phi$$

$$= \frac{\lambda^2}{4} v^4 - \frac{\lambda^2}{2} v^2 \phi^*\phi + \frac{\lambda^2}{4}(\phi^*\phi)^2$$

Masses: R: $\underbrace{m_R = \lambda v}_{\text{high energy}}$; I $\underbrace{m_I = 0}_{\text{low energy}}$

$$\frac{g}{2!} R I^2$$

$$g = -\frac{\lambda^2 v}{\sqrt{2}}$$

$$\frac{g'}{4!} I^4$$

$$V(\phi^*\phi) = \frac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$

$$\phi \to e^{i\theta}\phi$$

$$= \frac{\lambda^2}{4} v^4 - \frac{\lambda^2}{2} v^2 \phi^*\phi + \frac{\lambda^2}{4}\left(\phi^*\phi\right)^2$$

Masses: R: $\underbrace{m_R = \lambda v}_{\text{high energy}}$; I $\underbrace{m_I = 0}_{\text{low energy}}$

$\frac{g}{2!} R I^2$

$$g = -\frac{\lambda^2 v}{\sqrt{2}}$$

$\frac{g'}{4!} I^4$

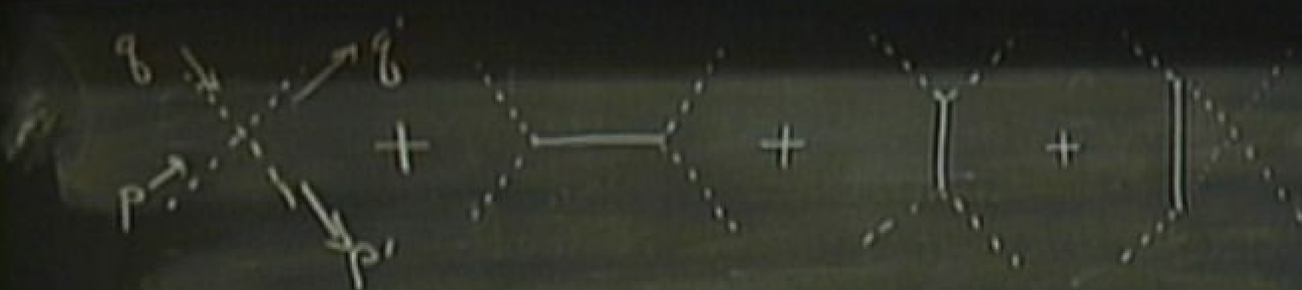
$$g' = 24\frac{\lambda^2}{16} = \frac{3\lambda^2}{2}$$

$$\mathcal{H} = \pi\dot{\phi} - \mathcal{L} = \dot{\phi}^*\dot{\phi} + \nabla\phi^*\nabla\phi + V(\phi^*\phi) \geq 0$$

$$\text{min: } \mathcal{H} = 0 \rightarrow \dot{\phi} = 0 \quad \nabla\phi = 0 \quad V(\phi^*\phi) = 0 \rightarrow \phi^*\phi = v^2$$

v

$4 \times 3 \times 2$



$$A(p, q, p', q') = i(2\pi)^4\delta^4(p+q-p'-q')\,\mathcal{A}$$

$$\mathcal{A} =$$

$$V(\phi^*\phi) = \frac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$

$$\phi \to e^{i\theta}\phi$$

$$= \frac{\lambda^2}{4}v^4 - \frac{\lambda^2}{2}v^2\phi^*\phi + \frac{\lambda^2}{4}(\phi^*\phi)^2$$

Masses: R: $\underbrace{m_R = \lambda v}_{\text{high energy}}$; I $\underbrace{m_I = 0}_{\text{low energy}}$

$\mathcal{L}_I$:

$-\frac{g}{2!} R I^2$

$\frac{g'}{4!} I^4$

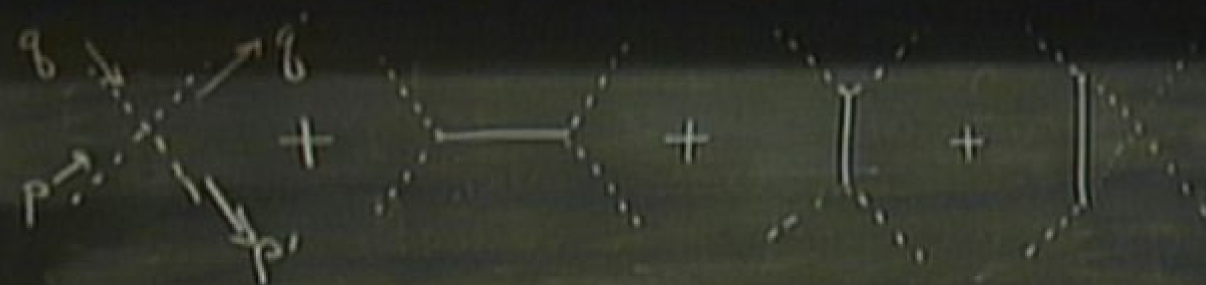
$g = \frac{\lambda^2 v}{\sqrt{2}}$

$g' = 24\frac{\lambda^2}{16} = \frac{3\lambda^2}{2}$

$$\mathcal{H} = \pi\dot{\phi} - \mathcal{L} = \dot{\phi}^*\dot{\phi} + \nabla\phi^*\nabla\phi + V(\phi^*\phi) \geq 0$$

min: $\mathcal{H} = 0 \to \dot{\phi} = 0 \quad \nabla\phi = 0 \quad V(\phi^*\phi) = 0 \to \phi^*\phi = v^2$

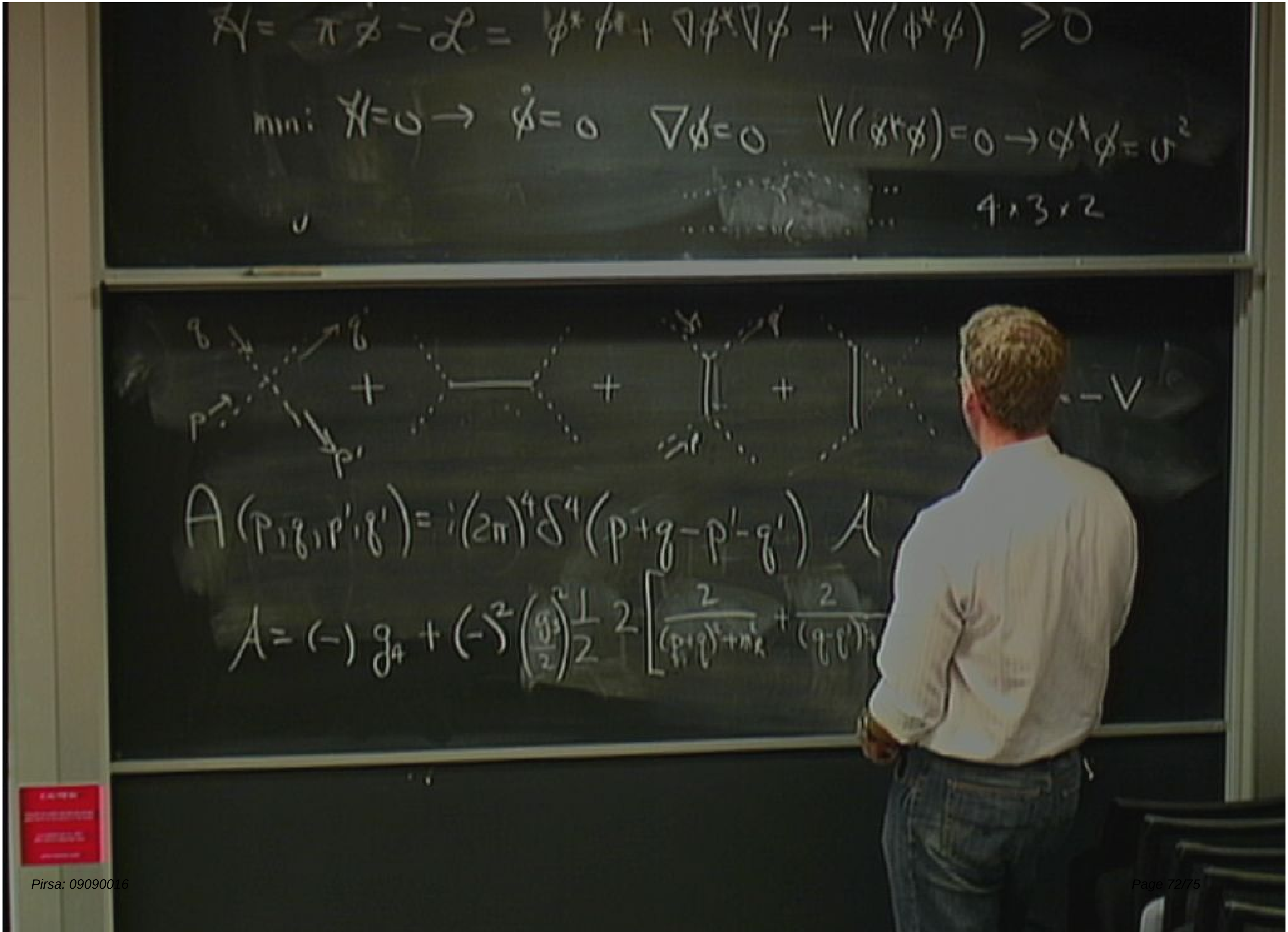
$4 \times 3 \times 2$

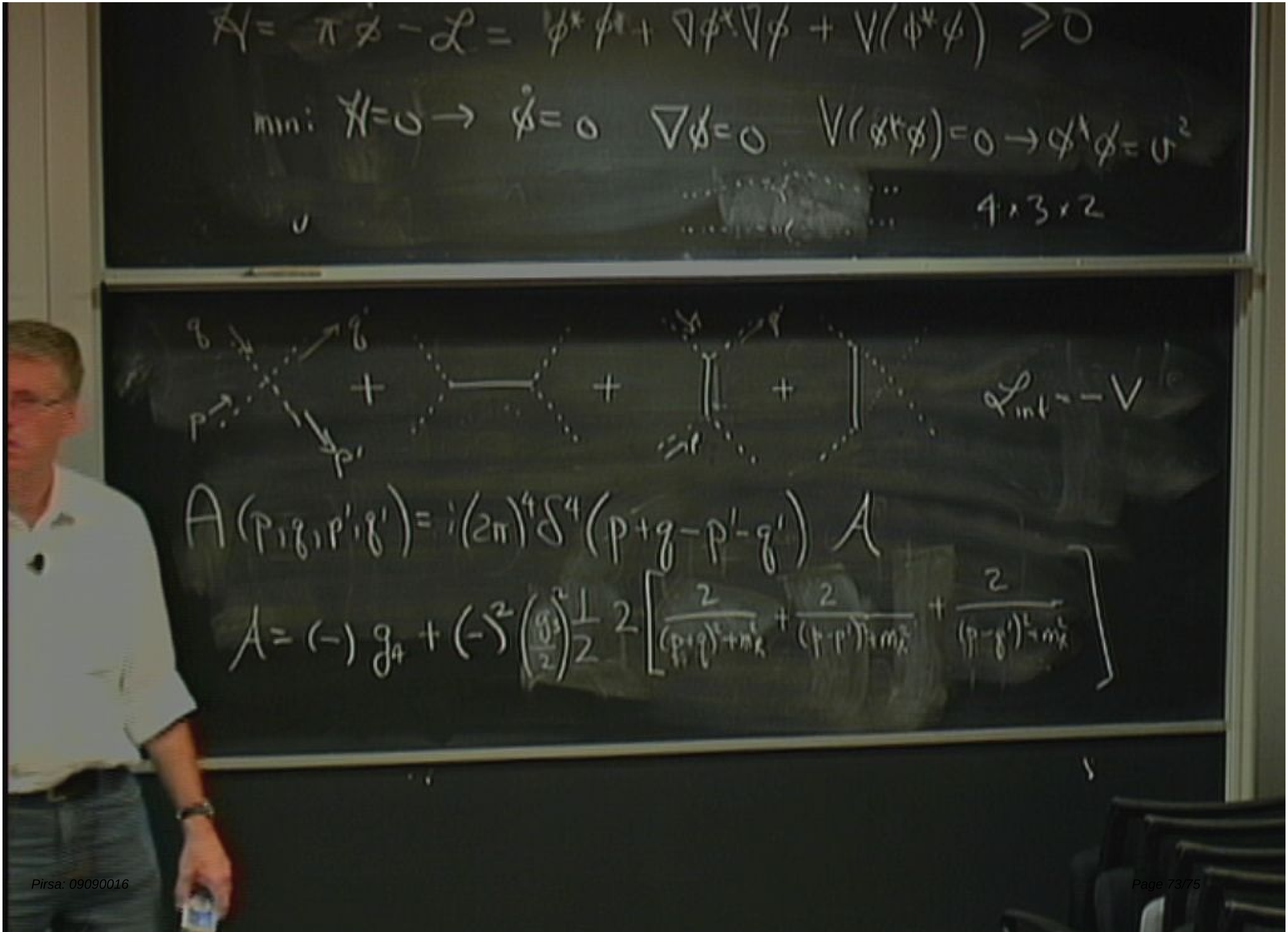


$$\mathcal{L}_{int} = -V$$

$$A(p,q,p',q') = i(2\pi)^4\delta^4(p+q-p'-q')\,\mathcal{A}$$

$$\mathcal{A} = (-)\,g_4 + (-)^2\,g_3^2\,[$$

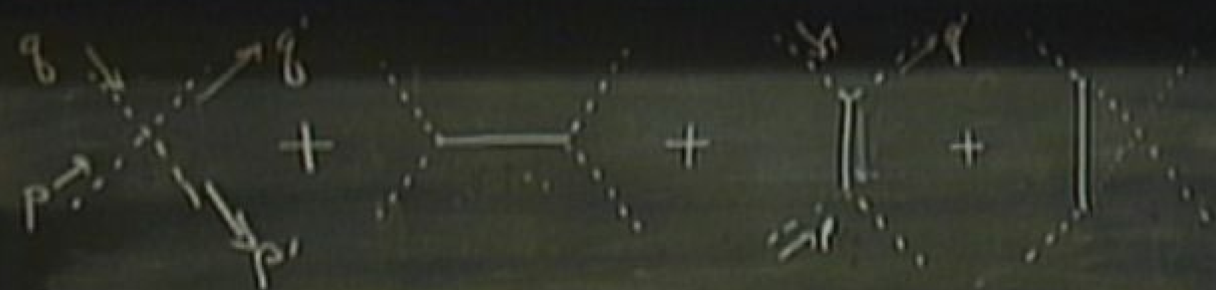

$\mathcal{H} = \pi\dot{\phi} - \mathcal{L} = \dot{\phi}^*\dot{\phi} + \nabla\phi^*\nabla\phi + V(\phi^*\phi) \geq 0$
min: $\mathcal{H}=0 \to \dot{\phi}=0 \quad \nabla\phi=0 \quad V(\phi^*\phi)=0 \to \phi^*\phi=v^2$
$4\times3\times2$
$A(p,q,p',q') = i(2\pi)^4\delta^4(p+q-p'-q')\,\mathcal{A}$


H = π φ̇ − L = φ̇* φ̇ + ∇φ*∇φ + V(φ*φ) ≥ 0
min: H = 0 → φ̇ = 0 ∇φ = 0 V(φ*φ) = 0 → φ*φ = v²
4 × 3 × 2
q p q' p'
L_int = −V
A(p, q, p', q') = i(2π)⁴ δ⁴(p + q − p' − q') A
A = (−) g₄ + (−)² (g₃/2)² 1/2 2 [2/((p+q)² + m_χ²) + 2/((p−p')² + m_χ²) + 2/((p−q')² + m_χ²)]

$$\mathcal{H} = \pi\dot{\phi} - \mathcal{L} = \dot{\phi}^*\dot{\phi} + \nabla\phi^*\nabla\phi + V(\phi^*\phi) \geq 0$$

$$\text{min: } \mathcal{H} = 0 \to \dot{\phi} = 0 \quad \nabla\phi = 0 \quad V(\phi^*\phi) = 0 \to \phi^*\phi = v^2$$

$4 \times 3 \times 2$



$$\mathcal{L}_{int} = -V$$

$$A(p,q,p',q') = -i(2\pi)^4\delta^4(p+q-p'-q')\,\mathcal{A}$$

$$\mathcal{A} = g_4 - g_3^2\left[\frac{1}{(p+q)^2+m_\chi^2} + \frac{1}{(p-p')^2+m_\chi^2} + \frac{1}{(p-q')^2+m_\chi^2}\right]$$

$$\lambda^2\left[\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{4}\left(\phi\phi - \mu^2\right)\right] \quad e$$

Calculate I scattering, in the limit where $E_{cm} \ll m_R$.

$$\sigma(\mathrm{I\,I}\to\mathrm{I\,I}) \simeq c\,\frac{E^8}{m_R^{10}} + \frac{\lambda E^2}{m_R^4} \quad (\ldots\text{al approximation}) \quad (\ldots\text{wers of } \lambda)$$