Title: Generating and detecting multi-qubit GHZ states in circuit QED

Date: Sep 22, 2009 04:00 PM

URL: http://pirsa.org/09090002

Abstract: I will present recent work [1] on preparation by measurement of Greenberger–Horne–Zeilinger (GHZ) states in circuit quantum electrodynamics. In particular, for the 3-qubit case, when employing a nonlinear filter on the recorded homodyne signal the selected states are found to exhibit values of the Bell–Mermin operator exceeding 2 under realistic conditions. I will discuss the potential of the dispersive readout to demonstrate a violation of the Mermin bound, and present a measurement scheme avoiding the necessity for full detector tomography.

[1] Lev S Bishop et al 2009 New J. Phys. 11 073040

Outline

- Circuit QED
  - Superconducting qubits
  - Transmission Line Resonator as ‘quantum bus’
- Preparation by measurement
  - Stochastic Master Equations
  - Nonlinear filtering
- Entangled state detection
  - Simplified detector tomography
  - Relation to Bell tests

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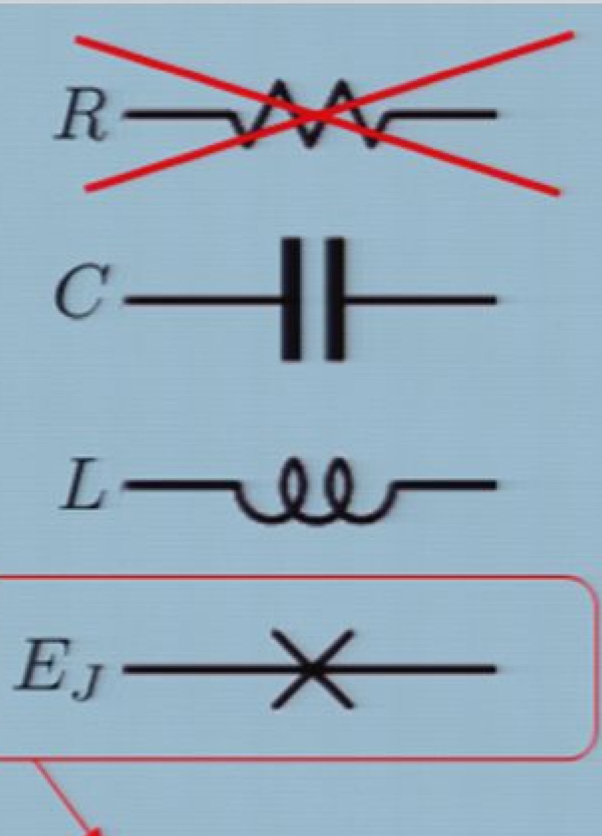
Quantum coherence in electrical circuits

- Avoid dissipation

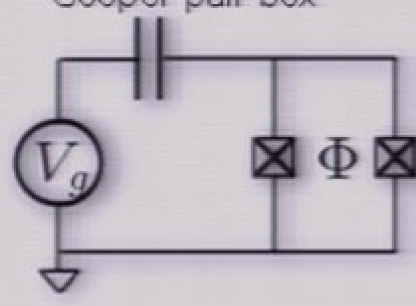
  → Superconducting circuits

Circuit elements:

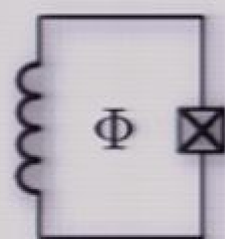


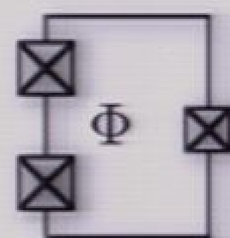
nonlinear, dissipationless element!

Cooper pair box



NEC Labs
Saclay
Yale
Chalmers





1-junction flux qubit

3-junction flux qubit

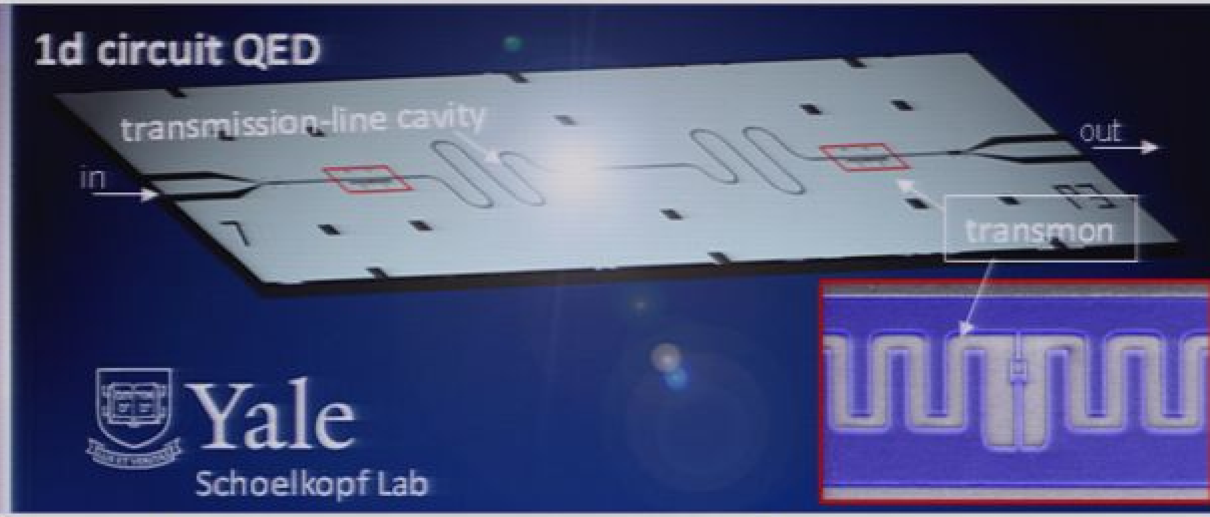
SUNY
Delft
MIT
UC Berkeley

flux-biased phase qubit

UCSB
NIST
U Maryland

Recent review: J Clarke, FK Wilhelm, Nature **453**, 1031 (2008)

From cavity QED to circuit QED



	atomic system	photonic system	coupling strength g/ω
cavity QED	alkali or Rydberg atom	3d optical or μwave cavity	$\sim 10^{-6}$
circuit QED	sc qubit	1d transmission-line resonator	$\sim 0.01 - 0.1$

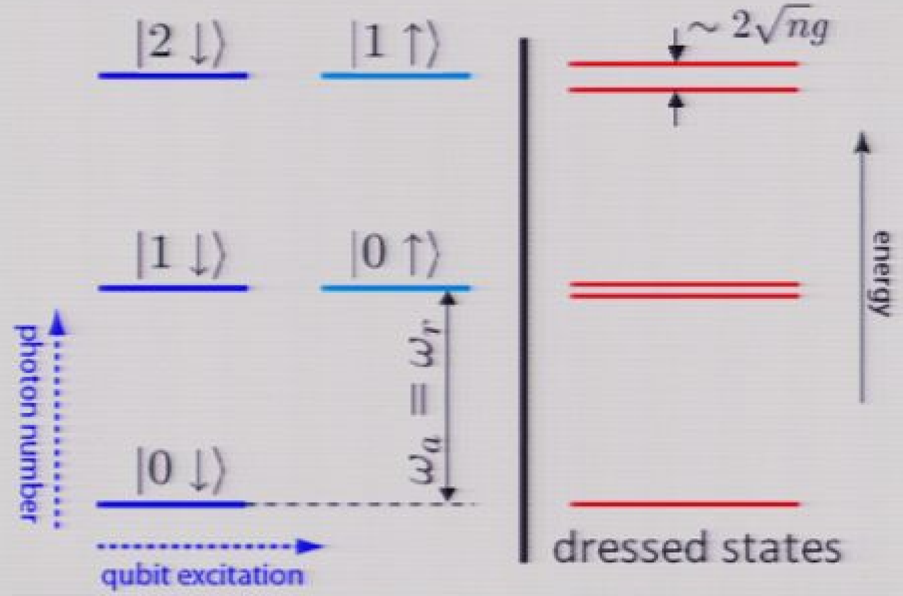
- Strong coupling, strongly dispersive regimes: easy with circuit QED
- Atom spatially fixed, no field inhomogeneity effects, etc
- Drive strength easily tunable over a wide power range
- Physics is described by the Jaynes-Cummings Hamiltonian (in RWA):

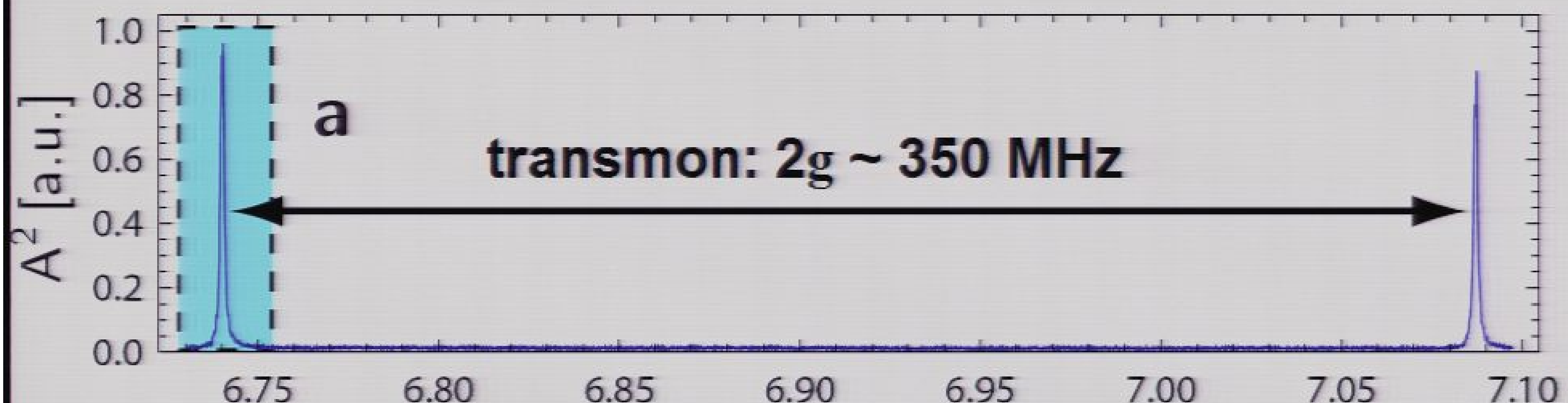
$$H = \hbar\omega_r a^\dagger a + \hbar\omega_q \sigma^+\sigma^- + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

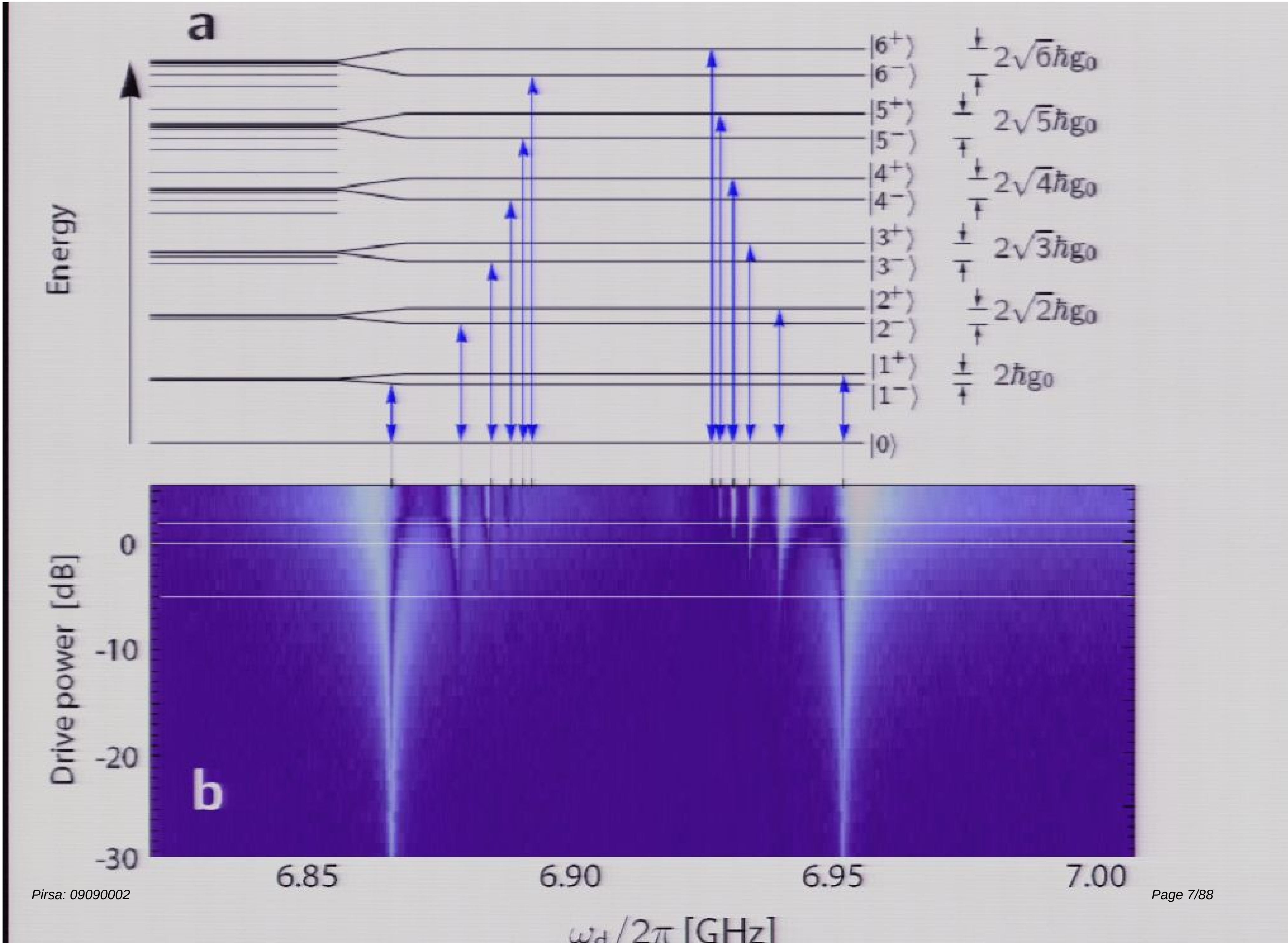
Strong coupling: vacuum Rabi splitting

Jaynes-Cummings Hamiltonian

$$\hat{H} = \hbar\omega_c \left(a^\dagger a + \frac{1}{2}\right) + \hbar\omega_a \hat{\sigma}_z$$
$$-\hbar g(a^\dagger \hat{\sigma}_- + \hat{\sigma}_+ a)$$







a
Energy
|6⁺⟩
|6⁻⟩
|5⁺⟩
|5⁻⟩
|4⁺⟩
|4⁻⟩
|3⁺⟩
|3⁻⟩
|2⁺⟩
|2⁻⟩
|1⁺⟩
|1⁻⟩
|0⟩
$2\sqrt{6}\hbar g_0$
$2\sqrt{5}\hbar g_0$
$2\sqrt{4}\hbar g_0$
$2\sqrt{3}\hbar g_0$
$2\sqrt{2}\hbar g_0$
$2\hbar g_0$
b
Drive power [dB]
0
-10
-20
-30
6.85
6.90
6.95
7.00
$\omega_d/2\pi$ [GHz]

Strong coupling: vacuum Rabi splitting

Jaynes-Cummings Hamiltonian

$$\hat{H} = \hbar\omega_c \left(a^\dagger a + \frac{1}{2} \right) + \hbar\omega_a \hat{\sigma}_z$$
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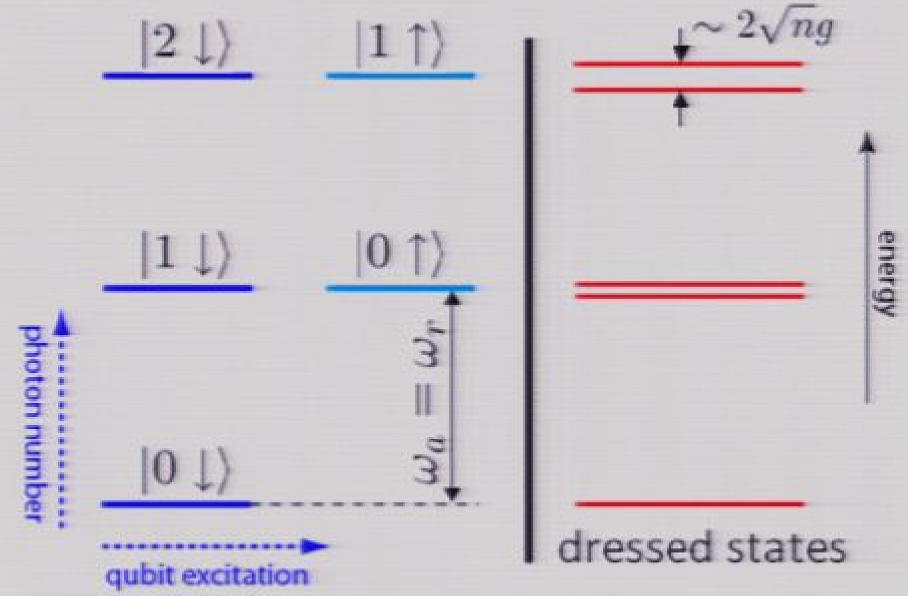

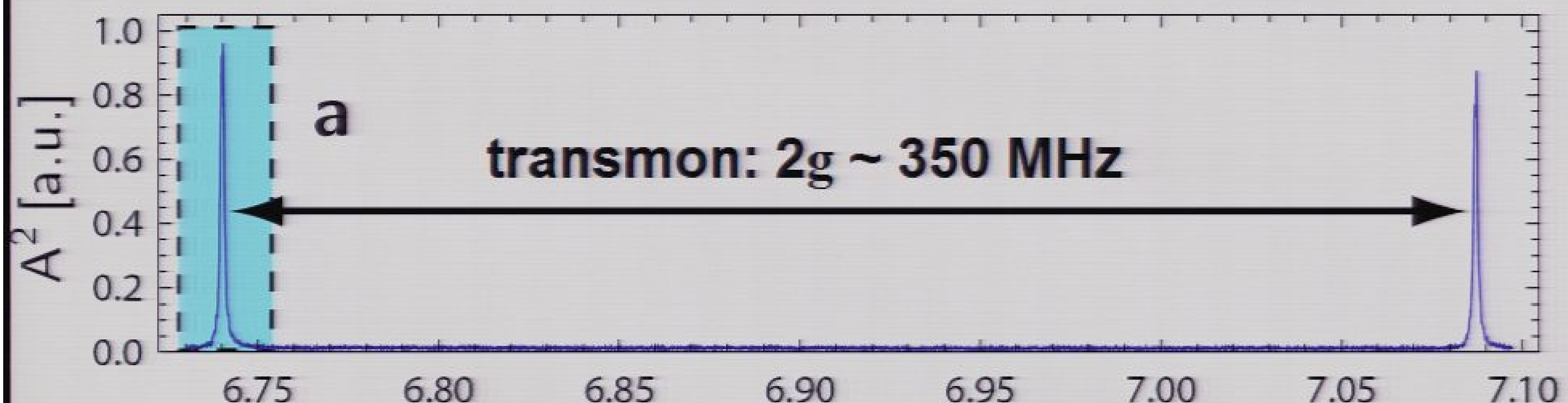

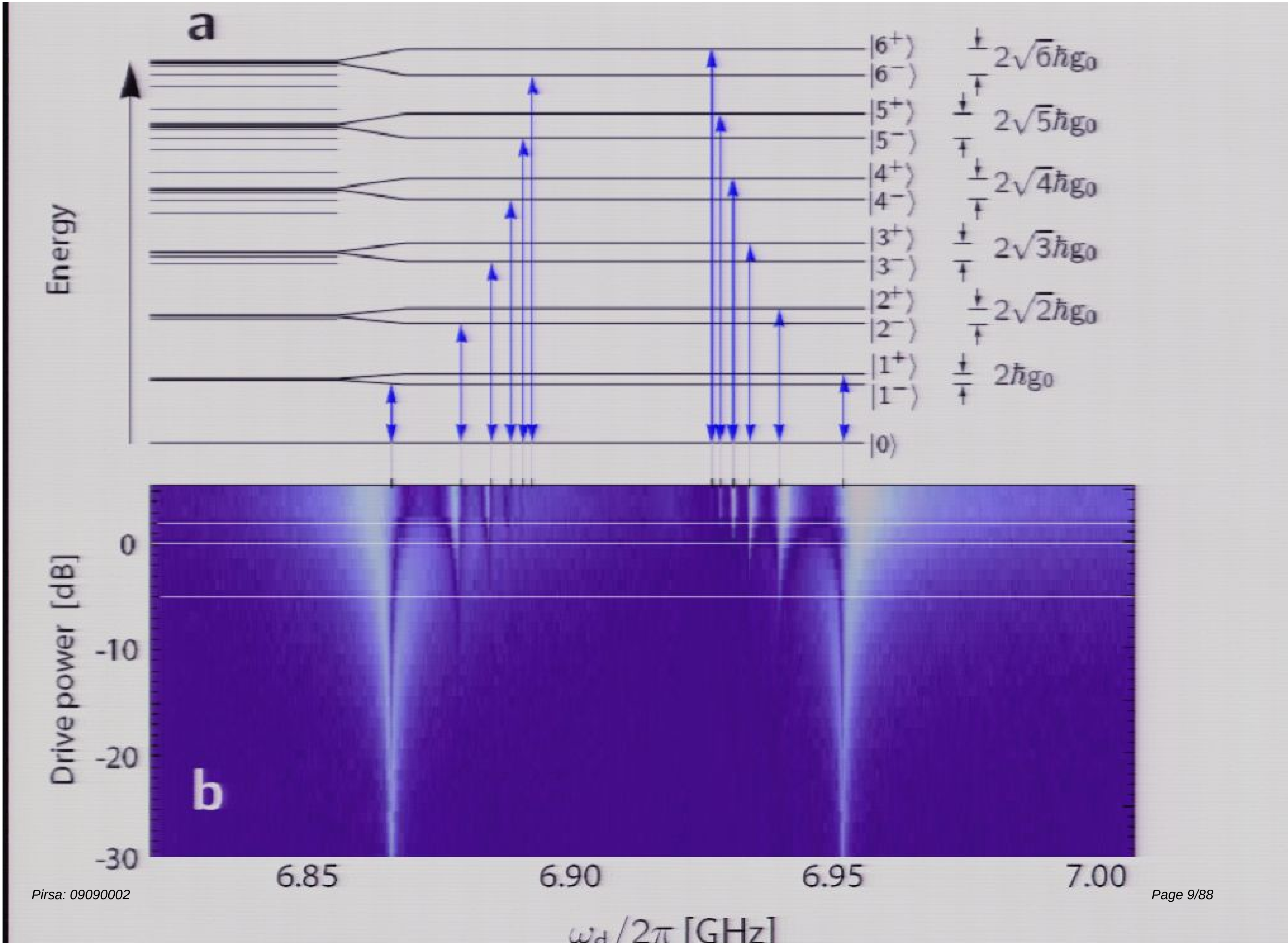
a
Energy
|6+⟩
|6−⟩
|5+⟩
|5−⟩
|4+⟩
|4−⟩
|3+⟩
|3−⟩
|2+⟩
|2−⟩
|1+⟩
|1−⟩
|0⟩
$2\sqrt{6}\hbar g_0$
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b
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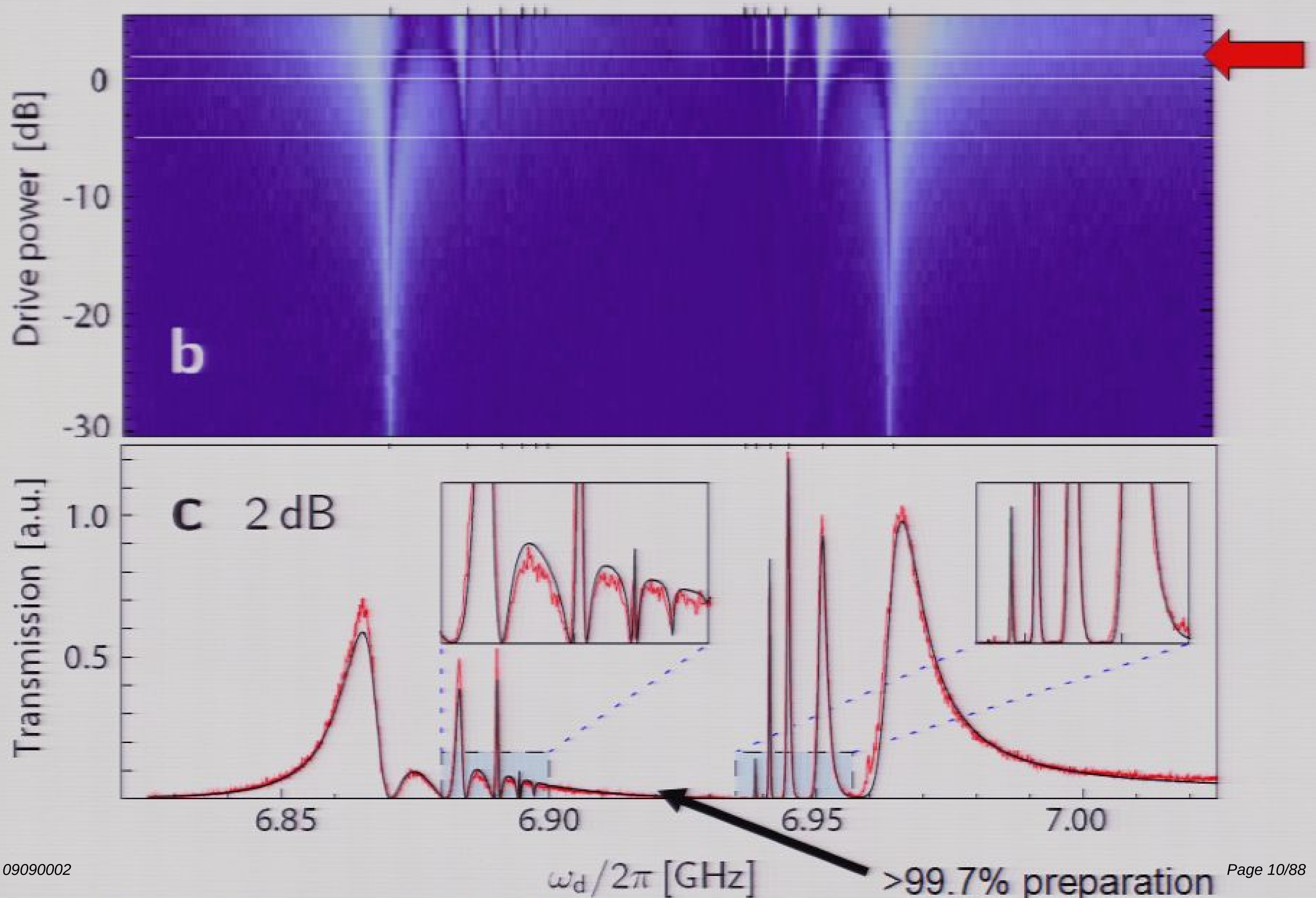
-30
6.85
6.90
6.95
7.00
$\omega_d/2\pi$ [GHz]

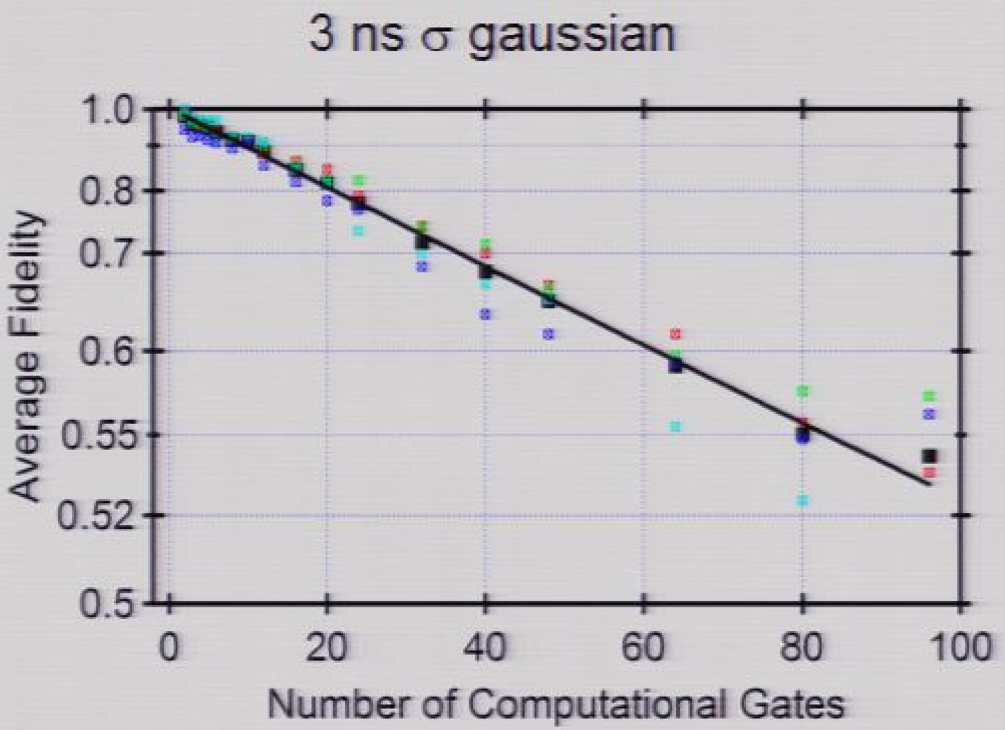
State preparation



>99.7% preparation fidelity

ishop et al. Nature Physics **5**, 105 (2009)

One-qubit rotations

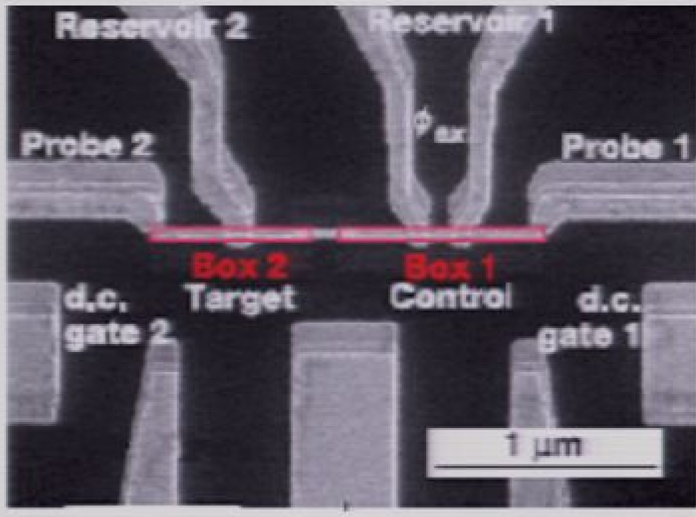


Carefully-shaped microwave pulses

Error per computational gate = 1.1%

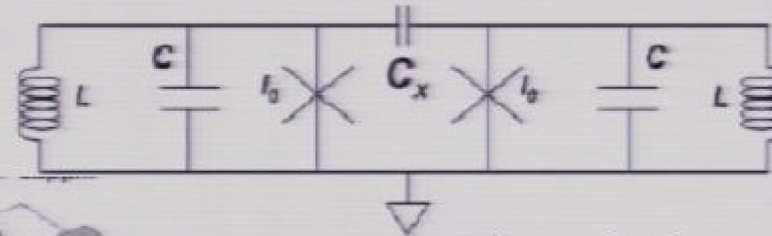
J. M. Chow *et al.* Phys. Rev. Lett. 102, 090502 (2009)

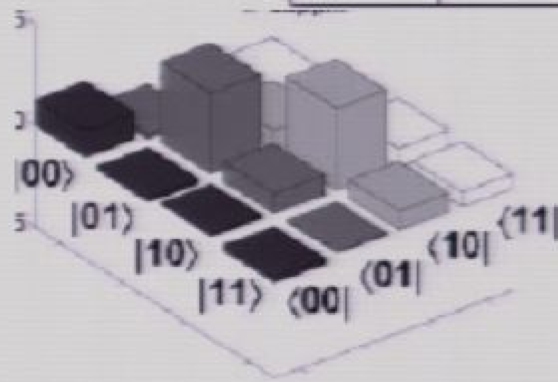
Coupling SC Qubits: Use a Circuit Element



Charge qubits: NEC 2003

a capacitor



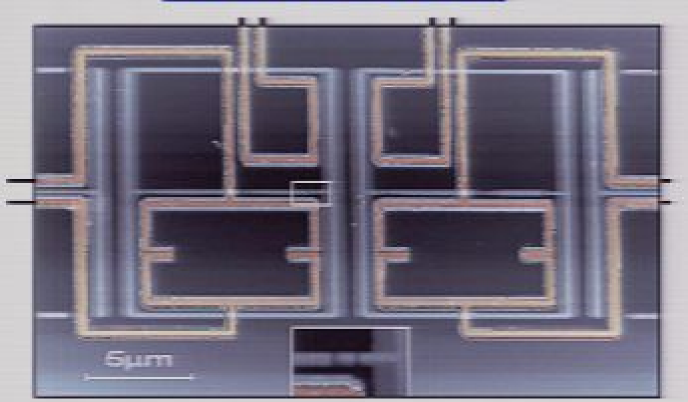


entangled states!

Concurrence ~ 55%

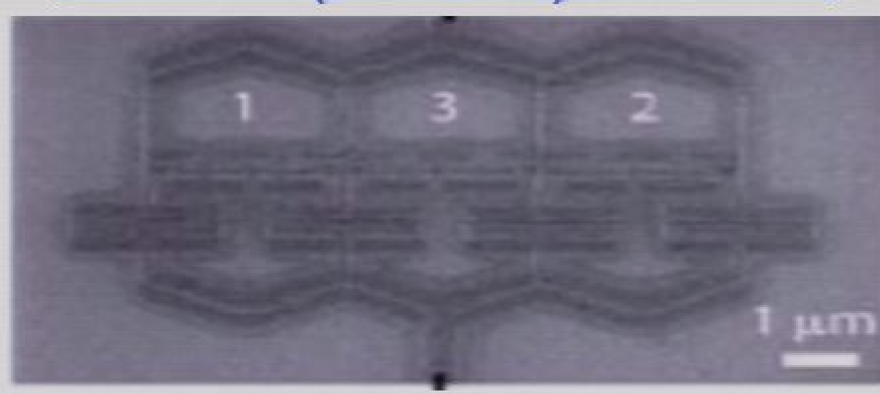
Phase qubits: UCSB 2006

an inductor



Flux qubits: Delft 2007

tunable (SQUID) element

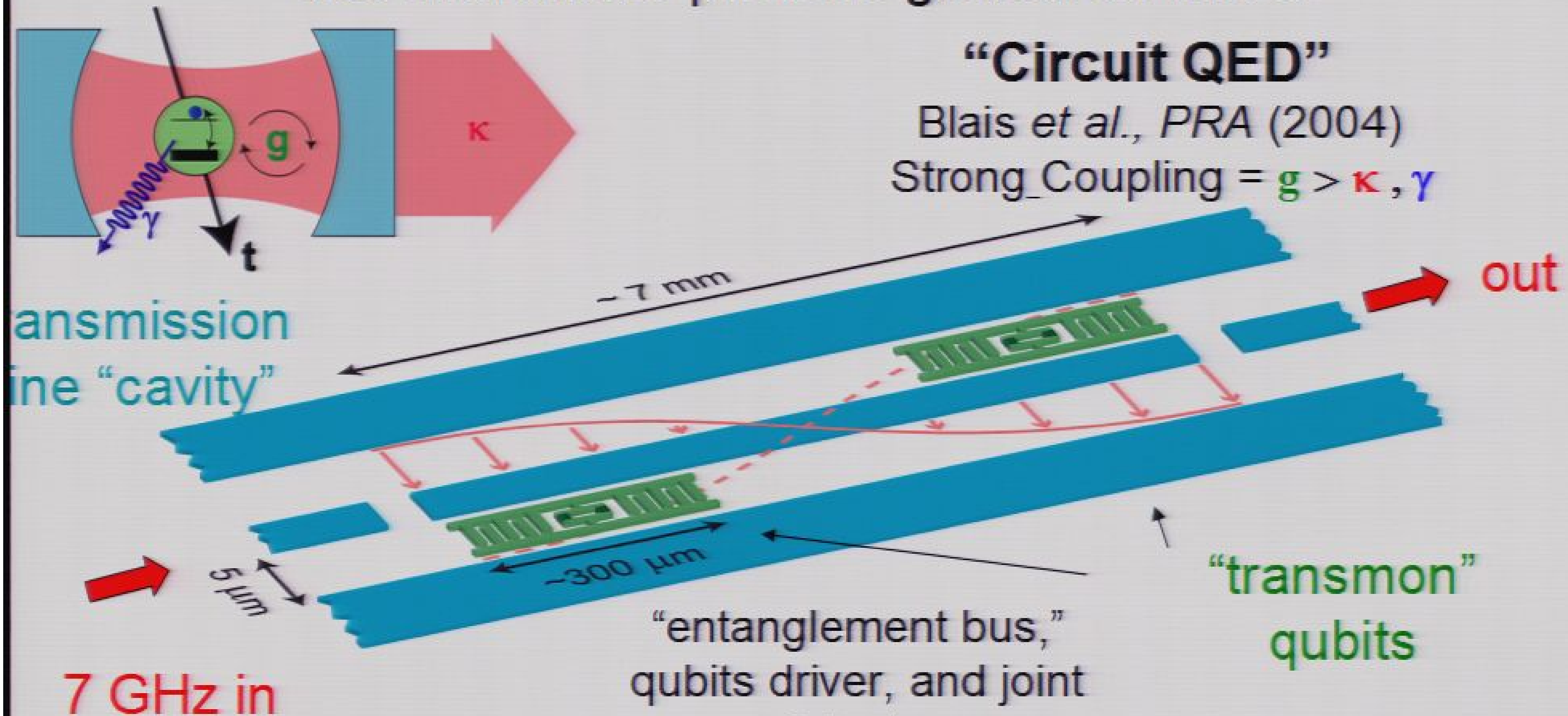


Flux qubits: Berkeley 2006, NEC 2007

Or tunable bus, Chalmers

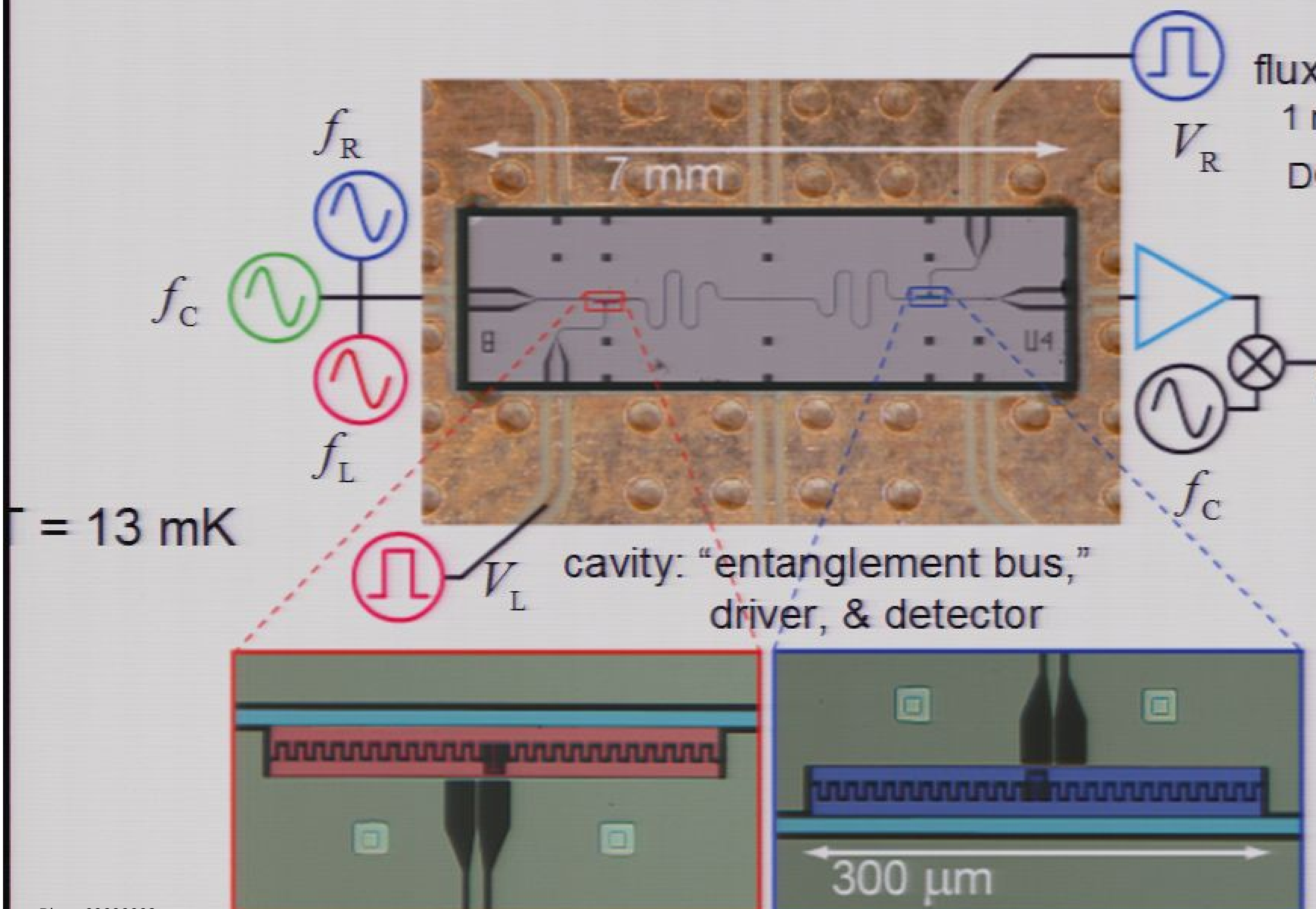
Qubits coupled with a quantum bus

use microwave photons guided on wires!



<u>Bus Experiments:</u> Majer *et al.*, *Nature*, 2007 (Charge qubits / Yale)
Sillanpää *et al.*, *Nature*, 2007 (Phase qubits / NIST)
DiCarlo *et al.*, *Nature*, 2009 (Charge qubits / Yale)

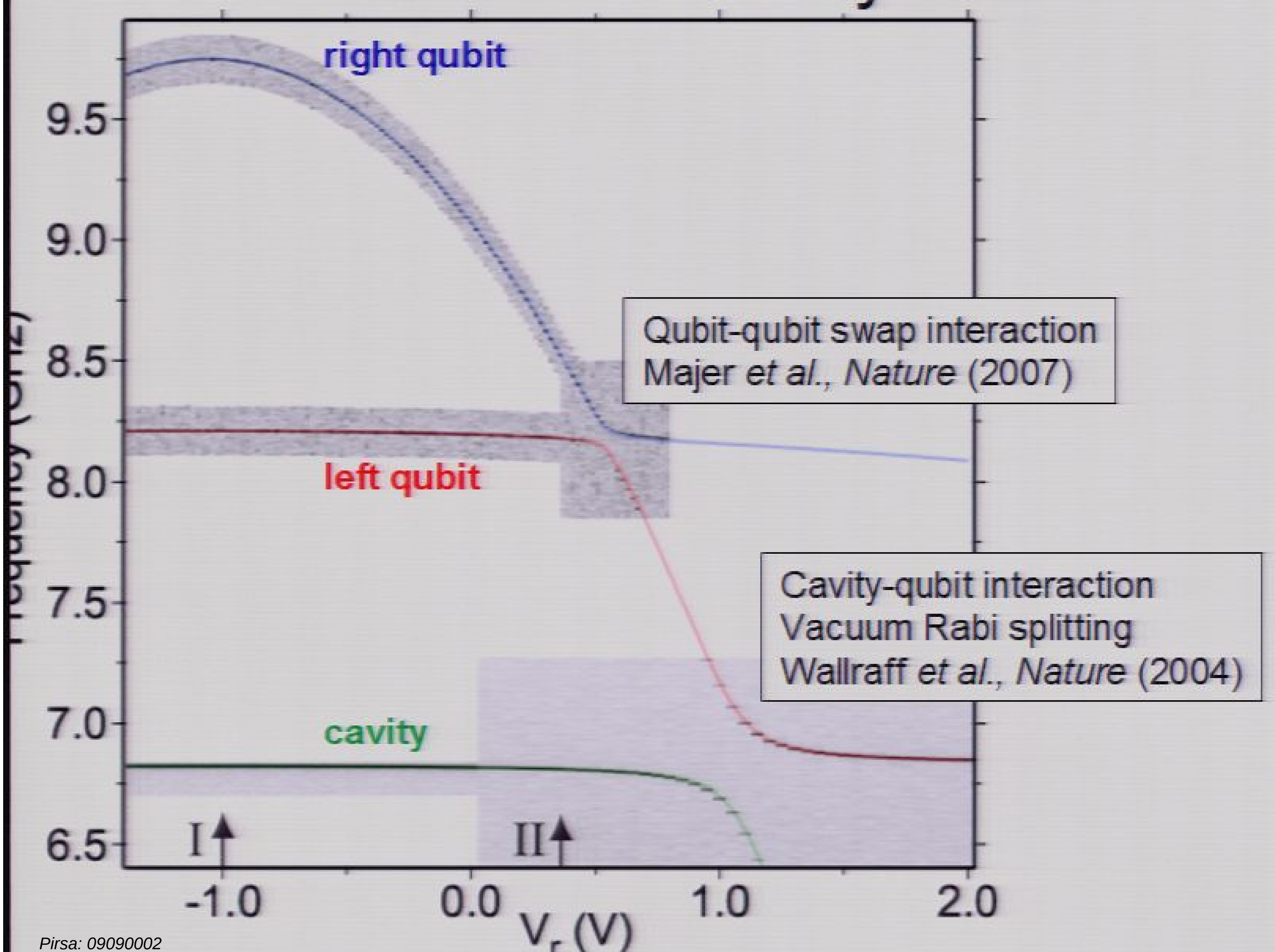
A two-qubit processor



Koch *et al.*, *PRA* 2007

Schreier et al., *PRB* 2008

Spectroscopy of qubits interacting with cavity



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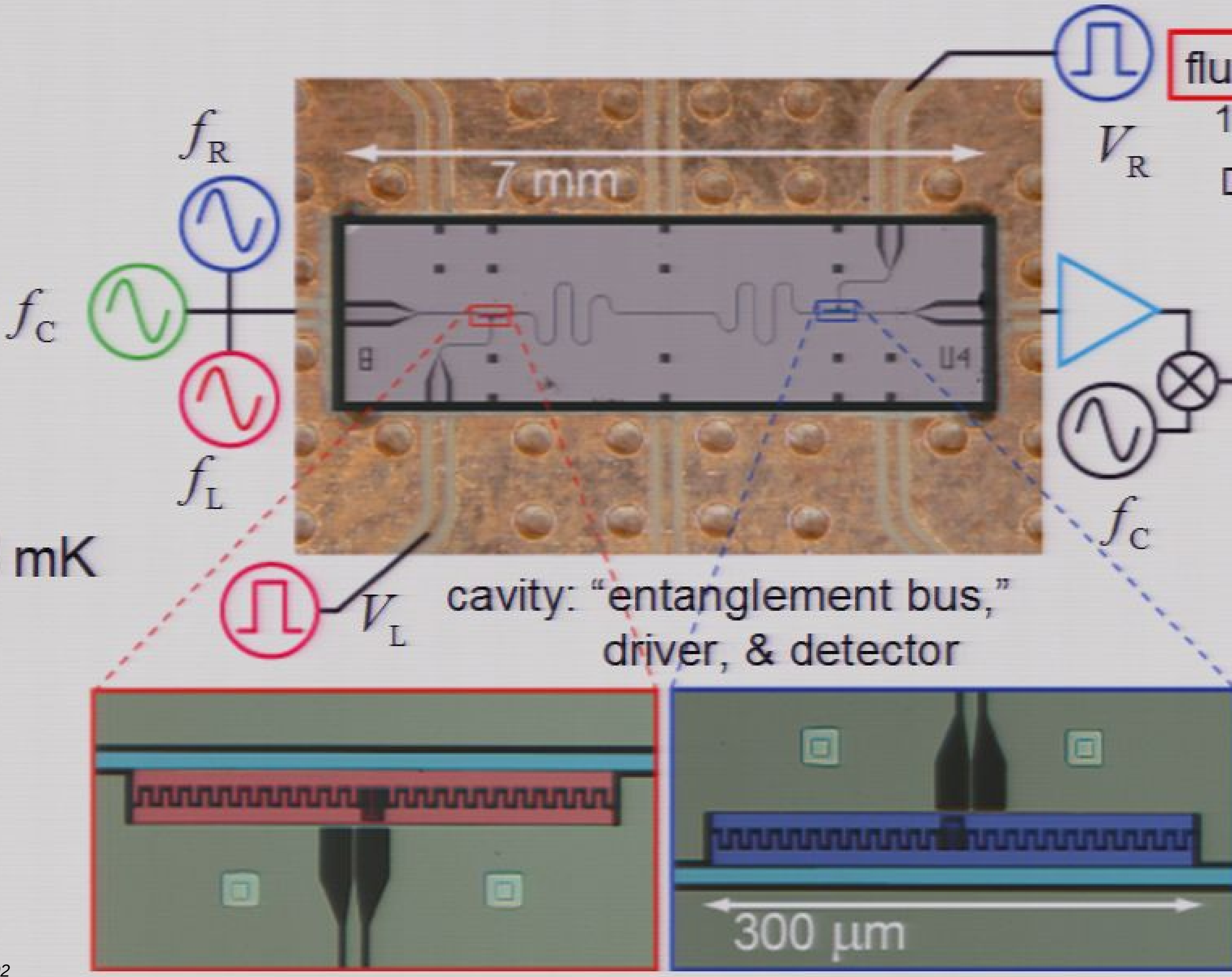


flux bias lines
1 ns resolution
DC - 2 GHz

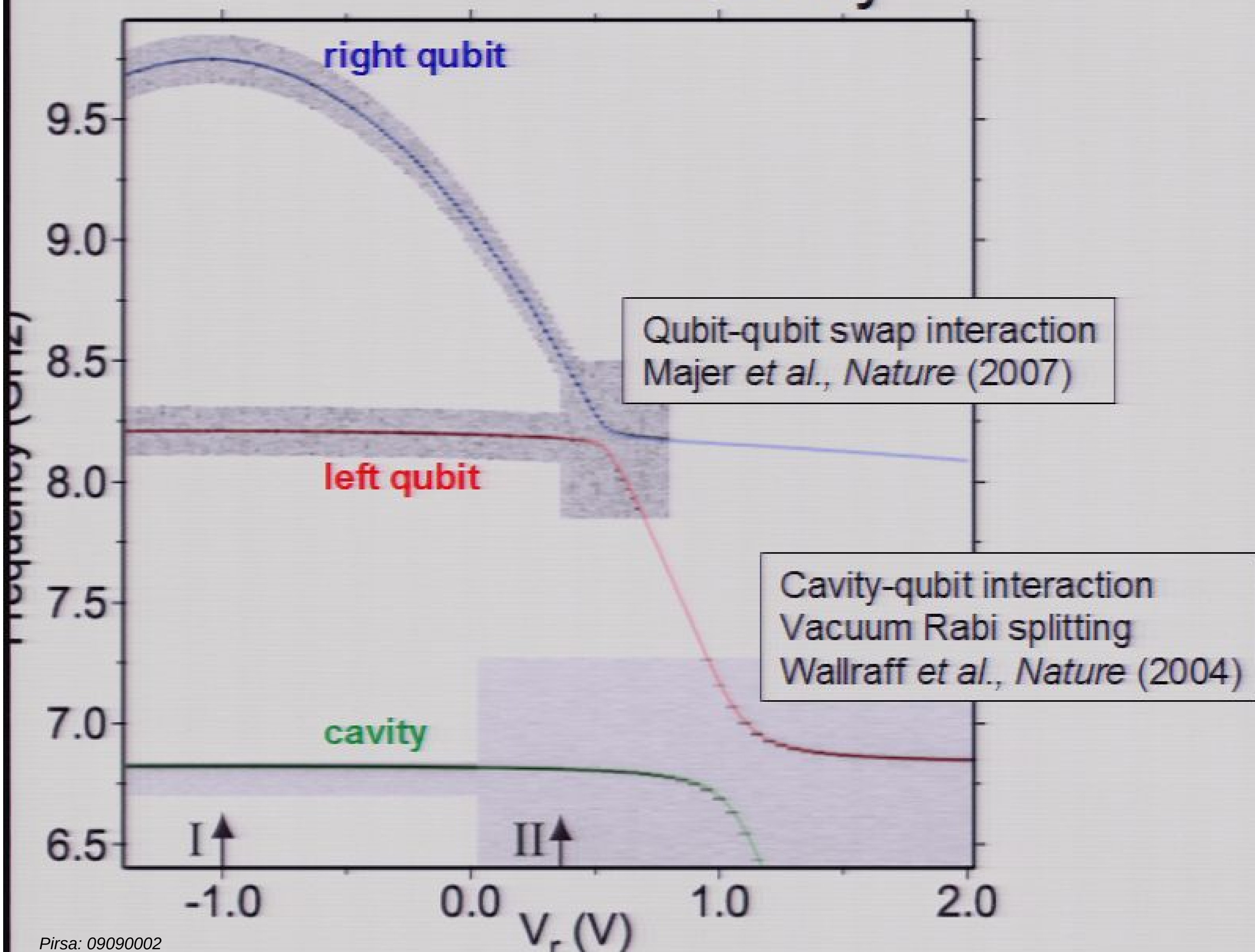
T = 13 mK

transmon qubits

Koch *et al.*, *PRA* 2007

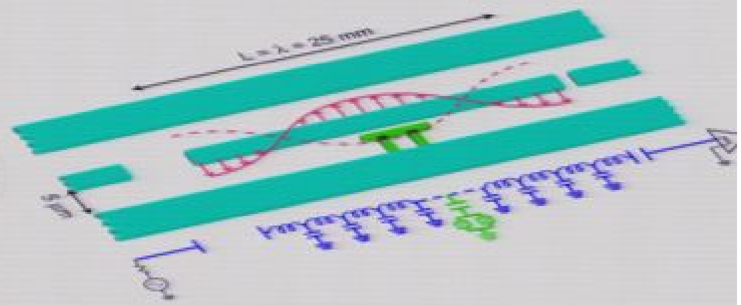
Schreier et al., *PRB* 2008

Spectroscopy of qubits interacting with cavity



Dispersive regime

Minimal model is Tavis-Cummings Hamiltonian



$$H = \omega_r a^\dagger a + \sum_j \frac{\omega_{q,j}}{2}\sigma_j^z + \sum_j g_j(a\sigma_j^+ + a^\dagger\sigma_j^-)$$

resonator qubit resonator- qubit coupling

In the dispersive regime $|g_j|/|\omega_{q,j} - \omega_r| = |g_j/\Delta_j| \ll 1$

$$H_{\text{eff}} = \omega_r a^\dagger a + \sum_j \frac{\omega_{q,j} - \chi_j}{2}\sigma_j^z + \sum_j \chi_j a^\dagger a \sigma_j^z$$

with dispersive shift

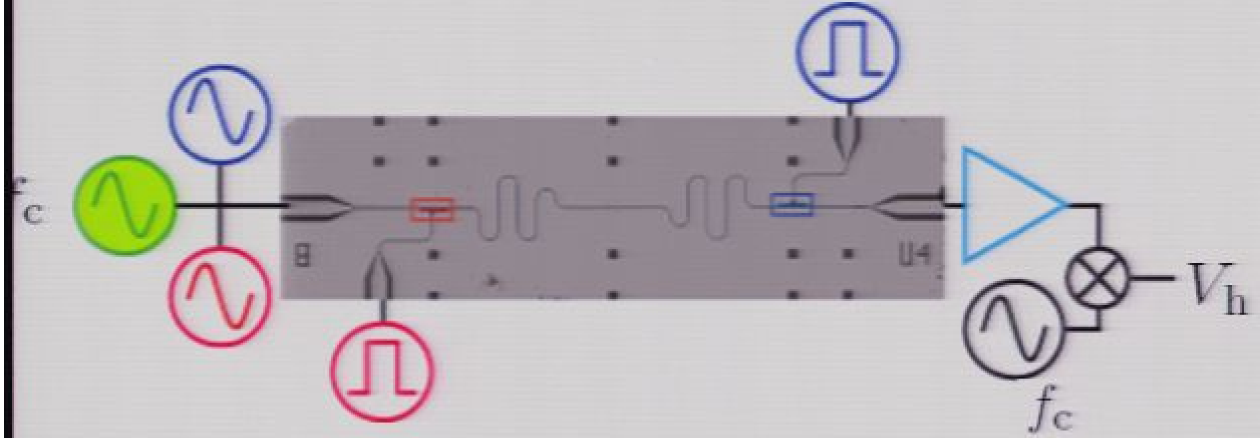
$$\chi_j = g_j^2/\Delta_j$$

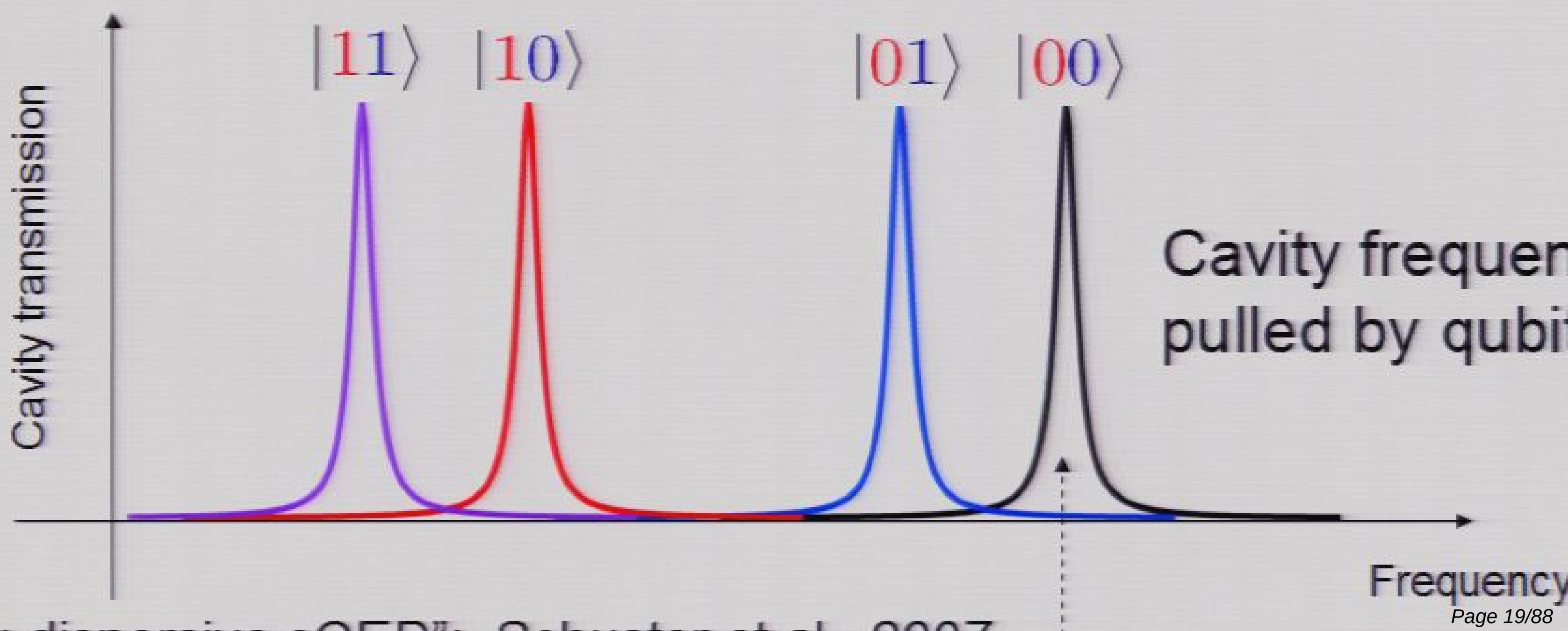
Lamb shift

AC stark shift

Blais et al, Phys Rev A **69**, 062320 (2004)
Wallraff et al, Nature **431**, 162 (2004)

Dispersive readout

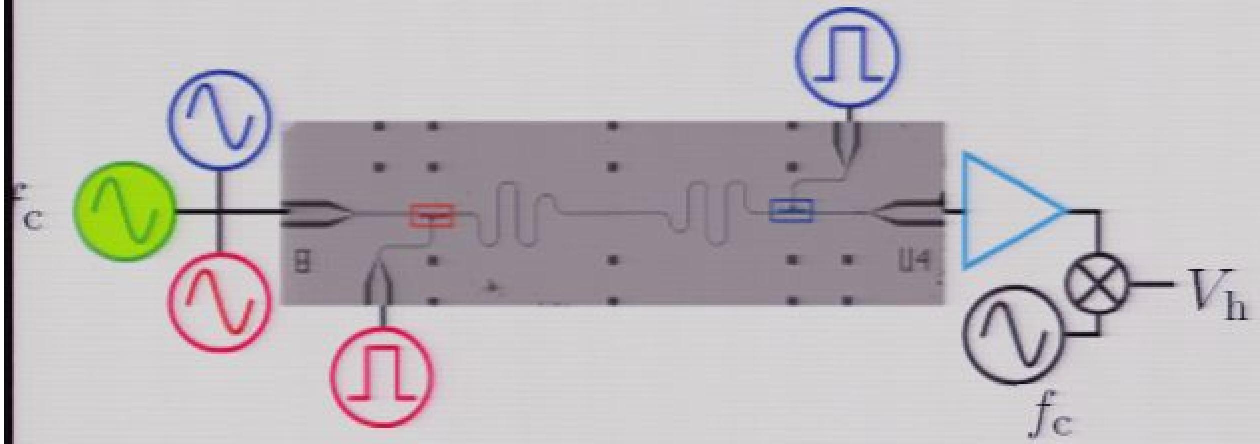


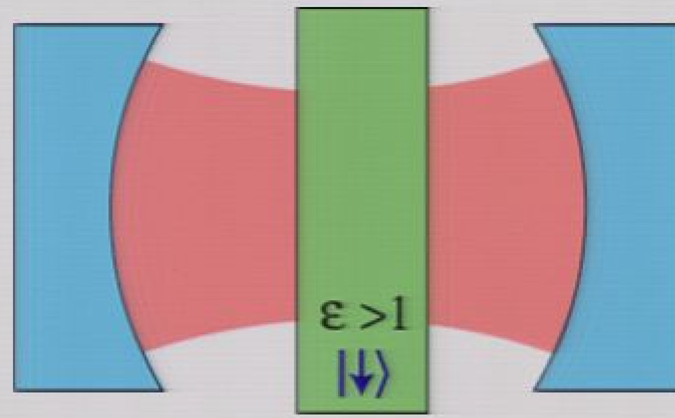


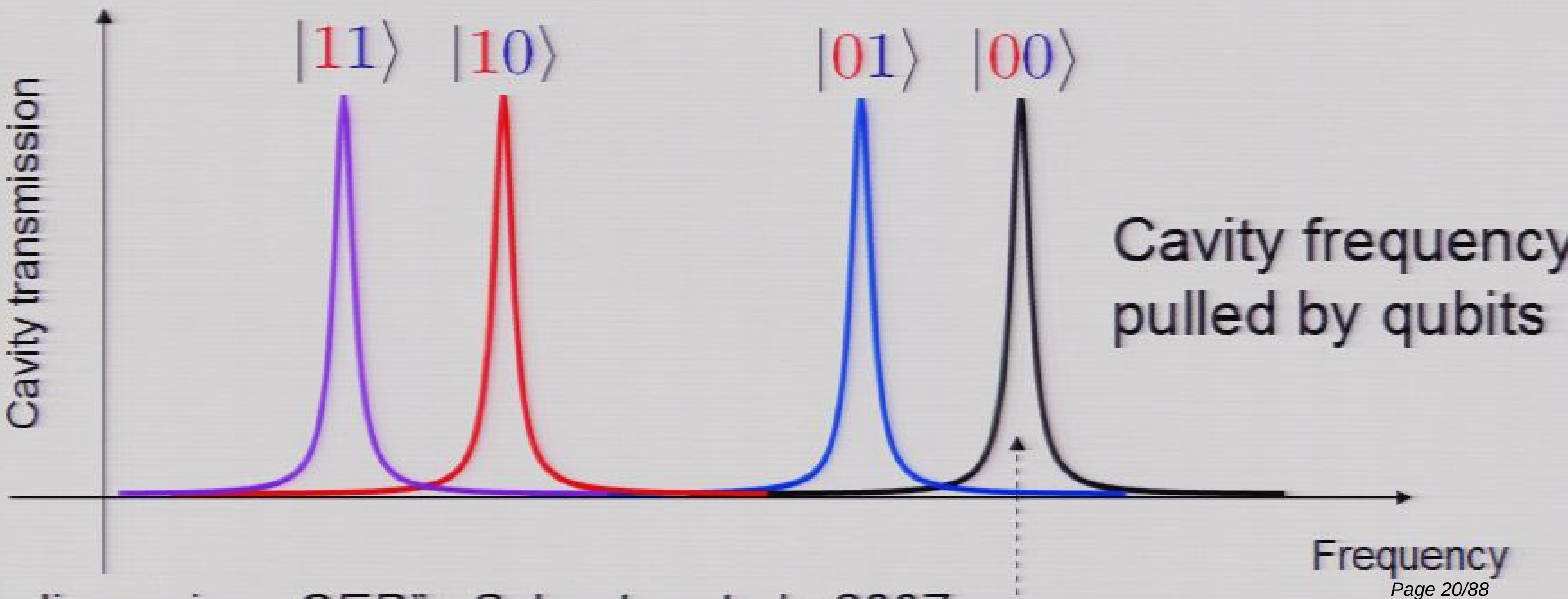
Cavity frequency pulled by qubits

"Strong dispersive cQED": Schuster et al., 2007

Dispersive readout

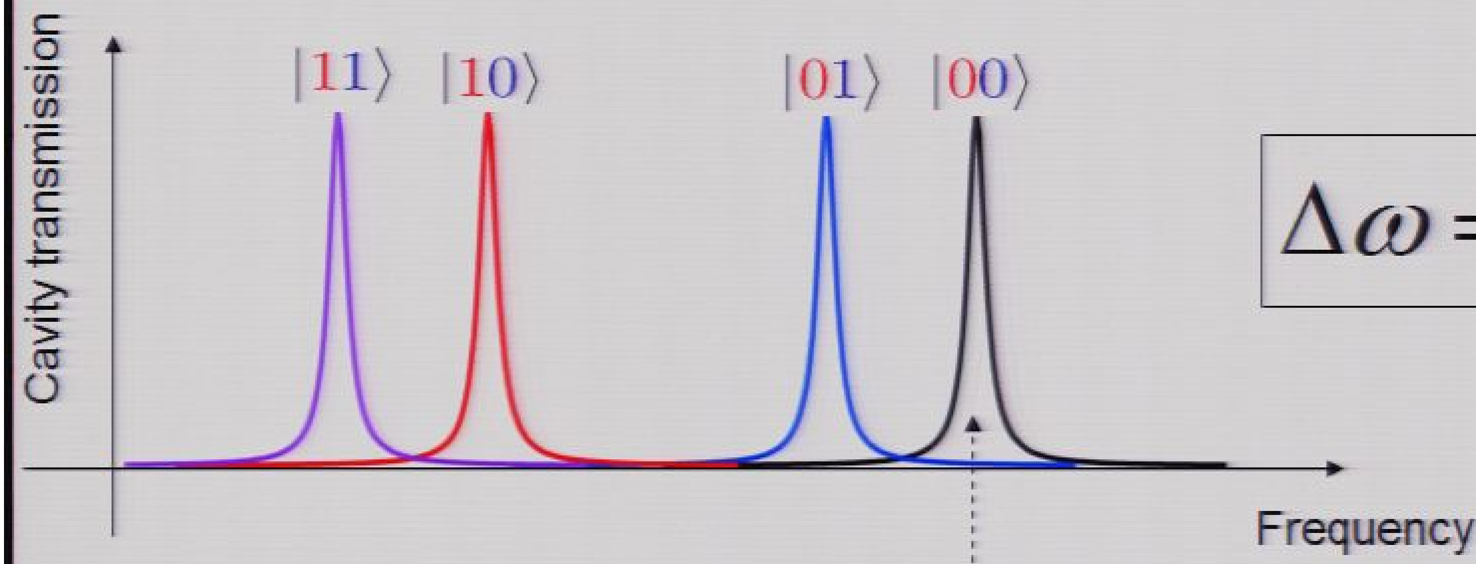






"Strong dispersive cQED": Schuster et al., 2007

Cavity Pull is linear in spin polarizations



$$\Delta\omega = a\sigma_{\mathrm{L}}^{z} + b\sigma_{\mathrm{R}}^{z}$$

Complex transmitted amplitude is non-linear in cavity pull:

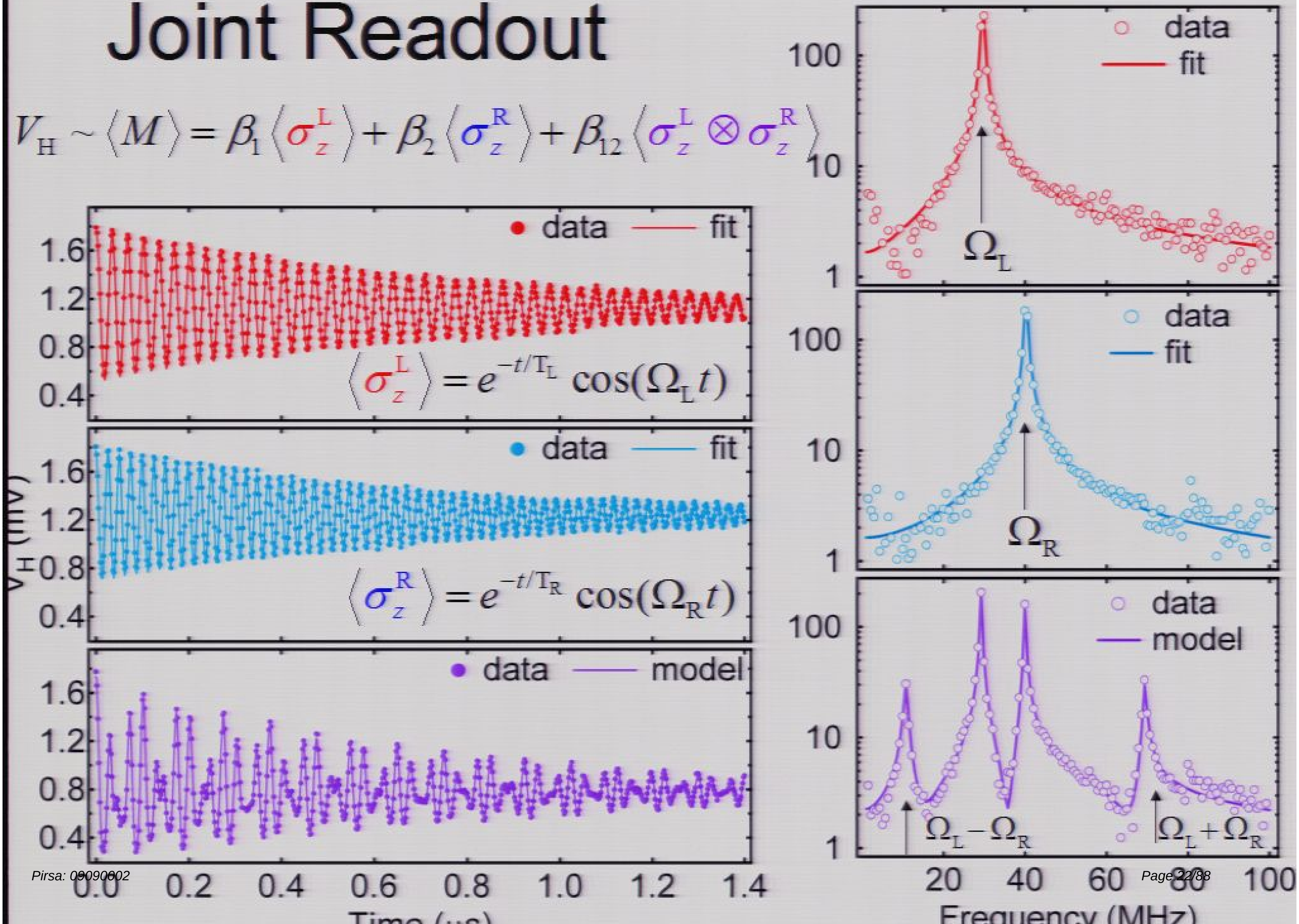
$$t = \frac{\kappa/2}{\omega_{\mathrm{drive}} - \omega_{\mathrm{cavity}} - \Delta\omega + i\kappa/2}$$

General non-linear function of two spins:

$$t = \beta_0 + \beta_1\,\sigma_{\mathrm{L}}^{z} + \beta_2\,\sigma_{\mathrm{R}}^{z} + \beta_{12}\,\sigma_{\mathrm{L}}^{z} \otimes \sigma_{\mathrm{R}}^{z}$$

Joint Readout

$$V_H \sim \langle M \rangle = \beta_1 \langle \sigma_z^L \rangle + \beta_2 \langle \sigma_z^R \rangle + \beta_{12} \langle \sigma_z^L \otimes \sigma_z^R \rangle$$



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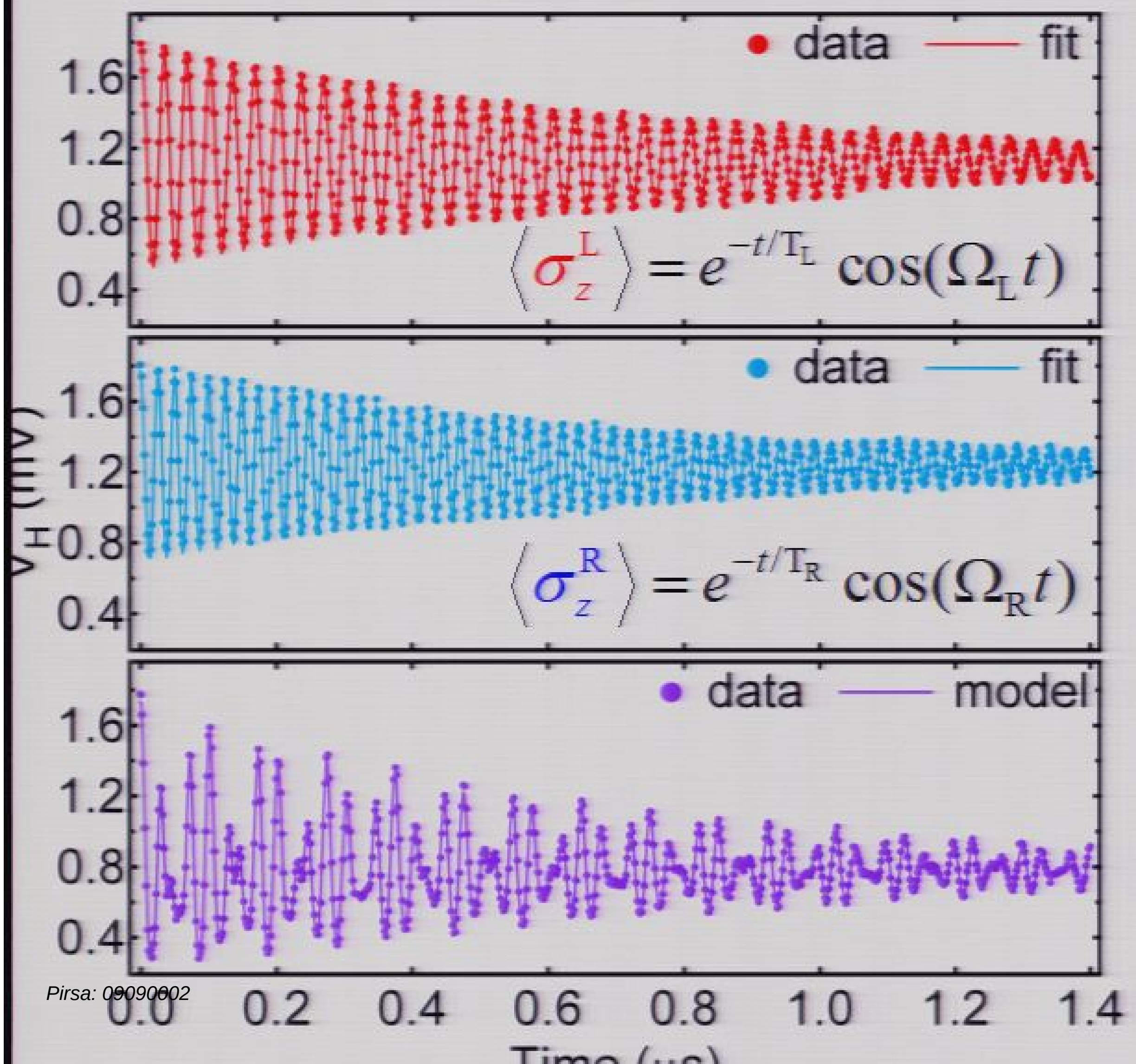
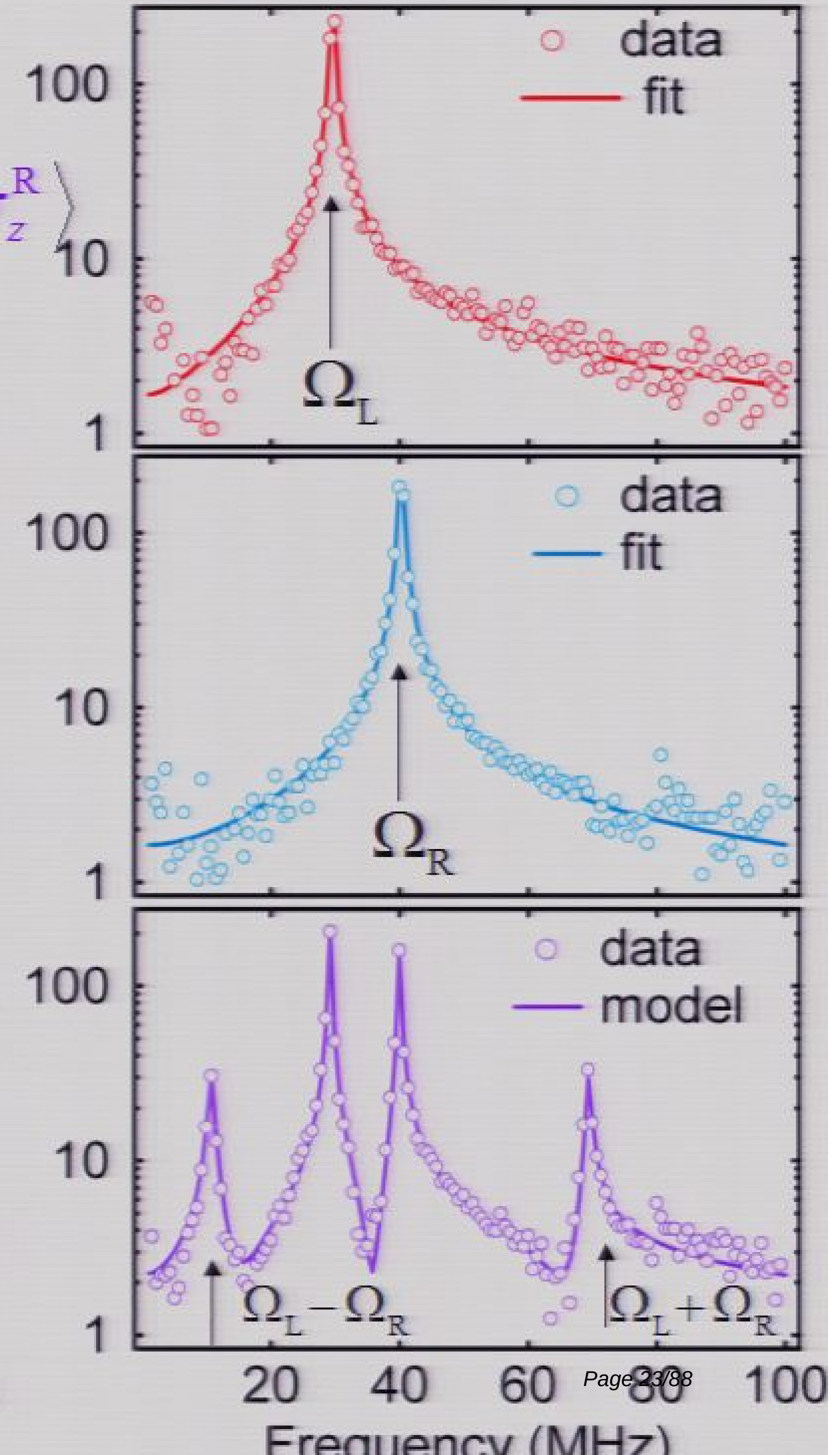
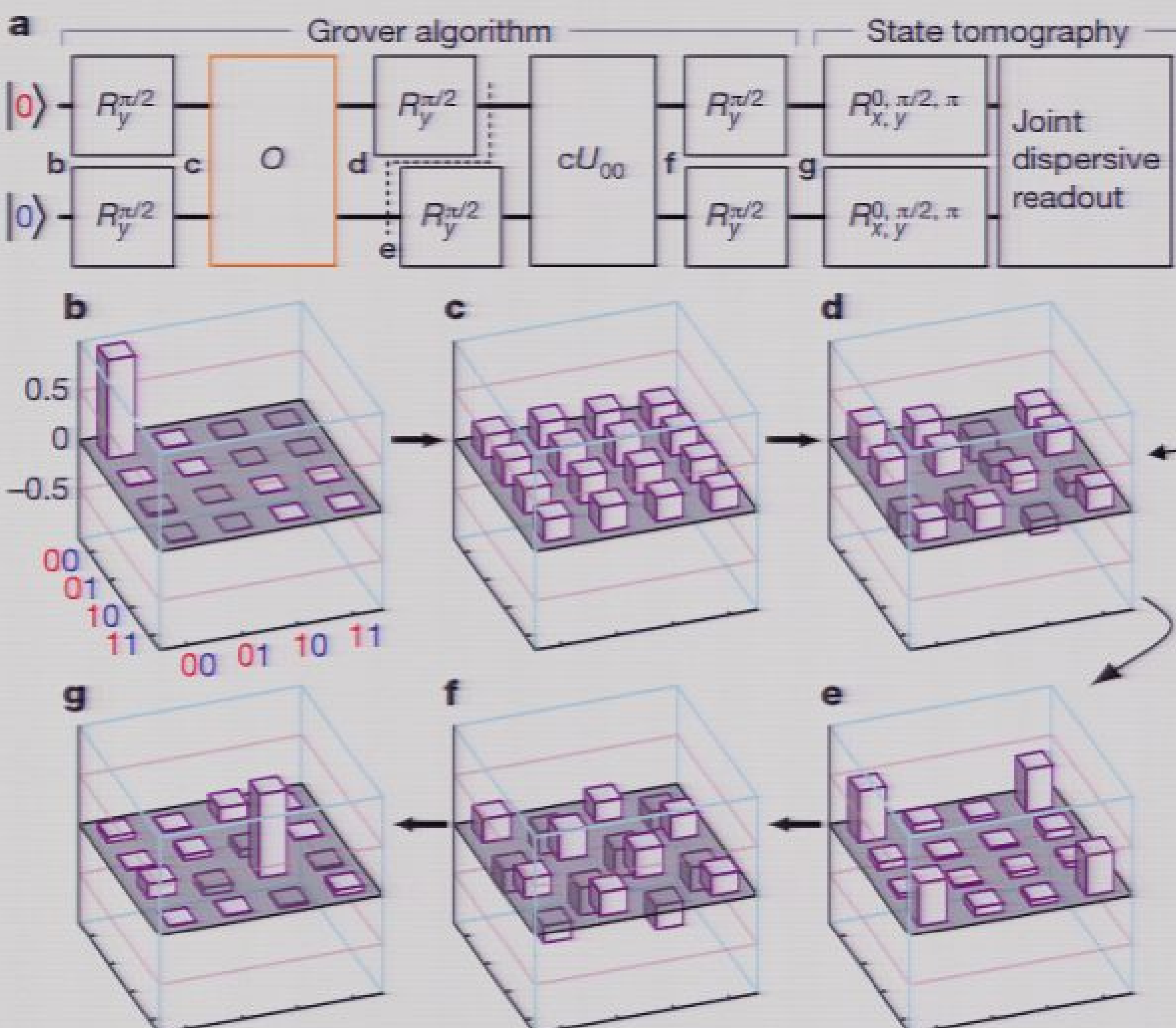




Running an algorithm

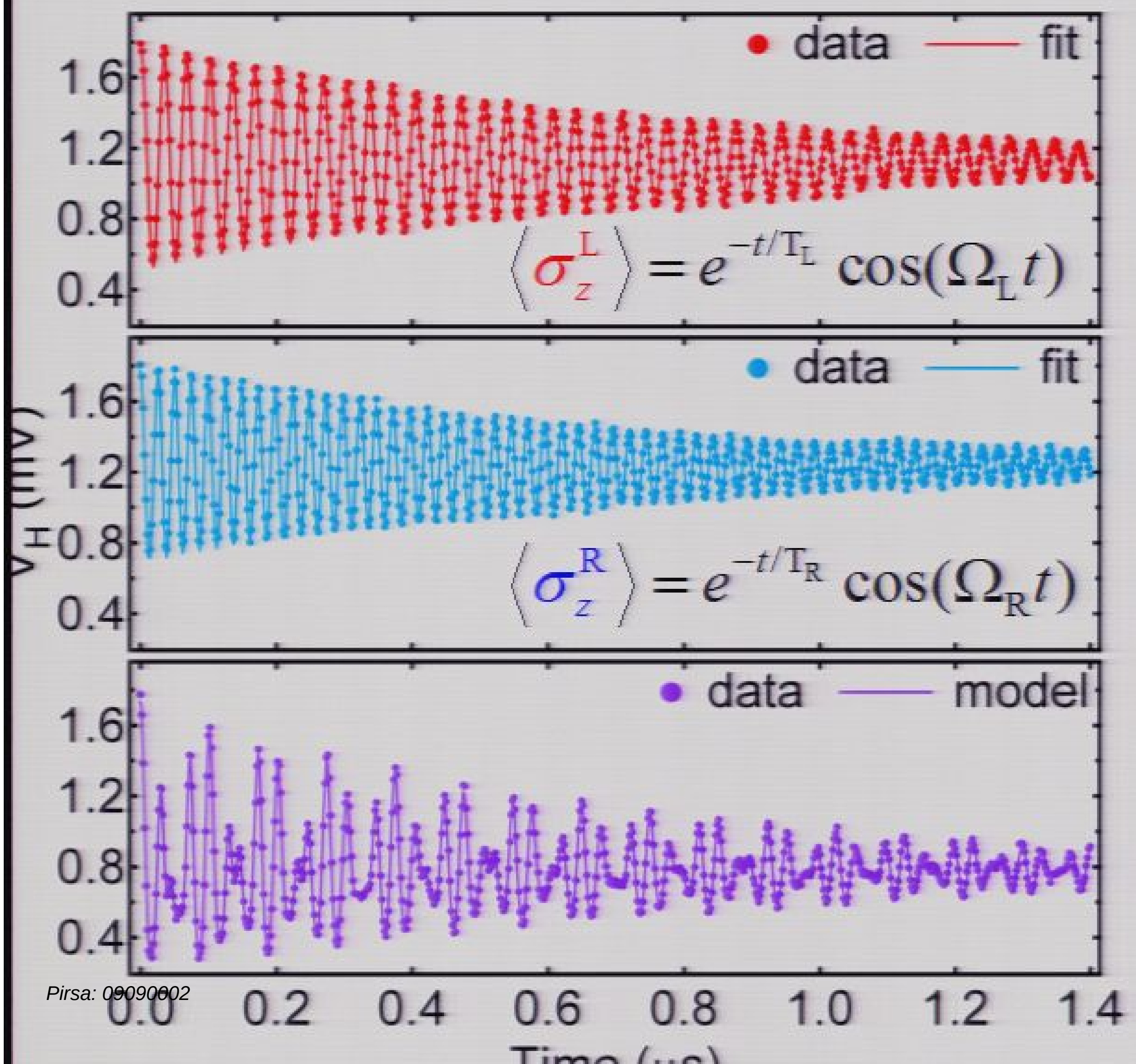
- DiCarlo *et al.*, Nature **460,** 240-244, (2009)

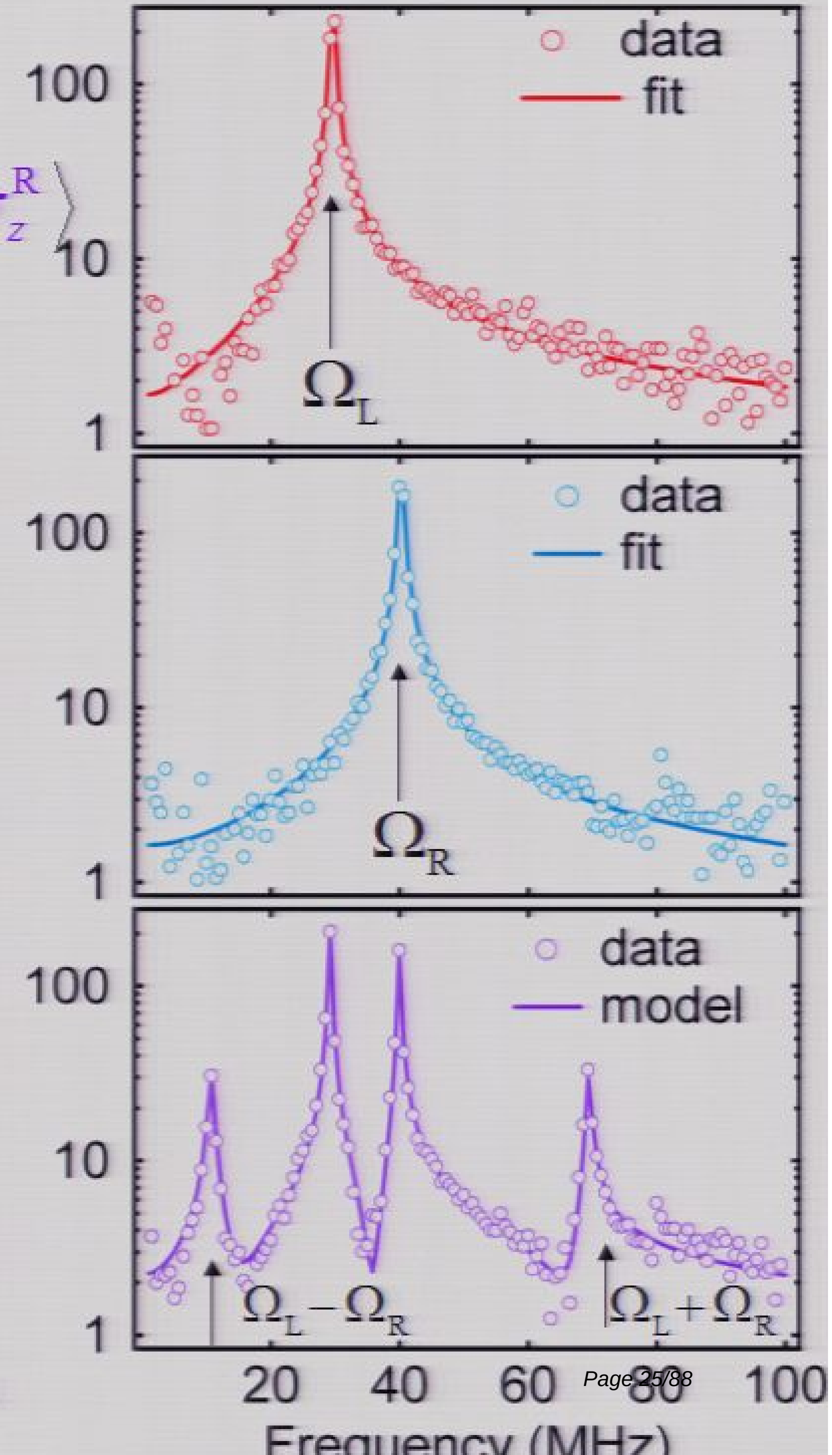


- Real part of 2-qubit density matrix

Joint Readout

$$V_{\mathrm{H}} \sim \langle M \rangle = \beta_1 \langle \sigma_z^{\mathrm{L}} \rangle + \beta_2 \langle \sigma_z^{\mathrm{R}} \rangle + \beta_{12} \langle \sigma_z^{\mathrm{L}} \otimes \sigma_z^{\mathrm{R}} \rangle$$





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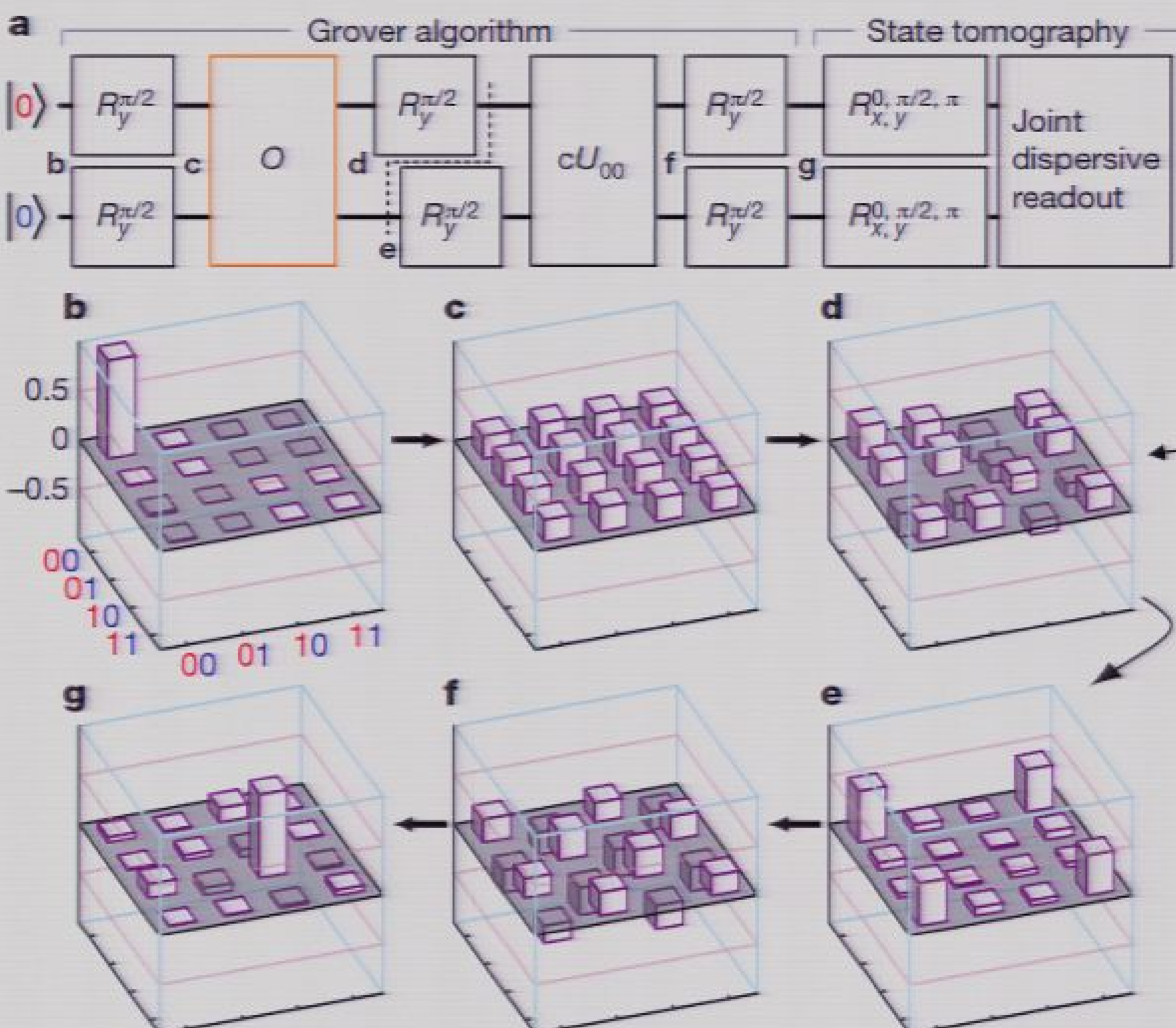
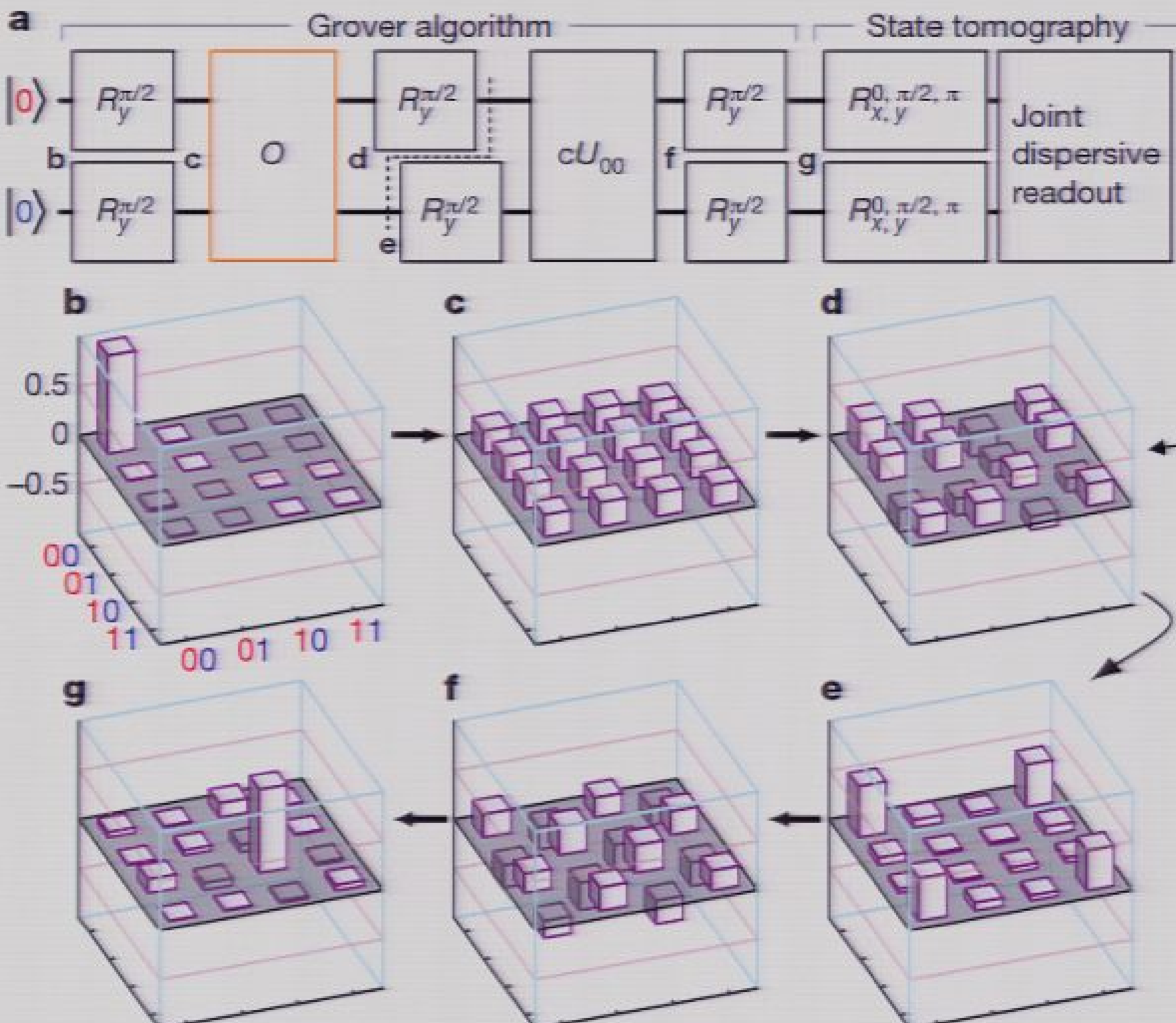


- Real part of 2-qubit density matrix

Running an algorithm

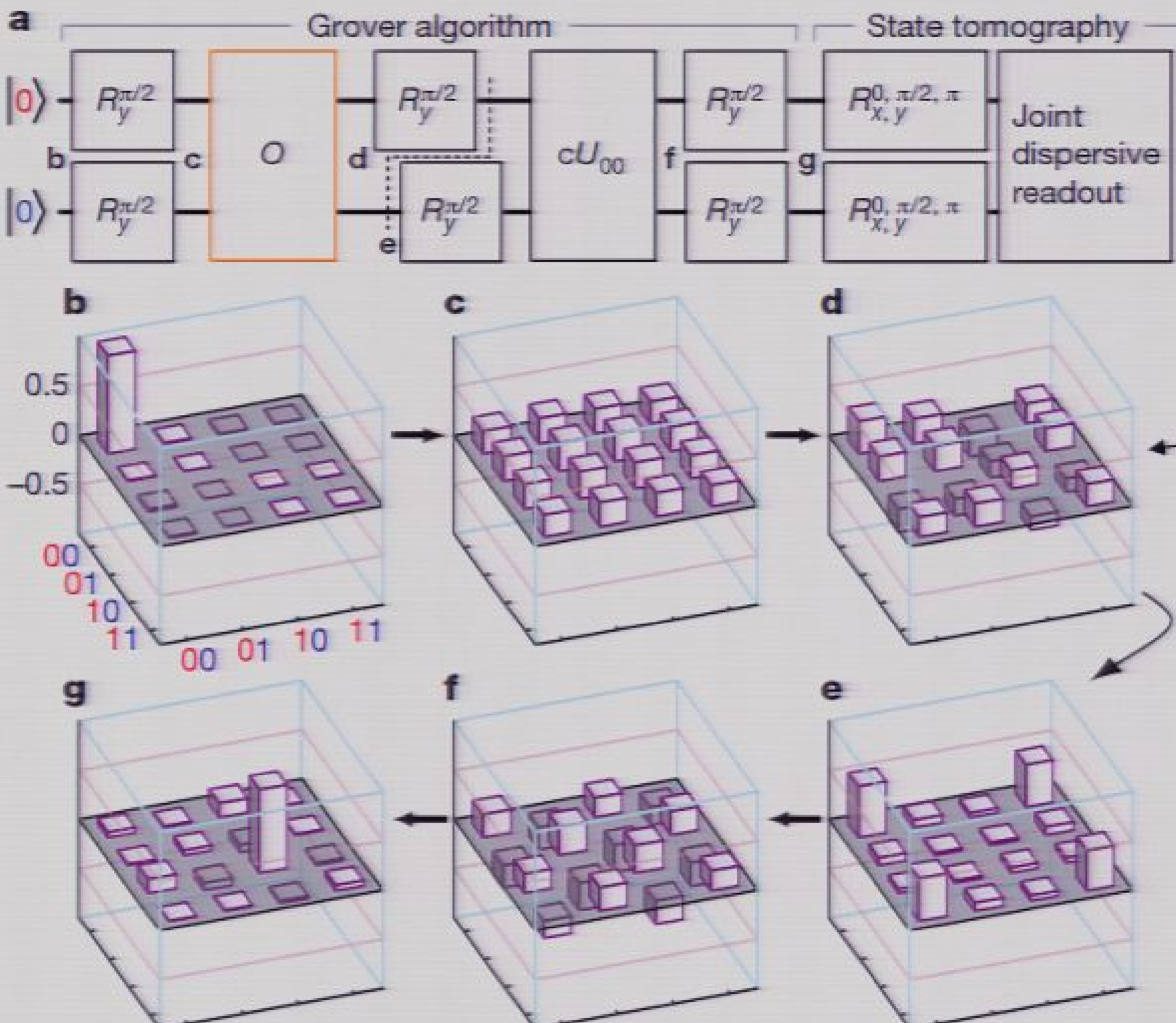
- DiCarlo *et al.*, Nature **460,** 240-244, (2009)



- Real part of 2-qubit density matrix
- Measured (not theory)

Running an algorithm

- DiCarlo *et al.*, Nature **460,** 240-244, (2009)



- Real part of 2-qubit density matrix
- Measured (not theory)
- 80% **algorithm** fidelity

Summary: Circuit QED building blocks

- Strong coupling to cavity bus gives…
- Protection from electromagnetic continuum
- Excellent state preparation
- Very good one-qubit gates
- Good two-qubit gates
- Useable (ensemble averaged) **joint** dispersive readout

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GHZ state

- GHZ state $|\text{GHZ}\rangle = (|\downarrow\downarrow \cdots \downarrow\rangle + |\uparrow\uparrow \cdots \uparrow\rangle)/\sqrt{2}$
- (mostly focus on 3-qubit case today)
- GHZ useful as resource state for quantum information
- Also useful for Bell-type tests
- Can prepare by various means, we focus on preparation by measurement

Preparation by measurement

- Initialize in the ground state $|\downarrow\downarrow\cdots\downarrow\rangle$
- Do one-qubit rotations $\rightarrow$ product state
  $|\Psi\rangle = (\alpha_1|\downarrow\rangle_1 + \beta_1|\uparrow\rangle_1) \otimes (\alpha_2|\downarrow\rangle_2 + \beta_2|\uparrow\rangle_2) \otimes \cdots$
- Perform a projective measurement that cannot distinguish certain states
- (Post-)select for result indicating interesting (eg. entangled) state $\Pi_i|\Psi\rangle$

Not a new idea:
Recently applied to circuit QED for making 2-qubit Bell states:
Hutchison *et al*, Can. J. Phys. **87**, 225 (2009).
We stole this technology for the 3 qubit case
As did others:
Helmer & Marquardt, Phys. Rev. A, **79**, 052328 (2009).

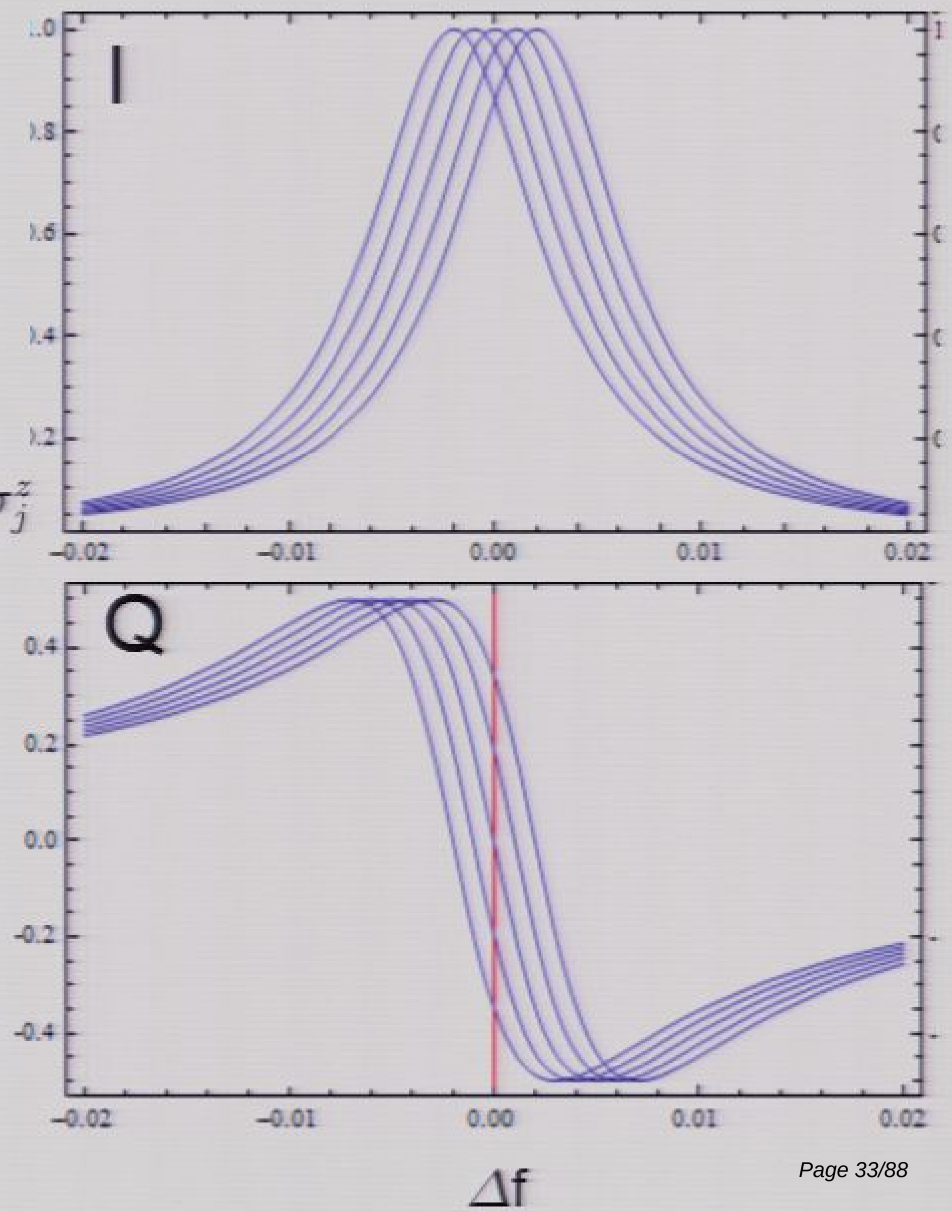
Dispersive read-out

Dispersive qubit-cavity coupling

$$\sum_j \chi_j a^\dagger a \sigma_j^z$$

leads to conditional resonator frequency

Transmission measurement yields $A = \sum_j \chi_j \sigma_j^z$



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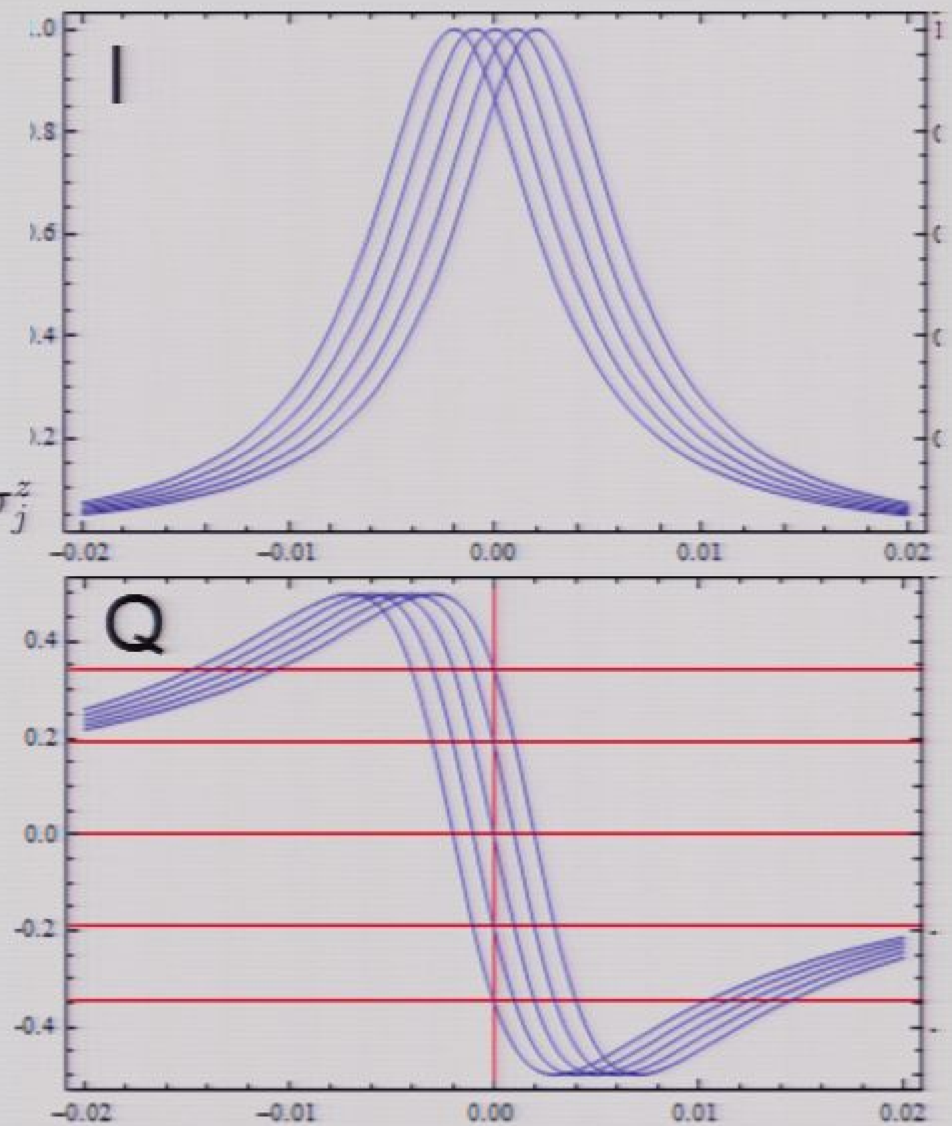

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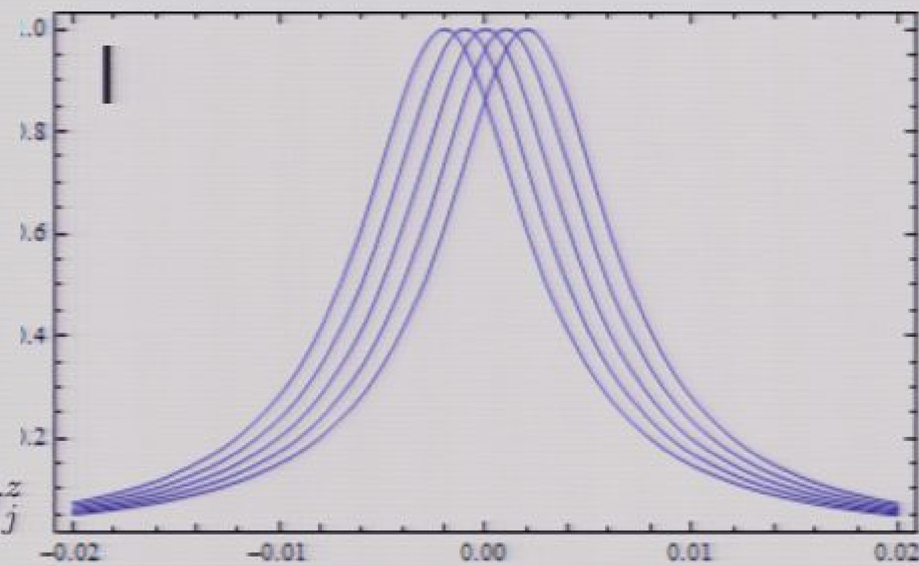

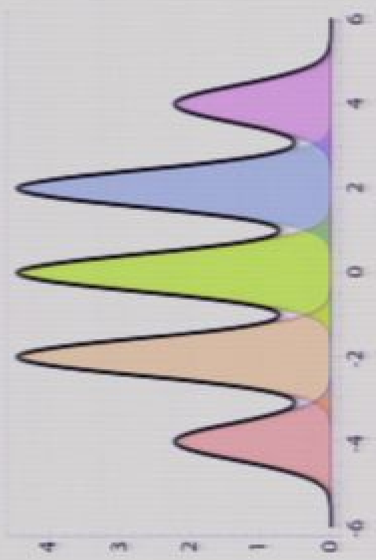

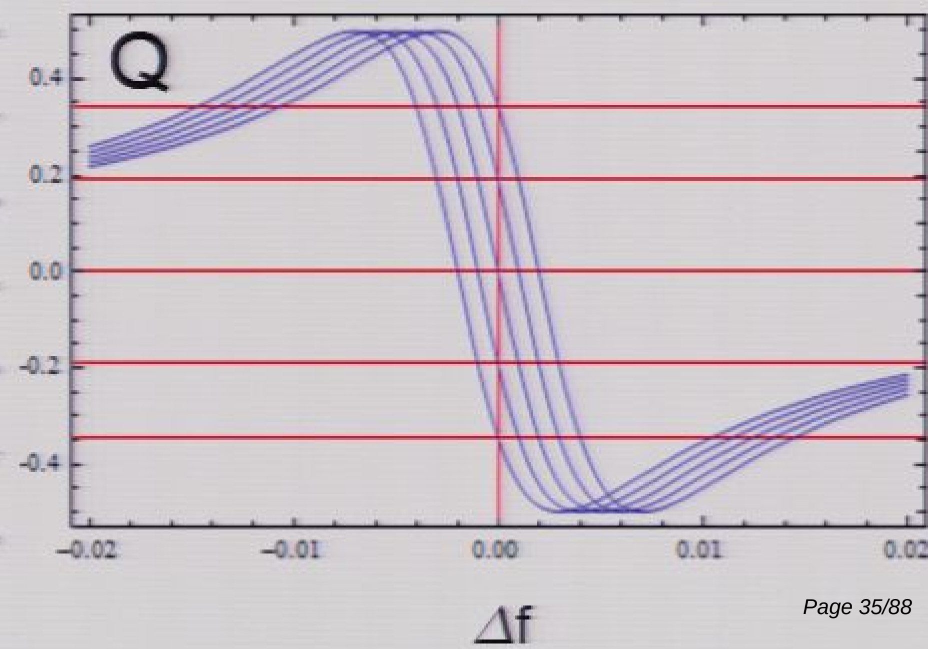

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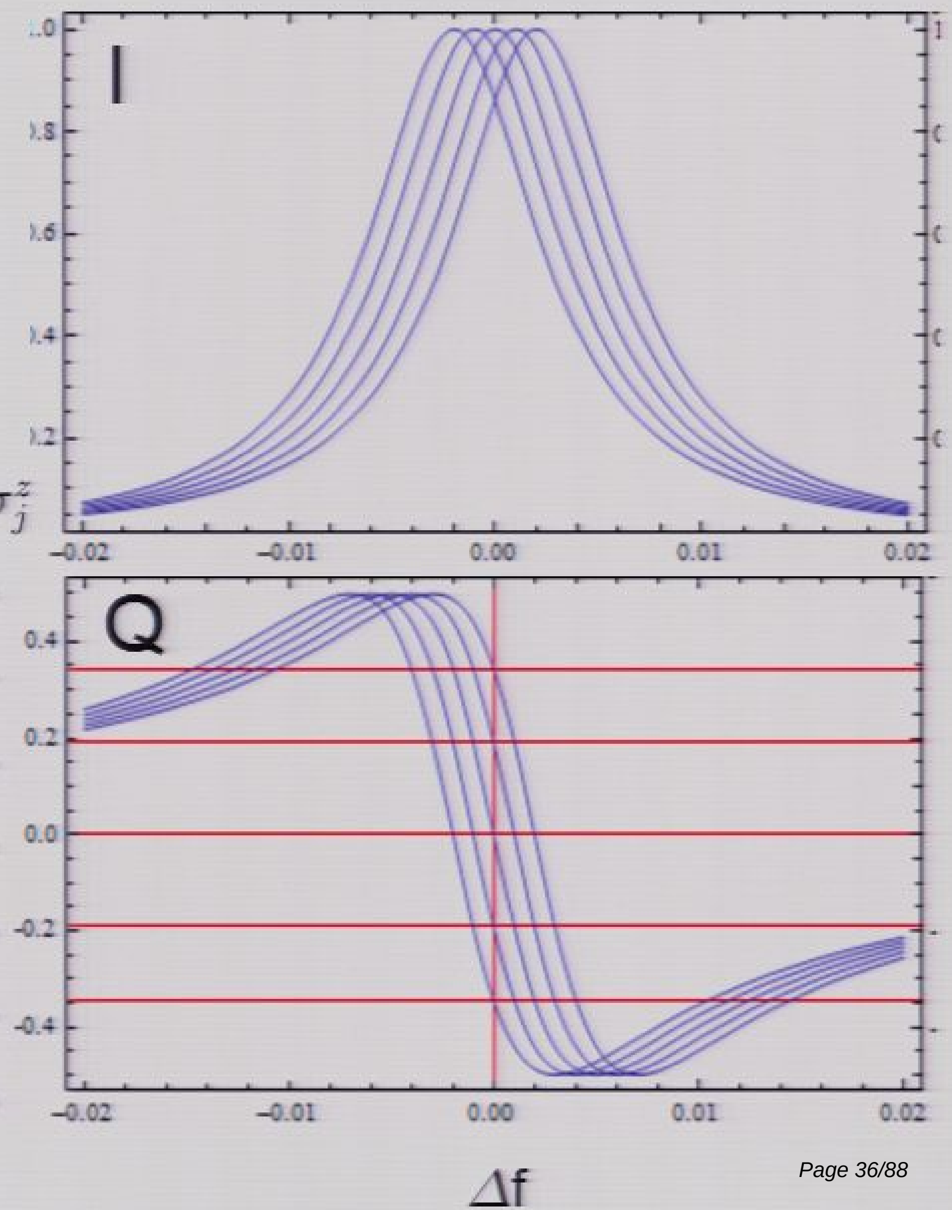
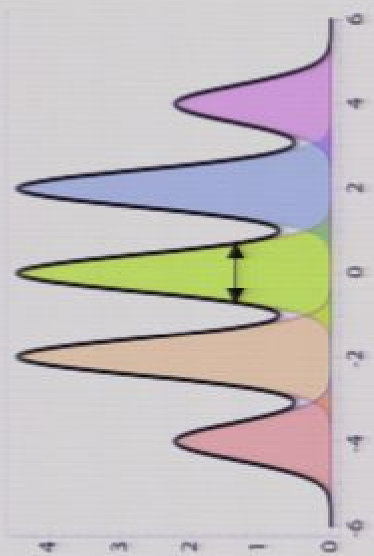
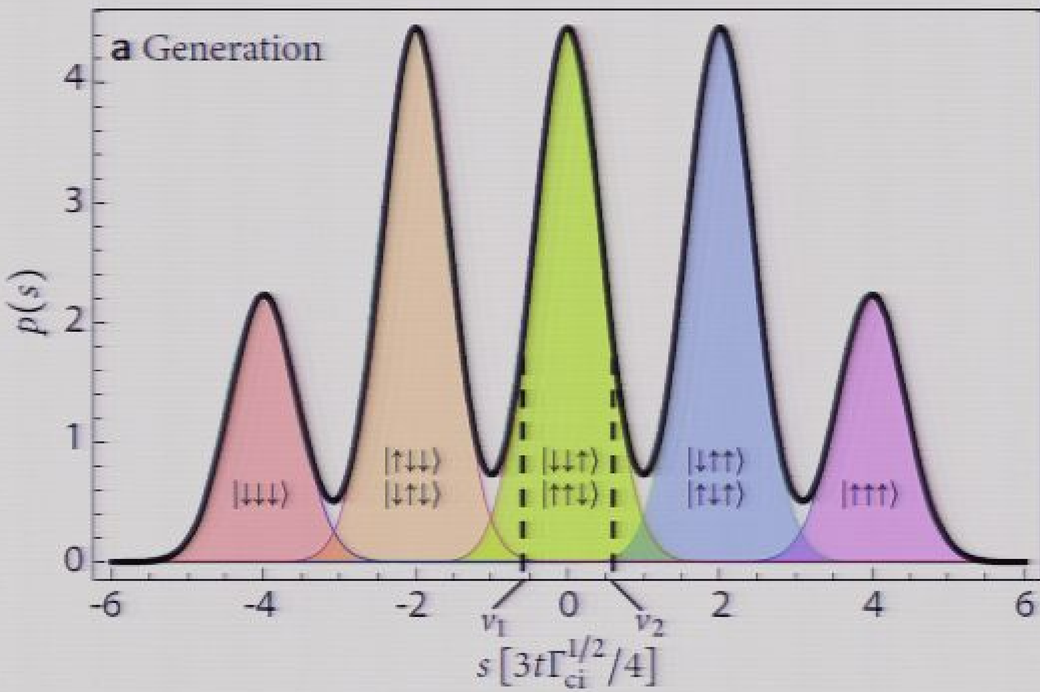


Width set by noise & integration time



Preparation by measurement



- Dispersive measurement with $\chi_1 : \chi_2 : \chi_3 = 1 : 1 : 2$
- Cannot distinguish $|\uparrow\uparrow\downarrow\rangle$ and $|\downarrow\downarrow\uparrow\rangle$
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GHZ Preparation protocol

- Initialize in the ground state $|\downarrow\downarrow\downarrow\rangle$
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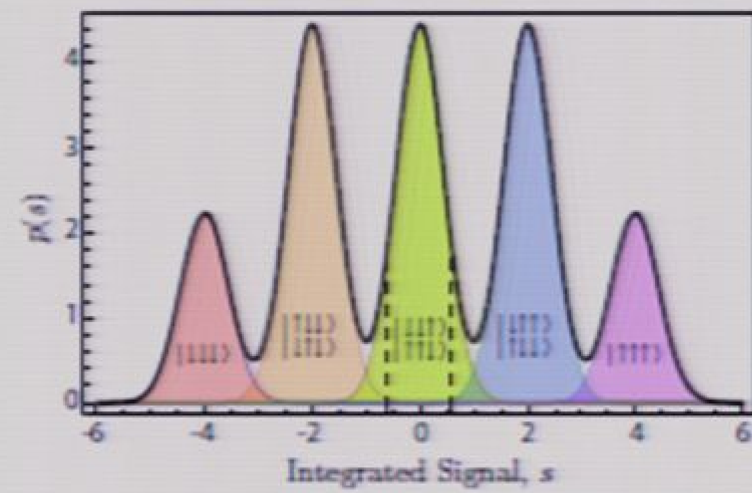

- (Post-)select for result '0' (probability 1/4)

$$|\text{pGHZ}\rangle = (|\uparrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle)/\sqrt{2}$$

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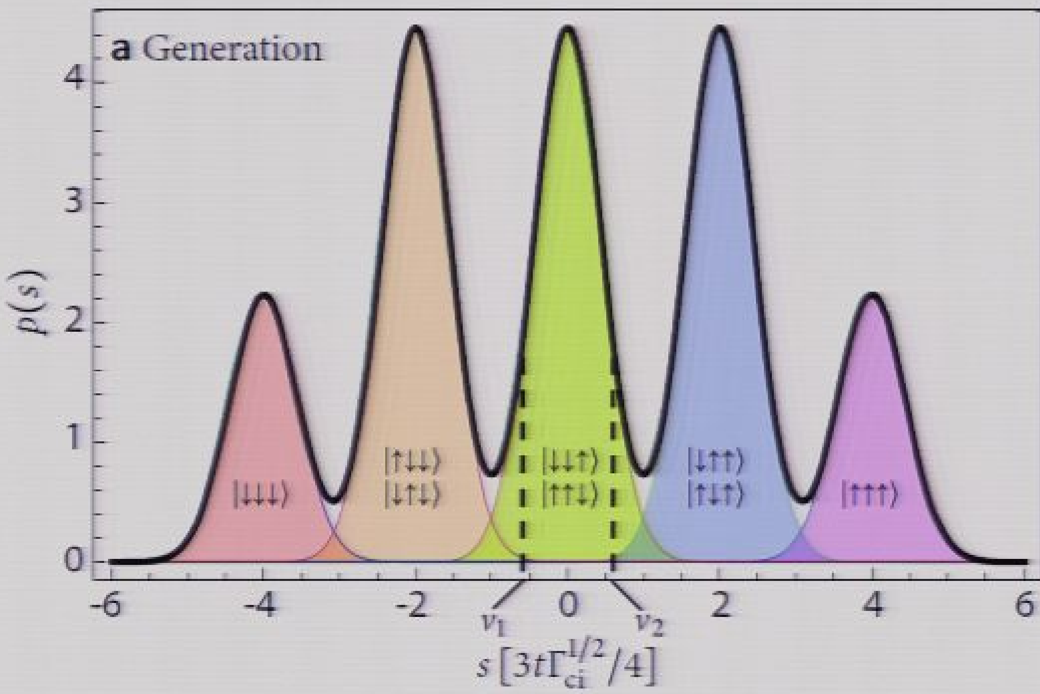

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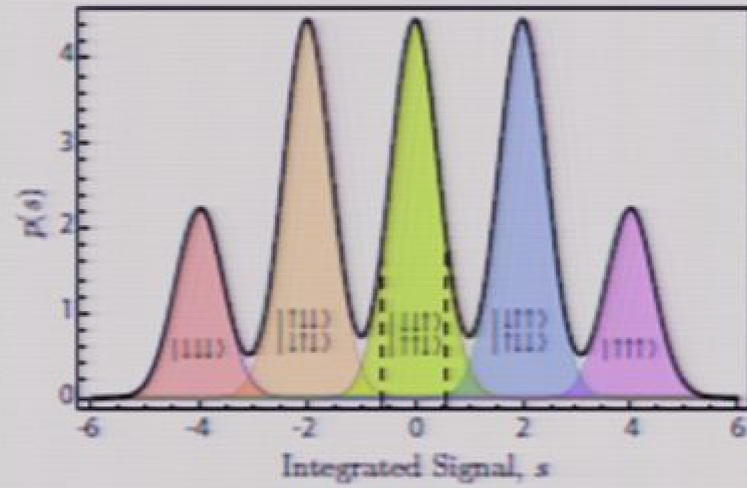

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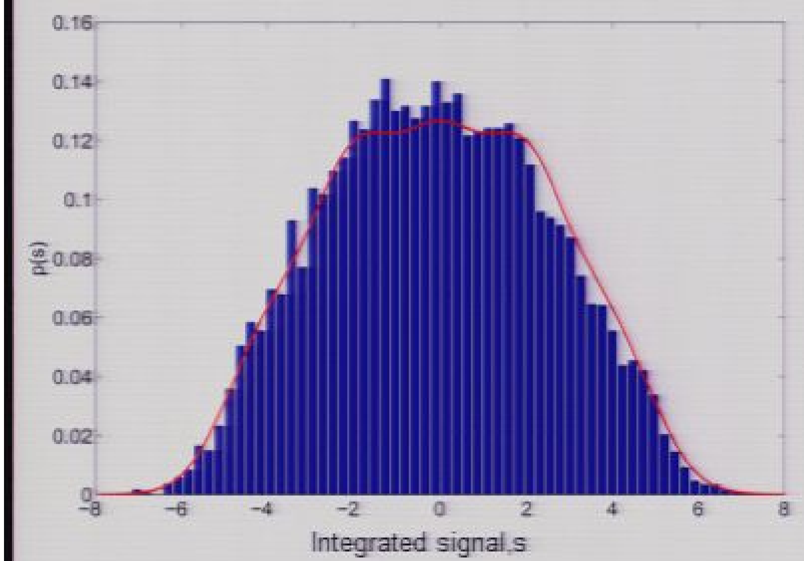

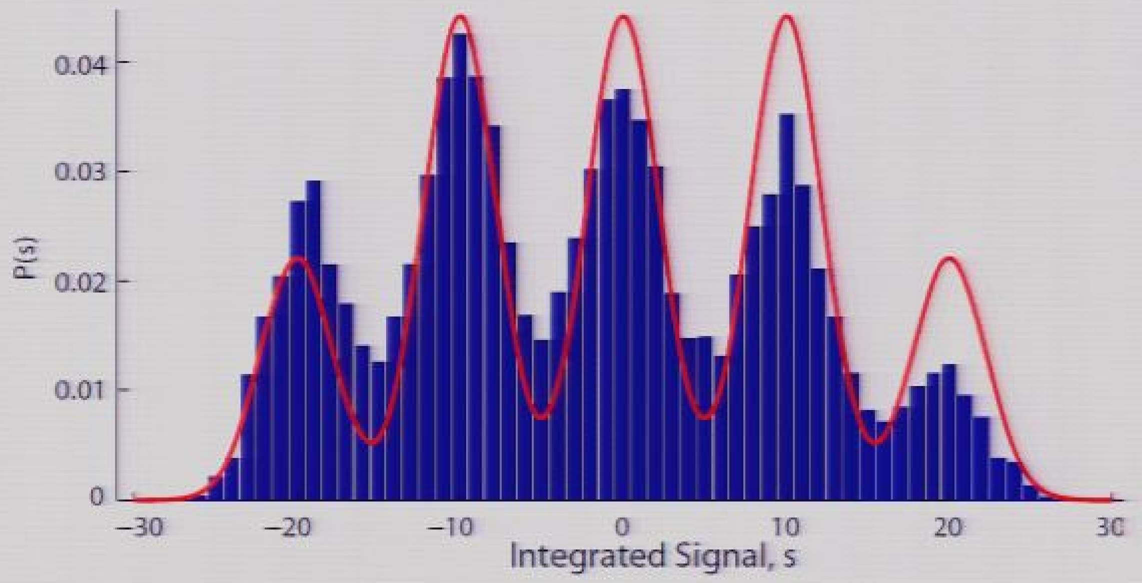

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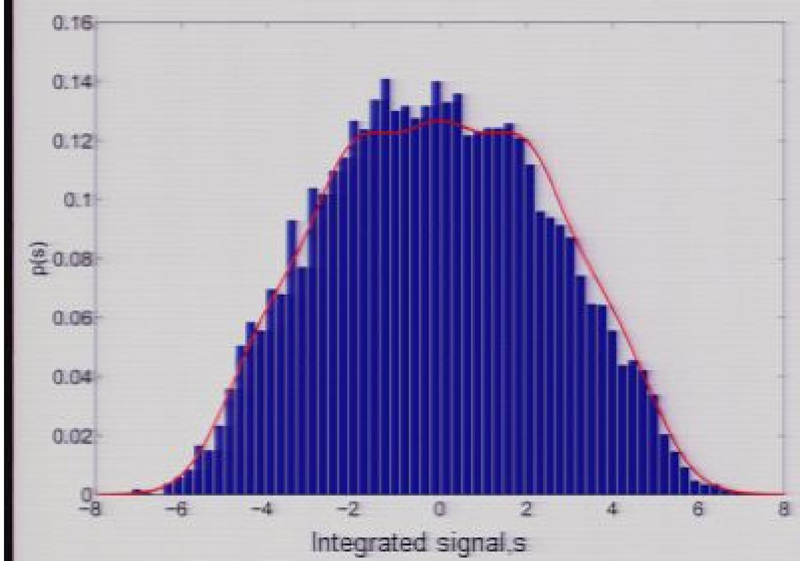

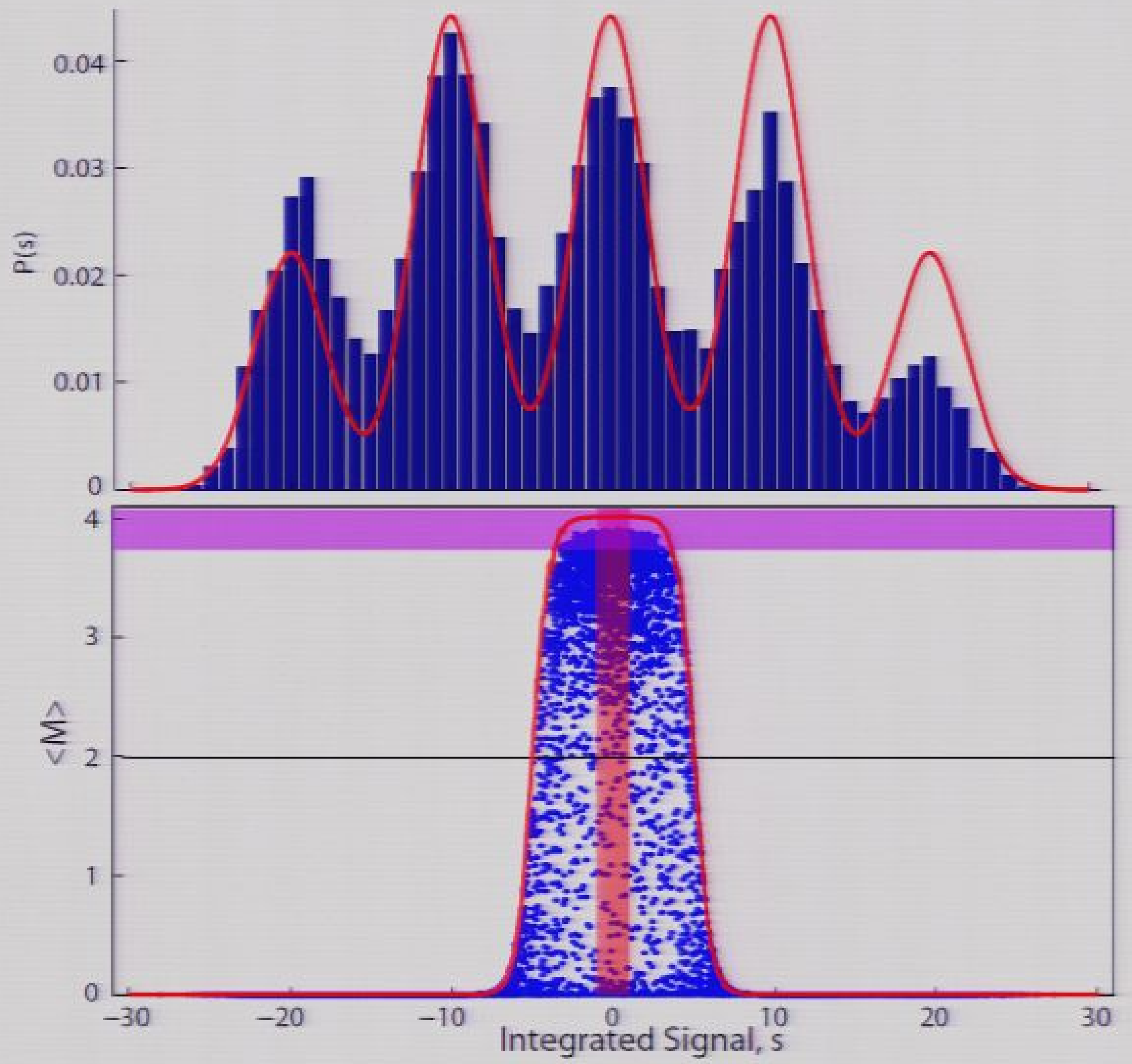

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$$\langle \mathrm{GHZ}|M|\mathrm{GHZ}\rangle = 4, \text{ classically: } \langle M\rangle < 2$$

Measurement model

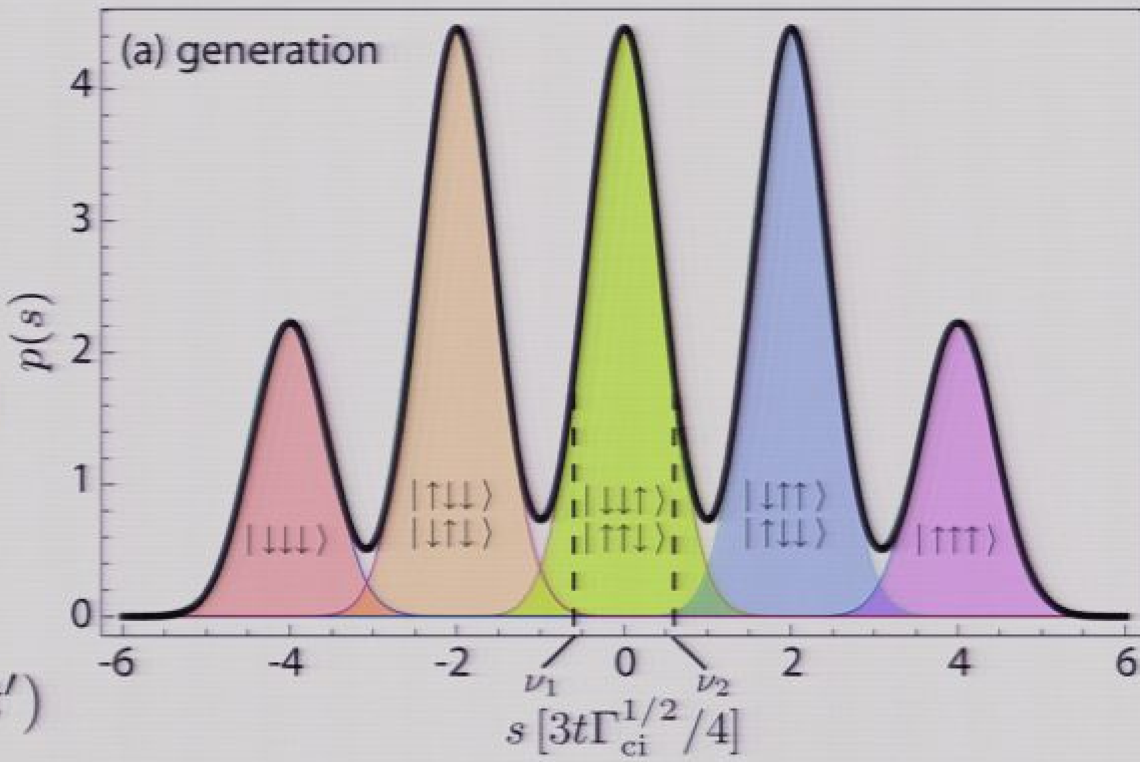
$$A = \chi\sigma_z^1 + \chi\sigma_z^3 + 2\chi\sigma_z^3$$

Homodyne signal

$$J(t) = \sqrt{\Gamma_{\rm ci}} \sum_j \langle \delta_j \sigma_j^z \rangle + \xi(t)$$

with $\langle \xi(t)\xi(t') \rangle = \delta(t - t')$

Integrated current $s = \int_0^t dt' \, J(t')$



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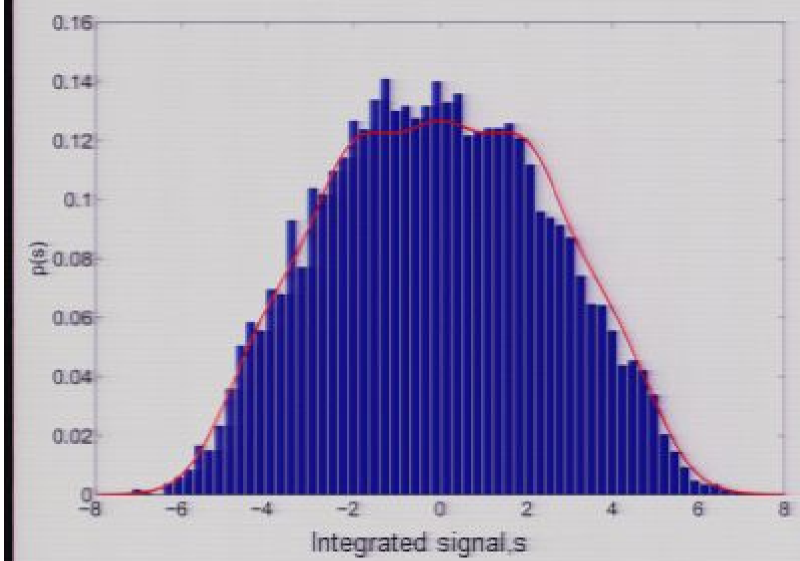

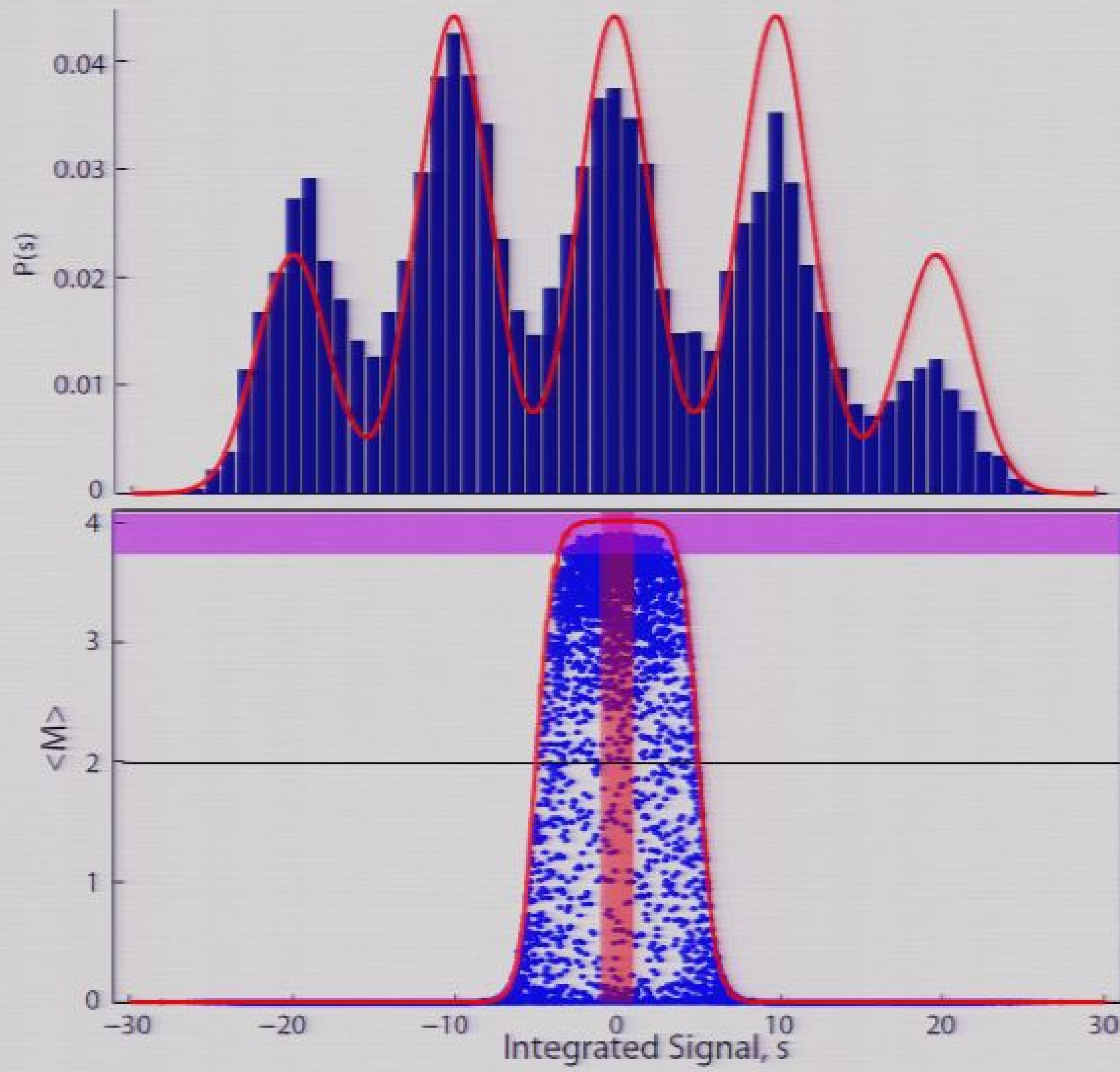

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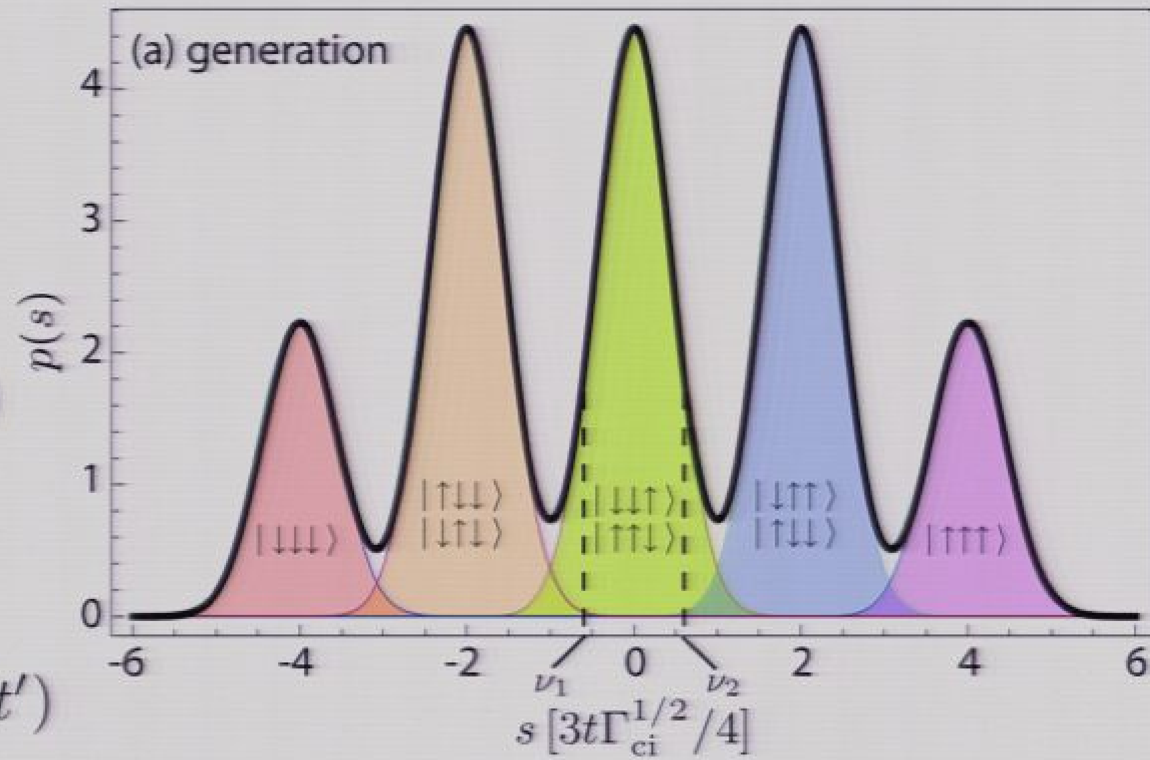

Dissipation model

Stochastic master equation for slaved cavity

$$\dot{\rho}_J = \sum_j (\gamma_{1j} + \gamma_{pj})\mathcal{D}[\sigma_j^-]\rho + \frac{\Gamma_\mathrm{d}}{2}\mathcal{D}[\sum_j \delta_j \sigma_j^z]\rho + \sqrt{\Gamma_\mathrm{ci}}\xi(t)\mathcal{M}[\sum_j \delta_j \sigma_j^z]\rho_J$$

qubit relaxation — measurement-induced dephasing — measurement innovation

with

$$\mathcal{M}[c]\rho_J = (c - \langle c \rangle)\rho_J/2 + \rho_J(c - \langle c \rangle)/2$$

and

$$\mathcal{D}[A]\rho = A\rho A^\dagger - \{A^\dagger A, \rho\}/2$$

with

$$\delta_j = \chi_j/\bar{\chi}$$
$$\bar{\chi} = \textstyle\sum_j \chi_j/N$$

Measurement rate $\Gamma_\mathrm{ci} = \eta\Gamma_\mathrm{m} = 64\eta\bar{\chi}^2|\epsilon|^2\kappa^{-3}$ and $\Gamma_d = \Gamma_\mathrm{ci}/2$

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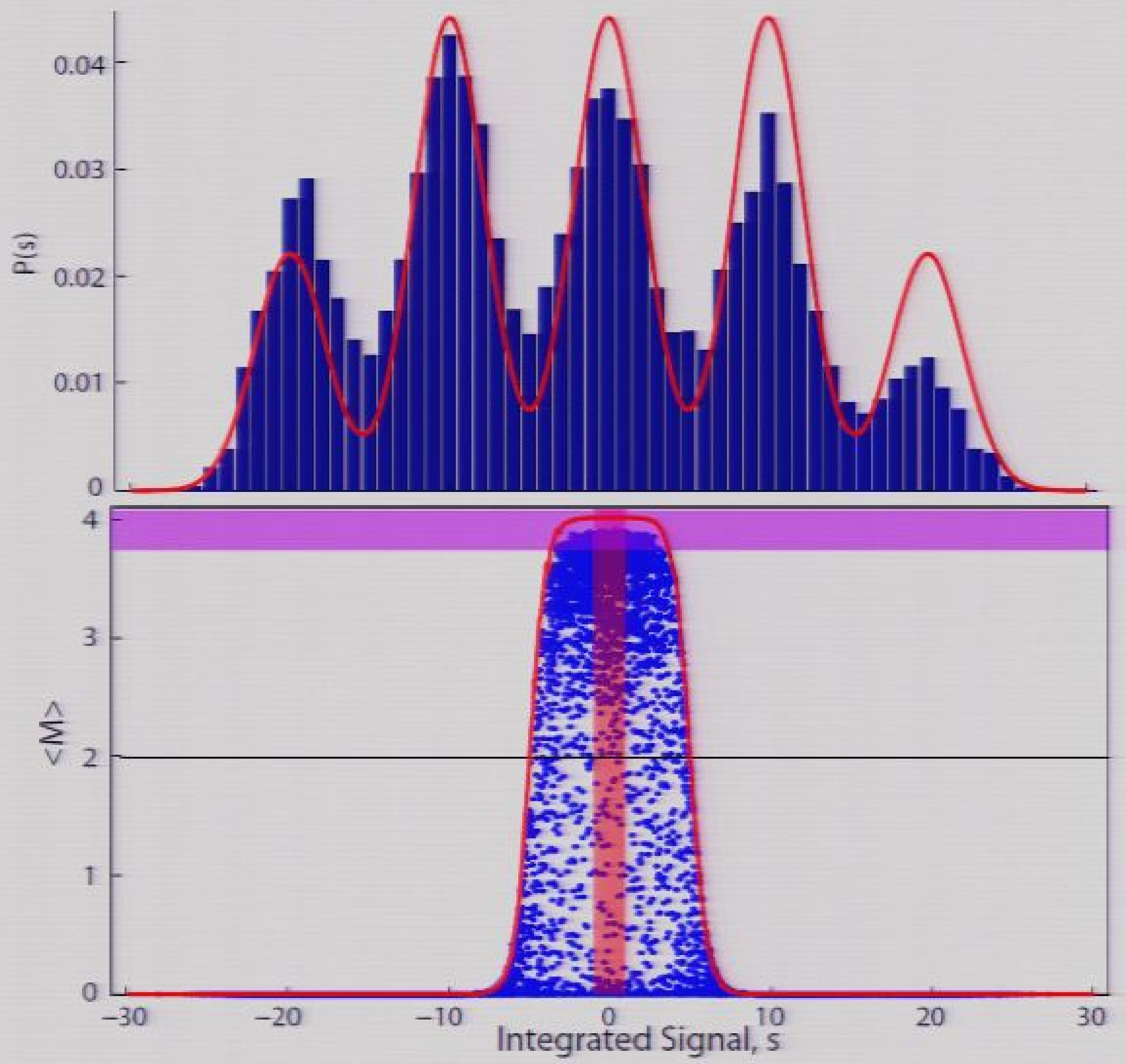
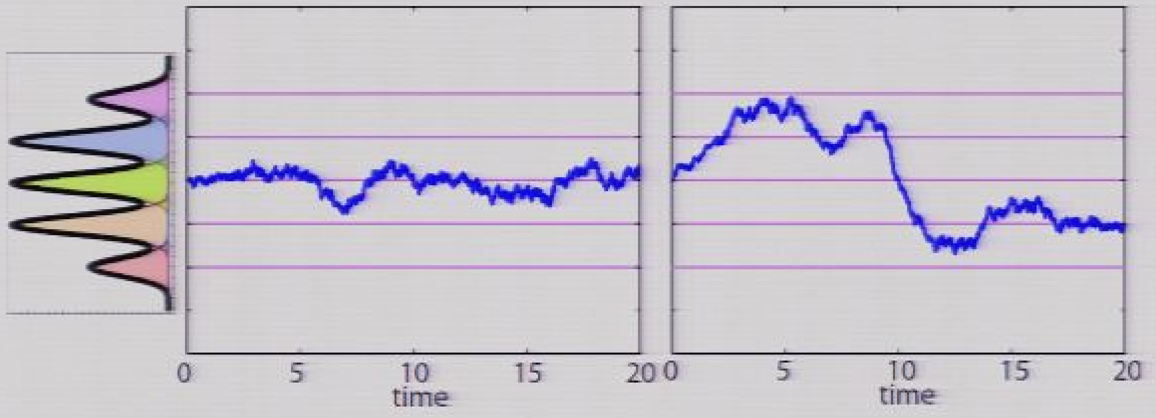
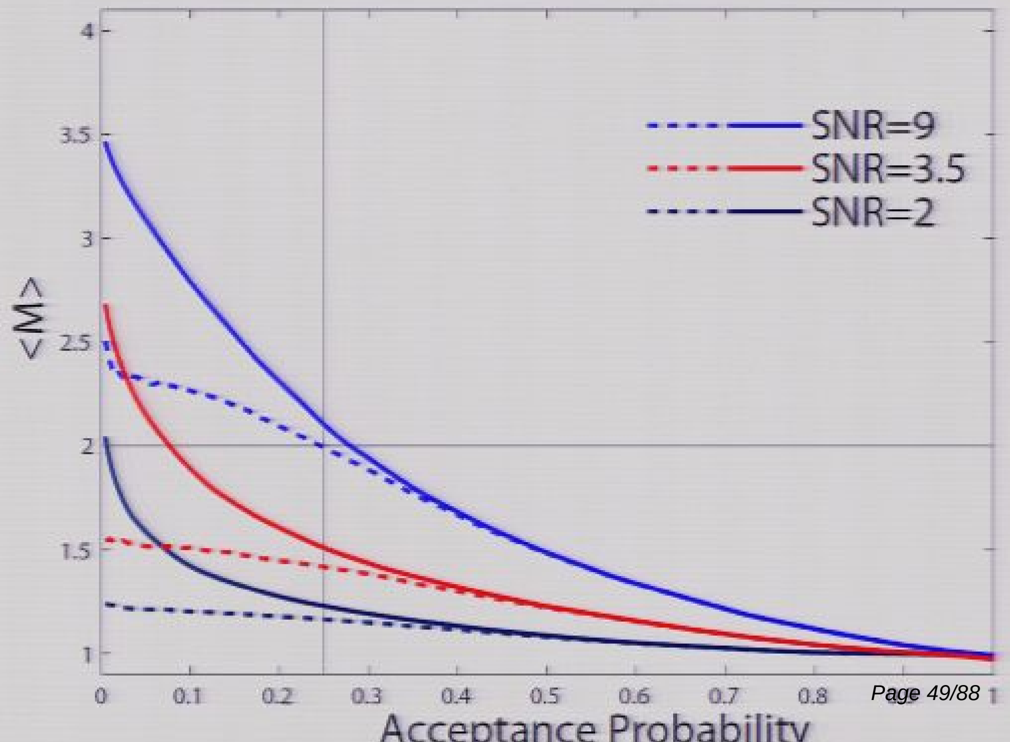
Simulations



Nonlinear filter

- Two traces each with mean ~ 0
- Much more information in measurement trace than just its mean
- Nonlinear filter uses whole measurement trace, along with system model, to estimate the state
- 3-qubit GHZ state is feasible this way





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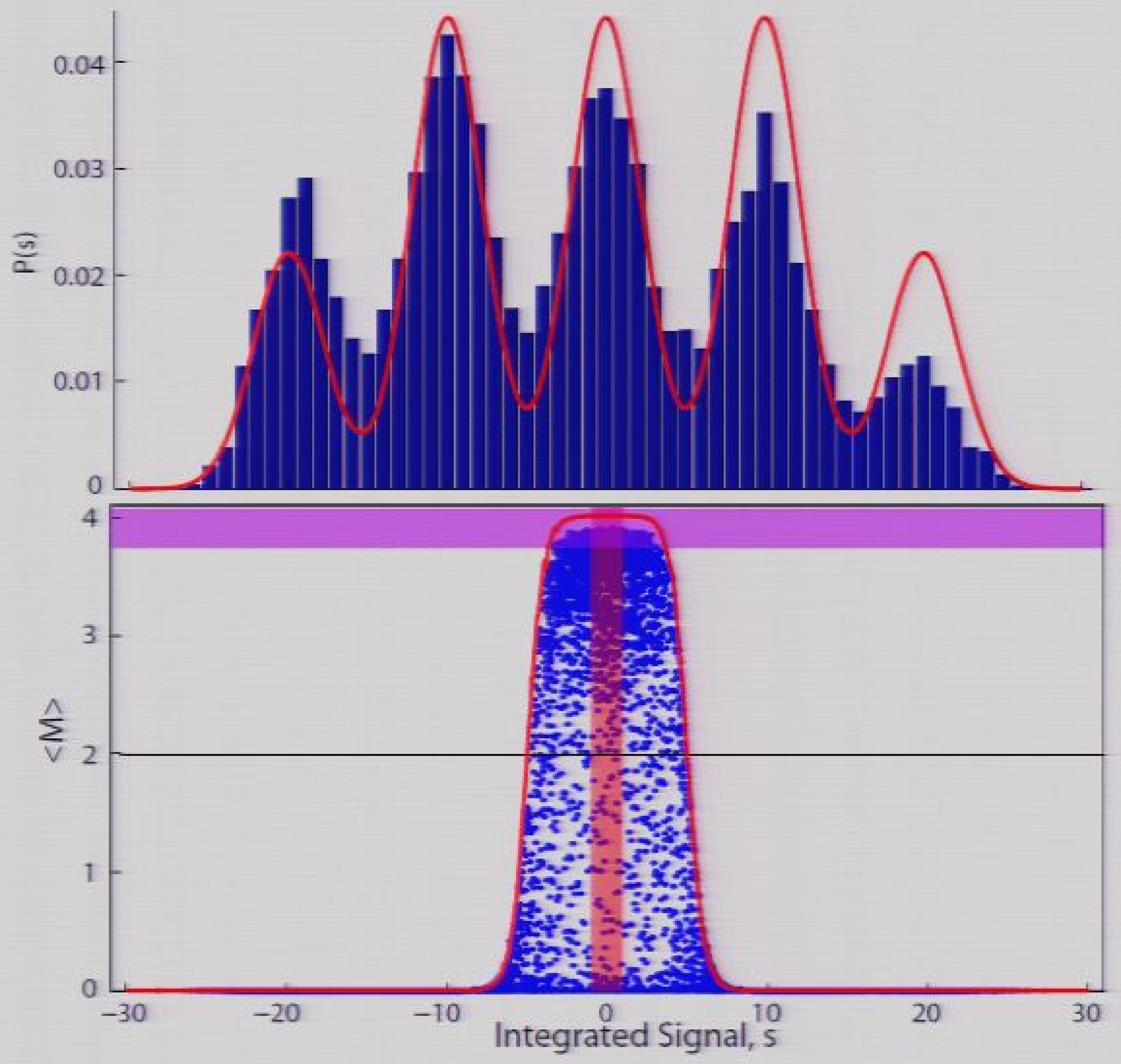

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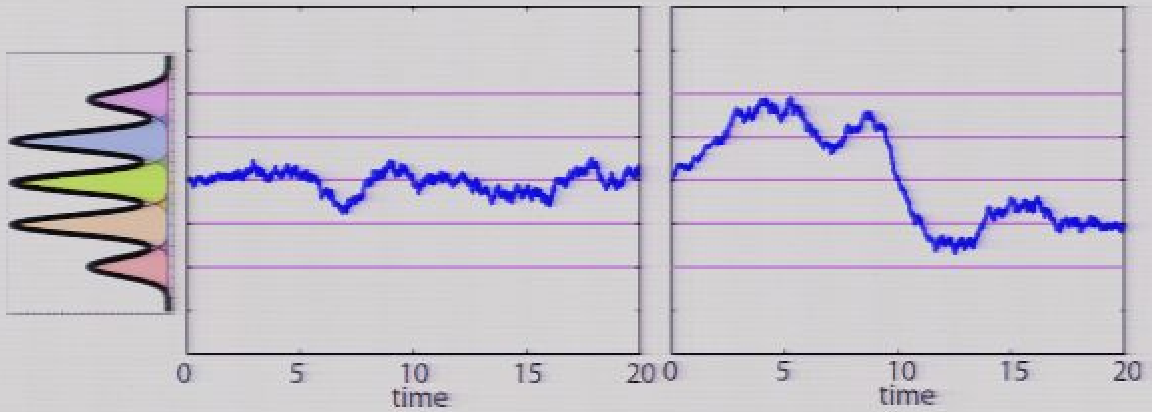

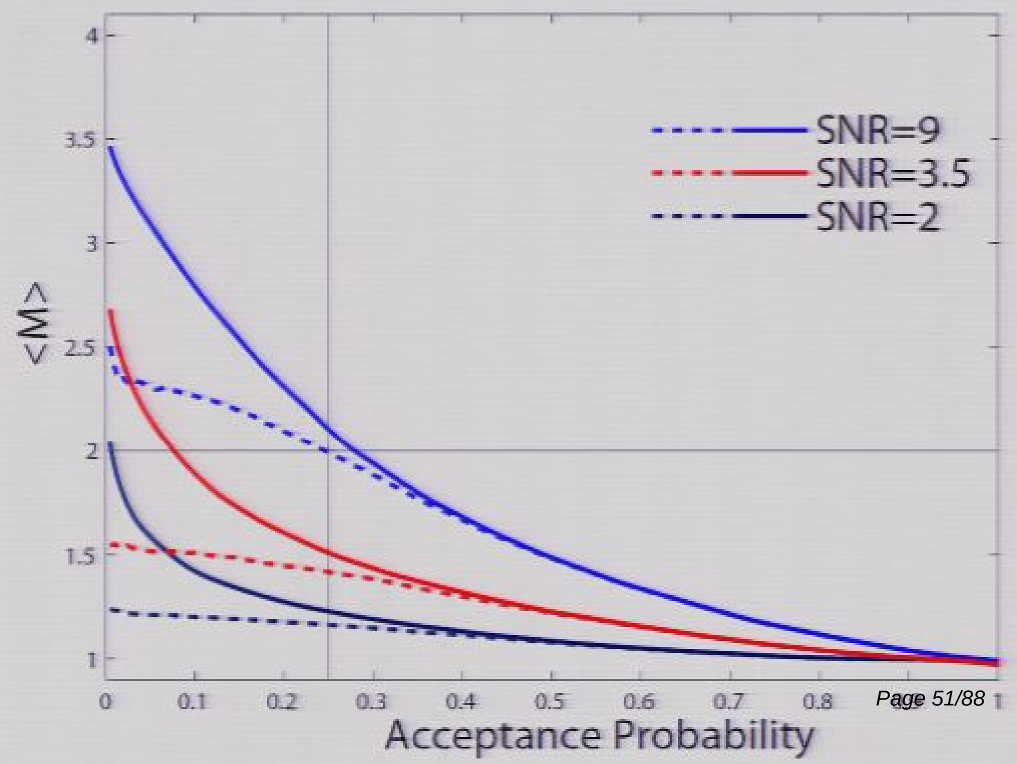

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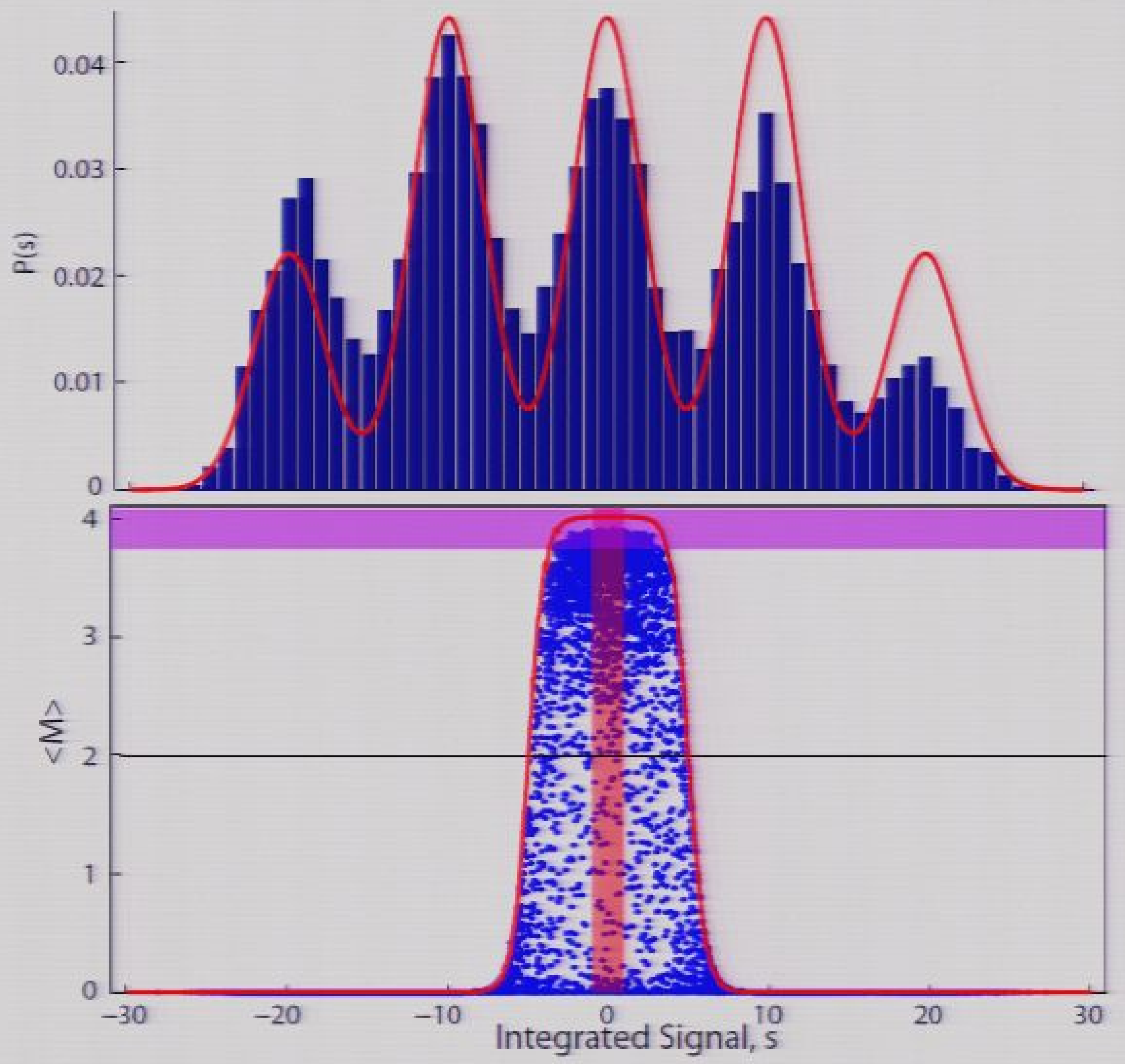

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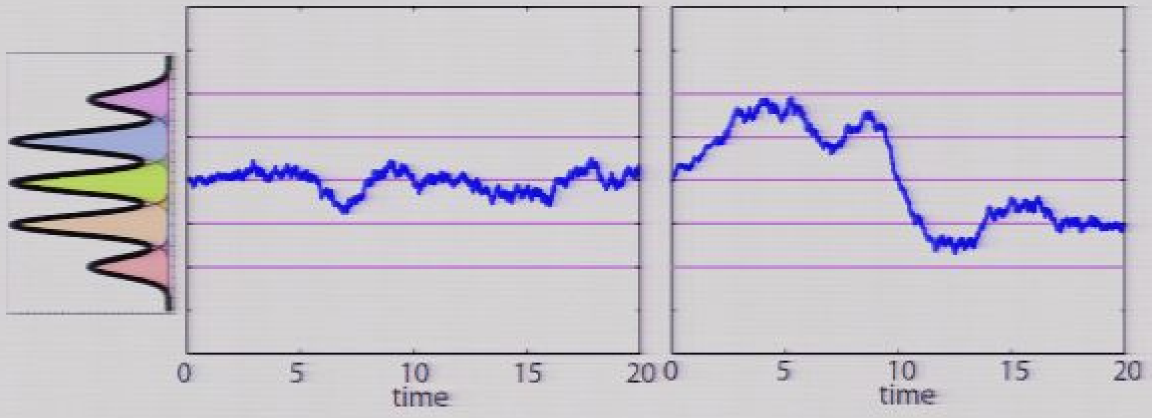

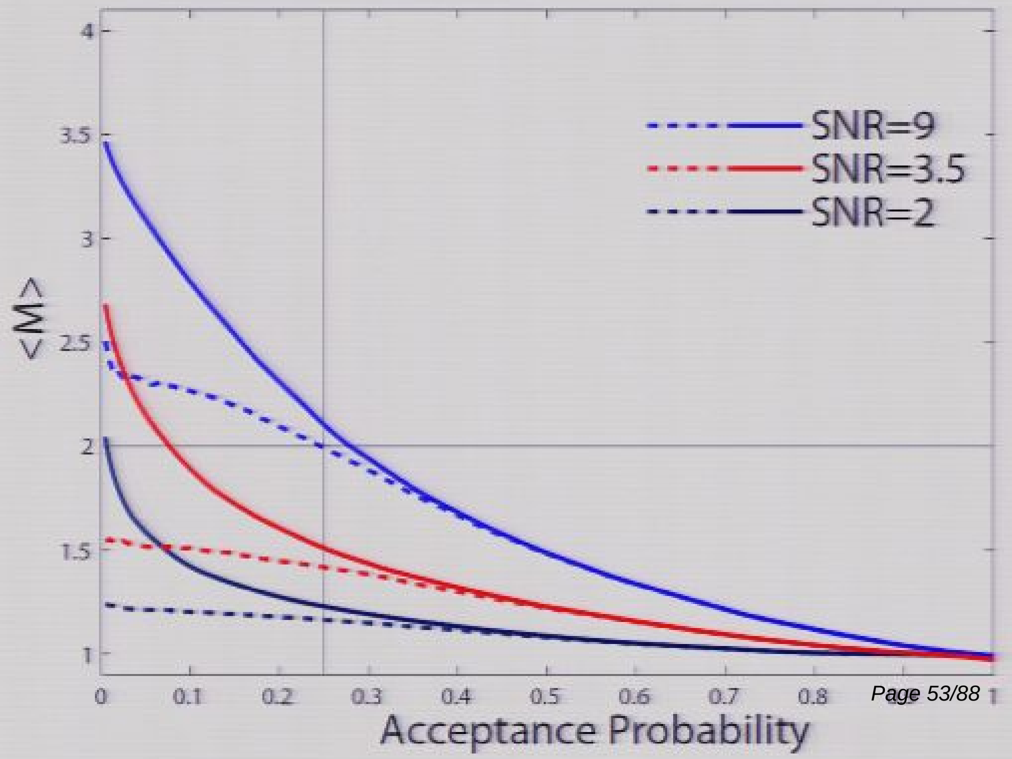

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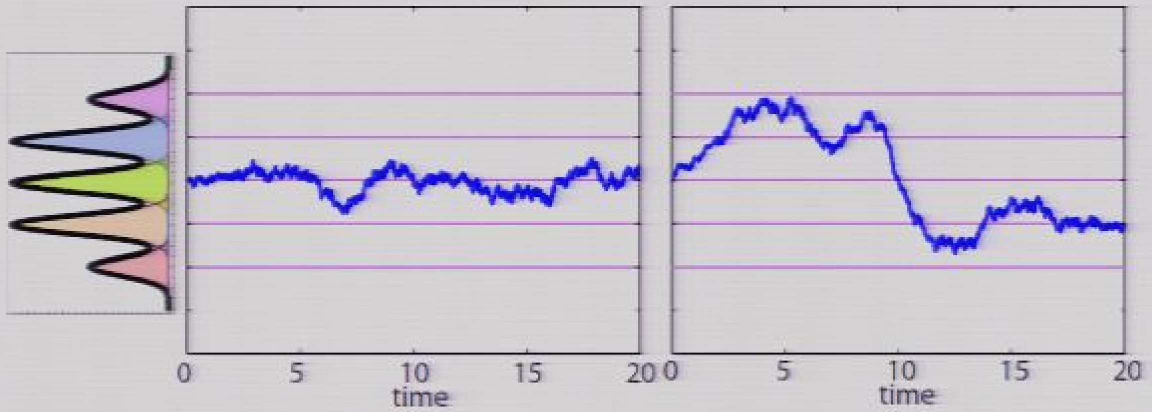

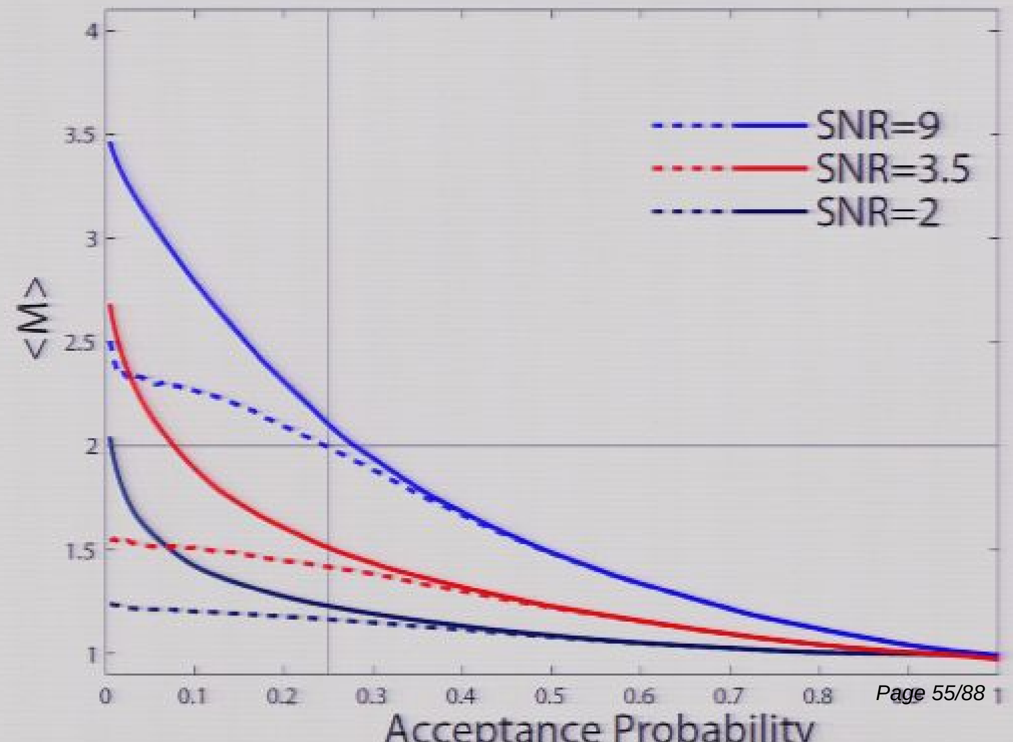

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Outline

- Circuit QED
  - Superconducting qubits
  - Transmission Line Resonator as ‘quantum bus’
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  - Nonlinear filtering
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  - Simplified detector tomography
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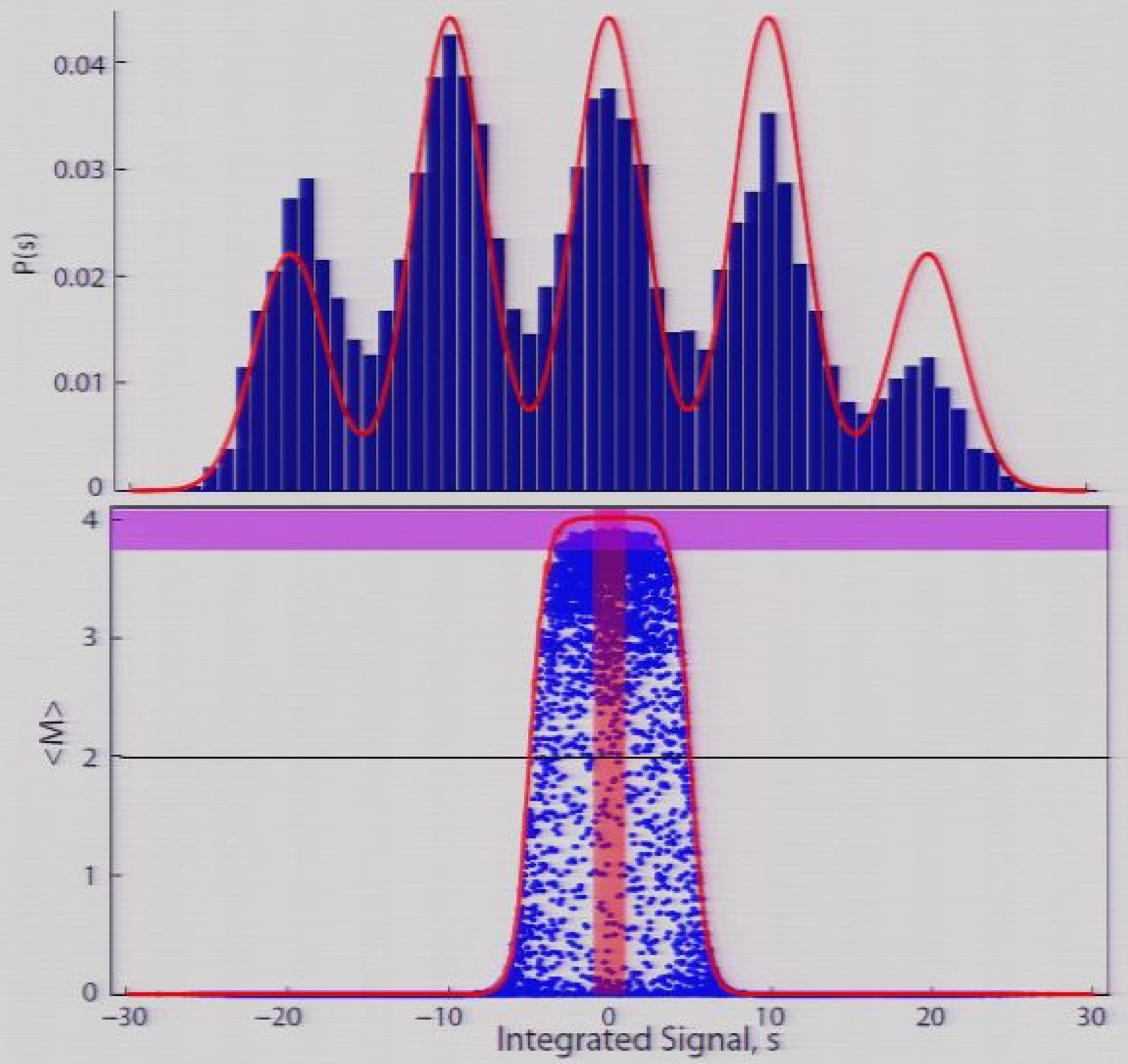

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Mermin operator

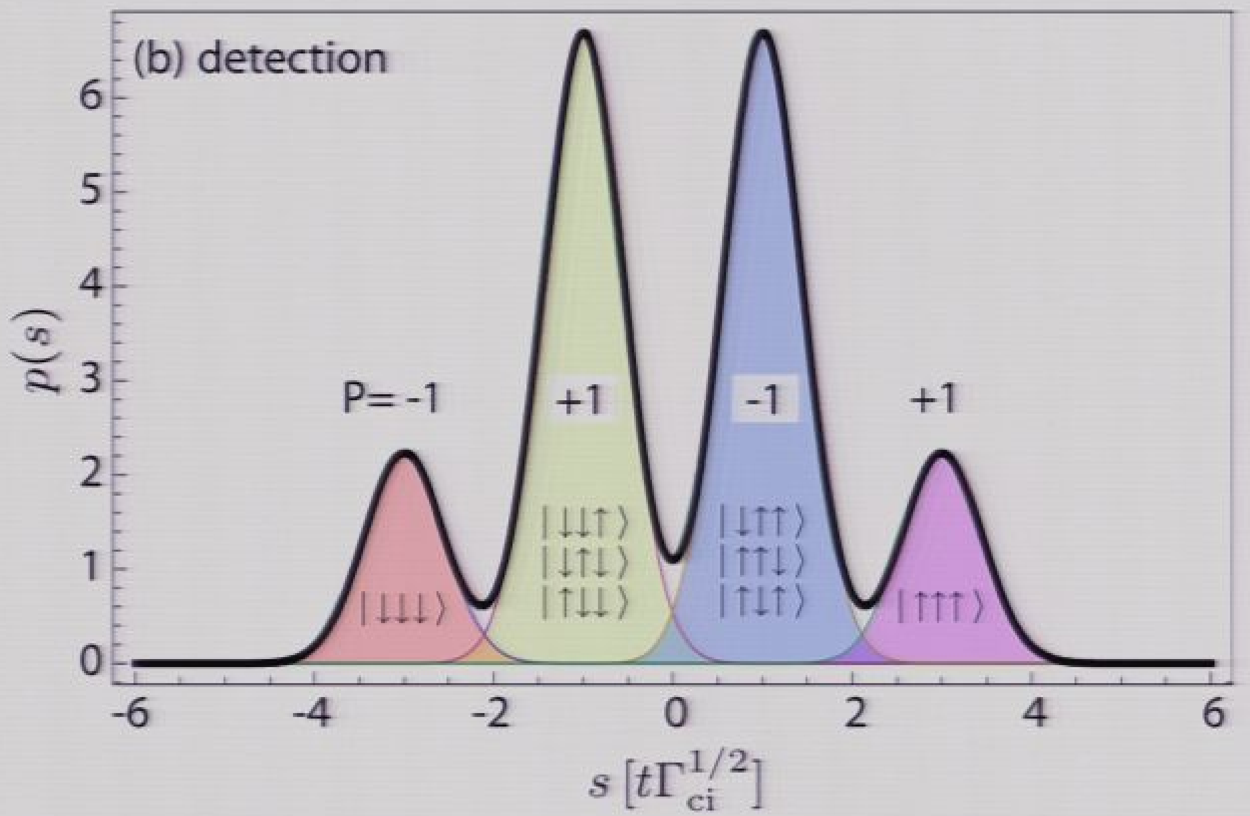
- Mermin operator $M = \sigma_1^x\sigma_2^x\sigma_3^x - \sigma_1^x\sigma_2^y\sigma_3^y - \sigma_1^y\sigma_2^x\sigma_3^y - \sigma_1^y\sigma_2^y\sigma_3^x$
- GHZ state is simultaneous eigenstate of these parity operators
- Ideally measure all the parities one after the other using a QND measurement
- Construct $\sigma_1^z\sigma_2^z\sigma_3^z$ and then use single-qubit rotations to generate the others
- Non-QND doesn't stop us measuring $\langle M \rangle$

Parity measurement

Mermin operator $M = \sigma_1^x\sigma_2^x\sigma_3^x - \sigma_1^x\sigma_2^y\sigma_3^y - \sigma_1^y\sigma_2^x\sigma_3^y - \sigma_1^y\sigma_2^y\sigma_3^x$

Step 1: Use single-qubit rotations

Step 2: Choose $A = \sum_j \sigma_j^z$



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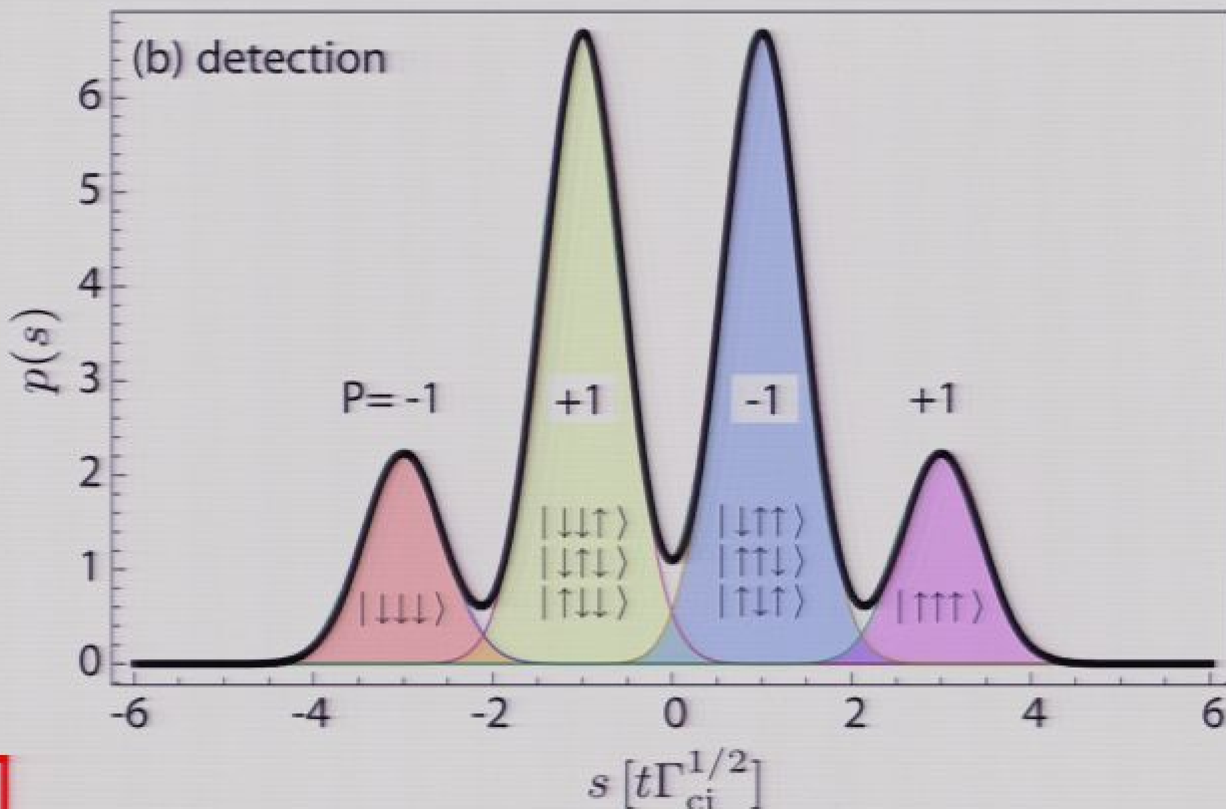
Step 3: Infer the parity $\prod_j \sigma_j^z$



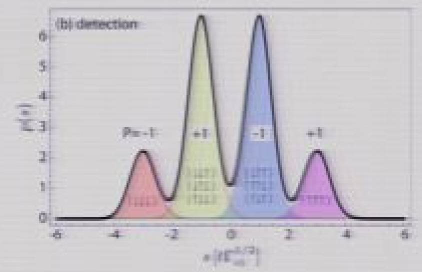
This choice reduces the number of peaks from 2^N to $N+1$!

Due to qubit relaxation there are false positive and false negatives so that we need detector tomography.

Measurement vs Preparation

- For preparation by measurement, don't care exactly **what** is measured
  - All that matters is the resulting ensemble
- For verifying the state, we certainly do care

The up-up-up trick



Idea: Decay **into** $|\Uparrow\rangle = |\uparrow\uparrow\cdots\uparrow\rangle$ is negligible.

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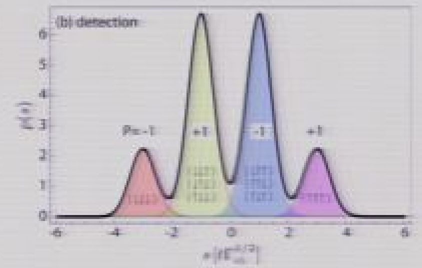

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$$E_1 = \alpha|\Uparrow\rangle\langle\Uparrow|$$
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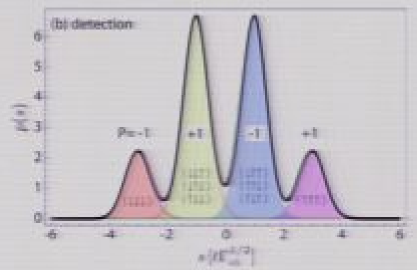

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Use more single-qubit rotations to determine each parity:

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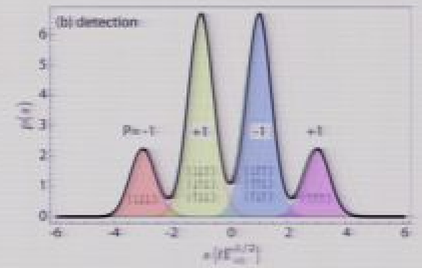

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$$F = f_{xxx} - f_{xyy} - f_{yxy} - f_{yyx} = \alpha\langle M\rangle \quad \text{thus} \quad \langle M\rangle = F/\alpha$$

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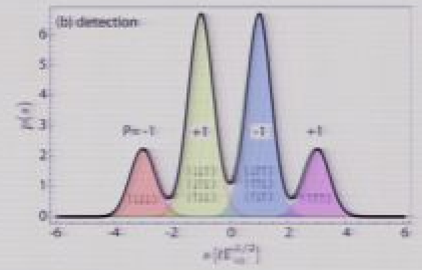

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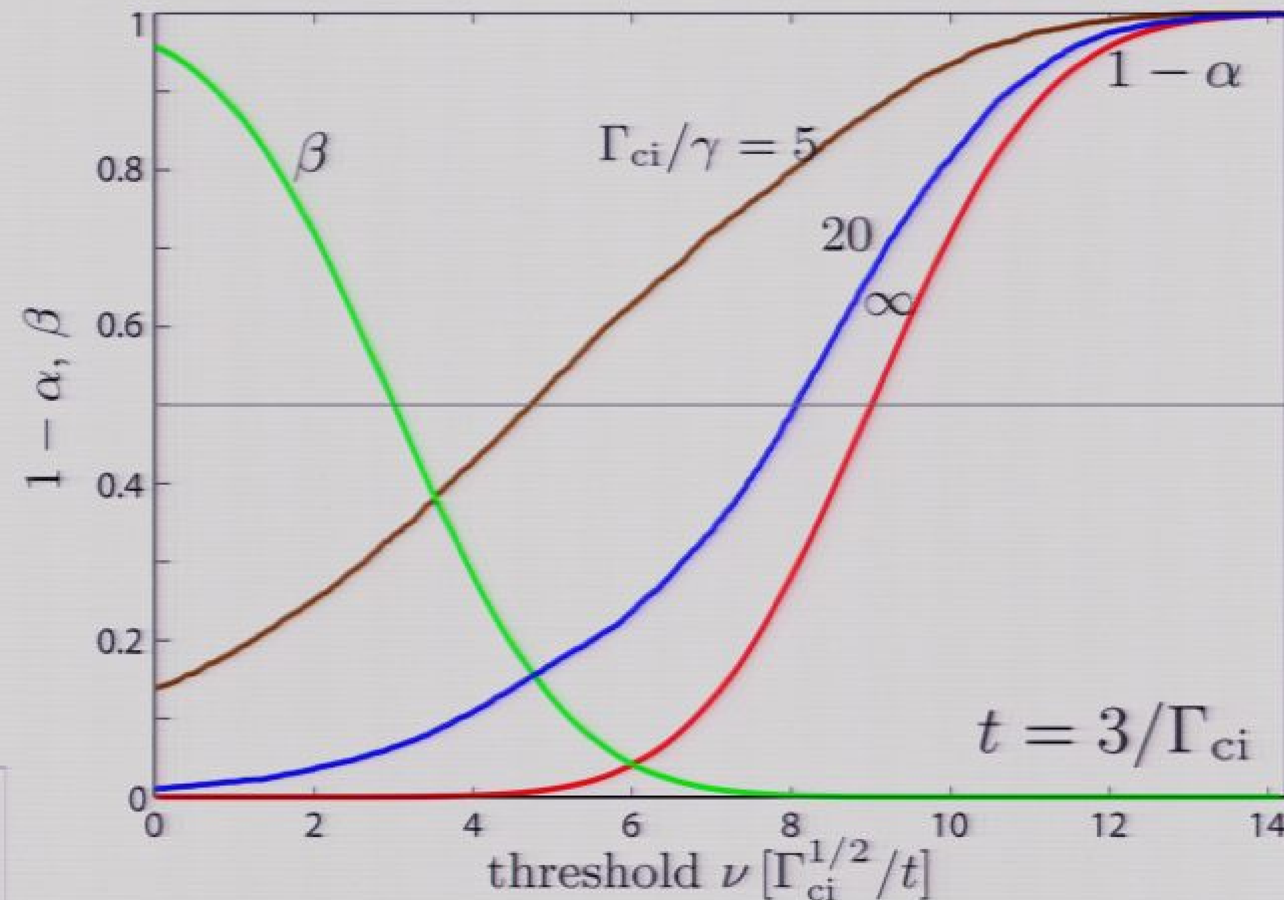
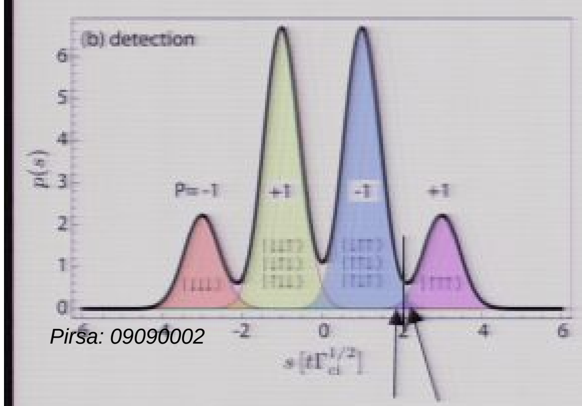
One parameter characterizes the measurement
Alternatively, we get lower bound on ⟨M⟩

The up-up-up trick

False negative probability $(1-\alpha)$

Worst-case false positive probability β





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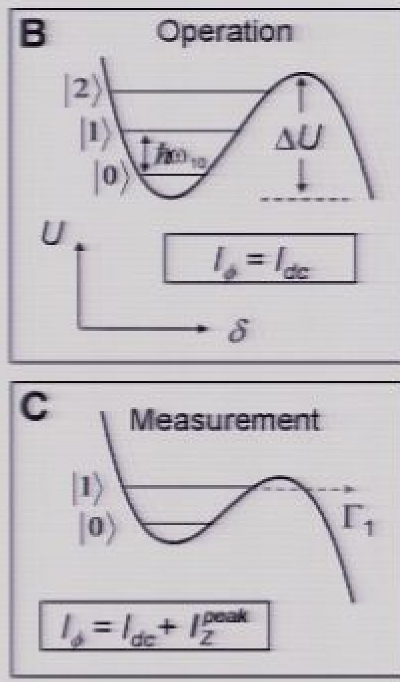
Phys Rev B **77**, 104502 2008



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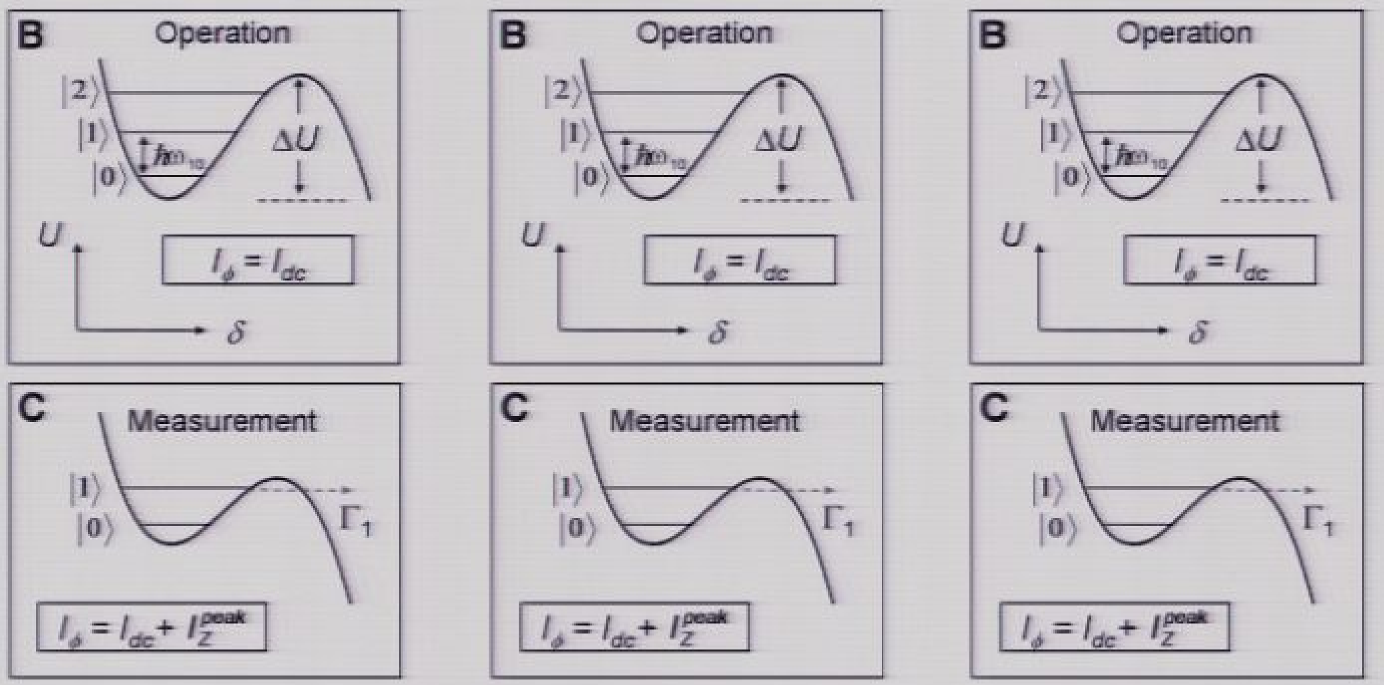
- Very similar to idea from Kofman and Korotkov for use with phase qubits

  Phys Rev B **77**, 104502 2008



Relation to Bell test

- **Not** a Bell test in the usual sense
  - Hard to avoid locality loophole given constraints of a dilution refridgerator
  - Dispersive readout inherently nonlocal
- Nevertheless, let's try to do something that is "about as hard as" a real Bell test
  - To prove we have a serious QIP platform
  - May be more convincing than a convoluted argument involving detector tomography, state tomography, nonlinear minimizations, etc

What is a Bell test?

- Alice and Bob (and Carol)
- Projective measurements, settings [rotations]:

  $\{A_i\}, \{B_i\}, \{C_i\}$
- Locality: implies $[A_i,B_k]=0$
  - Joint measurement factorizes: $F_{ijk}=A_i \otimes B_j \otimes C_k$
- Record settings and outcomes
- Get together and decide if **any** classical theory could reasonably describe the results
  - A probabilistic statement: quantum mechanics is vastly **more likely** than classical mechanics, given the observed results
- No requirement to use all the results:
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Bell tests with POVMs

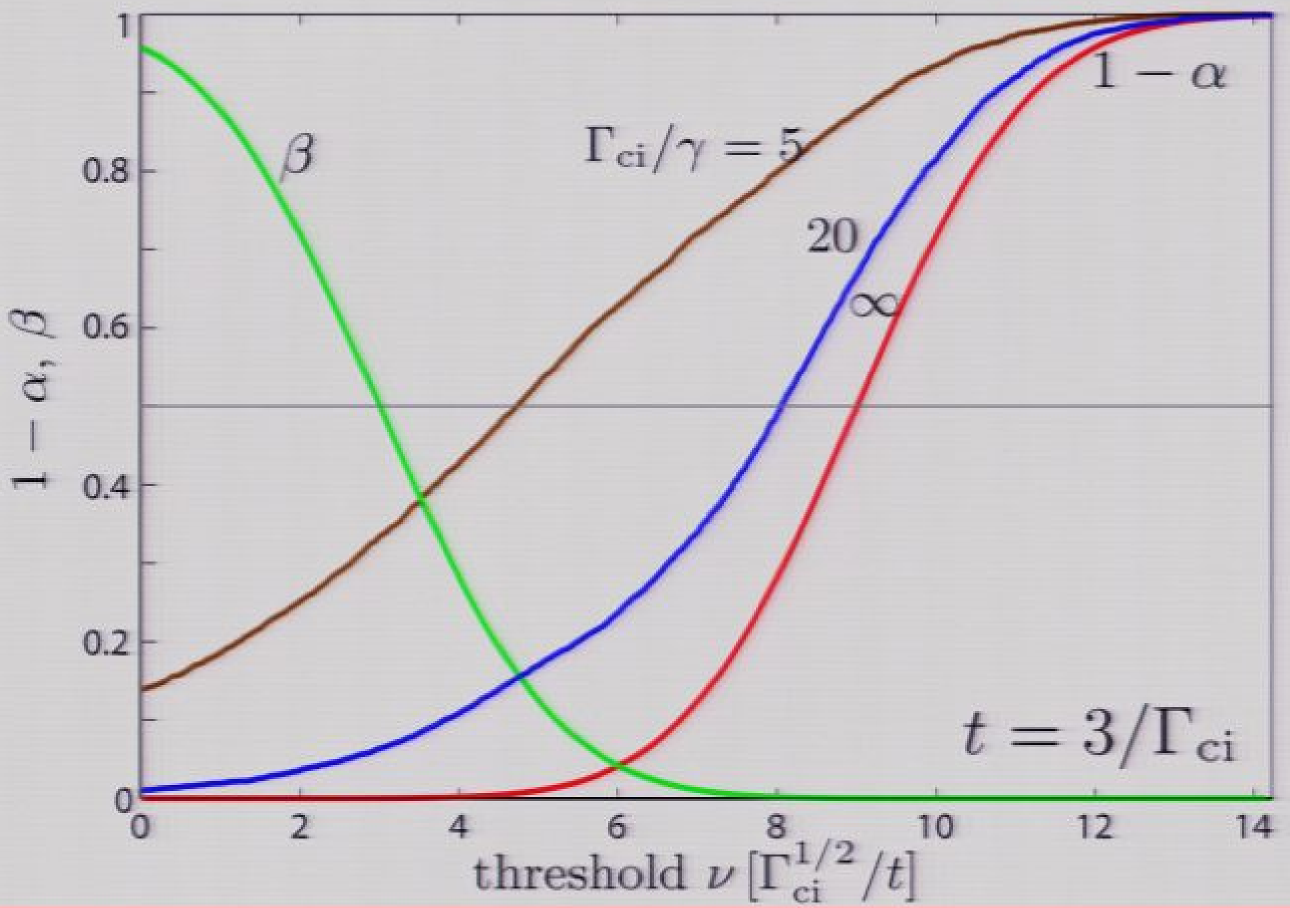
- Non-signalling: necessary requirement for the measurements to be local
- Within QM framework, it is equivalent:
  - D. Dieks, Phys. Rev. A **66**, 062104 (2002)
- Let's make sure that our measurement doesn't obviously signal
- Implies $F_{ijk} \to \{F^m_{ijk}\}, F^m_{ijk} = A^m_i \otimes B^m_j \otimes C^m_k$
  - Busch & Singh, Phys. Lett. A **249,** 10 (1998)
- The up-up-up effect is of exactly this form

  $F^{\Uparrow}_{zzz} = \alpha|\Uparrow\rangle\langle\Uparrow| = \alpha|\uparrow\rangle\langle\uparrow| \otimes |\uparrow\rangle\langle\uparrow| \otimes |\uparrow\rangle\langle\uparrow|$
  - For vanishing false-positive rate
  - Need good 1-qubit rotations

The up-up-up trick

False negative probability $(1-\alpha)$

Worst-case false positive probability β



Much stronger requirements on signal-to-noise ratio if don't ignore counts (detection loophole).

Conclusions

- Preparation by measurement has advantages for cQED
  - Different scaling properties
  - (2-qubit gates will improve…)
- Preparation of 3-qubit GHZ possible today
- Constructed measurement operator for which detector tomography is simple
- Bell-like test is still difficult

Lev S Bishop et al, *New J. Phys.* **11** (2009) 073040

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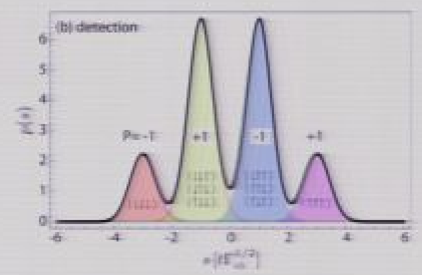


Idea: Decay **into** $|\Uparrow\rangle = |\uparrow\uparrow\cdots\uparrow\rangle$ is negligible.

POVM for "Is the signal in the right-most peak?" is:

$$E_1 = \alpha\,|\Uparrow\rangle\langle\Uparrow|$$

$$E_0 = \mathbb{1} - E_1$$

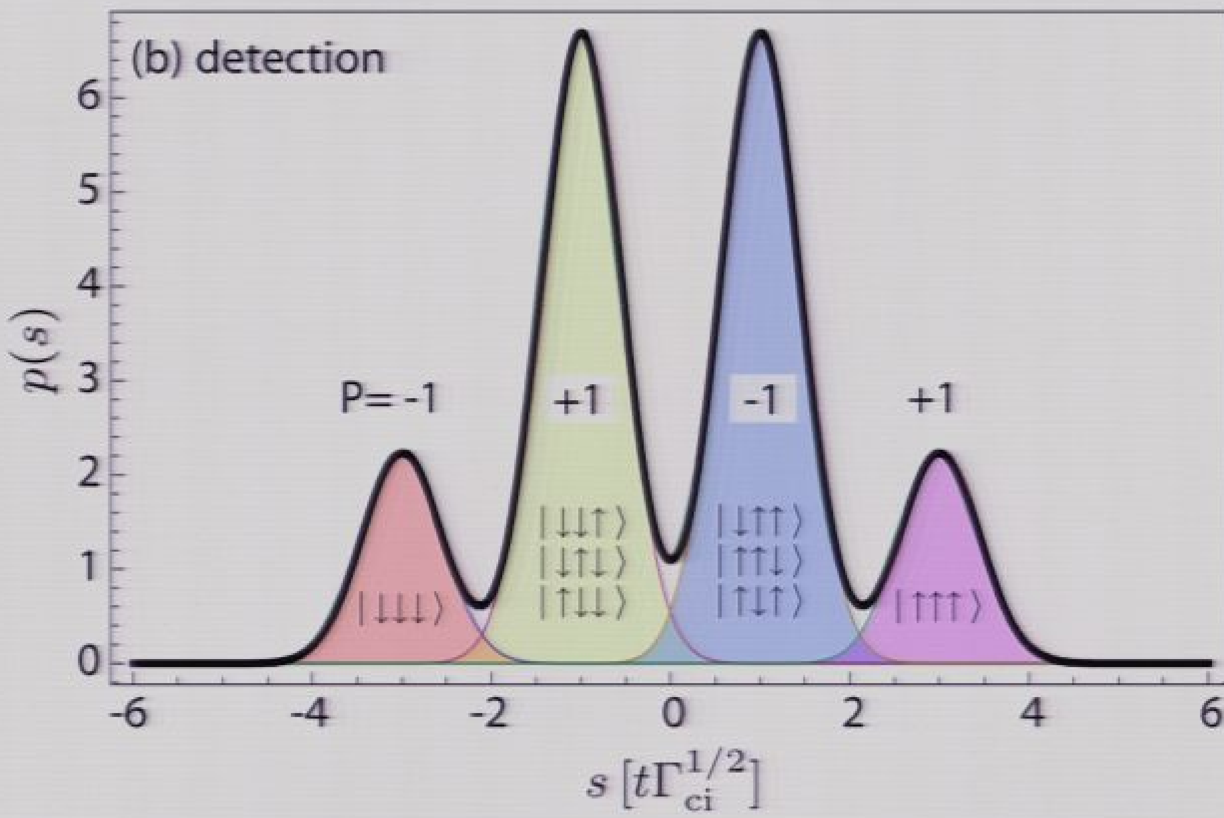
with $\alpha = P_{|\Uparrow\rangle}(s > \nu)$

Parity measurement

Mermin operator $M = \sigma_1^x\sigma_2^x\sigma_3^x - \sigma_1^x\sigma_2^y\sigma_3^y - \sigma_1^y\sigma_2^x\sigma_3^y - \sigma_1^y\sigma_2^y\sigma_3^x$

Step 1: Use single-qubit rotations

Step 2: Choose $A = \sum_j \sigma_j^z$



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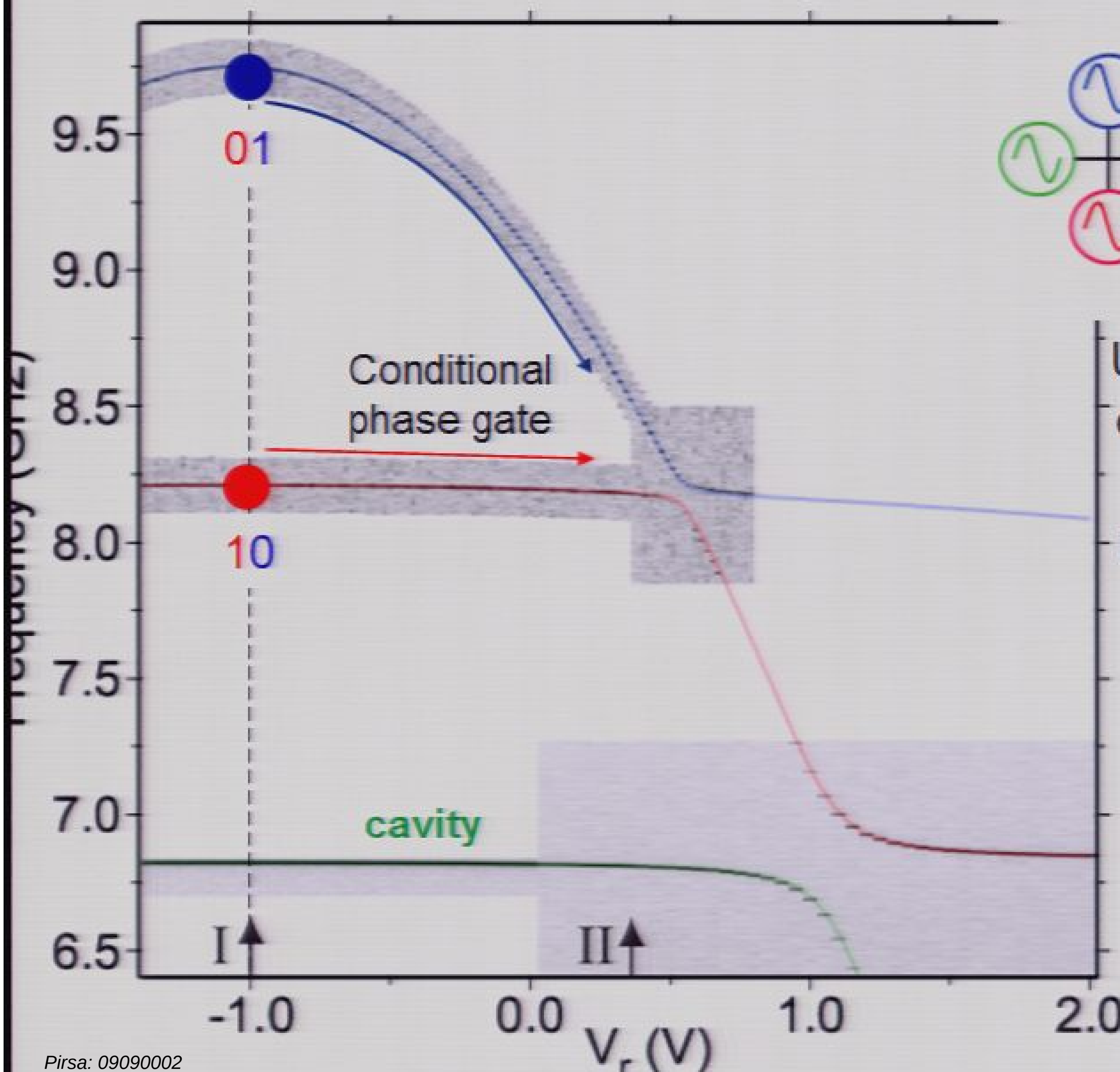
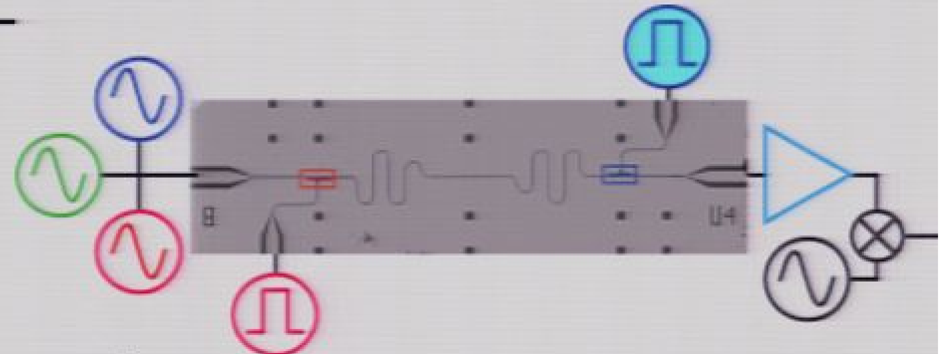
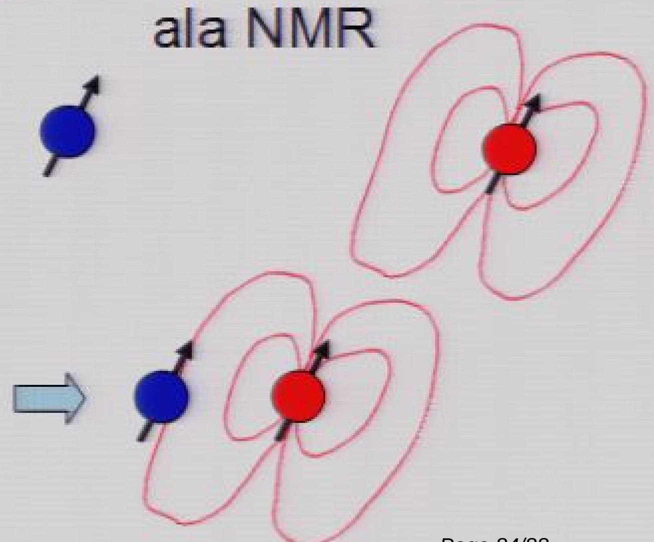
Two-qubit gate: turn on interactions





Use control lines to push qubits near a resonance:

A controlled z-z interaction ala NMR



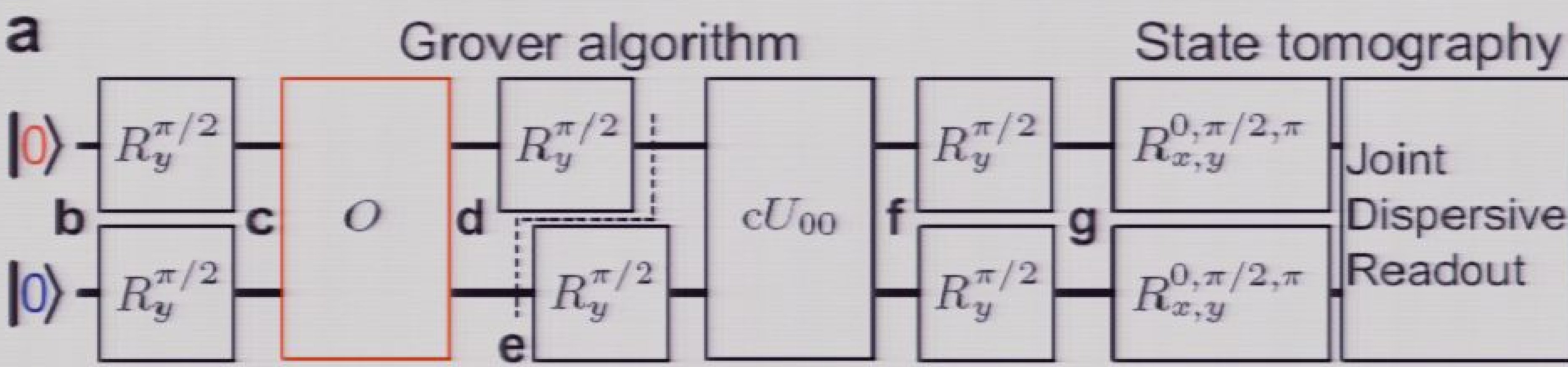
Grover's search algorithm

"unknown" unitary operation: →

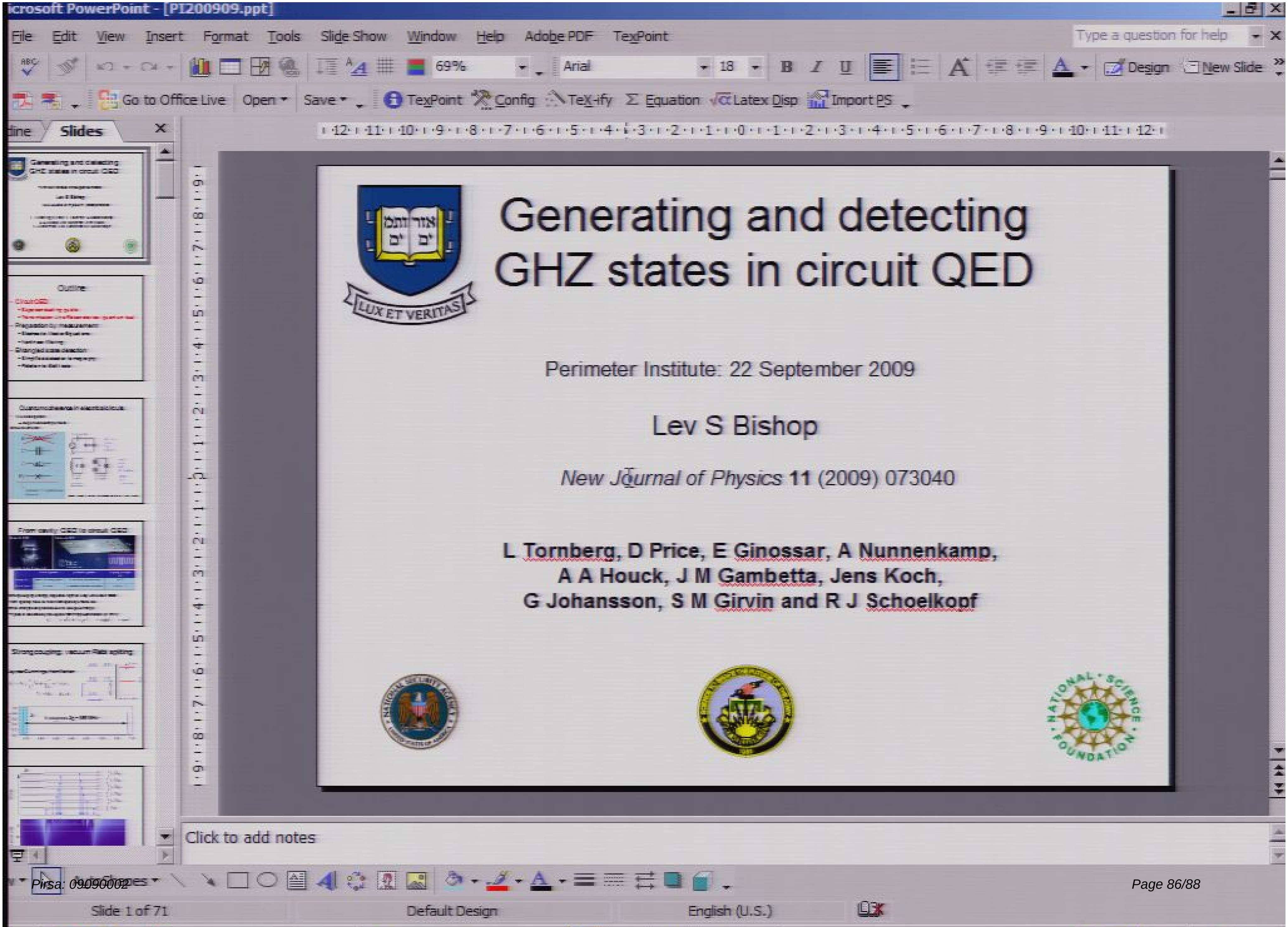
$$\bar{O}|\psi\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} |\psi\rangle$$

Challenge:
Find the location of the -1 !!!
(= queen)

Previously implemented in NMR: Chuang et al., 1998
Ion traps: Brickman et al., 2003
Linear optics: Kwiat et al., 2000



10 pulses w/ nanosecond resolution, total 104 ns duration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Perimeter Institute: 22 September 2009
Lev S Bishop
New Journal of Physics 11 (2009) 073040
L Tornberg, D Price, E Ginossar, A Nunnenkamp,
A A Houck, J M Gambetta, Jens Koch,
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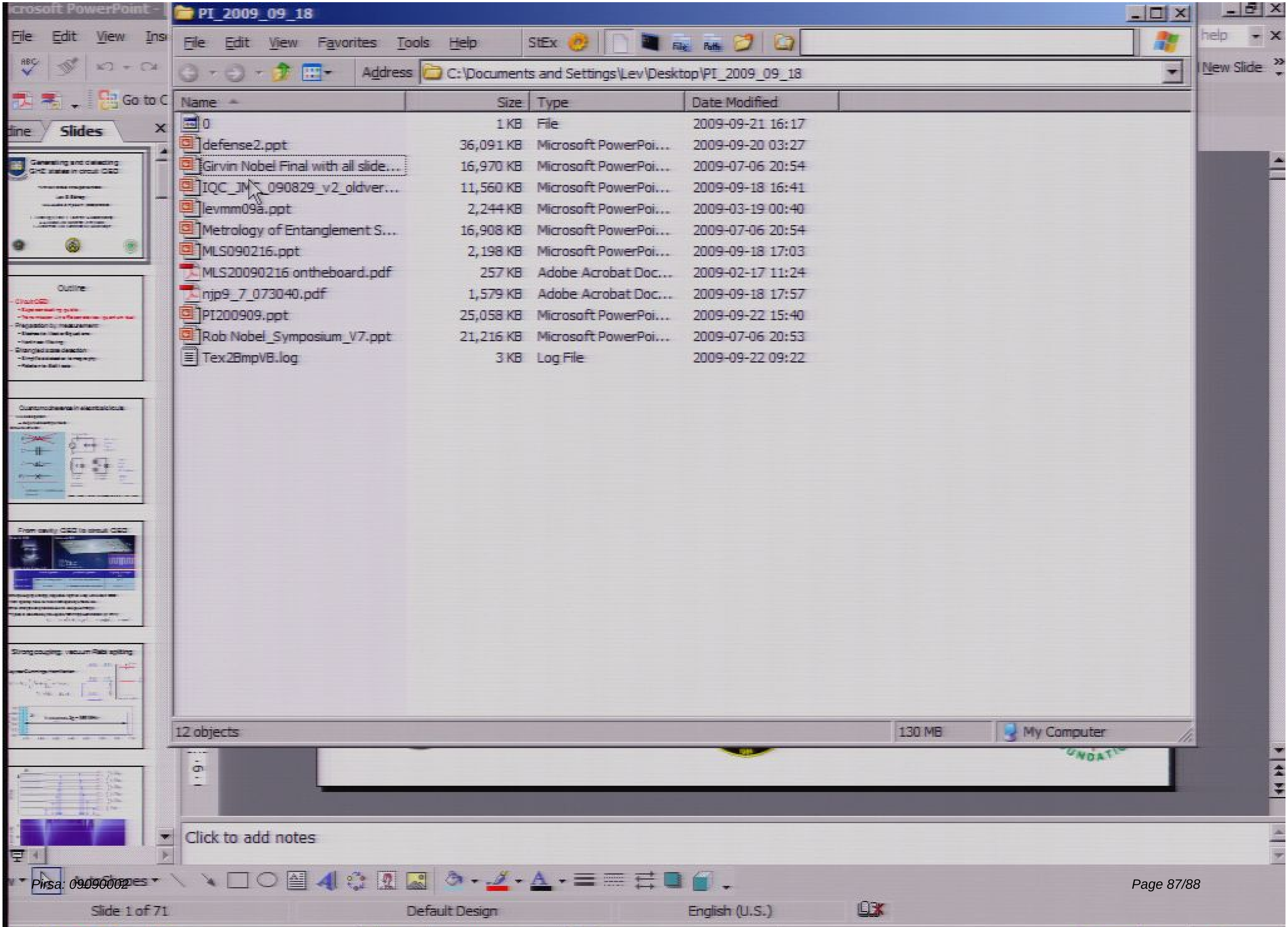
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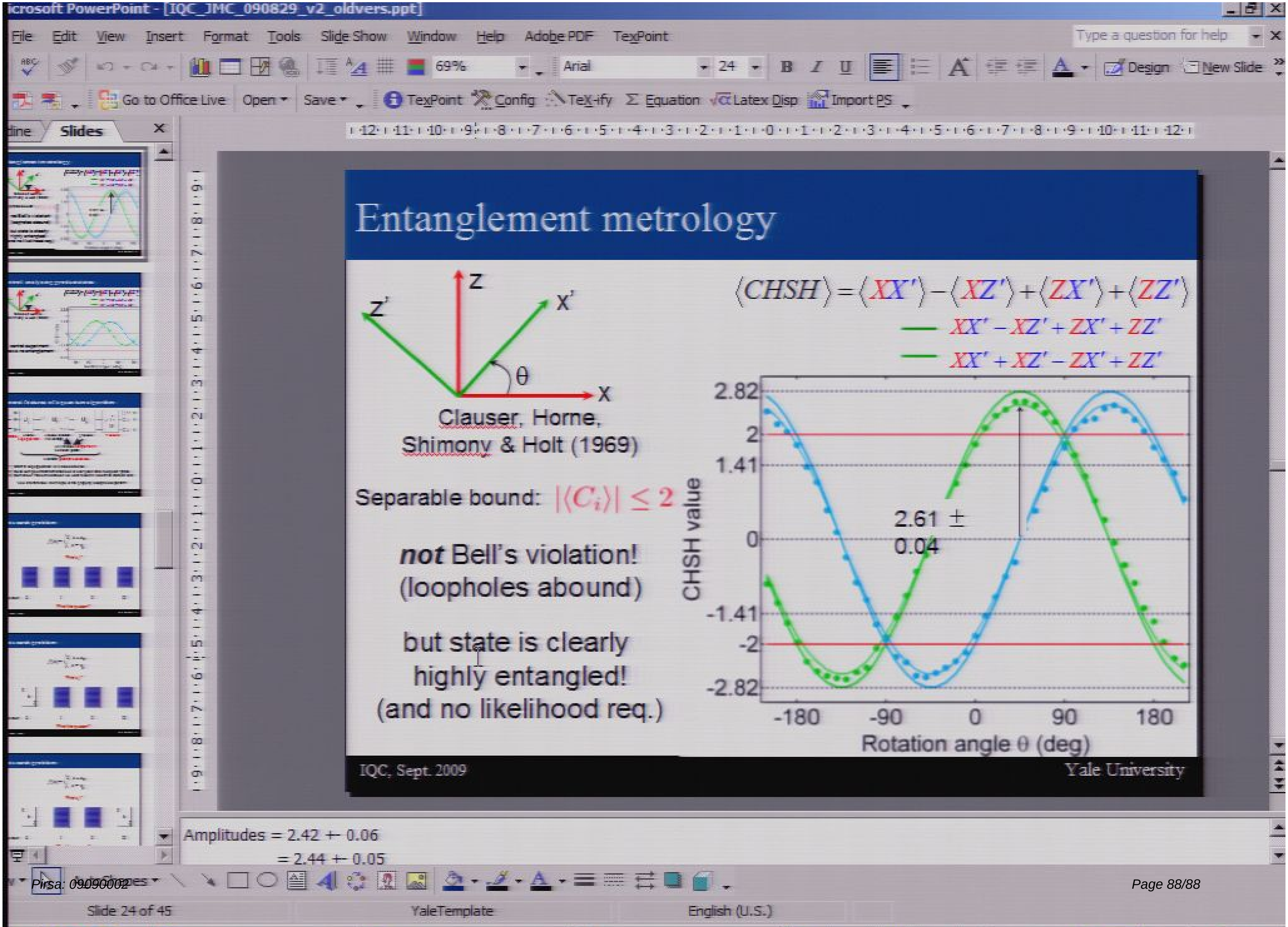
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Entanglement metrology
Z
Z'
X'
θ
X
Clauser, Horne, Shimony & Holt (1969)
Separable bound: $|\langle C_i \rangle| \le 2$
not Bell's violation! (loopholes abound)
but state is clearly highly entangled! (and no likelihood req.)
$\langle CHSH \rangle = \langle XX' \rangle - \langle XZ' \rangle + \langle ZX' \rangle + \langle ZZ' \rangle$
$XX' - XZ' + ZX' + ZZ'$
$XX' + XZ' - ZX' + ZZ'$
2.61 ± 0.04
CHSH value
2.82
2
1.41
0
-1.41
-2
-2.82
-180
-90
0
90
180
Rotation angle θ (deg)
IQC, Sept. 2009
Yale University
Amplitudes = 2.42 +- 0.06
= 2.44 +- 0.05
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