Title: Adiabatic Gate Teleportation and Topological Quantum Computing

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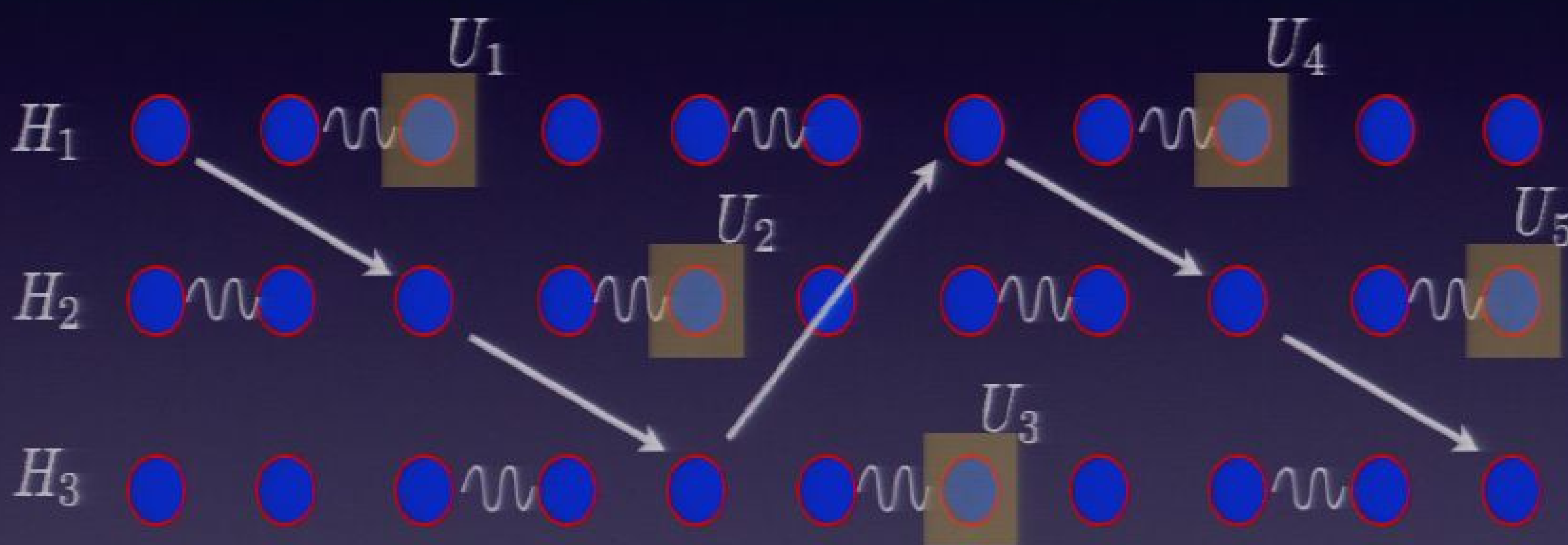
URL: http://pirsa.org/09090004

Abstract: TBA

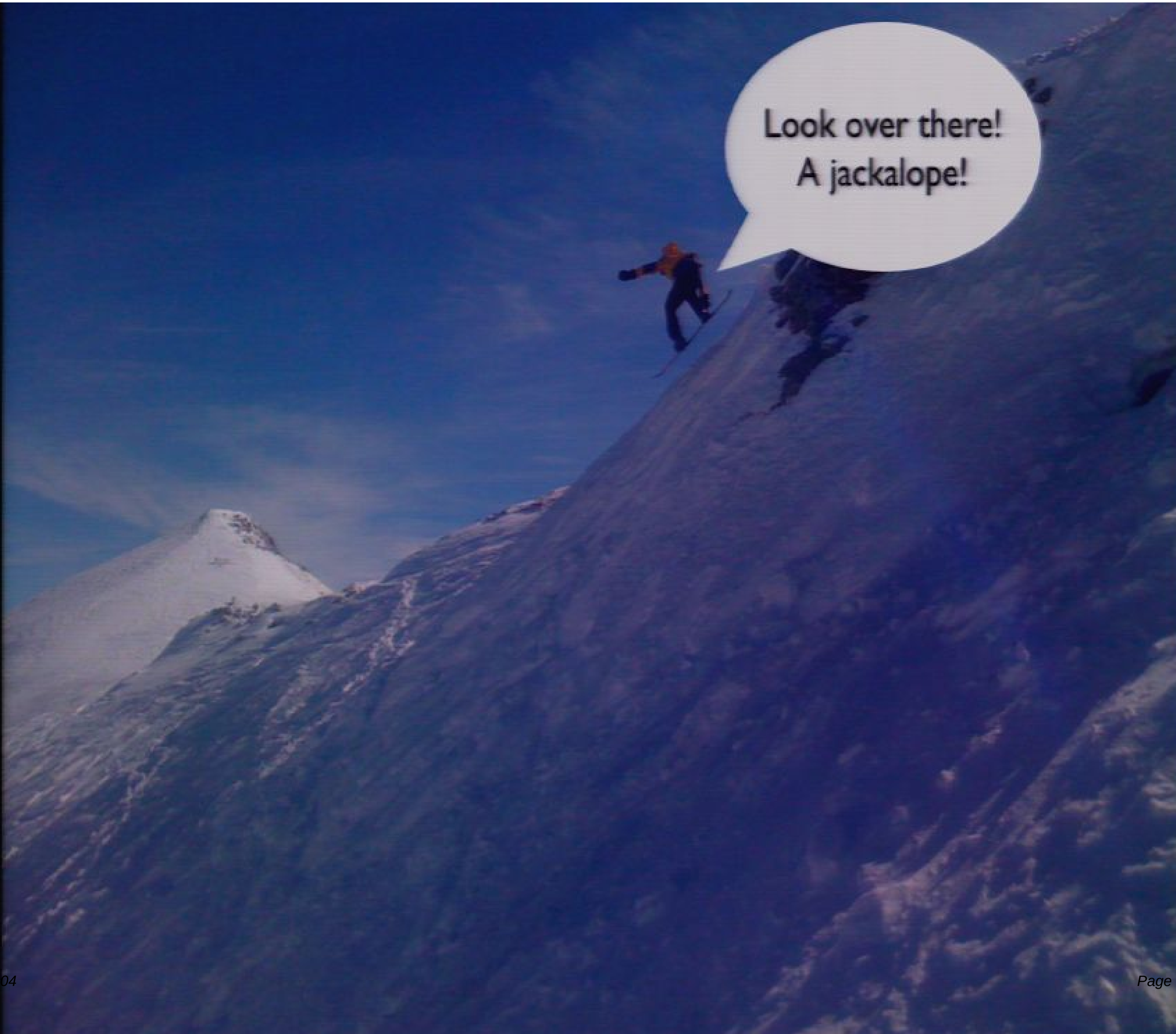
Adiabatic Gate Teleportation & Its Applications

Dave Bacon
Department of Computer Science & Engineering
University of Washington, Seattle, WA USA



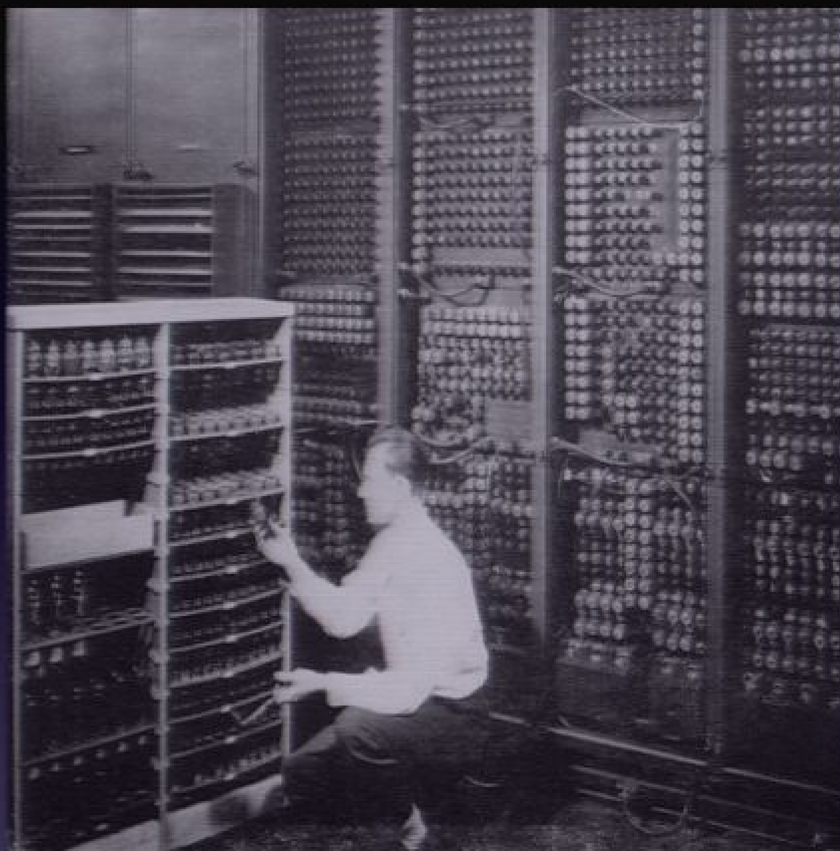
[joint work with Steve Flammia (Perimeter), Alice Neels (Washington), Andrew Landahl (Sandia Labs)]


Oops, I forgot to strap myself in!


Look over there!
A jackalope!

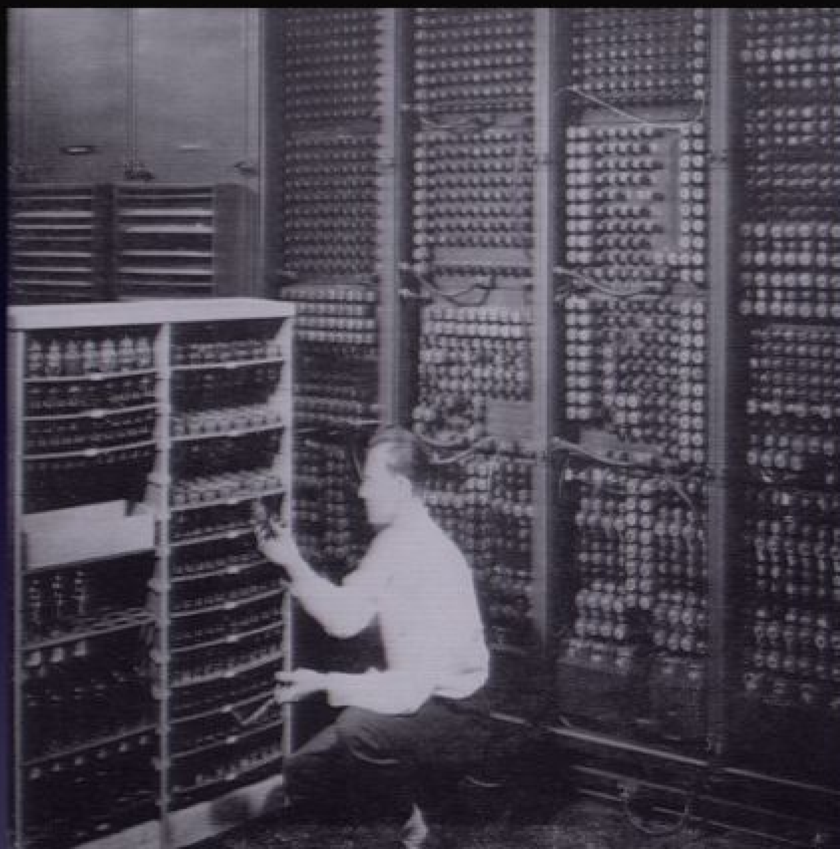

I'll bet most states are
too entangled to be used as
computational resources

ENIAC, 1946



- 17,468 vacuum tubes
- 7,200 crystal diodes
- 1,500 relays
- 70,000 resistors
- 10,000 capacitors
- ~5,000,000 hand-soldered joints

ENIAC, 1946



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~ one vacuum tube failure / day

(equivalent to ~ one transistor failure /second today)

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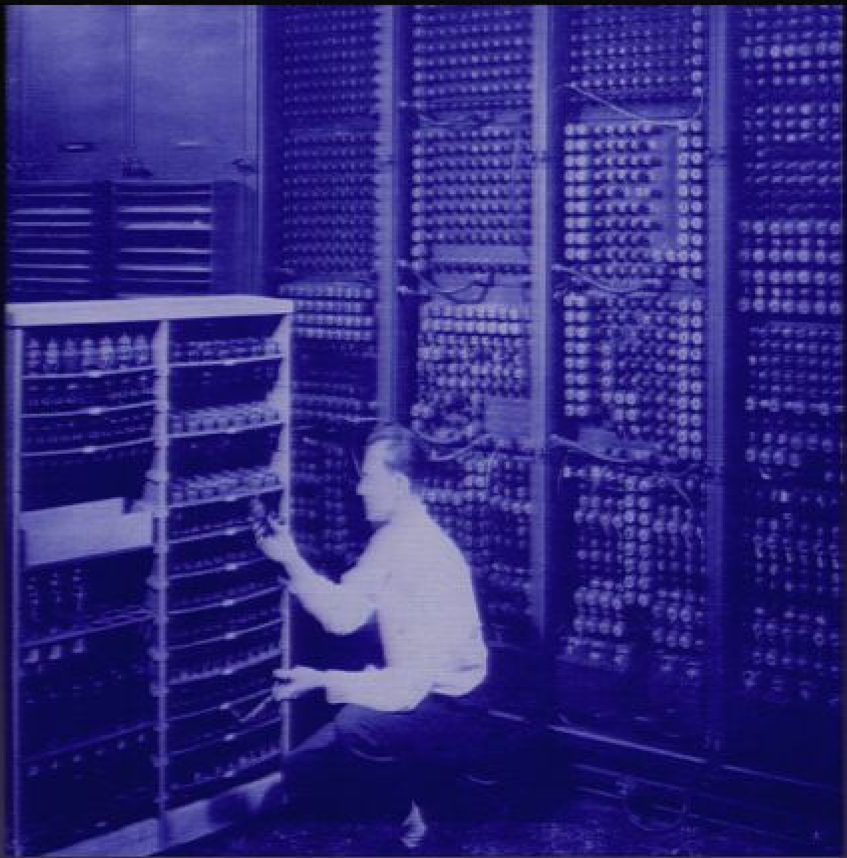

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Alternative History

Imagine: no transistors, no integrated circuits...

Alternative History

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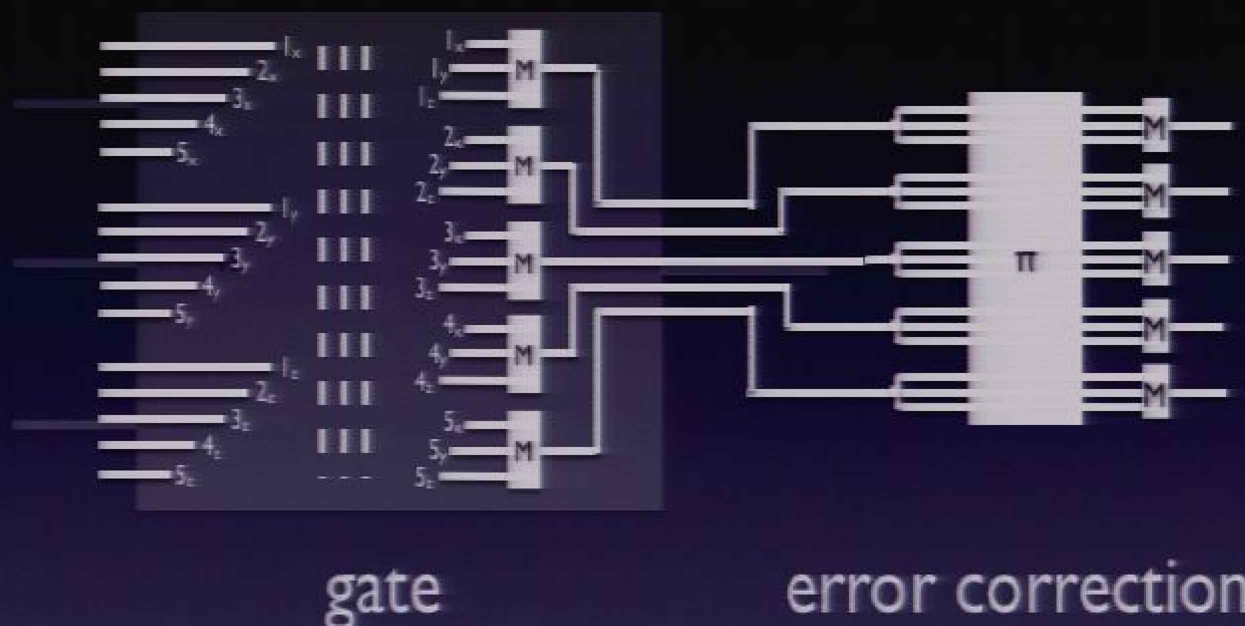
Could we build a computer out of unreliable components?

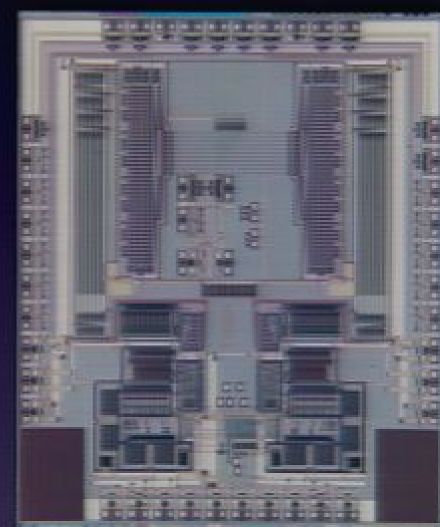
J. von Neumann, "Probabilistic Logics and the Synthesis of Reliable Organism from Unreliable Components" 1956

Von Neumann's Solution



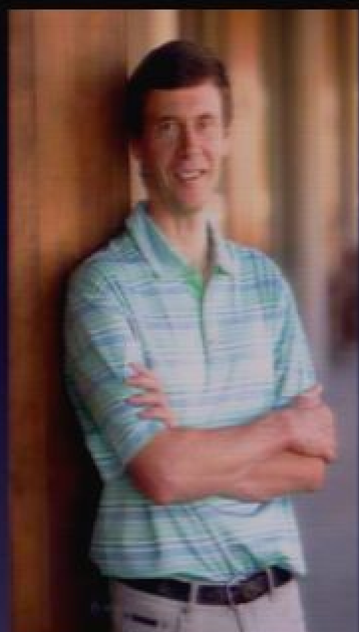
Theorem (von Neumann): A circuit of g gates can be made to fail with probability ε using O(g log (g/ε)) gates which fail with probability p, assuming $p<p_{th}$ (vN estimated p_{th} ~ 1%)

Engineering? We Don't Need Your Stickin' Engineering



There are distinct *physical* reasons why robust classical computation is possible

Kitaev's Idea



"Magnetism arise from spins of individual atoms. Each spin is quite sensitive to thermal fluctuations. But the spins interact with each other and tend to be oriented in the same direction. If some spin flips to the opposite direction, the interaction forces it to flip back to the direction of other spins. This process is quite similar to the standard error correction procedure for the repetition code. We may say that errors are being corrected at the physical level. Can we propose something similar in the quantum case? **Yes, but it is not so simple.**"

Alexei Kitaev

Kitaev, "Fault-tolerant quantum computation by anyons" (1997, published 2003)

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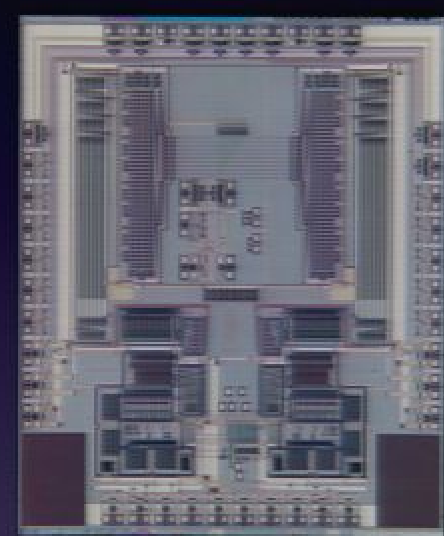

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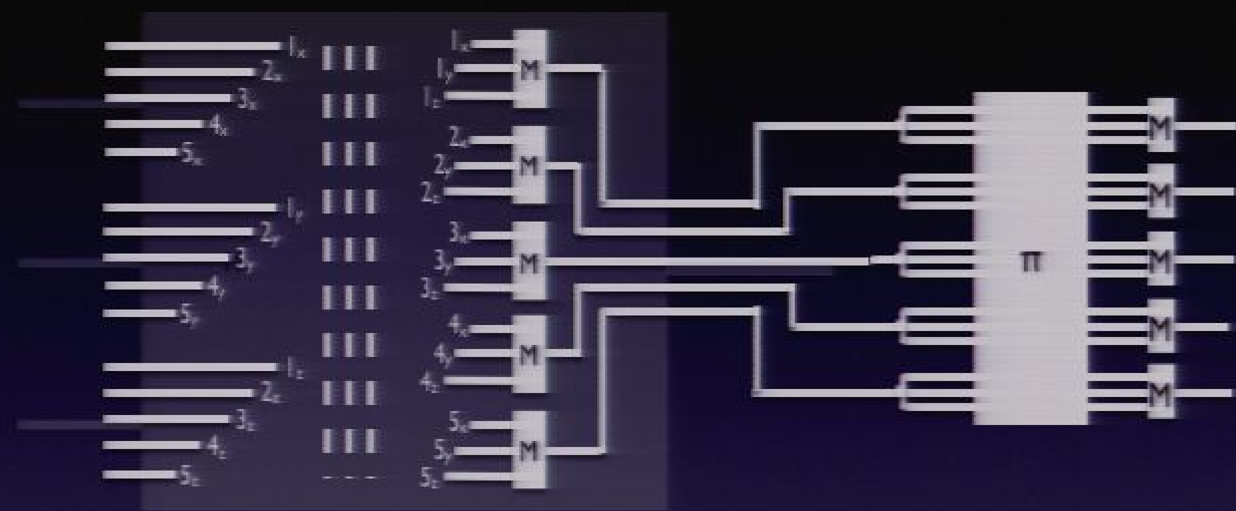

gate error correction

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Topological Quantum Computing

non-abelian Anyons used to build a universal quantum computer

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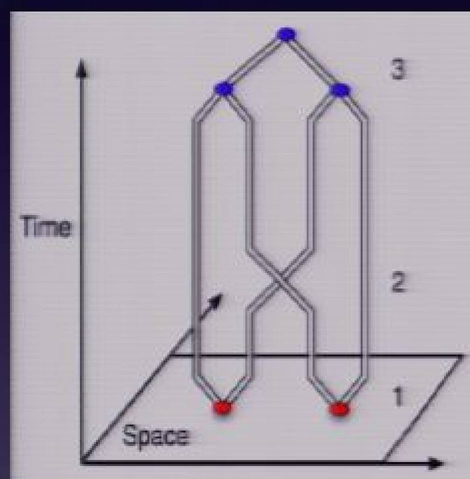
$$H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$



quantum computing using representations of braid group

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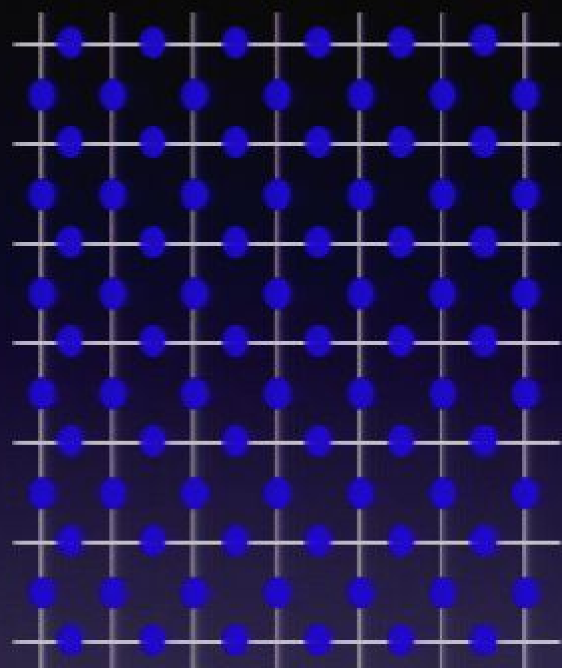
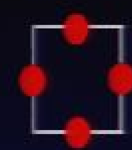
Kitaev's Toric System



periodic boundary conditions (torus)

plaquette operator



$$S_p = \prod_{e \in \delta P} Z_e$$

vertex operator

$$S_v = \prod_{e | v \in e} X_e$$

$$[S_p, S_v] = 0 \quad S_p^2 = S_v^2 = I$$

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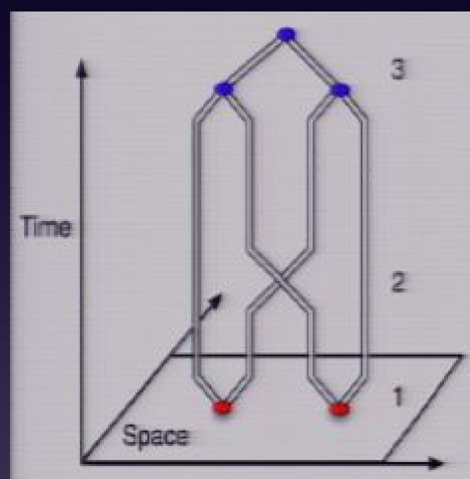

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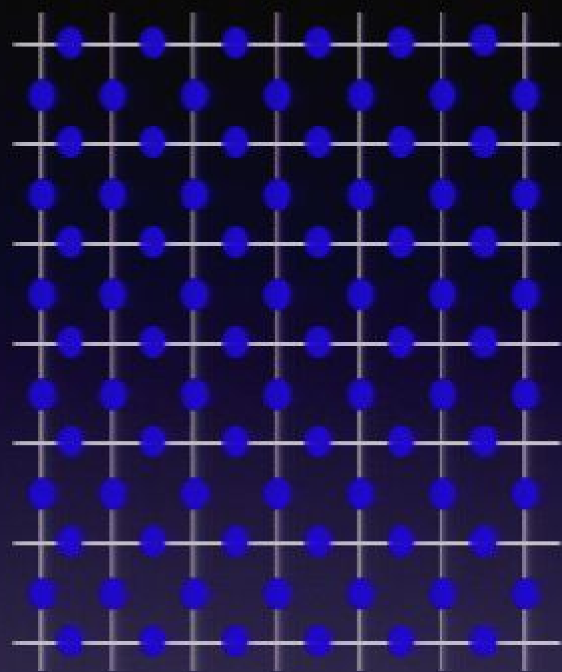

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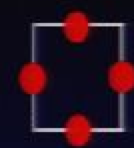

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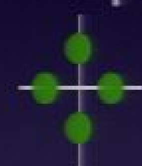

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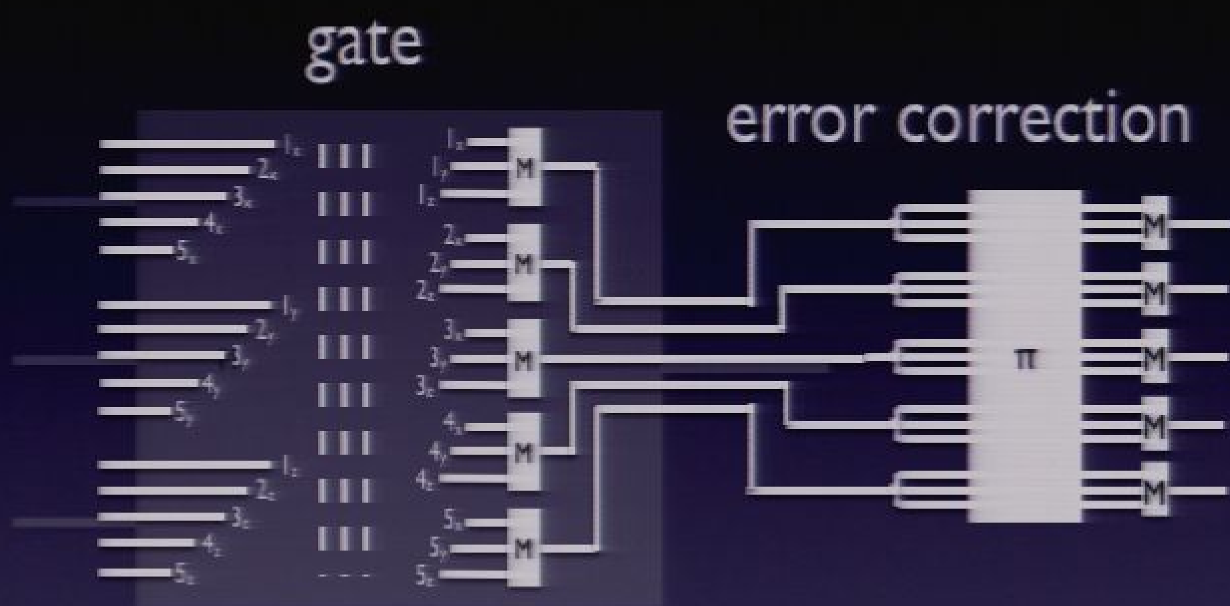
Controversy

Is topological quantum computing fault-tolerant?



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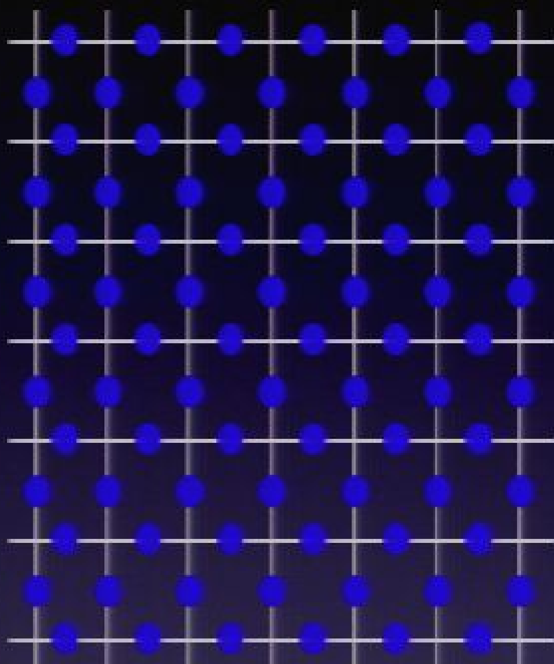

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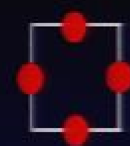

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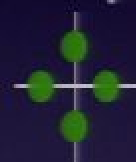

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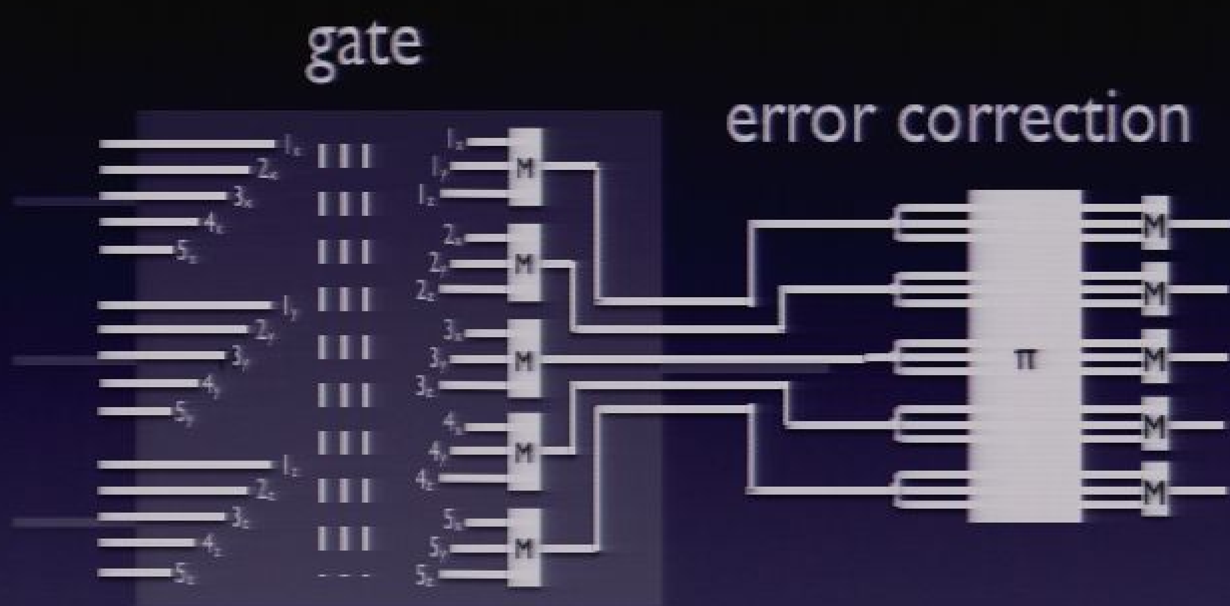

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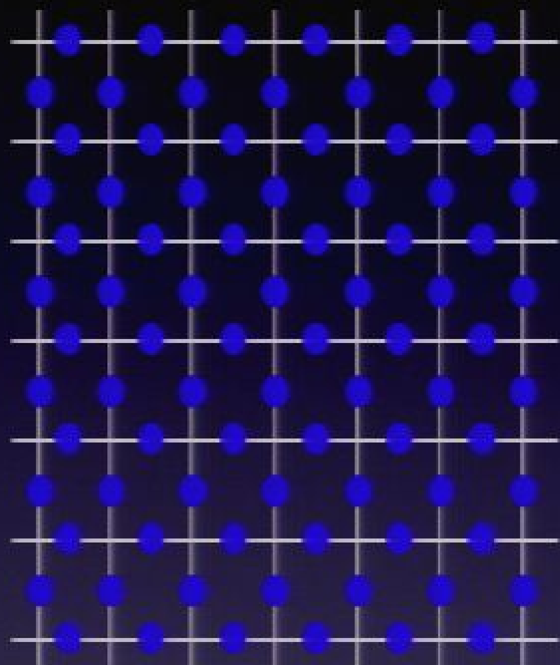

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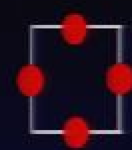

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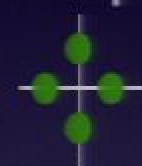

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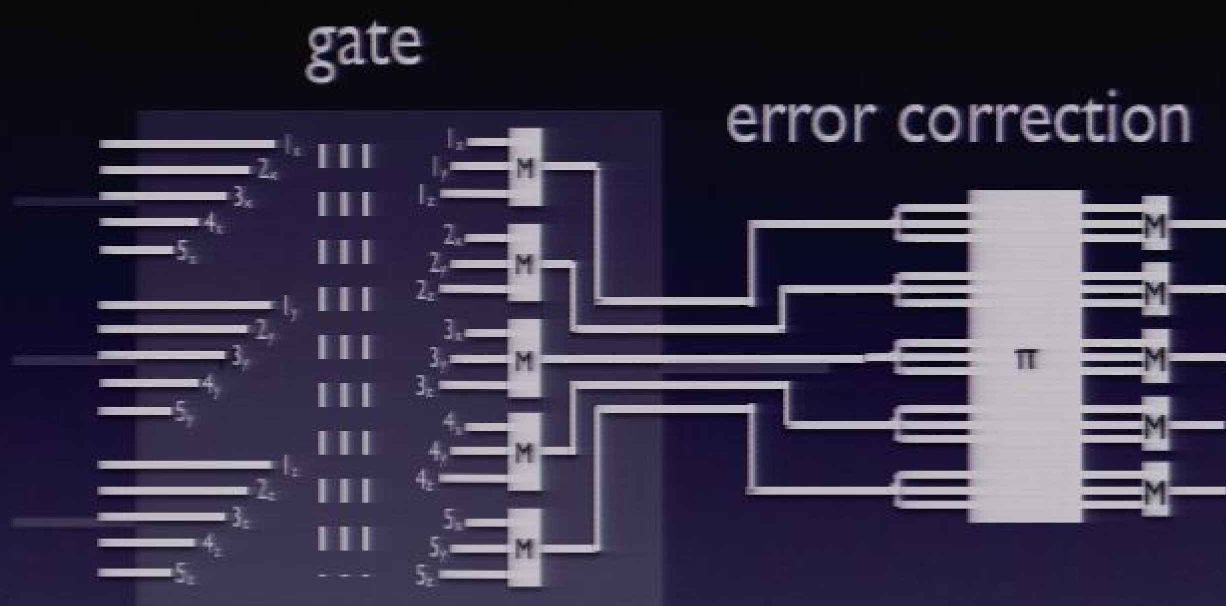

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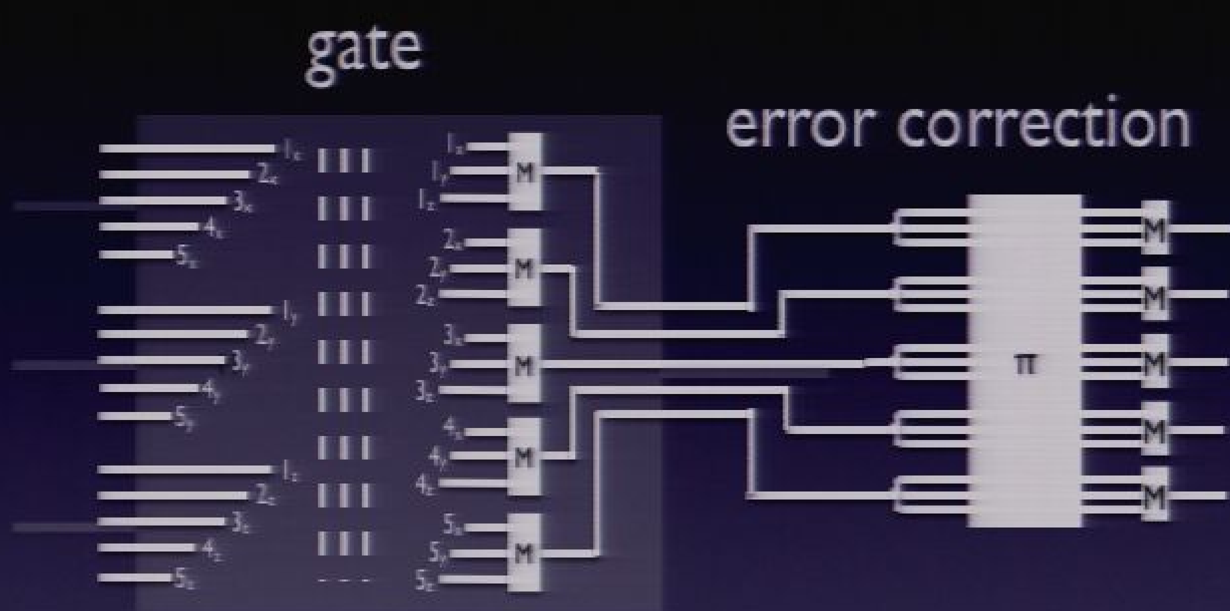

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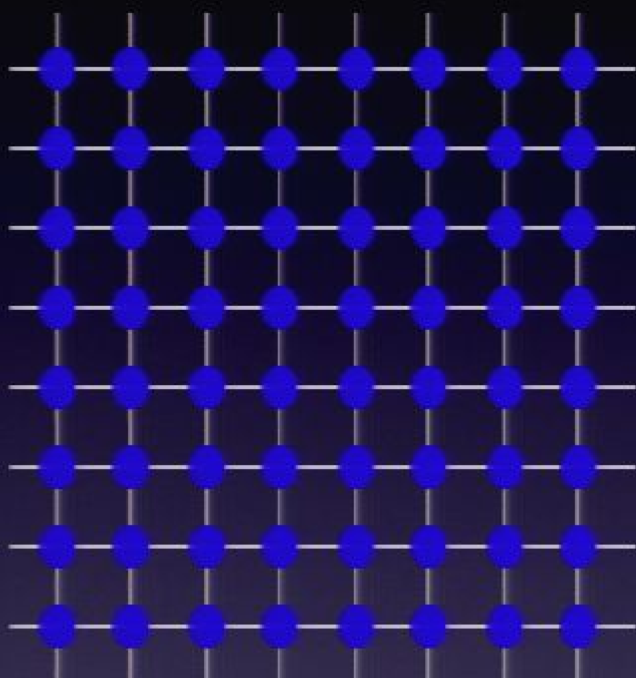
Cartoon Guide to Classical Physical Fault-Tolerance



Energy

$$E = -J \sum_{\langle i,j \rangle} s_i s_j$$

neighbors

total magnetization

$$M = \sum_i s_i$$

● $+1$ / -1 spin

$s_i \in \{+1, -1\}$

"0" as $M > N_0$

"1" as $M < -N_0$

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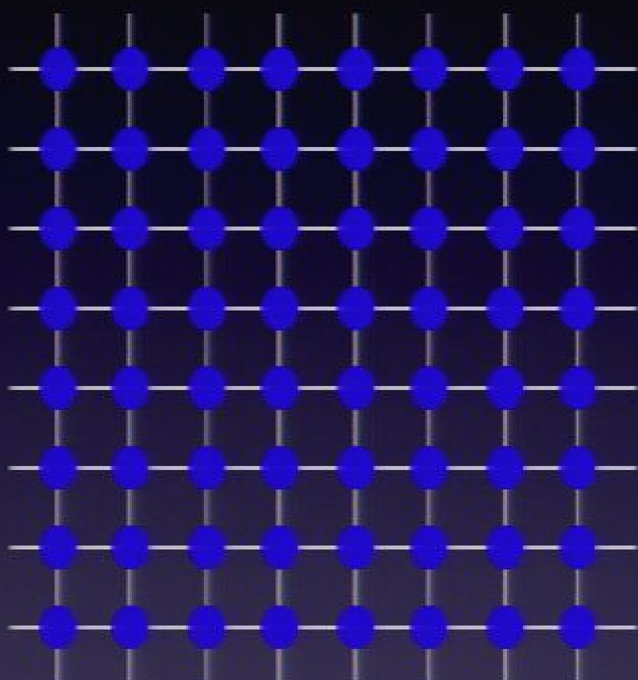

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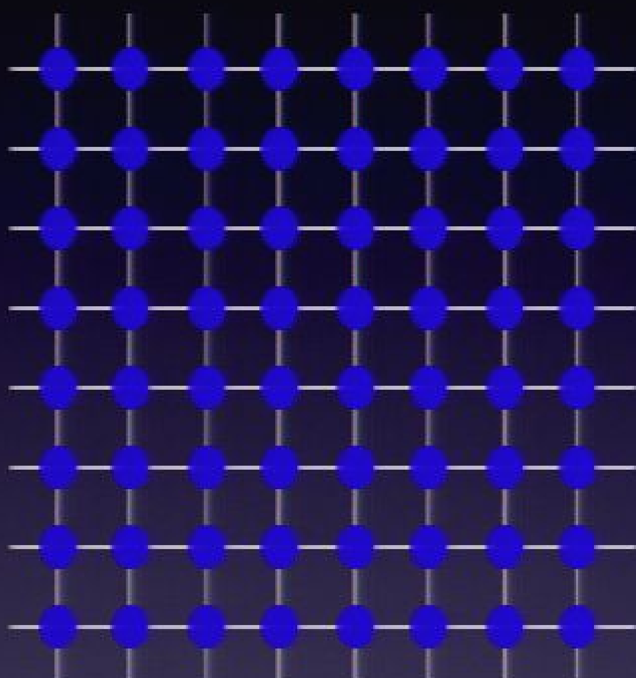

Simple noise model

1. Pick spin at random

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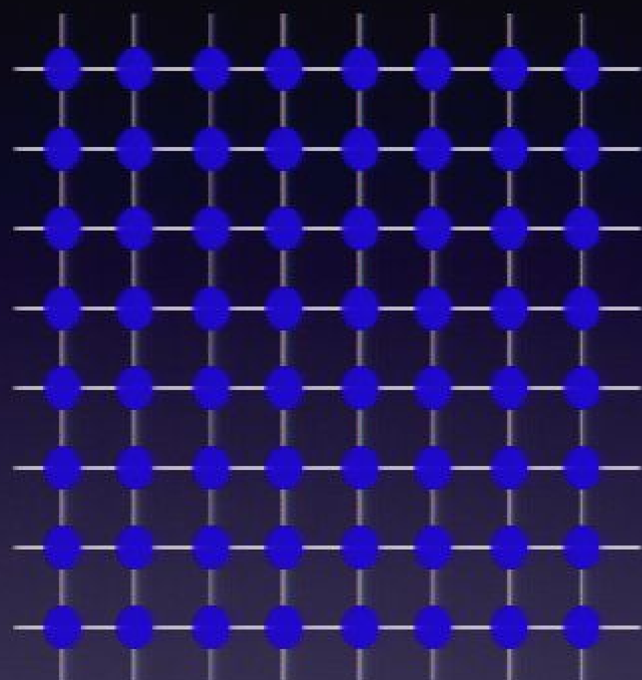

Simple noise model

1. Pick spin at random

2. If flipping spin would decrease energy
   a. flip spin

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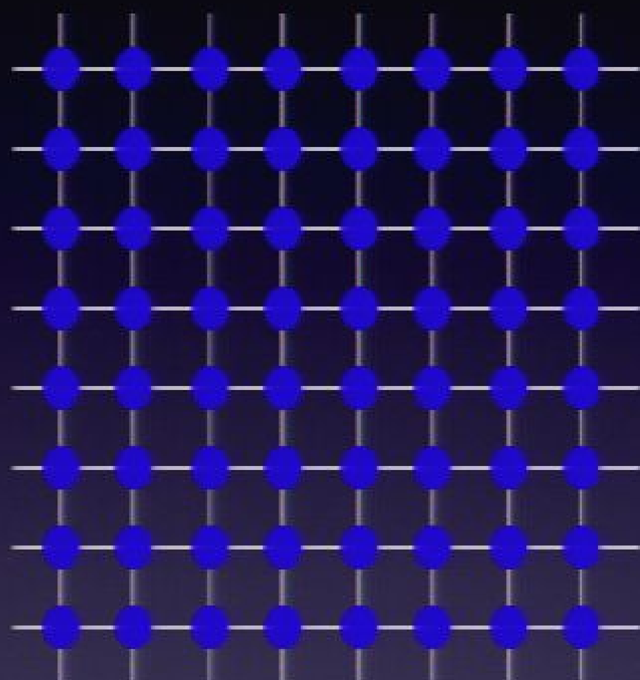


"bare" noise time step

N_r runs of 1-3

Simple noise model

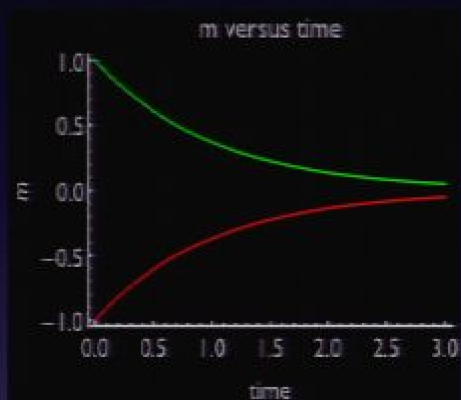
1. Pick spin at random
2. If flipping spin would decrease energy
   a. flip spin
3. If flipping spin would increase energy:
   a. flip spin with probability

$\exp(-\beta \Delta E) \qquad \beta = T^{-1}$

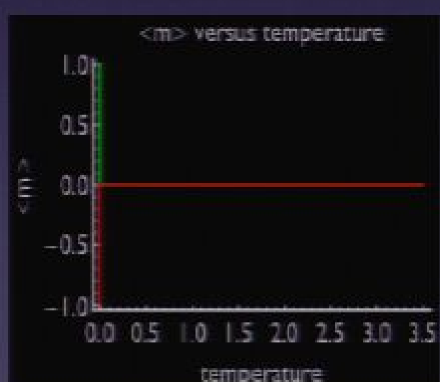
$\beta = T^{-1}$

Memory

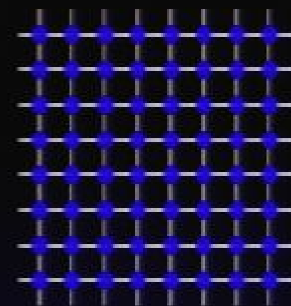
1D

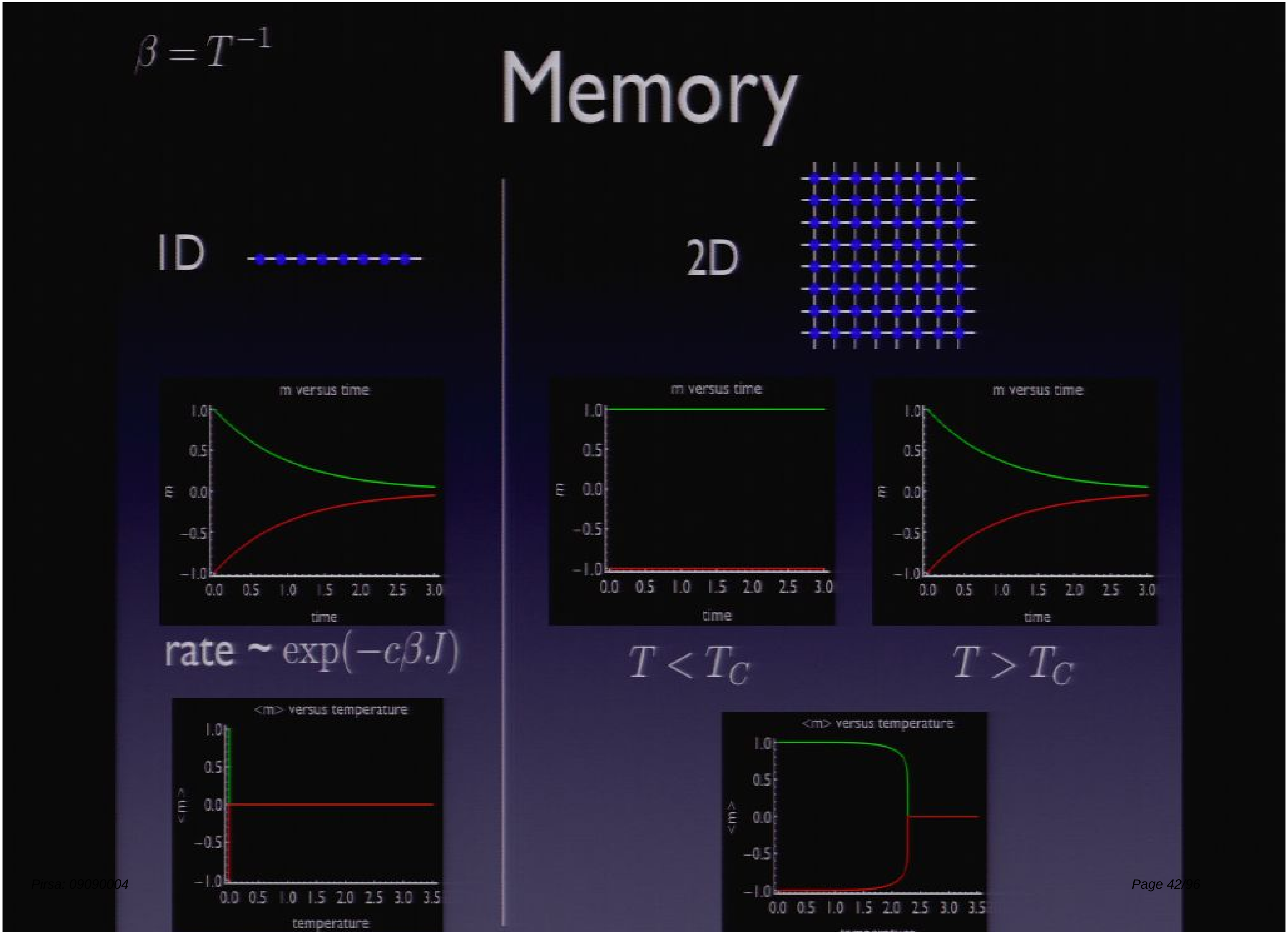


rate ~ $\exp(-c\beta J)$



2D

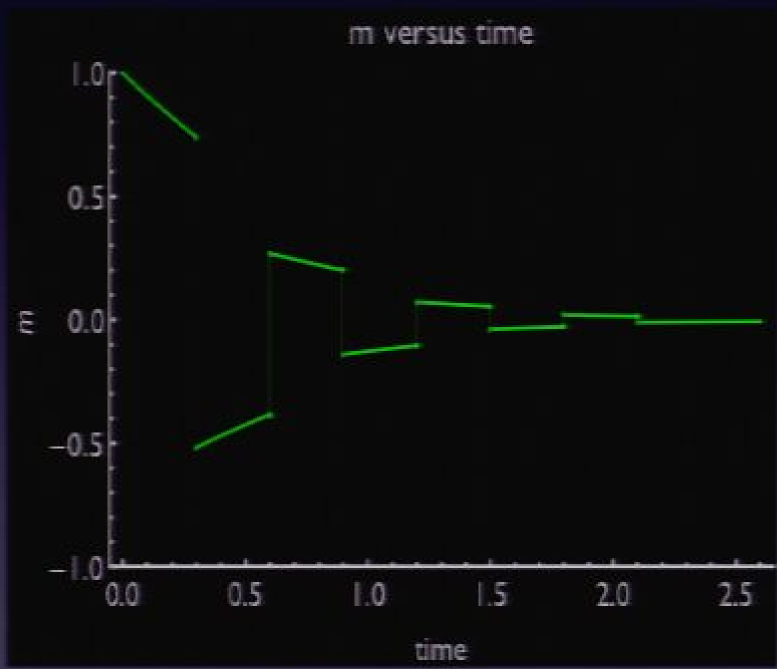




$\beta = T^{-1}$
Memory
1D
2D
m versus time
time
rate ~ $\exp(-c\beta J)$
$T < T_C$
$T > T_C$
<m> versus temperature
temperature

Fault-Tolerance

1D



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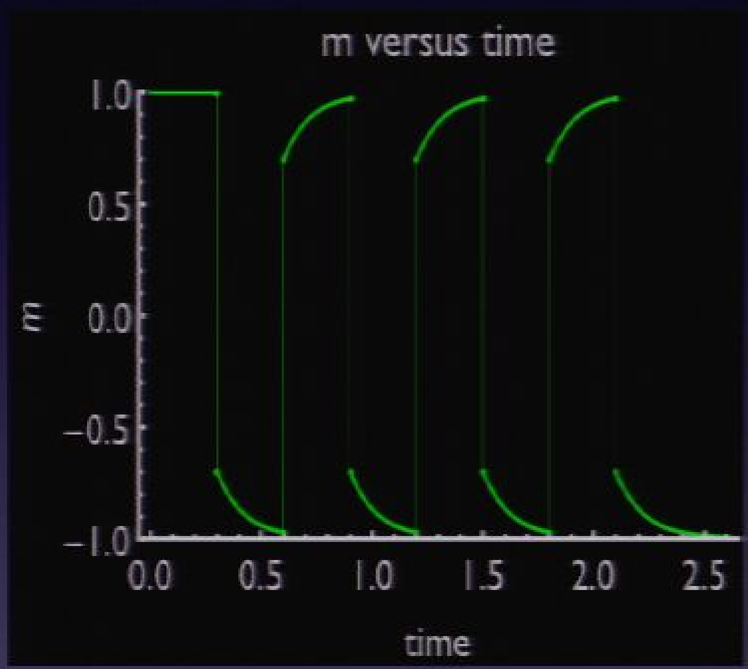
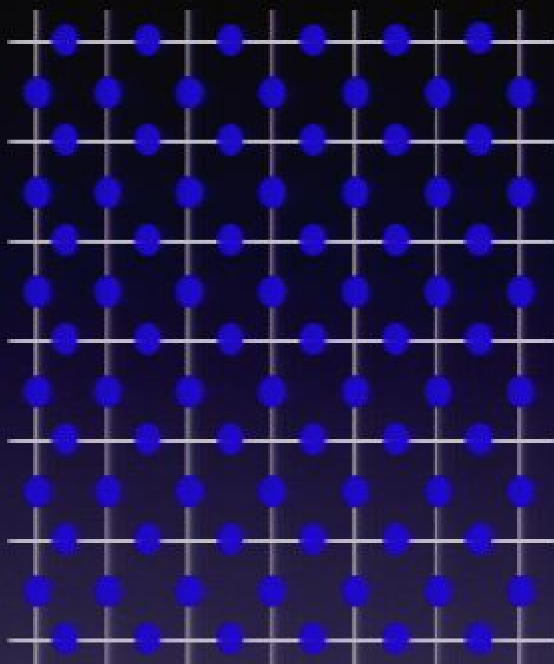


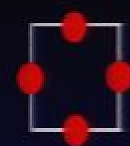
Resilience to "gate" of flipping spins: self-correcting

Toy Model / Strawman



periodic boundary conditions (torus)

plaquette operator



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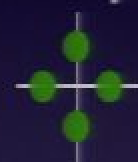

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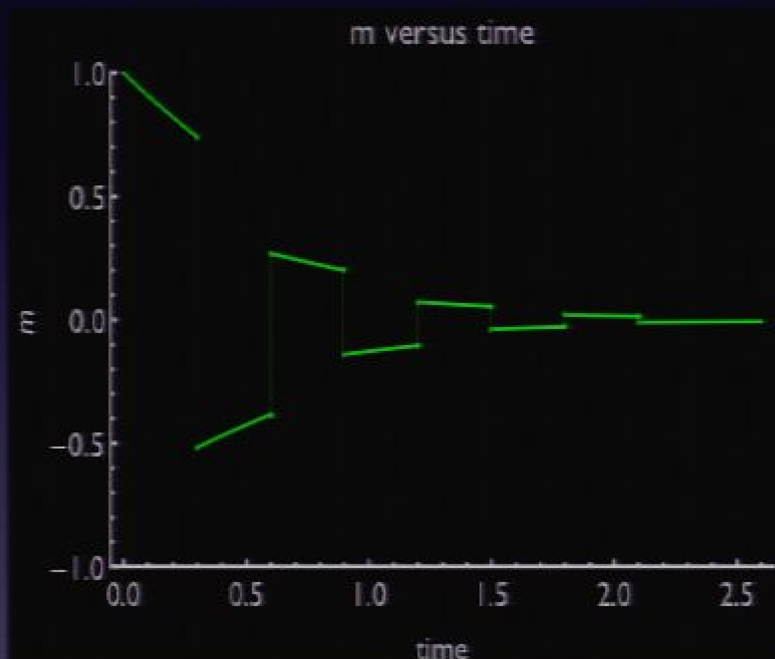
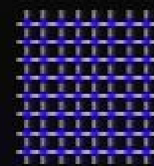
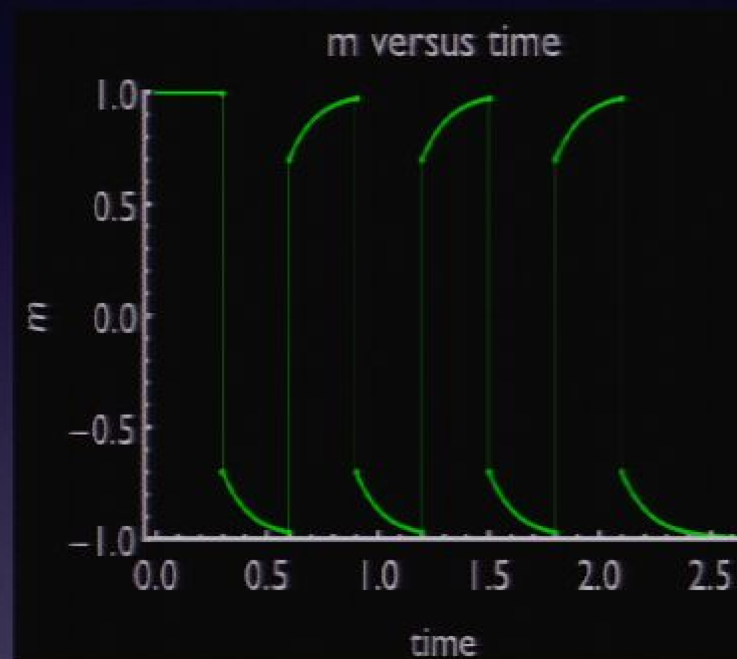
Fault-Tolerance

1D 



2D 



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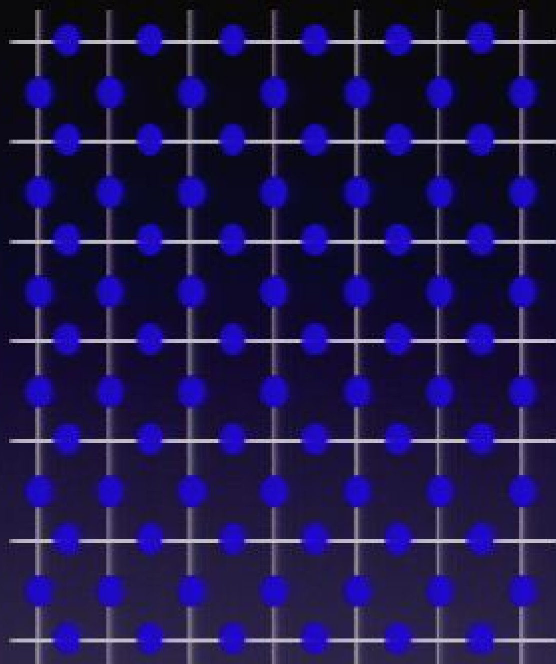

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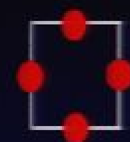

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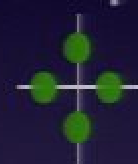

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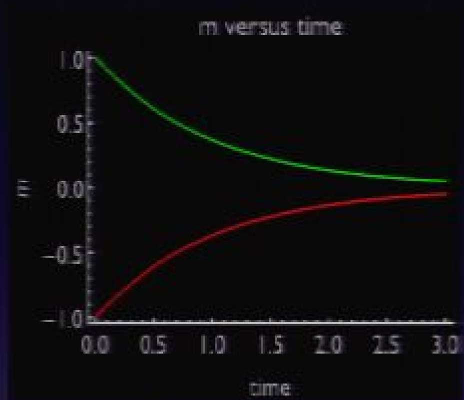
A Problem?

Toric code is like 1D Ising:



"On Thermalization in Kitaev's 2D model" R. Alicki, M. Fannes, and M. Horodecki, arXiv:0810.4584

Decay rate for information encoded into ground state is independent of system size, but proportional to

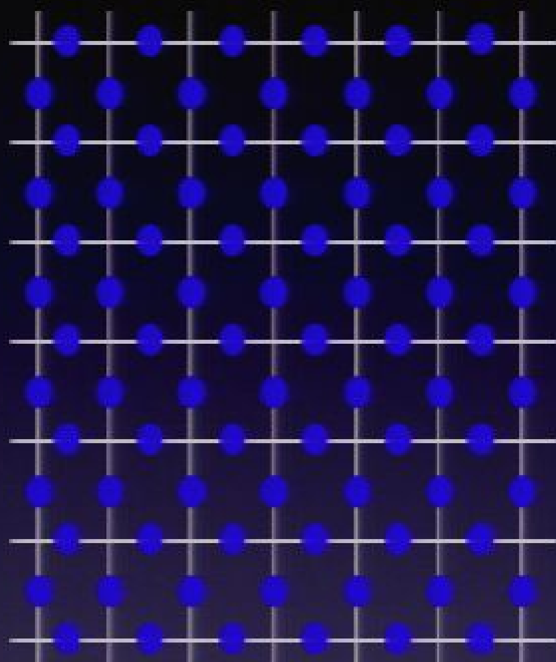
$$\exp(-c\beta J)$$

Contrast with 4D model

"On thermal stability of topological qubit in Kitaev's 4D model" R. Alicki, (M.+P. +R.) Horodecki, arXiv:0811.0033

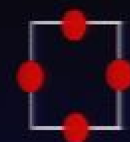
$$\exp(-cN)$$

Toy Model / Strawman



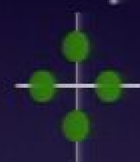
periodic boundary conditions (torus)

plaquette operator



$$S_p = \prod_{e \in \delta P} Z_e$$

vertex operator



$$S_v = \prod_{e | v \in e} X_e$$

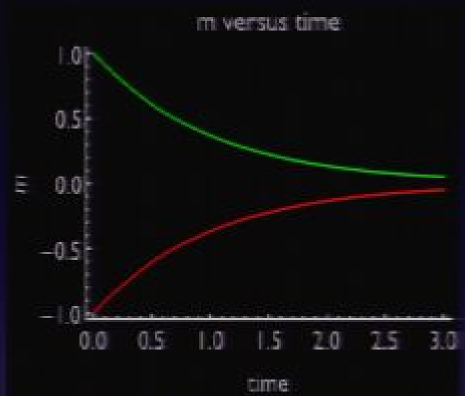
$$[S_p, S_v] = 0 \quad S_p^2 = S_v^2 = I$$

Kitaev Toric Code Hamiltonian:

$$H = -\frac{\Delta}{4}\left[\sum_p S_p + \sum_v S_v\right]$$

A Problem?

Toric code is like 1D Ising:



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Contrast with 4D model

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$$\exp(-cN)$$

Not a Problem?

Factor a 1000 bit number:

18819881292060796383869723946165043
98071635633794173827007633564229888
59715234665485319060606504743045317
38801130339671619969232120573403187
9550656996221305168759307650257059

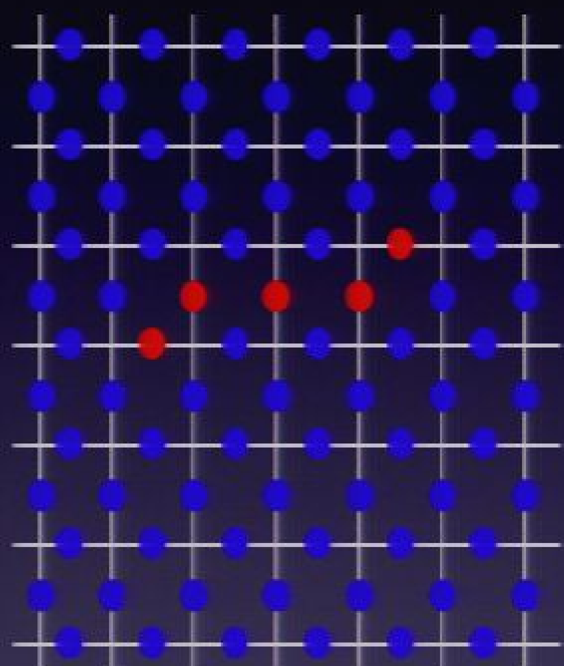
Naive algorithm: $\sim 10^9$ gates

Gap to temp ratio: ~20

Seems likely that exponential suppression as function of temp well worth working for

A Problem?

What about during computing?



Creation and manipulation of excited states require operators like:

$$X^{\otimes 5}$$

How to do this?

$$\$^{\otimes 5} \quad \text{not } X^{\otimes 5}$$

How do you "create" and "move" anyons without creating errors = creating extra anyons

TQC: Gates

"We now introduce into the system's Hamiltonian a scalar potential composed of many local "traps," each sufficient to capture exactly one quasiparticle. These traps may be created by impurities, by small gates, or by the potential created by tips of scanning microscopes. The quasiparticle's charge screens the potential introduced by the trap and the quasiparticle-tip combination cannot be observed by local measurements from far away. We denote the positions of these traps by $R_1,\ldots,R_k$, and assume that these positions are well spaced from each other compared to the microscopic length scales. A state with quasiparticles at these positions can be viewed as an excited state of the Hamiltonian without the trap potential or, alternatively, as the ground state in the presence of the trap potential...."

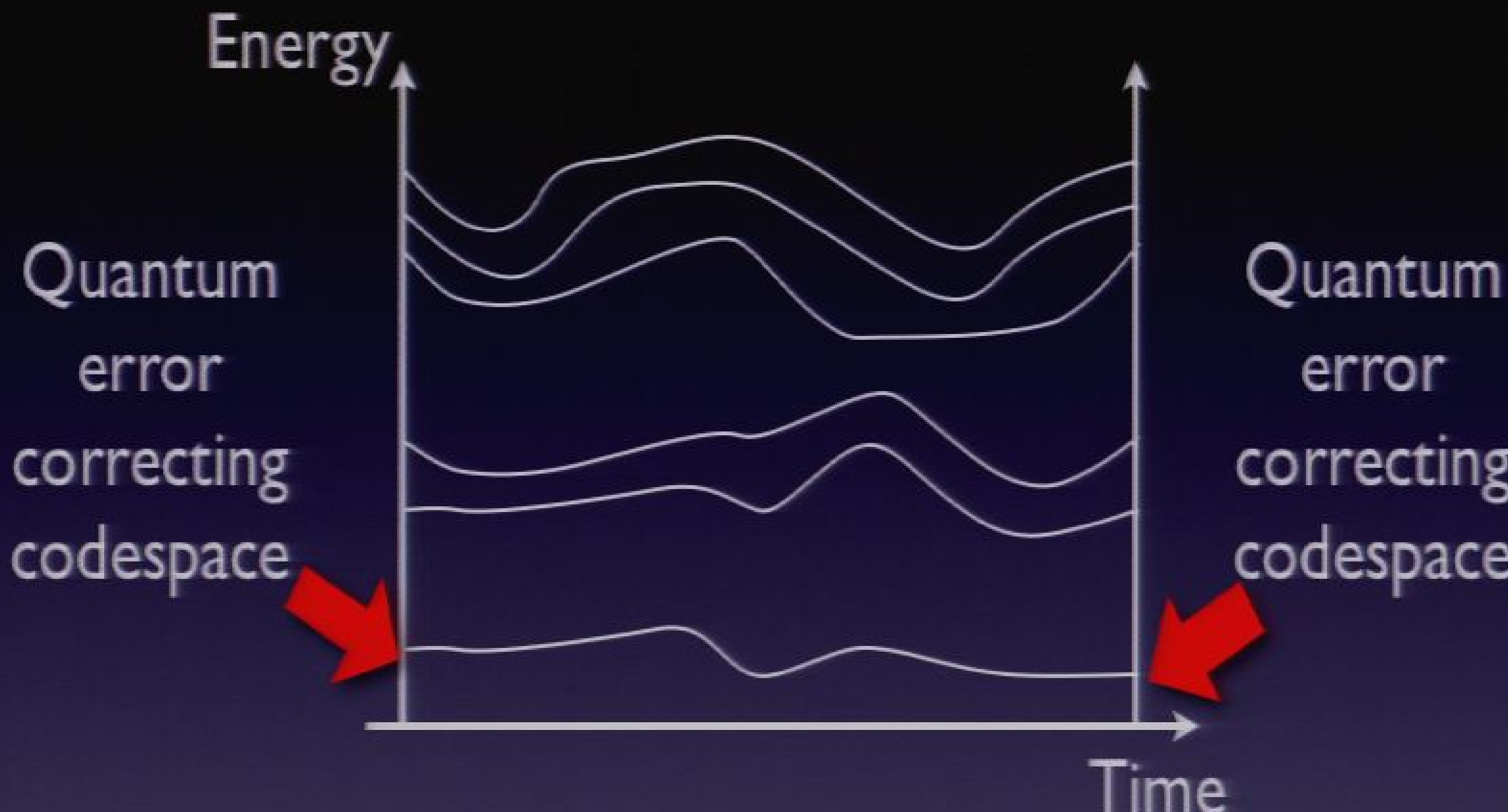
C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma "Non-Abelian anyons and topological quantum computation" Rev. Mod. Phys. 80, 1083 (2008).

TQC Problems?

How does one guarantee that one can trap one and only one anyon?

Every operation the previous paragraph needs to be demonstrated that it can be done with EXCEPTIONALLY small error if this is to be called physical fault-tolerance.

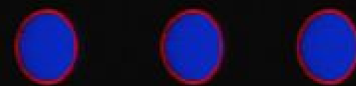
Adiabatic Dragging



Require: degenerate through entire evolution

“Open loop” holonomic evolution

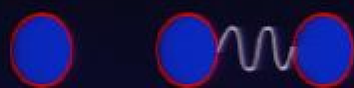
Adiabatic Teleportation



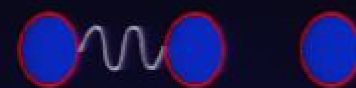
Initial Code

$$|\Phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Final Code

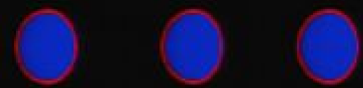


$|\psi\rangle \otimes |\Phi_+\rangle$



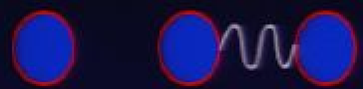
$|\Phi_+\rangle \otimes |\psi'\rangle$

Adiabatic Teleportation



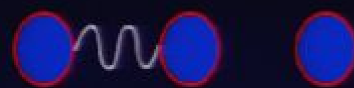
Initial Code

$$|\Phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



$$|\psi\rangle \otimes |\Phi_+\rangle$$

Final Code



$$|\Phi_+\rangle \otimes |\psi'\rangle$$

Initial Stabilizer

$$S_1 = Z_2 Z_3$$
$$S_2 = X_2 X_3$$

$$S_i|\psi\rangle \otimes |\Phi_+\rangle = |\psi\rangle \otimes |\Phi_+\rangle$$

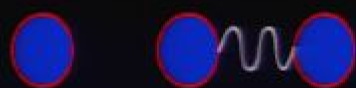
Final Stabilizer

$$S'_1 = Z_1 Z_2$$
$$S'_2 = X_1 X_2$$

$$S'_i|\Phi_+\rangle \otimes |\psi\rangle = |\Phi_+\rangle \otimes |\psi\rangle$$

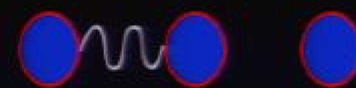
Adiabatic Teleportation

Initial Code



$|\psi\rangle \otimes |\Phi_+\rangle$

Final Code



$|\Phi_+\rangle \otimes |\psi'\rangle$

Initial Hamiltonian $\quad \Delta > 0 \quad$ Final Hamiltonian

$$H_i = -\Delta(X_2 X_3 + Z_2 Z_3)$$

$$H_i = -\Delta(S_1 + S_2)$$

$$H_f = -\Delta(X_1 X_2 + Z_1 Z_2)$$

$$H_i = -\Delta(S_1' + S_2')$$

Evolution: $H(t) = \left(1 - \frac{t}{T}\right) H_i + \frac{t}{T} H_f$

Adiabatic Teleportation

$$\bar{X}_1 = X_1X_2 \qquad \bar{Z}_1 = Z_2Z_3$$
$$\bar{X}_2 = X_2X_3 \qquad \bar{Z}_2 = Z_1Z_2$$
$$\bar{X}_3 = X_1X_2X_3 \qquad \bar{Z}_3 = Z_1Z_2Z_3$$

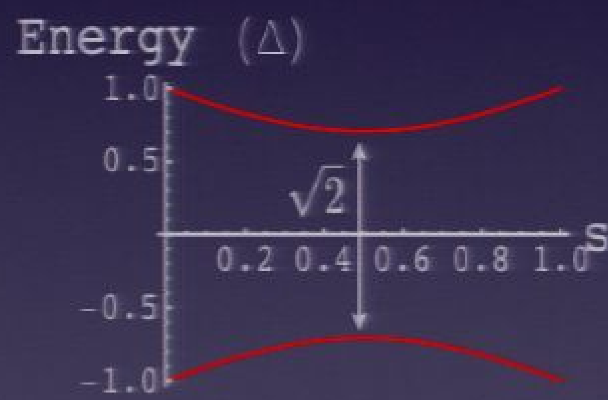
Initial Hamiltonian

$$H_i = -\Delta(\bar{Z}_1 + \bar{X}_2)$$

Final Hamiltonian

$$H_f = -\Delta(\bar{X}_1 + \bar{Z}_2)$$

$$H(s) = (1-s)H_i + sH_f$$



Adiabatic Teleportation

$$\begin{aligned} \bar{X}_1 &= X_1 X_2 & \bar{Z}_1 &= Z_2 Z_3 \\ \bar{X}_2 &= X_2 X_3 & \bar{Z}_2 &= Z_1 Z_2 \\ \bar{X}_3 &= X_1 X_2 X_3 & \bar{Z}_3 &= Z_1 Z_2 Z_3 \end{aligned}$$

Initial Hamiltonian

$$H_i = -\Delta(\bar{Z}_1 + \bar{X}_2)$$

$$X_1 = \bar{X}_2 \bar{X}_3$$

$$Z_1 = \bar{Z}_1 \bar{Z}_3$$

Final Hamiltonian

$$H_f = -\Delta(\bar{X}_1 + \bar{Z}_2)$$

$$X_3 = \bar{X}_1 \bar{X}_3$$

$$Z_3 = \bar{Z}_2 \bar{Z}_3$$

$$|\psi\rangle \otimes |\Phi_+\rangle \longrightarrow |\Phi_+\rangle \otimes |\psi\rangle$$

Adiabatic Gate Teleportation

$$H_i = -U_3\Delta(X_2X_3 + Z_2Z_3)U_3^\dagger \qquad H_f = -\Delta(X_1X_2 + Z_1Z_2)$$

Note: spectrum unchanged by unitary

$$|\psi\rangle \otimes |\Phi_+\rangle \longrightarrow |\Phi_+\rangle \otimes U_3|\psi\rangle$$

Example (Hadamard gate):

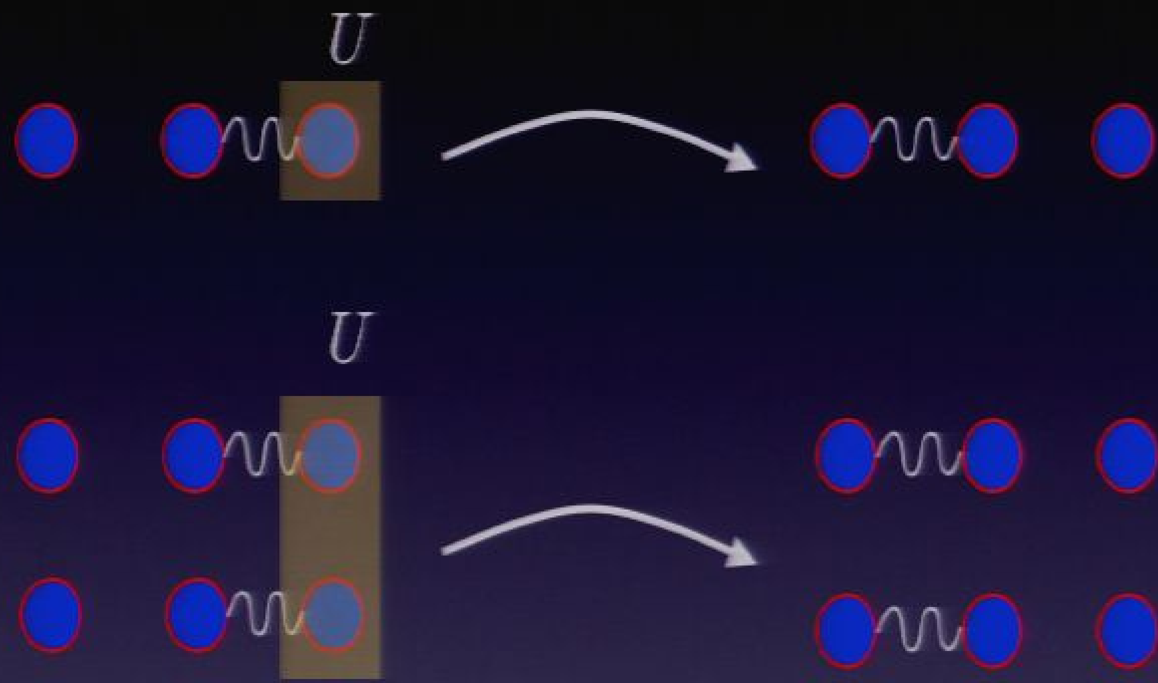
$$H_i = -\Delta(X_2Z_3 + Z_2X_3) \qquad H_f = -\Delta(X_1X_2 + Z_1Z_2)$$

$$|\psi\rangle \otimes |\Phi_+\rangle \longrightarrow |\Phi_+\rangle \otimes H|\psi\rangle$$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Let's Build a Computer

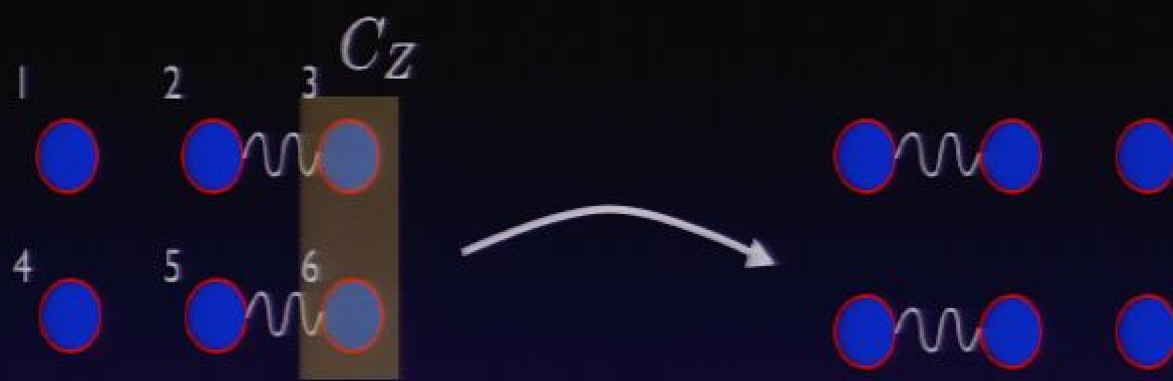
One qubit gates



Two qubit gates

Problem: initial Hamiltonian requires 3 qubit interactions

Example



$$H_i = -\Delta(X_2X_3Z_6 + Z_2Z_3 + Z_5Z_6 + Z_3X_5X_6)$$

To get 2 qubit interactions: perturbation theory gadgets

Gadgets

Replace 3,6 by encoded qubits: $|0_L\rangle = |00\rangle, |1_L\rangle = |11\rangle$

$$
\begin{aligned}
H_i &= -\lambda(X_2X_{3a} + Z_2Z_{3a}) - \lambda(X_5X_{6a} + Z_5Z_{6a}) \\
&\quad -\omega Z_{3a}Z_{3b} - \lambda(X_{3b}Z_{6b} + Z_{3b}X_{6b}) \qquad \lambda \ll \omega
\end{aligned}
$$

$$
\begin{aligned}
H_f &= -\lambda(X_1X_2 + Z_1Z_2) - \lambda(X_4X_5 + Z_4Z_5) \\
&\quad -\omega Z_{3a}Z_{3b}
\end{aligned}
$$

[S. D. Bartlett and T. Rudolph, Phys. Rev. A 74, 040302 (2006)]

Gadgets

$$H_i = -\lambda(X_2X_{3a} + Z_2Z_{3a}) - \lambda(X_5X_{6a} + Z_5Z_{6a})$$
$$-\omega Z_{3a}Z_{3b} - \lambda(X_{3b}Z_{6b} + Z_{3b}X_{6b})$$
$$H_f = -\lambda(X_1X_2 + Z_1Z_2) - \lambda(X_4X_5 + Z_4Z_5)$$
$$-\omega Z_{3a}Z_{3b}$$

Minimum gap: $O\left(\frac{\lambda^2}{\omega}\right)$ Initial ground state fidelity: $1 - O\left(\frac{\lambda^2}{\omega^2}\right)$

Variations on a Theme

Exchange interactions:

$$H = -\Delta(X_1 X_2 + Y_1 Y_2 + Z_1 Z_2)$$

Adiabatically preparing rotated Hamiltonians:

$$H_i = -\Delta(X_1 X_2 + Z_1 Z_2) \qquad H_f = -\Delta U_2 (X_1 X_2 + Z_1 Z_2) U_2^\dagger$$

Not all U's allow a gap

$$A = \frac{1}{2}\begin{bmatrix} 1 + i\sqrt{2} & 1 \\ 1 & -1 + 1\sqrt{2} \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{bmatrix}$$

gap: $\sqrt{2}\Delta$ gap: $\sqrt{2 - \sqrt{2}\Delta}$

Gadgets

$$
\begin{aligned}
H_i &= -\lambda(X_2 X_{3a} + Z_2 Z_{3a}) - \lambda(X_5 X_{6a} + Z_5 Z_{6a}) \\
&\quad -\omega Z_{3a} Z_{3b} - \lambda(X_{3b} Z_{6b} + Z_{3b} X_{6b}) \\
H_f &= -\lambda(X_1 X_2 + Z_1 Z_2) - \lambda(X_4 X_5 + Z_4 Z_5) \\
&\quad -\omega Z_{3a} Z_{3b}
\end{aligned}
$$

Minimum gap: $O\left(\frac{\lambda^2}{\omega}\right)$ Initial ground state fidelity: $1 - O\left(\frac{\lambda^2}{\omega^2}\right)$

Variations on a Theme

Exchange interactions:

$$H = -\Delta(X_1 X_2 + Y_1 Y_2 + Z_1 Z_2)$$

Adiabatically preparing rotated Hamiltonians:

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gap: $\sqrt{2}\Delta$ gap: $\sqrt{2 - \sqrt{2}}\Delta$

"Impossibility" Theorem

Is it possible to adiabatically SWAP with only one adiabatic interpolation?

$$H_i = \delta_1(|01\rangle\langle 01| + |11\rangle\langle 11|) + \delta_2|00\rangle\langle 00| + \delta_3|10\rangle\langle 10|$$

$$H_f = \gamma_1(|10\rangle\langle 10| + |11\rangle\langle 11|) + \gamma_2|00\rangle\langle 00| + \gamma_3|01\rangle\langle 01|$$

$$H(t) = \left(1 - \frac{t}{T}\right) H_i + \frac{t}{T} H_f$$

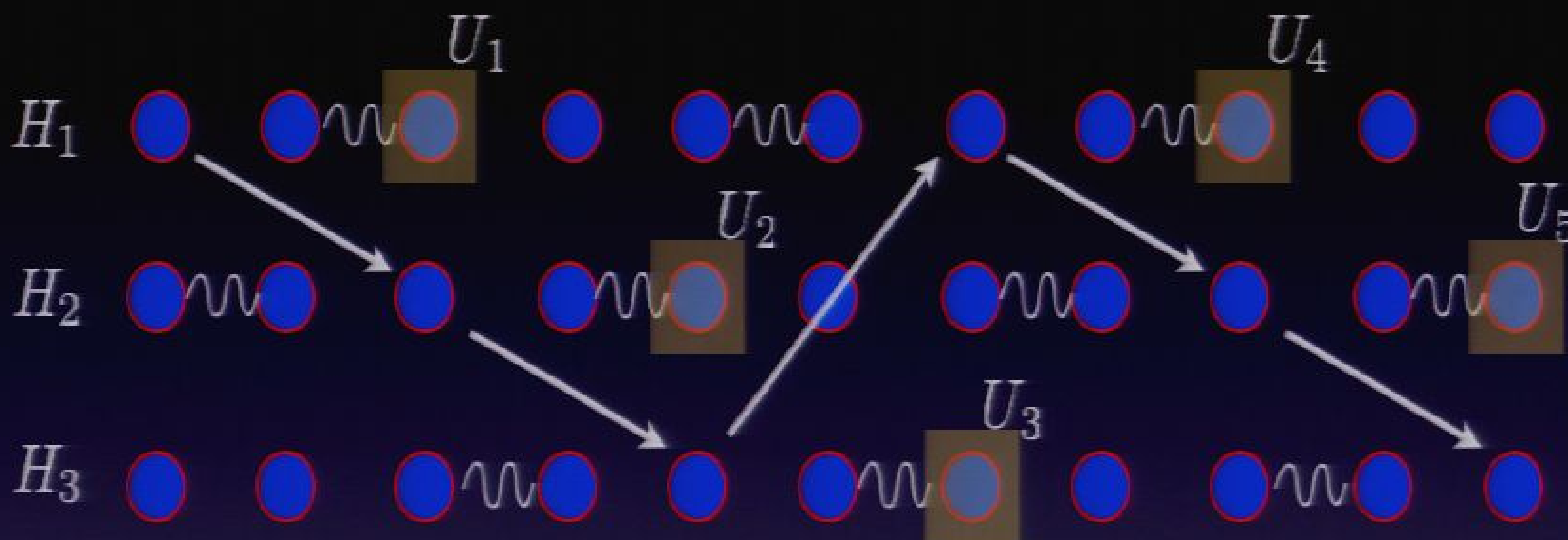
Diagonal in same basis so cannot transfer amplitude to perform SWAP

Gadgets

$$
\begin{aligned}
H_i &= -\lambda(X_2 X_{3a} + Z_2 Z_{3a}) - \lambda(X_5 X_{6a} + Z_5 Z_{6a}) \\
&\quad -\omega Z_{3a} Z_{3b} - \lambda(X_{3b} Z_{6b} + Z_{3b} X_{6b}) \\
H_f &= -\lambda(X_1 X_2 + Z_1 Z_2) - \lambda(X_4 X_5 + Z_4 Z_5) \\
&\quad -\omega Z_{3a} Z_{3b}
\end{aligned}
$$

Minimum gap: $O\left(\frac{\lambda^2}{\omega}\right)$ Initial ground state fidelity: $1 - O\left(\frac{\lambda^2}{\omega^2}\right)$

Architectures



Sequence of Hamiltonians: $H_1, H_2, H_3, H_1, H_2, H_3, \ldots$

Sequence of single qubit gates applied: $U_1, U_2, U_3, U_4, U_5, \ldots$

Generalizing to more than 1 qubit:

Universal QC by cycling between only 3 Hamiltonians

"Impossibility" Theorem

Is it possible to adiabatically SWAP with only one adiabatic interpolation?

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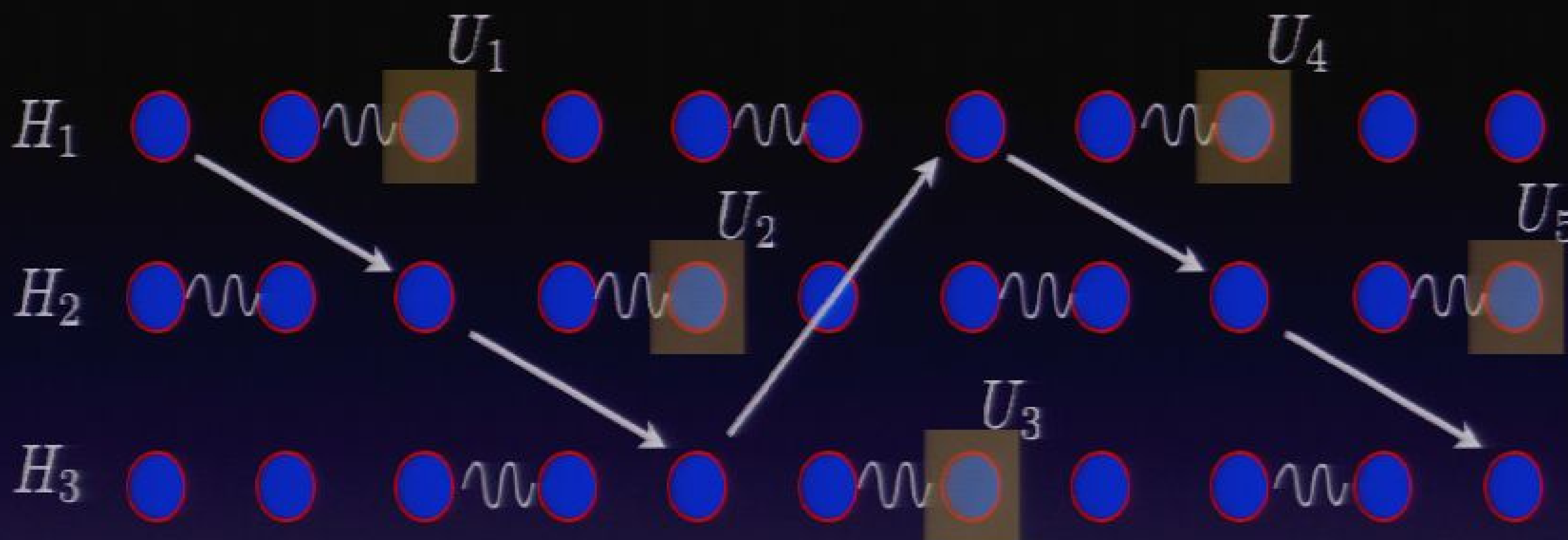
$$H_f = \gamma_1(|10\rangle\langle 10| + |11\rangle\langle 11|) + \gamma_2|00\rangle\langle 00| + \gamma_3|01\rangle\langle 01|$$

$$H(t) = \left(1 - \frac{t}{T}\right) H_i + \frac{t}{T} H_f$$

Diagonal in same basis so cannot transfer amplitude to perform SWAP

Architectures



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Sequence of single qubit gates applied: $U_1, U_2, U_3, U_4, U_5, \ldots$

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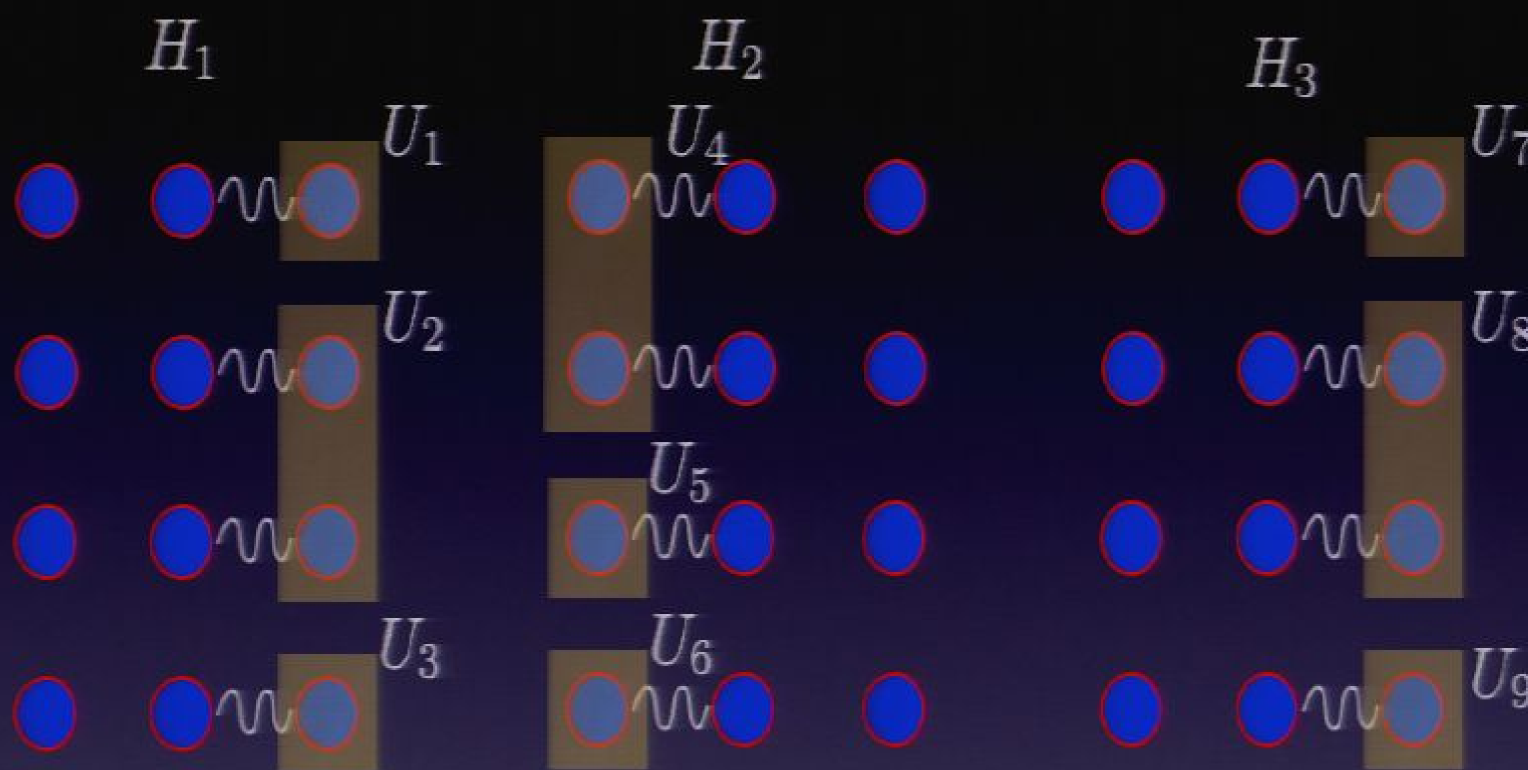
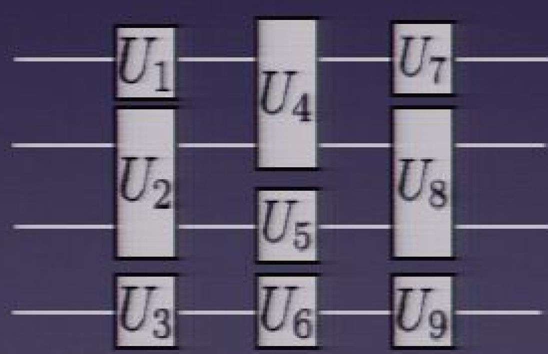




n qubit circuit

4n or 5n qubits

Comparison

Holonomic quantum computing, but open loop

open loop holomic QC:
[D. Kult, J. Aberg, E. Sjoqvist, Phys. Rev. A 74, 022106 (2006)]

Adiabatic universal quantum computing, but with piecewise evolution

D. Aharonov et al., in 45th Annual IEEE FOCS (2004)]

Path robustness in holonomy is subsystem dependent

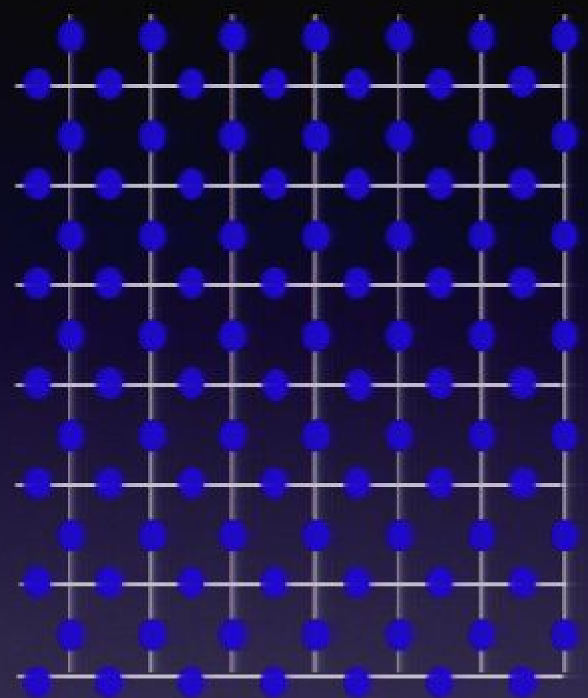
compare:
[O. Oreshkov, T.A. Brun, D.A. Lidar, Phys. Rev. Lett. 102, 070502 (2009)]

Piecewise keeps gap constant:

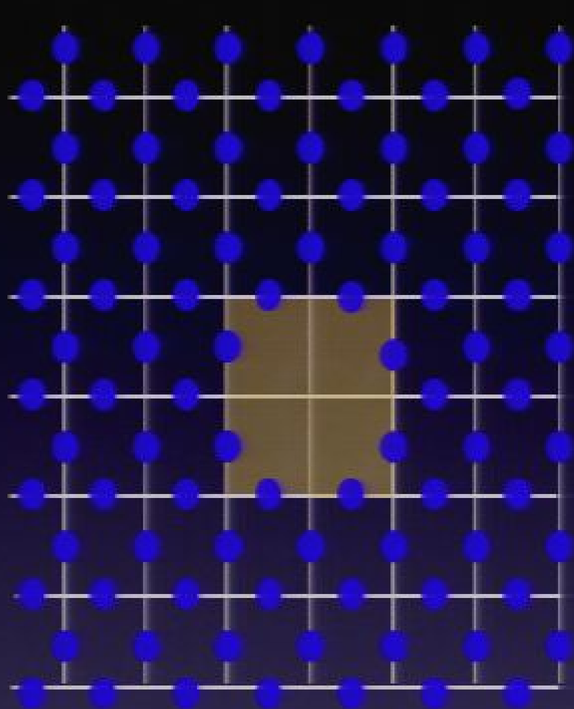
compare spin-1 chain transmission of:
[K. Eckert, O. Romero-Isart, and A. Sanpera, New J. Phys. 9, 155 (2007)]

Details: arXiv:0905.0901

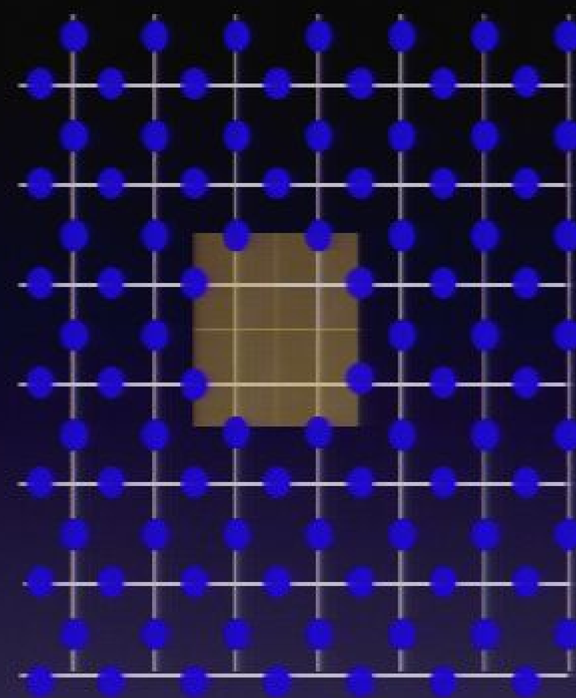
Back to TQC!



Encoded Qubits

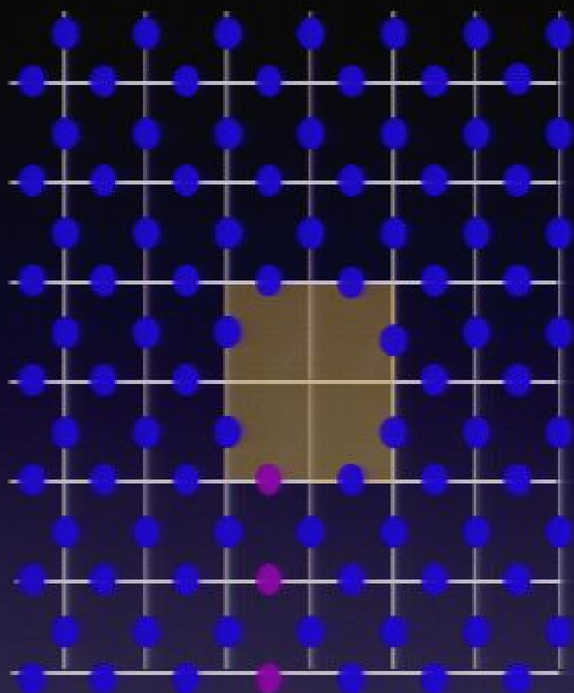


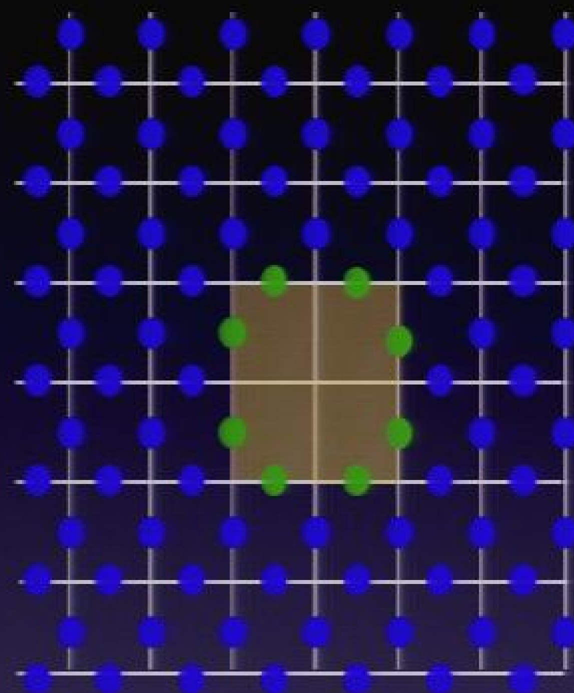
Smooth qubit



Rough qubit

Back to TQC!



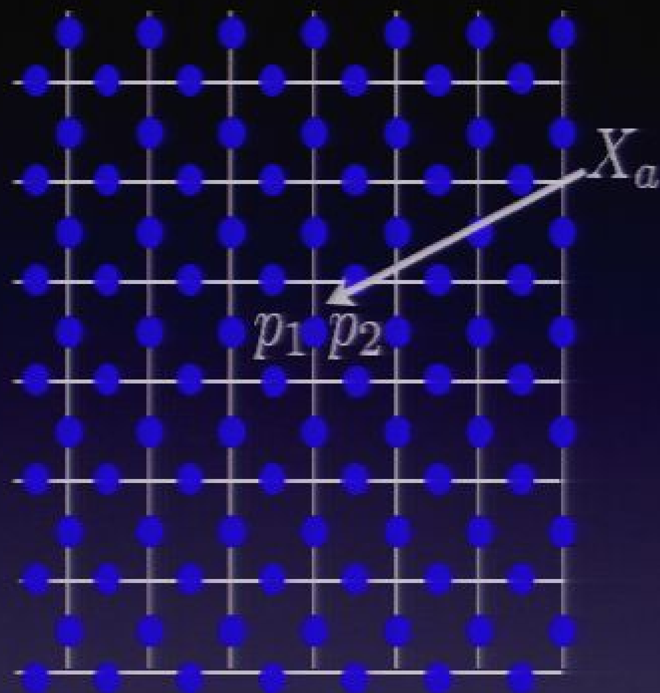


$$\bar{X} = X^{\otimes c_1}$$

$$\bar{Z} = Z^{\otimes c_2}$$

Smooth qubit

Prepare Smooth Puncture



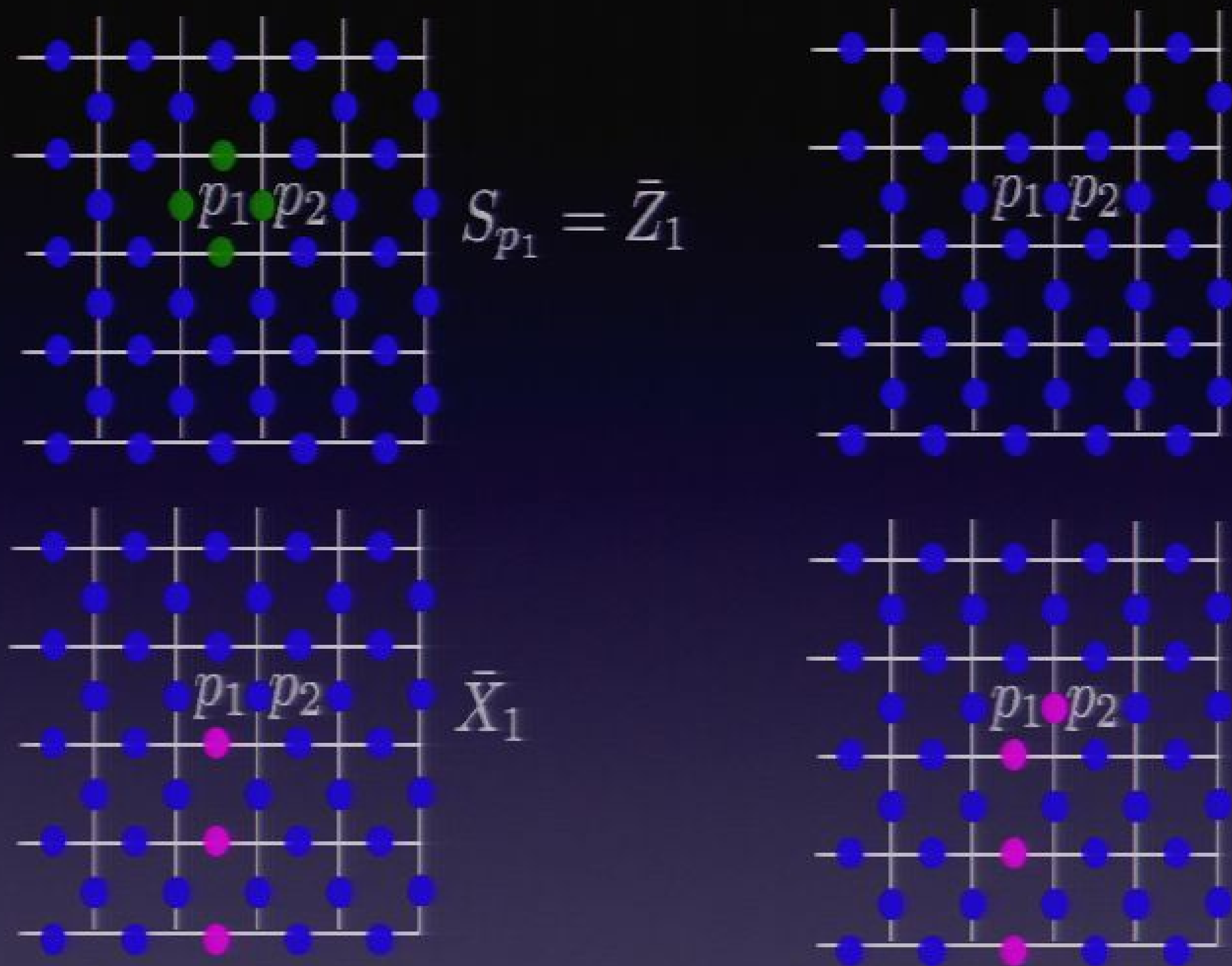
$$H_0$$

$$\downarrow$$

$$H_0 + \frac{\Delta}{4}(S_{p_1} + S_{p_2}) - \frac{\Delta}{2}X$$

Turn off 2 plaquette operators
adjacent to a qubit where X is turned on.

Prepare Smooth Puncture



$S_{p_1} = \bar{Z}_1$

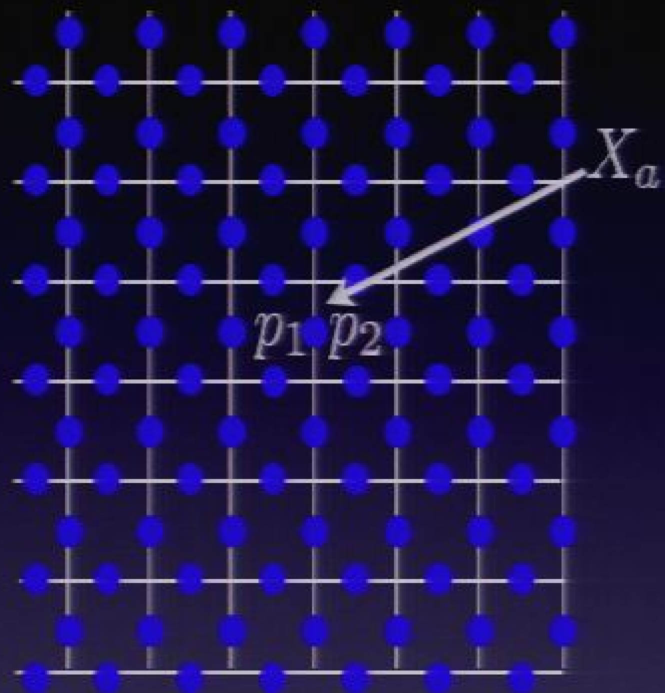
$S_{p_2} = \bar{Z}_2$

$\bar{X}_1$

$\bar{X}_2$

$$-\frac{\Delta}{4}(\bar{Z}_1 + \bar{Z}_2) \longrightarrow -\frac{\Delta}{2}\bar{X}_1\bar{X}_2$$

Prepare Smooth Puncture



$$H_0$$
$$\downarrow$$
$$H_0 + \frac{\Delta}{4}(S_{p_1} + S_{p_2}) + \frac{\Delta}{2}X_a$$
$$-\frac{\Delta}{4}(\bar{Z}_1 + \bar{Z}_2)$$
$$\downarrow$$
$$-\frac{\Delta}{2}\bar{X}_1\bar{X}_2$$

$\bar{Z}_1\bar{Z}_2$ commutes with H(t)

$+1$ eigenspace of $\bar{Z}_1\bar{Z}_2$

$$-\frac{\Delta}{2}X \longrightarrow -\frac{\Delta}{2}Z$$

end up in +1 e.v. of smooth qubit encoded Z

Prepare Smooth Puncture

$$H_0$$

$$\downarrow$$

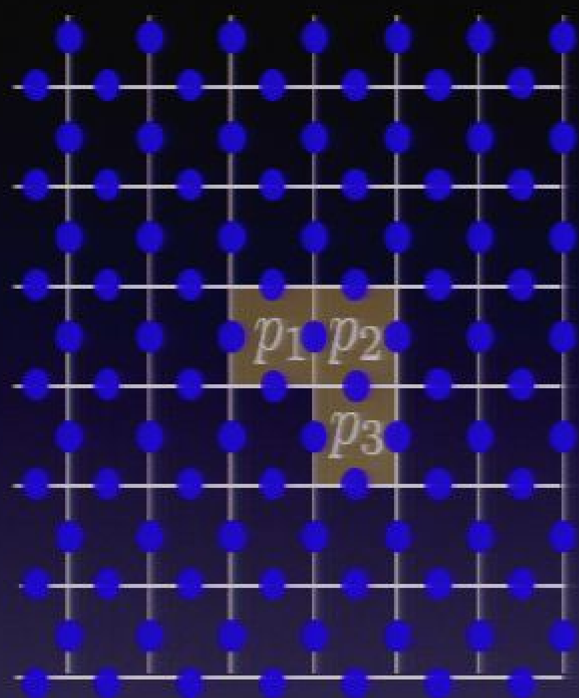
$$H_0 + \frac{\Delta}{4}(S_{p_1} + S_{p_2}) + \frac{\Delta}{2}X_a$$

$$\downarrow$$

$$H_0 + \frac{\Delta}{4}(S_{p_1} + S_{p_2}) - \frac{\Delta}{2}X_a + \frac{\Delta}{4}(S_{v_1} + S_{v_2}) - \frac{\Delta}{4}(S'_{v_1} + S'_{v_2}))$$

Smooth qubit in +1 eigenvalue of encoded Z

Deform Smooth Puncture



Turn on

$$-\frac{\Delta}{2}X_b$$

While turning off

$$-\frac{\Delta}{4}S_{p_3}$$

+ relevant vertex operators

Rough qubits: Ditto

Rough qubit prepare +1 e.v. of encoded X

Toolkit

1. Create small smooth qubit in +1 e.v. of Z
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3. Enlarge, move both punctures
4. Braid puncture: this give a C_X target = rough

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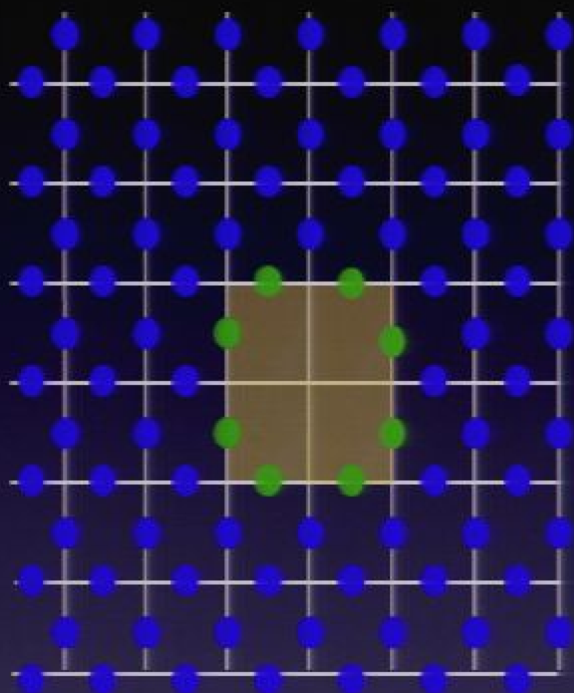
5. Measure encoded X,Z of qubits
6. Create magic states

Requires distillation.

Measurement



$$\bar{Z} = Z^{\otimes c_2}$$

Simple, right?

But how to perform why Hamiltonian still on?

Note: destructive Pauli measurements are tolerant to errors because (1) complementary errors are large and (2) we can remove region being measured

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5+6 not tolerant to errors in this model

Summary

Adiabatically deforming between different Hamiltonians which have stabilizer codewords as energy eigenstates yields interesting protocols like AGT

Using AGT as a motivator you can understand how topological quantum computing can or cannot robustly compute (and thus can provide rigorous error bounds.)

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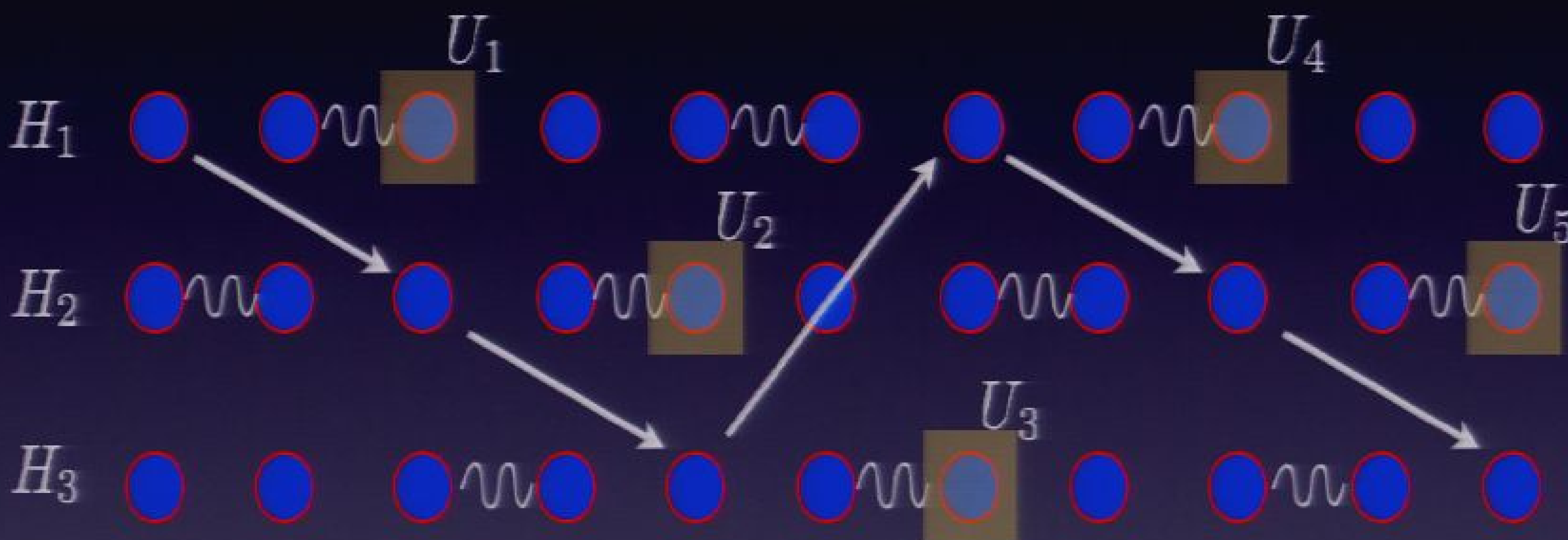
Adiabatic Gate Teleportation

Dave Bacon
University of Washington, Seattle, WA USA



[joint work with Steve Flammia (Perimeter), Alice Neels (Washington), Andrew Landahl (Sandia Labs)]

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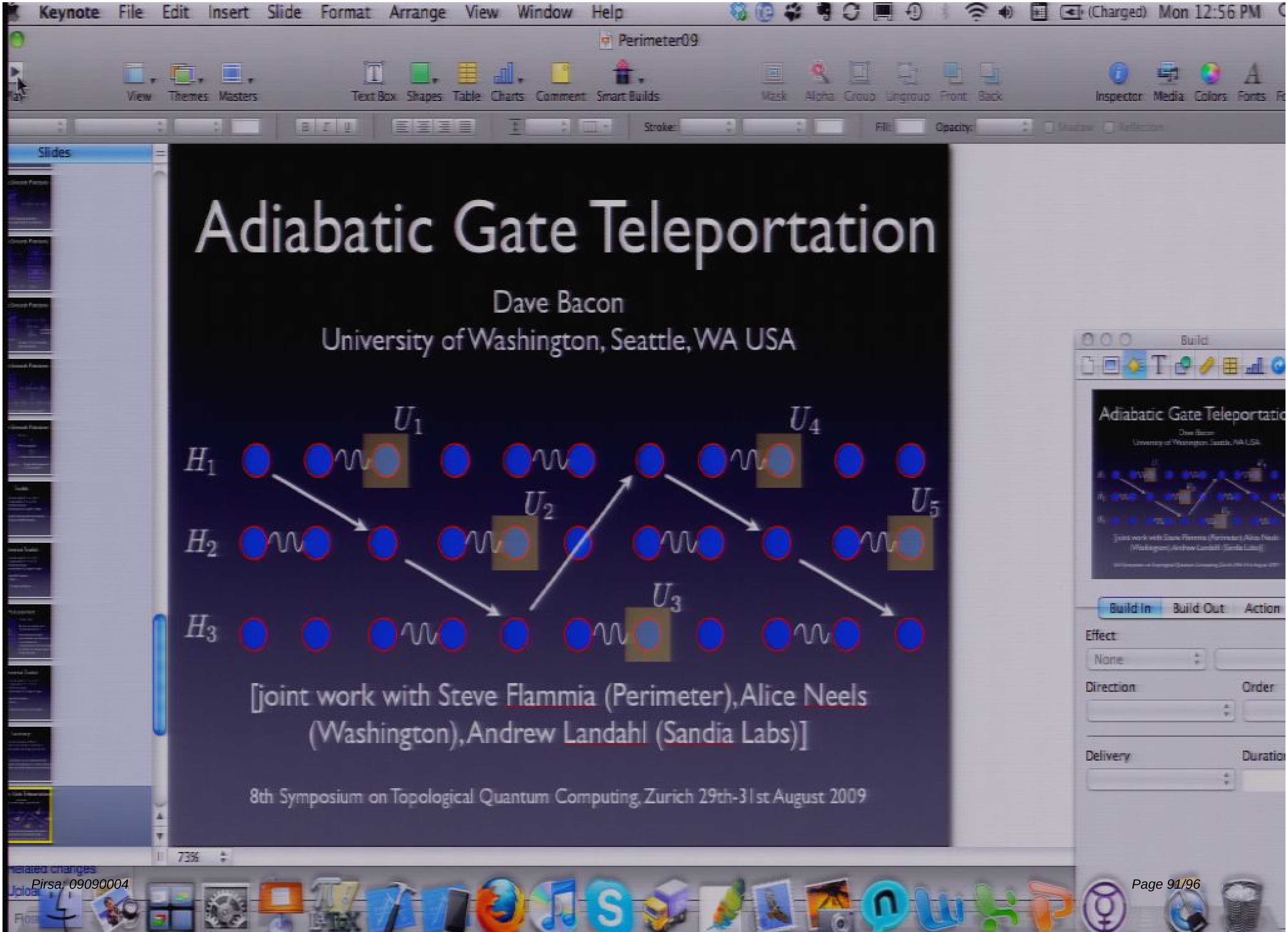
Adiabatic Gate Teleportation
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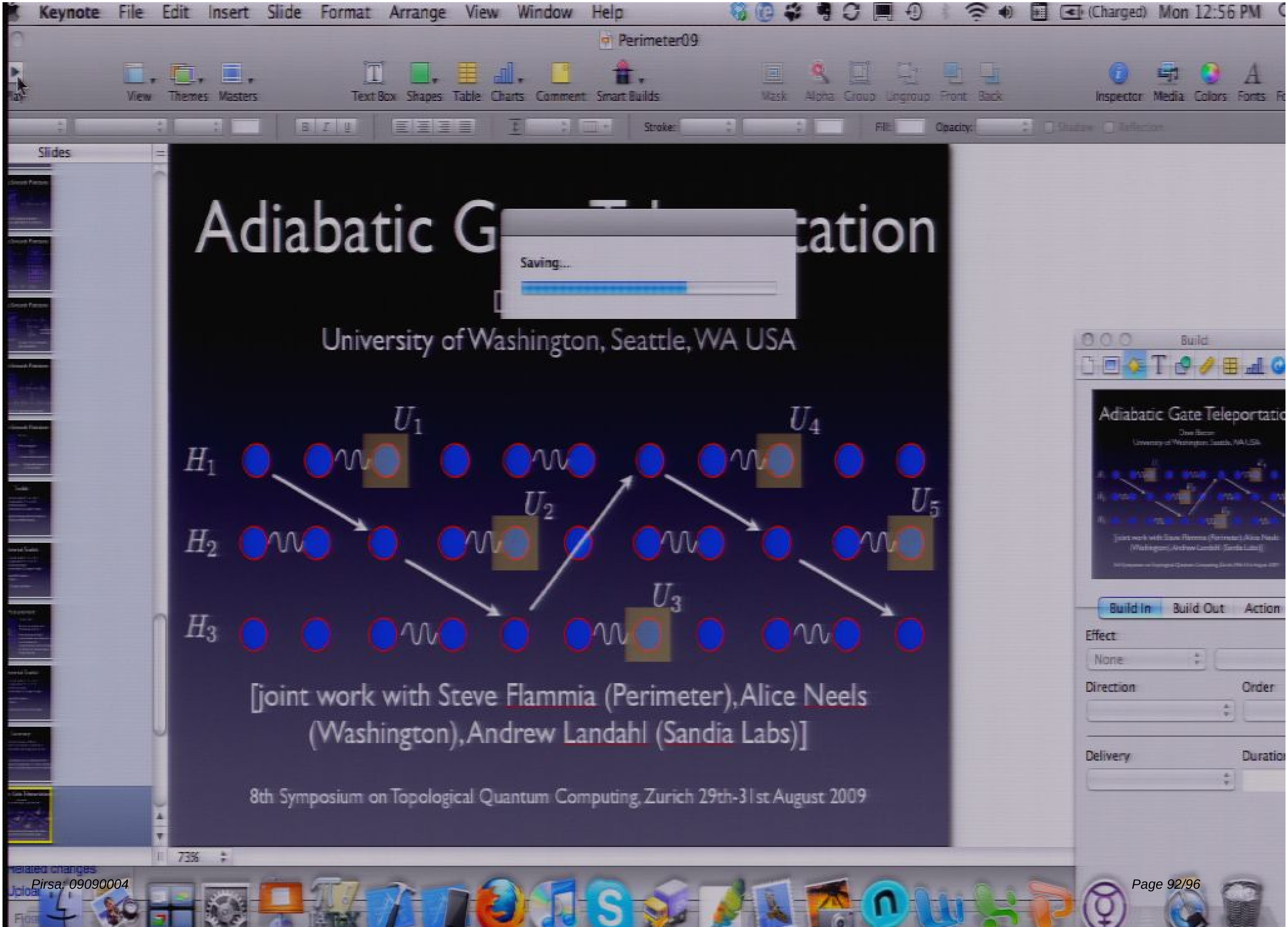
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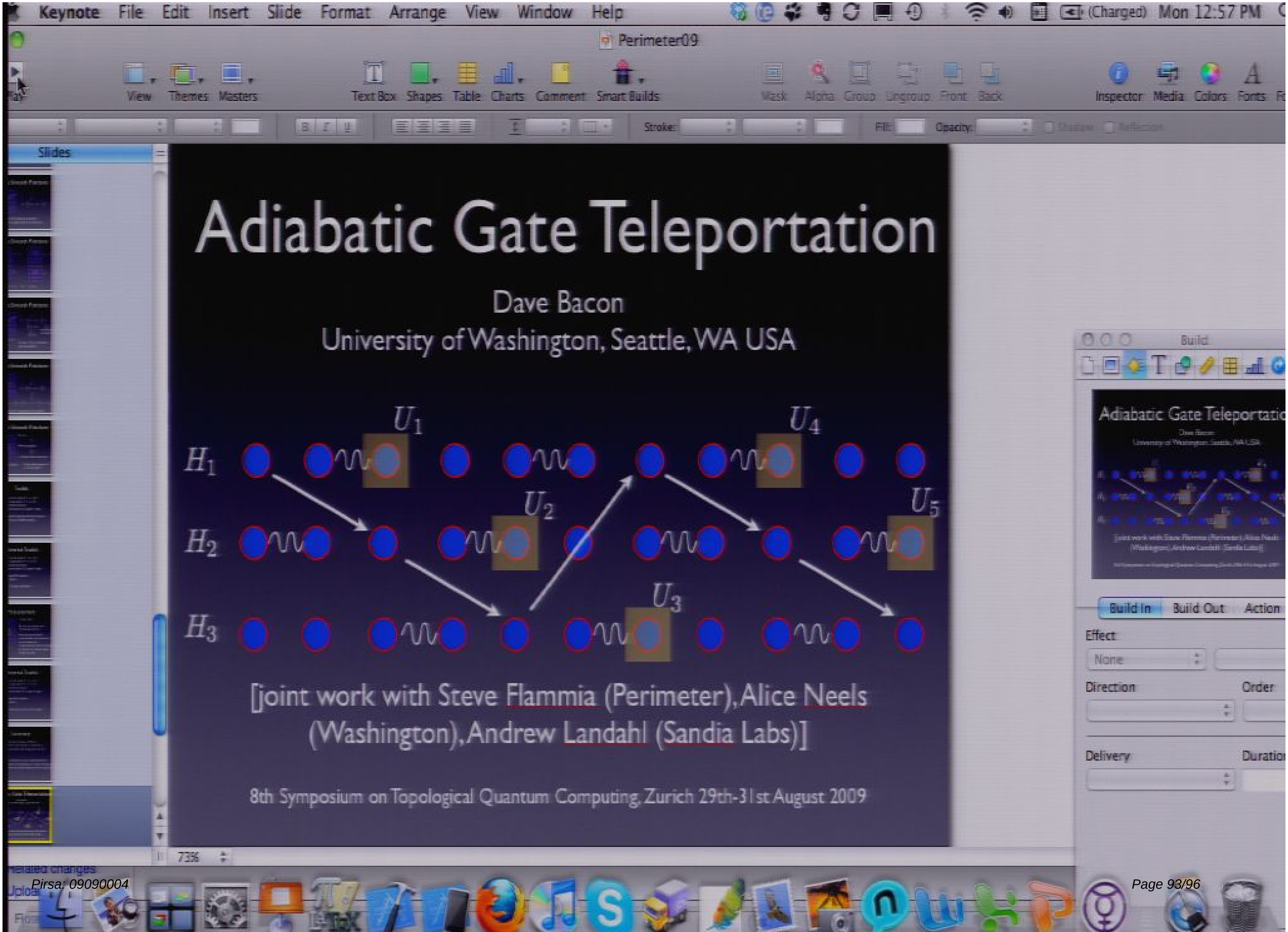
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Adiabatic Gate Teleportation
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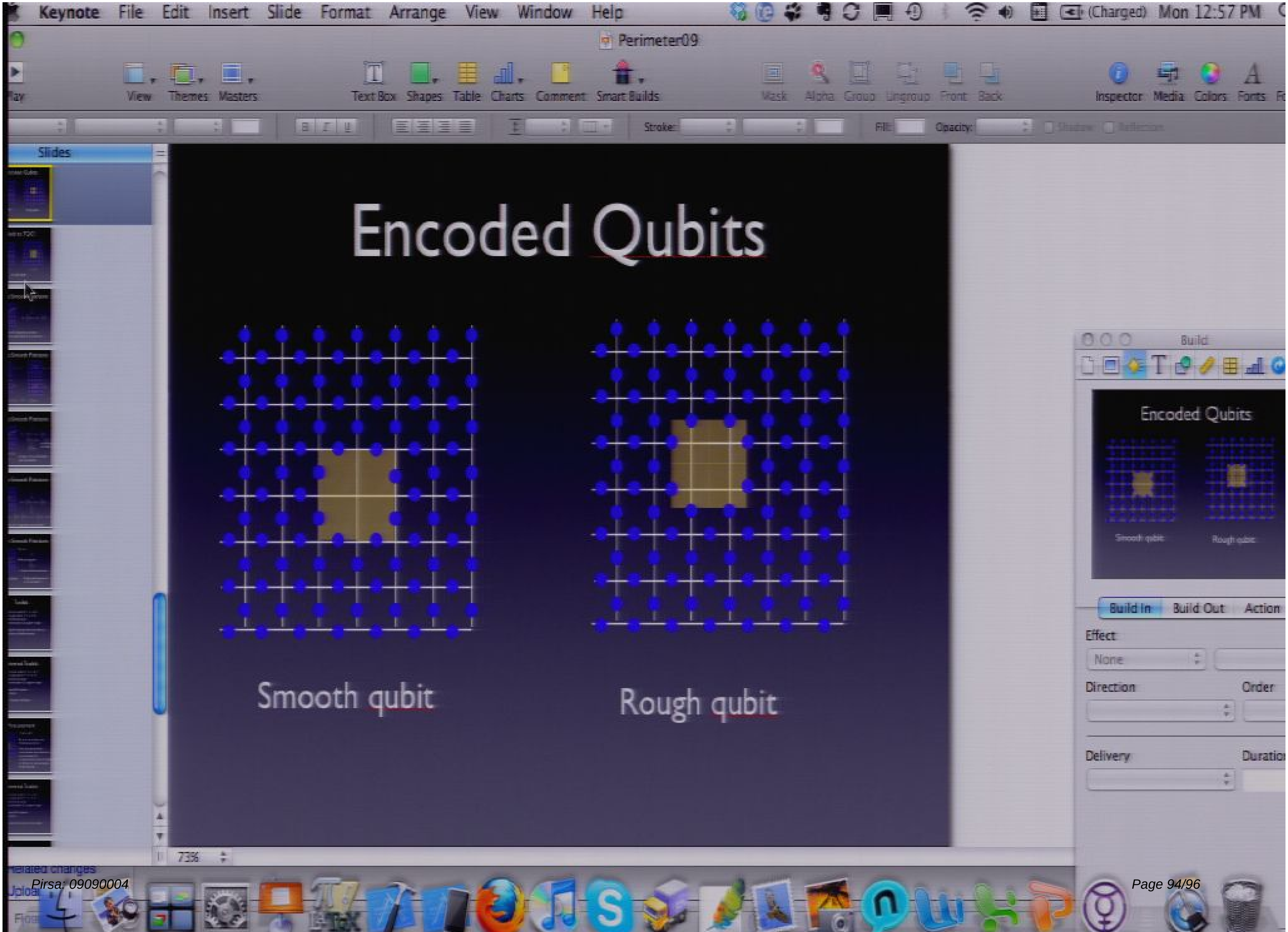
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Comparison

Holonomic quantum computing, but open loop

open loop holomic QC:

[D. Kult, J. Aberg, E. Sjoqvist, Phys. Rev. A 74, 022106 (2006)]

Adiabatic universal quantum computing, but with piecewise evolution

D. Aharonov et al., in 45th Annual IEEE FOCS (2004)]

Path robustness in holonomy is subsystem dependent

compare:

[O. Oreshkov, T.A. Brun, D.A. Lidar, Phys. Rev. Lett. 102, 070502 (2009)]

Piecewise keeps gap constant:

compare spin-1 chain transmission of:

[K. Eckert, O. Romero-Isart, and A. Sanpera, New J. Phys. 9, 155 (2007)]

Details: arXiv:0905.0901

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