Title: Generating scale-invariant fluctuations without inflation

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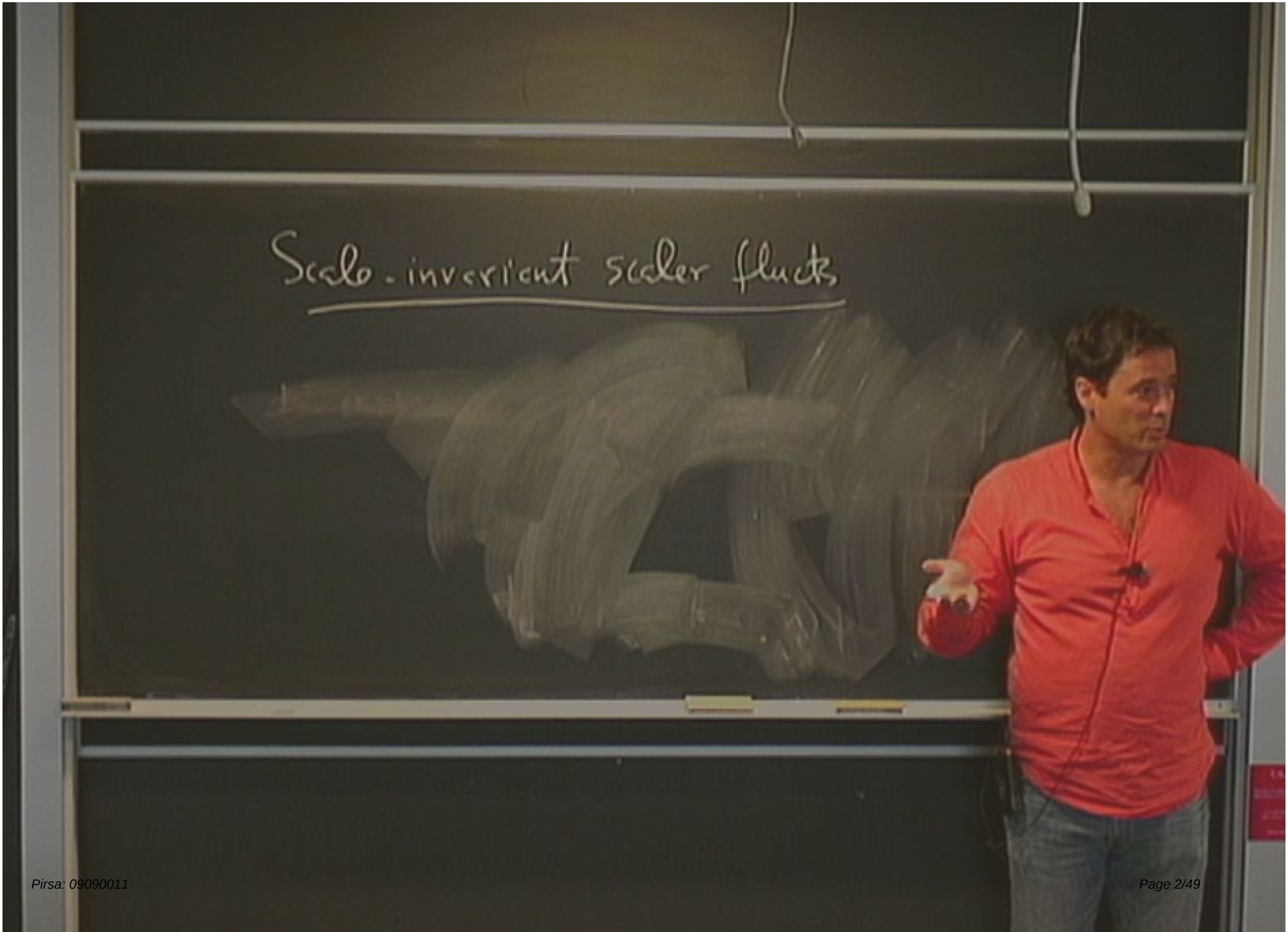
Abstract: I describe a number of techniques that allow for the generation of (near) scale-invariant fluctuations in the early Universe without inflation or ekpyrosis. The basic ingredient is a decaying maximal speed of propagation, for which a Universal law is found. Connections are made with k-essence, the cuscaton, and the DBI action. However the simplest realizations result from bimetric theories and deformed dispersion relations and DSR. A number of implications to theories of quantum gravity are discussed.


Scale-invariant scalar flucts

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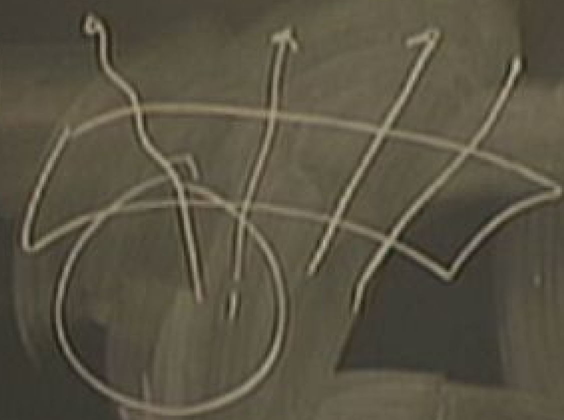
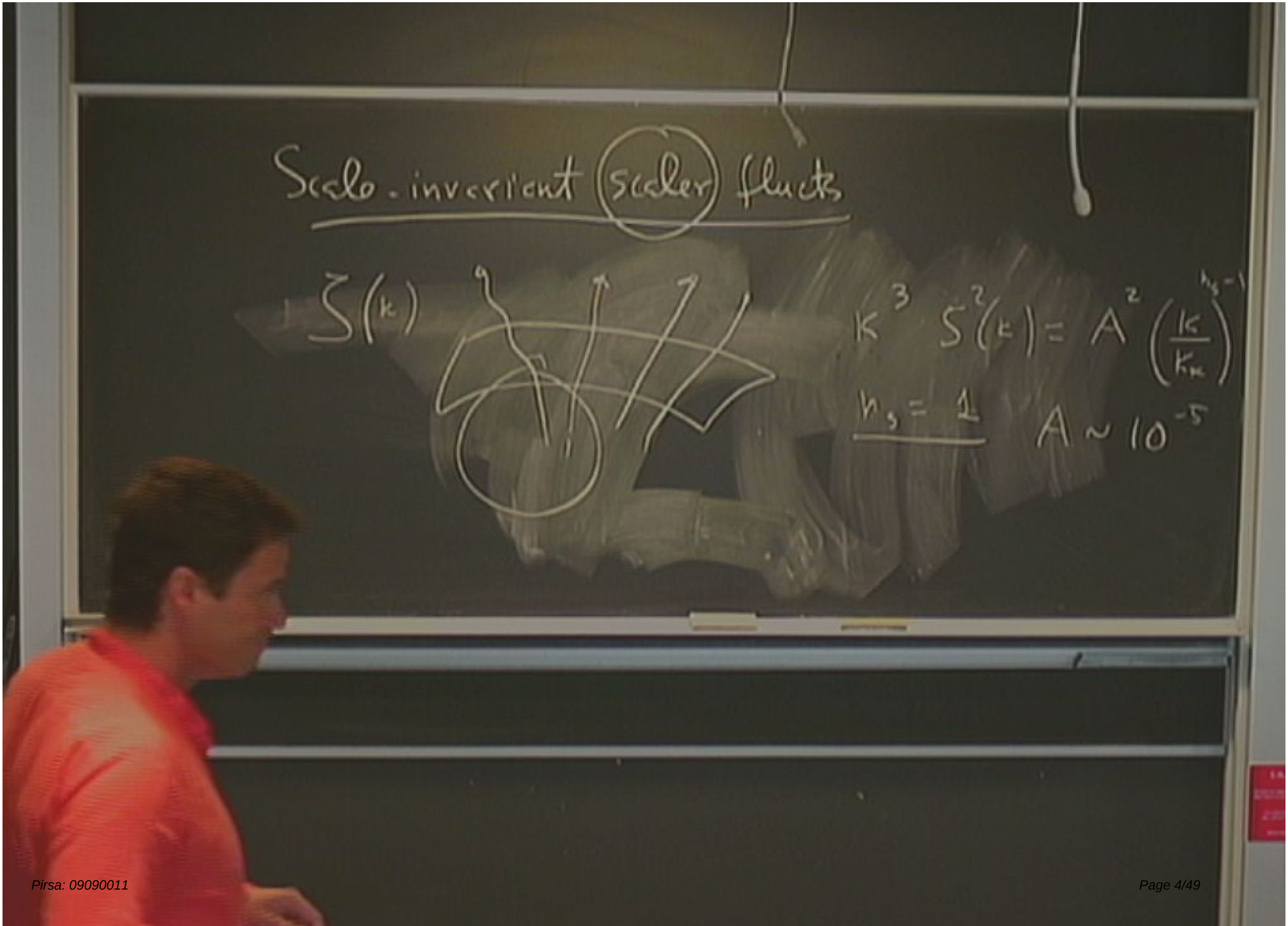
$\zeta(k)$



$$k^3\,\zeta^2(k) = A^2\left(\frac{k}{k_*}\right)^{n_s-1}$$


Scale-invariant scalar flucts
$\zeta(k)$
$k^3 \zeta^2(k) = A^2 \left(\frac{k}{k_*}\right)^{n_s - 1}$
$n_s = 1$
$A \sim 10^{-5}$

$$\zeta = \frac{v}{z}$$

v - 2nd quant QFT

$z \, \alpha$

$$\zeta = \frac{v}{z} \qquad v - \text{2nd quant QFT} \qquad z \propto R \propto \eta^{\frac{2}{1+3w}}$$

$$v'' + \left[k^2 \right.$$

$$\zeta = \frac{v}{z}$$

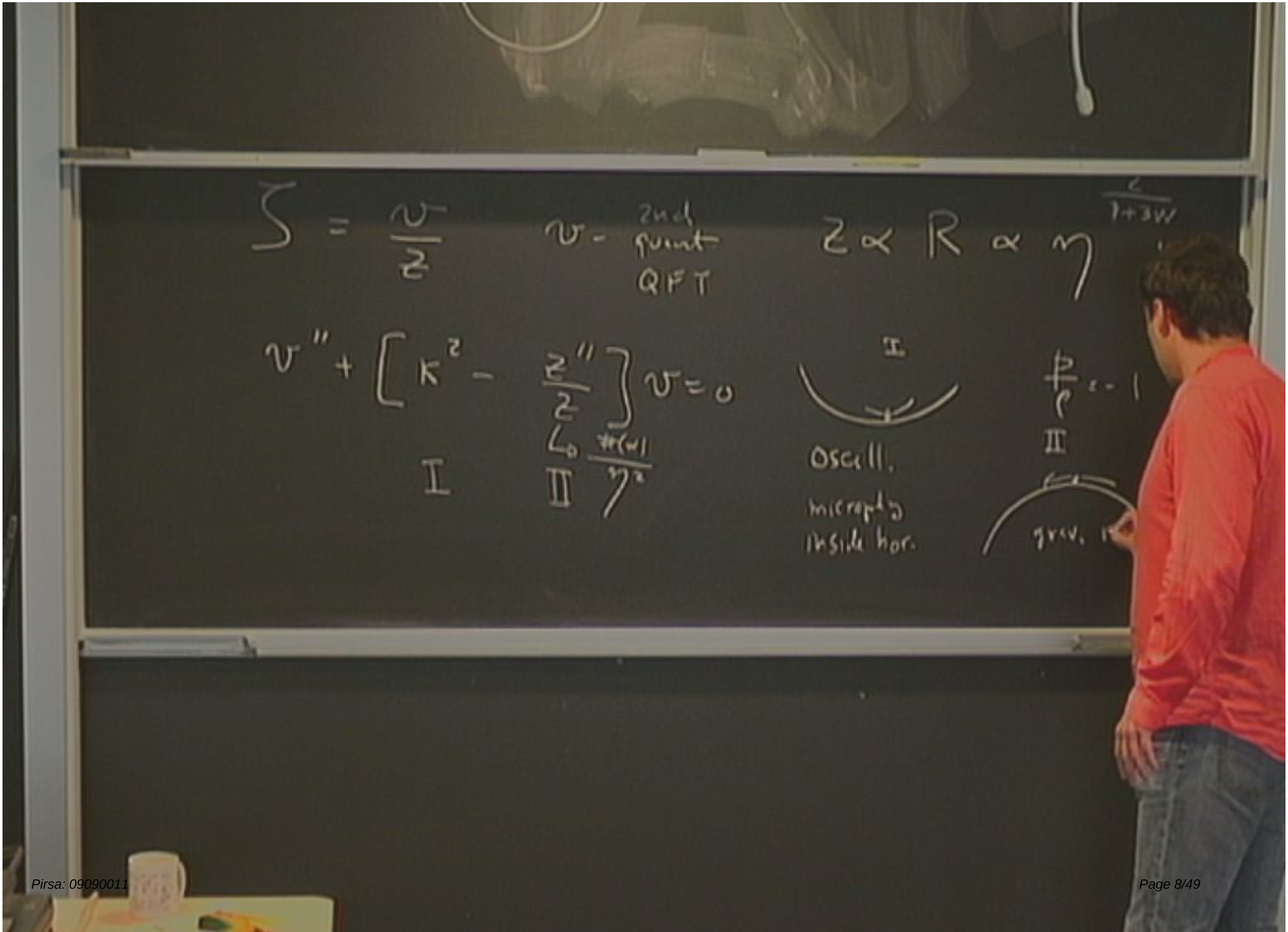
v - 2nd quant QFT

$$z \propto R \propto \eta^{\frac{2}{1+3w}}$$

$$v'' + \left[k^2 - \frac{z''}{z} \right] v = 0$$

I

II $\hookrightarrow \frac{\nu(\nu+1)}{\eta^2}$


ζ = v/z
v - 2nd quant QFT
z ∝ R ∝ η^(2/(1+3w))
v'' + [k² - z''/z] v = 0
I
II
Oscill.
inside hor.
grav.

$$\zeta = \frac{v}{z} \qquad v - \text{2nd quant QFT} \qquad z \propto R \propto \eta^{\frac{2}{1+3w}} \;\; \hookrightarrow \frac{p}{\rho}$$

$$v'' + \left[k^2 - \frac{z''}{z} \right] v = 0$$

$$\frac{z''}{z} \hookrightarrow \frac{\#(w)}{\eta^2}$$

I $\qquad$ II

I

Oscill.

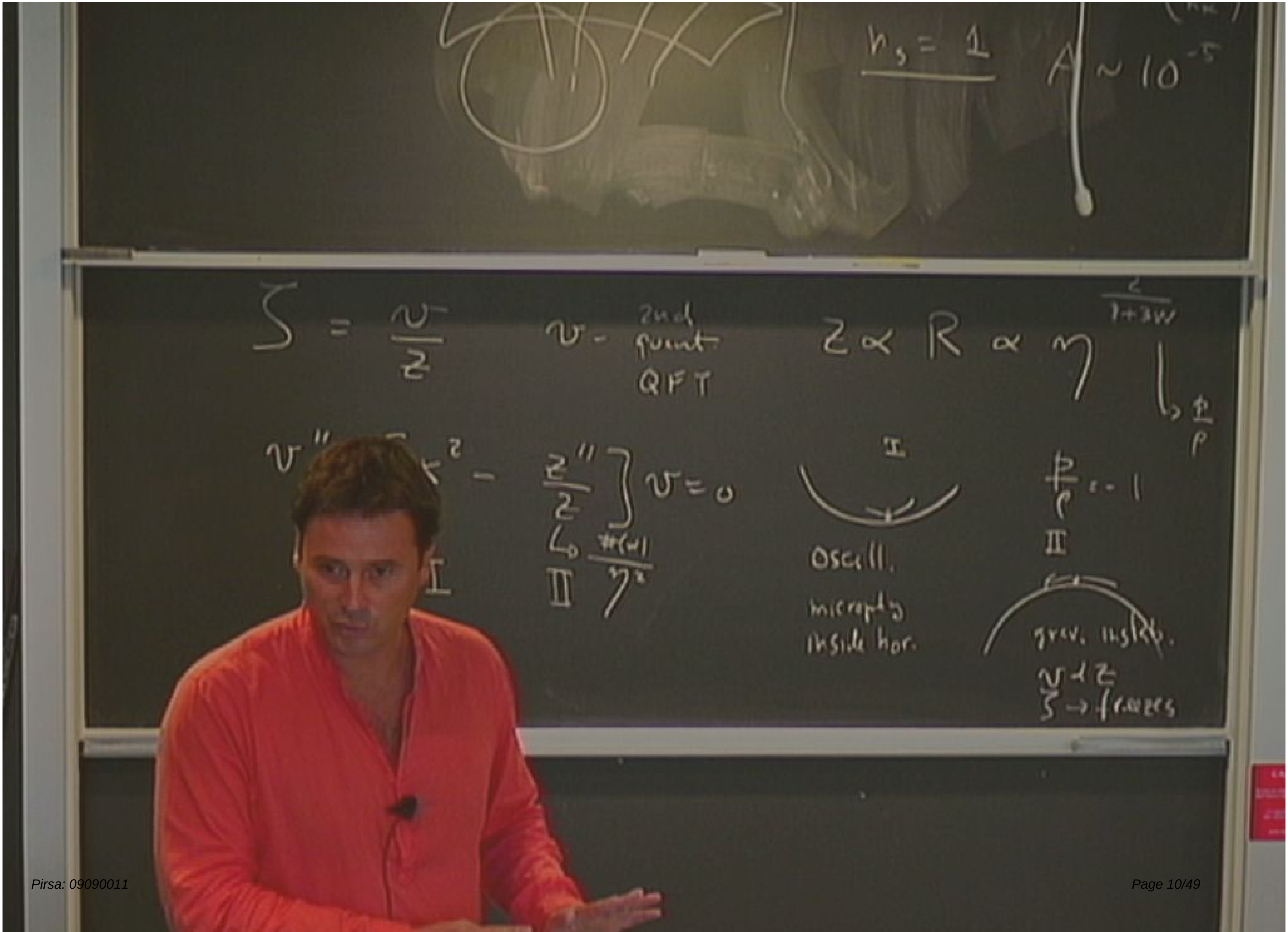
microphys

inside hor.

$\frac{p}{\rho} < -1$

II

grav. instab.


n_s = 1
A ~ 10^-5
ζ = v / z
v - 2nd quant. QFT
z ∝ R ∝ η^(2/(1+3w))
v'' + [k² - z''/z] v = 0
oscill.
microphy inside hor.
p/ρ
grav. instab.
v ∝ z
ζ → freezes

Big Bang

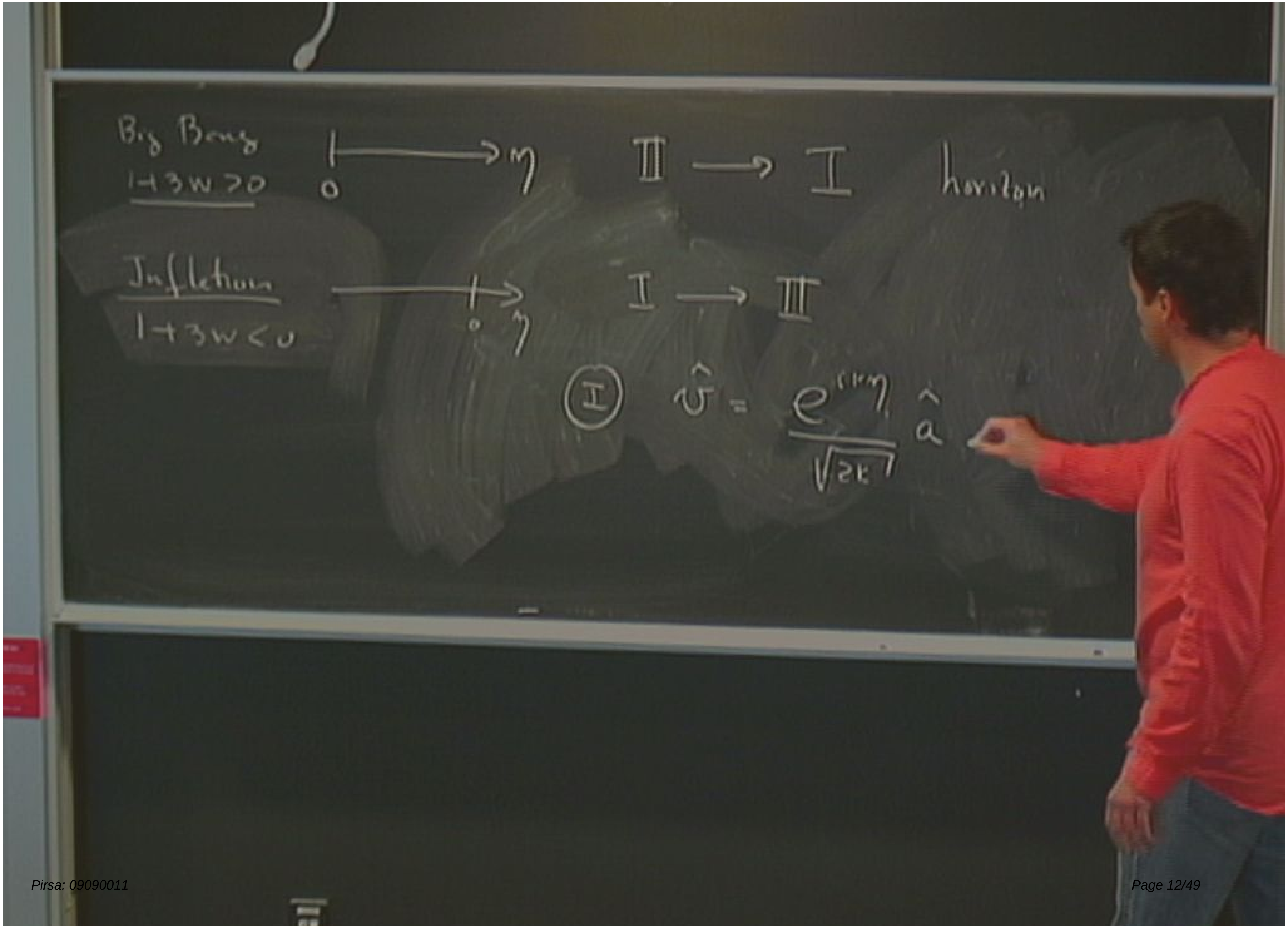
$1+3w>0$

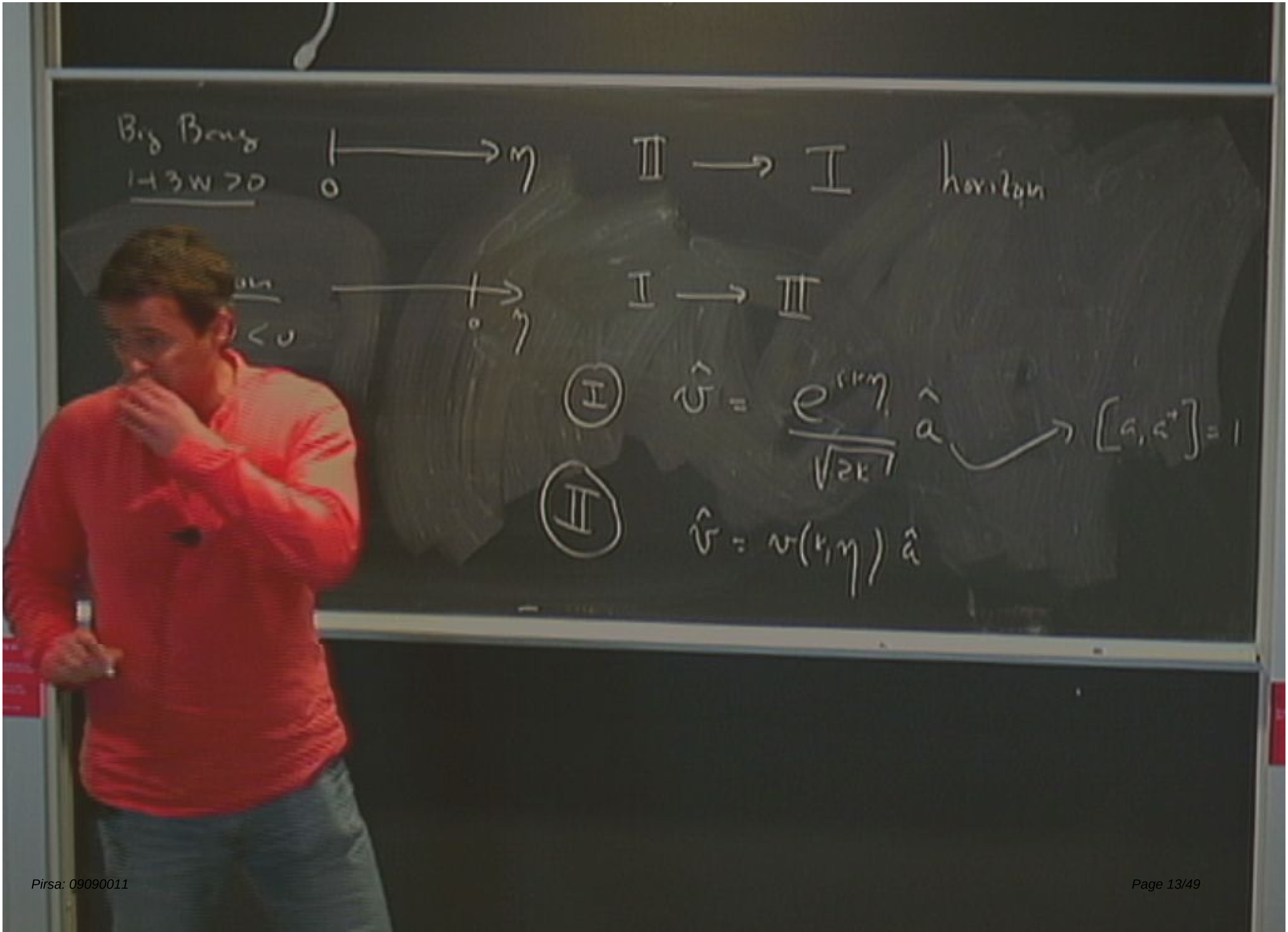
$\underset{0}{|}\!\longrightarrow \eta$ $\quad$ III $\longrightarrow$ I $\quad$ horizon

Inflation

$1+3w<0$

$\longrightarrow\!\!\!\!\!\!\underset{0}{|}\!\!\!\longrightarrow \eta$


Big Bang
1+3w>0
Inflation
1+3w<0
III → I
I → III
horizon
I
$\hat{v} = \frac{e^{ik\eta}}{\sqrt{2k}}\hat{a}$


Big Bang
1+3w>0
0
η
III → I
horizon
I → III
0 η
I
II

Big Bang $\quad 0 \longmapsto \eta \qquad \mathrm{II} \longrightarrow \mathrm{I} \qquad$ horizon

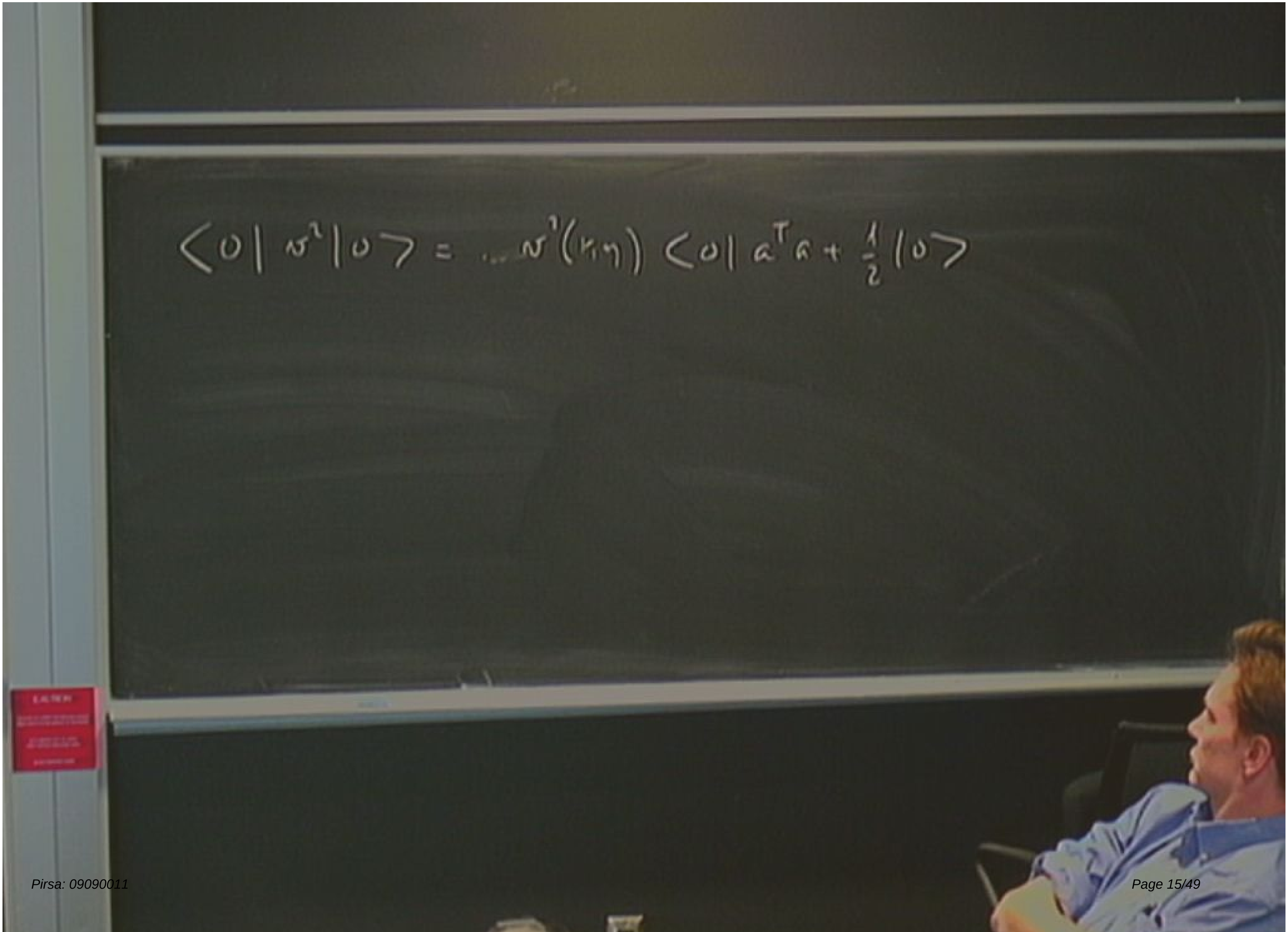
$1+3w>0$

Inflation $\quad \longrightarrow 0 \;\; \eta \qquad \mathrm{I} \longrightarrow \mathrm{II}$

$1+3w<0$

$$\mathrm{I} \qquad \hat{v} = \frac{e^{ik\eta}}{\sqrt{2k}}\,\hat{a} \longrightarrow [a, a^{\dagger}] = 1$$

$$\mathrm{II} \qquad \hat{v} = v(k,\eta)\,\hat{a} \qquad |0\rangle$$


$\langle 0| v^2 |0\rangle = \ldots v^2(k,\eta) \langle 0| a^\dagger a + \frac{1}{2} |0\rangle$

Big Bang $\quad \underset{0}{\longmapsto} \eta \qquad \mathrm{II} \longrightarrow \mathrm{I} \qquad$ horizon

$1+3w>0$

Inflation $\quad \longrightarrow \underset{0}{+} \eta \qquad \mathrm{I} \longrightarrow \mathrm{III}$

$1+3w<0$

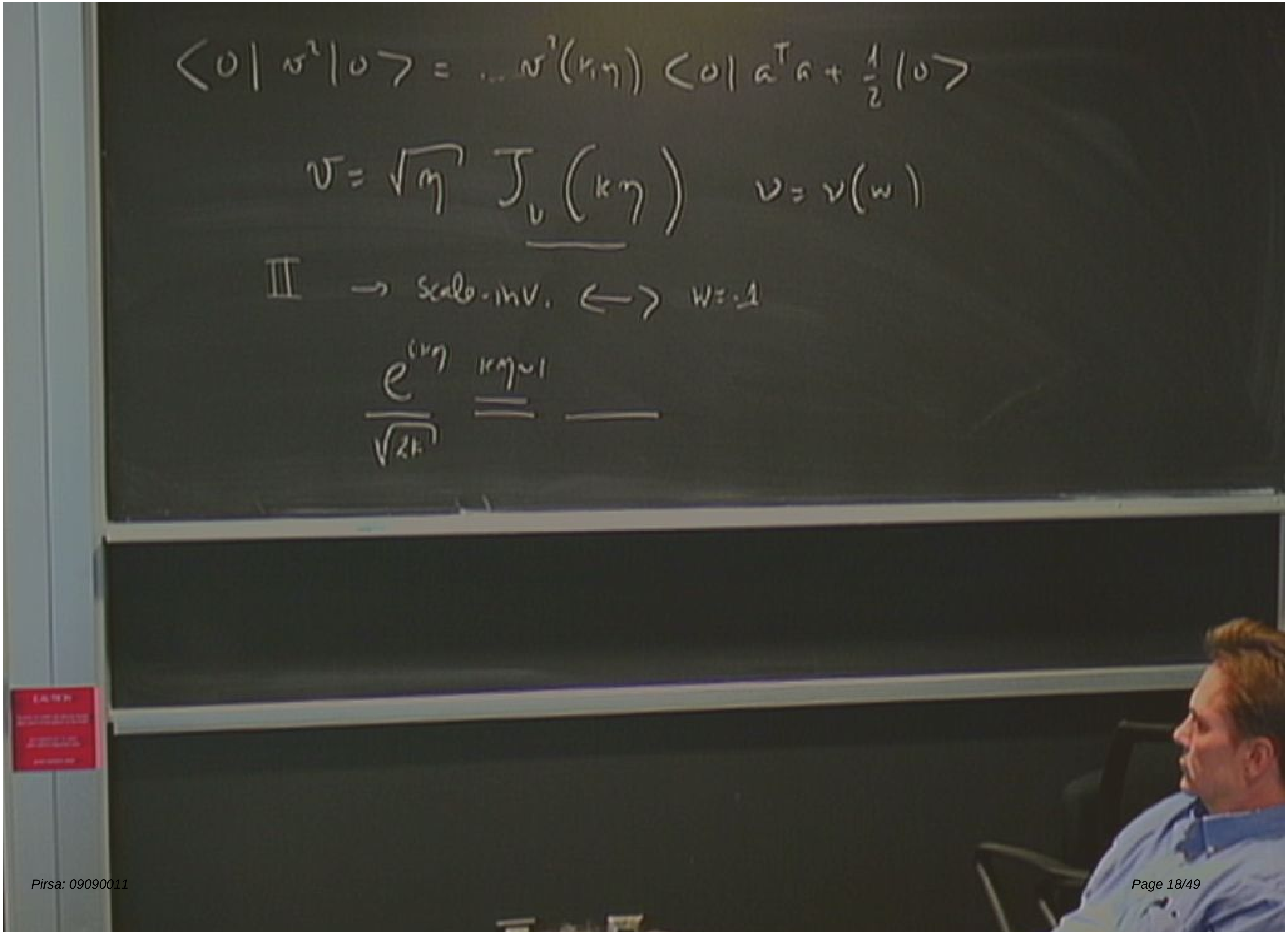
$\mathrm{I} \quad \hat{v} = \dfrac{e^{ik\eta}}{\sqrt{2k}} \hat{a} \longrightarrow [a, a^{*}] = 1$

$\mathrm{II} \quad \hat{v} = v(k, \eta)\, \hat{a} \qquad |0\rangle$

$$\langle 0 | v^2 | 0 \rangle = \ldots v^2(k,\eta) \langle 0 | a^\dagger a + \tfrac{1}{2} | 0 \rangle$$

$$v = \sqrt{\eta}\, \underline{J_\nu(k\eta)} \qquad \nu = \nu(w)$$

III $\rightarrow$ scale-inv. $\longleftrightarrow$ $w = -1$


$\langle 0 | v^2 | 0 \rangle = \dots v^2(k,\eta) \langle 0 | a^\dagger a + \frac{1}{2} | 0 \rangle$
$v = \sqrt{\eta}\, J_\nu(k\eta)$ $\nu = \nu(w)$
III $\rightarrow$ scale-inv. $\longleftrightarrow$ $w = -1$
$\frac{e^{ik\eta}}{\sqrt{2k}} \overset{k\eta \sim 1}{=}$

Big Bang $\quad \underset{0}{\vdash}\!\longrightarrow \eta \qquad \mathrm{II} \longrightarrow \mathrm{I} \qquad$ horizon

$1+3w>0$

Inflation $\quad \longrightarrow\!\!\!\!\!\!\underset{0}{+}\!\!\longrightarrow \eta \qquad \mathrm{I} \longrightarrow \mathrm{II}$

$1+3w<0$

$$\mathrm{I} \qquad \hat{v} = \frac{e^{ik\eta}}{\sqrt{2k}}\,\hat{a} \longrightarrow [a, a^\dagger] = 1$$

$$\mathrm{II} \qquad \hat{v} = v(k,\eta)\,\hat{a} \qquad |0\rangle$$

$$\zeta = \frac{v}{z} \qquad v - \text{2nd quant. QFT} \qquad z \propto \frac{R}{c_s} \propto \eta^{\frac{2}{1+3w}}$$

$\hookrightarrow \frac{p}{\rho}$

$$v'' + \left[c_s^2 k^2 - \frac{z''}{z}\right] v = 0$$

$\frac{z''}{z} \hookrightarrow \frac{\nu(\nu+1)}{\eta^2}$

I — II

I

oscill.

microphys.

inside hor.

$\frac{p}{\rho} = -1$

II

grav. instab.

$v \propto z$

$\zeta \to$ freezes

$C_s \propto \eta^{-\alpha} \quad \alpha > 1$

Big Bang

$\underline{1+3w>0}$

$0 \longmapsto \eta \qquad \mathrm{II} \rightleftarrows \mathrm{I} \qquad$ horizon

$\underline{\text{Inflation}}$

$\cancel{1+3w<0}$

$\longrightarrow 0\ \eta \qquad \mathrm{I} \longrightarrow \mathrm{II}$

$\mathrm{I} \qquad \hat{v} = \dfrac{e^{ik\eta}}{\sqrt{2k}}\,\hat{a} \longrightarrow [a, a^{+}] = 1$

$\mathrm{II} \qquad \hat{v} = v(k,\eta)\,\hat{a} \qquad |0\rangle$

$C_s \propto \eta^{-\alpha}$ $\alpha > 1$

Big Bang

$1+3w>0$

$0 \longmapsto \eta$ III $\rightleftarrows$ I horizon

Inflation

$1+3w<0$

$\longrightarrow \eta$ (at 0) I $\longrightarrow$ III

I $\hat{v} = \frac{e^{i \int ...}}{\sqrt{2k c_s}} \hat{a}$ $\longrightarrow [a, a^\dagger] = 1$

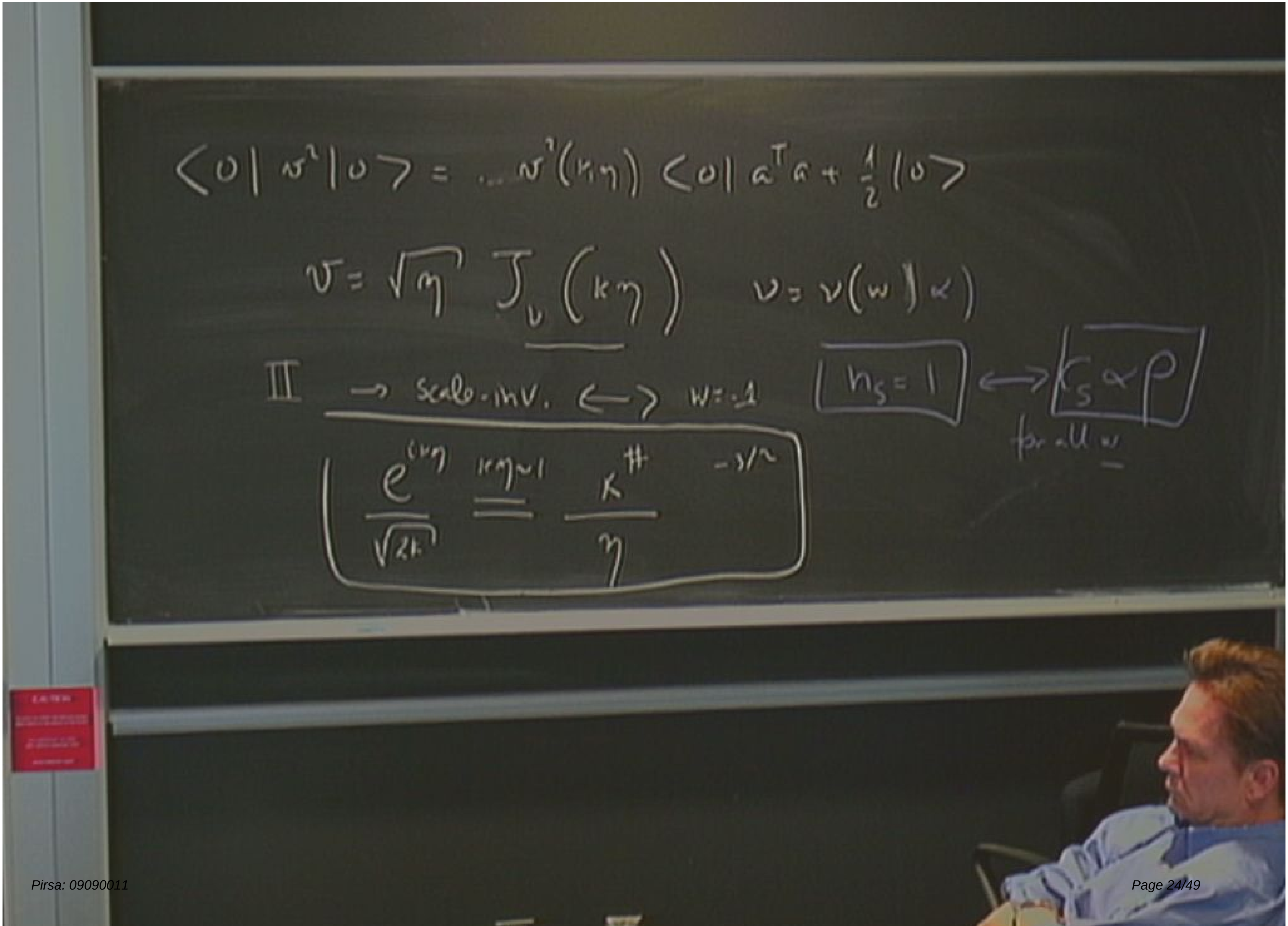
III $\hat{v} = v(k, \eta)\, \hat{a}$ $|0\rangle$

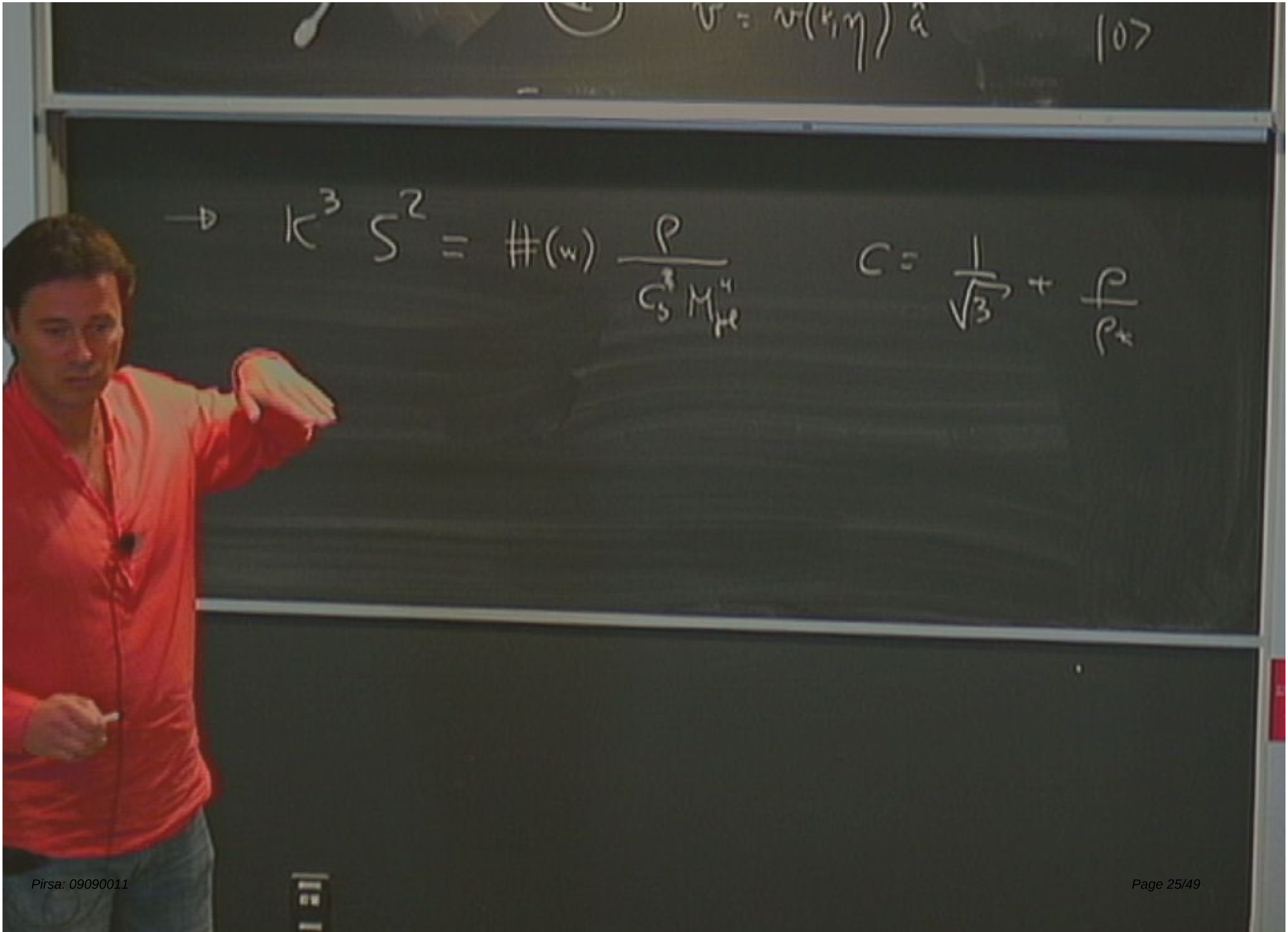
$$\langle 0 | v^2 | 0 \rangle = \ldots v^2(k,\eta) \langle 0 | a^\dagger a + \tfrac{1}{2} | 0 \rangle$$

$$v = \sqrt{\eta}\, \underline{J_\nu(k\eta)} \qquad \nu = \nu(w, \kappa)$$

$$\mathrm{III} \;\to\; \underline{\text{scale-inv.} \;\Longleftrightarrow\; w = -1}$$

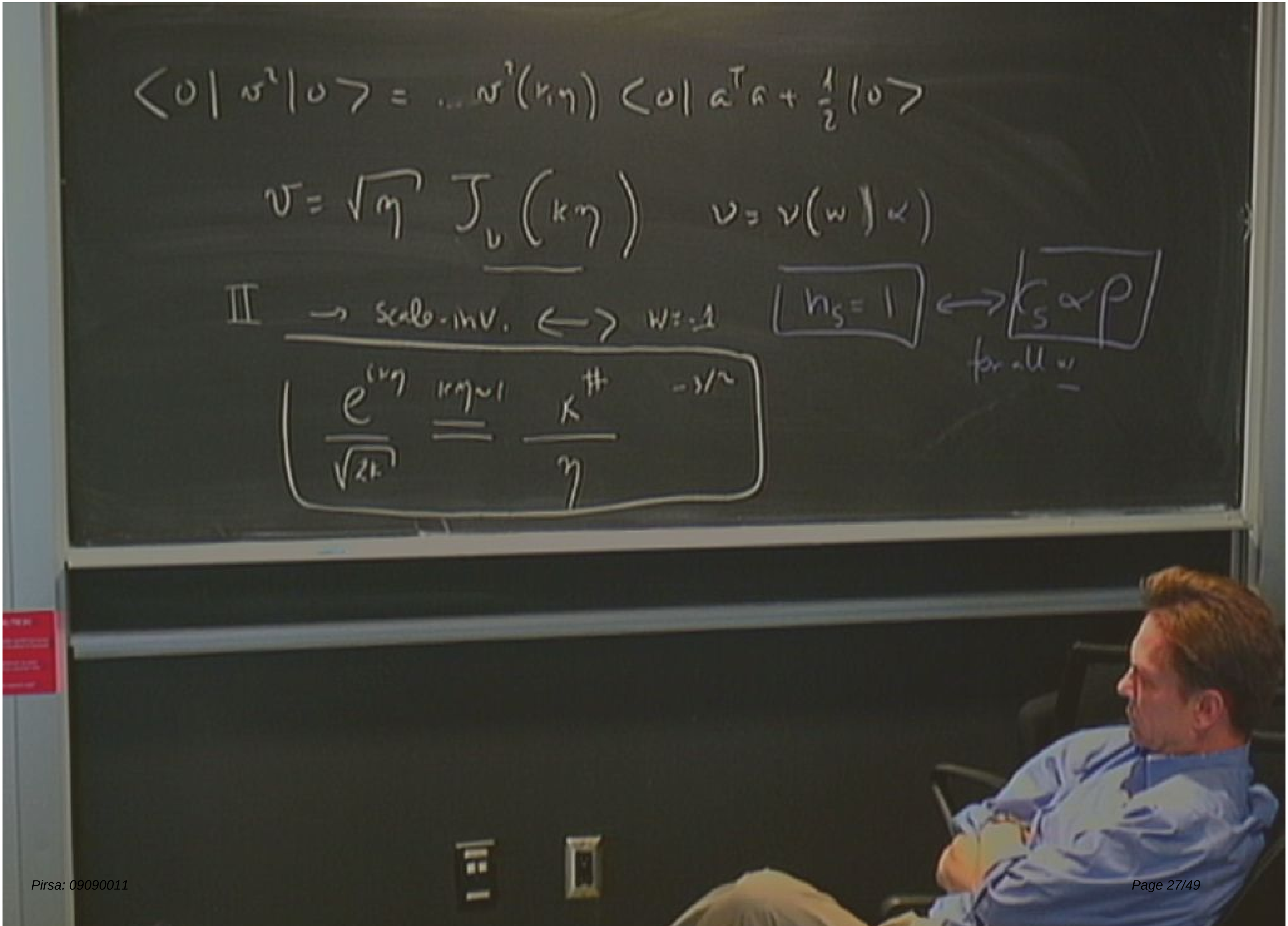
$$\boxed{\frac{e^{ik\eta}}{\sqrt{2k}} \overset{k\eta \sim 1}{=\!=} \frac{k^{\#}}{\eta} \qquad {}^{-3/2}}$$


$\langle 0| v^2 |0\rangle = \dots v^2(k\eta) \langle 0| a^\dagger a + \frac{1}{2} |0\rangle$
$v = \sqrt{\eta}\, J_\nu(k\eta)$
$\nu = \nu(w | \kappa)$
III → scale-inv. ⟷ $w = -1$
$\boxed{n_s = 1}$ ⟷ $\boxed{k_s \propto \rho}$
for all w
$\frac{e^{ik\eta}}{\sqrt{2k}} \overset{k\eta \sim 1}{=} \frac{k^{\#}}{\eta}$
$-3/2$


$v = v(k,\eta)\,\hat{a}$
$|0\rangle$
$\to k^3 S^2 = \#(w) \frac{\rho}{c_s^3 M_{pl}^4}$
$c = \frac{1}{\sqrt{3}} + \frac{\rho}{\rho_*}$

$v = v(k,\eta)\,\hat{a}$ $\qquad |0\rangle$

$$\to \quad k^3 \zeta^2 = \#(w)\,\frac{\rho}{c_s^3 M_{pl}^4} \qquad c = \frac{1}{\sqrt{3}} + \frac{\rho}{\rho_*}$$


⟨0| v²|0⟩ = ... v²(k,η) ⟨0| a†a + 1/2 |0⟩
v = √η J_ν (kη)
ν = ν(w)
II → scale-inv. ⟷ w = -1
n_s = 1 ⟷ k_s ∝ ρ
for all w
e^{ikη}/√(2k)
kη~1
k^#/η
-3/2

II $\quad \hat{v} = v(k,\eta)\,\hat{a} \qquad |0\rangle$

$$\to \quad k^3 \zeta^2 = \#(w)\,\frac{\rho}{c_s^3 M_{pl}^4} \qquad c = c_0\left(1 + \frac{\rho}{\rho_*}\right)$$

$$\to \quad |0\rangle \;?$$

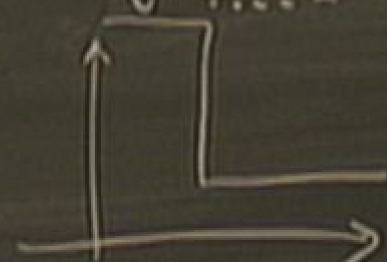
$v^2 \ldots\ldots$ as by $\langle T|\, a^\dagger a + \frac{1}{2}\,|T\rangle$

$$\hookrightarrow \frac{1}{e^{k/T}-1} = \frac{T}{k}$$

II $\quad \hat{v} = v(k,\eta)\,\hat{a} \qquad |0\rangle$

$$\rightarrow \quad k^3 \zeta^2 = \#(w)\,\frac{\rho}{c_s^3 M_{pl}^4} \qquad c = c_0\left(1 + \frac{\rho}{\rho_*}\right)$$

$$\rightarrow \quad |0\rangle \;?$$



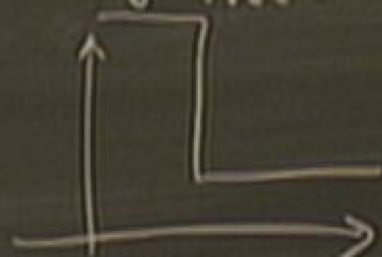
as by $\langle T | a^\dagger a + \frac{1}{2} | T\rangle$

$$\hookrightarrow \frac{1}{e^{k/T}-1} = \frac{T}{k}$$

(II) $\hat{v} = v(k,\eta)\,\hat{a}$ $|0\rangle$

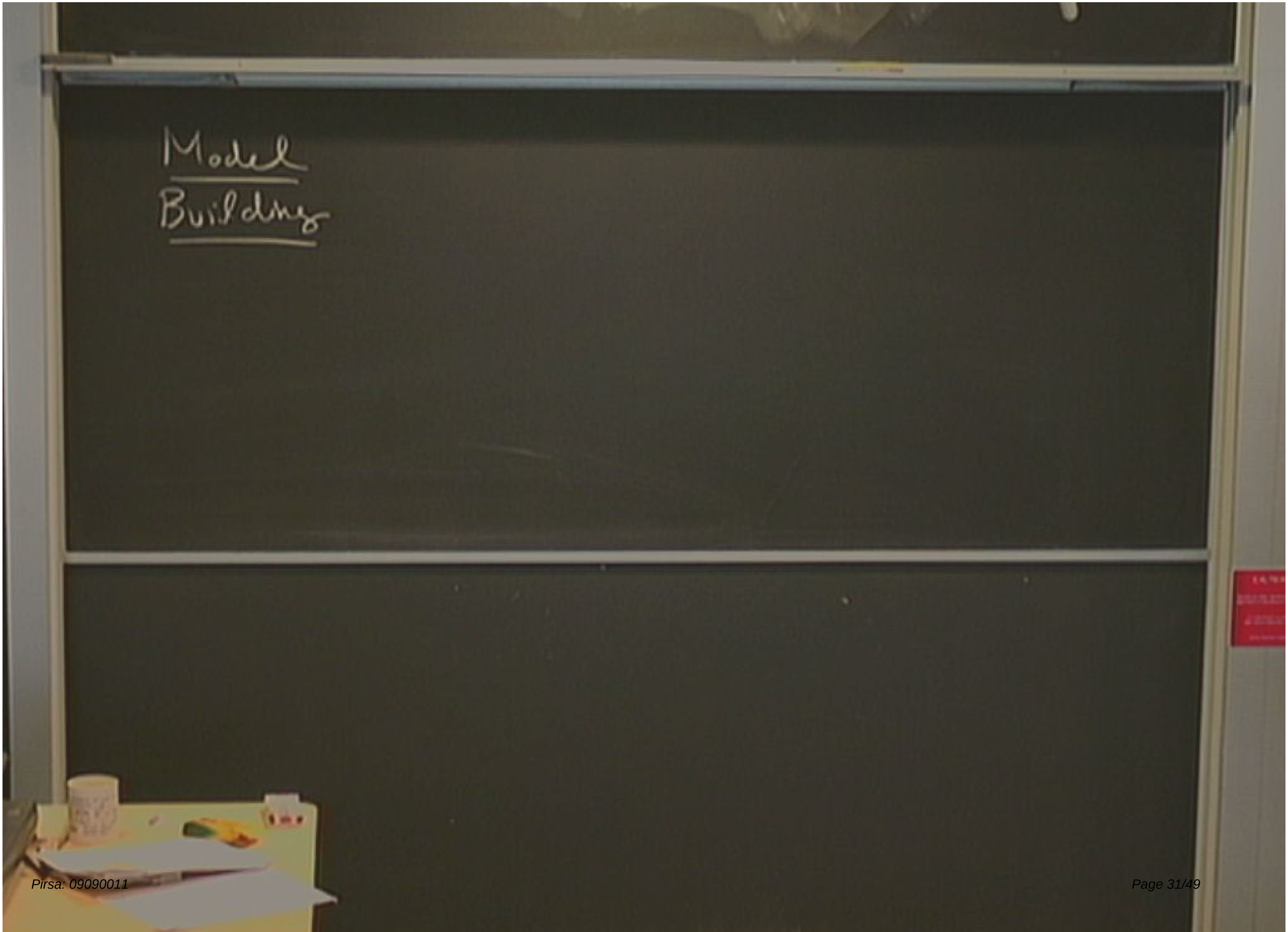
$\rightarrow k^3 \zeta^2 = \#(w) \dfrac{\rho}{c_s^3 M_{pl}^4}$ $C = C_0\left(1 + \dfrac{\rho}{\rho_*}\right)$

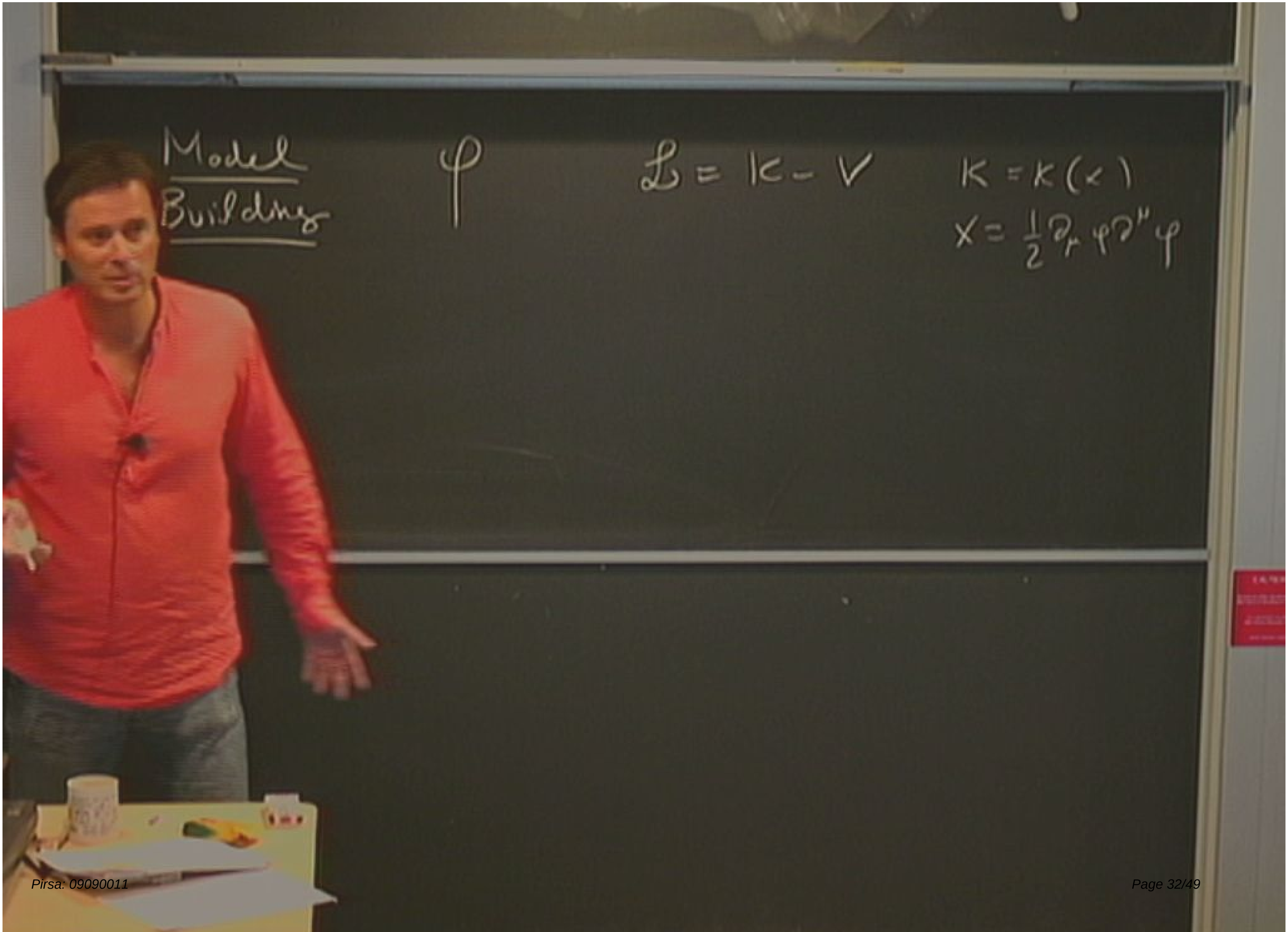
$\rightarrow |0\rangle$?

$v^2 \ldots\ldots$ 

$\langle T | a^\dagger a + \frac{1}{2} | T \rangle$

$\hookrightarrow \dfrac{1}{e^{k/T} - 1} = \dfrac{T}{k}$


Model
Building


Model Building
φ
$\mathcal{L} = K - V$
$K = K(X)$
$X = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi$

Model Building

φ

$$\mathcal{L} = K - V \qquad K = K(X)$$

$$X = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi$$

$$\begin{cases} p = K - V \\ \rho = 2X K_{,X} - K + V \end{cases}$$

$$c_s^2 = \frac{K_{,X}}{2K_{,XX}X + K_{,X}}$$

Model Building

φ

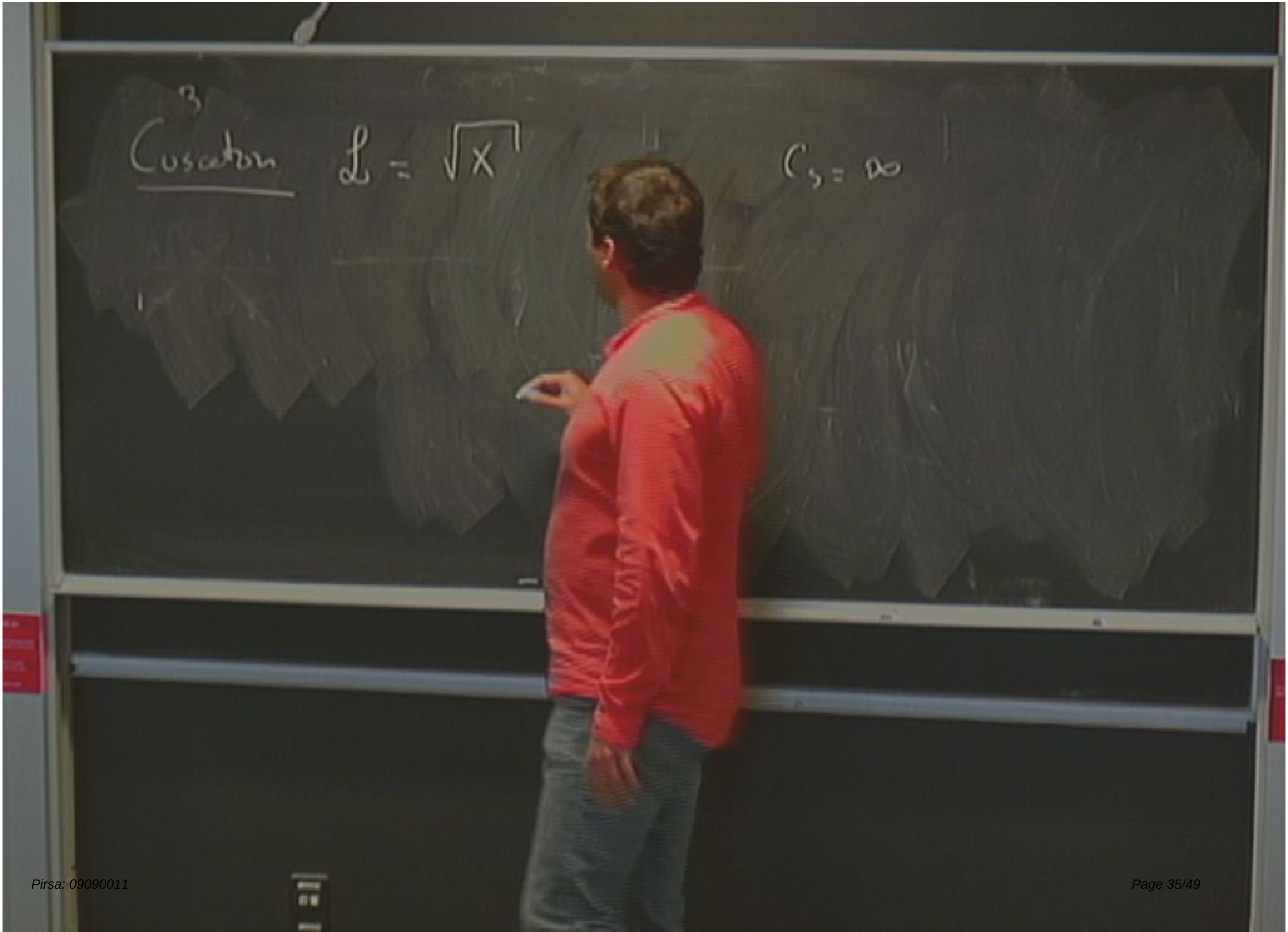
$\mathcal{L} = K - V$

$K = K(X)$

$X = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi$

$$w \Leftarrow \begin{cases} p = K - V \\ \rho = 2X K_{,X} - K + V \end{cases}$$

$$c_s^2 = \frac{K_{,X}}{2K_{,XX}X + K_{,X}}$$


Cuscaton
$\mathcal{L} = \sqrt{X}$
$c_s = \infty$

3 **Cuscuton** $\mathcal{L} = \sqrt{X} - \frac{1}{2}m^2\varphi^2$ $c_s = \infty$

$\hookrightarrow w = \ldots$

3

Cuscaton $\mathcal{L} = \sqrt{X} - \frac{1}{2} m^2 \varphi^2 \quad c_s = \infty$

$\hookrightarrow w = \ldots$

$\mathcal{L} \to \mathcal{L} + X^\beta \quad \beta < \frac{1}{2} \quad X \gg 1$

3

Cuscaton $\mathcal{L} = \sqrt{X} - \frac{1}{2}m^2\varphi^2$ $\quad c_s = \infty$

$\hookrightarrow w = \ldots$

$\mathcal{L} \to \mathcal{L} + X^\beta$

$\beta < \frac{1}{2} \qquad X \gg 1$

$\beta = -\frac{1}{2}$

$\boxed{c_s \propto \rho}$

$$\mathcal{L} = \frac{1}{f}\sqrt{1+2fX} - \frac{1}{f} - \frac{1}{2}m^2\varphi^2 + \cdots$$

$\cancel{f<0}$ $\quad f>0$

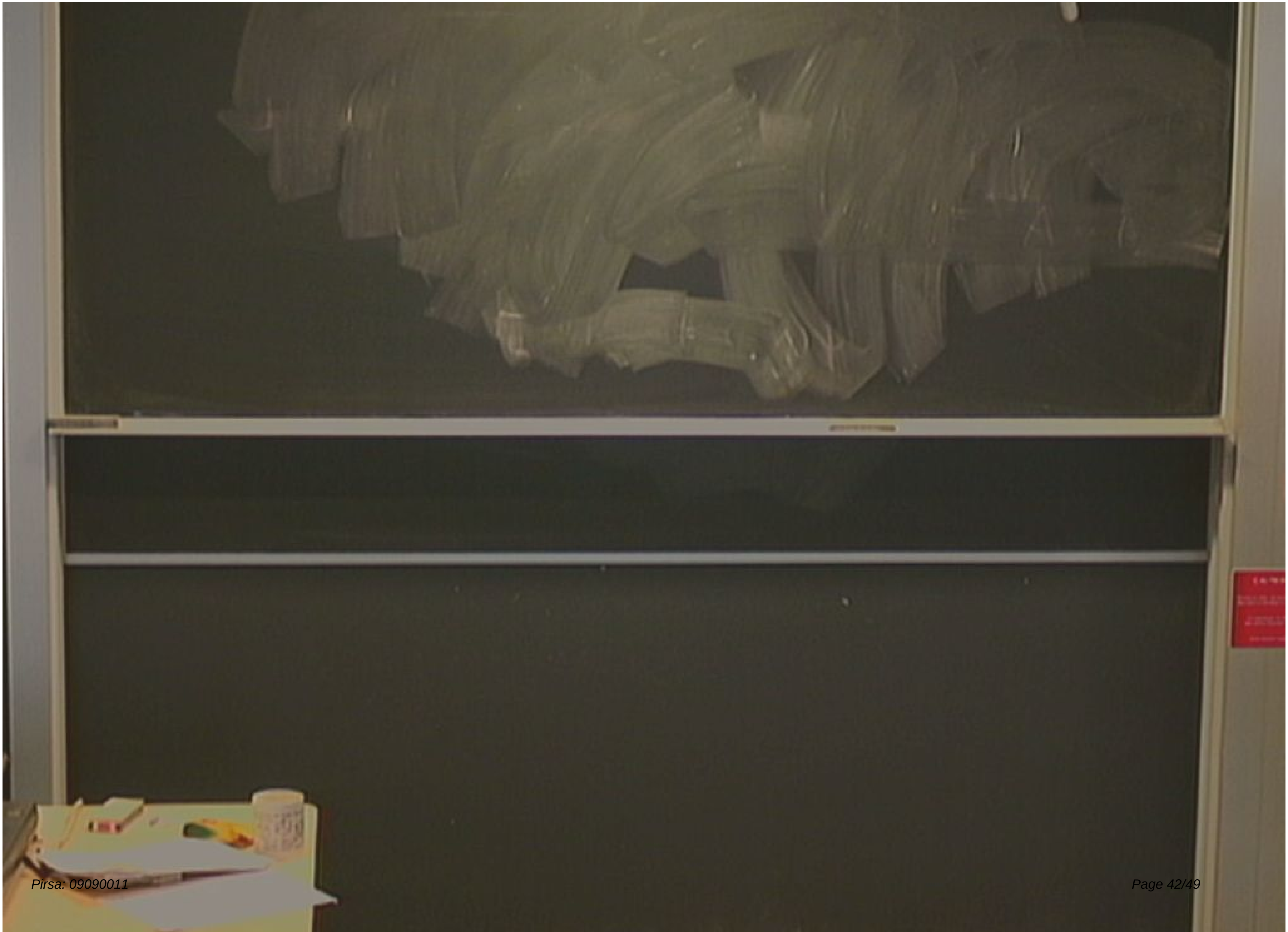
$$\mathcal{L} = \frac{1}{f}\sqrt{1+2fX} - \frac{1}{f} - \frac{1}{2}m^2\varphi^2 + \dots$$

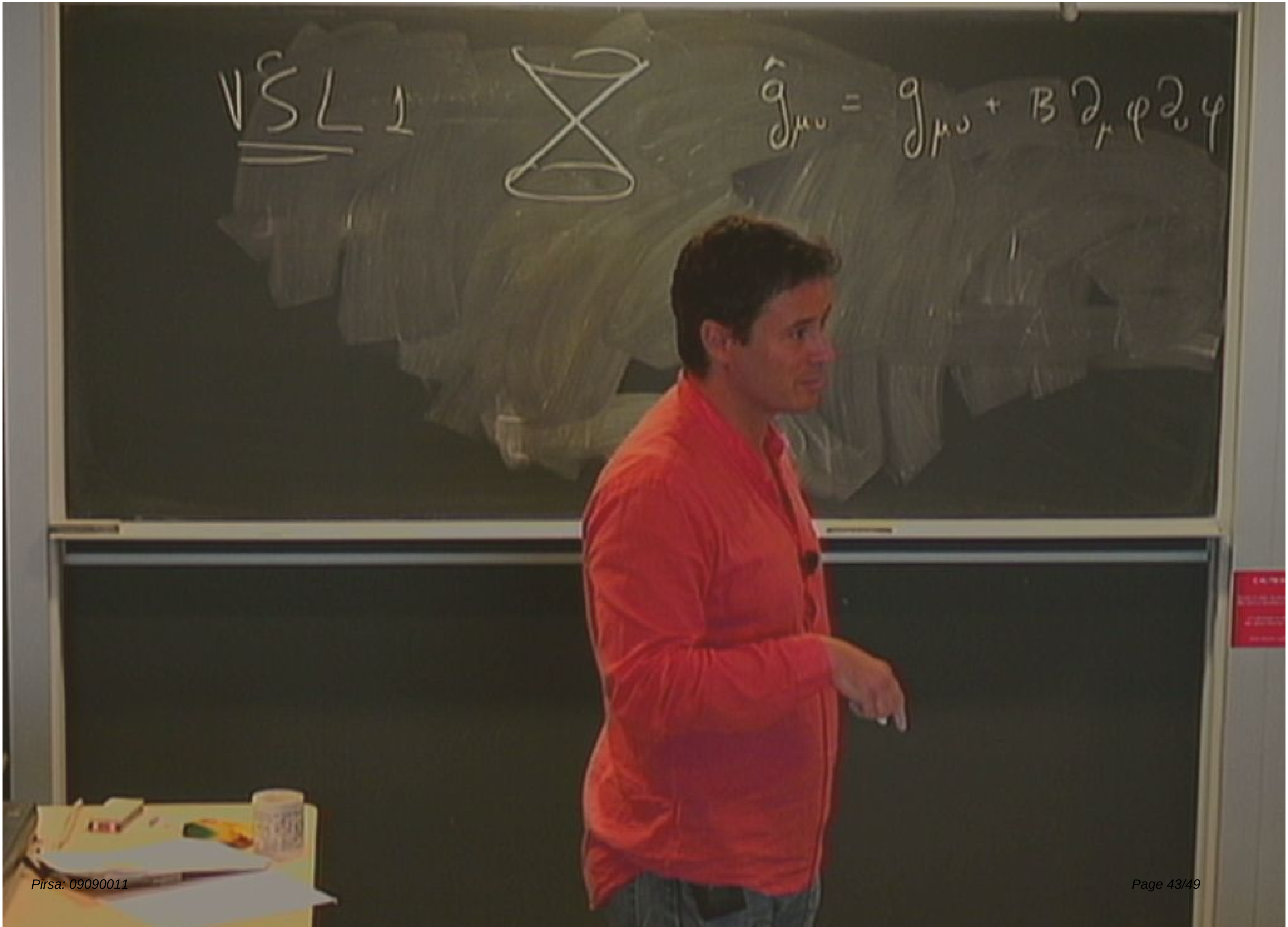
$\cancel{f<0}$ $\quad f>0 \qquad X \gg 1$

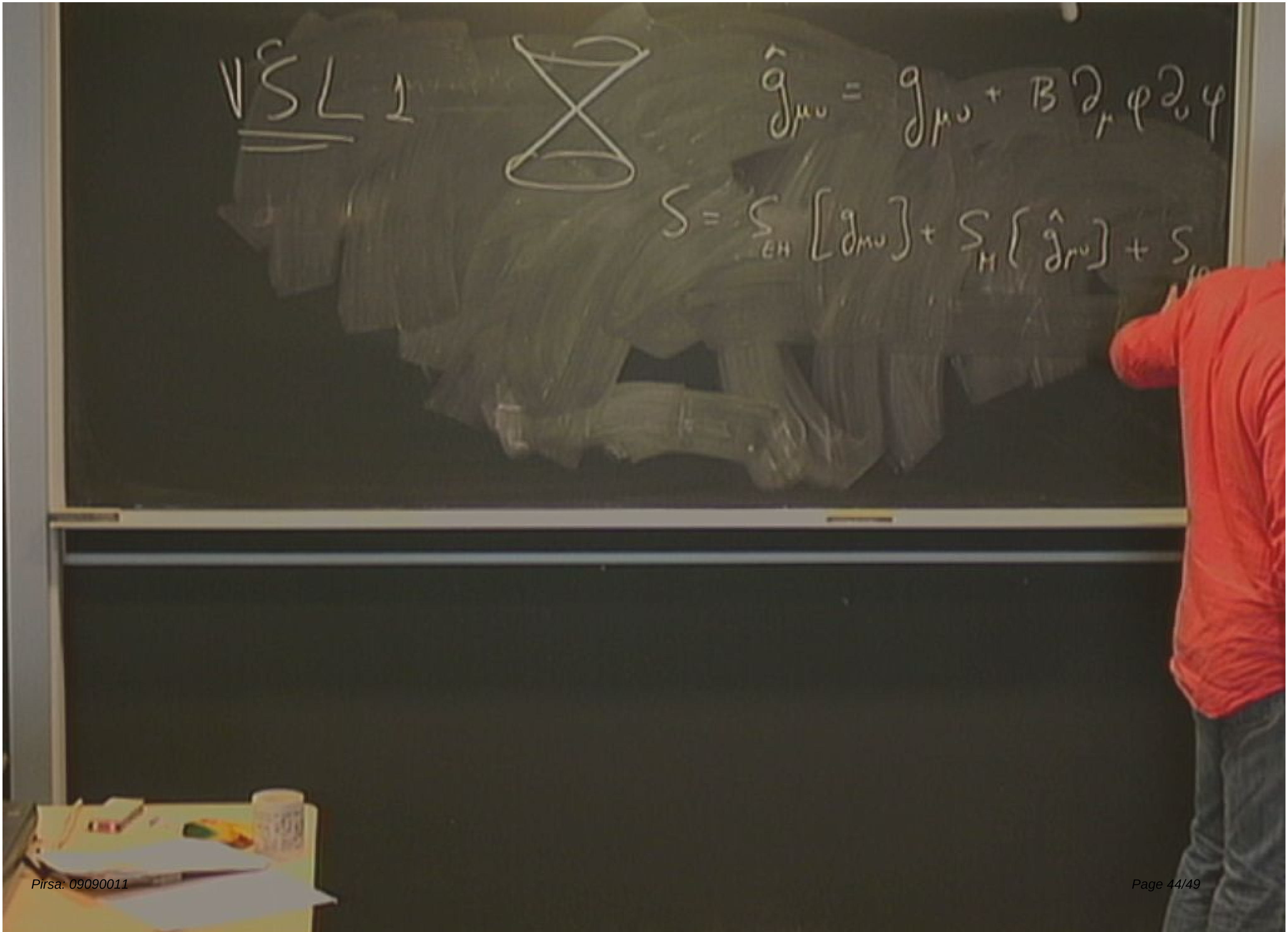
$+ - - -$

$$\mathcal{L} = \frac{1}{f}\sqrt{1+2fX} - \frac{1}{f} - \frac{1}{2}m^2\varphi^2$$

$$\cancel{f<0} \qquad f>0 \qquad\qquad X \gg 1$$




VSL 1
$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\partial_\mu\varphi\partial_\nu\varphi$


VSL 1
$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\,\partial_\mu\varphi\,\partial_\nu\varphi$
$S = S_{EH}[g_{\mu\nu}] + S_M[\hat{g}_{\mu\nu}] + S$

VSL 1

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\,\partial_\mu\varphi\,\partial_\nu\varphi$$

$$S = S_{EH}[g_{\mu\nu}] + S_M[\hat{g}_{\mu\nu}] + S_\varphi$$

$$B = B(\varphi, x)$$

$$S_{\varphi} = \int d^4x \sqrt{-\hat{g}}\ \frac{1}{B} - \int d^4x \sqrt{-g}\ \frac{1}{B}$$

$$S_\varphi = \int d^4x \sqrt{-\hat{g}}\ \frac{1}{B} - \int d^4x \sqrt{-g}\ \frac{1}{B}$$

$B = \text{const}$

VSL 2

$$K_i = U L_{0i} U^{-1} \qquad U(E,p) = \ldots$$

$\hookrightarrow E_p$ invariant

$$c = \frac{dE}{dp} = c(p)$$

K

$$K_{ph} = \frac{K}{R}$$

$$v'' + \left[c_s^2 k^2 - \frac{z''}{z}\right] v = 0$$

$$n_s = 1 \Longleftrightarrow E^2 = p^2\left(1 + (\lambda p)^2\right)^2 \qquad c_s = c_s\left(\frac{K}{R}\right)$$

VW

$$E \propto p^3$$