Title: Aspects of Horava-Lifshitz Cosmology

Date: Sep 15, 2009 02:00 PM

URL: http://pirsa.org/09090008

Abstract: <div id="Cleaner">In the first half of this talk I will review the basic idea of the power-counting renormalizable theory of gravitation recently proposed by Horava. In the second half I will talk about some cosmological implications of the theory. In particular, I will show that the anisotropic scaling with a dynamical critical exponent z=3 leads to generation of scale-invariant cosmological perturbations and that the absence of local Hamiltonian constraint leads to a component similar to cold dark matter as integration ""constant"".

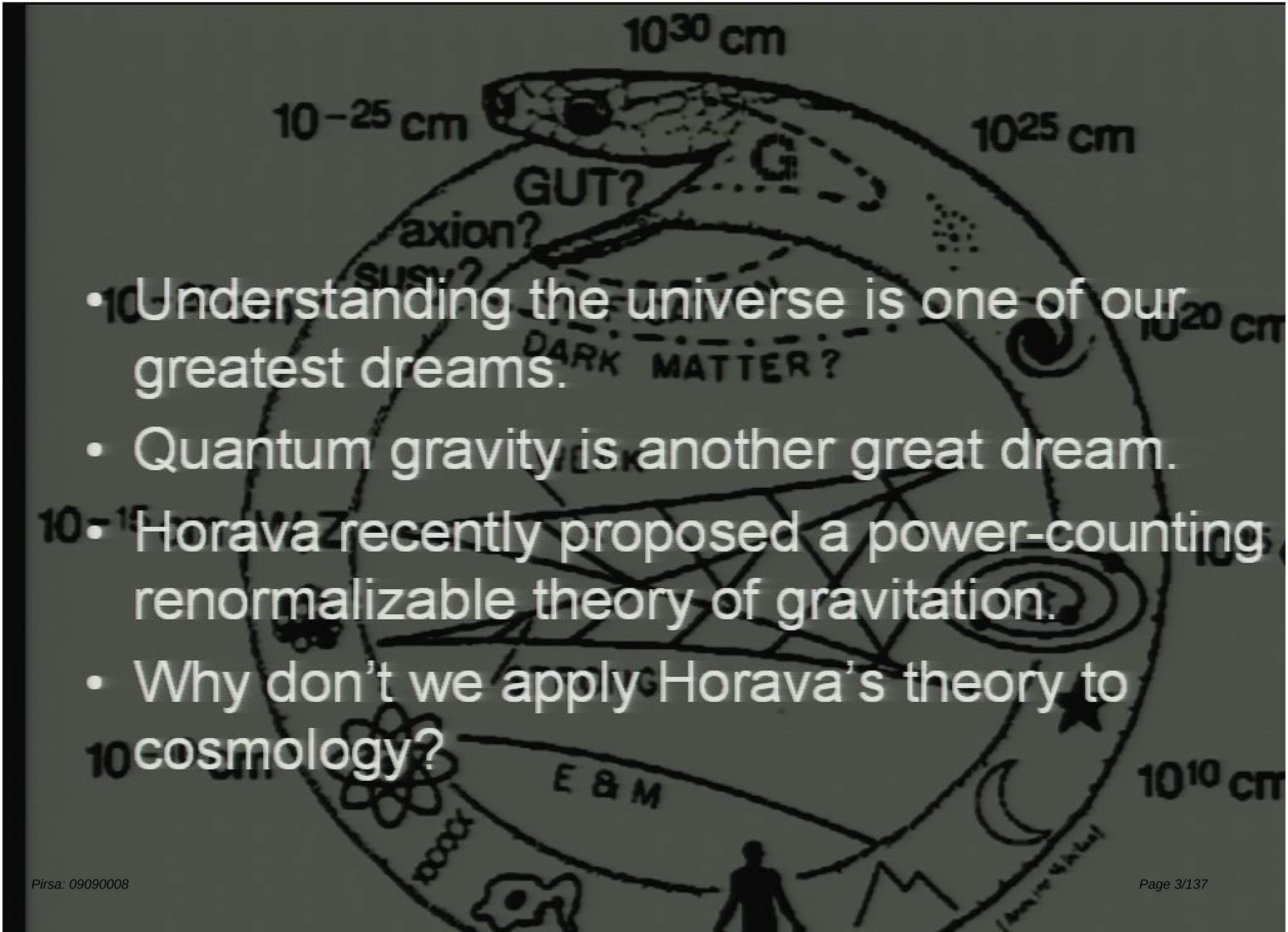
IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Aspects of Horava-Lifshitz cosmology

arXiv:0904.2190 [hep-th]
* arXiv:0905.0055 [hep-th]
arXiv:0905.3563 [hep-th]
arXiv:0906.5069 [hep-th]
+ arXiv:0909.2149 [astro-ph.CO]

Shinji Mukohyama (IPMU, U of Tokyo)

* w/ K.Nakayama, F.Takahashi and S.Yokoyama
+ w/ S.Maeda and T.Shiromizu



- Understanding the universe is one of our greatest dreams.
- Quantum gravity is another great dream.
- Horava recently proposed a power-counting renormalizable theory of gravitation.
- Why don't we apply Horava's theory to cosmology?

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

- **Scaling dim of ϕ**

  $t \to b\, t$ ($E \to b^{-1} E$)

  $x \to b\, x$

  $\phi \to b^s \phi$

  $1+3-2+2s = 0$

  $s = -1$

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(1+3+ns)}$$

- **Scaling dim of ϕ**

  $t \rightarrow b\,t$ $(E \rightarrow b^{-1} E)$

  $x \rightarrow b\,x$

  $\phi \rightarrow b^s \phi$

  $1+3-2+2s = 0$

  $s = -1$

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(1+3+ns)}$$

- Scaling dim of ϕ

  $t \rightarrow b\,t$ ($E \rightarrow b^{-1}E$)

  $x \rightarrow b\,x$

  $\phi \rightarrow b^s \phi$

  $1+3-2+2s = 0$

  $s = -1$

- Renormalizability

  $$n \leq 4$$

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

- Scaling dim of ϕ

  t → b t (E → b^{-1}E)

  x → b x

  $\phi \rightarrow b^s \phi$

  1+3-2+2s = 0

  s = -1

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(1+3+ns)}$$

- Renormalizability

  $$n \leq 4$$

- Gravity is highly non linear and thus non-renormalizable

Abandon Lorentz symmetry?

$$I \supset \int dt\,dx^3\,\dot{\phi}^2$$

Abandon Lorentz symmetry?

$$I \supset \int dt\, dx^3\, \dot{\phi}^2$$

- Anisotropic scaling

  $t \rightarrow b^z t$ ($E \rightarrow b^{-z} E$)

  $x \rightarrow b\, x$

  $\phi \rightarrow b^s \phi$

  $z+3-2z+2s = 0$

  $s = -(3-z)/2$

Abandon Lorentz symmetry?

$$I \supset \int dt dx^3 \dot{\phi}^2$$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(z+3+ns)/z}$$

- Anisotropic scaling

  $t \rightarrow b^z t$ $(E \rightarrow b^{-z} E)$

  $x \rightarrow b x$

  $\phi \rightarrow b^s \phi$

  $z+3-2z+2s = 0$

  $s = -(3-z)/2$

- $s = 0$ if $z = 3$

Abandon Lorentz symmetry?

$$I \supset \int dt dx^3 \dot{\phi}^2$$

- Anisotropic scaling

  $t \rightarrow b^z t$ ($E \rightarrow b^{-z} E$)

  $x \rightarrow b x$

  $\phi \rightarrow b^s \phi$

  $z+3-2z+2s = 0$

  $s = -(3-z)/2$

- $s = 0$ if $z = 3$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(z+3+ns)/z}$$

- For $z = 3$, any nonlinear interactions are renormalizable!

Abandon Lorentz symmetry?

$$I \supset \int dt dx^3 \dot{\phi}^2$$

- Anisotropic scaling

  t → b^z t (E→b^{-z}E)

  x → b x

  ϕ → b^s ϕ

  z+3-2z+2s = 0

  s = -(3-z)/2

- s = 0 if z = 3

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(z+3+ns)/z}$$

- For z = 3, any nonlinear interactions are renormalizable!
- Gravity becomes renormalizable!?

Scalar with z=3

free part
$$I_\phi = \frac{1}{2}\int dt d^3x \left(\dot{\phi}^2 + \phi O \phi\right)$$

$$O = \frac{\Delta^3}{M^4} - \frac{\kappa\Delta^2}{M^2} + c_\phi^2 \Delta - m_\phi^2$$

UV: z=3 IR: z=1

- **UV: z=3** , renormalizable nonlinear theory

  RG flow

- **IR: z=1** , familiar Lorentz invariant theory

Note: we need a mechanism to make "limits of speed" of different species to be the same.
c.f. Iengo, Russo, and Serone (2009)

Abandon Lorentz symmetry?

$$I \supset \int dt dx^3 \dot{\phi}^2$$

- Anisotropic scaling
  t → b^z t (E→b^{-z}E)
  x → b x
  ϕ → b^s ϕ
  z+3-2z+2s = 0
  s = -(3-z)/2
- s = 0 if z = 3

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(z+3+ns)/z}$$

- For z = 3, any nonlinear interactions are renormalizable!
- Gravity becomes renormalizable!?

Scalar with z=3

free part $$I_\phi = \frac{1}{2}\int dt d^3x\left(\dot{\phi}^2 + \phi O \phi\right)$$

$$O = \underbrace{\frac{\Delta^3}{M^4}}_{\text{UV: } z=3} - \frac{\kappa\Delta^2}{M^2} + \underbrace{c_\phi^2\Delta - m_\phi^2}_{\text{IR: } z=1}$$

- **UV: z=3** , renormalizable nonlinear theory

  ⬇ RG flow

- **IR: z=1** , familiar Lorentz invariant theory

Note: we need a mechanism to make “limits of speed” of different species to be the same.
c.f. Iengo, Russo, and Serone (2009)
$\delta c = 0$ is an IR fixed point but approach seems slow

Scalar with z=3

free part $$I_\phi = \frac{1}{2}\int dt d^3x \left(\dot{\phi}^2 + \phi O \phi\right)$$

$$O = \underbrace{\frac{\Delta^3}{M^4}}_{\text{UV: z=3}} - \frac{\kappa\Delta^2}{M^2} + \underbrace{c_\phi^2 \Delta - m_\phi^2}_{\text{IR: z=1}}$$

- **UV: z=3** , renormalizable nonlinear theory

  ⬇ RG flow

- **IR: z=1** , familiar Lorentz invariant theory

Note: we need a mechanism to make "limits of speed" of different species to be the same.

c.f. Iengo, Russo, and Serone (2009)

$\delta c = 0$ is an IR fixed point but approach seems slow

Horava-Lifshitz gravity

Horava (2009)

- Basic quantities:
  lapse $N(t)$, shift $N^i(t,x)$, 3d spatial metric $g_{ij}(t,x)$
- ADM metric (emergent in the IR)
  $ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
- Foliation-preserving deffeomorphism
  $t \rightarrow t'(t), \quad x^i \rightarrow x'^i(t,x^j)$
- Anisotropic scaling with z=3 in UV
  $t \rightarrow b^z t, \quad x^i \rightarrow b\, x^i$
- Ingredients in the action

$$Ndt \qquad \sqrt{g}d^3x \qquad g_{ij} \qquad D_i \qquad R_{ij}$$

$$K_{ij} = \frac{1}{2N}\left(\partial_t g_{ij} - D_i N_j - D_j N_i\right)$$

($C_{ijkl} = 0$ in 3d)

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g} d^3x \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

c.f. $\lambda = 1$ for GR

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad D_i R_{jk} D^i R^{jk} \quad D_i R D^i R \quad R_i^j R_j^k R_k^i \quad R R_i^j R_j^i \quad R^3 \quad \right]$$

c.f. $D_i R_{jk} D^j R^{ki}$ is written in terms of other term

Horava-Lifshitz gravity

Horava (2009)

- Basic quantities:
  lapse $N(t)$, shift $N^i(t,x)$, 3d spatial metric $g_{ij}(t,x)$
- ADM metric (emergent in the IR)
  $ds^2 = -N^2 dt^2 + g_{ij}\,(dx^i + N^i dt)(dx^j + N^j dt)$
- Foliation-preserving deffeomorphism
  $t \rightarrow t'(t), \quad x^i \rightarrow x'^i(t,x^j)$
- Anisotropic scaling with z=3 in UV
  $t \rightarrow b^z\, t, \quad x^i \rightarrow b\, x^i$
- Ingredients in the action

$$N dt \qquad \sqrt{g}\, d^3x \qquad g_{ij} \qquad D_i \qquad R_{ij}$$

$$K_{ij} = \frac{1}{2N}\left(\partial_t g_{ij} - D_i N_j - D_j N_i\right) \qquad (C_{ijkl} = 0 \text{ in 3d })$$

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g} d^3x \left(K_{ij}K^{ij} - \lambda K^2 \right)$$

c.f. $\lambda = 1$ for GR

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad D_i R_{jk} D^i R^{jk} \qquad D_i R D^i R \right.$$

$$\left. R_i^j R_j^k R_k^i \qquad R R_i^j R_j^i \qquad R^3 \quad \right]$$

c.f. $D_i R_{jk} D^j R^{ki}$ is written in terms of other term

Horava-Lifshitz gravity

Horava (2009)

- Basic quantities:
  lapse $N(t)$, shift $N^i(t,x)$, 3d spatial metric $g_{ij}(t,x)$
- ADM metric (emergent in the IR)
  $ds^2 = -N^2dt^2 + g_{ij}\,(dx^i + N^i dt)(dx^j + N^j dt)$
- Foliation-preserving deffeomorphism
  $t \rightarrow t'(t), \quad x^i \rightarrow x'^i(t,x^j)$
- Anisotropic scaling with z=3 in UV
  $t \rightarrow b^z\, t, \quad x^i \rightarrow b\, x^i$
- Ingredients in the action

$$Ndt \qquad \sqrt{g}d^3x \qquad g_{ij} \qquad D_i \qquad R_{ij}$$

$$K_{ij} = \frac{1}{2N}\left(\partial_t g_{ij} - D_i N_j - D_j N_i\right)$$

(C_{ijkl} = 0 in 3d)

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g} d^3x \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

c.f. $\lambda = 1$ for GR

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad D_i R_{jk} D^i R^{jk} \quad D_i R D^i R$$

$$R_i^j R_j^k R_k^i \quad R R_i^j R_j^i \quad R^3 \quad]$$

c.f. $D_i R_{jk} D^j R^{ki}$ is written in terms of other term

Relevant deformations (with parity)

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R_i^j R_j^i \quad R^2 \quad \right]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R \quad \right]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad 1 \quad \right]$$

IR action with z=1

- **UV: z=3** , power-counting renormalizability

  ⬇ RG flow

- **IR: z=1** , recovers familiar GR iff $\lambda \to 1$ (provided there is "Vainshtein effect")

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g} d^3x \left(\overbrace{K_{ij}K^{ij} - \lambda K^2}^{\text{kinetic term}} + \underbrace{R - 2\Lambda}_{\text{IR potential}} \right)$$

note:
Renormalizability has not been proved.
RG flow has not yet been investigated.

Relevant deformations (with parity

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R_i^j R_j^i \quad\quad R^2 \quad \right]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad\quad R \quad\quad \right]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad\quad 1 \quad\quad \right]$$

Relevant deformations (with parity)

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R_i^j R_j^i \quad R^2 \quad \right]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R \quad \right]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad 1 \quad \right]$$

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g} d^3x \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

c.f. $\lambda = 1$ for GR

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad D_i R_{jk} D^i R^{jk} \quad D_i R D^i R \quad R_i^j R_j^k R_k^i \quad R R_i^j R_j^i \quad R^3 \quad \right]$$

c.f. $D_i R_{jk} D^j R^{ki}$ is written in terms of other term

Horava-Lifshitz gravity

Horava (2009)

- Basic quantities:
  lapse $N(t)$, shift $N^i(t,x)$, 3d spatial metric $g_{ij}(t,x)$
- ADM metric (emergent in the IR)
  $ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
- Foliation-preserving deffeomorphism
  $t \rightarrow t'(t), \quad x^i \rightarrow x'^i(t,x^j)$
- Anisotropic scaling with z=3 in UV
  $t \rightarrow b^z t, \quad x^i \rightarrow b\, x^i$
- Ingredients in the action

$$Ndt \qquad \sqrt{g}d^3x \qquad g_{ij} \qquad D_i \qquad R_{ij}$$

$$K_{ij} = \frac{1}{2N}\left(\partial_t g_{ij} - D_i N_j - D_j N_i\right)$$

($C_{ijkl} = 0$ in 3d)

Scalar with z=3

free part $I_\phi = \frac{1}{2}\int dt d^3x \left(\dot{\phi}^2 + \phi O \phi\right)$

$$O = \underbrace{\frac{\Delta^3}{M^4}}_{\text{UV: } z=3} - \frac{\kappa\Delta^2}{M^2} + \underbrace{c_\phi^2\Delta - m_\phi^2}_{\text{IR: } z=1}$$

- **UV: z=3** , renormalizable nonlinear theory

  ⬇ RG flow

- **IR: z=1** , familiar Lorentz invariant theory

Note: we need a mechanism to make “limits of speed” of different species to be the same.

c.f. Iengo, Russo, and Serone (2009)

Horava-Lifshitz gravity

Horava (2009)

- Basic quantities:
  lapse $N(t)$, shift $N^i(t,x)$, 3d spatial metric $g_{ij}(t,x)$
- ADM metric (emergent in the IR)
  $ds^2 = -N^2 dt^2 + g_{ij} (dx^i + N^i dt)(dx^j + N^j dt)$
- Foliation-preserving deffeomorphism
  $t \rightarrow t'(t), \quad x^i \rightarrow x'^i(t,x^j)$
- Anisotropic scaling with z=3 in UV
  $t \rightarrow b^z t, \quad x^i \rightarrow b\, x^i$
- Ingredients in the action

$Ndt \quad \sqrt{g}d^3x \quad g_{ij} \quad D_i \quad R_{ij}$

$K_{ij} = \frac{1}{2N}\left(\partial_t g_{ij} - D_i N_j - D_j N_i\right)$ ($C_{ijkl} = 0$ in 3d)

Relevant deformations (with parity)

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R_i^j R_j^i \qquad R^2 \quad \right]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R \quad \right]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad 1 \quad \right]$$

IR action with z=1

- **UV: z=3** , power-counting renormalizability

  ⬇ RG flow

- **IR: z=1** , recovers familiar GR iff $\lambda \to 1$
  (provided there is “Vainshtein effect”)

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g}\, d^3x \left(\overbrace{K_{ij}K^{ij} - \lambda K^2}^{\text{kinetic term}} + \underbrace{R - 2\Lambda}_{\text{IR potential}} \right)$$

note:
Renormalizability has not been proved.
RG flow has not yet been investigated.

Relevant deformations (with parity)

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad R_i^j R_j^i \quad R^2 \quad]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad R \quad]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x [\quad 1 \quad]$$

IR action with z=1

- **UV: z=3** , power-counting renormalizability

  ⬇ RG flow

- **IR: z=1** , recovers familiar GR iff $\lambda \rightarrow 1$
  (provided there is "Vainshtein effect")

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g} d^3x \left(\overbrace{K_{ij}K^{ij} - \lambda K^2}^{\text{kinetic term}} + \underbrace{R - 2\Lambda}_{\text{IR potential}} \right)$$

note:
Renormalizability has not been proved.
RG flow has not yet been investigated.

Propagating d.o.f.

- Minkowski + perturbation
  $N = 1$, $N^i = 0$, $g_{ij} = \delta_{ij} + h_{ij}$
- Residual guage freedom =
  time-independent spatial diffeo.
- Momentum constraint
  $$\partial_t \partial_i H_{ij} = 0 \qquad H_{ij} \equiv h_{ij} - \lambda h \delta_{ij}$$
- Fix the residual guage freedom by setting
  $\partial_i H_{ij} = 0$ at some fixed time surface.
- Decompose H_{ij} into trace and traceless parts
  TT part : 2 d.o.f. (usual tensor graviton)
  Trace part : 1 d.o.f. (scalar graviton)

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g} d^3x \left(K_{ij}K^{ij} - \lambda K^2 \right)$$

c.f. $\lambda = 1$ for GR

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad D_i R_{jk} D^i R^{jk} \quad D_i R D^i R$$

$$R_i^j R_j^k R_k^i \quad R R_i^j R_j^i \quad R^3 \quad]$$

c.f. $D_i R_{jk} D^j R^{ki}$ is written in terms of other term

Relevant deformations (with parity

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R_i^j R_j^i \quad\quad R^2 \quad \right]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R \quad \right]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad 1 \quad \right]$$

IR action with z=1

- **UV: z=3** , power-counting renormalizability

  ⬇ RG flow

- **IR: z=1** , recovers familiar GR iff $\lambda \to 1$

  (provided there is "Vainshtein effect")

kinetic term

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g}\, d^3x \left(\overbrace{K_{ij}K^{ij} - \lambda K^2}^{\text{kinetic term}} + \underbrace{R - 2\Lambda}_{\text{IR potential}} \right)$$

IR potential

note:

Renormalizability has not been proved.

RG flow has not yet been investigated.

Propagating d.o.f.

- Minkowski + perturbation
  $N = 1$, $N^i = 0$, $g_{ij} = \delta_{ij} + h_{ij}$
- Residual guage freedom =
  time-independent spatial diffeo.
- Momentum constraint
  $$\partial_t \partial_i H_{ij} = 0 \qquad\qquad H_{ij} \equiv h_{ij} - \lambda h \delta_{ij}$$
- Fix the residual guage freedom by setting
  $\partial_i H_{ij} = 0$ at some fixed time surface.
- Decompose H_{ij} into trace and traceless parts
  TT part : 2 d.o.f. (usual tensor graviton)
  Trace part : 1 d.o.f. (scalar graviton)

IR action with z=1

- **UV: z=3** , power-counting renormalizability

  ⬇ RG flow

- **IR: z=1** , recovers familiar GR iff λ → 1 (provided there is “Vainshtein effect”)

kinetic term

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g} d^3x \left(\overbrace{K_{ij}K^{ij} - \lambda K^2}^{\text{kinetic term}} + \underbrace{R - 2\Lambda}_{\text{IR potential}} \right)$$

IR potential

note:
Renormalizability has not been proved.
RG flow has not yet been investigated.

Propagating d.o.f.

- Minkowski + perturbation
  $N = 1$, $N^i = 0$, $g_{ij} = \delta_{ij} + h_{ij}$
- Residual guage freedom =
  time-independent spatial diffeo.
- Momentum constraint
  $$\partial_t \partial_i H_{ij} = 0 \qquad H_{ij} \equiv h_{ij} - \lambda h \delta_{ij}$$
- Fix the residual guage freedom by setting
  $\partial_i H_{ij} = 0$ at some fixed time surface.
- Decompose H_{ij} into trace and traceless parts
  TT part : 2 d.o.f. (usual tensor graviton)
  Trace part : 1 d.o.f. (scalar graviton)

Scalar graviton and λ → 1

$$h_{ij} = \tilde{H}_{ij} + \frac{1-\lambda}{2(1-3\lambda)} H \delta_{ij} - \frac{\partial_i \partial_j}{2\partial^2} H$$

- In the limit λ → 1, the scalar graviton H becomes pure gauge. So, it decouples.
- However, its kinetic term will vanish

$$I_{kin} \sim \int dt d^3x \left[(\partial_t \tilde{H}_{ij})^2 + \frac{\lambda - 1}{2(3\lambda - 1)} (\partial_t H)^2 \right]$$

  and may have strong self-coupling.
- This is not a problem if there is "Vainshtein effect", since HL gravity is supposed to be UV complete.

Propagating d.o.f.

- Minkowski + perturbation
  $N = 1,\ N^i = 0,\ g_{ij} = \delta_{ij} + h_{ij}$
- Residual guage freedom =
  time-independent spatial diffeo.
- Momentum constraint
  $$\partial_t \partial_i H_{ij} = 0 \qquad\qquad H_{ij} \equiv h_{ij} - \lambda h \delta_{ij}$$
- Fix the residual guage freedom by setting
  $\partial_i H_{ij} = 0$ at some fixed time surface.
- Decompose H_{ij} into trace and traceless parts
  TT part : 2 d.o.f. (usual tensor graviton)
  Trace part : 1 d.o.f. (scalar graviton)

Scalar graviton and λ → 1

$$h_{ij} = \tilde{H}_{ij} + \frac{1-\lambda}{2(1-3\lambda)} H \delta_{ij} - \frac{\partial_i \partial_j}{2\partial^2} H$$

- In the limit λ → 1, the scalar graviton H becomes pure gauge. So, it decouples.
- However, its kinetic term will vanish

$$I_{kin} \sim \int dt d^3x \left[(\partial_t \tilde{H}_{ij})^2 + \frac{\lambda - 1}{2(3\lambda - 1)} (\partial_t H)^2 \right]$$

  and may have strong self-coupling.
- This is not a problem if there is "Vainshtein effect", since HL gravity is supposed to be UV complete.

Propagating d.o.f.

- Minkowski + perturbation
  $N = 1,\ N^i = 0,\ g_{ij} = \delta_{ij} + h_{ij}$
- Residual guage freedom =
  time-independent spatial diffeo.
- Momentum constraint
  $\partial_t \partial_i H_{ij} = 0 \qquad H_{ij} \equiv h_{ij} - \lambda h \delta_{ij}$
- Fix the residual guage freedom by setting
  $\partial_i H_{ij} = 0$ at some fixed time surface.
- Decompose H_{ij} into trace and traceless parts
  TT part : 2 d.o.f. (usual tensor graviton)
  Trace part : 1 d.o.f. (scalar graviton)

IR action with z=1

- **UV: z=3** , power-counting renormalizability

  ⬇ RG flow

- **IR: z=1** , recovers familiar GR iff $\lambda \rightarrow 1$
  (provided there is "Vainshtein effect")

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g}\, d^3x \left(\overbrace{K_{ij}K^{ij} - \lambda K^2}^{\text{kinetic term}} + \underbrace{R - 2\Lambda}_{\text{IR potential}} \right)$$

note:
Renormalizability has not been proved.
RG flow has not yet been investigated.

Propagating d.o.f.

- Minkowski + perturbation
  $N = 1$, $N^i = 0$, $g_{ij} = \delta_{ij} + h_{ij}$
- Residual guage freedom =
  time-independent spatial diffeo.
- Momentum constraint

$$\partial_t \partial_i H_{ij} = 0 \qquad\qquad H_{ij} \equiv h_{ij} - \lambda h \delta_{ij}$$

- Fix the residual guage freedom by setting
  $\partial_i H_{ij} = 0$ at some fixed time surface.
- Decompose H_{ij} into trace and traceless parts
  TT part : 2 d.o.f. (usual tensor graviton)
  Trace part : 1 d.o.f. (scalar graviton)

Scalar graviton and λ → 1

$$h_{ij} = \tilde{H}_{ij} + \frac{1-\lambda}{2(1-3\lambda)} H\delta_{ij} - \frac{\partial_i \partial_j}{2\partial^2} H$$

- In the limit λ → 1, the scalar graviton H becomes pure gauge. So, it decouples.
- However, its kinetic term will vanish

$$I_{kin} \sim \int dt d^3x \left[(\partial_t \tilde{H}_{ij})^2 + \frac{\lambda - 1}{2(3\lambda - 1)} (\partial_t H)^2 \right]$$

and may have strong self-coupling.

- This is not a problem if there is "Vainshtein effect", since HL gravity is supposed to be UV complete.

Note

- There are at least four versions of the theory
- Only the version without the detailed balance condition with the projectability condition has a potential to be theoretically consistent and cosmologically viable.
- Horava's original proposal was with the projectability condition and with/without the detailed balance condition.
- Note, however, that there are still many fundamental issues to be addressed in the future for this version.

Note

- Imposing local Hamiltonian constraint would result in theoretical inconsistencies and phenomenological obstacles.
- “Strong coupling in Horava gravity” by C.Charmousis, et.al., arXiv:0905.2579
  “A trouble with Horava-Lifshitz gravity” by M.Li and Y.Pang, arXiv:0905.2751
- Those problems disappear once we notice that there is no local Hamiltonian constraint.
  (c.f. section 5 of arXiv:0905.3563)

Scalar graviton and λ → 1

$$h_{ij} = \tilde{H}_{ij} + \frac{1-\lambda}{2(1-3\lambda)} H\delta_{ij} - \frac{\partial_i \partial_j}{2\partial^2} H$$

- In the limit λ → 1, the scalar graviton H becomes pure gauge. So, it decouples.
- However, its kinetic term will vanish

$$I_{kin} \sim \int dt d^3x \left[(\partial_t \tilde{H}_{ij})^2 + \frac{\lambda - 1}{2(3\lambda - 1)} (\partial_t H)^2 \right]$$

and may have strong self-coupling.

- This is not a problem if there is "Vainshtein effect", since HL gravity is supposed to be UV complete.

Note

- There are at least four versions of the theory
- Only the version without the detailed balance condition with the projectability condition has a potential to be theoretically consistent and cosmologically viable.
- Horava's original proposal was with the projectability condition and with/without the detailed balance condition.
- Note, however, that there are still many fundamental issues to be addressed in the future for this version.

Scalar graviton and λ → 1

$$h_{ij} = \tilde{H}_{ij} + \frac{1-\lambda}{2(1-3\lambda)} H\delta_{ij} - \frac{\partial_i \partial_j}{2\partial^2} H$$

- In the limit λ → 1, the scalar graviton H becomes pure gauge. So, it decouples.
- However, its kinetic term will vanish

$$I_{kin} \sim \int dt d^3x \left[(\partial_t \tilde{H}_{ij})^2 + \frac{\lambda - 1}{2(3\lambda - 1)} (\partial_t H)^2 \right]$$

and may have strong self-coupling.

- This is not a problem if there is "Vainshtein effect", since HL gravity is supposed to be UV complete.

Note

- There are at least four versions of the theory
- Only the version without the detailed balance condition with the projectability condition has a potential to be theoretically consistent and cosmologically viable.
- Horava's original proposal was with the projectability condition and with/without the detailed balance condition.
- Note, however, that there are still many fundamental issues to be addressed in the future for this version.

Scalar graviton and λ → 1

$$h_{ij} = \tilde{H}_{ij} + \frac{1-\lambda}{2(1-3\lambda)} H\delta_{ij} - \frac{\partial_i \partial_j}{2\partial^2} H$$

- In the limit λ → 1, the scalar graviton H becomes pure gauge. So, it decouples.
- However, its kinetic term will vanish

$$I_{kin} \sim \int dt d^3x \left[(\partial_t \tilde{H}_{ij})^2 + \frac{\lambda - 1}{2(3\lambda - 1)} (\partial_t H)^2 \right]$$

and may have strong self-coupling.
- This is not a problem if there is "Vainshtein effect", since HL gravity is supposed to be UV complete.

Note

- There are at least four versions of the theory
- Only the version without the detailed balance condition with the projectability condition has a potential to be theoretically consistent and cosmologically viable.
- Horava's original proposal was with the projectability condition and with/without the detailed balance condition.
- Note, however, that there are still many fundamental issues to be addressed in the future for this version.

Relevant deformations (with parity)

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R_i^j R_j^i \quad R^2 \quad \right]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R \quad \right]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad 1 \quad \right]$$

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g} d^3x \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

c.f. $\lambda = 1$ for GR

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad D_i R_{jk} D^i R^{jk} \quad D_i R D^i R \quad R_i^j R_j^k R_k^i \quad R R_i^j R_j^i \quad R^3 \quad \right]$$

c.f. $D_i R_{jk} D^j R^{ki}$ is written in terms of other term

Relevant deformations (with parity)

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\qquad R_i^j R_j^i \qquad R^2 \qquad \right]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\qquad R \qquad \right]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x \left[\qquad 1 \qquad \right]$$

Propagating d.o.f.

- Minkowski + perturbation
  $N = 1,\ N^i = 0,\ g_{ij} = \delta_{ij} + h_{ij}$
- Residual guage freedom =
  time-independent spatial diffeo.
- Momentum constraint
  $$\partial_t \partial_i H_{ij} = 0 \qquad H_{ij} \equiv h_{ij} - \lambda h \delta_{ij}$$
- Fix the residual guage freedom by setting
  $\partial_i H_{ij} = 0$ at some fixed time surface.
- Decompose H_{ij} into trace and traceless parts
  TT part : 2 d.o.f. (usual tensor graviton)
  Trace part : 1 d.o.f. (scalar graviton)

Note

- There are at least four versions of the theory
- Only the version without the detailed balance condition with the projectability condition has a potential to be theoretically consistent and cosmologically viable.
- Horava's original proposal was with the projectability condition and with/without the detailed balance condition.
- Note, however, that there are still many fundamental issues to be addressed in the future for this version.

Projectability condition

- Infinitesimal tr. $\delta t = f(t)$, $\delta x^i = \zeta^i(t, x^j)$

$$\delta g_{ij} = \partial_i \zeta^k g_{jk} + \partial_j \zeta^k g_{ik} + \zeta^k \partial_k g_{ij} + f \dot{g}_{ij}$$

$$\delta N_i = \partial_i \zeta^j N_j + \zeta^j \partial_j N_i + \dot{\zeta}^j g_{ij} + \dot{f} N_i + f \dot{N}_i$$

$$\delta N = \zeta^i \partial_i N + \dot{f} N + f \dot{N}$$

- Space-independent N cannot be transformed to space-dependent N.
- N is gauge d.o.f. associated with the space-independent time reparametrization.
- It is natural to restrict N to be space-independent.
- Consequently, Hamiltonian constraint is an equation integrated over a whole space.

Note

- Imposing local Hamiltonian constraint would result in theoretical inconsistencies and phenomenological obstacles.
- "Strong coupling in Horava gravity"
  by C.Charmousis, et.al., arXiv:0905.2579
  "A trouble with Horava-Lifshitz gravity"
  by M.Li and Y.Pang, arXiv:0905.2751
- Those problems disappear once we notice that there is no local Hamiltonian constraint.
  (c.f. section 5 of arXiv:0905.3563)

Projectability condition

- Infinitesimal tr. $\delta t = f(t)$, $\delta x^i = \zeta^i(t,x^j)$

$$\delta g_{ij} = \partial_i \zeta^k g_{jk} + \partial_j \zeta^k g_{ik} + \zeta^k \partial_k g_{ij} + f \dot{g}_{ij}$$

$$\delta N_i = \partial_i \zeta^j N_j + \zeta^j \partial_j N_i + \dot{\zeta}^j g_{ij} + \dot{f} N_i + f \dot{N}_i$$

$$\delta N = \zeta^i \partial_i N + \dot{f} N + f \dot{N}$$

- Space-independent N cannot be transformed to space-dependent N.
- N is gauge d.o.f. associated with the space-independent time reparametrization.
- It is natural to restrict N to be space-independent.
- Consequently, Hamiltonian constraint is an equation integrated over a whole space.

Note

- Imposing local Hamiltonian constraint would result in theoretical inconsistencies and phenomenological obstacles.
- "Strong coupling in Horava gravity"
  by C.Charmousis, et.al., arXiv:0905.2579
  "A trouble with Horava-Lifshitz gravity"
  by M.Li and Y.Pang, arXiv:0905.2751
- Those problems disappear once we notice that there is no local Hamiltonian constraint.
  (c.f. section 5 of arXiv:0905.3563)

Black holes with N=N(t)?

- Schwarzschild BH in PG coordinate

$$ds^2 = -dt_P^2 + \left(dr \pm \sqrt{\frac{2m}{r}}dt_P\right)^2 + r^2 d\Omega^2$$

exact sol for $\lambda = 1$

- Gaussian normal coordinate

$$ds^2 = -dt_G^2 + \cdots$$

approx sol for $\lambda = 1$

Lemaitre reference frame
Doran coordinate

- Star with $\lambda > 1$ and $\lambda \to 1$

work in progress with K.Izumi and K.Takahashi

Note

- Imposing local Hamiltonian constraint would result in theoretical inconsistencies and phenomenological obstacles.
- "Strong coupling in Horava gravity"
  by C.Charmousis, et.al., arXiv:0905.2579
  "A trouble with Horava-Lifshitz gravity"
  by M.Li and Y.Pang, arXiv:0905.2751
- Those problems disappear once we notice that there is no local Hamiltonian constraint.
  (c.f. section 5 of arXiv:0905.3563)

Black holes with N=N(t)?

- Schwarzschild BH in PG coordinate

$$ds^2 = -dt_P^2 + \left(dr \pm \sqrt{\frac{2m}{r}}dt_P\right)^2 + r^2 d\Omega^2$$

exact sol for $\lambda = 1$

- Gaussian normal coordinate

$$ds^2 = -dt_G^2 + \cdots$$

approx sol for $\lambda = 1$

Lemaitre reference frame
Doran coordinate

- Star with $\lambda > 1$ and $\lambda \to 1$

work in progress with K.Izumi and K.Takahashi

Horava-Lifshitz cosmology

- It is interesting to investigate cosmological implications, in parallel with fundamental issues such as renormalizability and RG flow.
- Higher curvature terms lead to regular bounce (Calcagni 2009, Brandenberger 2009).
- Higher curvature terms ($1/a^6$, $1/a^4$) might make the flatness problem milder (Kiritsis&Kofinas 2009).
- The z=3 scaling leads to scale-invariant cosmological perturbations in non-inflationary epoch (Mukohyama 2009) and peculiar scaling of ρ_{rad} (Mukohyama, Nakayama, Takahashi, Yokoyama 2009).
- Absence of local Hamiltonian constraint leads to CDM as integration "constant" (Mukohyama 2009).
- Constant time hypersurfaces do not form caustics (Mukohyama 2009).
- New mechanism for generation of primordial magnetic seed field (Maeda, Mukohyama, Shiromizu 2009).

Black holes with N=N(t)?

- Schwarzschild BH in PG coordinate

$$ds^2 = -dt_P^2 + \left(dr \pm \sqrt{\frac{2m}{r}} dt_P\right)^2 + r^2 d\Omega^2$$

exact sol for $\lambda = 1$

- Gaussian normal coordinate

$$ds^2 = -dt_G^2 + \cdots$$

approx sol for $\lambda = 1$

Lemaitre reference frame
Doran coordinate

- Star with $\lambda > 1$ and $\lambda \rightarrow 1$

work in progress with K.Izumi and K.Takahash

Horava-Lifshitz cosmology

- It is interesting to investigate cosmological implications, in parallel with fundamental issues such as renormalizability and RG flow.
- Higher curvature terms lead to regular bounce (Calcagni 2009 Brandenberger 2009).
- Higher curvature terms ($1/a^6$, $1/a^4$) might make the flatness problem milder (Kiritsis&Kofinas 2009).
- The z=3 scaling leads to scale-invariant cosmological perturbations in non-inflationary epoch (Mukohyama 2009) and peculiar scaling of ρ_{rad} (Mukohyama, Nakayama, Takahashi, Yokoyama 2009).
- Absence of local Hamiltonian constraint leads to CDM as integration "constant" (Mukohyama 2009).
- Constant time hypersurfaces do not form caustics (Mukohyama 2009).
- New mechanism for generation of primordial magnetic seed field (Maeda, Mukohyama, Shiromizu 2009).

Black holes with N=N(t)?

- Schwarzschild BH in PG coordinate

$$ds^2 = -dt_P^2 + \left(dr \pm \sqrt{\frac{2m}{r}}dt_P\right)^2 + r^2 d\Omega^2$$

exact sol for $\lambda = 1$

- Gaussian normal coordinate

$$ds^2 = -dt_G^2 + \cdots$$

approx sol for $\lambda = 1$

Lemaitre reference frame
Doran coordinate

- Star with $\lambda > 1$ and $\lambda \rightarrow 1$

work in progress with K.Izumi and K.Takahash

Horava-Lifshitz cosmology

- It is interesting to investigate cosmological implications, in parallel with fundamental issues such as renormalizability and RG flow.
- Higher curvature terms lead to regular bounce (Calcagni 2009; Brandenberger 2009).
- Higher curvature terms ($1/a^6$, $1/a^4$) might make the flatness problem milder (Kiritsis&Kofinas 2009).
- The z=3 scaling leads to scale-invariant cosmological perturbations in non-inflationary epoch (Mukohyama 2009) and peculiar scaling of ρ_{rad} (Mukohyama, Nakayama, Takahashi, Yokoyama 2009).
- Absence of local Hamiltonian constraint leads to CDM as integration "constant" (Mukohyama 2009).
- Constant time hypersurfaces do not form caustics (Mukohyama 2009).
- New mechanism for generation of primordial magnetic seed field (Maeda, Mukohyama, Shiromizu 2009).

Scale-invariant cosmological perturbations from Horava-Lifshitz gravity without inflation

arXiv:0904.2190 [hep-th]

c.f. Basic mechanism is common for “Primordial magnetic field from non-inflationary cosmic expansion in Horava-Lifshitz gravity”, arXiv:0909.2149 [astro-th.CO] with S.Maeda and T.Shiromizu.

Usual story with z=1

Usual story with z=1

- $\omega^2 >> H^2$: oscillate

  $\omega^2 << H^2$: freeze

  oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

Usual story with z=1

- $\omega^2 >> H^2$: oscillate

  $\omega^2 << H^2$: freeze

  oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

- Scaling law

  $t \rightarrow b\, t$ ($E \rightarrow b^{-1}E$)

  $x \rightarrow b\, x$ ⇒ $\delta\phi \propto E \sim H$

  $\phi \rightarrow b^{-1}\, \phi$

  Scale-invariance requires almost const. H, i.e. inflation.

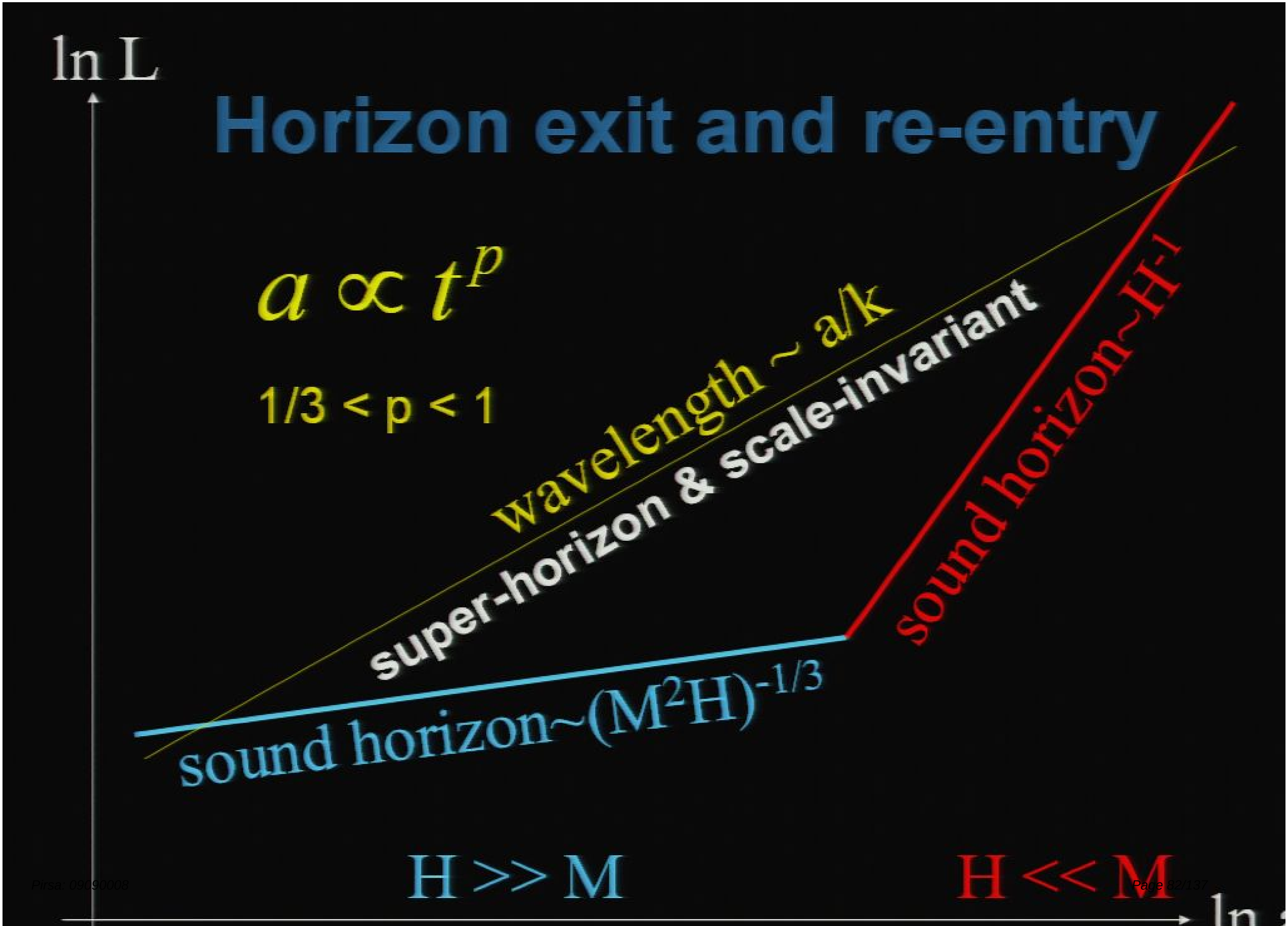
UV fixed point with z=3

UV fixed point with z=3

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$
  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$
  OK for $a \sim t^p$ with $p > 1/3$

UV fixed point with z=3

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$
  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$
  OK for $a \sim t^p$ with $p > 1/3$
- Scaling law
  $t \to b^3 t$ ($E \to b^{-3} E$)
  $x \to b x$
  $\phi \to b^0 \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$
  Scale-invariant fluctuations!



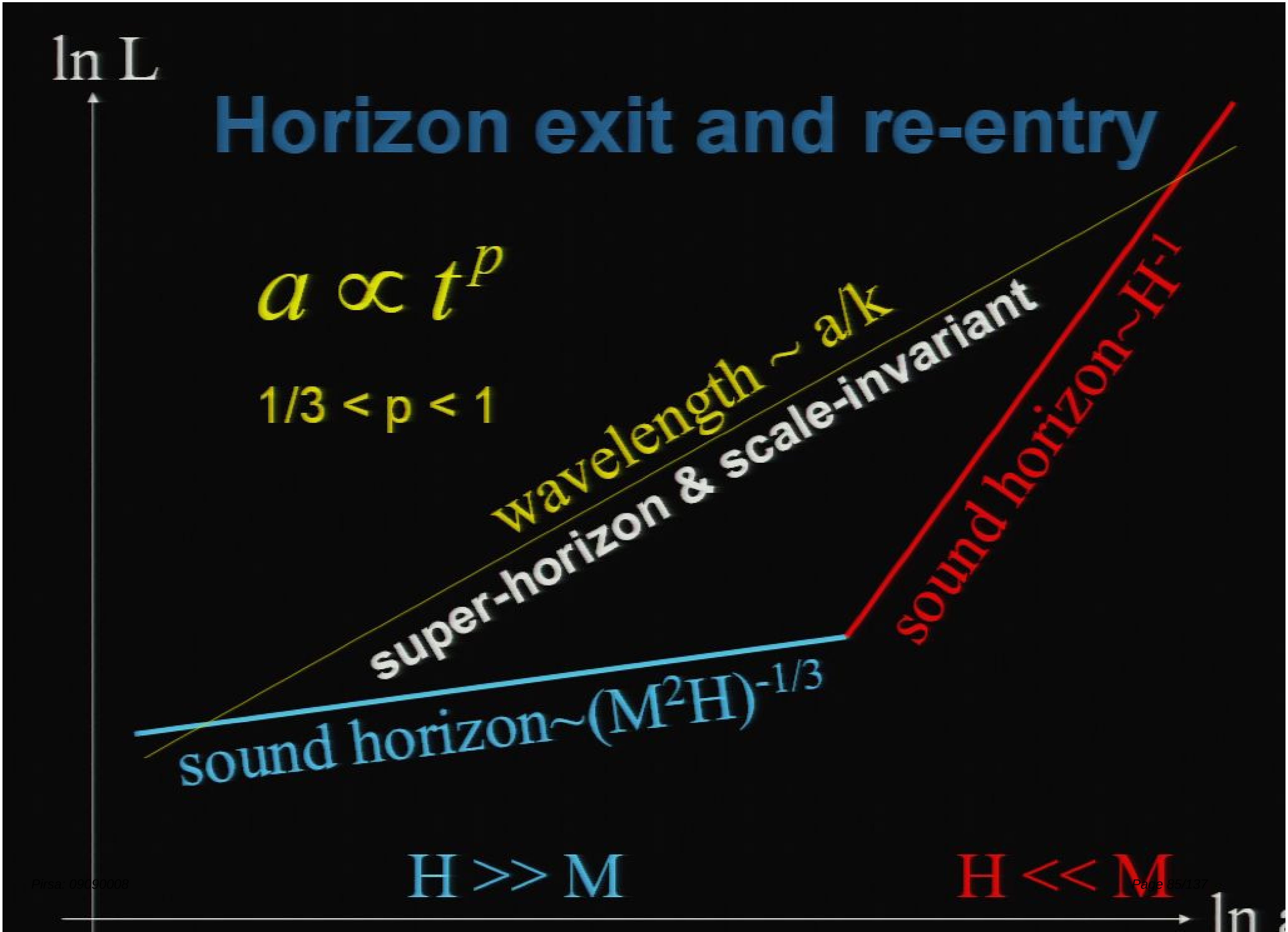
Horizon exit and re-entry
ln L
$a \propto t^p$
1/3 < p < 1
wavelength ~ a/k
super-horizon & scale-invariant
sound horizon~H^{-1}
sound horizon~(M^{2}H)$^{-1/3}$
H >> M
H << M
ln a

UV fixed point with z=3

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$
  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$
  OK for $a \sim t^p$ with $p > 1/3$
- Scaling law
  $t \to b^3\, t$ ($E \to b^{-3} E$)
  $x \to b\, x$
  $\phi \to b^0\, \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$
  Scale-invariant fluctuations!

UV fixed point with z=3

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$
  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$
  OK for $a \sim t^p$ with $p > 1/3$
- Scaling law
  $t \rightarrow b^3\, t$ ($E \rightarrow b^{-3}E$)
  $x \rightarrow b\, x$
  $\phi \rightarrow b^0\, \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$
  Scale-invariant fluctuations!


ln L
Horizon exit and re-entry
a ∝ t^p
1/3 < p < 1
wavelength ~ a/k
super-horizon & scale-invariant
sound horizon~H^-1
sound horizon~(M^2H)^-1/3
H >> M
H << M
ln a

UV fixed point with z=3

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$
  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$
  OK for $a \sim t^p$ with $p > 1/3$
- Scaling law
  $t \rightarrow b^3 t$ ($E \rightarrow b^{-3}E$)
  $x \rightarrow b\,x$
  $\phi \rightarrow b^0 \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$
  Scale-invariant fluctuations!

Usual story with z=1

- $\omega^2 >> H^2$: oscillate

  $\omega^2 << H^2$: freeze

  oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

- Scaling law

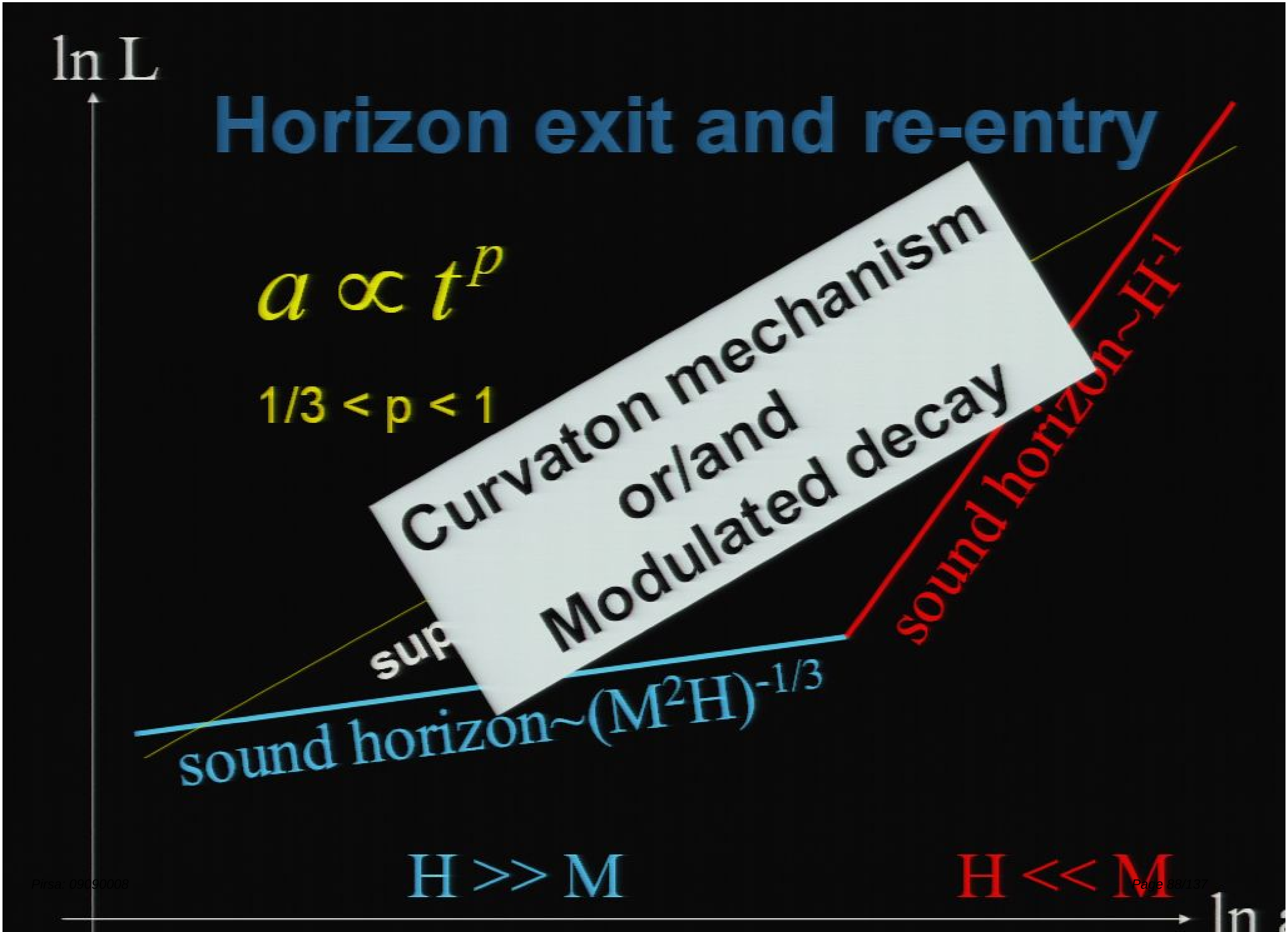
  $t \rightarrow b\,t$ ($E \rightarrow b^{-1}E$)

  $x \rightarrow b\,x$

  $\phi \rightarrow b^{-1}\,\phi$

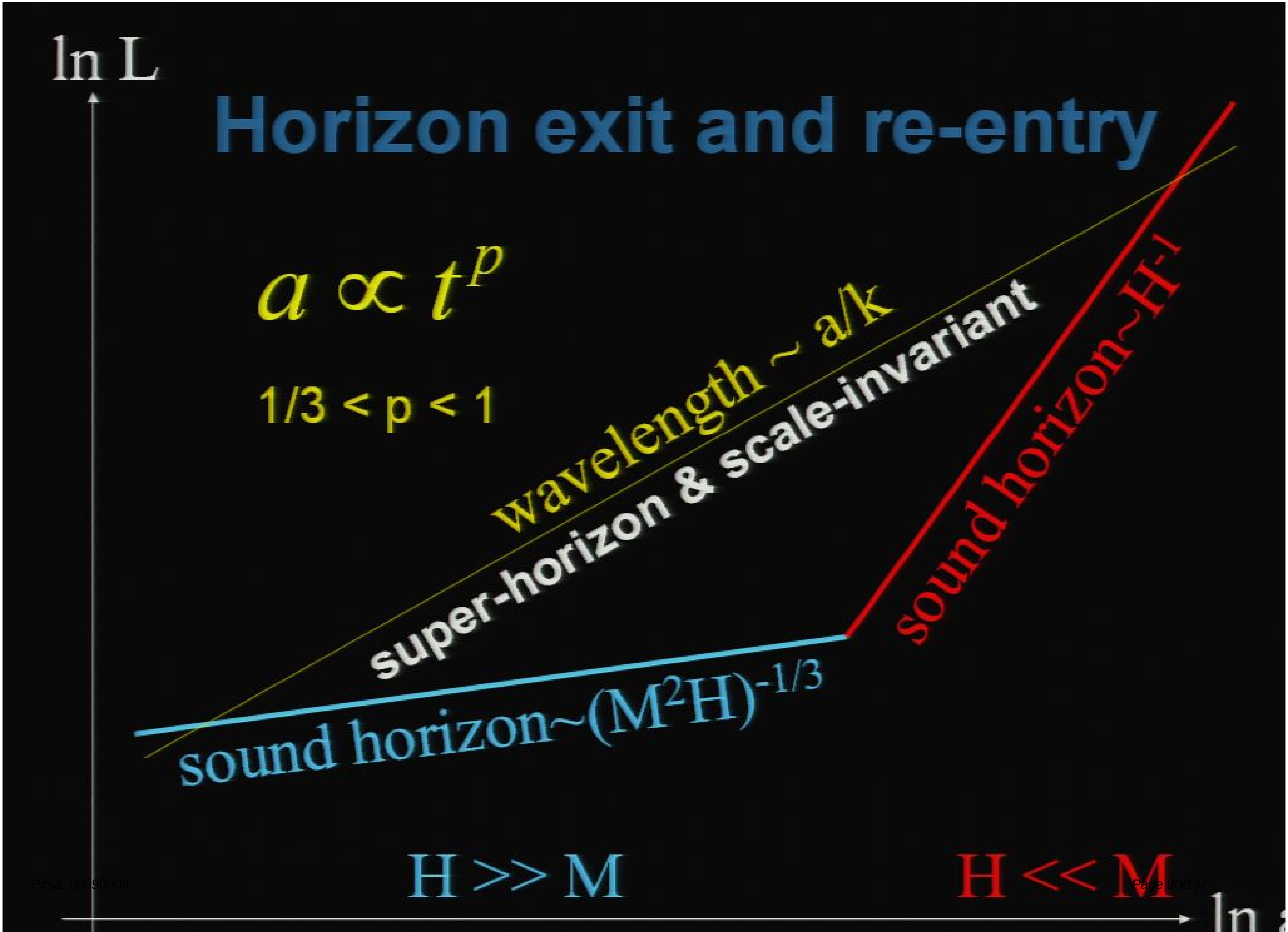
  $$\Rightarrow \quad \delta\phi \propto E \sim H$$

  Scale-invariance requires almost const. H, i.e. inflation.

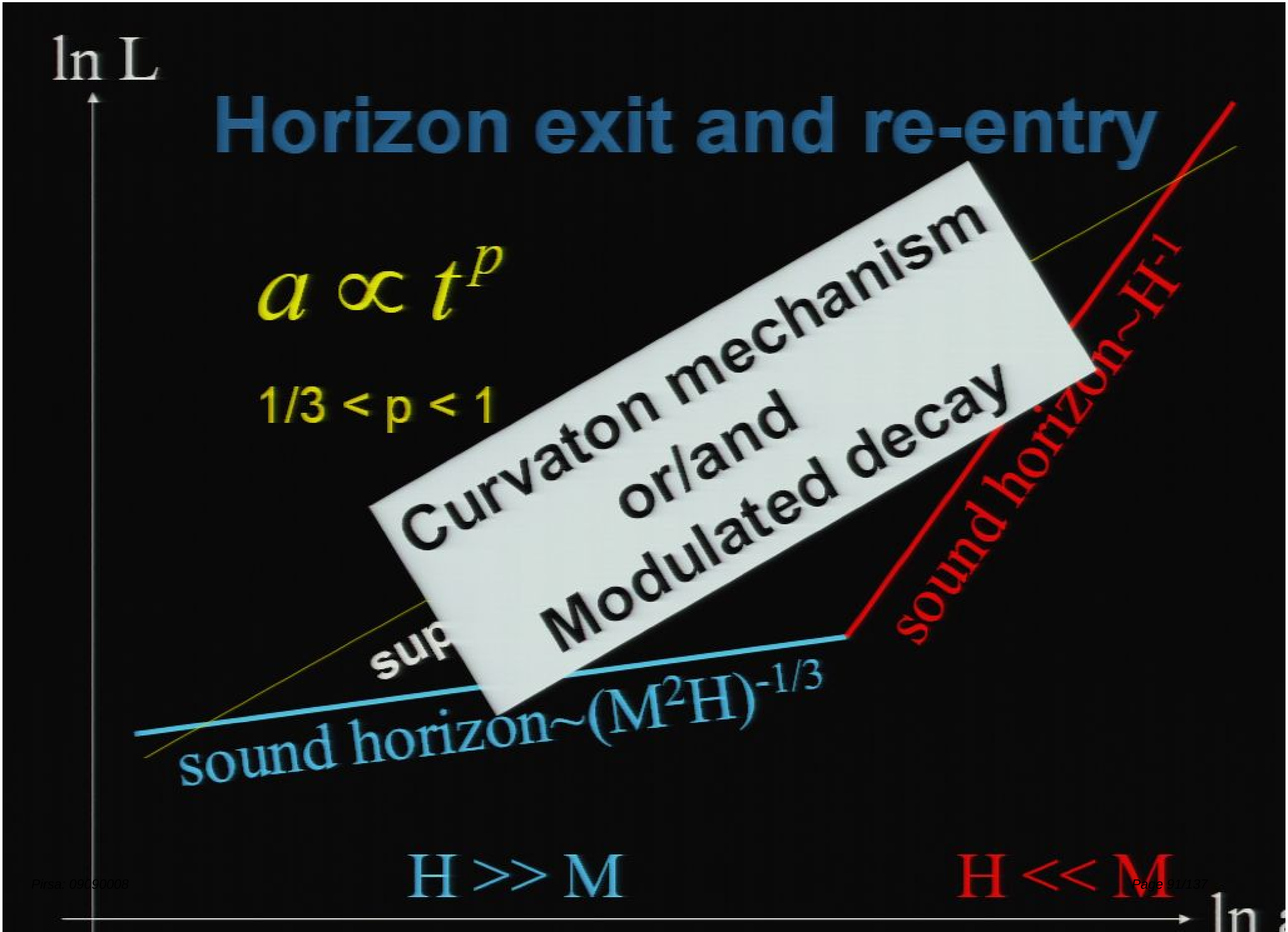

ln L
Horizon exit and re-entry
a ∝ t^p
1/3 < p < 1
Curvaton mechanism
or/and
Modulated decay
sup
sound horizon~(M^2H)^-1/3
sound horizon~H^-1
H >> M
H << M
ln a

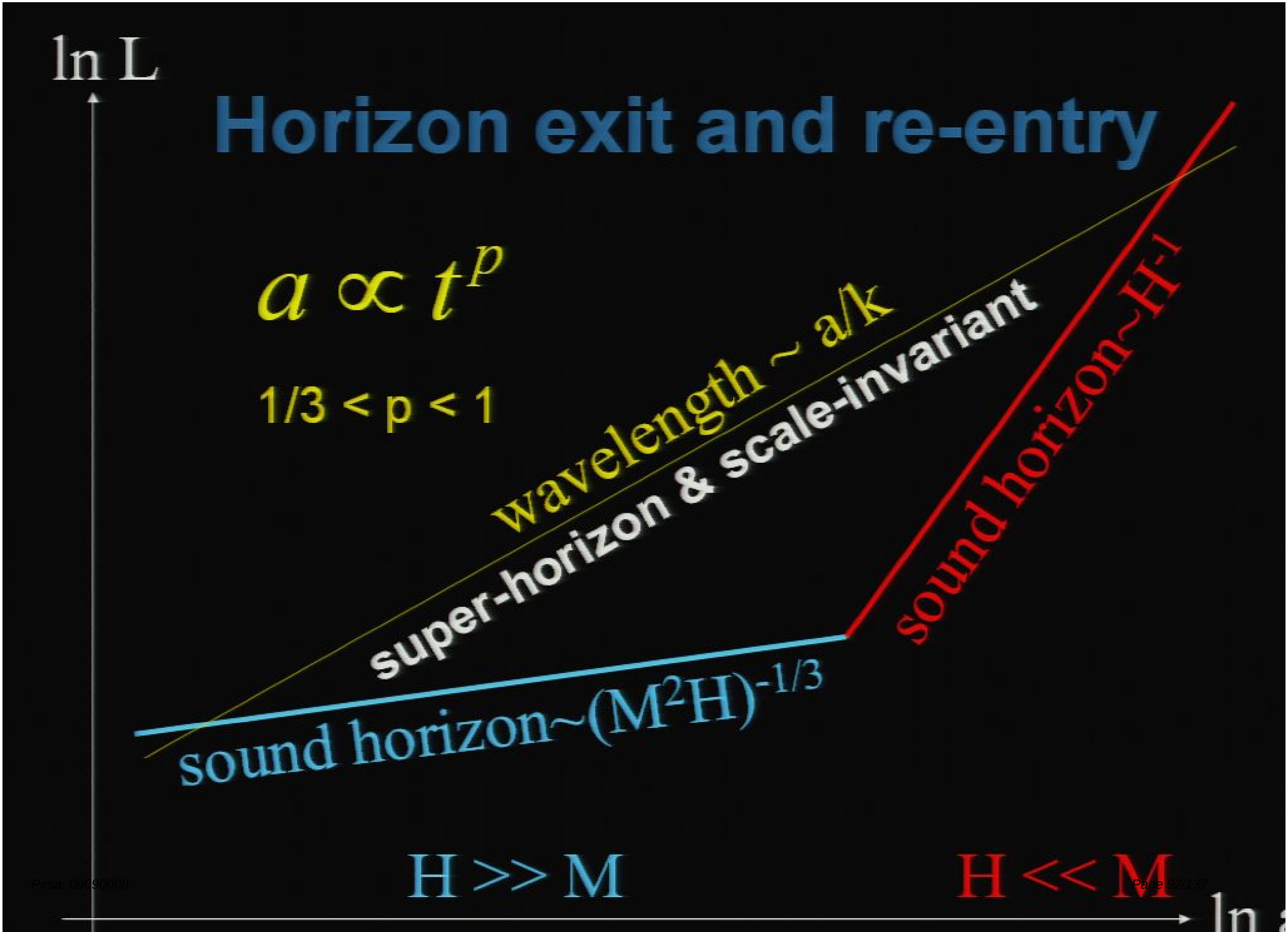
UV fixed point with z=3

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$
  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$
  OK for $a \sim t^p$ with $p > 1/3$
- Scaling law
  $t \to b^3 t$ ($E \to b^{-3}E$)
  $x \to b\, x$
  $\phi \to b^0 \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$
  Scale-invariant fluctuations!



ln L
Horizon exit and re-entry
a ∝ t^p
1/3 < p < 1
wavelength ~ a/k
super-horizon & scale-invariant
sound horizon~H^-1
sound horizon~(M^2H)^-1/3
H >> M
H << M
ln a


ln L
Horizon exit and re-entry
a ∝ t^p
1/3 < p < 1
Curvaton mechanism
or/and
Modulated decay
sup
sound horizon~(M²H)^-1/3
sound horizon~H^-1
H >> M
H << M
ln a


Horizon exit and re-entry
ln L
a ∝ t^p
1/3 < p < 1
wavelength ~ a/k
super-horizon & scale-invariant
sound horizon~(M²H)^-1/3
sound horizon~H^-1
H >> M
H << M
ln a

UV fixed point with z=3

Usual story with z=1

- $\omega^2 >> H^2$: oscillate

  $\omega^2 << H^2$: freeze

  oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

- Scaling law

  $t \to b\, t$ ($E \to b^{-1}E$)

  $x \to b\, x$

  $\phi \to b^{-1}\, \phi$

  $$\Rightarrow \quad \delta\phi \propto E \sim H$$

  Scale-invariance requires almost const. H, i.e. inflation.

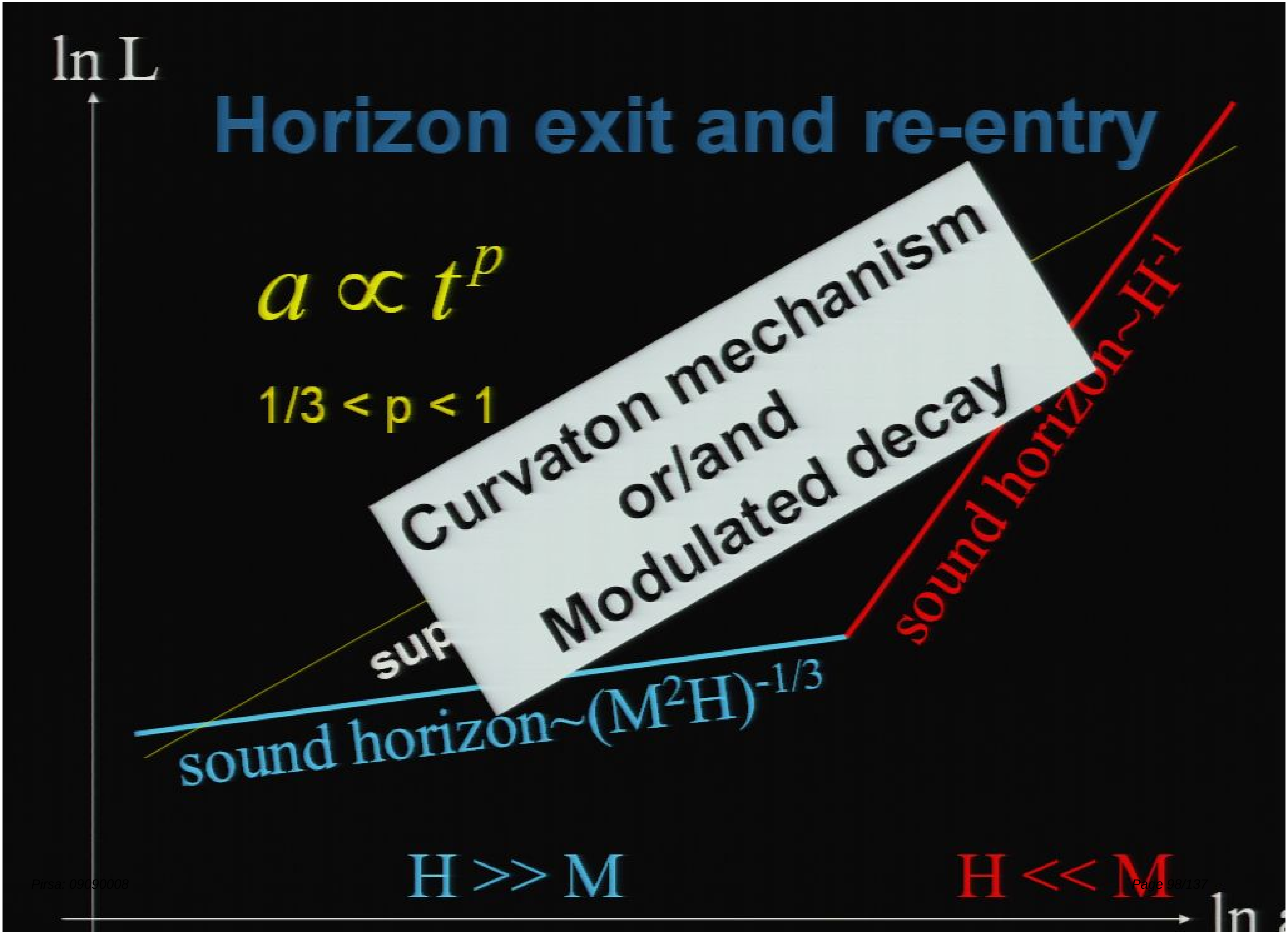
UV fixed point with z=3

UV fixed point with z=3

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$
  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$
  OK for $a \sim t^p$ with $p > 1/3$

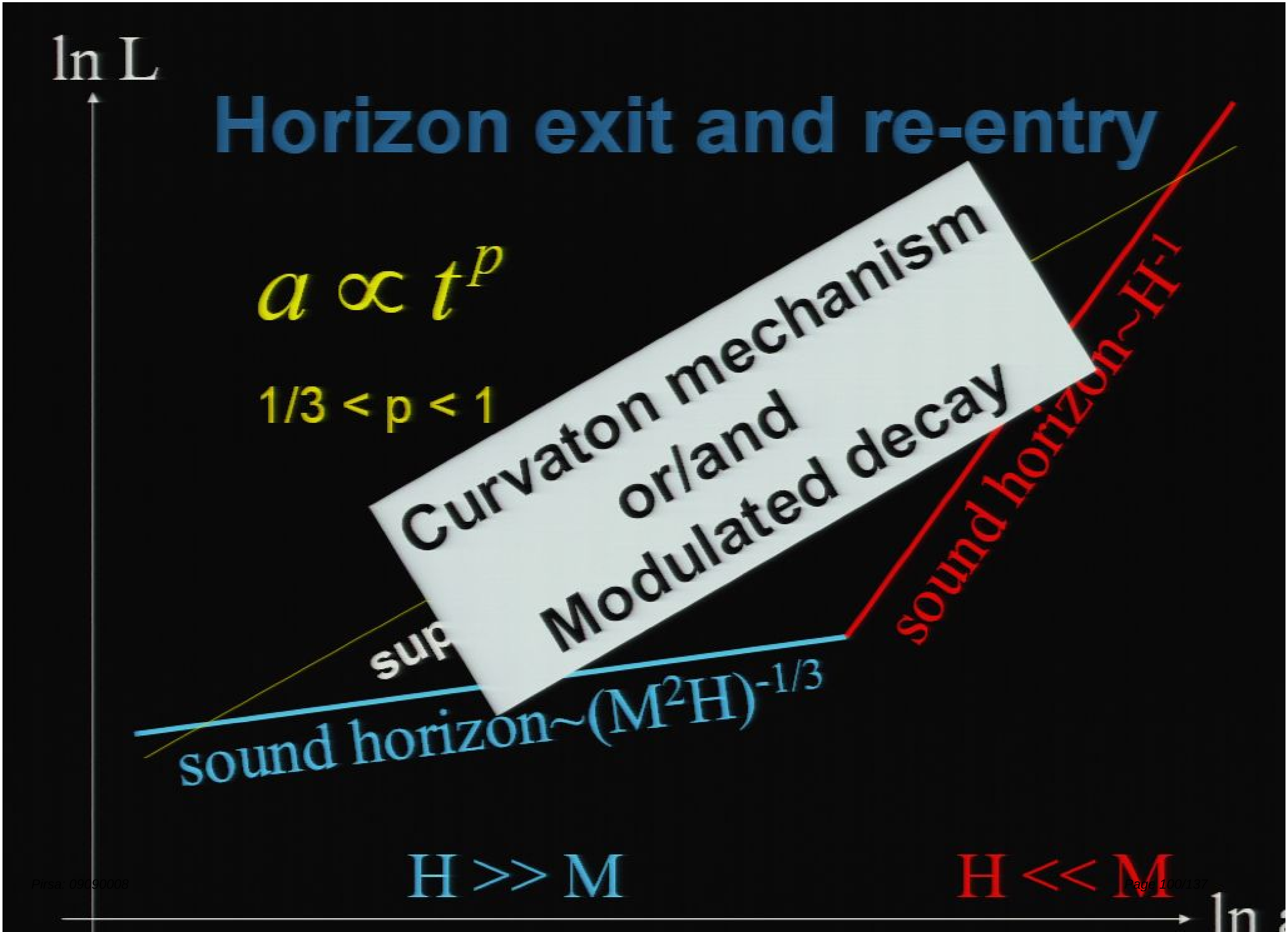
Dark matter as integration constant in Horava-Lifshitz gravity

arXiv:0905.3563 [hep-th]


Horizon exit and re-entry
ln L
ln a
$a \propto t^p$
1/3 < p < 1
Curvaton mechanism
or/and
Modulated decay
sound horizon~$(M^2H)^{-1/3}$
sound horizon~H^{-1}
H >> M
H << M

Dark matter as integration constant in Horava-Lifshitz gravity

arXiv:0905.3563 [hep-th]

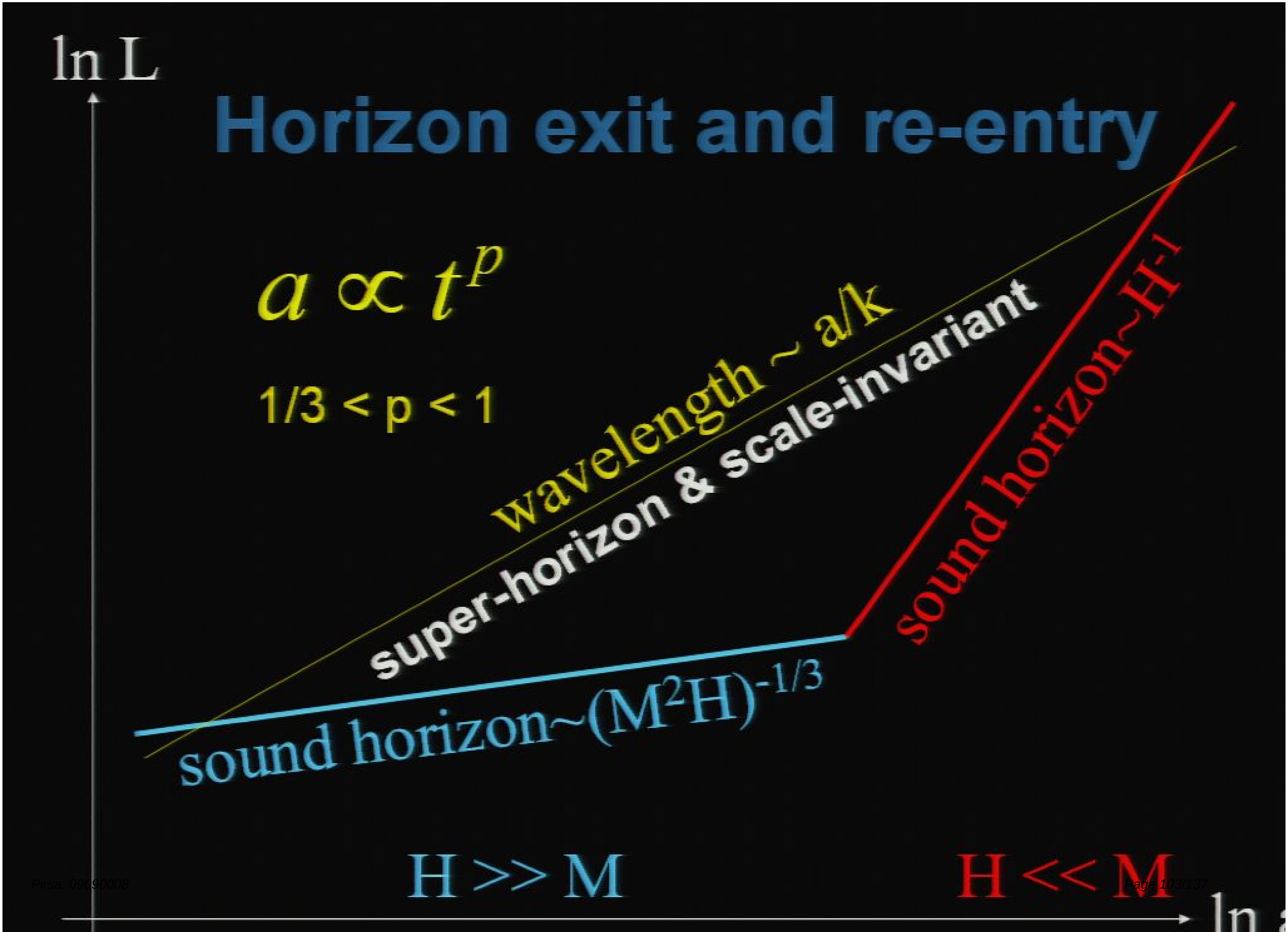

ln L
Horizon exit and re-entry
$a \propto t^p$
1/3 < p < 1
Curvaton mechanism
or/and
Modulated decay
sound horizon~$(M^2H)^{-1/3}$
sound horizon~H^{-1}
H >> M
H << M

UV fixed point with z=3

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$
  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$
  OK for $a \sim t^p$ with $p > 1/3$
- Scaling law
  $t \to b^3 t$ ($E \to b^{-3}E$)
  $x \to b\,x$
  $\phi \to b^0\,\phi$ $\Rightarrow$ $\delta\phi \propto E^0 \sim H^0$
  Scale-invariant fluctuations!

UV fixed point with z=3

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$
  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$
  OK for $a \sim t^p$ with $p > 1/3$
- Scaling law
  $t \to b^3\, t \quad (E \to b^{-3}E)$
  $x \to b\, x$
  $\phi \to b^0\, \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$
  Scale-invariant fluctuations!



Horizon exit and re-entry
ln L
a ∝ t^p
1/3 < p < 1
wavelength ~ a/k
super-horizon & scale-invariant
sound horizon~(M²H)^-1/3
sound horizon~H^-1
H >> M
H << M
ln a

Dark matter as integration constant in Horava-Lifshitz gravity

arXiv:0905.3563 [hep-th]

Structure of GR

- 4D diffeomorphism →
  4 constraints = 1 Hamiltonian + 3 momentum
  @ each time @ each point
- Constraints are preserved by dynamical equations.
- We can solve dynamical equations, provided that constraints are satisfied at initial time.

Structure of GR

- 4D diffeomorphism →
  4 constraints = 1 Hamiltonian + 3 momentum
  @ each time @ each point
- Constraints are preserved by dynamical equations.
- We can solve dynamical equations, provided that constraints are satisfied at initial time.

Structure of HL gravity

- Foliation-preserving diffeomorphism
  = 3D spatial diffeormorphism
  + space-independent time reparametrization
- 3 local constraints + 1 global constraint
  = 3 momentum @ each time @ each point
  + 1 Hamiltonian @ each time integrated
- Constraints are preserved by dynamical equations.
- We can solve dynamical equations, provided that constraints are satisfied at initial time.

IR limit of HL gravity

$$\frac{1}{16\pi G_N}\int N dt\sqrt{g}\,d^3x\left(K_{ij}K^{ij}-\lambda K^2+R-2\Lambda\right)$$

- Looks like GR iff λ = 1. So, we assume that λ = 1 is an IR fixed point of RG flow.
- Global Hamiltonian constraint

$$\int d^3x\sqrt{g}(G^{(4)}_{\mu\nu}+\Lambda g^{(4)}_{\mu\nu}-8\pi G_N T_{\mu\nu})n^\mu n^\nu = 0$$

$$n_\mu dx^\mu = -N dt, \quad n^\mu\partial_\mu = \frac{1}{N}(\partial_t - N^i\partial_i)$$

- Momentum constraint & dynamical eq

$$(G^{(4)}_{i\mu}+\Lambda g^{(4)}_{i\mu}-8\pi G_N T_{i\mu})n^\mu = 0$$

$$G^{(4)}_{ij}+\Lambda g^{(4)}_{ij}-8\pi G_N T_{ij} = 0$$

Dark matter as integration constant

- Def. $T^{HL}_{\mu\nu}$ $\quad G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu} \right)$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad\qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g}\rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K\rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$

Summary

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.

Dark matter as integration constant

- Def. $T^{HL}{}_{\mu\nu}$ $\quad G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu}\right)$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g}\rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K\rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$

IR limit of HL gravity

$$\frac{1}{16\pi G_N}\int N dt\sqrt{g}d^3x\left(K_{ij}K^{ij}-\lambda K^2+R-2\Lambda\right)$$

- Looks like GR iff λ = 1. So, we assume that λ = 1 is an IR fixed point of RG flow.
- Global Hamiltonian constraint

$$\int d^3x\sqrt{g}(G^{(4)}_{\mu\nu}+\Lambda g^{(4)}_{\mu\nu}-8\pi G_N T_{\mu\nu})n^\mu n^\nu=0$$

$$n_\mu dx^\mu=-Ndt,\quad n^\mu\partial_\mu=\frac{1}{N}(\partial_t-N^i\partial_i)$$

- Momentum constraint & dynamical eq

$$(G^{(4)}_{i\mu}+\Lambda g^{(4)}_{i\mu}-8\pi G_N T_{i\mu})n^\mu=0$$

$$G^{(4)}_{ij}+\Lambda g^{(4)}_{ij}-8\pi G_N T_{ij}=0$$

Dark matter as integration constant

- Def. $T^{HL}_{\mu\nu}$ $\quad G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu} \right)$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad\qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K \rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$

IR limit of HL gravity

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g} d^3x \left(K_{ij}K^{ij} - \lambda K^2 + R - 2\Lambda\right)$$

- Looks like GR iff λ = 1. So, we assume that λ = 1 is an IR fixed point of RG flow.
- Global Hamiltonian constraint

$$\int d^3x \sqrt{g}(G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} - 8\pi G_N T_{\mu\nu}) n^\mu n^\nu = 0$$

$$n_\mu dx^\mu = -N dt, \quad n^\mu \partial_\mu = \frac{1}{N}(\partial_t - N^i \partial_i)$$

- Momentum constraint & dynamical eq

$$(G^{(4)}_{i\mu} + \Lambda g^{(4)}_{i\mu} - 8\pi G_N T_{i\mu}) n^\mu = 0$$

$$G^{(4)}_{ij} + \Lambda g^{(4)}_{ij} - 8\pi G_N T_{ij} = 0$$

Dark matter as integration constant

- Def. $T^{HL}_{\mu\nu}$ $G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu}\right)$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K\rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$

Micro to Macro

- Overall behavior of smooth $T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu$ is like pressureless dust.
- Microscopic lumps of ρ^{HL} can collide and bounce. (cf. early universe bounce [Calcagni 2009, Brandenberger 2009])
- Group of microscopic lumps with collisions and bounces → When coarse-grained, can it mimic a cluster of particles with velocity dispersion?
- Dispersion relation of matter fields defined in the rest frame of "dark matter"
  → Any astrophysical implications?

Dark matter as integration constant

- Def. $T^{HL}_{\mu\nu}$ $\quad G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu} \right)$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad\qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K \rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$

IR limit of HL gravity

$$\frac{1}{16\pi G_N}\int N dt\sqrt{g}\,d^3x\left(K_{ij}K^{ij}-\lambda K^2+R-2\Lambda\right)$$

- Looks like GR iff λ = 1. So, we assume that λ = 1 is an IR fixed point of RG flow.
- Global Hamiltonian constraint

$$\int d^3x\sqrt{g}(G^{(4)}_{\mu\nu}+\Lambda g^{(4)}_{\mu\nu}-8\pi G_N T_{\mu\nu})n^\mu n^\nu=0$$

$$n_\mu dx^\mu=-Ndt,\quad n^\mu\partial_\mu=\frac{1}{N}(\partial_t-N^i\partial_i)$$

- Momentum constraint & dynamical eq

$$(G^{(4)}_{i\mu}+\Lambda g^{(4)}_{i\mu}-8\pi G_N T_{i\mu})n^\mu=0$$

$$G^{(4)}_{ij}+\Lambda g^{(4)}_{ij}-8\pi G_N T_{ij}=0$$

Dark matter as integration constant

- Def. $T^{HL}{}_{\mu\nu}$ $G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu} \right)$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K \rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$

Micro to Macro

- Overall behavior of smooth $T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu$ is like pressureless dust.
- Microscopic lumps of ρ^{HL} can collide and bounce. (cf. early universe bounce [Calcagni 2009, Brandenberger 2009])
- Group of microscopic lumps with collisions and bounces → When coarse-grained, can it mimic a cluster of particles with velocity dispersion?
- Dispersion relation of matter fields defined in the rest frame of "dark matter"
  → Any astrophysical implications?

Summary

Micro to Macro

- Overall behavior of smooth $T^{HL}{}_{\mu\nu} = \rho^{HL} n_\mu n_\nu$ is like pressureless dust.
- Microscopic lumps of ρ^{HL} can collide and bounce. (cf. early universe bounce [Calcagni 2009, Brandenberger 2009])
- Group of microscopic lumps with collisions and bounces → When coarse-grained, can it mimic a cluster of particles with velocity dispersion?
- Dispersion relation of matter fields defined in the rest frame of "dark matter"
  → Any astrophysical implications?

Dark matter as integration constant

- Def. $T^{HL}_{\mu\nu}$ $\quad G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu} \right)$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad\qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K \rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$

Summary

Summary

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.

Summary

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- While there are many fundamental issues to be addressed, it is interesting to investigate cosmological implications.

Summary

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- While there are many fundamental issues to be addressed, it is interesting to investigate cosmological implications.
- The z=3 scaling leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p > 1/3$

Summary

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- While there are many fundamental issues to be addressed, it is interesting to investigate cosmological implications.
- The z=3 scaling leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p>1/3$
- The lack of local Hamiltonian constraint leads to “dark matter” as an integration constant.

Open problems

Open problems

- Renormalizability beyond power-counting

Open problems

- Renormalizability beyond power-counting
- RG flow: is $\lambda = 1$ an IR fixed point ?

Open problems

- Renormalizability beyond power-counting
- RG flow: is $\lambda = 1$ an IR fixed point ?
- Embedding into an unified theory : can we get a common "limit of speed" ?

Open problems

- Renormalizability beyond power-counting
- RG flow: is $\lambda = 1$ an IR fixed point ?
- Embedding into an unified theory : can we get a common "limit of speed" ?
- Are there vDVZ discontinuity and Vainshtein effect? Unlike massive gravity case, Vainshtein effect can be trusted because of "UV completeness".

Open problems

- Renormalizability beyond power-counting
- RG flow: is $\lambda = 1$ an IR fixed point ?
- Embedding into an unified theory : can we get a common “limit of speed” ?
- Are there vDVZ discontinuity and Vainshtein effect? Unlike massive gravity case, Vainshtein effect can be trusted because of “UV completeness”.
- How to setup initial condition for “dark matter”?

Open problems

- Renormalizability beyond power-counting
- RG flow: is $\lambda = 1$ an IR fixed point ?
- Embedding into an unified theory : can we get a common “limit of speed” ?
- Are there vDVZ discontinuity and Vainshtein effect? Unlike massive gravity case, Vainshtein effect can be trusted because of “UV completeness”.
- How to setup initial condition for “dark matter”?
- Spectral tilt from anomalous dimension?

Open problems

- Renormalizability beyond power-counting
- RG flow: is $\lambda = 1$ an IR fixed point ?
- Embedding into an unified theory : can we get a common "limit of speed" ?
- Are there vDVZ discontinuity and Vainshtein effect? Unlike massive gravity case, Vainshtein effect can be trusted because of "UV completeness".
- How to setup initial condition for "dark matter"?
- Spectral tilt from anomalous dimension?
- Can we solve the flatness problem without inflation?

Open problems

- Renormalizability beyond power-counting
- RG flow: is $\lambda = 1$ an IR fixed point ?
- Embedding into an unified theory : can we get a common "limit of speed" ?
- Are there vDVZ discontinuity and Vainshtein effect? Unlike massive gravity case, Vainshtein effect can be trusted because of "UV completeness".
- How to setup initial condition for "dark matter"?
- Spectral tilt from anomalous dimension?
- Can we solve the flatness problem without inflation?
- …
- There are many things to do!