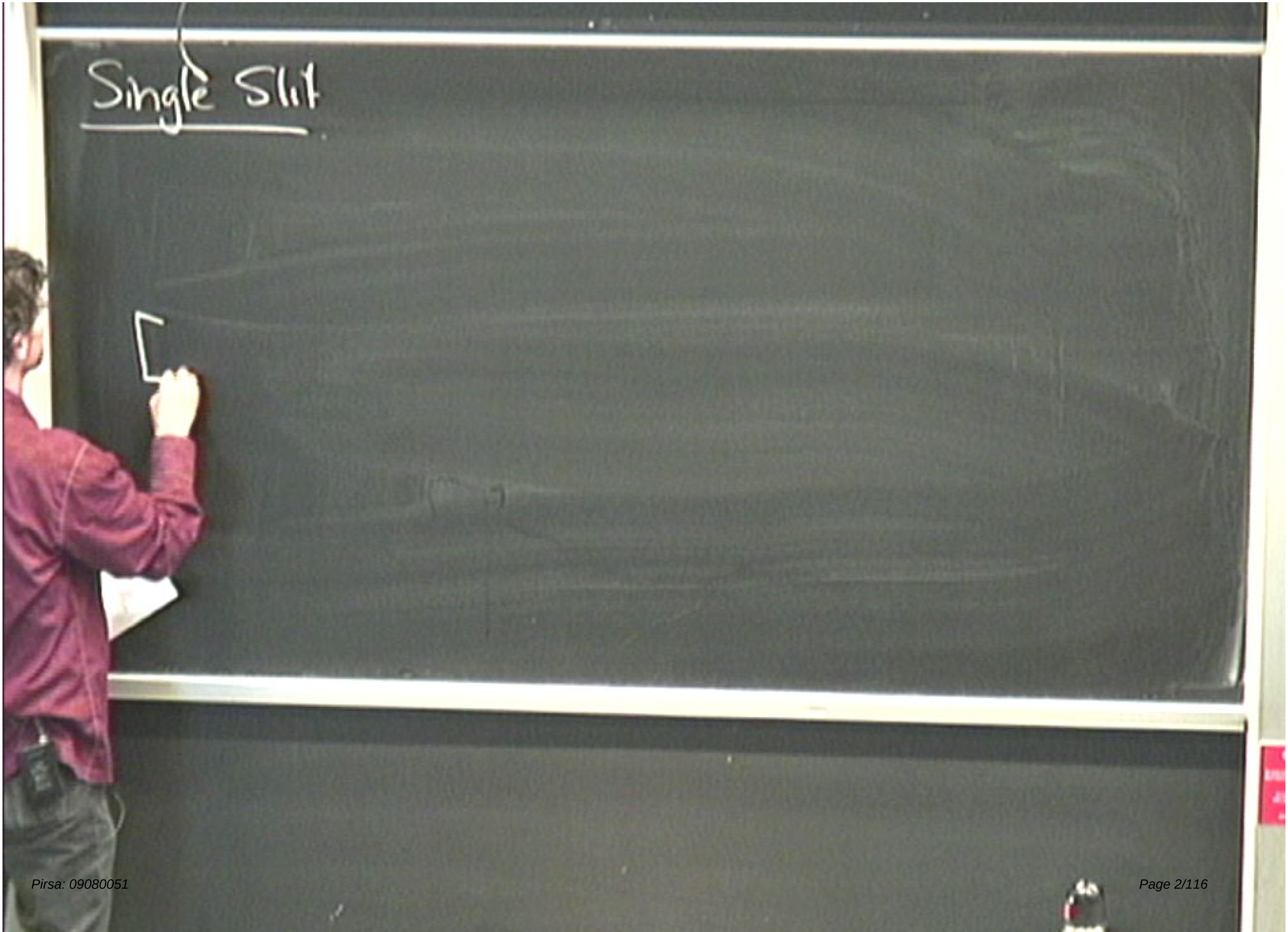
Title: Quantum 1

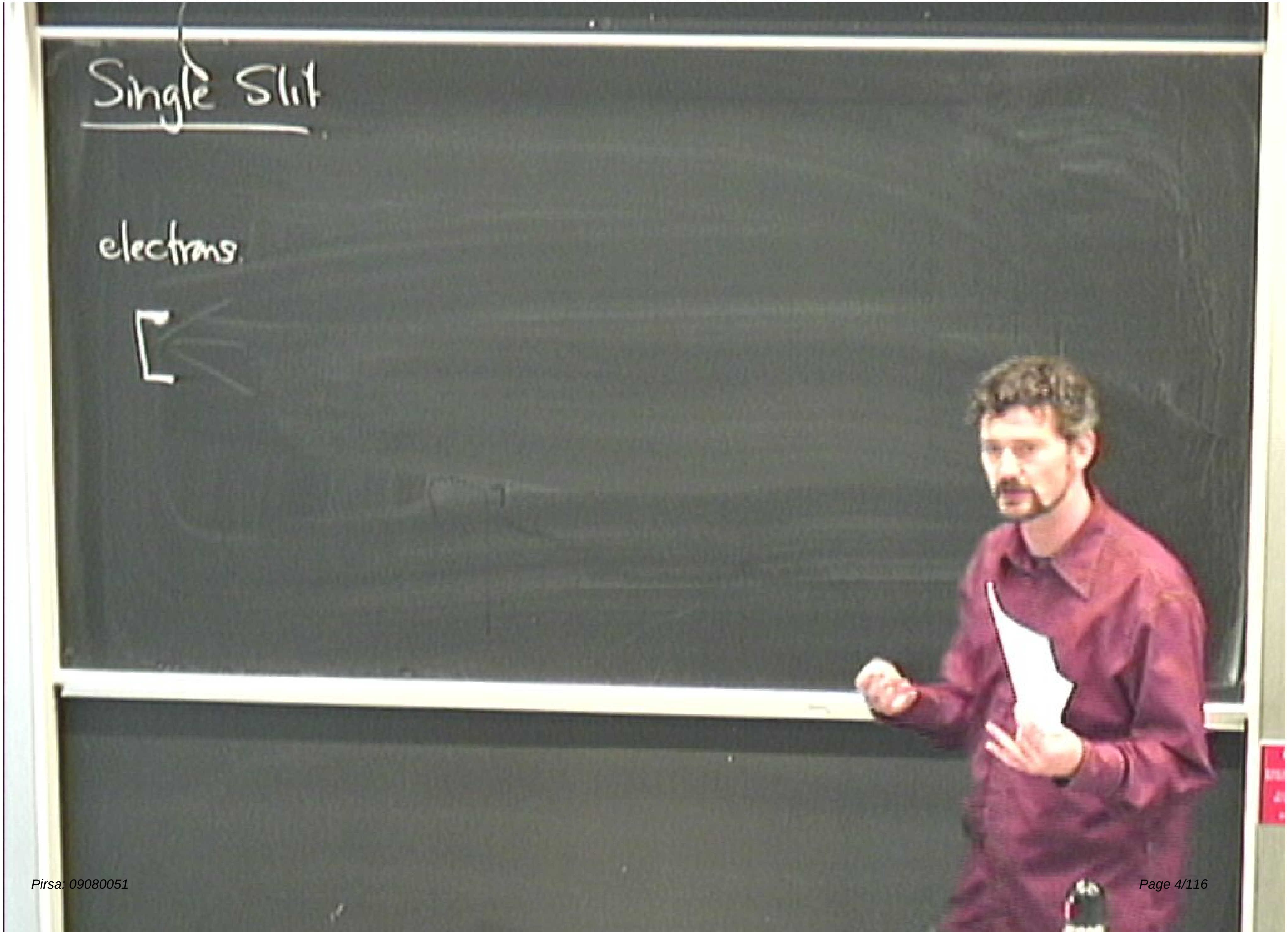
Date: Aug 11, 2009 09:00 AM

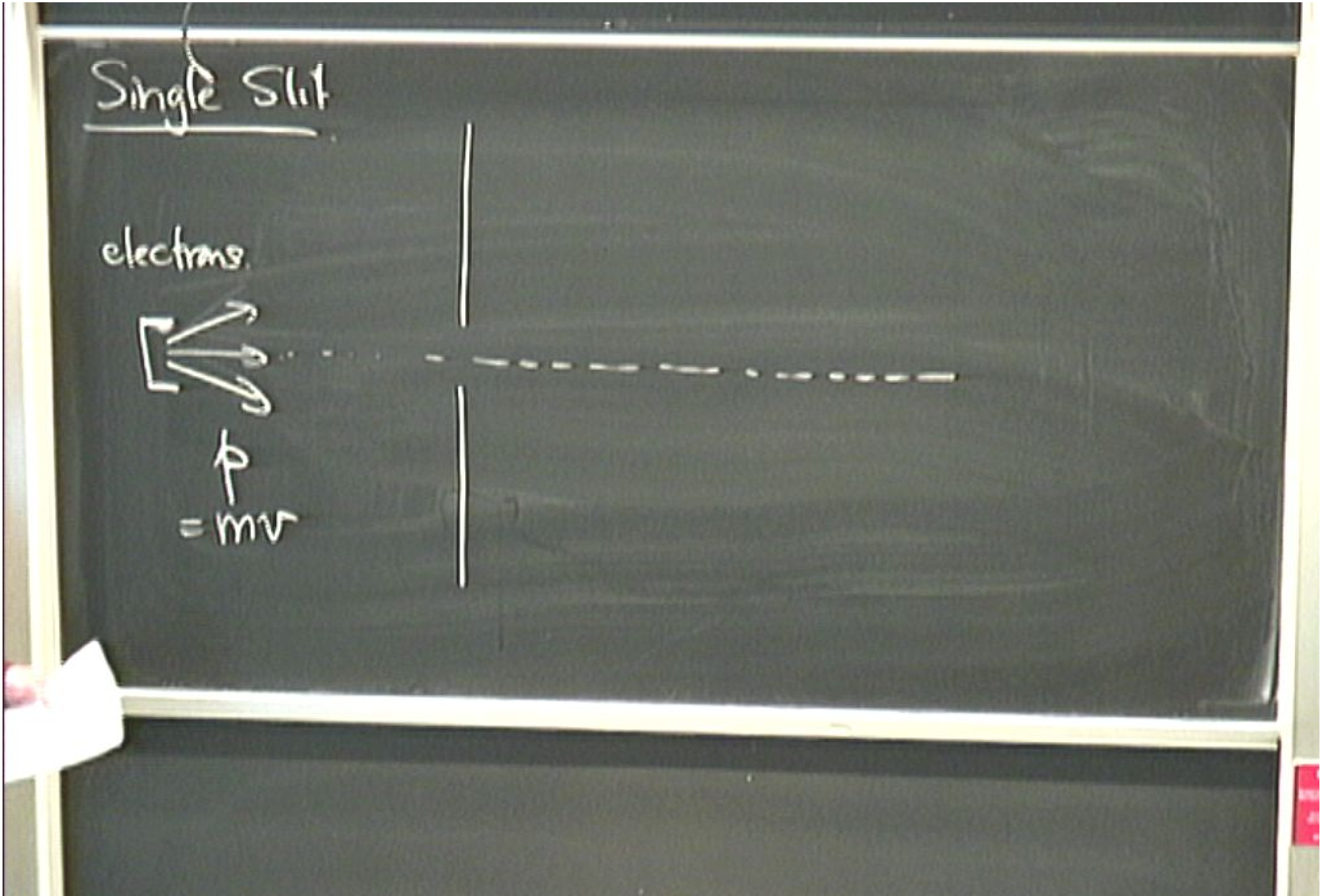
URL: http://pirsa.org/09080051

Abstract: Single and double slit experiments as motivation for the de Broglie relation. Application to the particle in a box problem: superposition principle and energy quantization.

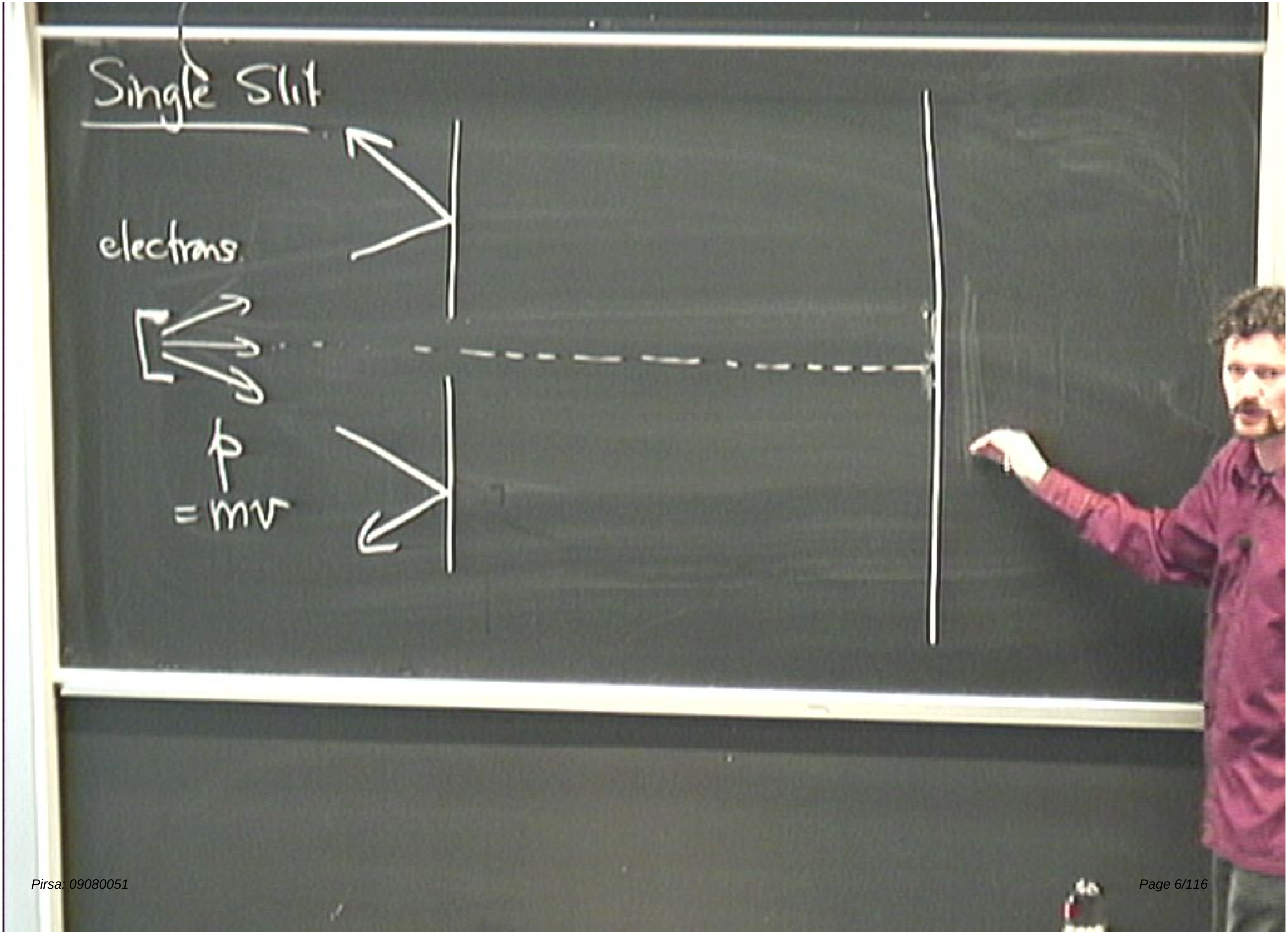

Single Slit

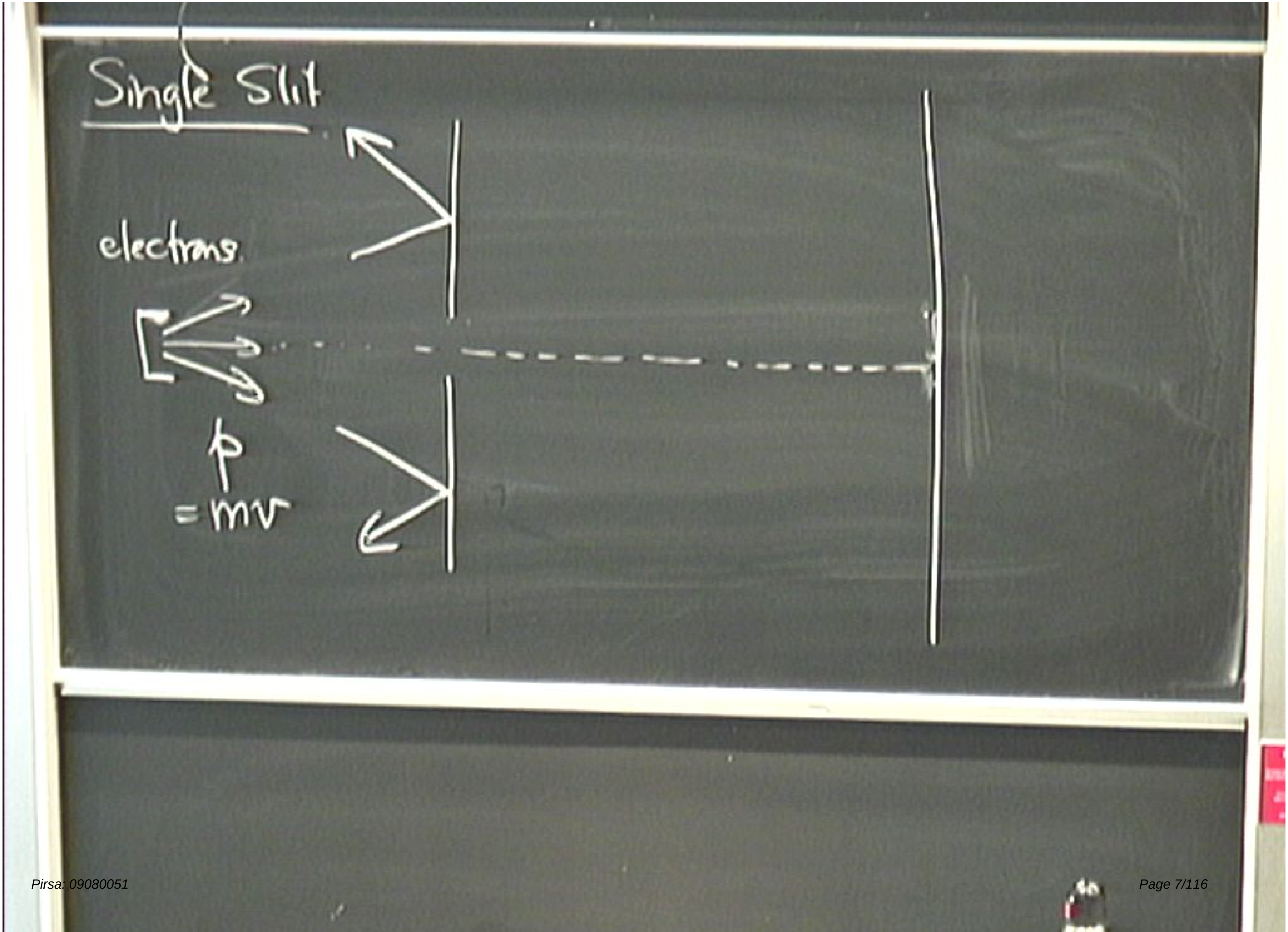
Single Slit

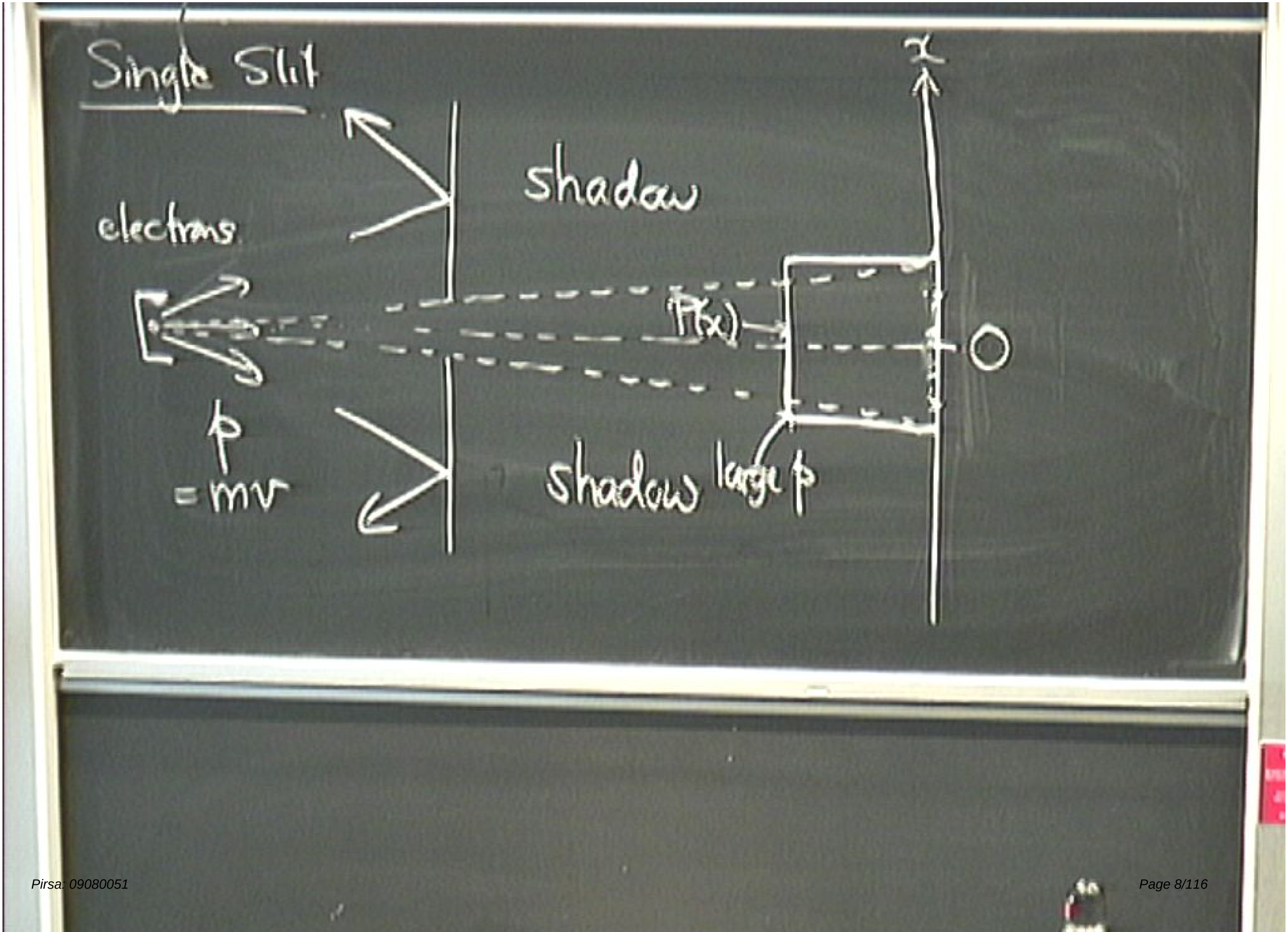

Single Slit
electrons

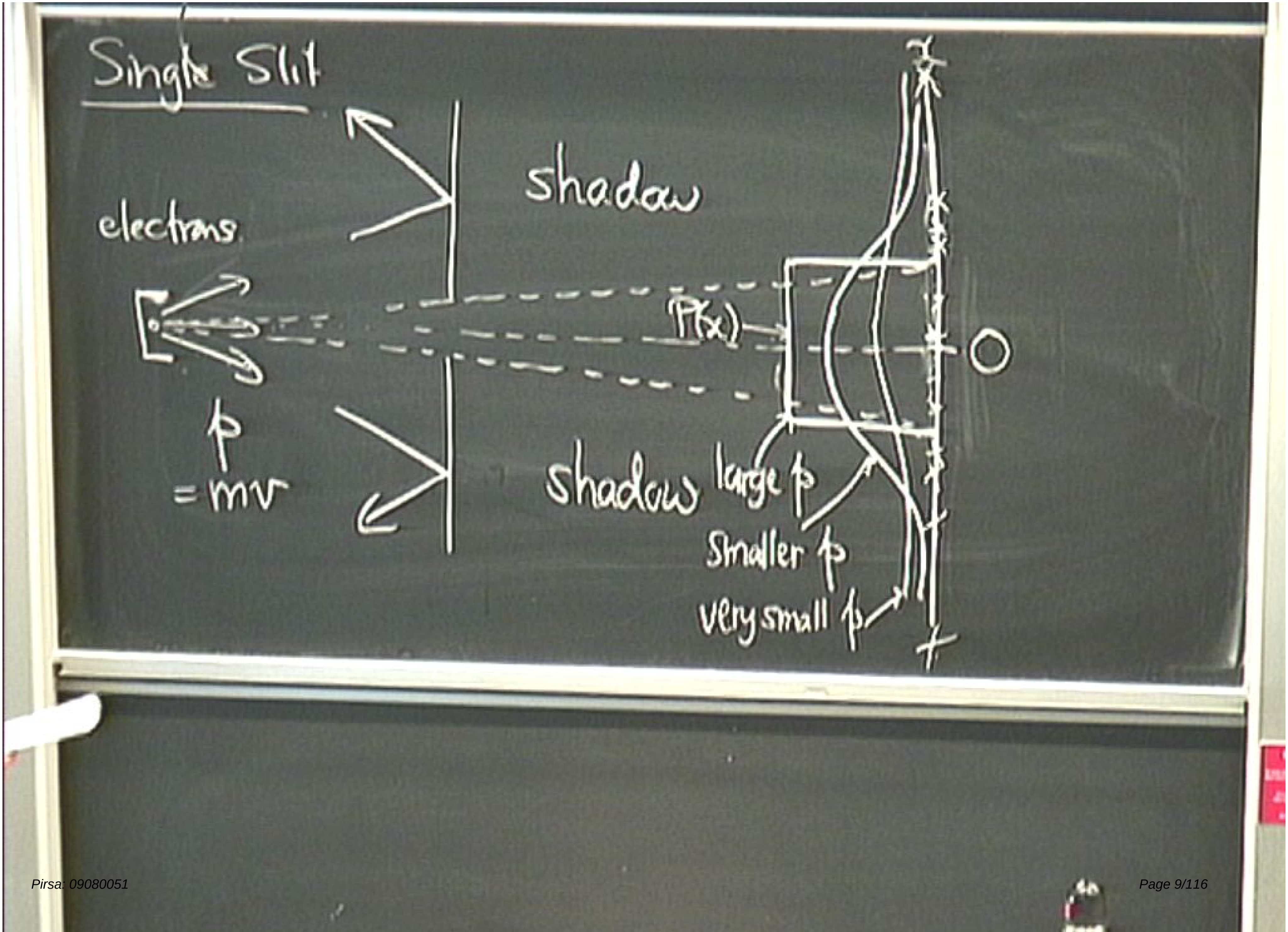


Single Slit
electrons
p
= mv


Single Slit
electrons
p
= mv


Single Slit
electrons
p
= mv

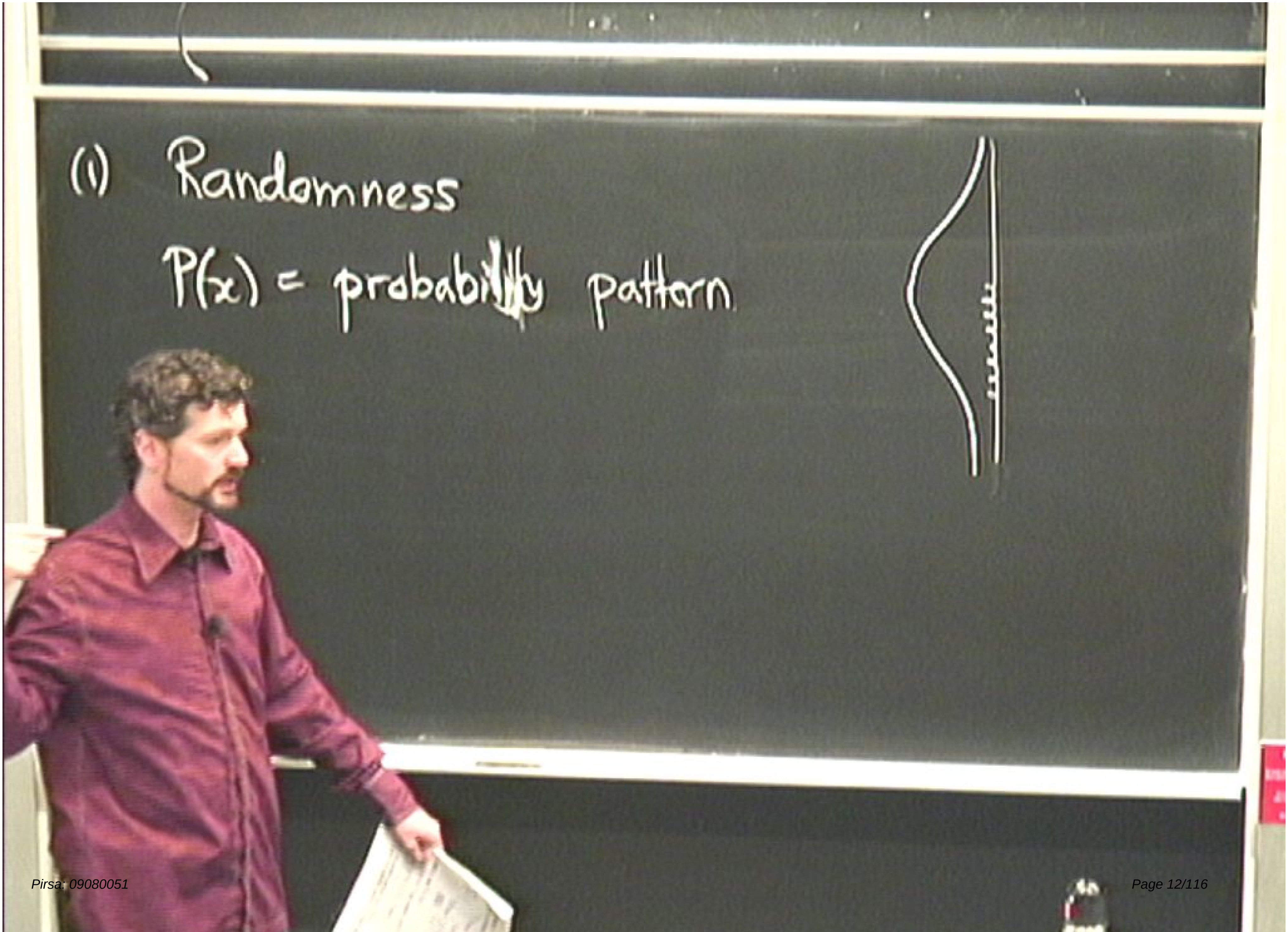

Single Slit
electrons
p
= mv
shadow
shadow
P(x)
large p
x
0

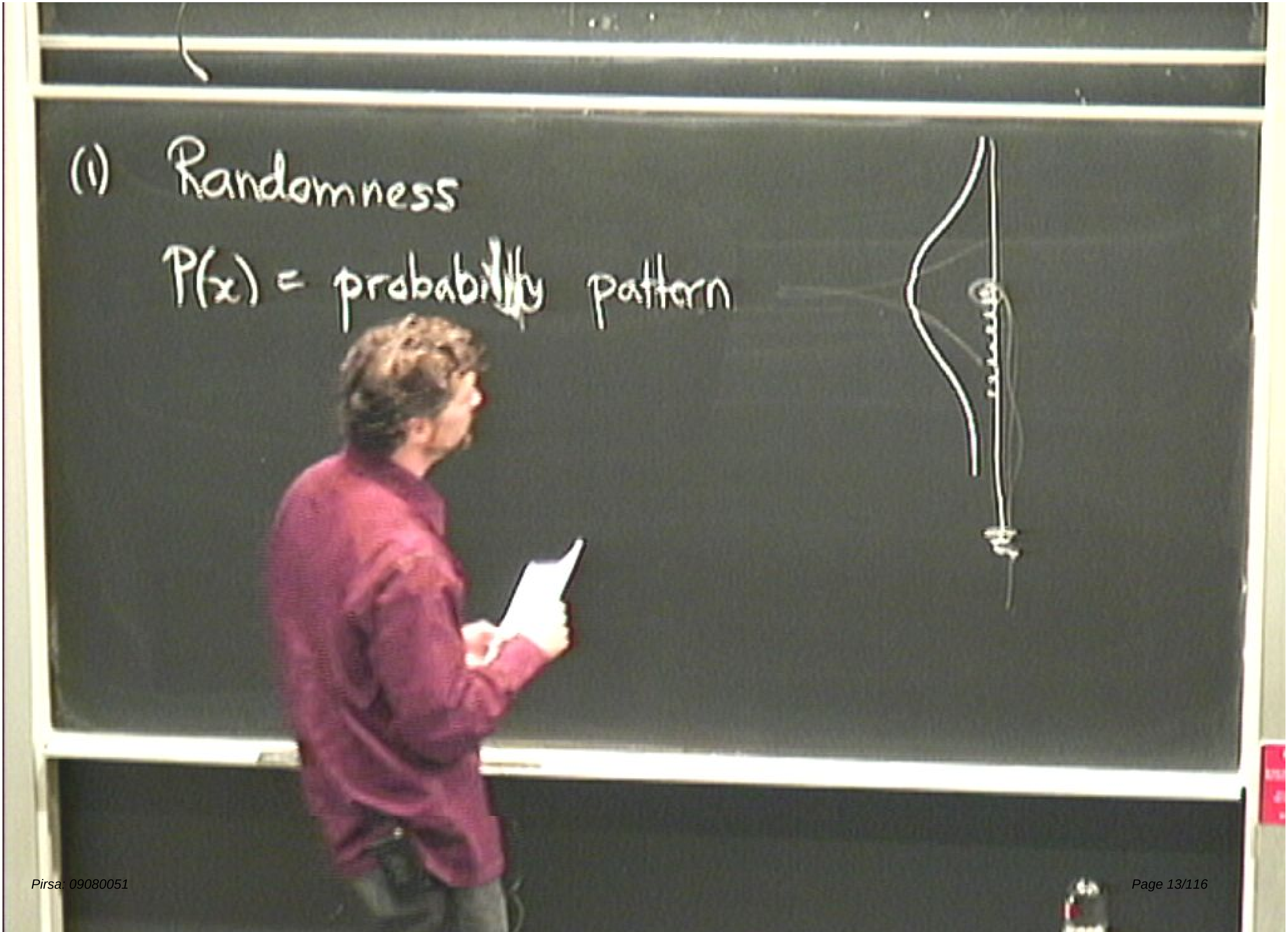

Single Slit
electrons
p
= mv
shadow
shadow
P(x)
large p
Smaller p
very small p
x
O

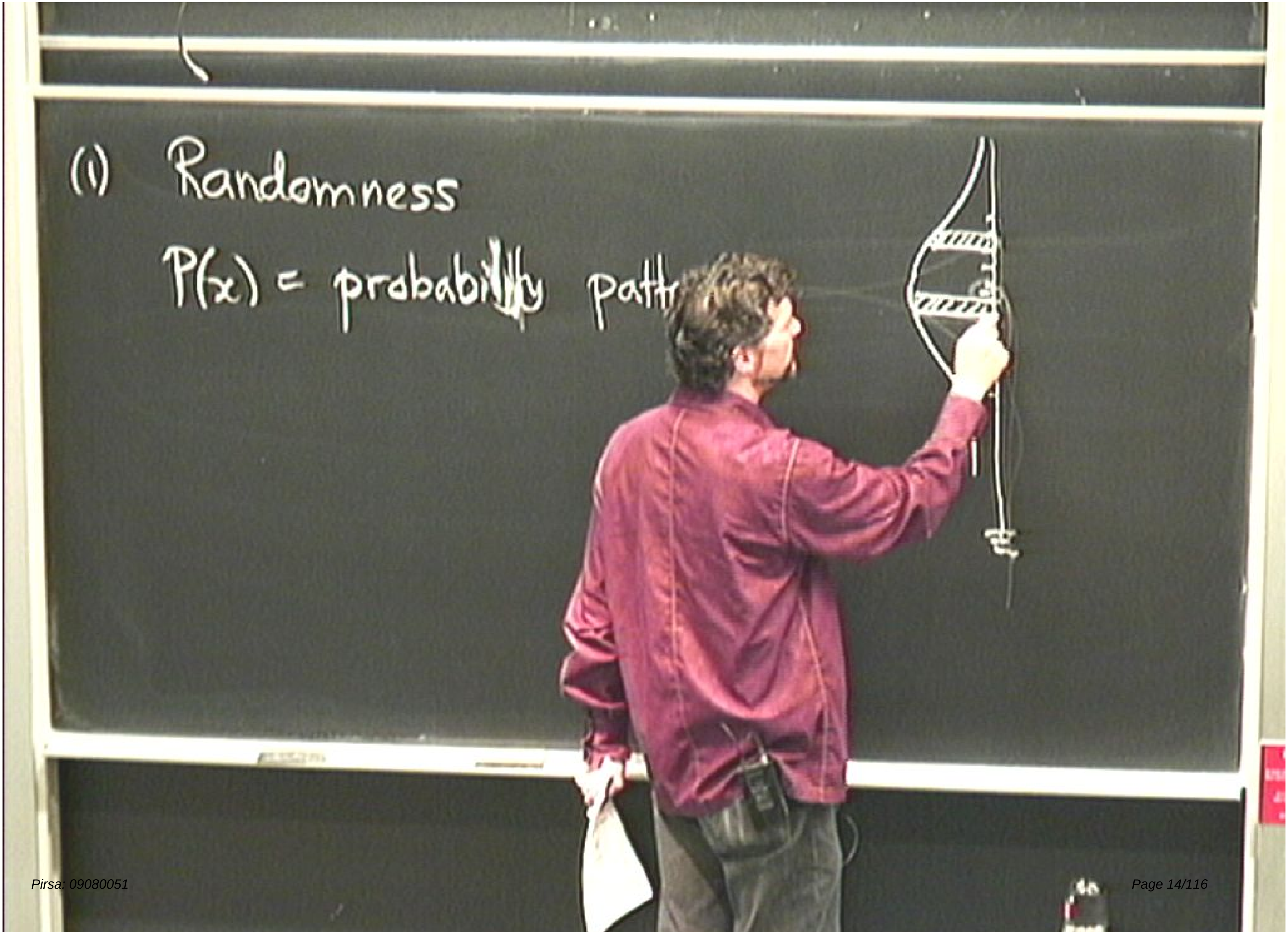
(1) Randomness

(1) Randomness

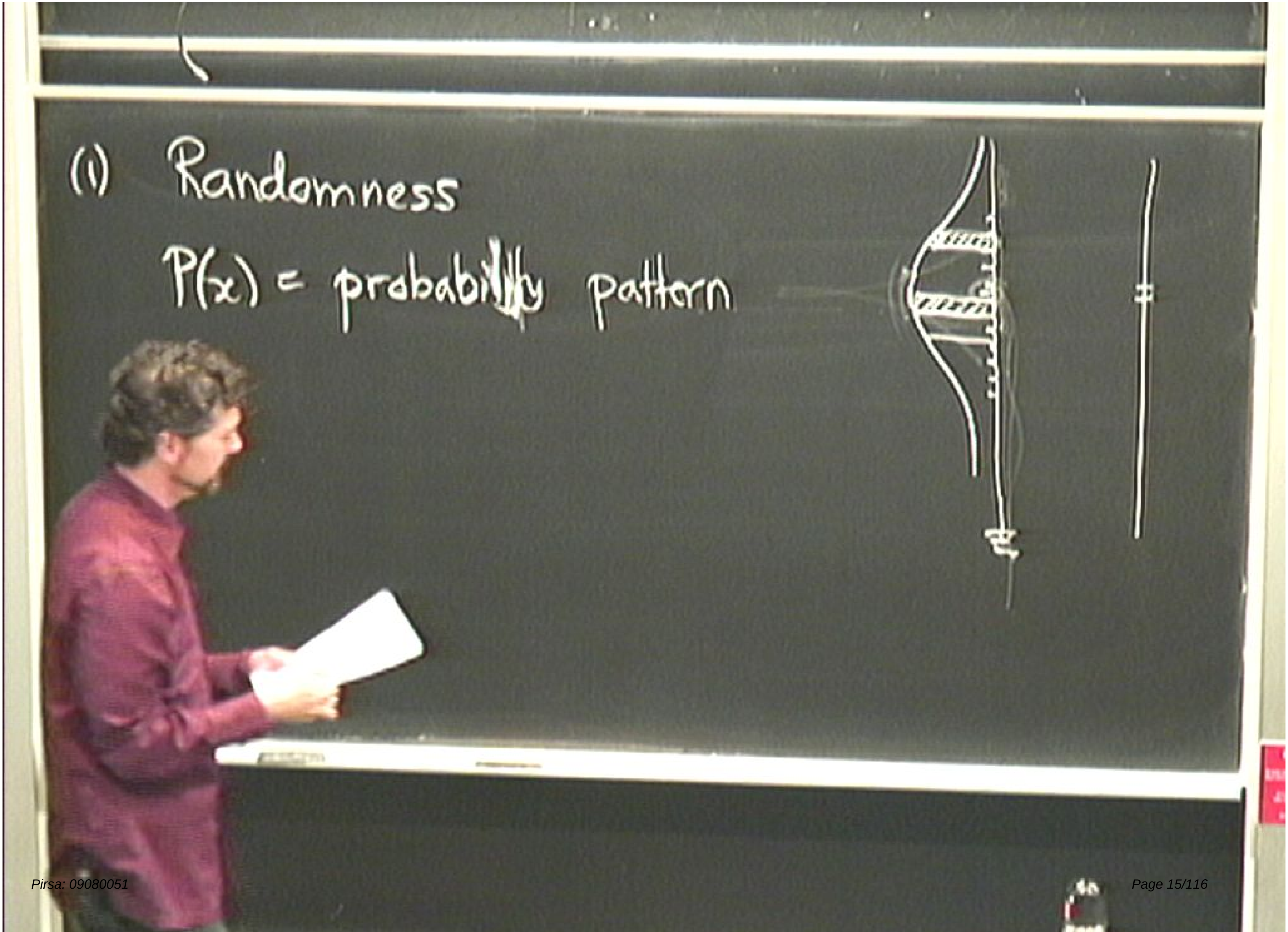
$P(x)$ = probability pattern

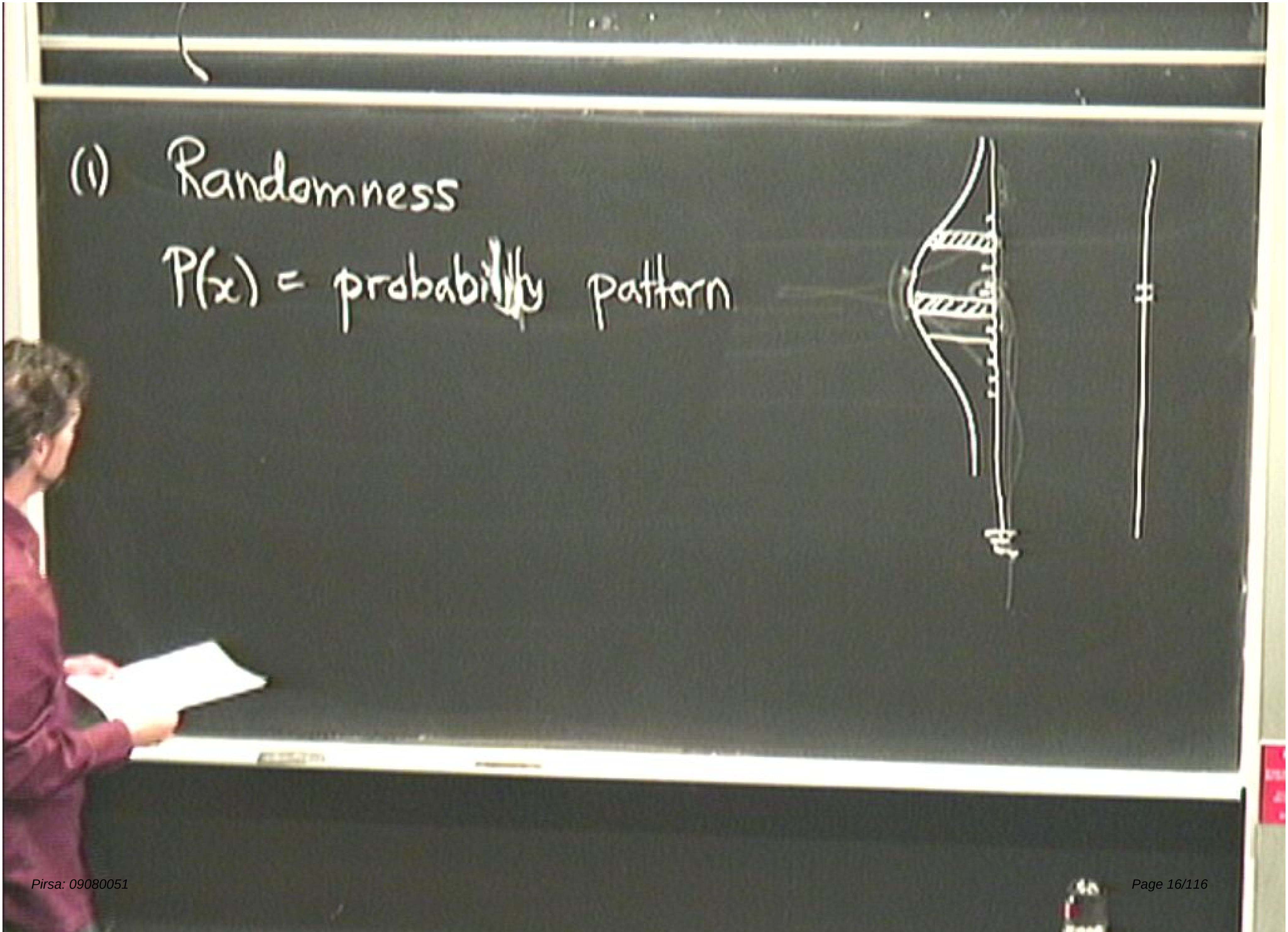

(1) Randomness
P(x) = probability pattern


(1) Randomness
P(x) = probability pattern



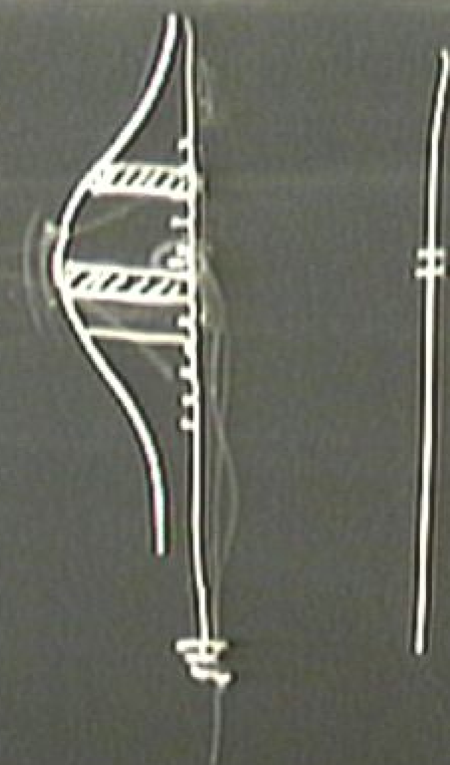
(1) Randomness
P(x) = probability


(1) Randomness
P(x) = probability pattern


(1) Randomness
P(x) = probability pattern

(1) Randomness

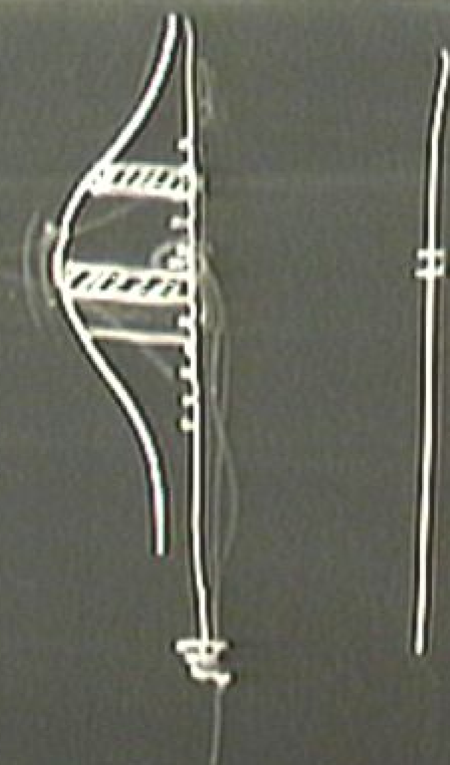
$P(x)$ = probability pattern



(2)

(1) Randomness

$P(x)$ = probability pattern

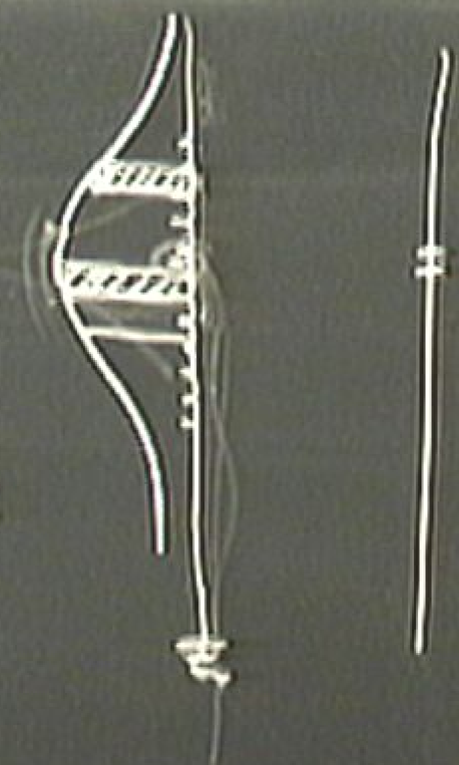


(2)

(1) Randomness

$P(x)$ = probability pattern

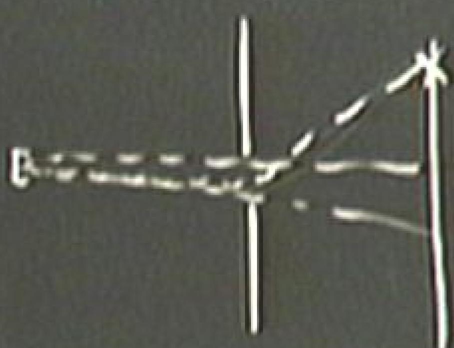
(2) Spreading of $P(x)$ as p is reduced

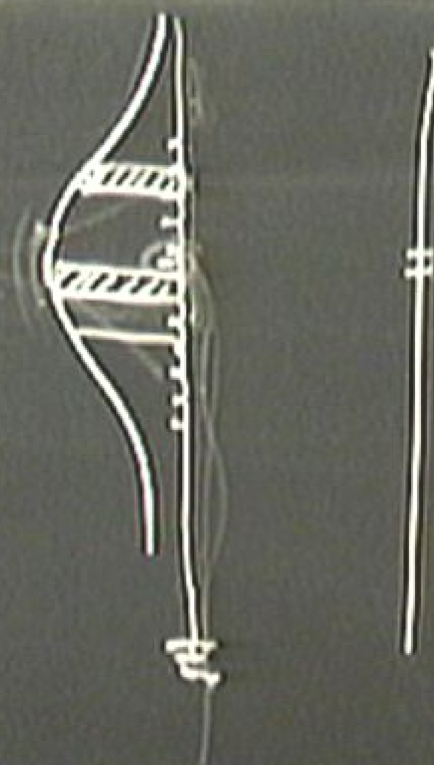


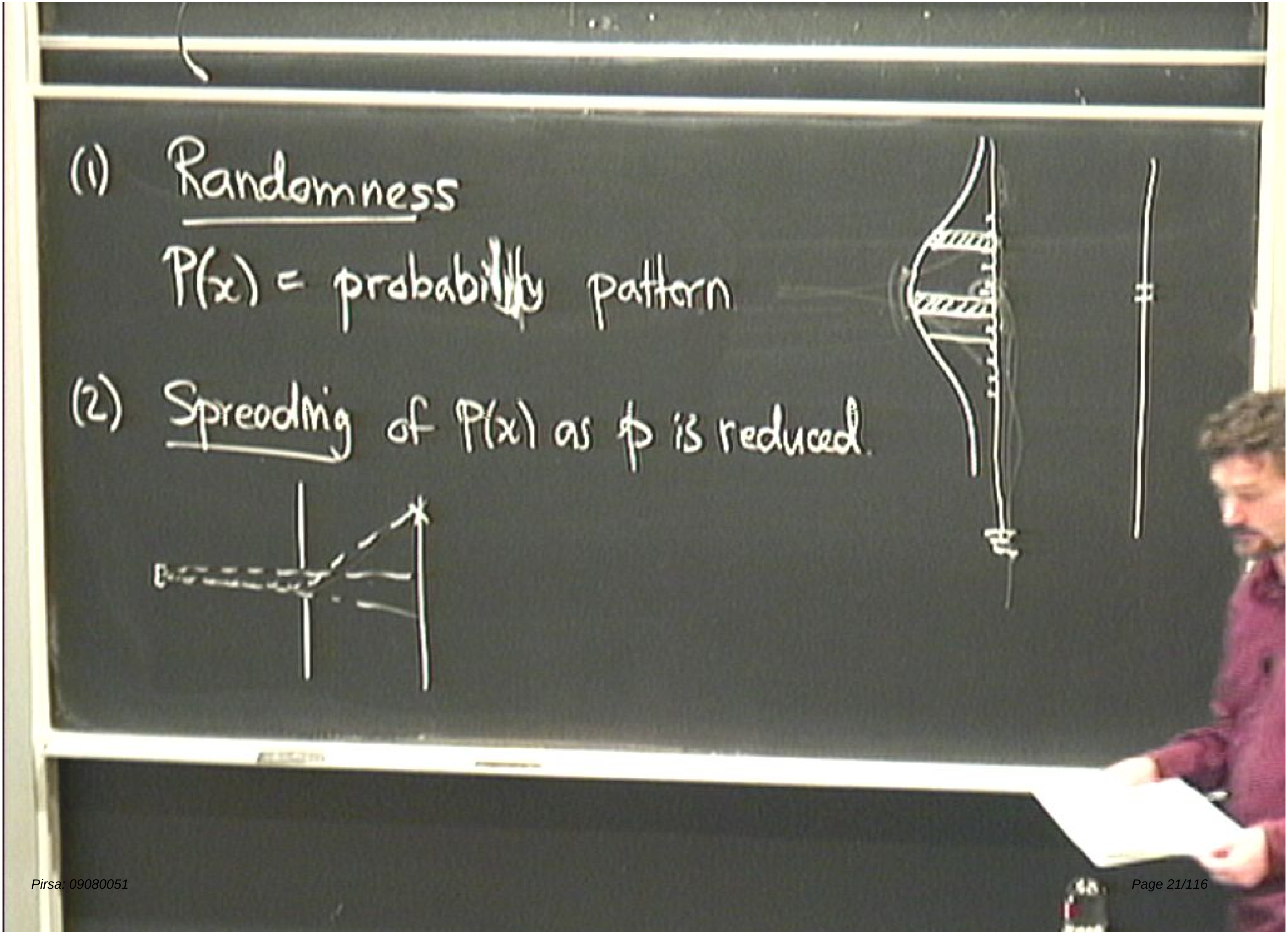
(1) Randomness

$P(x)$ = probability pattern

(2) Spreading of $P(x)$ as p is reduced



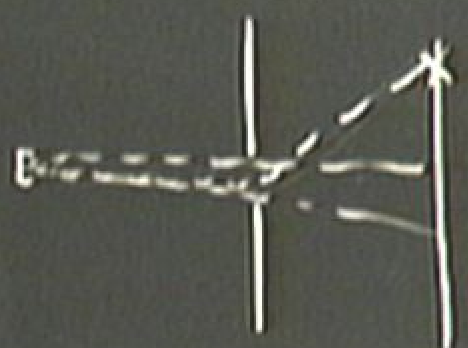


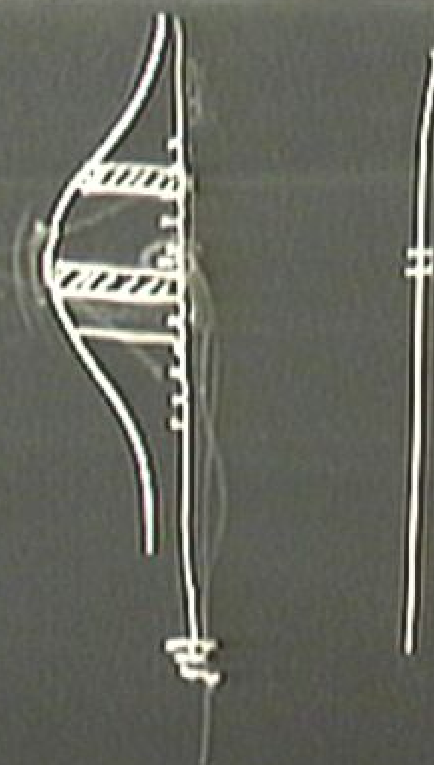

(1) Randomness
P(x) = probability pattern
(2) Spreading of P(x) as p is reduced

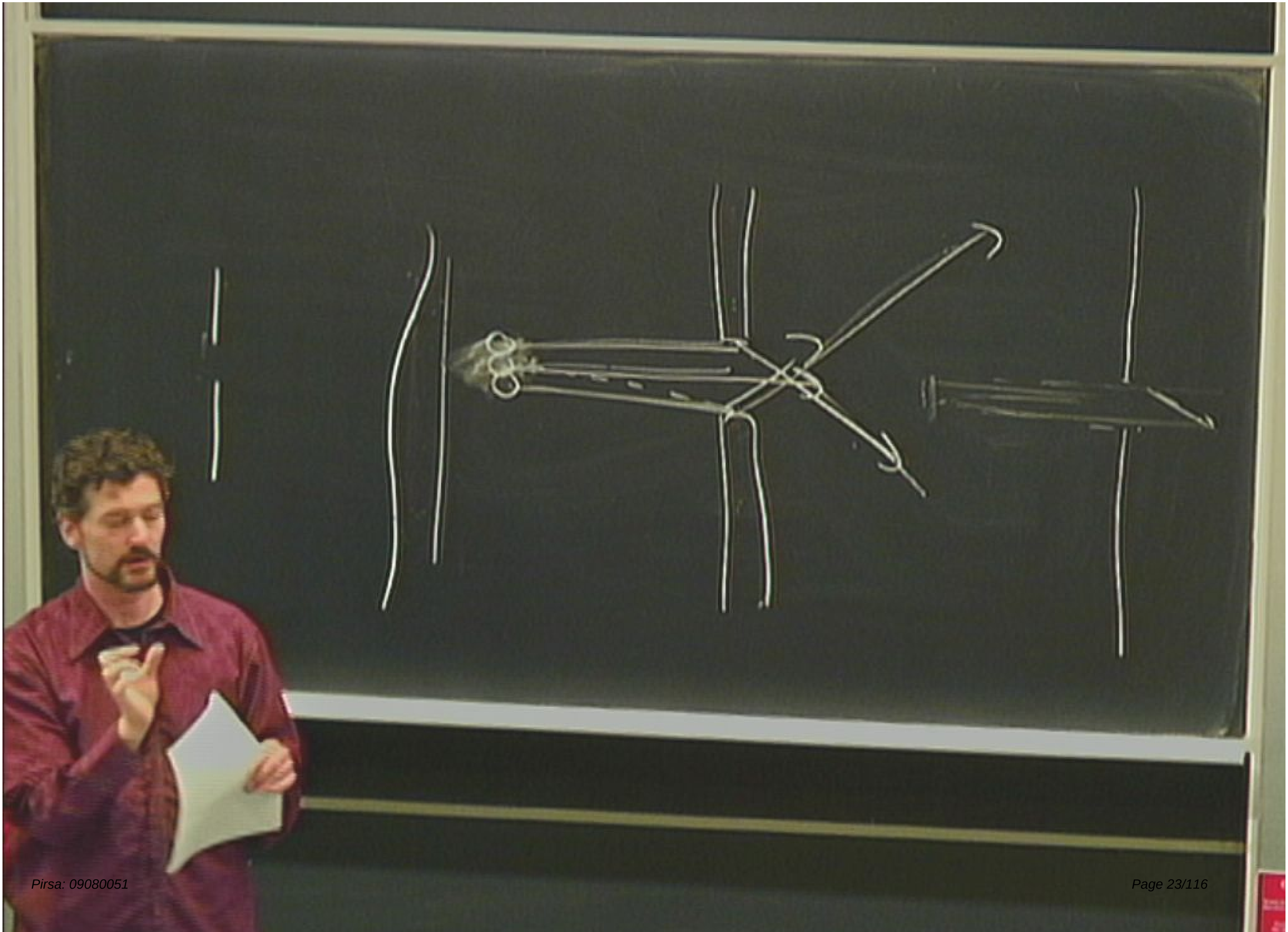
(1) Randomness

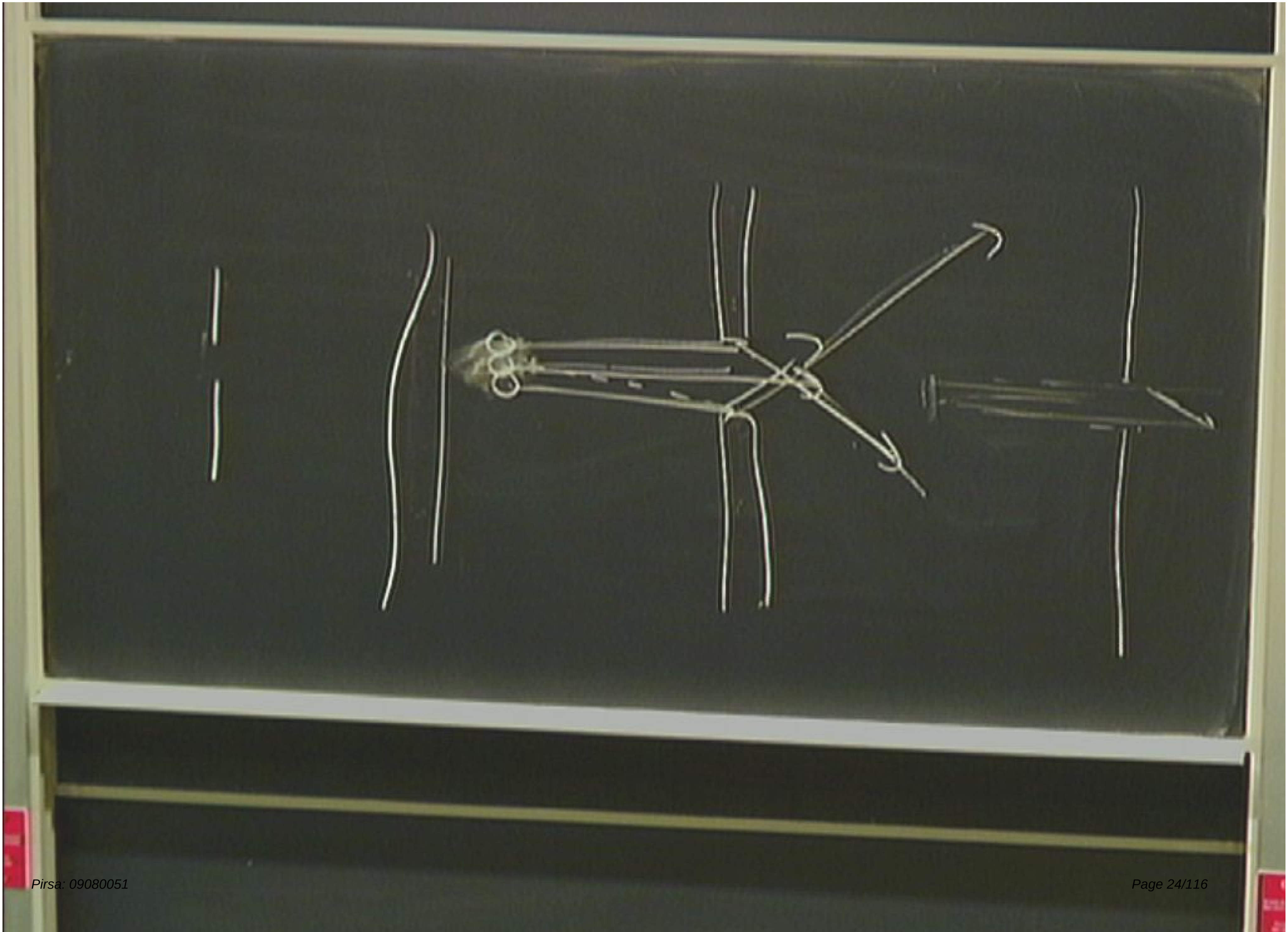
$P(x)$ = probability pattern

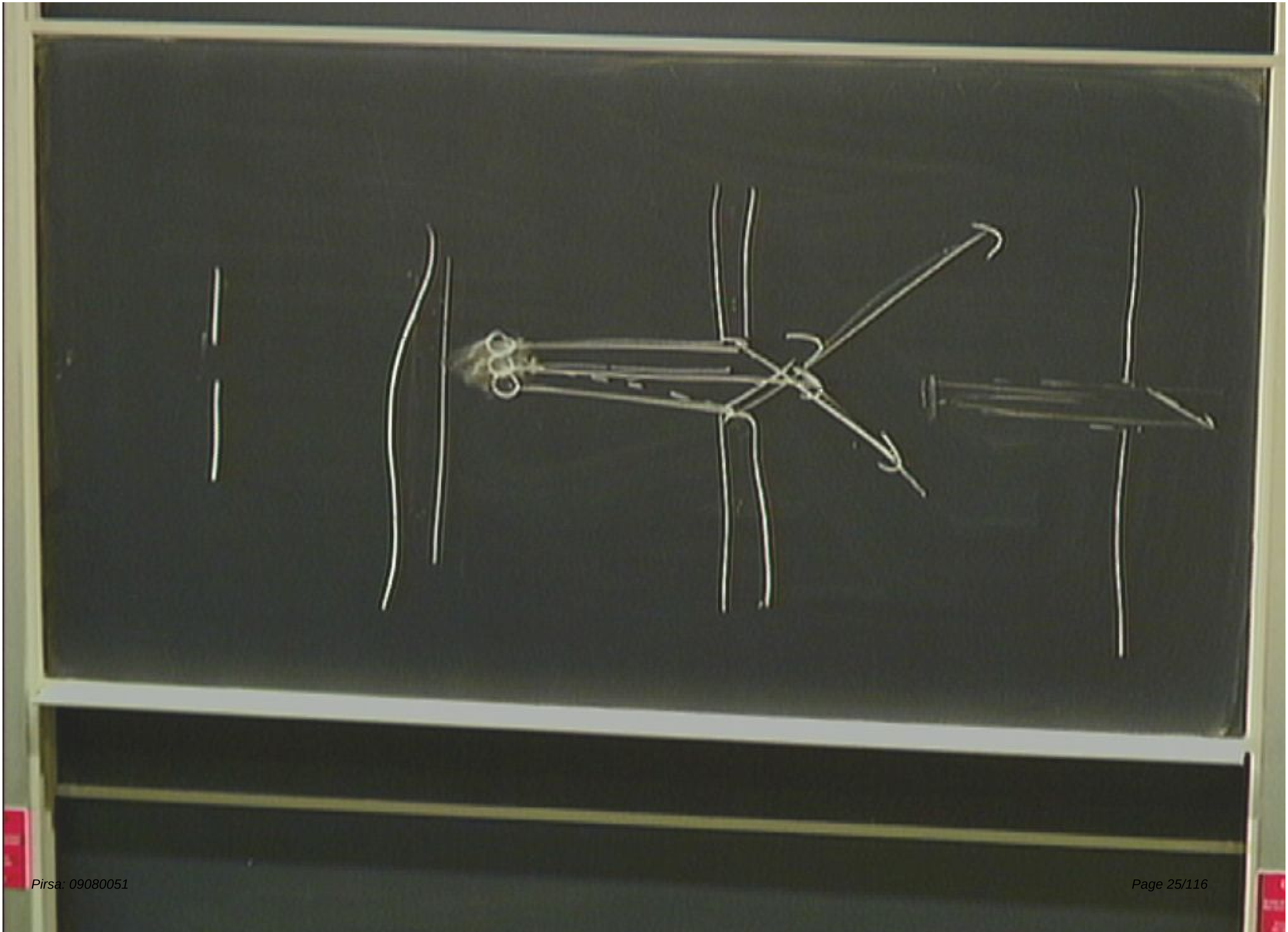
(2) Spreading of $P(x)$ as p is reduced



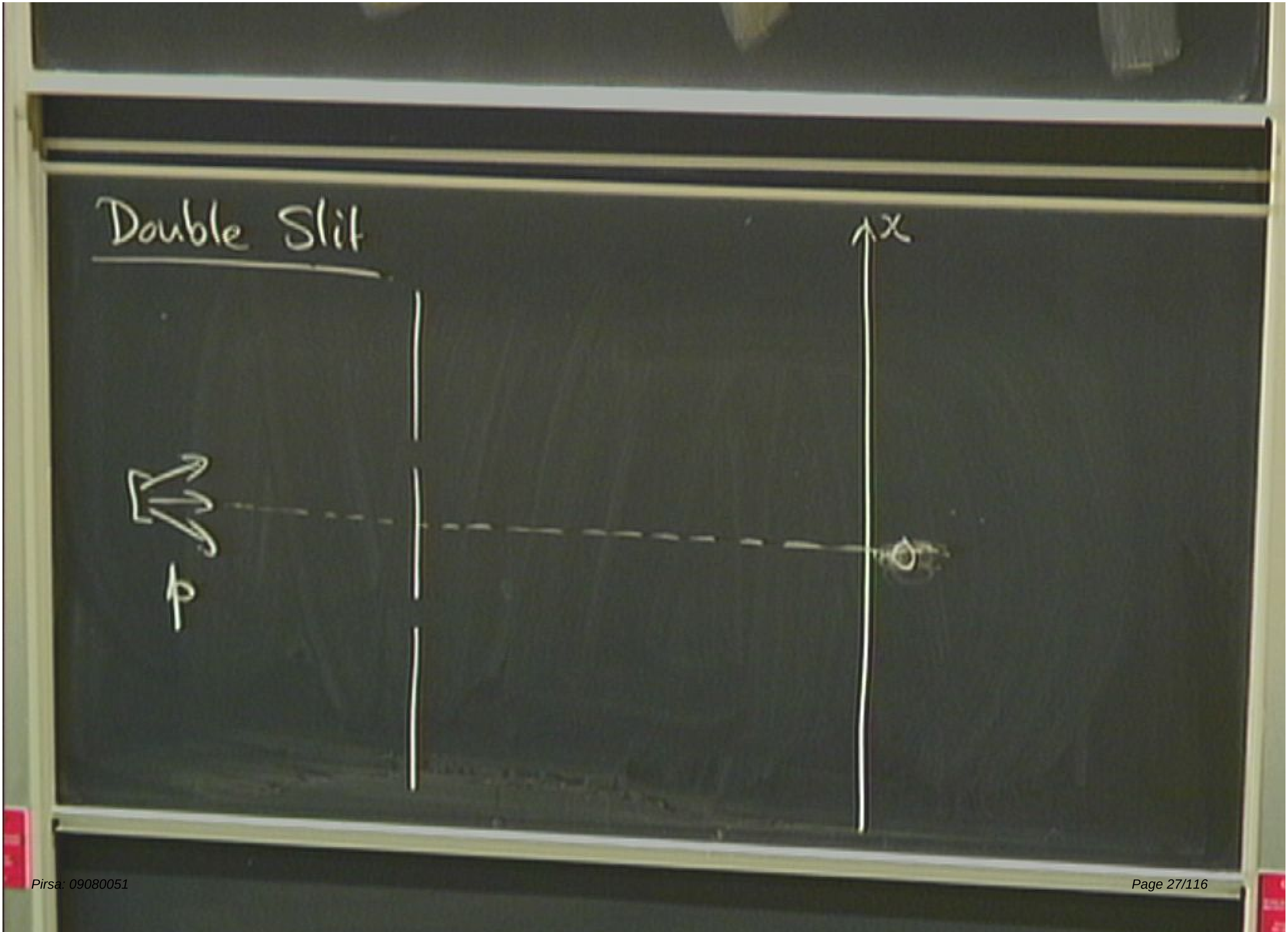


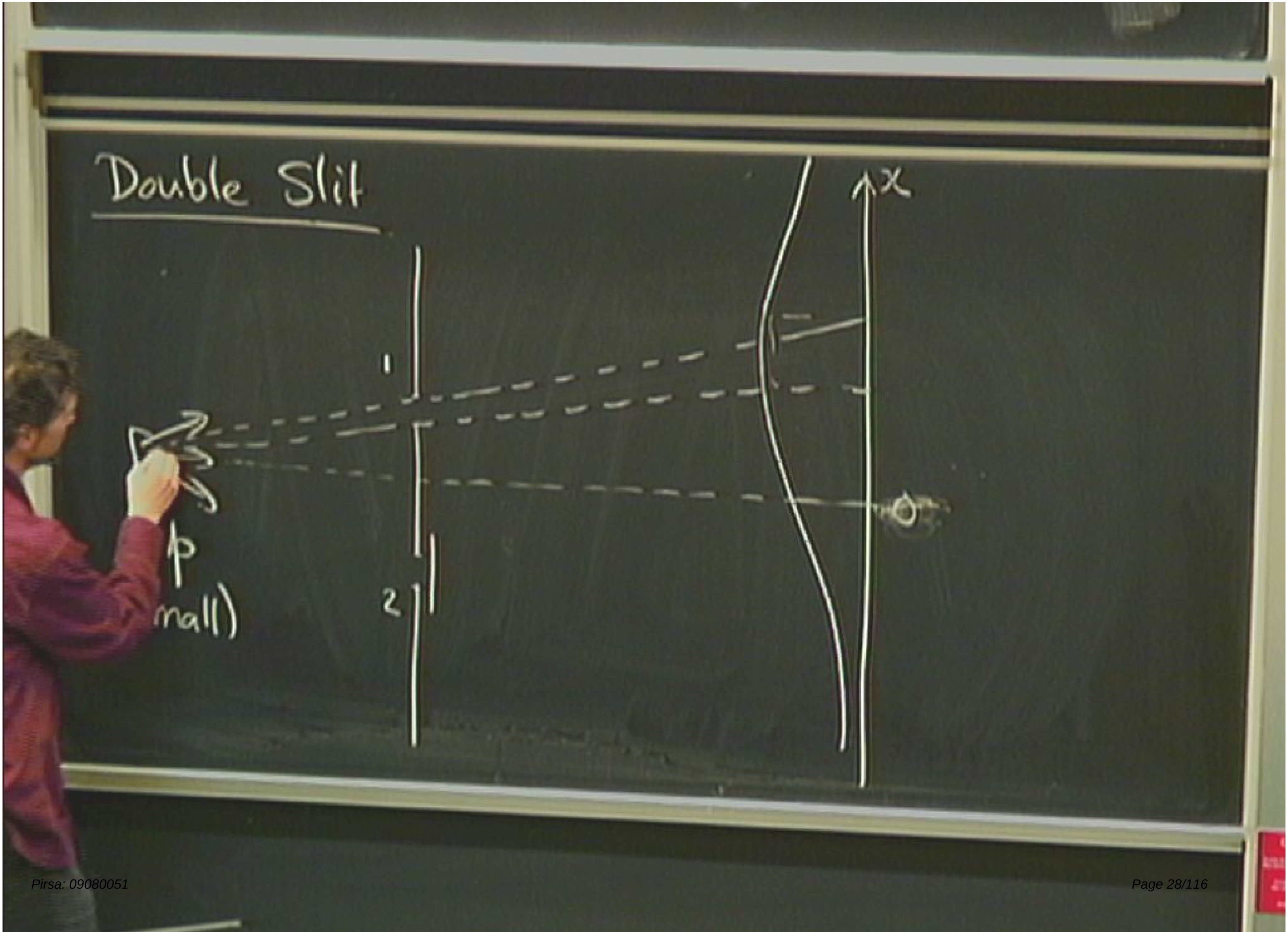


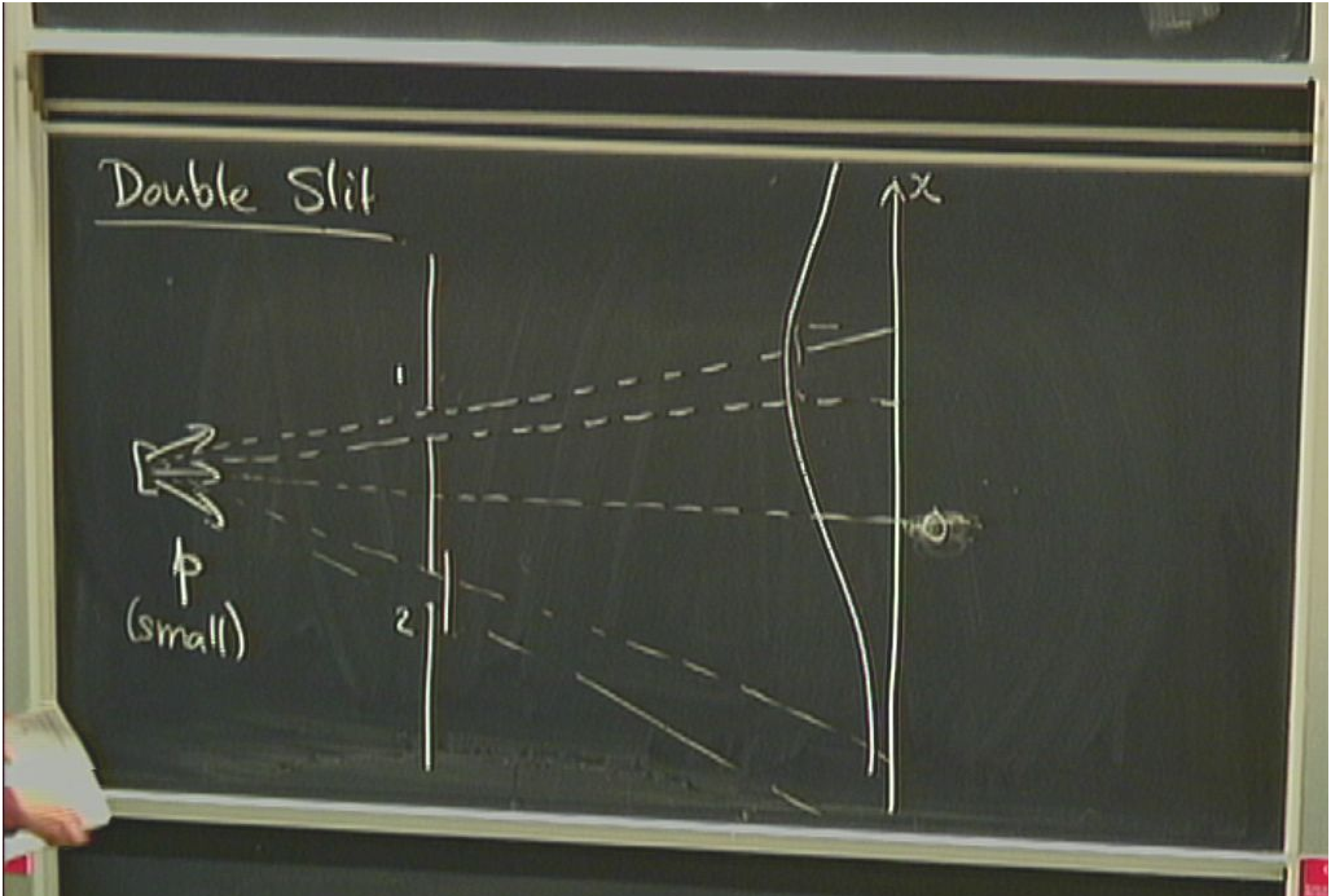




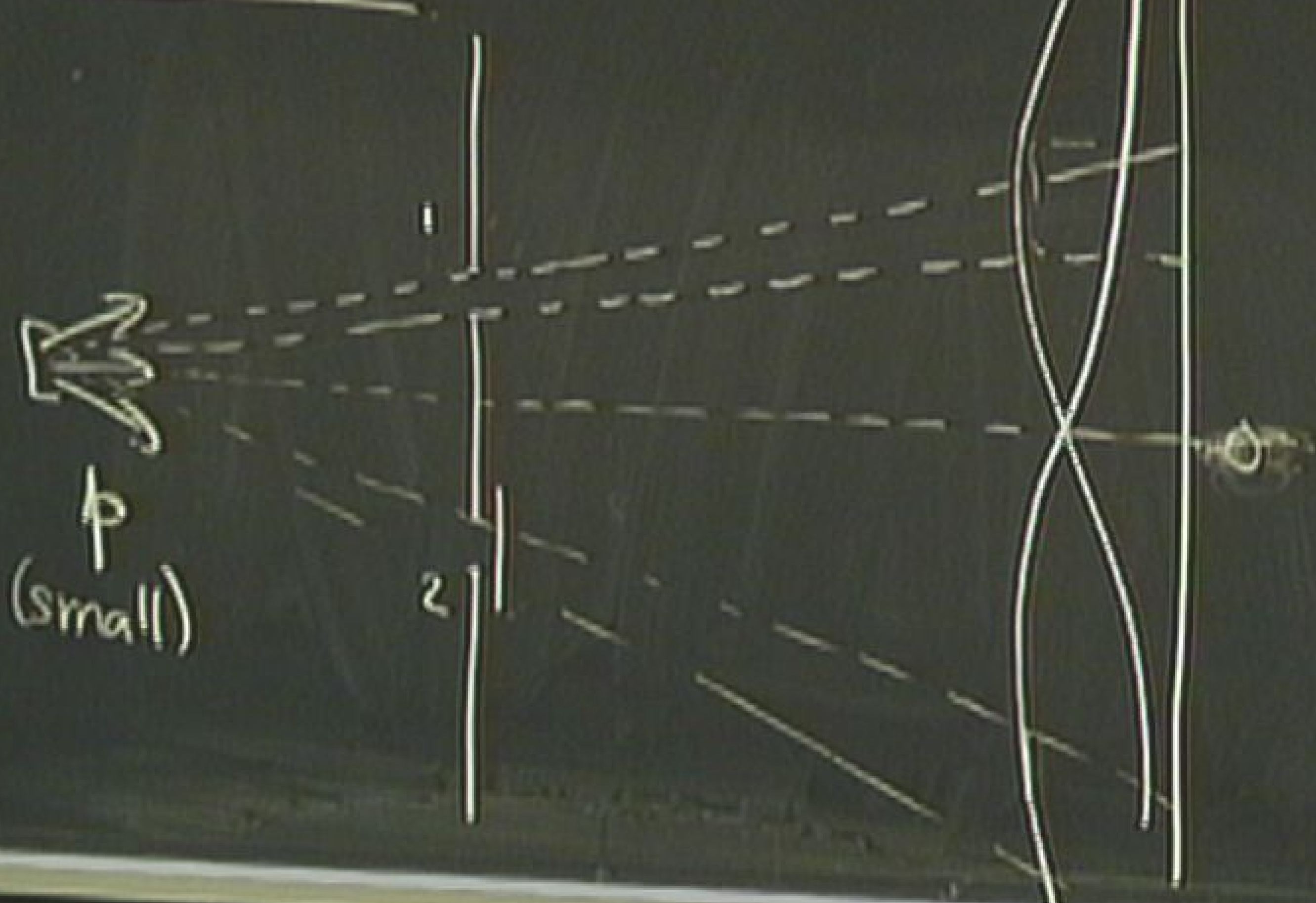
Double Slit

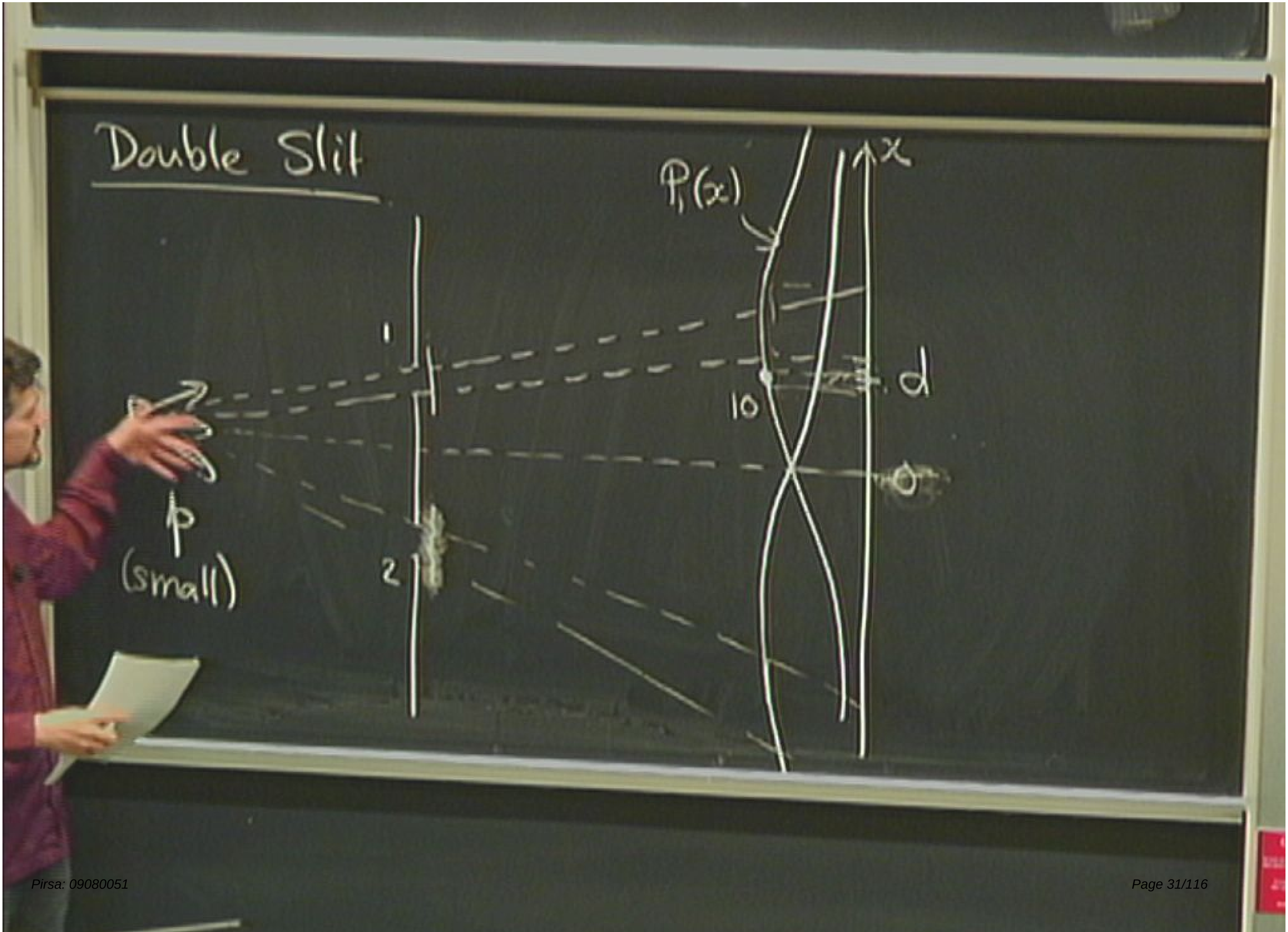

Double Slit
x
p

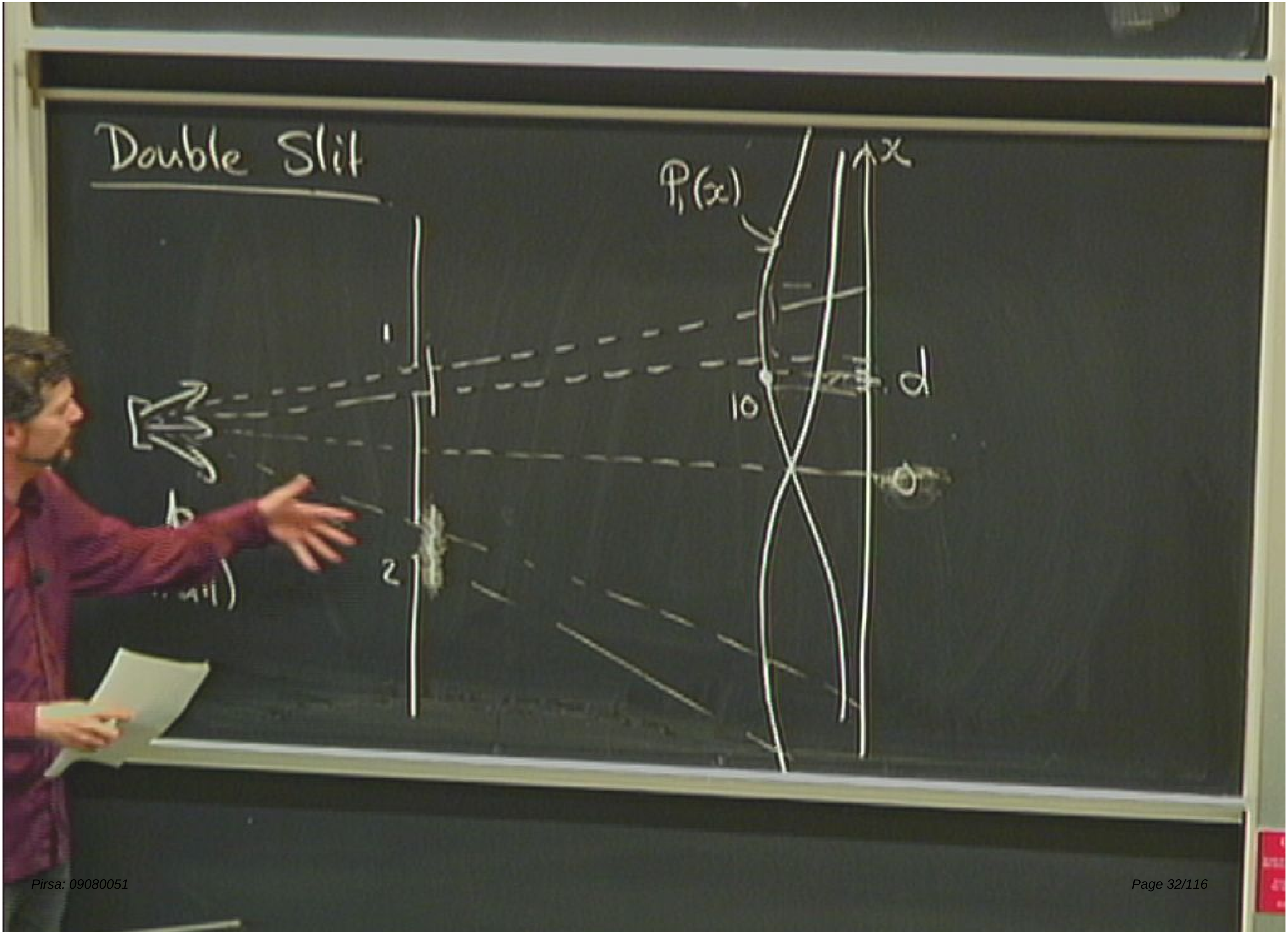

Double Slit
x
1
2
p
mall)

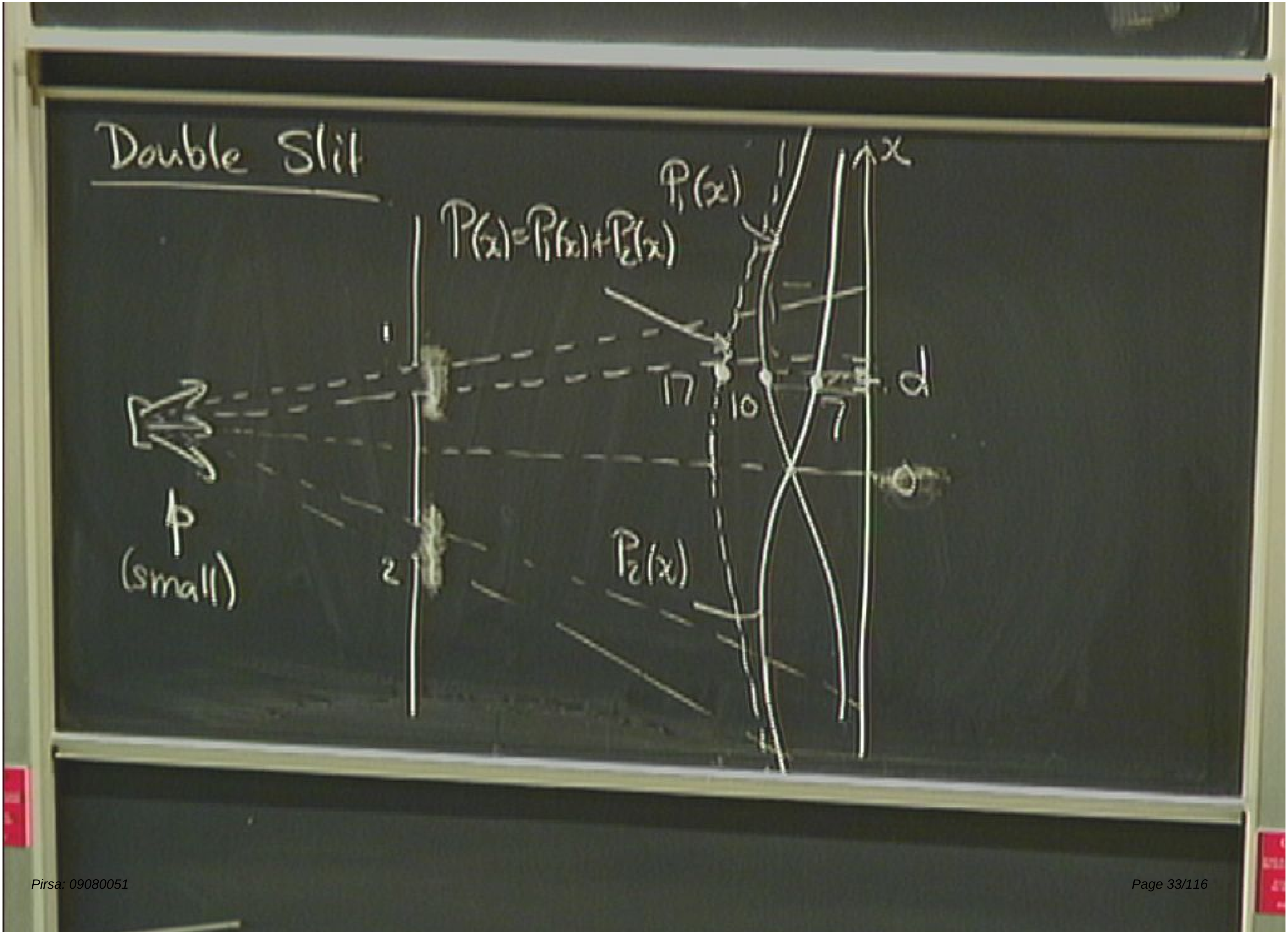

Double Slit
1
2
x
p
(small)

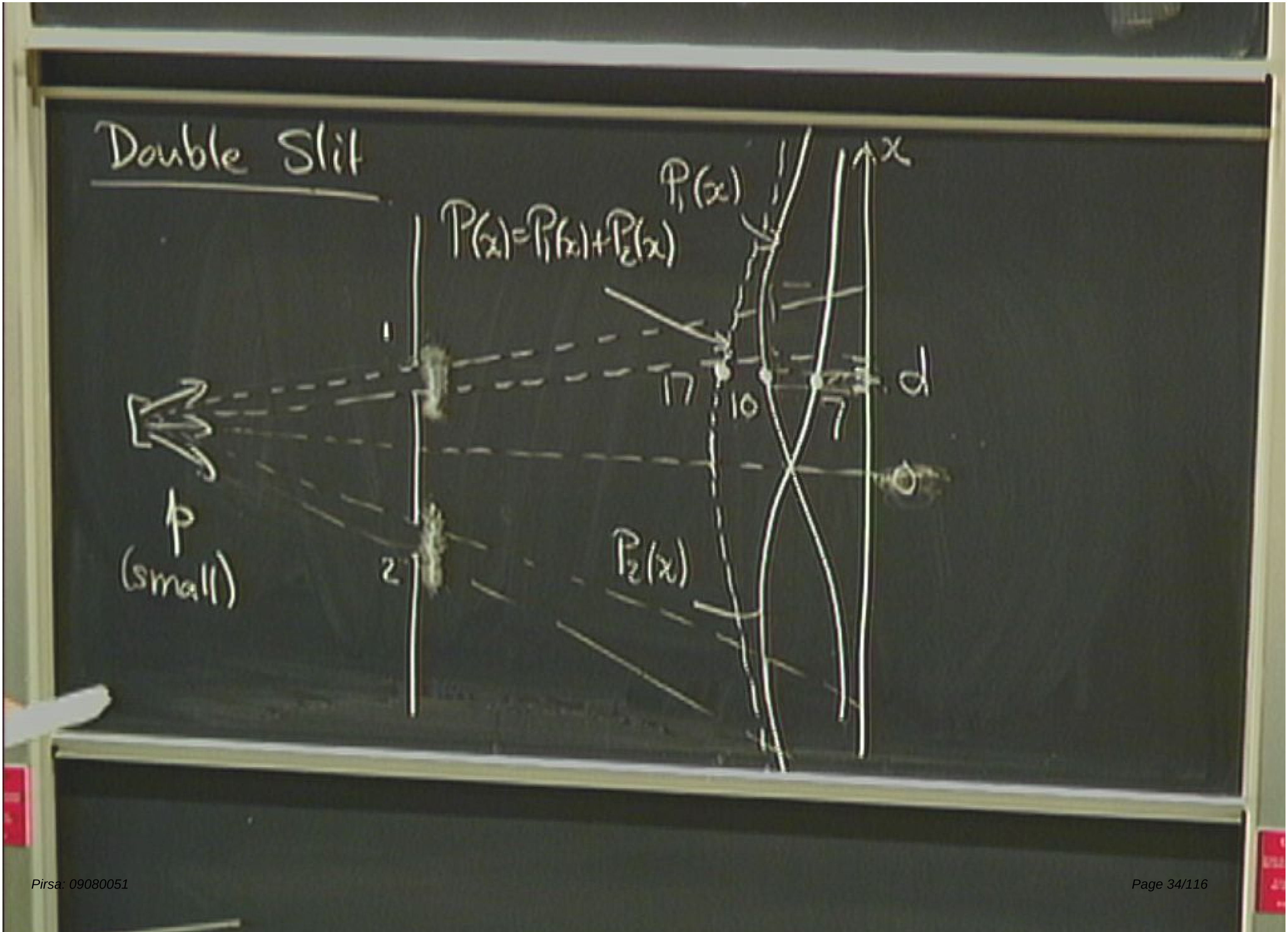
Double Slit

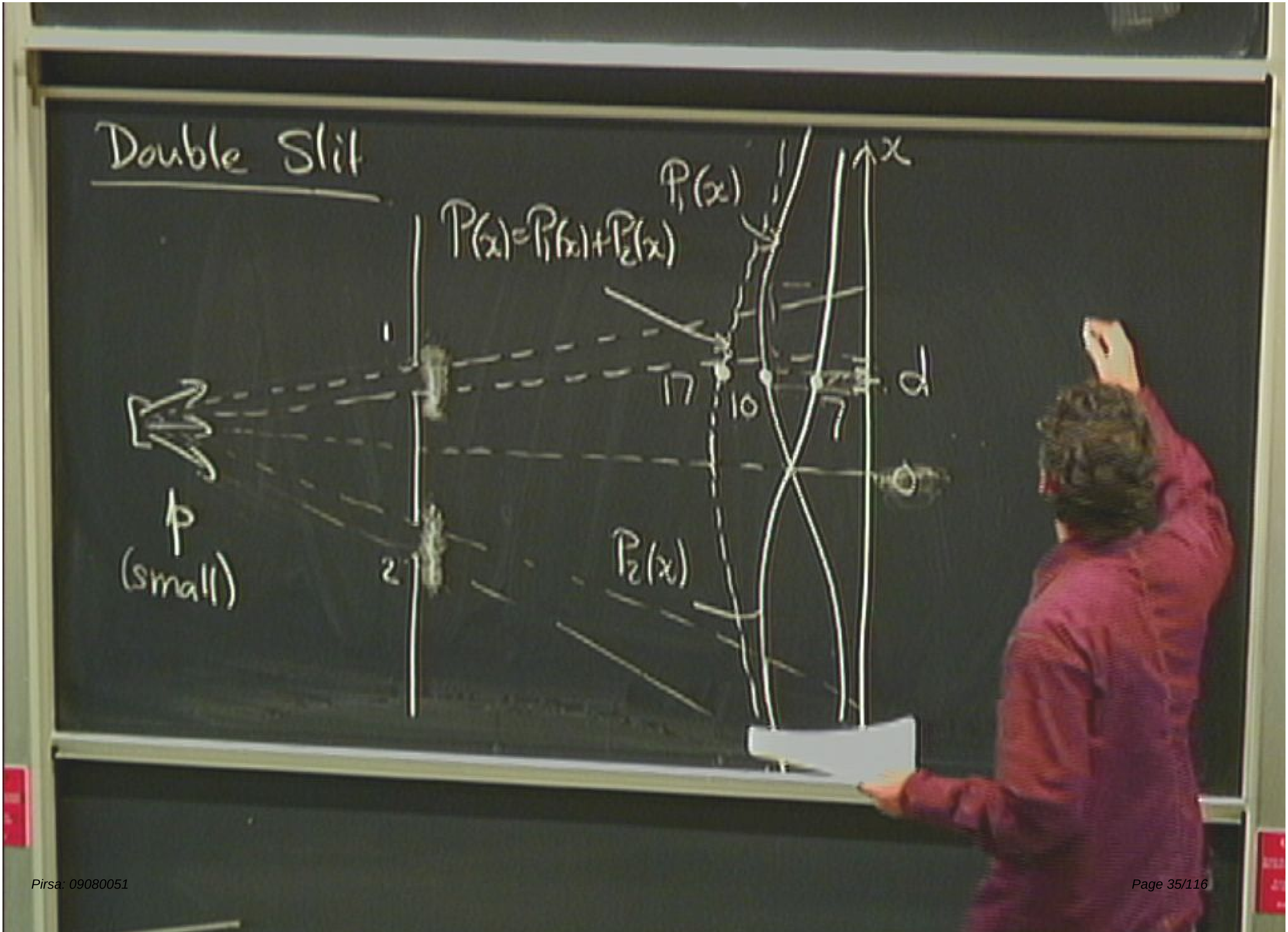


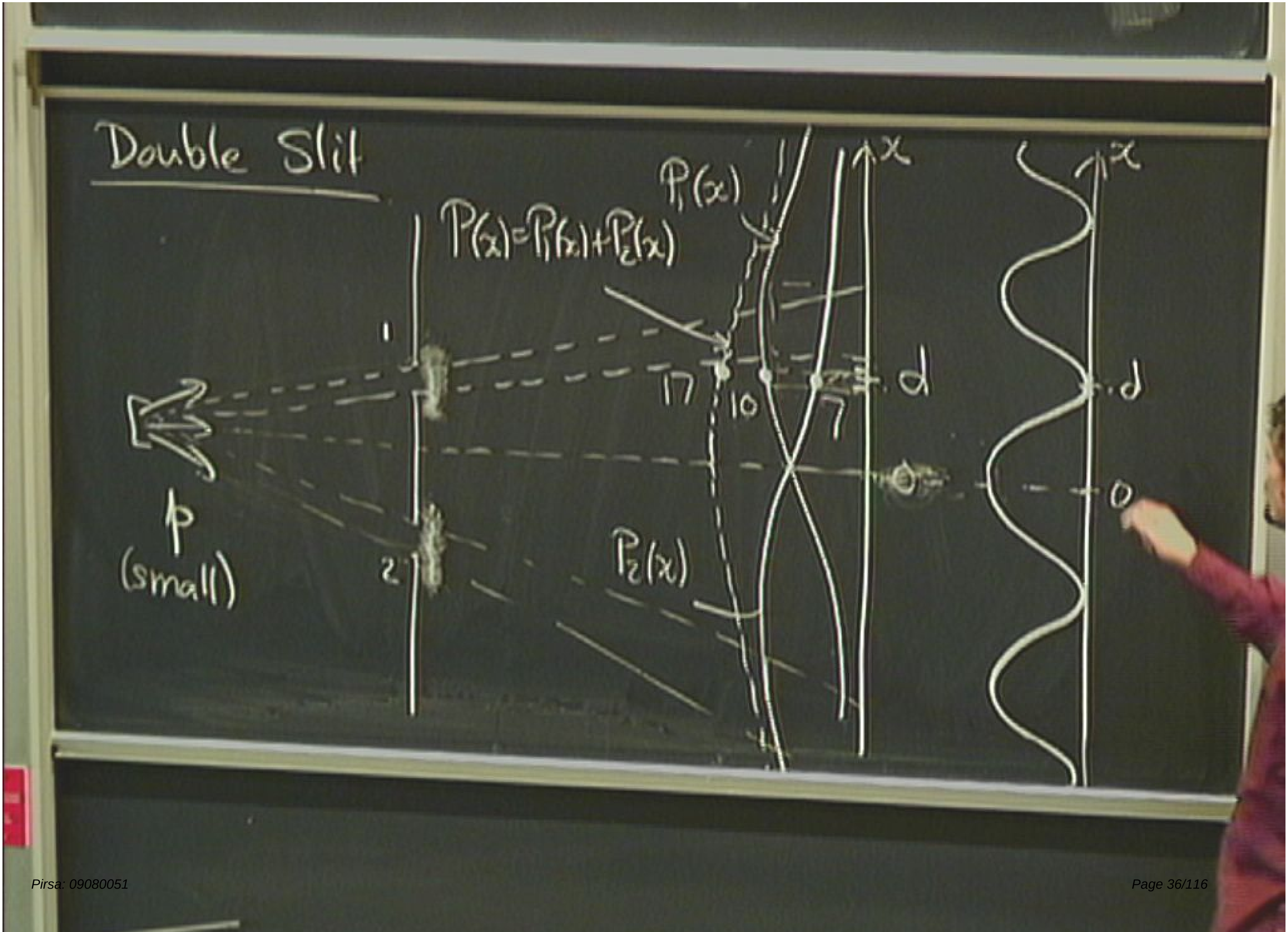

Double Slit
$P_1(x)$
x
1
2
10
d
0
p
(small)

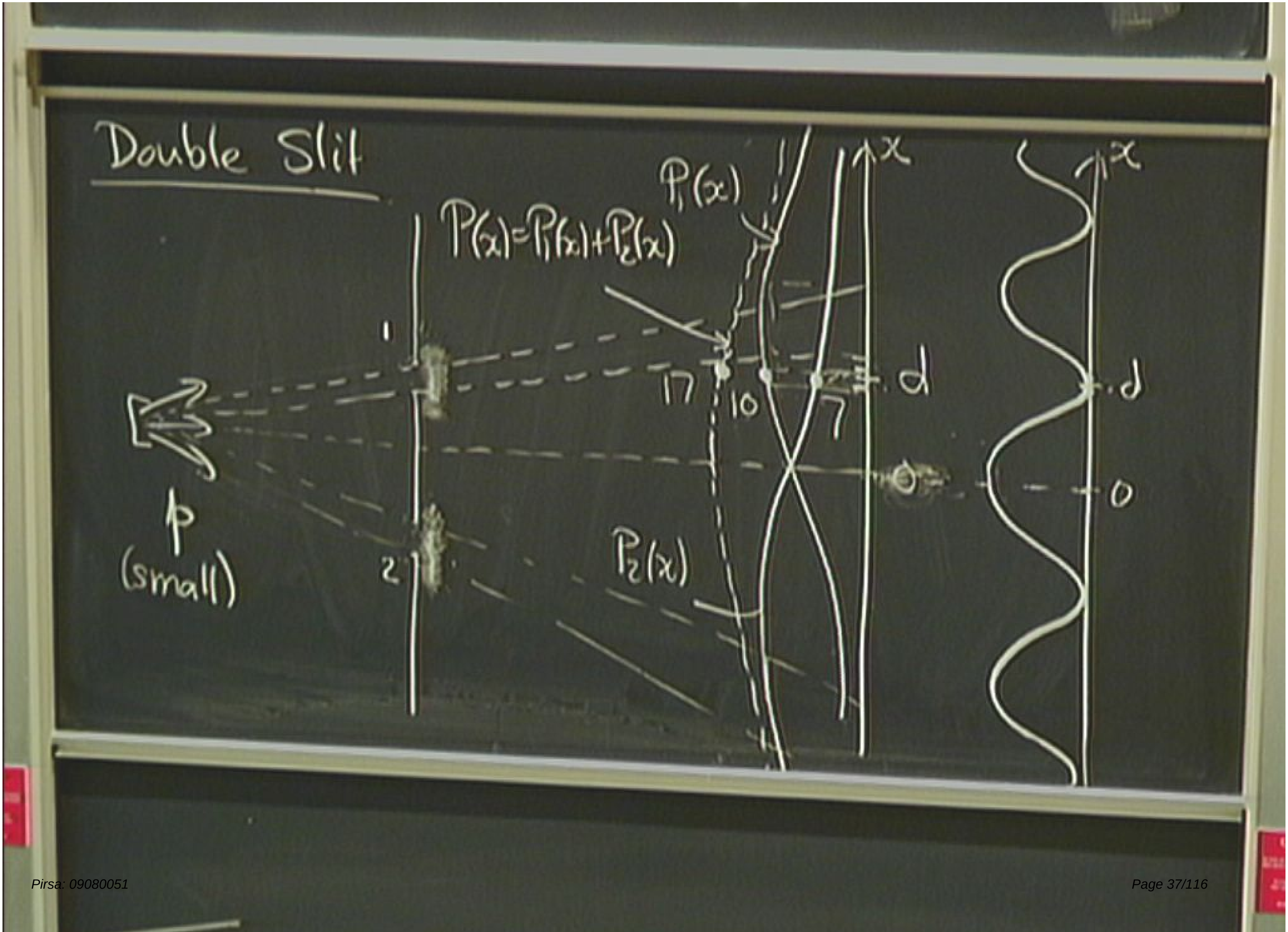

Double Slit
$P_1(x)$
x
1
2
10
d

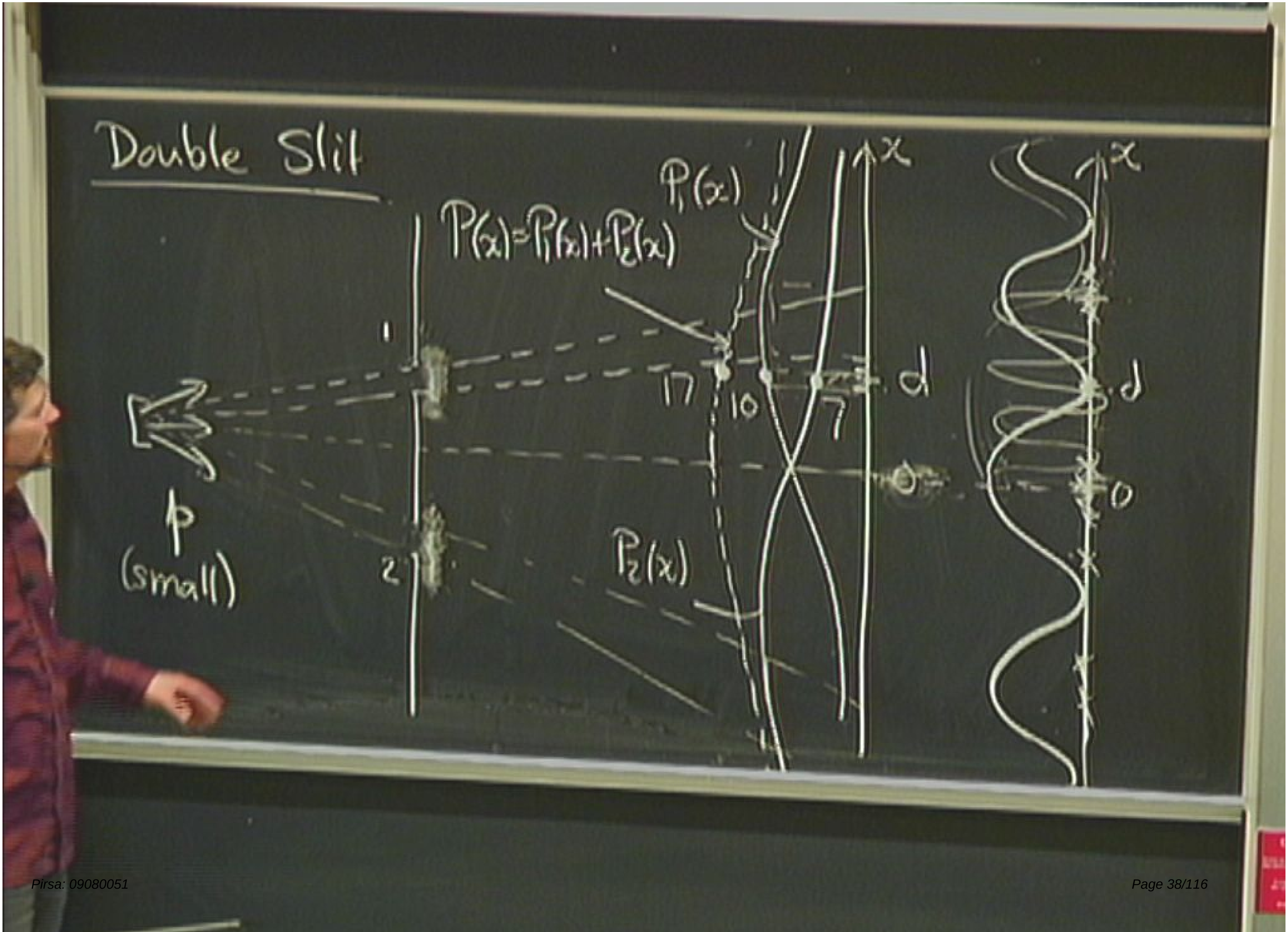

Double Slit
P(x)=P1(x)+P2(x)
P1(x)
x
1
2
17
10
7
d
0
p
(small)
P2(x)

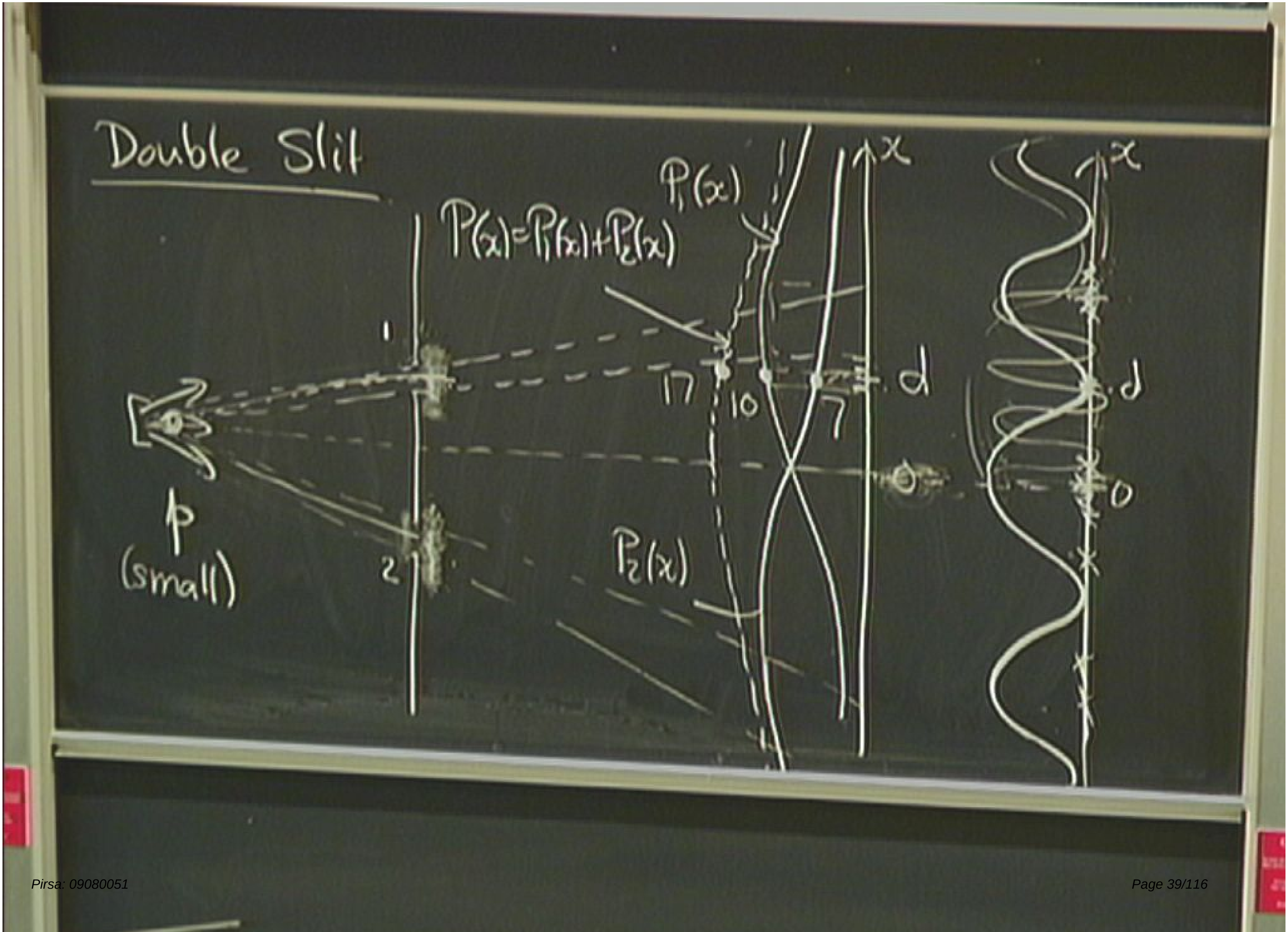

Double Slit
P(x)=P1(x)+P2(x)
P1(x)
x
1
17
10
7
d
0
p
(small)
2
P2(x)


Double Slit
$P(x) = P_1(x) + P_2(x)$
$P_1(x)$
$P_2(x)$
1
2
p
(small)
17
10
7
d
x

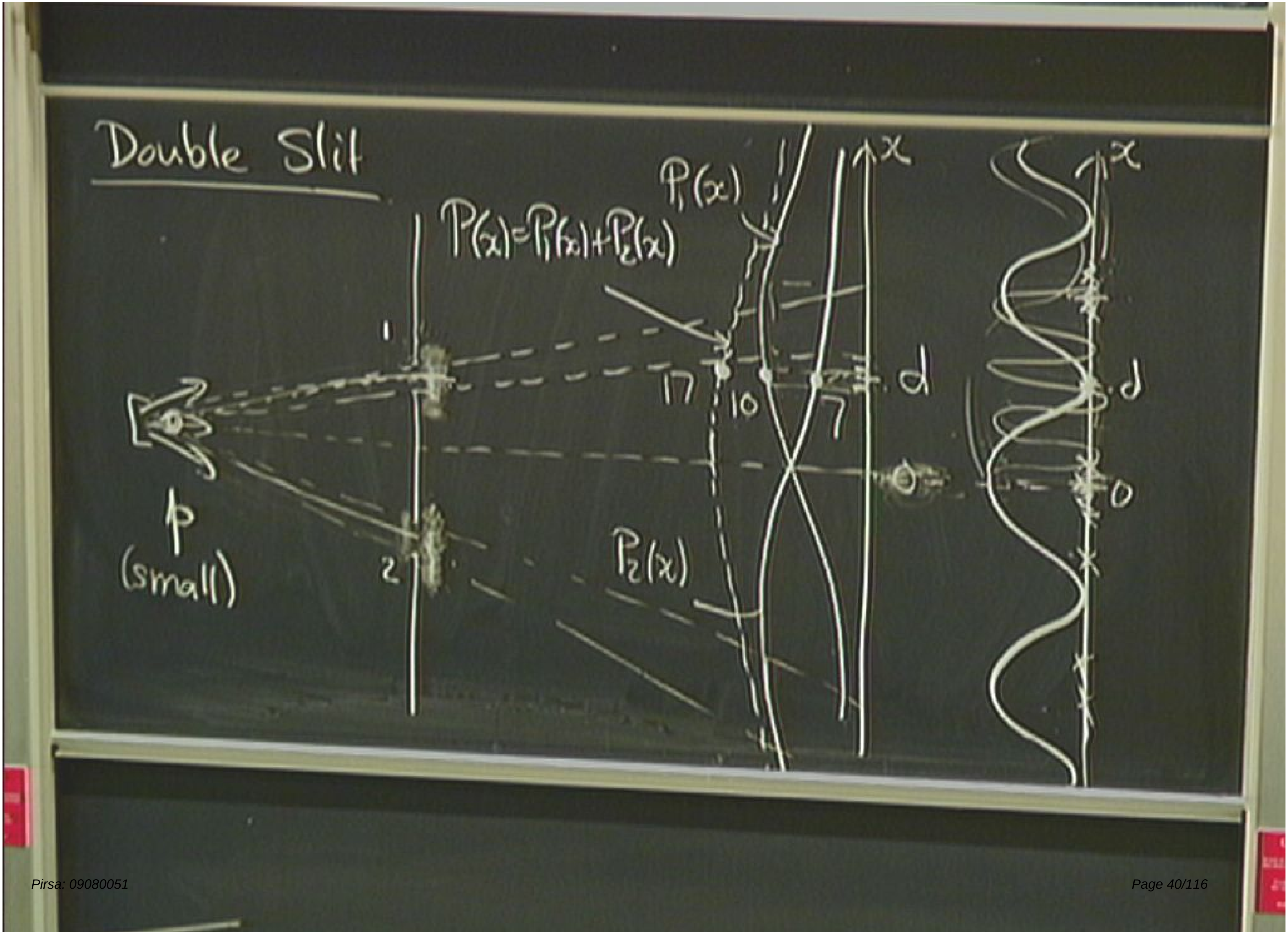

Double Slit
P(x)=P1(x)+P2(x)
P1(x)
P2(x)
x
1
2
17
10
7
d
0
p
(small)

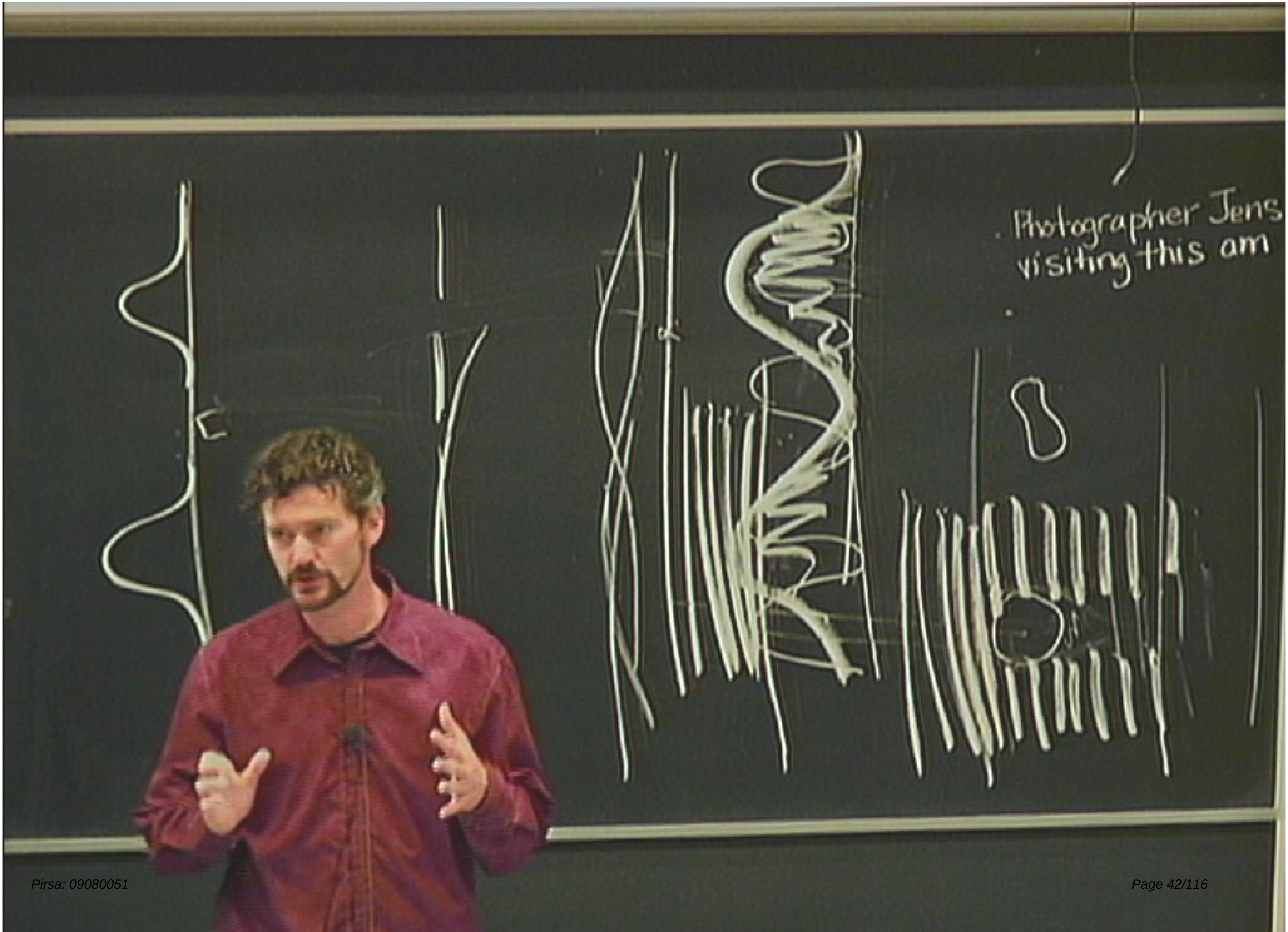

Double Slit
P(x)=P1(x)+P2(x)
P1(x)
P2(x)
1
2
p
(small)
17
10
7
d
0
x

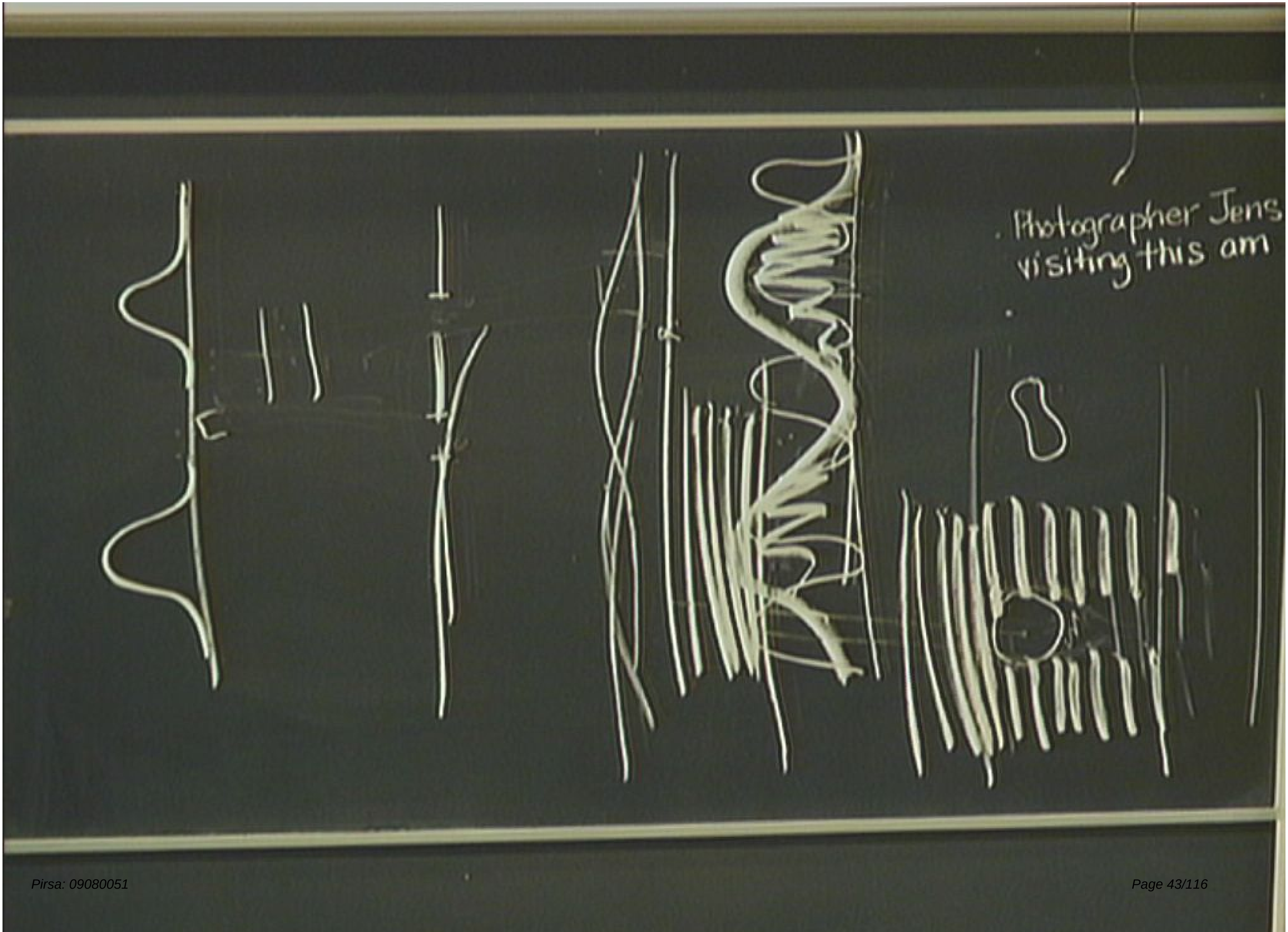

Double Slit
P(x)=P1(x)+P2(x)
P1(x)
P2(x)
1
2
p
(small)
17
10
7
d
0
x

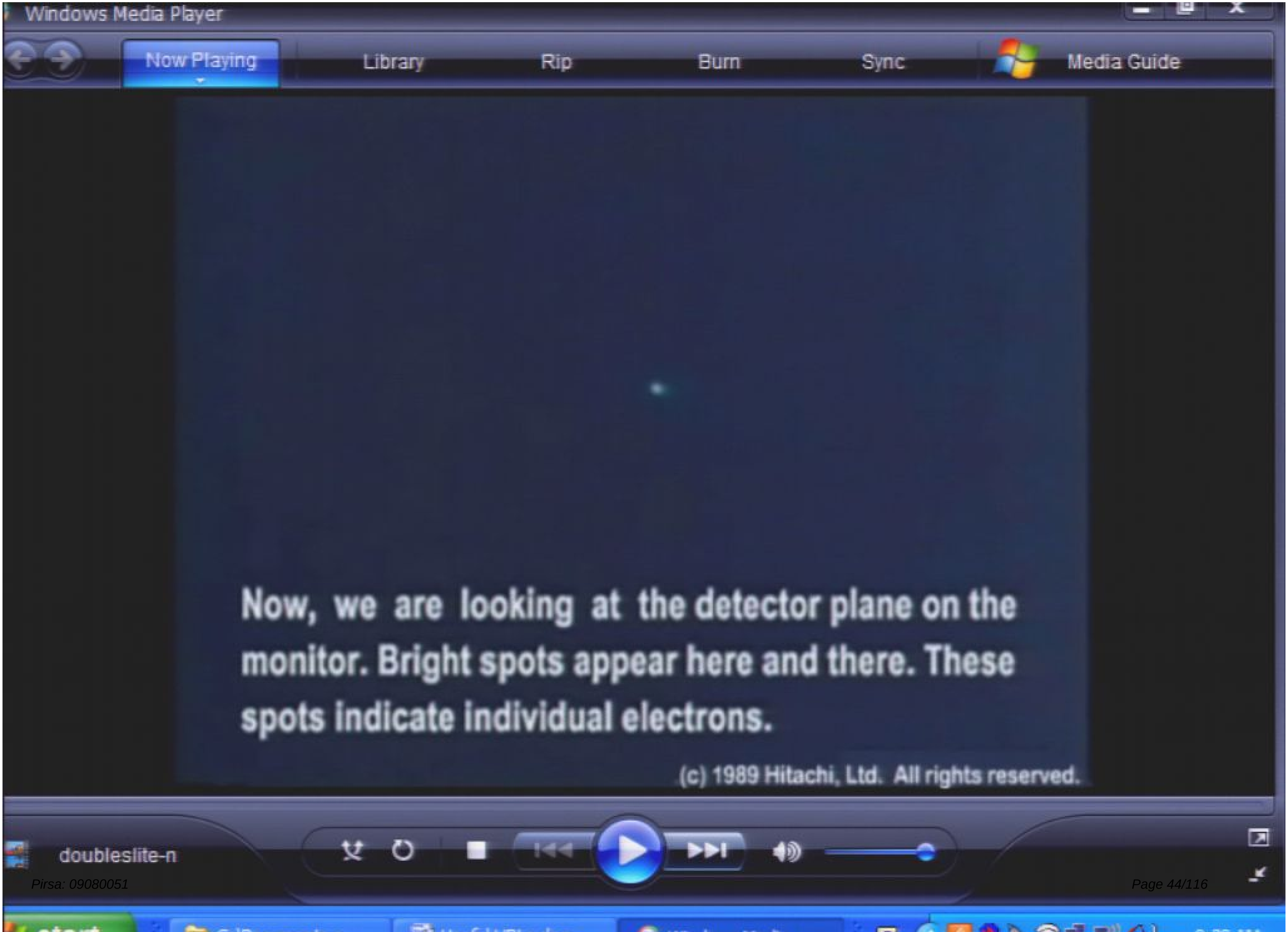


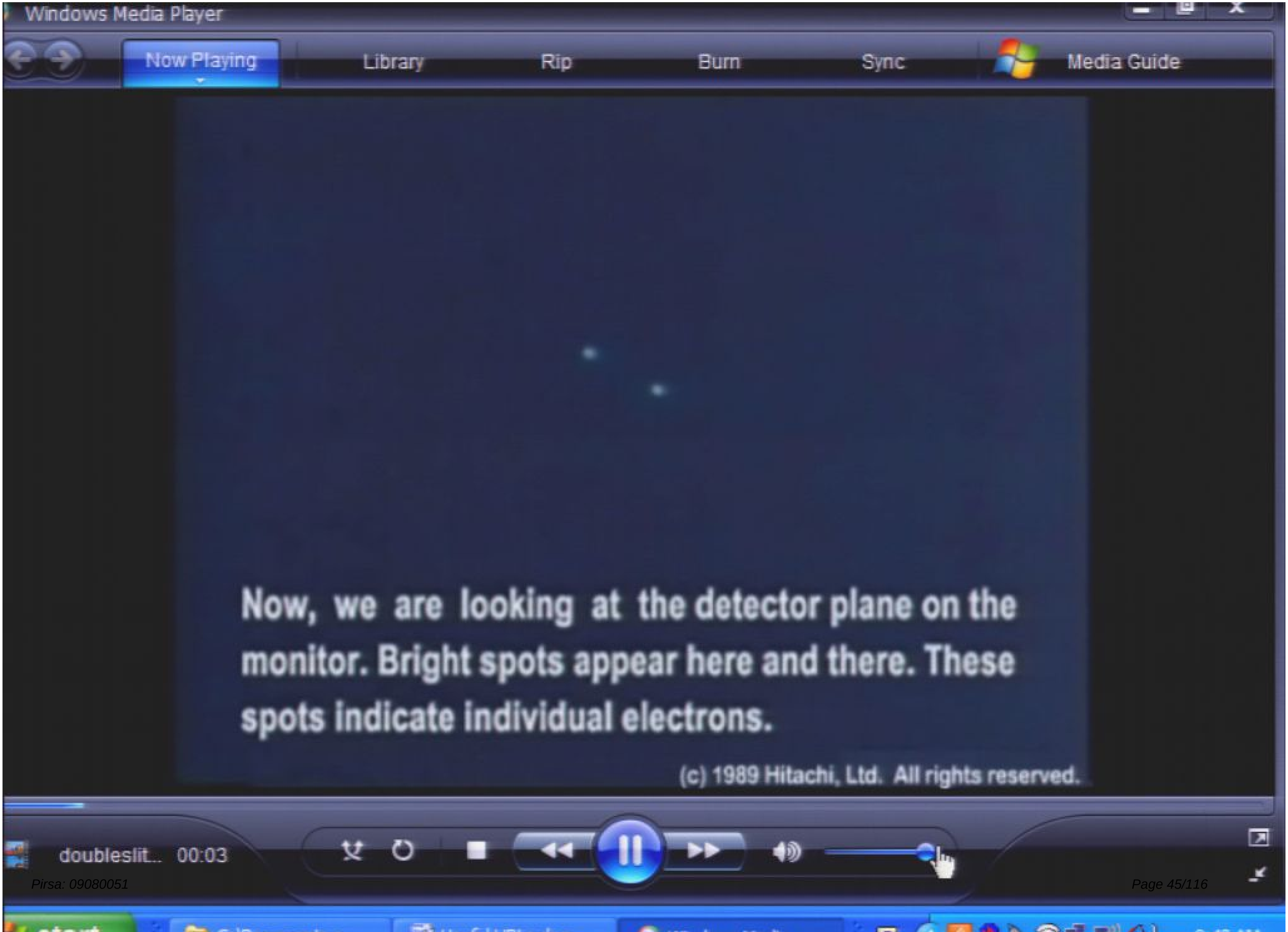
Double Slit
P(x)=P1(x)+P2(x)
P1(x)
P2(x)
1
2
p
(small)
17
10
7
d
0
x

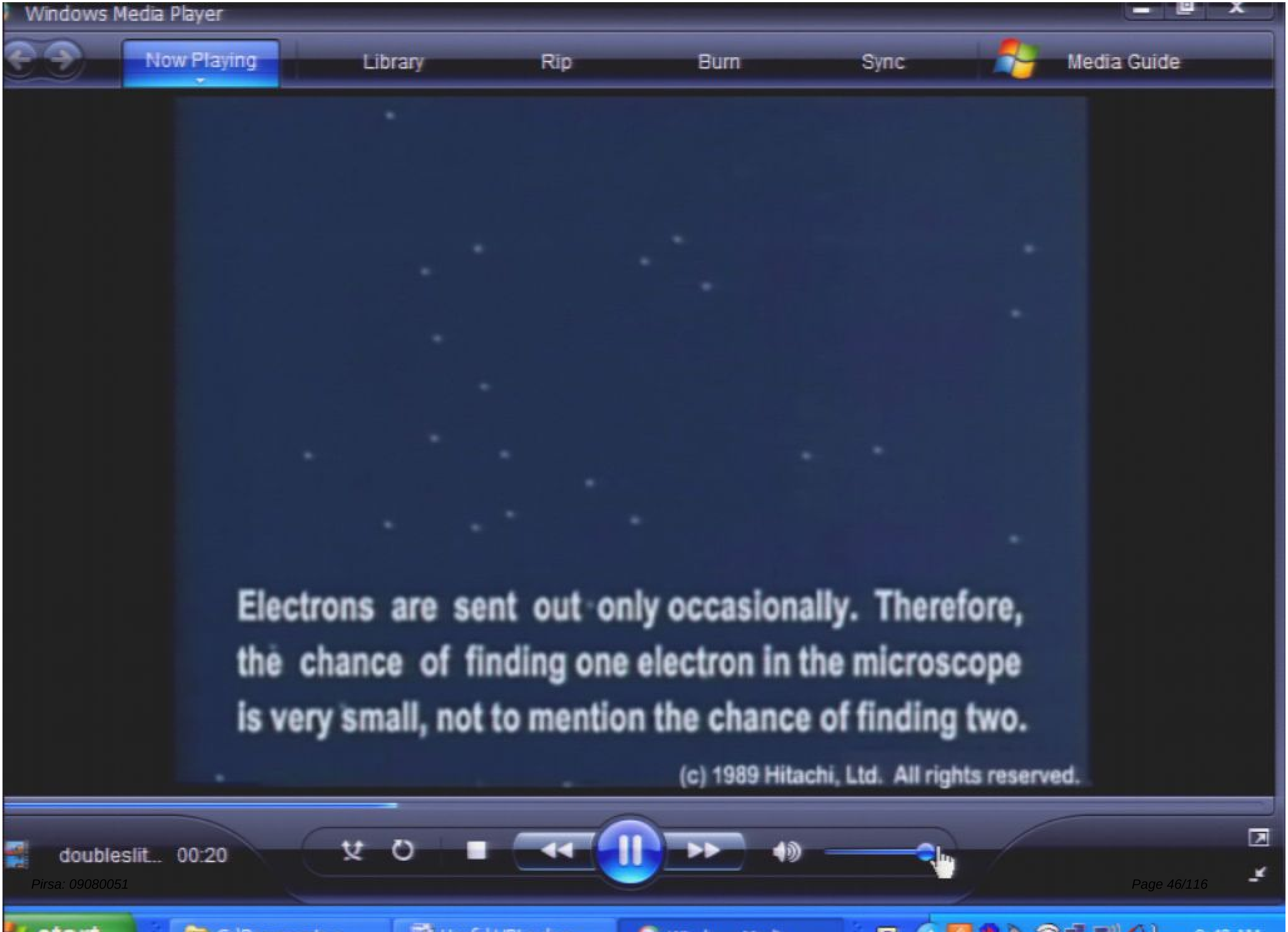

Double Slit
P(x)=P1(x)+P2(x)
P1(x)
P2(x)
1
2
p
(small)
17
10
7
d
0
x

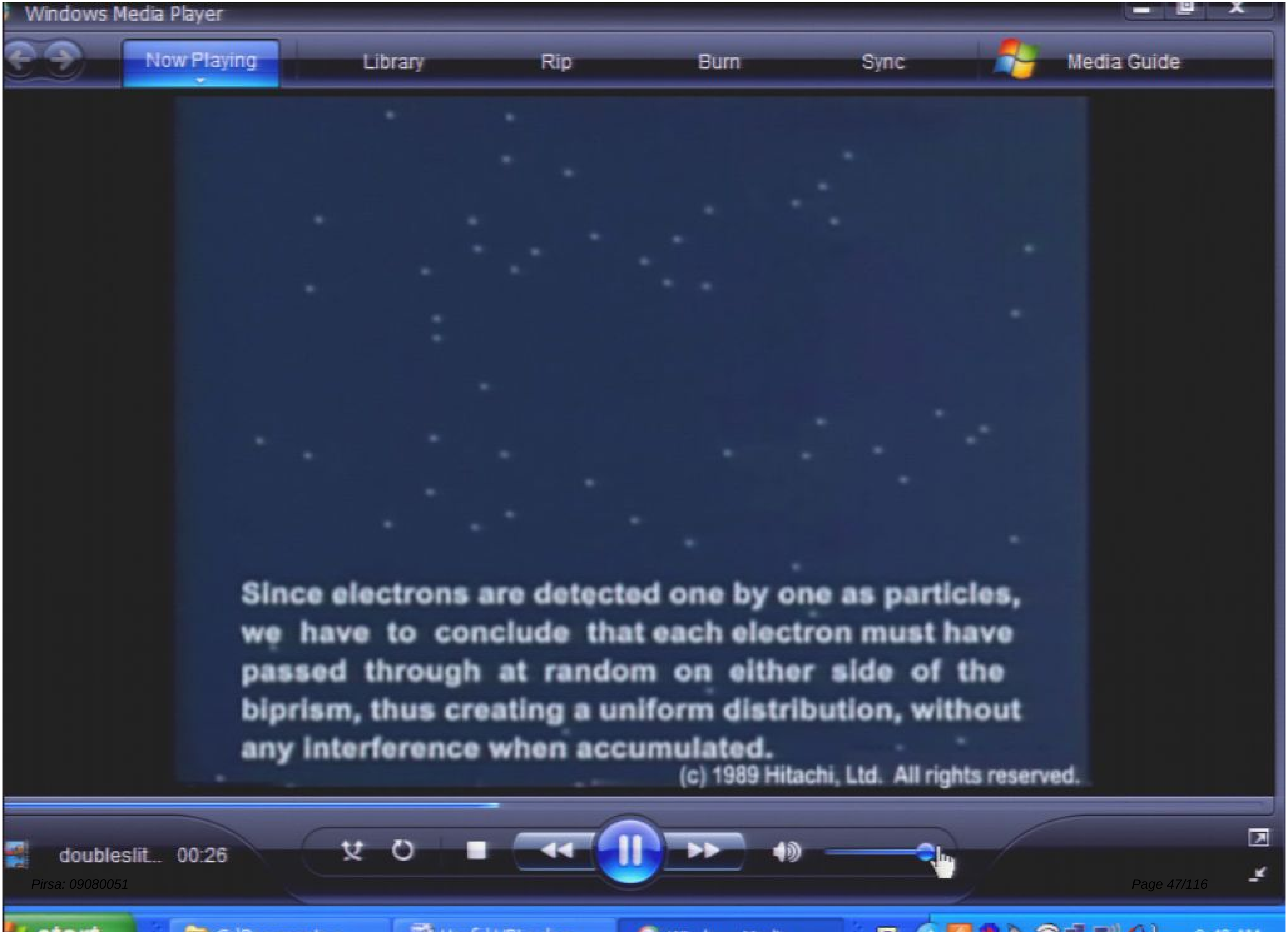

Photographer Jens
visiting this am


Photographer Jens
visiting this am

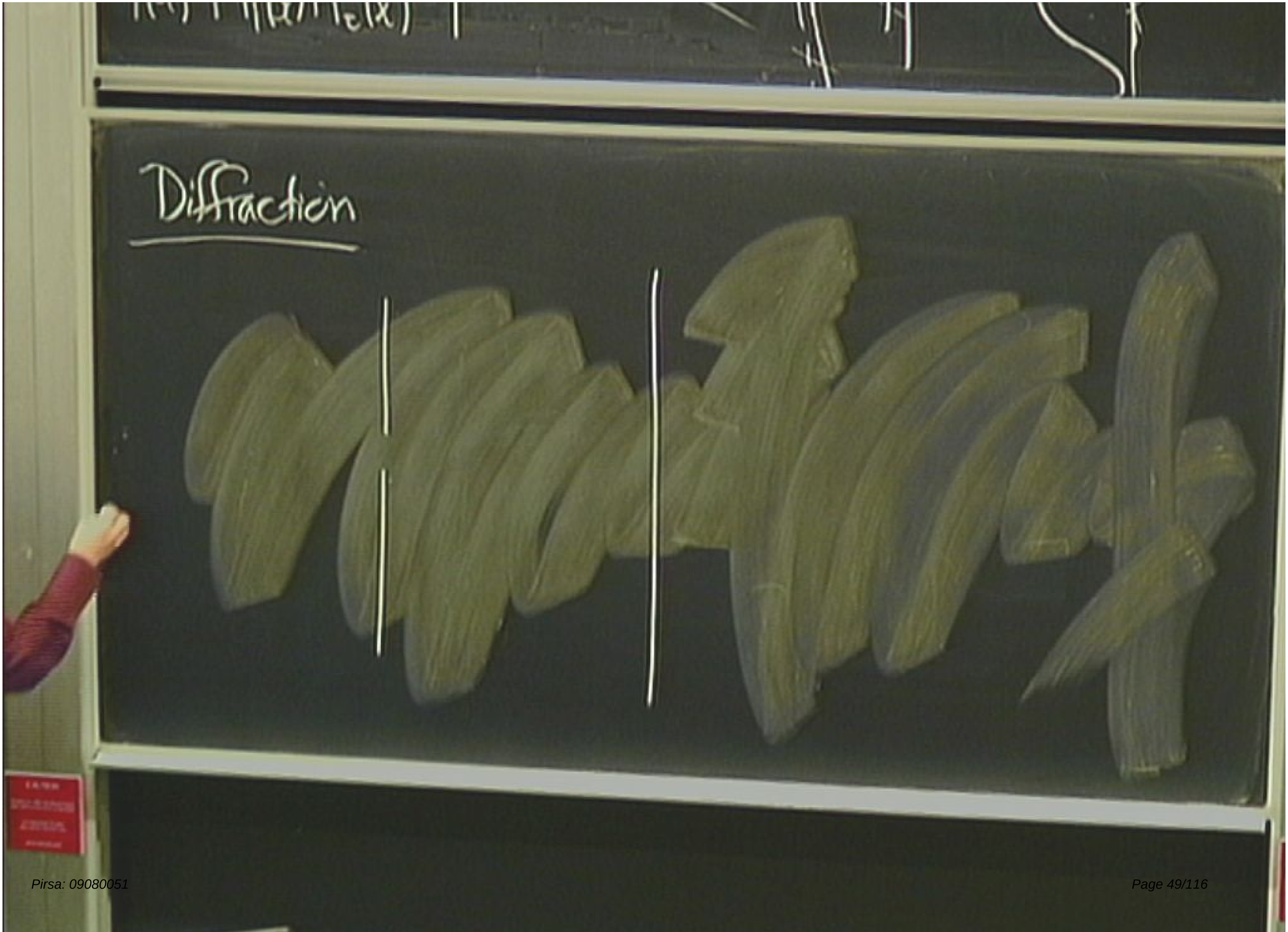

Windows Media Player
Now Playing
Library
Rip
Burn
Sync
Media Guide
Now, we are looking at the detector plane on the monitor. Bright spots appear here and there. These spots indicate individual electrons.
(c) 1989 Hitachi, Ltd. All rights reserved.
doubleslite-n

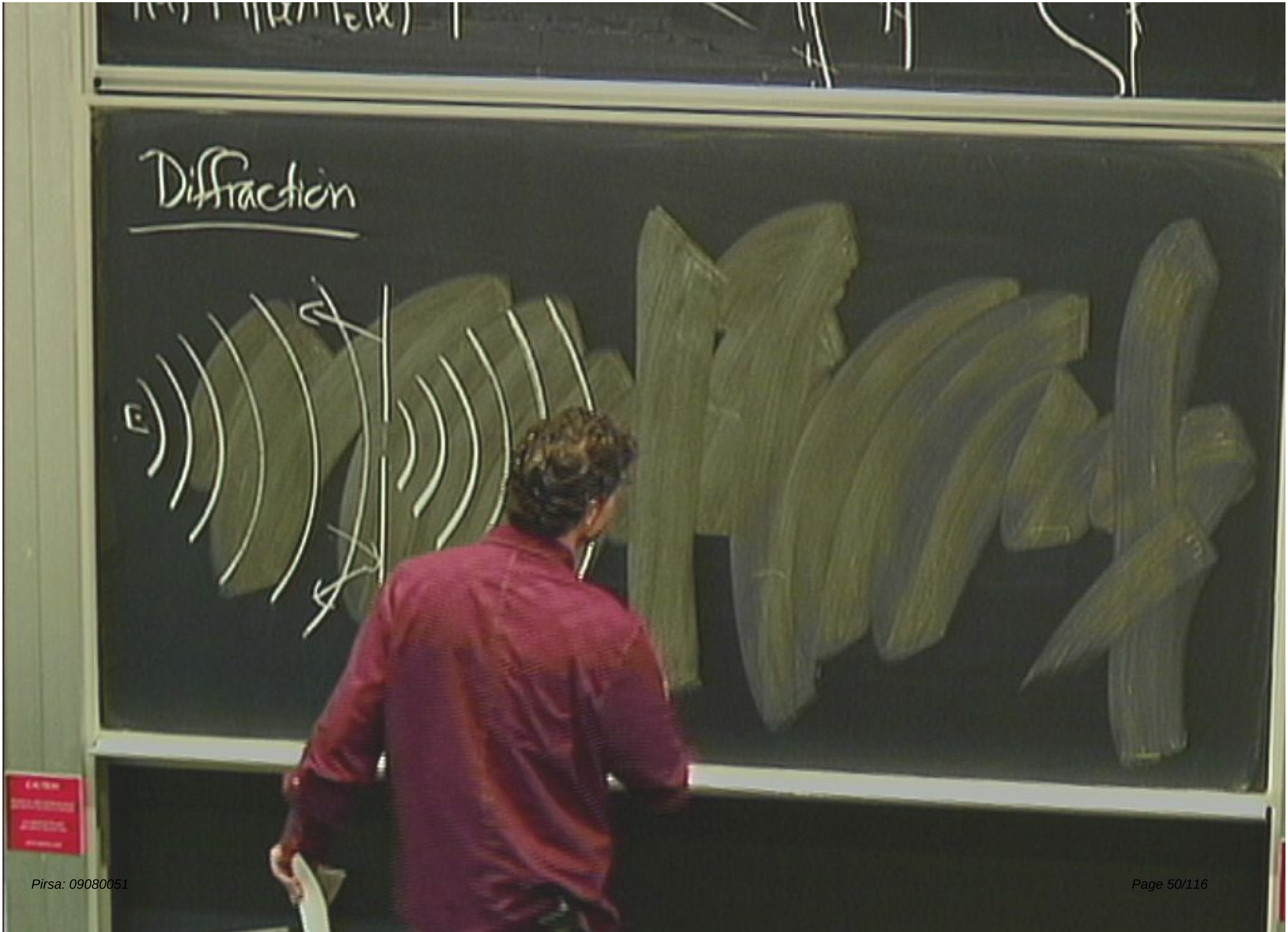

Windows Media Player
Now Playing
Library
Rip
Burn
Sync
Media Guide
Now, we are looking at the detector plane on the monitor. Bright spots appear here and there. These spots indicate individual electrons.
(c) 1989 Hitachi, Ltd. All rights reserved.
doubleslit... 00:03

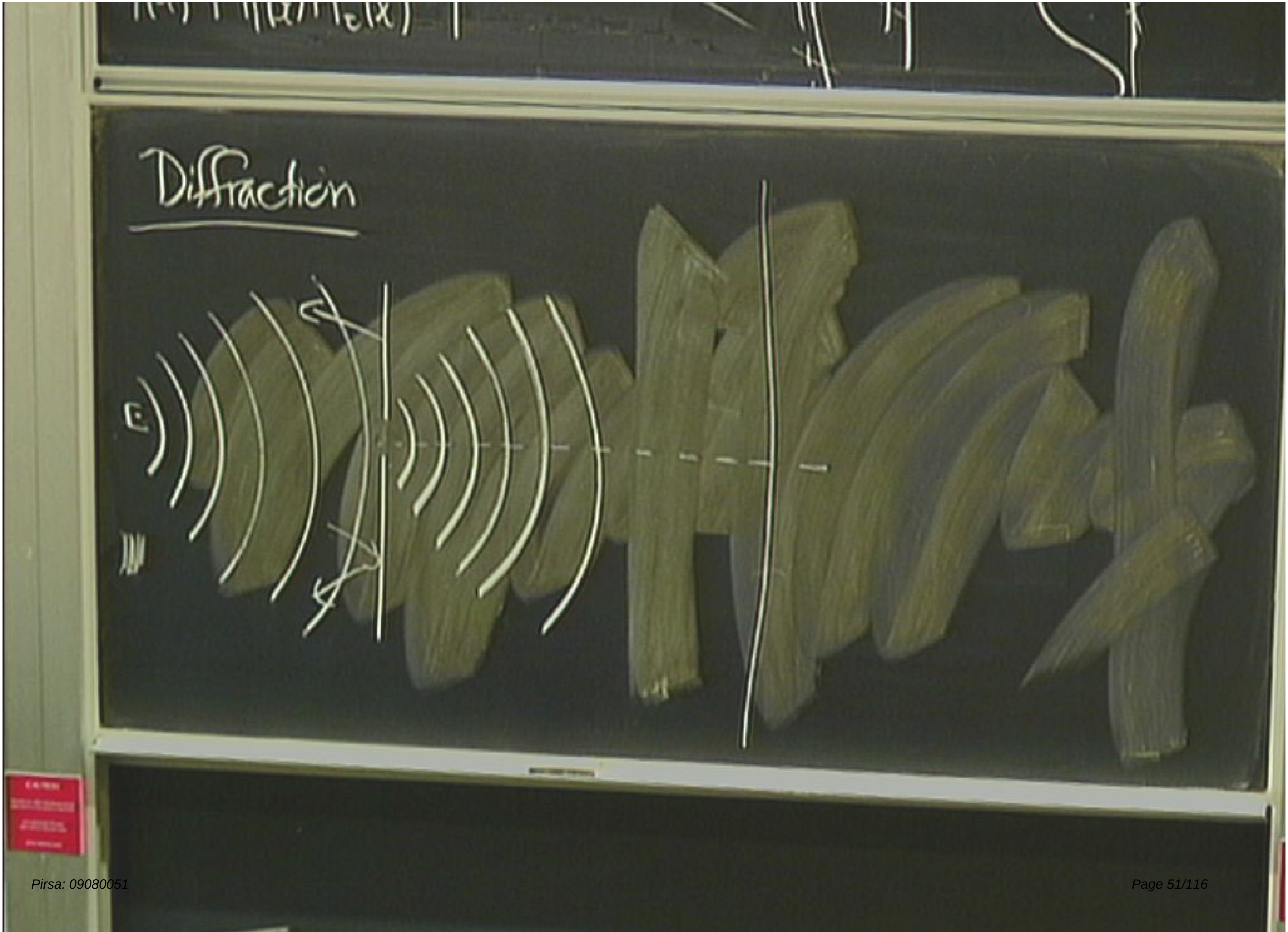

Windows Media Player
Now Playing
Library
Rip
Burn
Sync
Media Guide
Electrons are sent out only occasionally. Therefore, the chance of finding one electron in the microscope is very small, not to mention the chance of finding two.
(c) 1989 Hitachi, Ltd. All rights reserved.
doubleslit... 00:20

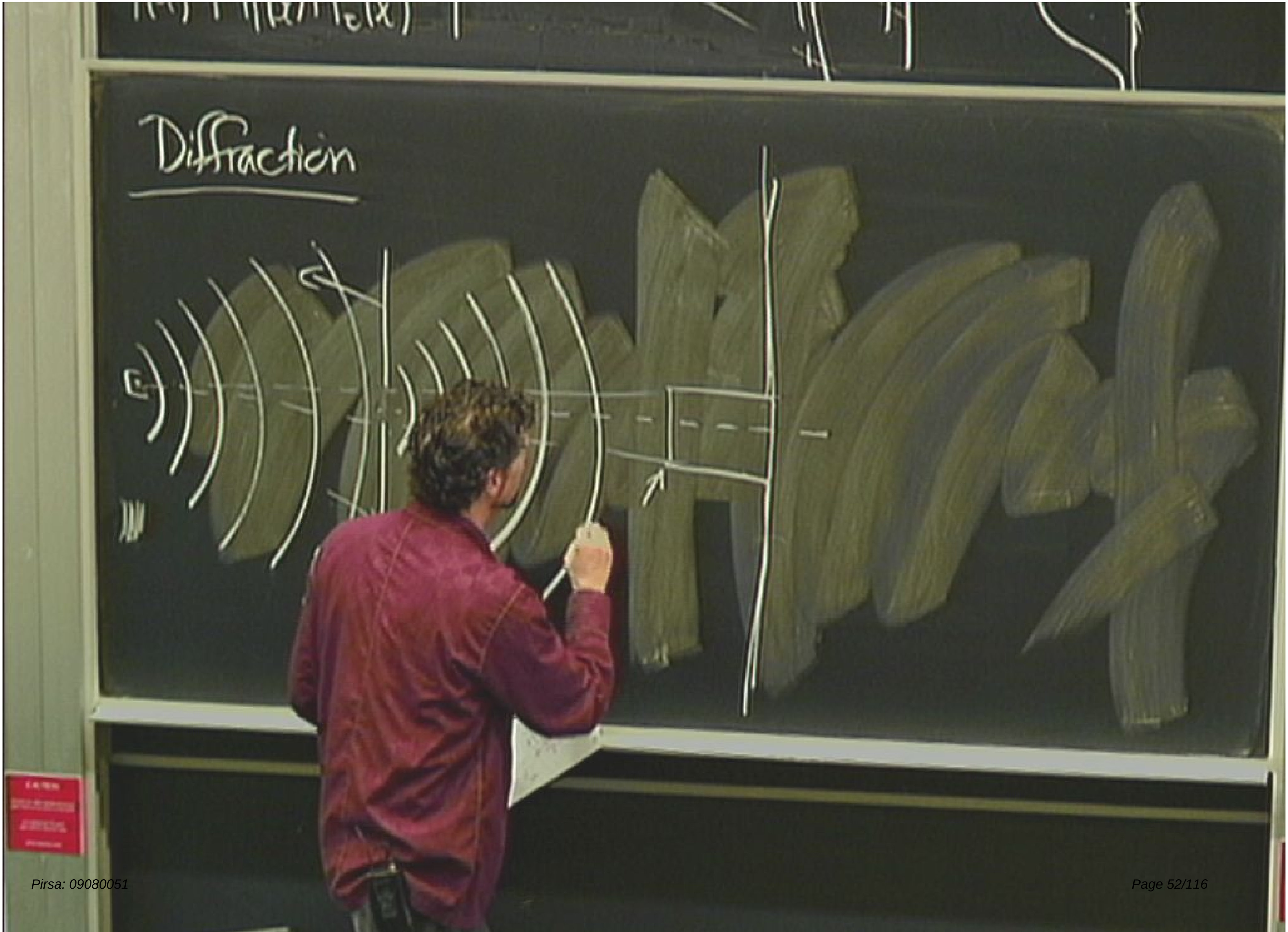

Windows Media Player
Now Playing
Library
Rip
Burn
Sync
Media Guide
Since electrons are detected one by one as particles, we have to conclude that each electron must have passed through at random on either side of the biprism, thus creating a uniform distribution, without any interference when accumulated.
(c) 1989 Hitachi, Ltd. All rights reserved.
doubleslit... 00:26

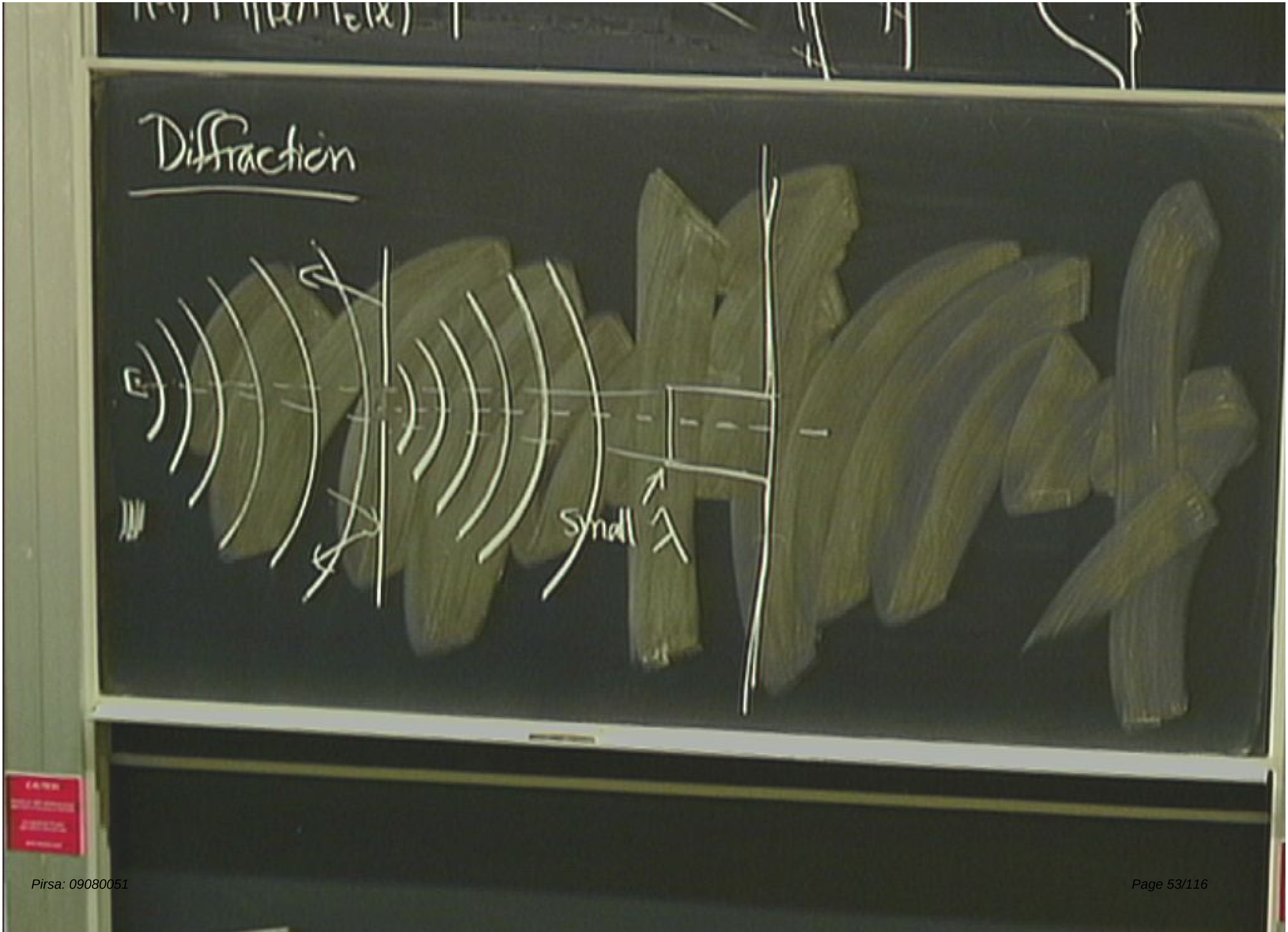

Windows Media Player
Now Playing
Library
Rip
Burn
Sync
Media Guide
Since this experiment lasted for more than 30 minutes,
I have sped the move up. Interference fringes are
now clearly visible.
(c) 1989 Hitachi, Ltd. All rights reserved.


Diffraction

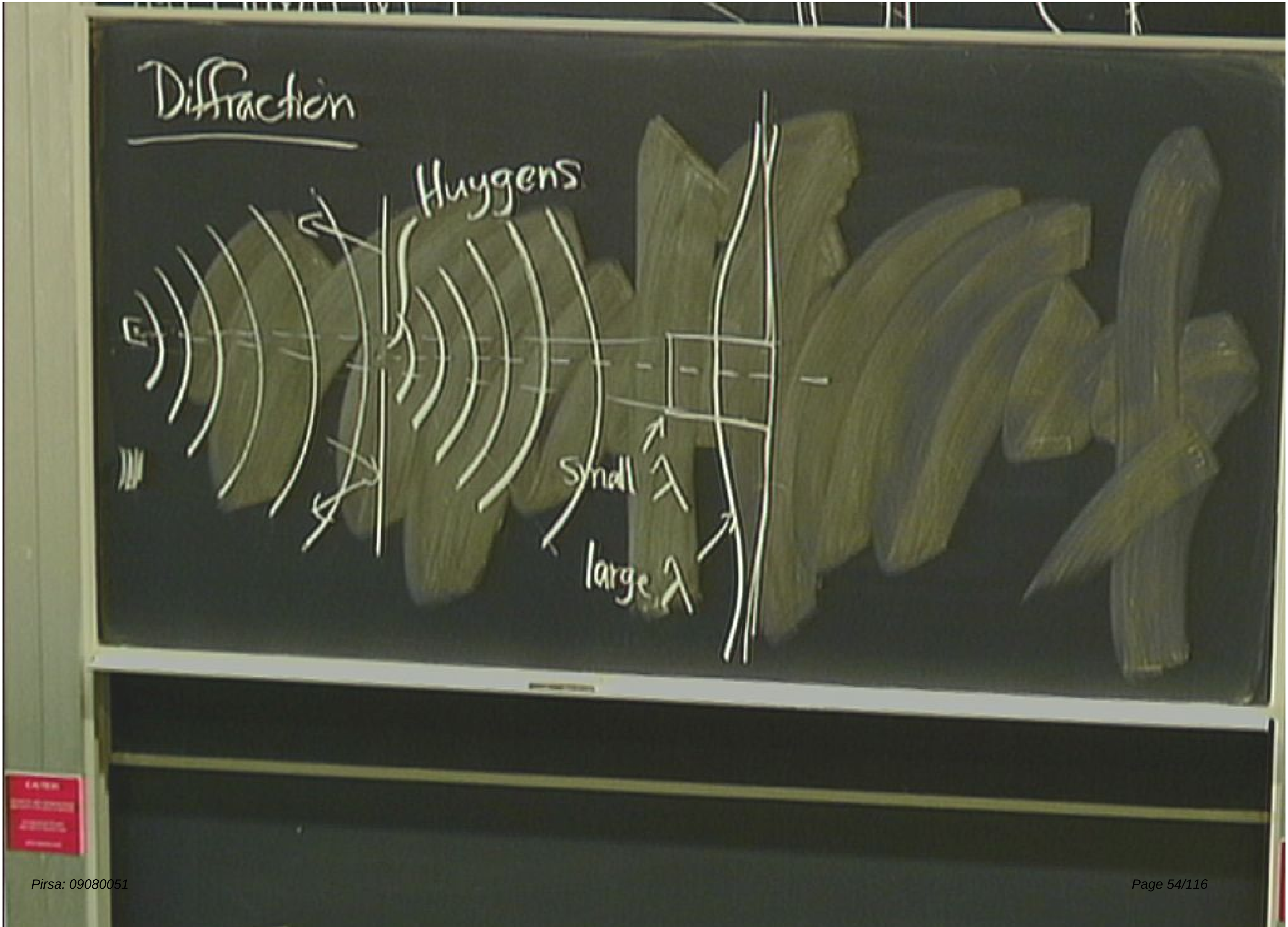

Diffraction

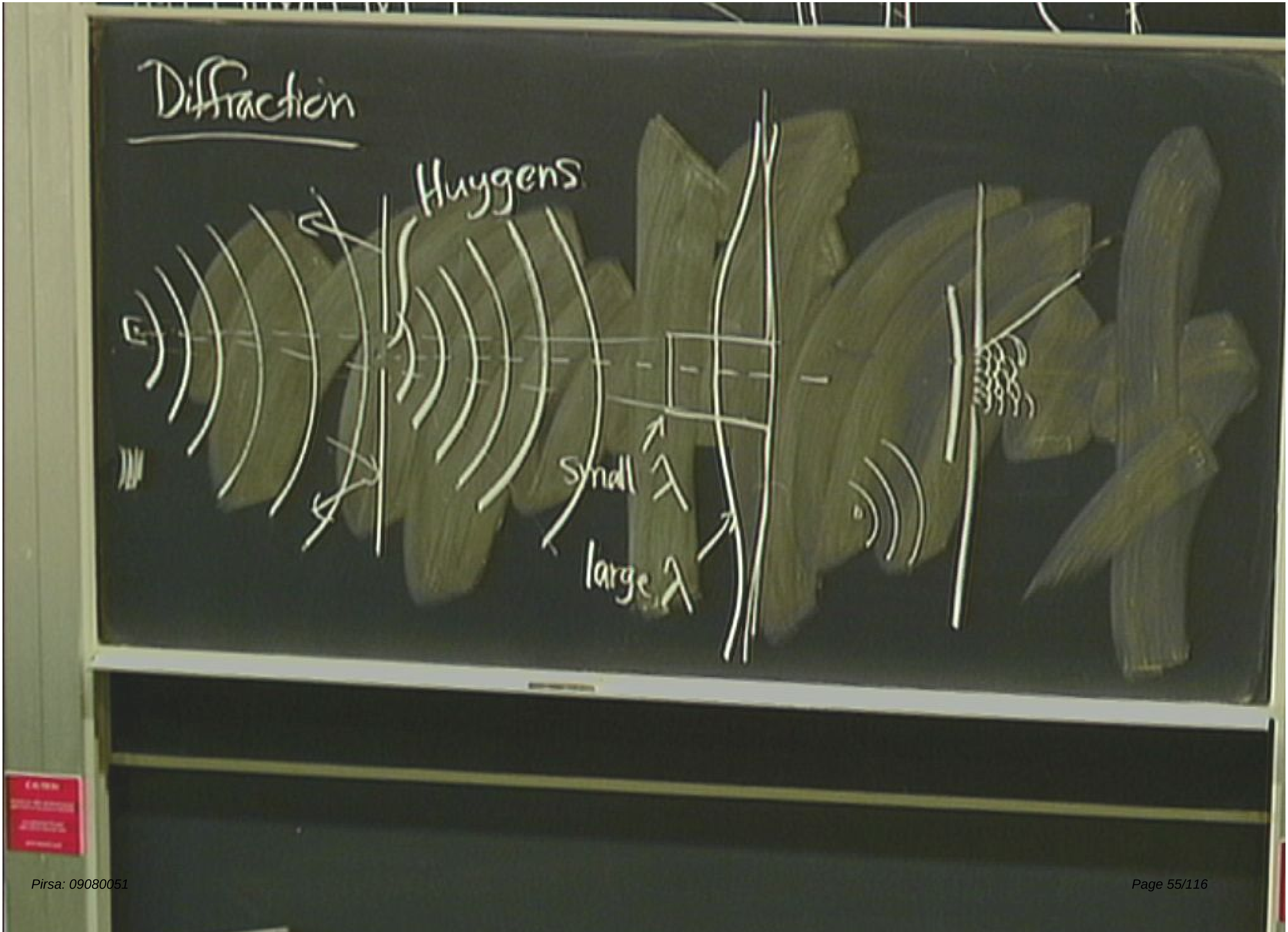

Diffraction


Diffraction

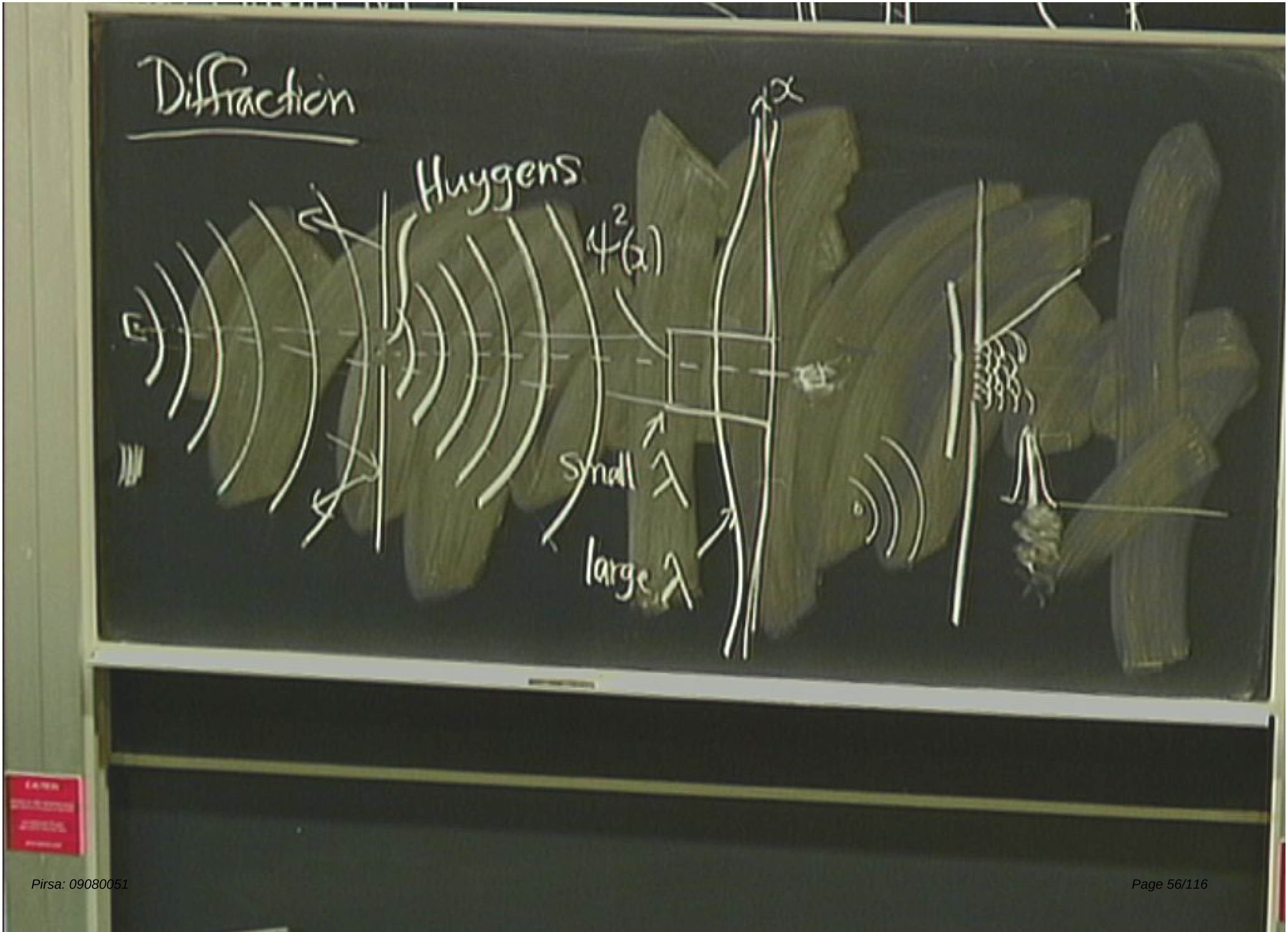


Diffraction
Small λ

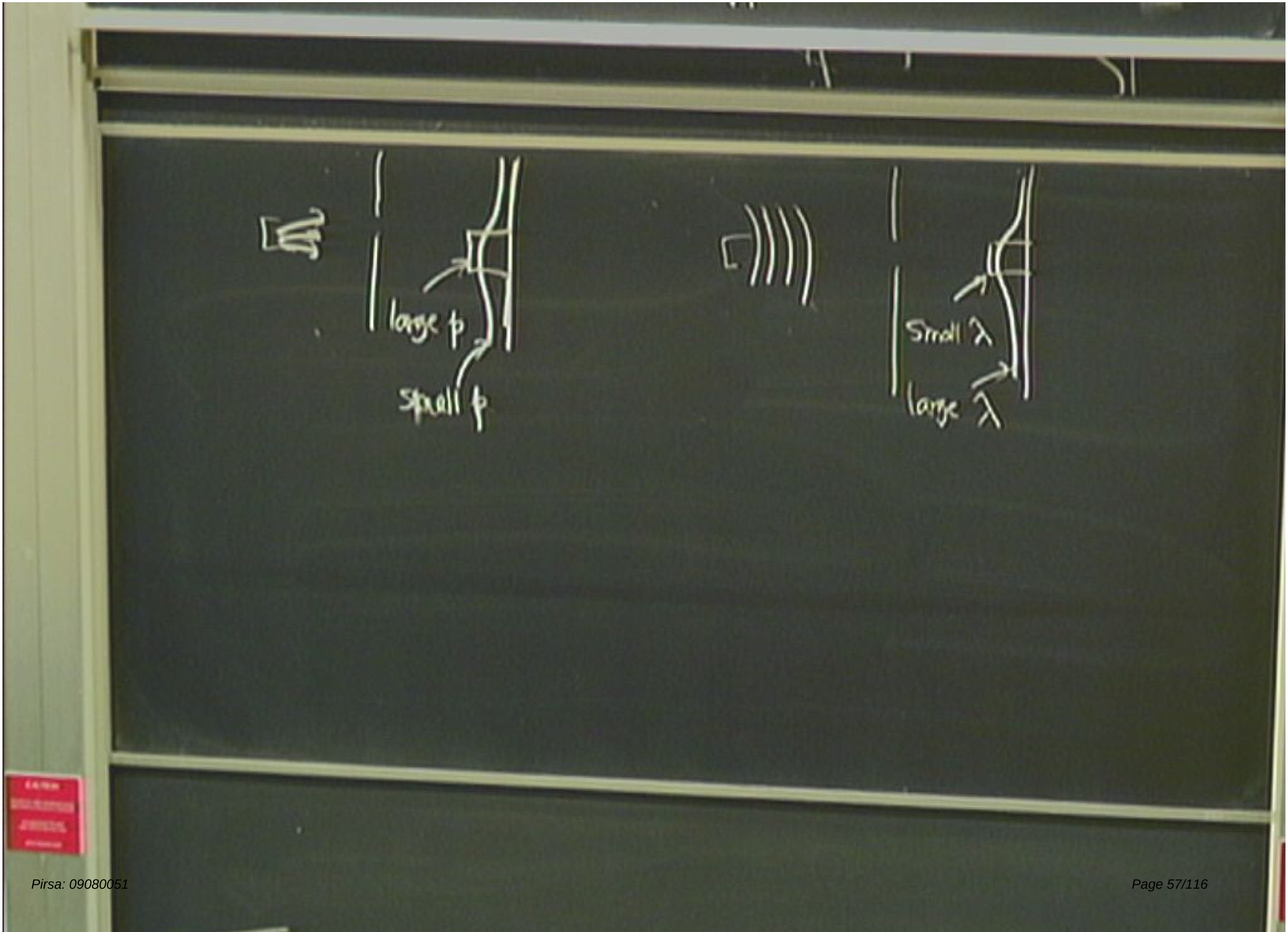

Diffraction
Huygens
small λ
large λ

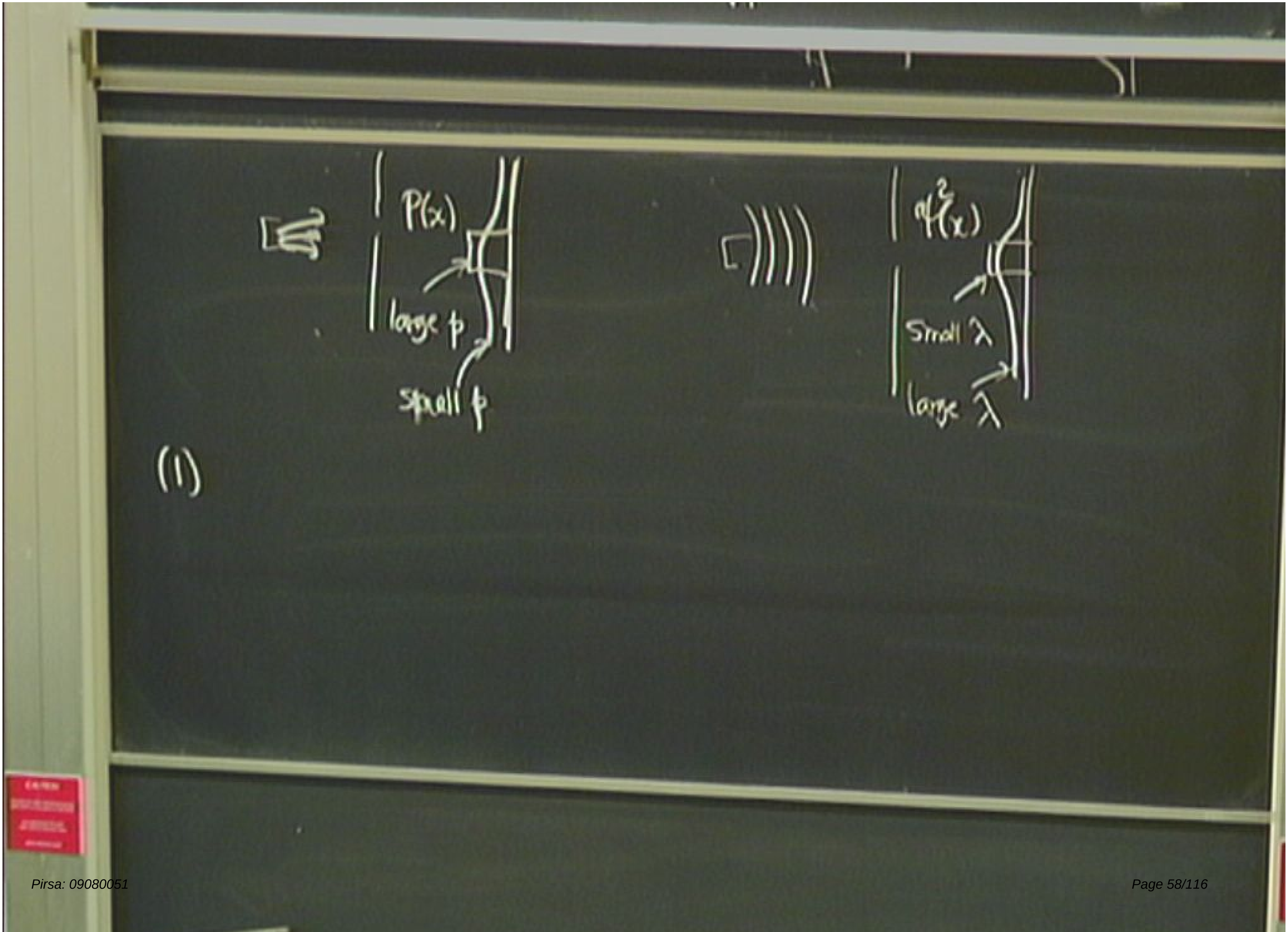


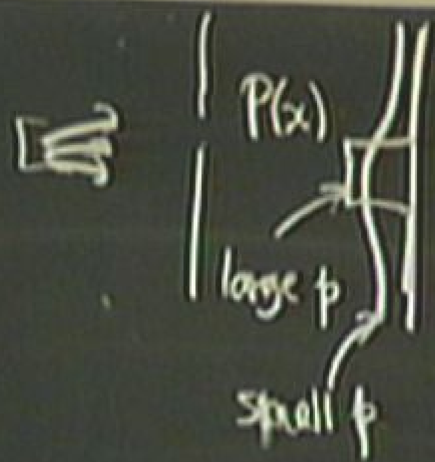
Diffraction
Huygens
small λ
large λ

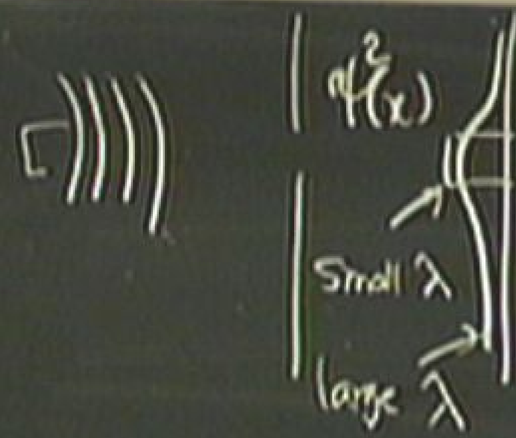


Diffraction
Huygens
$\psi^2(x)$
x
small λ
large λ


large p
small p
small λ
large λ

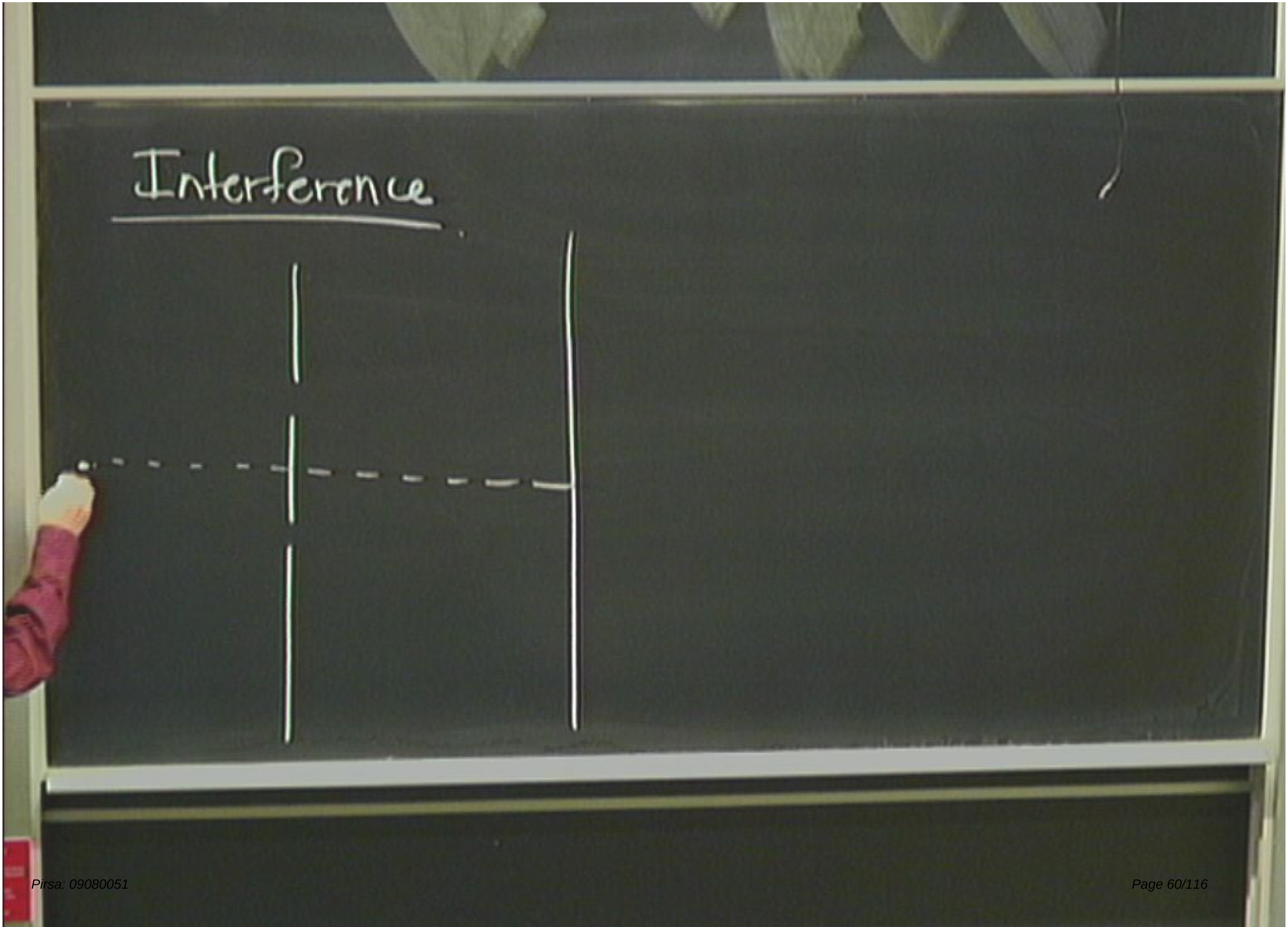

P(x)
large p
small p
(1)
ψ²(x)
small λ
large λ

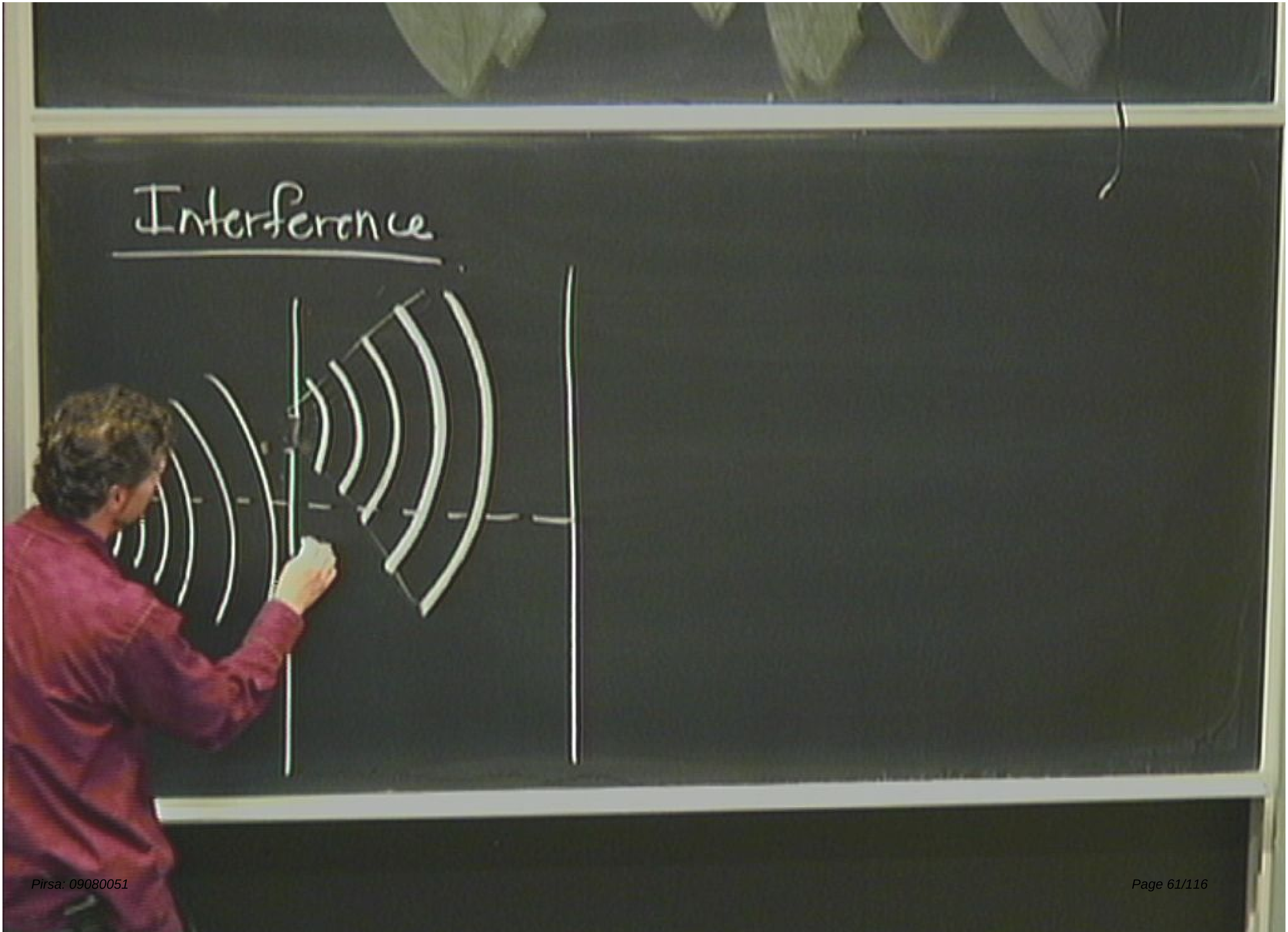


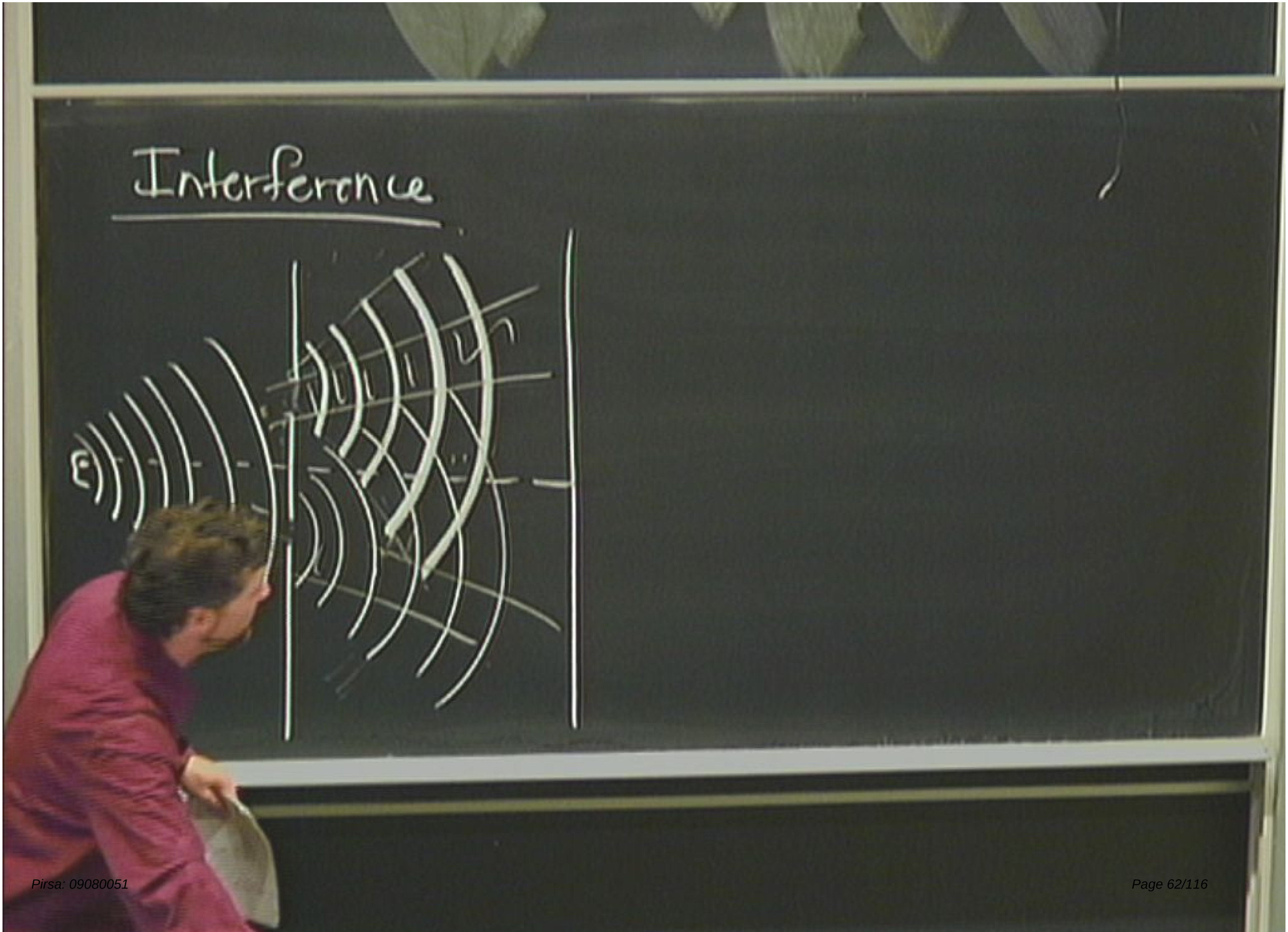


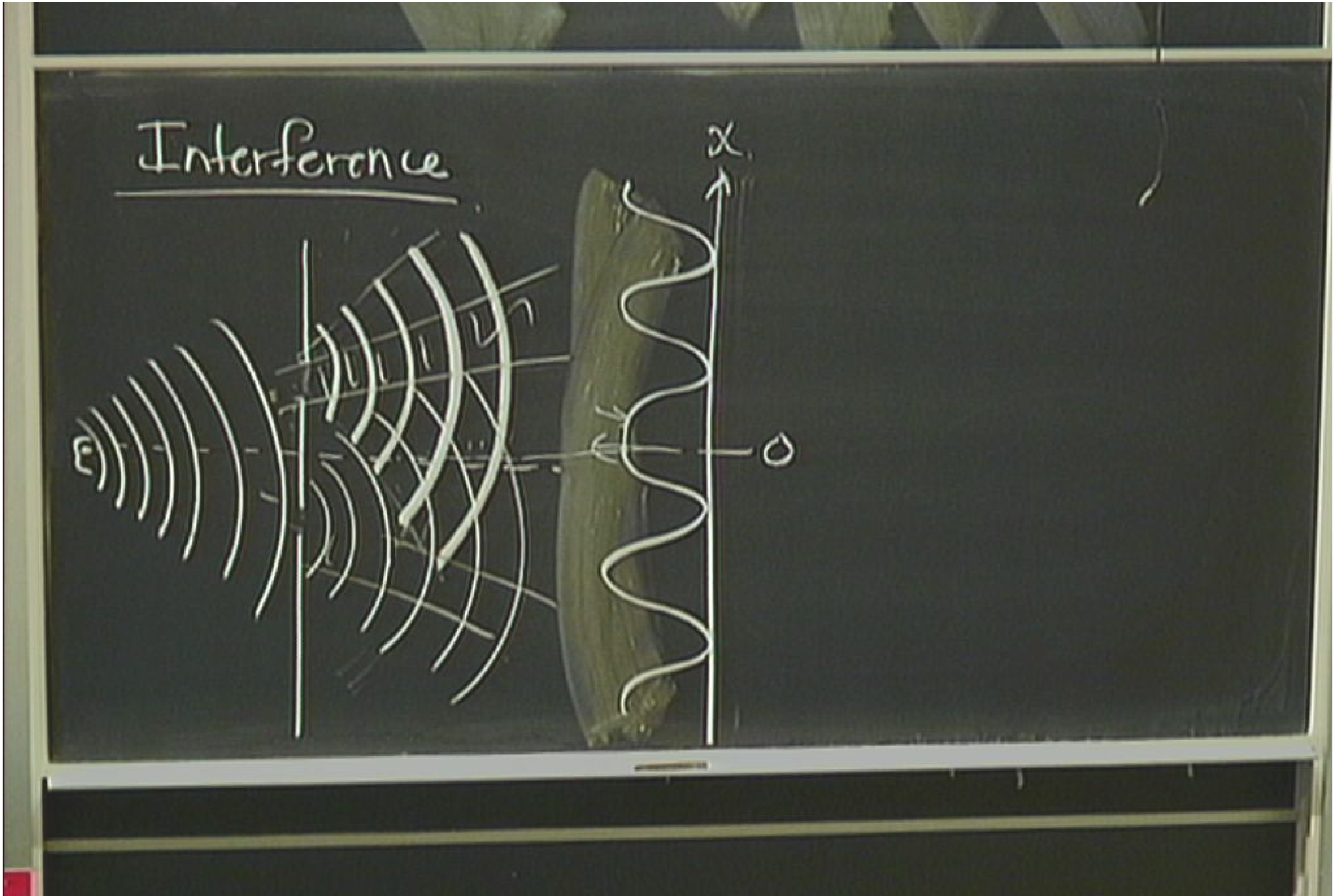
(1) identify $\psi^2(x)$ with $P(x)$

$\uparrow$ wave intensity $\qquad$ $\uparrow$ particle probability

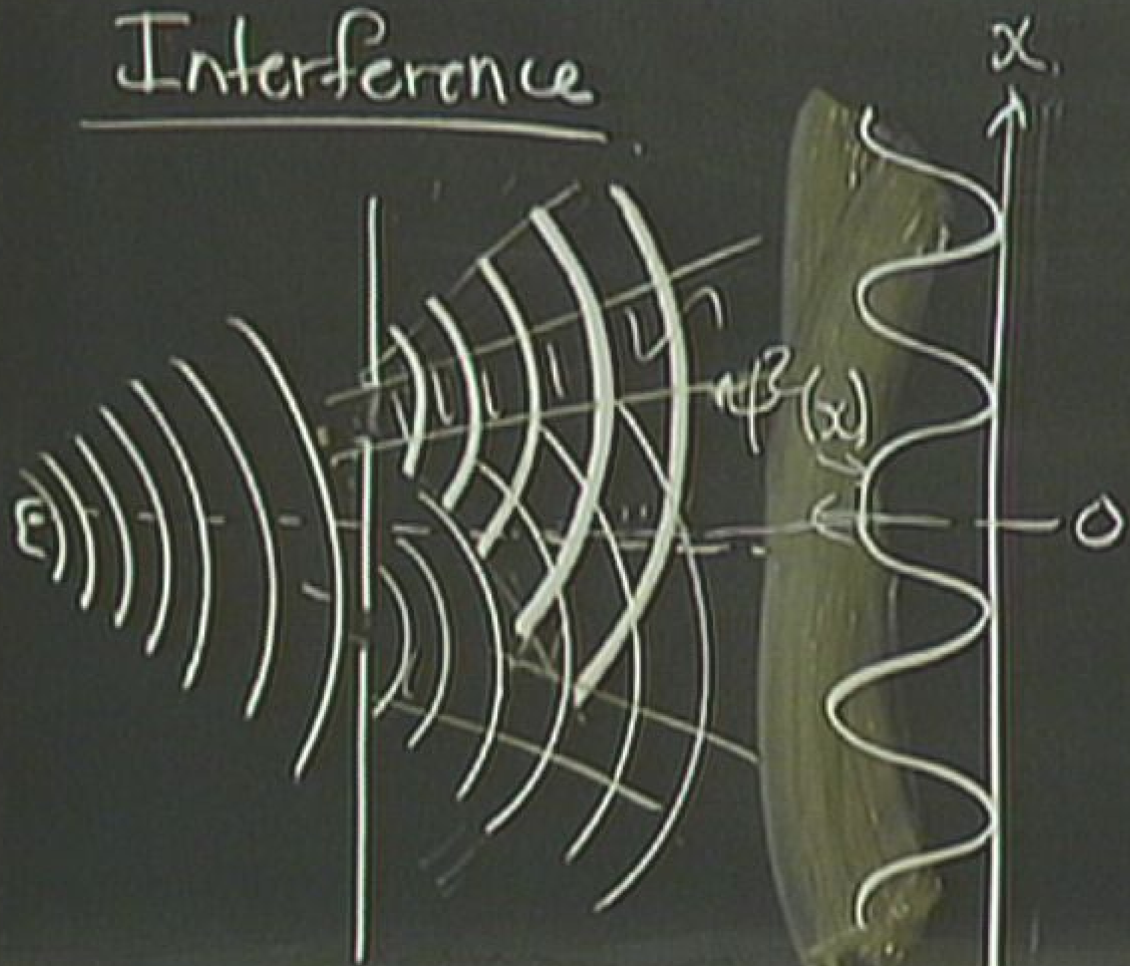

Interference

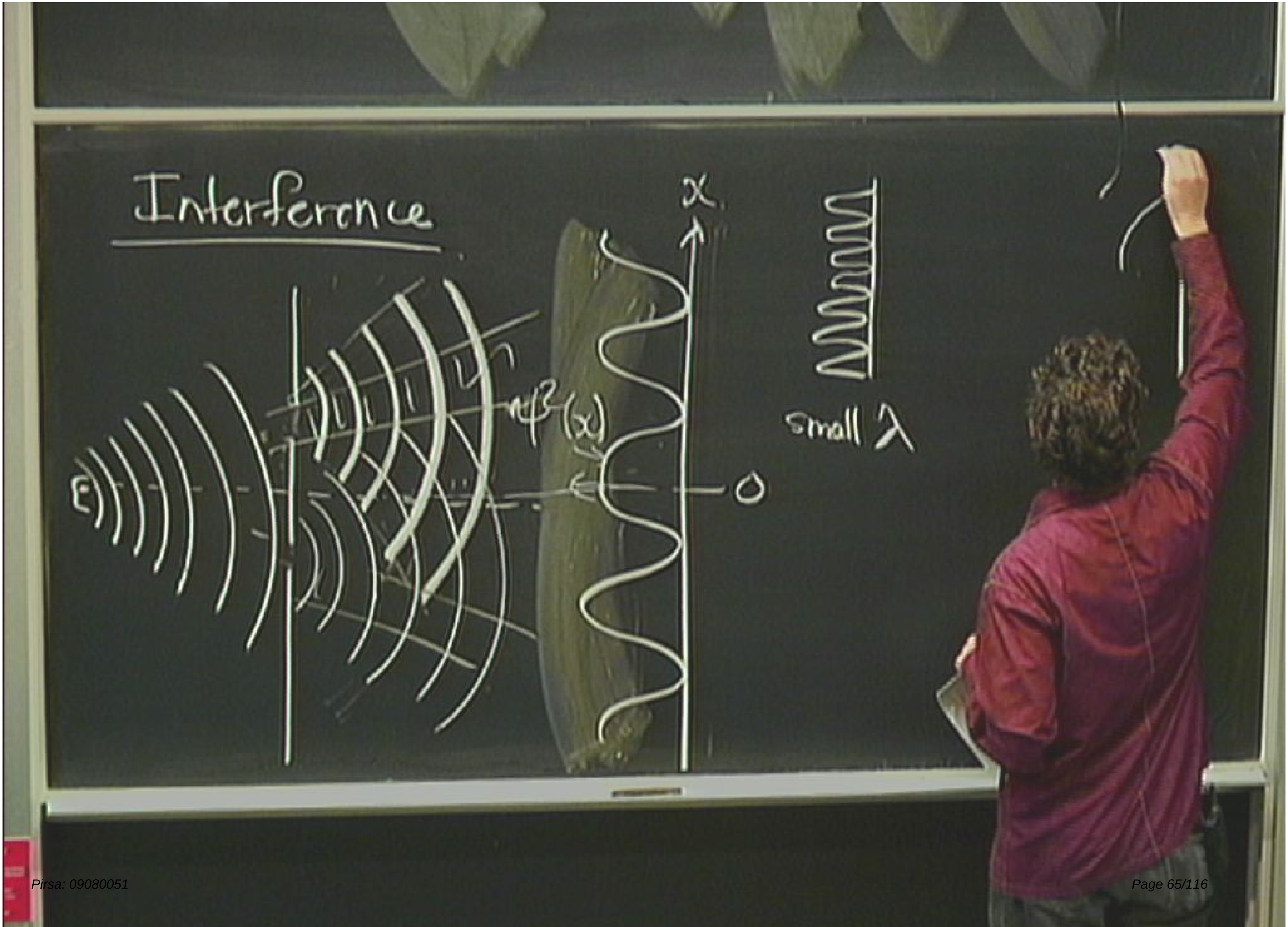

Interference

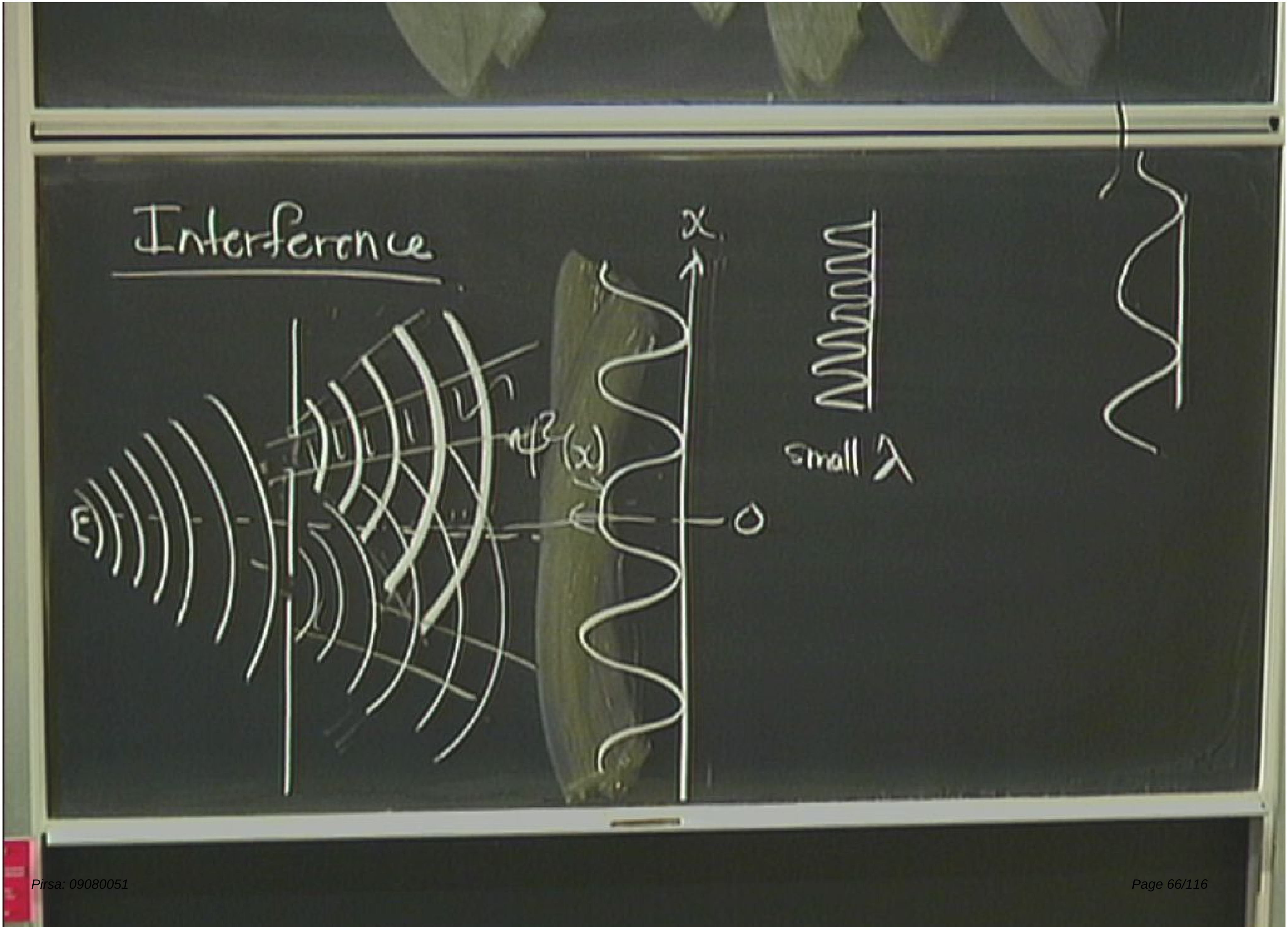

Interference

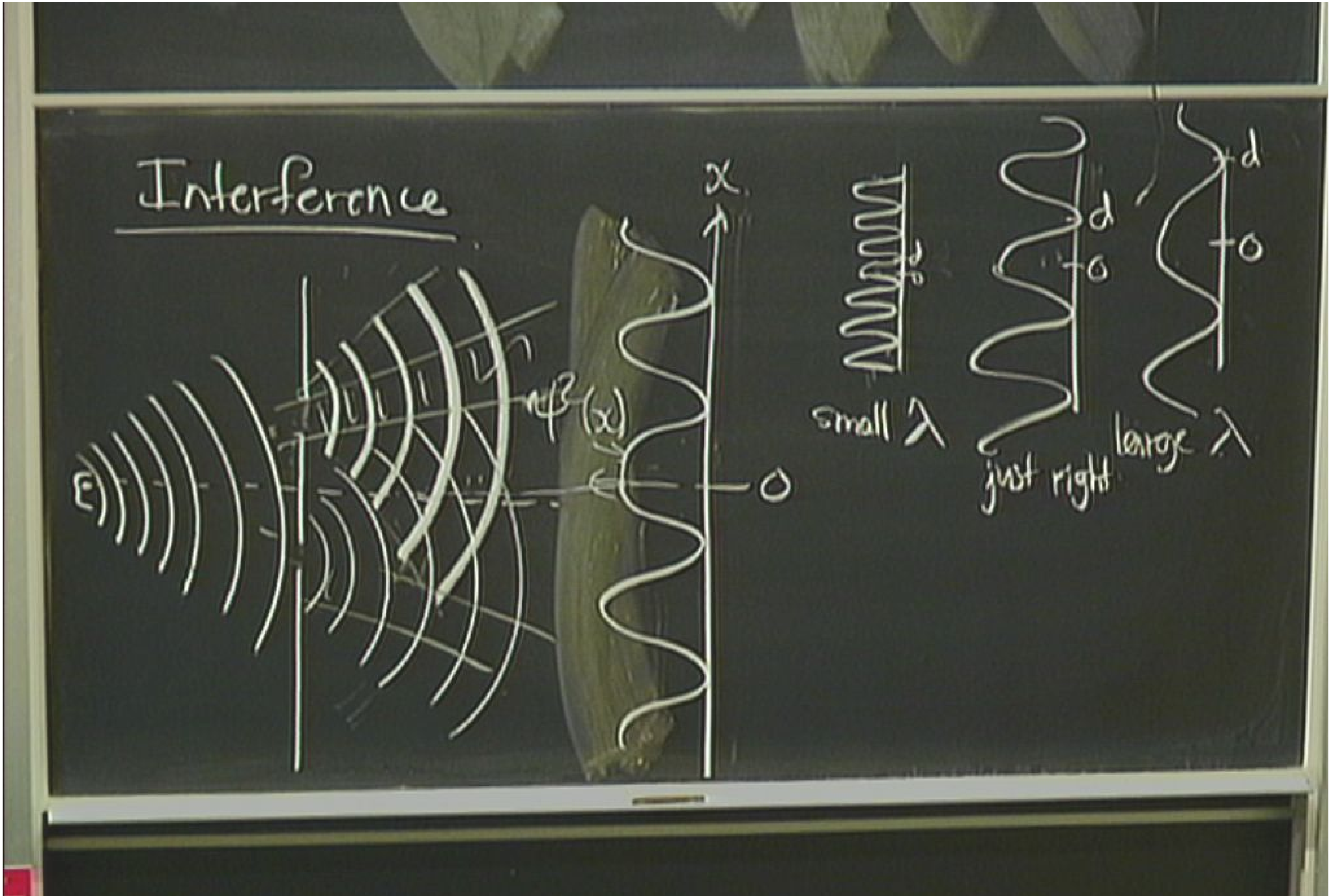

Interference
x
0

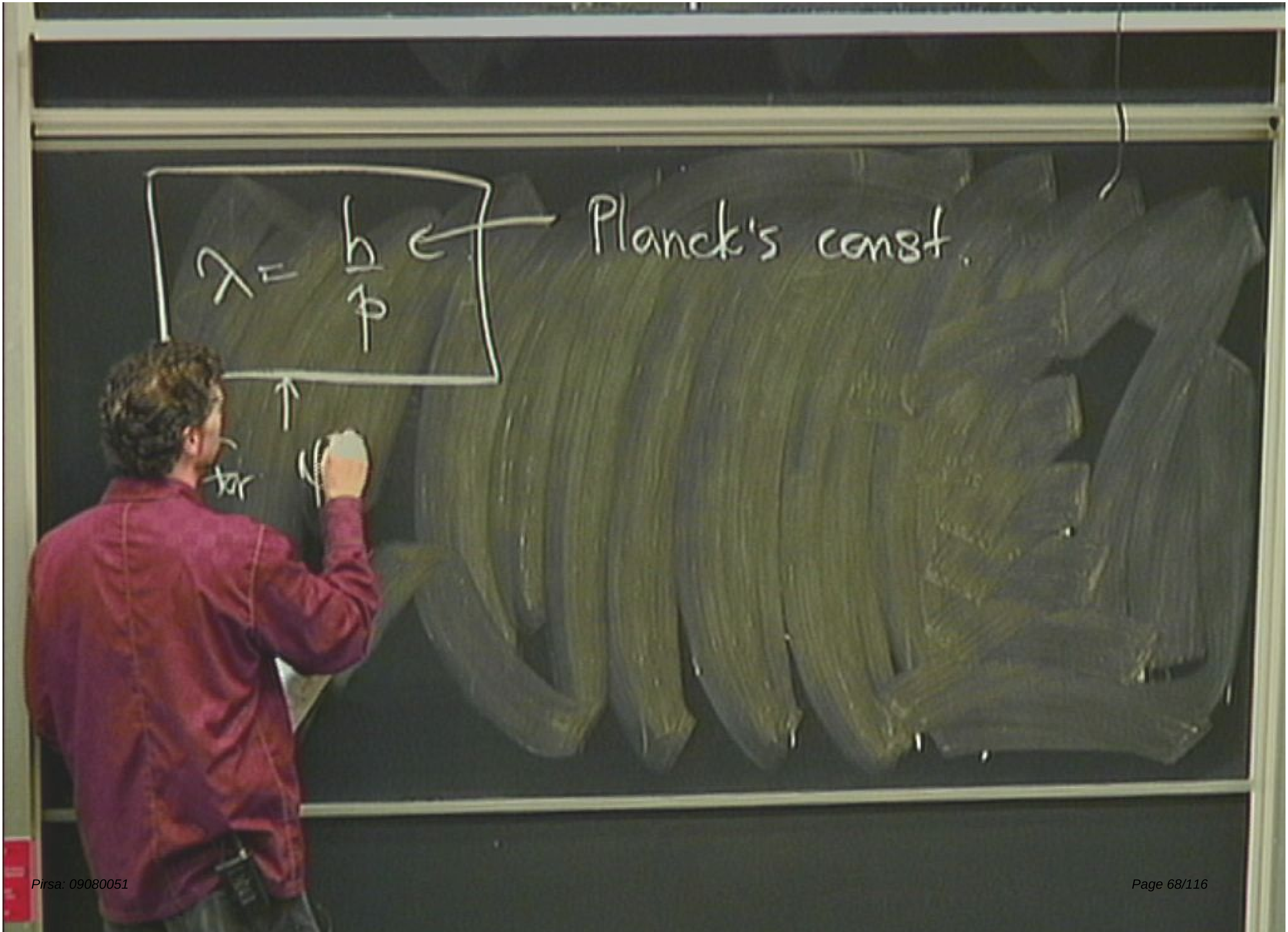
Interference




Interference
x
$n\psi^2(x)$
0
small λ


Interference
x
ψ²(x)
0
small λ


Interference
x
ψ(x)
0
small λ
just right
large λ
d
0

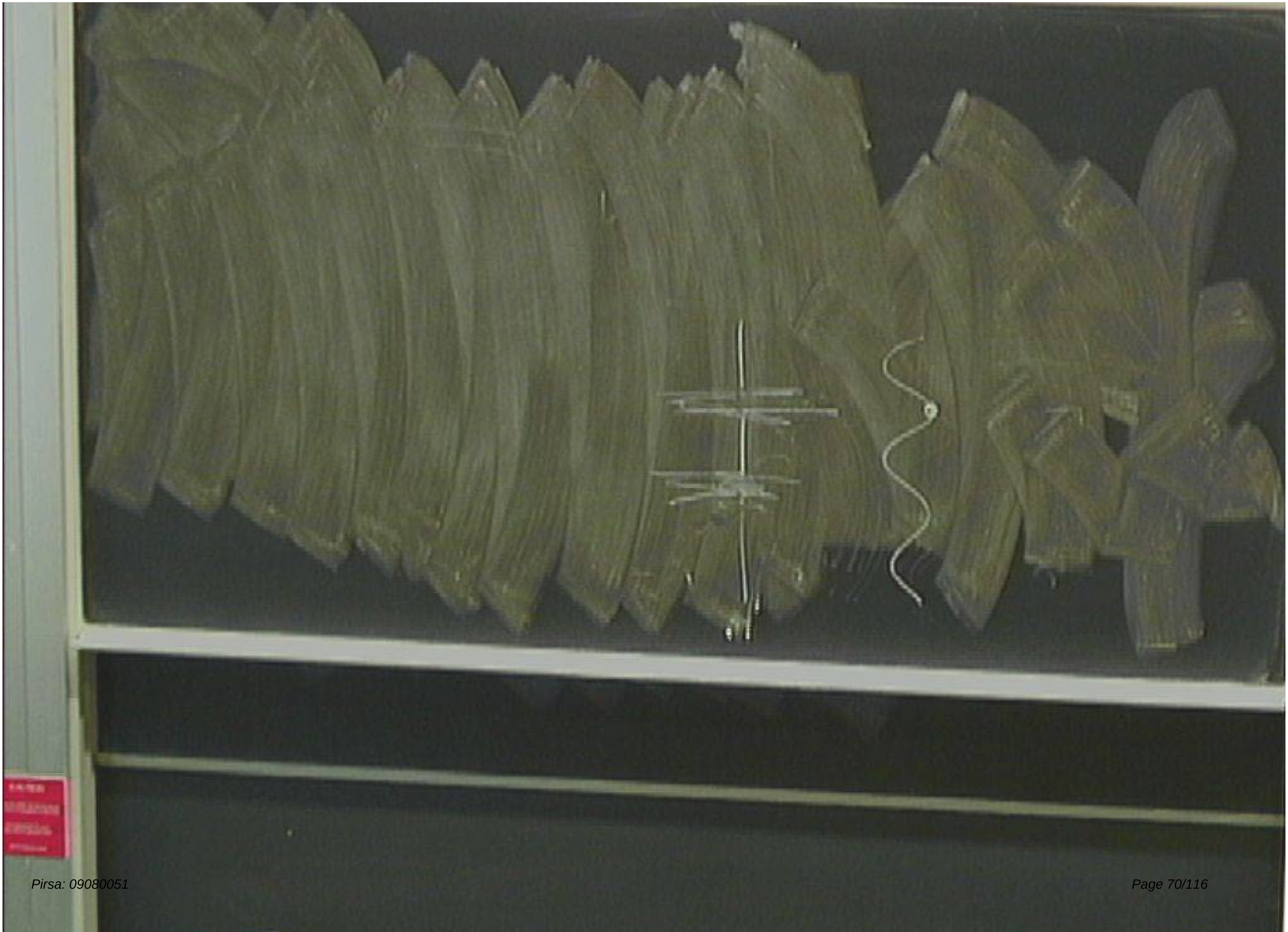

λ = h/p
Planck's const.

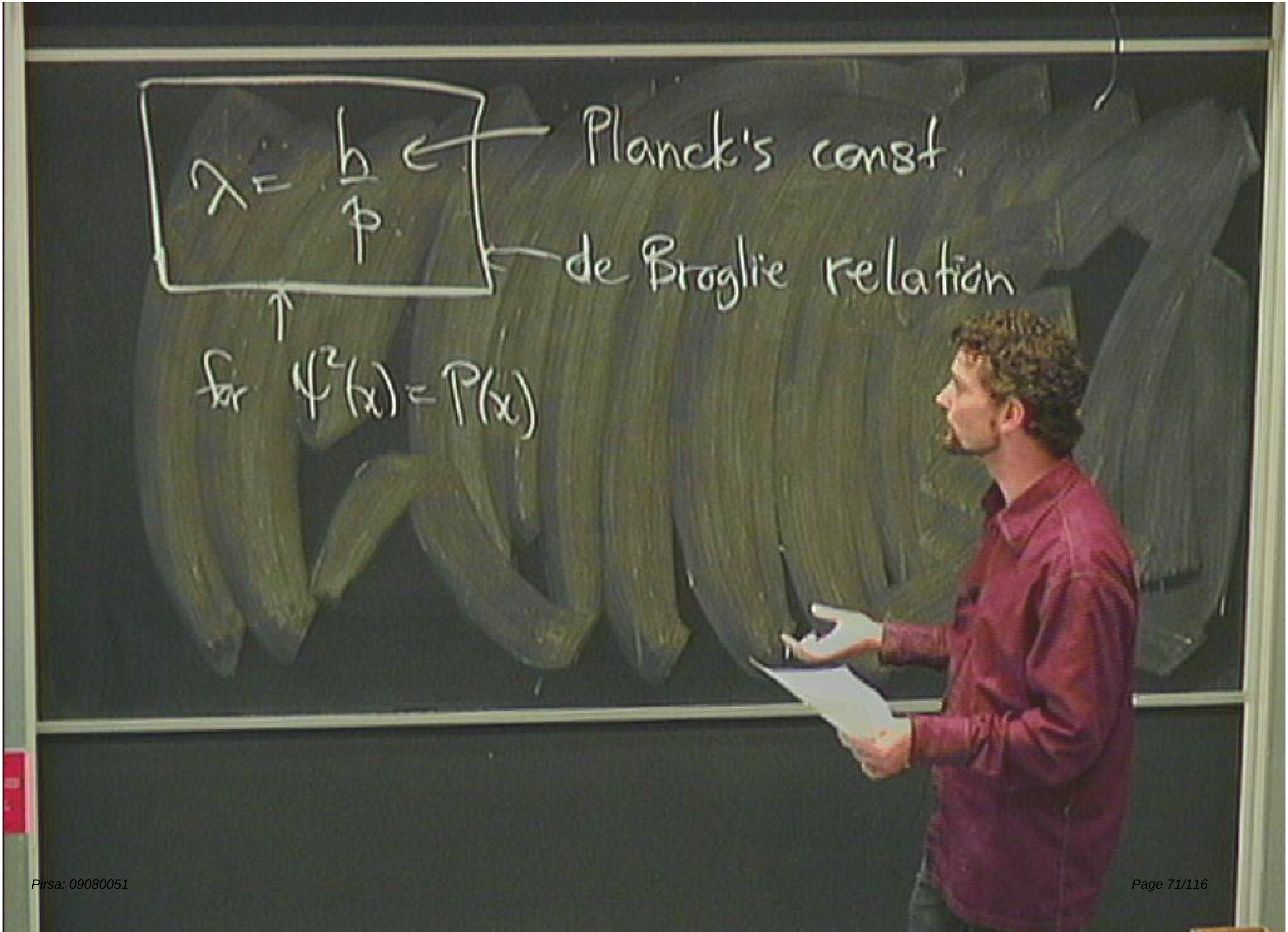
$$\boxed{\lambda = \frac{h}{p}}$$

Planck's const.

de Broglie relation

for $\psi^2(x) = P(x)$





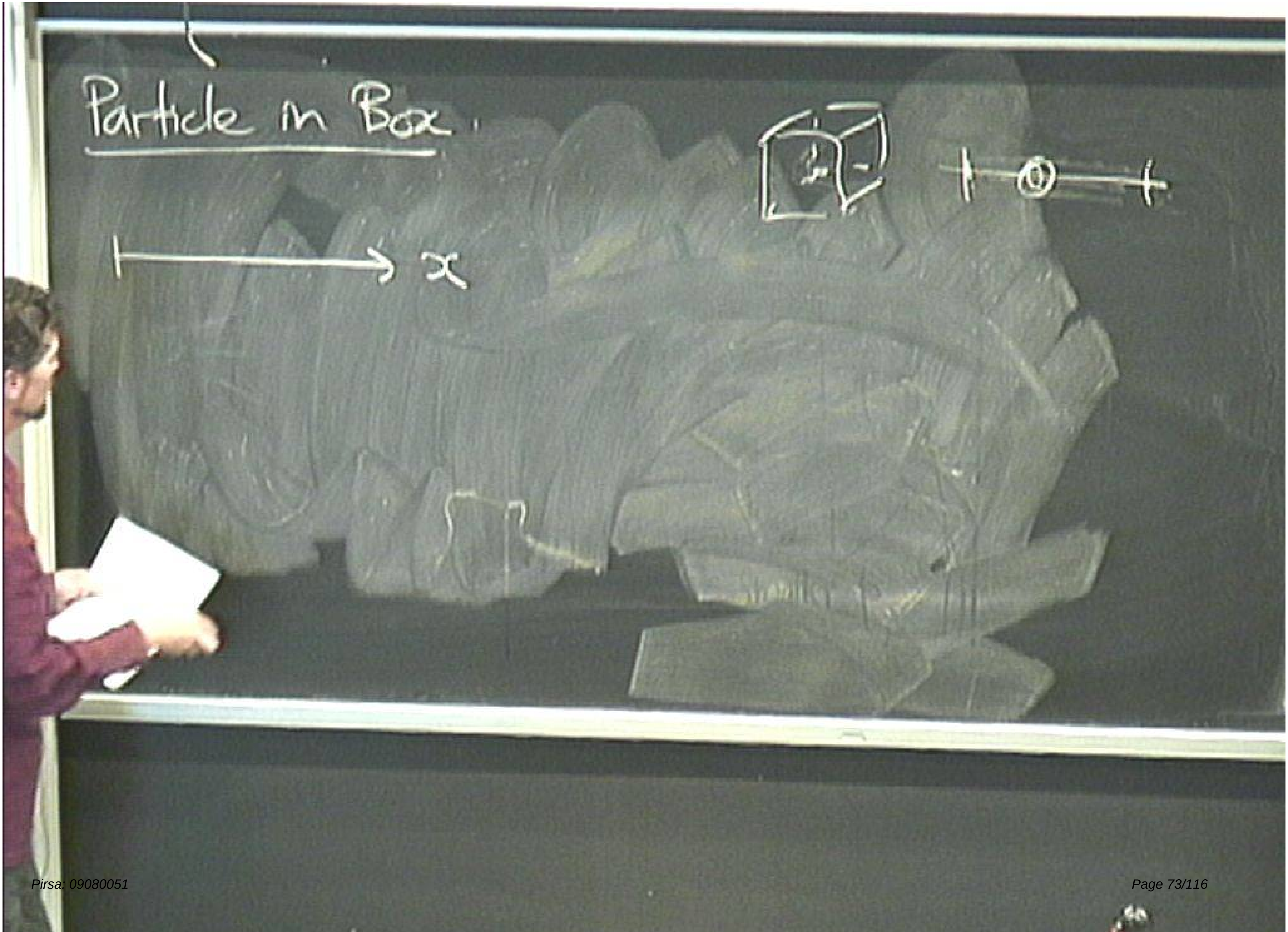
$\lambda = \frac{h}{p}$
Planck's const.
de Broglie relation
for $\psi^2(x) = P(x)$

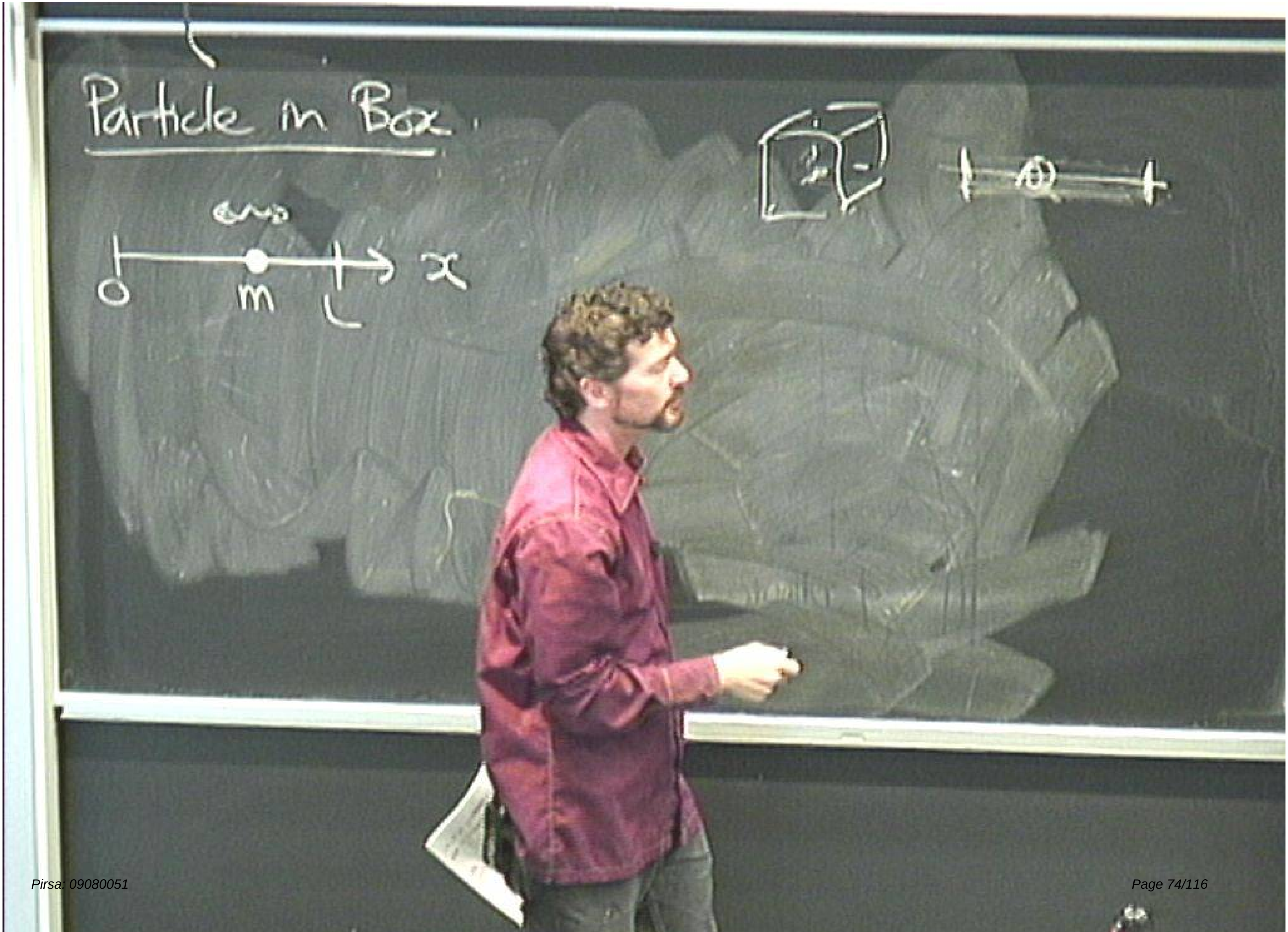
$$\boxed{\lambda = \frac{h}{p}}$$ ← Planck's const.

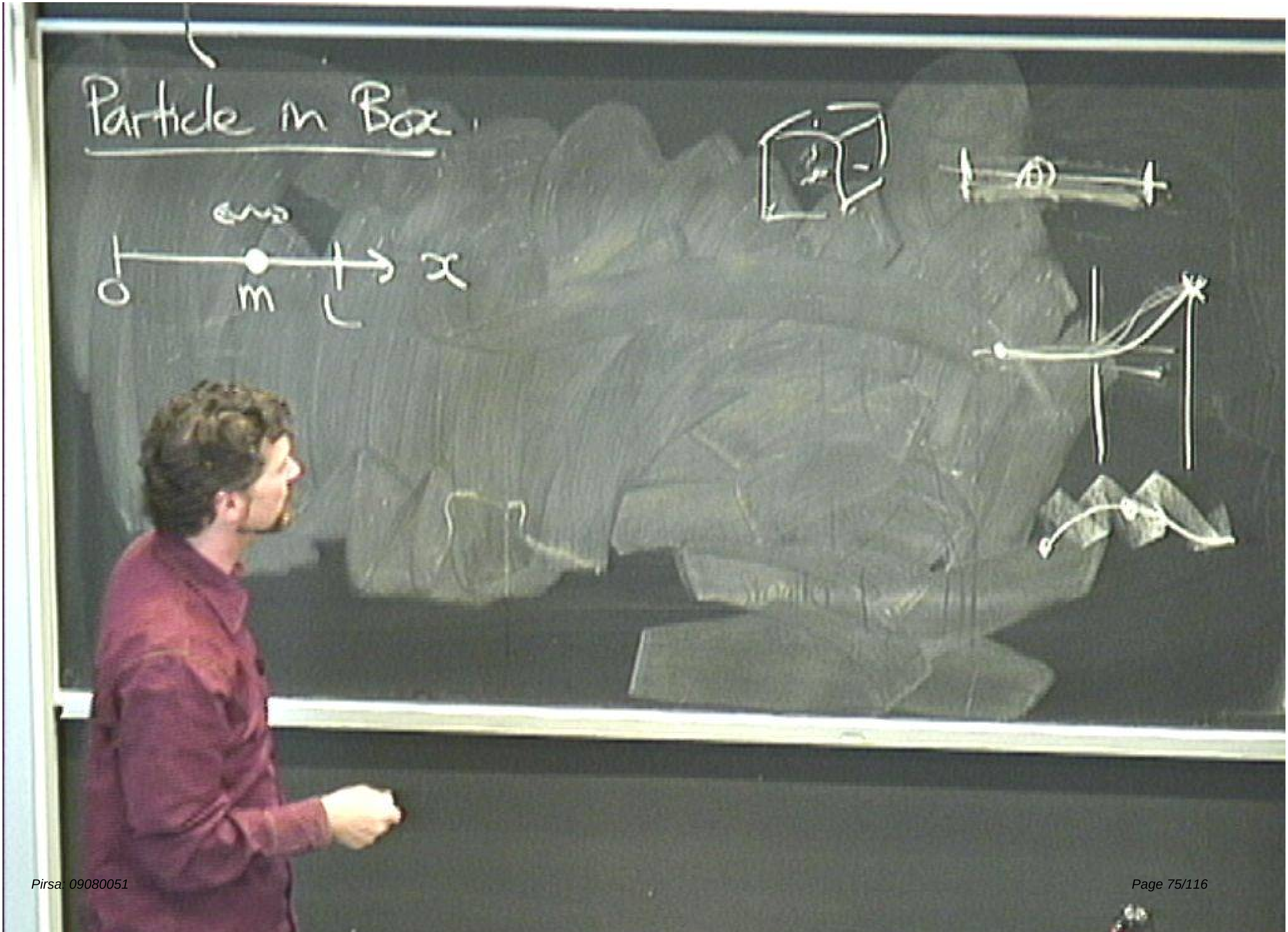
de Broglie relation

for $\psi^2(x) = P(x)$

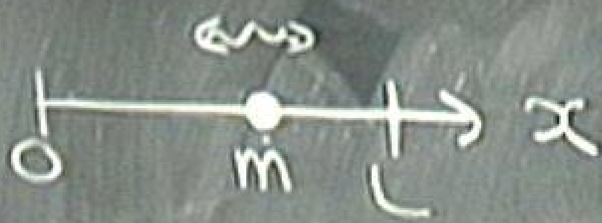
6.0×10^{-34} J·s.


Particle in Box.
x

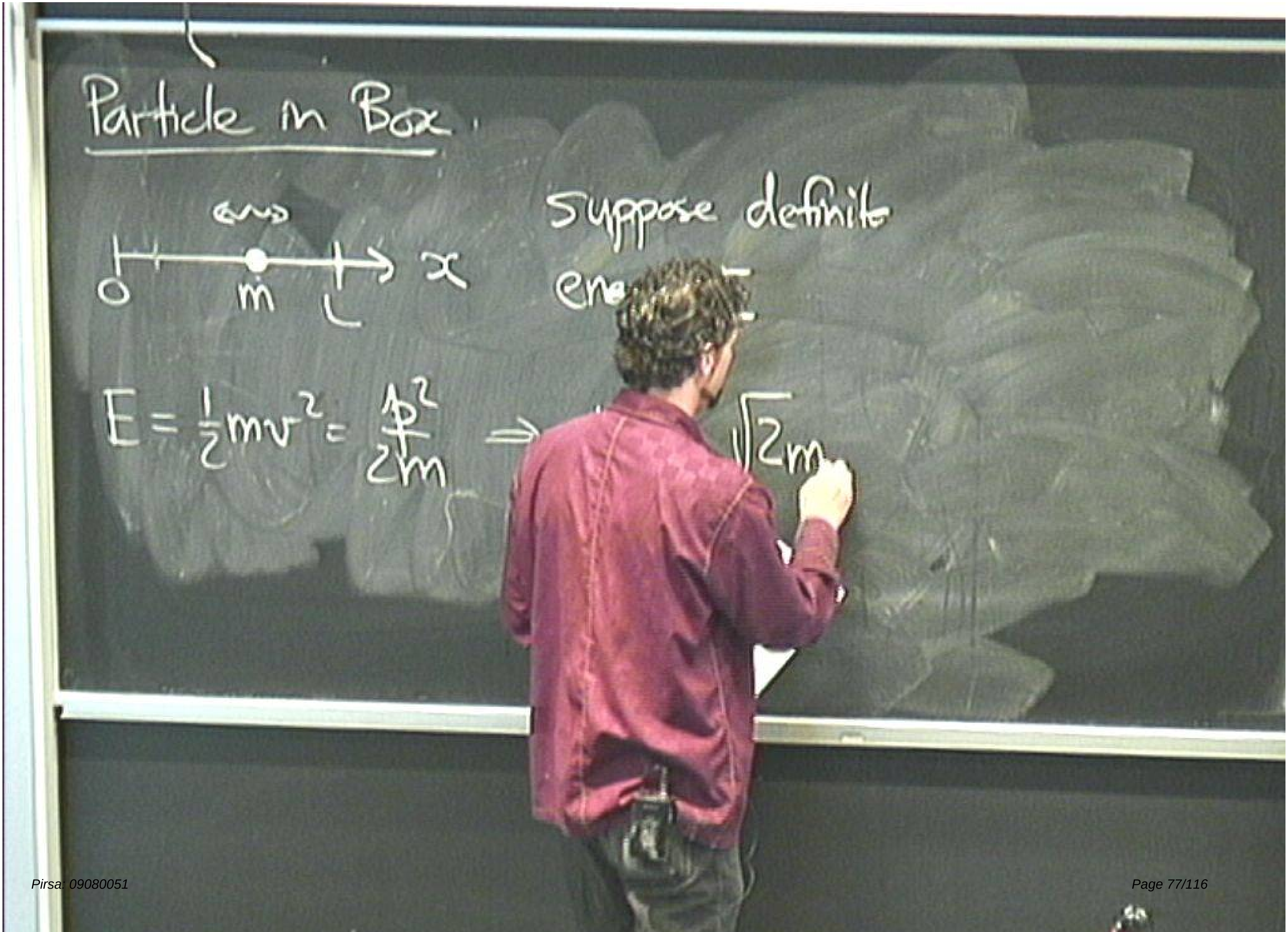

Particle in Box.
0
m
L
x


Particle in Box
0
m
L
x

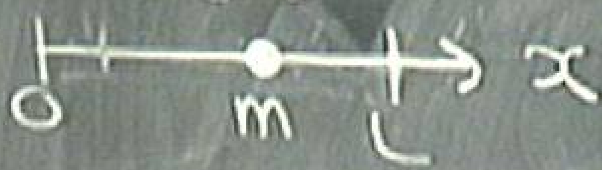
Particle in Box.



Suppose definite energy E


Particle in Box
ends
0
m
L
x
suppose definite
$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \Rightarrow$
$\sqrt{2m}$

Particle in Box



Suppose definite energy E

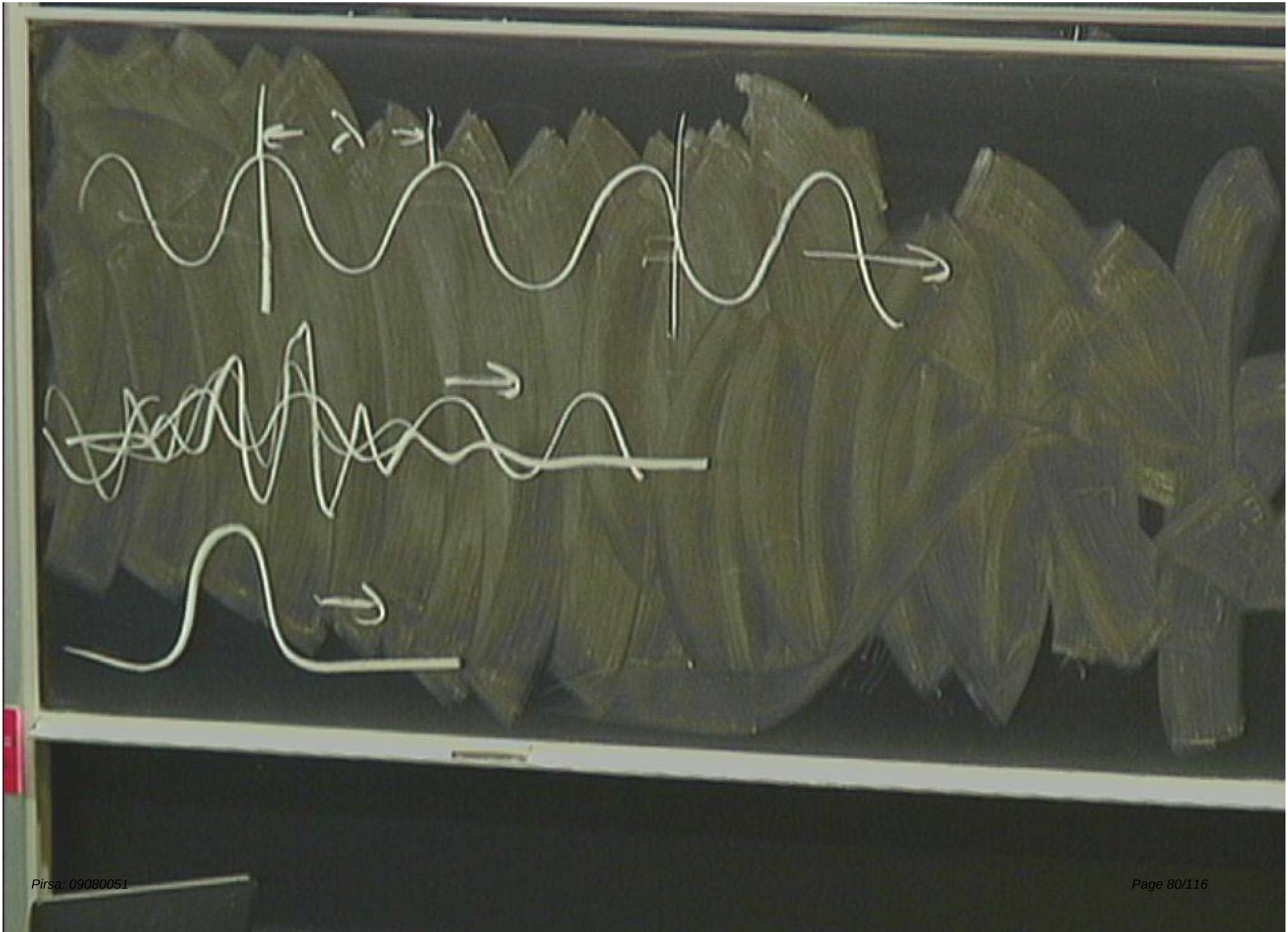
$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \Rightarrow p = \pm\sqrt{2mE} \quad \text{(right/left)}$$

Particle in Box

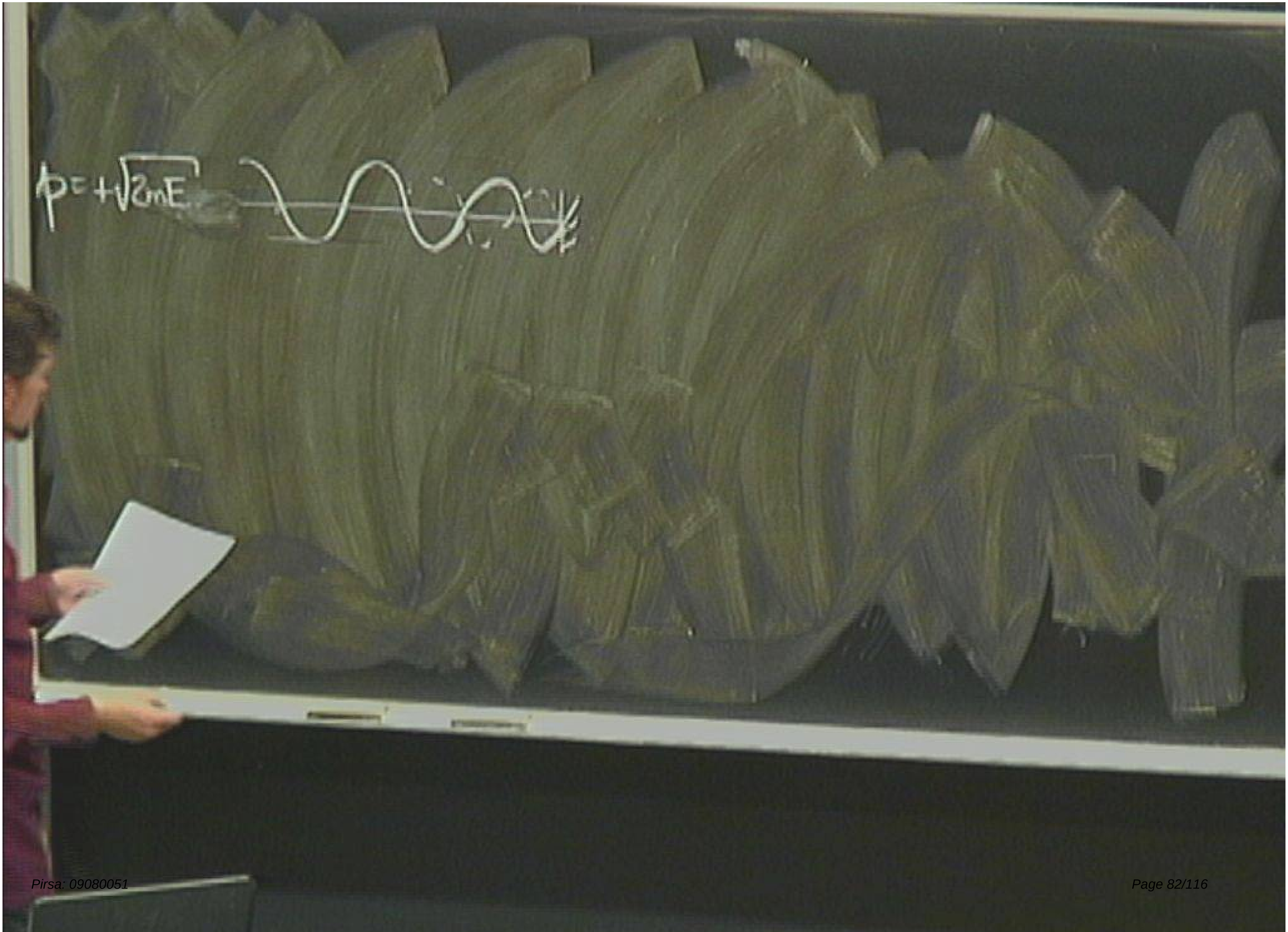
Suppose definite energy E

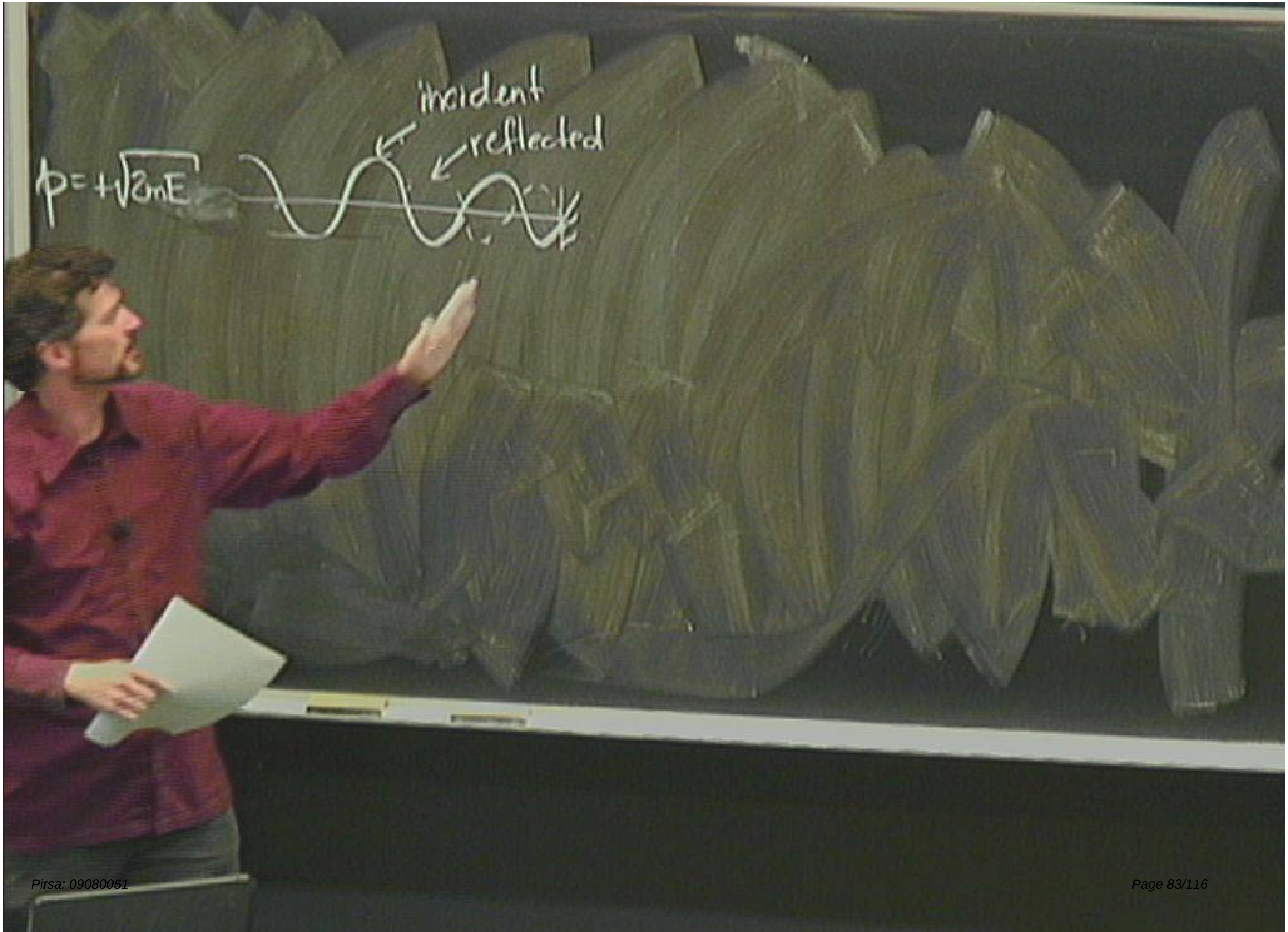
$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \Rightarrow p = \pm\sqrt{2mE} \quad \text{(right/left)}$$

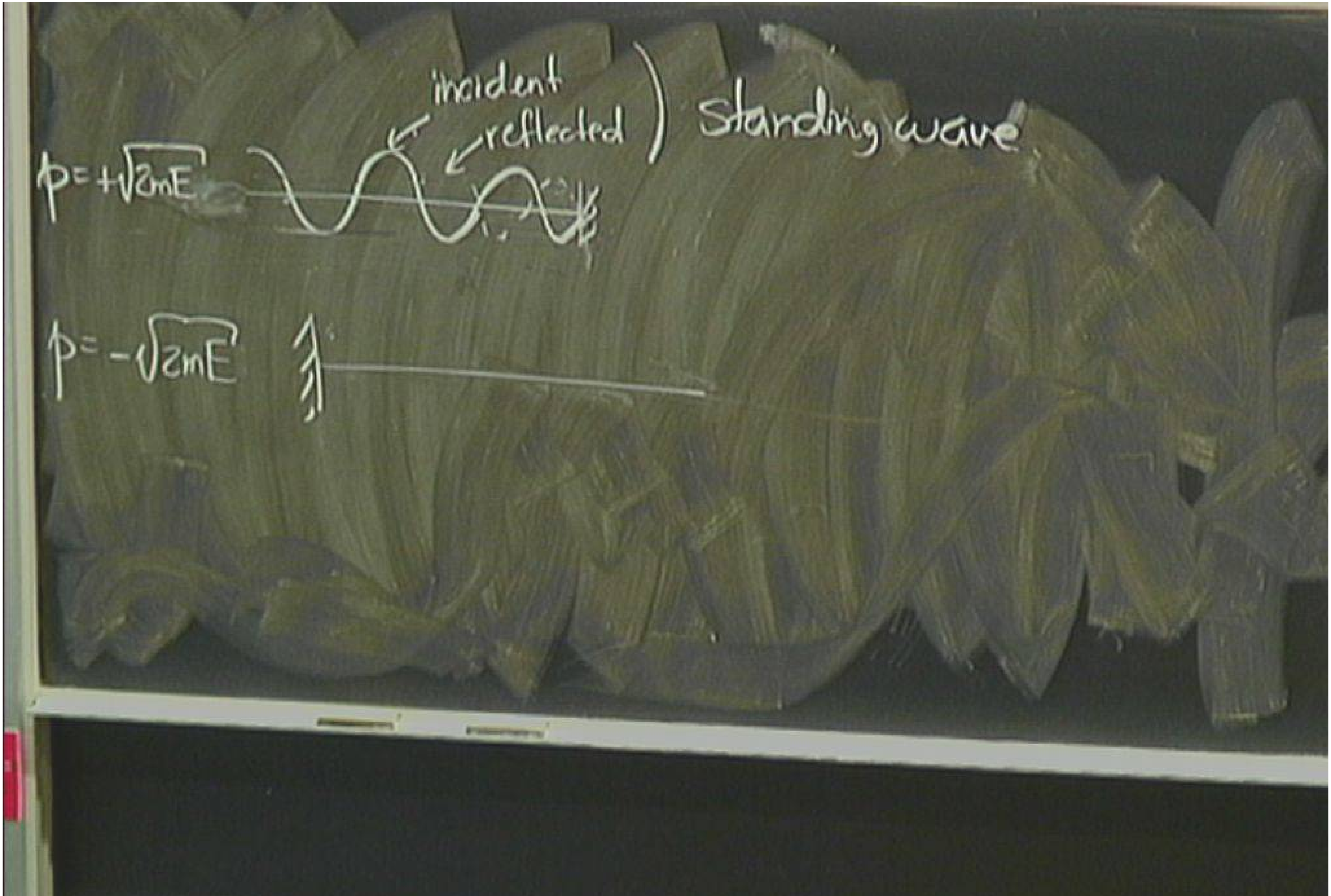
$$\lambda = \frac{h}{\sqrt{2mE}}$$

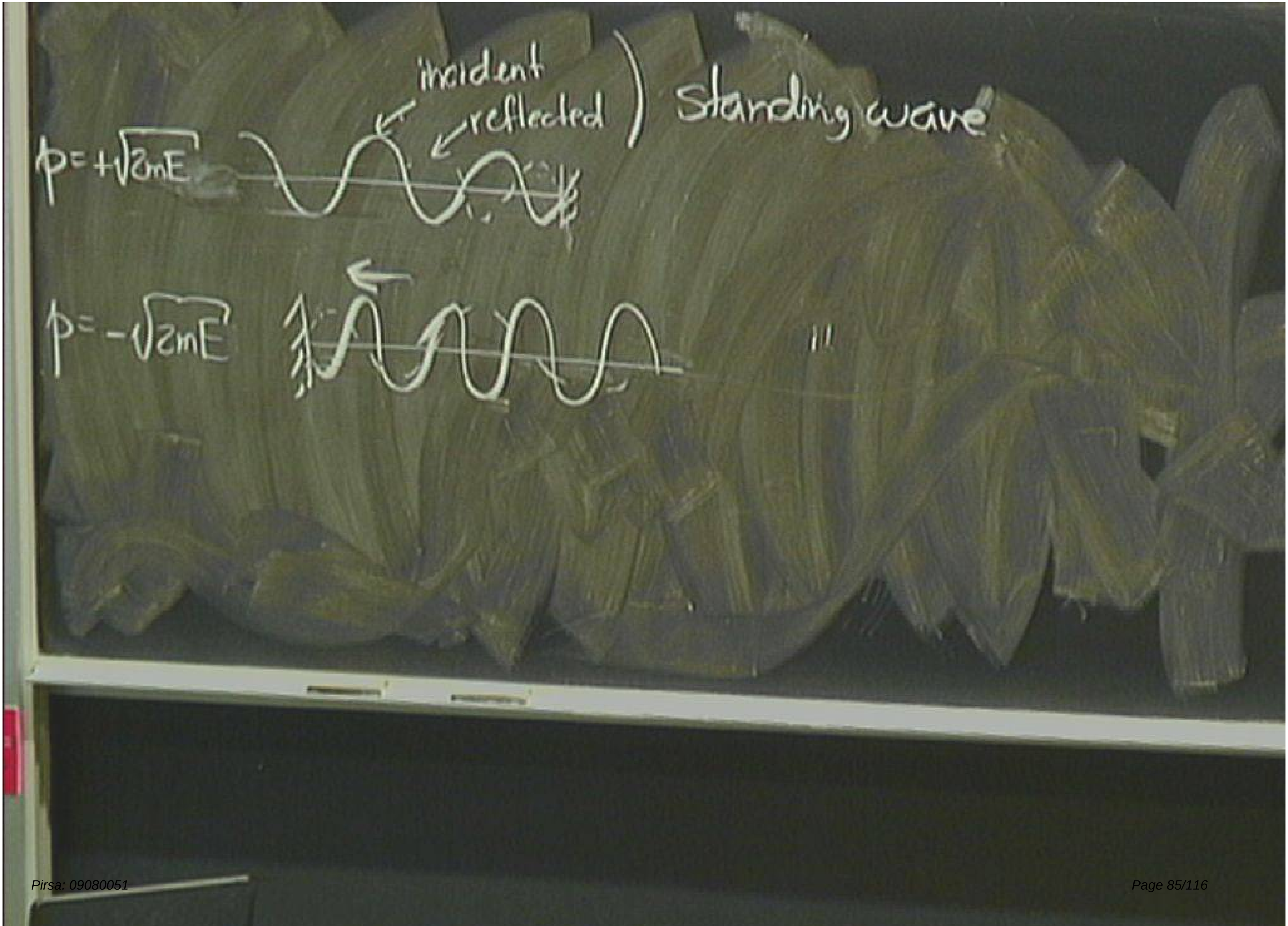

λ


p= +√2mE

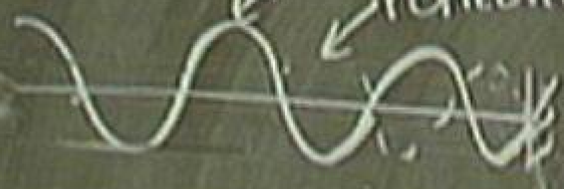

$p = +\sqrt{2mE}$


incident
reflected
$p = +\sqrt{2mE}$


incident
reflected
standing wave
$p = +\sqrt{2mE}$
$p = -\sqrt{2mE}$


incident
reflected
standing wave
p = +√2mE
p = −√2mE
"

$p = +\sqrt{2mE}$

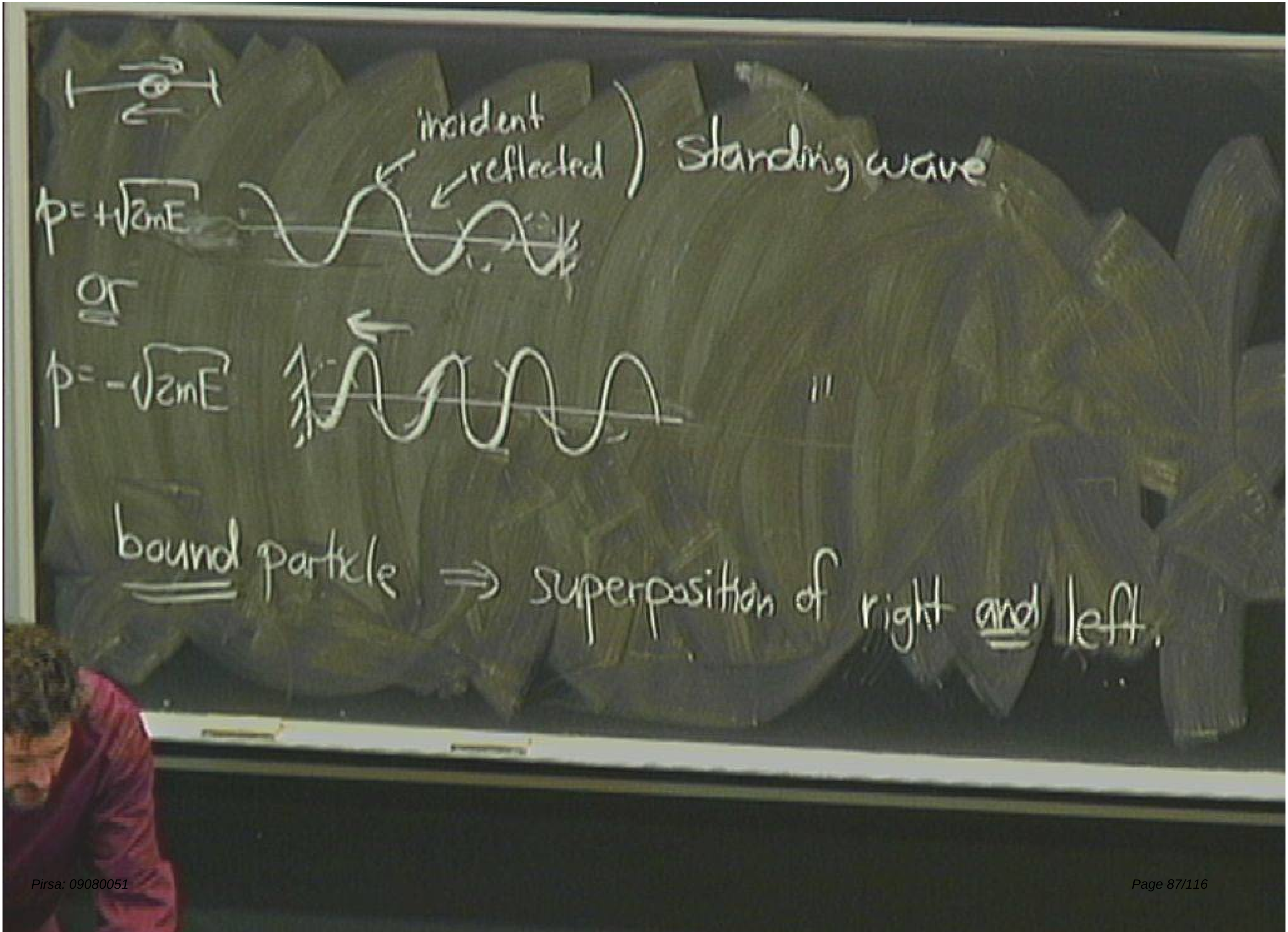


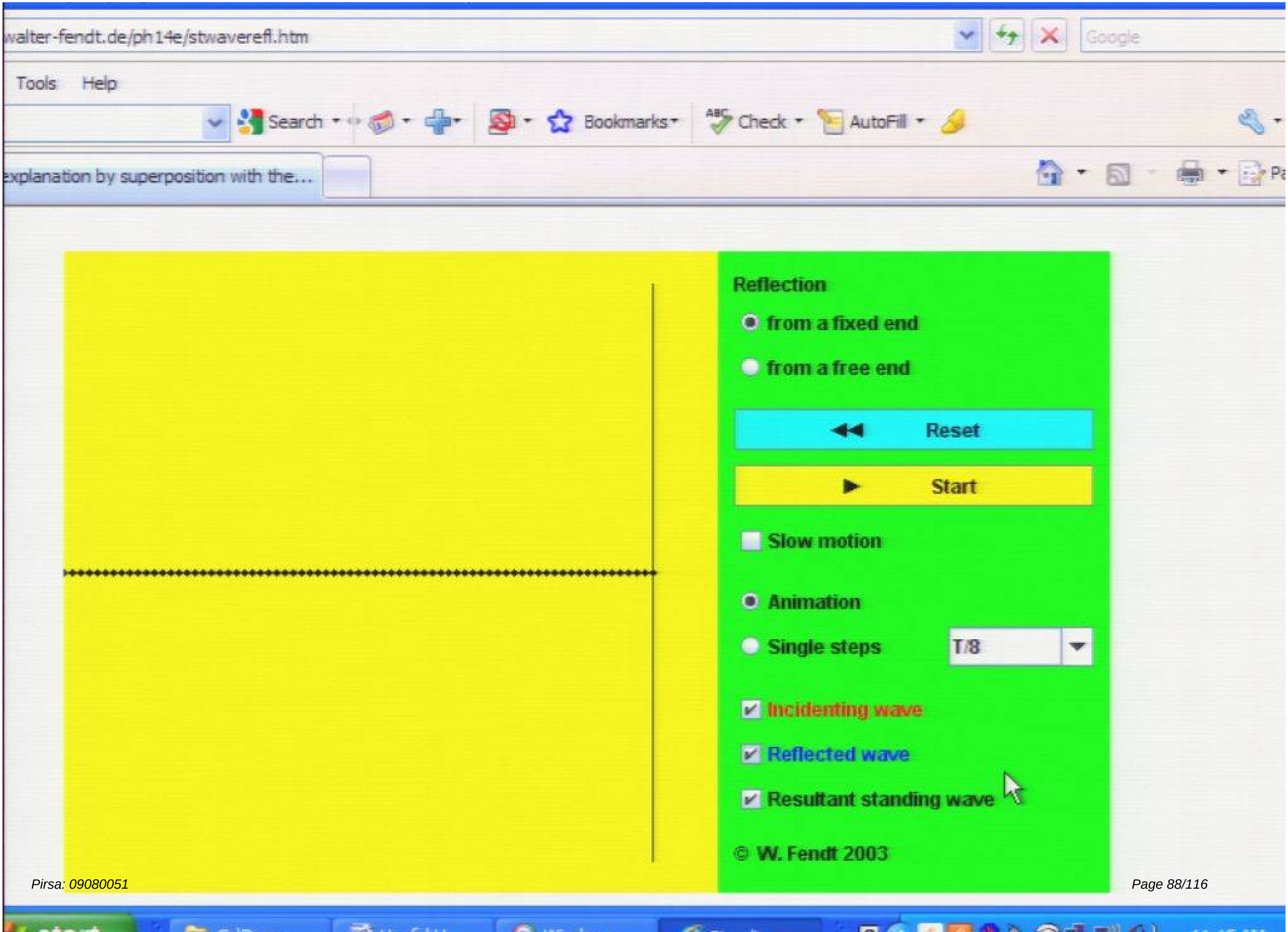
standing wave

$p = -\sqrt{2mE}$

"

bound particle $\Rightarrow$ superposition of right **and** left.


incident
reflected
standing wave
$p = +\sqrt{2mE}$
or
$p = -\sqrt{2mE}$
"
bound particle $\Rightarrow$ superposition of right and left.


walter-fendt.de/ph14e/stwaverefl.htm
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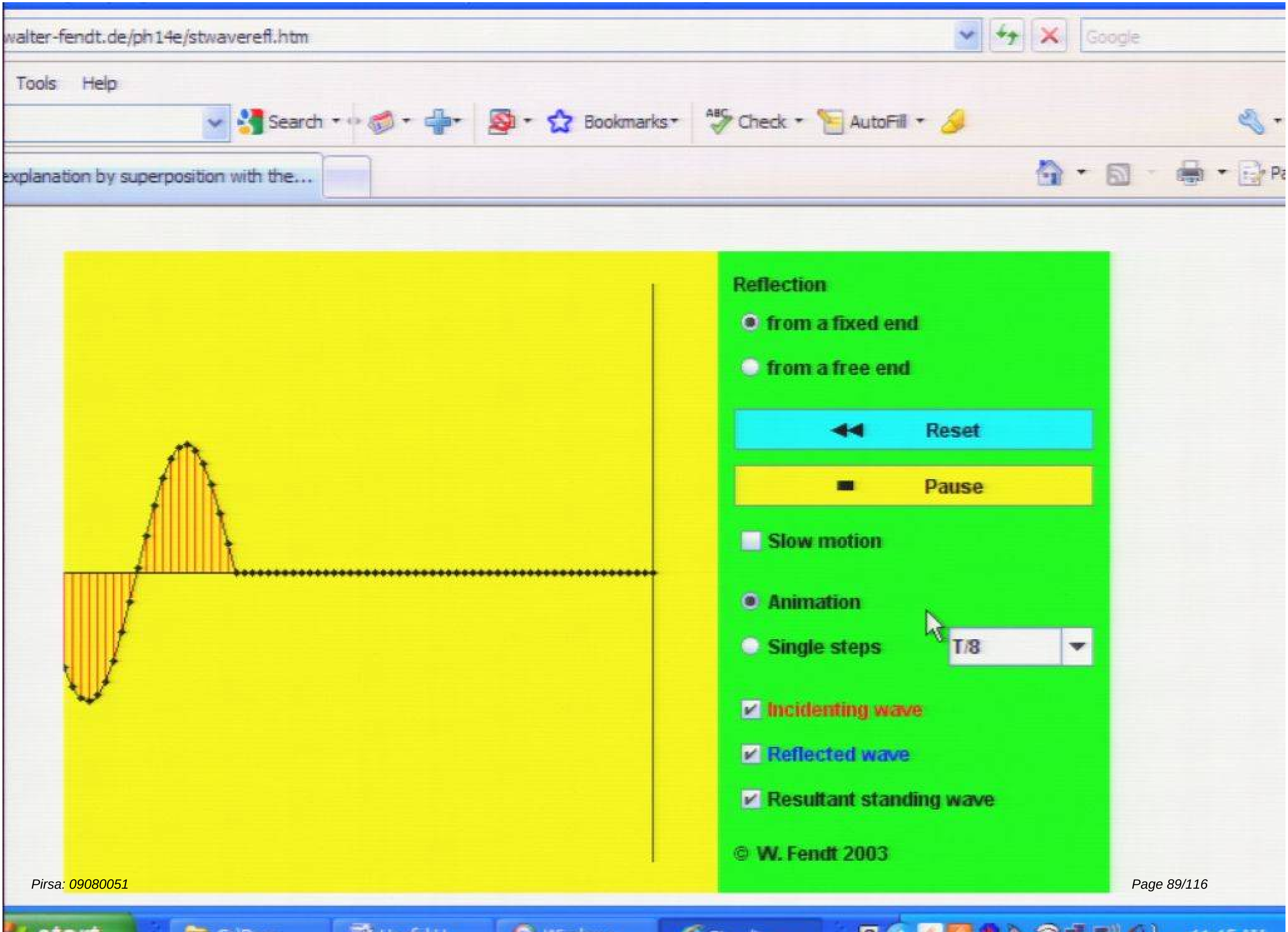
Check
AutoFill
explanation by superposition with the...
Reflection
from a fixed end
from a free end
Reset
Start
Slow motion
Animation
Single steps
T/8
Incidenting wave
Reflected wave
Resultant standing wave
© W. Fendt 2003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explanation by superposition with the...
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from a fixed end
from a free end
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Pause
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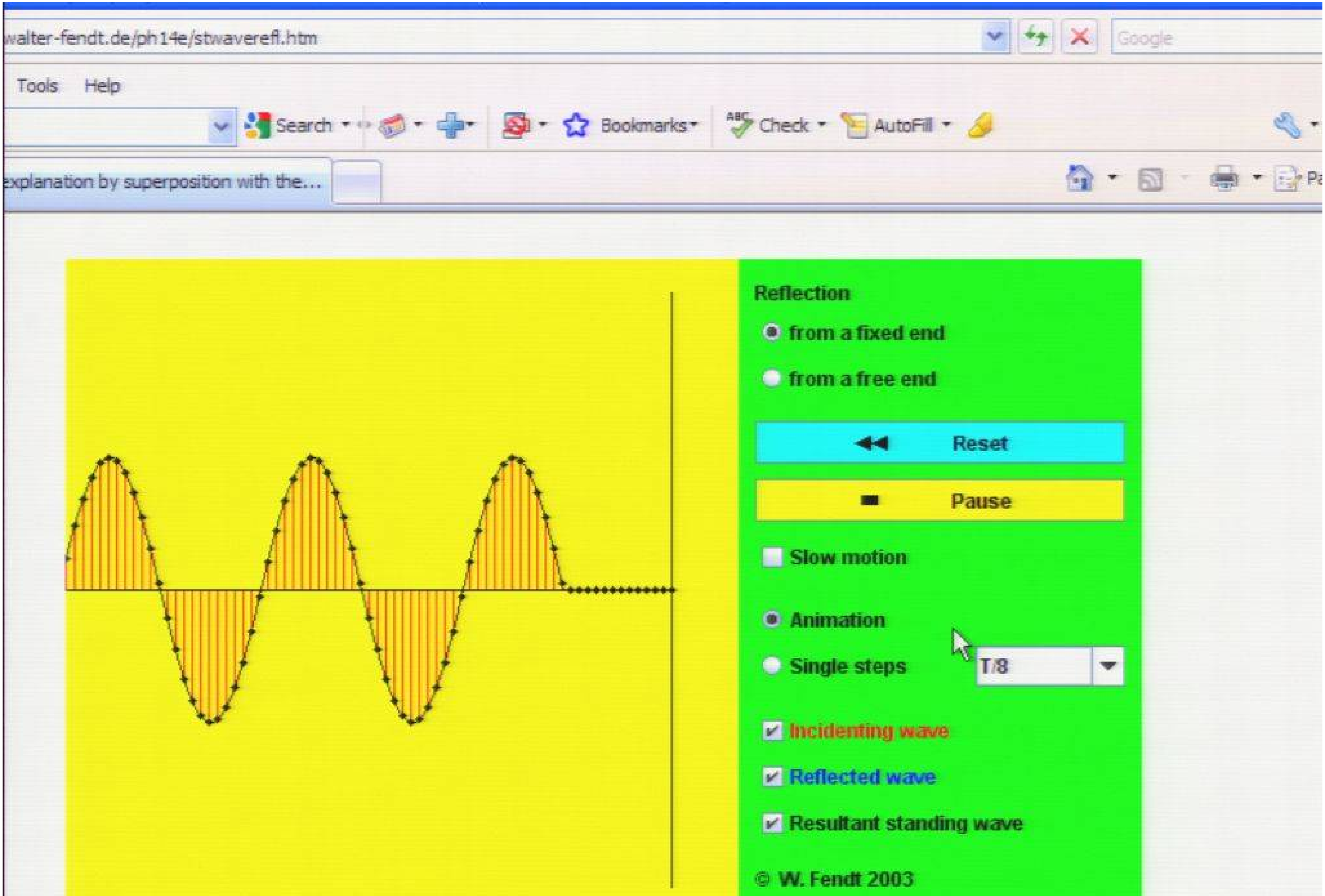
Reflected wave
Resultant standing wave
© W. Fendt 2003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Resultant standing wave
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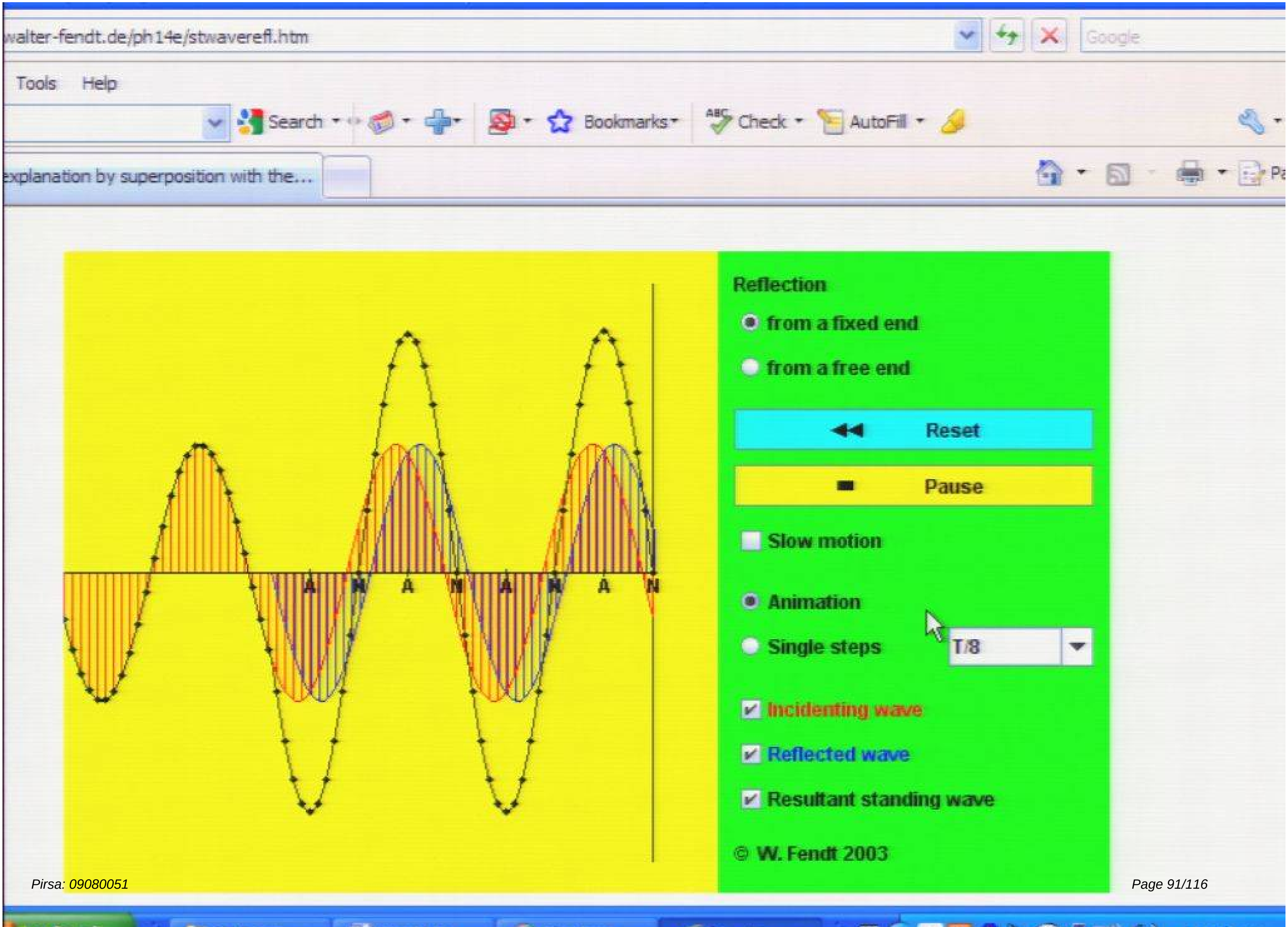
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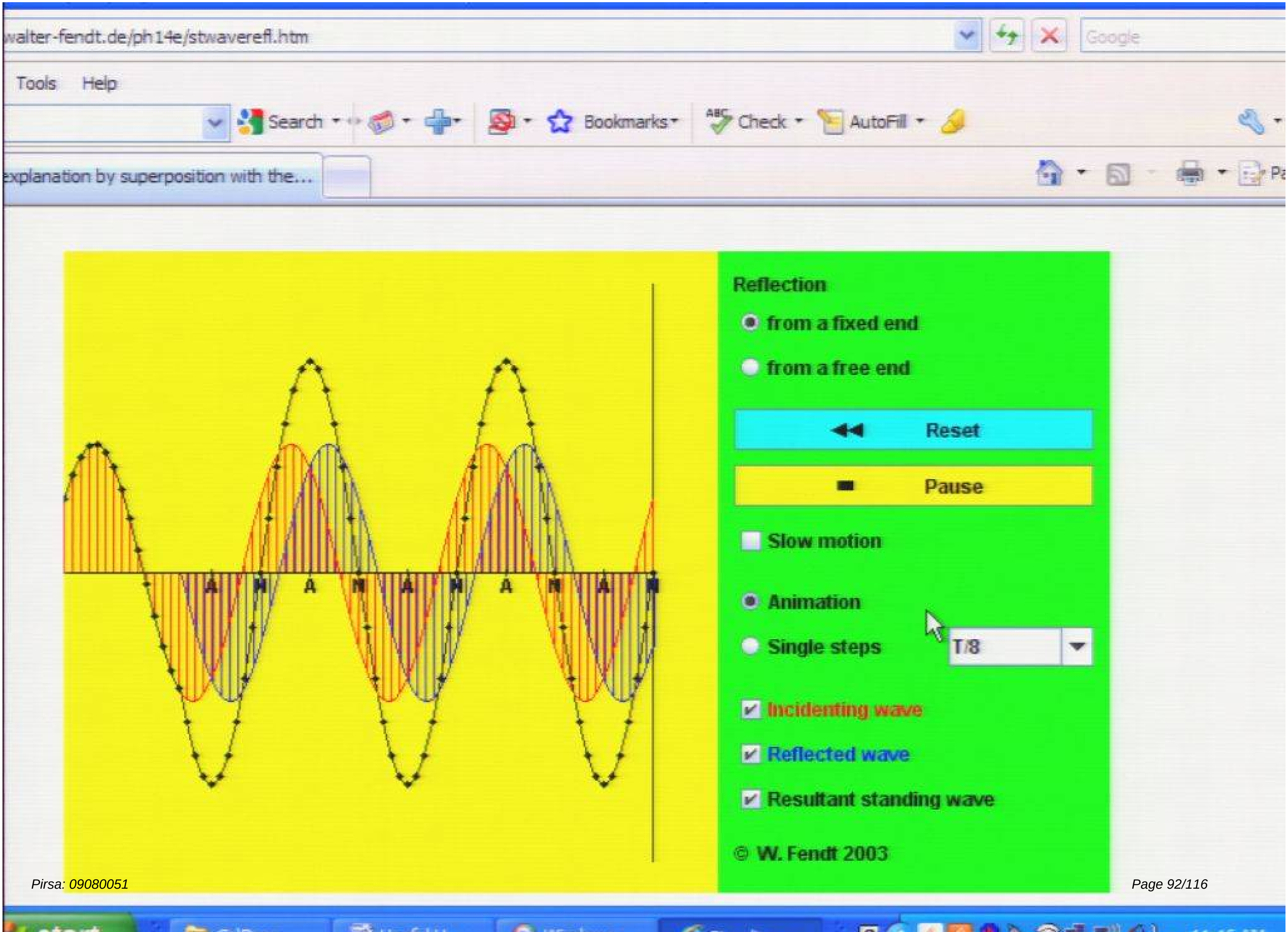
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A N A N A N A N A N
Reflection
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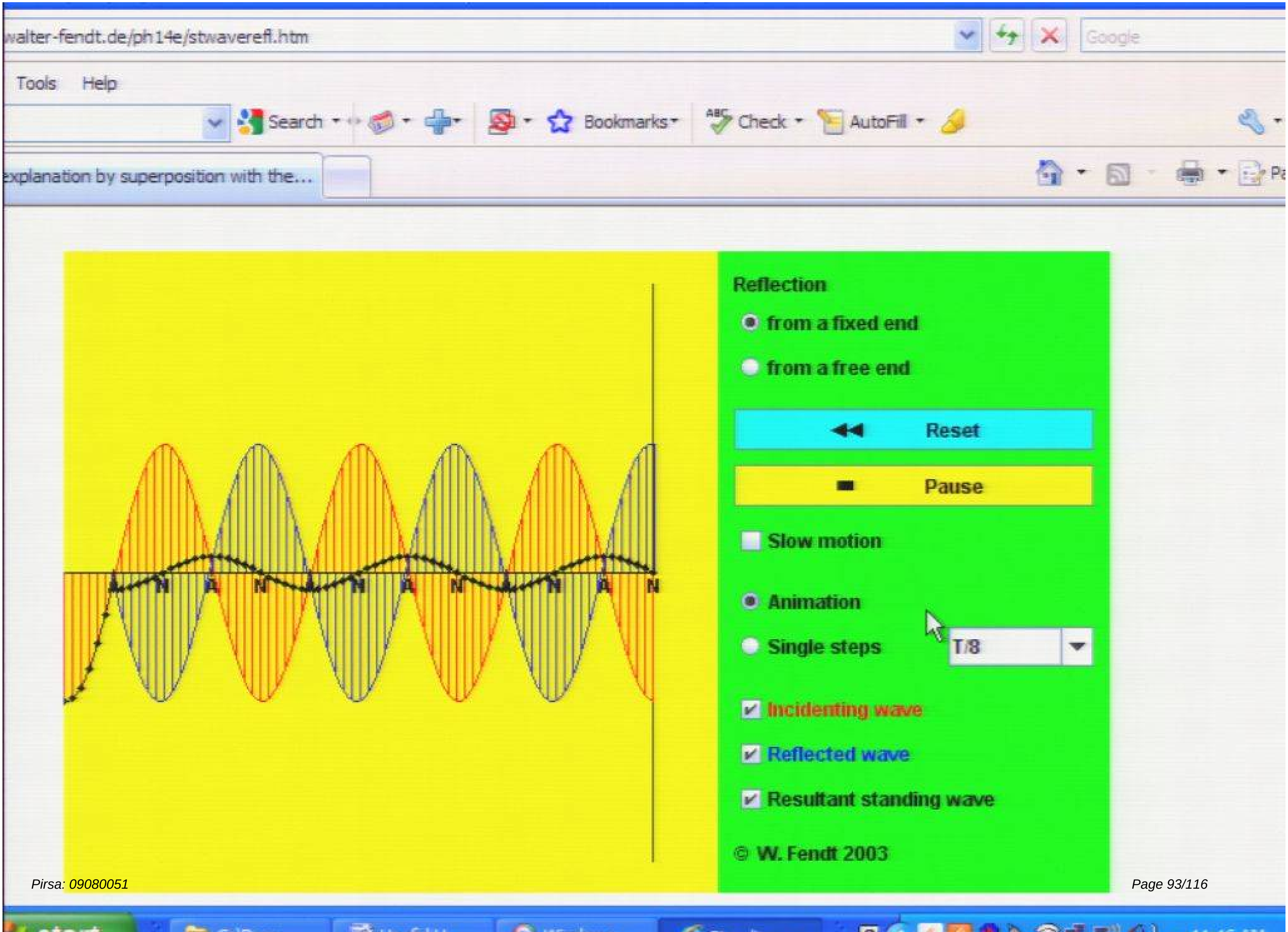
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Incidenting wave
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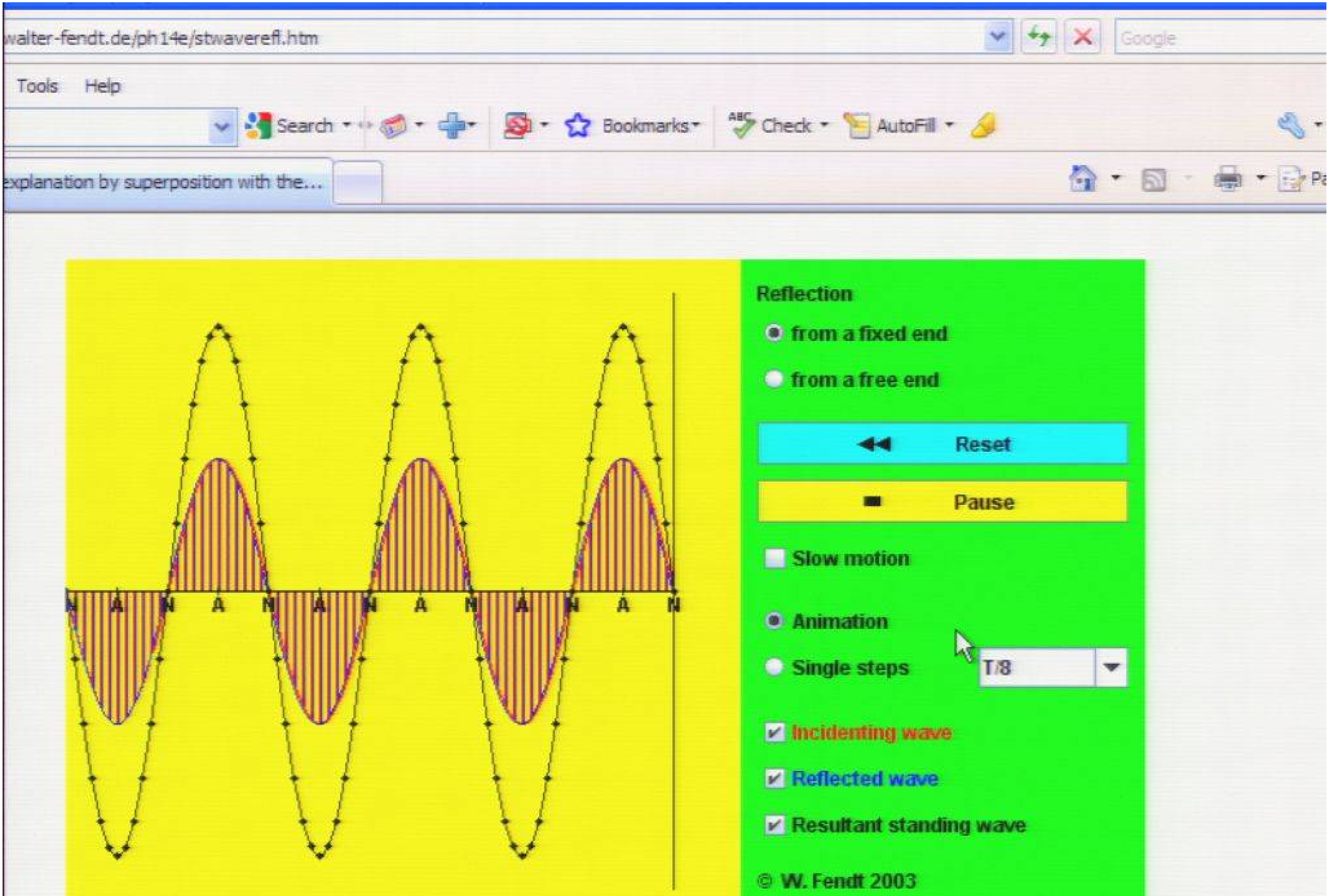
walter-fendt.de/ph14e/stwaverefl.htm
Google
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N A N A N A N A N A N A N
Reflection
from a fixed end
from a free end
Reset
Pause
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Animation
Single steps
T/8
Incidenting wave
Reflected wave
Resultant standing wave
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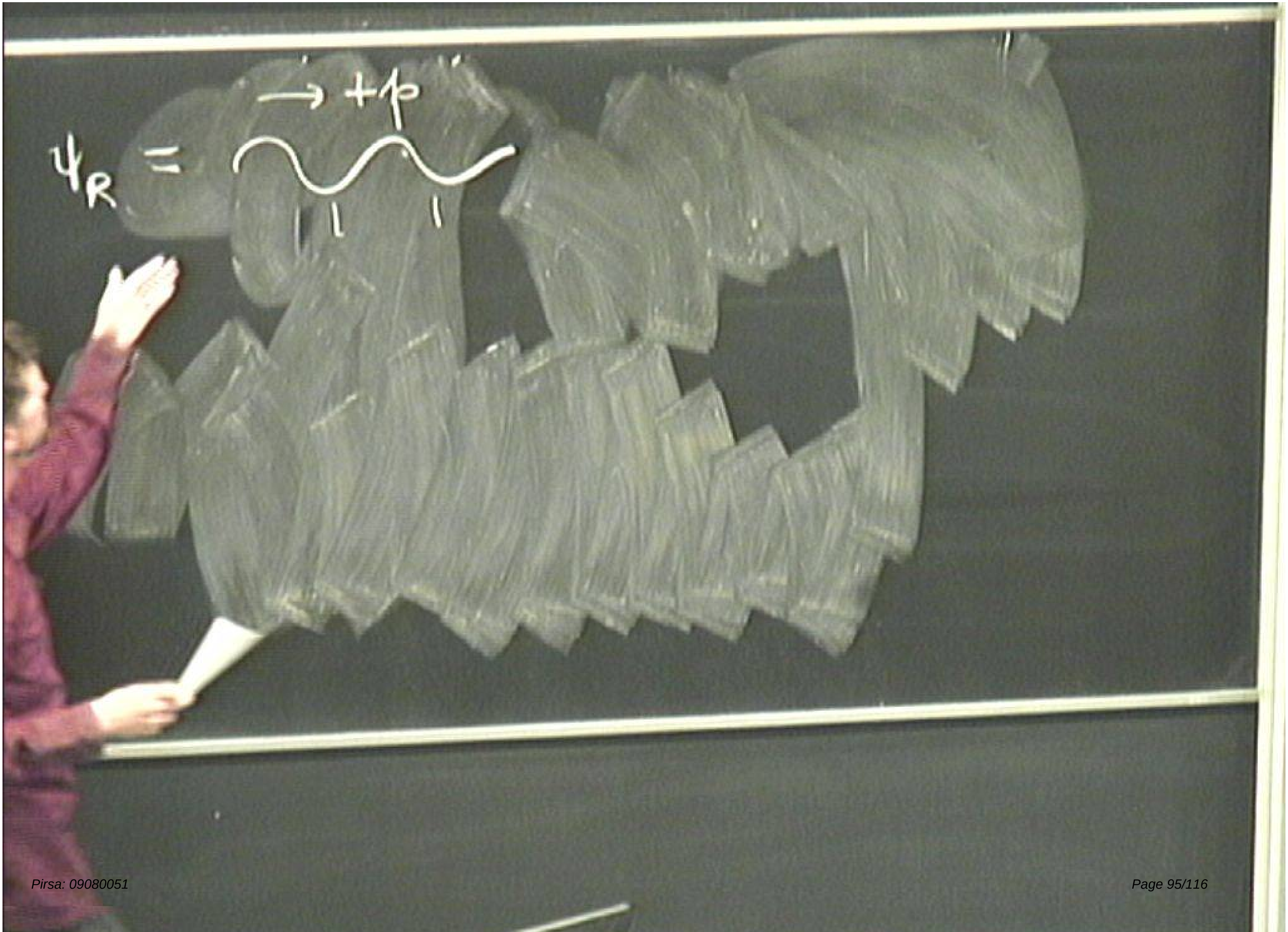
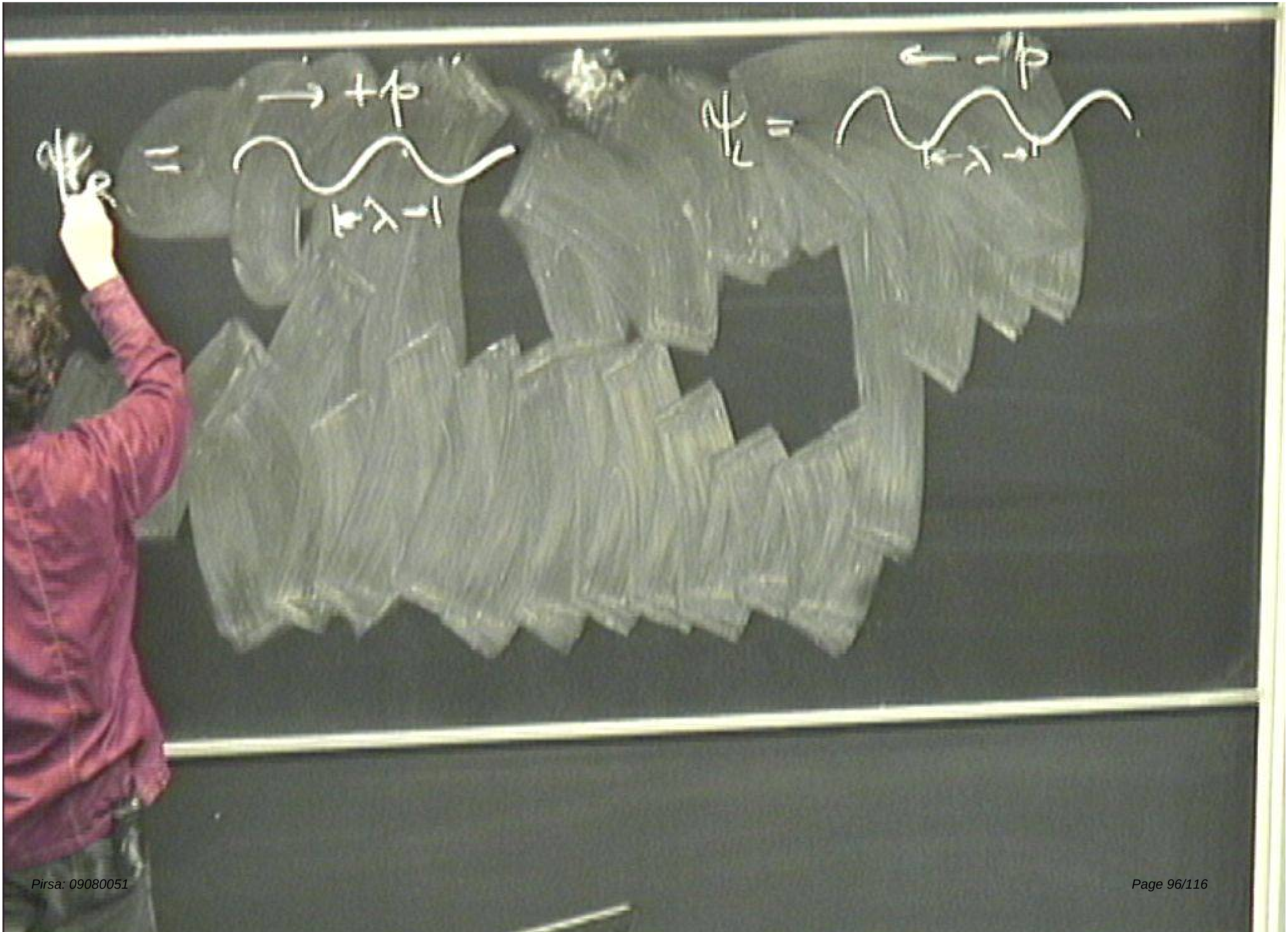
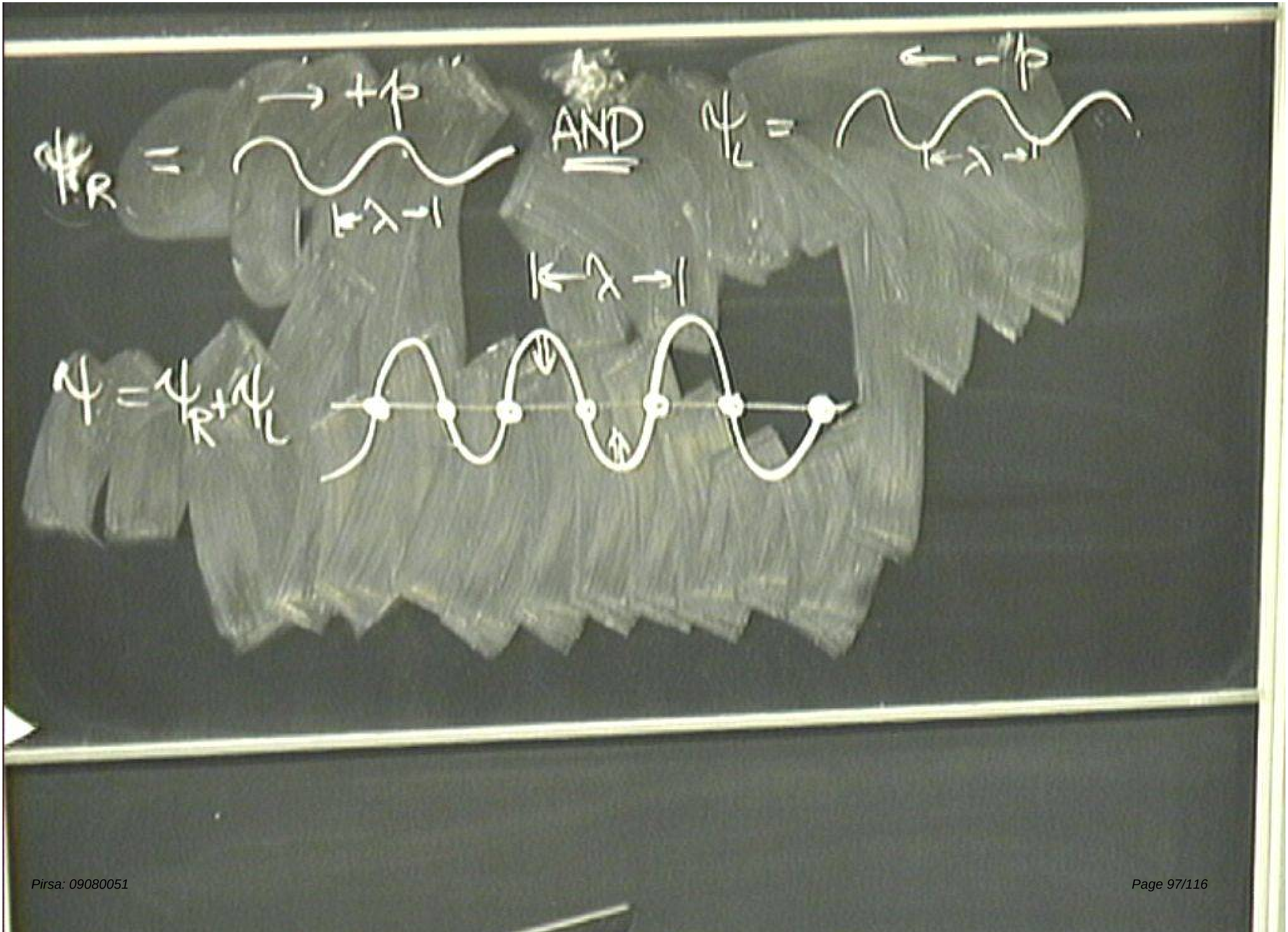
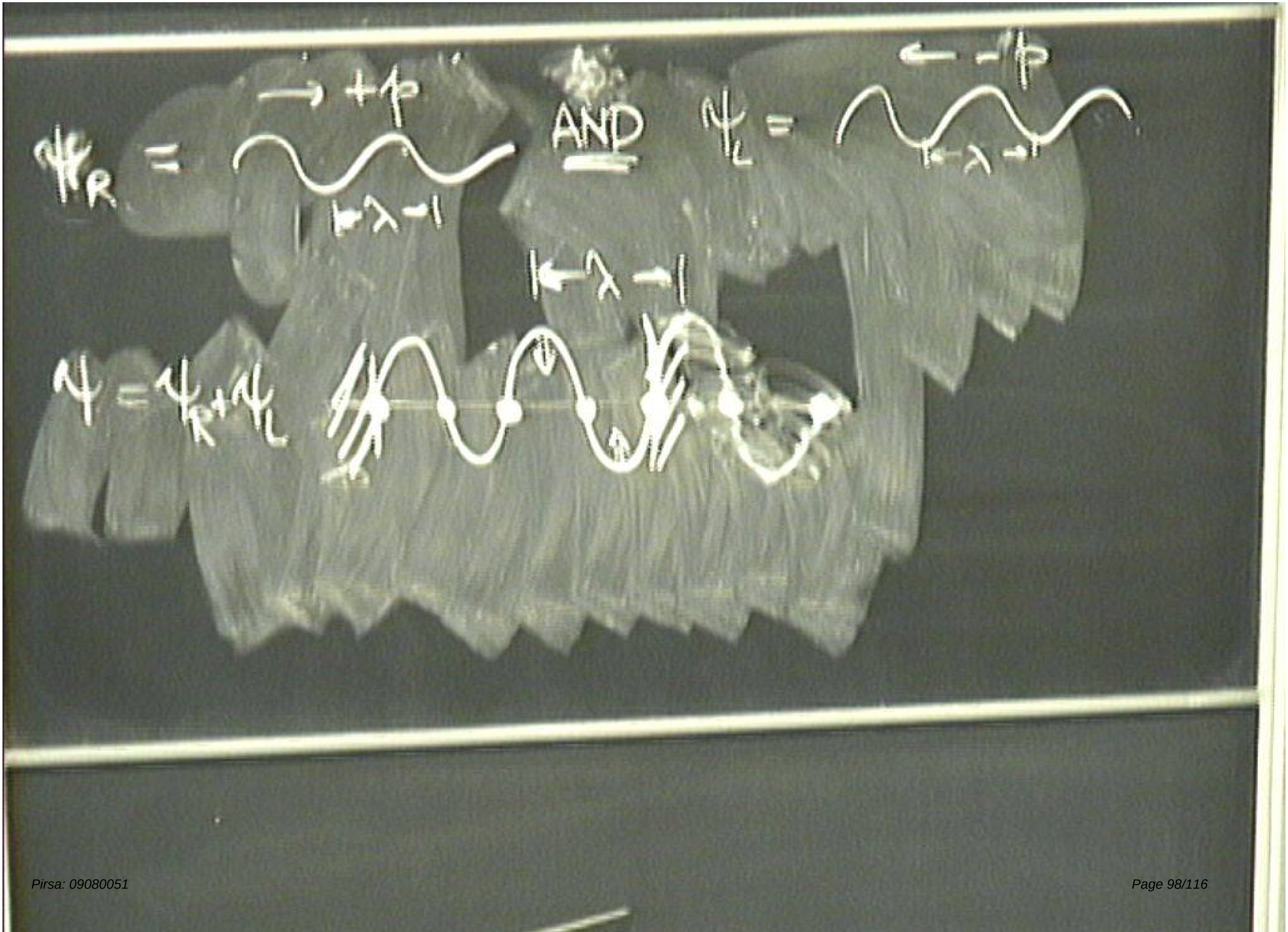
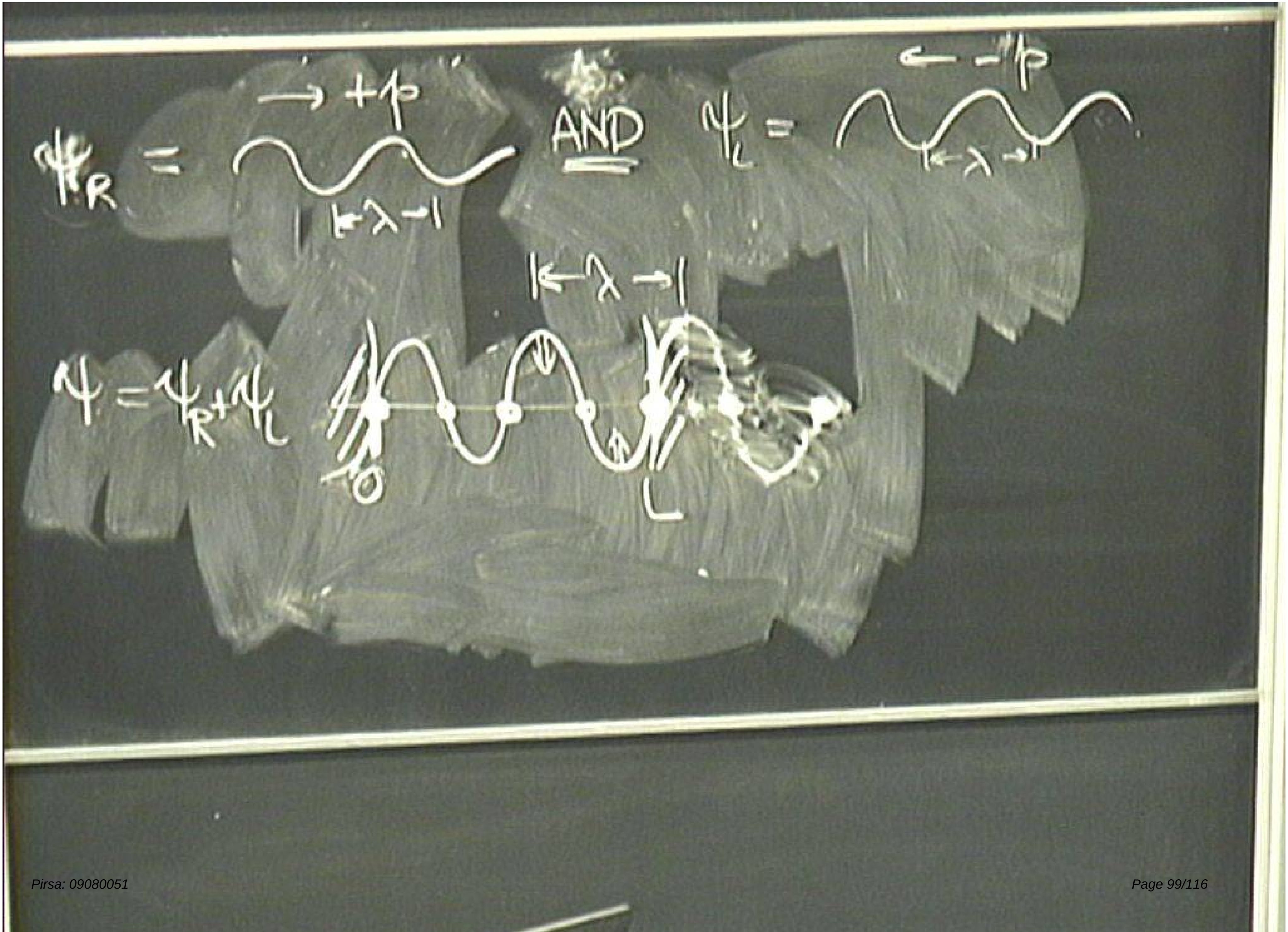

$\psi_R =$
$\rightarrow +p$

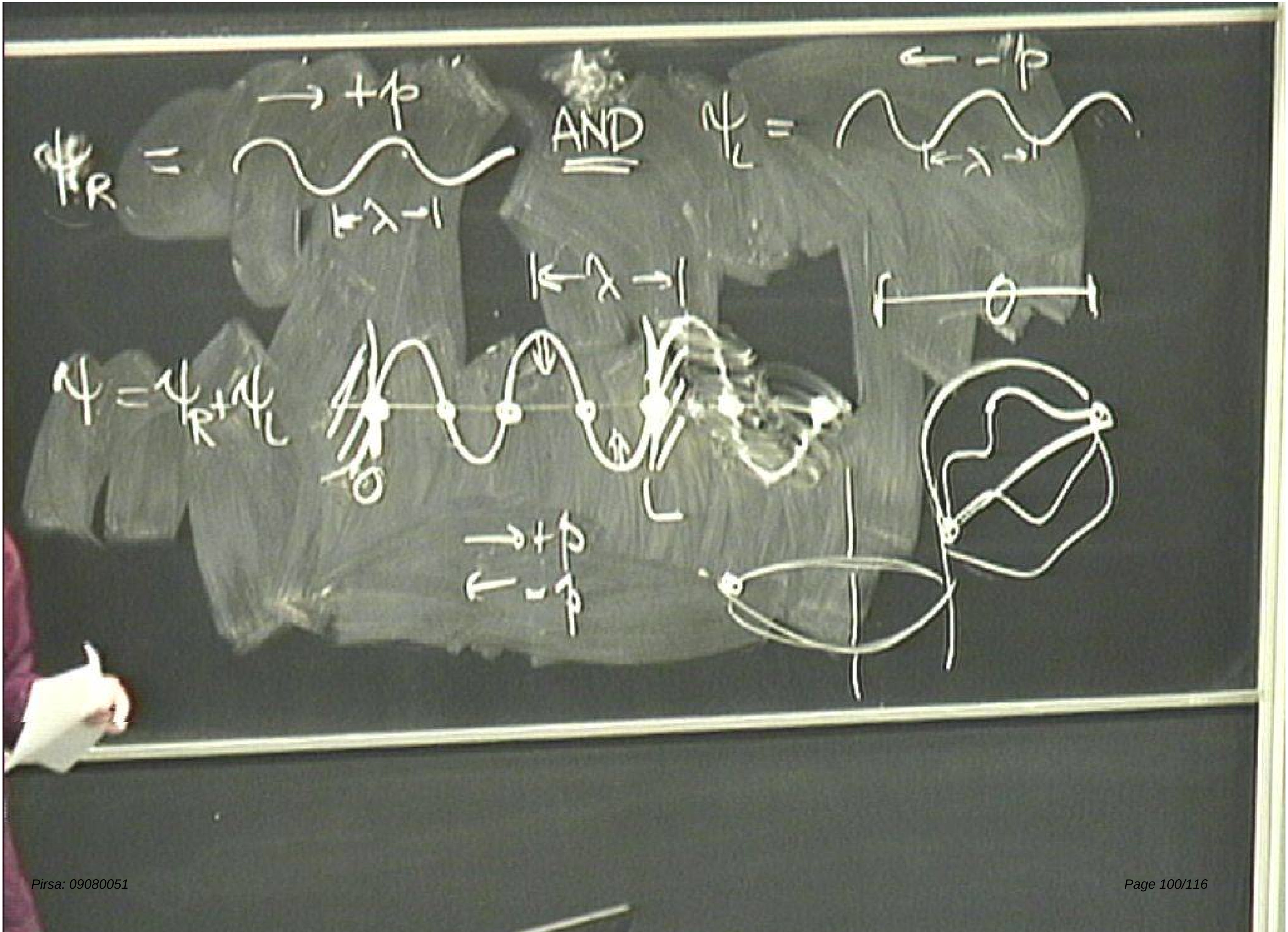

→ +p
ψ_R =
|← λ →|
← −p
ψ_L =
|← λ →|

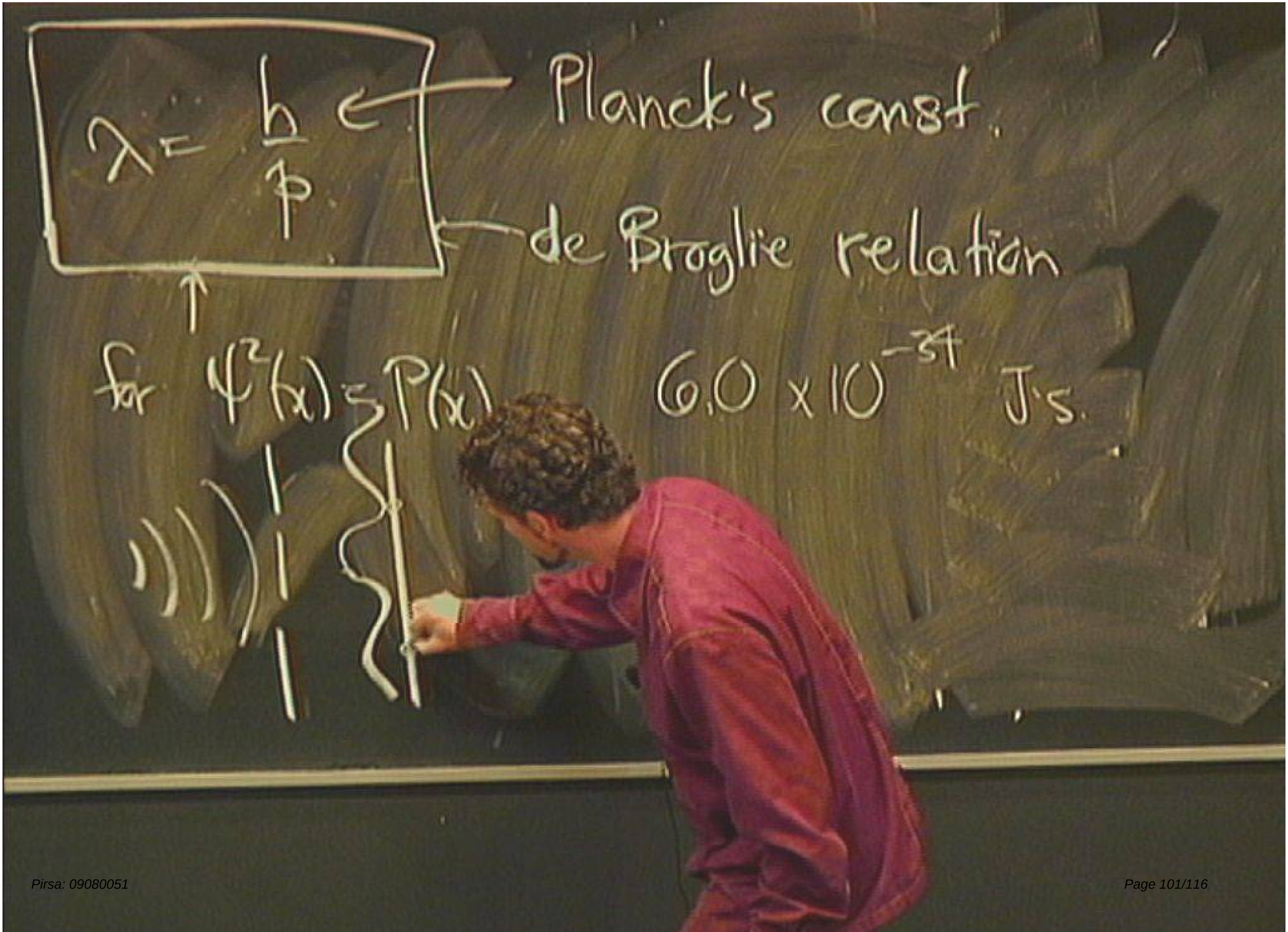


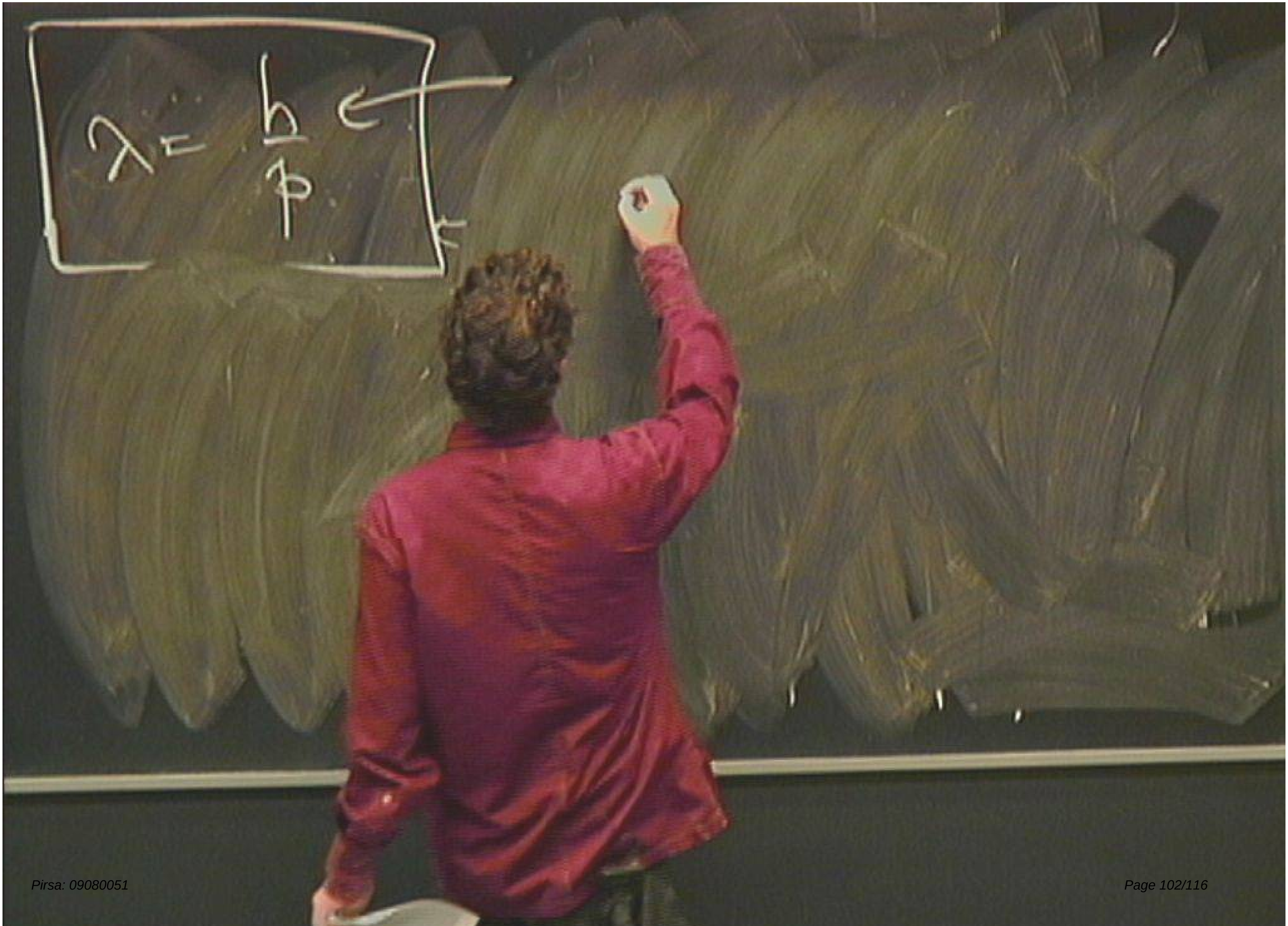
→ +p
ψ_R =
|←λ→|
AND
← −p
ψ_L =
|←λ→|
|←λ→|
ψ = ψ_R + ψ_L

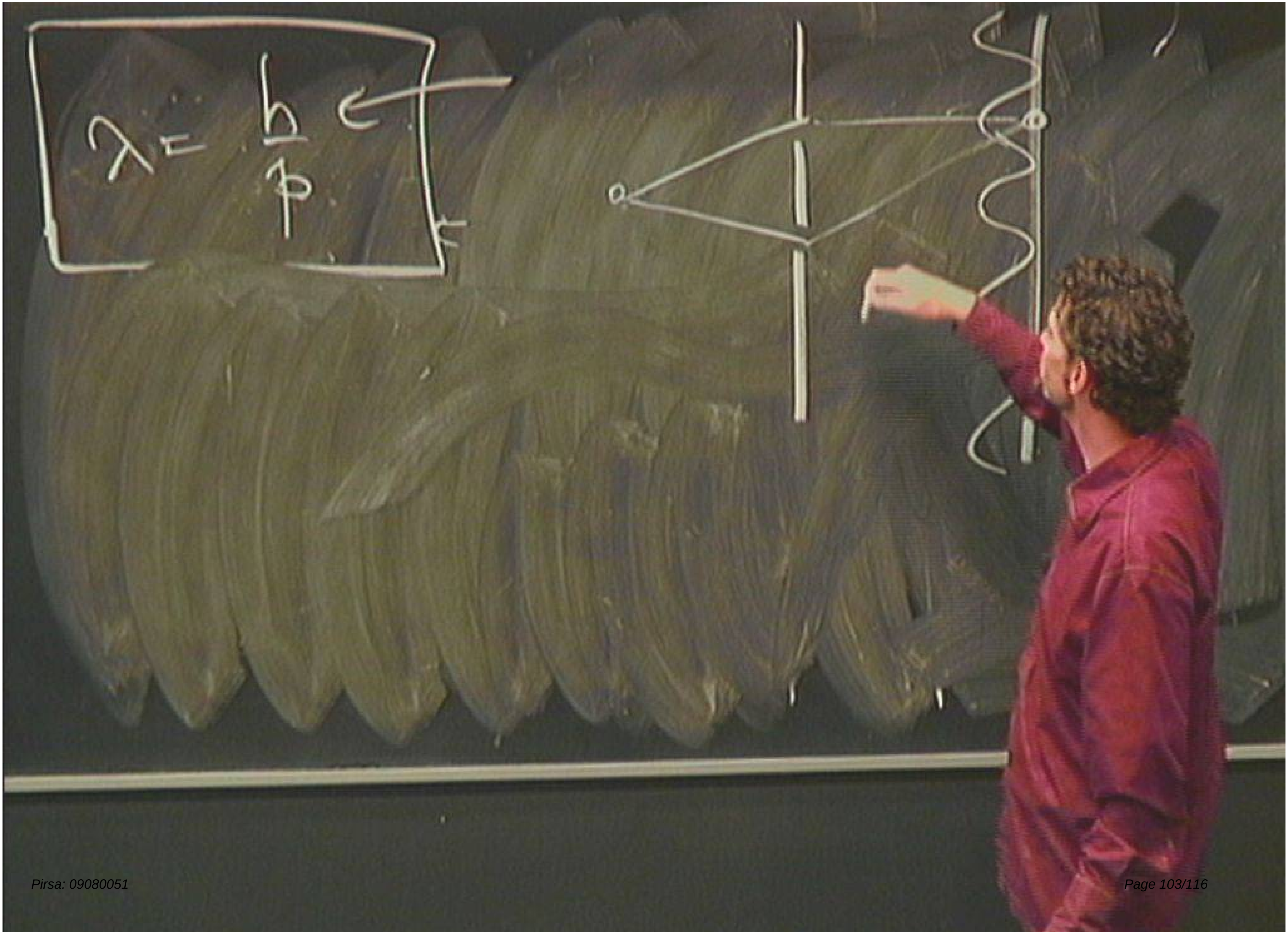

→ +p
ψR =
|←λ→|
AND
ψL =
← −p
|←λ→|
|←λ→|
ψ = ψR + ψL

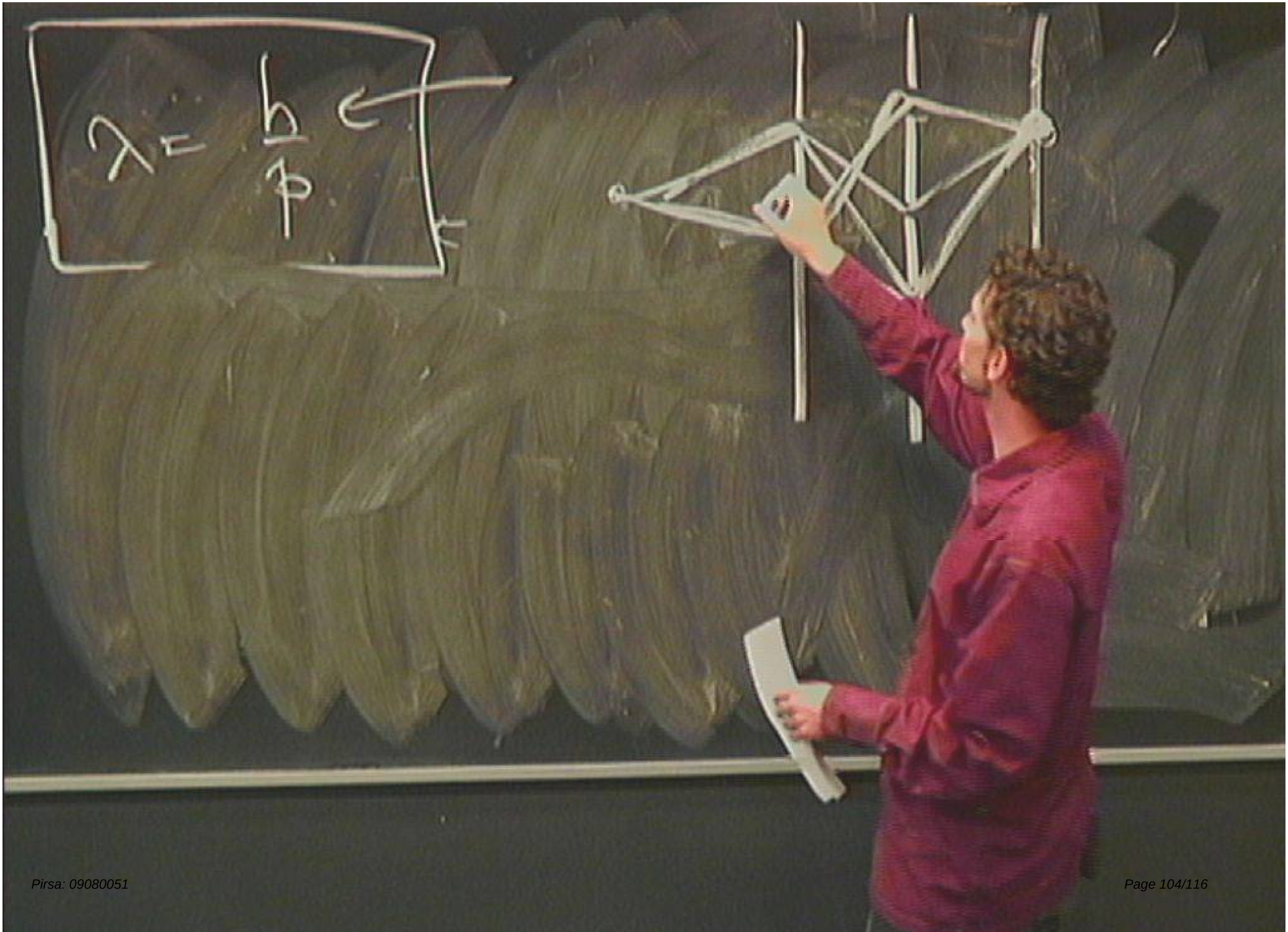

→ +p
ψR =
|←λ→|
AND
ψL =
← −p
|←λ→|
|←λ→|
ψ = ψR + ψL
0
L

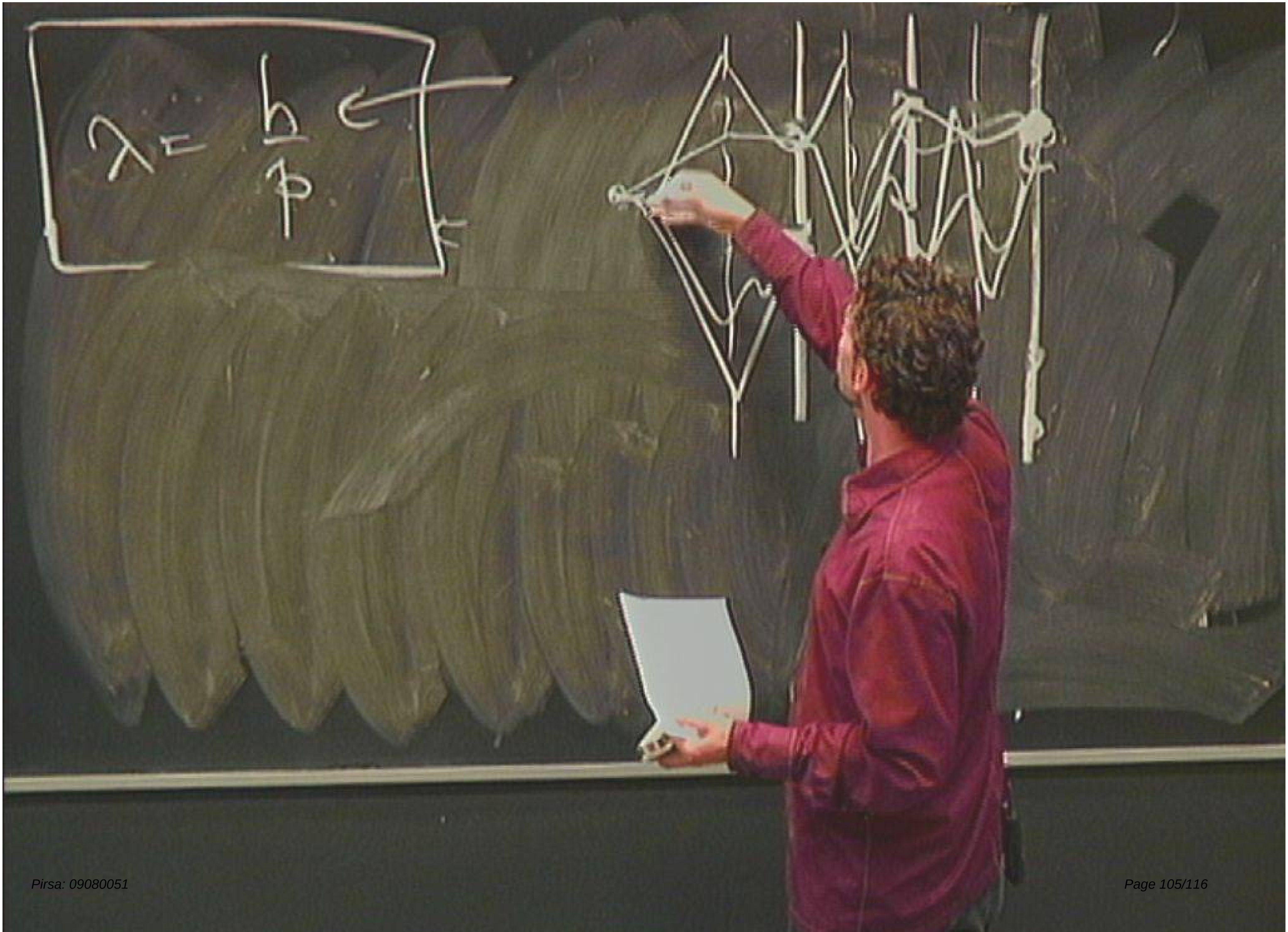

→ +p
ψR =
|← λ →|
AND
← −p
ψL =
|← λ →|
|← λ →|
ψ = ψR + ψL
0
L
→ +p
← −p

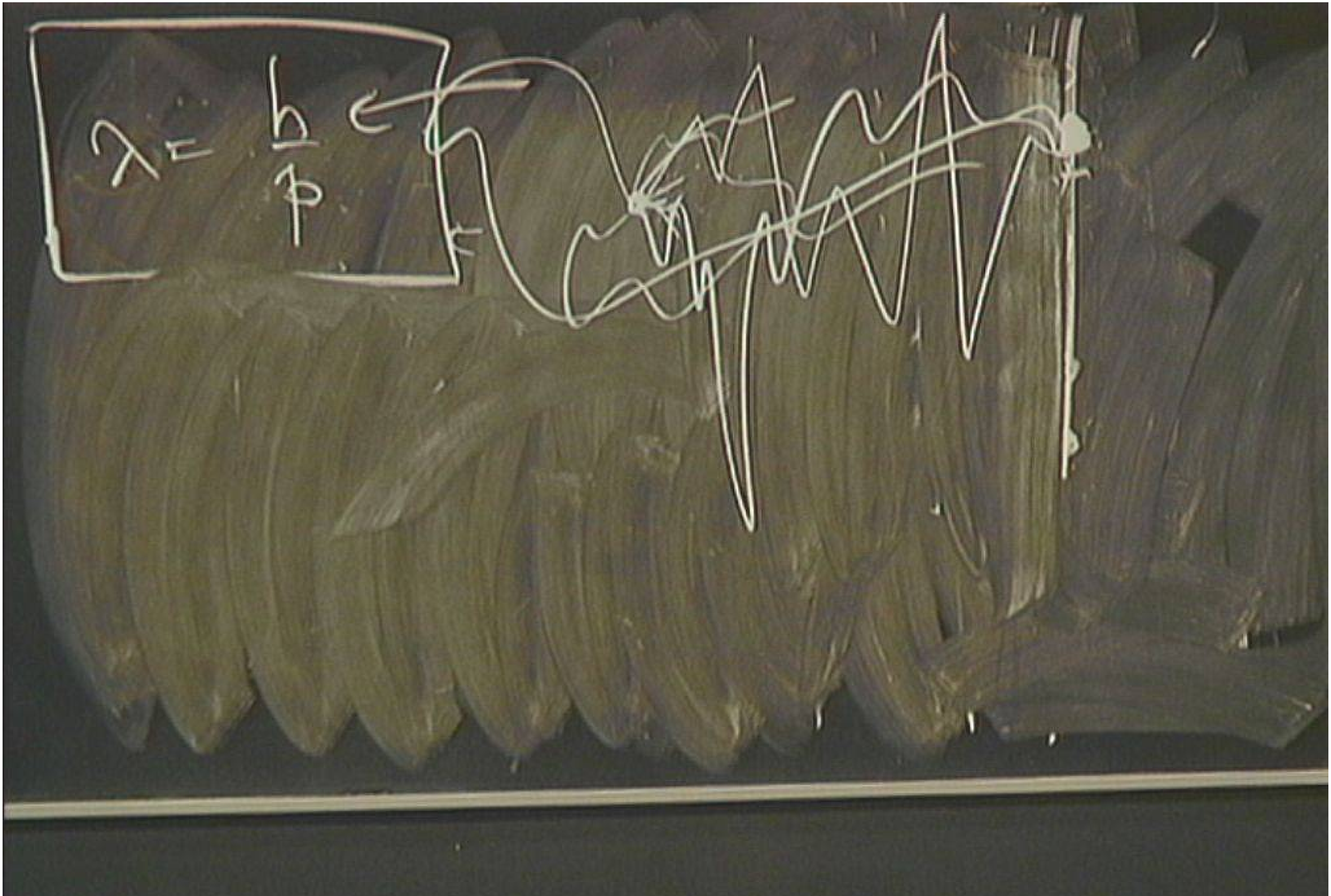

$\lambda = \frac{h}{p}$
Planck's const.
de Broglie relation
for $\psi^2(x) = P(x)$
6.0×10^{-34} J's.


$\lambda = \frac{h}{p}$

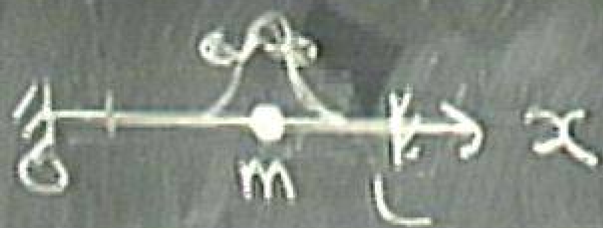

$\lambda = \frac{h}{p}$


λ = h/p


λ = h/p


$\lambda = \frac{h}{p}$

Particle in Box

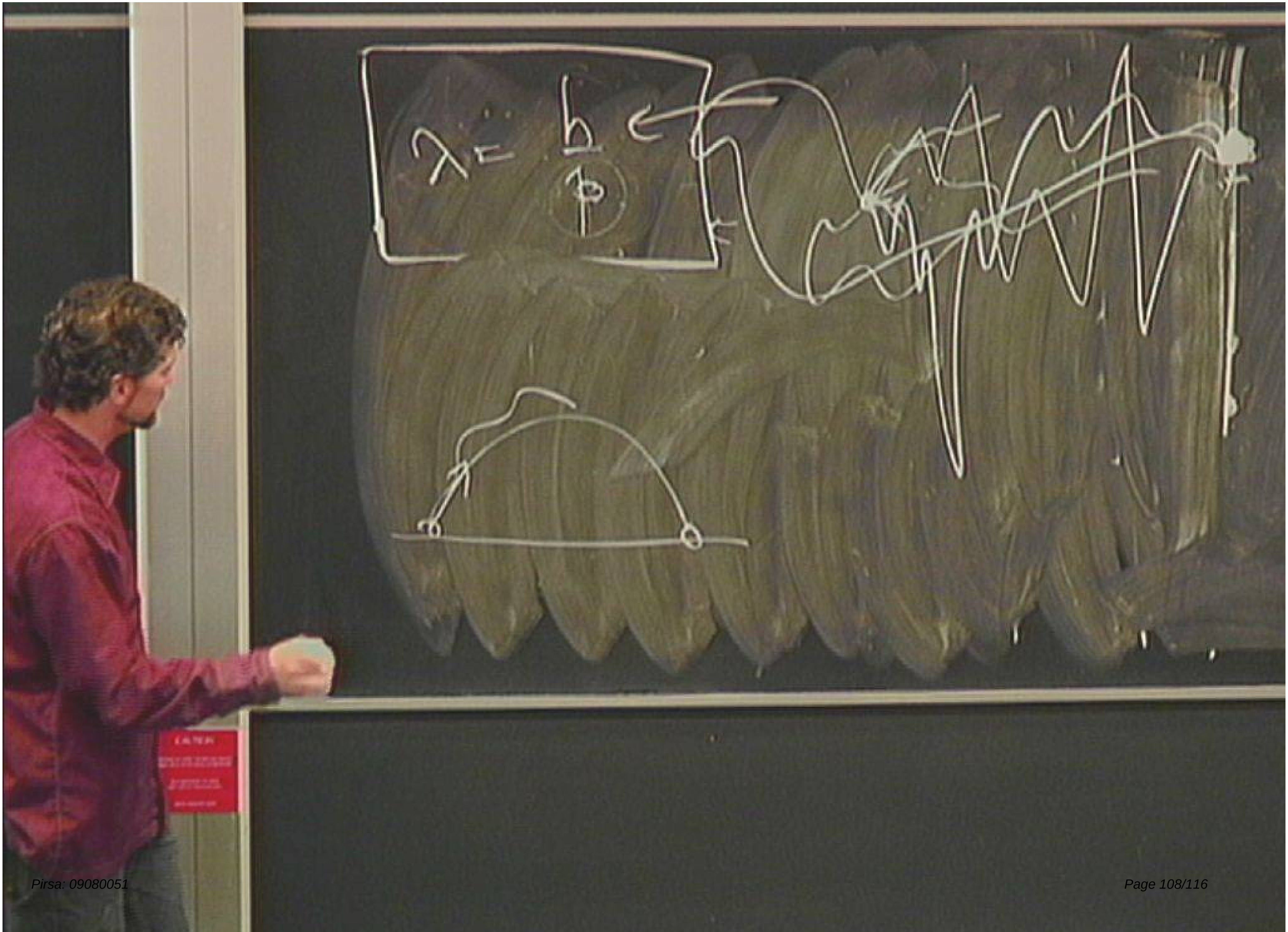


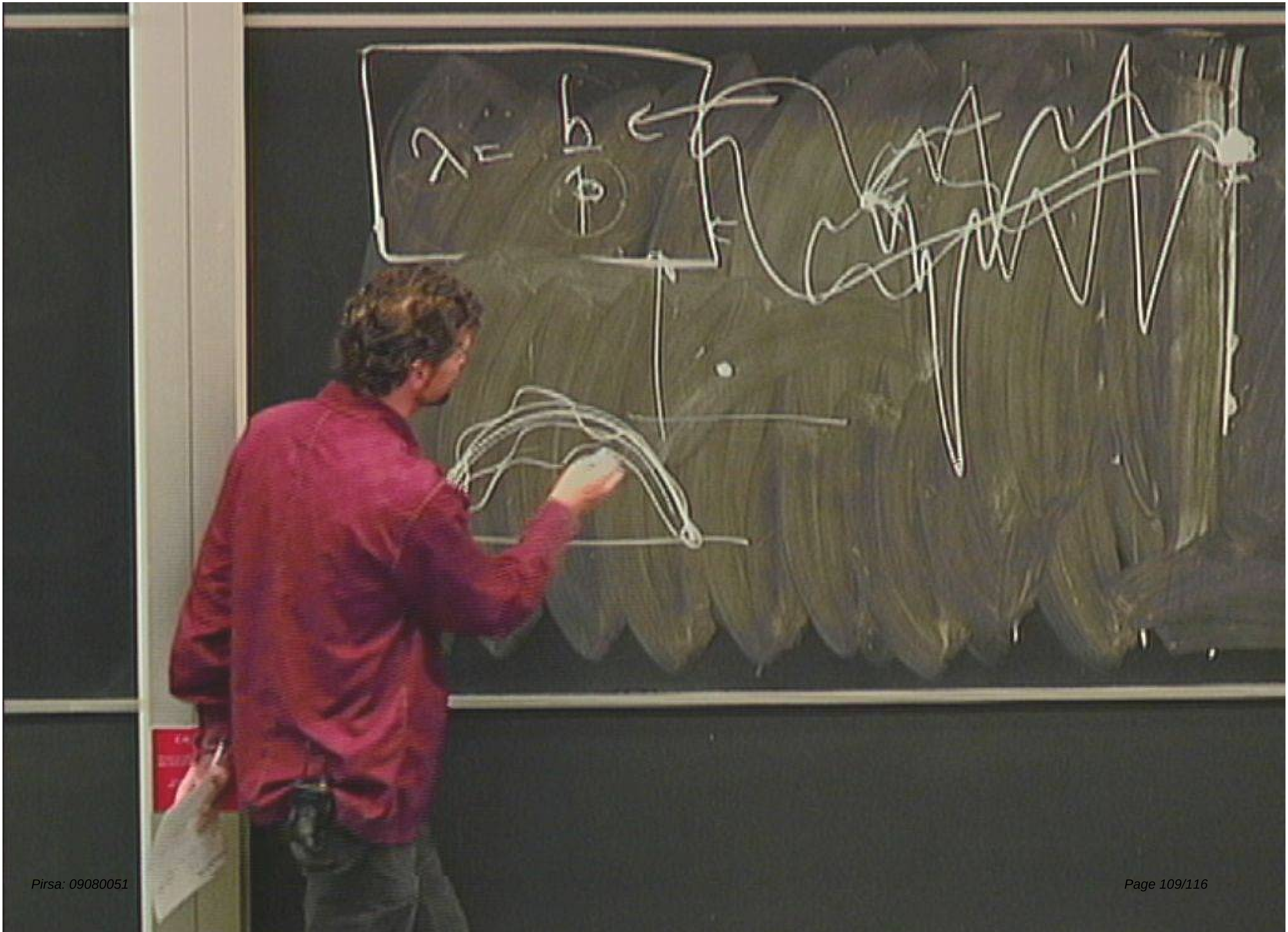
suppose definite energy E

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \Rightarrow p = \pm\sqrt{2mE}$$

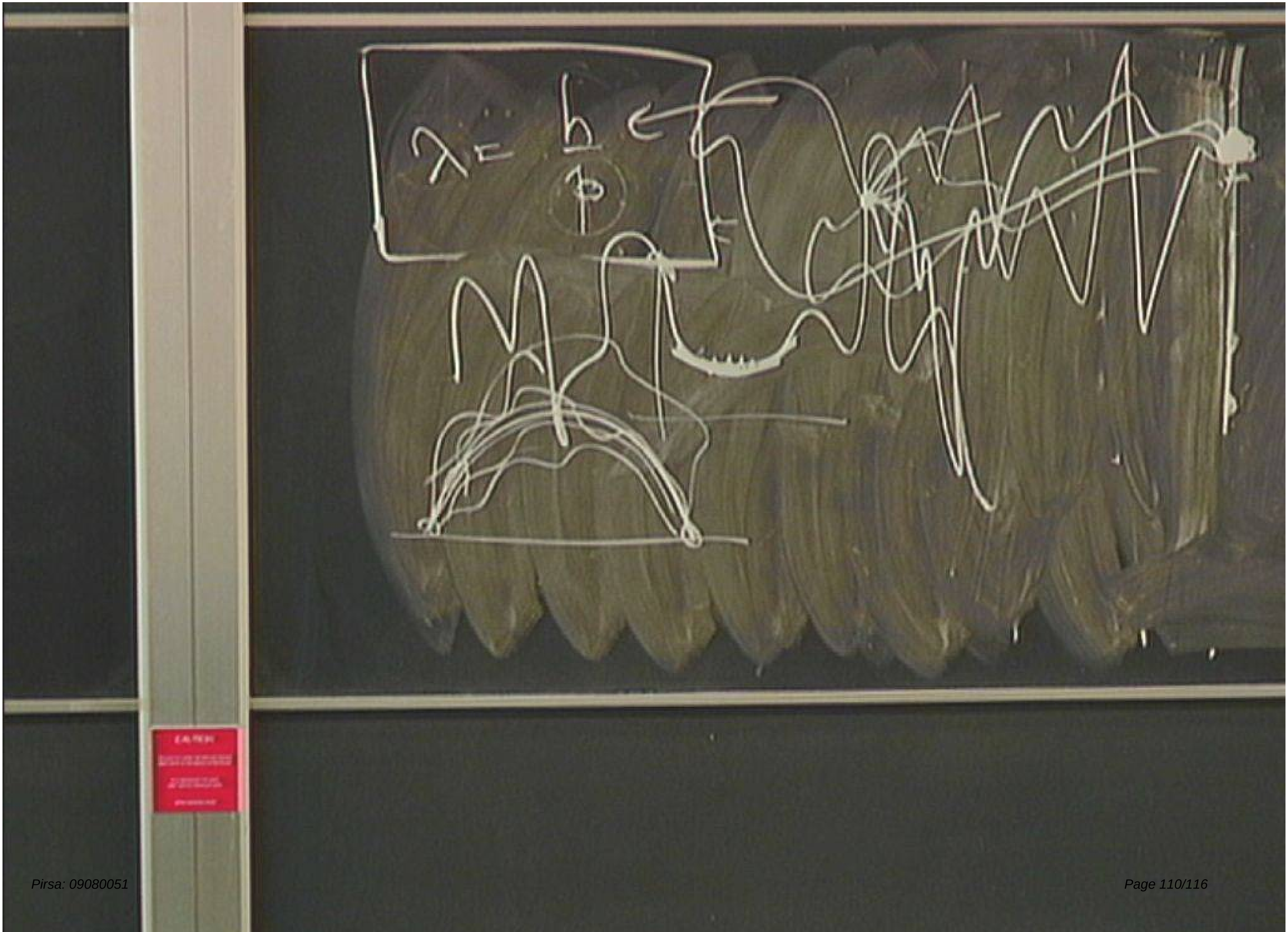
(right/left)

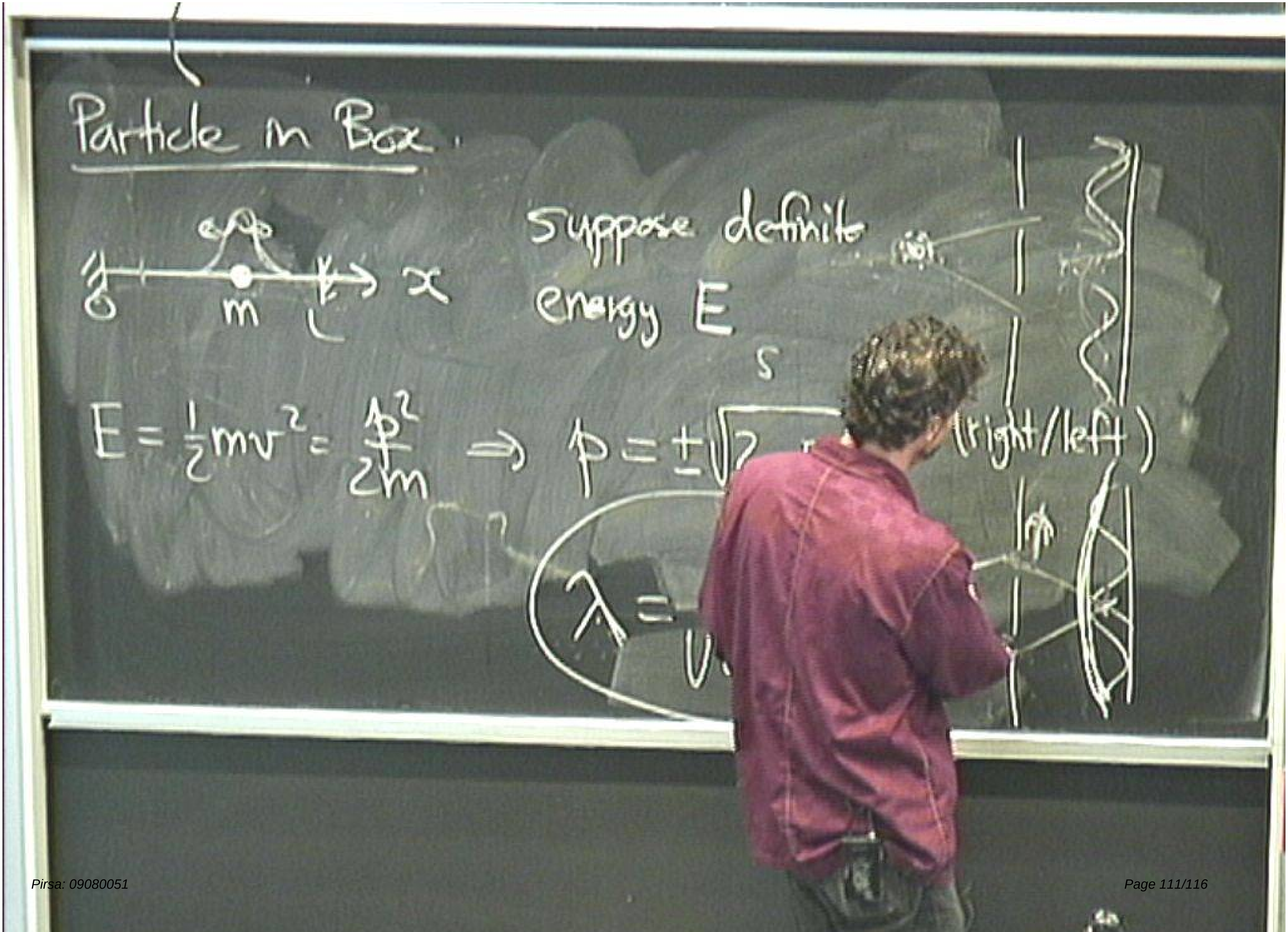
$$\lambda = \frac{h}{\sqrt{2mE}}$$


λ = h / p

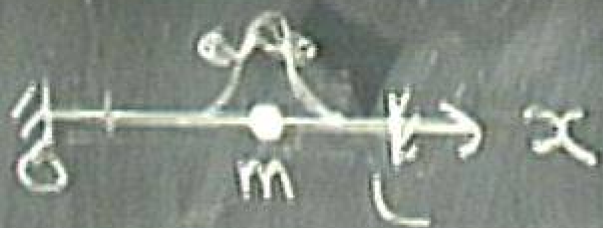


$\lambda = \frac{h}{p}$


λ = h/p


Particle in Box
0
m
L
x
suppose definite
energy E
E = ½mv² = p²/2m ⇒ p = ±√2
(right/left)
λ =

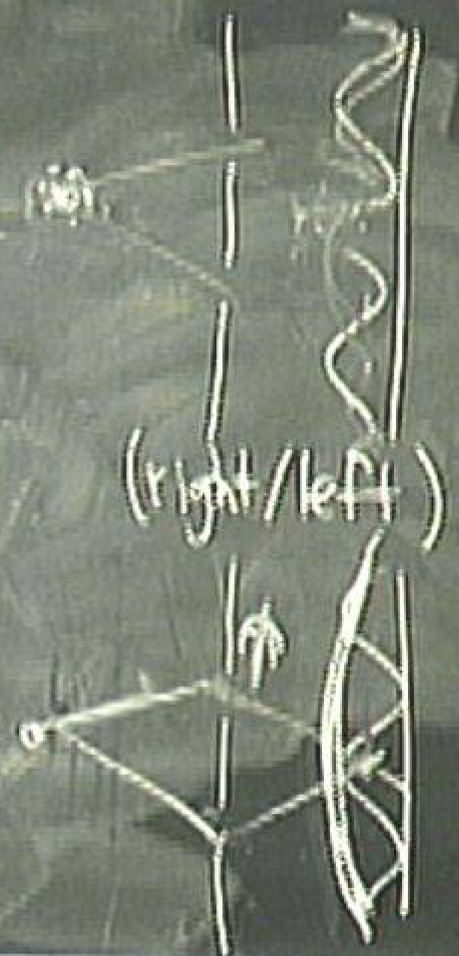
Particle in Box



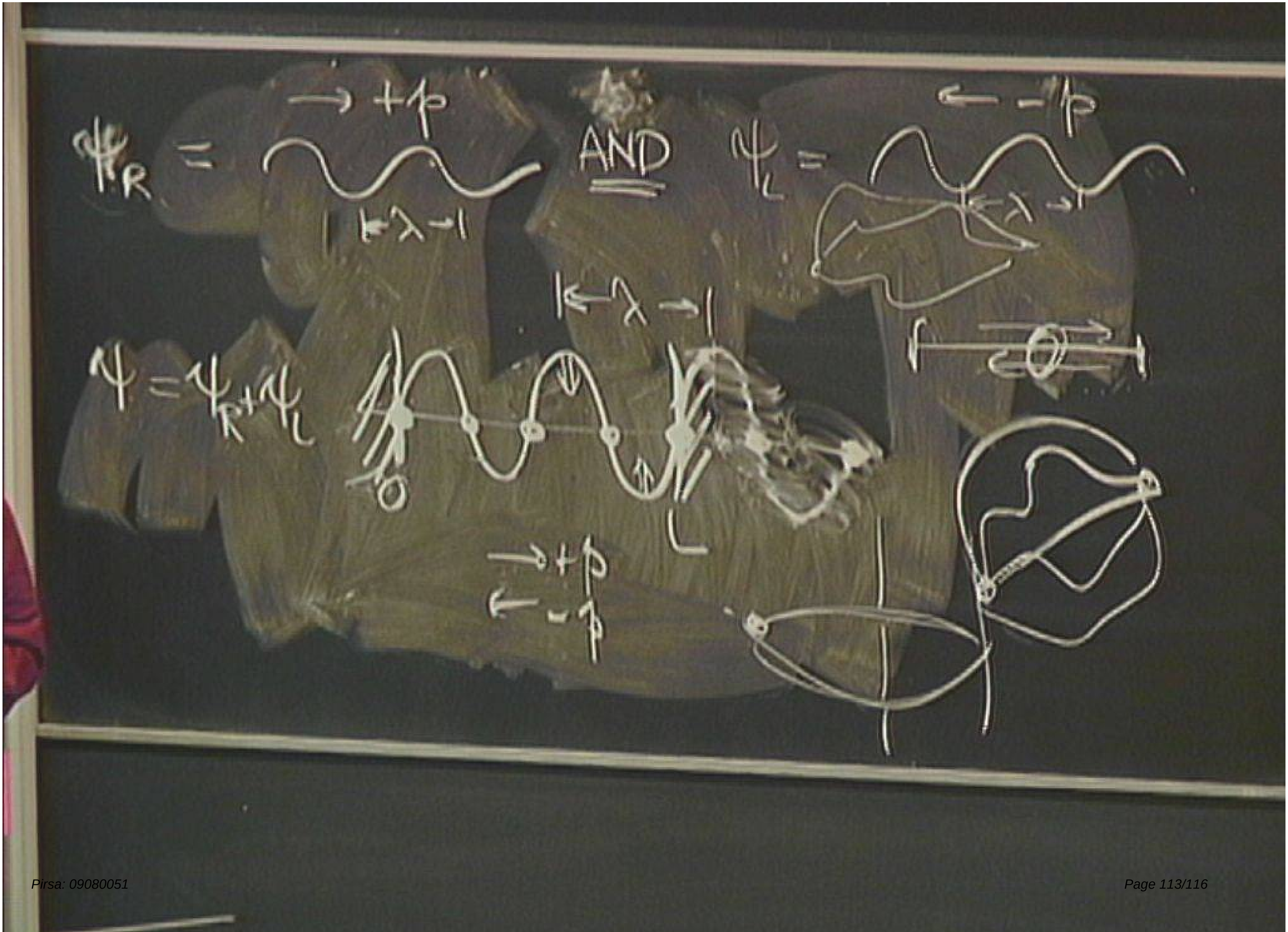
Suppose definite energy E

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \Rightarrow p = \pm\sqrt{2mE}$$

(right/left)



$$\lambda = \frac{h}{\sqrt{2mE}}$$


→ +p
ψR =
|←λ→|
AND
ψL =
← −p
|←λ→|
|←λ→|
ψ = ψR + ψL
0
L
→ +p
← −p