Title: Causal Set Cosmology

Date: Sep 08, 2009 02:00 PM

URL: http://pirsa.org/09090001

Abstract: The standard model of cosmology has some puzzles/problems such as the cosmological constant problem and the horizon problem which according to many stem from our lack of understanding of the very early universe. This in turn means that almost none of the theories of quantum gravity are at a stage where anything substantial can be said about observational cosmology.
In the past few years Causal Set theory has proved itself different in this case where a possible solution to the Cosmological constant problem was proposed. Now some work in progress has also shown that some models of Causal Set dynamics give exponential expansion in the early universe. I hope to discuss both of these exciting prospects but this talk will mainly focus on the first proposal.

Causal Set Cosmology

MAQBOOL AHMED

Centre for Advanced Mathematics
and Physics

National University of Sciences
and technology

Outline

- Problems of the Standard Model
- Causal Set Idea
  - Classical Sequential Growth
- Causal Set Solution to Λ prob.
- Causal Set Solution to the Horizon problem
- Conclusions

Successes of the Standard Model

- Cosmological Nucleosynthesis
- CMB
- Expansion of the universe becomes natural
- Framework for Structure Formation

Problems (/Challenges/Opportunities)

- The Origin
- CMB is homogeneous!
- Why flat?
- $\Lambda_g \simeq 0$
- Is $\eta \neq 0$? Why?
- Primordial(?) Fluctuations
- Dark matter is .....
-
-
-

A Causal Set (or causet) is a partially ordered set with the following properties

- irreflexive ($x \nless x$)
- transitive ($x < y$ and $y < z \Rightarrow x < z$)
- locally finite

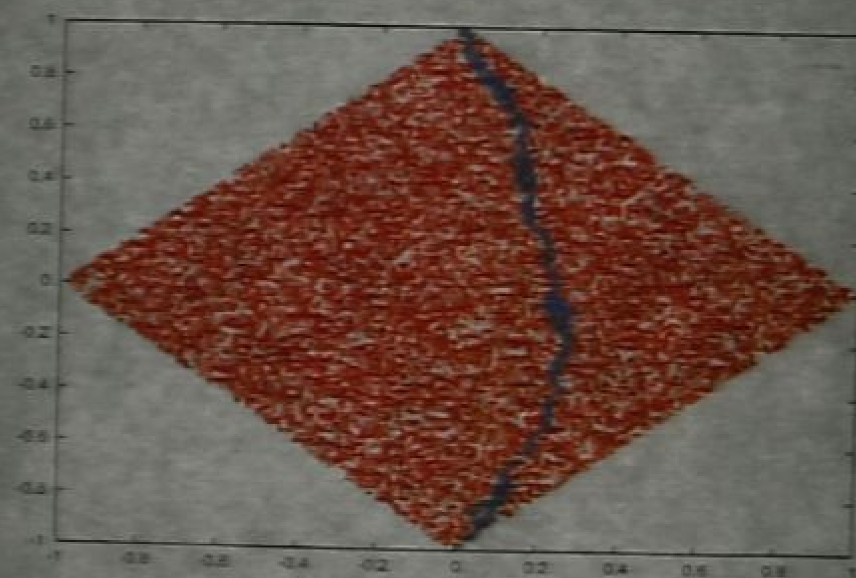
$$|\{z \mid x < z < y\}| < \infty$$

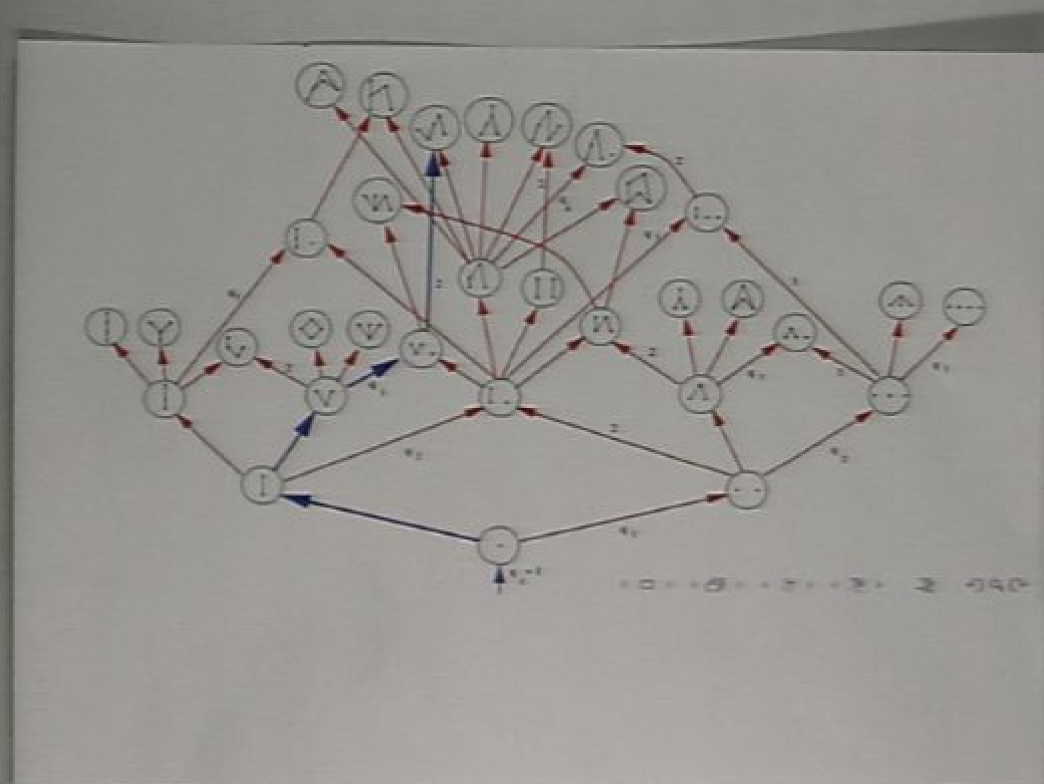
Causal Structure	$\prec$
Geometrical quan. like prop. time and 4-volume	Simple process of counting causet ele
V	N
τ	L

Timelike Distance Definitions

Length L of *longest* chain between x and y

$$d(x, y) := L$$





Sequential Growth

Dynamics

- Stochastic process
- Probabilities are assigned based on
  - Internal temporality
  - Discrete general covariance
  - Bell causality

Causet Solution to the

Λ_g Problem

– Local Finiteness

– $\Delta V \, \Delta \Lambda \sim 1$

– $\Delta V \sim \pm\sqrt{N} \sim \pm\sqrt{V}$

If $\langle \Lambda \rangle = 0$

$$\Delta \Lambda \sim \frac{1}{\Delta V} \sim \pm \frac{1}{\sqrt{V}}$$

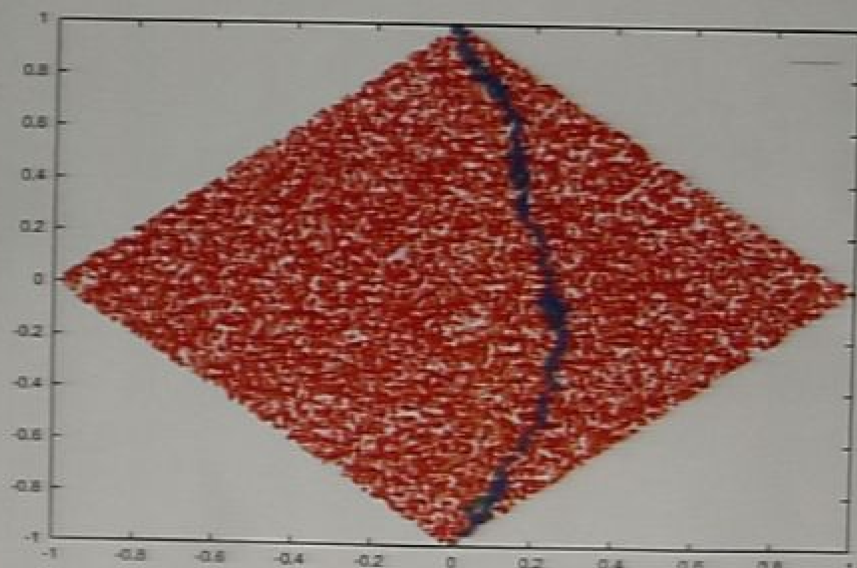
Let $V \sim ($[illegible]

$\Delta \Lambda \sim H^2 \sim \rho_{critical}$

Timelike Distance Definitions

Length L of *longest* chain between x and y

$$d(x,y) := L$$



Causet Solution to the Λ_g Problem

- Local Finiteness
- $\Delta V \, \Delta\Lambda \sim 1$
- $\Delta V \sim \pm\sqrt{N} \sim \pm\sqrt{V}$

If $\langle \Lambda \rangle = 0$

Let $V \sim (H^{-})^4$

Λ_g Problem

- Local Finiteness

- $\Delta V \, \Delta \Lambda \sim 1$

- $\Delta V \sim \pm\sqrt{N} \sim \pm\sqrt{V}$

If $\langle \Lambda \rangle = 0$

$$\Delta \Lambda \sim \frac{1}{\Delta V} \sim \pm \frac{1}{\sqrt{V}}$$

Let $V \sim (H^{-})^{4}$

$\Delta \Lambda \sim H^{2} \sim \rho_{critical}$

and then write

$$\rho_{\Lambda,i+1} = \frac{S_{i+1}}{N_{i+1}} = \frac{S_i + \alpha\xi_{i+1}\sqrt{\delta N_i}}{N_i + \delta N_i}. \qquad (5)$$

Here α is an unknown dimensionless parameter which governs the dynamics of the theory; ξ_{i+1} is a random number with mean 0 and standard deviation 1; and S_0 is set to zero at some very early time t_0. We then expand the universe according to

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}(\rho_{matter} + \rho_{radiation} + \rho_\Lambda), \qquad (6)$$

recompute the new space-time volume and repeat.

Figure 2 shows the evolution of the energy density in one such realization. During the radiation era, ρ_Λ scales roughly as a^{-4}, while during the matter era it scales as a^{-3}. Thus at all times it is comparable to the ambient energy density. If the recipe we have devised for implementing the ideas of causal set theory and unimodular gravity is an accurate approximation to the ultimate quantum theory, then these modifications of GR do indeed lead to an *Everpresent* Λ, a cosmological term which is always with us [13].

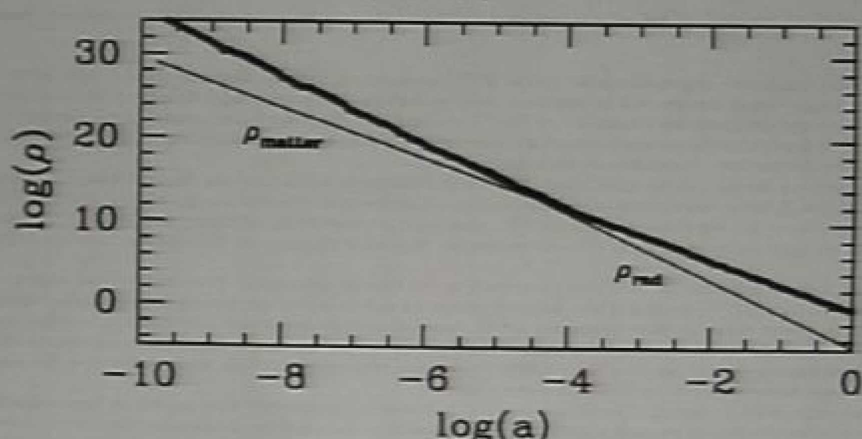
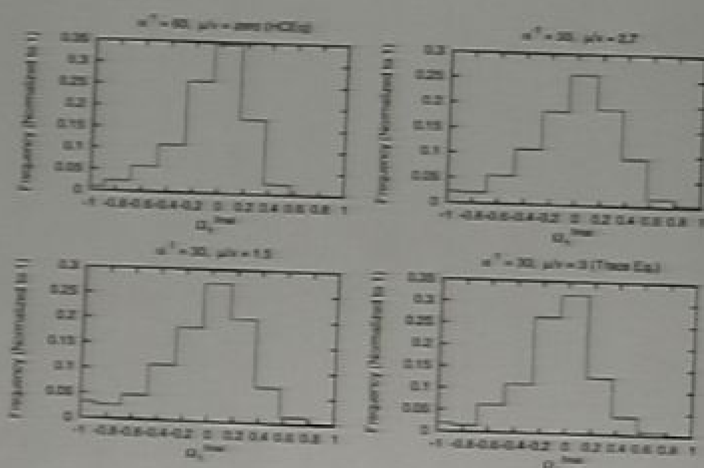


FIG. 2: Evolution of the energy densities in the universe. The thick curve is the absolute value of the energy density in the cosmology constant. The fluctuating ρ_Λ is always of order the ambient density, be it radiation (early on) or matter (later). Here the dimensionless parameter α which governs the amplitude of the fluctuations has been set to 0.01.

Hidden in the gross structure of Figure 2 are the fluctuations about this average scaling. These fluctuations are crucial if the theory is to describe the real universe for two reasons: First, there cannot be too much excess energy at $a \sim 10^{-9}$ or else the successful predictions of Big Bang Nucleosynthesis (BBN) will be destroyed. Second, if ρ_Λ scales exactly as matter today, it will not have the correct equation of state to account for the cosmological observations. Figure 3 shows the ratio of the energy density in Λ to the total energy density as a function of the scale factor for another realization, this time with a slightly larger value of α. This ratio, Ω_Λ, fluctuates about zero with an amplitude of order 0.5 (as we will shortly see, this amplitude is a function of α). In this particular realization, Λ accounts for over fifty percent of the energy density today and changes very little going back to redshift $z = 1$ ($a = 0.5$); thus it behaves recently as a true cosmological constant, and therefore satisfies the observed cosmological constraints.

In half the realizations, ρ_Λ will be positive today. Whether or not it is positive enough to explain the observations then becomes a question of probability.



Why is the universe so large when it is not so old?

Inflationary Soln.

There is a phase of exponential expansion in the early universe.

(Caused by a scalar field displaced from its true min.)

Causet Idea

There are models in CSG that have an initial random tree like era with exponential

[illegible]

era of de Sitter like expansion.

Why is the universe so large when it is not so old?

Inflationary Solu.

There is a phase of exponential expansion in the early universe.

(Caused by a scalar field displaced from its true min.)

Causet Idea

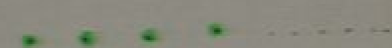
There are models in CSG that have an initial random tree like era with exponential growth followed by another era of de Sitter like expansion.

Percolation

$$P_r(C_n \to C_{n+1}) \propto \sum_{k=0}^{\varpi - m} \binom{\varpi - m}{k} t_{k+m}$$

$$t_n = \left(\frac{p}{1-p}\right)^n \qquad \left(\begin{array}{c}\text{transitive}\\ \text{percolation}\end{array}\right)$$

- For $p \lll 1$



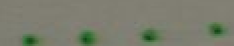
- Force each element to pick a parent.

Percolation

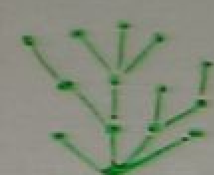
$$Pr(C_n \to C_{n+1}) \propto \sum_{k=0}^{\varpi - m} \binom{\varpi - m}{k} t_{k+m}$$

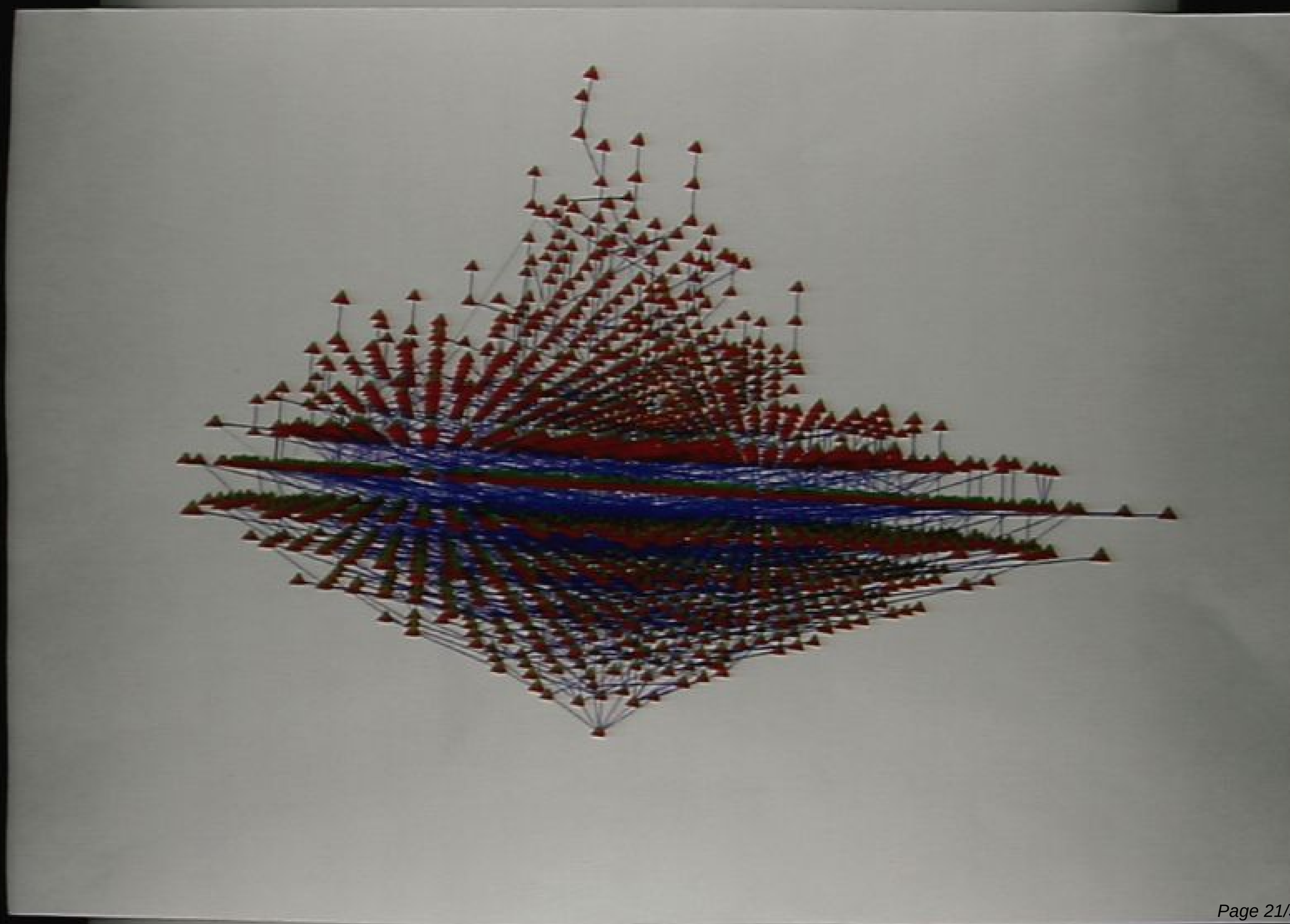
$$t_n = \left(\frac{p}{1-p}\right)^n \qquad \text{(transitive percolation)}$$

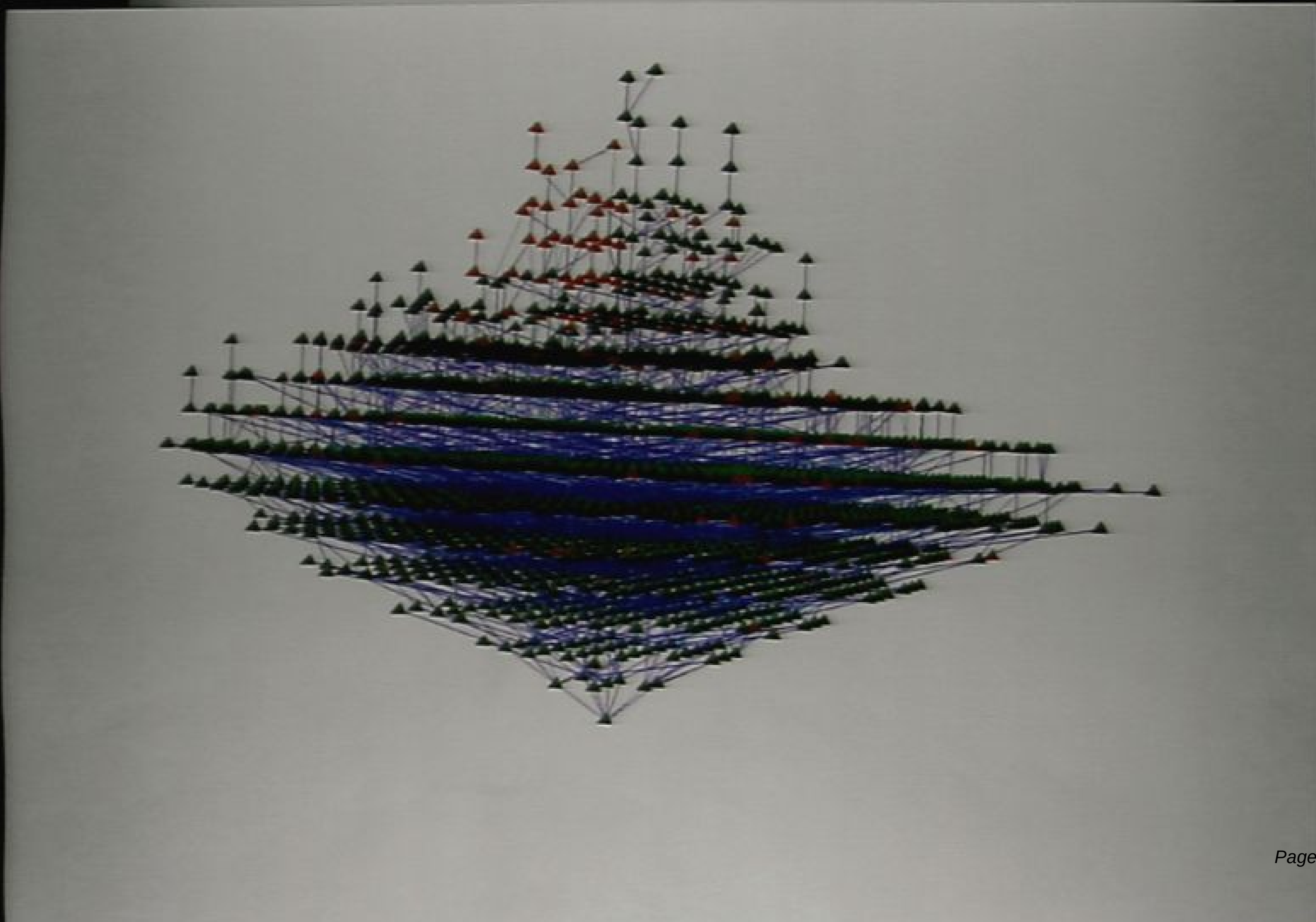
- For $p \lll 1$

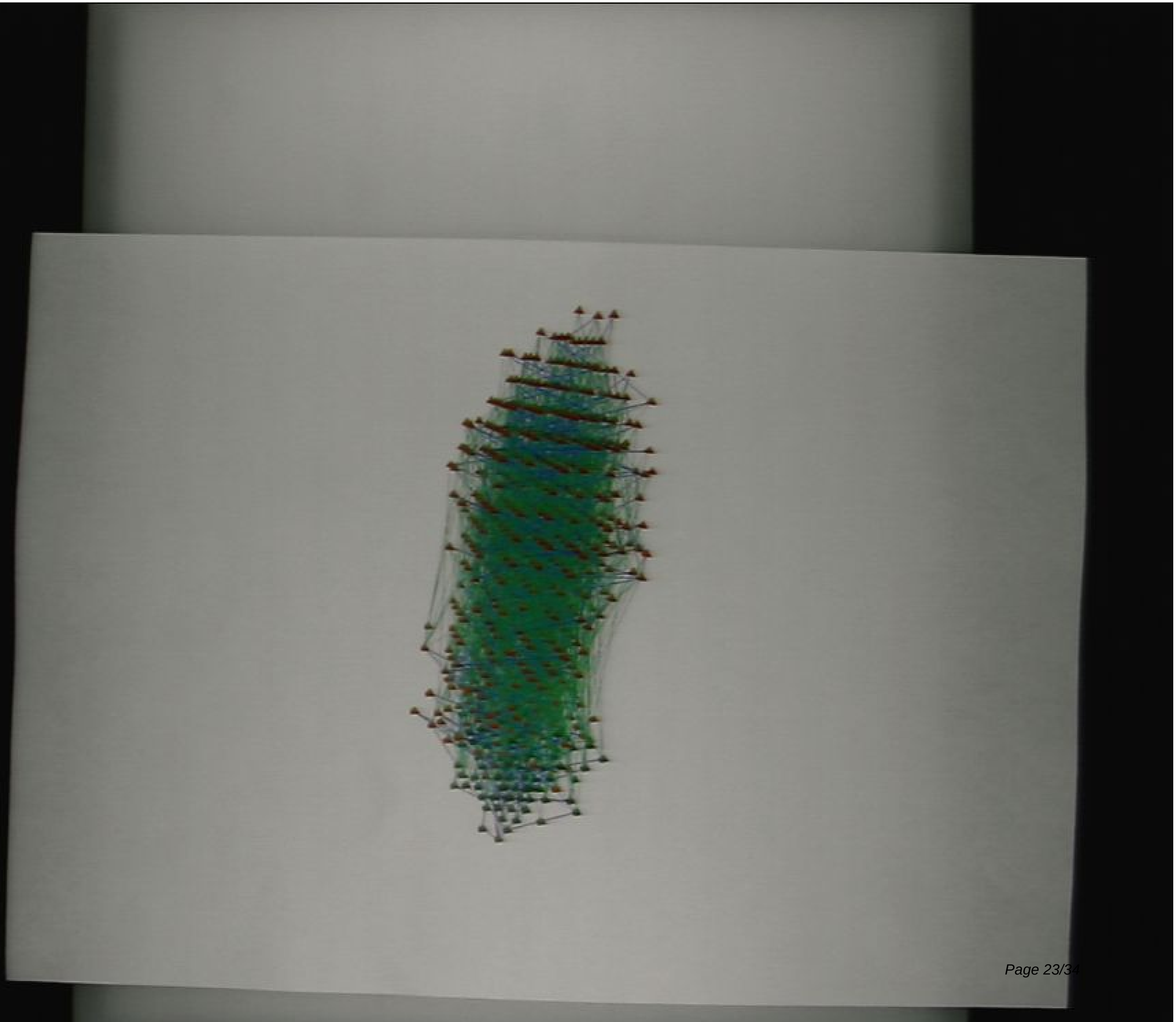


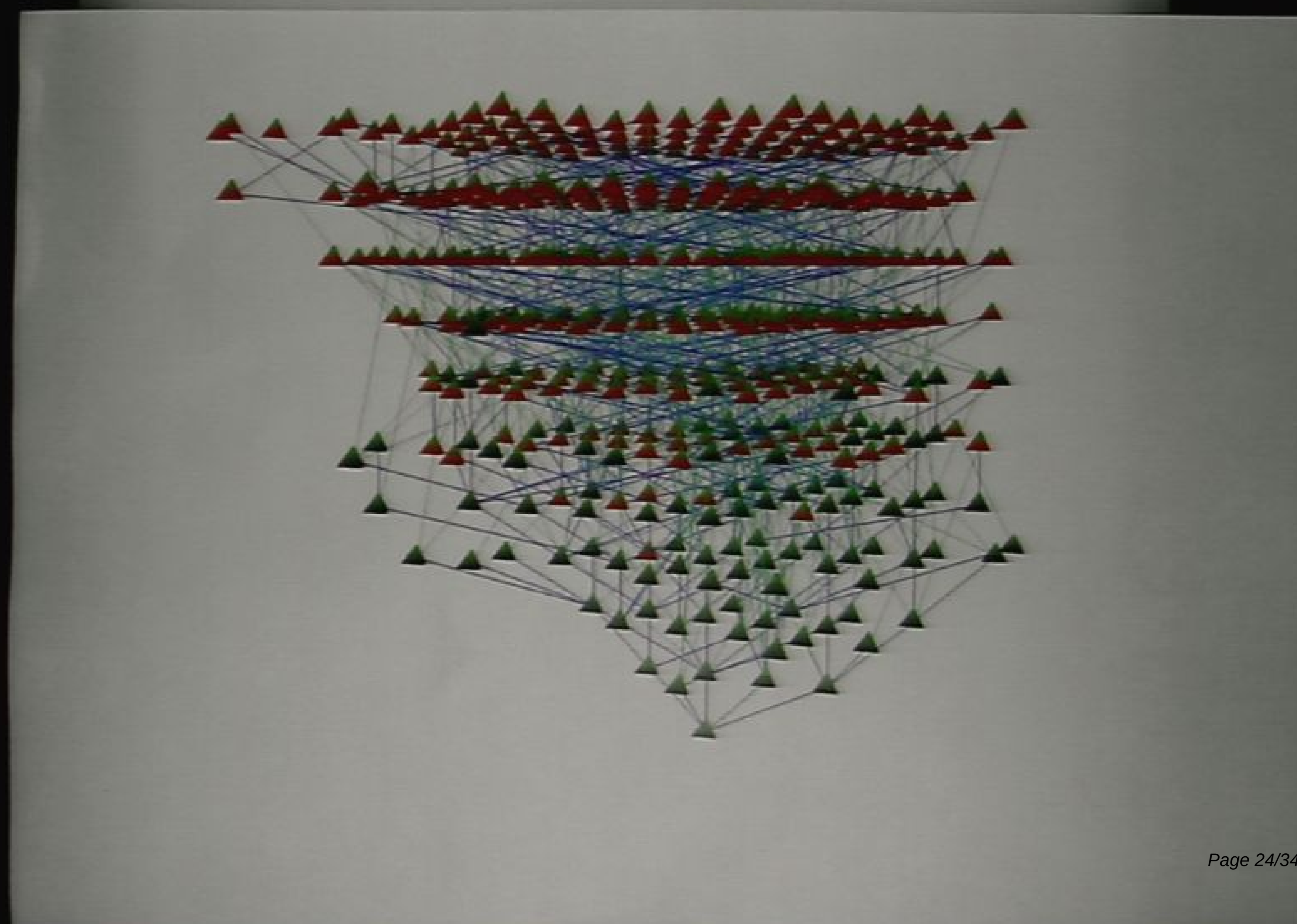
- Force each element to pick a parent.





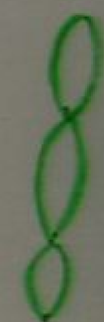






Originary Percolation

- homogeneous



- Appears inhomog. in time due to random fluctuations

- size $\sim \frac{1}{\rho}$

de Sitter ?

$$ds^2 = -dt^2 + e^{2t/\ell}\left(dr^2 + r^2 d\Omega_D^2\right)$$

Does the early universe of CSG (with orig. percolation) resemble de Sitter ?

– $V_0 \longleftrightarrow N_0$

$\tau \longleftrightarrow L$

– $$V_0 = c_D\, \ell^{D+1}\left[\ln\cosh^2\frac{\tau}{2\ell} + \sum_{i=1}^{D}\frac{(-1)^{i+1}}{i}\binom{D}{i}\left\{T_+^i + T_-^i - 2\right\}\right]$$

odd D

$$= c_D\, \ell^{D+1}\left[\frac{\tau}{\ell} + \sum_{i=1}^{D}\frac{(-1)^i}{i}\binom{D}{i}\left\{T_+^i - T_-^i\right\}\right]$$

even D

de Sitter ?

$$ds^2 = -dt^2 + e^{2t/\ell}\left(dr^2 + r^2 d\Omega_D^2\right)$$

Does the early universe of CSG (with orig. percolation) resemble de Sitter?

$-\quad V_\Diamond \longleftrightarrow N_\Diamond$

$\tau \longleftrightarrow L$

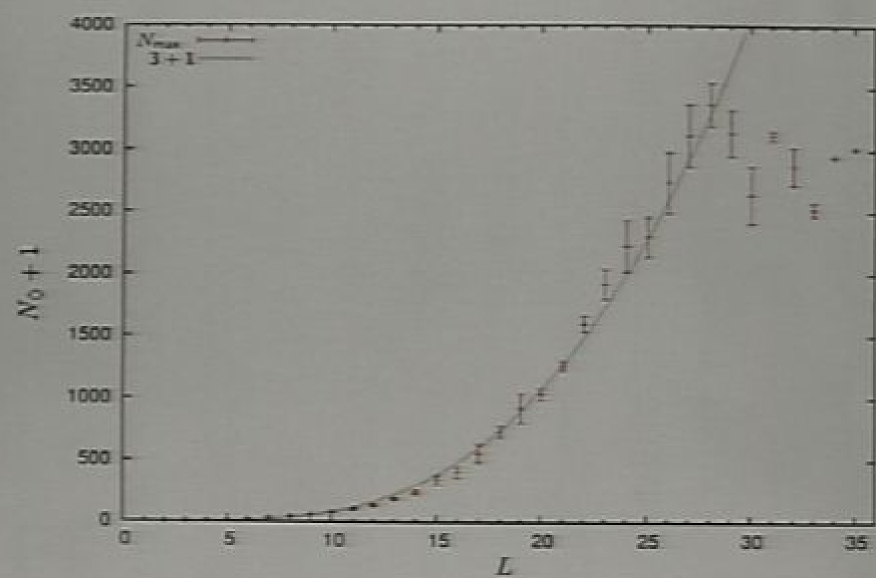
$$-\ V_0 = C_D\,\ell^{D+1}\left[\ln\cosh^2\frac{\tau}{2\ell} + \sum_{i=1}^{D}\frac{(-1)^{i+1}}{i}\binom{D}{i}\left\{T_+^{\,i} + T_-^{\,i} - 2\right\}\right] \quad \text{odd } D$$

$$= C_D\,\ell^{D+1}\left[\frac{\tau}{\ell} + \sum_{i=1}^{D}\frac{(-1)^{i}}{i}\binom{D}{i}\left\{T_+^{\,i} - T_-^{\,i}\right\}\right] \quad \text{even } D$$

$$T_+ = 1 + \tanh\frac{\tau}{2\ell}$$

$$T_- = 1 - \tanh\frac{\tau}{2\ell}$$

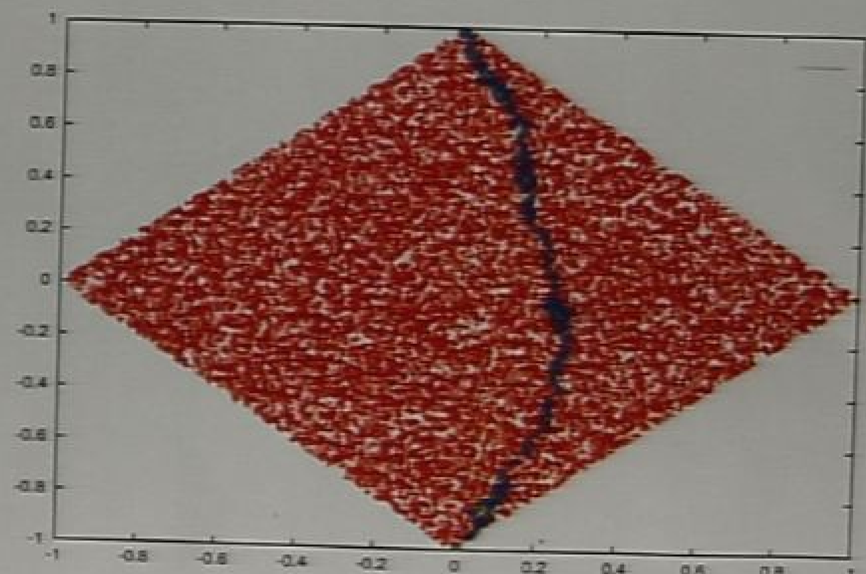
$p = .0002, N = 30000$



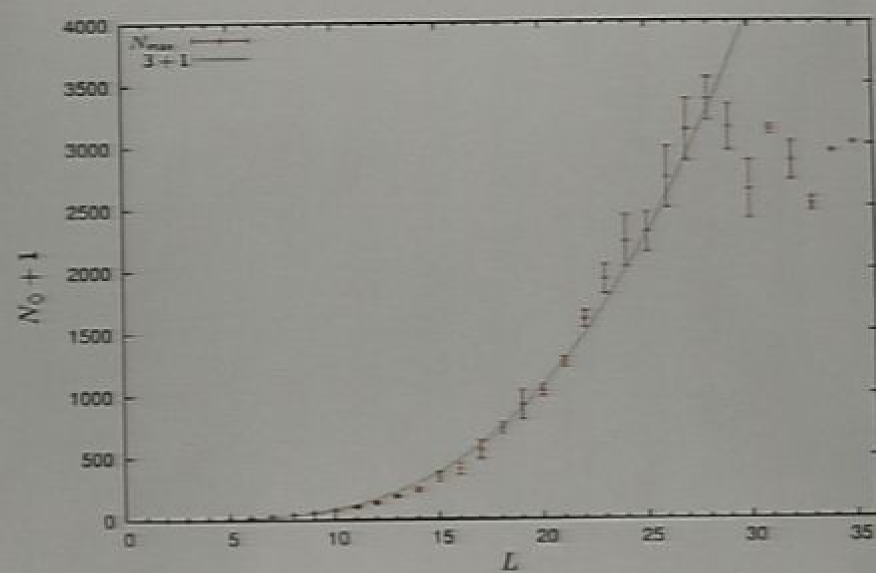
Timelike Distance Definitions

Length L of *longest* chain between x and y

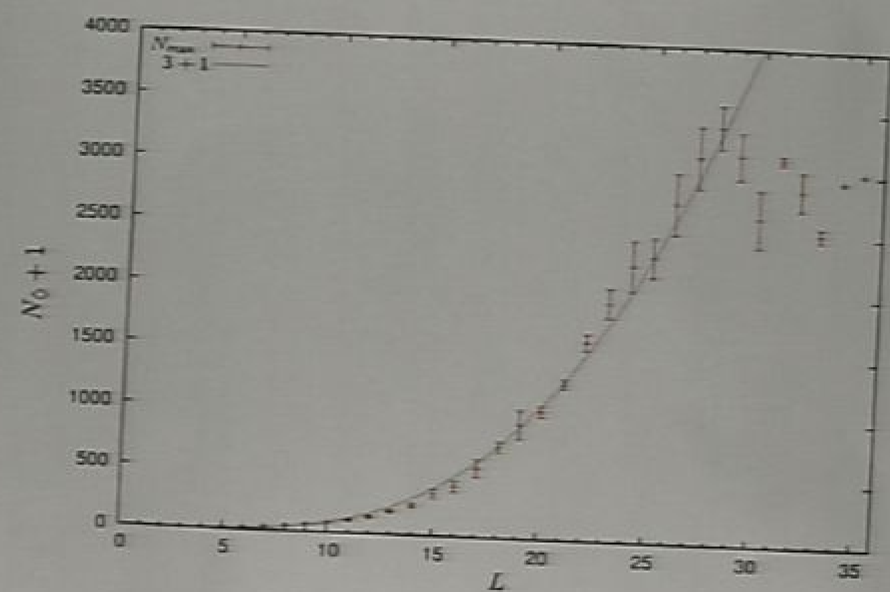
$$d(x, y) := L$$



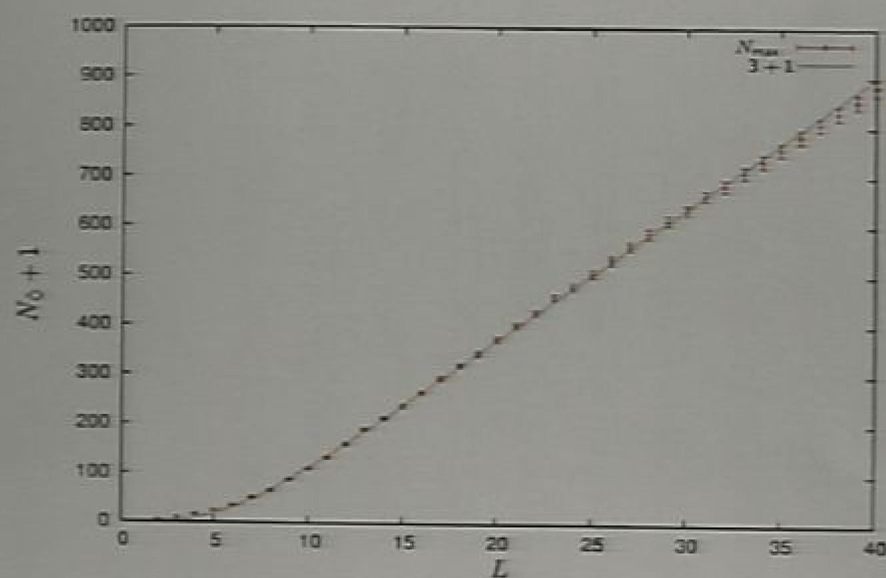
$p = .0002, N = 30000$



$p = .0002, N = 30000$



Much larger $p = .02$, $N = 3000$



$$mL = \tau$$

$$V = \ell^4 N$$

Conclusions

CSG is a simple model for growing causal sets based on three very clear and philosophically very satisfying principles. And yet it yields a universe that not only has many desirable cosmological properties but also solves many of the standard model's problems.

- It has a begining but the "initial singularity" is not a problem anymore
- Shortly after big bang (the last post) the universe undergoes a de Sitter like expansion to something large and homogeneous
- A "natural" explanation of the small non-zero value of the cosmological constant (with the potential to satisfy the obs.)
- Since the energy density in Λ is always comparable to the total energy density, it also solves the "why Now?" puzzle.
- No need to fine tune anywhere.