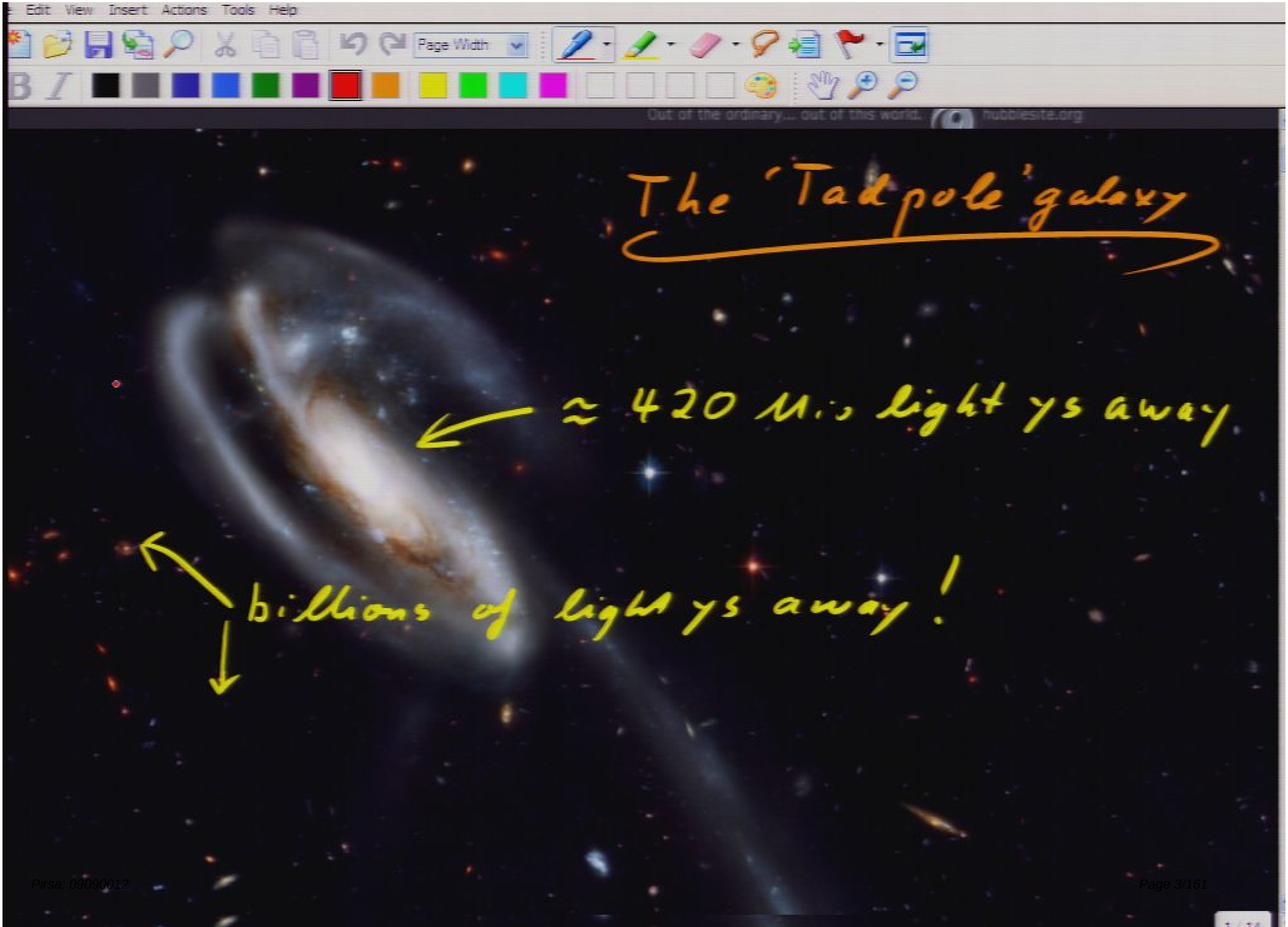
Title: General Relativity for Cosmology - Lecture 1

Date: Sep 21, 2009 04:00 PM

URL: http://pirsa.org/09090012

Abstract:

www.math.uwaterloo.ca/~akempf/AMATH875-F09.shtml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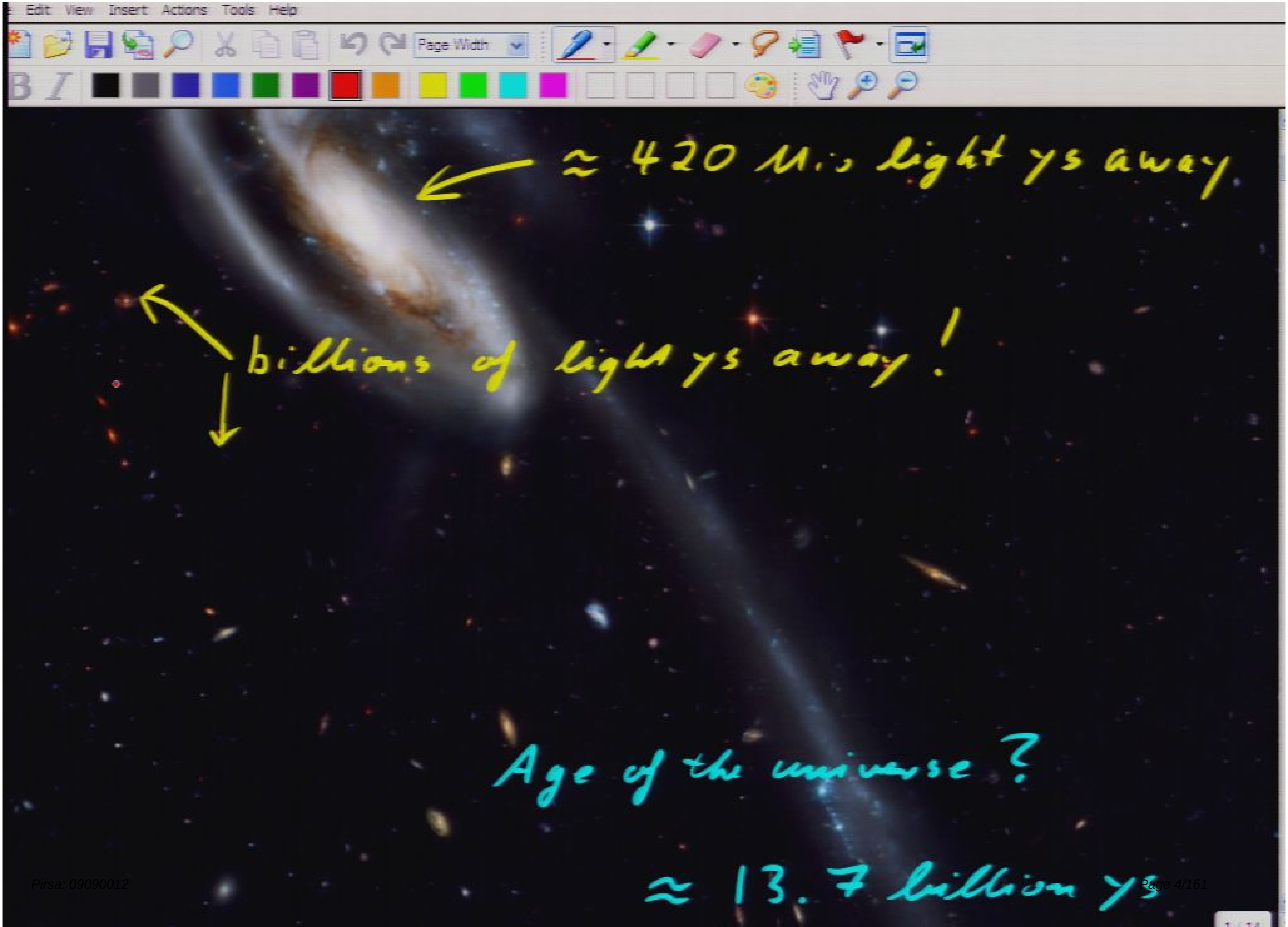
Out of the ordinary... out of this world.
hubblesite.org
The 'Tadpole' galaxy
≈ 420 Mio light ys away
billions of light ys away!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Age of the universe?
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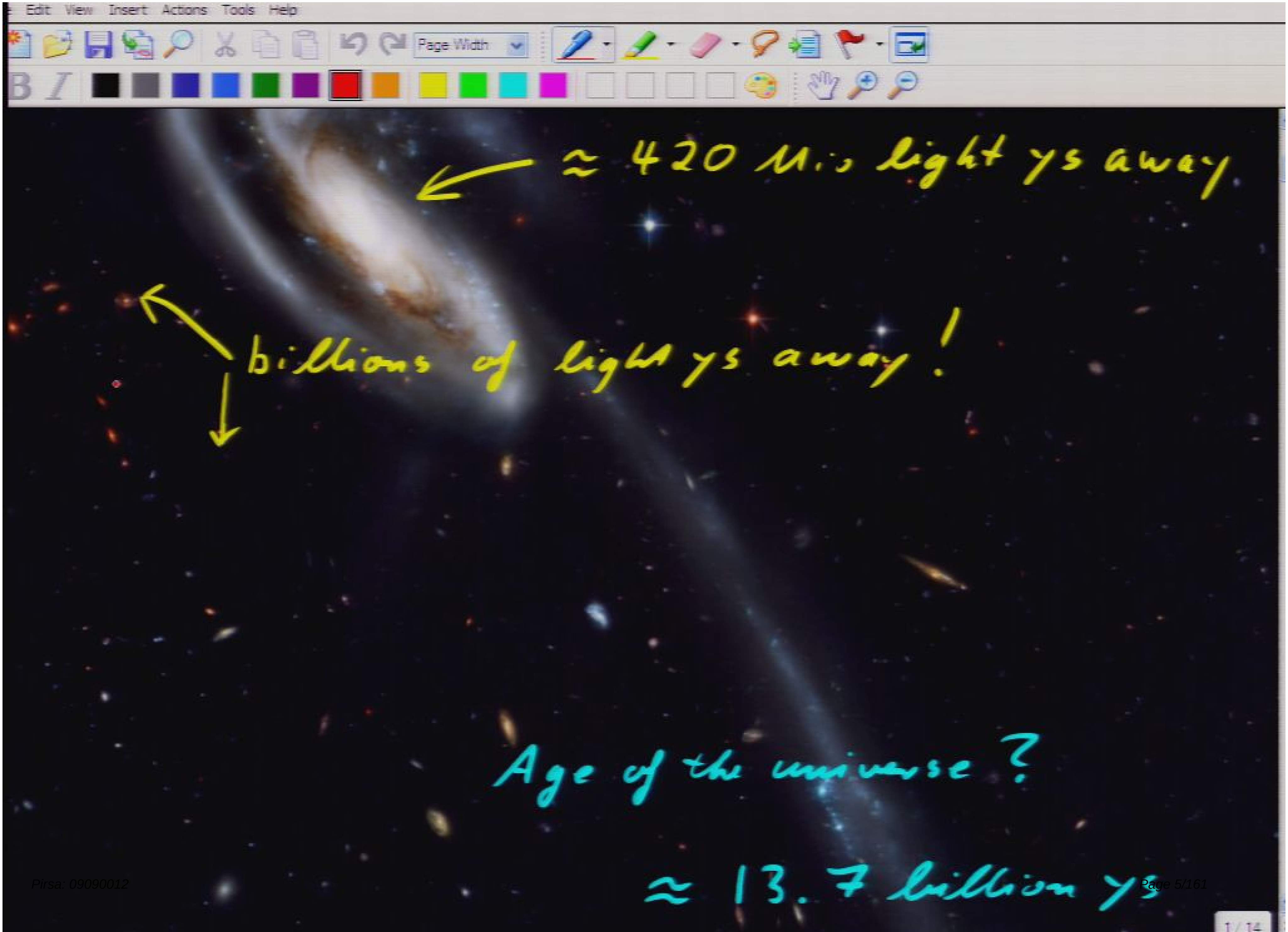
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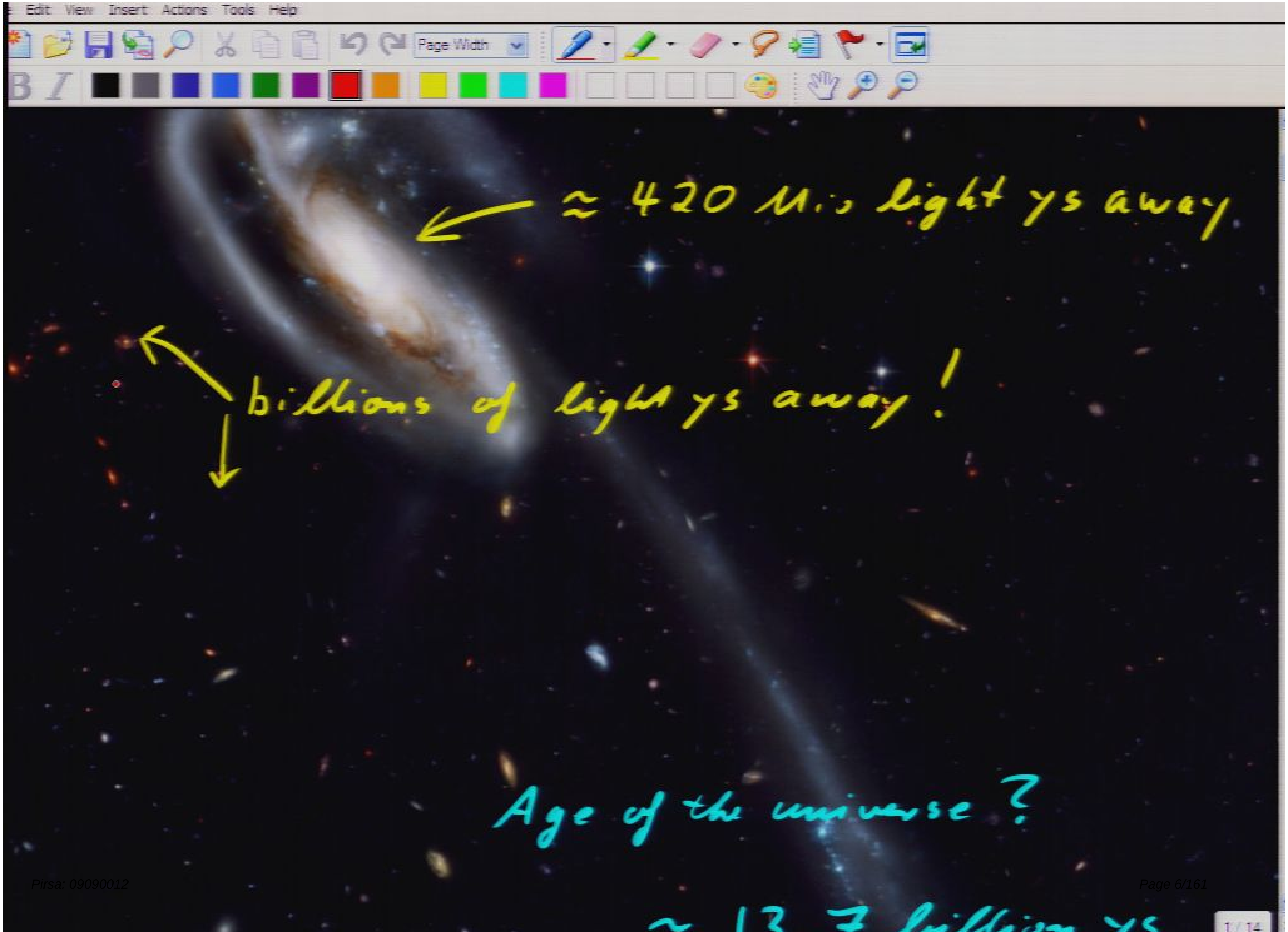
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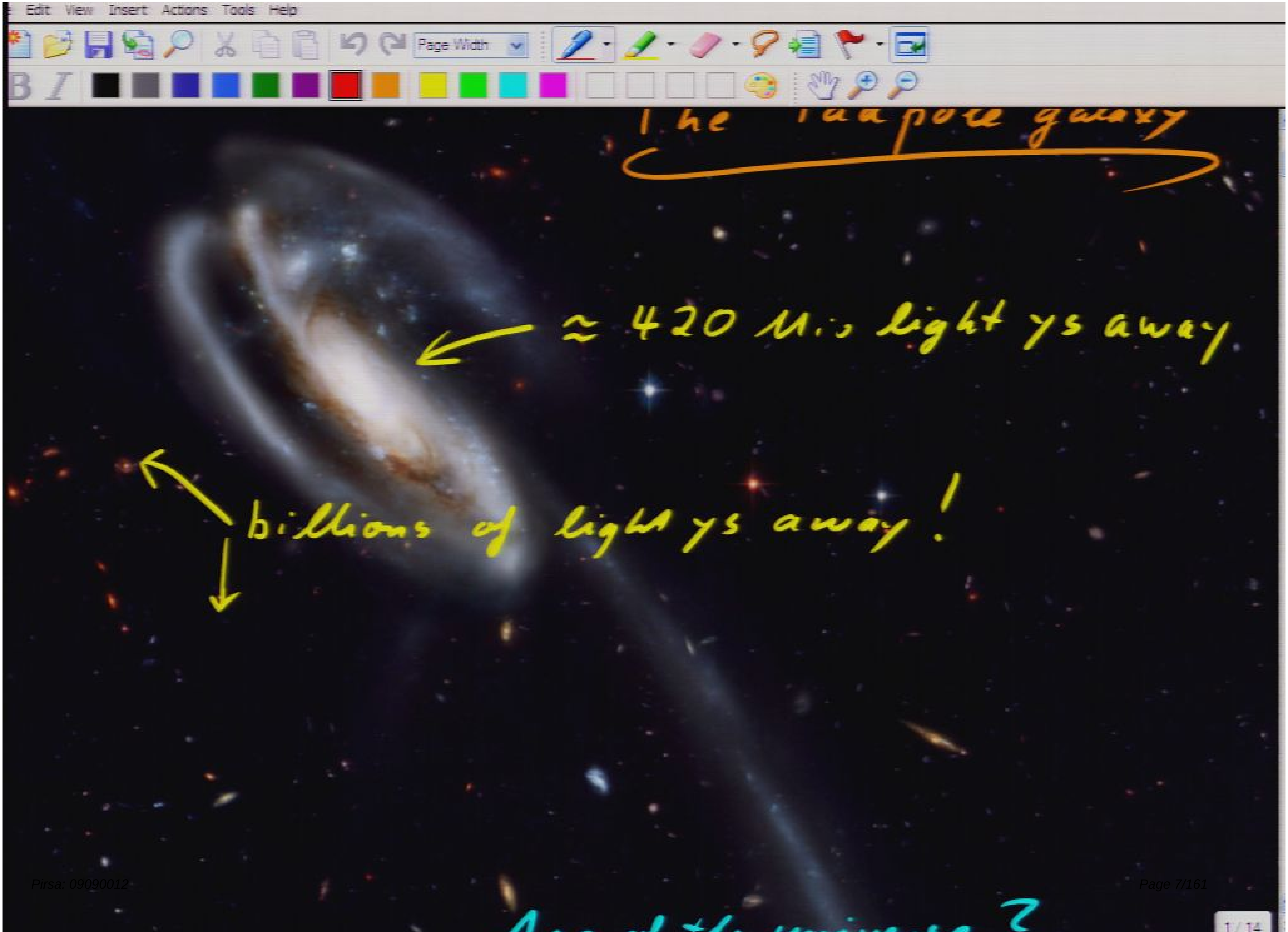
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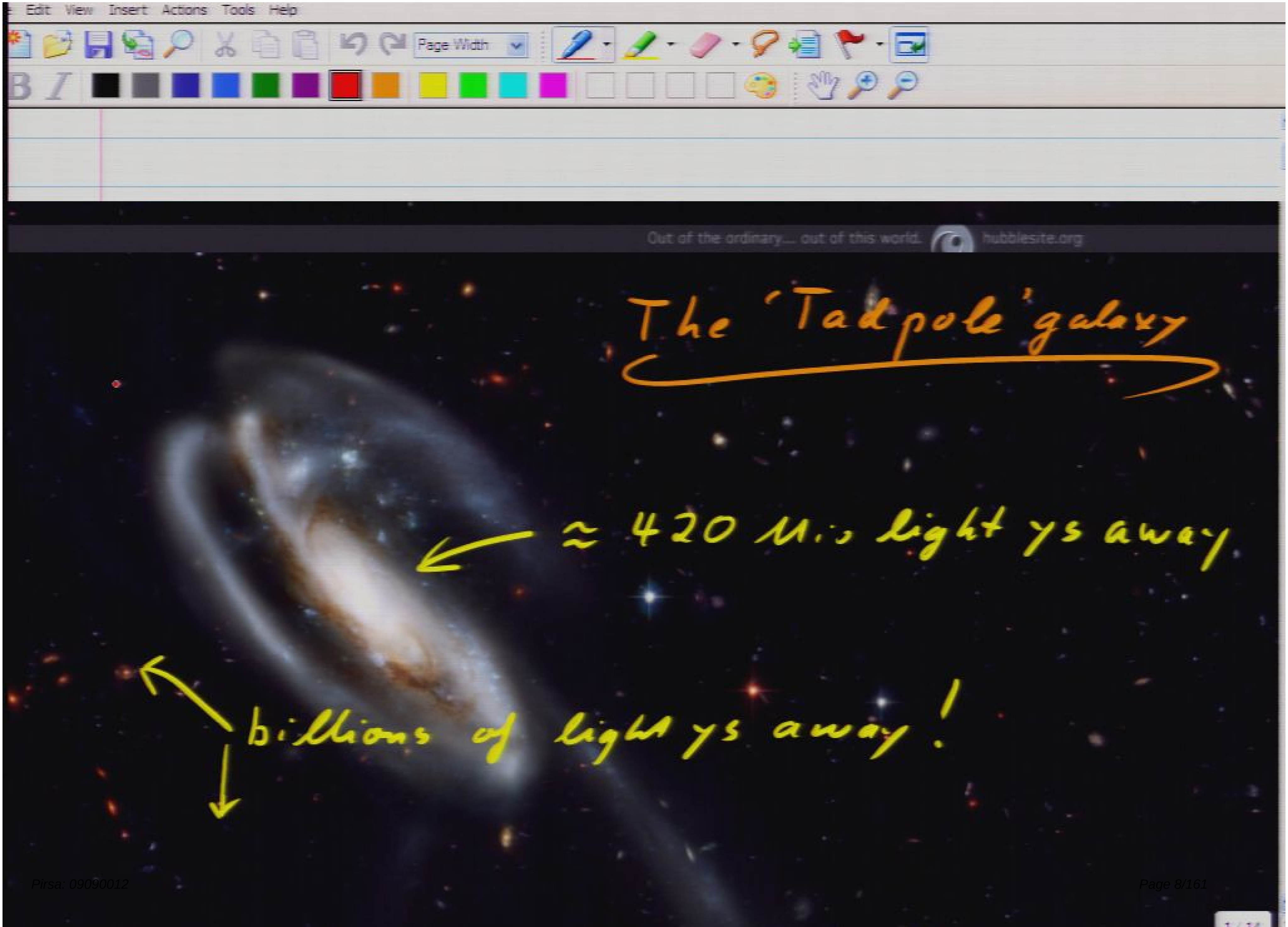
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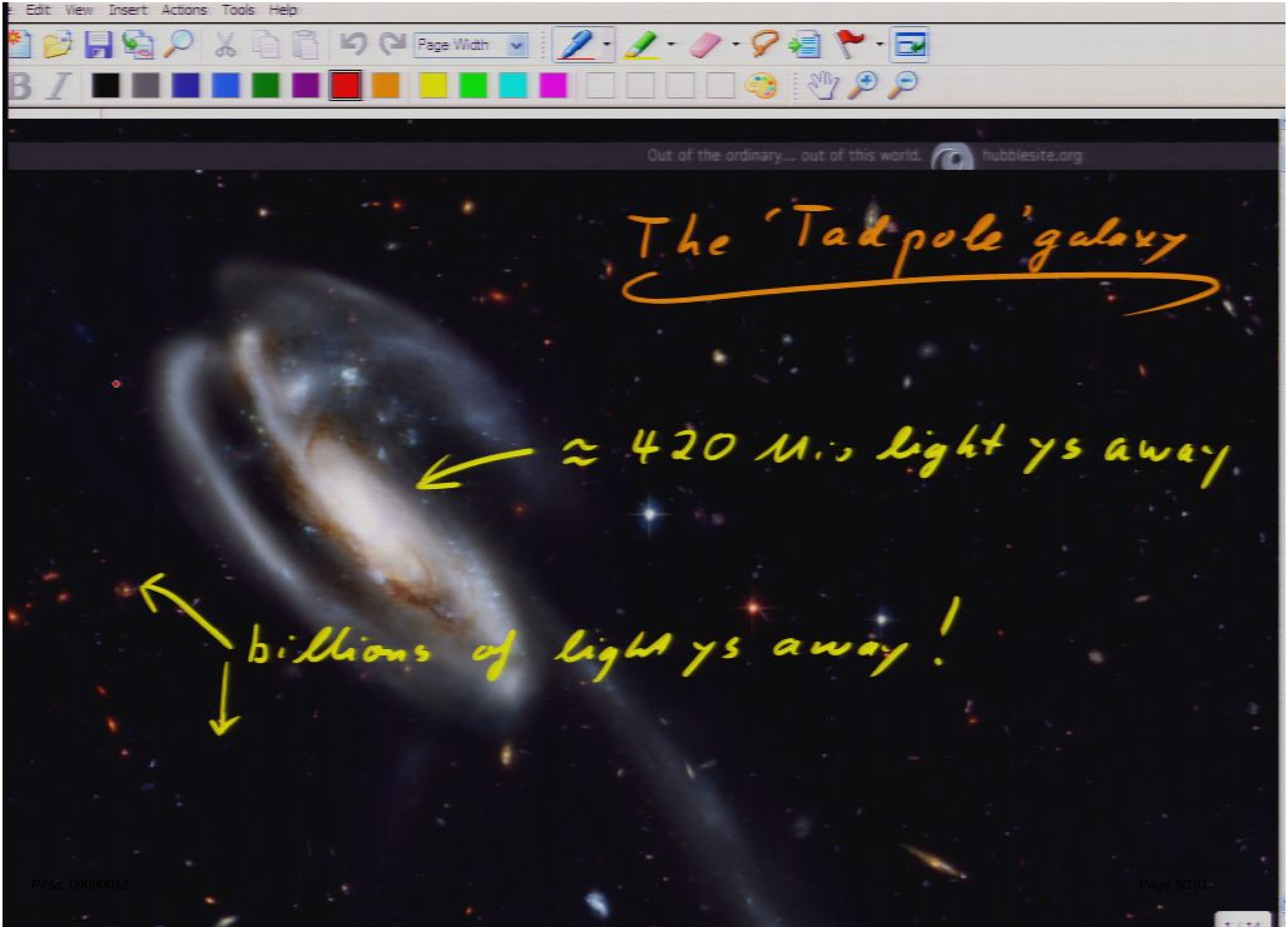
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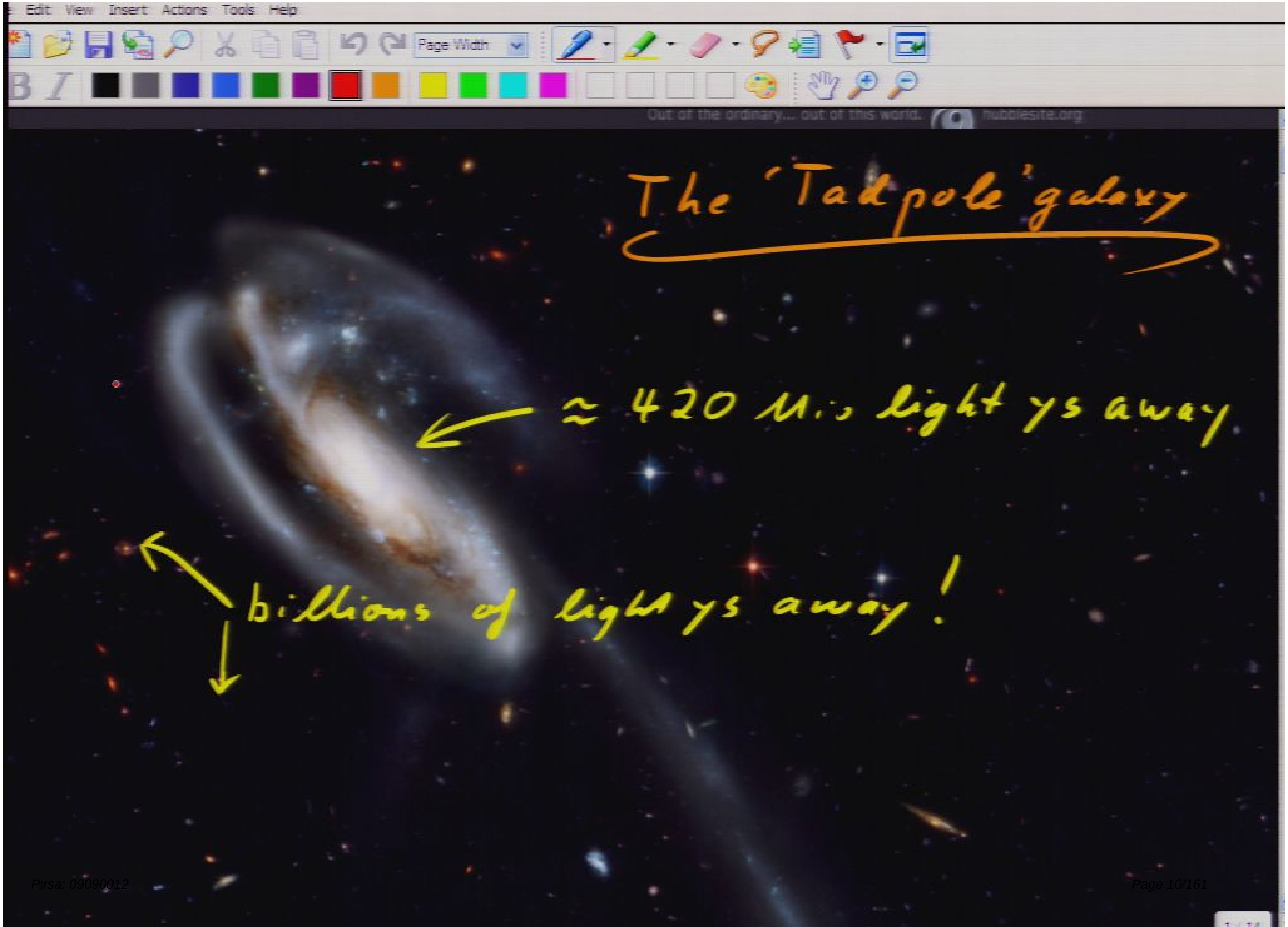
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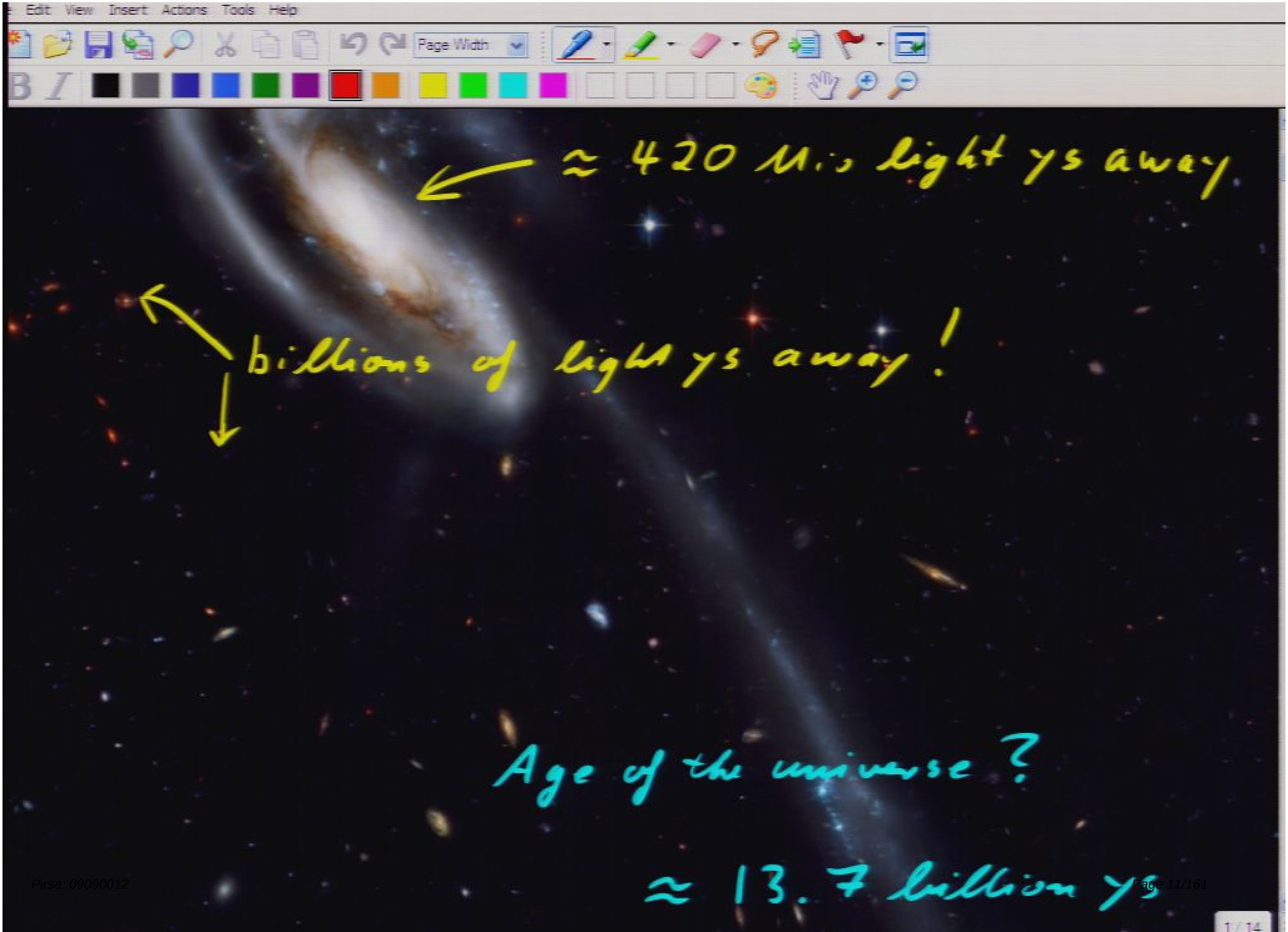
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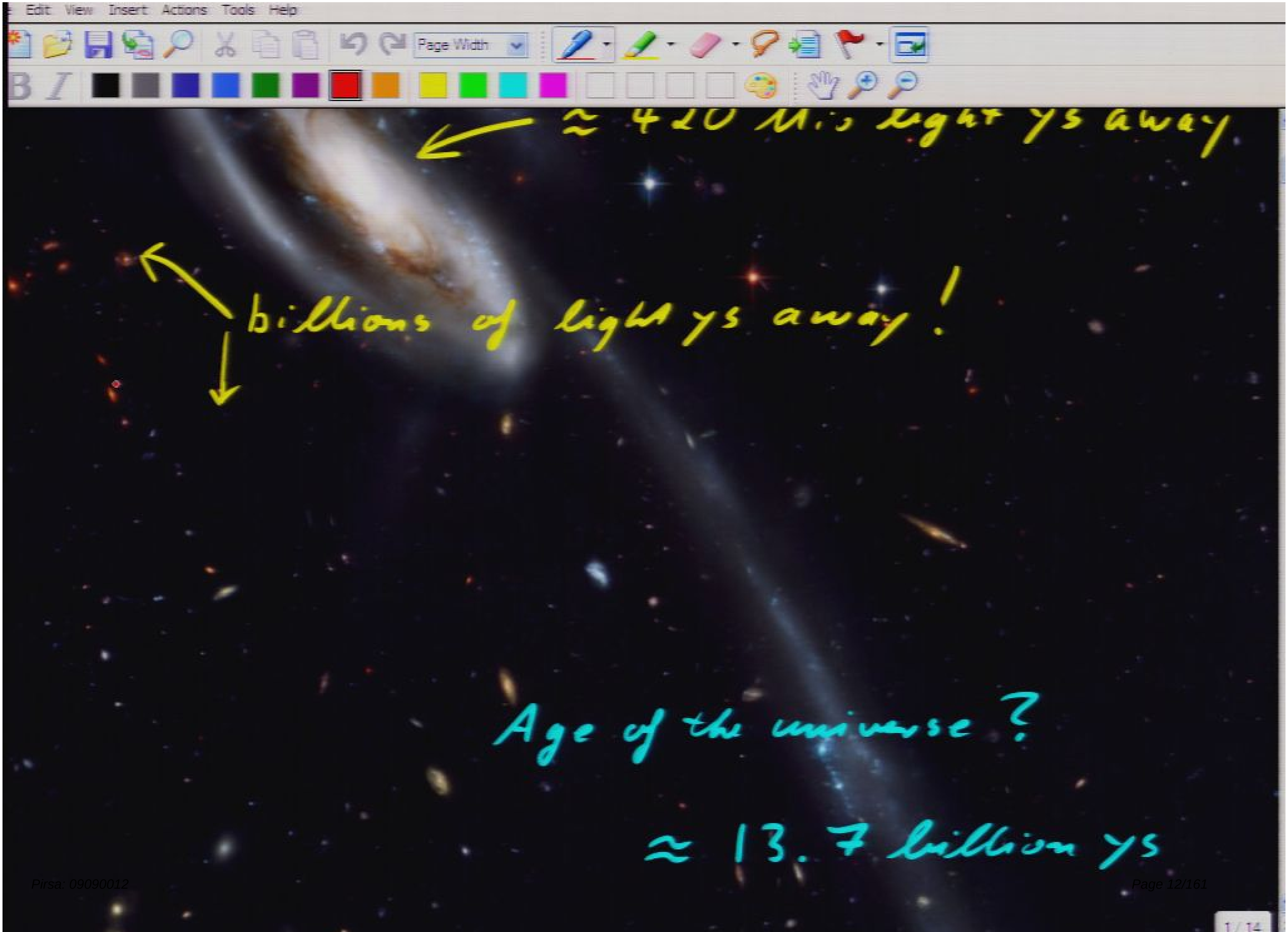

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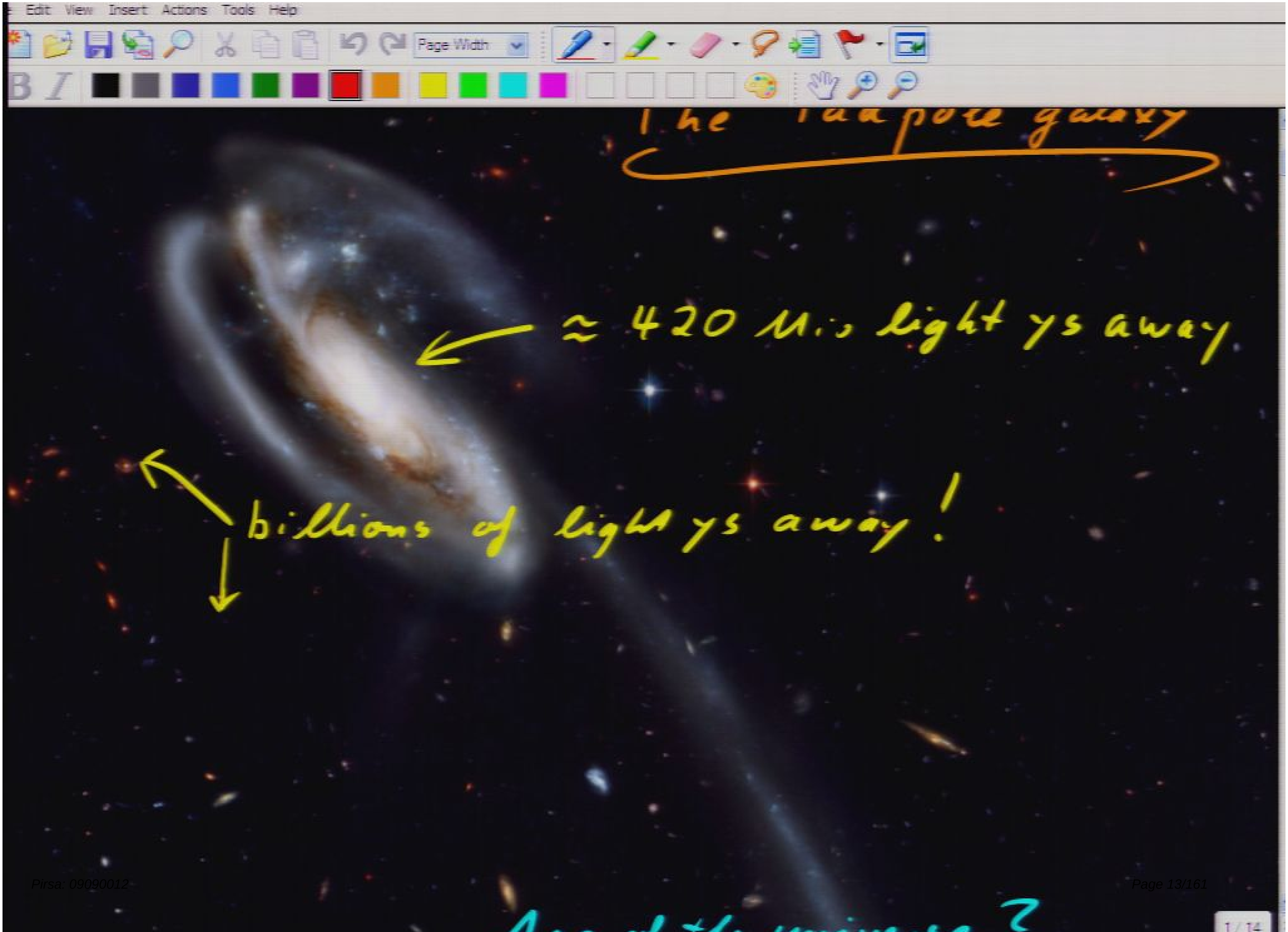
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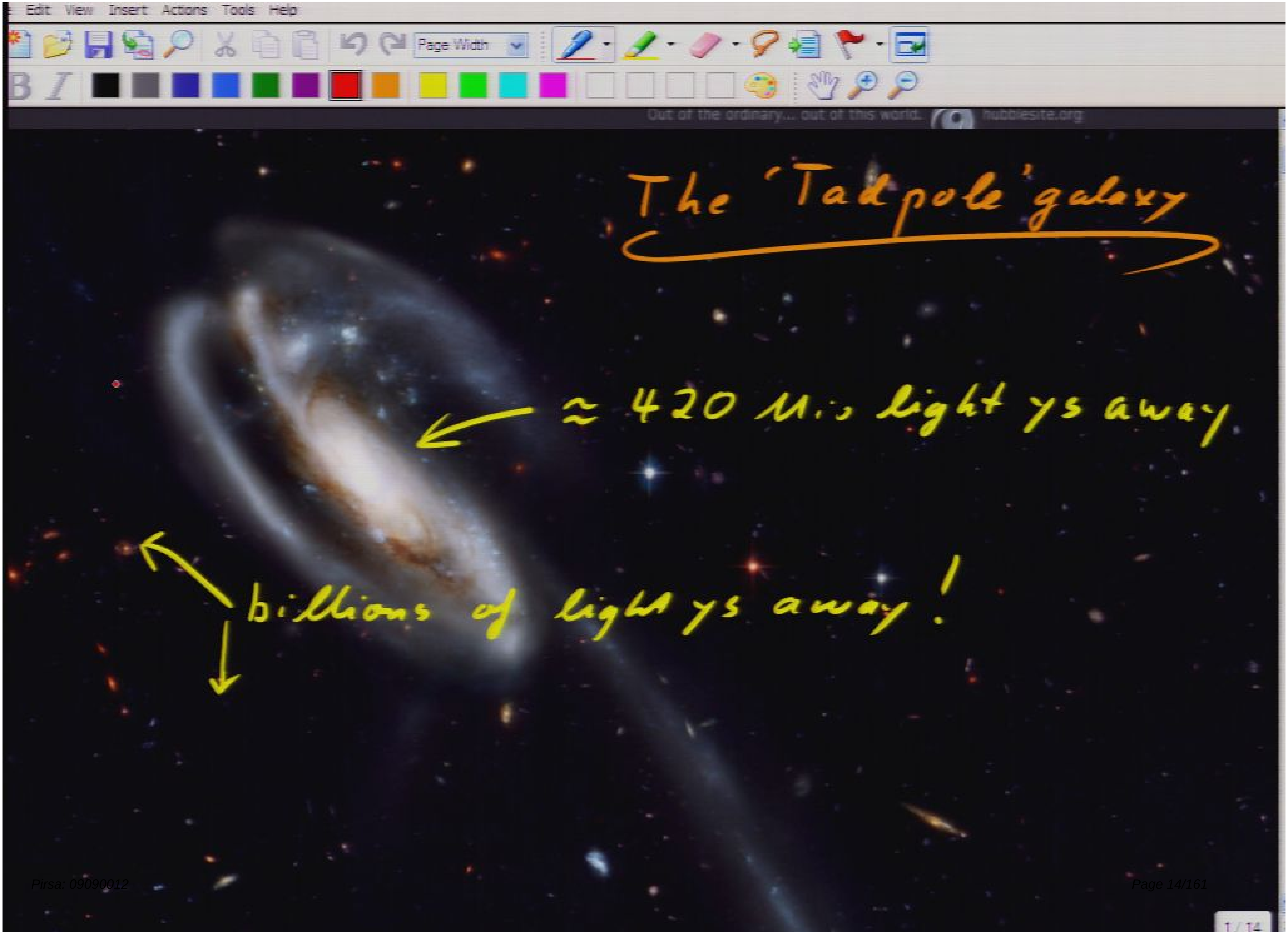
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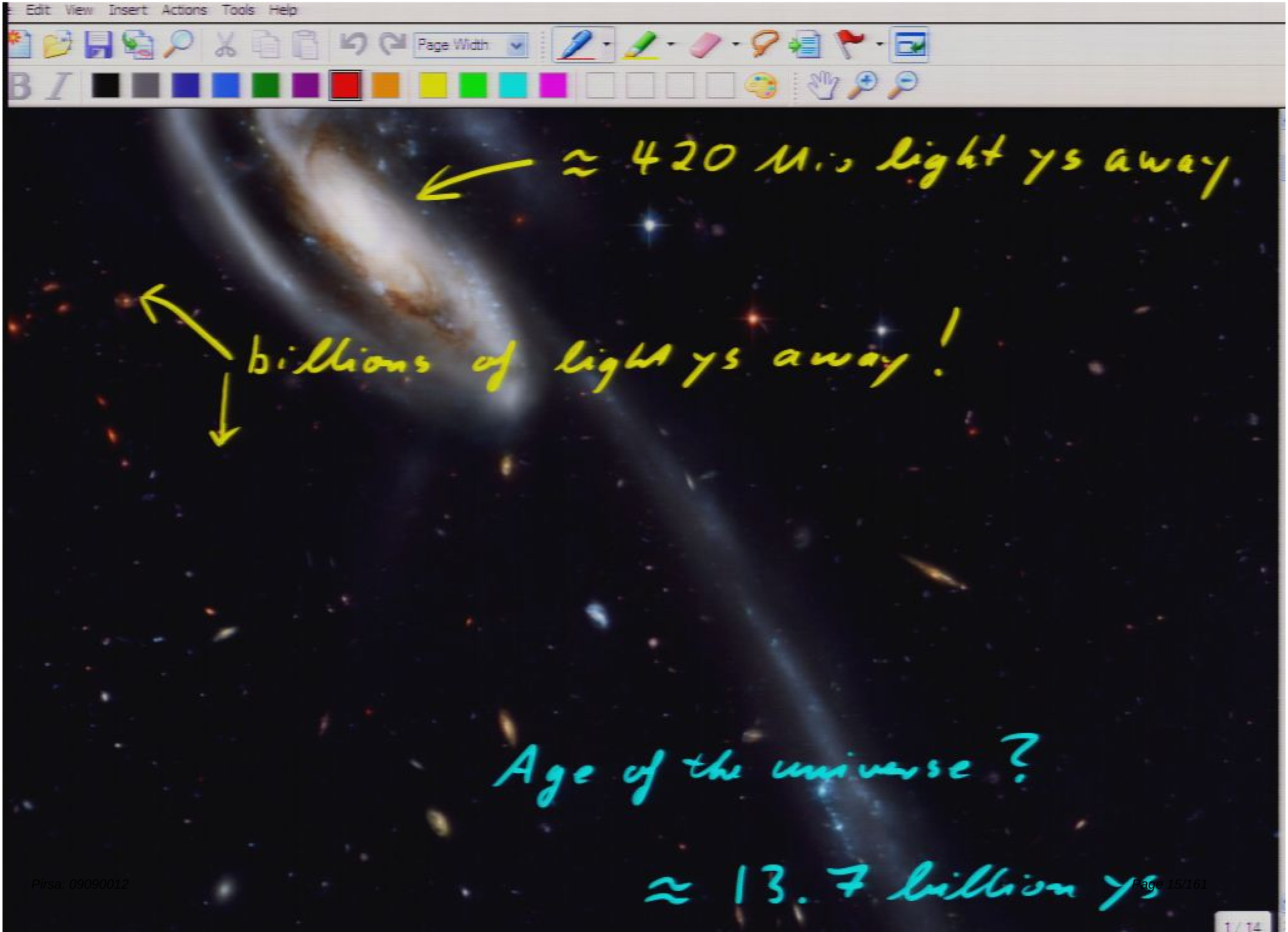
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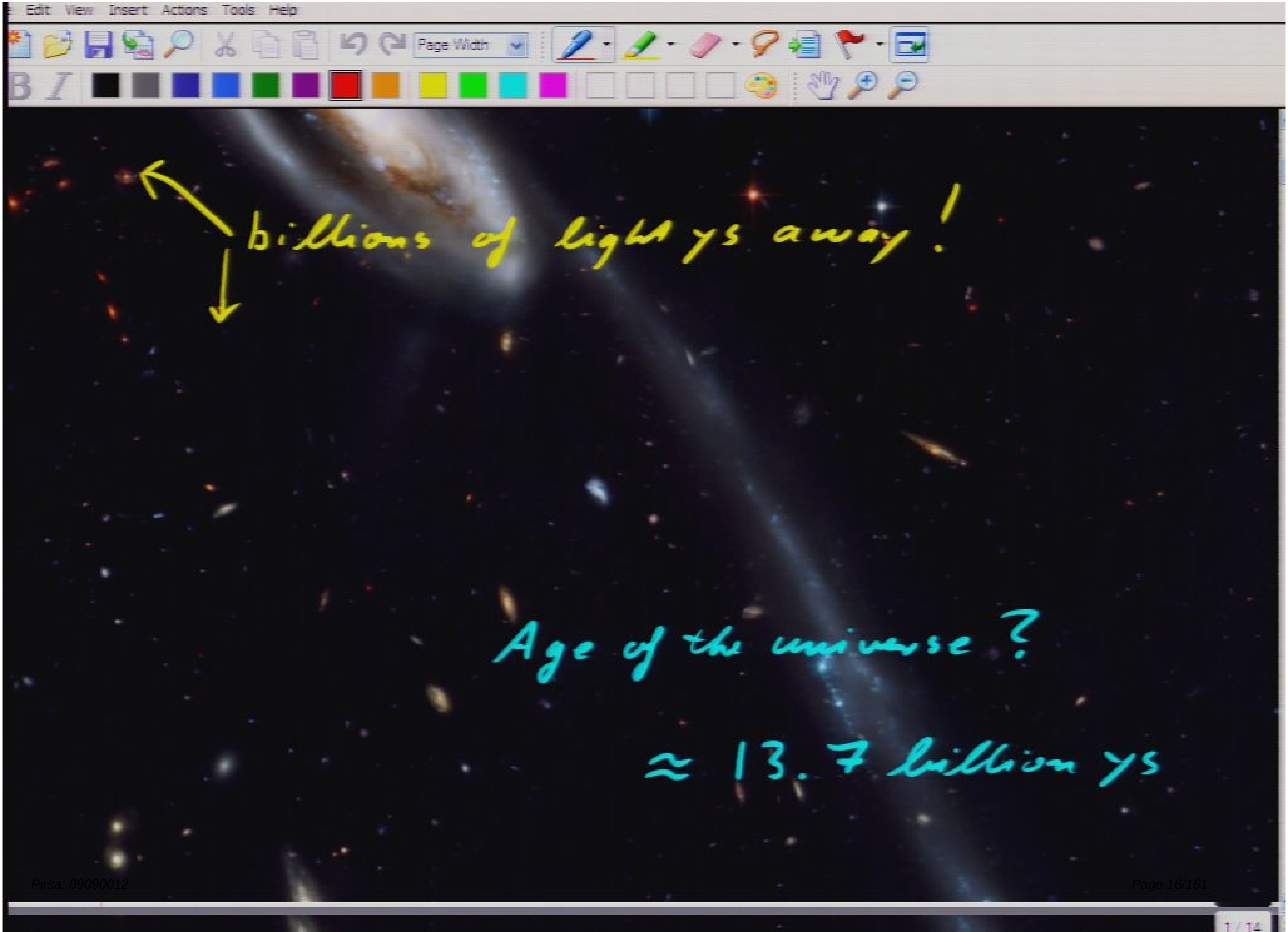
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General Relativity for Cosmology

Fall 2009

AMATH 875 / PHYS 786

Instructor: A. Kempf, (MC6071, ext. 5462)

Prerequisite: Introductory general relativity, e.g., AMATH 675.

Time and venue: Mon + Thu 4-5:20pm, Bob room at PI,
Video-linked to:
* Univ. of Waterloo, Room MC6091
* Univ. of Guelph, Room Rozanski106,
* McMaster Univ., Room ABB131

Office hours: by arrangement

Literature: Detailed lecture notes will be freely available electronically. Also, see the texts listed below.

Outline:

This course begins by introducing the differential geometry of Lorentzian manifolds from scratch and then builds up quickly to the advanced framework in terms of differential forms and the vielbein formalism. These methods are then used to define general relativity, also as a gauge theory. We then study some of general relativity's deeper properties, such as the formalism of spinors, and aspects of the causal structure and singularities. One key goal is to lay the foundations for students who wish to proceed to studies in quantum gravity. We then apply general relativity to cosmological models and to cosmological perturbation theory. Thereby, we are covering the theory of cosmic inflation which is very successful in predicting in

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Lecture Notes:

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In the meanwhile, here are those of F07, in pdf format: 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22

(You may also be interested in viewing the colloquium that I gave at Perimeter Institute on Sep.16, 2009:
"Spacetime can be simultaneously continuous and discrete, the same way that information can".
For the recording, see here.)

Additional literature

Our introduction to the differential geometry for General relativity uses material mainly from these three texts, while our notation conforms with that of the text by Straumann:

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symmetric between A and B,
as it must be by the principle
of relativity.

- B can use above calculation
and concludes that A's wrist watch
lags behind his coordinate system's
clocks that it passes.

is accelerating, so that above calculation fails.

Note: It seemed a small problem

because can always choose to work in

is accelerating, so that above calculation fails.

Note: It seemed a small problem

because can always choose to work in

an inertial coordinate system - and in it

the laws of nature take the usual form.

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2. J. Stewart, *Advanced General Relativity*, Cambridge (1991)
3. S. Hawking, G.F.R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge (1973)

Good references are also:

1. Scott Dodelson, *Modern Cosmology*, Academic Press, San Diego, (2003)
2. A.R. Liddle, D.H. Lyth, *Cosmological Inflation and Large-Scale Structure*, CUP (2000)
3. G.F.R. Ellis and J. Wainwright, *Dynamical Systems in Cosmology*, CUP (1997)
4. R. M. Wald, *General Relativity*, University of Chicago Press (1984)
5. H. Stephani, *General Relativity*, Cambridge University Press (CUP) (1982)

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We will cover Sakharov's "induced gravity" argument. Find the original (very short) paper here: Sakharov, and this review.

Here are links to general online reviews:

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You may also wish to have a look at these free only resources:

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Context:

This course is one in a group of four related graduate courses whose curricula have been coordinated so as to optimally complement another:

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These courses can be taken in arbitrary sequence and no course is a pre- or anti- requisite for another.

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University of Waterloo
Waterloo, Ontario
Canada N2L 3G1
Phone: (519) 888-4567 ext. 2700
Fax: (519) 746-4319

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This is assigned reading!

- The differential geometry of Riemann ...

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Review: Special Relativity

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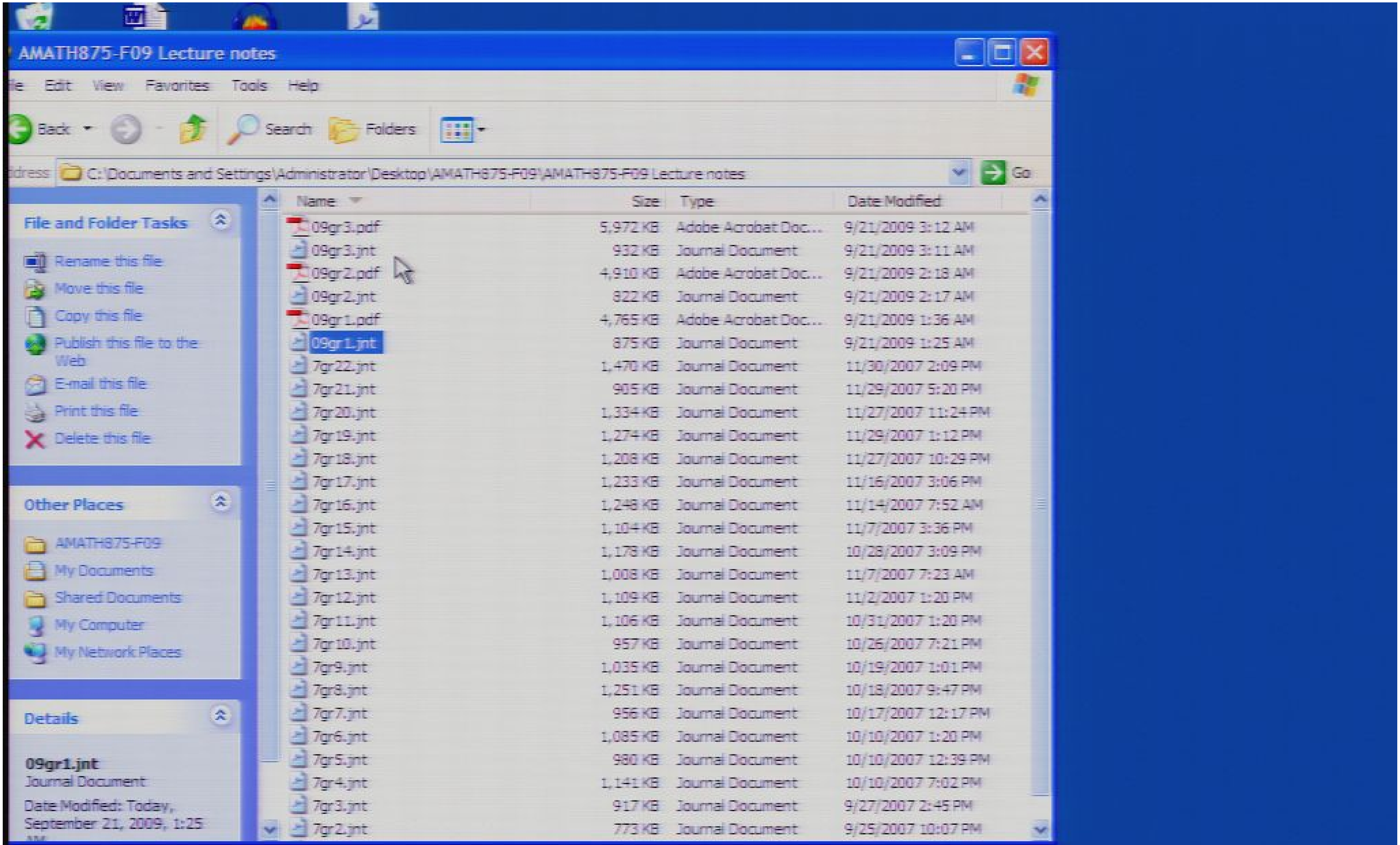
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Note Title 9/18/2005

GR for Cosmology, Fall 09, Achim Kempf, Lecture 2

What had Einstein achieved with special relativity?

- Laws of nature took the same form in all inertial, cartesian coordinate systems (i.e. in all coordinate systems that are a freely moving rectangular arrangement of equal length rods with synchronized clocks at the vertices).

- He could deduce the form of the laws of nature in an arbitrary coordinate system

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Note Title 9/18/2005

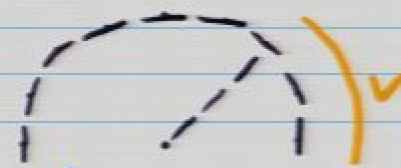
What had Einstein achieved with special relativity?

- Laws of nature took the same form in all inertial, cartesian coordinate systems (i.e. in all coordinate systems that are a freely moving rectangular arrangement of equal length rods with synchronized clocks at the vertices).

- He could deduce the form of the laws of nature in an arbitrary coordinate system rest may not be synchronizable.

(Recall that as the travelling twin accelerates he cannot keep his cds's clocks synchronized)

E.g.: Consider, built from equally-made rods, in an inertial cartesian cds:

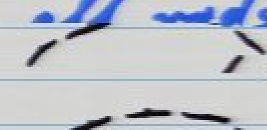


$$\frac{\text{Circumference}}{\text{Diameter}} = \pi$$

fit around.

→ The radial rods get thinner but stay the same length. Thus, as many as usual fit on the radius.

→ But now consider this in a rotating cds in which all rods are at rest.



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other direction.

2. Rather than seeing this as the complete downfall of special relativity, Einstein conjectured:

Even in the presence of gravity, an inertial, cartesian coordinate system can be constructed around every point (event), with arbitrary precision, at least in a suitably small neighborhood of that point. And, the laws of special relativity hold in that local coordinate system.

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pulled differently.

3. Einstein remembered a similar problem earlier faced by Riemann:

- Euclidean geometry is nice and simple, but how to describe a smooth but curved manifold?
- Riemann solved the problem after observing that at each point such manifolds are flat to any arbitrary precision - within a sufficiently small neighborhood.

⇒ Einstein's strategy:

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Pseudo-Riemannian Differential Geometry

A. Differentiable Manifolds

(Riemann ≈ 1850s, Poincaré ≈ 1890s, Whitney ≈ 1930s...)

Def: An n-dimensional topological Manifold, M, is a Hausdorff space which is locally homeomorphic to $\mathbb{R}^n$.

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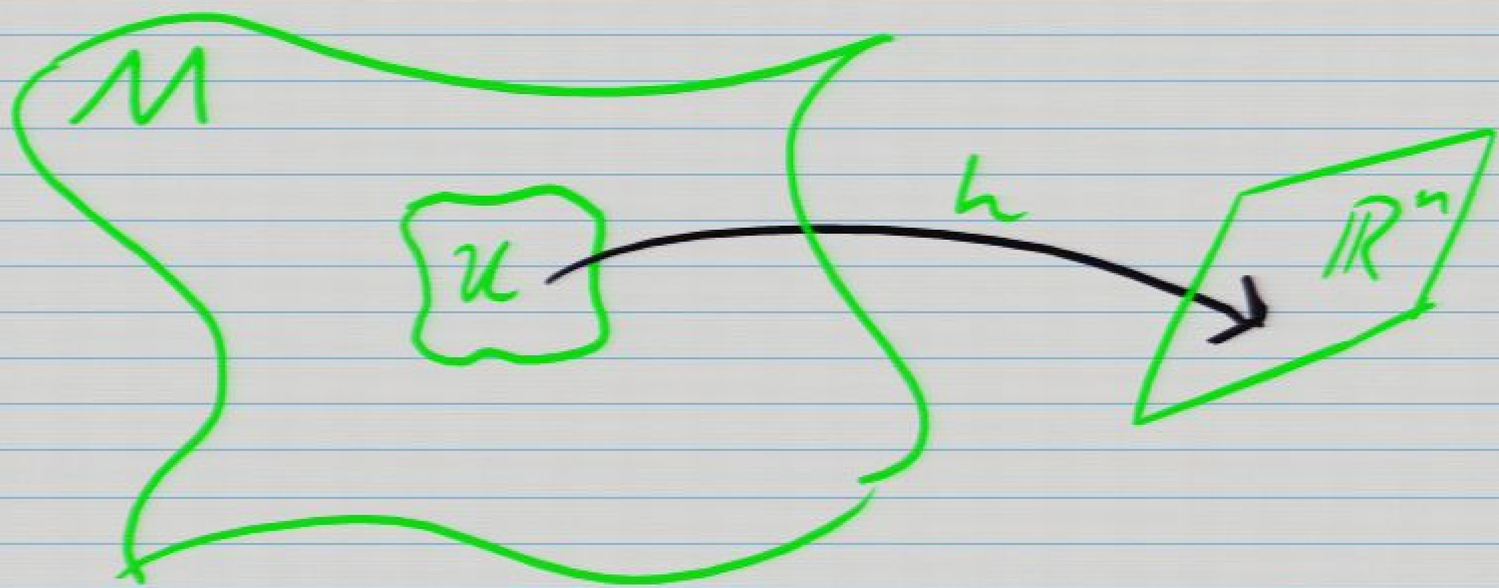

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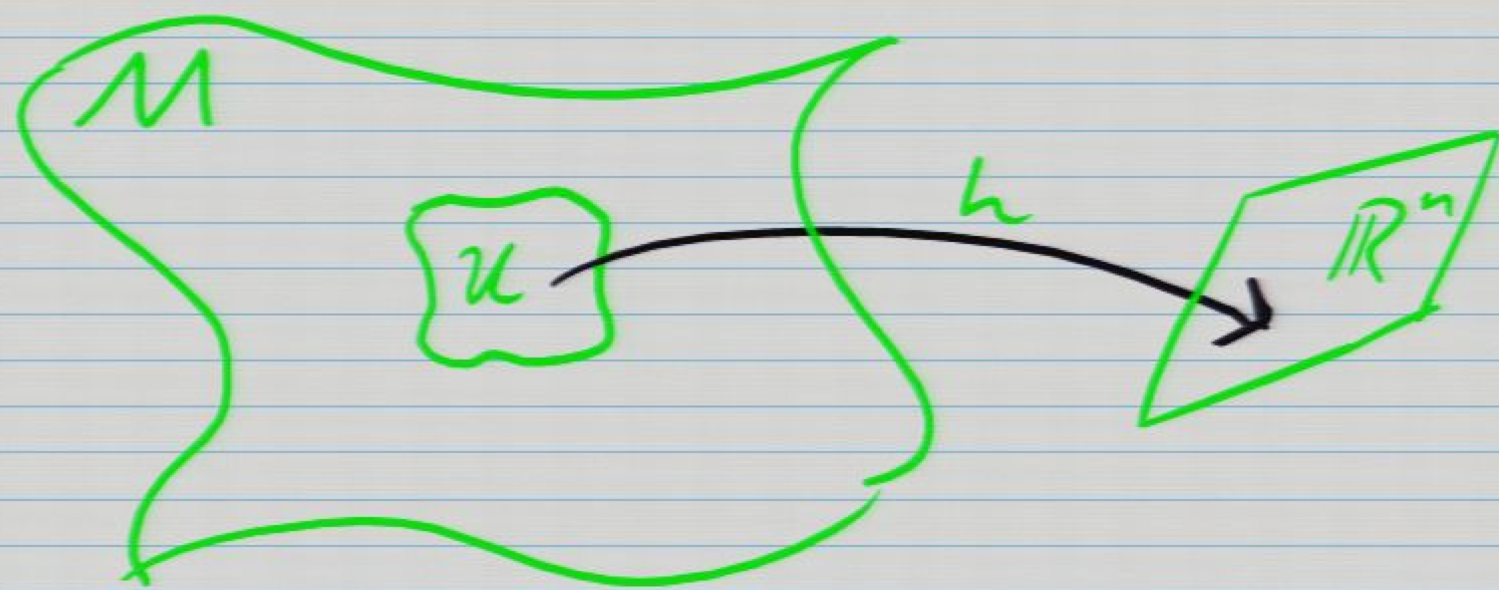

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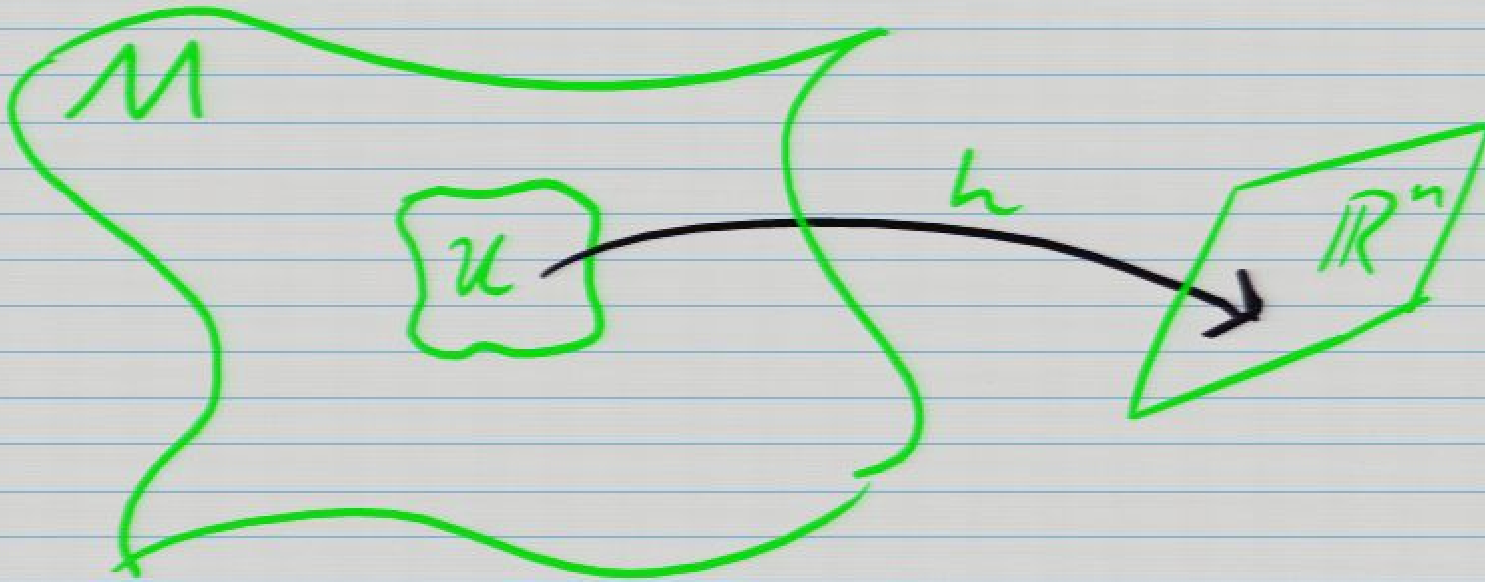

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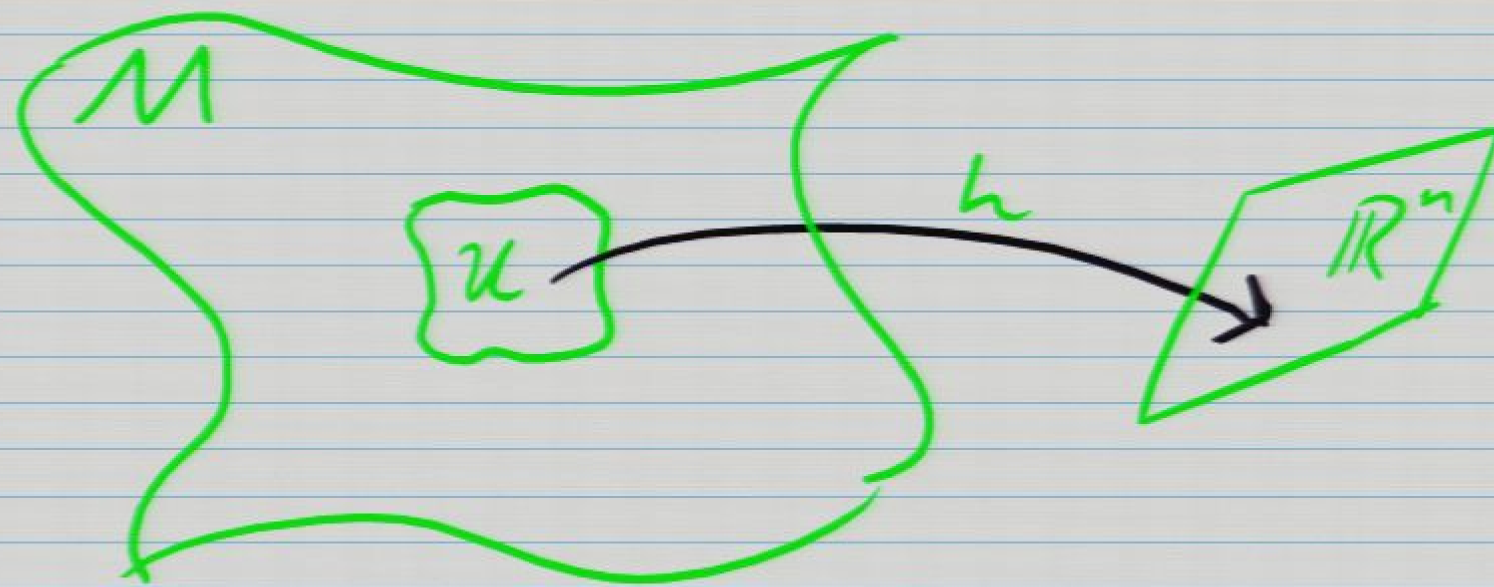

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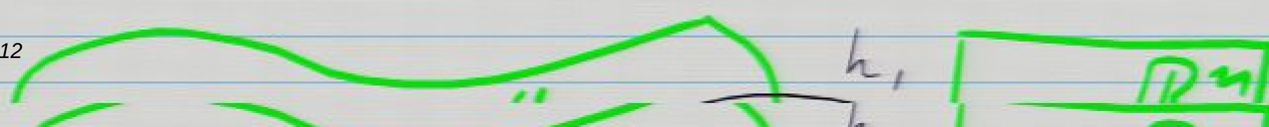

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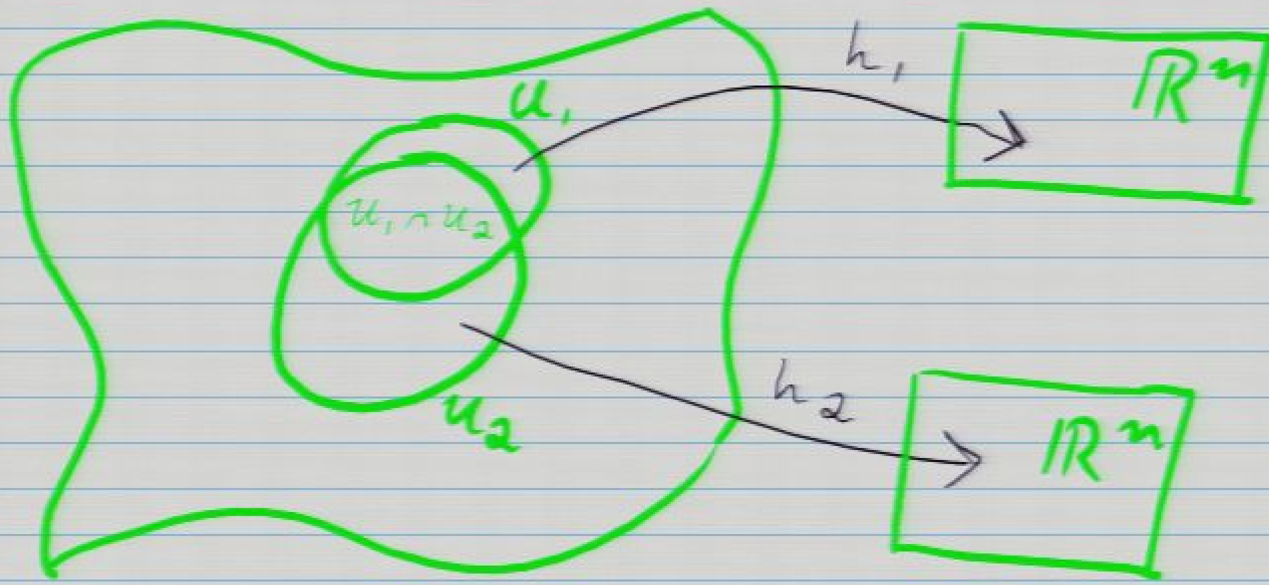

Then, $h_{12} : h_2 \circ h_1^{-1}$ is a continuous change of coordinates map $h_{12} : \mathbb{R}^n \to \mathbb{R}^n$.

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Crucial: For maps $\mathbb{R}^n \to \mathbb{R}^n$ we know what differentiability means!

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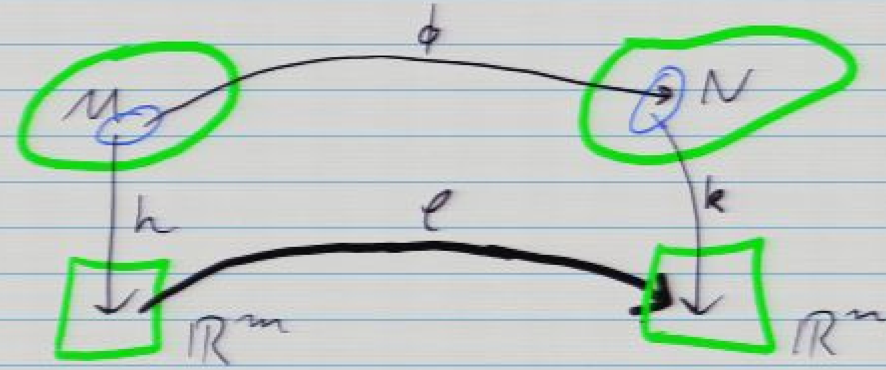

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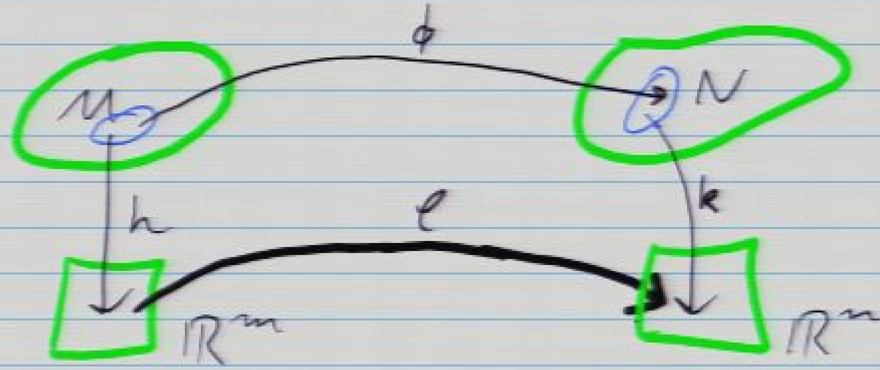

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Def: If ϕ is invertible, it is called an **embedding.**

short for differentiable

Def: If $M \subset N$ are diffable manifolds and the inclusion $\phi : M \rightarrow N$ is an embedding, then M is called a **submanifold** of N.

Note: there are slightly varying definitions in the literature

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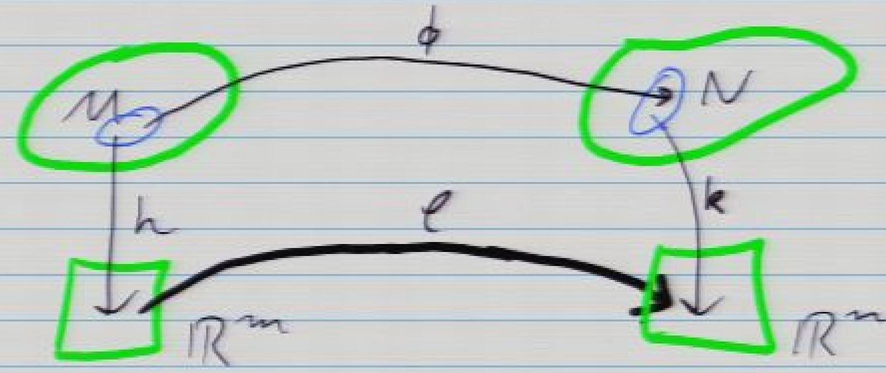

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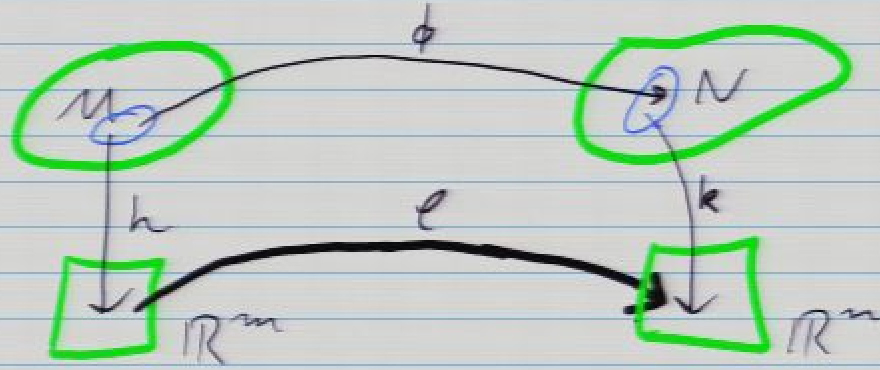

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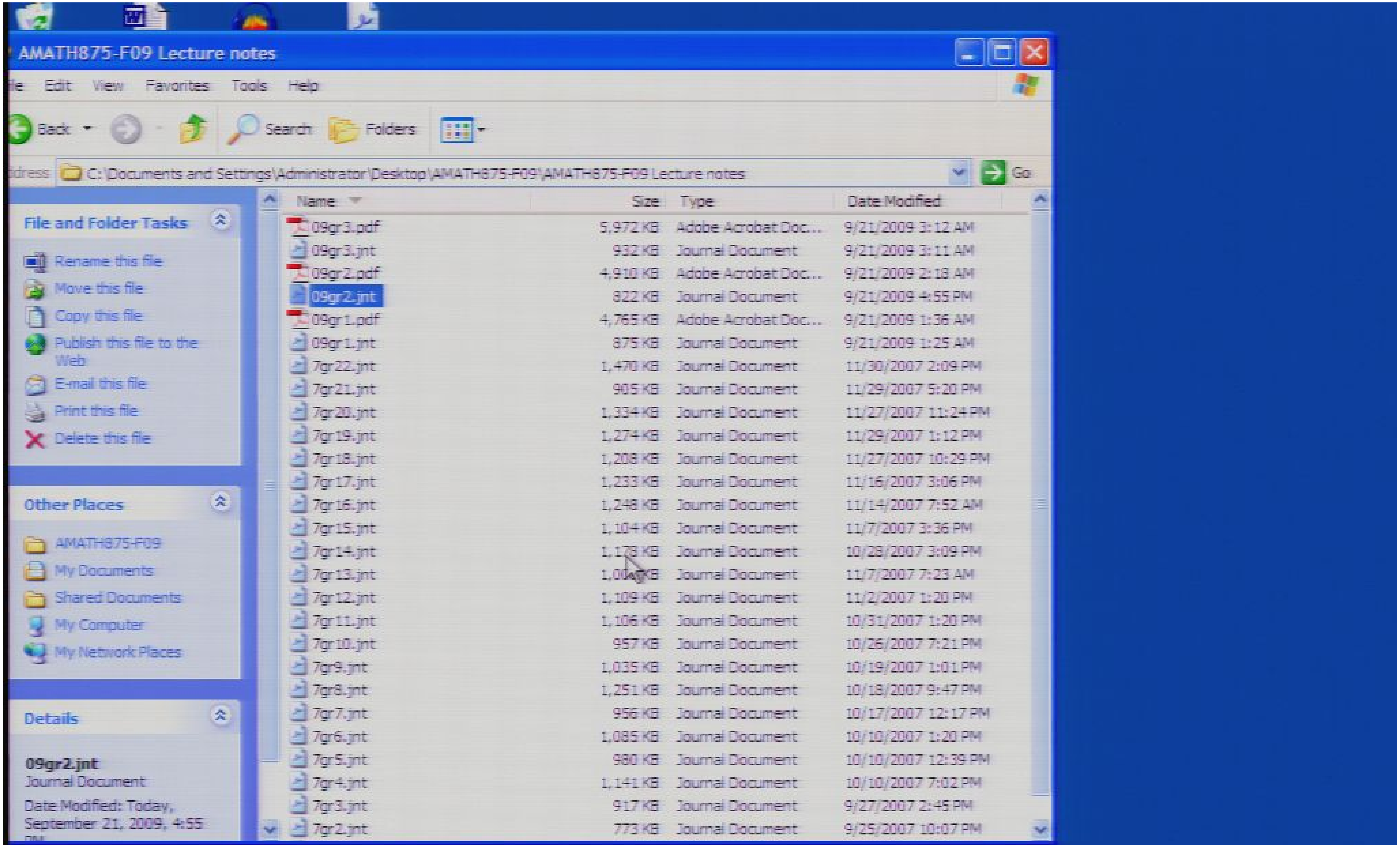
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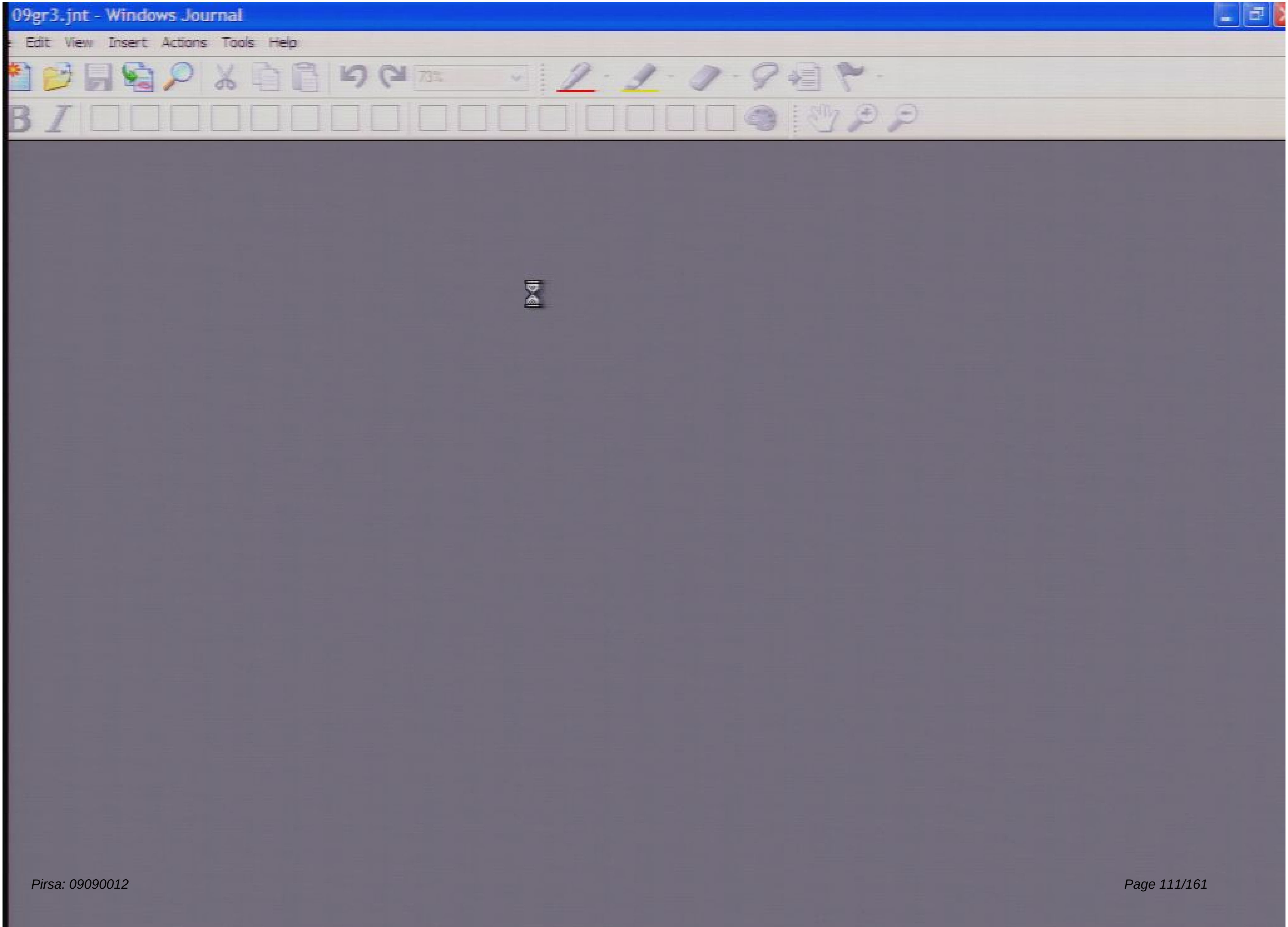
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E.g. $\eta = \frac{\partial}{\partial x^i}\Big|_{x=q}$ is the image

of some abstract $\xi \in T_p(M)$, for fixed φ.

Notation: $\xi =$ "$\frac{\partial}{\partial x^i}\Big|_{x=p}$"

symbolic notation

Question:

If we hold p and $\xi \in T(p)$ fixed,

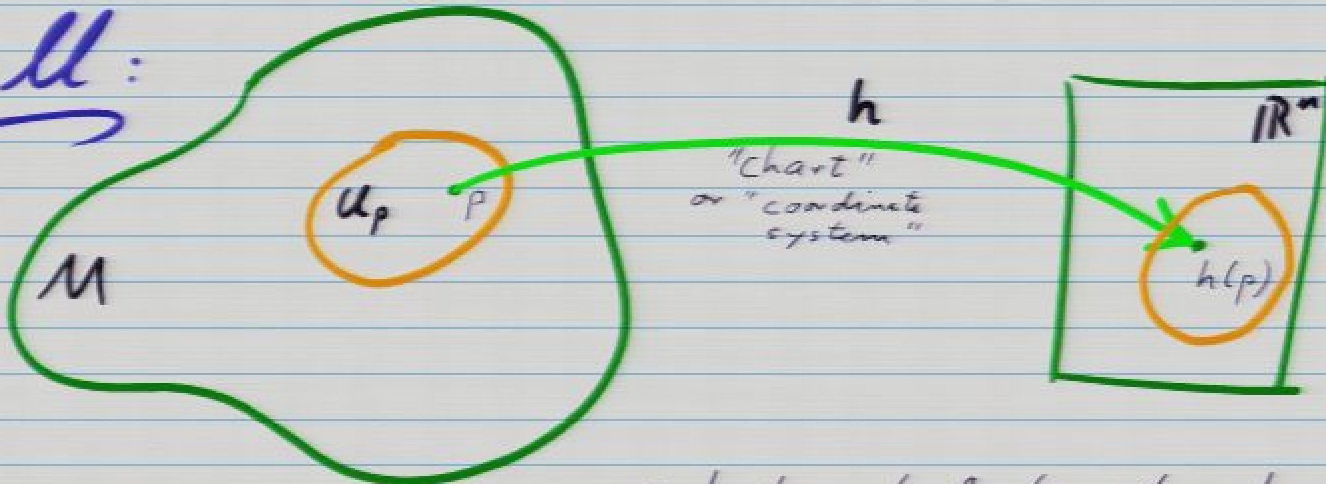
how do the numbers $(x^1, \ldots, x^n)$

and $(\eta^1, \ldots, \eta^n)$ change when we

change the chart?

GR for Cosmology, Fall 09, Achim Kempf, Lecture 3

Recall:



→ charts are tools to get a handle at the otherwise nameless abstract points of the manifold.

Problem:

How to define the abstract "Tangent space, $T_p(M)$,"

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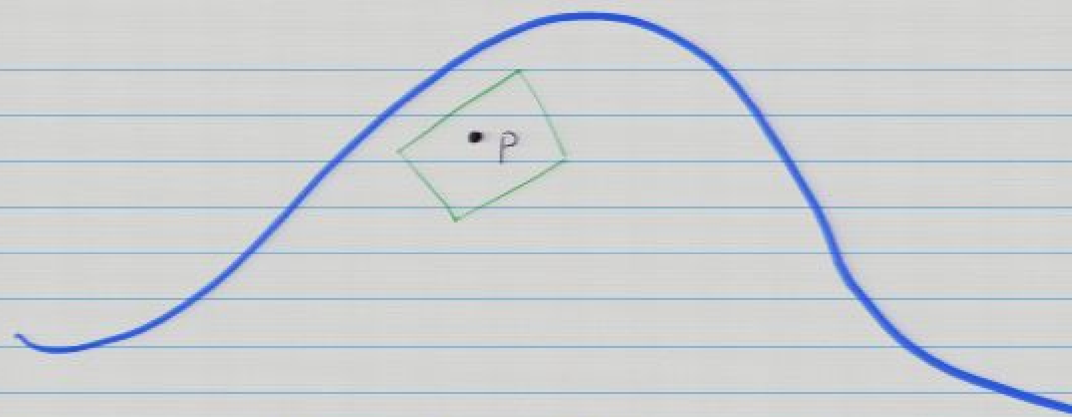
abstract points of the manifold.

Problem: How to define the abstract "Tangent space, $T_p(M)$," of a diffable mfld at a point p?

Intuition:



E.g. 2 dim manifold has 2 dim vector space of tangent vectors.

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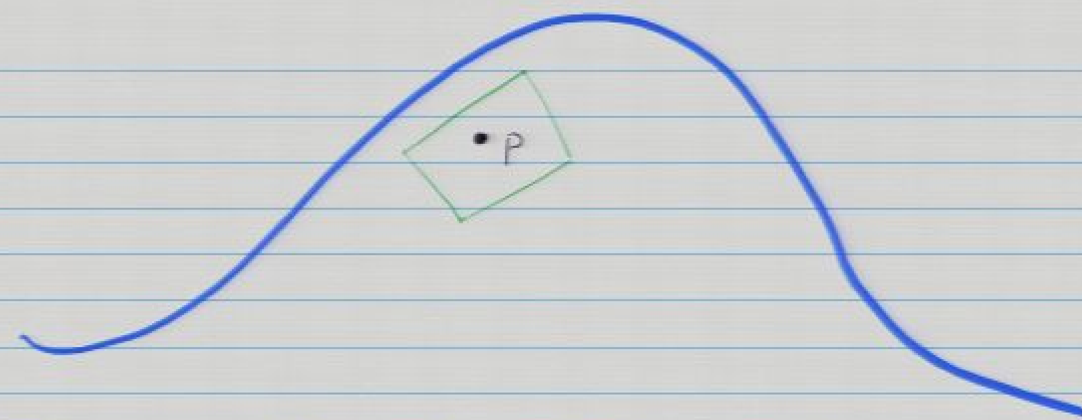

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→ Proper definition should imply:

An n-dim mfld possesses for every point p an n-dim vector space of tangent vectors.

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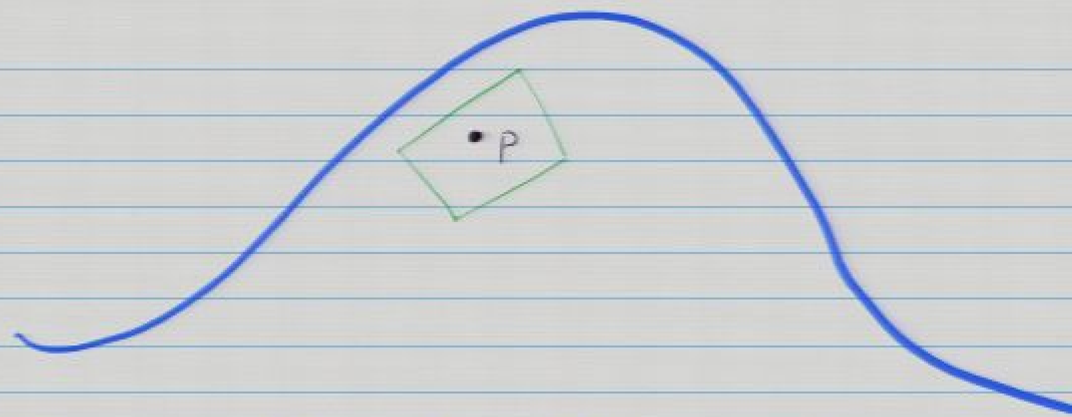

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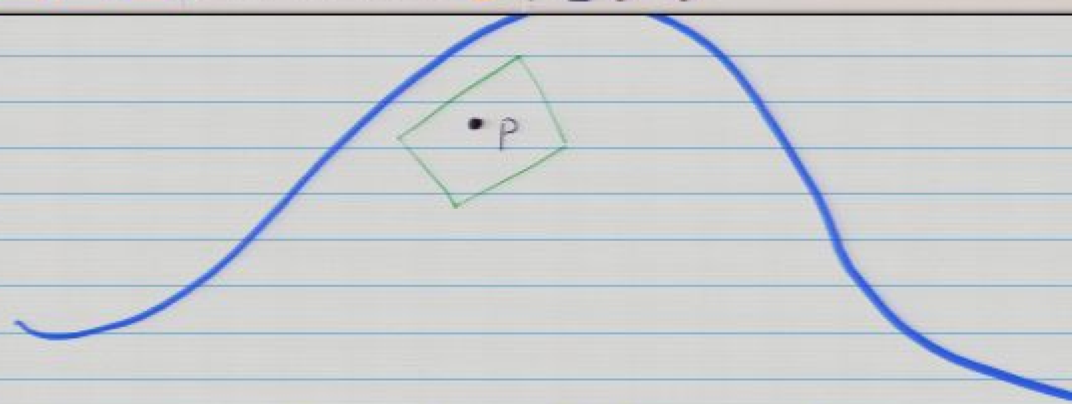

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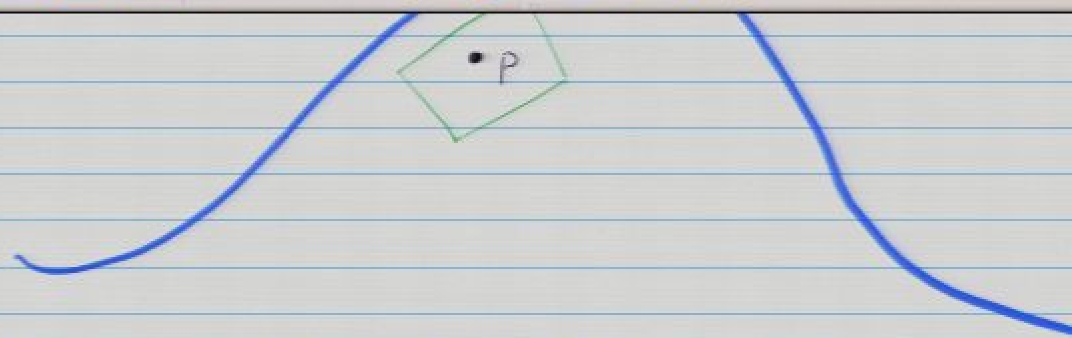

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3 equivalent definitions of $T_p(M)$:

1. "Algebraic" definition of $T_p(M)$

lengthy and abstract but modern and powerful!

Idea: A tangent vector can denote a directional derivative $\Rightarrow$ recognizable by Leibniz rule of derivatives:

$$(fg)' = f'g + fg'$$

2. "Ph... " definition of $T_p(M)$

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Idea: The elements of $T_p(M)$ are to be vectors $\Rightarrow$ recognizable by how their components change with charts.

3. "Geometric definition of $T_p(M)$"

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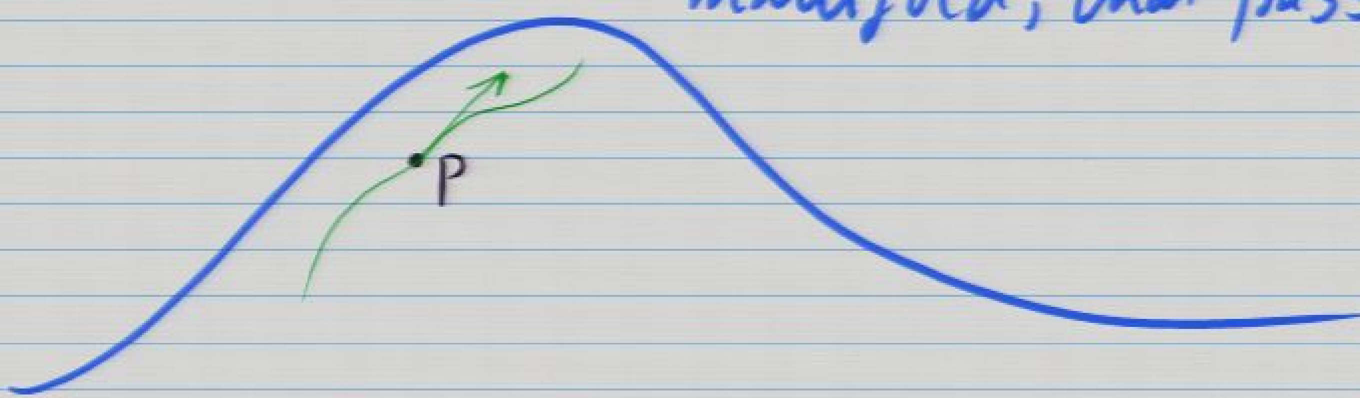
Idea: The elements of $T_p(M)$ are to be actual tangent vectors of one-dim paths in the manifold, that pass through p.



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The 3 defs are equivalent, but:

One tends to need all 3 occasionally!

$\rightsquigarrow$ we will do all 3:

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Namely: In the case of $M = \mathbb{R}^n$, any tangent vector ξ at a point p can be viewed as a directional 1st derivative: $\xi = \sum_{i=1}^{n} \xi_i \frac{\partial}{\partial x^i}\Big|_{x=p}$

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We'll try to identify tangent vectors by this property.

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$$\Psi \in \bar{\phi}_p \text{ if } \exists\, U_p\ \forall q \in U_p : \ \phi(q) = \Psi(q)$$

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"there exists"

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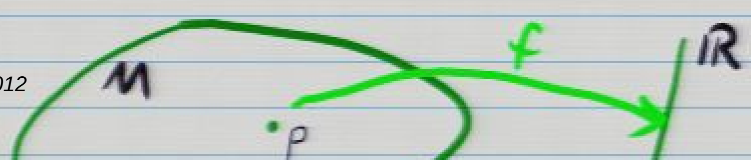
Intuition:

The elements of $\overline{\Phi_p}$ ought to be all those functions $\psi : M \to N$ which possess the same 1st derivatives at p.

Note that the definition does not even assume existence of derivatives, but the intuition holds for diff'able functions.

For example:

Consider germs of functions f:



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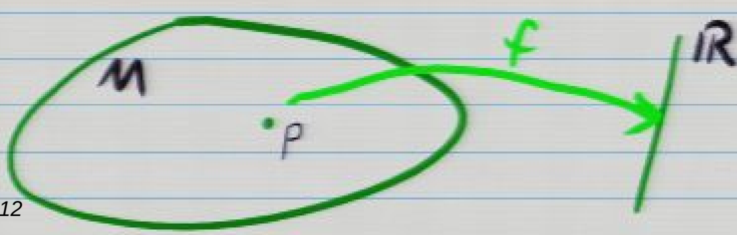

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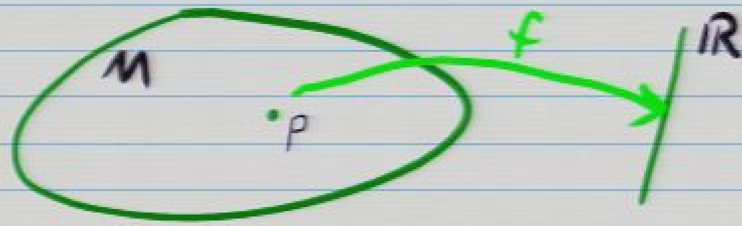

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- To specify a germ, it suffices to specify an arbitrary one of its functions.
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Recall idea: The elements of $T(p)$ are to be 1st derivatives $\Rightarrow$ recognizable by Leibniz rule.

Definition: The tangent space $T_p(M)$ is the set of "derivations" of $\mathcal{F}(p)$, i.e. the set of linear maps $\xi: \mathcal{F}(p) \to \mathbb{R}$ which

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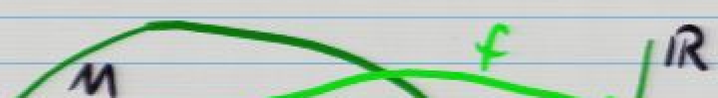

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$\Rightarrow$ Characteristic property "Leibniz rule":

We'll try to identify tangent vectors by this property.

$$\boxed{\xi(fg) = \xi(f)\,g + f\,\xi(g)}$$

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Definition: The tangent space $T_p(M)$ is the set of "derivations" of $\mathcal{F}(p)$, i.e. the set of linear maps $\xi : \mathcal{F}(p) \to \mathbb{R}$ which obey: (Leibniz rule for differentiable functions g, f)

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Remark:

- this definition is abstract enough not only for arbitrary diff'able mflds!

- this definition (as derivations of the algebra of functions) is also suitable

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$$\xi\left(\bar{f}_p \bar{g}_p\right) = \xi\left(\bar{f}_p\right) \cdot \underbrace{\bar{g}_p(p)}_{= g(p)} + \underbrace{\bar{f}_p(p)}_{= f(p)} \xi\left(\bar{g}_p\right)$$

Remark:

- this definition is abstract enough not only for arbitrary diff'able mflds!

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Recall idea: The elements of $T(p)$ are to be 1st derivatives $\Rightarrow$ recognizable by Leibniz rule.

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$$\xi(f_p g_p) = \xi(f_p)\cdot \underbrace{g_p(p)}_{=g(p)} + \underbrace{f_p(p)}_{=f(p)}\xi(g_p)$$

Remark:

- this definition is abstract enough not only for arbitrary diffable mflds!
- this definition (as derivations of the algebra of functions) is also suitable for "Noncommutative Geometry": There, (Quantum Gravity) the algebra of functions $F(p)$ is noncommutative.

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Properties of $T_p(M)$:

Simple example: a constant function c:

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