Title: Generating scale-invariant fluctuations without inflation

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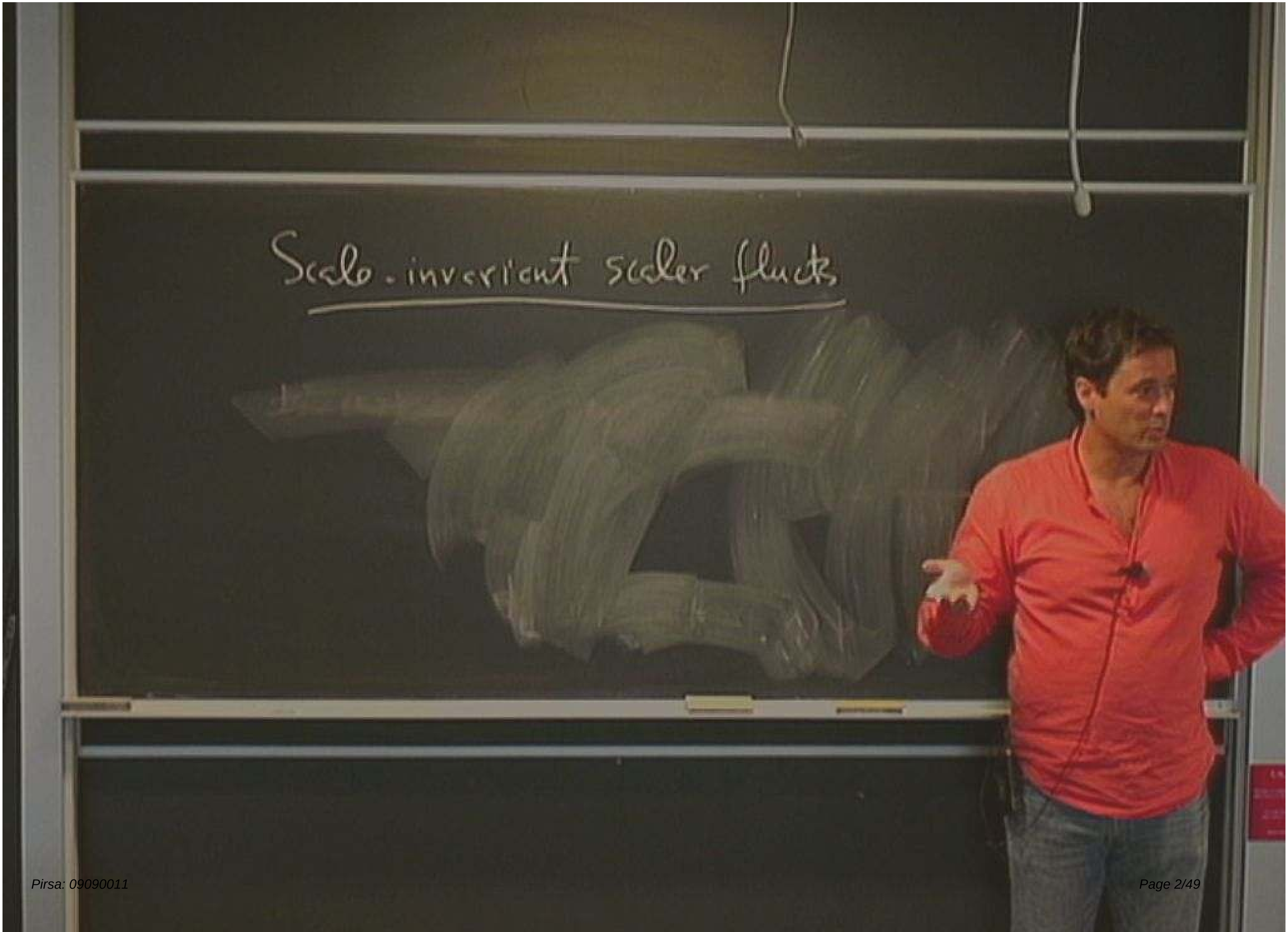
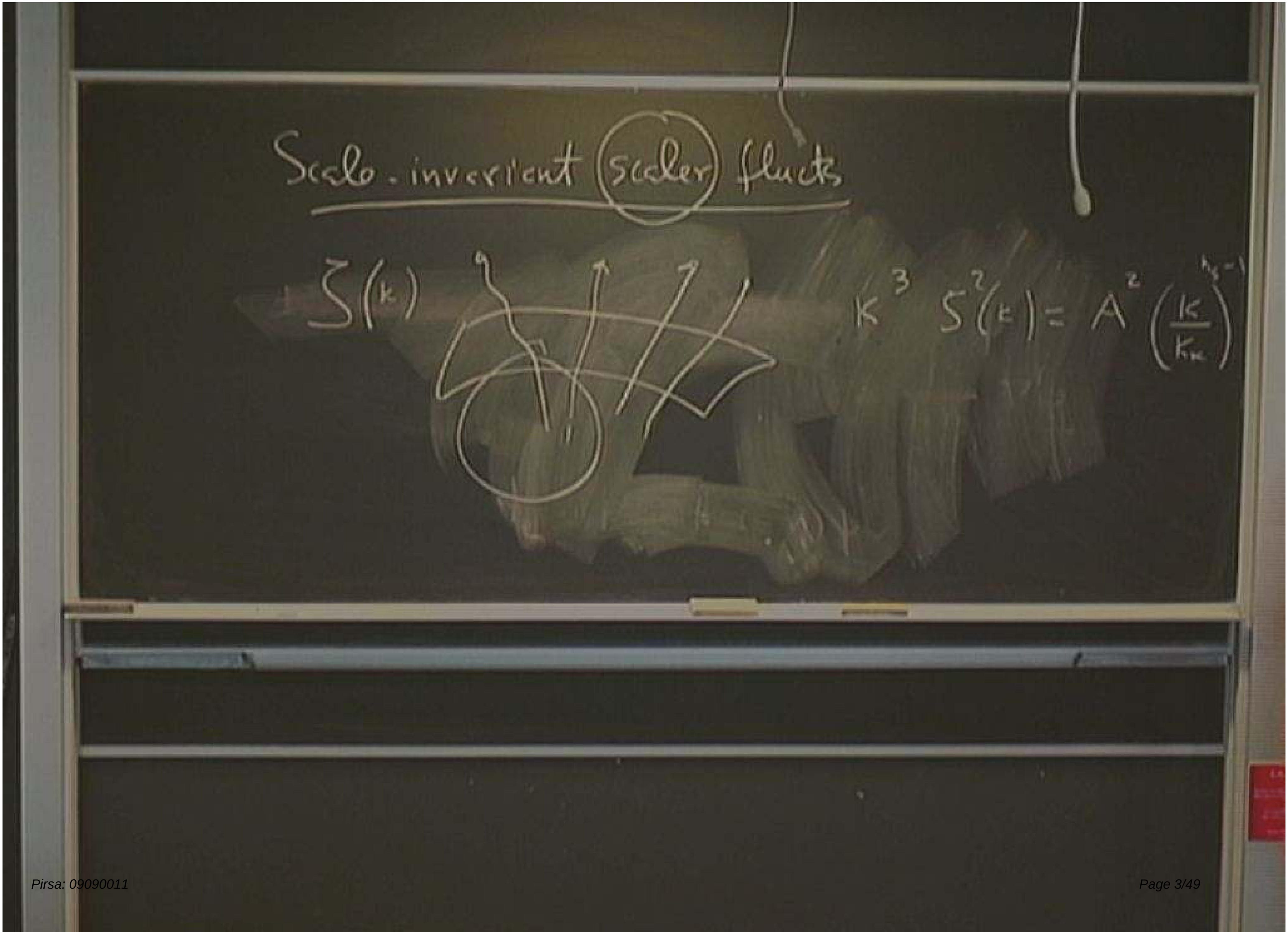
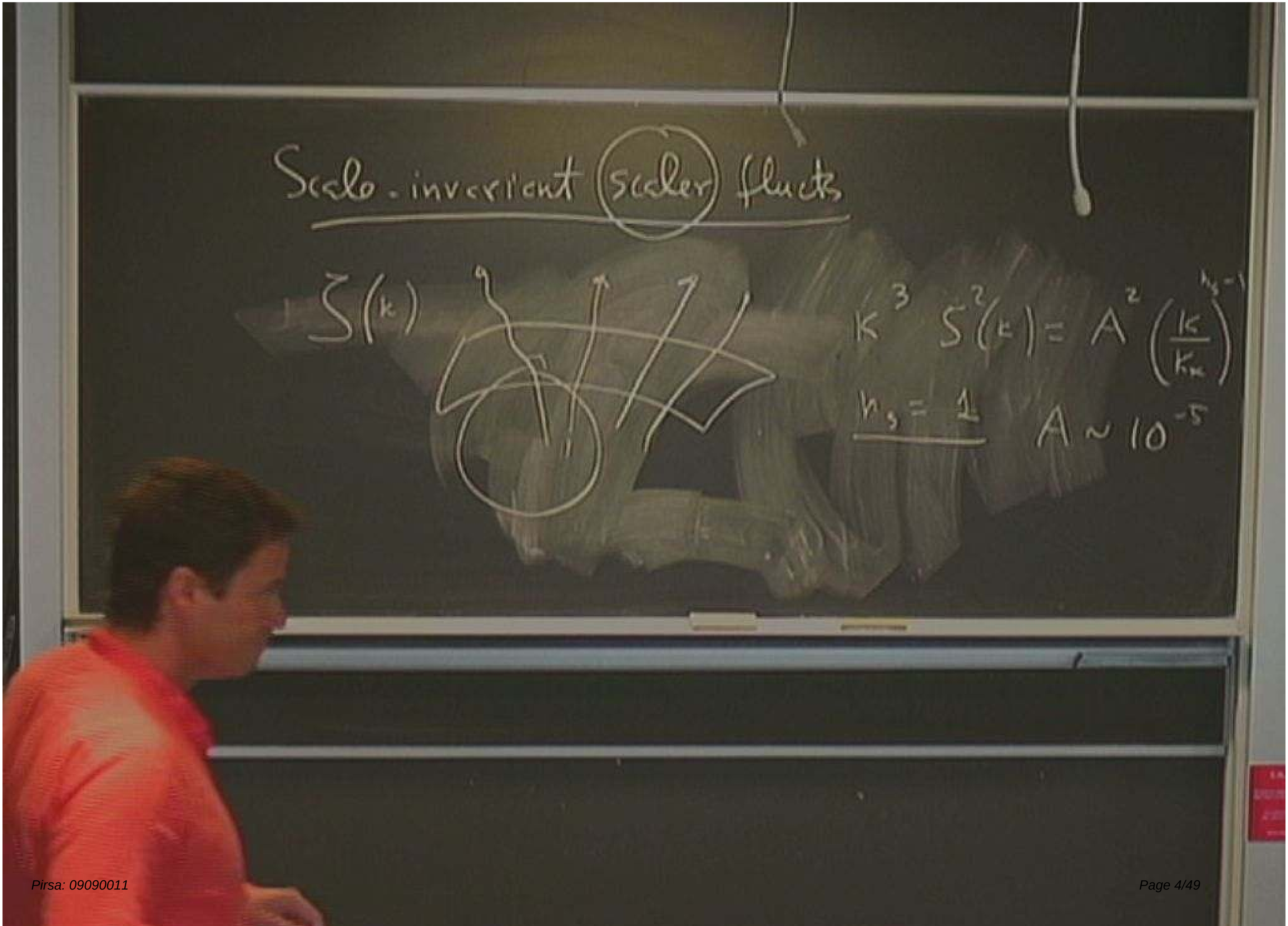
Abstract: I describe a number of techniques that allow for the generation of (near) scale-invariant fluctuations in the early Universe without inflation or ekpyrosis. The basic ingredient is a decaying maximal speed of propagation, for which a Universal law is found. Connections are made with k-essence, the cuscaton, and the DBI action. However the simplest realizations result from bimetric theories and deformed dispersion relations and DSR. A number of implications to theories of quantum gravity are discussed.


Scale-invariant scalar flucts


Scale-invariant scalar flucts
$\zeta(k)$
$k^3 \zeta^2(k) = A^2 \left(\frac{k}{k_*}\right)^{n_s-1}$


Scale-invariant scalar flucts
$\zeta(k)$
$k^3 \zeta^2(k) = A^2 \left(\frac{k}{k_*}\right)^{n_s-1}$
$n_s = 1$
$A \sim 10^{-5}$

$$\zeta = \frac{v}{z}$$

$v$ - 2nd quant QFT

$z \propto$

$$\zeta = \frac{v}{z}$$

$v$ - 2nd quant QFT

$$z \propto R \propto \eta^{\frac{2}{1+3w}}$$

$$v'' + [k^2$$

$$\zeta = \frac{v}{z}$$

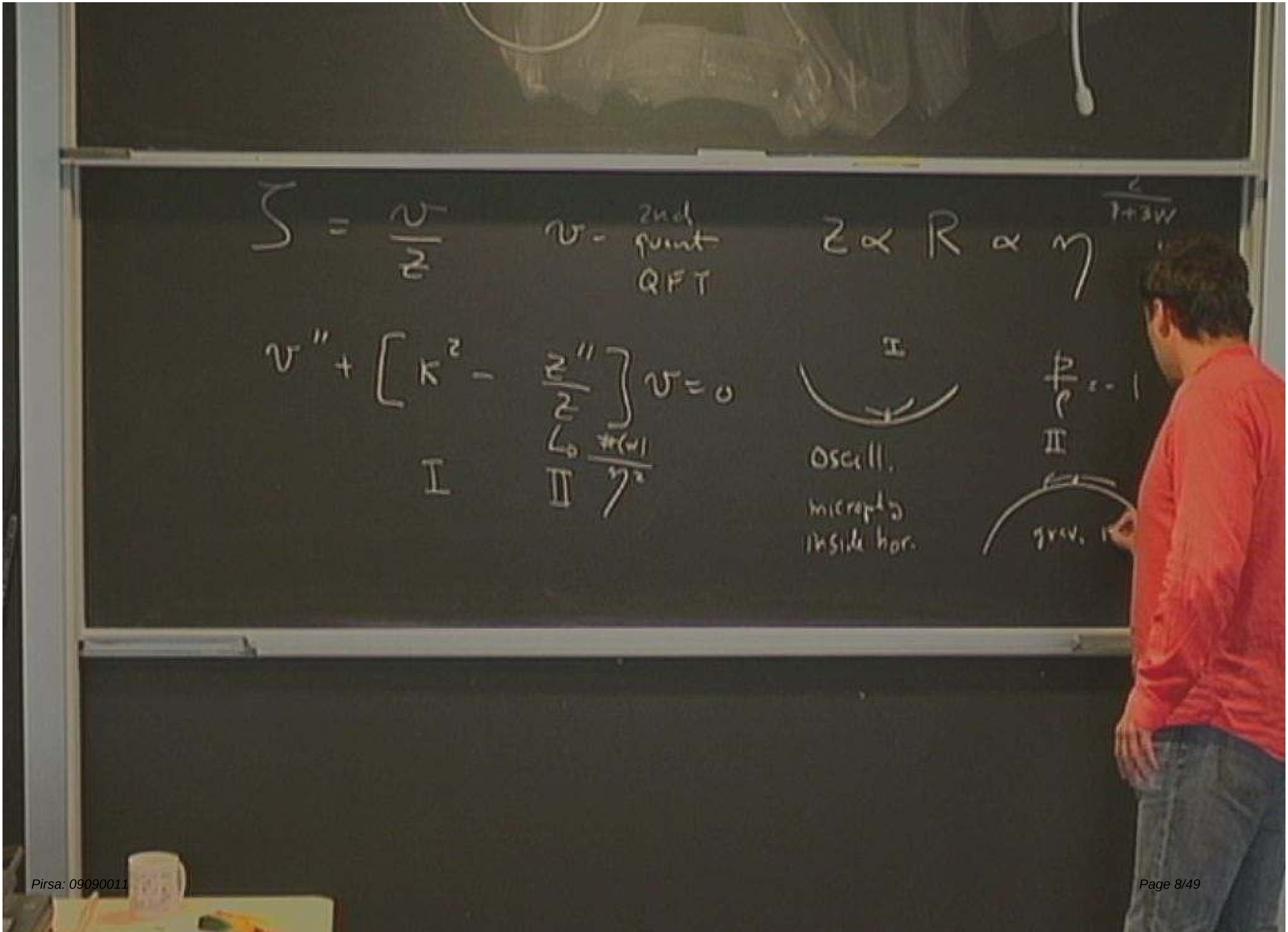
$v$ - 2nd quant QFT

$$z \propto R \propto \eta^{\frac{2}{1+3w}}$$

$$v'' + \left[k^2 - \frac{z''}{z}\right] v = 0$$

$\frac{z''}{z} \hookrightarrow \frac{f(w)}{\eta^2}$

I    II



$\zeta = \frac{v}{z}$
$v$ - 2nd quant QFT
$z \propto R \propto \eta^{\frac{2}{1+3w}}$
$v'' + \left[k^2 - \frac{z''}{z}\right] v = 0$
I
II
oscill.
inside hor.
grav.

$\zeta = \frac{v}{z}$ $\quad v$ - 2nd quant QFT $\quad z \propto R \propto \eta^{\frac{2}{1+3w}}$ $\quad \hookrightarrow \frac{p}{\rho}$

$$v'' + \left[ k^2 - \frac{z''}{z} \right] v = 0$$

I $\quad$ II $\quad \hookrightarrow \frac{\#(w)}{\eta^2}$

I

Oscill.

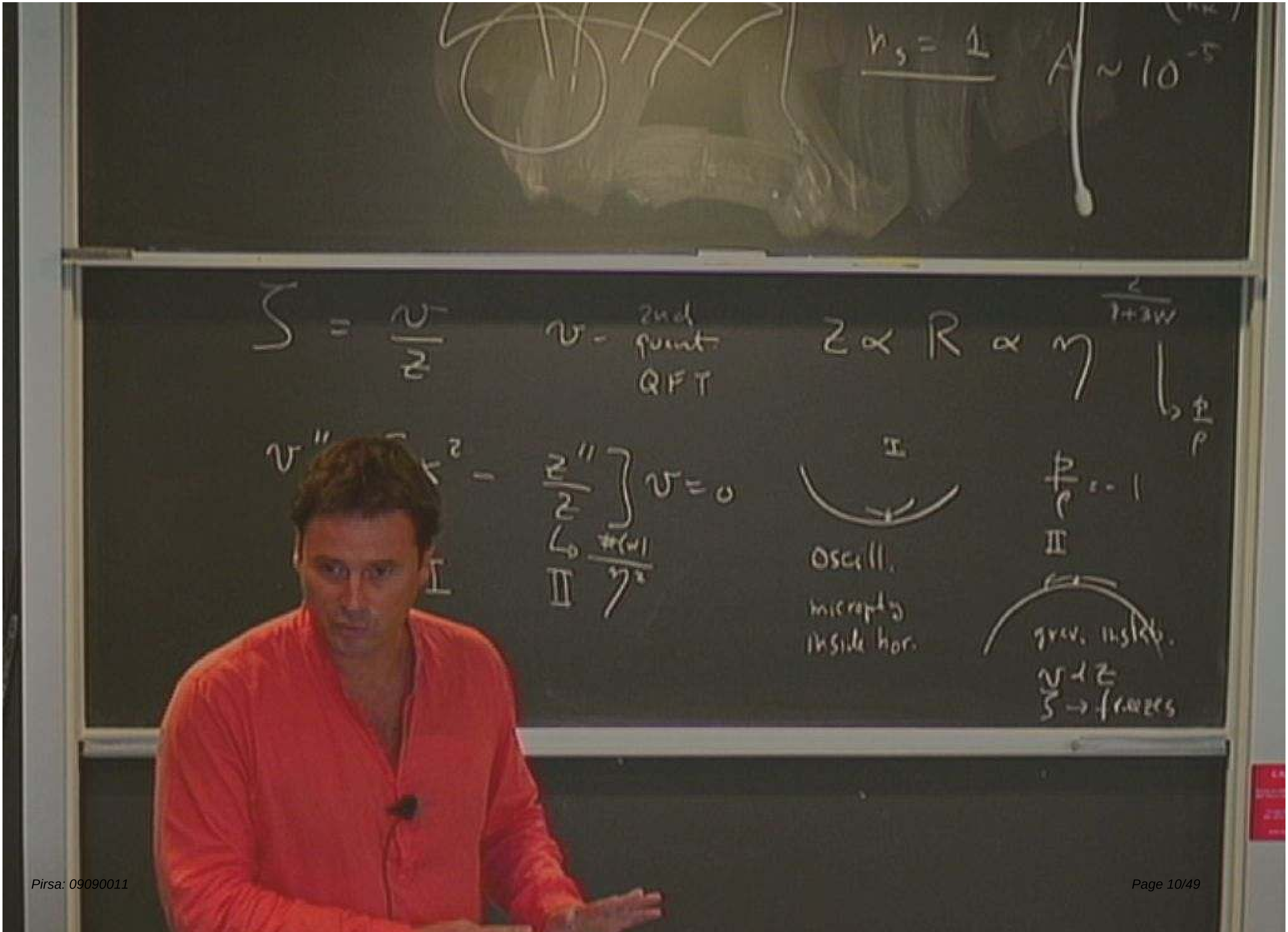
microphys

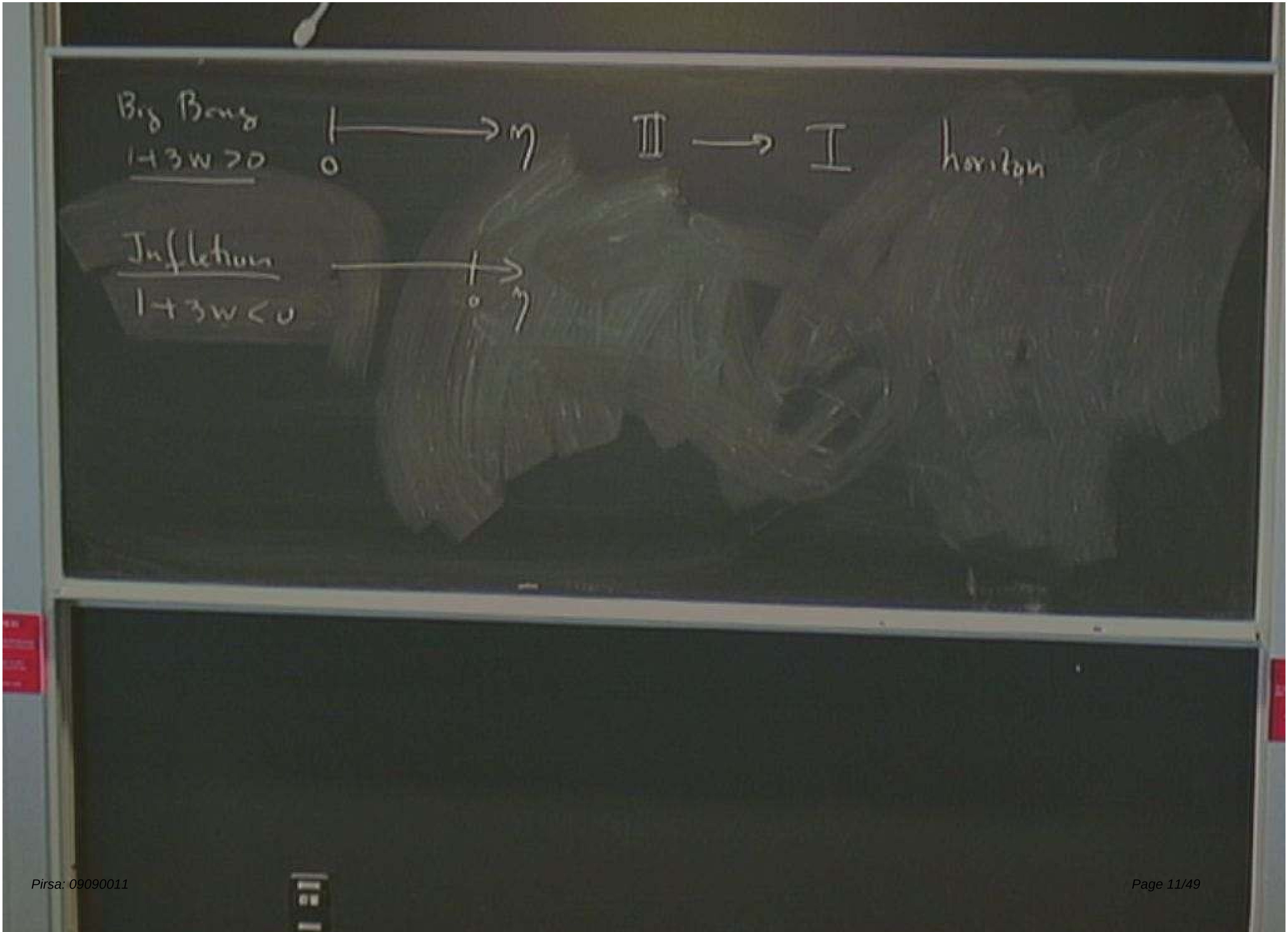
inside hor.

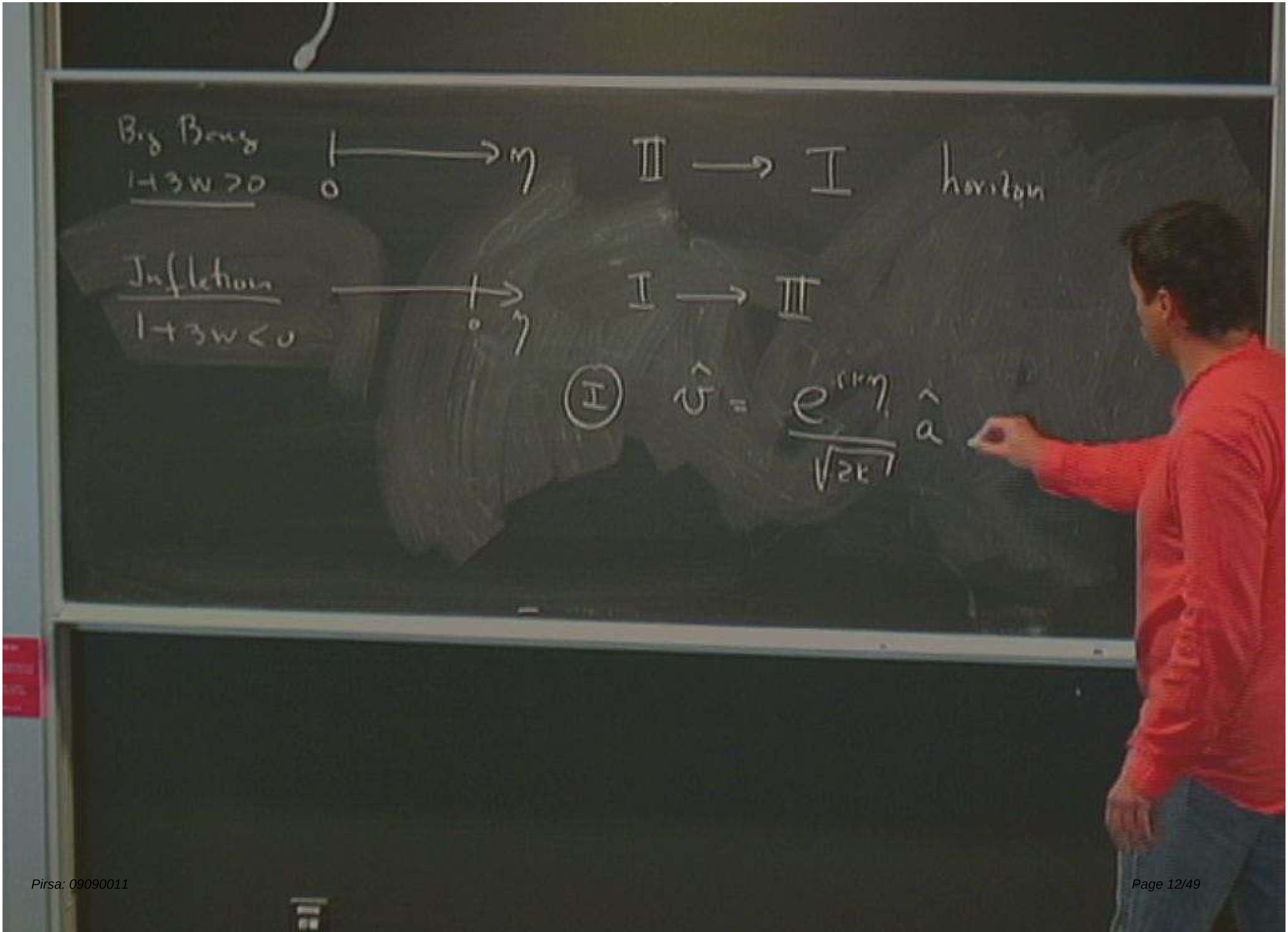
$\frac{p}{\rho} < -1$

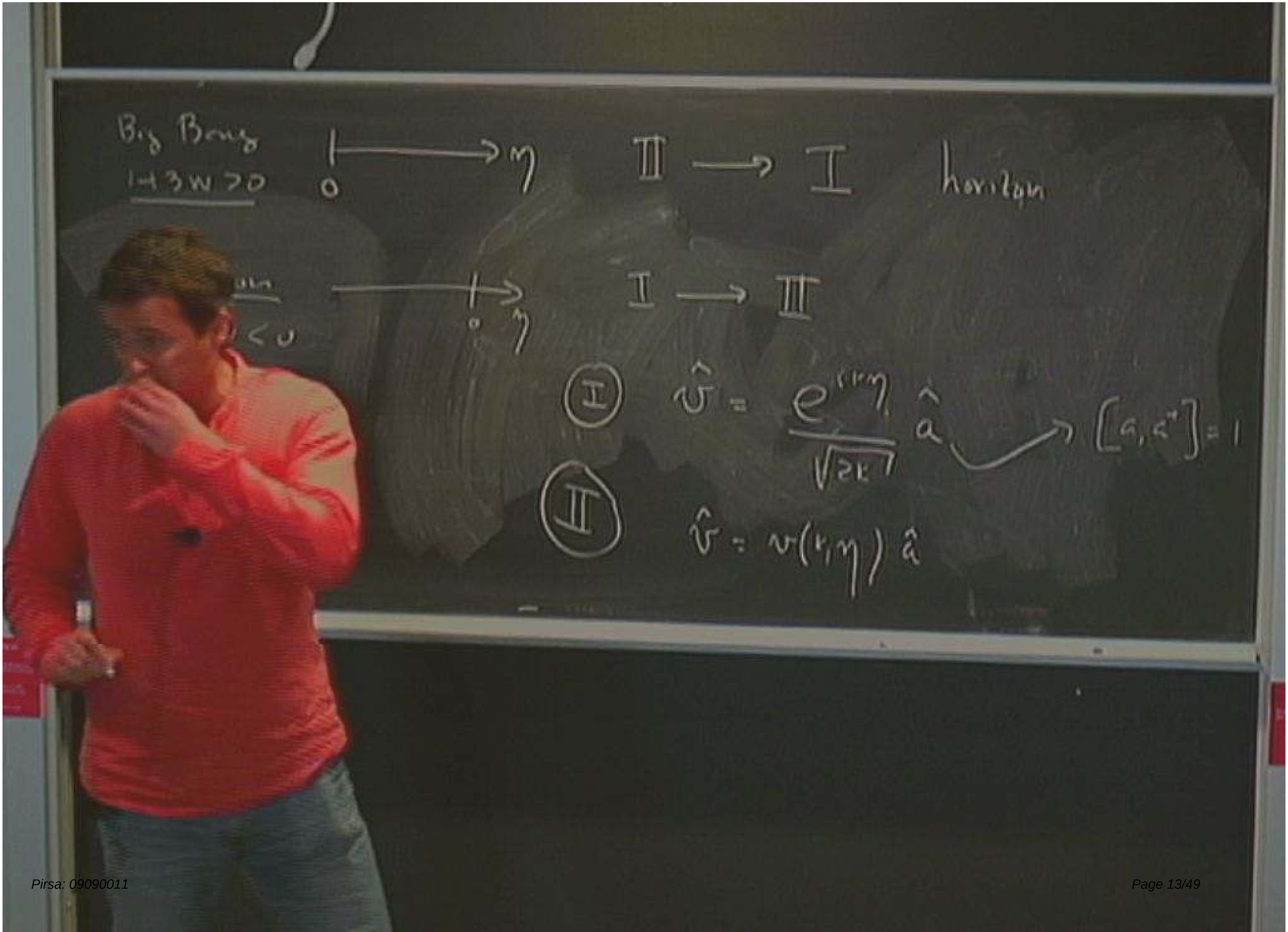
II

grav. instab.


n_s = 1
A ~ 10^-5
ζ = v/z
v - 2nd quant. QFT
z ∝ R ∝ η^(2/(1+3w))
v'' + [k² - z''/z] v = 0
I
II
oscill.
microphy.
inside hor.
grav. instab.
v ∝ z
ζ → freezes


Big Bang
1+3w>0
0
η
III → I
horizon
Inflation
1+3w<0
0
η


Big Bang
1+3w>0
0 η
III → I
horizon
Inflation
1+3w<0
0 η
I → III
I $\hat{v} = \frac{e^{ik\eta}}{\sqrt{2k}} \hat{a}$


Big Bang
1+3w>0
horizon
[a, a†] = 1

Big Bang

$1+3w>0$

$\underset{0}{\vert}\!\longrightarrow \eta$ $\qquad$ $\mathrm{II} \longrightarrow \mathrm{I}$ $\qquad$ horizon

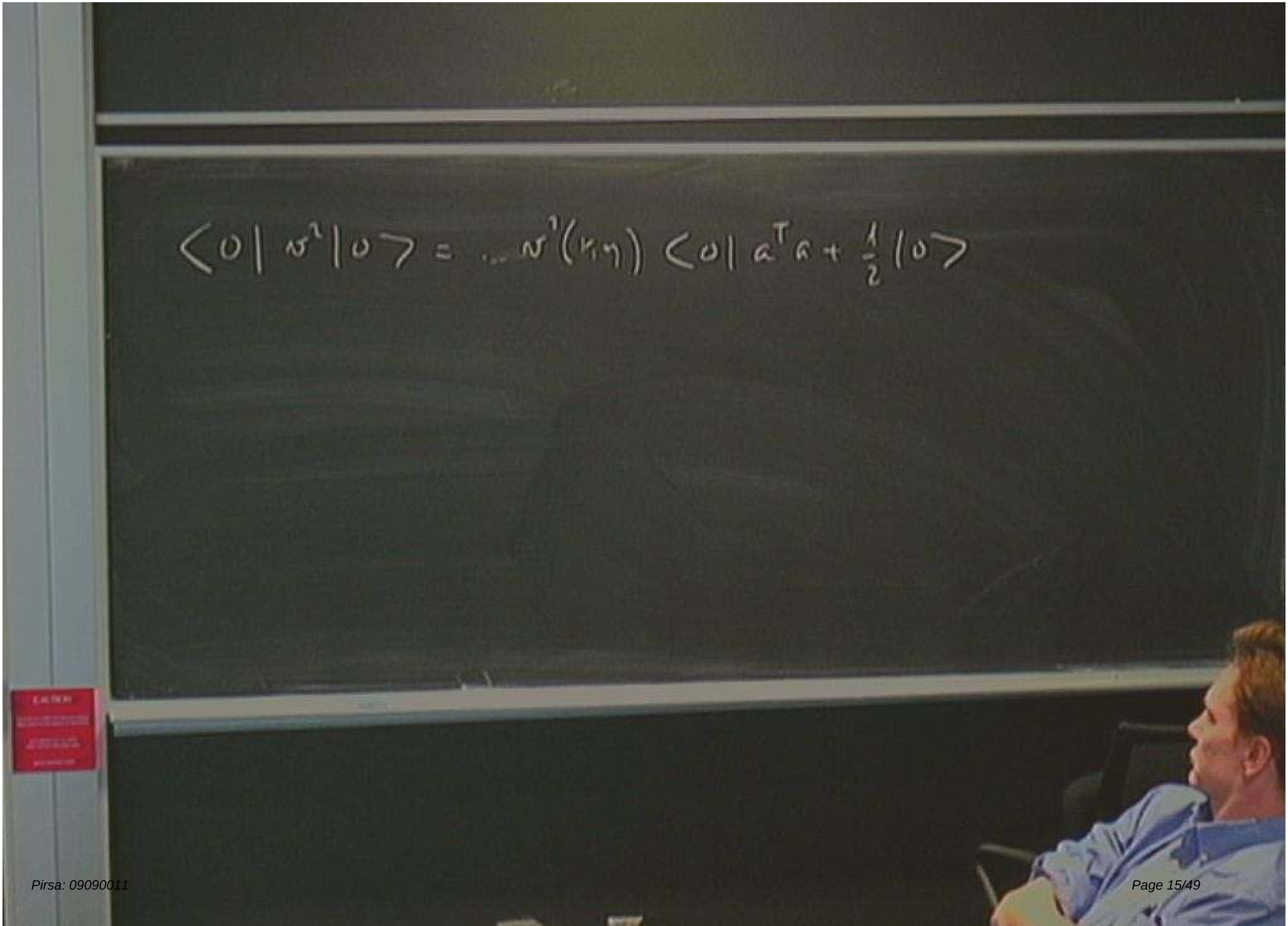
Inflation

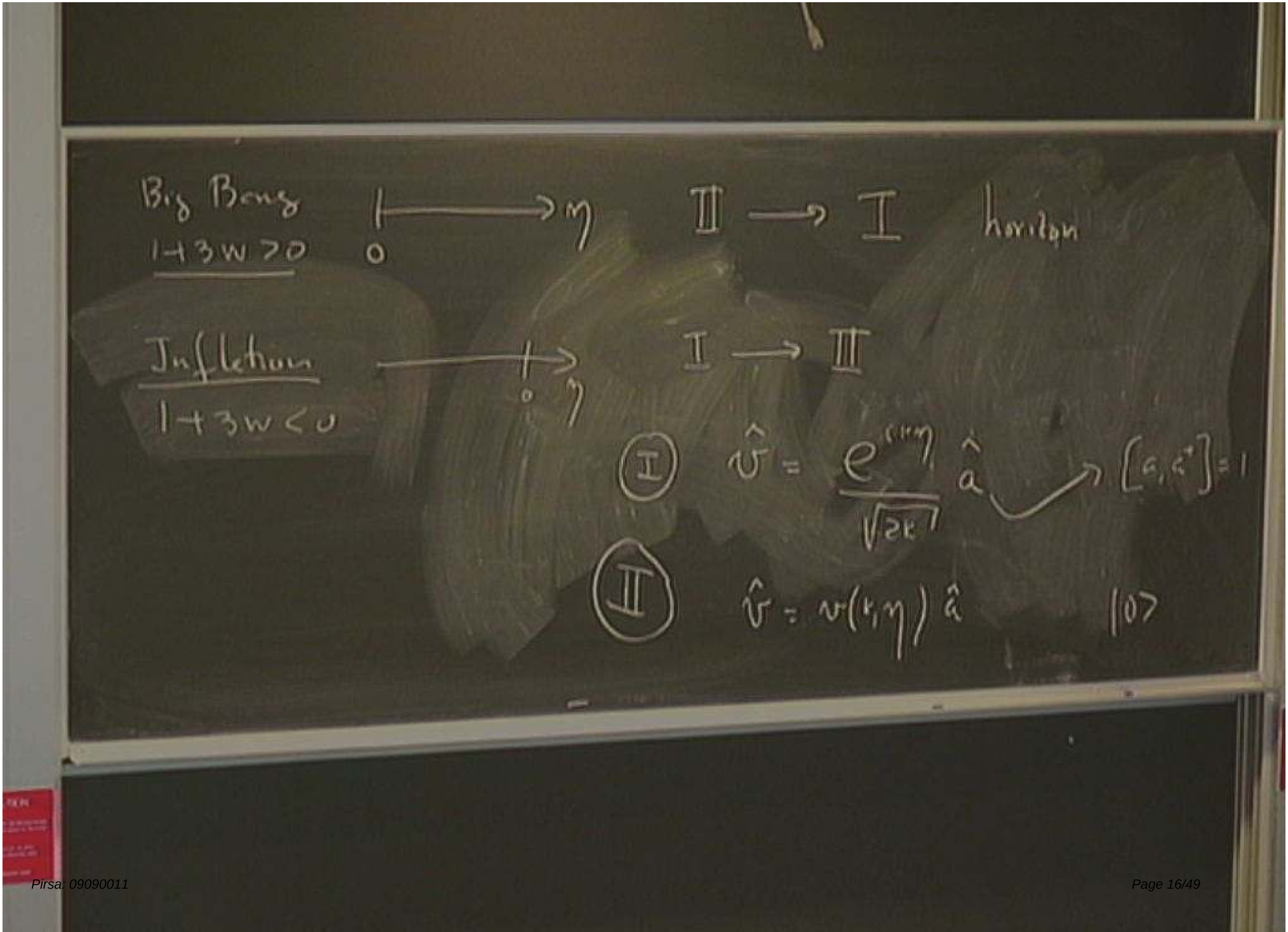
$1+3w<0$

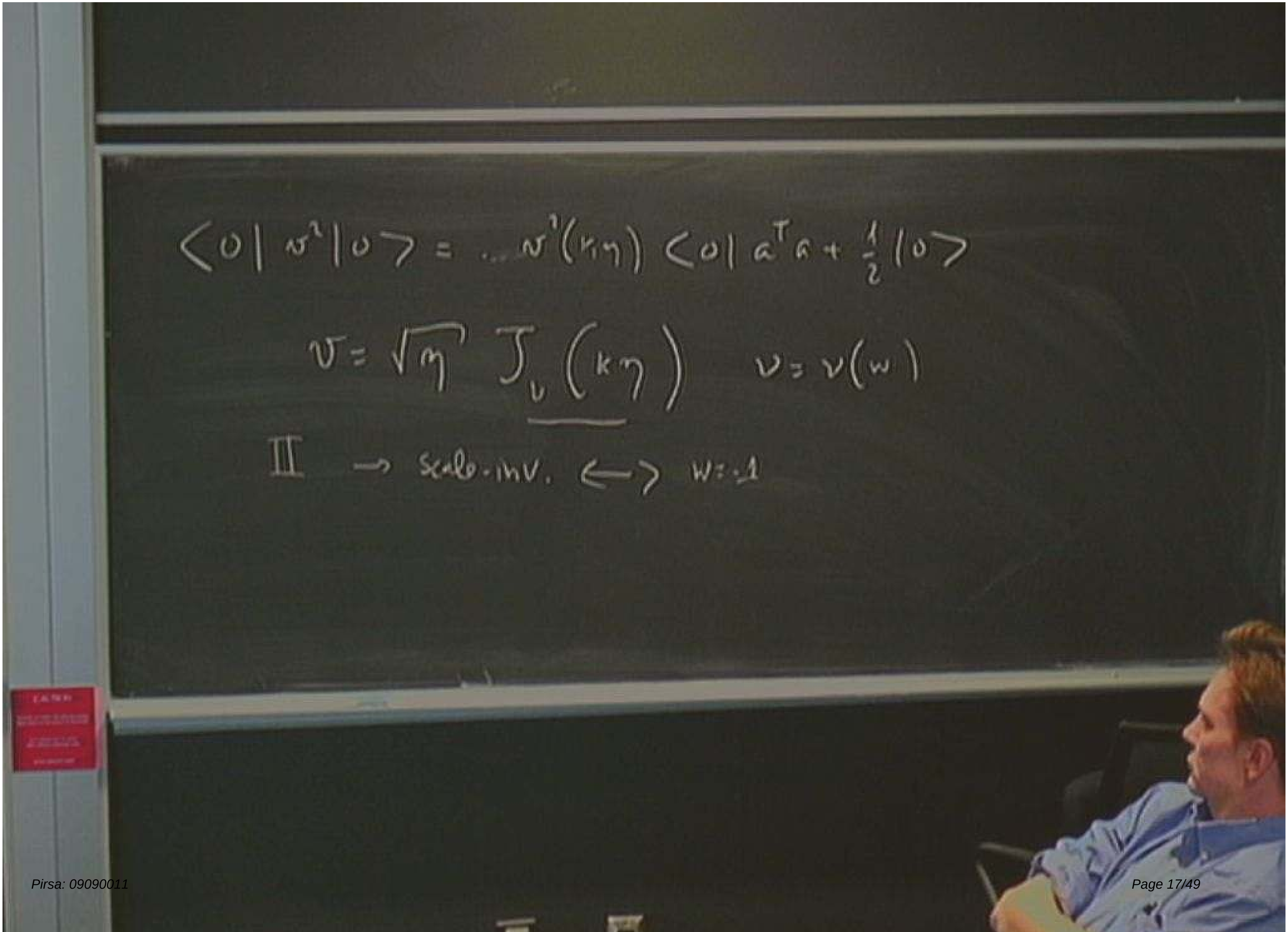
$\longrightarrow\!\underset{0}{\vert}\!\rightarrow \eta$ $\qquad$ $\mathrm{I} \longrightarrow \mathrm{II}$

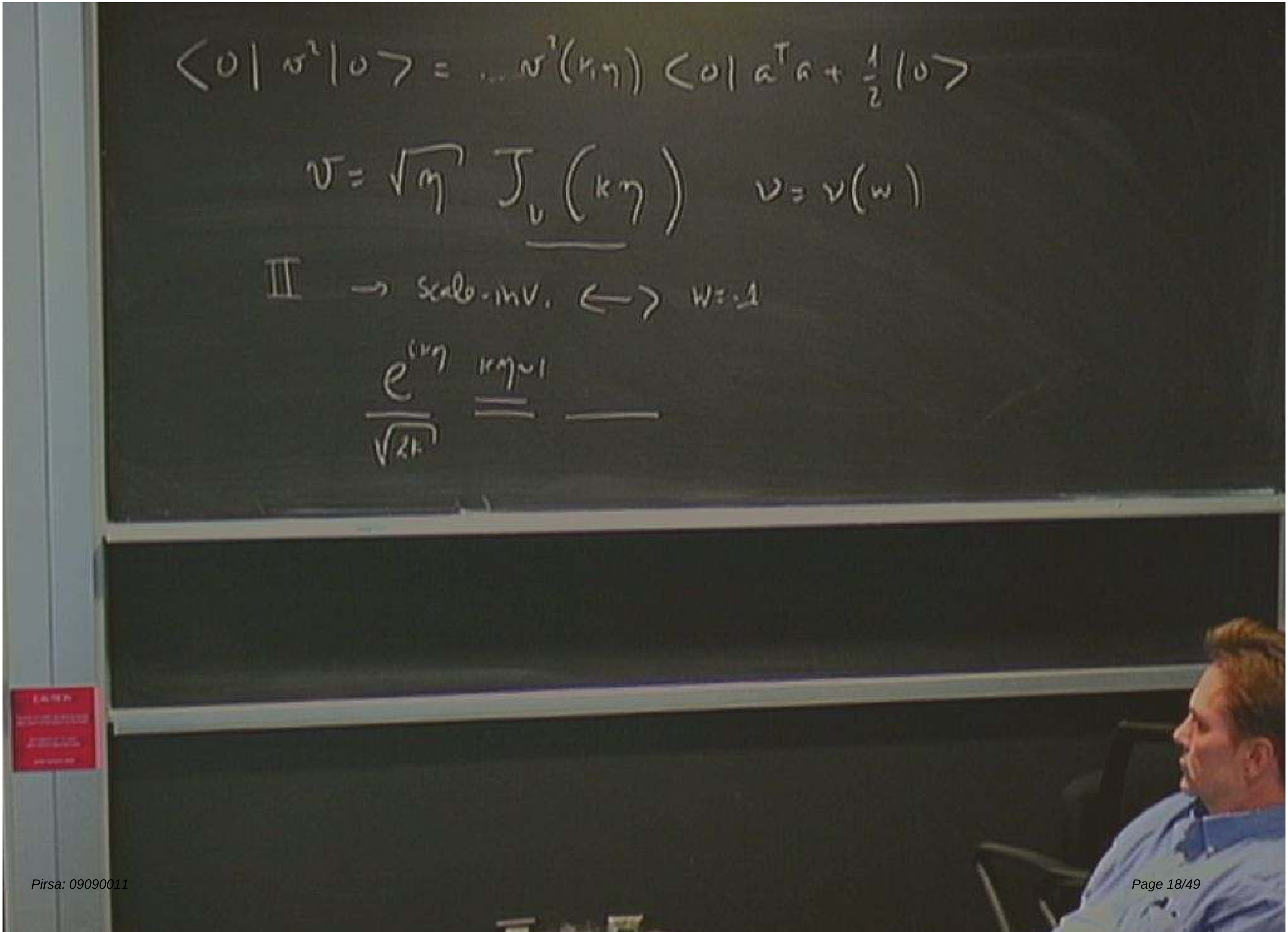
$\mathrm{I}$ $\quad \hat{v} = \dfrac{e^{ik\eta}}{\sqrt{2k}}\,\hat{a} \longrightarrow [a, a^\dagger] = 1$

$\mathrm{II}$ $\quad \hat{v} = v(k,\eta)\,\hat{a}$ $\qquad |0\rangle$


$\langle 0| v^2 |0\rangle = \dots v^2(k,\eta) \langle 0| a^\dagger a + \frac{1}{2} |0\rangle$


Big Bang
1+3w>0
0 η
II → I
horizon
Inflation
1+3w<0
0 η
I → II
I v̂ = e^{ikη}/√(2k) â
[a, a†] = 1
II v̂ = v(k,η) â
|0>


$\langle 0| v^2 |0\rangle = \ldots v^2(k,\eta) \langle 0| a^\dagger a + \frac{1}{2} |0\rangle$
$v = \sqrt{\eta}\, J_\nu(k\eta)$
$\nu = \nu(w)$
III $\rightarrow$ scale-inv. $\longleftrightarrow$ $w = -1$



$\langle 0 | v^2 | 0 \rangle = \ldots v^2(k,\eta) \langle 0 | a^\dagger a + \frac{1}{2} | 0 \rangle$
$v = \sqrt{\eta}\, J_\nu(k\eta)$ $\nu = \nu(w)$
III $\rightarrow$ scale-inv. $\longleftrightarrow$ $w = -1$
$\frac{e^{ik\eta}}{\sqrt{2k}} \overset{k\eta \sim 1}{=}$

Big Bang $\longmapsto \eta$ 0

$1+3w>0$

$\mathrm{II} \to \mathrm{I}$ horizon

Inflation $\longrightarrow$ 0 $\eta$

$1+3w<0$

$\mathrm{I} \to \mathrm{III}$

$\mathrm{I}$ $\hat{v} = \frac{e^{ik\eta}}{\sqrt{2k}} \hat{a} \to [a, a^{+}] = 1$

$\mathrm{II}$ $\hat{v} = v(k, \eta)\, \hat{a}$ $|0\rangle$

$$\zeta = \frac{v}{z} \qquad v - \text{2nd quant. QFT} \qquad z \propto \frac{R}{c_s} \propto \eta^{\frac{2}{1+3w}} \;\hookrightarrow \frac{p}{\rho}$$

$$v'' + \left[ c_s^2 k^2 - \frac{z''}{z} \right] v = 0$$

$$\text{I} \qquad \text{II} \;\; \hookrightarrow \frac{\nu(\nu+1)}{\eta^2}$$

I

Oscill.

microphys.

inside hor.

$\frac{p}{\rho} \simeq -1$

II

grav. instab.

$v \propto z$

$\zeta \rightarrow$ freezes

Big Bang $\quad c_s \propto \eta^{-\alpha} \quad \alpha > 1$

$1+3w>0 \quad \underset{0}{\vdash}\!\longrightarrow \eta \qquad \mathrm{III} \rightleftarrows \mathrm{I}$ horizon

Inflation

$\cancel{1+3w<0} \quad \longrightarrow \underset{0}{|} \eta \qquad \mathrm{I} \longrightarrow \mathrm{III}$

$\mathrm{I} \quad \hat{v} = \frac{e^{ik\eta}}{\sqrt{2k}} \hat{a} \longrightarrow [a, a^\dagger] = 1$

$\mathrm{II} \quad \hat{v} = v(k,\eta)\, \hat{a} \qquad |0\rangle$

Big Bang $c_s \propto \eta^{-\alpha}$ $\alpha > 1$

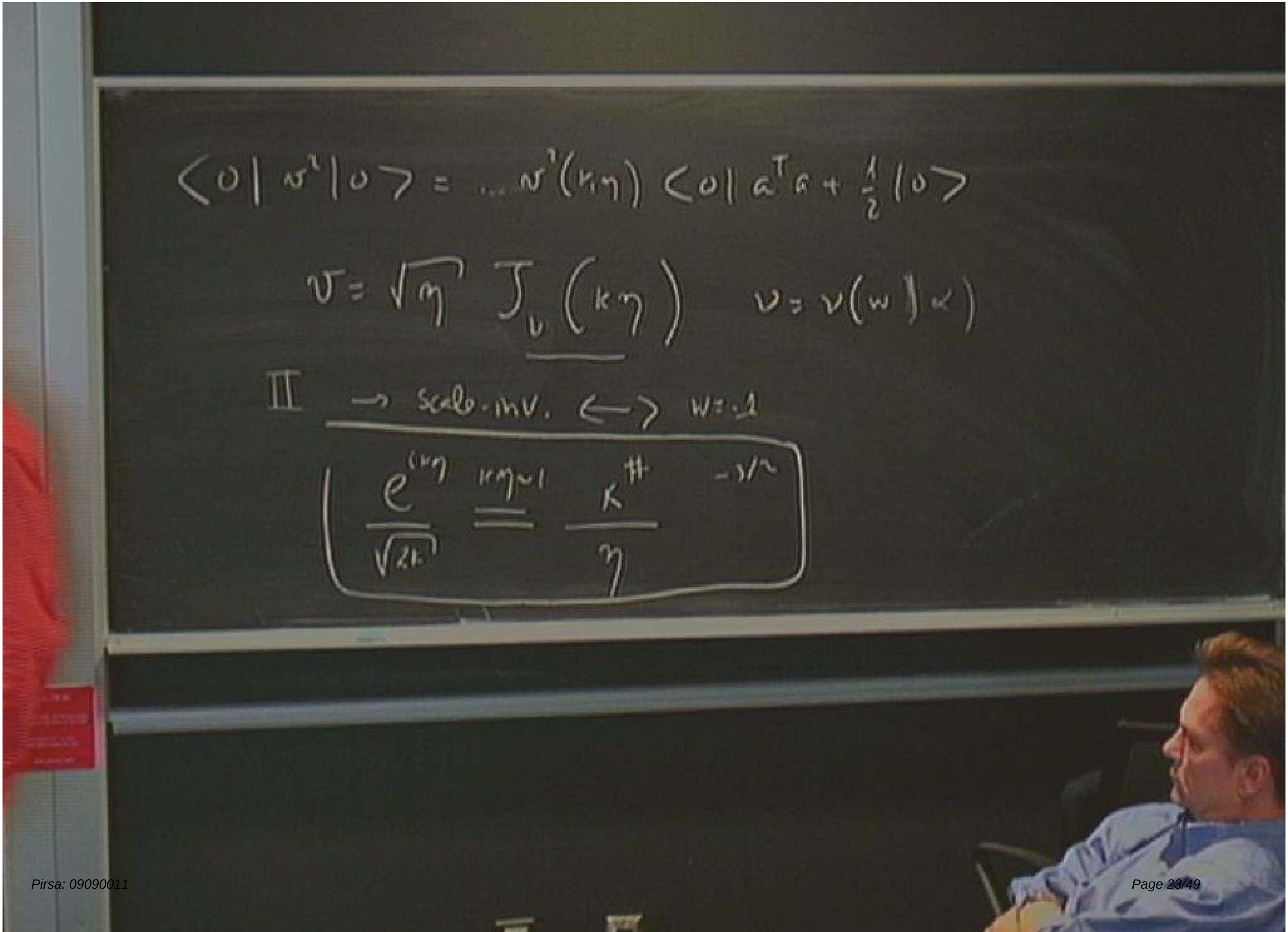
$\underline{1+3w>0}$ $\quad 0 \longrightarrow \eta \quad$ III $\rightleftarrows$ I $\quad$ horizon

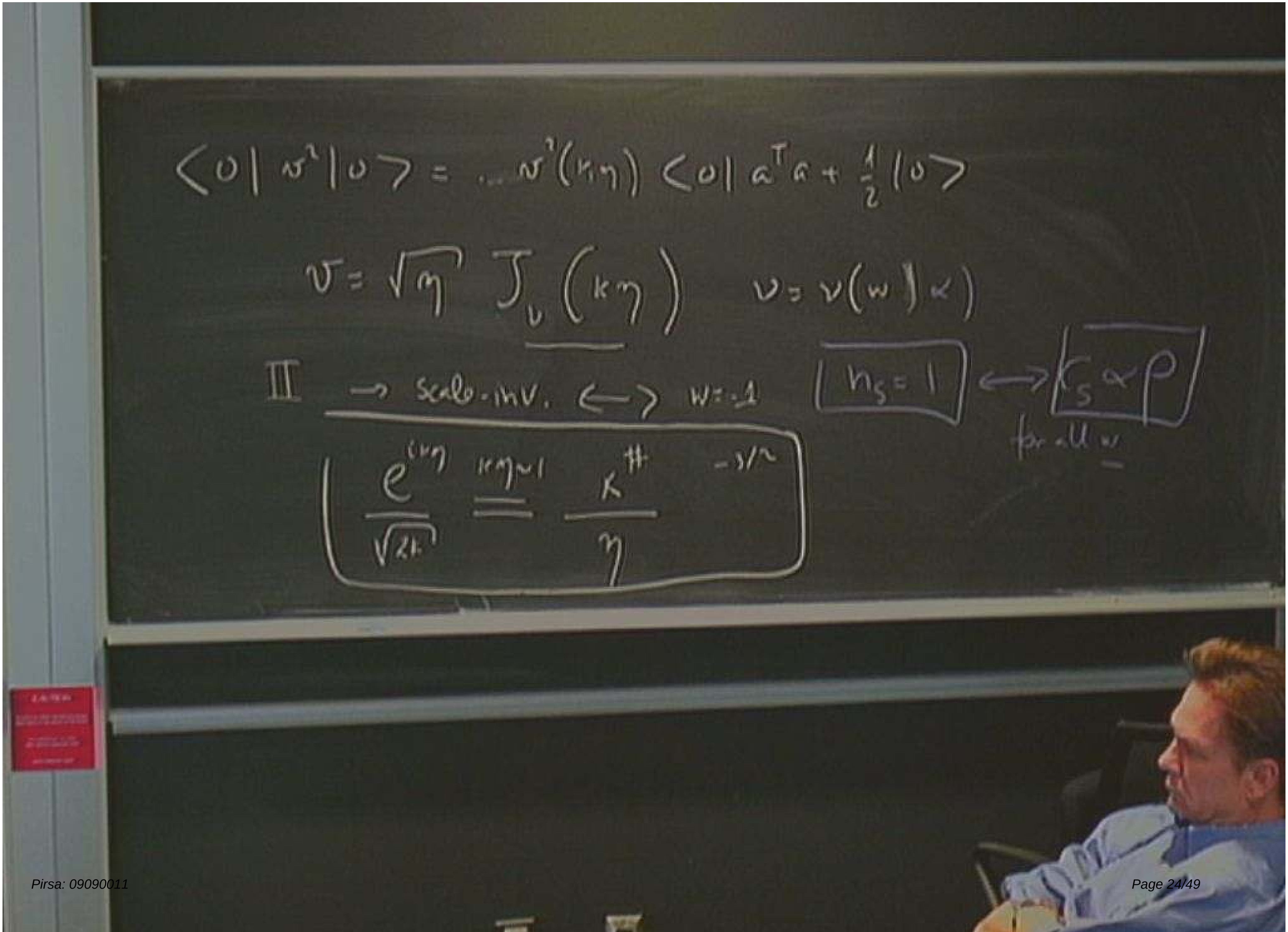
$\underline{\text{Inflation}}$

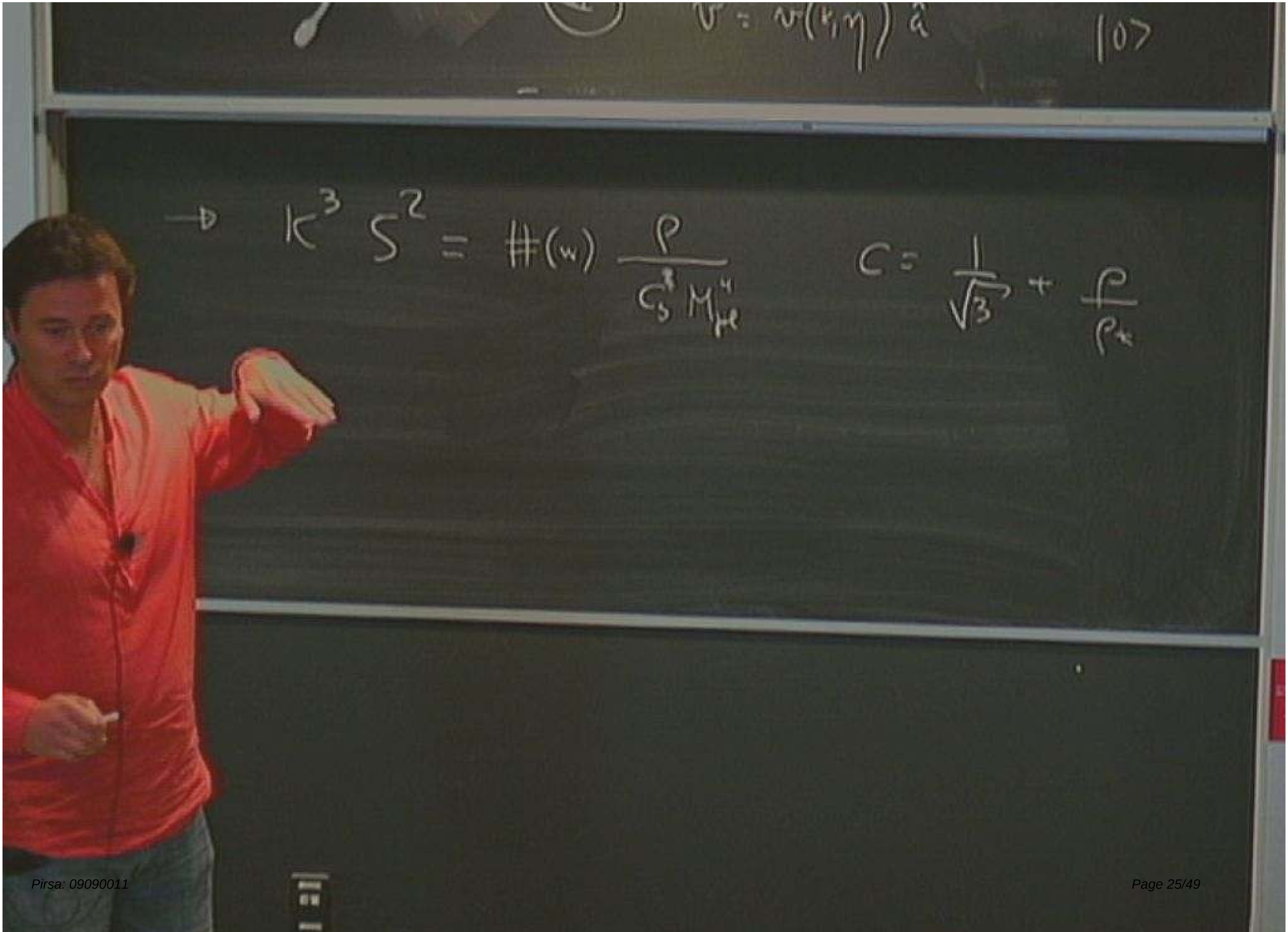
$1+3w<0$ $\quad \longrightarrow \eta$ (0) $\quad$ I $\longrightarrow$ III

I $\quad \hat{v} = \dfrac{e^{i\int k c_s d\eta}}{\sqrt{2kc_s}}\,\hat{a} \longrightarrow [a, a^\dagger] = 1$

II $\quad \hat{v} = v(k,\eta)\,\hat{a}$ $\quad |0\rangle$

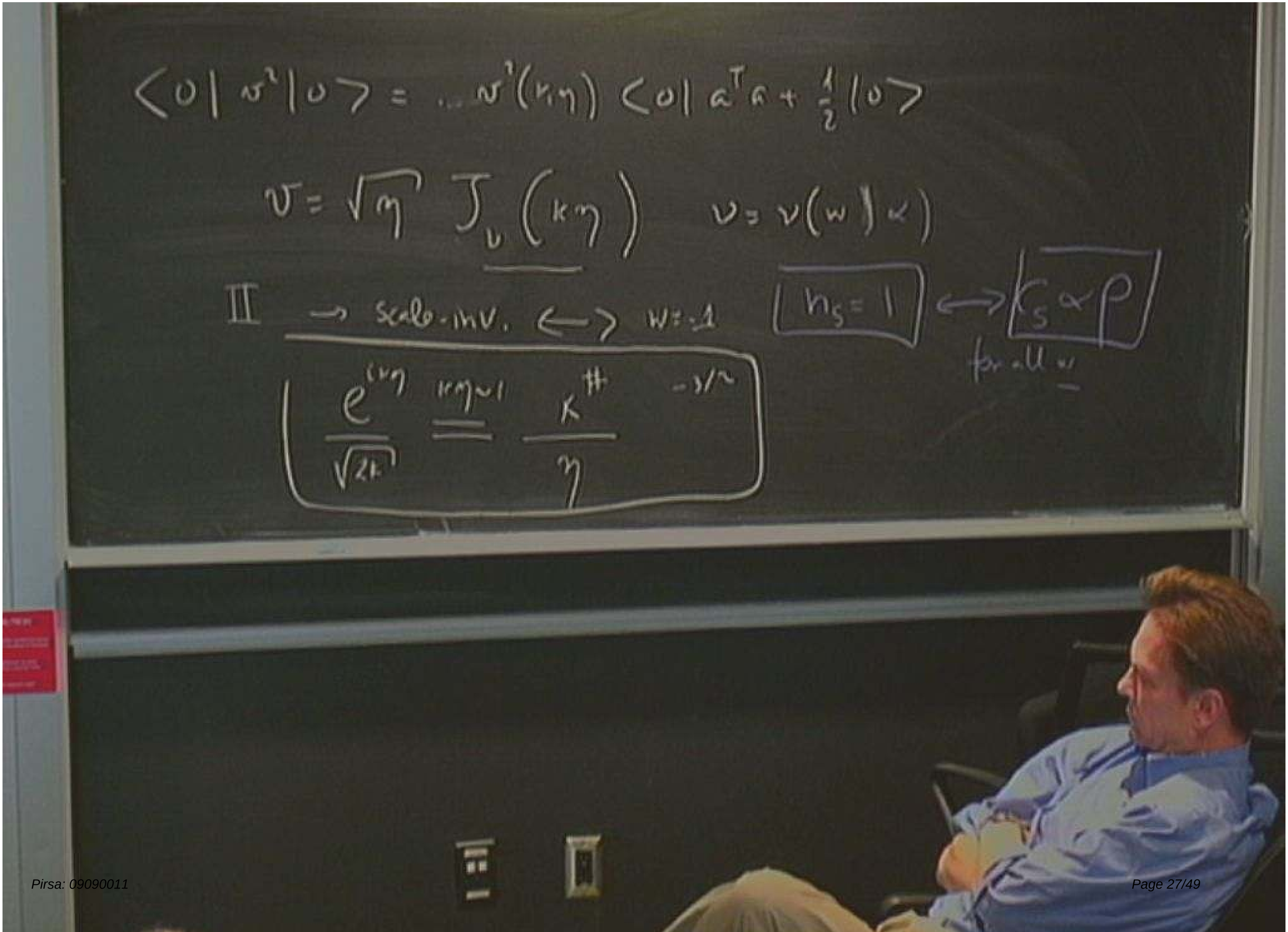

⟨0| v² |0⟩ = ... v²(kη) ⟨0| a†a + 1/2 |0⟩
v = √η J_ν (kη)
ν = ν(w | α)
III → scale inv. ⟷ w = -1
e^{ikη} / √2k
kη~1
k^# / η
-3/2


⟨0| v² |0⟩ = ... v²(kη) ⟨0| a†a + 1/2 |0⟩
v = √η J_ν (kη)
ν = ν(w) κ)
III → scale-inv. ⟷ w = -1
n_s = 1 ⟷ k_s ∝ ρ
for all w
e^{ikη} / √2k
kη~1
k^# / η
-3/2

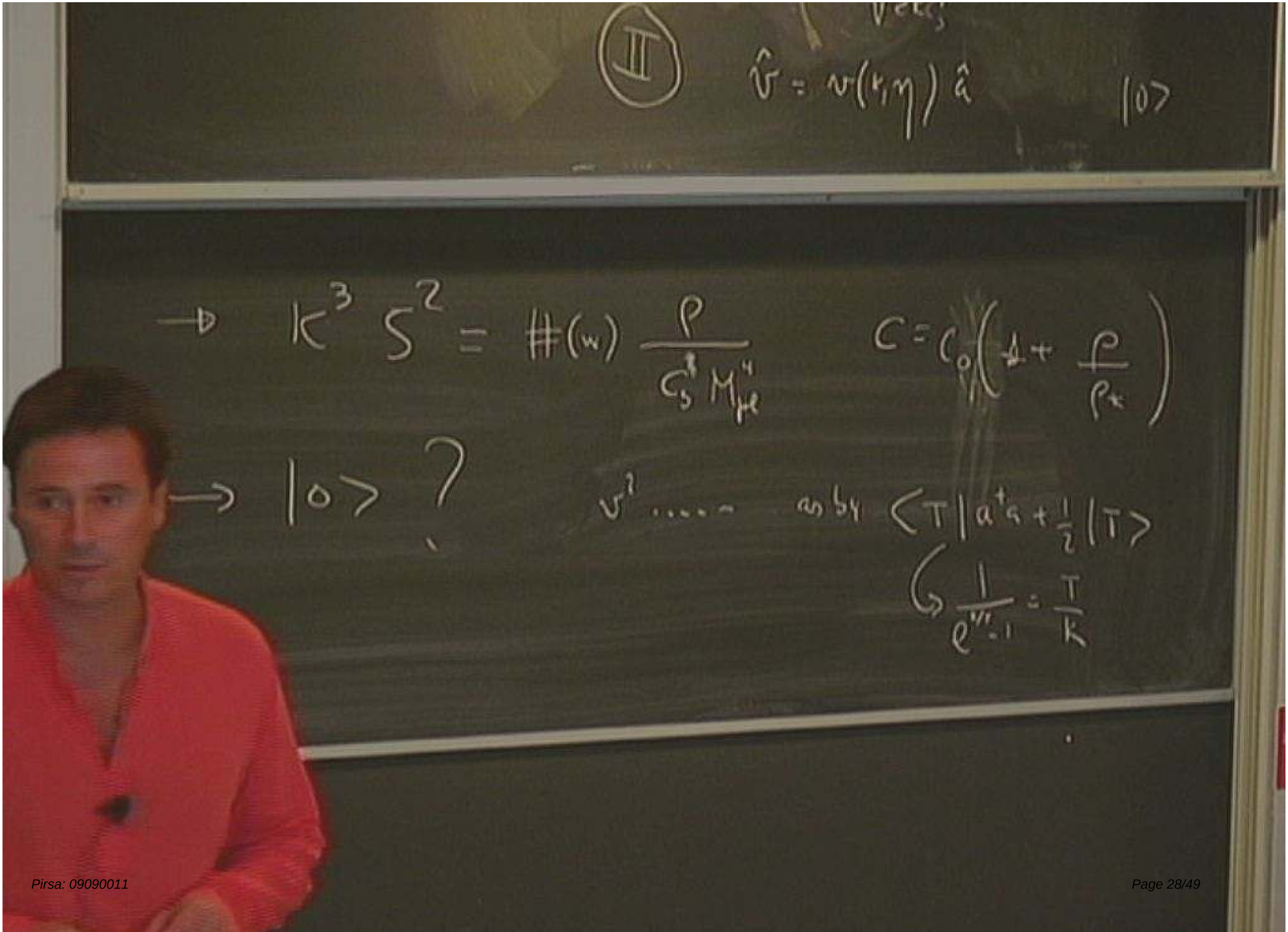

v = v(k,η) â
|0⟩
→ k³ S² = #(w) ρ / (c_s³ M_pl⁴)
c = 1/√3 + ρ/ρ_*

$v = v(k,\eta)\,\hat{a}$ $\quad |0\rangle$

$$\to \quad k^3 S^2 = \#(w)\,\frac{\rho}{c_s^3 M_{pl}^4} \qquad c = \frac{1}{\sqrt{3}} + \frac{\rho}{\rho_*}$$



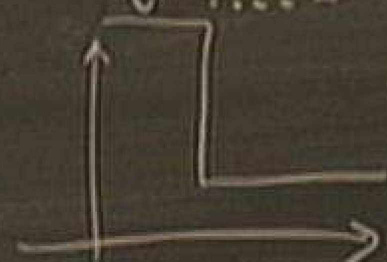
<0| v² |0> = ... v²(k,η) <0| a†a + 1/2 |0>
v = √η J_ν (kη)   ν = ν(w | κ)
II → scale-inv. ⟷ w = -1
n_s = 1 ⟷ K_S ∝ ρ
for all w
e^{ikη} / √2k   kη~1   k^# / η   -3/2


II
$\hat{v} = v(k,\eta)\,\hat{a}$
$|0\rangle$
$\rightarrow k^3 \zeta^2 = \#(w) \frac{\rho}{c_s^3 M_{pl}^4}$
$C = C_0\left(1 + \frac{\rho}{\rho_*}\right)$
$\rightarrow |0\rangle$ ?
$v^2$ .....
$\langle T | a^\dagger a + \frac{1}{2} | T \rangle$
$\frac{1}{e^{w/T} - 1} \approx \frac{T}{k}$

(II) $\hat{v} = v(k,\eta)\,\hat{a}$ $\qquad |0\rangle$

$$\to \quad k^3 \zeta^2 = \#(w)\,\frac{\rho}{c_s^3 M_{pl}^4} \qquad c = c_0\left(1 + \frac{\rho}{\rho_*}\right)$$

$$\to \quad |0\rangle \; ?$$




as by $\langle T|\, a^\dagger a + \frac{1}{2} \,|T\rangle$

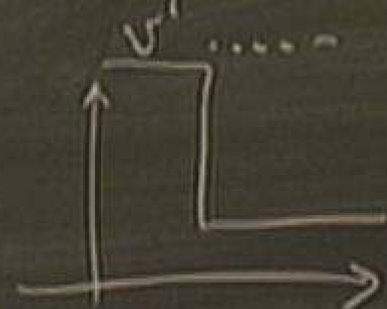
$$\hookrightarrow \frac{1}{e^{\omega/T}-1} = \frac{T}{k}$$

II

$$\hat{v} = v(k,\eta)\,\hat{a} \qquad |0\rangle$$

$$\rightarrow \quad k^3\,\zeta^2 = \#(w)\,\frac{\rho}{c_s^{3} M_{pl}^4} \qquad C = C_0\left(\cancel{1} + \frac{\rho}{\rho_*}\right)$$

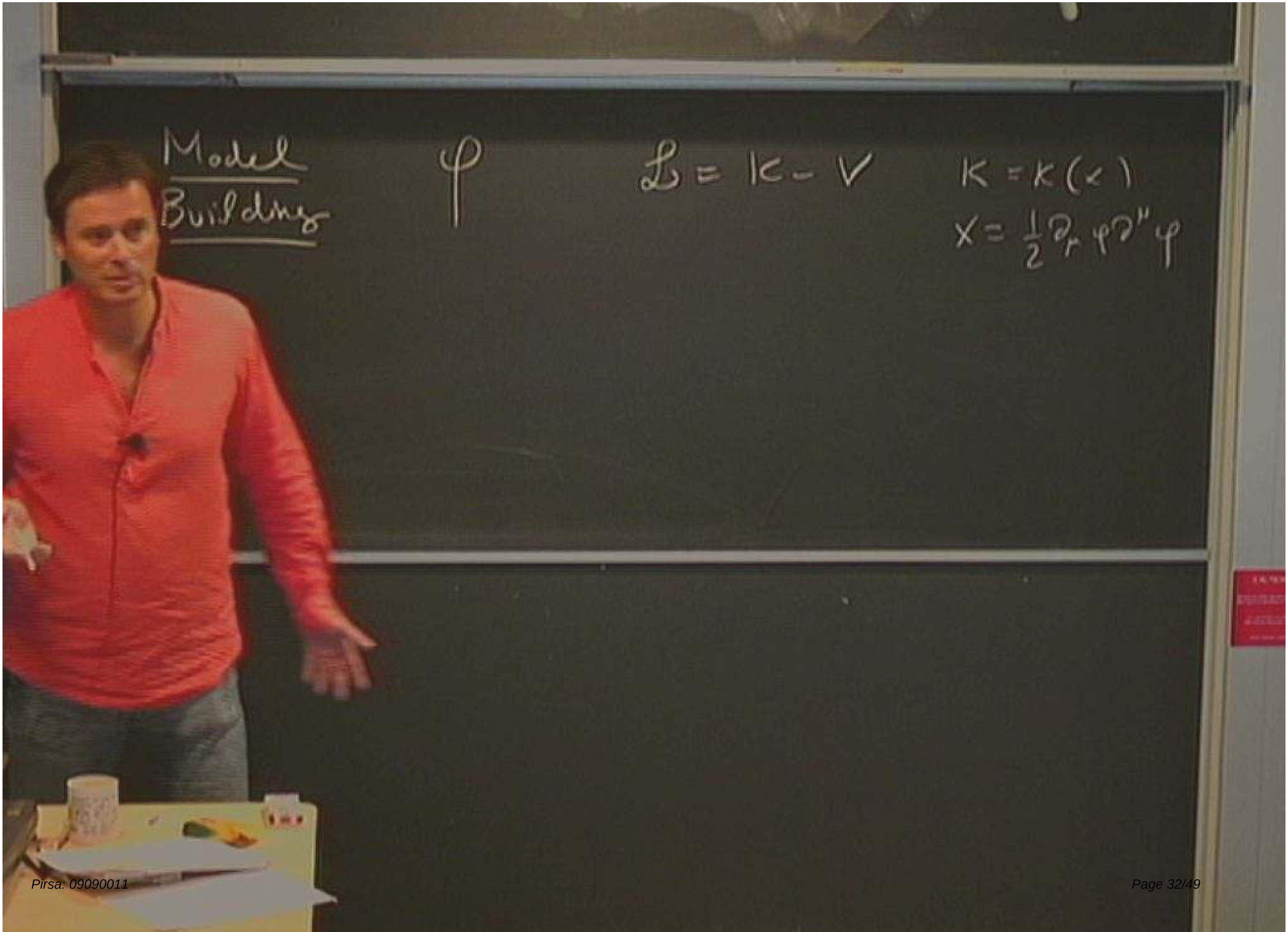
$$\rightarrow \quad |0\rangle \; ?$$




an by $\langle T | a^\dagger a + \frac{1}{2} | T \rangle$

$$\frac{1}{e^{\cdots}-1} = \frac{T}{k}$$

Model Building


Model Building
$\varphi$
$\mathcal{L} = K - V$
$K = K(X)$
$X = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi$

Model Building

$\varphi$

$$\mathcal{L} = K - V \qquad K = K(X)$$

$$X = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi$$

$$\begin{cases} p = K - V \\ \rho = 2X K_{,X} - K + V \end{cases}$$

$$c_s^2 = \frac{K_{,X}}{2K_{,XX}X + K_{,X}}$$

Model Building

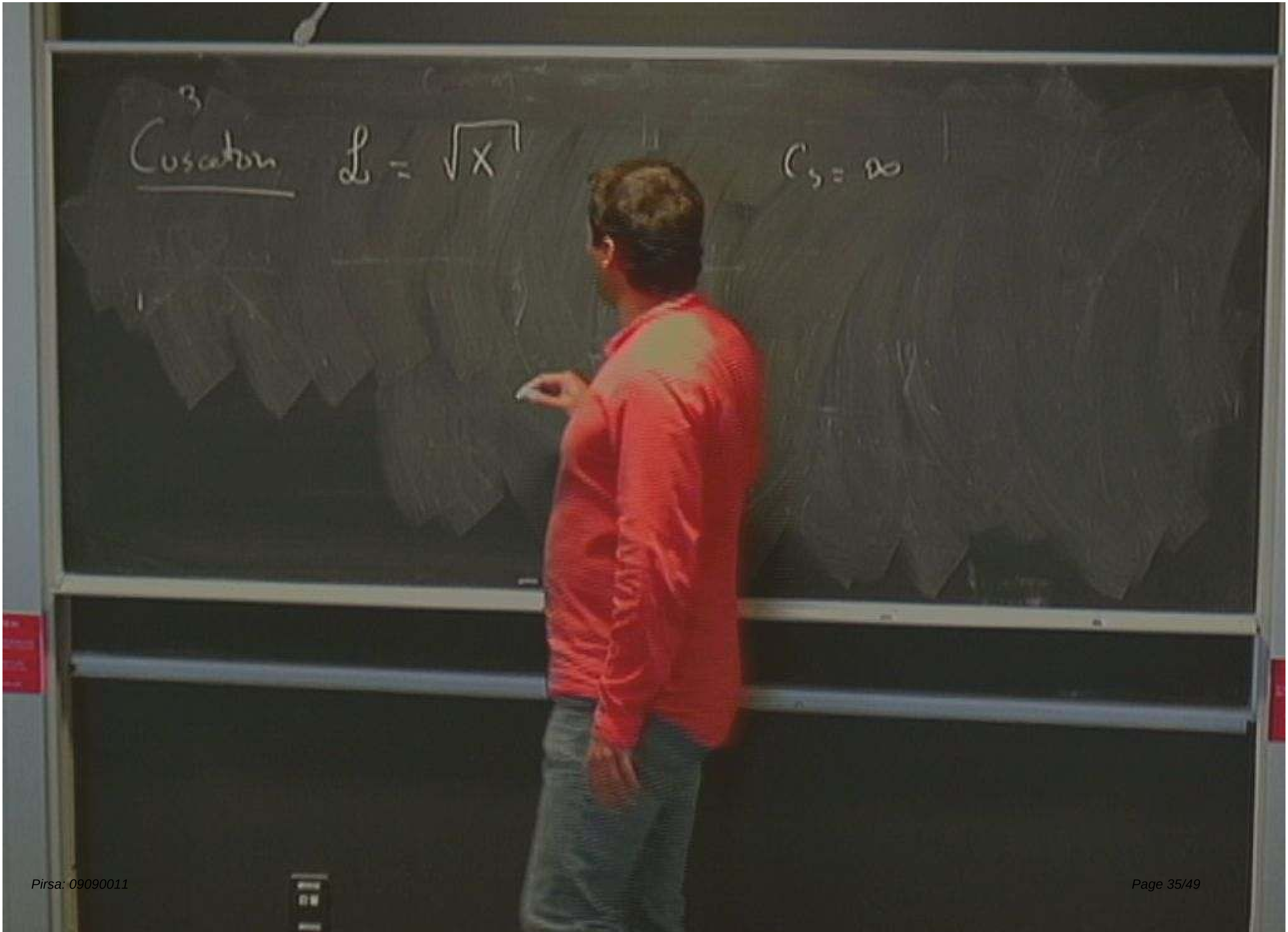
$\varphi$

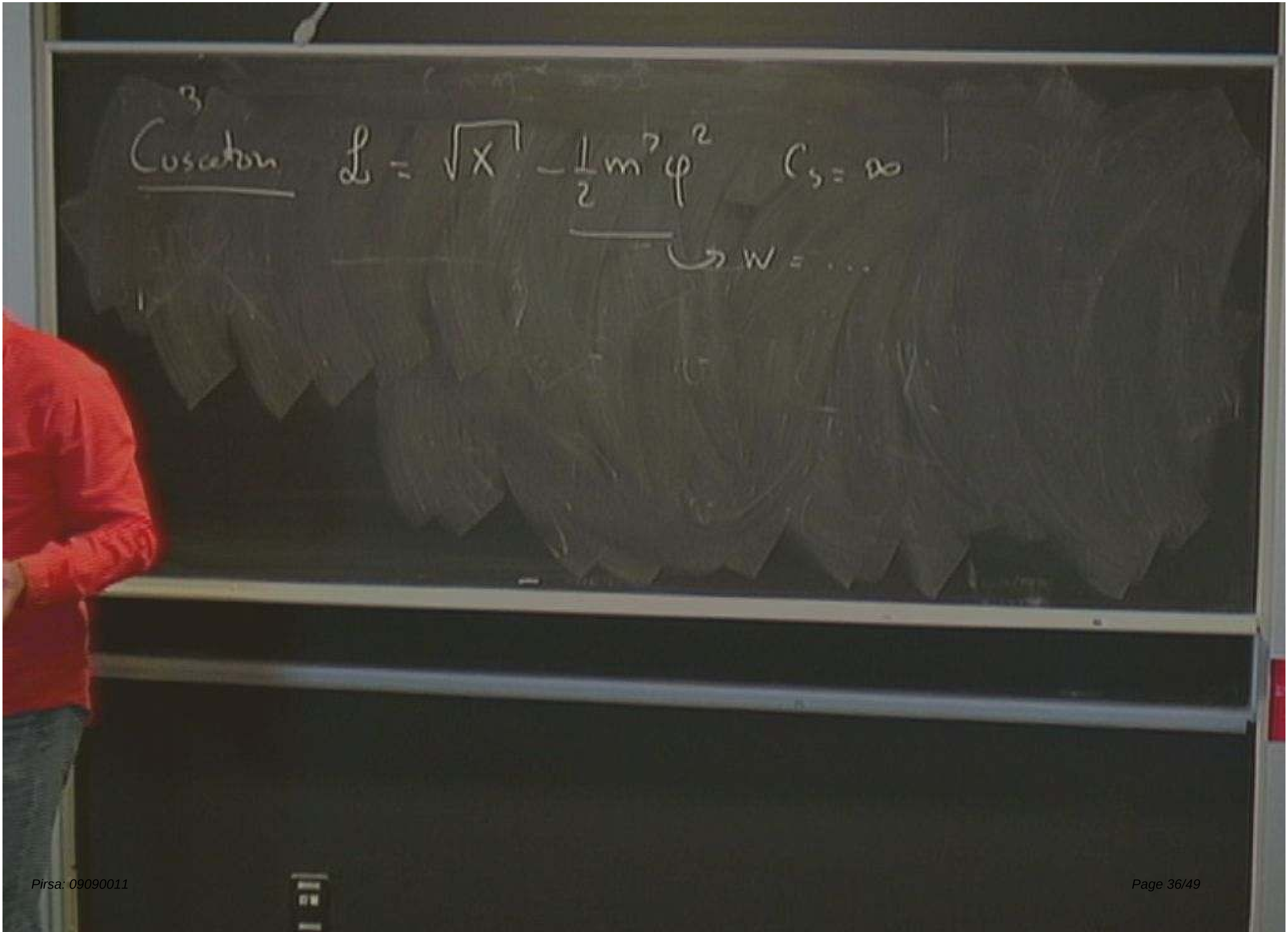
$$\mathcal{L} = K - V \qquad K = K(X)$$

$$X = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi$$

$$w \Leftarrow \begin{cases} p = K - V \\ \rho = 2XK_{,X} - K + V \end{cases}$$

$$c_s^2 = \frac{K_{,X}}{2K_{,XX}X + K_{,X}}$$


Cuscuton
$\mathcal{L} = \sqrt{X}$
$c_s = \infty$


3
Cuscaton $\mathcal{L} = \sqrt{X} - \frac{1}{2}m^2\varphi^2$ $c_s = \infty$
$w = \dots$

Cuscaton[3] $\mathcal{L} = \sqrt{X} - \frac{1}{2} m^2 \varphi^2$ $\quad c_s = \infty$

$\hookrightarrow w = \dots$

$\mathcal{L} \to \mathcal{L} + X^{\beta}$ $\quad \beta < \frac{1}{2} \quad X \gg 1$

3

**Cuscaton** $\mathcal{L} = \sqrt{X} - \frac{1}{2} m^2 \varphi^2$ $\quad c_s = \infty$

$\hookrightarrow W = \dots$

$\mathcal{L} \to \mathcal{L} + X^{\beta}$

$\beta < \frac{1}{2}$ $\quad X \gg 1$

$\beta = -\frac{1}{2}$

$\boxed{c_s \propto \rho}$

$$\mathcal{L} = \frac{1}{f}\sqrt{1+2fX} - \frac{1}{f} - \frac{1}{2}m^2\varphi^2$$

$+ \ldots$

$\cancel{f<0}$ $f>0$

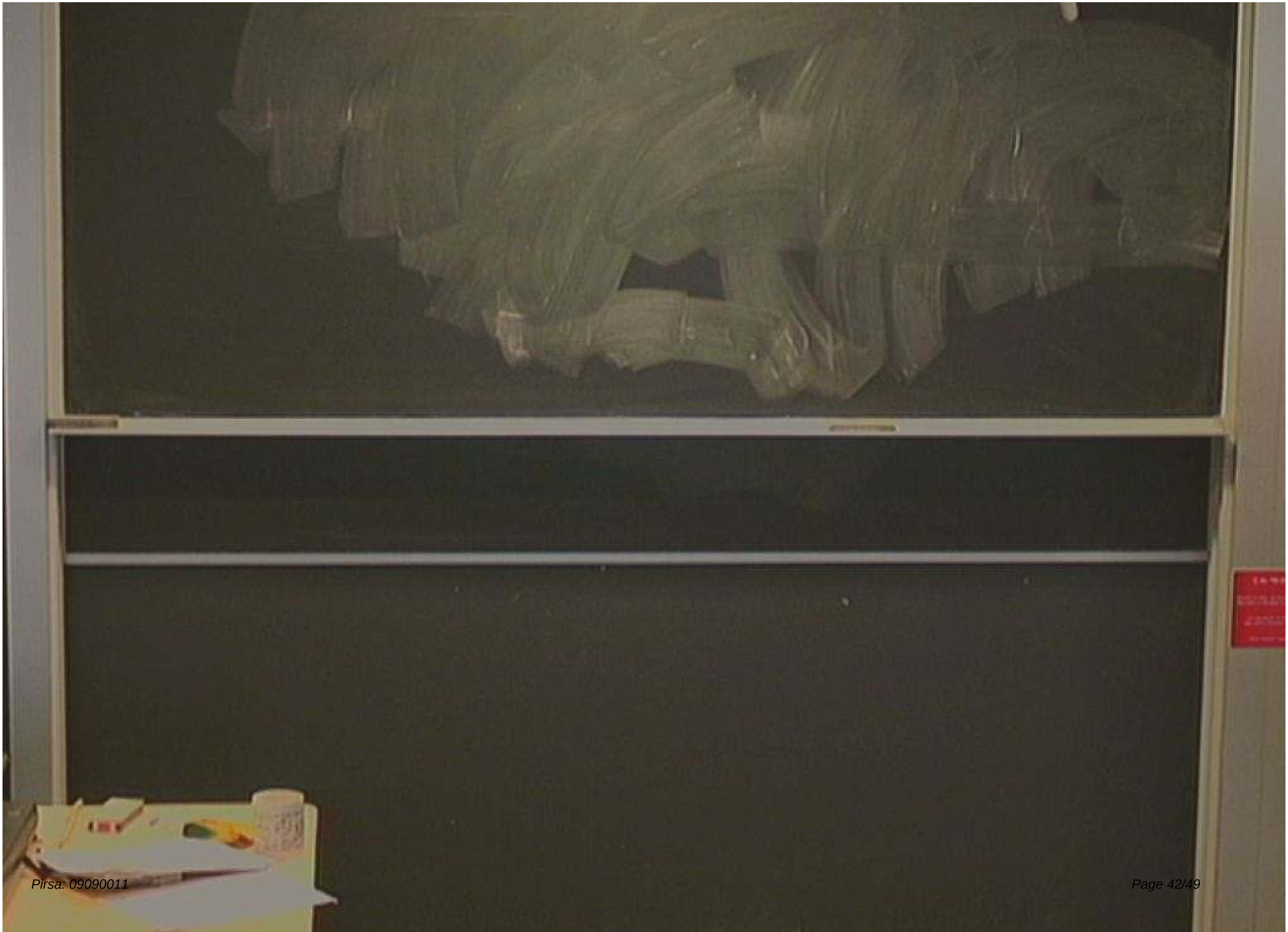
$$\mathcal{L} = \frac{1}{f}\sqrt{1+2fX} - \frac{1}{f} - \frac{1}{2}m^2\varphi^2$$

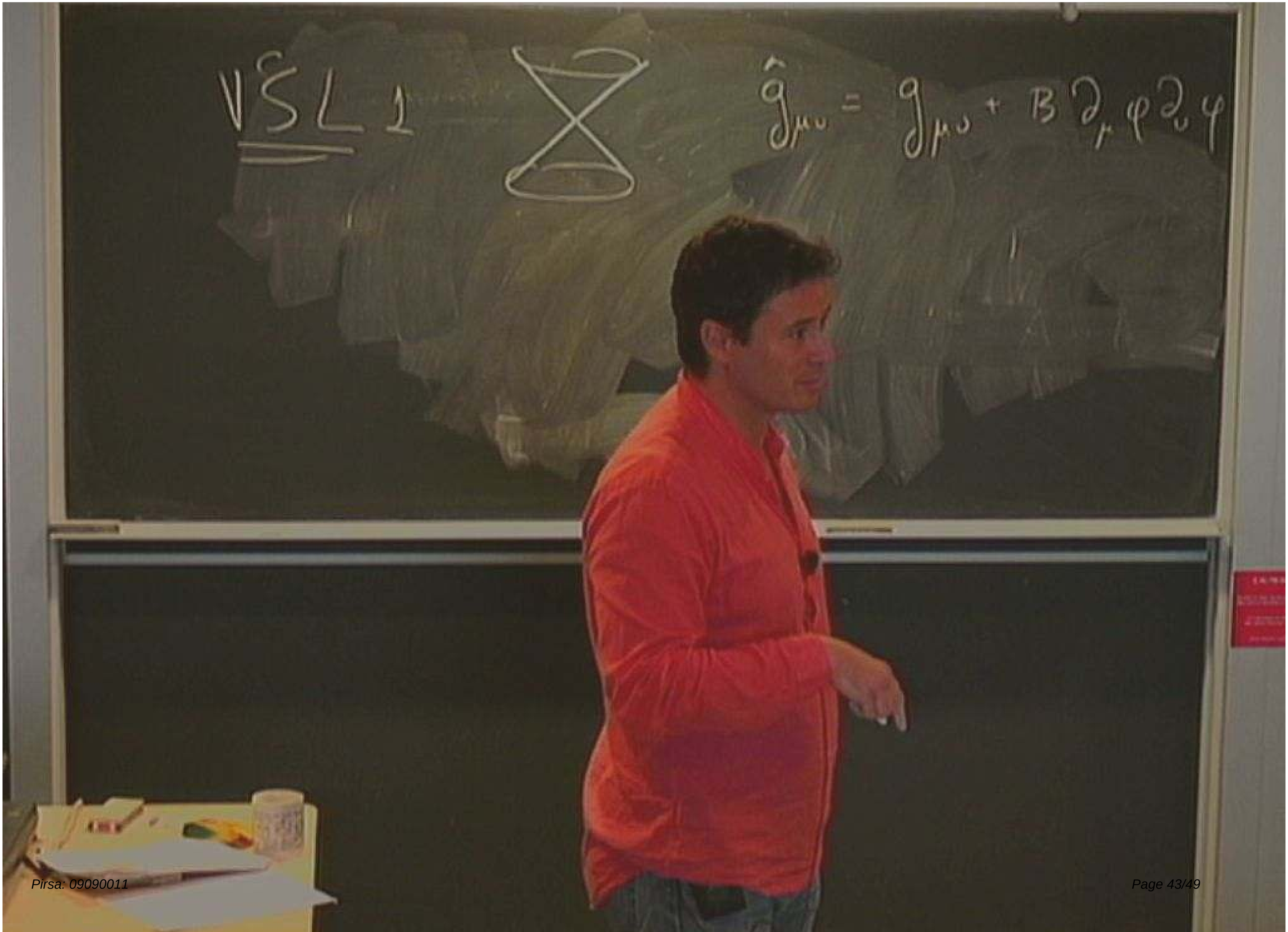
$+ \ldots$

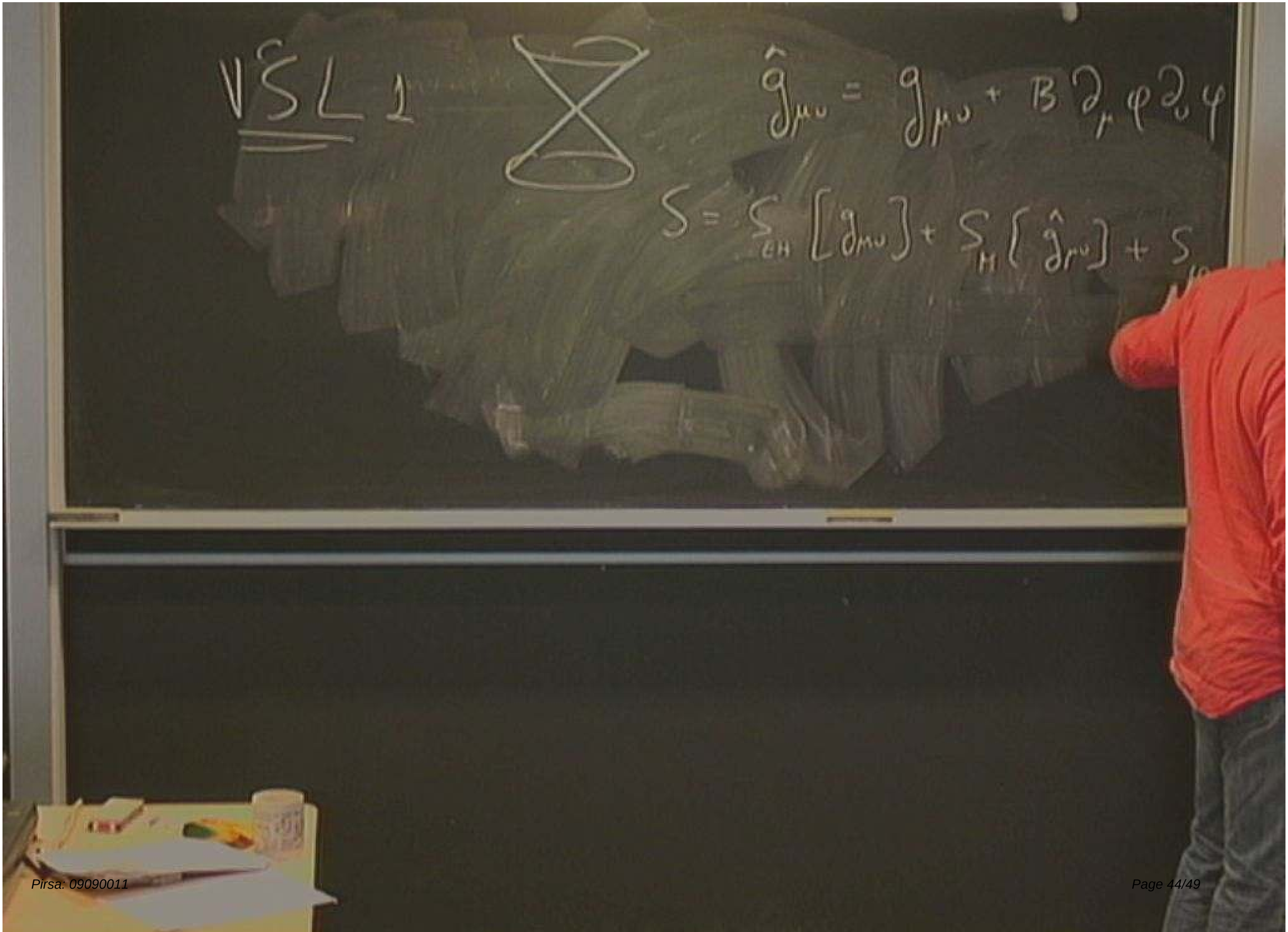
$\cancel{f<0}$ $f>0$ $X \gg 1$

$$\mathcal{L} = \frac{1}{f}\sqrt{1+2fX} - \frac{1}{f} - \frac{1}{2}m^2\varphi^2 + \dots$$

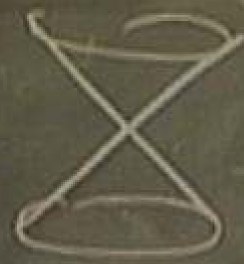
$\cancel{f<0}$ $\quad f>0 \qquad X \gg 1$




VSL 1
$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\,\partial_\mu \varphi\, \partial_\nu \varphi$


VSL 1
$\hat{g}_{\mu\nu} = g_{\mu\nu} + B \partial_\mu \varphi \partial_\nu \varphi$
$S = S_{EH}[g_{\mu\nu}] + S_M[\hat{g}_{\mu\nu}] + S$

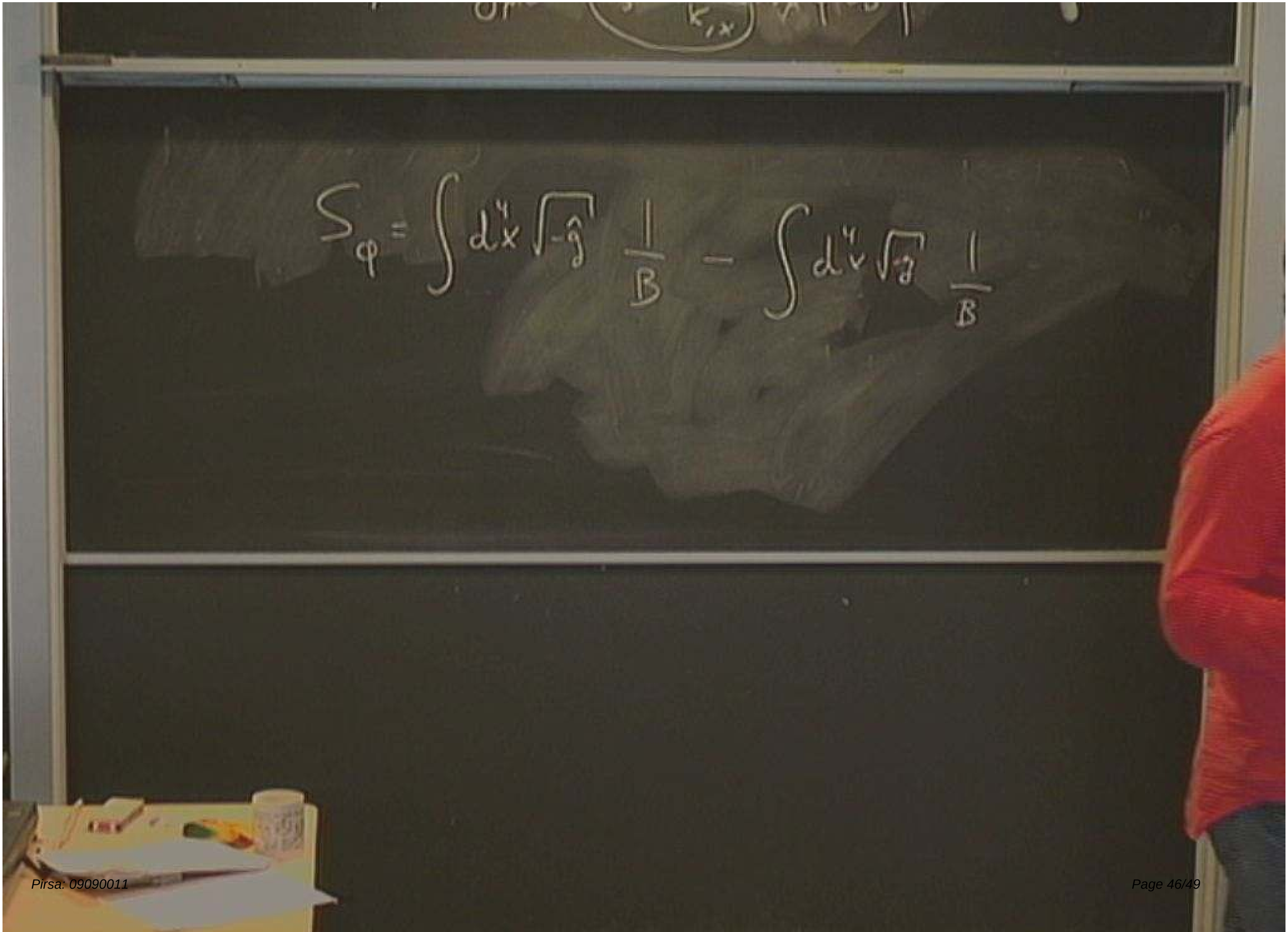
VSL 1



$$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\,\partial_\mu\varphi\,\partial_\nu\varphi$$

$$S = S_{EH}[g_{\mu\nu}] + S_M[\hat{g}_{\mu\nu}] + S_\varphi$$

$$B = B(\varphi, x)$$


$S_\varphi = \int d^4x \sqrt{-\hat{g}}\ \frac{1}{B} - \int d^4x \sqrt{-g}\ \frac{1}{B}$

$$S_\varphi = \int d^4x \sqrt{-\hat{g}}\, \frac{1}{B} - \int d^4x \sqrt{-g}\, \frac{1}{B}$$

$B = \text{const}$

VSL 2

$K_i = U L_{0i} U^{-1}$ $U(E, p) = \ldots$

$\hookrightarrow E_p$ invariant

$$c = \frac{dE}{dp} = c(p)$$

$K$ $\quad K_{ph} = \frac{k}{R}$ $\quad v'' + \left[c_s^2 k^2 - \frac{z''}{z}\right] v = 0$

$n_s = 1 \Longleftrightarrow E^2 = p^2(1 + (\lambda p)^2)^2 \quad c_s = c_s\left(\frac{k}{R}\right)$

$VW$

$E \propto p^3$