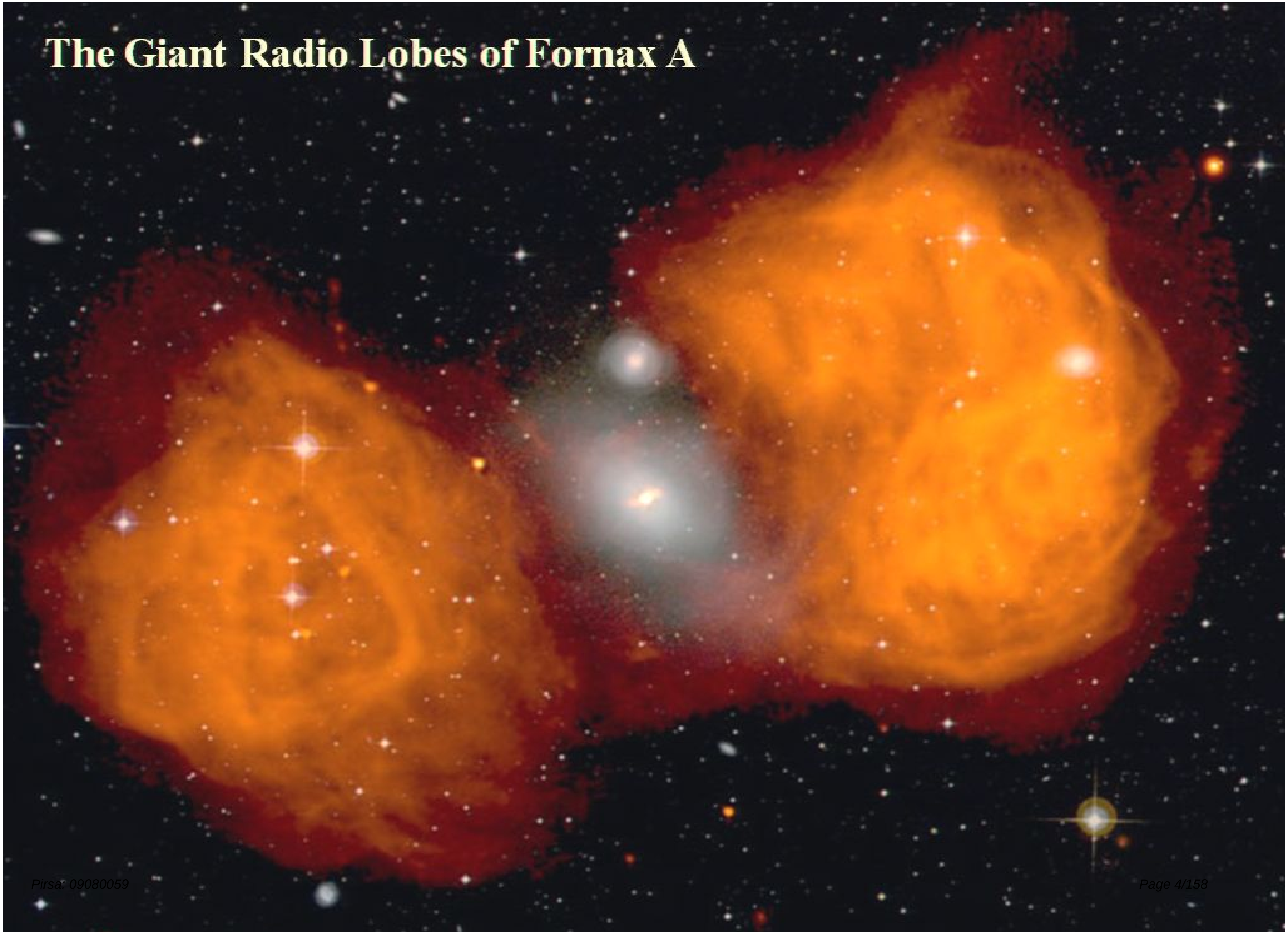
Title: General Relativity 1

Date: Aug 19, 2009 02:30 PM

URL: http://pirsa.org/09080059

Abstract: The refinement of our understanding of Space and Time through thought experiments; starting with Newtonian ideas and ending with the mathematical adventures of Einstein and other prominent scientists as they contemplate the structure of stars.


Sunset Over Gusev Crater


The Giant Radio Lobes of Fornax A


Black Holes

A huge great enormous thing, like — like nothing. A huge big — well, like a — I don't know — like an enormous big nothing ...

Piglet describes the Heffalump,
in *Winnie the Pooh* by A.A. Milne

Dark stars

- **Rev. John Michell (1783)**

  A British born "natural philosopher" dared to combine the corpuscular description of light with Newton's gravitation laws to predict what large compact stars should look like.

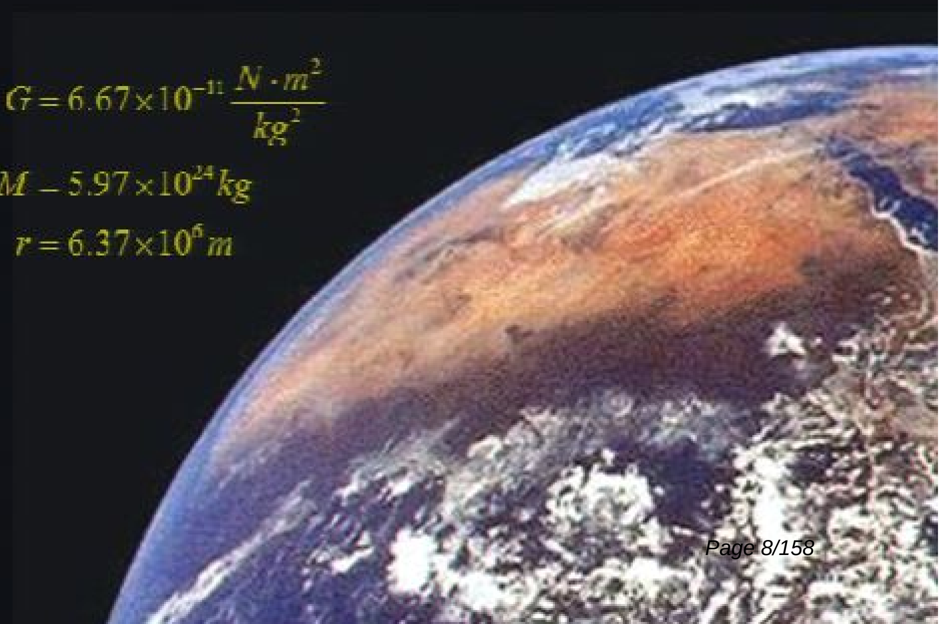
- He showed that a star, that has the same density of the sun, but 500 time as big, would have such a gravity, that "All light emitted from such a body would be made to return towards it". He said we wouldn't be able to see such a body, but we sure will feel it's gravitational pull.

- We could fly close to this "Dark star" and look around and describe the features of the object.

- A novelty, world lost interest when light was shown to be waves in 1803 by Thomas Young

Calculation of Escape Velocity for Earth

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$



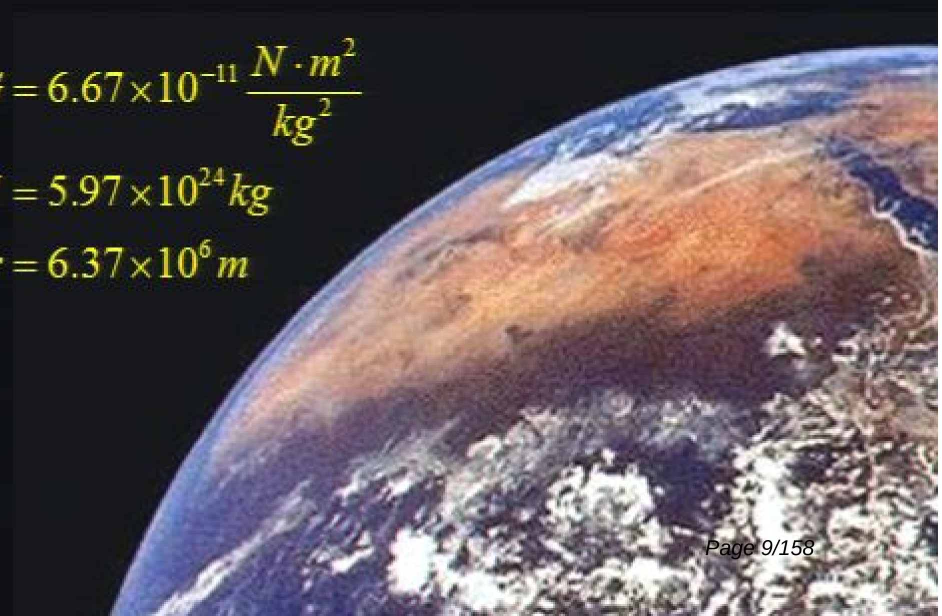
Calculation of Escape Velocity for Earth

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

$$G = 6.67\times10^{-11}\,\frac{N\cdot m^2}{kg^2}$$

$$M = 5.97\times10^{24}\,kg$$

$$r = 6.37\times10^{6}\,m$$



Calculate Escape Velocity

Calculation of Escape Velocity for Earth

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

$$\frac{1}{2}v^2 = \frac{GM}{r}$$

$$v = \sqrt{2\frac{\left(6.67\times10^{-11}\right)\left(5.97\times10^{24}\right)}{6.37\times10^{6}}}$$

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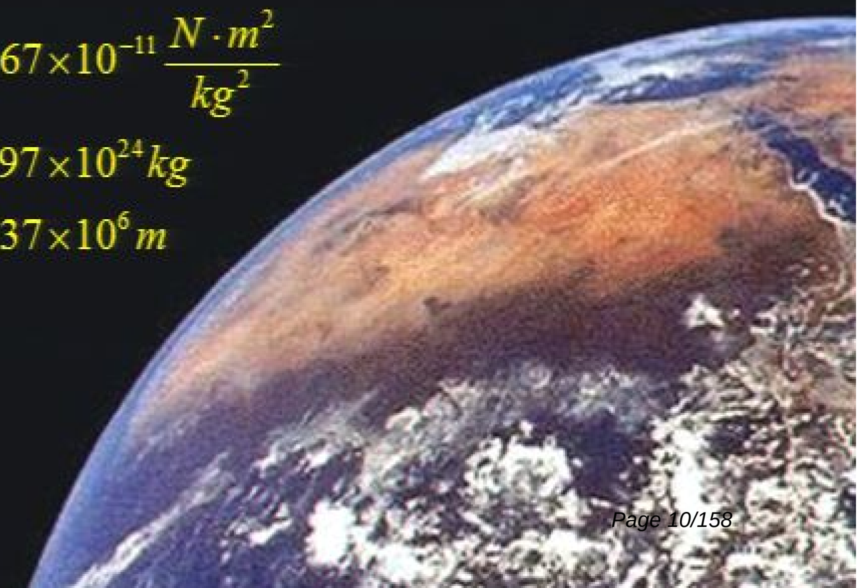

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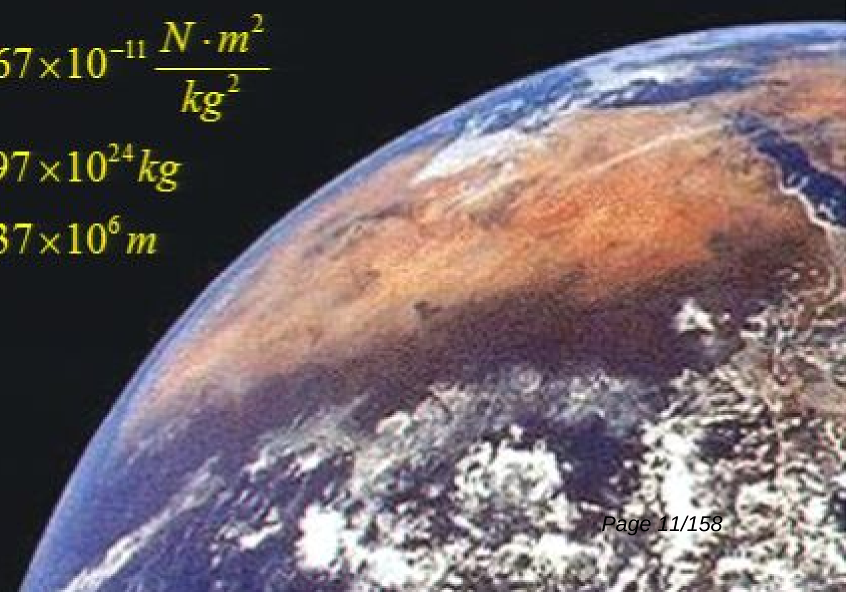

$$v \approx 11181 m/s$$

11.2 km/s

What is Earth's r when escape velocity is 3×10^8 m/s

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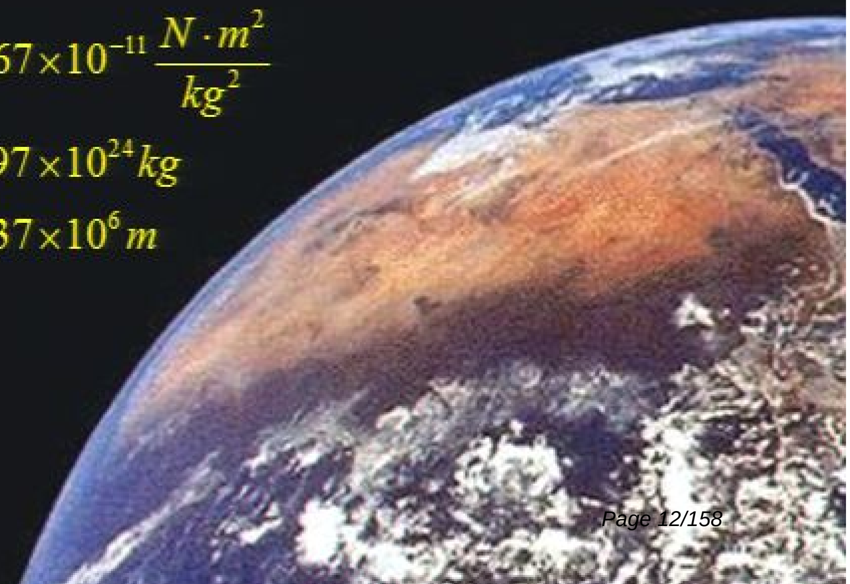

What is Earth's r when escape velocity is $3x10^8$ m/s

~8.9 mm

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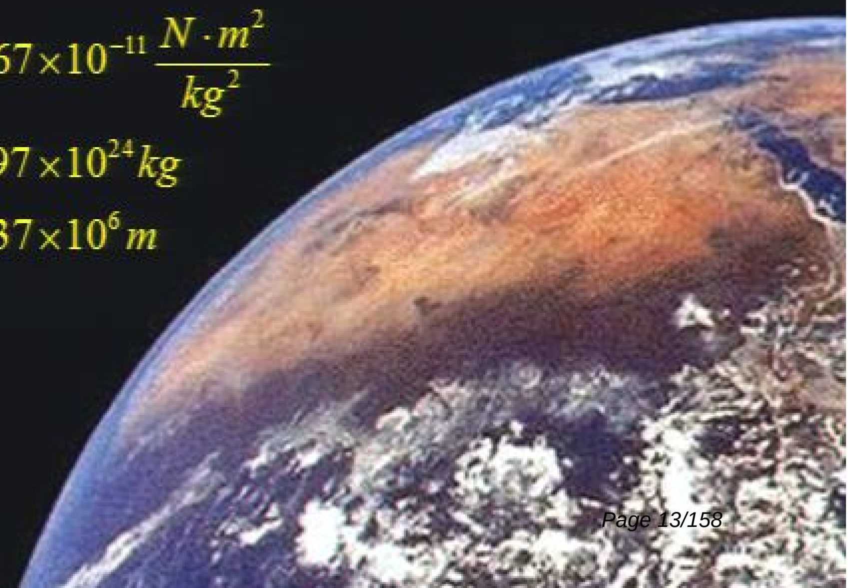
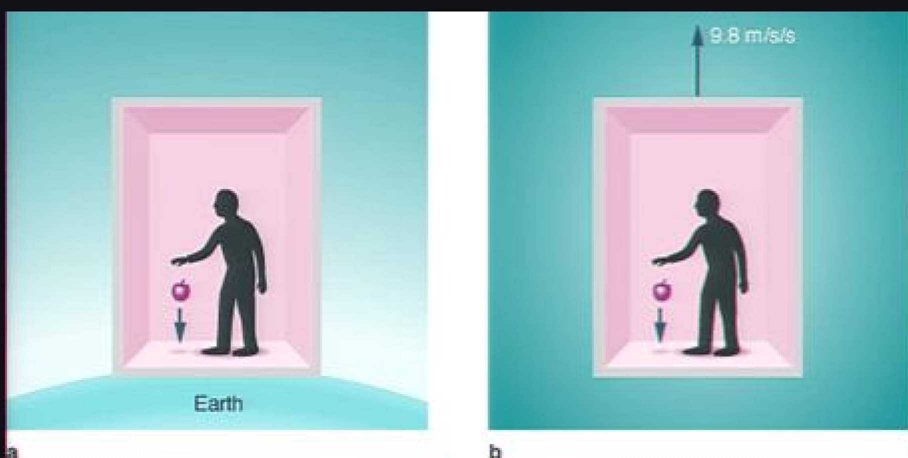
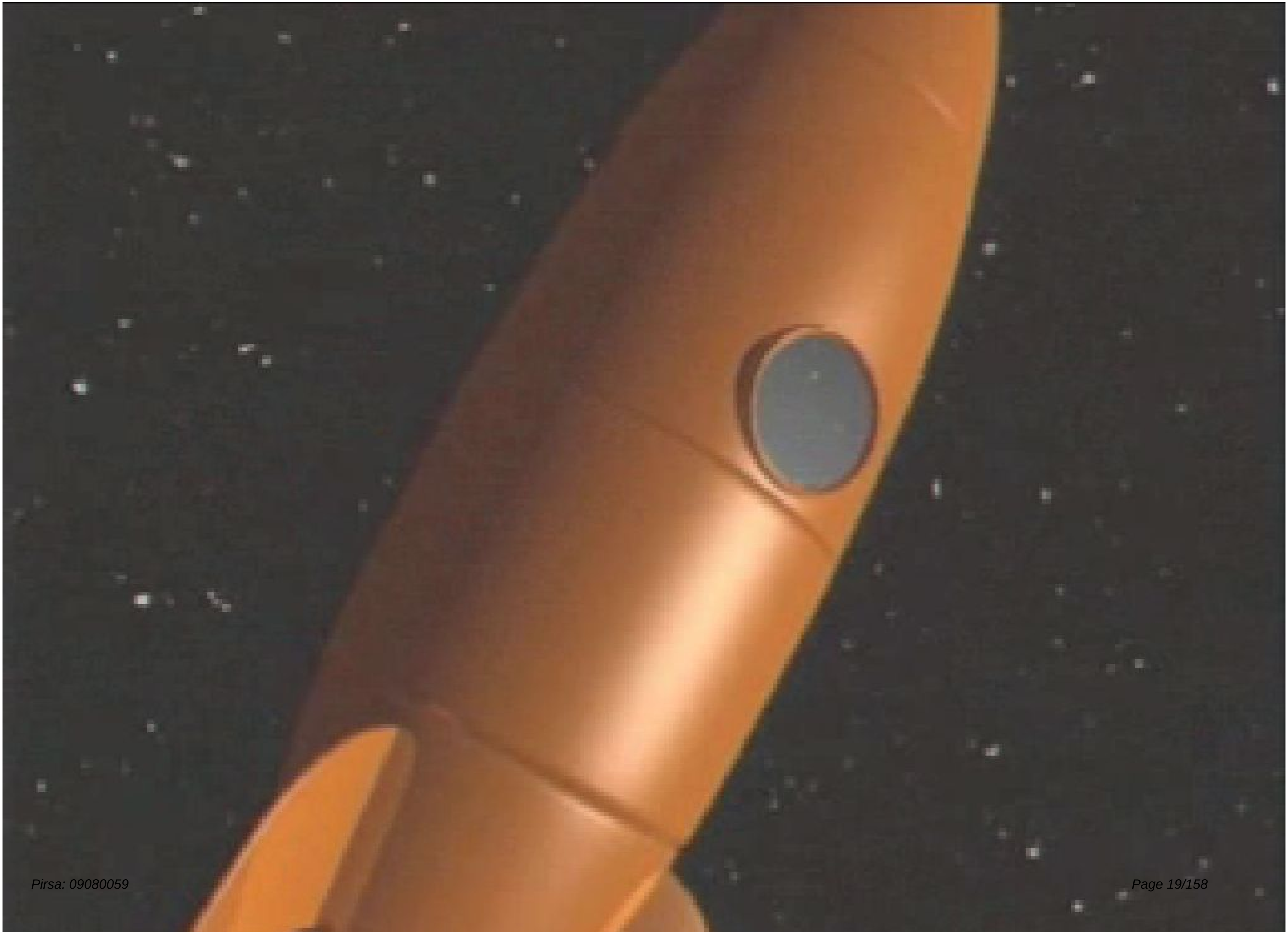
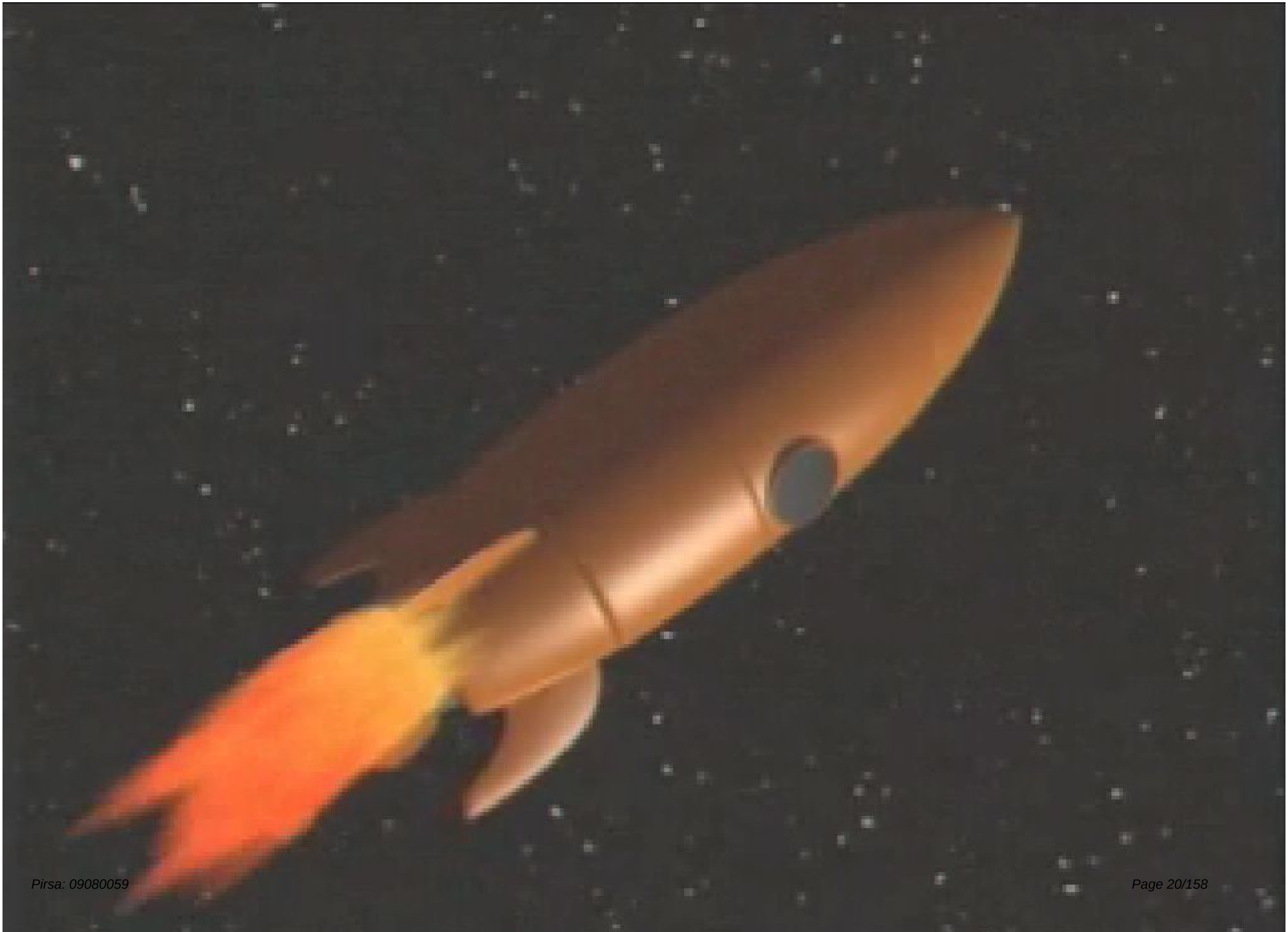


Einstein's Equivalence Principle

- There is no experiment that you can perform that will distinguish these two diagrams







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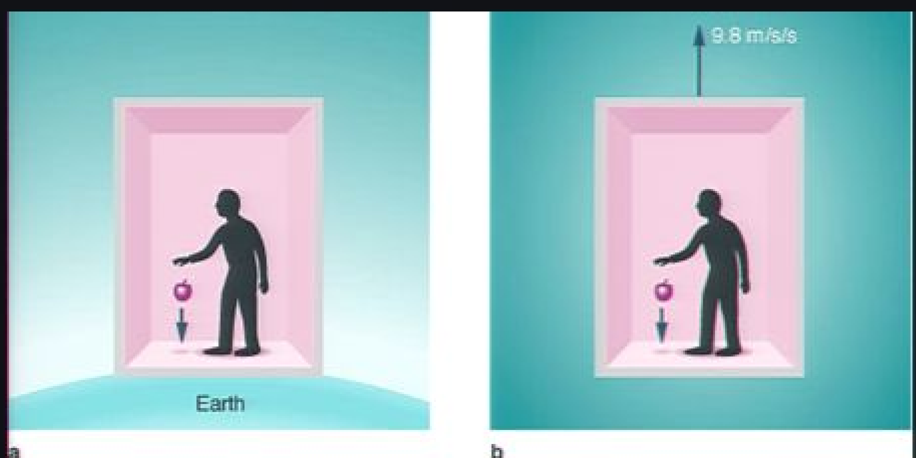
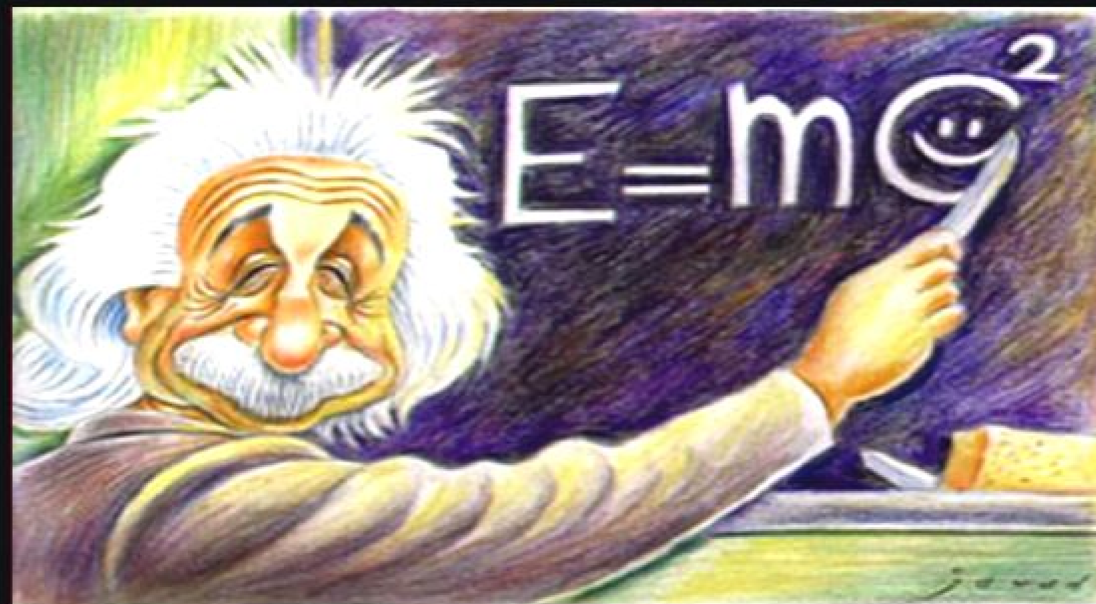


Gravity as a Curvature of Spacetime

The early 1900's changed the way gravity is looked at. Einstein didn't think of gravity as a force between objects, but as a curving of "straight lines" due to mass. Light always follows these straight lines.
Time also slows down near masses (space and time are different parts of "spacetime", which is what gets bent).



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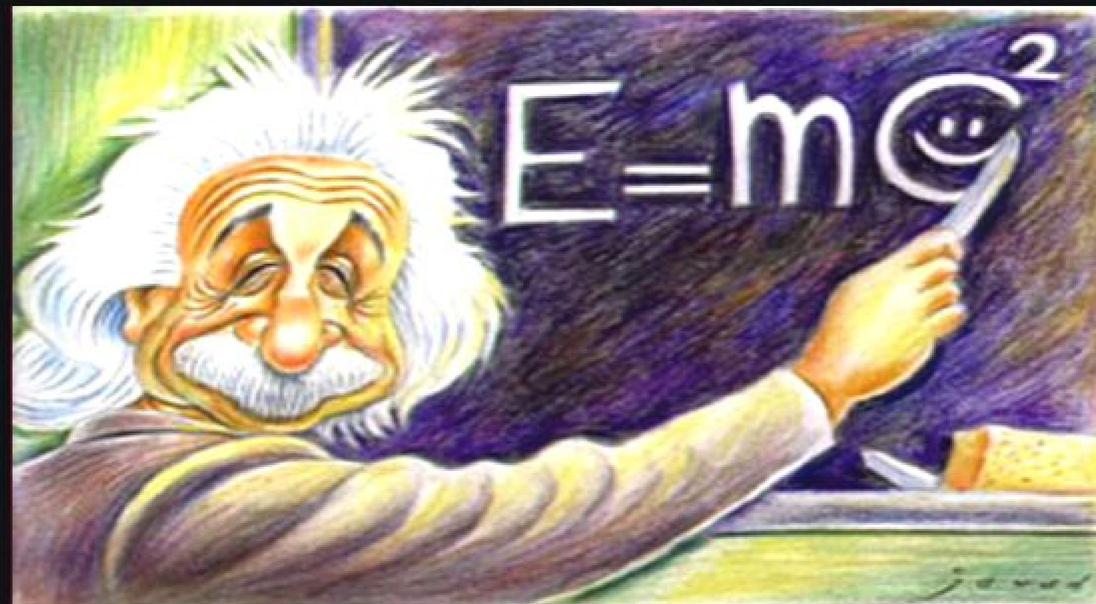

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Encodes geometry (curving) of Space Time

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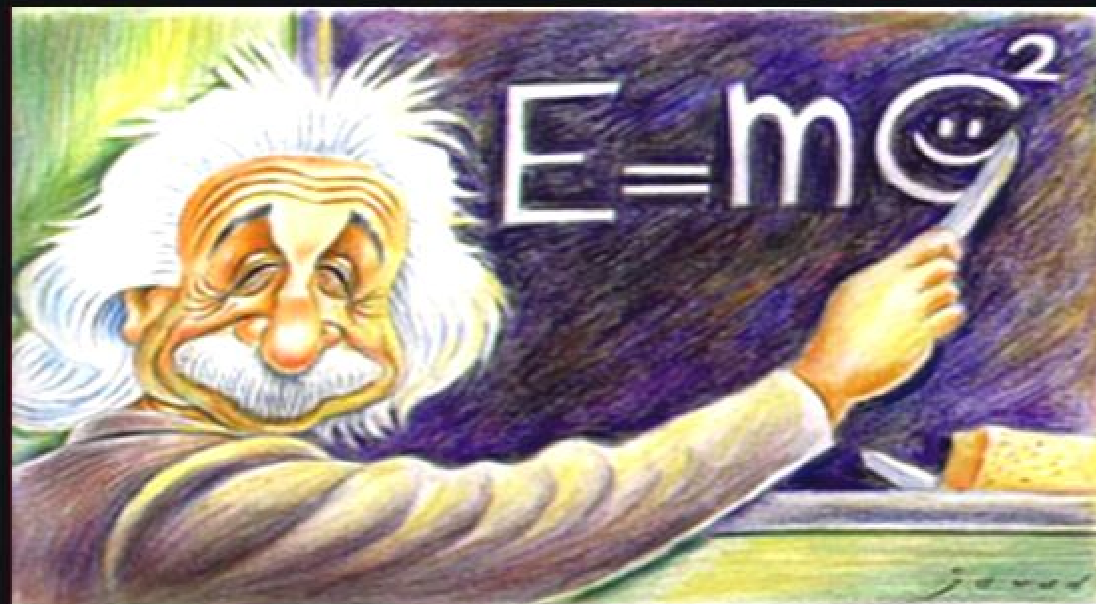

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Encodes geometry (curving) of Space Time

Cosmological Constant

Encodes the distribution of matter and its motion in Space Time

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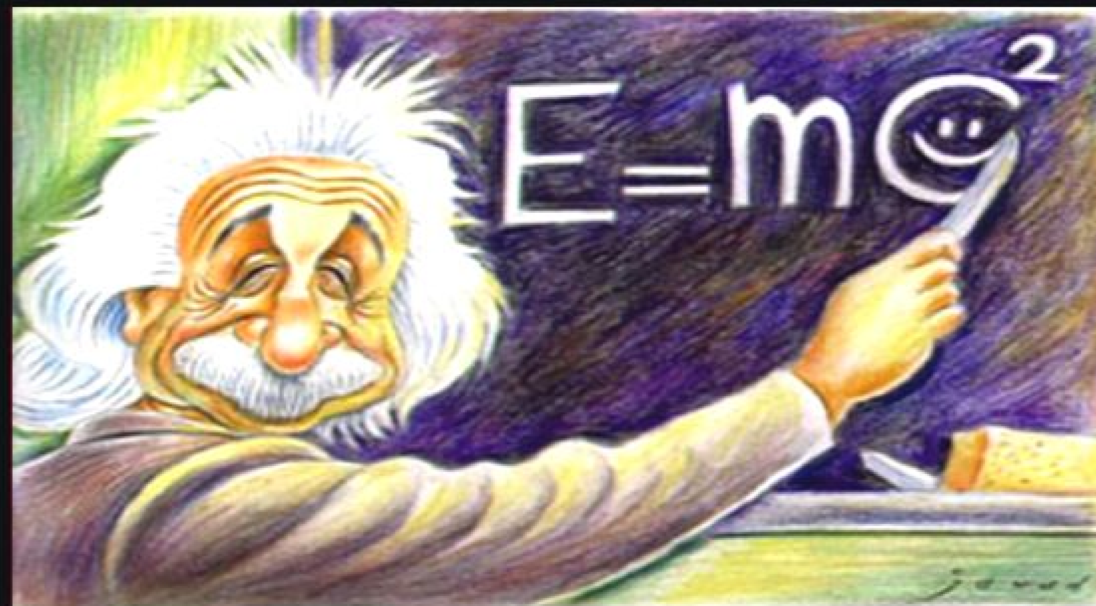

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Encodes the distribution of matter and its motion in Space Time

Einstein Field Equation: another dissection

- Generally speaking, Einstein field equation: $G_{\mu\nu} = 8\pi T_{\mu\nu}$

is coupled elliptic-hyperbolic nonlinear partial differential equations for the metric components.

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"metric components" – components of the metric tensor $g_{\mu\nu}$

Let's Review

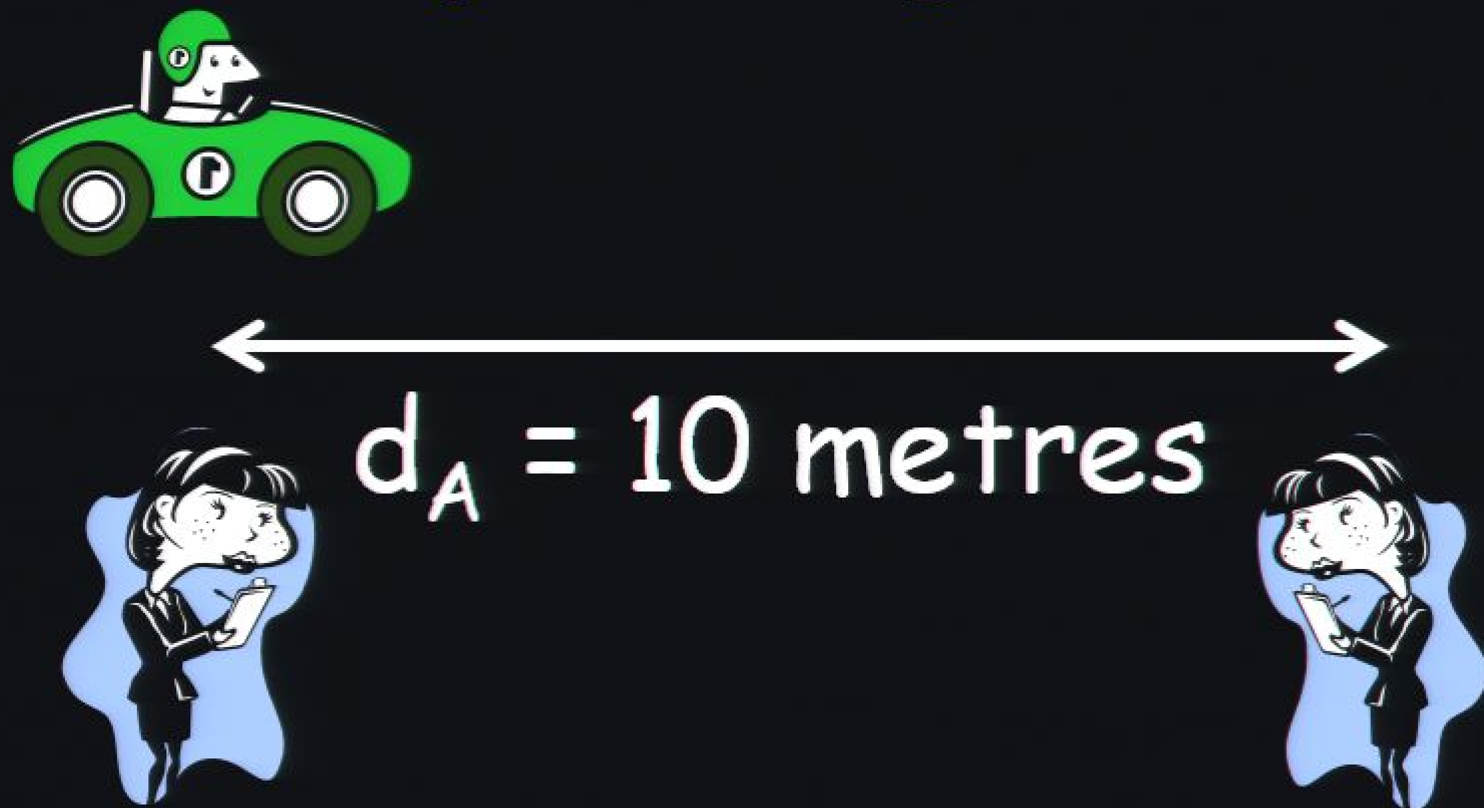
SPACETIME

Space Diagram

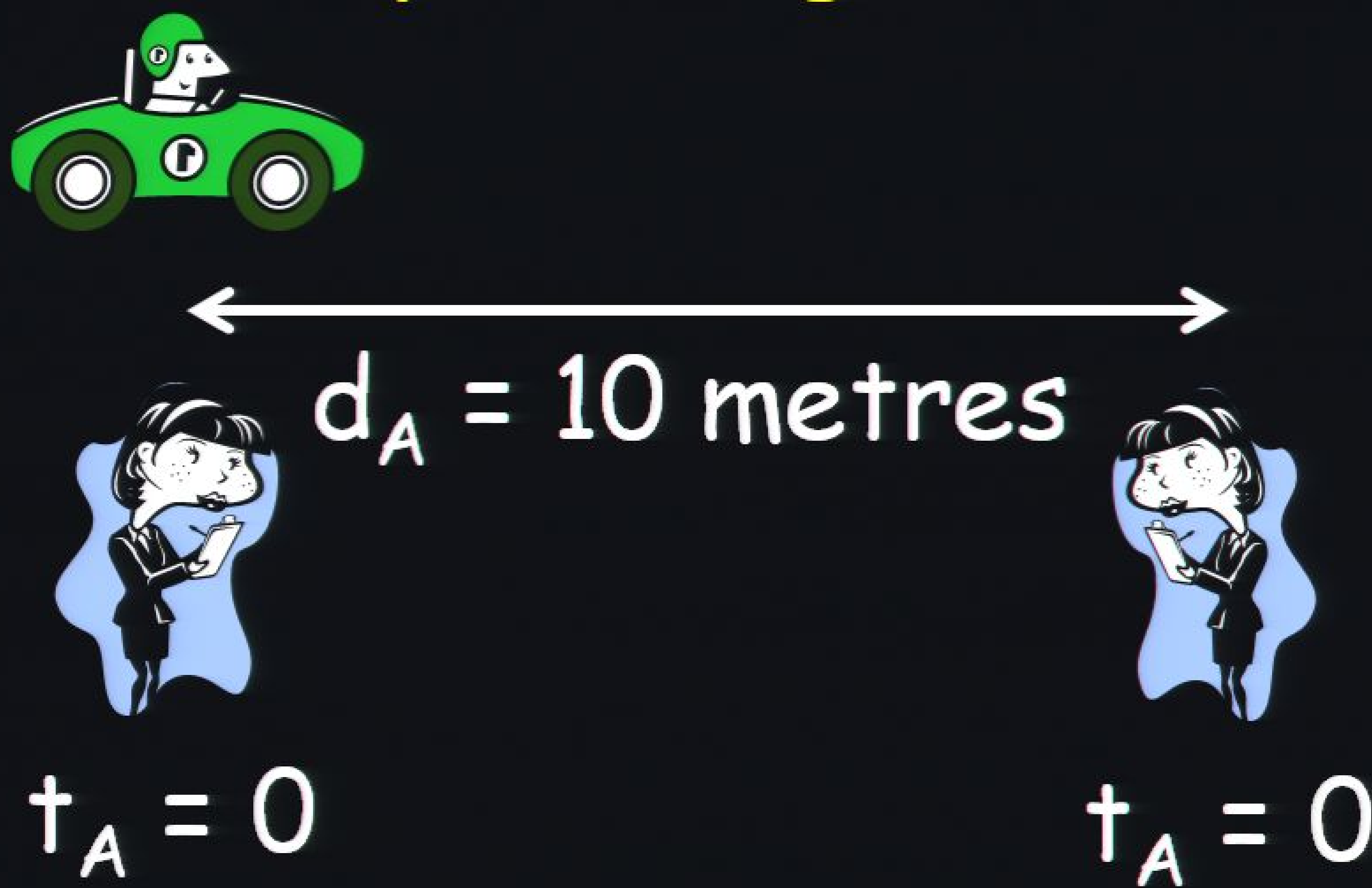
Space Diagram

Alice's twin sister, Alice

Space Diagram



Space Diagram



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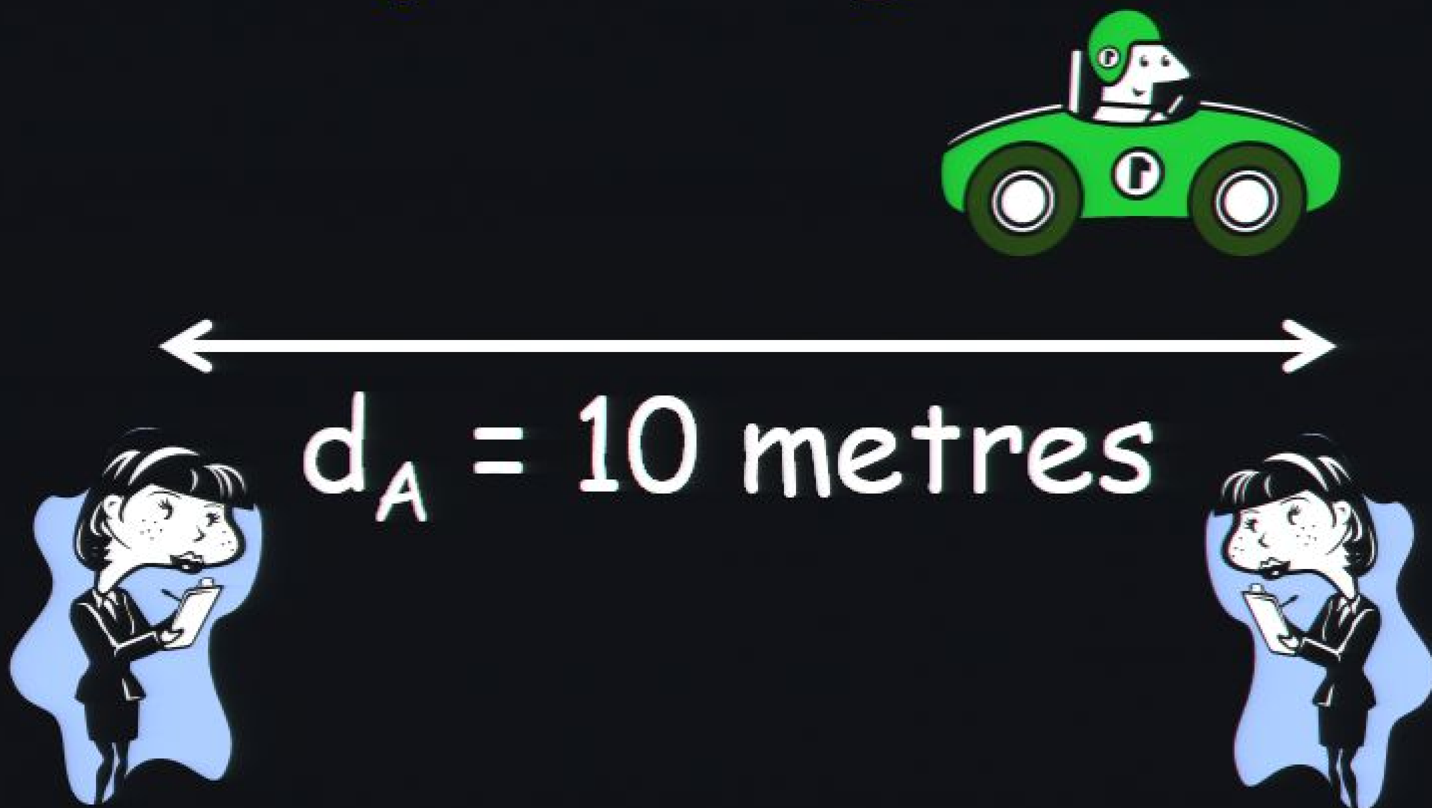
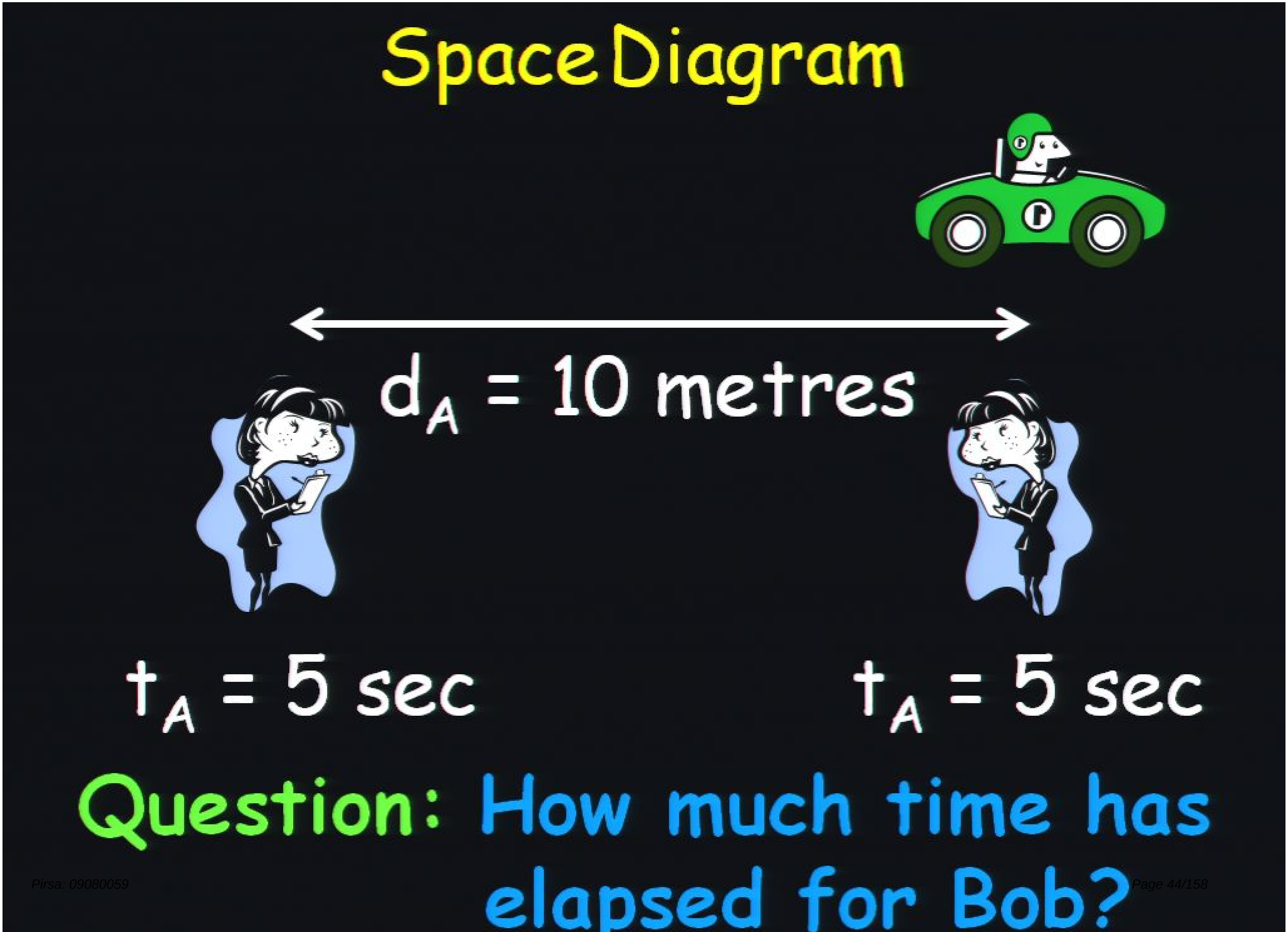


Space Diagram

d_A = 10 metres

t_A = 5 sec

t_A = 5 sec


Space Diagram
d_A = 10 metres
t_A = 5 sec
t_A = 5 sec
Question: How much time has elapsed for Bob?

Draw a “Spacetime Diagram”

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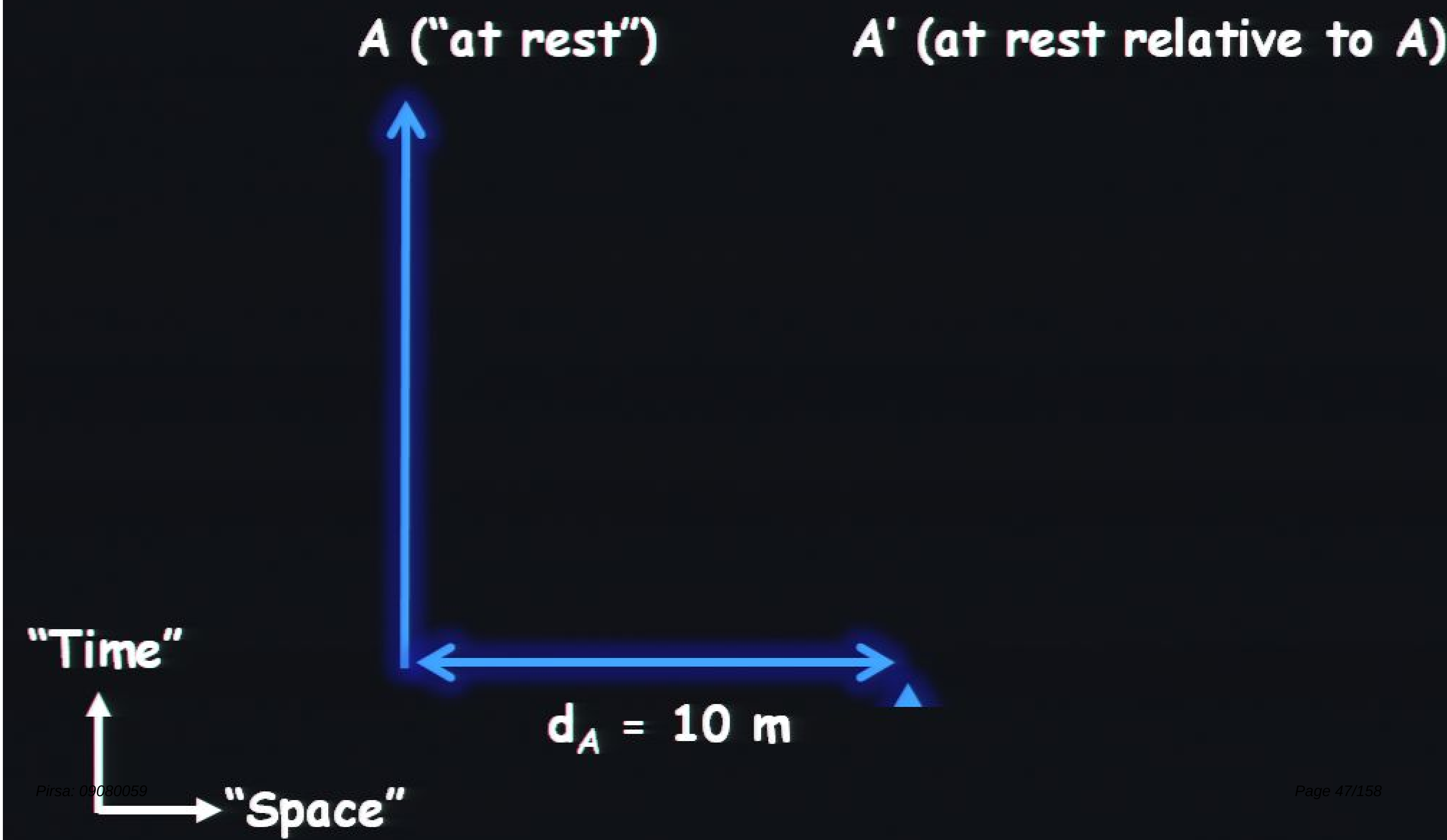

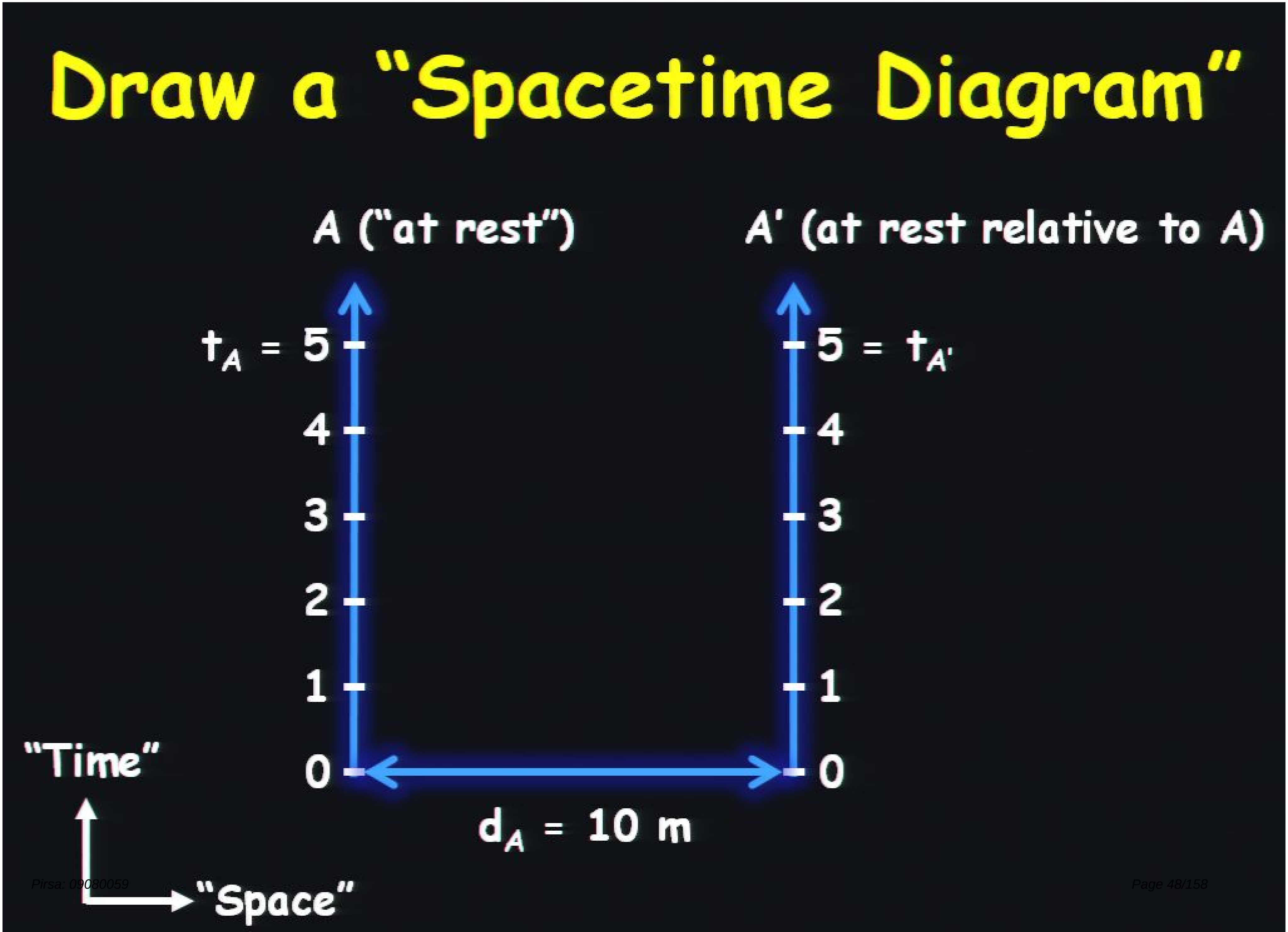

Draw a "Spacetime Diagram"
A ("at rest")
A' (at rest relative to A)
t_A = 5
4
3
2
1
0
5 = $t_{A'}$
4
3
2
1
0
"Time"
d_A = 10 m
"Space"

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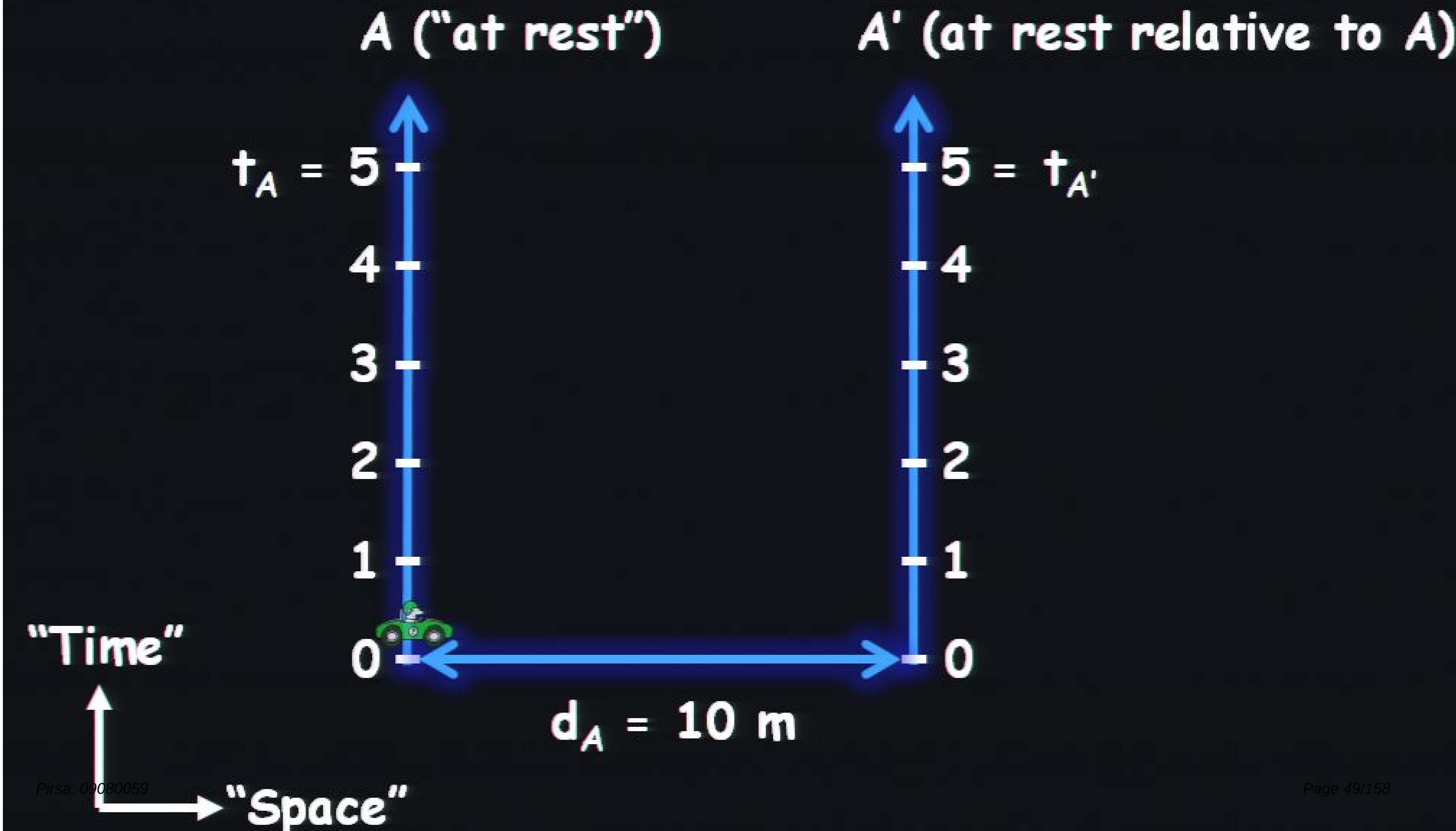

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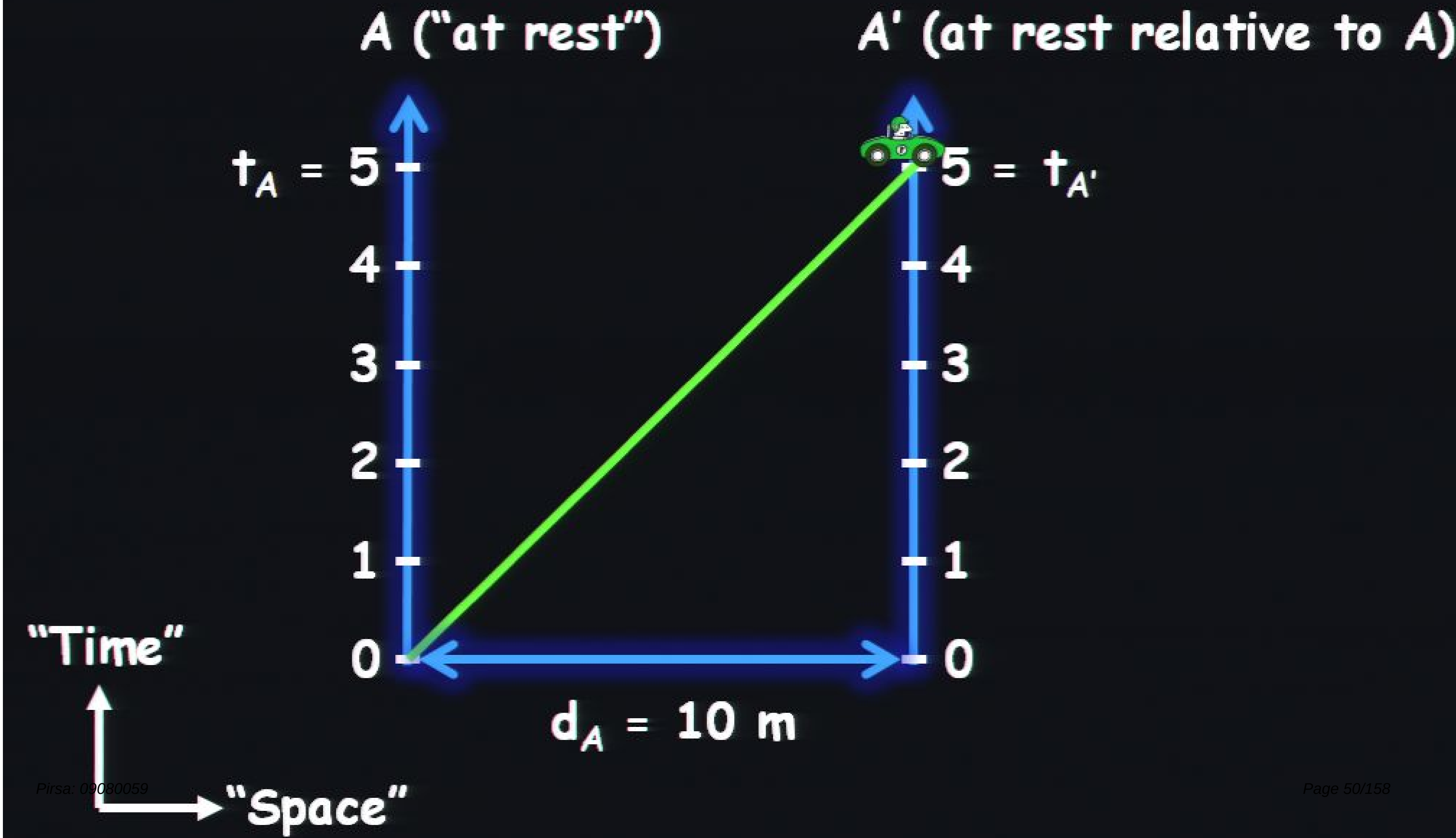

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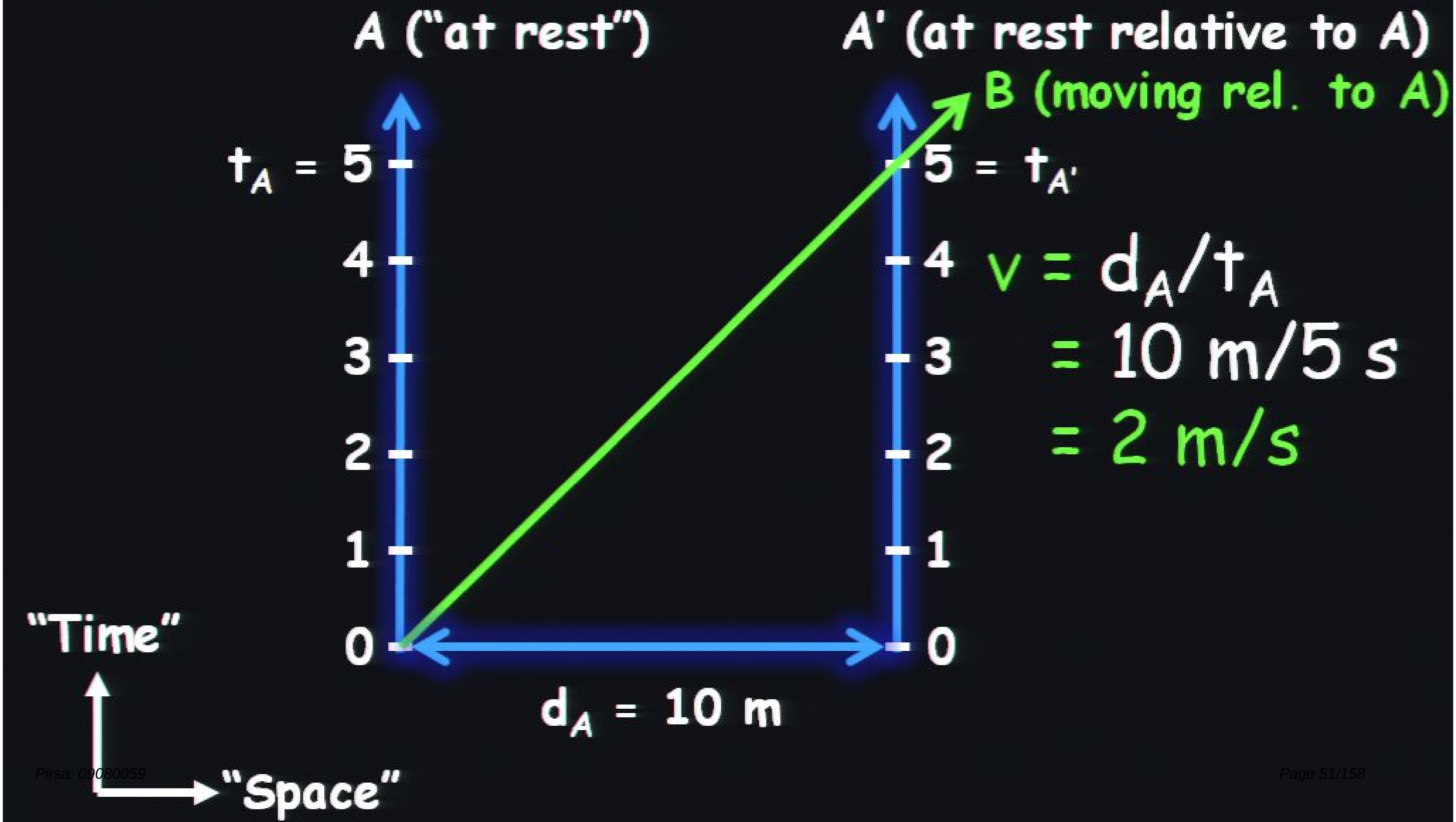


$v = d_A/t_A$

$= 10\ m/5\ s$

$= 2\ m/s$

Let's Have Spacetime Fun!

Sketch spacetime diagrams for each:

1: Bob at rest relative to Alice

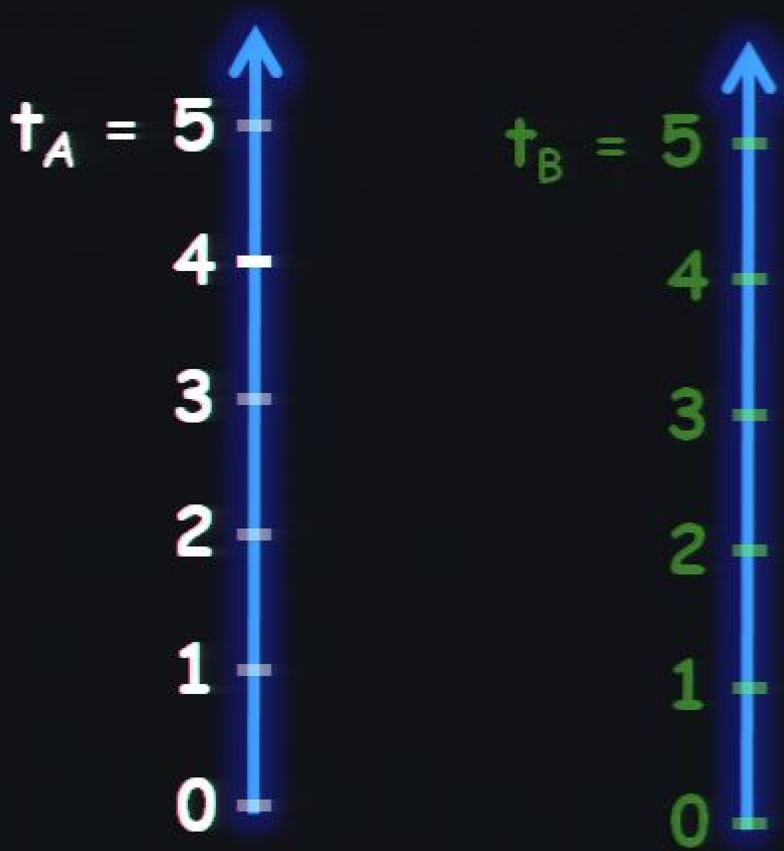
2: Alice tossing a baseball up

3: Bob moving Fast

4: Bob moving Slow

5: The Earth orbiting about the Sun

Bob at rest relative to Alice



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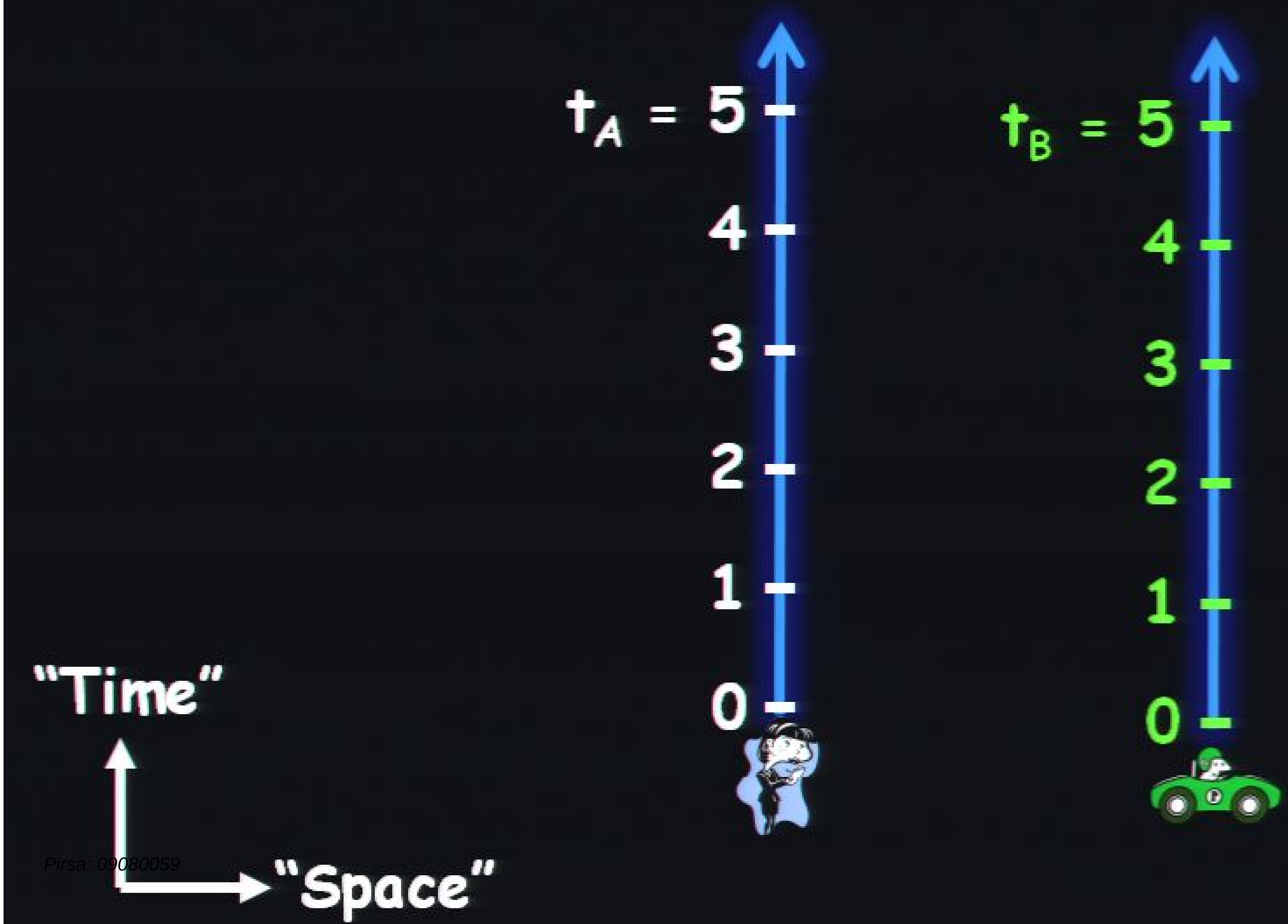
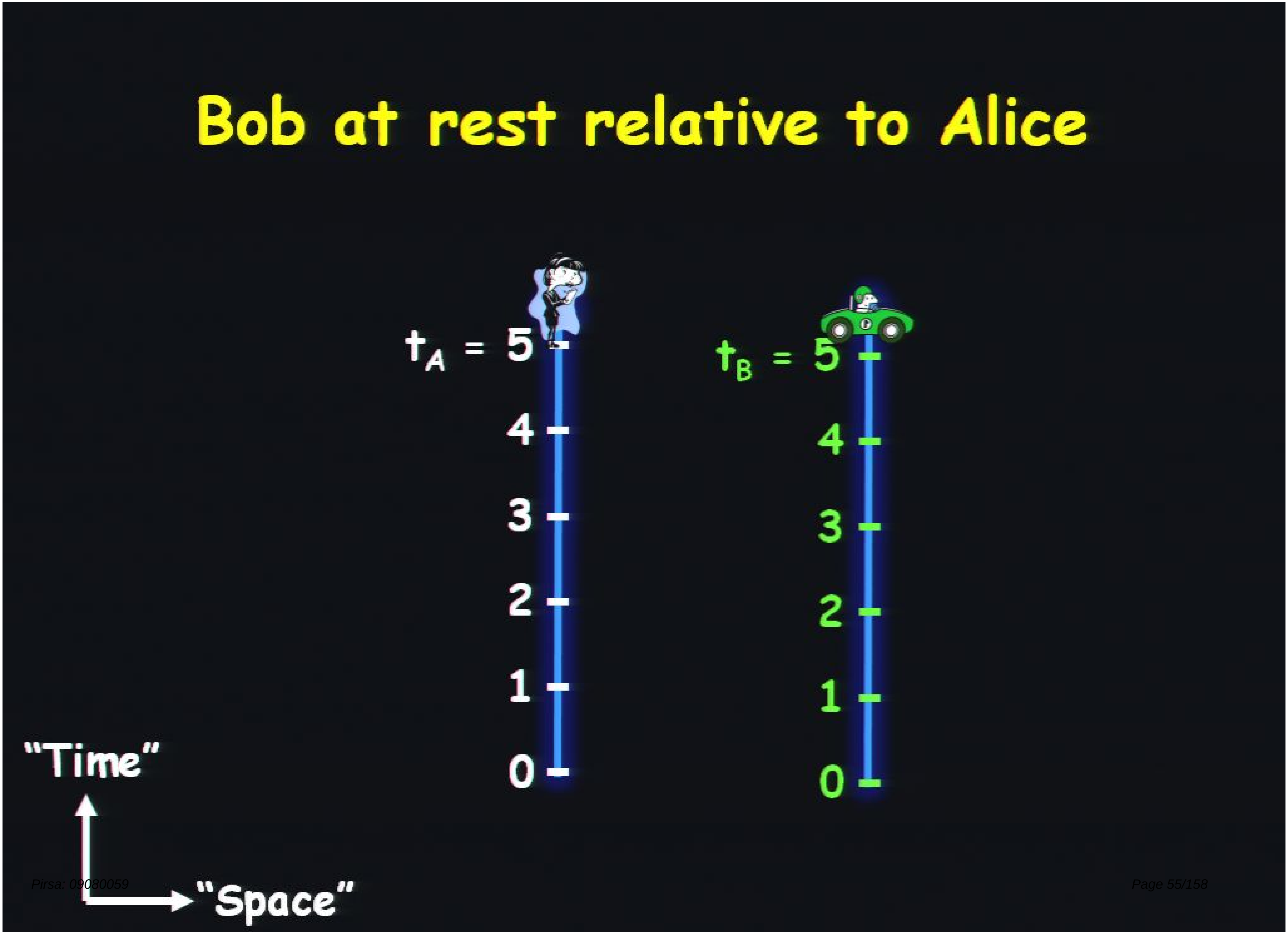
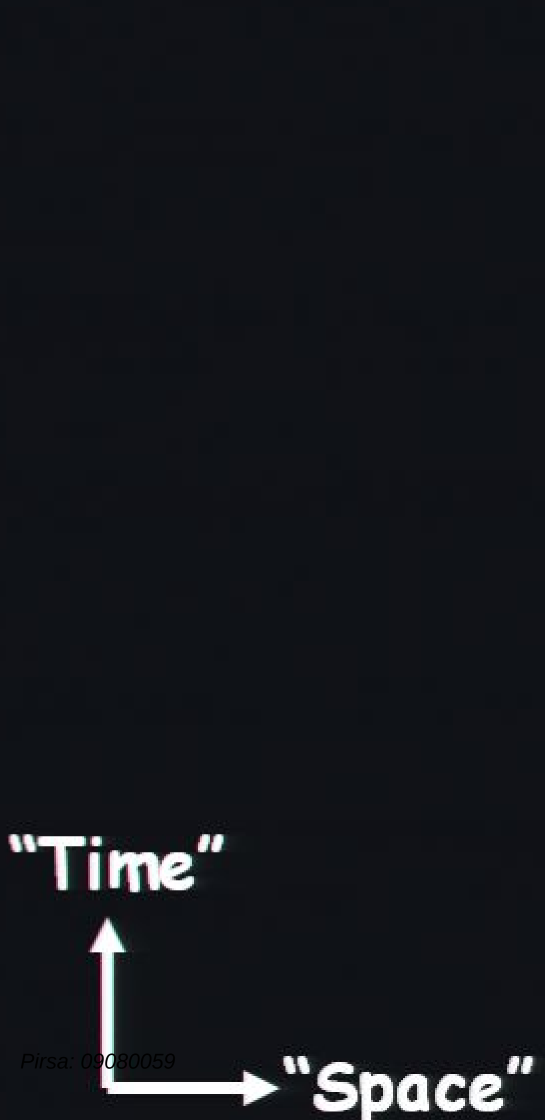



Bob at rest relative to Alice
t_A = 5
4
3
2
1
0
t_B = 5
4
3
2
1
0
"Time"
"Space"

Alice Tossing a Baseball Up

Alice Tossing a Baseball Up



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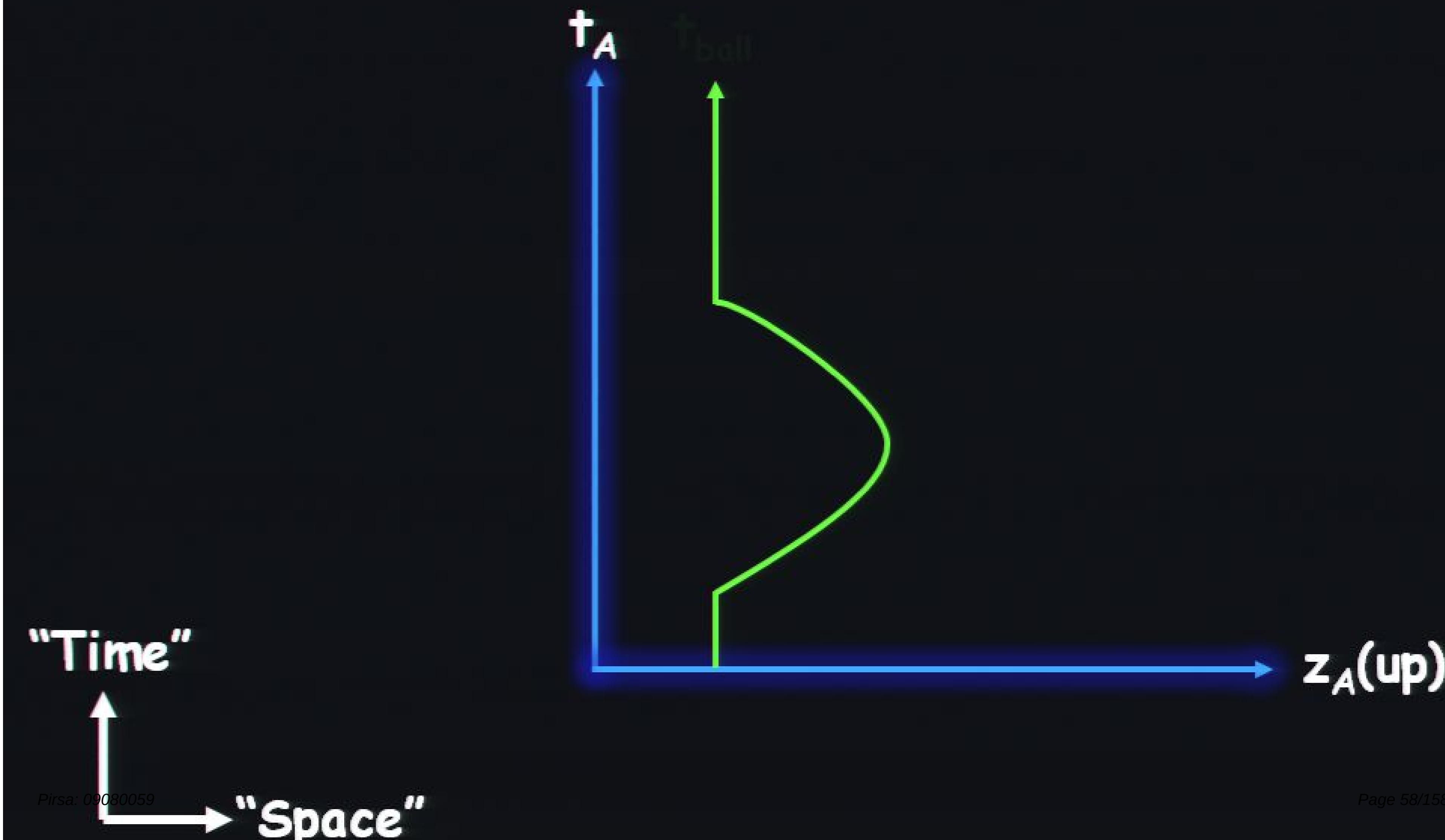

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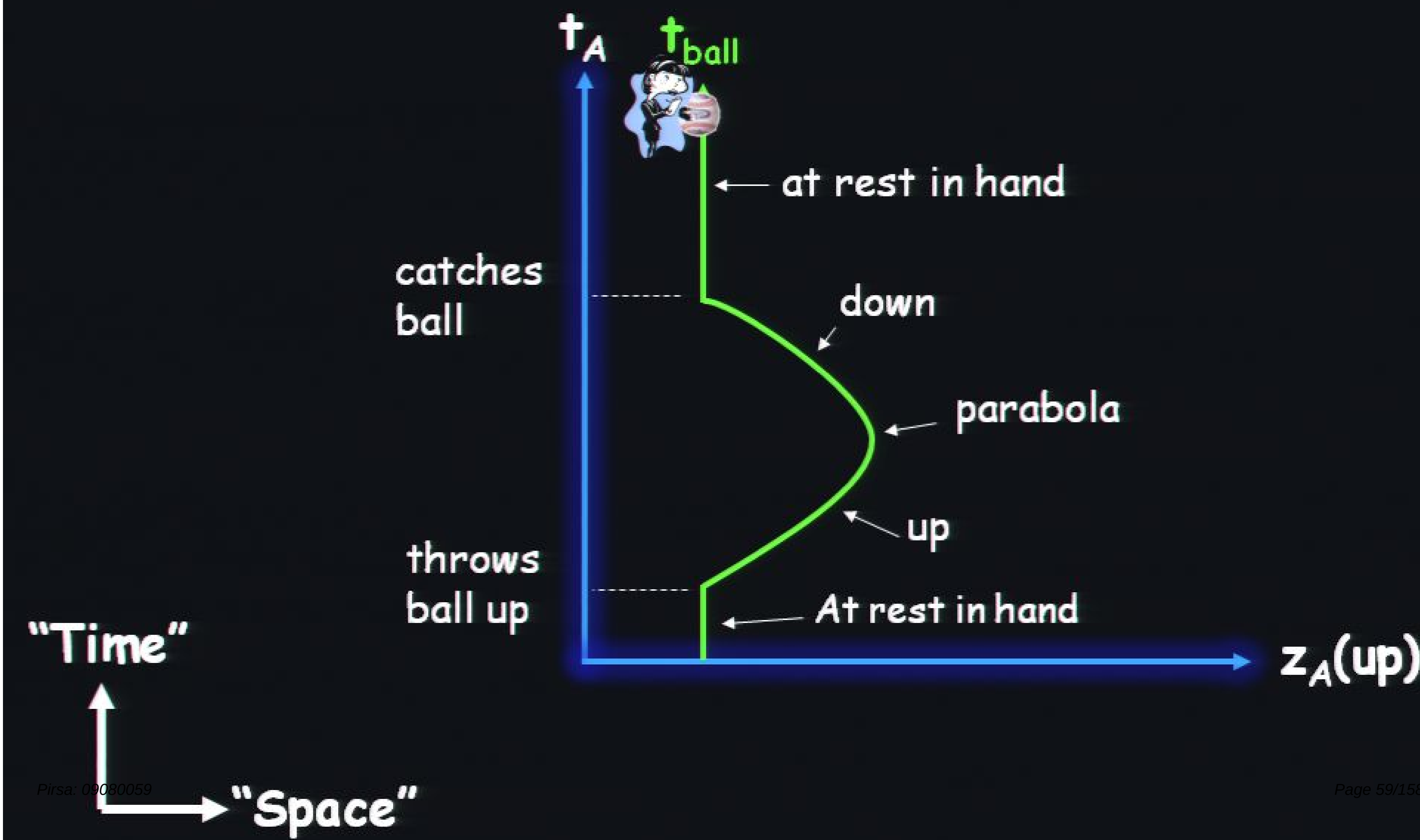
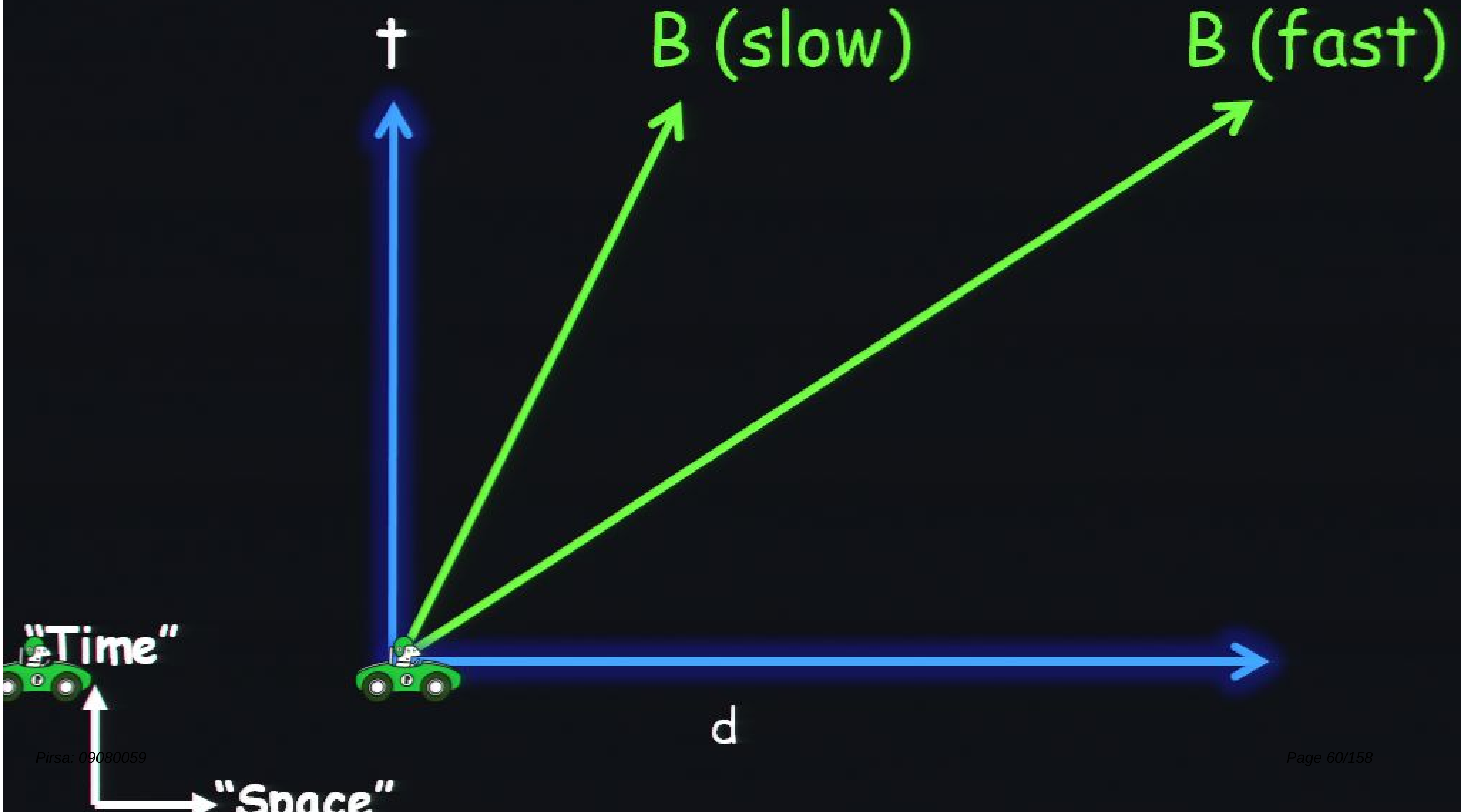


Bob Moving Fast and Slow



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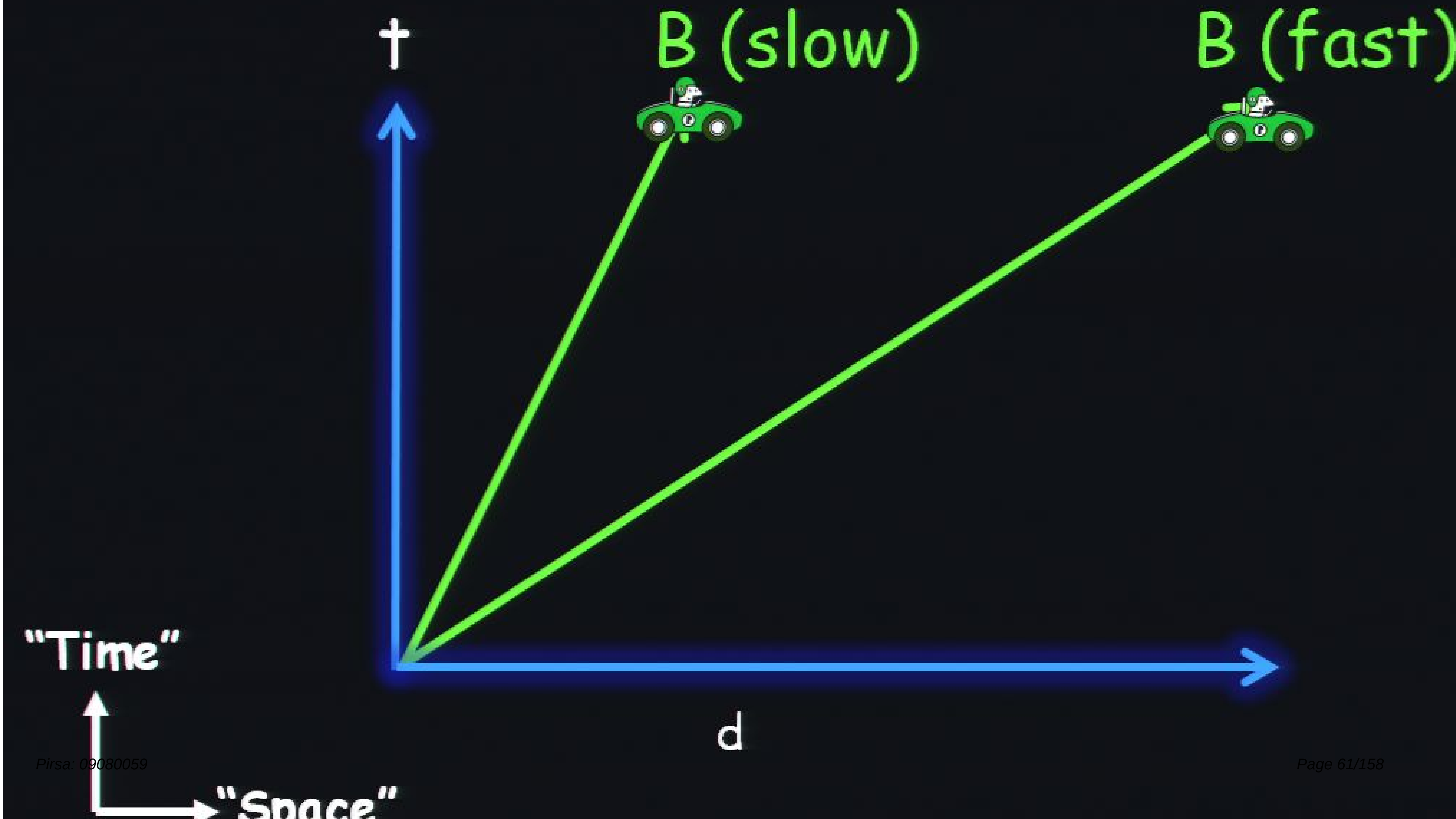
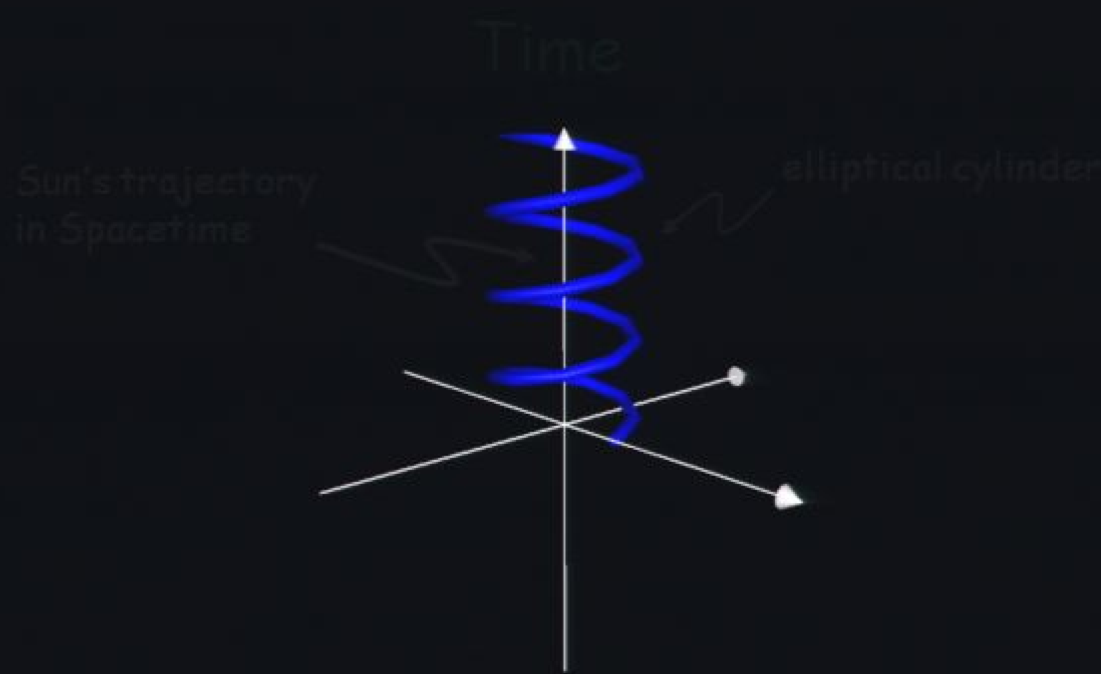


Earth Orbiting the Sun



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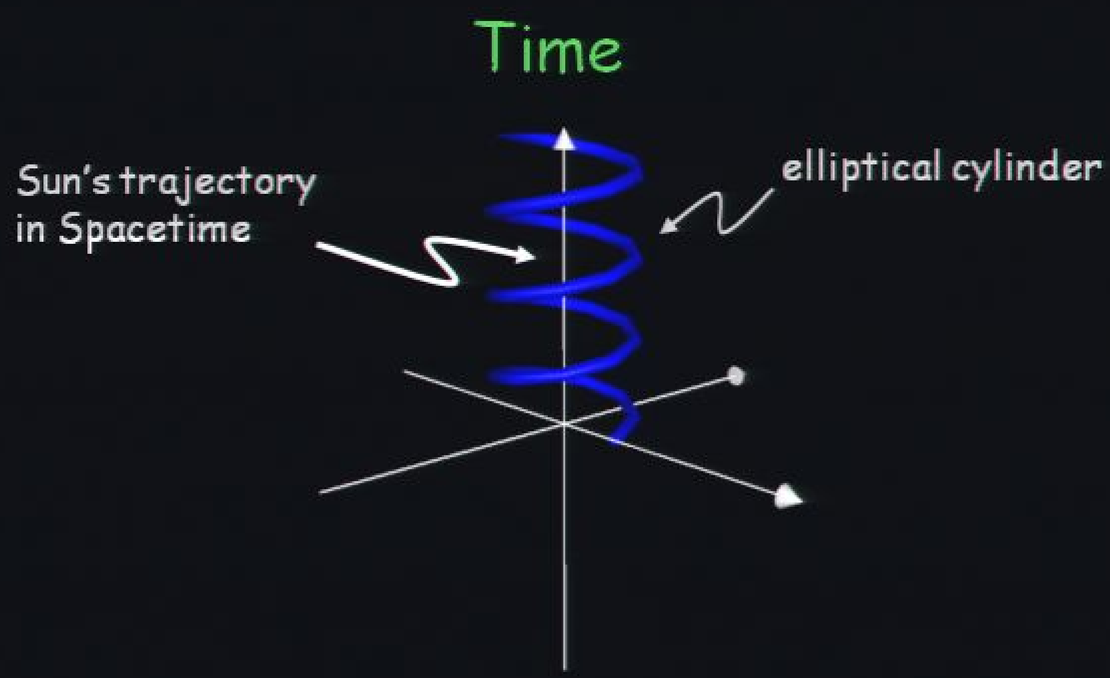

y

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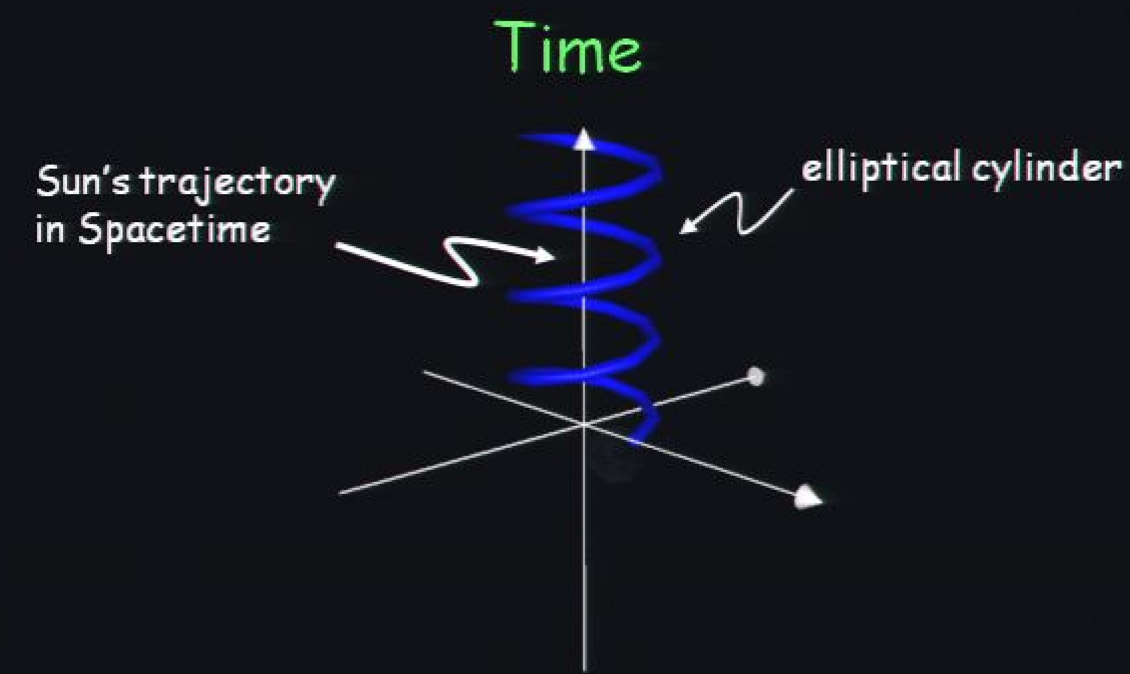
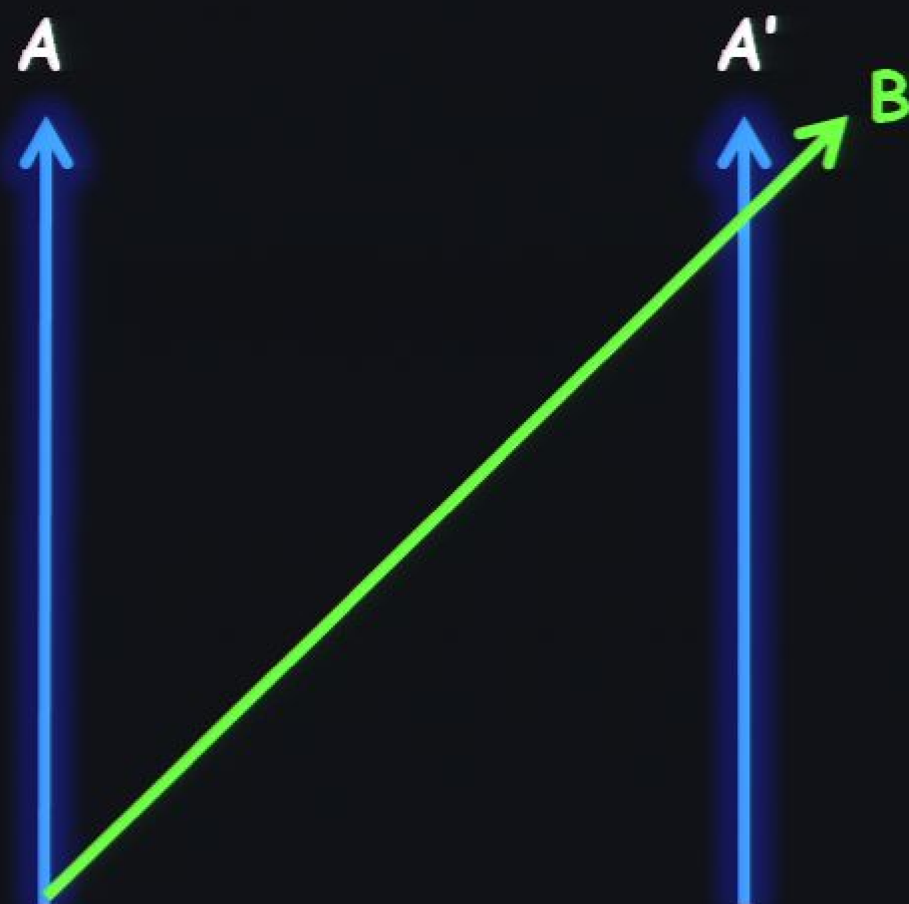


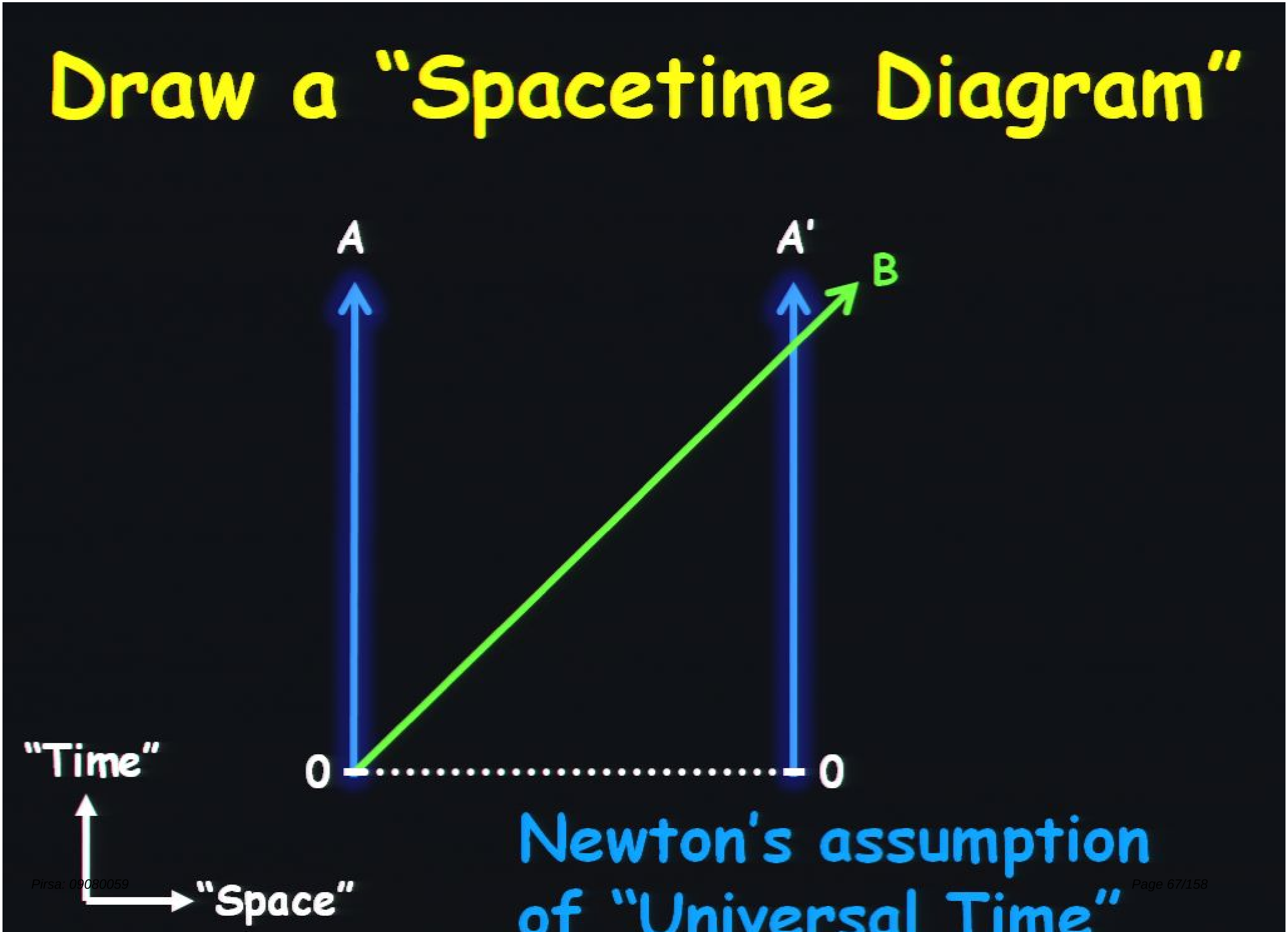
Earth's Trajectory

Back to Bob and Alice

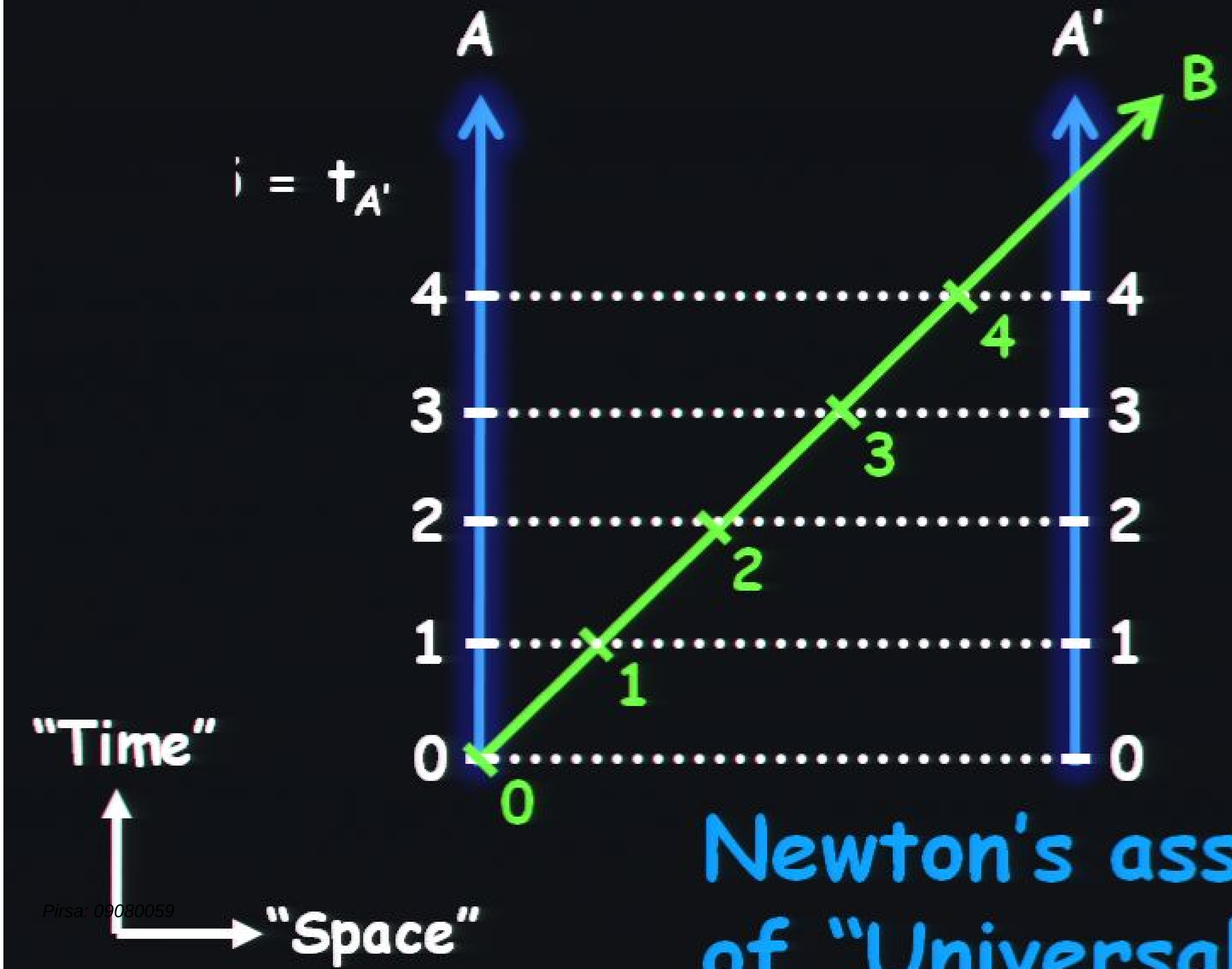
Draw a "Spacetime Diagram"



How do we indicate the ticking of B's clock?


Draw a "Spacetime Diagram"
A
A'
B
"Time"
0
0
"Space"
Newton's assumption
of "Universal Time"

Draw a “Spacetime Diagram”



Newton’s assumption of “Universal Time”

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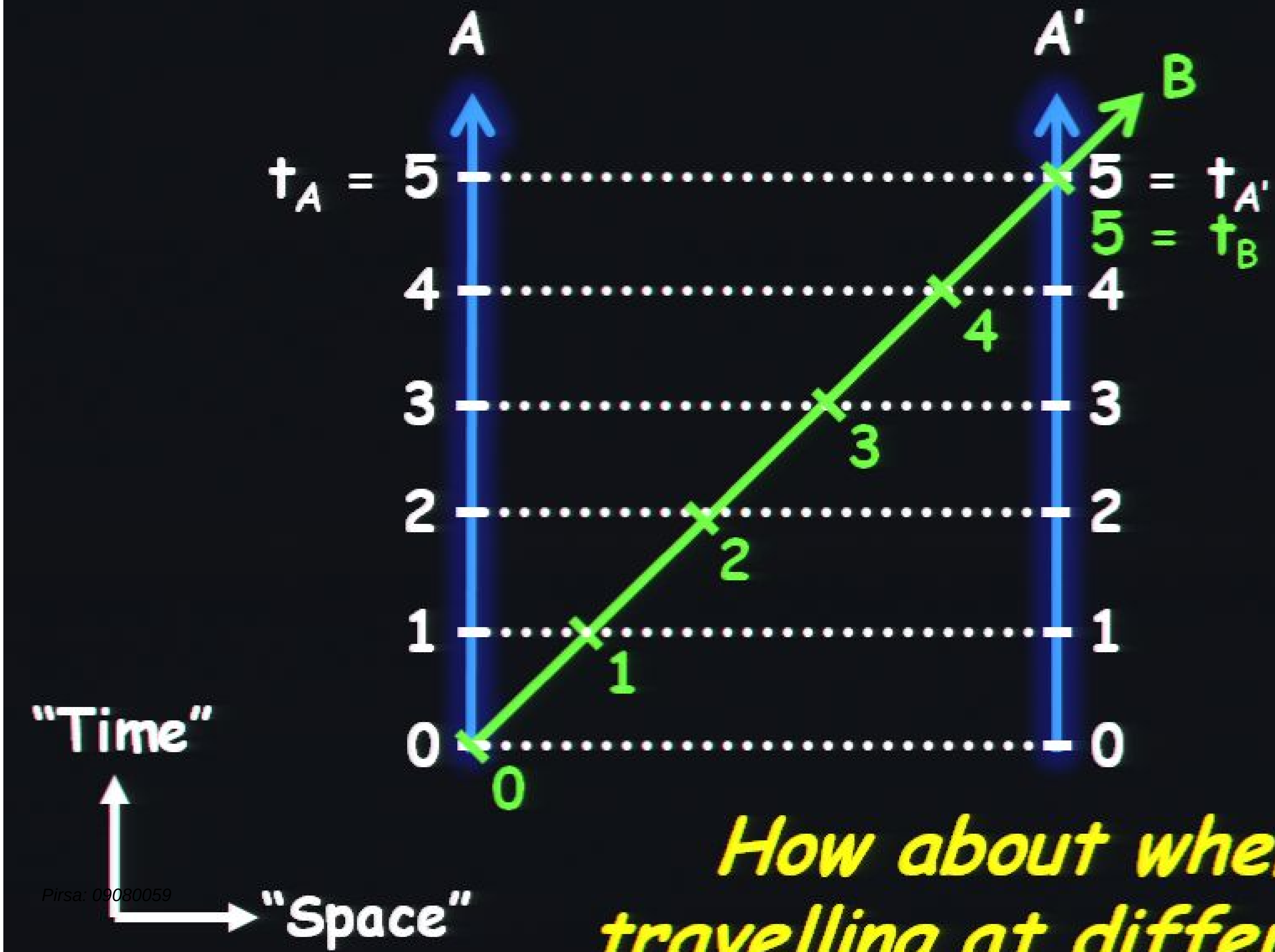
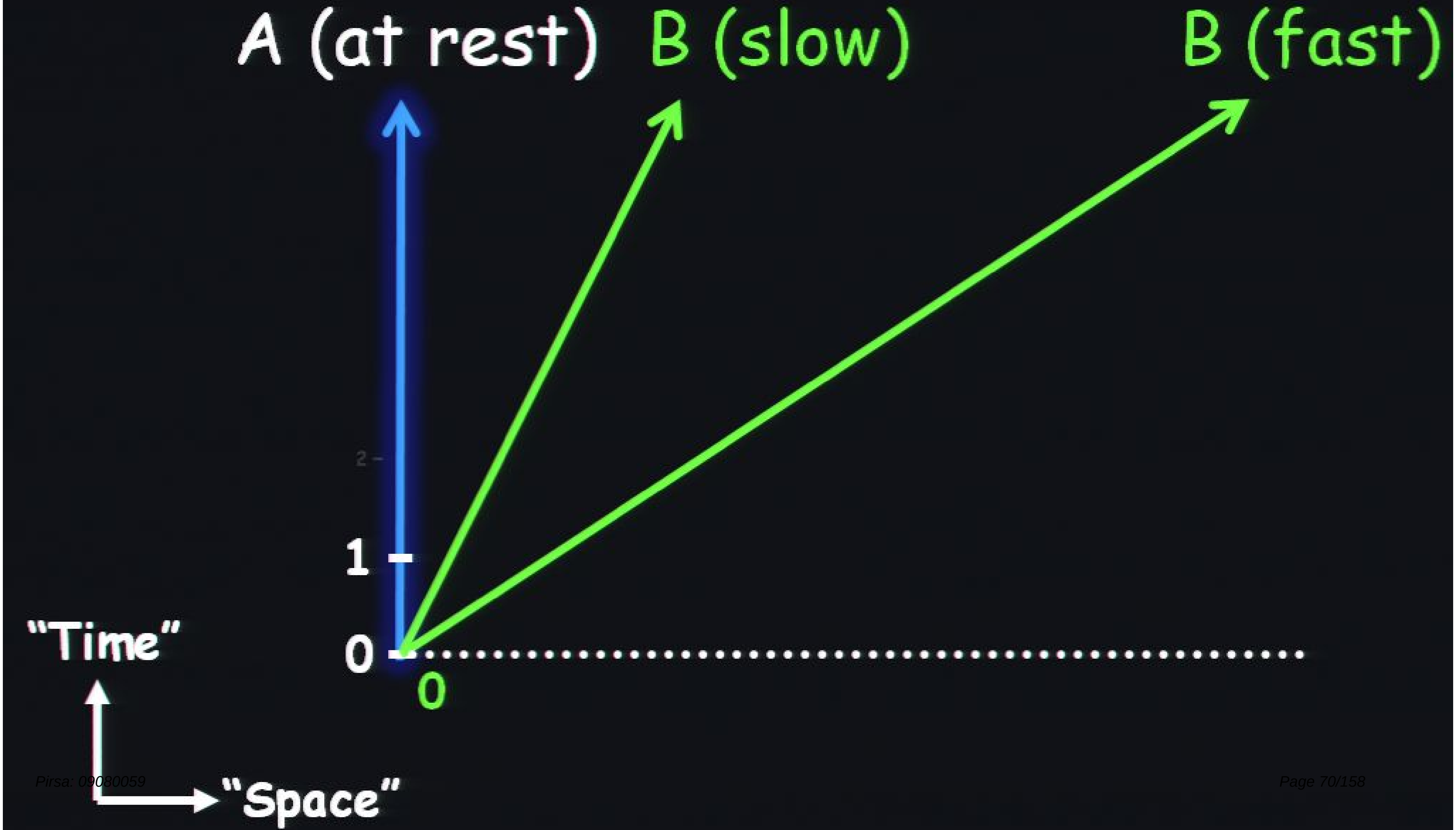
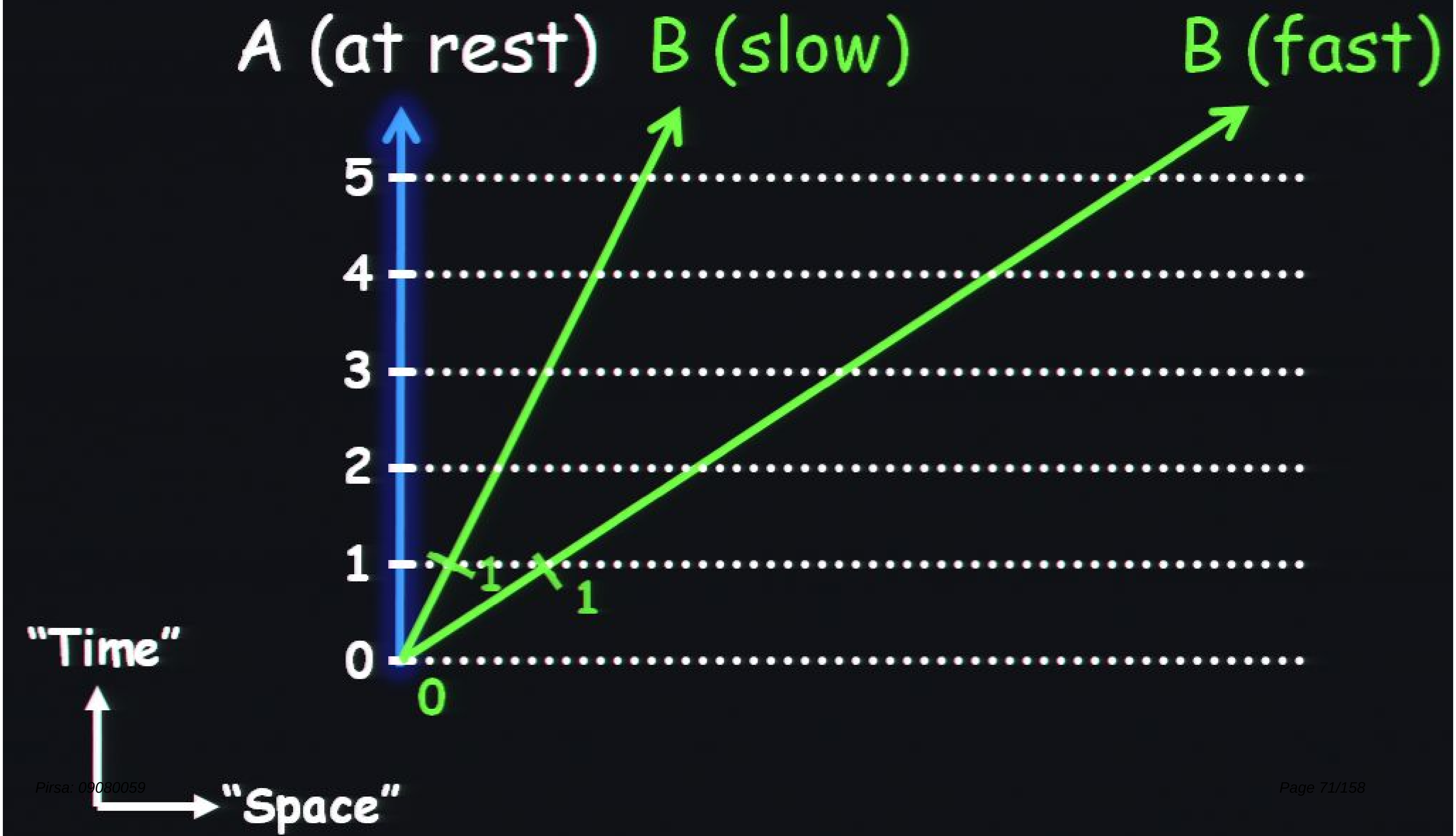


How about when Bob is travelling at different speeds

Newton's "Universal Time"



Newton's "Universal Time"



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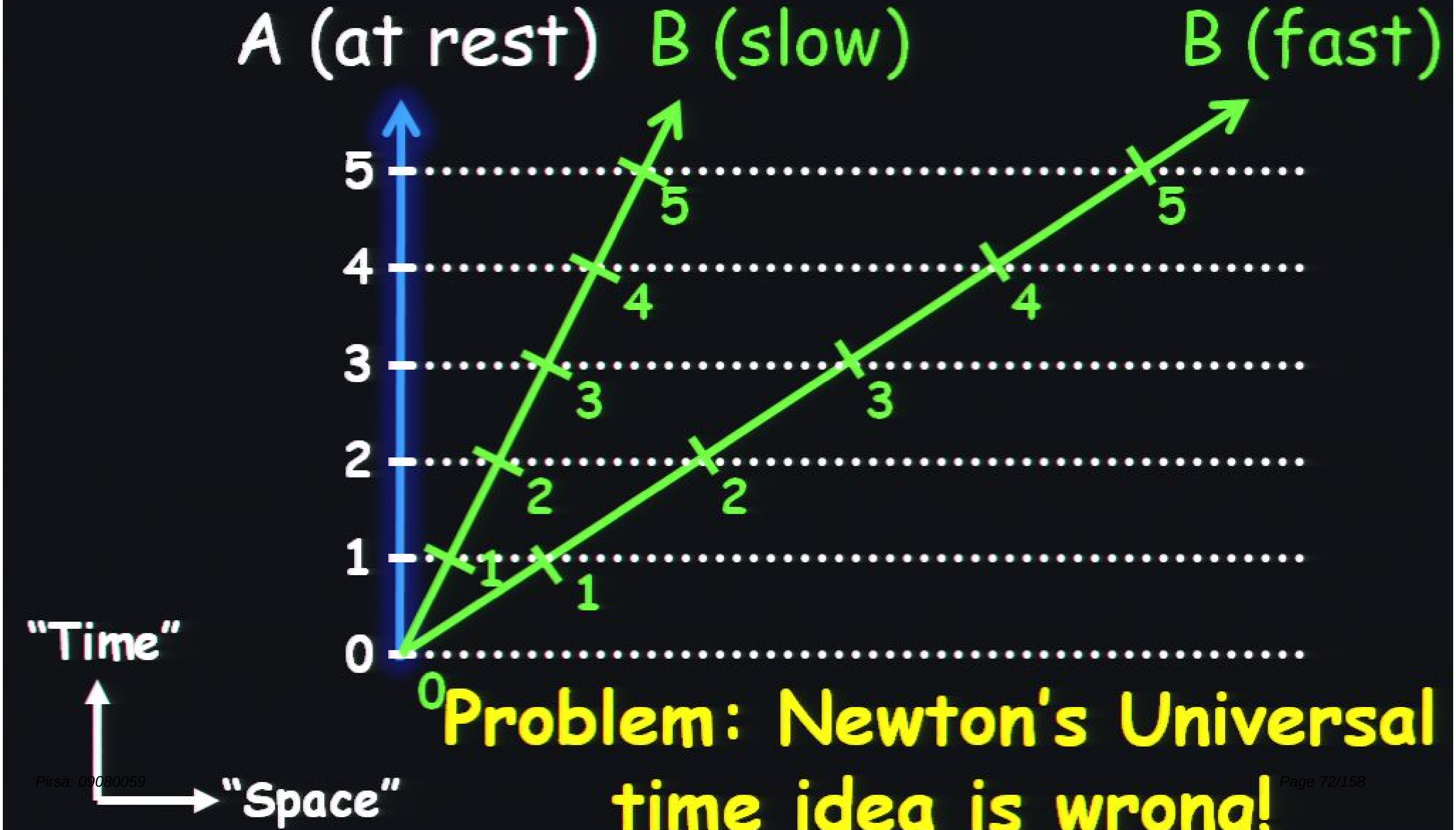
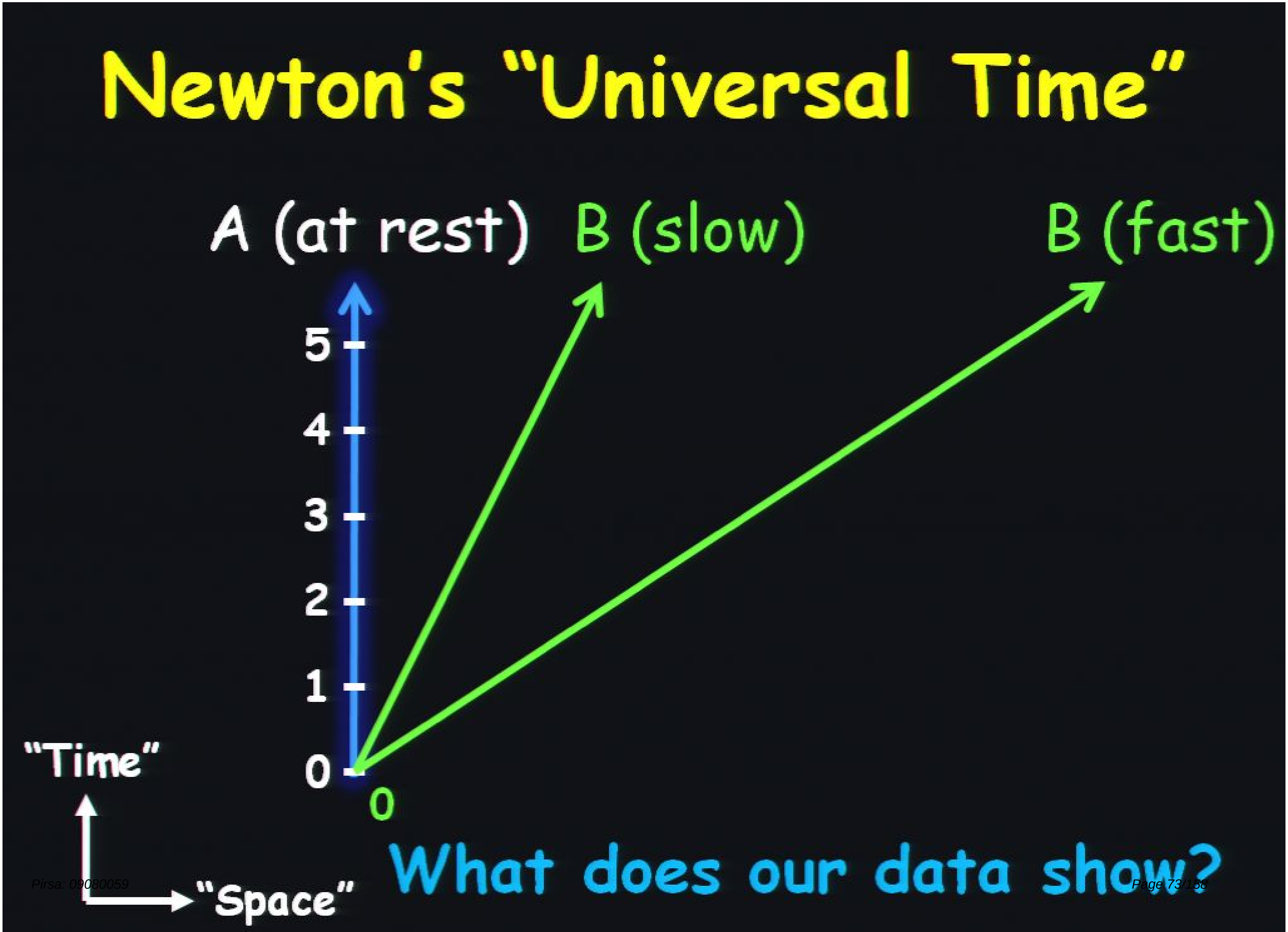
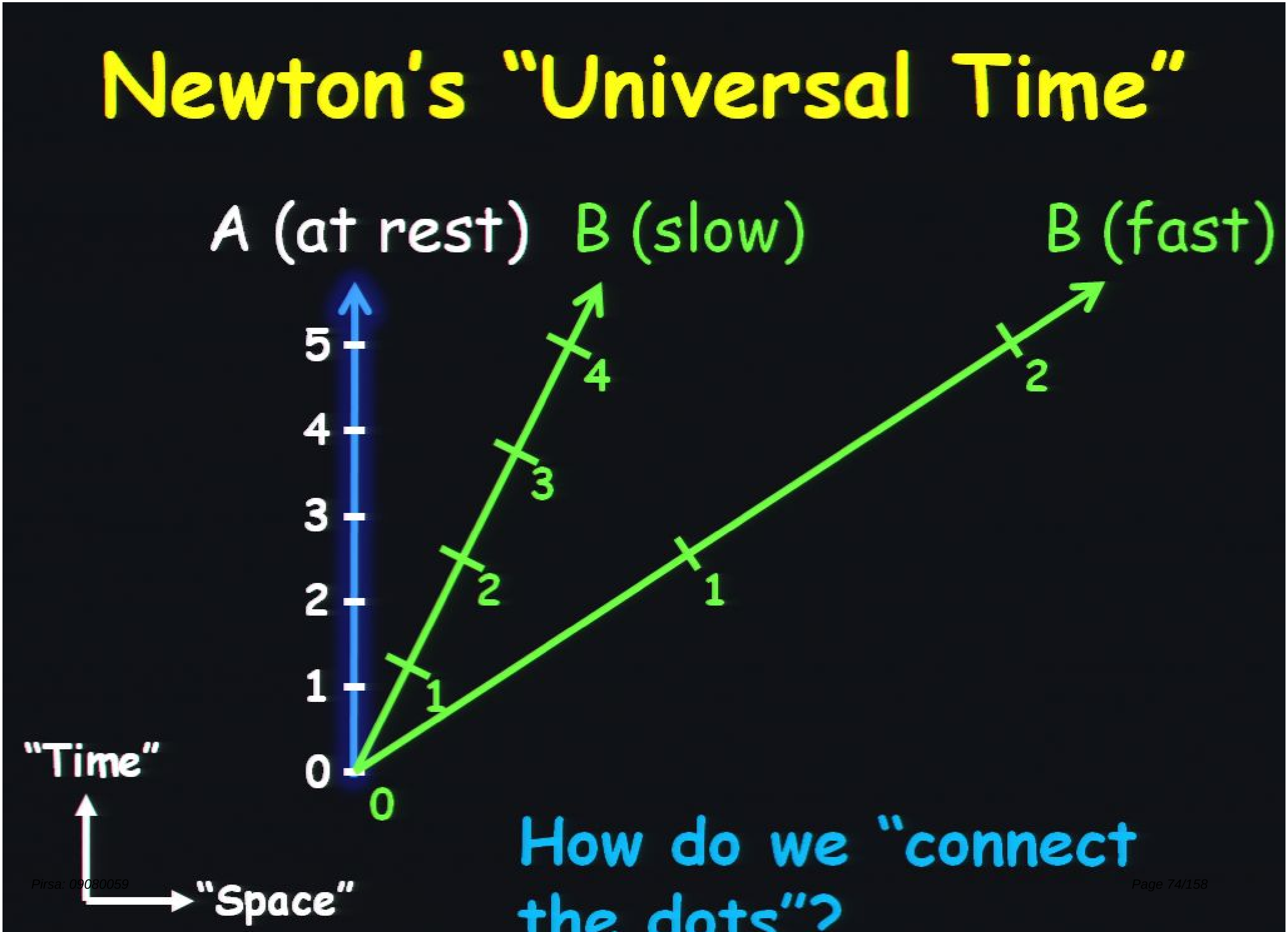


Problem: Newton's Universal time idea is wrong!


Newton's "Universal Time"
A (at rest)
B (slow)
B (fast)
5
4
3
2
1
0
0
"Time"
"Space"
What does our data show?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B (slow)
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4
3
2
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0
4
3
2
1
2
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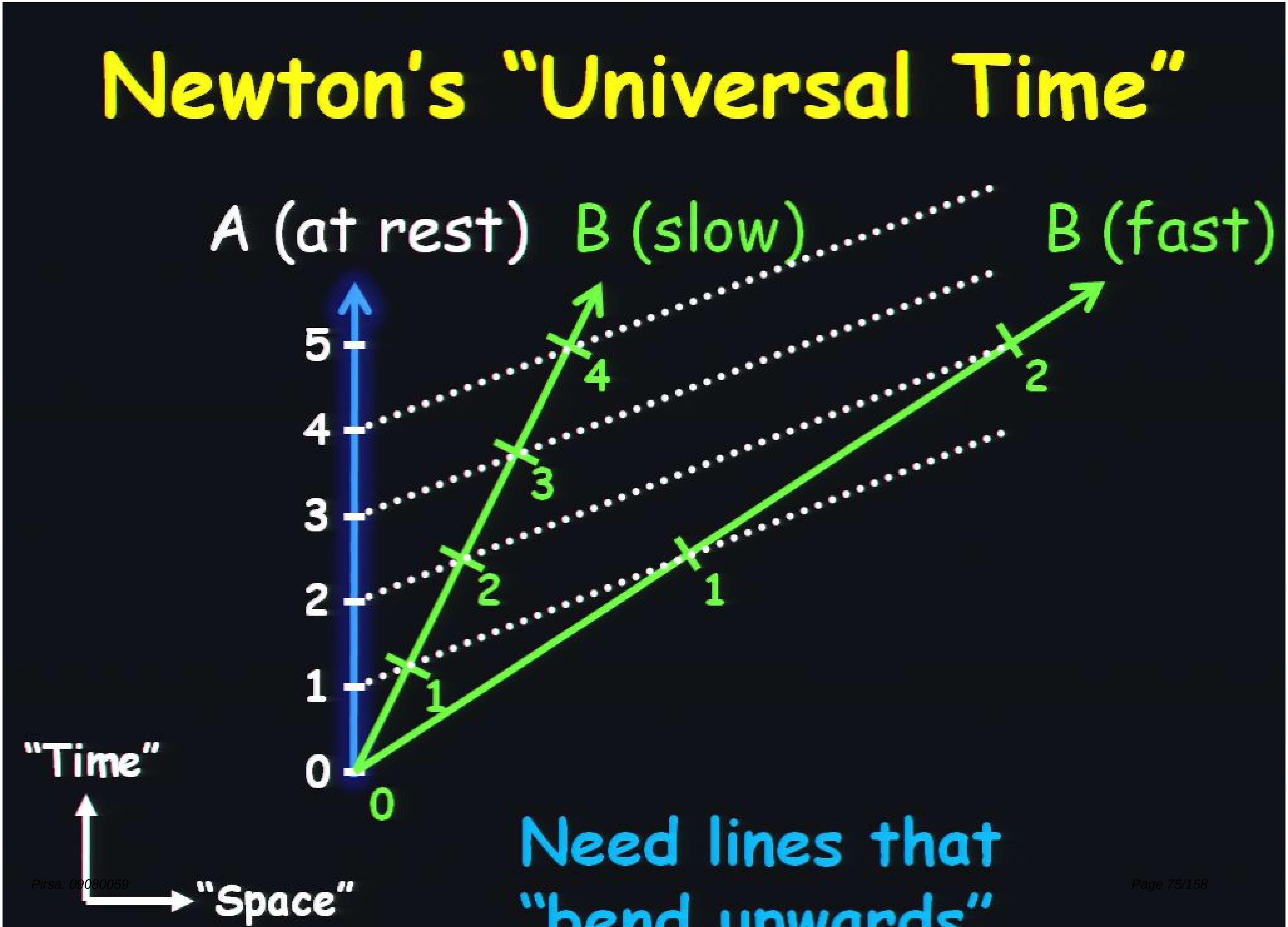
0
"Time"
"Space"
How do we "connect
the dots"?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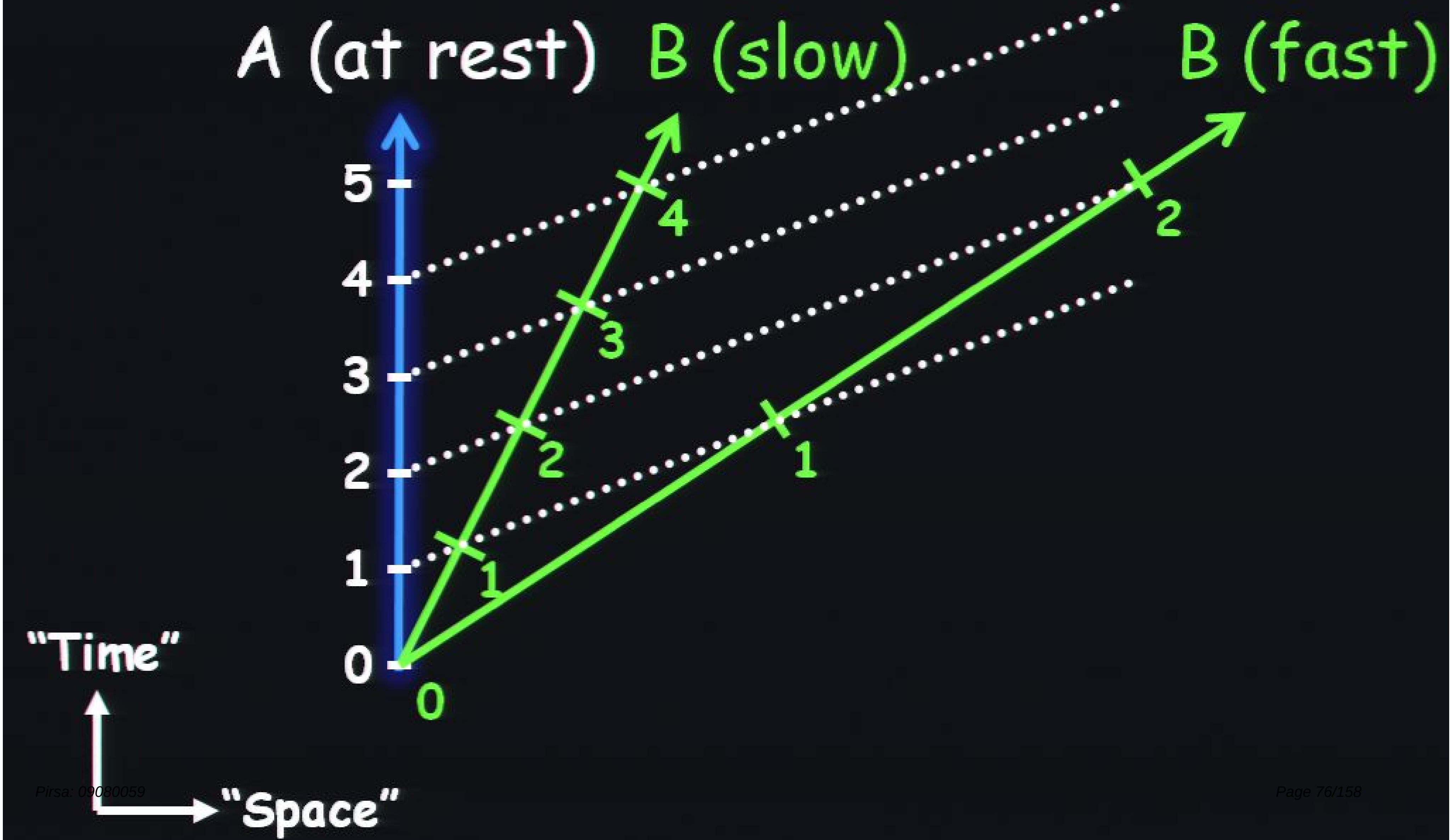
2
1
0
4
3
2
1
0
2
1
"Time"
"Space"
Need lines that
"bend upwards"

Newton's "Universal Time"



The Geometry of Space

A (walking N)

The Geometry of Space

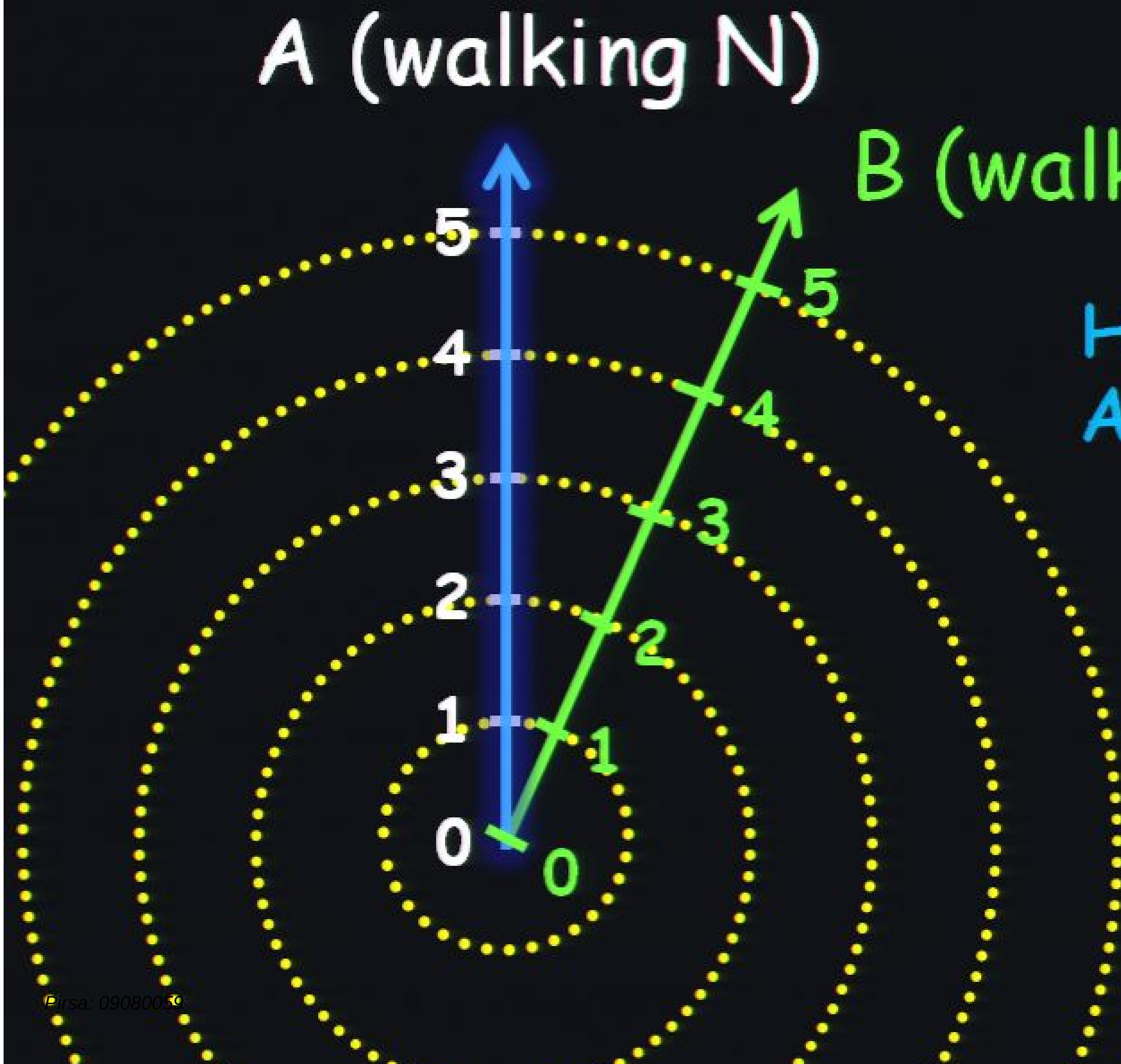
A (walking N)

The Geometry of Space

How do we transfer A's tick marks?

Let's Try CIRCLES!

The Geometry of Space



How do we transfer A's tick marks?

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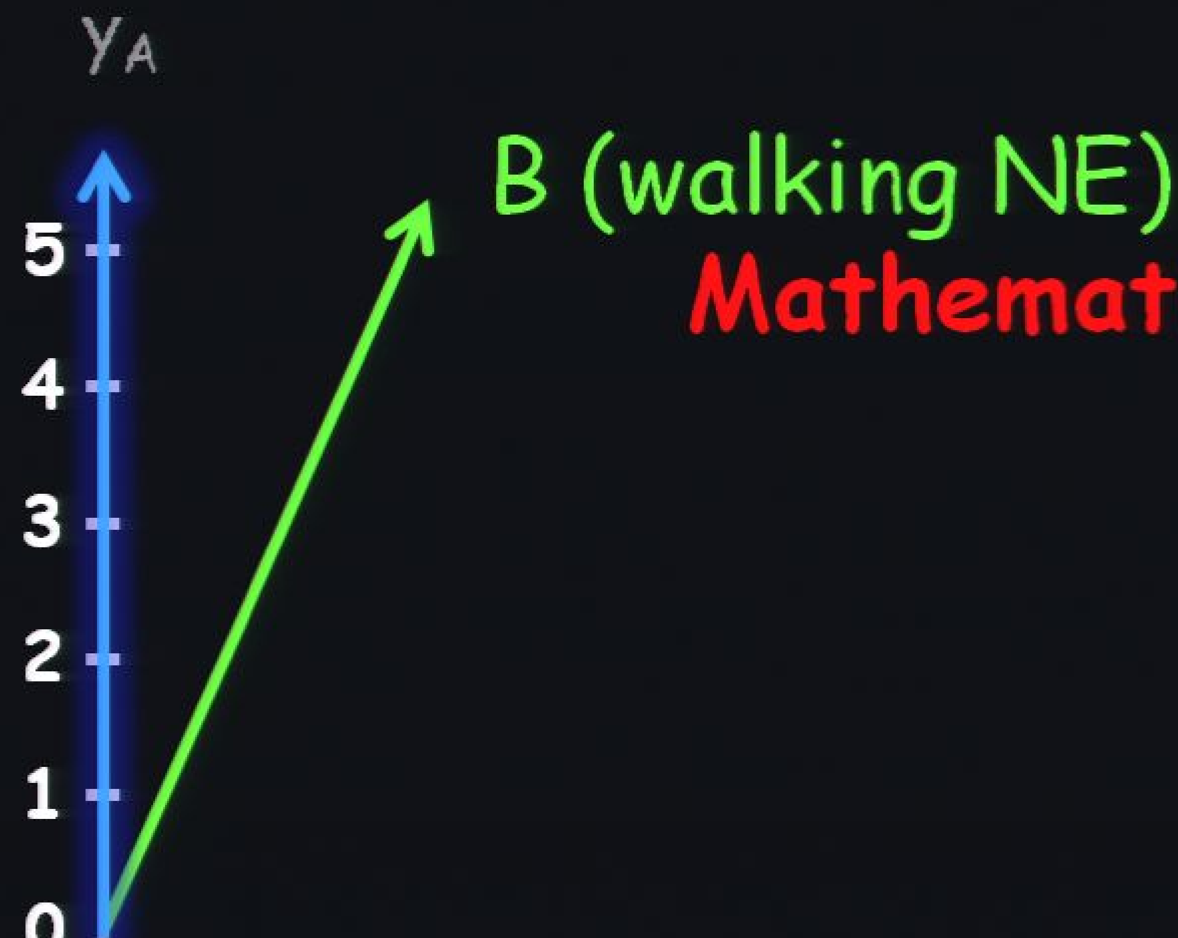
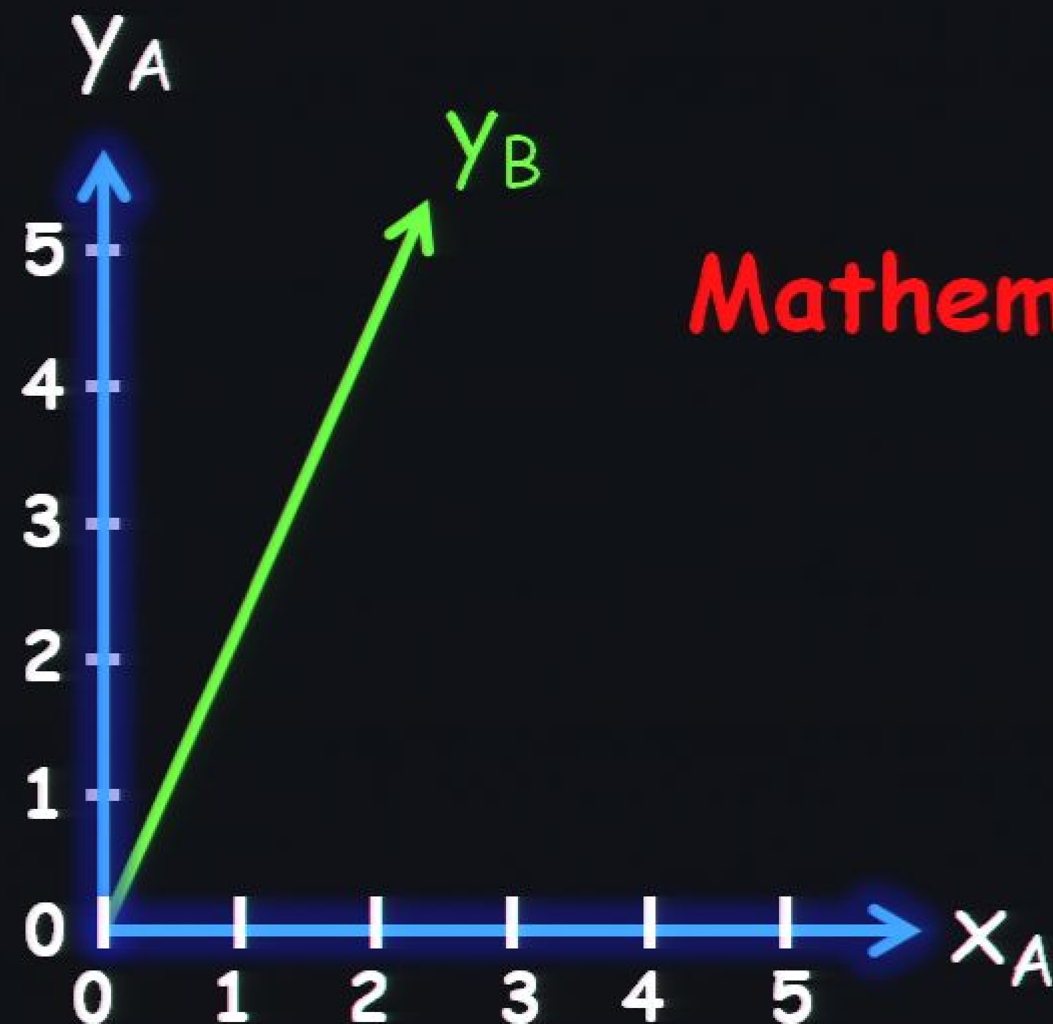


Mathematically...

The Geometry of Space



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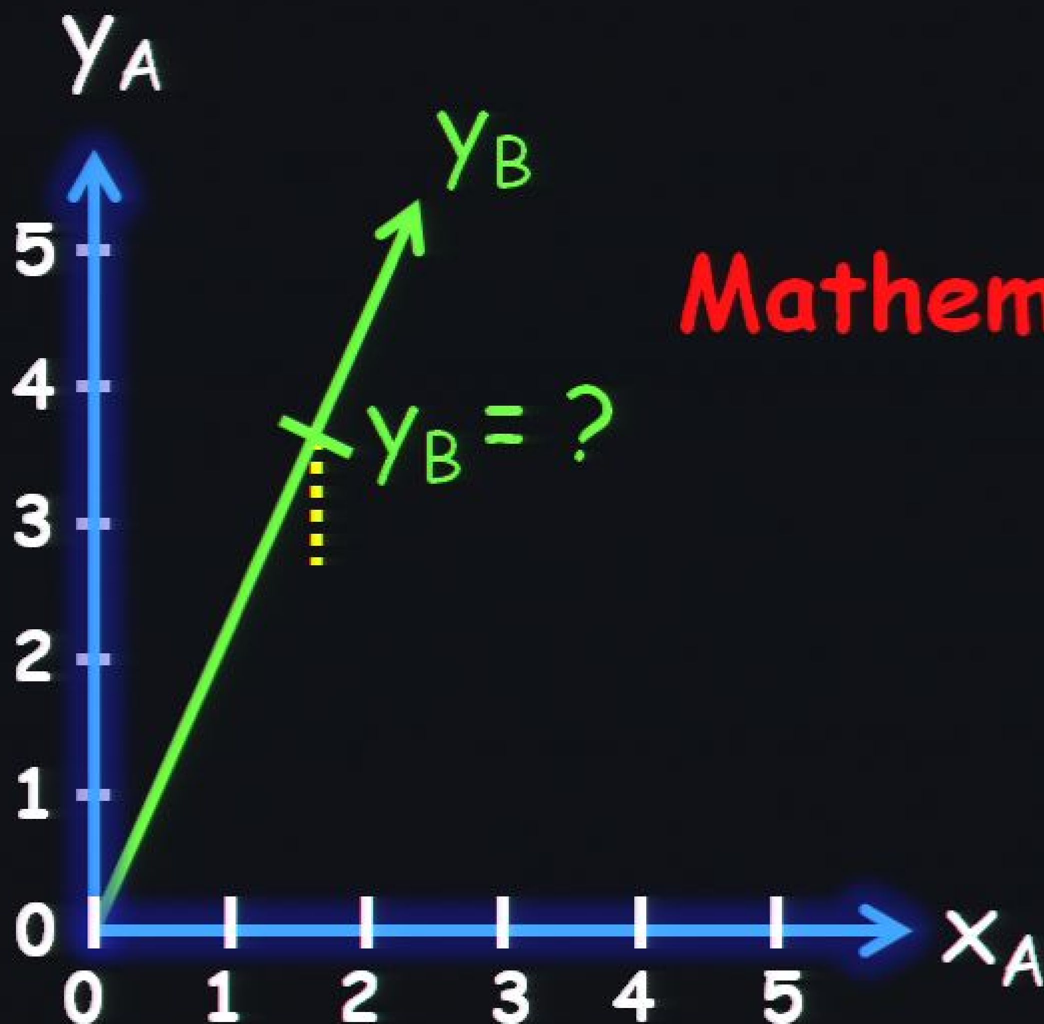

Mathematically...

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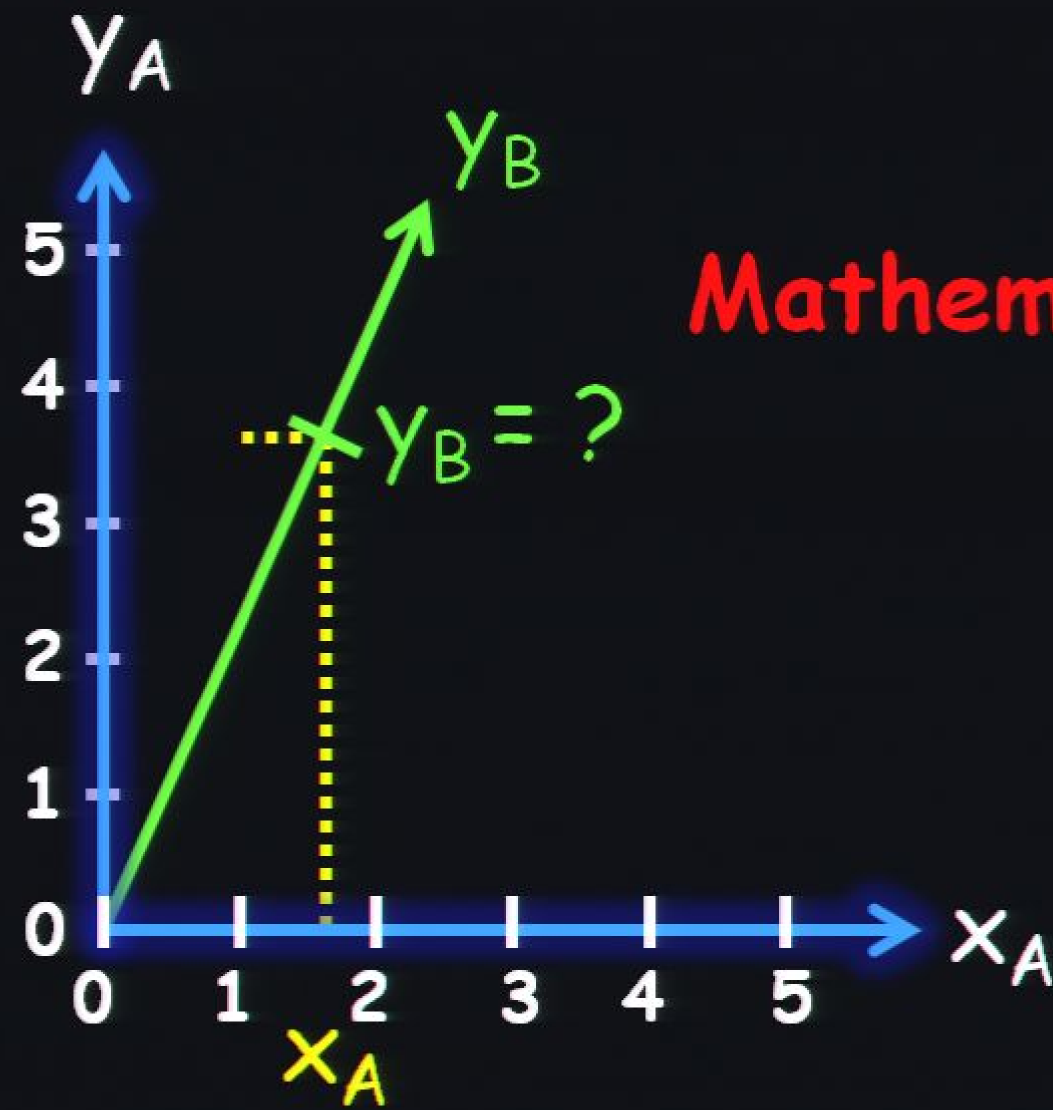
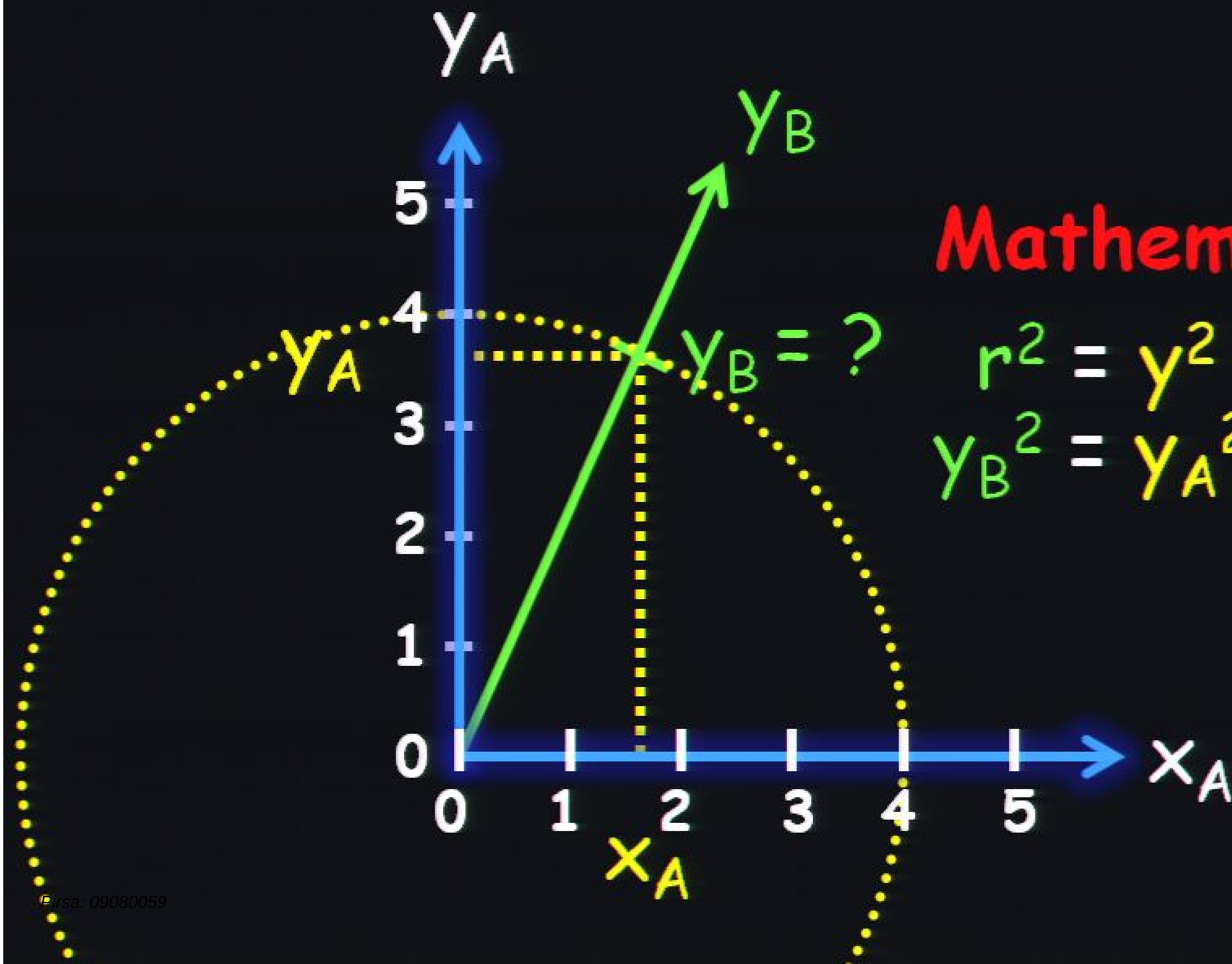


Mathematically…

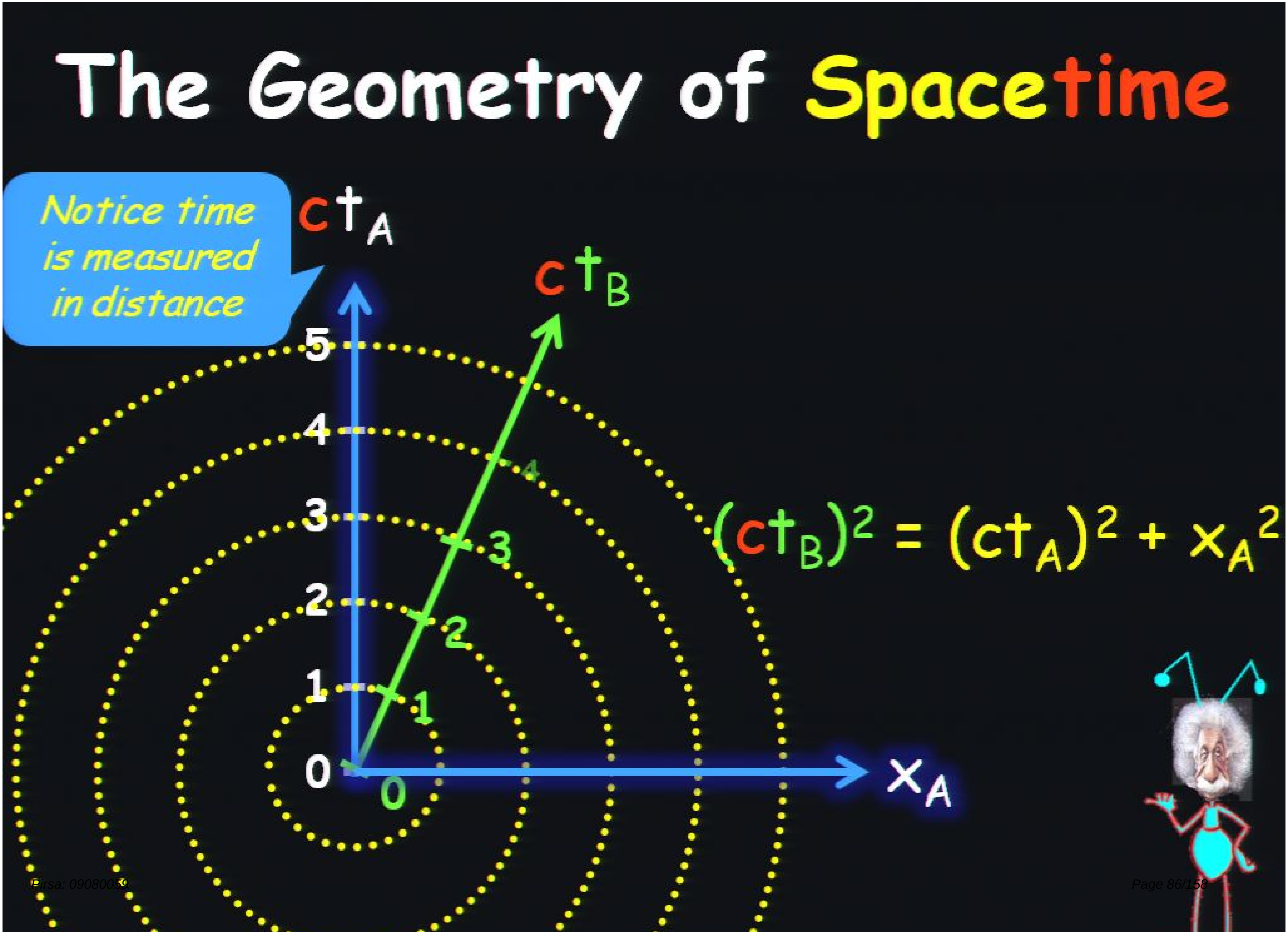
The Geometry of Space



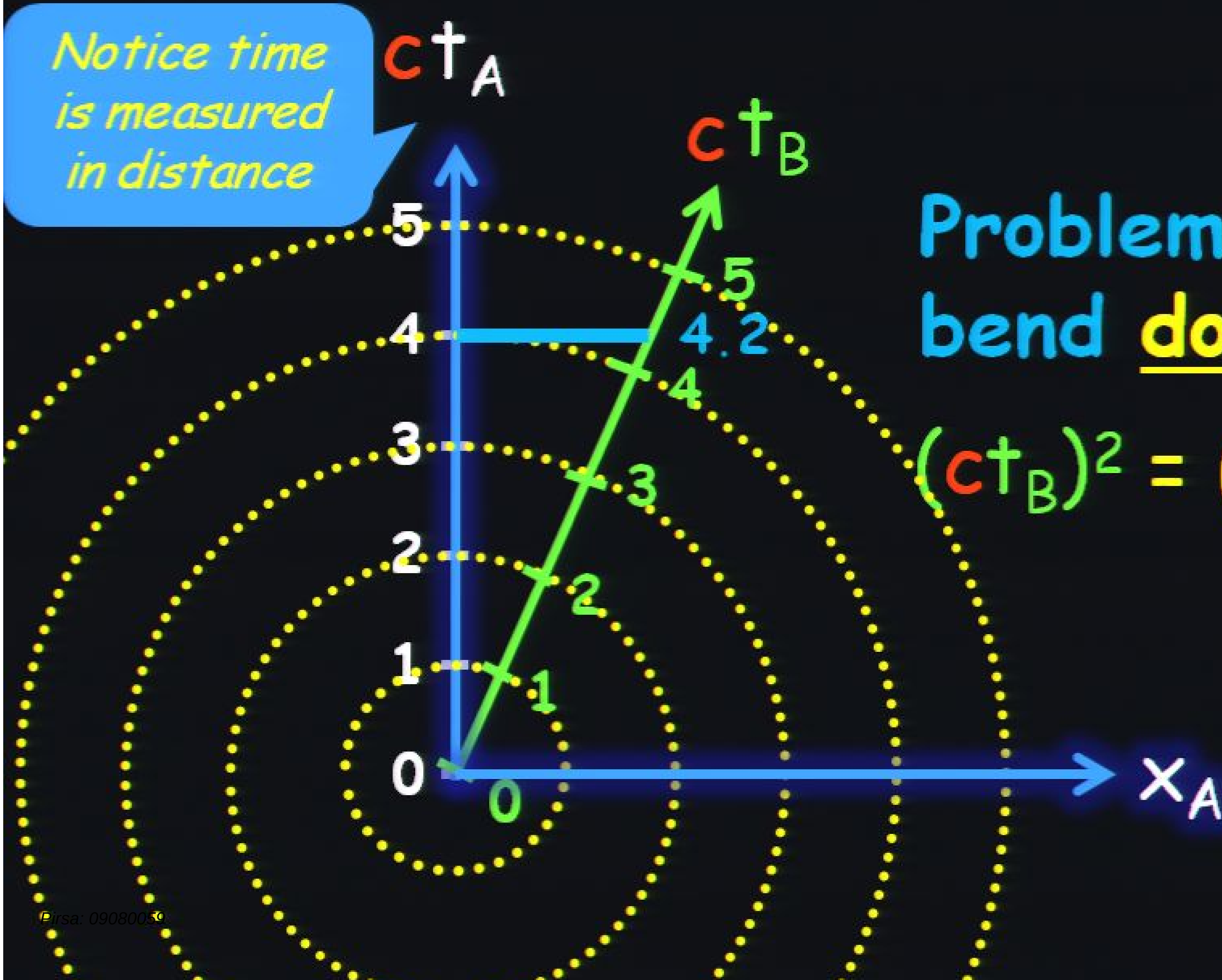
Mathematically…

$$r^2 = y^2 + x^2$$

$$y_B^2 = y_A^2 + x_A^2$$


The Geometry of Spacetime
Notice time is measured in distance
ct_A
ct_B
5
4
3
2
1
0
xA
(ct_B)^2 = (ct_A)^2 + x_A^2

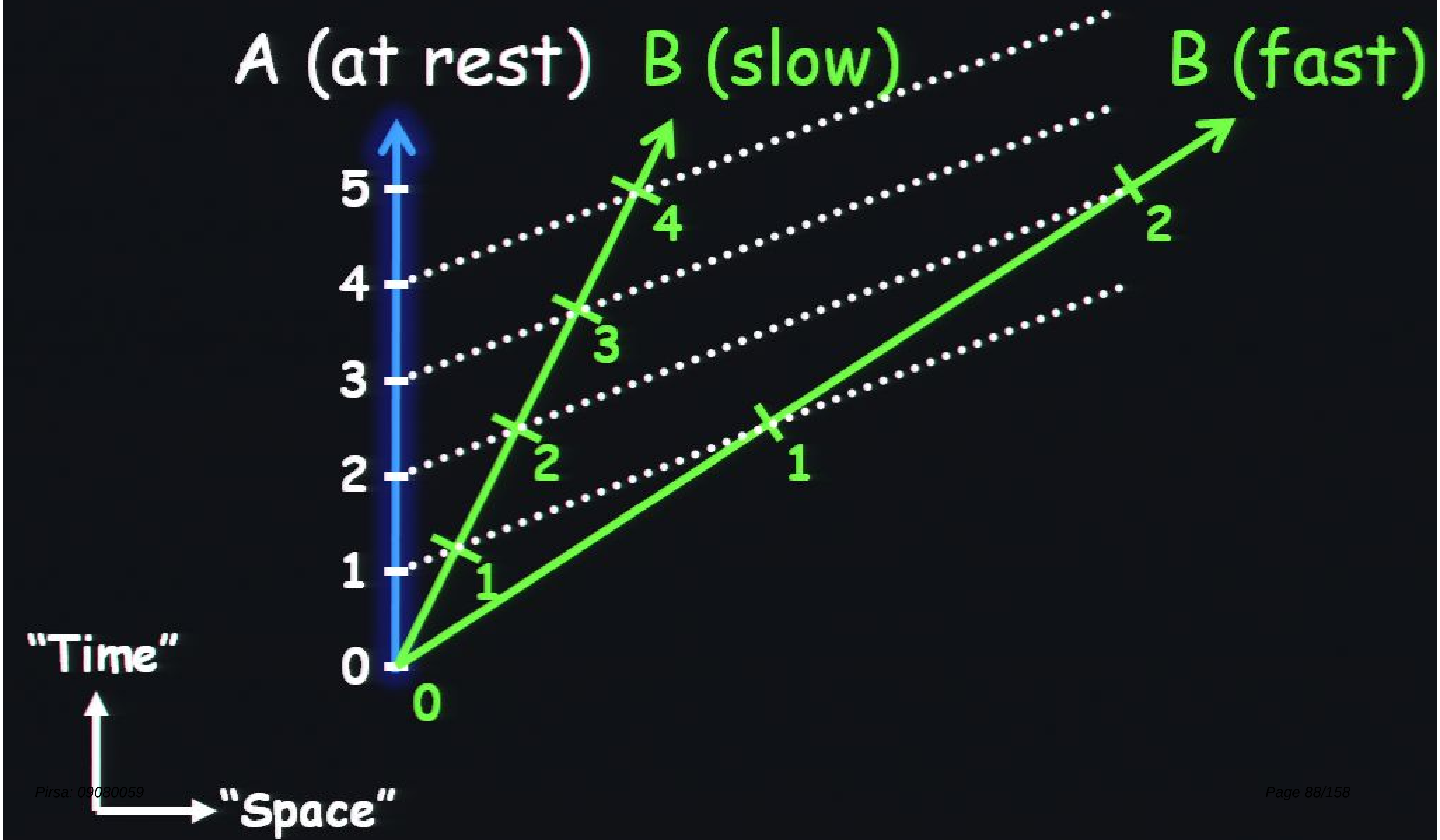
The Geometry of Spacetime

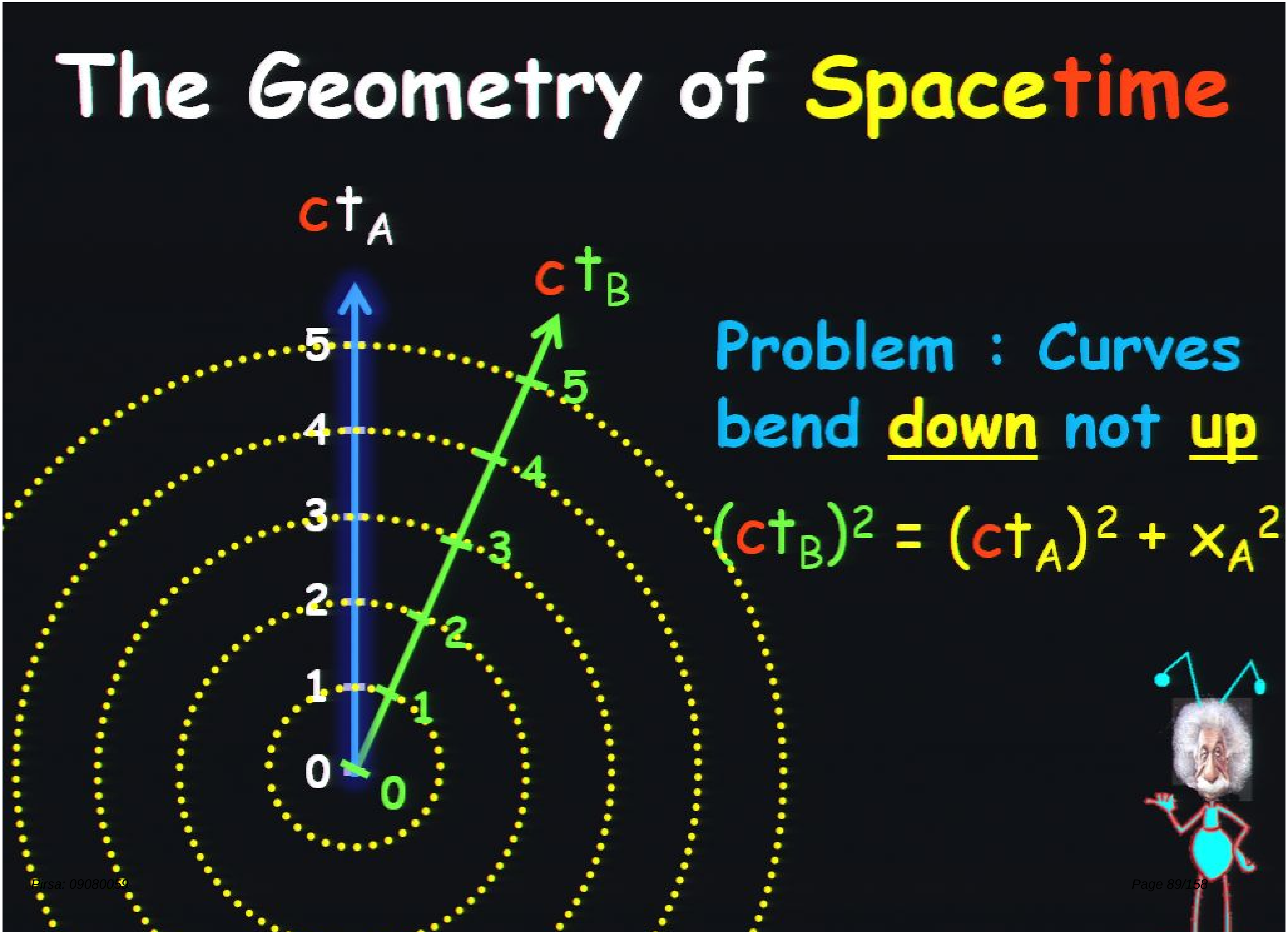


Problem : Curves bend <u>down</u> not <u>up</u>

$$(ct_B)^2 = (ct_A)^2 + x_A^2$$

Experimental Data:





The Geometry of Spacetime
ct_A
ct_B
Problem : Curves bend down not up
$(ct_B)^2 = (ct_A)^2 + x_A{}^2$
0
1
2
3
4
5
0
1
2
3
4
5

The Geometry of Spacetime

Try hyperbolas instead of circles:

$$(ct_B)^2 = (ct_A)^2 + x_A^2$$

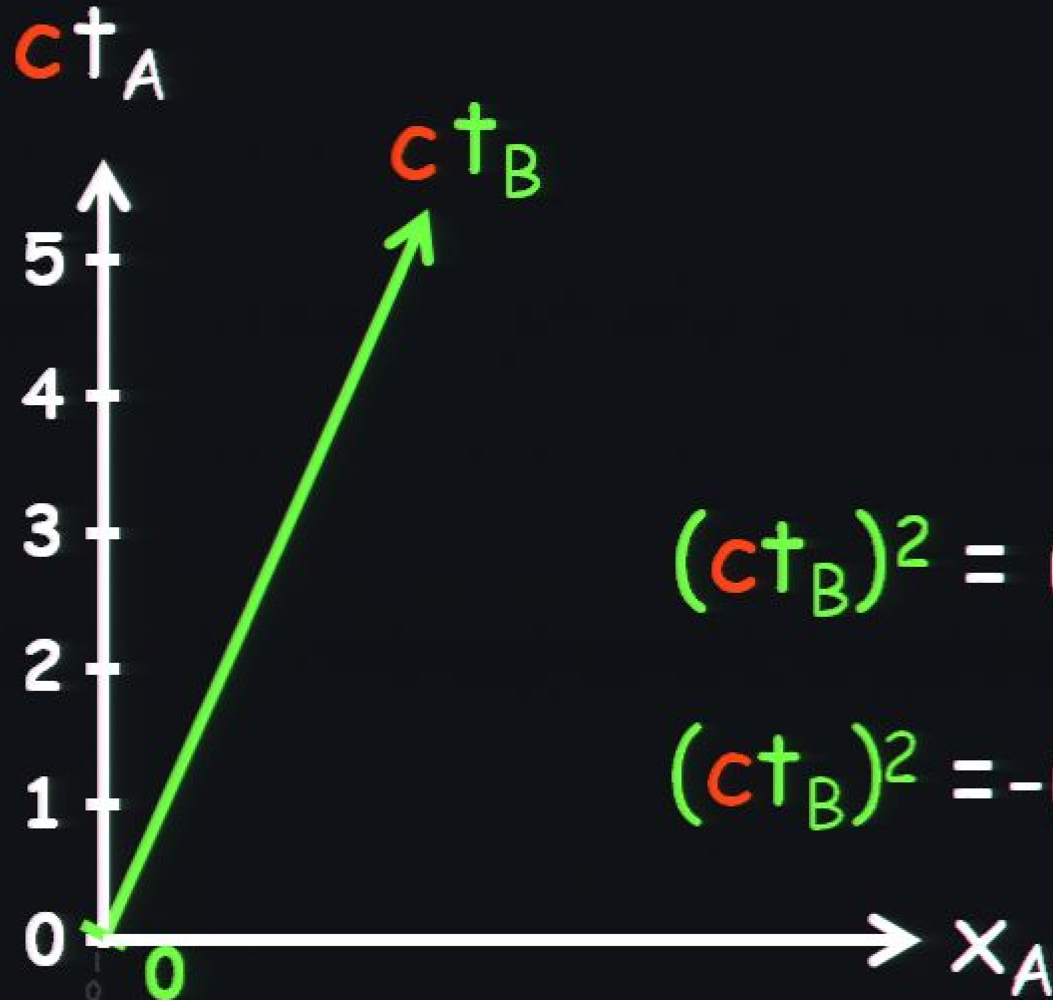
The Geometry of Spacetime

Try <u>hyperbolas</u> instead of circles:

$$(ct_B)^2 = (ct_A)^2 - x_A^{\,2}$$

Minus sign

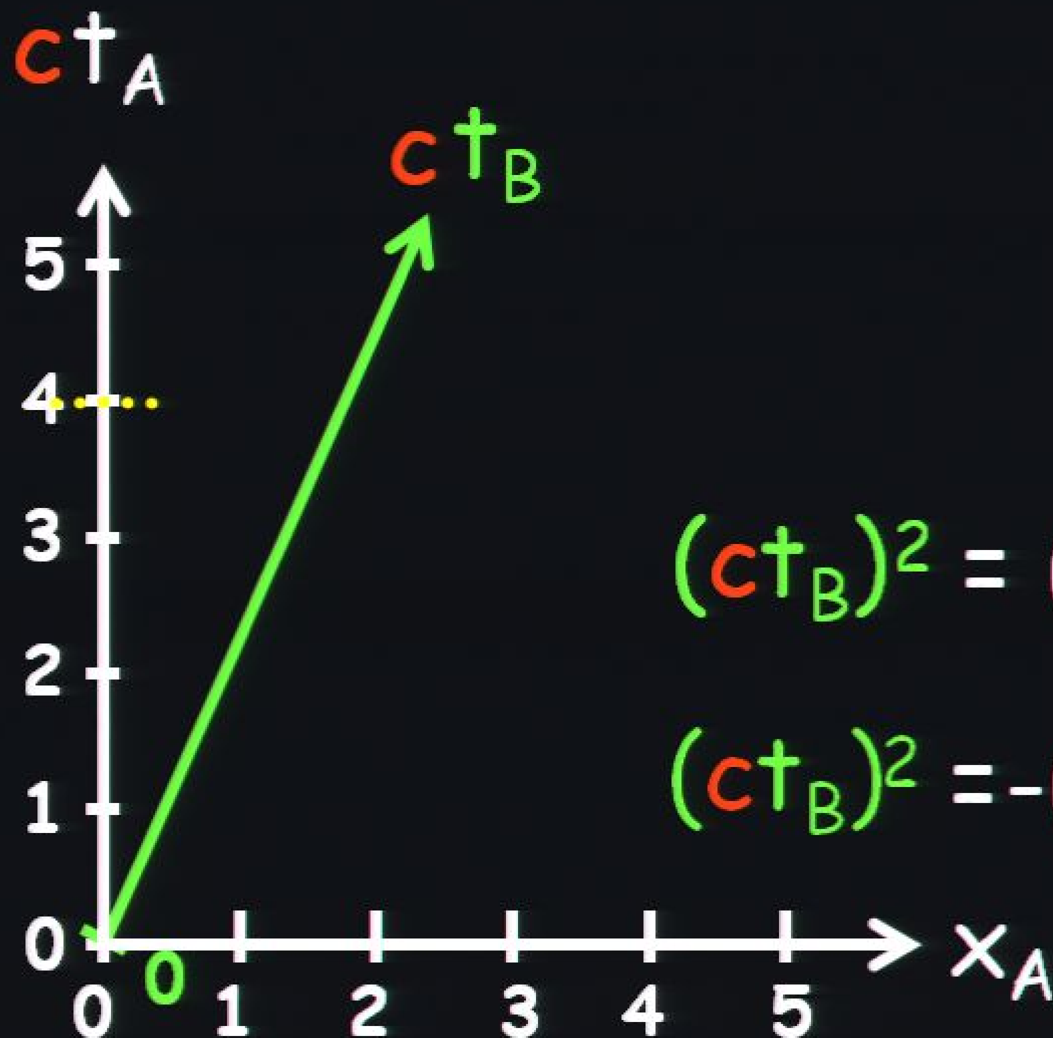
The Geometry of Spacetime



$$(ct_B)^2 = (ct_A)^2 - x_A^2$$

$$(ct_B)^2 = -(ct_A)^2 + x_A^2$$

The Geometry of Spacetime



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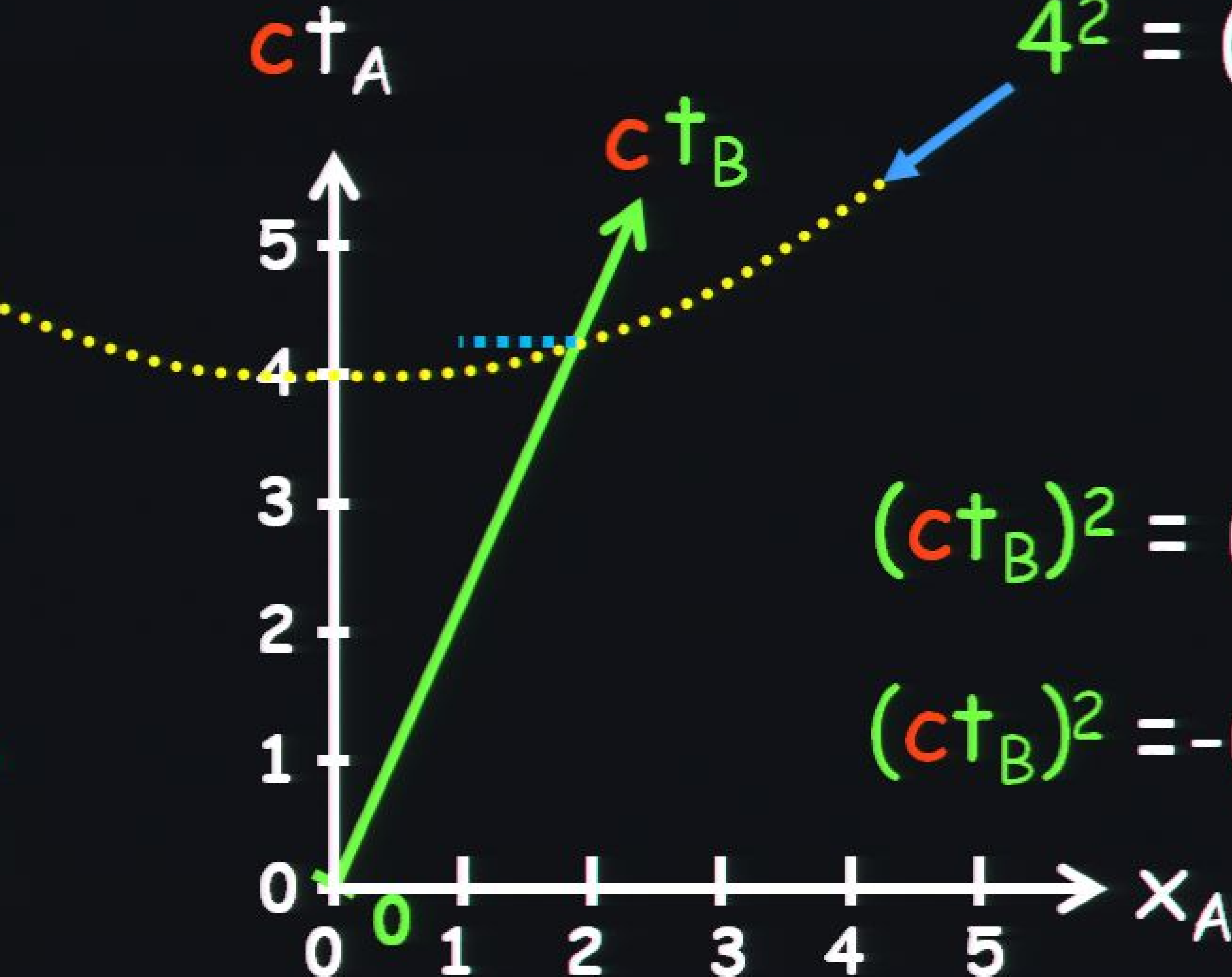
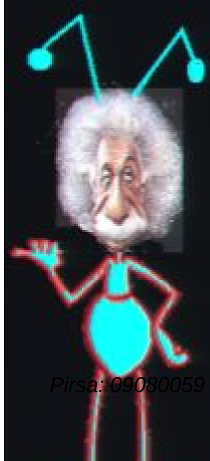
The Geometry of Spacetime



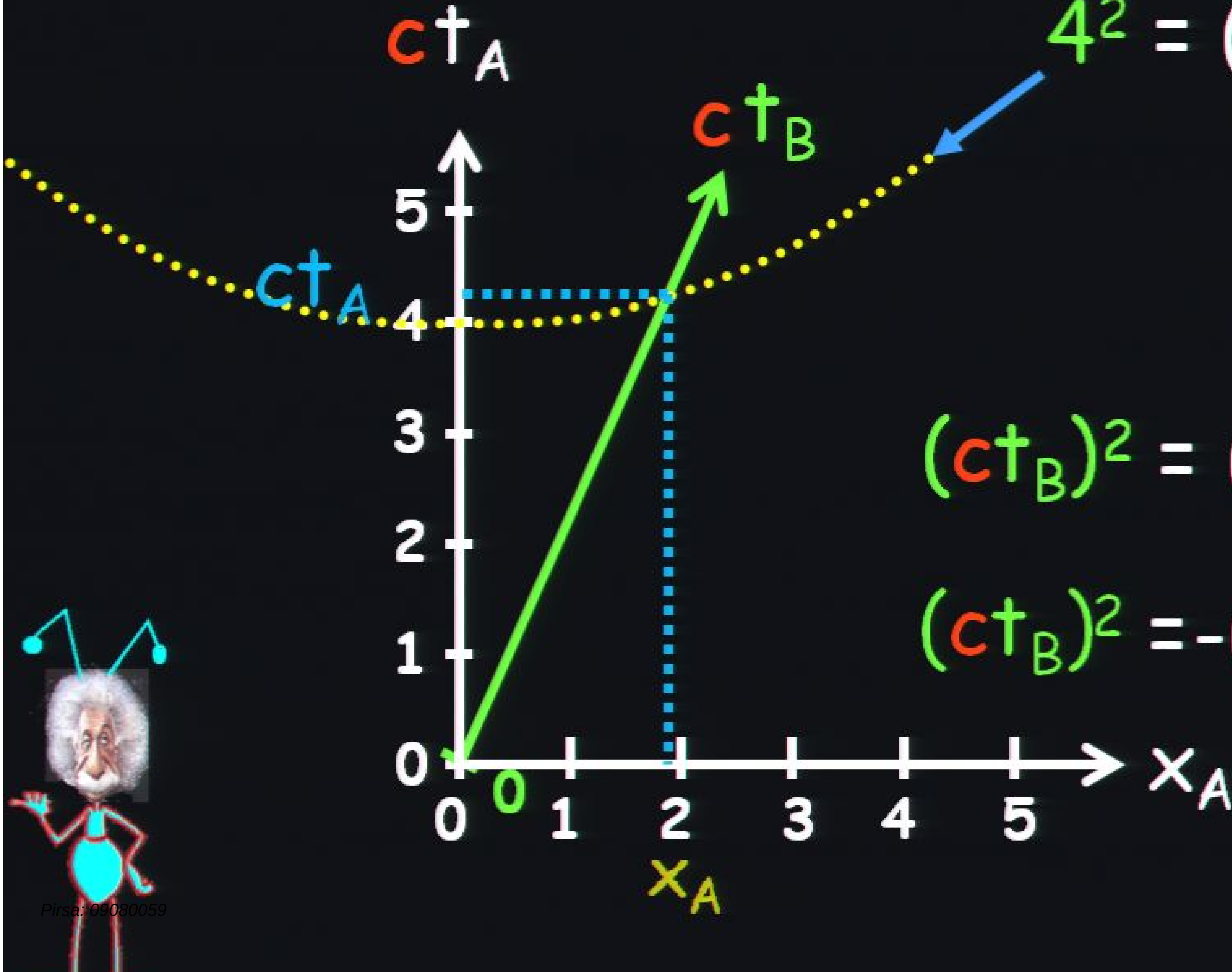
$$4^2 = (ct_A)^2 - x_A^2$$

$$(ct_B)^2 = (ct_A)^2 - x_A^2$$

$$(ct_B)^2 = -(ct_A)^2 + x_A^2$$



The Geometry of Spacetime



$$4^2 = (ct_A)^2 - x_A{}^2$$

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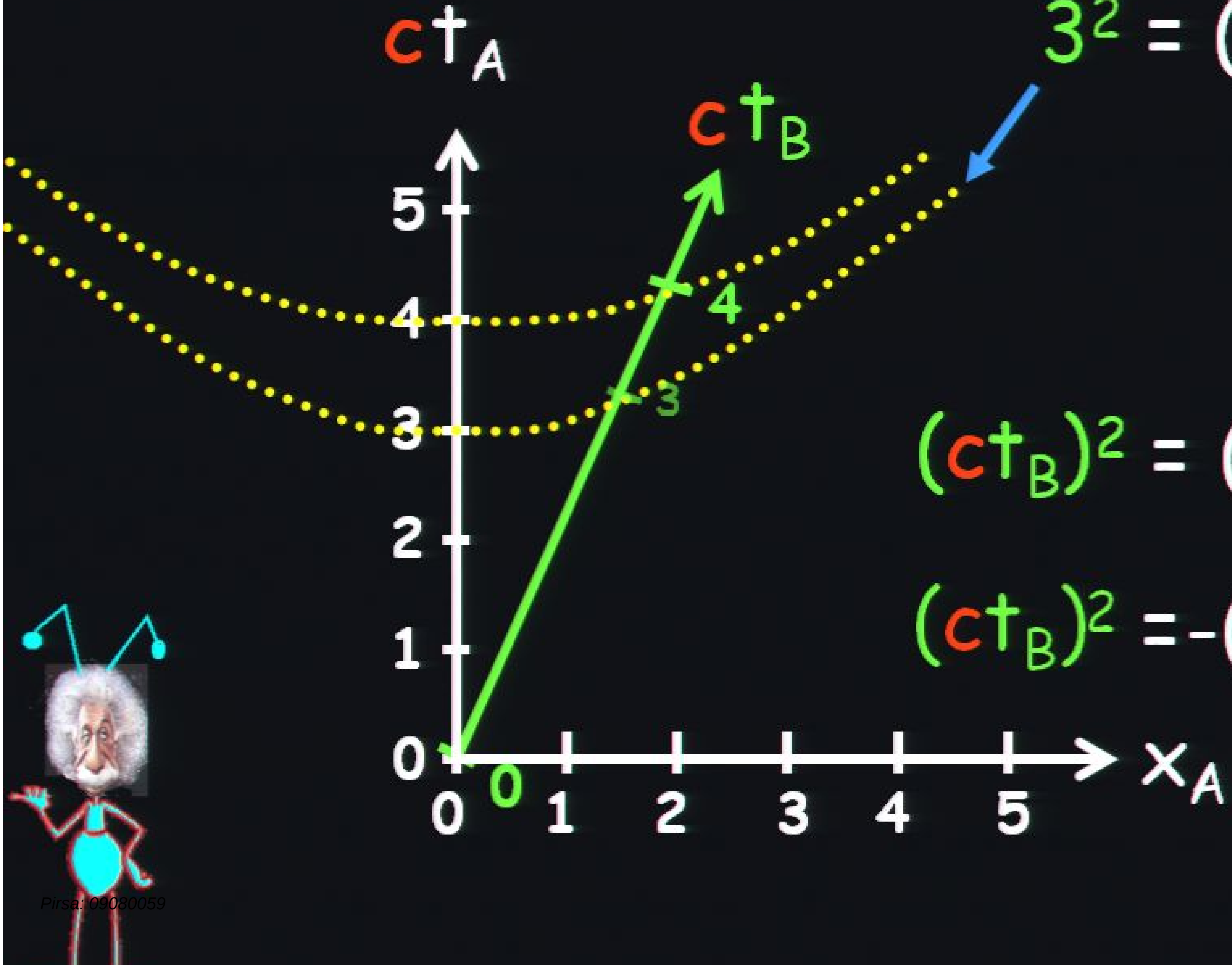
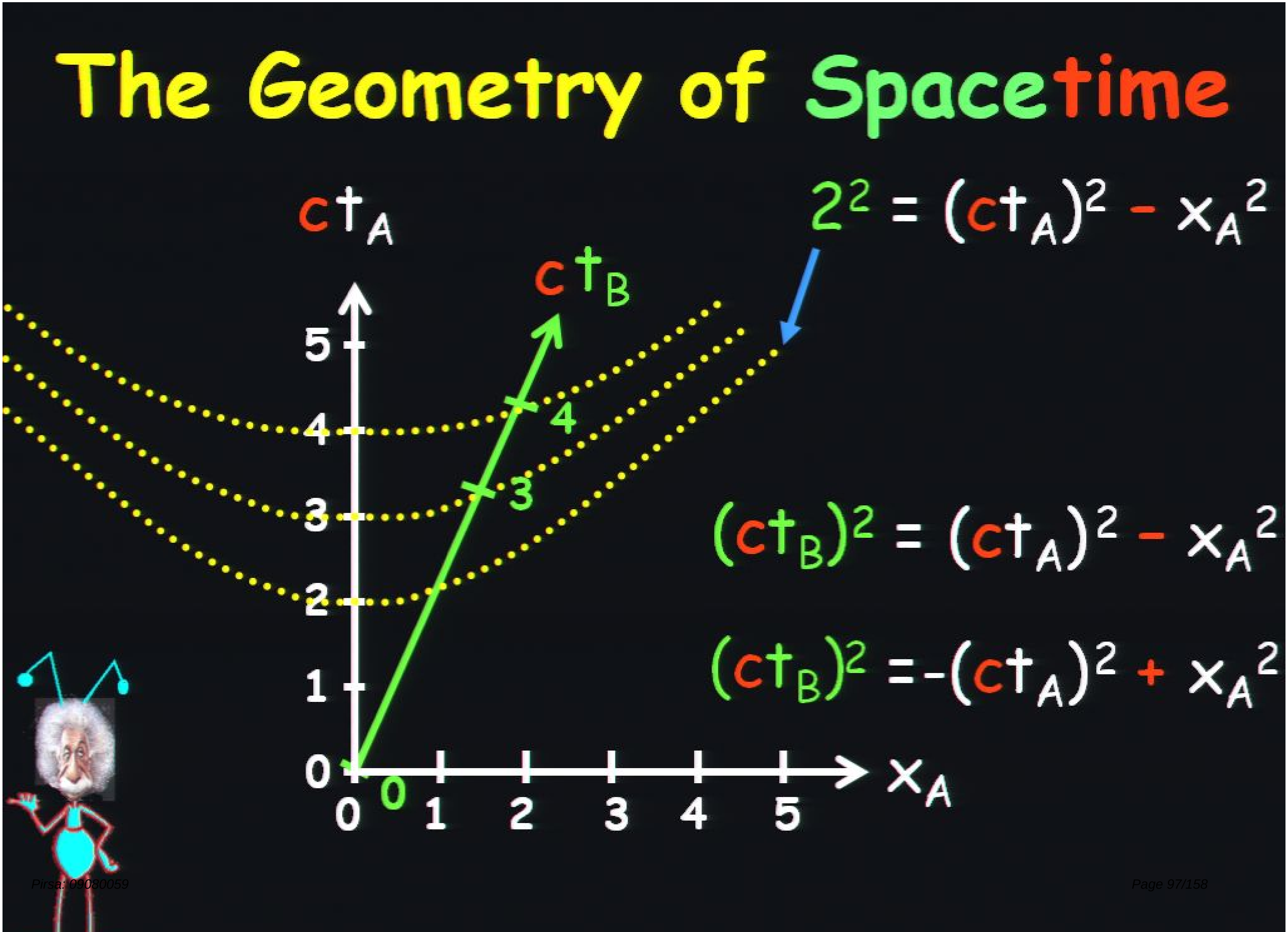
The Geometry of Spacetime

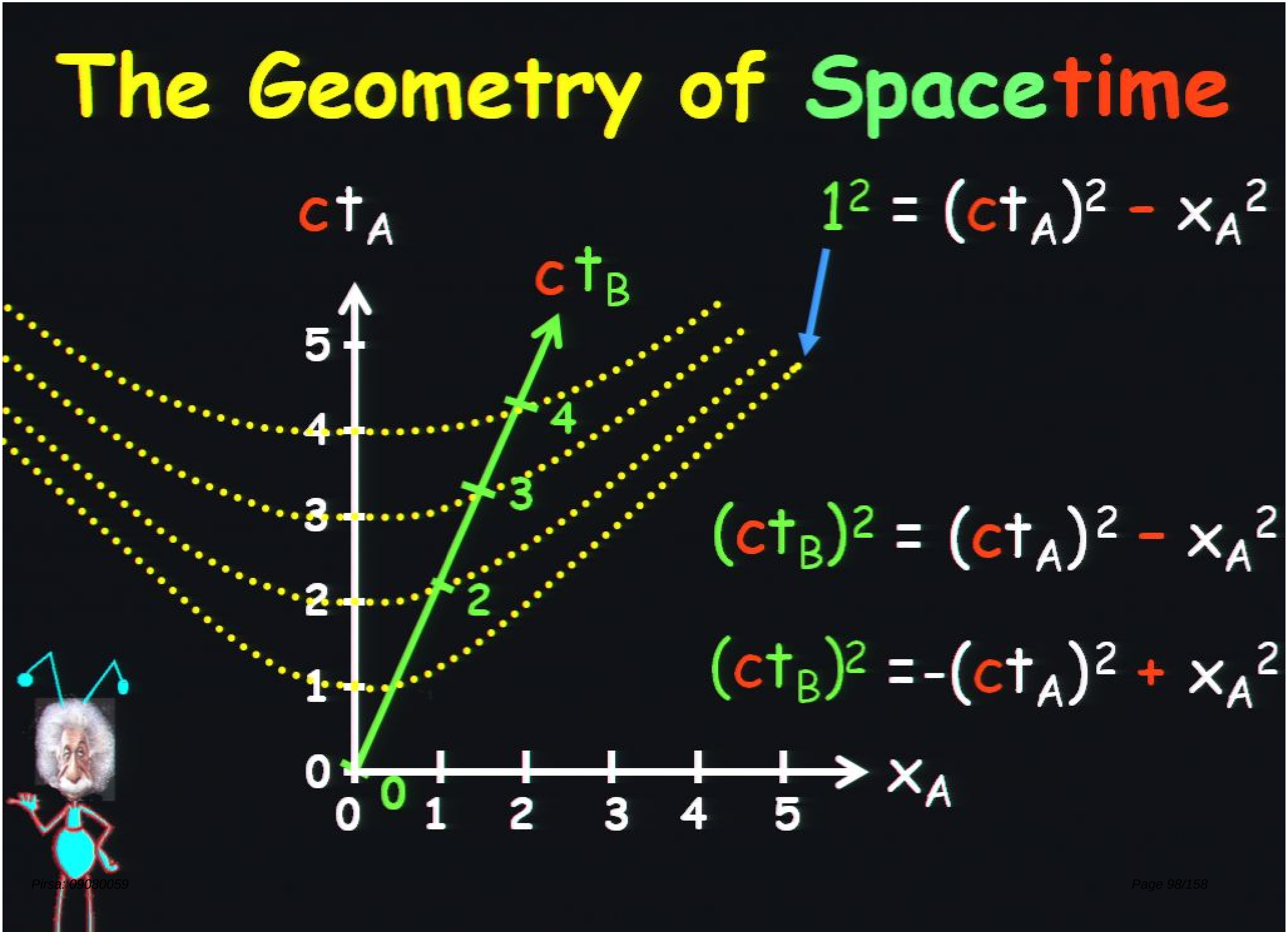


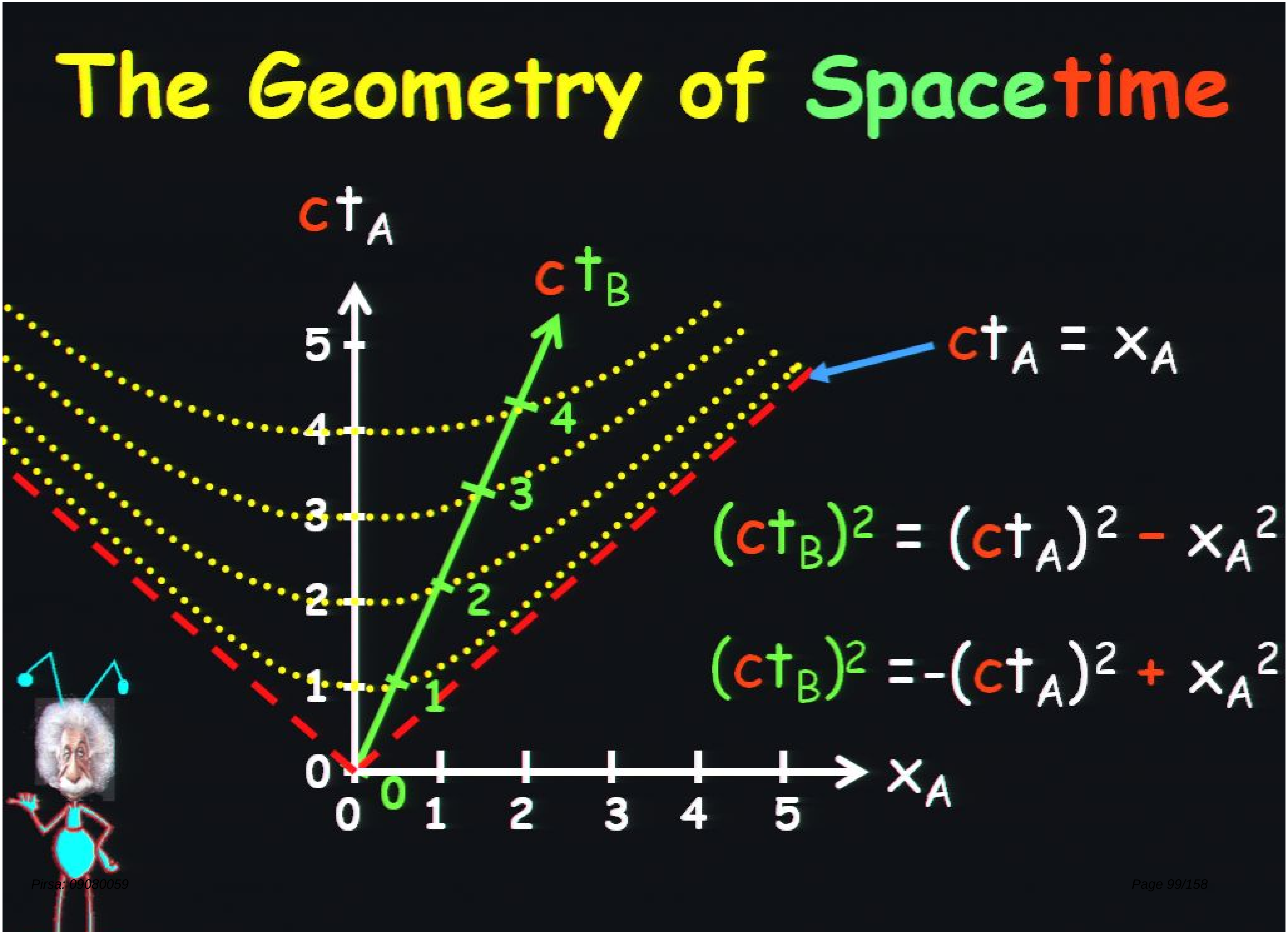
$$3^2 = (ct_A)^2 - x_A{}^2$$

$$(ct_B)^2 = (ct_A)^2 - x_A{}^2$$

$$(ct_B)^2 = -(ct_A)^2 + x_A{}^2$$


The Geometry of Spacetime
$2^2 = (ct_A)^2 - x_A{}^2$
ct_A
ct_B
5
4
3
2
1
0
4
3
0
0 1 2 3 4 5
x_A
$(ct_B)^2 = (ct_A)^2 - x_A{}^2$
$(ct_B)^2 = -(ct_A)^2 + x_A{}^2$

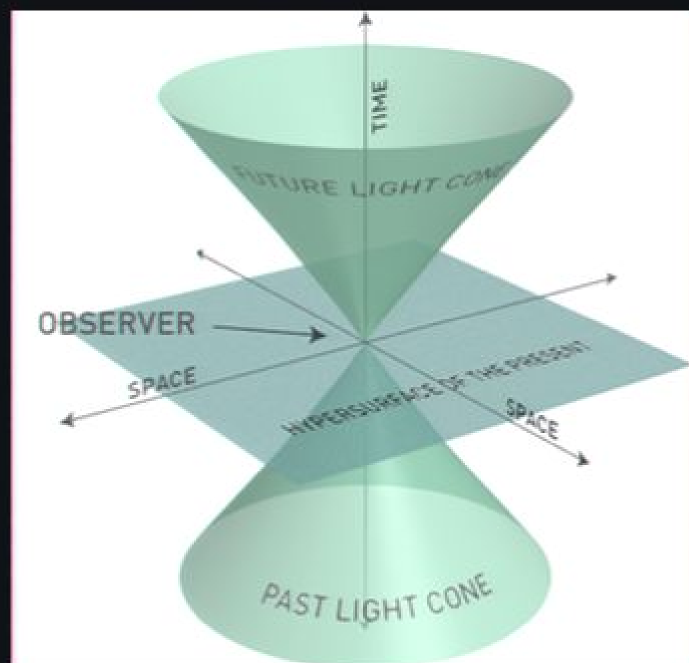

The Geometry of Spacetime
$1^2 = (ct_A)^2 - x_A{}^2$
ct_A
ct_B
$(ct_B)^2 = (ct_A)^2 - x_A{}^2$
$(ct_B)^2 = -(ct_A)^2 + x_A{}^2$
x_A
0 1 2 3 4 5
0 1 2 3 4 5
0 2 3 4



The Geometry of Spacetime
ctA
ctB
ctA = xA
(ctB)2 = (ctA)2 - xA2
(ctB)2 =-(ctA)2 + xA2
5
4
3
2
1
0
0 1 2 3 4 5
xA

Spacelike, Null, Timelike

Special relativity implies that all matter must move at less than or equal to the speed of light.



Einstein's Spacetime

- *Define a mathematical tool that handles both Space and Time* $P(t,x,y,z)$

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$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2$$ *Cartesian coordinates*

$$(\Delta s)^2 = (\Delta r)^2 + r^2 (\Delta \phi)^2$$ *Polar coordinates*

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- *Example of two dimensional Minkowski Space*

$$(\Delta s)^2 = -(\Delta t)^2 + (\Delta x)^2$$ *Usual representation*

$$(\Delta s)^2 = -(\Delta t)^2 + t^2 (\Delta \phi)^2$$ *Milne representation*

The Geodesics

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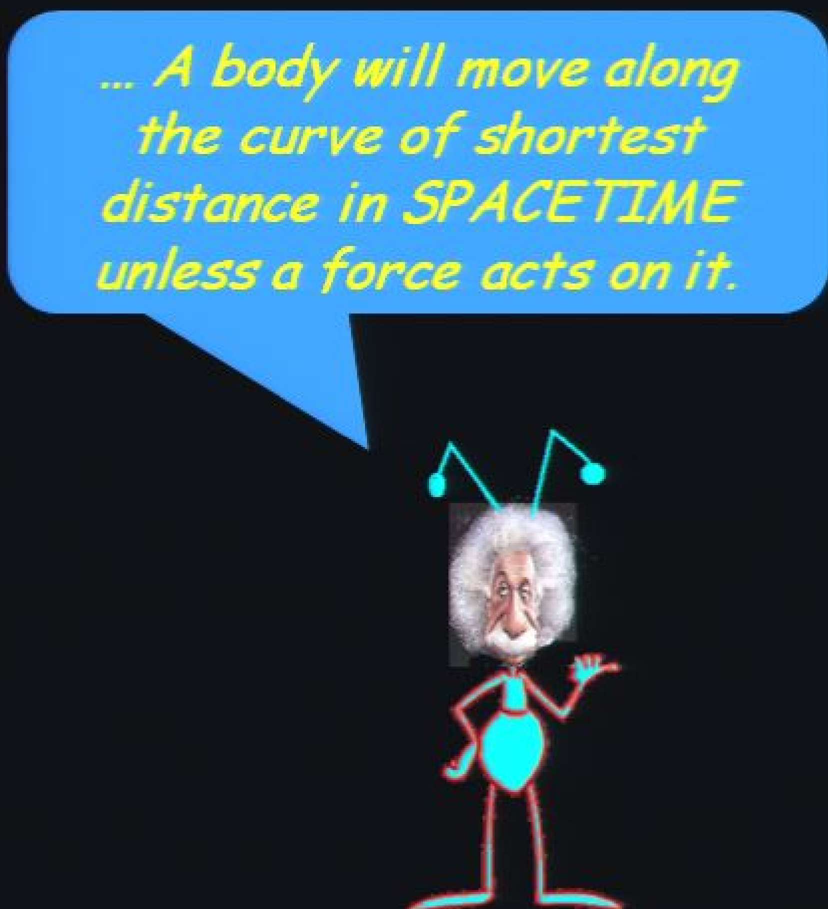

The Geodesics

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… A body will move along the curve of shortest distance in SPACETIME unless a force acts on it.

… The shortest distance between two points in general space isn't generally a straight line.

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Curves of shortest distance are known in relativistic jargon as geodesics.

You are familiar with the Geodesic

The pilot does not need to turn the plane to fly from Toronto to Rome

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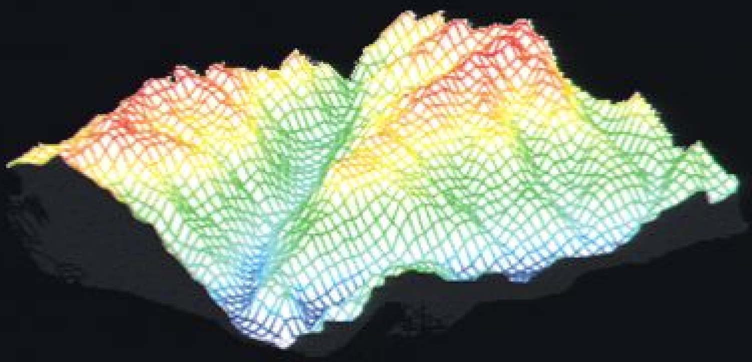
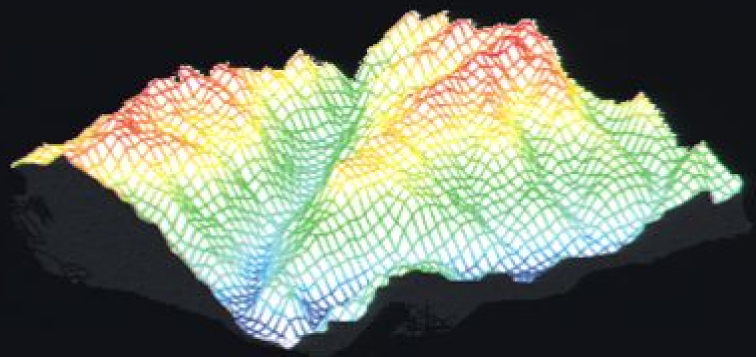


Geodesics are very difficult to calculate in general. Imagine surveying a complex landscape with hills and valleys. How is one to calculate the shortest distance over this terrain?



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...and I had to do it in four dimensions!, Yep, that *time* thing.

It would by very easy to do this is we looked at the landscape from above. Then I could use $ds^2=dx^2+dy^2$

...but I needed the distance in *SPACETIME*, so I used a mathematical quantity which converts the flat map distances into actual distances on our curved space. This is the *METRIC* of the space. Denoted by "*g*".

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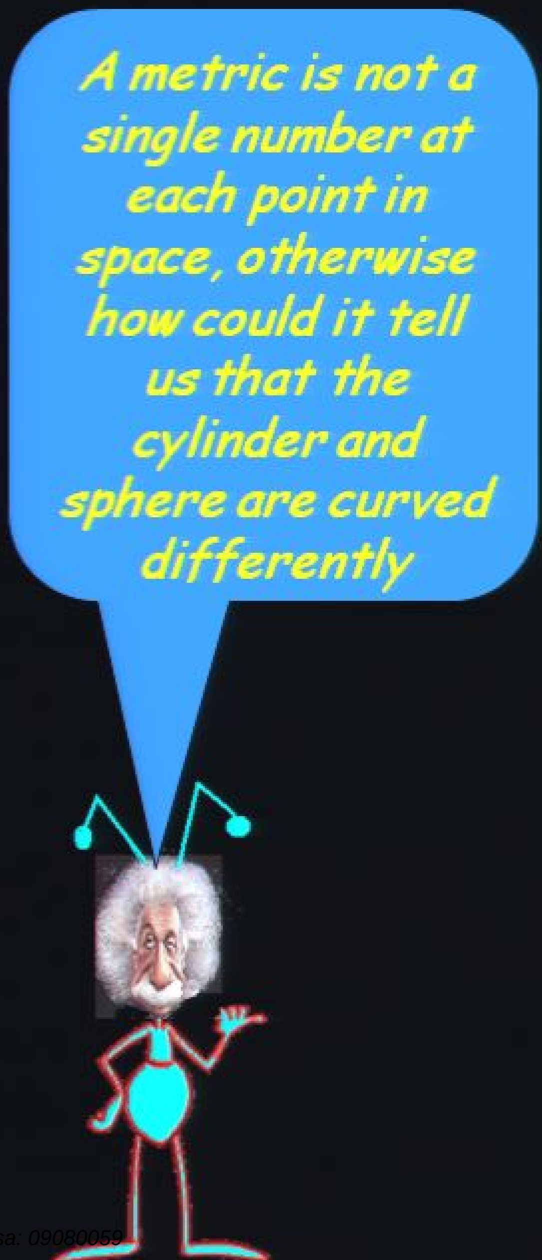
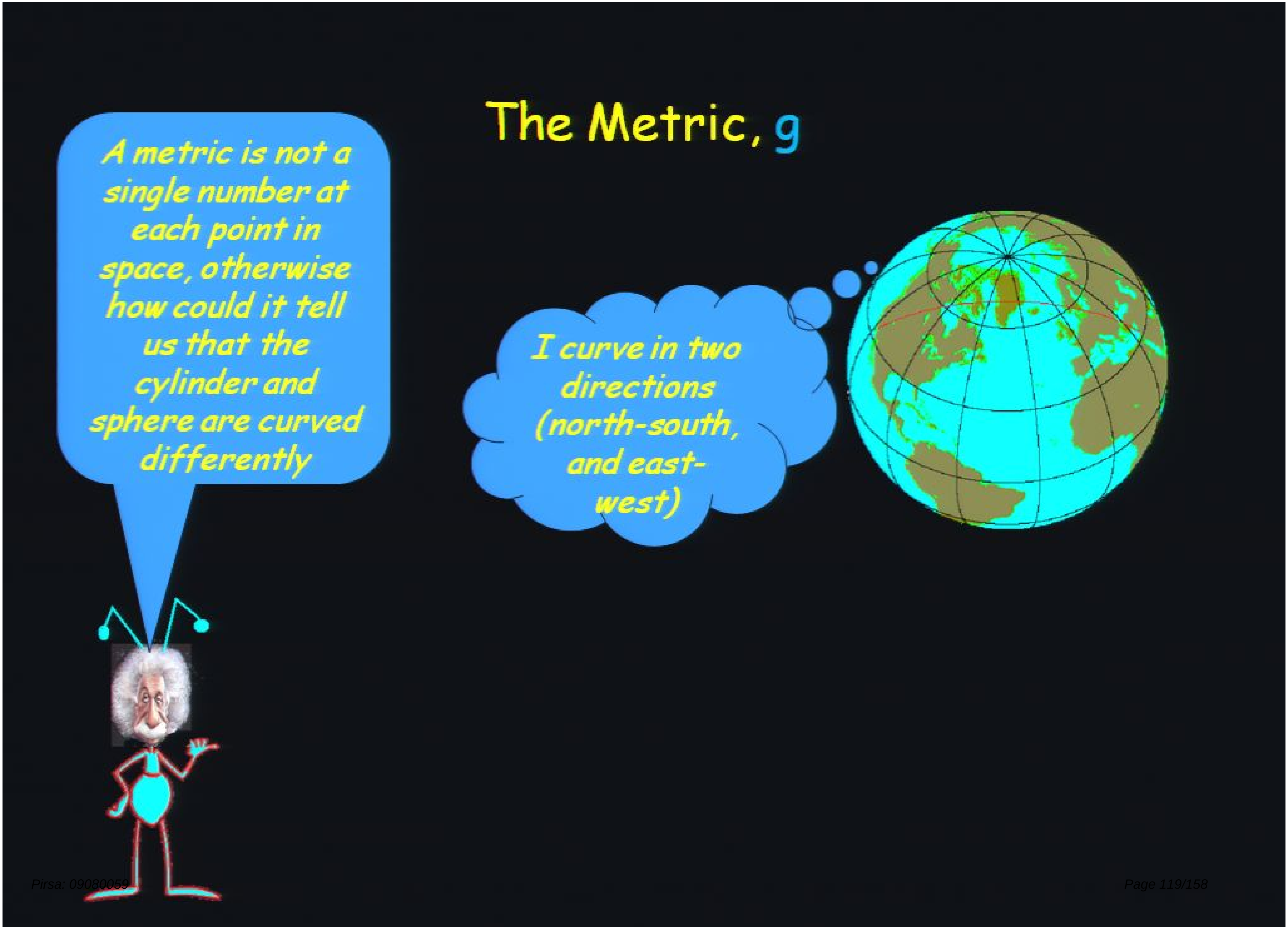
It may cost you more to take a taxi in New York than it does in Waterloo to travel the same distance.

The "taxi metric" can depend on time, it can depend on location.

The Metric, g

The Metric, g


A metric is not a single number at each point in space, otherwise how could it tell us that the cylinder and sphere are curved differently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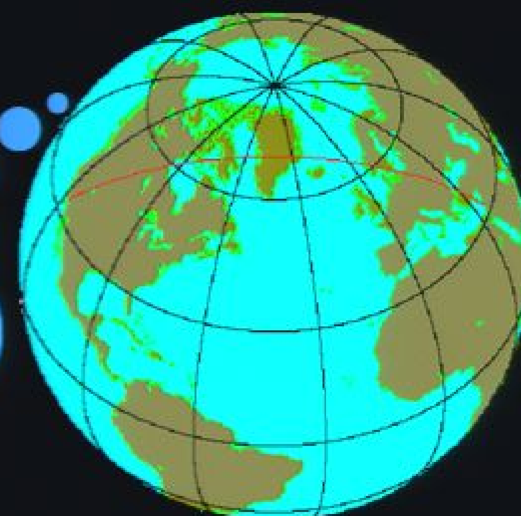
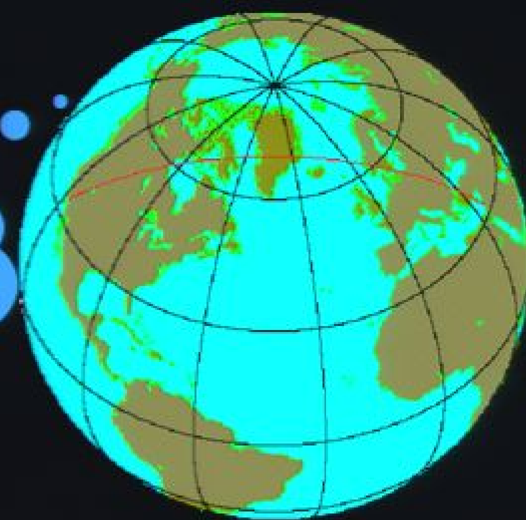


Me, I only bend in east -west.

The Metric, g

A metric is not a single number at each point in space, otherwise how could it tell us that the cylinder and sphere are curved differently

My mama calls me an intrinsic curvature



My mama calls me my little extrinsic curvature

So, for these geometries we need 2 numbers to uniquely specify the curvature of the surface

What about g, in Four Dimensions?

What about g, in Four Dimensions?

We will designate two numbers representing the metric, g_{xx} and g_{yy}, to show that they are associated with curvature in the x and y direction.

In four dimensions, it works out that we need 10

I needed another mathematical tool to help me keep the numbers straight, and yet allow me to do calculations.

The Tensor (in Spacetime)

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2- tensor is a matrix of 4x4=16 numbers

$$B_{ij} = \begin{pmatrix} 3 & -1 & 17 & 2 \\ 7 & 99 & 0 & 34 \\ 1000 & 3 & 0 & -1 \\ 4 & -2.5 & 7 & -12.3 \end{pmatrix}$$

The Tensor (in Spacetime)

With these tools, I can finally write down how the Geometry of SpaceTime is affected by mass or is it how mass is affected by the geometry of SpaceTime

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The Einstein field equation (EFE) is usually written in the form

$$G_{ij} = \frac{8\pi G}{c^4} T_{ij} + \Lambda g_{ij}$$

Einstein's Tensor

Stress-Energy Tensor

Metric Tensor

$$\begin{pmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{pmatrix} = \frac{8\pi G}{c^4} \begin{pmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{pmatrix} + \Lambda \begin{pmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{pmatrix}$$

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R$$

Curvature of Space Time

describes the density and flux of energy and momentum in spacetime

The EFE is a tensor equation relating a set of symmetric 4 x 4 tensors. Einstein's equations are actually 16 equations in the form: $G_{11} = \frac{8\pi G}{c^4} T_{11} + \Lambda g_{11}$

If you sit down and write down the Ricci tensor for a general case of a 2-dimensional space with axial symmetry, you would get something like this:

$$R_{\eta\eta} = -\frac{2a^2 \frac{\partial\psi}{\partial\theta} \cot\theta}{\delta\,\psi} + \frac{2ac \frac{\partial\psi}{\partial\eta} \cot\theta}{\delta\,\psi} + \frac{a \frac{\partial c}{\partial\eta} \cot\theta}{\delta} - \frac{\frac{\partial a}{\partial\eta} c \cot\theta}{2\delta} - \frac{a \frac{\partial a}{\partial\theta} \cot\theta}{2\delta} - \frac{2a^2 \frac{\partial^2\psi}{\partial\theta^2}}{\delta\,\psi}$$

$$-\frac{2a^2 (\frac{\partial\psi}{\partial\theta})^2}{\delta\,\psi^2} + \frac{4ac \frac{\partial\psi}{\partial\eta} \frac{\partial\psi}{\partial\theta}}{\delta\,\psi^2} - \frac{a^2 \frac{\partial d}{\partial\theta} \frac{\partial\psi}{\partial\theta}}{\delta\, d\,\psi} + \frac{ac \frac{\partial d}{\partial\eta} \frac{\partial\psi}{\partial\theta}}{\delta\, d\,\psi} + \frac{2a \frac{\partial c}{\partial\eta} \frac{\partial\psi}{\partial\theta}}{\delta\,\psi} - \frac{\frac{\partial a}{\partial\eta} c \frac{\partial\psi}{\partial\theta}}{\delta\,\psi}$$

$$-\frac{3a \frac{\partial a}{\partial\theta} \frac{\partial\psi}{\partial\theta}}{\delta\,\psi} - \frac{2a^2 c \frac{\partial c}{\partial\theta} \frac{\partial\psi}{\partial\theta}}{\delta^2\psi} + \frac{2a^2 b \frac{\partial c}{\partial\eta} \frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{a^2 \frac{\partial b}{\partial\eta} c \frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{a \frac{\partial a}{\partial\eta} bc \frac{\partial\psi}{\partial\theta}}{\delta^2\psi} + \frac{a^3 \frac{\partial b}{\partial\theta} \frac{\partial\psi}{\partial\theta}}{\delta^2\psi}$$

$$+\frac{a^2 \frac{\partial a}{\partial\theta} b \frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{2ab \frac{\partial^2\psi}{\partial\eta^2}}{\delta\,\psi} - \frac{2 \frac{\partial^2\psi}{\partial\eta^2}}{\psi} + \frac{4ac \frac{\partial^2\psi}{\partial\eta\partial\theta}}{\delta\,\psi} - \frac{2ab(\frac{\partial\psi}{\partial\eta})^2}{\delta\,\psi^2} + \frac{6(\frac{\partial\psi}{\partial\eta})^2}{\psi^2}$$

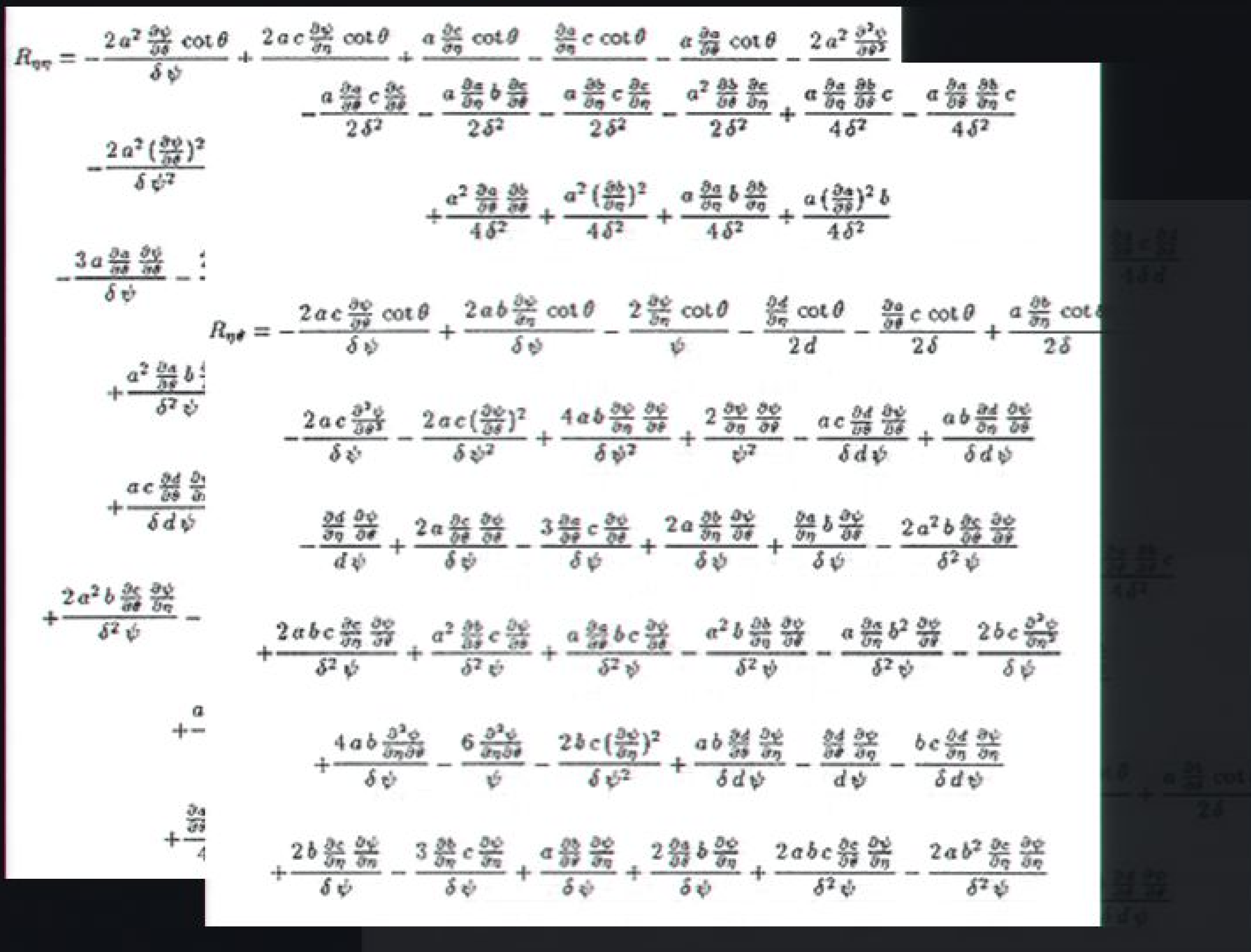
$$+\frac{ac \frac{\partial d}{\partial\theta} \frac{\partial\psi}{\partial\eta}}{\delta\, d\,\psi} - \frac{ab \frac{\partial d}{\partial\eta} \frac{\partial\psi}{\partial\eta}}{\delta\, d\,\psi} - \frac{2c \frac{\partial c}{\partial\eta} \frac{\partial\psi}{\partial\eta}}{\delta\,\psi} + \frac{\frac{\partial a}{\partial\theta} c \frac{\partial\psi}{\partial\eta}}{\delta\,\psi} - \frac{2a \frac{\partial b}{\partial\eta} \frac{\partial\psi}{\partial\eta}}{\delta\,\psi} + \frac{\frac{\partial a}{\partial\eta} b \frac{\partial\psi}{\partial\eta}}{\delta\,\psi}$$

$$+\frac{2a^2 b \frac{\partial c}{\partial\theta} \frac{\partial\psi}{\partial\eta}}{\delta^2\psi} - \frac{2abc \frac{\partial c}{\partial\eta} \frac{\partial\psi}{\partial\eta}}{\delta^2\psi} - \frac{a^2 \frac{\partial b}{\partial\theta} c \frac{\partial\psi}{\partial\eta}}{\delta^2\psi} - \frac{a \frac{\partial a}{\partial\theta} bc \frac{\partial\psi}{\partial\eta}}{\delta^2\psi} + \frac{a^2 b \frac{\partial b}{\partial\eta} \frac{\partial\psi}{\partial\eta}}{\delta^2\psi} + \frac{a \frac{\partial a}{\partial\eta} b^2 \frac{\partial\psi}{\partial\eta}}{\delta^2\psi}$$

$$+\frac{a \frac{\partial c}{\partial\eta} \frac{\partial d}{\partial\theta}}{2\delta\, d} - \frac{\frac{\partial a}{\partial\eta} c \frac{\partial d}{\partial\theta}}{4\delta\, d} - \frac{a \frac{\partial a}{\partial\theta} \frac{\partial d}{\partial\theta}}{4\delta\, d} - \frac{\frac{\partial^2 d}{\partial\eta^2}}{2d} + \frac{(\frac{\partial d}{\partial\eta})^2}{4d^2} - \frac{c \frac{\partial c}{\partial\eta} \frac{\partial d}{\partial\eta}}{2\delta\, d}$$

$$+\frac{\frac{\partial a}{\partial\theta} c \frac{\partial d}{\partial\eta}}{4\delta\, d} + \frac{\frac{\partial a}{\partial\eta} b \frac{\partial d}{\partial\eta}}{4\delta\, d} + \frac{a \frac{\partial^2 c}{\partial\eta\partial\theta}}{\delta} - \frac{a \frac{\partial^2 b}{\partial\eta^2}}{2\delta} - \frac{a \frac{\partial^2 a}{\partial\theta^2}}{2\delta} + \frac{ac \frac{\partial c}{\partial\eta} \frac{\partial c}{\partial\theta}}{\delta^2}$$

... and just a little bit more.



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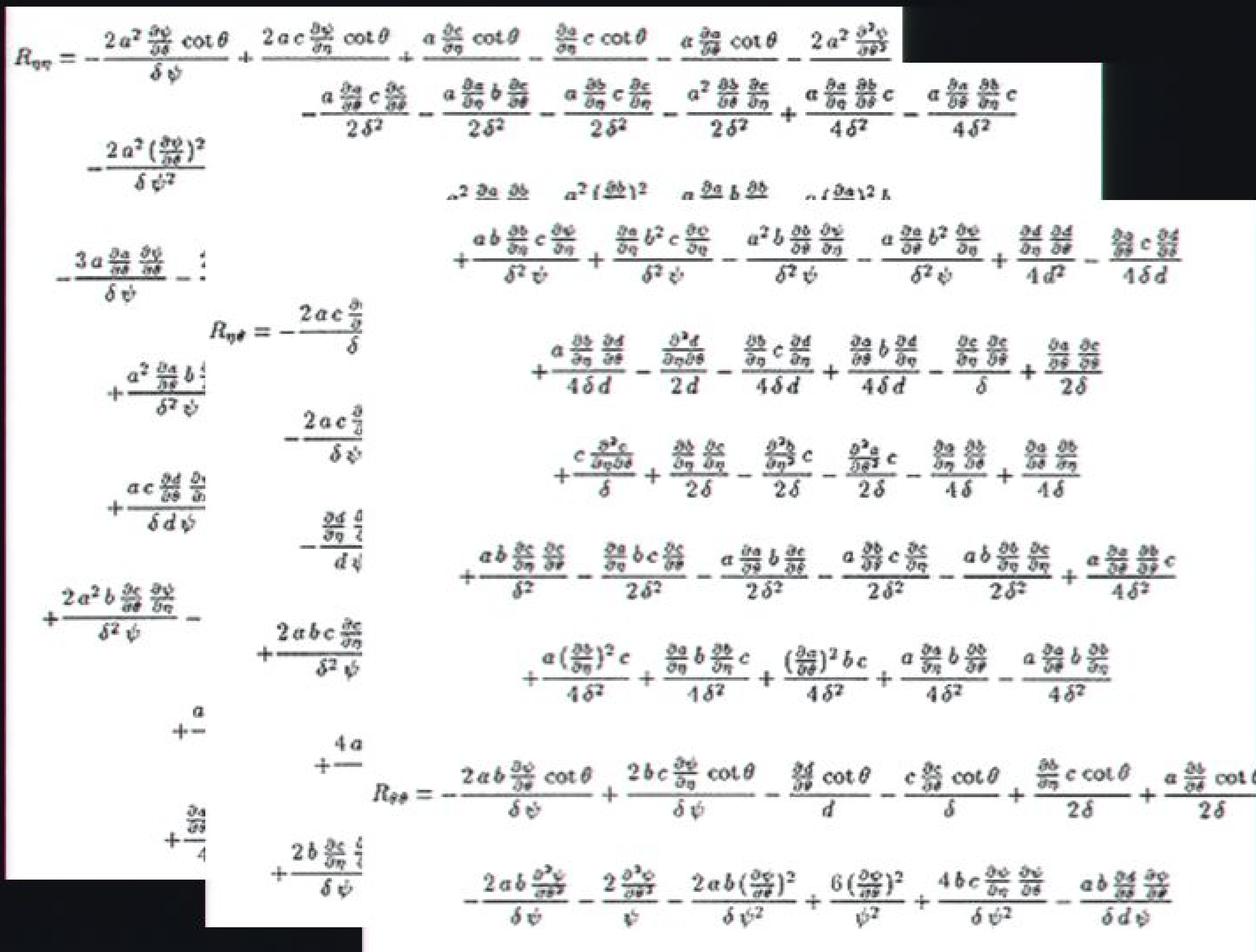
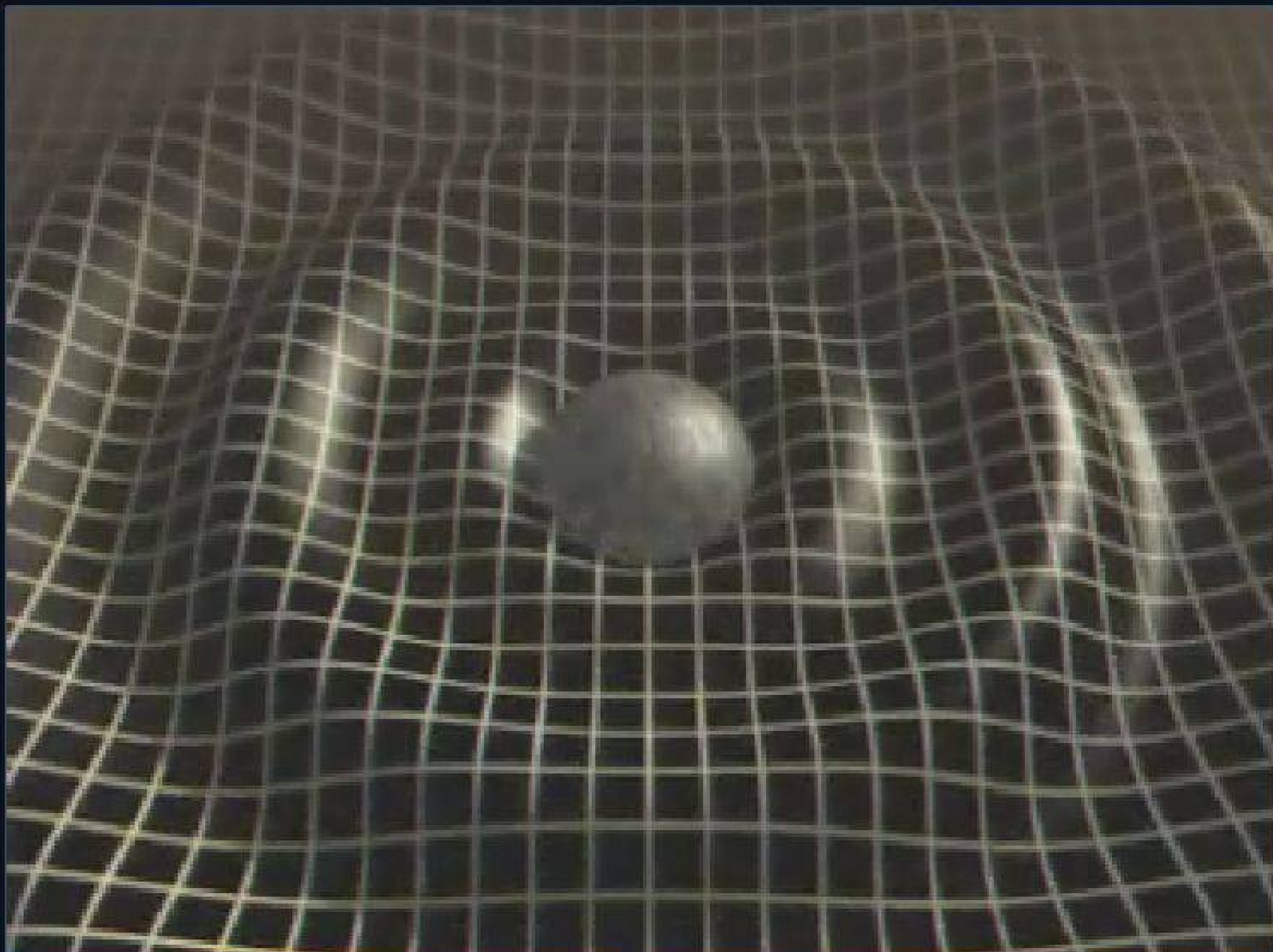


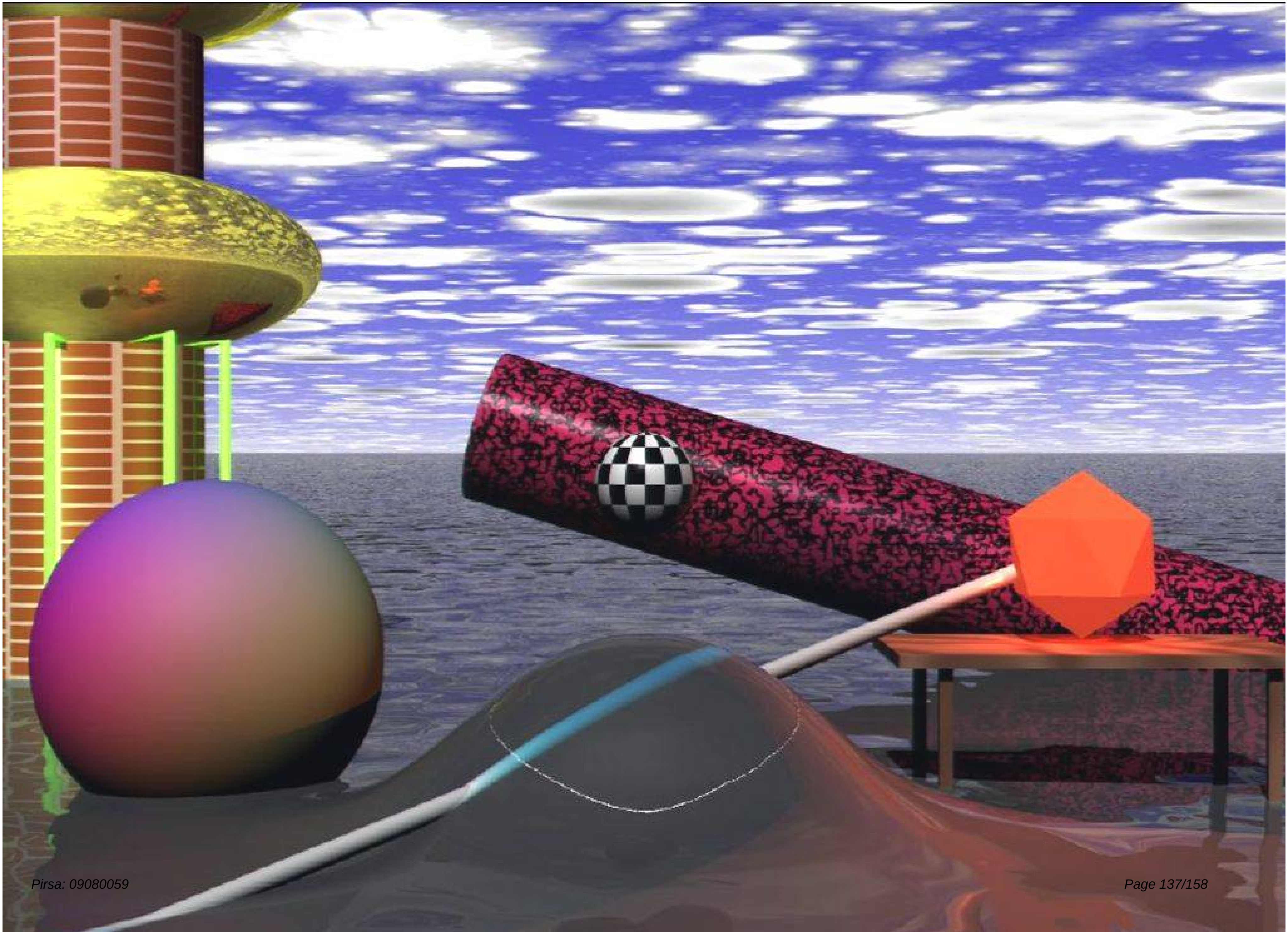
This is a general expression for **Ricci tensor** R_{mn} in only two dimensions, with axial symmetry.

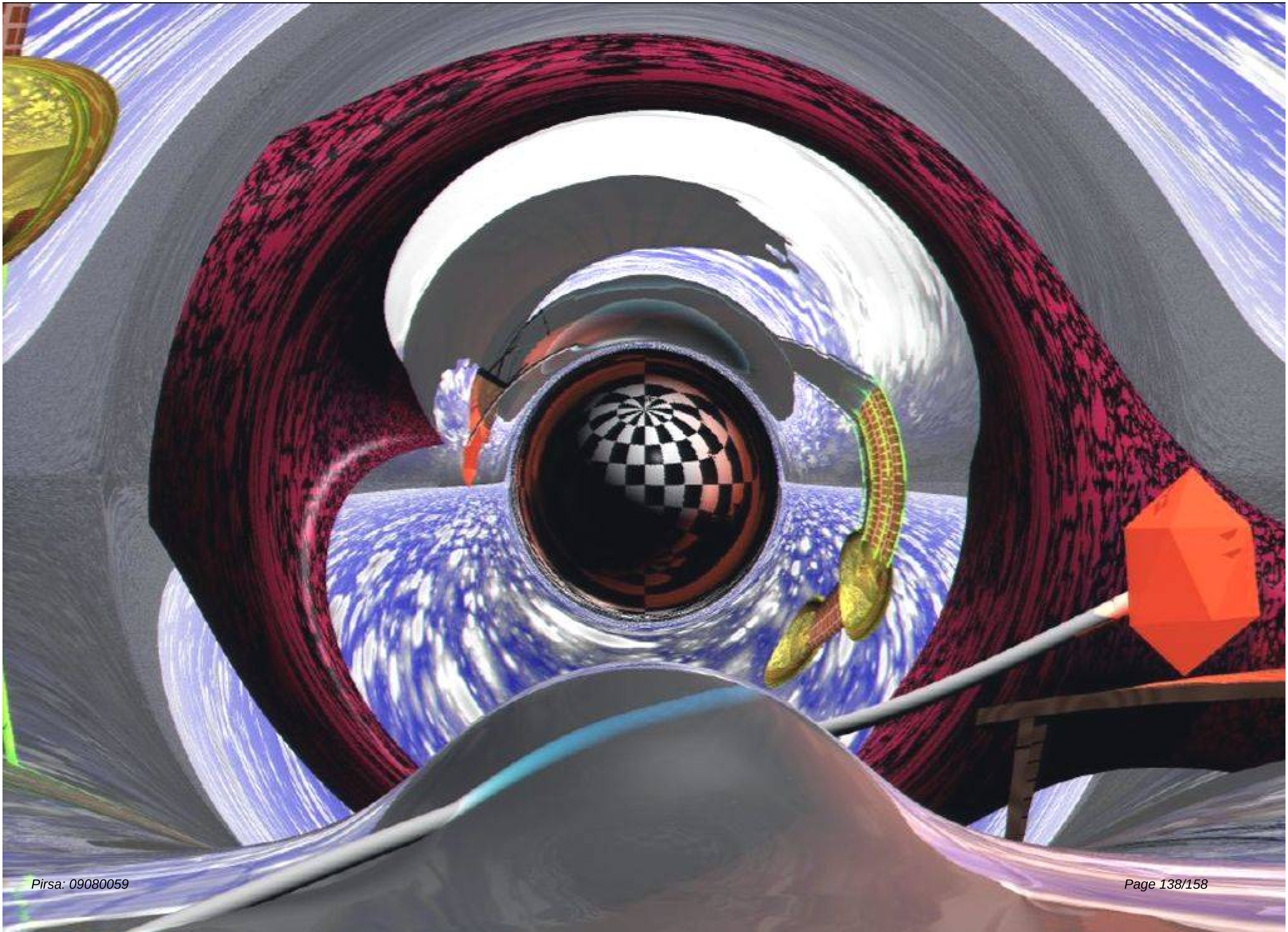
Just try to imagine all of three dimensions of space plus one of time!

What does all this say?

Curvature

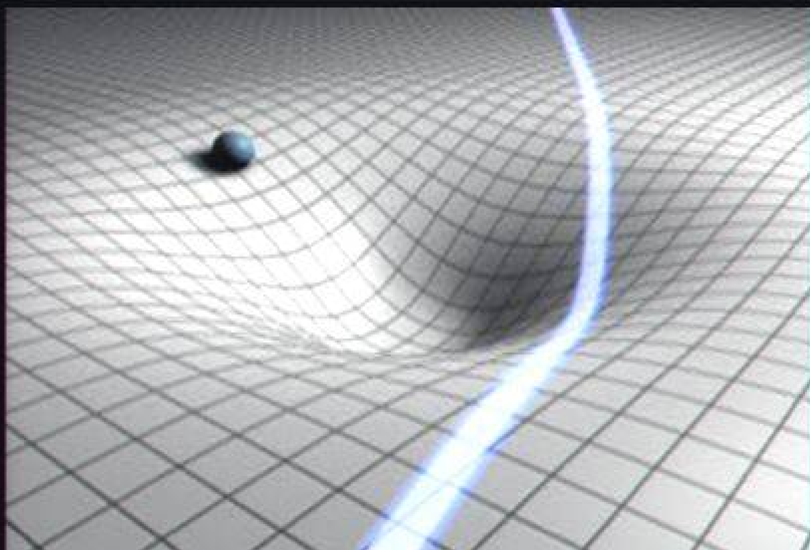


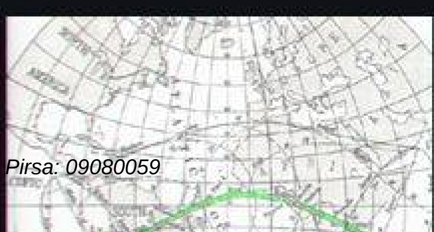


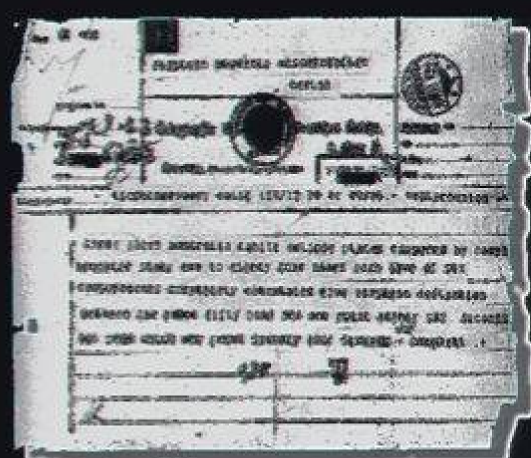


1919 Verification

Tiny island of Principe, nestled in the crook of Africa's Gulf of Guinea







The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.

1919 Verification

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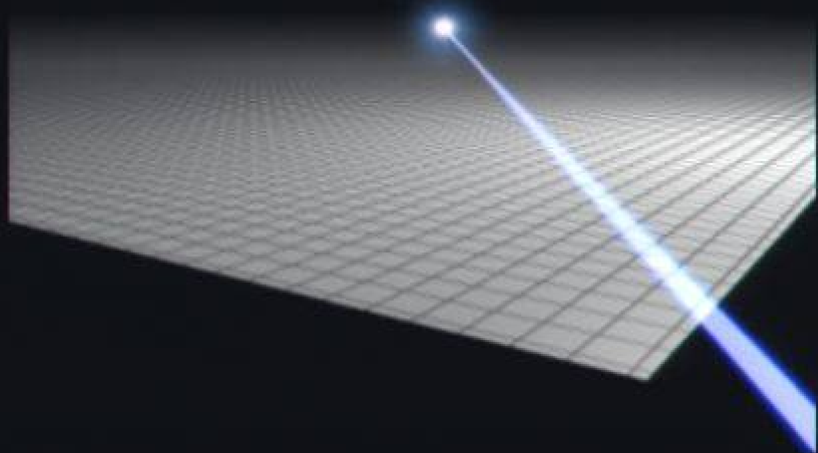
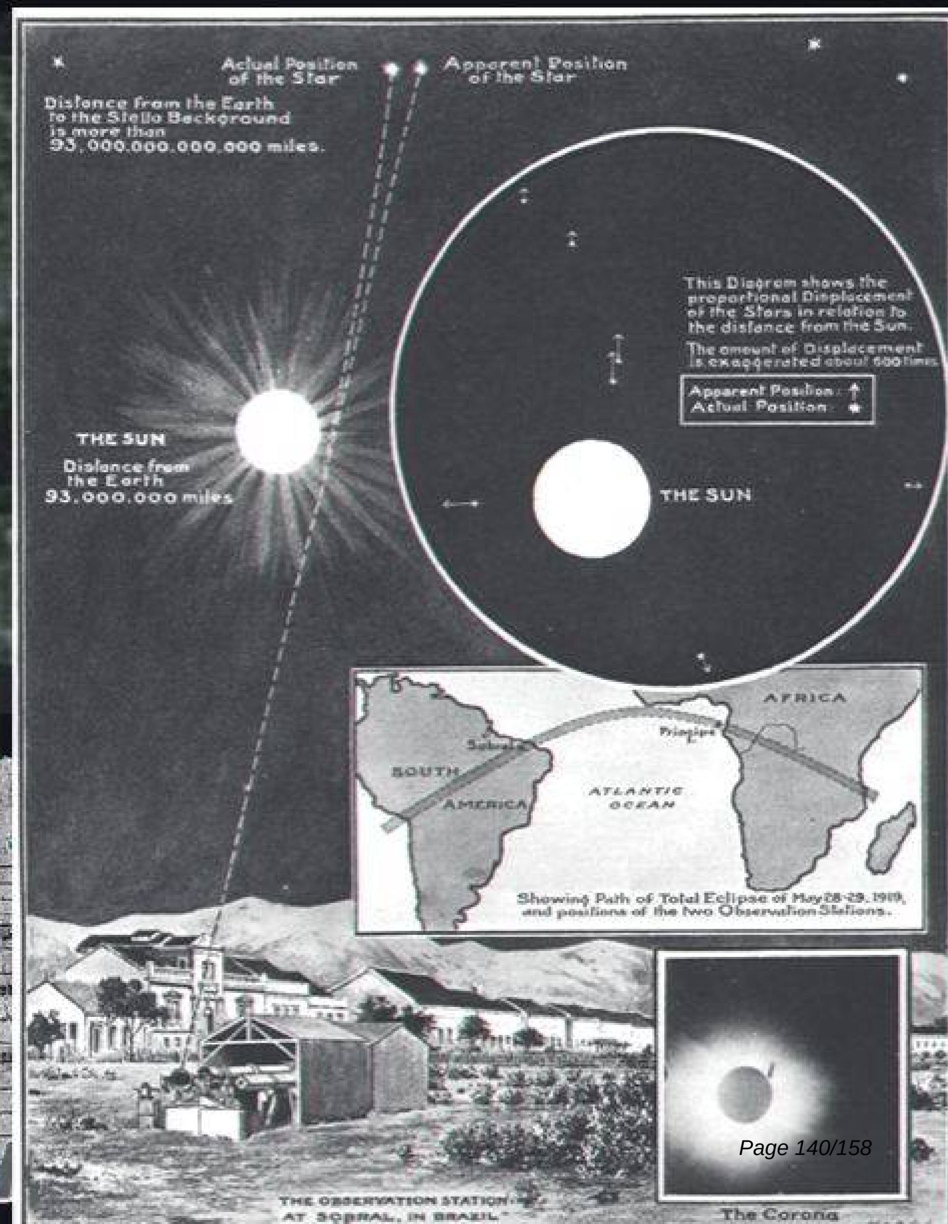
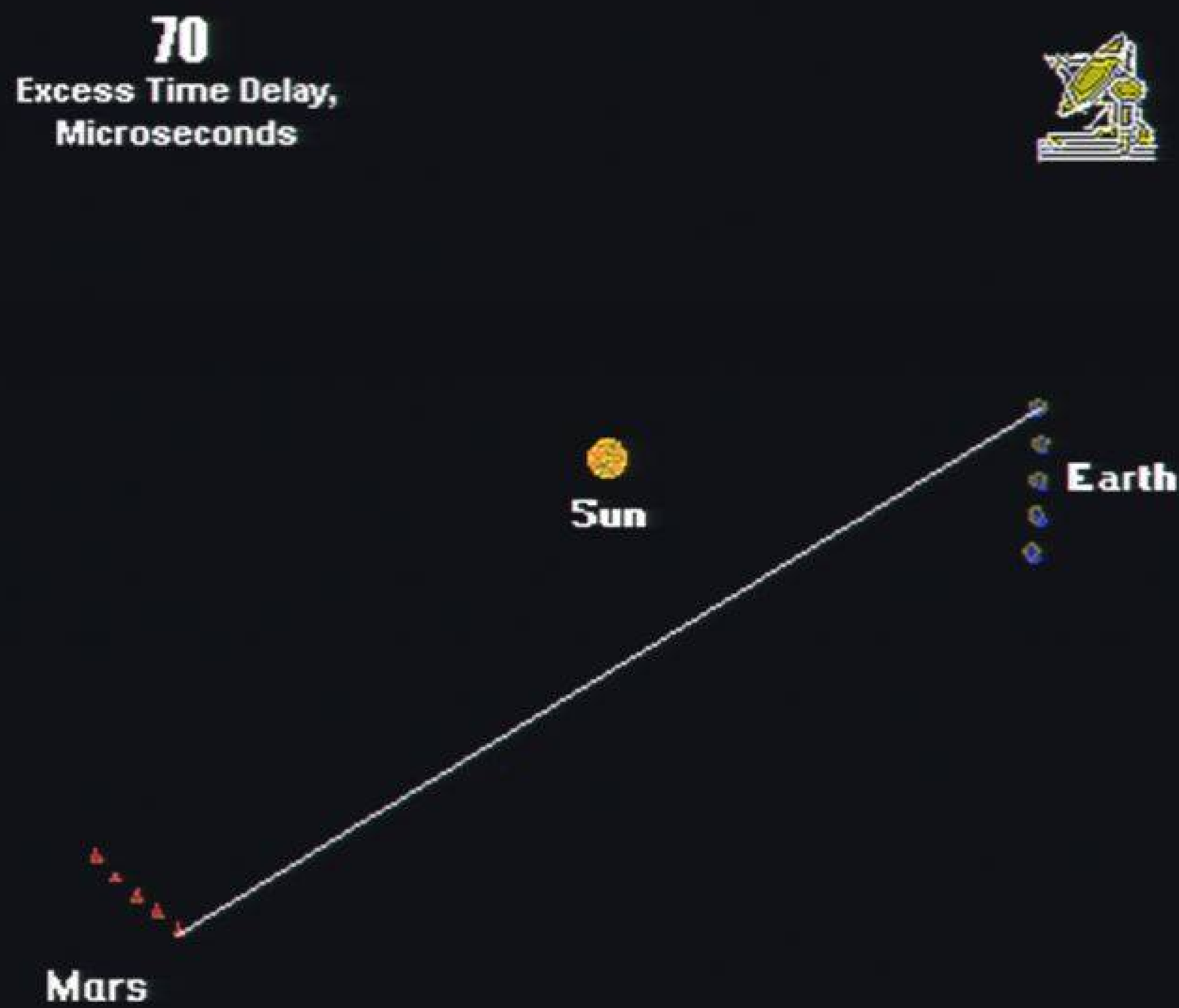




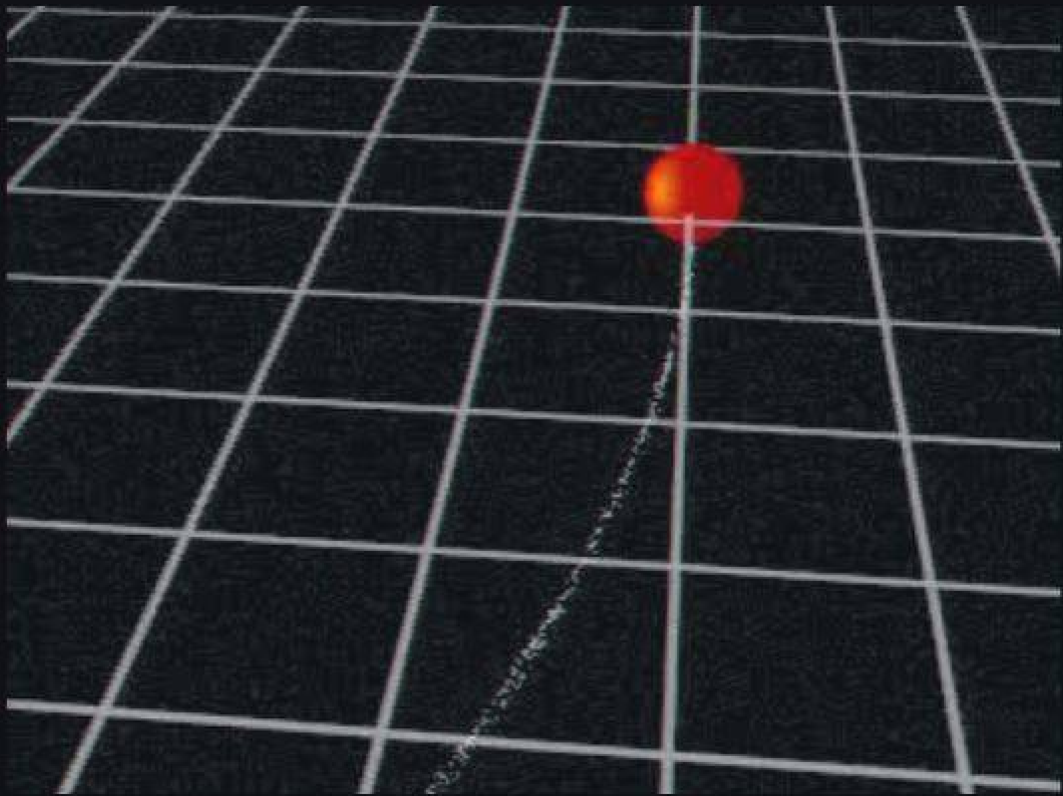
About 1.30 when the partial phase was well advanced, we began to get glimpses of the Sun, at 1.55 we could see the crescent (through the cloud) almost continuously and large patches of clear sky appearing. We had to carry out our programme of photographs in faith. I did not see the eclipse, being too busy changing plates, except for one glance to make sure it had begun.... We took 16 photographs ... but the cloud has interfered very much with the star images.

General Relativity Test (1976)



Mercury's Precession of axis

- Mercury's precession did not agree with Newton's Laws
- Out by 43 arc seconds per century.
- Einstein's Theory accounted for this 43 arc seconds



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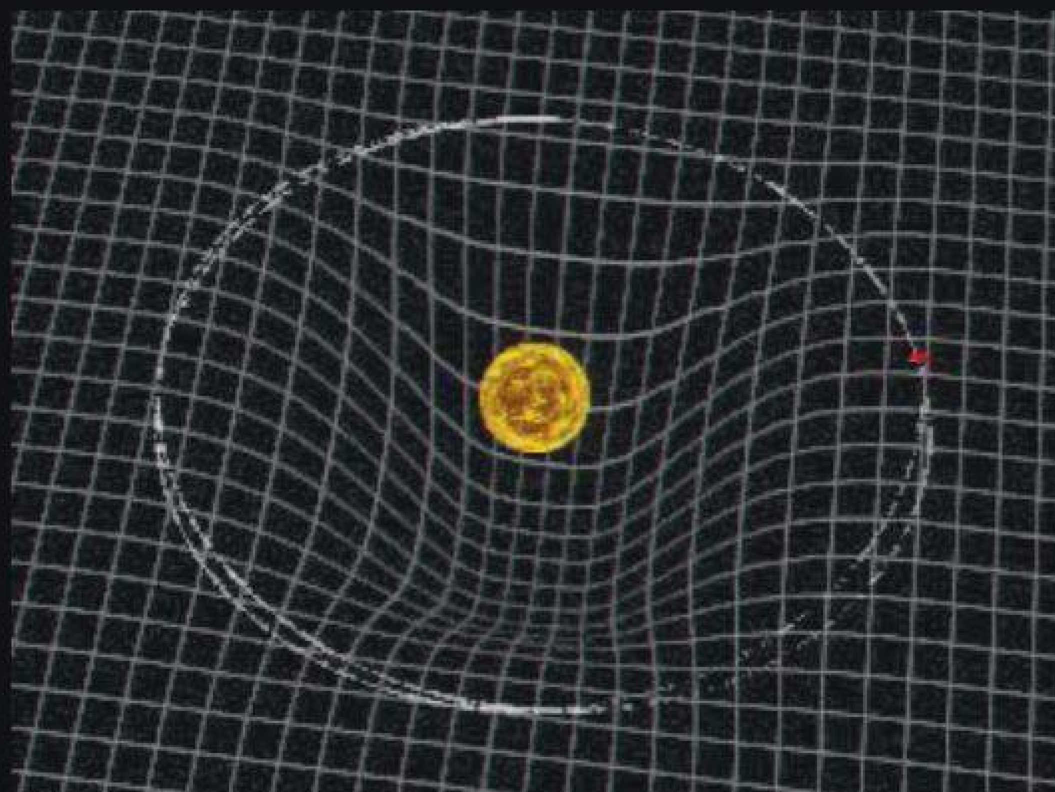


Calculation of Schwarzschild radius

What to do with the Field Equations

Calculation of Schwarzschild radius

- In 1916 Karl Schwarzschild discovers a solution of the Einstein field equation, which describes a nonspinning, uncharged spherical body.
- Did this when serving in the German Army on the Russian front of World War I
- Only required a few days to solve equation and describe spacetime curvature.
- Einstein presented solution on behalf of Schwarzschild to the Academy of Sciences.
- Schwarzschild died on the front 4 months later.

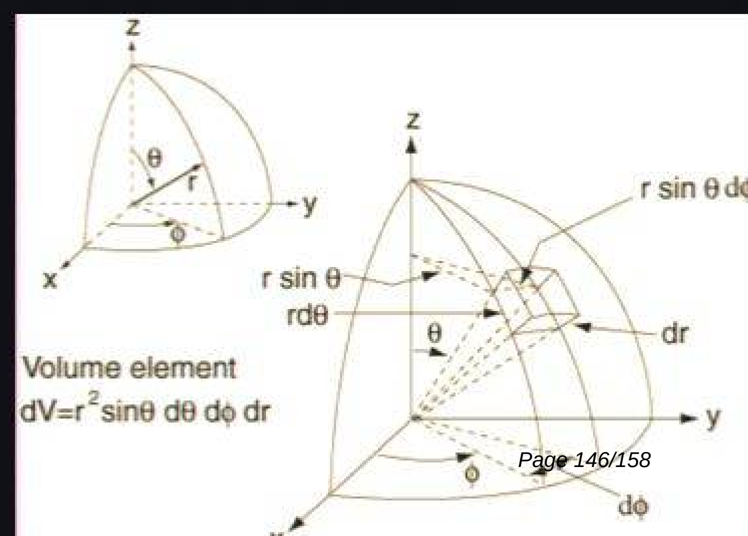
The Schwarzschild Radius

$$d\sigma^2 = -\left(1-\frac{r_s}{r}\right)c^2dt^2 + \frac{dr^2}{\left(1-\frac{r_s}{r}\right)} + r^2\left(d\theta^2 + \sin^2(\theta)\,d\phi^2\right)$$

Curvature factor

$$r_s = \frac{2GM}{c^2}$$

r, θ, ϕ are the polar coordinates



Schwarzschild Metric

$$d\sigma^2 = -\left(1-\frac{r_s}{r}\right)c^2dt^2 + \frac{dr^2}{\left(1-\frac{r_s}{r}\right)} + r^2\left(d\theta^2 + \sin^2(\theta)\,d\phi^2\right)$$

$$d\tau^2 = \left(1-\frac{r_s}{r}\right)c^2dt^2 - \frac{dr^2}{\left(1-\frac{r_s}{r}\right)} - r^2\left(d\theta^2 + \sin^2(\theta)\,d\phi^2\right)$$

τ is the proper time (time measured by a clock moving along path)

σ is the proper distance (distance measured by a clock moving along path)

t is the time coordinate (measured by a far away stationary observer)

r is the radial coordinate (circumference of a circle centered on star divided by 2π)

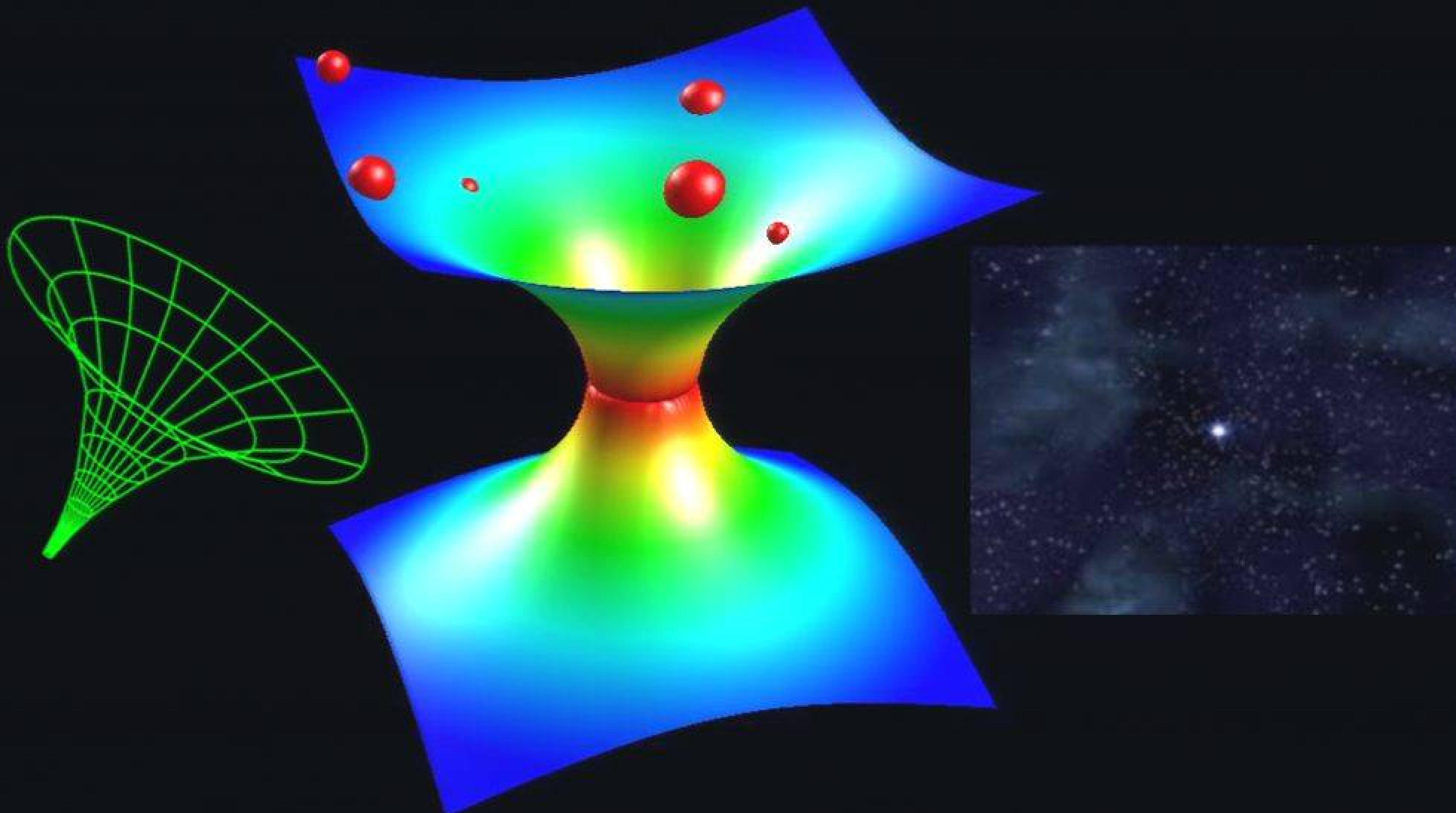
r_s is the Schwarzschild radius

Schwarzchild radii for different objects

Object	Mass	R_S
Atom	10^{-26} kg	10^{-51} cm
Human Being	70 kg	10^{-23} cm
Earth	6.0×10^{24} kg	0.89 cm
Sun	2.0×10^{30} kg	3.0 km
Galaxy	$10^{11}\ M_S$	10^{-2} l.y.
Universe (if closed)	$10^{23}\ M_S$	10^{10} l.y.

$$r_s = \frac{2GM}{c^2}$$

Embedding Diagrams

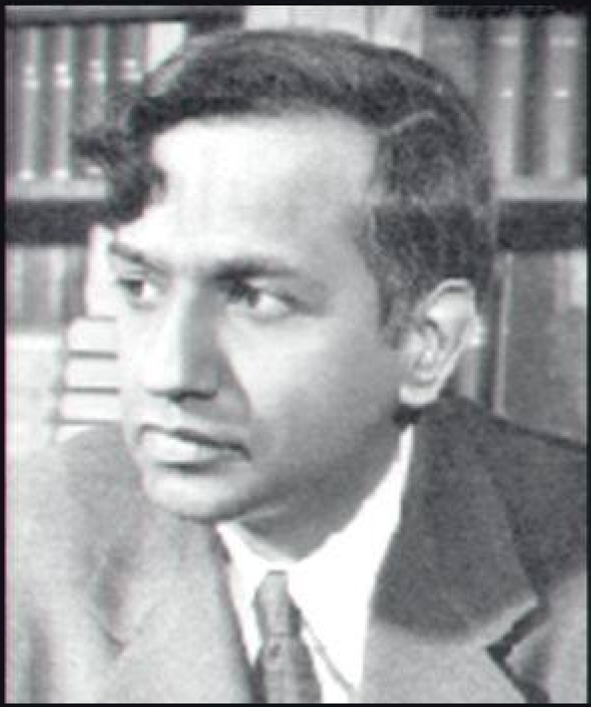


Emphasis on Spatial rather than temporal interpretations

Sir Arthur Eddington

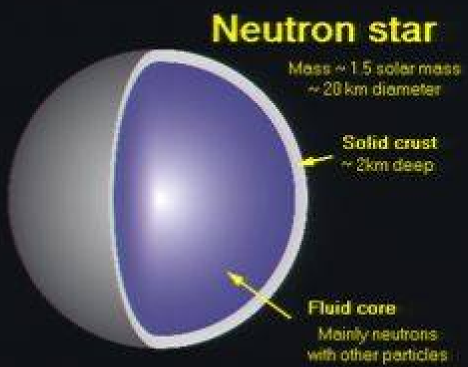
- 1926 Book – The internal constitution of the Stars
- Early proponent of Einstein's Theory of General Relativity (next to Einstein best expert on General Relativity)
- Poses the mystery of white dwarfs and attacks the reality of black holes predicted by Schwarzschild.
- Believed White Dwarf was last state in a stars life (rock Star)
- Paradox with White Dwarf

Subrahmanyan Chandrasekhar

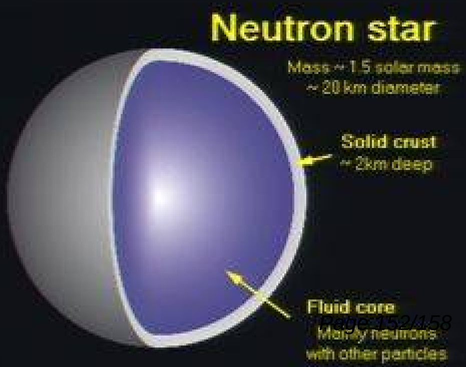


- *Idolized Eddington, resolved Eddington's paradox*
- *In 1930 he showed that there is a maximum mass for White Dwarfs*
- *1935 Eddington attacks his work. "Chandra" left the field of Blackholes until 1970's*
- *Nobel Prize in Physics 1983*

Walter Baade and Fritz Zwicky



- *Identifies the process of a supernovae, predicted that this collapse strips the atoms of their electrons, packing the nuclei together as a neutron star .*
- *Neutron stars would not be verified observably until 1968.*
- *Identified the galaxies associated with cosmic radio sources.*
- *Still something was missing that took a star from fusion to supernovae.*

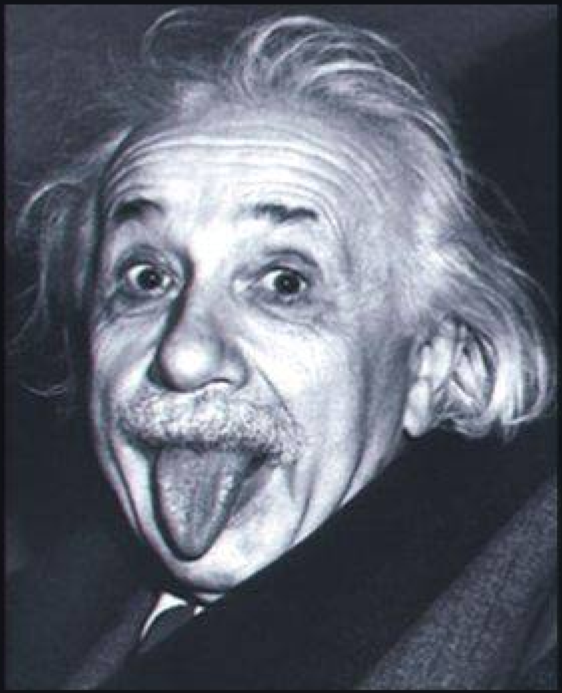


Robert J. Oppenheimer

- *Showed that there is a maximum mass for a neutron star from 1.5 to about 3 solar masses (1938).*
- *In a highly idealized calculation, showed that an imploding star forms a black hole.*
- *Led the American atomic bomb project.*
- *Which provided the opportunity to experimentally verify and test theories (too expensive for the universities) and the development of the atomic bombs which mimic the power source for the sun to come up with the mathematics and understanding of stellar mechanics*
- *Major battle with Wheeler.*

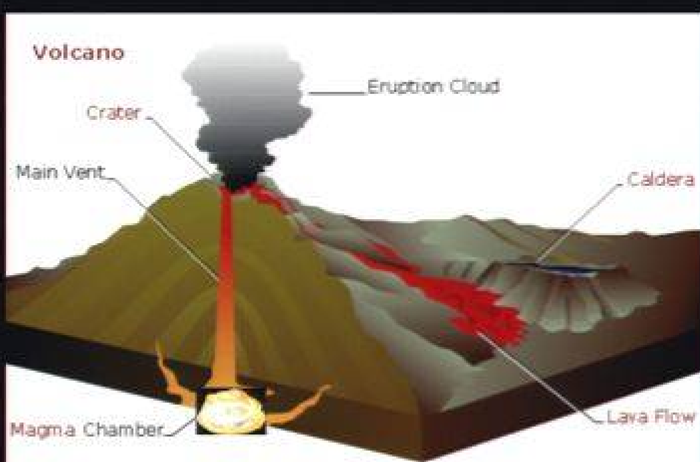
Robert J. Oppenheimer

In 1939 Einstein wrote a paper about his concerns about Oppenheimer's paper and the Schwarzschild radius and states "Schwarzschild singularities do not exist in physical reality". He demonstrated that a collapsing star is unstable when it reaches the Schwarzschild radius, which ended up being mute since that star collapses into a singularity there anyway.



Yakov Zel'dovich

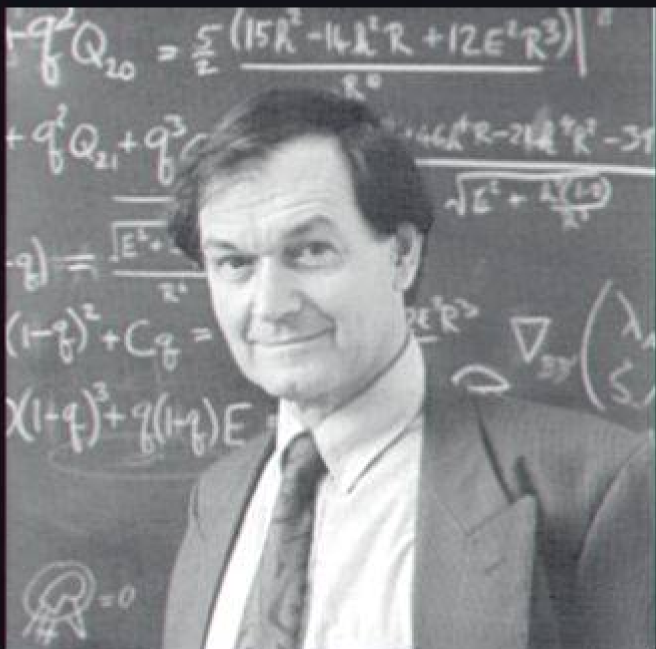
- *Soviet counterpart to Oppenheimer.*
- *Developed the theory of nuclear chain reactions. (1939)*
- *Lead theorist on USSR atomic bomb (1945)*
- *Creates black hole research team (1962).*
- *Super massive black holes power Quasars (1960's).*



John Wheeler

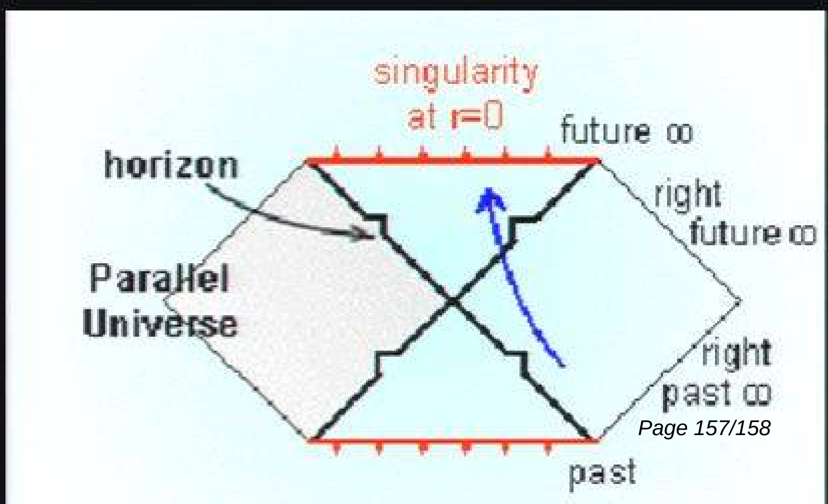
- With Bohr develops the theory of nuclear fission.
- Completes a catalog cold, dead stars firming up evidence of destiny of dead stars. (1957)
- Major battle with Oppenheimer about existence of black holes. (1957)
- Retracted argument and became the leading proponent of black hole. (1960)
- Coined the phrase "Black Hole".
- Coined the phrase "a Black Hole has no hair" (1968).

Roger Penrose

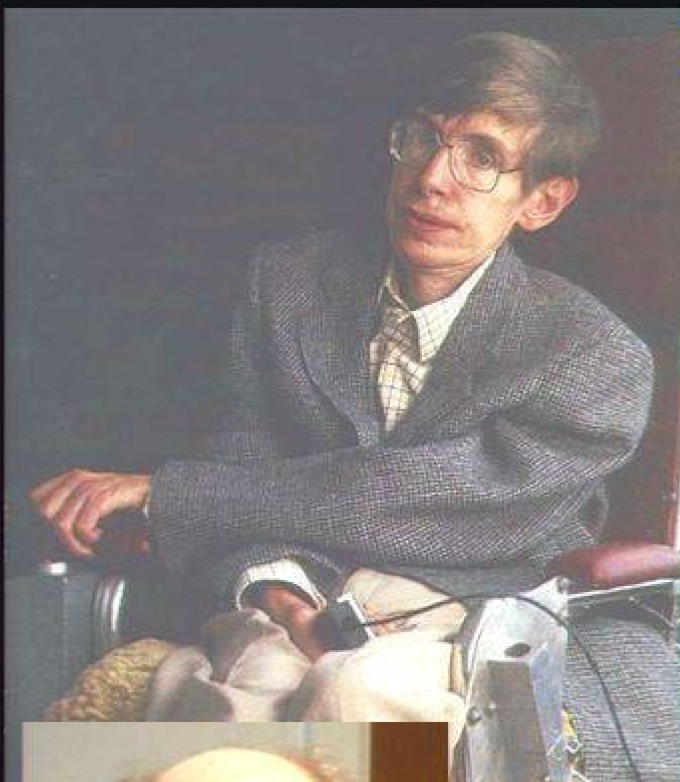


- Speculated black holes loose their hair by radiating it away.
- Discovered that spinning black holes store energy in space outside their horizon (1969).
- Discovered surface area of black holes must increase.
- Proved that black holes must have singularities at their core (1964).
- Proposed cosmic censorship conjecture (1969).

Topology

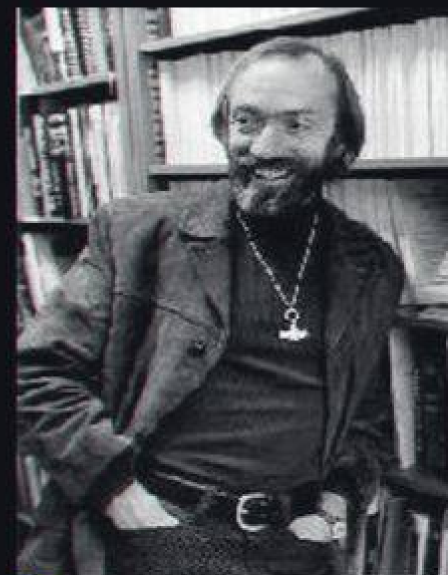


The Blackhole Stars Today



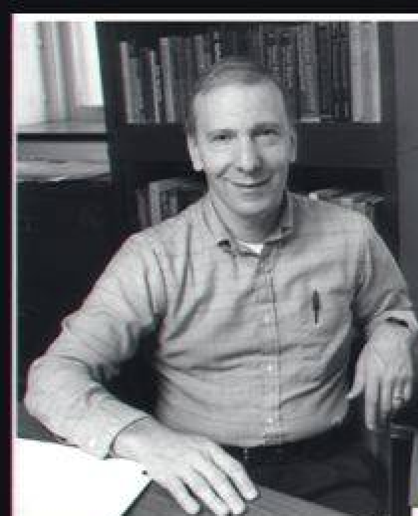
Hawking

Bekenstein



Thorne

Susskind



Werner Israel

Robert Wald