Title: Math Primer

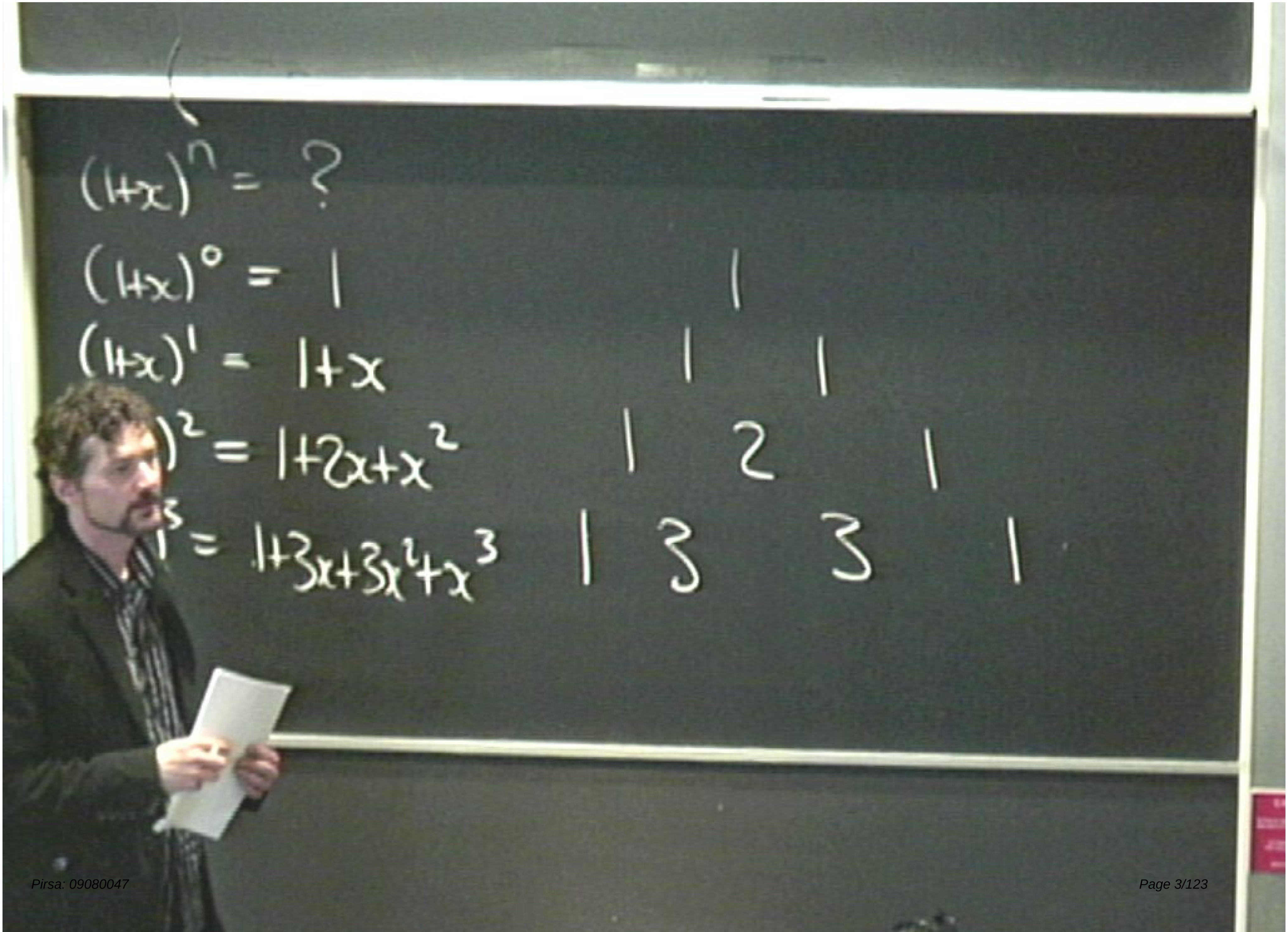
Date: Aug 10, 2009 09:30 AM

URL: http://pirsa.org/09080047

Abstract: An introduction to the mathematics necessary to fully appreciate the ISSYP relativity and quantum lectures. Binomial theorem, series expansions of common functions, complex numbers, and real and complex waves.

$(1+x)^n = ?$

$(1+x)^0 =$


$(1+x)^n = ?$
$(1+x)^0 = 1$
1
$(1+x)^1 = 1+x$
$1 \quad 1$
$)^2 = 1+2x+x^2$
$1 \quad 2 \quad 1$
$= 1+3x+3x^2+x^3$
$1 \quad 3 \quad 3 \quad 1$

$$(1+x)^n = \ ?$$

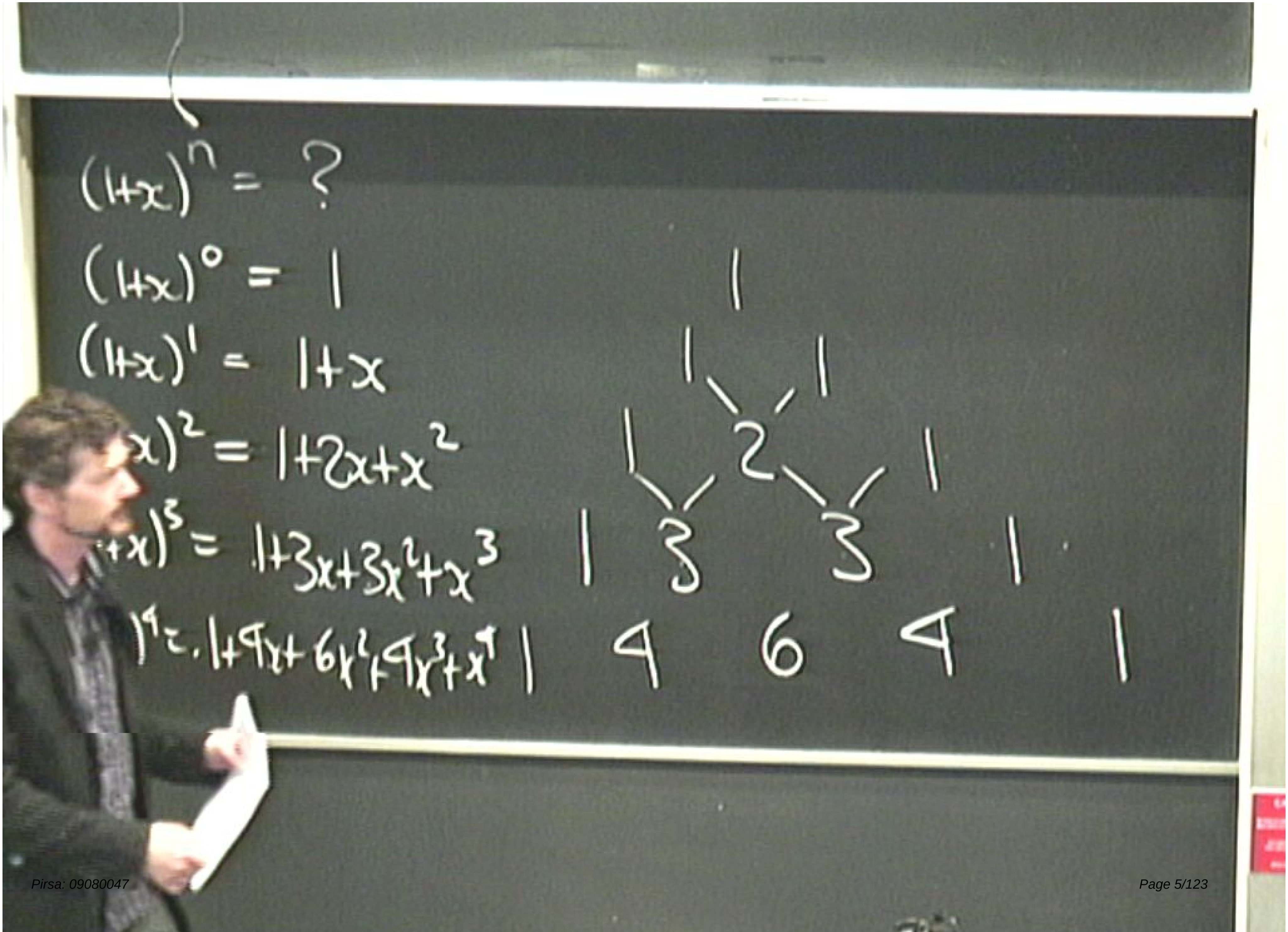
$$(1+x)^0 = 1$$

$$(1+x)^1 = 1+x$$

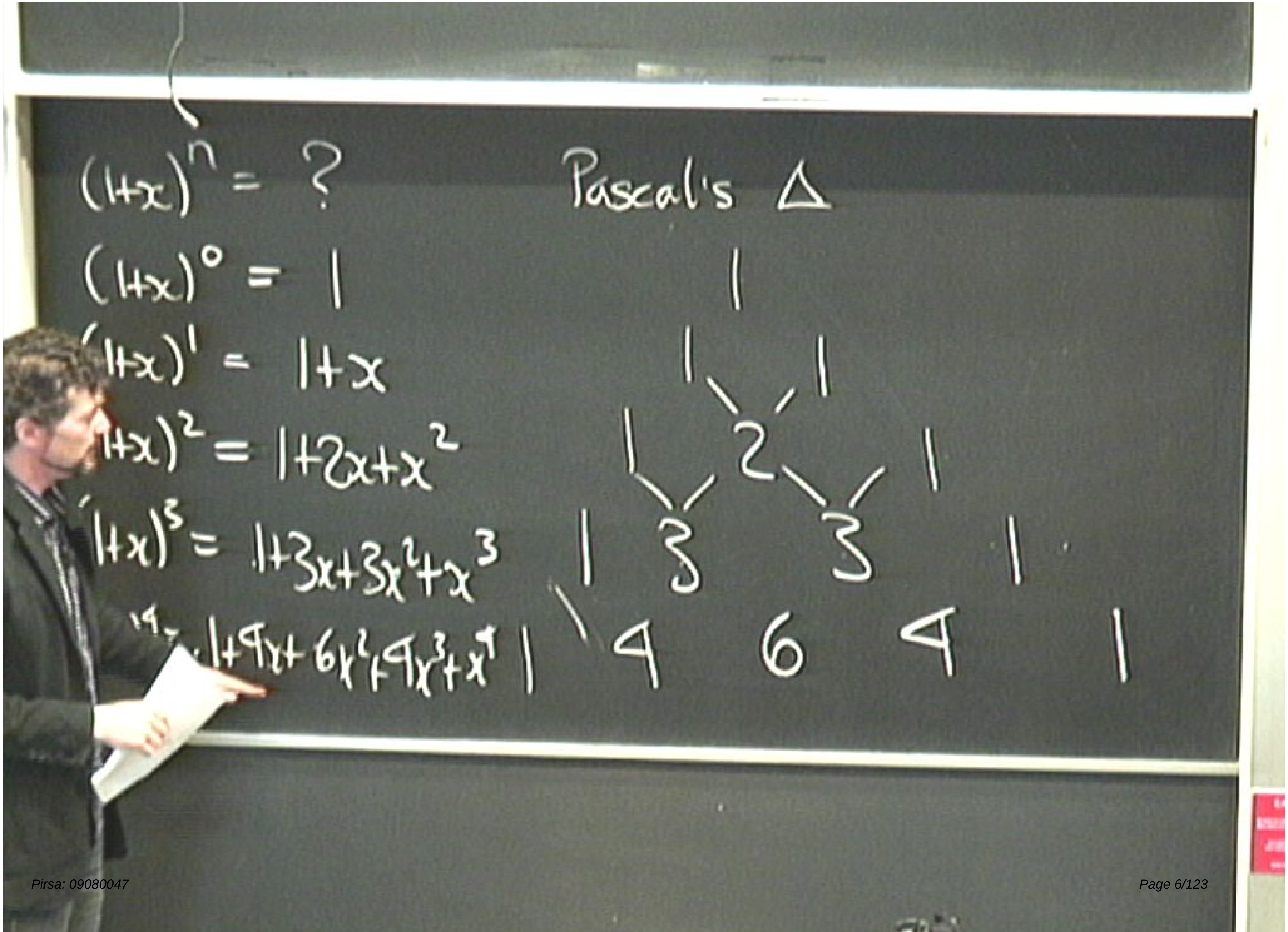
$$(1+x)^2 = 1+2x+x^2$$

$$(1+x)^3 = 1+3x+3x^2+x^3$$

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$



(1+x)^n = ?
(1+x)^0 = 1
(1+x)^1 = 1+x
x)^2 = 1+2x+x^2
+x)^3 = 1+3x+3x^2+x^3
)^4 = 1+4x+6x^2+4x^3+x^4
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1


$(1+x)^n = ?$
Pascal's Δ
$(1+x)^0 = 1$
$(1+x)^1 = 1+x$
$(1+x)^2 = 1+2x+x^2$
$(1+x)^3 = 1+3x+3x^2+x^3$
$1+4x+6x^2+4x^3+x^4$
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$(1+x)^n = ?$$

$$(1+x)^0 = 1$$

$$(1+x)^1 = 1+x$$

$$(1+x)^2 = 1+2x+x^2$$

$$(1+x)^3 = 1+3x+3x^2+x^3$$

$$(1+x)^4 = 1+4x+6x^2+4x^3+x^4$$

Pascal's Δ

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

$$(1+x)^n = 1 + nx$$

$$(1+x)^n = 1 + nx + \frac{n}{1}\frac{(n-1)}{2}x^2 +$$

$$(1+x)^n = 1 + \frac{n}{1} x + \frac{n}{1}\frac{(n-1)}{2} x^2 + \frac{n}{1}\frac{(n-1)}{2}\frac{(n-2)}{3} x^3 + \cdots$$

$$(1+x)^n = 1 + \frac{n}{1}x + \frac{n}{1}\frac{(n-1)}{2}x^2 + \frac{n}{1}\frac{(n-1)}{2}\frac{(n-2)}{3}x^3 + \cdots$$

$n = 0, 1, 2, 3, 4 \ldots$

$$(1+x)^n = 1 + nx + \frac{n}{1}\frac{(n-1)}{2}x^2 + \frac{n}{1}\frac{(n-1)}{2}\frac{(n-2)}{3}x^3 + \cdots$$

$n = 0, 1, 2, 3, 4 \ldots \longrightarrow$ series terminates.

$n = -1$

$$(1+x)^{-1} \overset{?}{=} 1 + \frac{(-1)}{1}x + \frac{(-1)}{1}\frac{(-1-1)}{2}x^2 + \frac{(-1)}{1}\frac{(-1-1)}{2}\frac{(-1-2)}{3}x^3 + \cdots$$

$$= 1 - x$$

$$(1+x)^n = 1 + nx + \frac{n}{1}\frac{(n-1)}{2}x^2 + \frac{n}{1}\frac{(n-1)}{2}\frac{(n-2)}{3}x^3 + \cdots$$

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e.g. $n = -1$

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$$= 1 - x + x^2 - x^3 + x^4 \mp \cdots$$

$$(1+x)^n = 1 + \frac{n}{1}x + \frac{n}{1}\frac{(n-1)}{2}x^2 + \frac{n}{1}\frac{(n-1)}{2}\frac{(n-2)}{3}x^3 + \cdots$$

$(n = 0, 1, 2, 3, 4 \ldots) \rightarrow$ series terminates.

e.g. $n = -1$

$$(1+x)^{-1} \stackrel{?}{=} 1 + \frac{(-1)}{1}x + \frac{(-1)}{1}\frac{(-1-1)}{2}x^2 + \overset{-1}{\frac{(-1)}{1}}\,\overset{-1}{\frac{(-1-1)}{2}}\,\overset{-1}{\frac{(-1-2)}{3}}x^3 + \cdots$$

$$= 1 - x + x^2 - x^3 + x^4 \mp \cdots$$

$$(1+x)^n = 1 + \frac{n}{1}x + \frac{n}{1}\frac{(n-1)}{2}x^2 + \frac{n}{1}\frac{(n-1)}{2}\frac{(n-2)}{3}x^3 + \cdots$$

$(n = 0, 1, 2, 3, 4 \ldots) \longrightarrow$ series terminates.

e.g. $n = -1$

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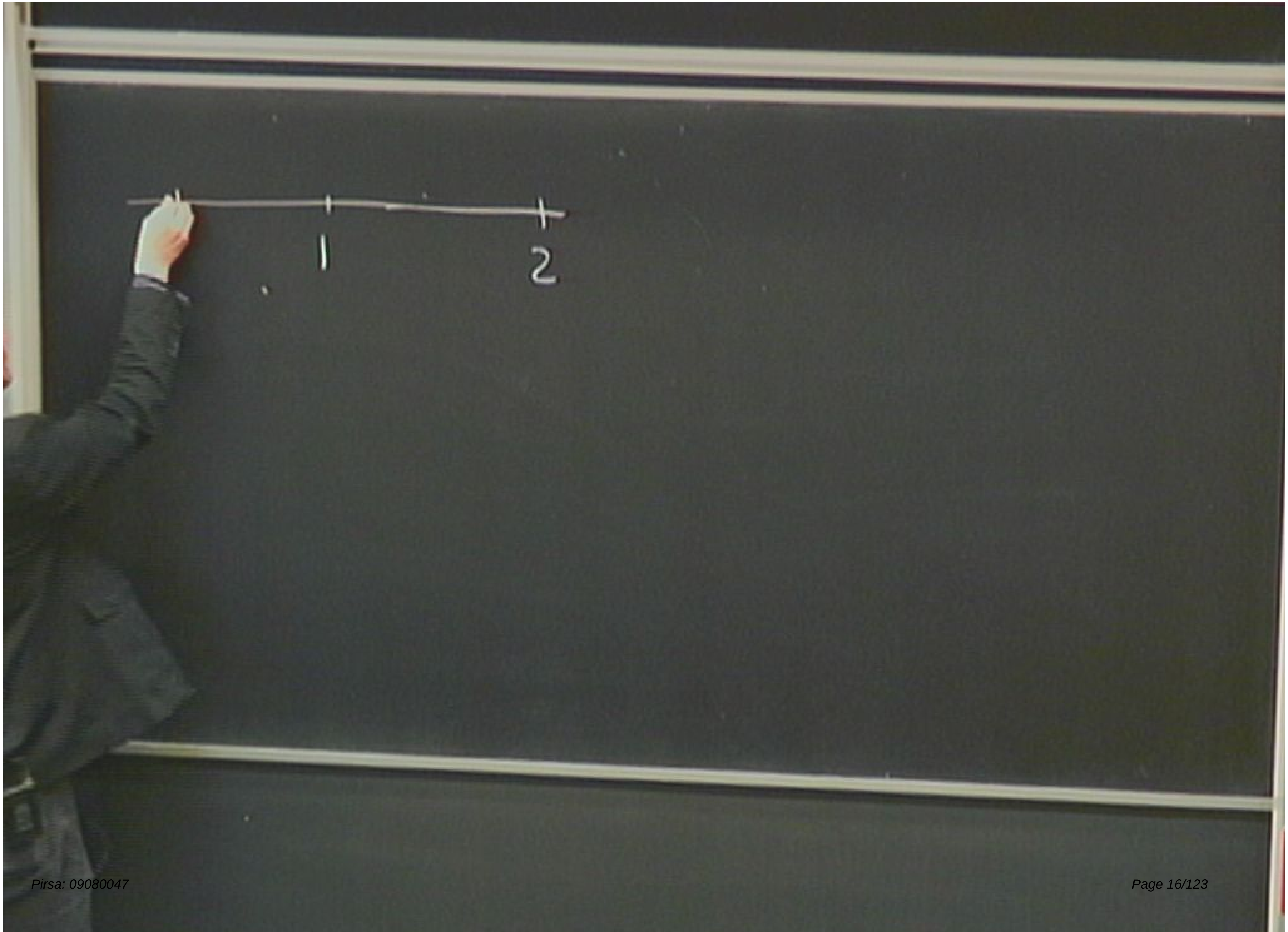
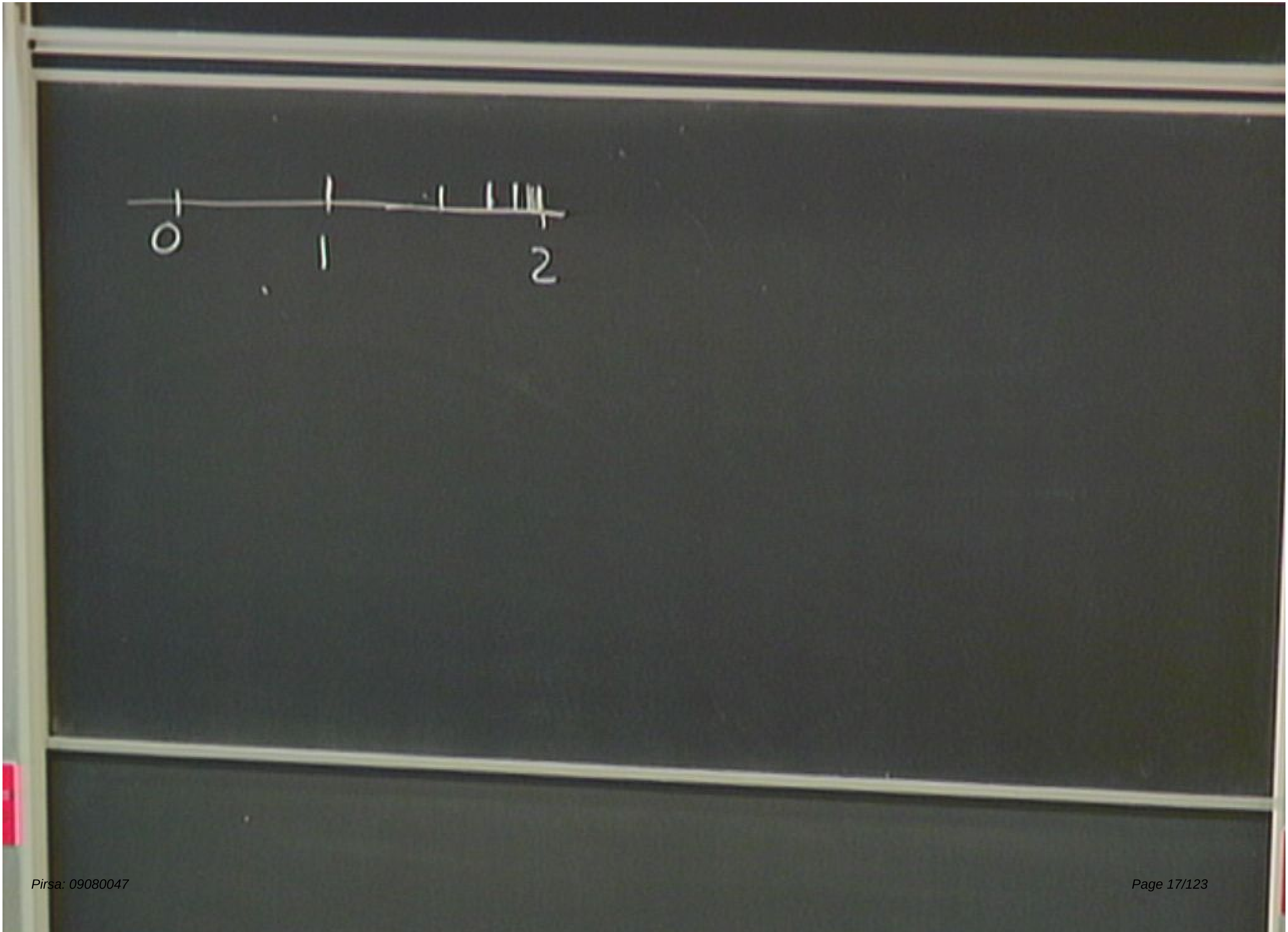
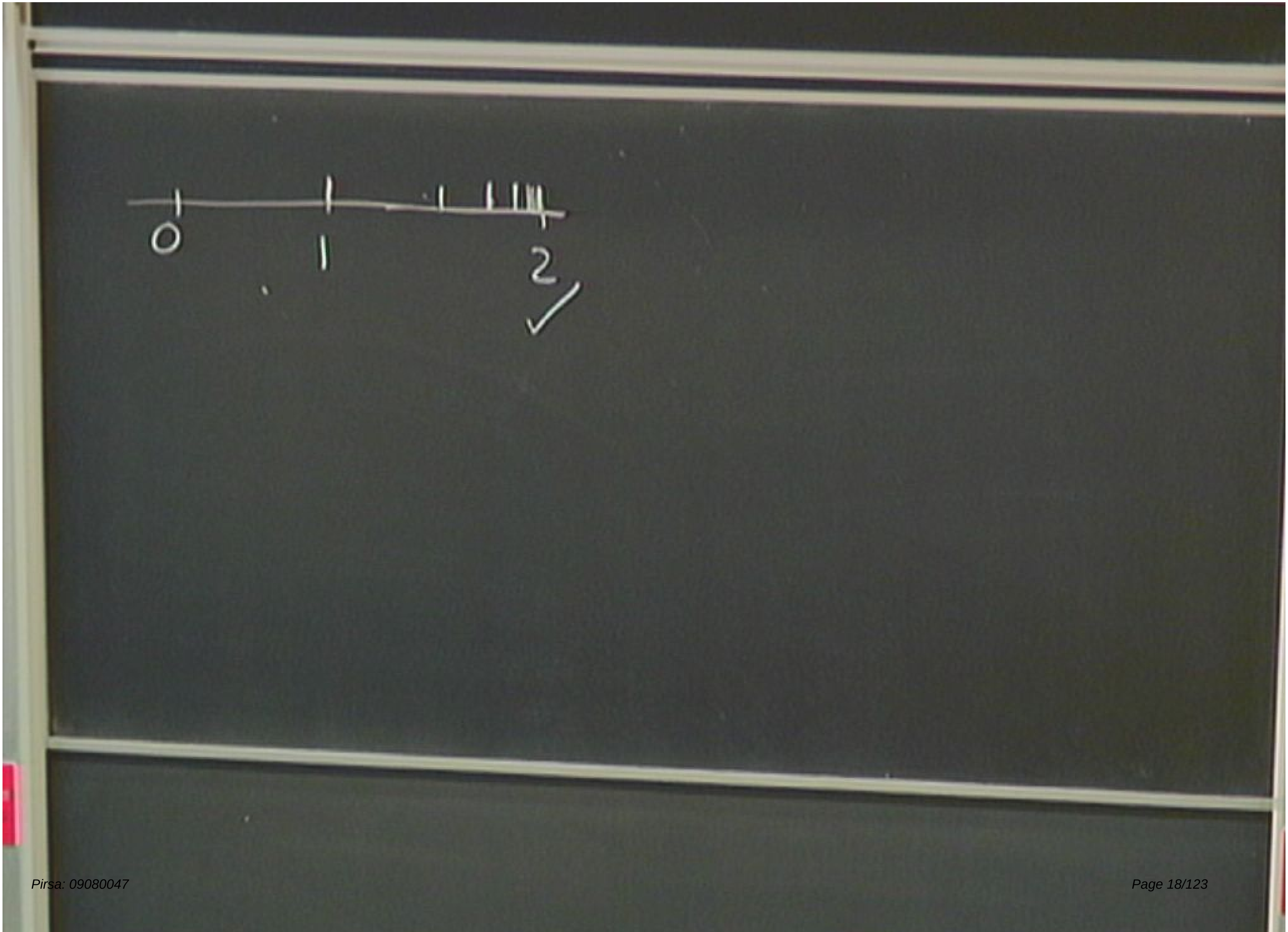
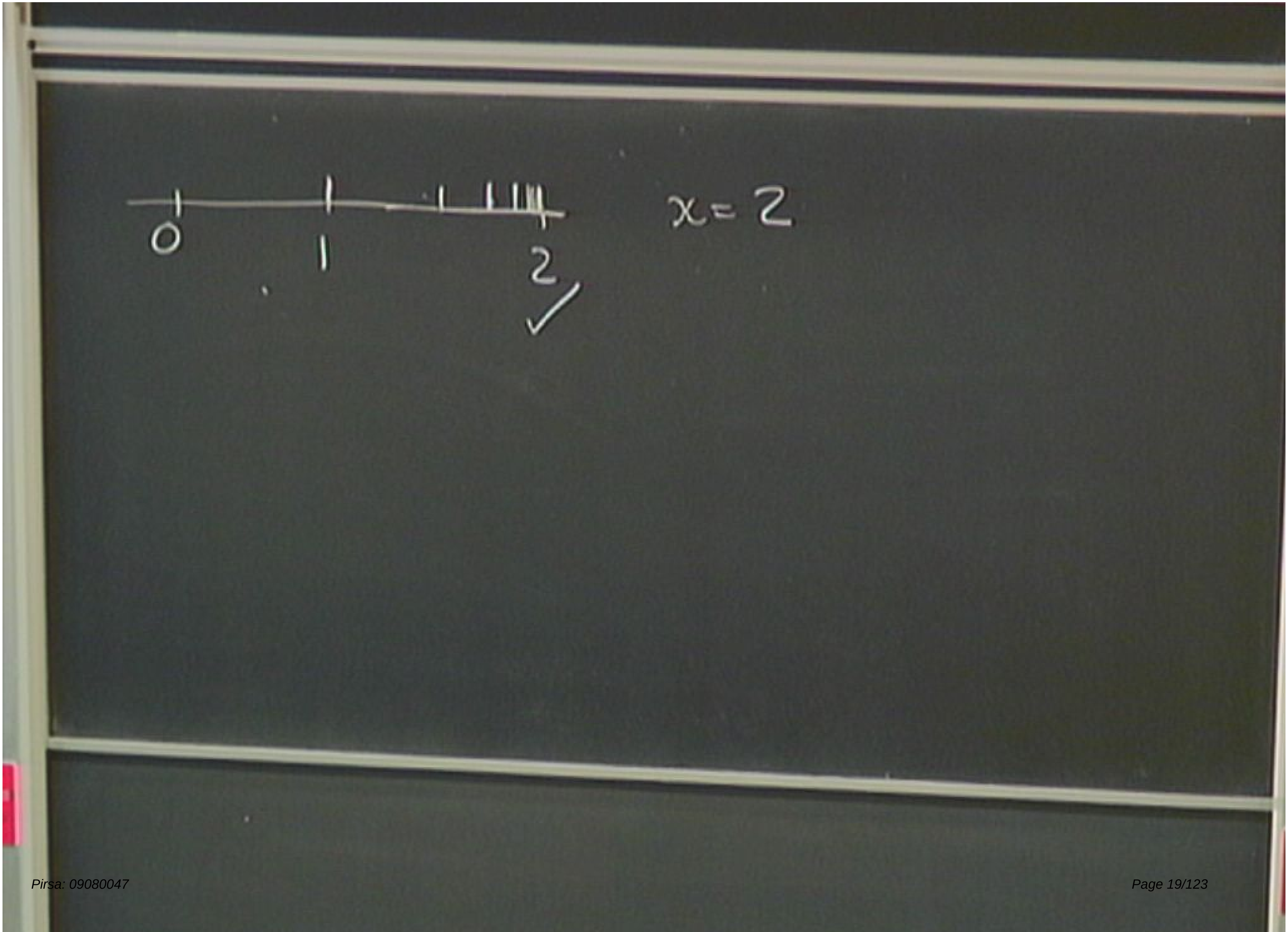
$$= 1 - x + x^2 - x^3 + x^4 \mp \cdots \longrightarrow \text{infinite series.}$$


1
2


0
1
2


0
1
2


0
1
2
x = 2

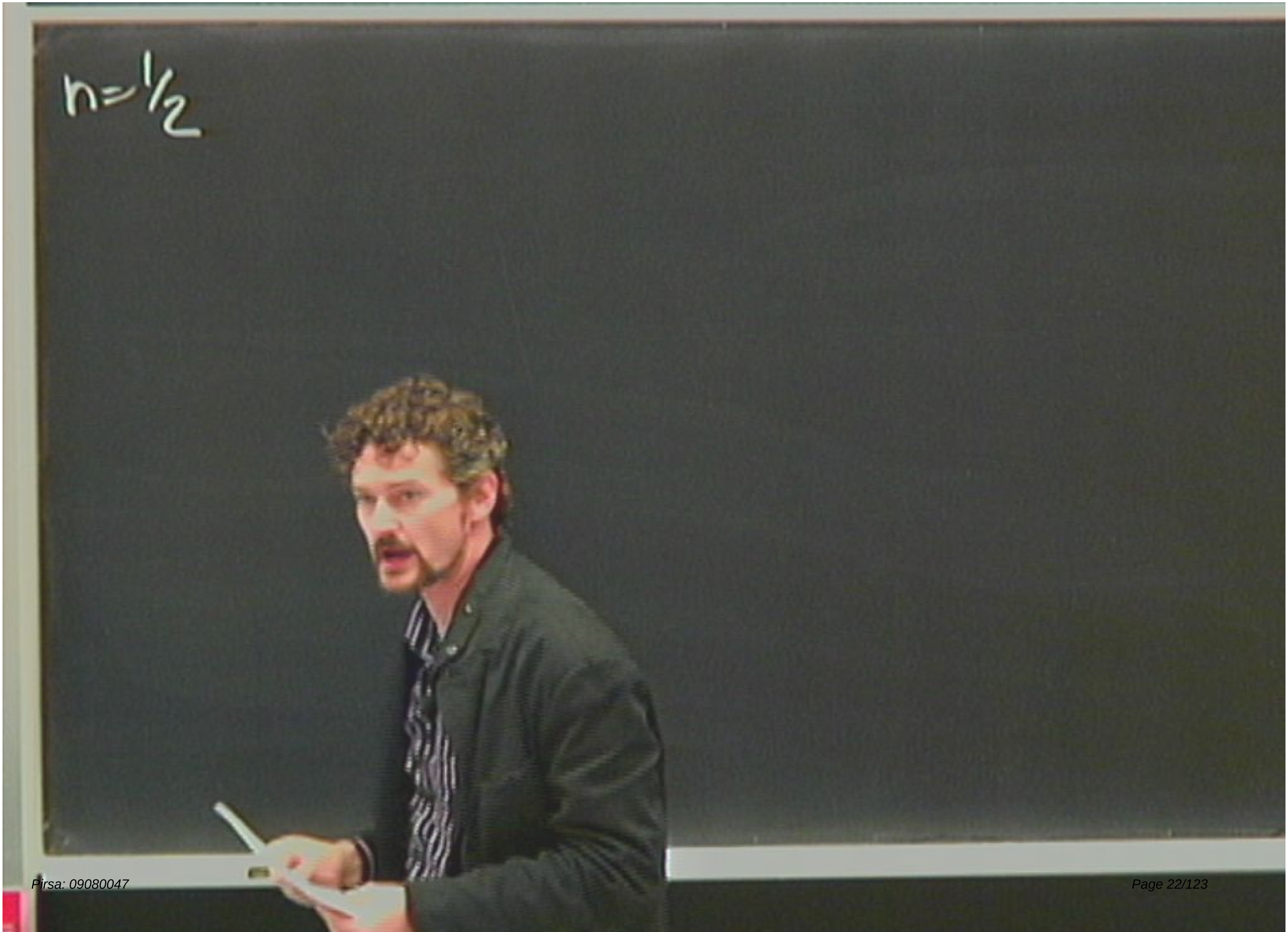
$x = 2$

$$(1+x)^{-1} = \frac{1}{1+x}$$

$x = 2$

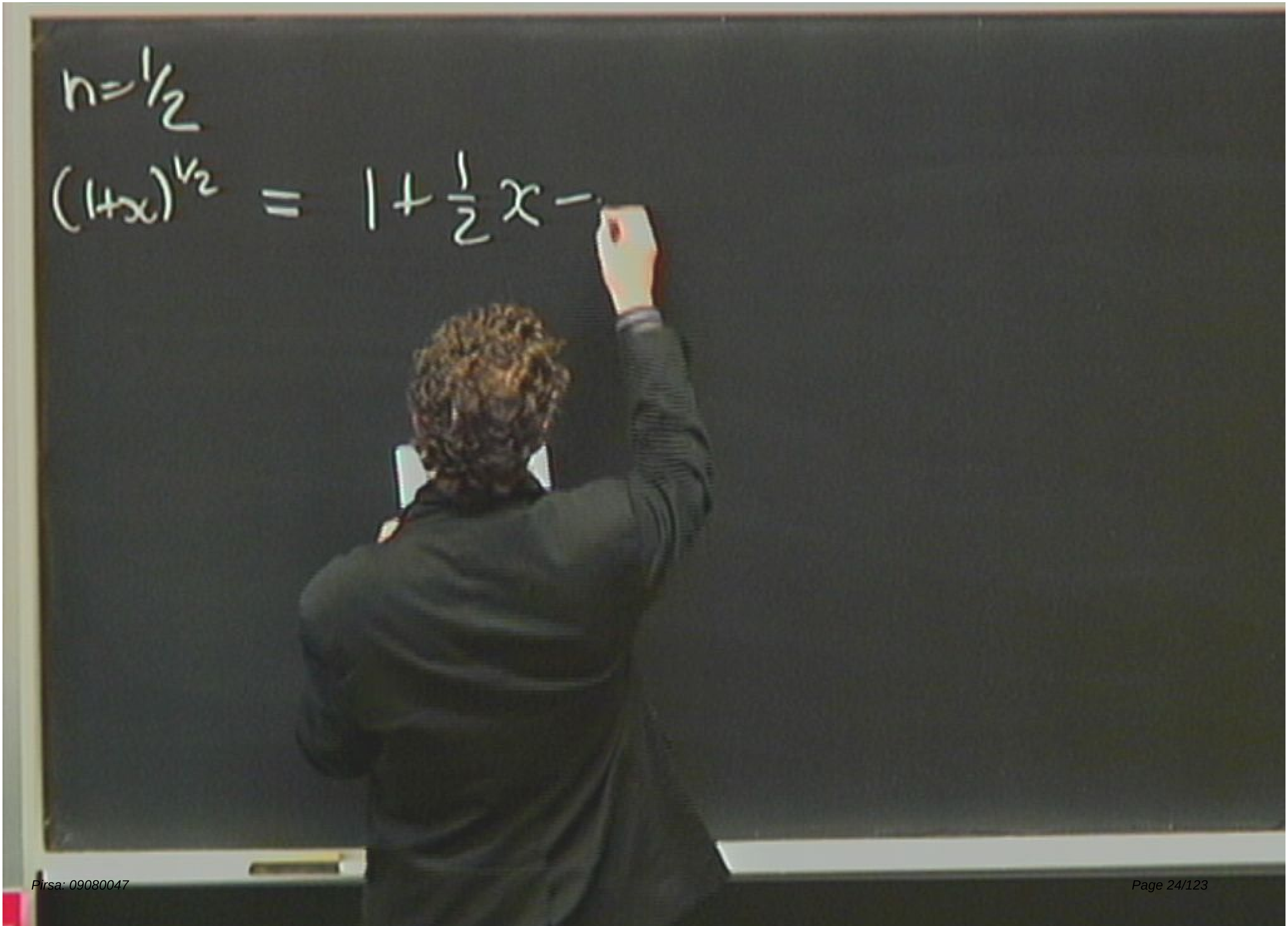
$$(1+x)^{-1} = \frac{1}{1+x}$$

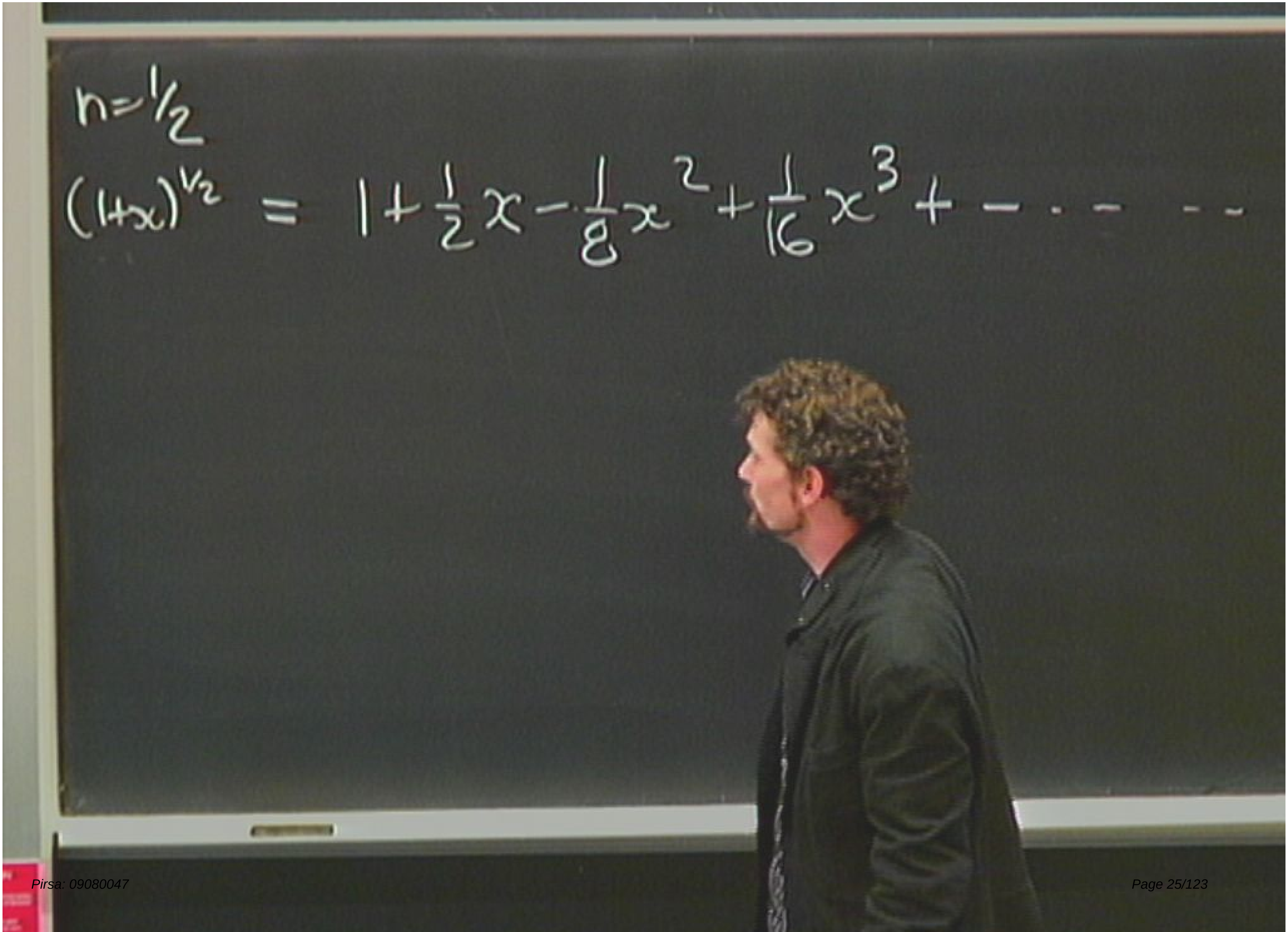
$$\begin{array}{r|l} & 1 - x + x^2 \ldots \\ \hline 1+x & 1 \\ & \underline{1+x} \\ & -x \\ & \underline{-x - x^2} \\ & +x^2 \end{array}$$


n=1/2

$n = 1/2$

$(1+x)^{1/2} =$


$n = 1/2$
$(1+x)^{1/2} = 1 + \frac{1}{2}x -$


$n = 1/2$
$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + - \cdots - \cdots$

$n = \frac{1}{2}$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + - \cdots - \; - -$$

$$(1+x)^{1/2}(1+x)^{1/2} =$$

$n = 1/2$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + - \cdots$$

$$(1+x)^{1/2}(1+x)^{1/2} = 1+x$$

$$\left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots\right)\left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots\right)$$

$n = 1/2$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + - \cdots - \cdots$$

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$$\left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots\right)\left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots\right)$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + - \cdot$$

$n = 1/2$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \cdots$$

$$(1+x)^{1/2}(1+x)^{1/2} = 1+x$$

$$\left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots\right)\left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots\right)$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots$$
$$+ \frac{1}{2}x + \frac{1}{4}x^2 + \cdots$$

$$n = \tfrac{1}{2}$$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + - \cdots$$

$$(1+x)^{1/2}(1+x)^{1/2} = 1+x$$

$$\left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots\right)\left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots\right)$$

$$\begin{aligned} &= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots \\ &\qquad + \frac{1}{2}x + \frac{1}{4}x^2 + \cdots \\ &\qquad\qquad - \frac{1}{8}x^2 + \cdots \end{aligned}$$

$n = 1/2$

$$(1+x)^{1/2} = 1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + \tfrac{1}{16}x^3 + - \cdots$$

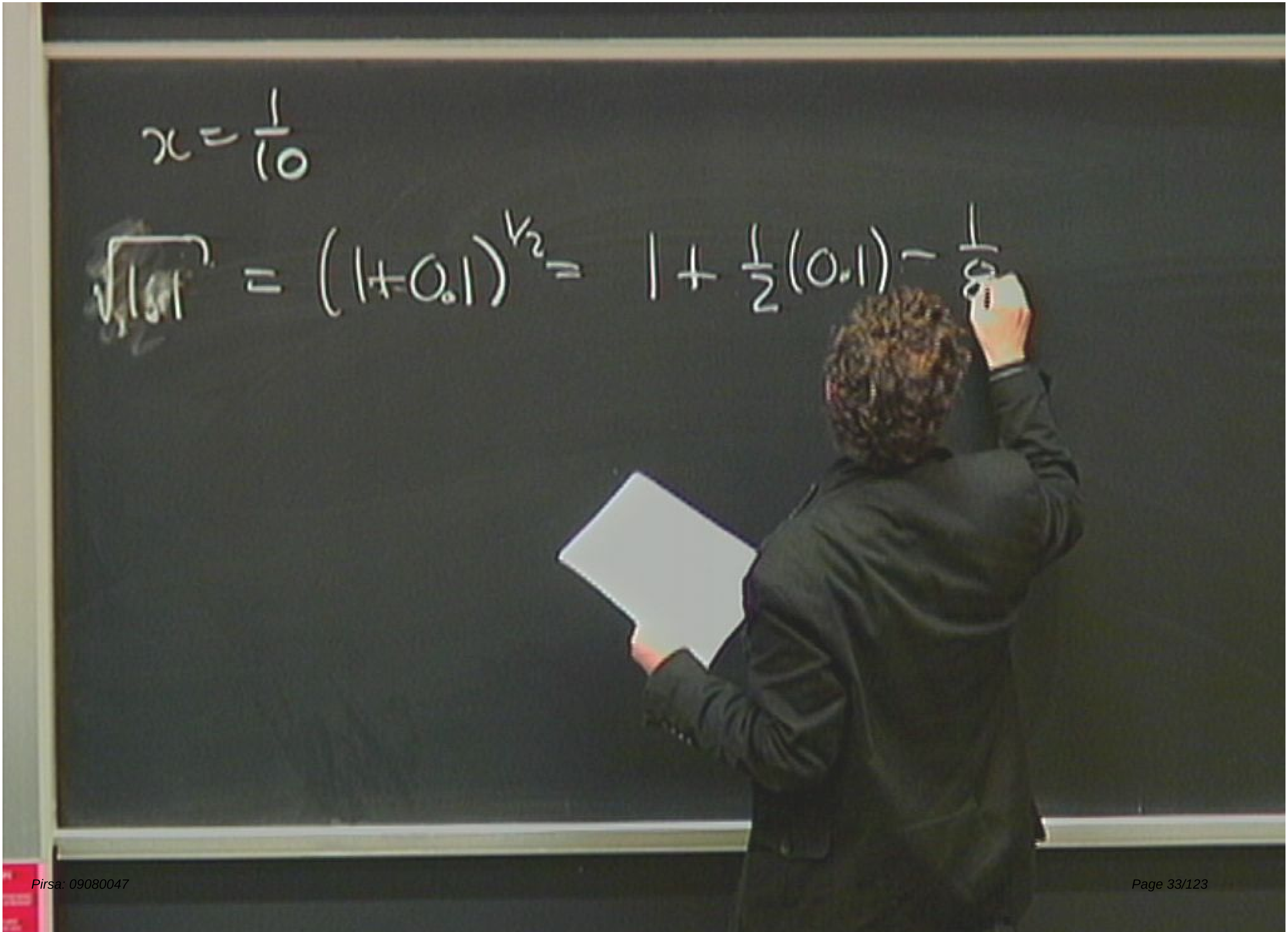
$$(1+x)^{1/2}(1+x)^{1/2} = 1+x$$

$$\left(1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + \cdots\right)\left(1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + \cdots\right)$$

$$\begin{aligned} = 1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 &+ \cdots \qquad = 1 + x + 0 + 0 + 0 \cdots \\ + \tfrac{1}{2}x + \tfrac{1}{4}x^2 &+ \cdots \\ - \tfrac{1}{8}x^2 &+ \cdots \end{aligned}$$

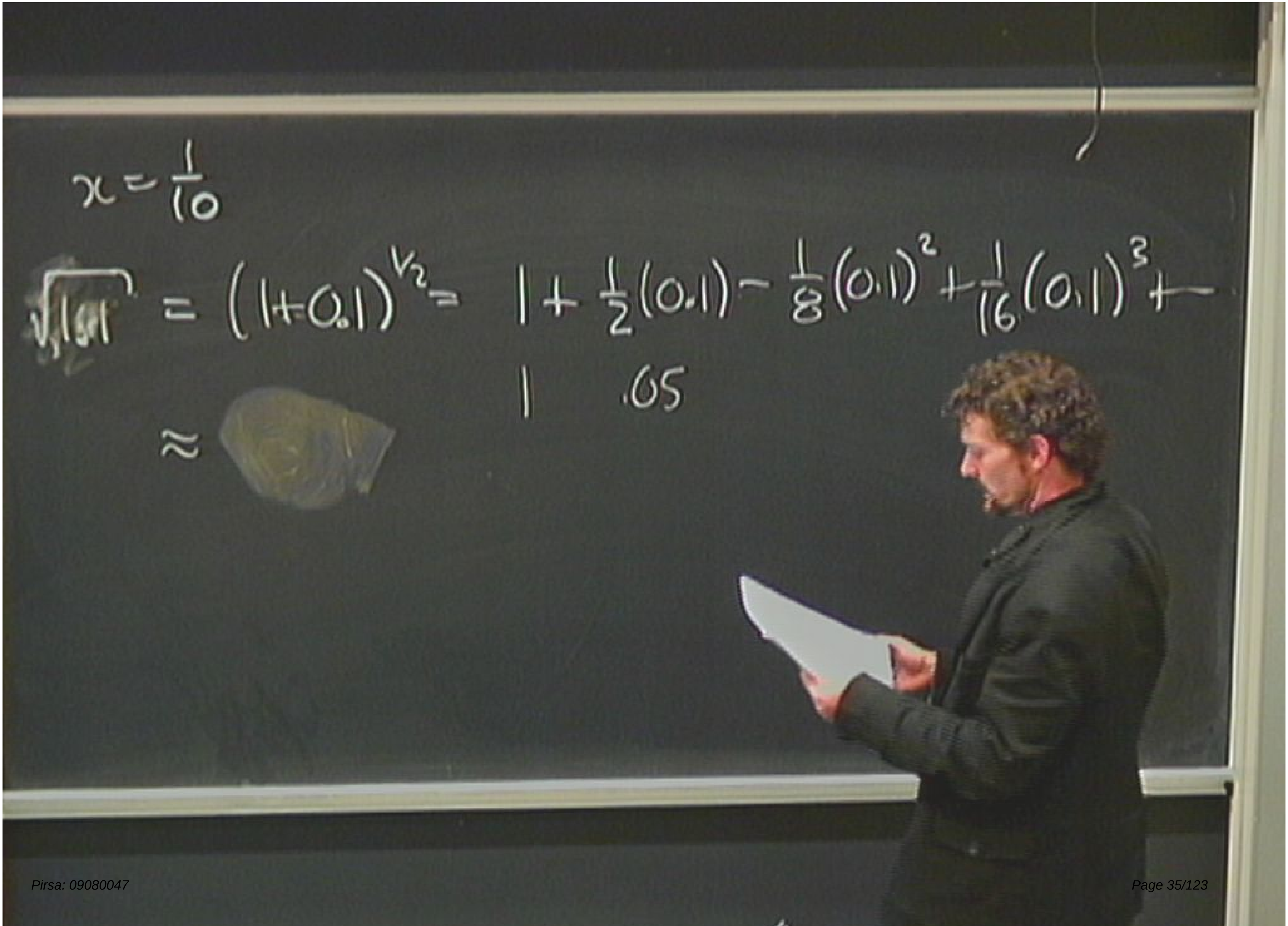
$$x = \frac{1}{10}$$

$$\sqrt{1.1} = (1+0.1)^{1/2}$$


$x = \frac{1}{10}$
$\sqrt{1.1} = (1+0.1)^{1/2} = 1 + \frac{1}{2}(0.1) - \frac{1}{8}$

$$x = \frac{1}{10}$$

$$\sqrt{1.1} = (1+0.1)^{1/2} = 1 + \frac{1}{2}(0.1) - \frac{1}{8}(0.1)^2$$

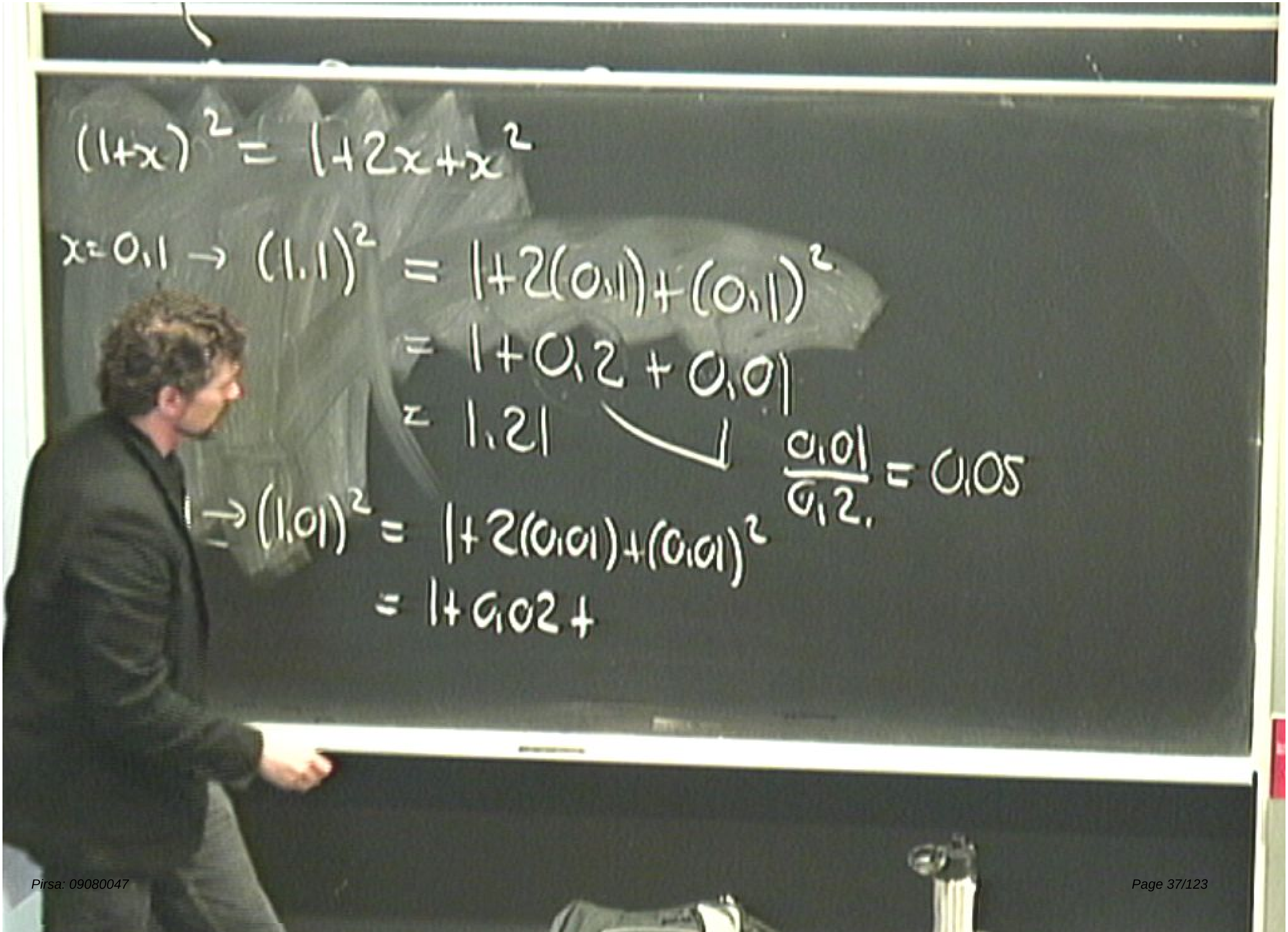

$x = \frac{1}{10}$
$\sqrt{1.1} = (1+0.1)^{1/2} = 1 + \frac{1}{2}(0.1) - \frac{1}{8}(0.1)^2 + \frac{1}{16}(0.1)^3 + \dots$
$1 \quad .05$
$\approx$

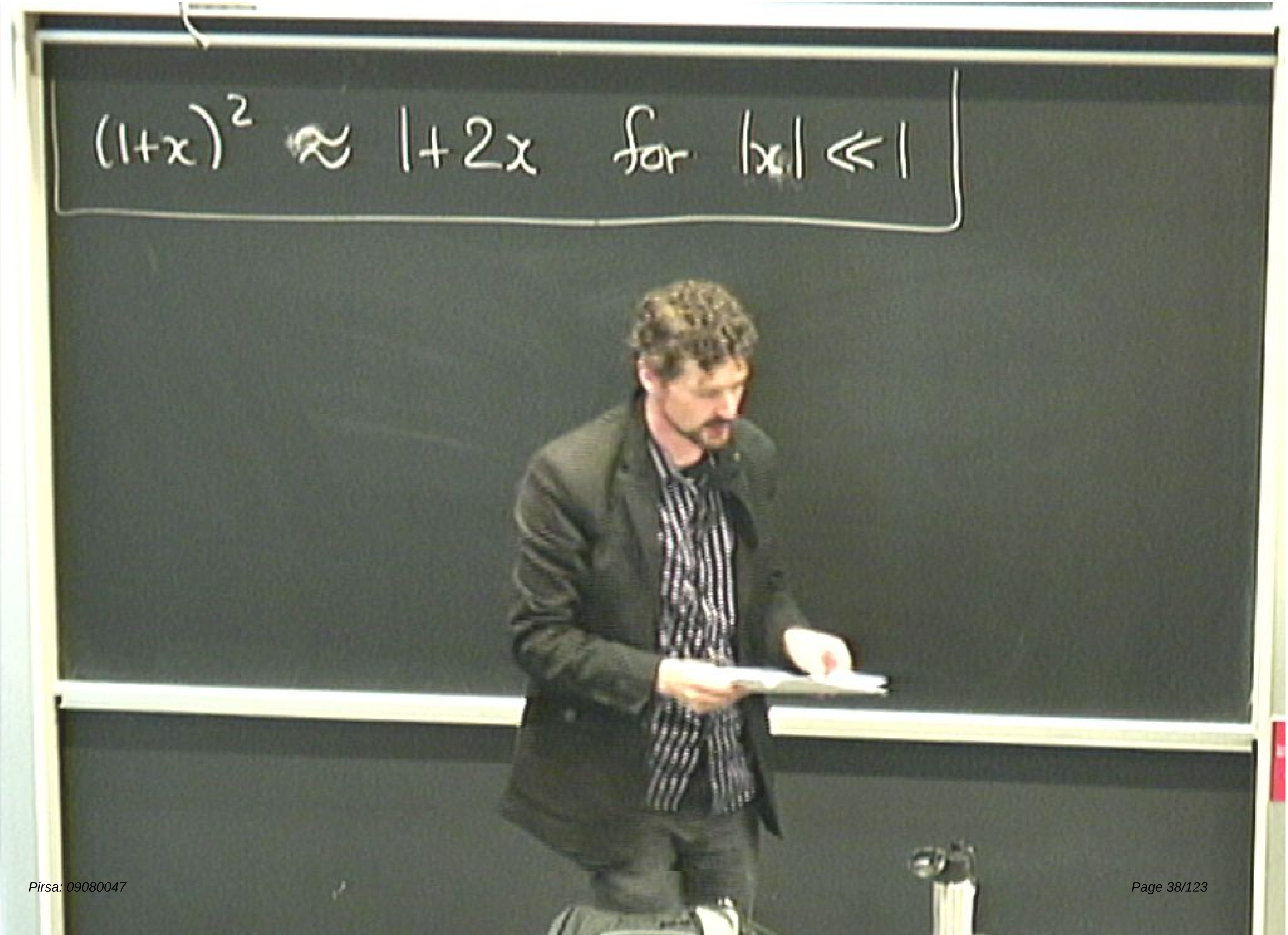
$x = \frac{1}{10}$

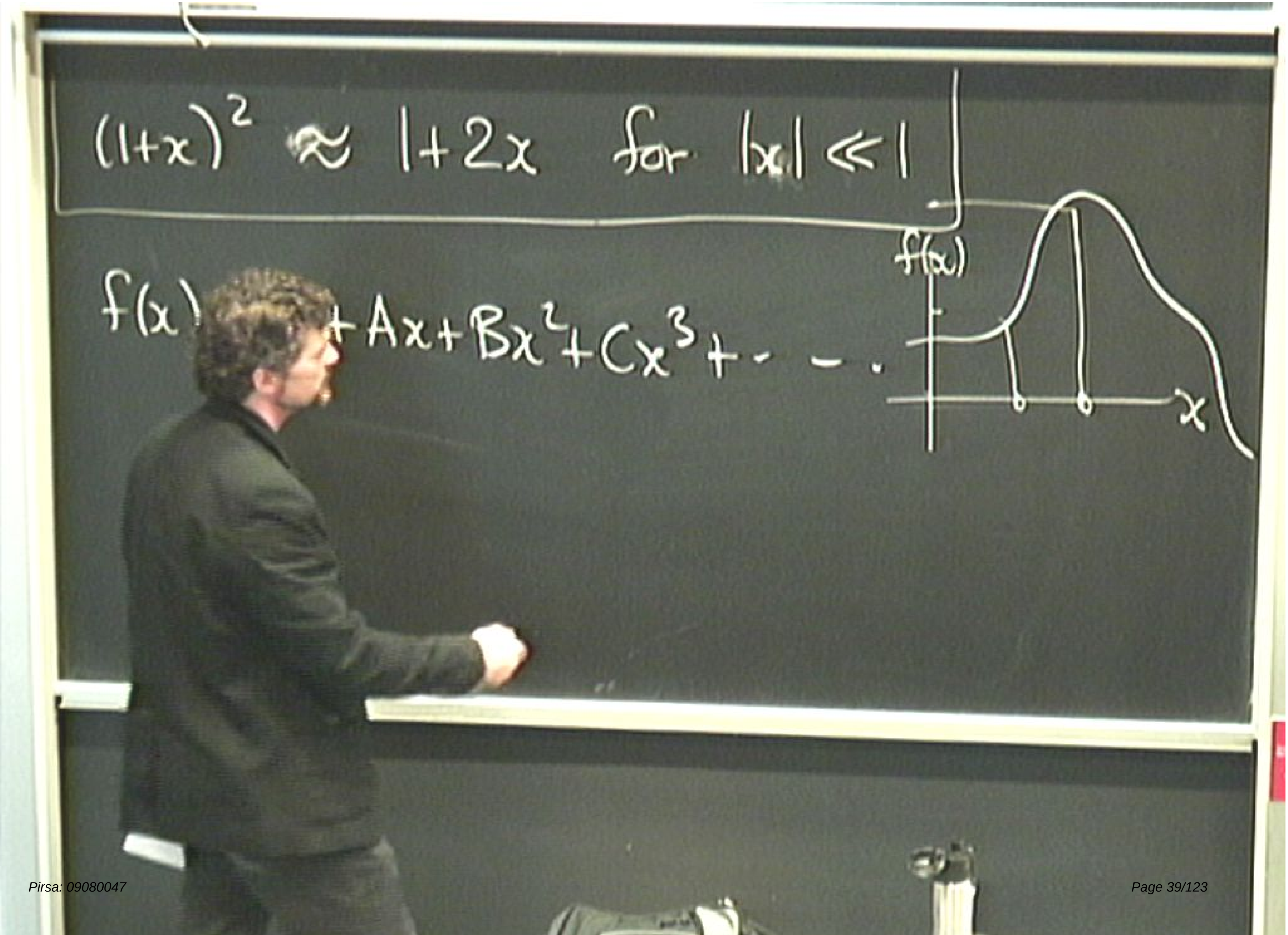
$$\sqrt{1.1} = (1+0.1)^{1/2} = 1 + \frac{1}{2}(0.1) - \frac{1}{8}(0.1)^2 + \frac{1}{16}(0.1)^3 + \cdots$$

$1 \quad .05 \quad 0.00125 \quad 0.0000625$

≈ 1.05

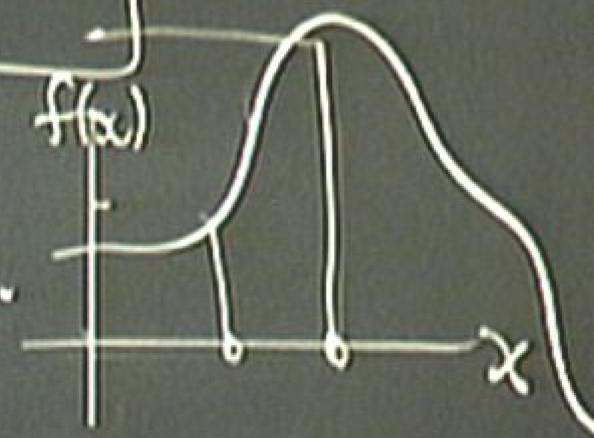

$(1+x)^2 = 1+2x+x^2$
$x=0{,}1 \to (1{,}1)^2 = 1+2(0{,}1)+(0{,}1)^2$
$= 1+0{,}2+0{,}01$
$= 1{,}21$
$\frac{0{,}01}{0{,}2} = 0{,}05$
$1 \to (1{,}01)^2 = 1+2(0{,}01)+(0{,}01)^2$
$= 1+0{,}02+$


$(1+x)^2 \approx 1+2x$ for $|x| \ll 1$


$(1+x)^2 \approx 1+2x$ for $|x| \ll 1$
$f(x)$
$+Ax+Bx^2+Cx^3+\ldots$
$f(x)$
x

$$\boxed{(1+x)^2 \approx 1+2x \quad \text{for } |x| \ll 1}$$

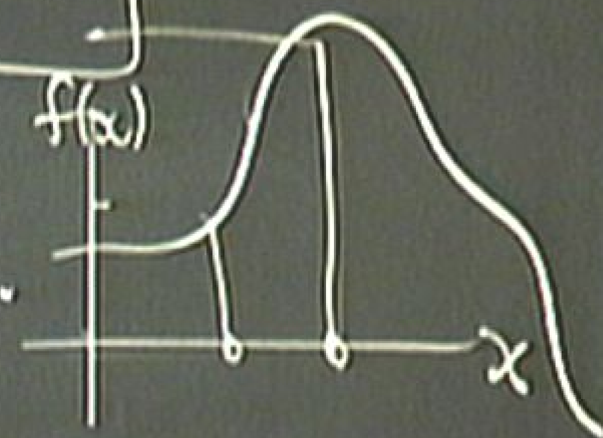
$$f(x) = 1 + Ax + Bx^2 + Cx^3 + \cdots$$



$$\boxed{(1+x)^2 \approx 1+2x \quad \text{for } |x| \ll 1}$$

$$f(x) = 1 + Ax + \cancel{Bx^2} + \cancel{Cx^3} + \cdots$$

$$\approx 1 + Ax \quad \text{for } |x| \ll 1$$

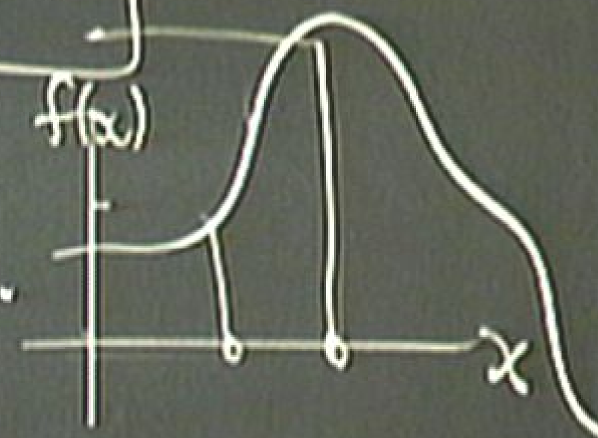


$$(1+x)^n$$

$$(1+x)^2 \approx 1+2x \quad \text{for } |x| \ll 1$$

$$f(x) = 1 + Ax + \cancel{Bx^2} \cancel{+ Cx^3} + \cdots$$

$$\approx 1 + Ax \quad \text{for } |x| \ll 1$$

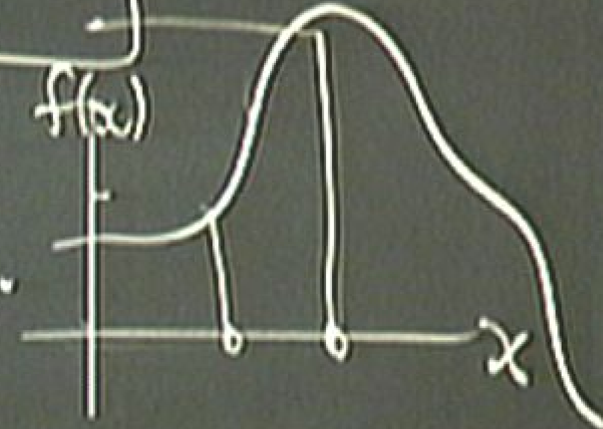


$$(1+x)^n \approx$$

$$\boxed{(1+x)^2 \approx 1+2x \quad \text{for } |x| \ll 1}$$

$$f(x) = 1 + Ax + \cancel{Bx^2} \cancel{+ Cx^3} + \cdots$$

$$\approx 1 + Ax \quad \text{for } |x| \ll 1$$



$$\boxed{(1+x)^n \approx 1+nx \quad \text{for } |x| \ll 1}$$

$$= 1 + \frac{n}{1} x + \frac{n}{1}\frac{(n-1)}{2} x^2 + \frac{n}{1}\frac{(n-1)}{2}\frac{(n-2)}{3} x^3 + \cdots$$

$(n = 0, 1, 2, 3, 4 \ldots) \longrightarrow$ series terminates.

e.g. $n = -1$ $\qquad x = -1/2$

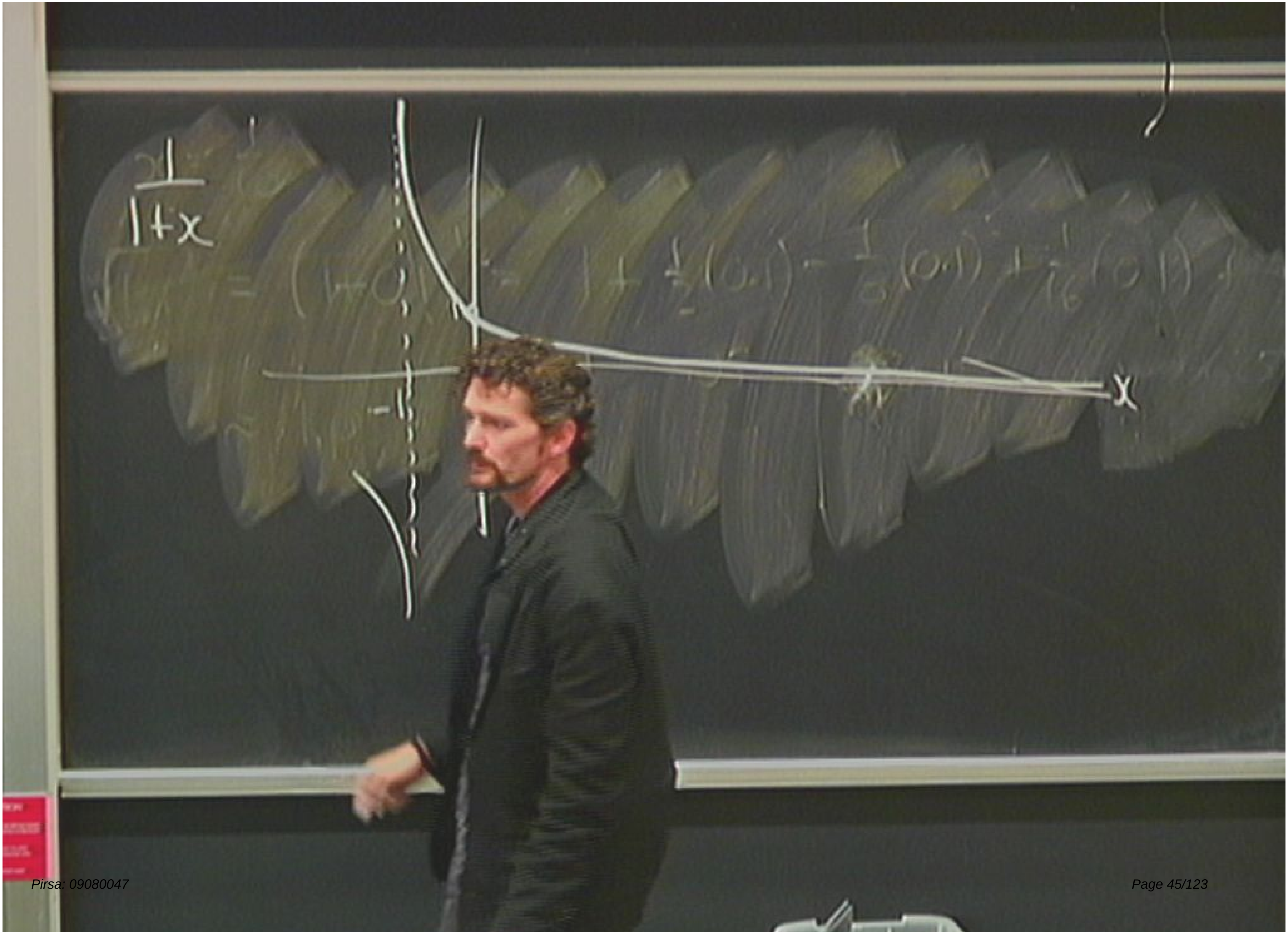
$$(1+x)^{-1} \stackrel{?}{=} 1 + \frac{(-1)}{1} x + \frac{(-1)}{1}\frac{(-1-1)}{2} x^2 + \frac{(-1)}{1}\frac{(-1-1)}{2}\frac{(-1-2)}{3} x^3 + \cdots$$

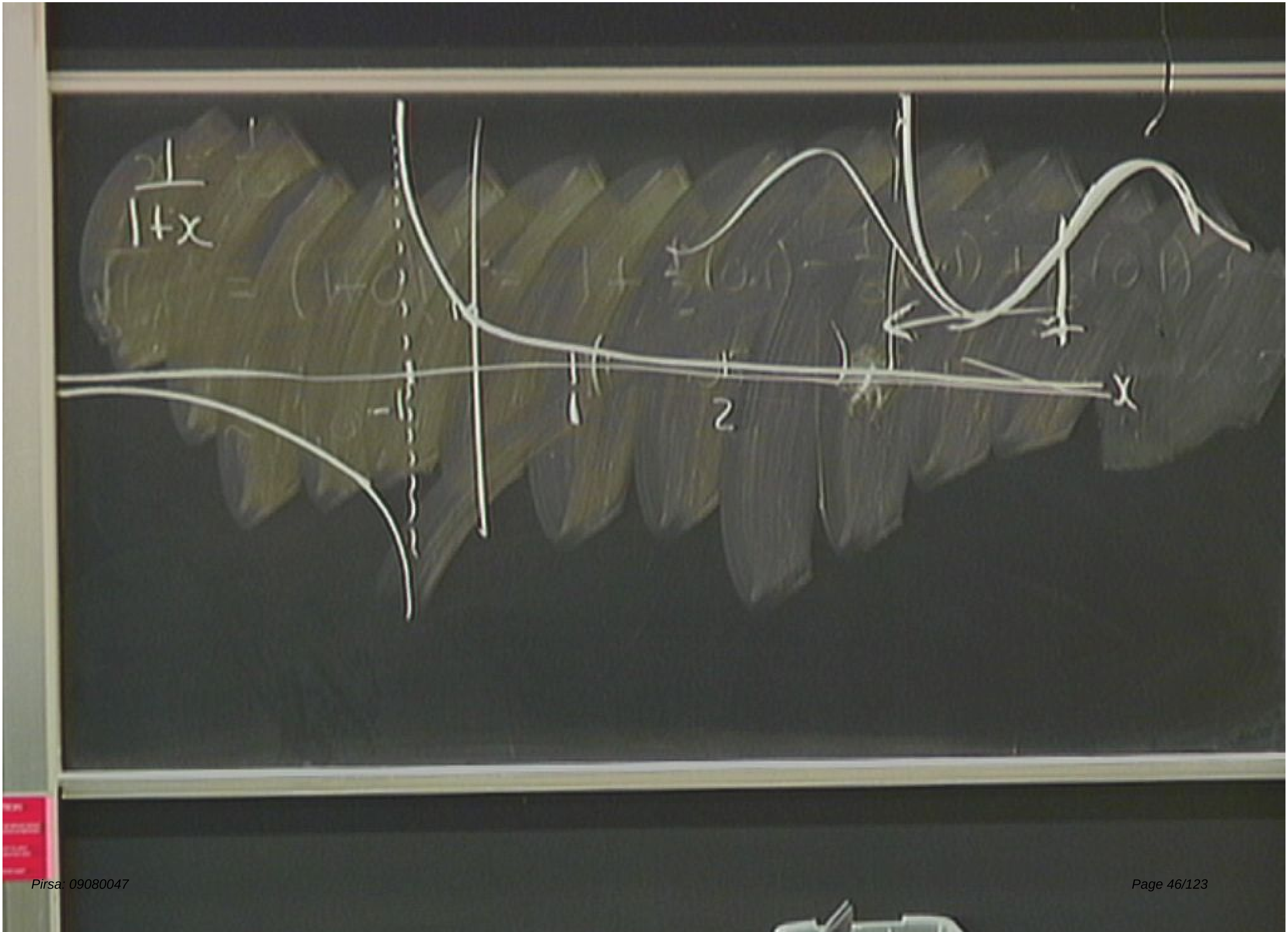
$x = -\frac{1}{2}$

2

$$\Rightarrow 1 - x + x^2 - x^3 + x^4 \mp \cdots \longrightarrow \text{infinite series.}$$

$$1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \cdots$$


1
1+x
x



$\frac{1}{1+x}$
-1
1
2
x

$$E^2 = m^2c^4 + p^2c^2$$

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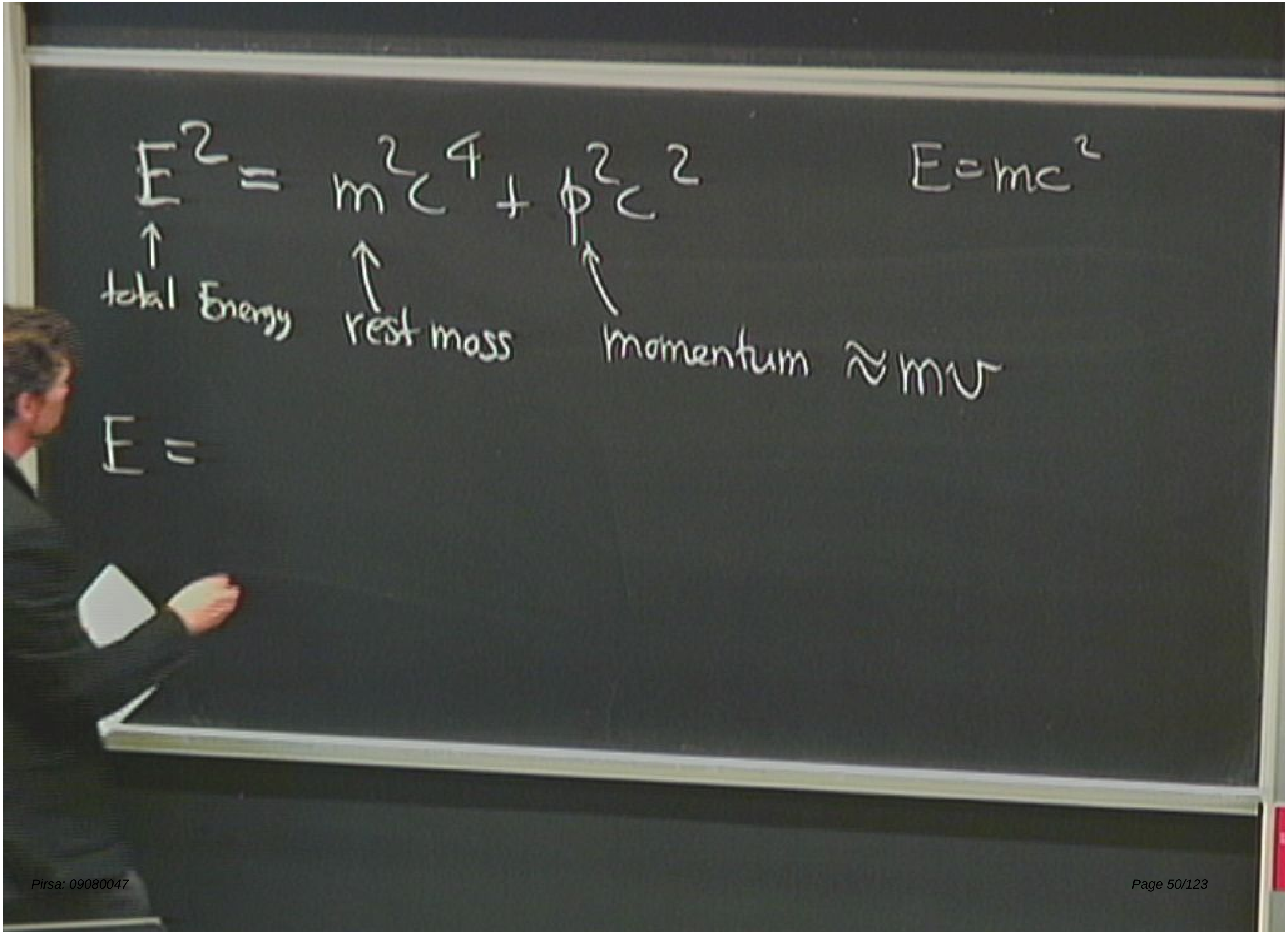
total Energy

rest mass

momentum $\approx mv$


$E^2 = m^2c^4 + p^2c^2$
$E = mc^2$
total Energy
rest mass
momentum $\approx mv$
$E =$

$$E^2 = m^2c^4 + p^2c^2$$

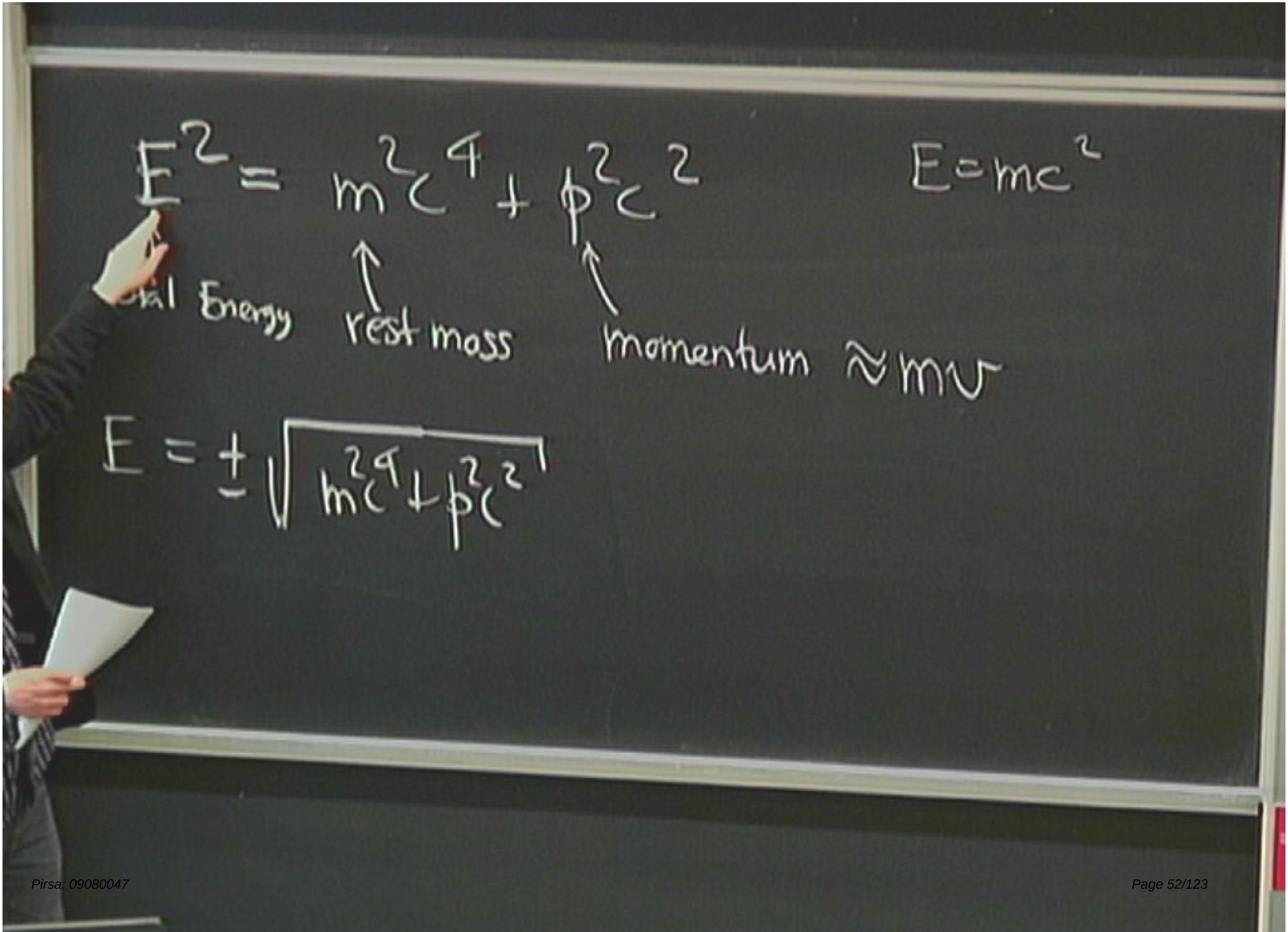
$$E = mc^2$$

total Energy

rest mass

momentum $\approx mv$

$$E = \sqrt{m^2c^4 + p^2c^2}$$


$E^2 = m^2c^4 + p^2c^2$
$E = mc^2$
tal Energy
rest mass
momentum $\approx mv$
$E = \pm\sqrt{m^2c^4 + p^2c^2}$

$$E^2 = m^2c^4 + p^2c^2$$

$E = mc^2$

total Energy

rest mass

momentum $\approx mv$

$$E = \pm\sqrt{m^2c^4 + p^2c^2}$$

$$E^2 = m^2c^4 + p^2c^2 \qquad E = mc^2$$

total Energy — rest mass — momentum $\approx mv$

$$E = \sqrt{m^2c^4 + p^2c^2} = \sqrt{(m^2c^4)\left(1 + \frac{p^2c^2}{m^2c^4}\right)}$$

$$= mc^2\left(1 + \frac{p^2c^2}{m^2c^4}\right)^{1/2}$$

$$p \approx mv$$

$$x = \frac{p^2c^2}{m^2c^4} = \frac{\cancel{m^2}v^2\cancel{c^2}}{\cancel{m^2}c^{\cancel{4}2}} = \left(\frac{v}{c}\right)^2$$

$$p \approx mv \qquad \frac{v}{c} \approx 0.0000027$$

$$x = \frac{p^2c^2}{m^2c^4} = \frac{\cancel{m^2}v^2\cancel{c^2}}{\cancel{m^2}c^{\cancel{4}2}} = \left(\frac{v}{c}\right)^2$$

$$p \approx mv \qquad \frac{v}{c} \approx 0.0000027$$

$$x = \frac{p^2c^2}{m^2c^4} = \frac{\cancel{m^2}v^2\cancel{c^2}}{\cancel{m^2}c^{\cancel{4}\,2}} = \left(\frac{v}{c}\right)^2$$

$$E \approx mc^2\left(1+\left(\frac{v}{c}\right)^2\right)^{1/2}$$

$$\approx mc^2\left(1+\frac{1}{2}\left(\frac{v}{c}\right)^2\right) = mc^2 +$$

$$p \approx mv \qquad \frac{v}{c} \approx 0.000027$$

$$x = \frac{p^2c^2}{m^2c^4} = \frac{\cancel{m^2}v^2\cancel{c^2}}{\cancel{m^2}c^{\cancel{4}2}} = \left(\frac{v}{c}\right)^2$$

$$E \approx mc^2\left(1+\left(\frac{v}{c}\right)^2\right)^{1/2}$$

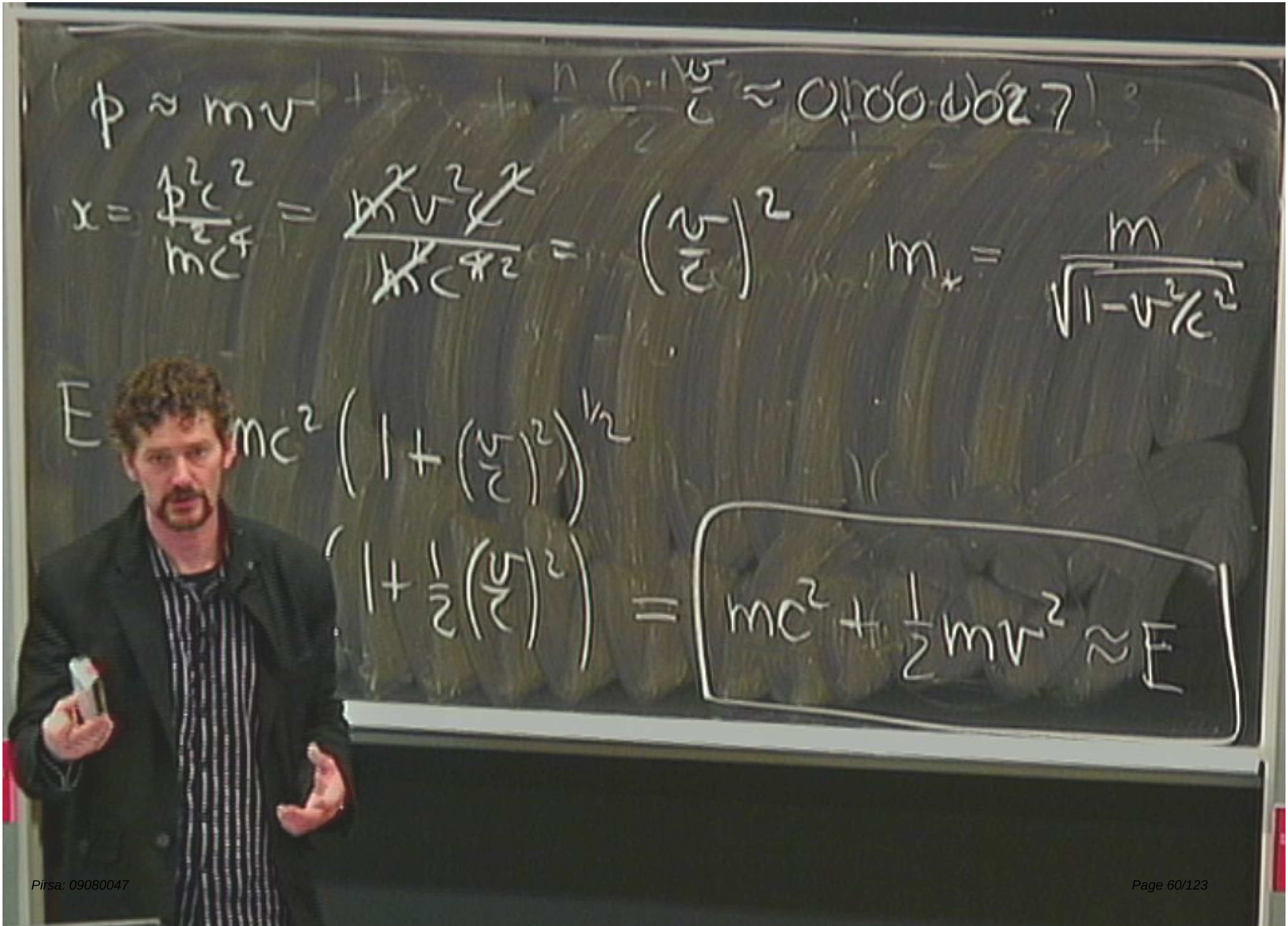
$$\approx mc^2\left(1+\frac{1}{2}\left(\frac{v}{c}\right)^2\right) = \boxed{mc^2+\frac{1}{2}mv^2 \approx E}$$

$p \approx mv$ $\qquad \frac{v}{c} \approx 0.0000027$

$$x = \frac{p^2c^2}{m^2c^4} = \frac{\cancel{m^2}v^2\cancel{c^2}}{\cancel{m^2}c^{\cancel{4}2}} = \left(\frac{v}{c}\right)^2$$

$$E \approx mc^2\left(1+\left(\frac{v}{c}\right)^2\right)^{1/2}$$

$$\approx mc^2\left(1+\frac{1}{2}\left(\frac{v}{c}\right)^2\right) = \boxed{mc^2 + \frac{1}{2}mv^2 \approx E}$$


p ≈ mv
≈ 0.000 0027
x = p²c² / m²c⁴ = m²v²c² / m²c⁴ = (v/c)²
m* = m / √(1 − v²/c²)
E = mc² (1 + (v/c)²)^½
(1 + ½ (v/c)²) = mc² + ½ mv² ≈ E

$$p \approx mv \qquad \frac{v}{c} \approx 0.0000027$$

$$x = \frac{p^2c^2}{m^2c^4} = \frac{\cancel{m^2}v^2\cancel{c^2}}{\cancel{m^2}c^{\cancel{4}\,2}} = \left(\frac{v}{c}\right)^2 \qquad m_* = \frac{m}{\sqrt{1-v^2/c^2}}$$

$$E \approx mc^2\left(1+\left(\frac{v}{c}\right)^2\right)^{1/2}$$

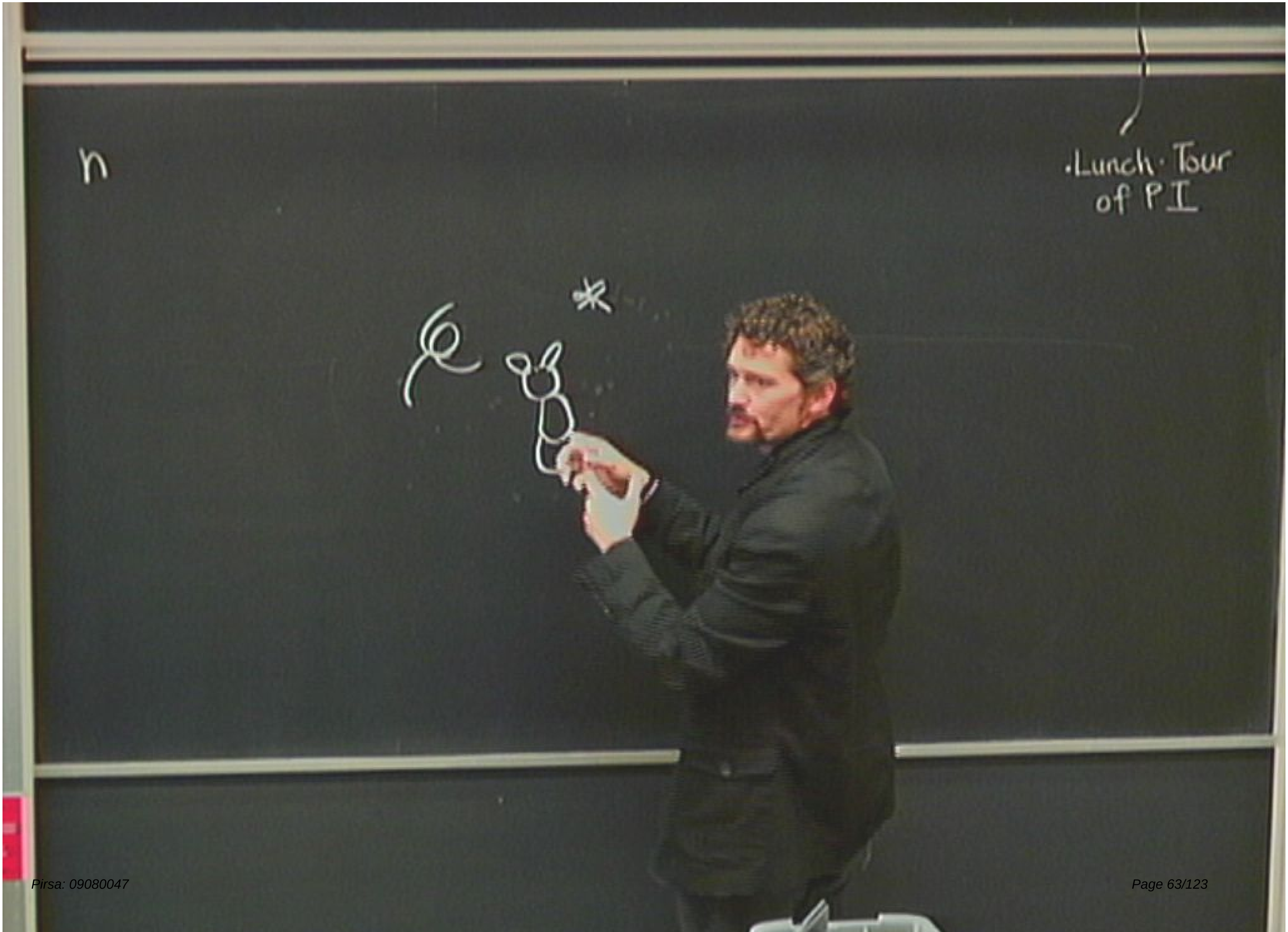
$$\approx mc^2\left(1+\frac{1}{2}\left(\frac{v}{c}\right)^2\right) = \boxed{mc^2+\frac{1}{2}mv^2 \approx E}$$

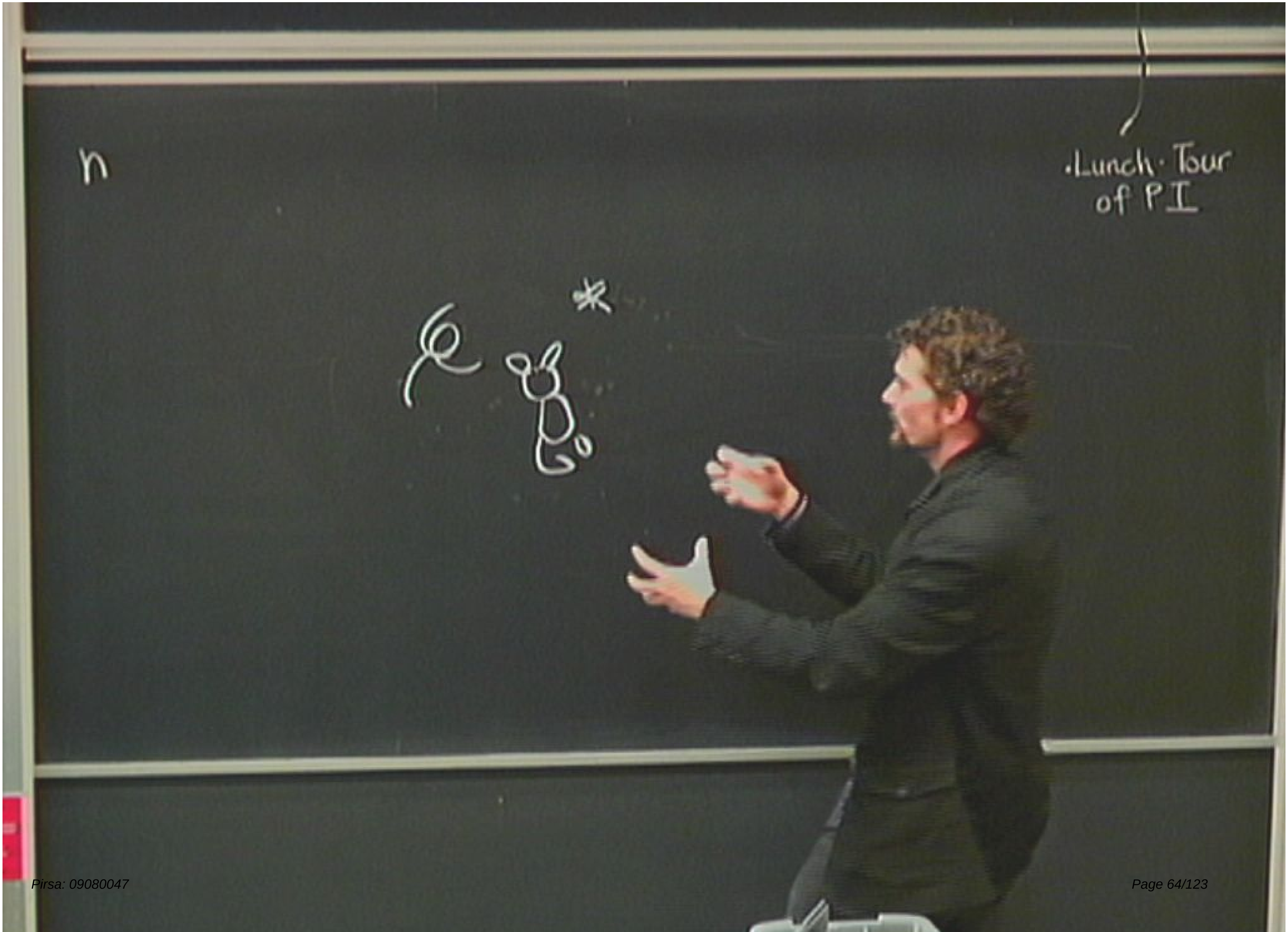
$$p \approx mv \qquad \frac{v}{c} \approx 0.0000027$$

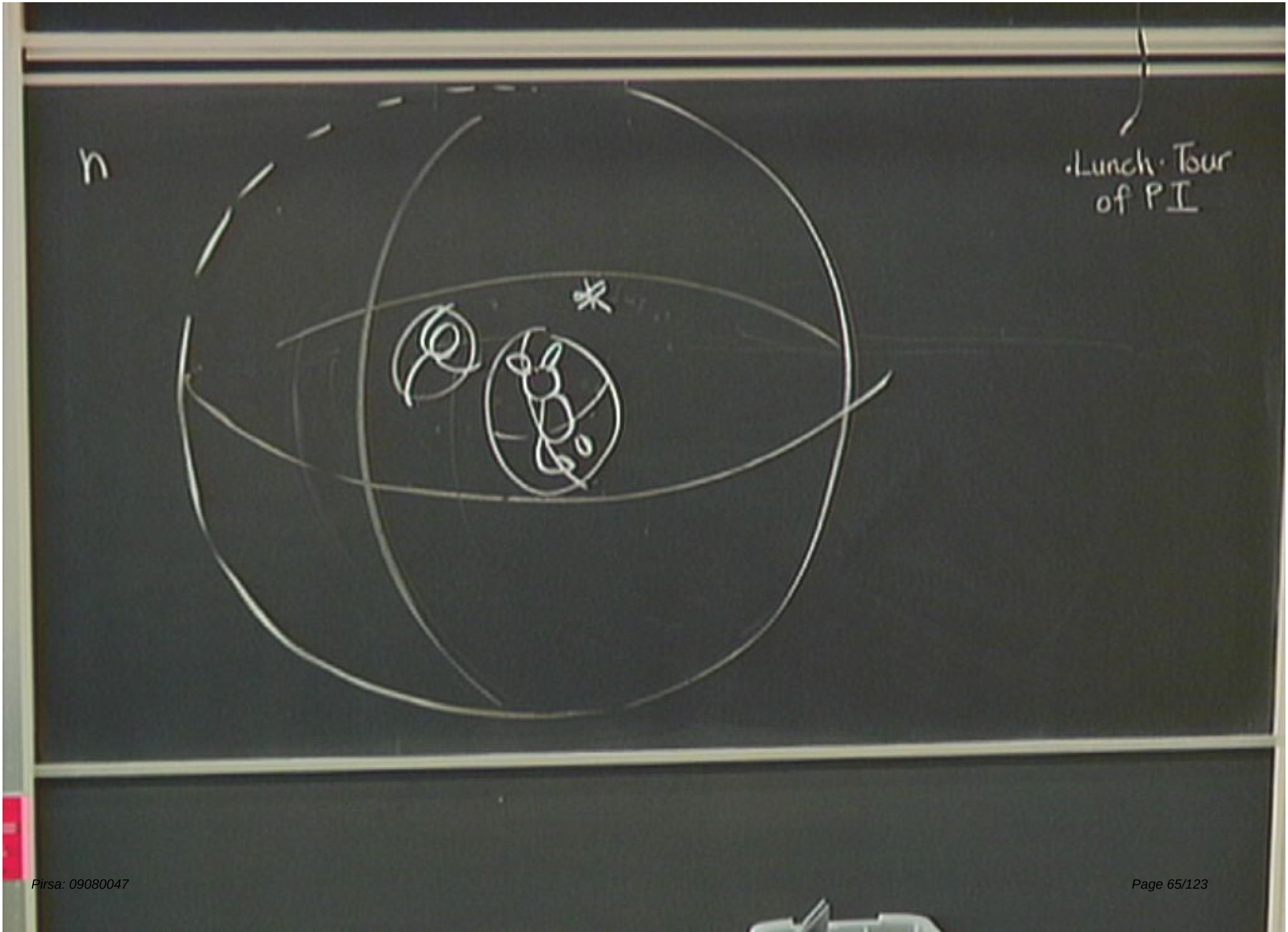
$$x = \frac{p^2c^2}{m^2c^4} = \frac{\cancel{m^2}v^2\cancel{c^2}}{\cancel{m^2}c^{\cancel{4}2}} = \left(\frac{v}{c}\right)^2 \qquad m_* = \frac{m}{\sqrt{1 - v^2/c^2}}$$

$$E \approx mc^2\left(1 + \left(\frac{v}{c}\right)^2\right)^{1/2}$$

$$\approx mc^2\left(1 + \frac{1}{2}\left(\frac{v}{c}\right)^2\right) = \boxed{mc^2 + \frac{1}{2}mv^2 \approx E}$$


n
•Lunch • Tour
of PI


n
·Lunch · Tour
of PI


n
·Lunch · Tour of PI

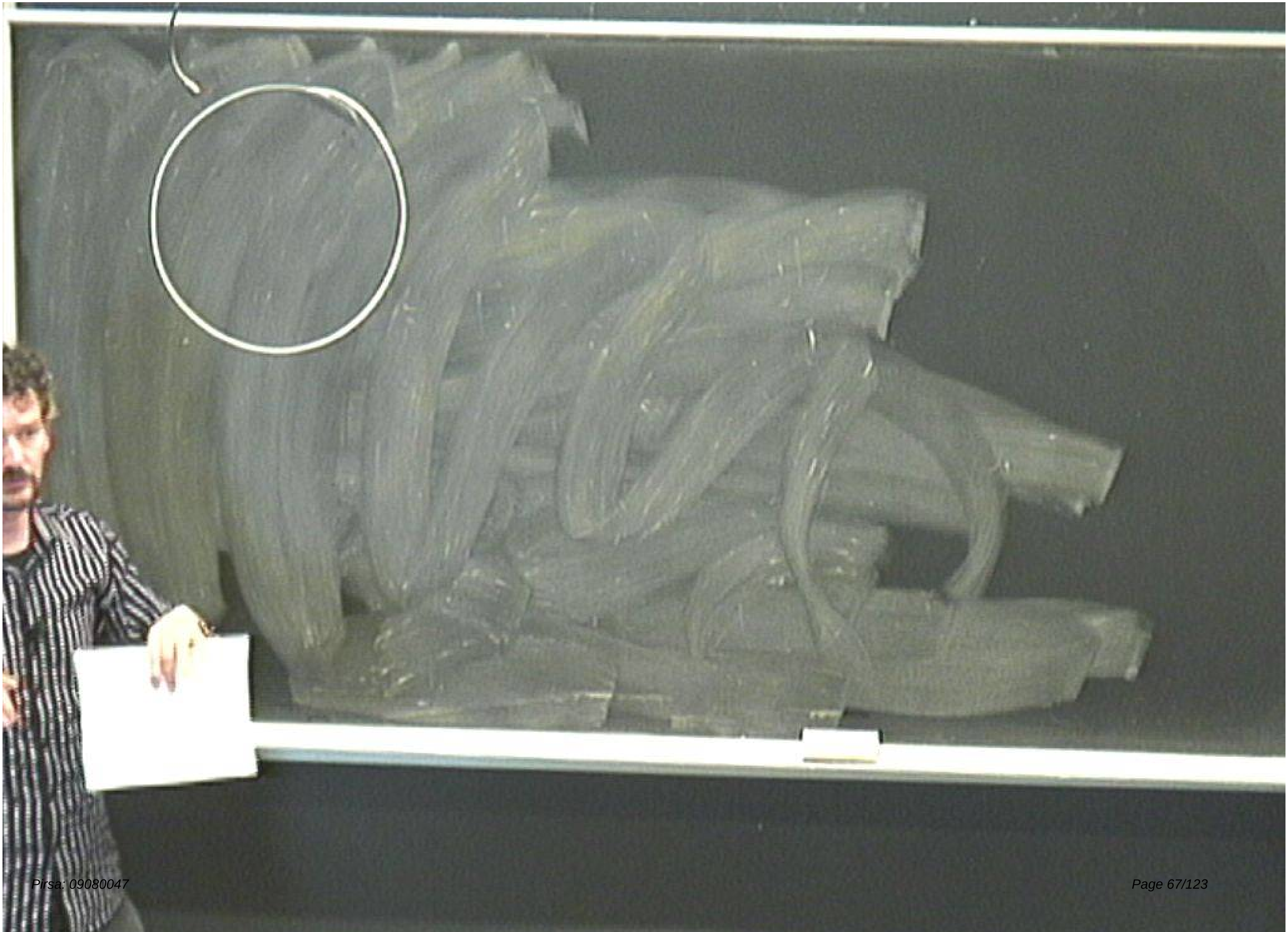
$$(1+x)^n = 1 + \frac{n}{1}x + \frac{n}{1}\frac{(n-1)}{2}x^2 + \frac{n}{1}\frac{(n-1)}{2}\frac{(n-2)}{3}x^3 + \ldots$$

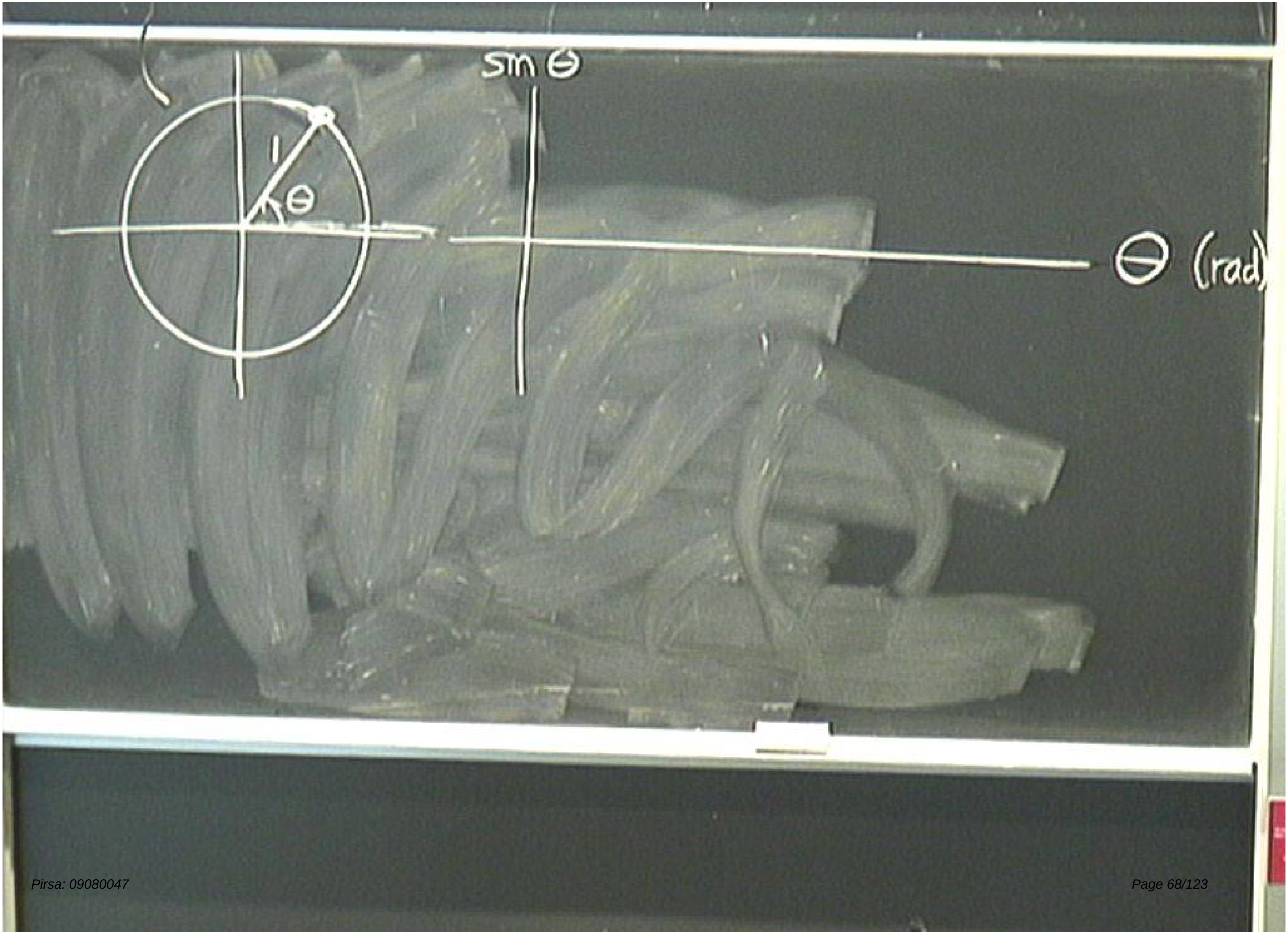
$$f(x) = a + bx + cx^2 + \ldots$$

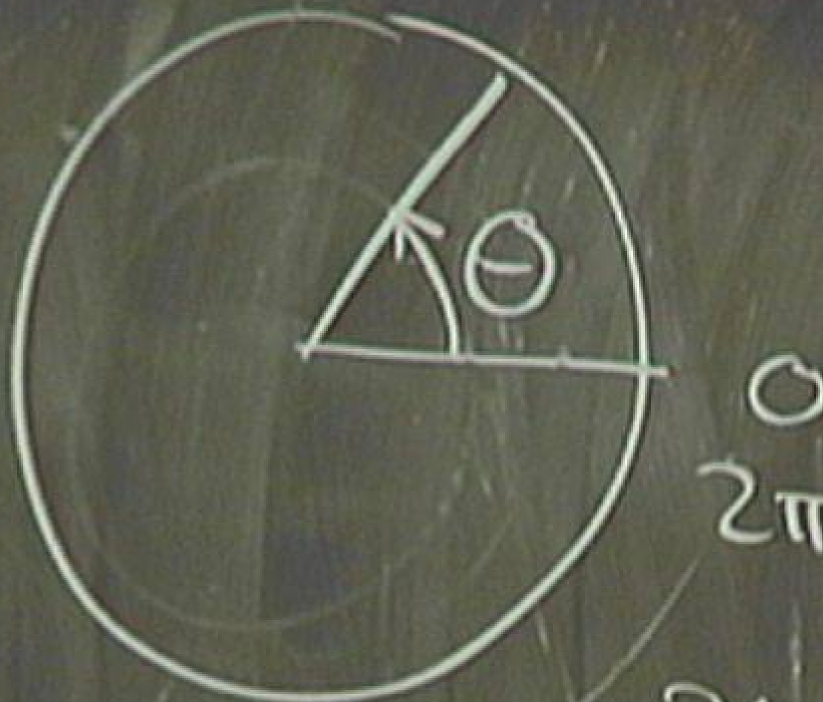
$\sin(x)$

$\cos(x)$

e^x

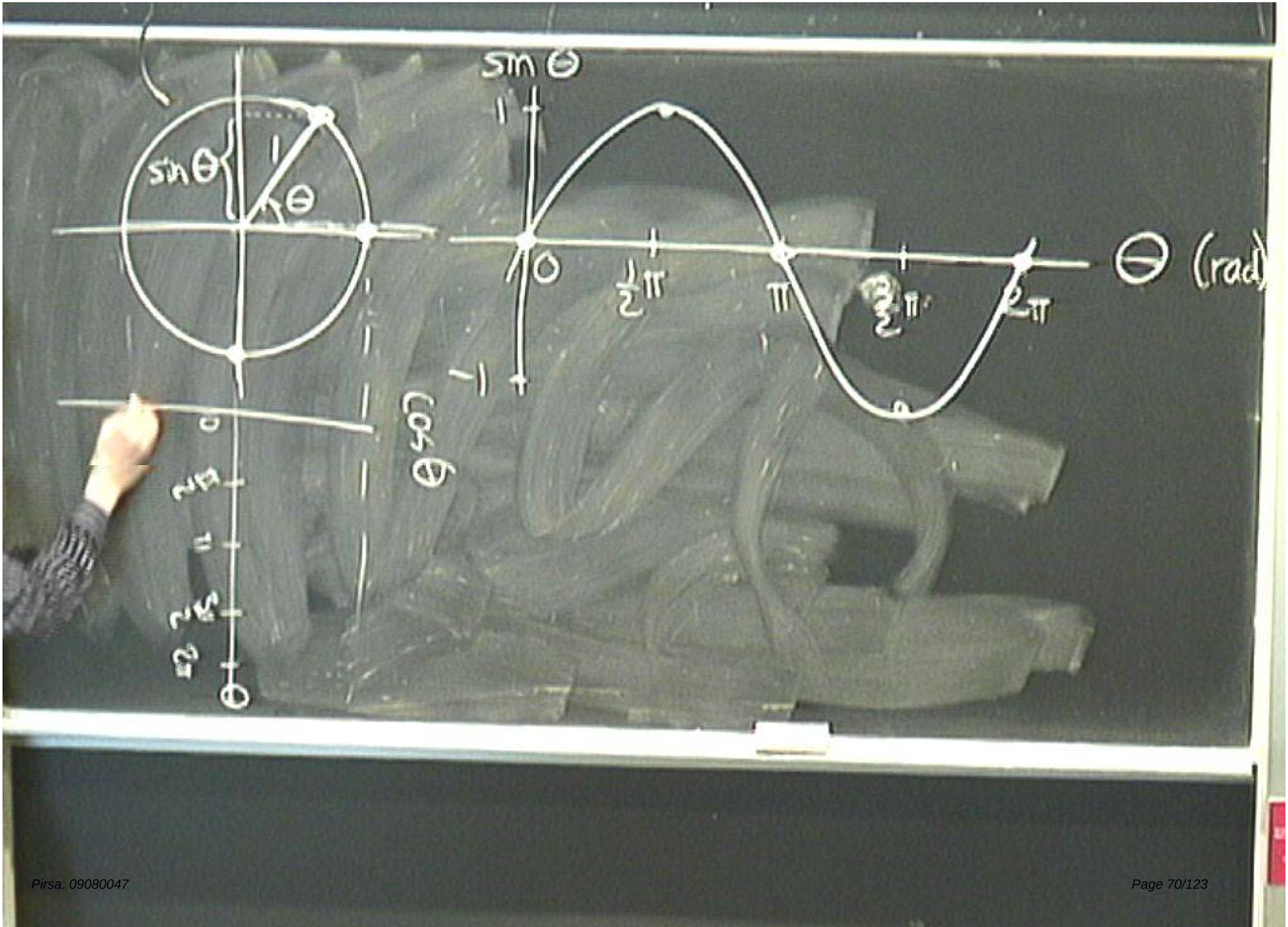



sin θ
1
θ
θ (rad)



$$2\pi \approx 6.28$$

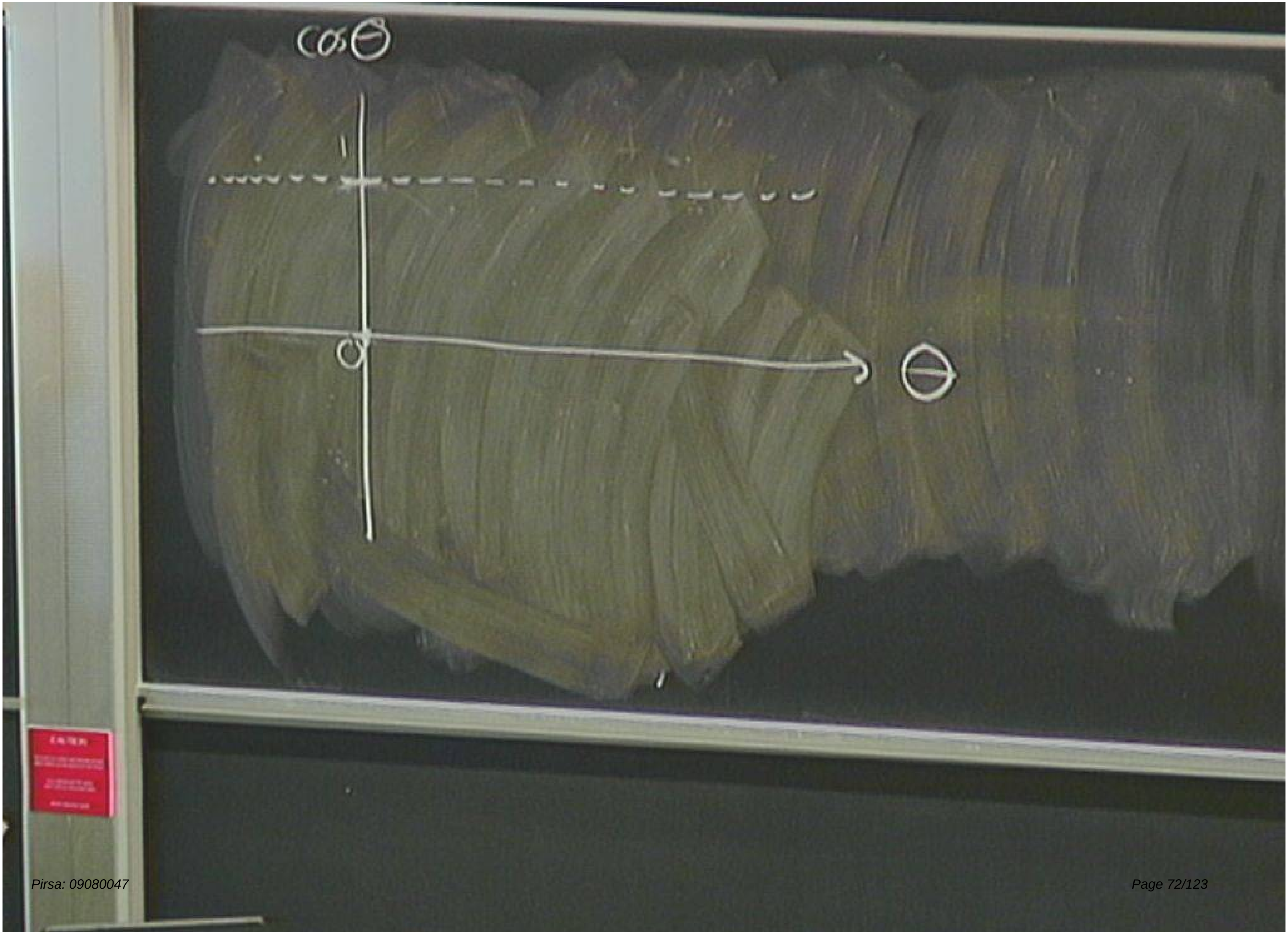
$$360^\circ = 2\pi \text{ rad.}$$

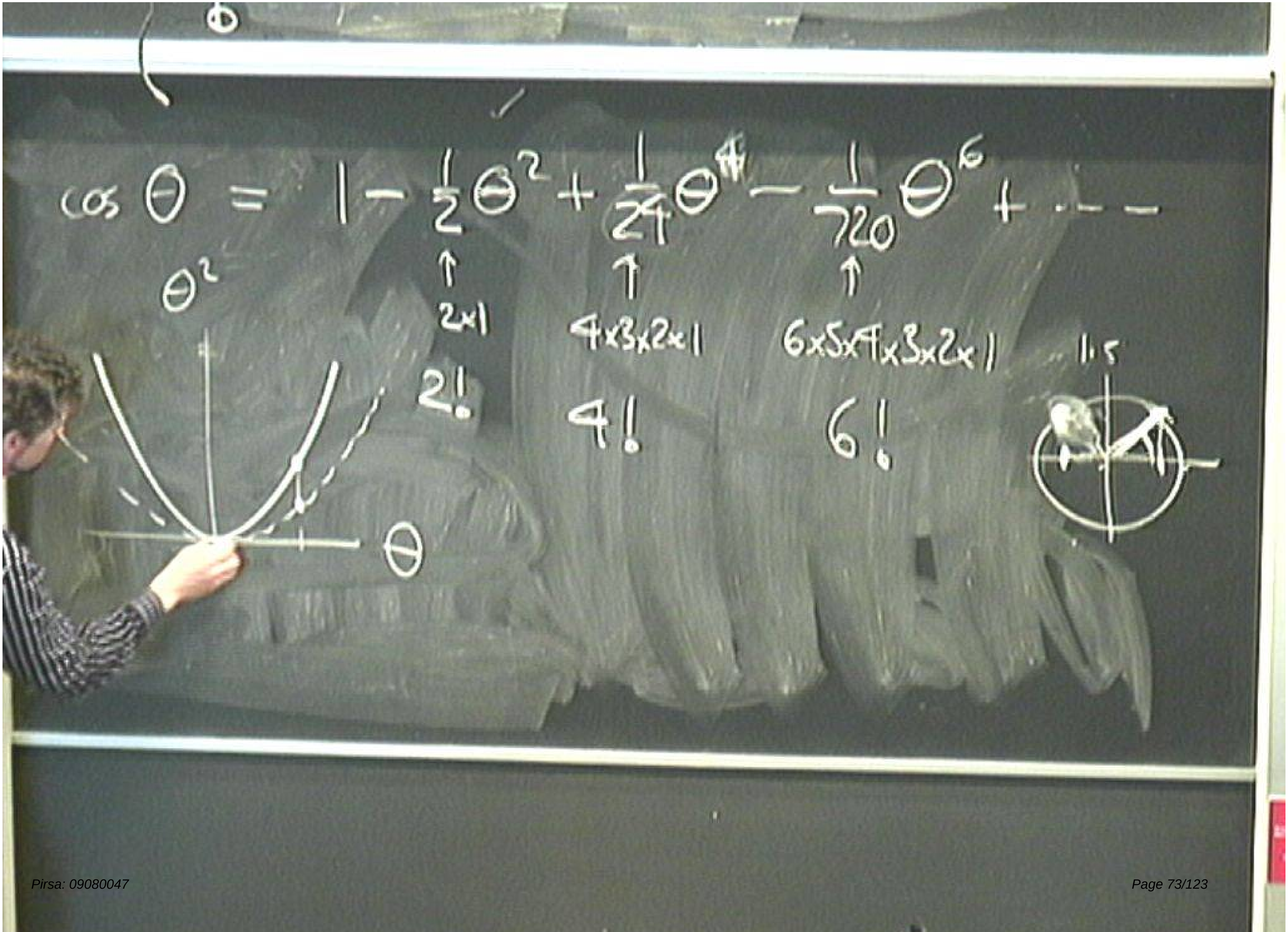


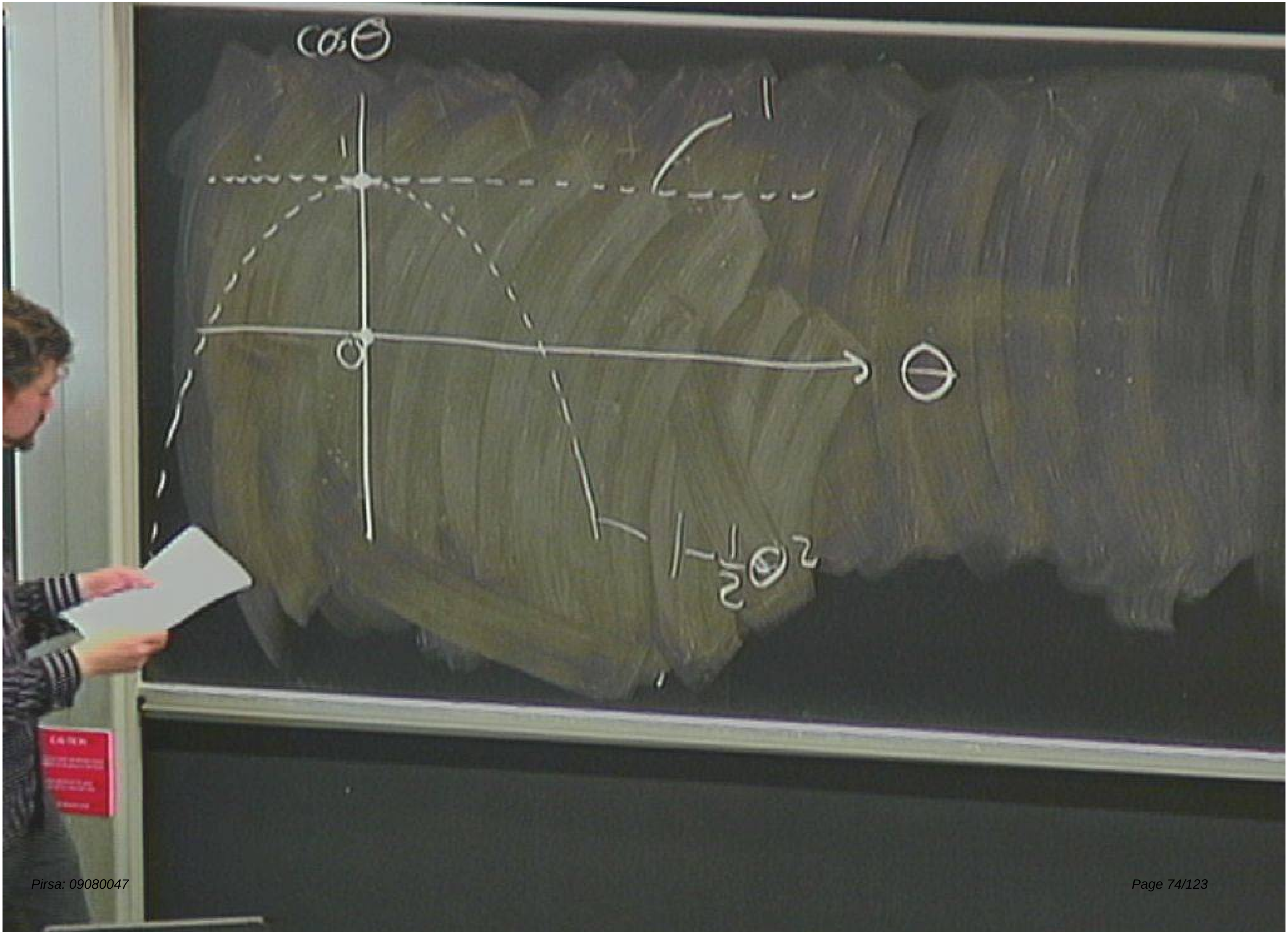
sin θ
1
θ
sin θ
1
0
½π
π
3/2π
2π
θ (rad)
-1
cos θ

$$\cos\theta = 1 - \frac{1}{2}\theta^2 + \frac{1}{24}\theta^4 - \frac{1}{720}\theta^6 + \cdots$$

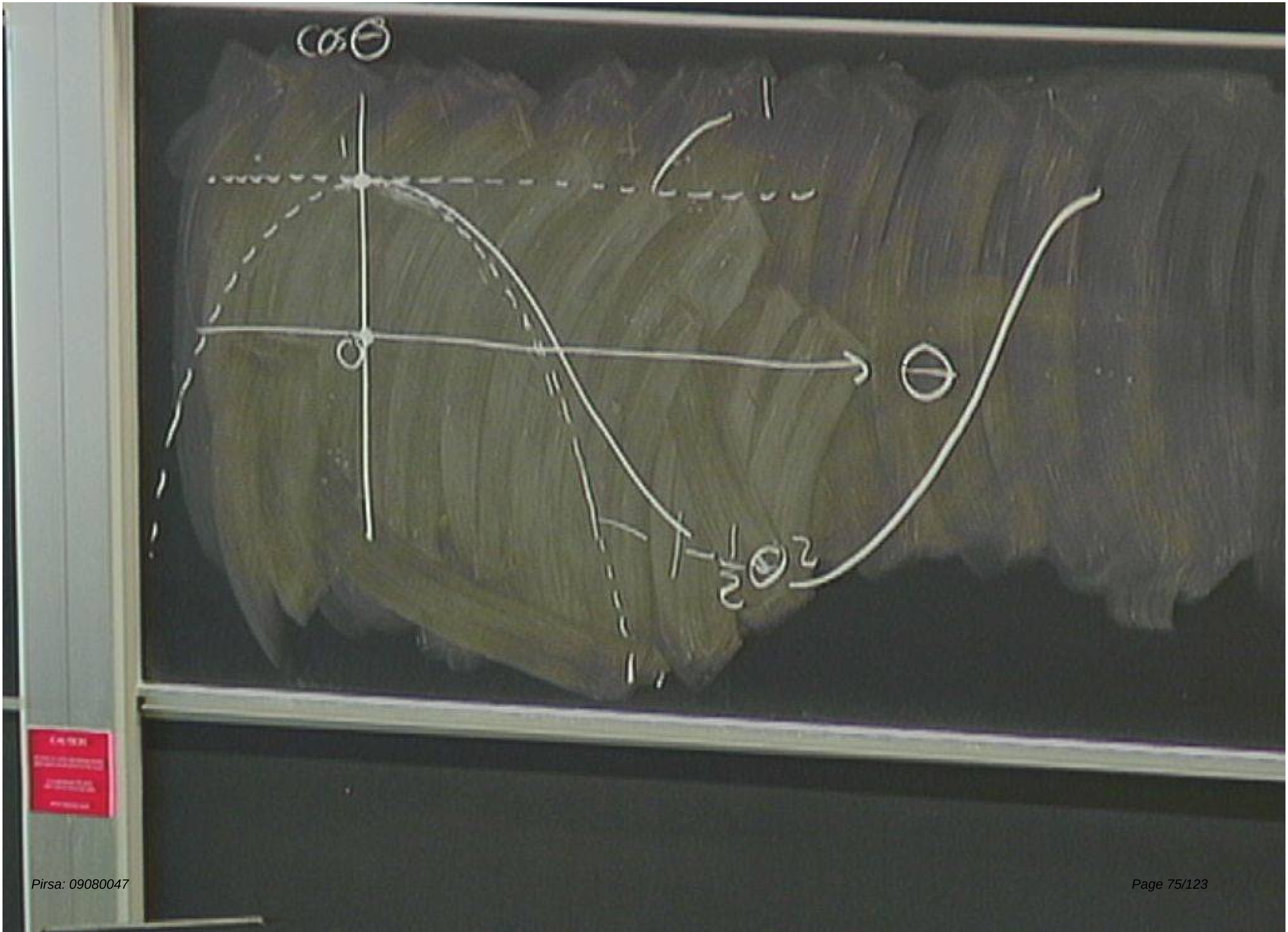
$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 2\times 1 & 4\times 3\times 2\times 1 & 6\times 5\times 4\times 3\times 2\times 1 \\ 2! & 4! & 6! \end{array}$$

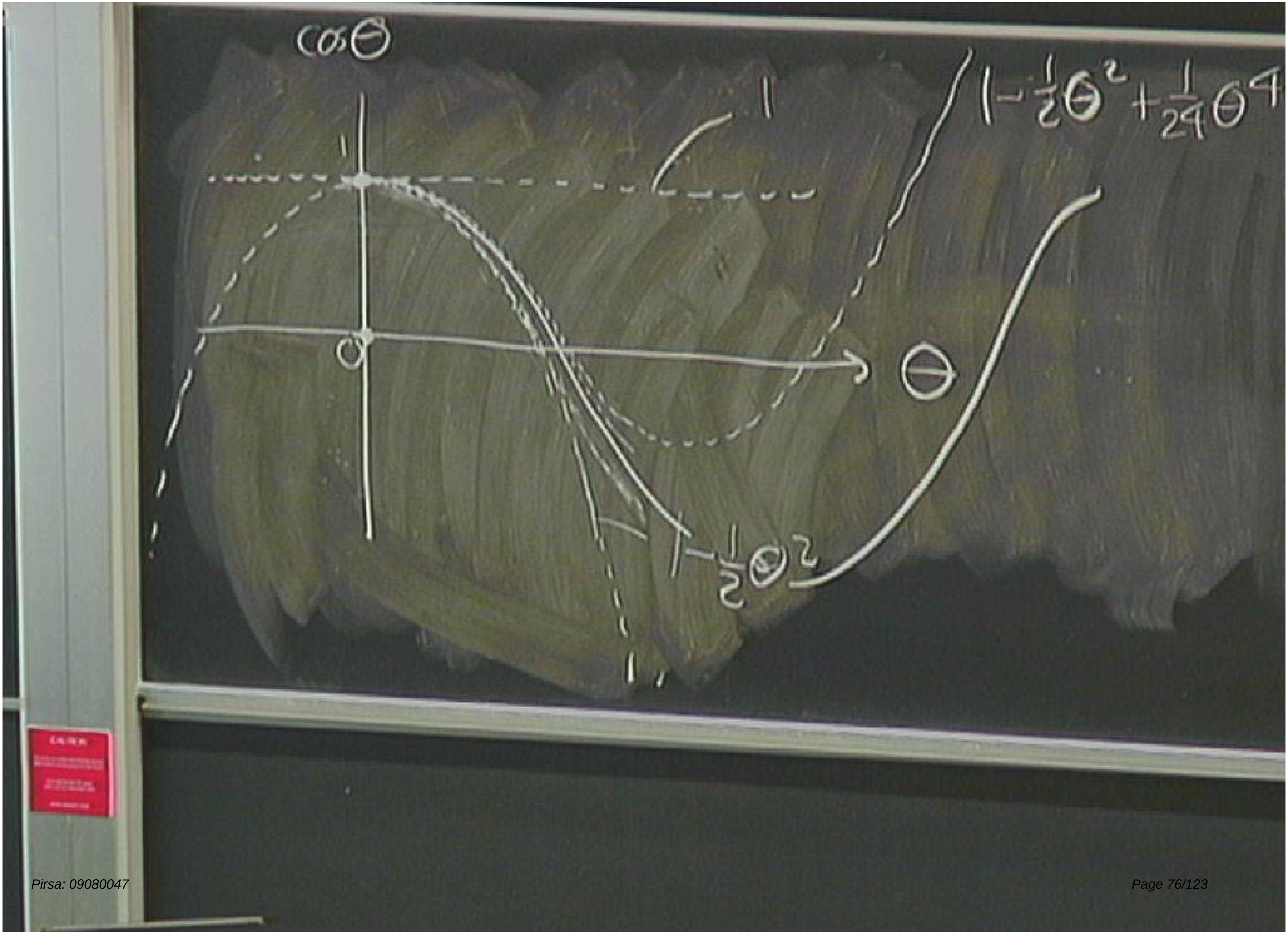

cos Θ
1
0
Θ

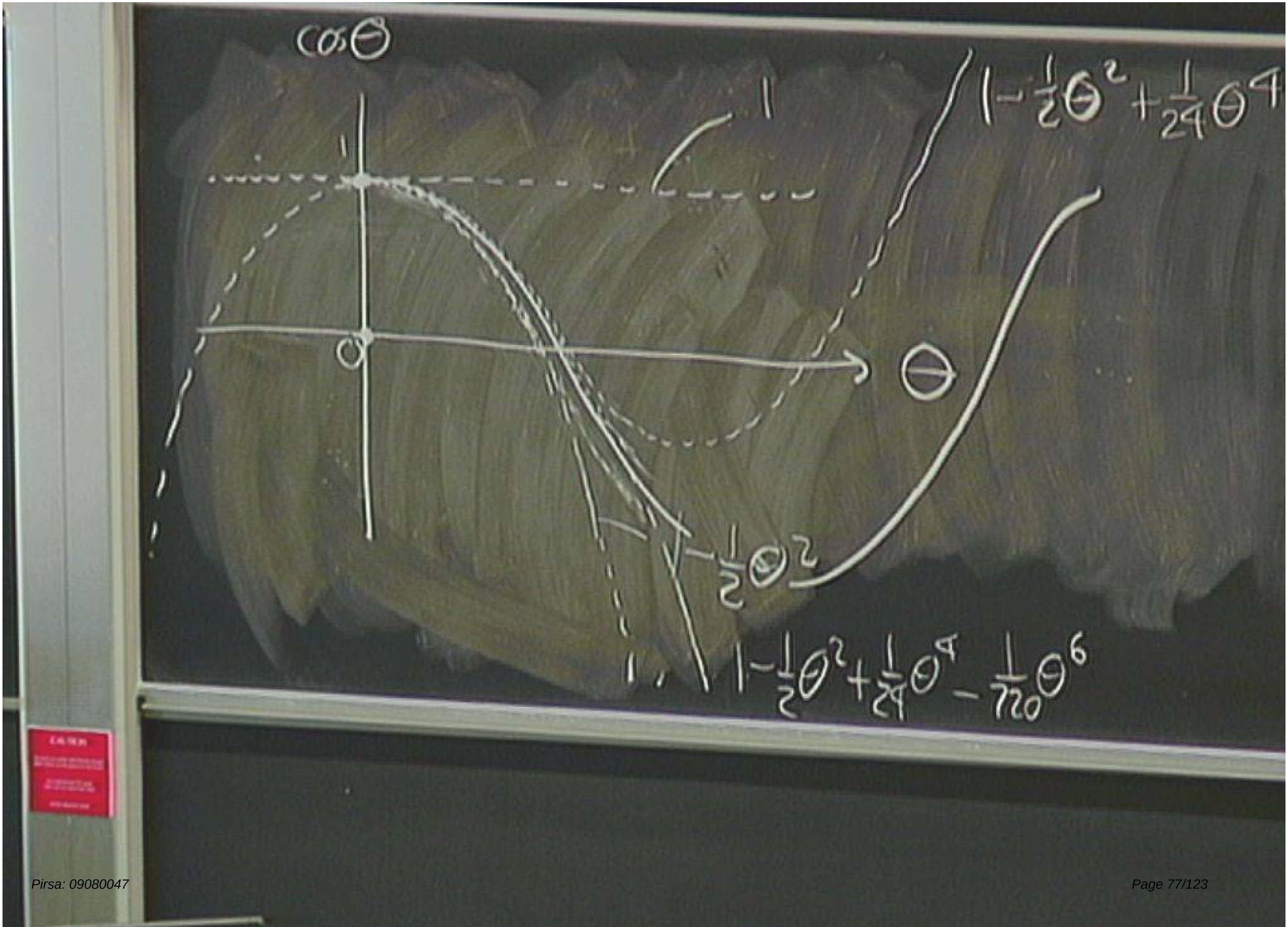

cos θ = 1 − 1/2 θ² + 1/24 θ⁴ − 1/720 θ⁶ + - - -
2x1
4x3x2x1
6x5x4x3x2x1
2!
4!
6!
θ²
θ
1.5



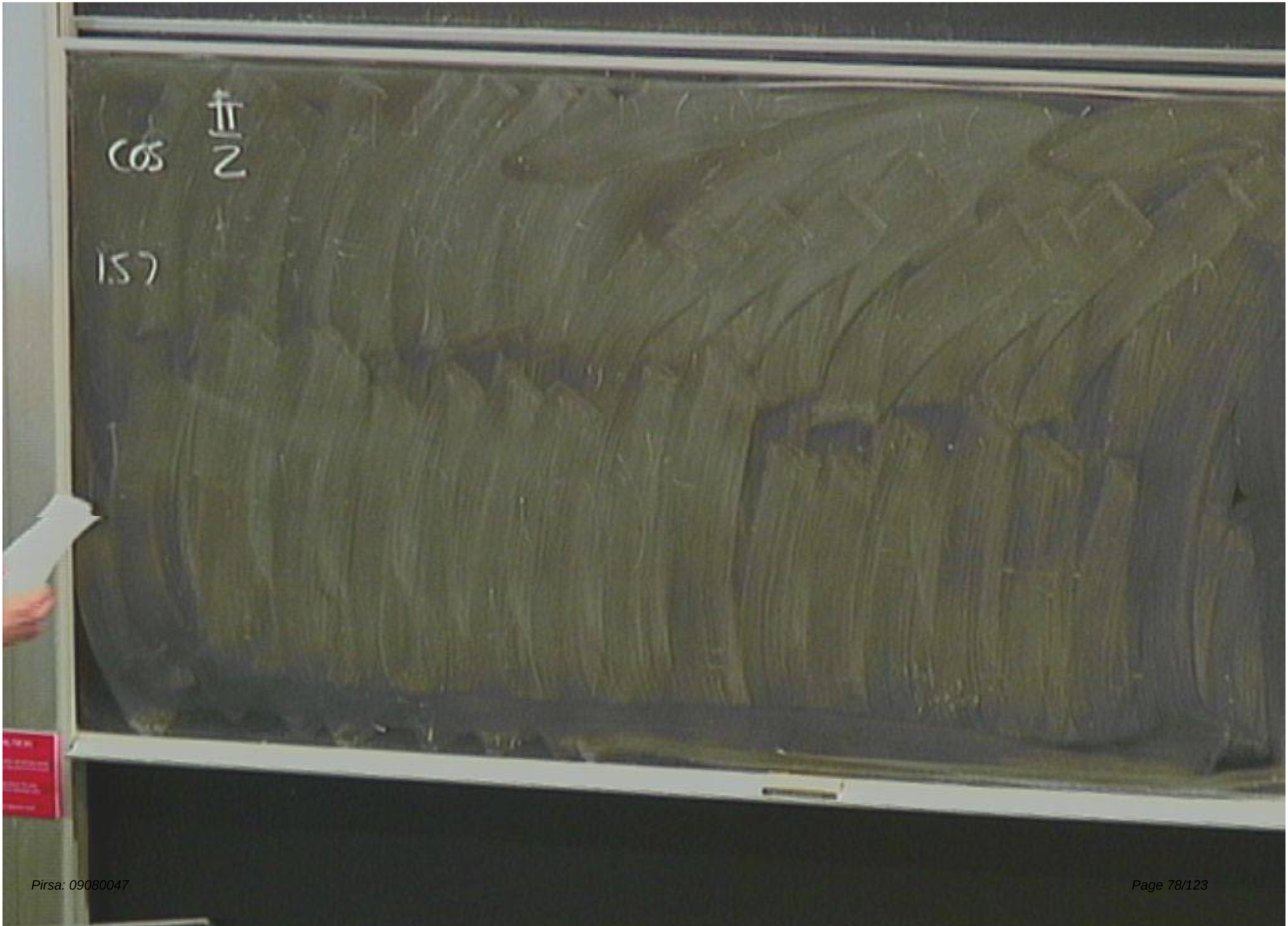
cos θ
1
1
0
θ
1 − ½θ²


cos Θ
1
0
Θ
1 − ½ Θ²


cos Θ
1
1
Θ
1 − ½Θ² + 1/24 Θ⁴
1 − ½Θ²
0



$\cos\theta$
1
0
θ
1
$1-\frac{1}{2}\theta^2+\frac{1}{24}\theta^4$
$1-\frac{1}{2}\theta^2$
$1-\frac{1}{2}\theta^2+\frac{1}{24}\theta^4-\frac{1}{720}\theta^6$


cos $\frac{\pi}{2}$
1.57

$$\cos \frac{\pi}{2} = 0$$

$$1 - \frac{1}{2}\left(\frac{\pi}{2}\right)^2 = -0.23$$

$$\cos\frac{\pi}{2} = 0$$

$$1 - \frac{1}{2}\left(\frac{\pi}{2}\right)^2 = -0.23$$

$$1 - \frac{1}{2}\left(\frac{\pi}{2}\right)^2 + \frac{1}{24}\left(\frac{\pi}{2}\right)^4 =$$

$$\cos\frac{\pi}{2} = 0$$

$$1 - \frac{1}{2}\left(\frac{\pi}{2}\right)^2 = -0.23$$

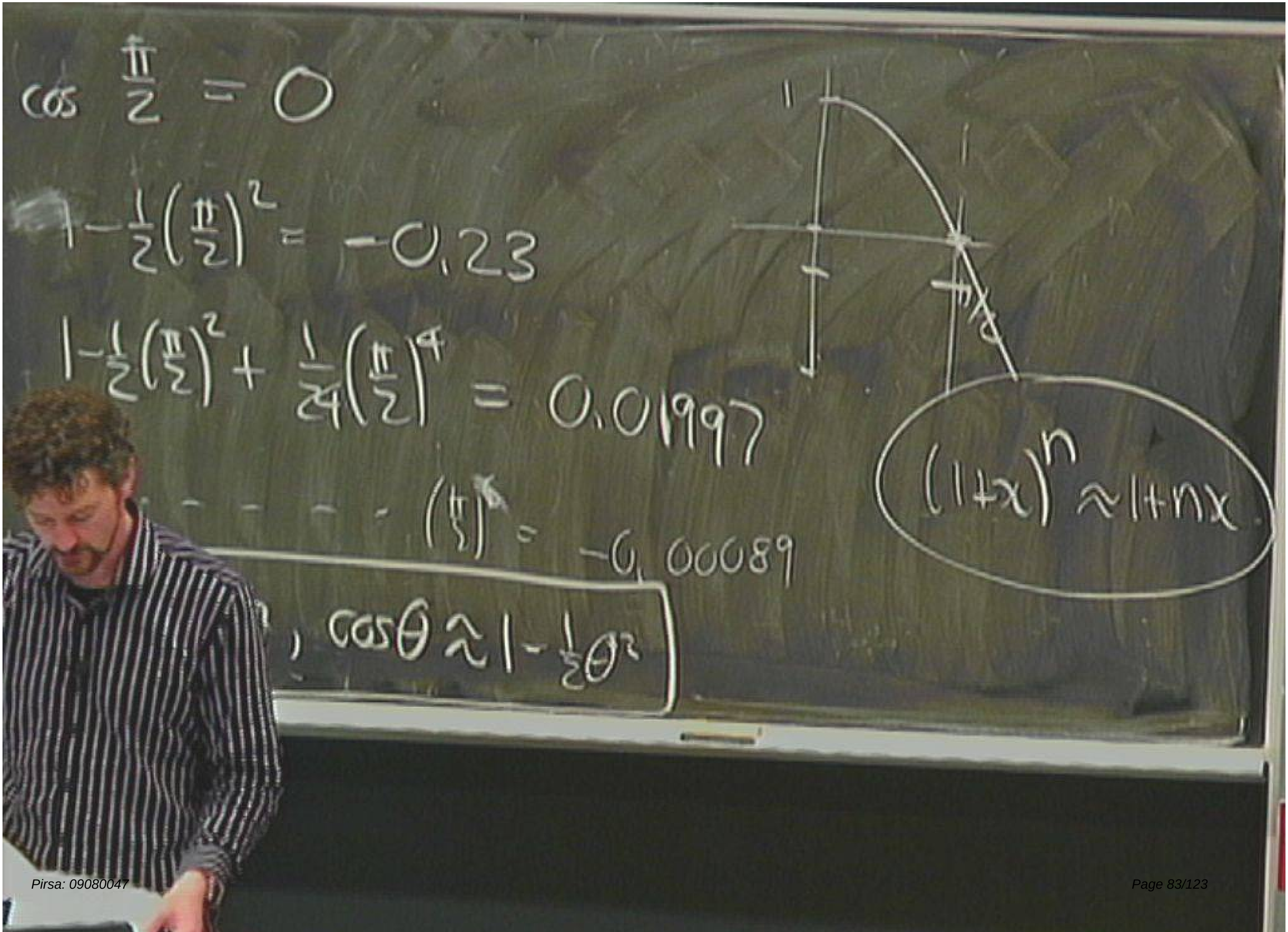
$$1 - \frac{1}{2}\left(\frac{\pi}{2}\right)^2 + \frac{1}{24}\left(\frac{\pi}{2}\right)^4 = 0.01997$$

$$\cos\frac{\pi}{2} = 0$$

$$1 - \frac{1}{2}\left(\frac{\pi}{2}\right)^2 = -0.23$$

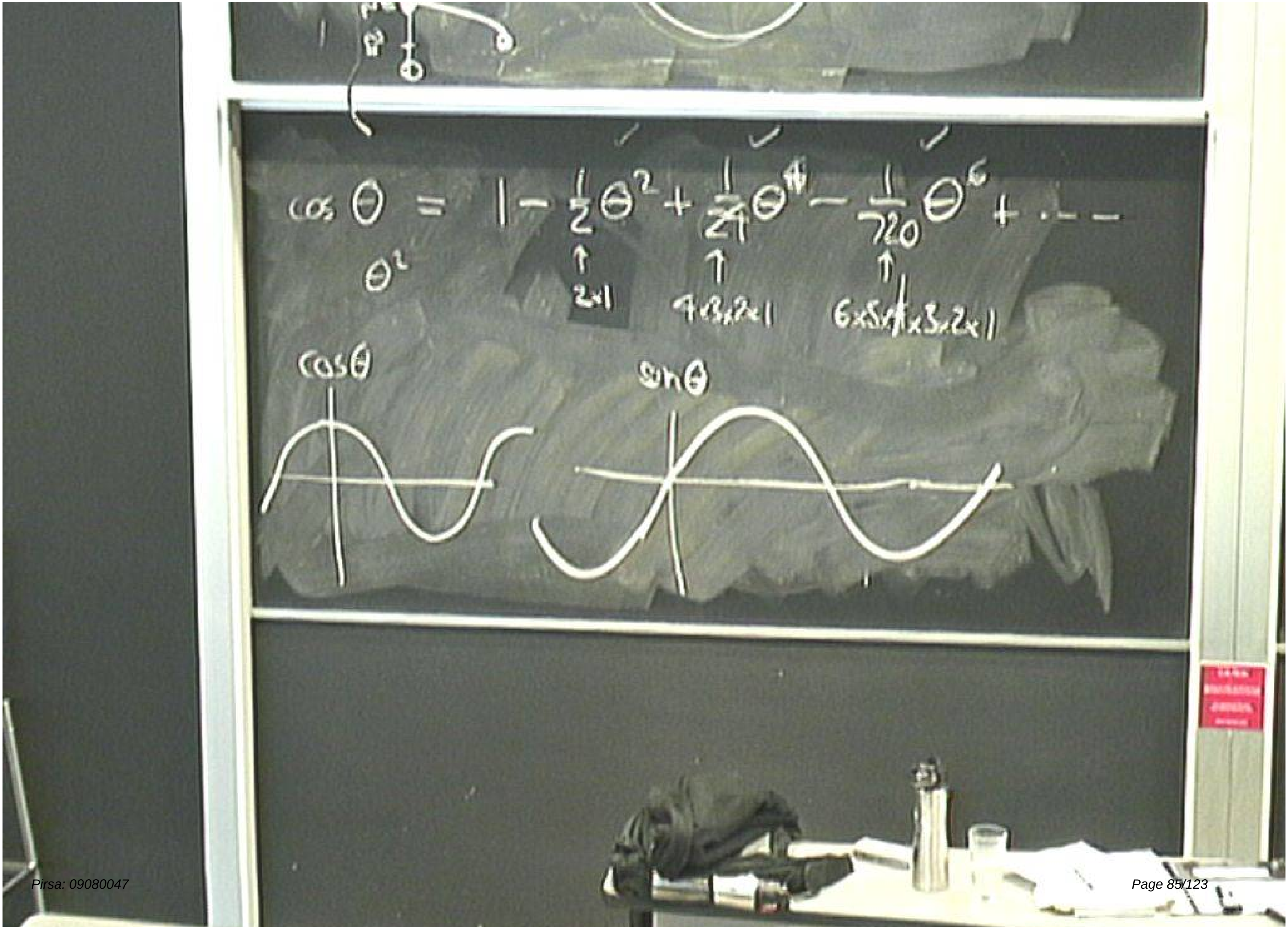
$$1 - \frac{1}{2}\left(\frac{\pi}{2}\right)^2 + \frac{1}{24}\left(\frac{\pi}{2}\right)^4 = 0.01997$$

$$1 \ldots - - - - \left(\frac{\pi}{2}\right) = -0.00089$$

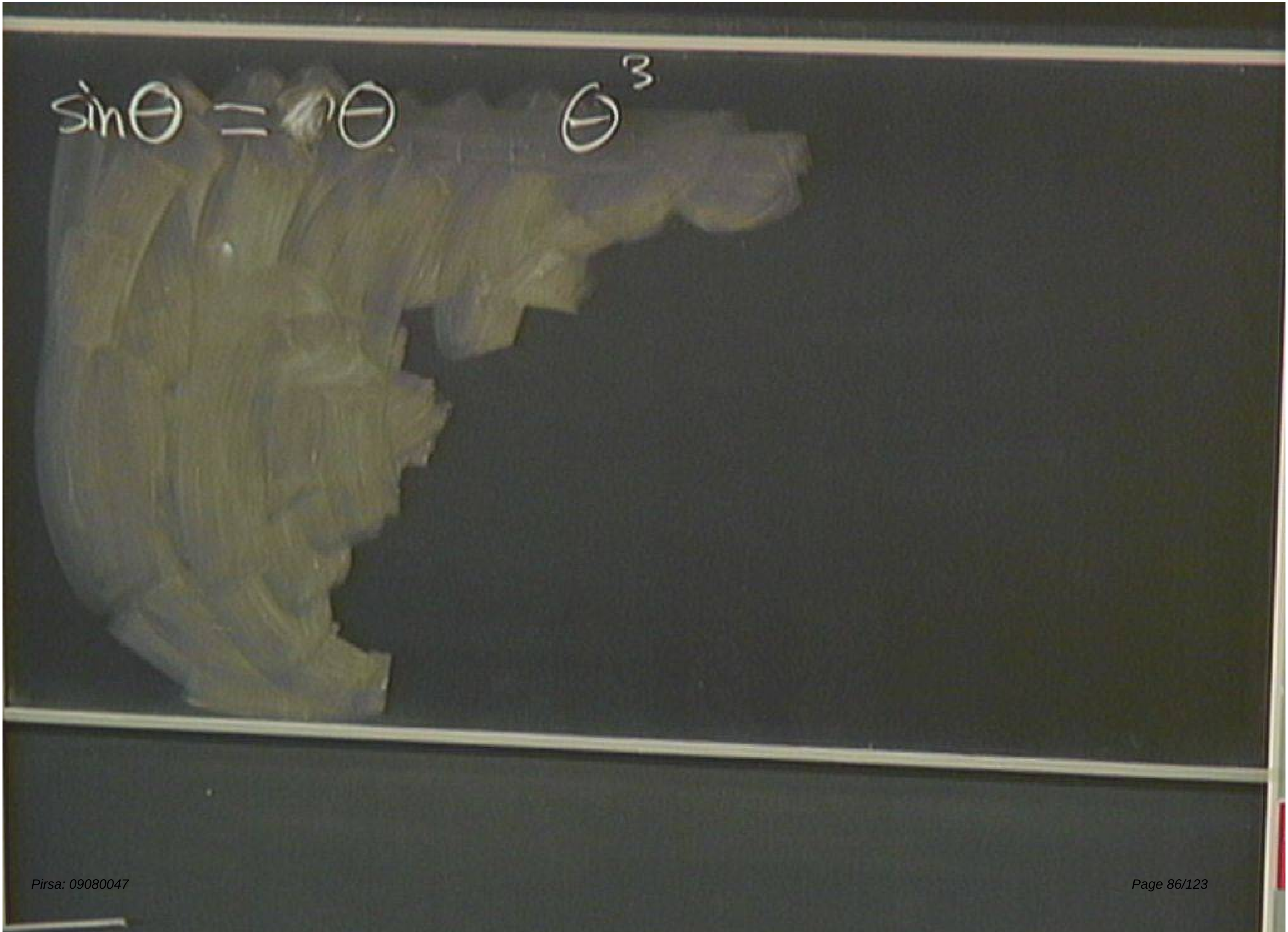


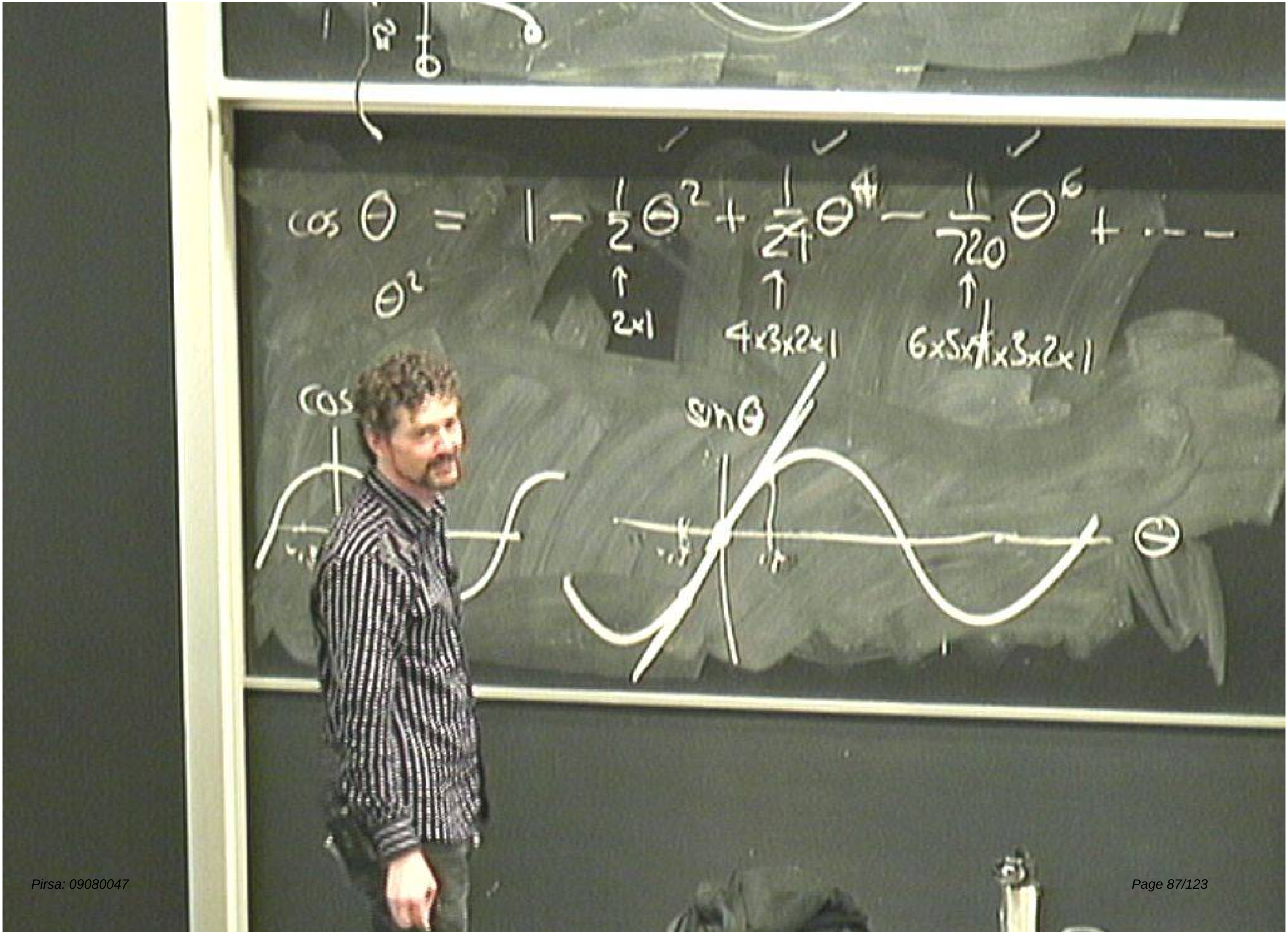
$\cos\frac{\pi}{2} = 0$
$1 - \frac{1}{2}\left(\frac{\pi}{2}\right)^2 = -0.23$
$1 - \frac{1}{2}\left(\frac{\pi}{2}\right)^2 + \frac{1}{24}\left(\frac{\pi}{2}\right)^4 = 0.01997$
$\left(\frac{\pi}{2}\right)^6 = -0.00089$
$\cos\theta \approx 1 - \frac{1}{2}\theta^2$
$(1+x)^n \approx 1+nx$
1
$\pi/2$

$\sin\theta =$



$\cos\theta = 1 - \frac{1}{2}\theta^2 + \frac{1}{24}\theta^4 - \frac{1}{720}\theta^6 + \ldots$
θ^2
2×1
$4\times3\times2\times1$
$6\times5\times4\times3\times2\times1$
$\cos\theta$
$\sin\theta$


sinΘ = Θ
Θ³


cos θ = 1 − 1/2 θ² + 1/24 θ⁴ − 1/720 θ⁶ + . . .
θ²
2×1
4×3×2×1
6×5×4×3×2×1
cos
sin θ
θ

$$\boxed{\sin\theta = \theta - \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 - + \cdots}$$

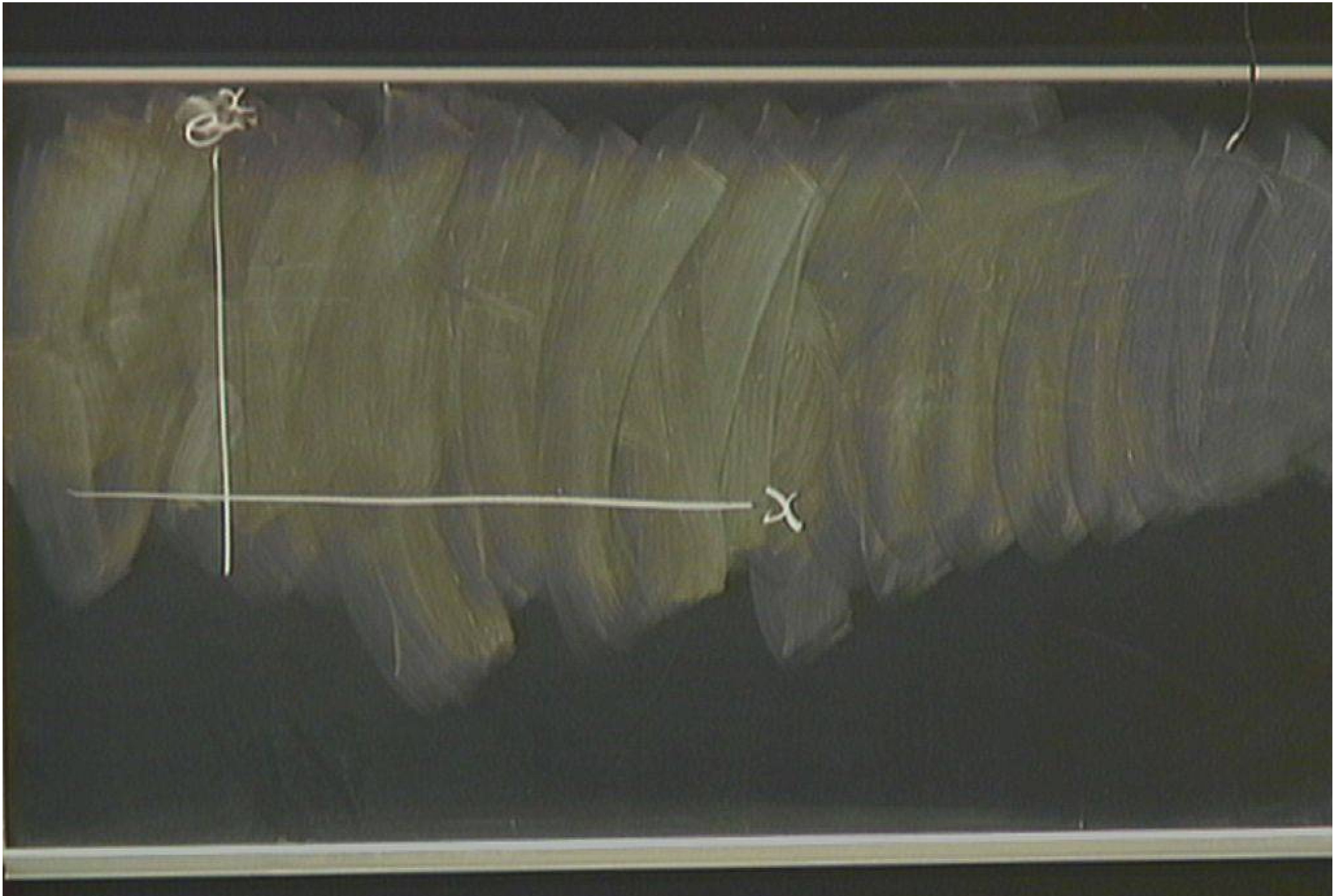
$$\boxed{\cos\theta = 1 - \frac{1}{2!}\theta^2 + \frac{1}{4!}\theta^4 - \frac{1}{6!}\theta^6 + - \cdots}$$

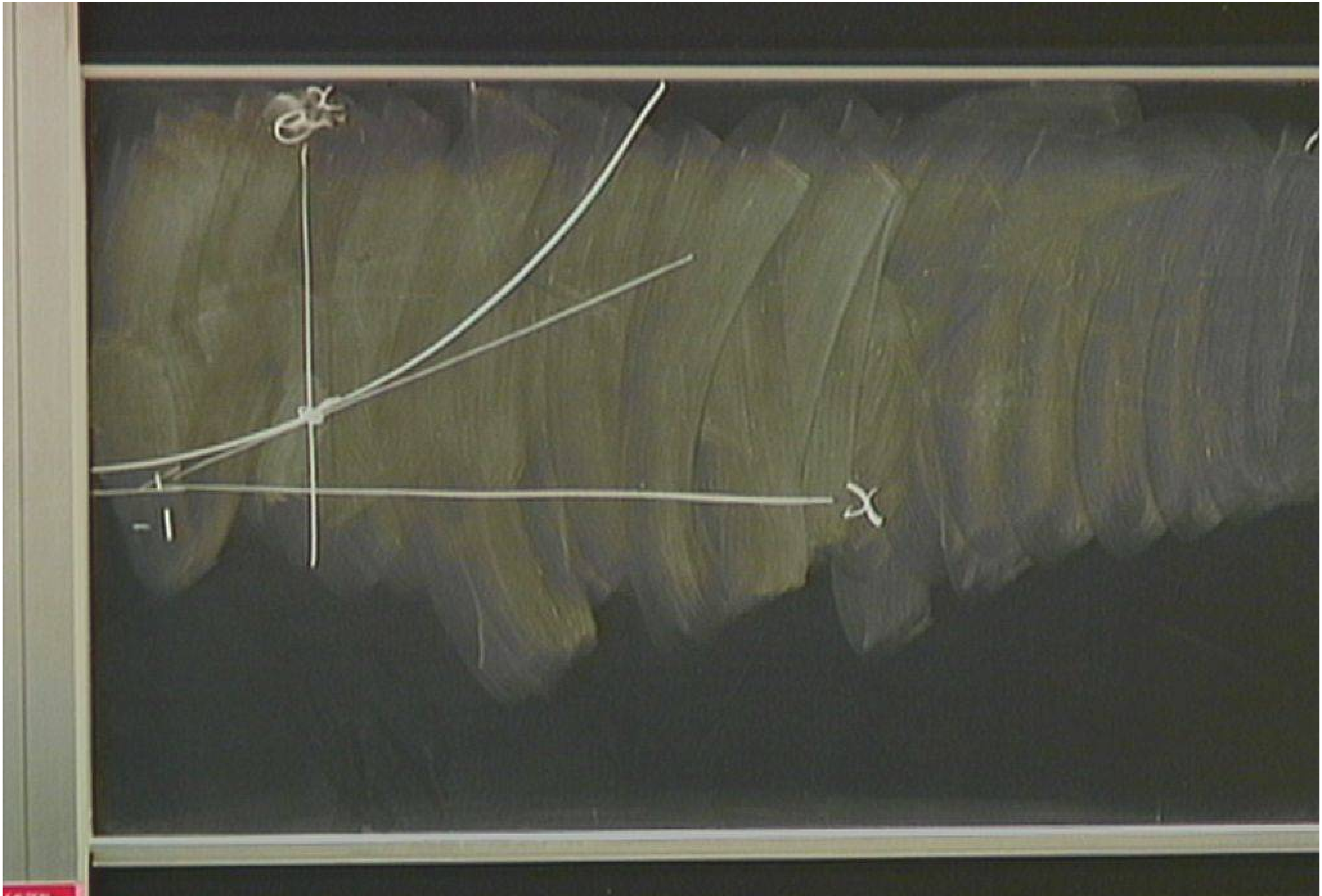
$$e^x = 1$$

$$\boxed{\sin\theta = \theta - \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 - + \cdots}$$

$$\boxed{\cos\theta = 1 - \frac{1}{2!}\theta^2 + \frac{1}{4!}\theta^4 - \frac{1}{6!}\theta^6 + - \cdots}$$

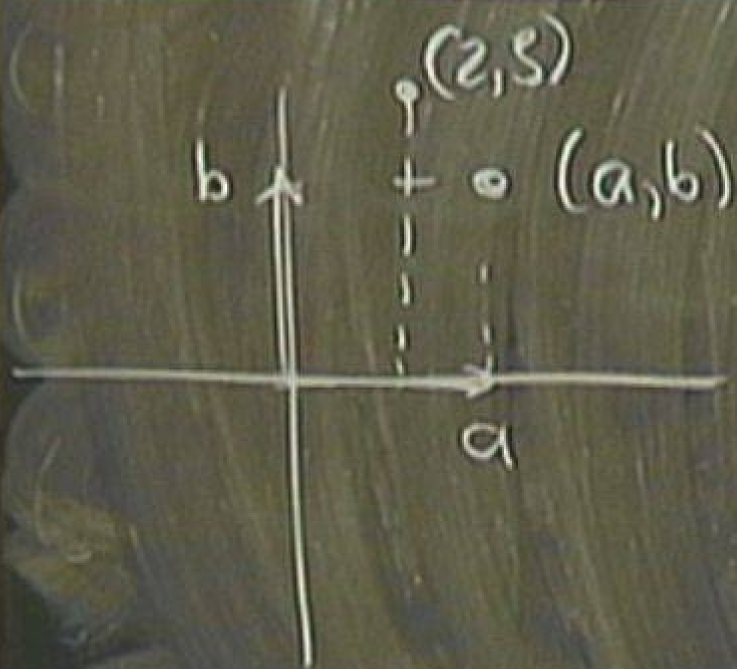
$$\boxed{e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \cdots}$$

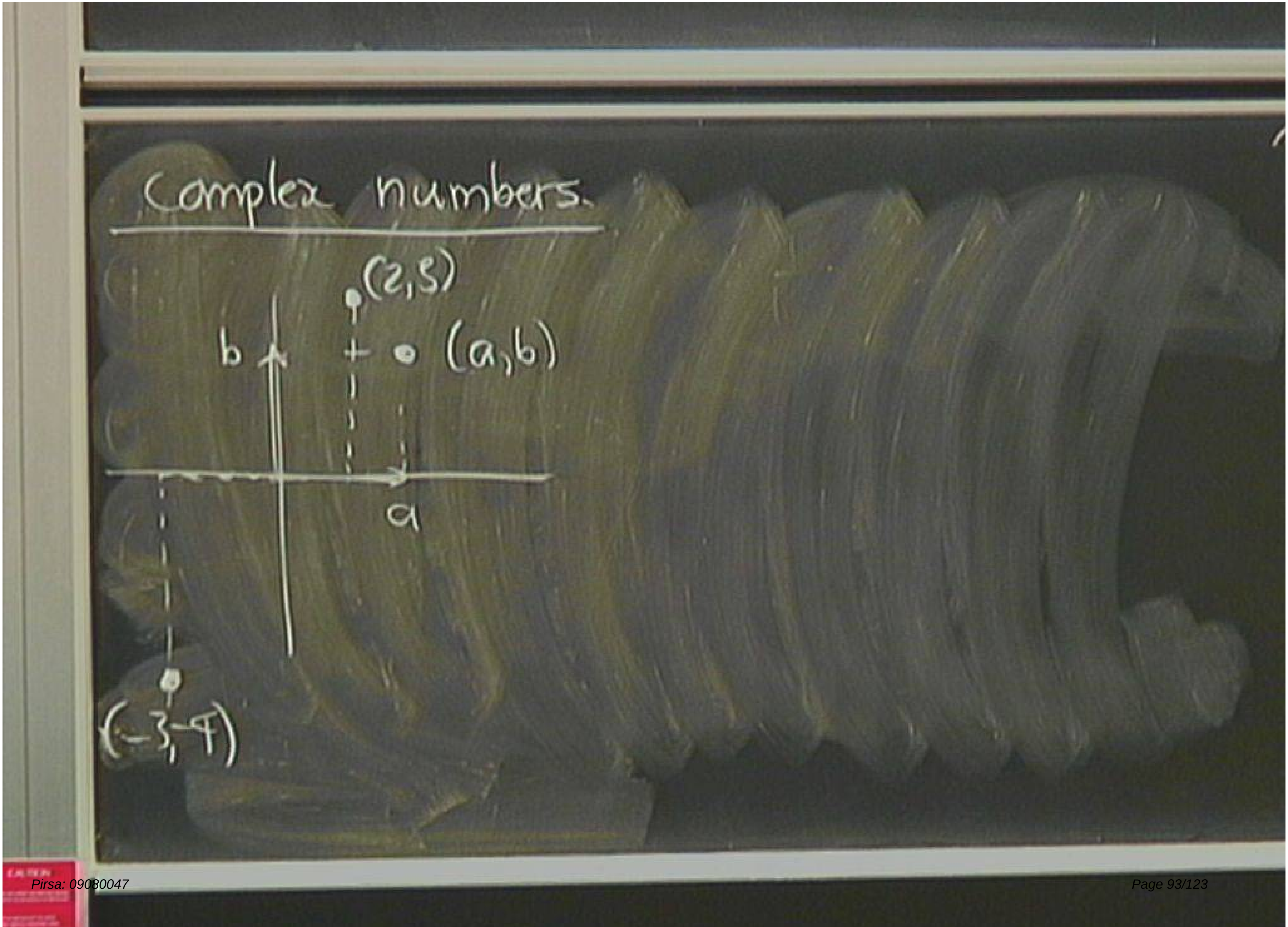

e^x
x

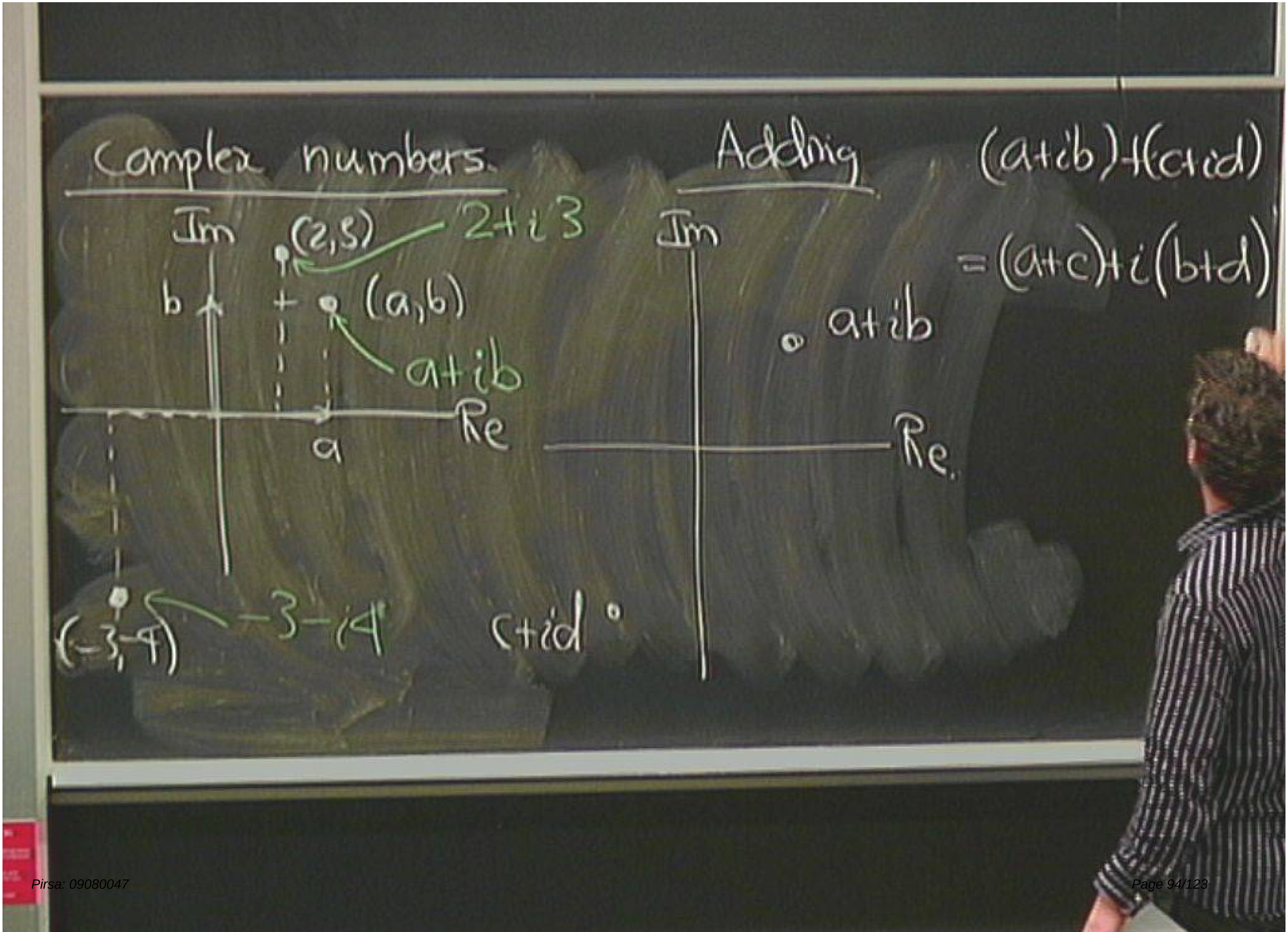


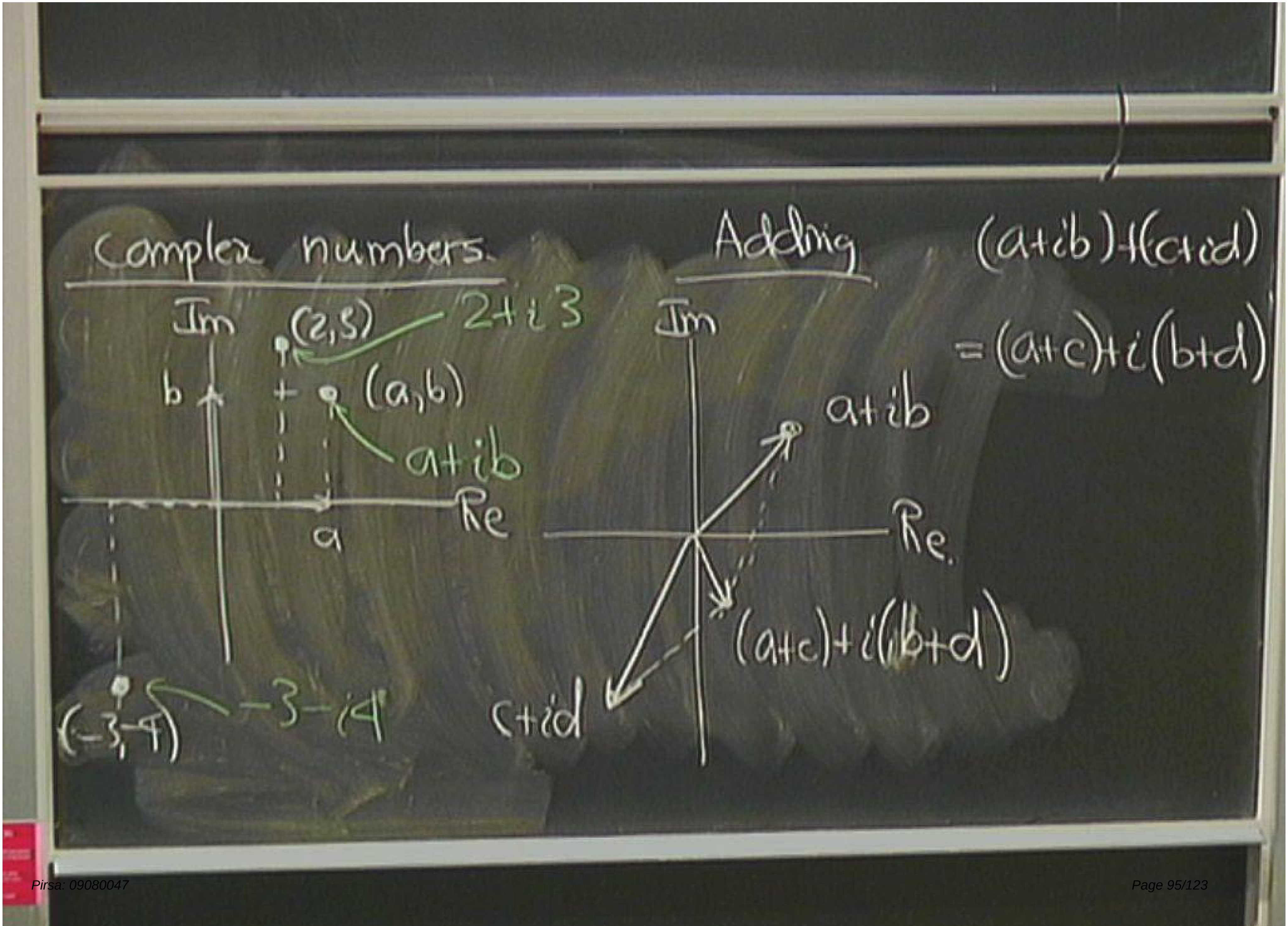
e^x
x
-1

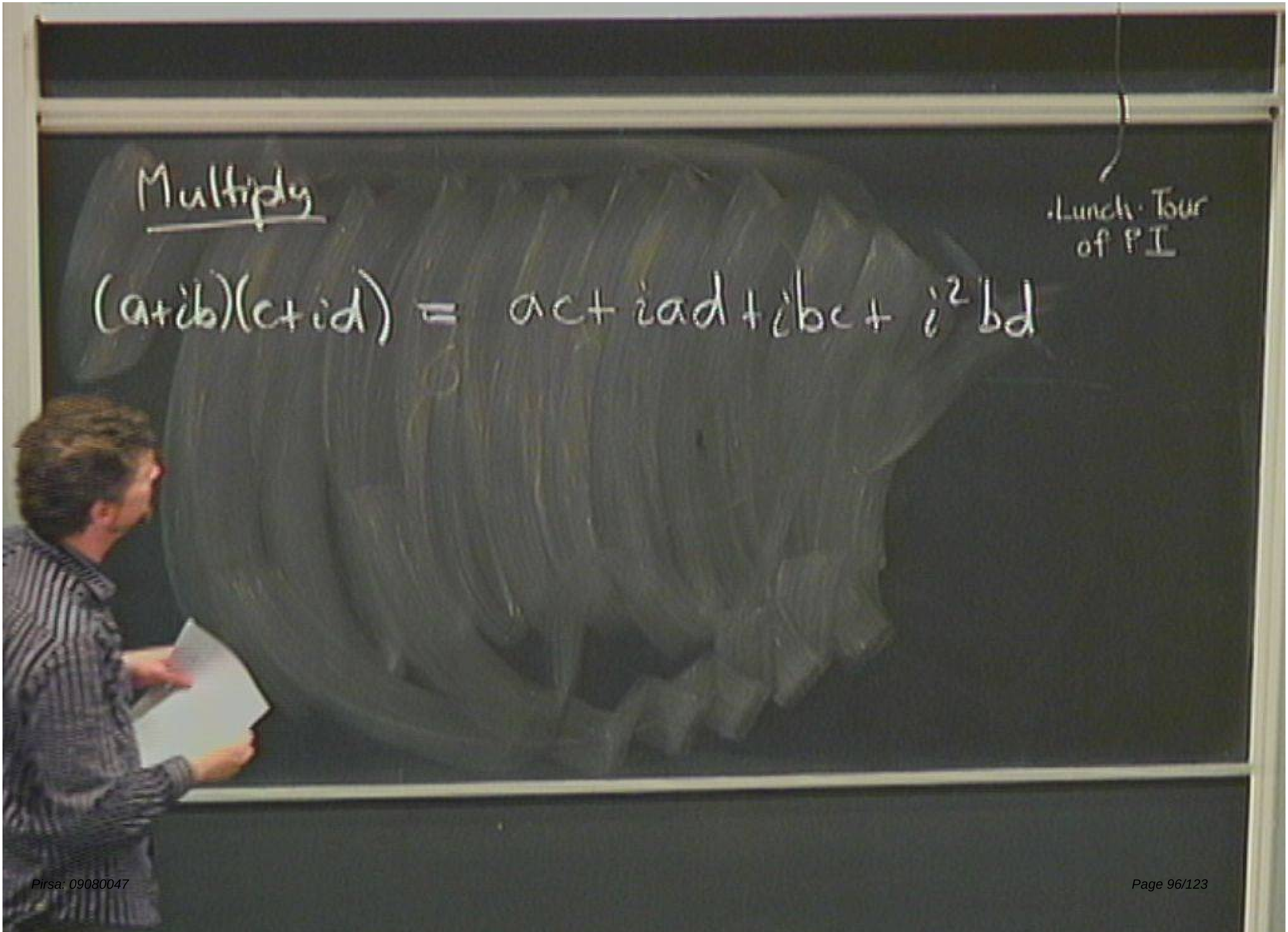
Complex numbers.

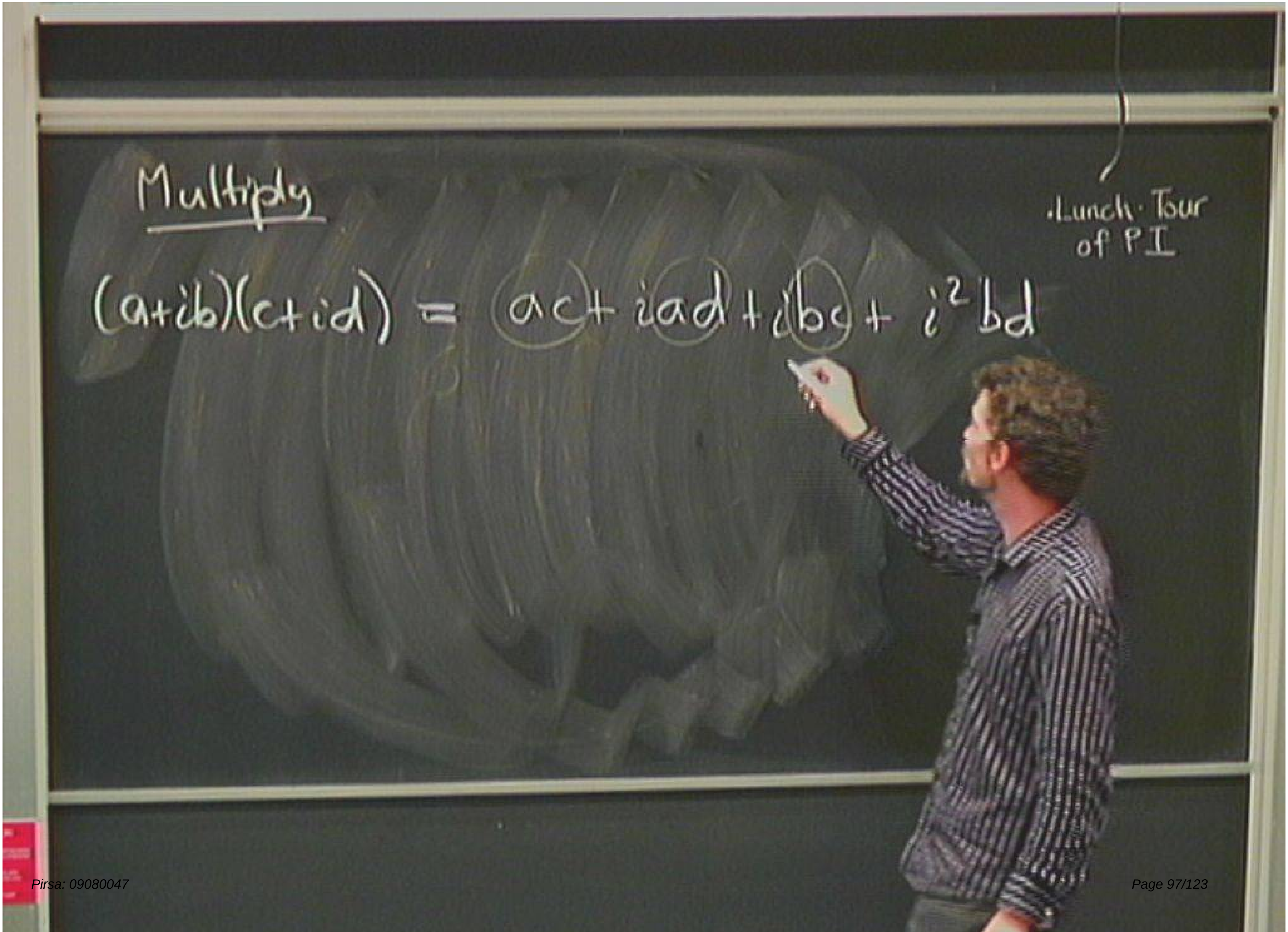



Complex numbers.
(2,3)
b
(a,b)
a
(-3,-4)


Complex numbers.
Im
(2,3)
2+i3
b
(a,b)
a+ib
Re
a
(-3,-4)
-3-i4
c+id
Adding
Im
a+ib
Re.
(a+ib)+(c+id)
=(a+c)+i(b+d)


Complex numbers
Im
(2,3)
2+i3
b
(a,b)
a+ib
Re
a
(-3,-4)
-3-i4
Adding
(a+ib)+(c+id)
=(a+c)+i(b+d)
Im
a+ib
Re
(a+c)+i(b+d)
c+id


Multiply
$(a+ib)(c+id) = ac + iad + ibc + i^2bd$
·Lunch· Tour of PI


Multiply
$(a+ib)(c+id) = ac + iad + ibc + i^2bd$
·Lunch · Tour of PI

Multiply

$$(a+ib)(c+id) = ac + iad + ibc + i^2 bd$$

define :

.Lunch · Tour of PI

Multiply

Lunch · Tour of PI

$$(a+ib)(c+id) = ac + iad + ibc + i^2 bd$$

$$\boxed{\text{define: } \quad i^2 = -1}$$ (imaginary unit)

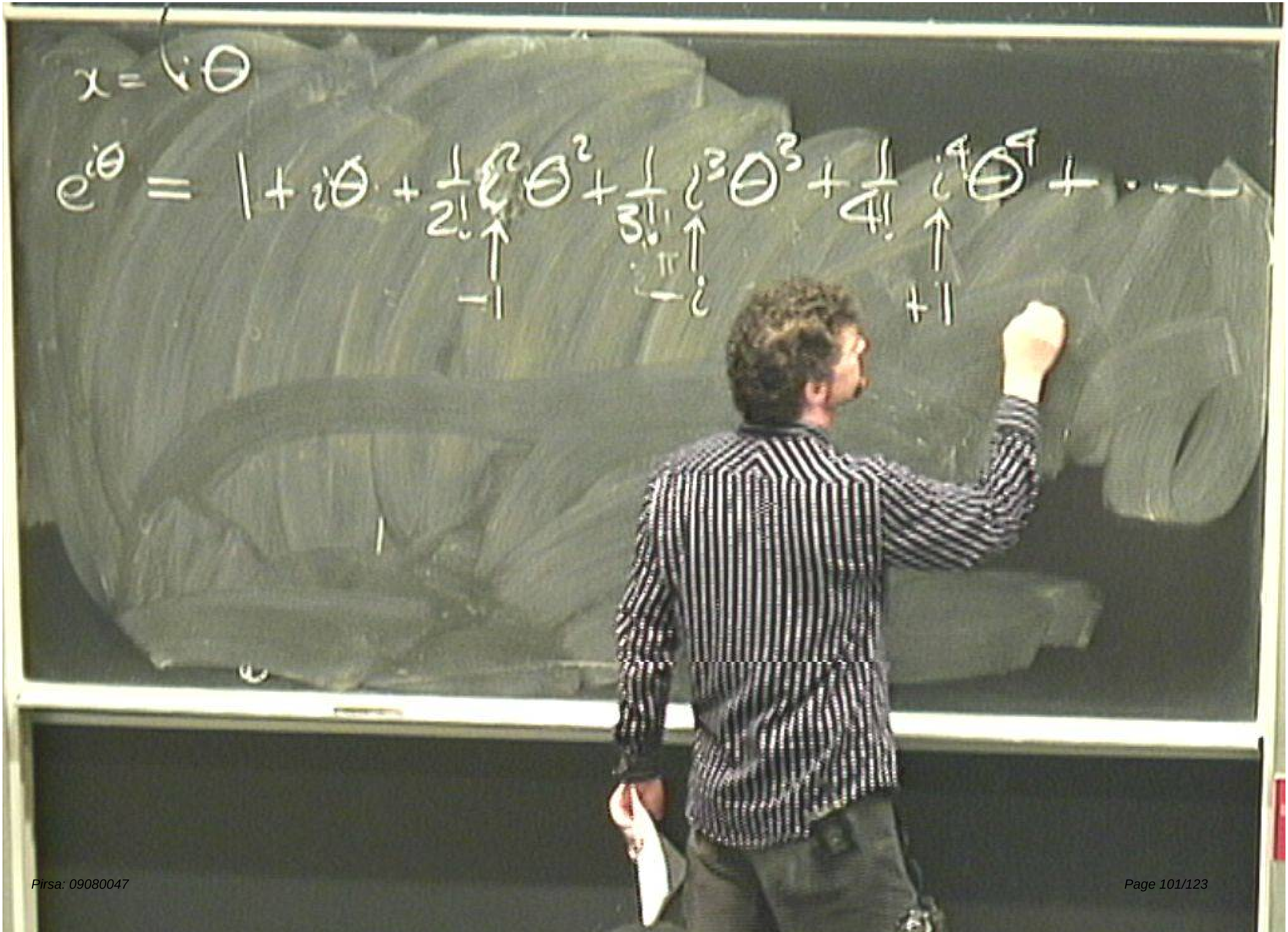
Lunch · Tour of PI

Multiply

$$(a+ib)(c+id) = ac + iad + ibc + i^2 bd$$

$$\boxed{\text{define: } i^2 = -1}$$ (imaginary unit)

$$= (ac-bd) + i(ad+bc)$$

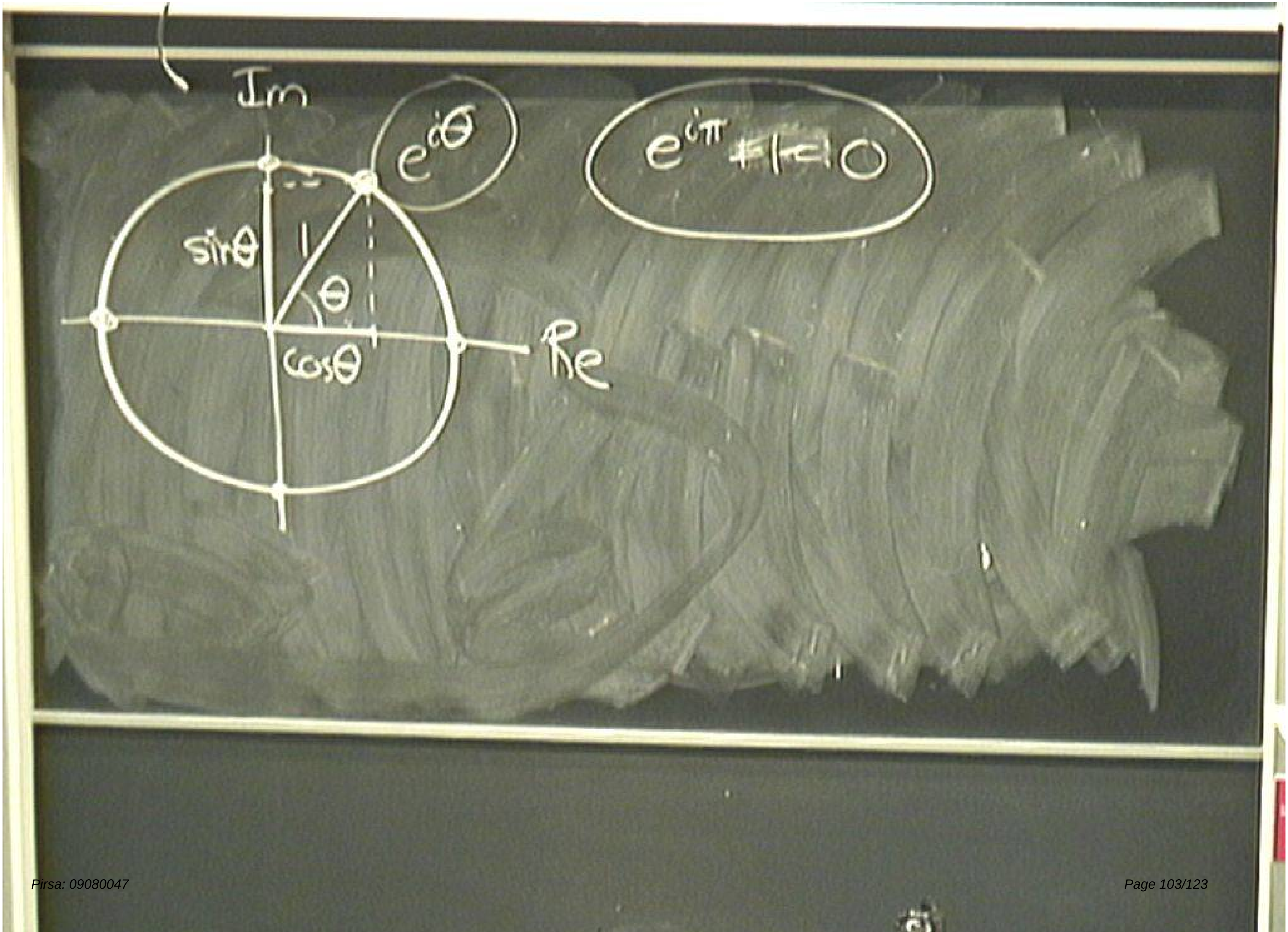

$x = i\theta$
$e^{i\theta} = 1 + i\theta + \frac{1}{2!} i^2 \theta^2 + \frac{1}{3!} i^3 \theta^3 + \frac{1}{4!} i^4 \theta^4 + \cdots$
-1
$-i$
$+1$

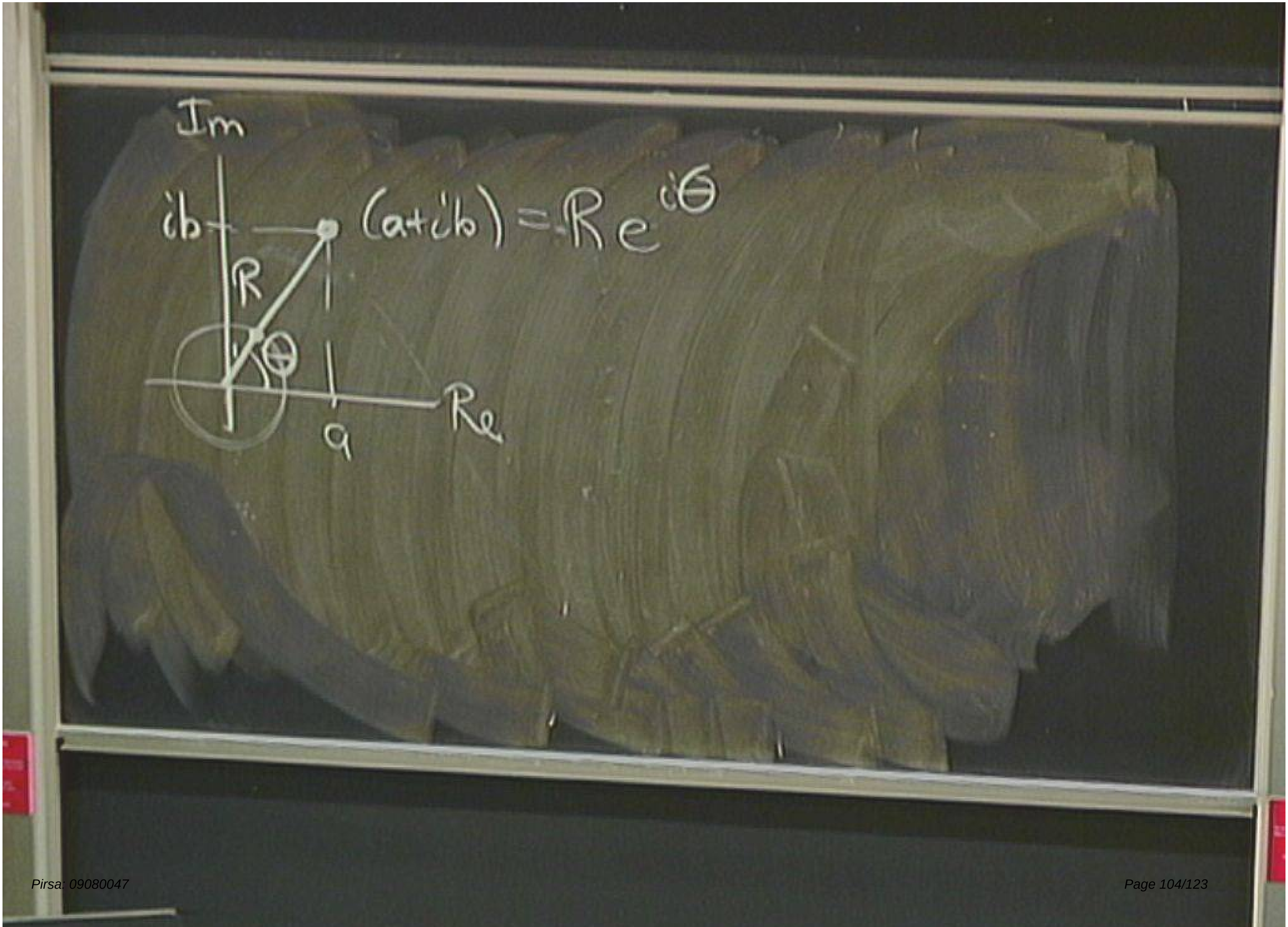
$x = i\theta$

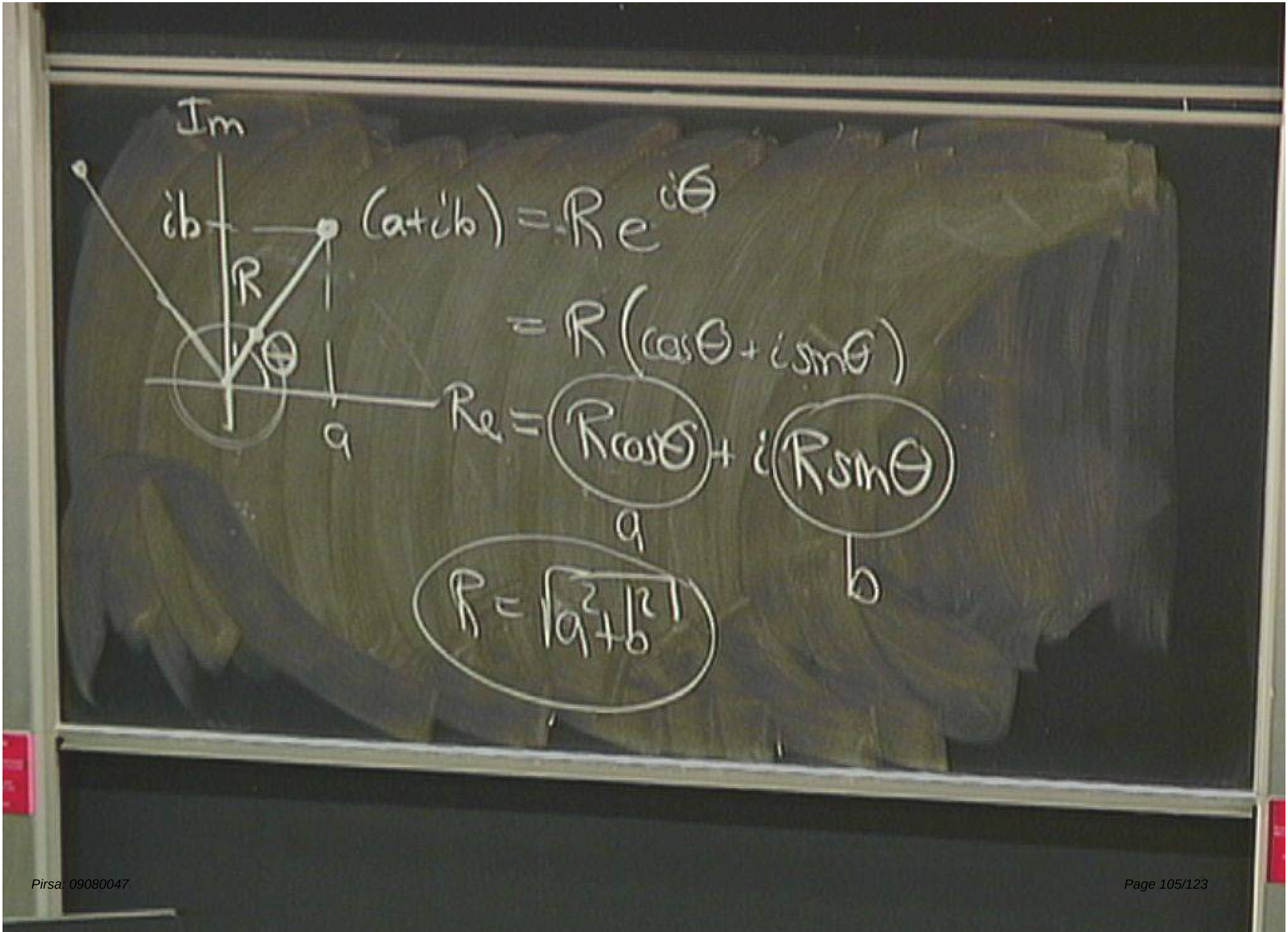
$$e^{i\theta} = \underset{\substack{\uparrow \\ 1}}{1} + \underset{\substack{\uparrow \\ i}}{i}\theta + \frac{1}{2!}\underset{\substack{\uparrow \\ -1}}{i^2}\theta^2 + \frac{1}{3!}\underset{\substack{\uparrow \\ -i}}{i^3}\theta^3 + \frac{1}{4!}\underset{\substack{\uparrow \\ +1}}{i^4}\theta^4 + \cdots$$

$$= \left\{1 - \frac{1}{2!}\theta^2 + \frac{1}{4!}\theta^4 \cdots\right\} + i\left\{\theta - \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 \cdots\right\}$$

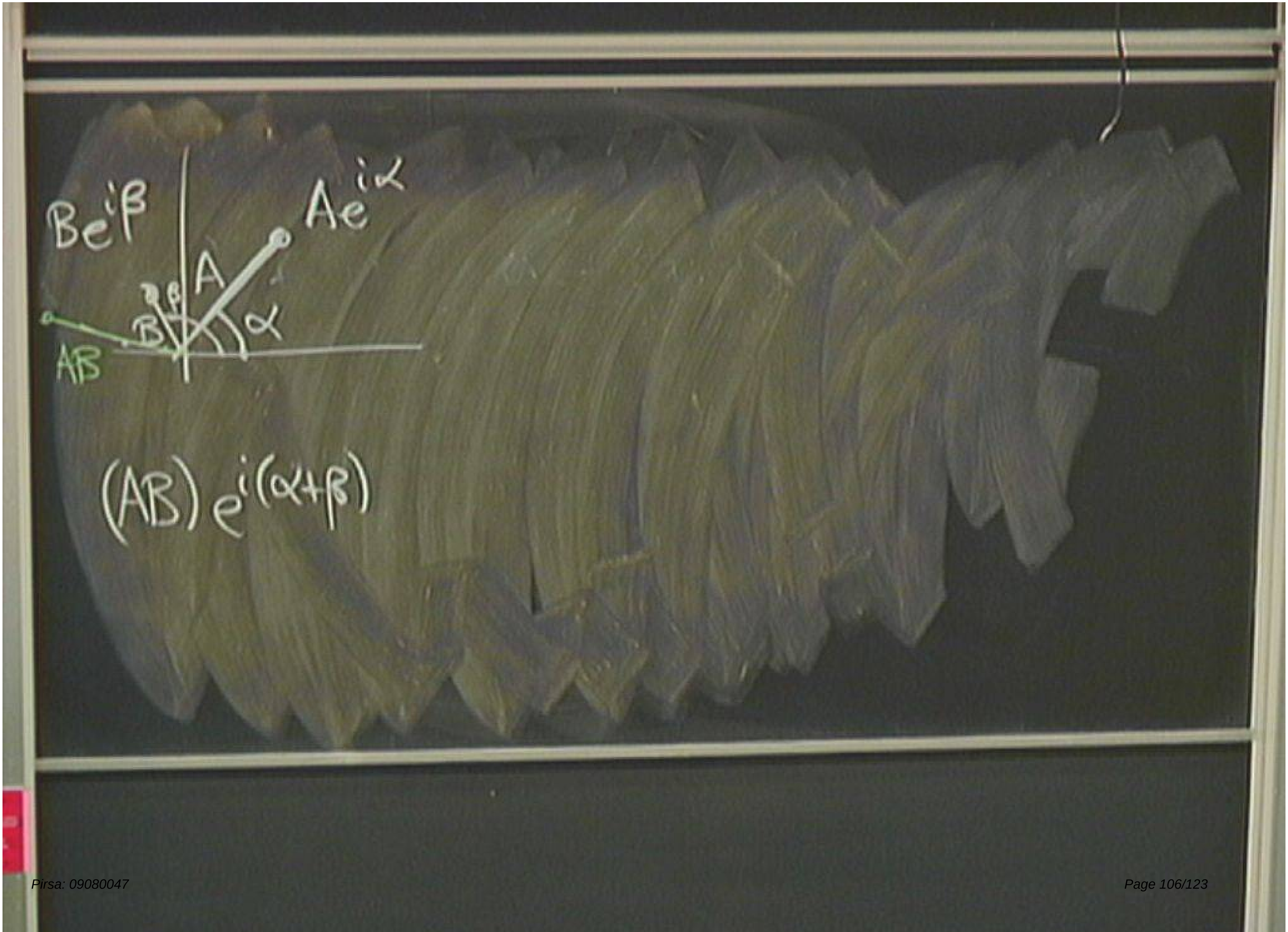
$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

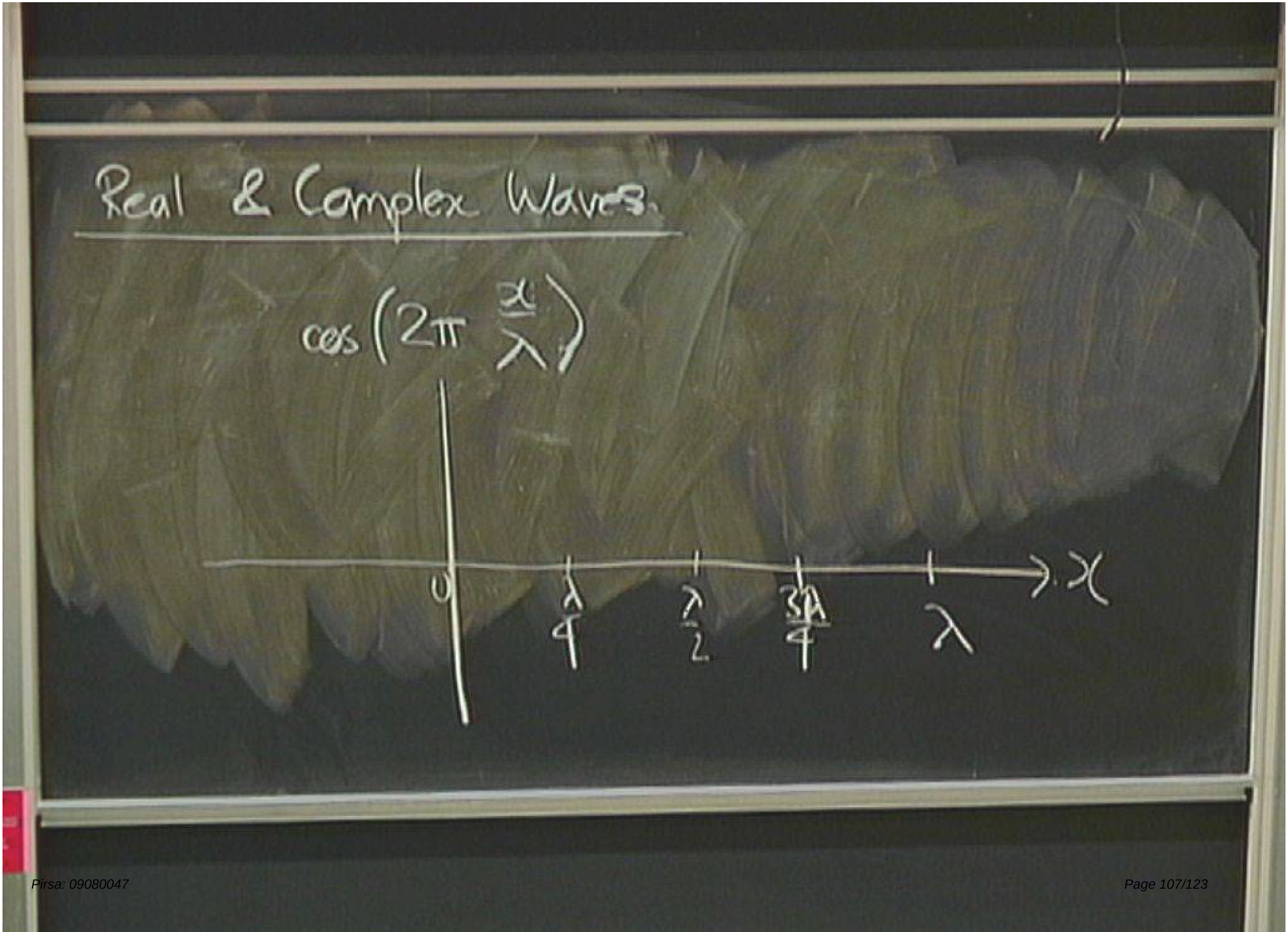

Im
e^{iθ}
e^{iπ}+1=0
sinθ
1
θ
cosθ
Re

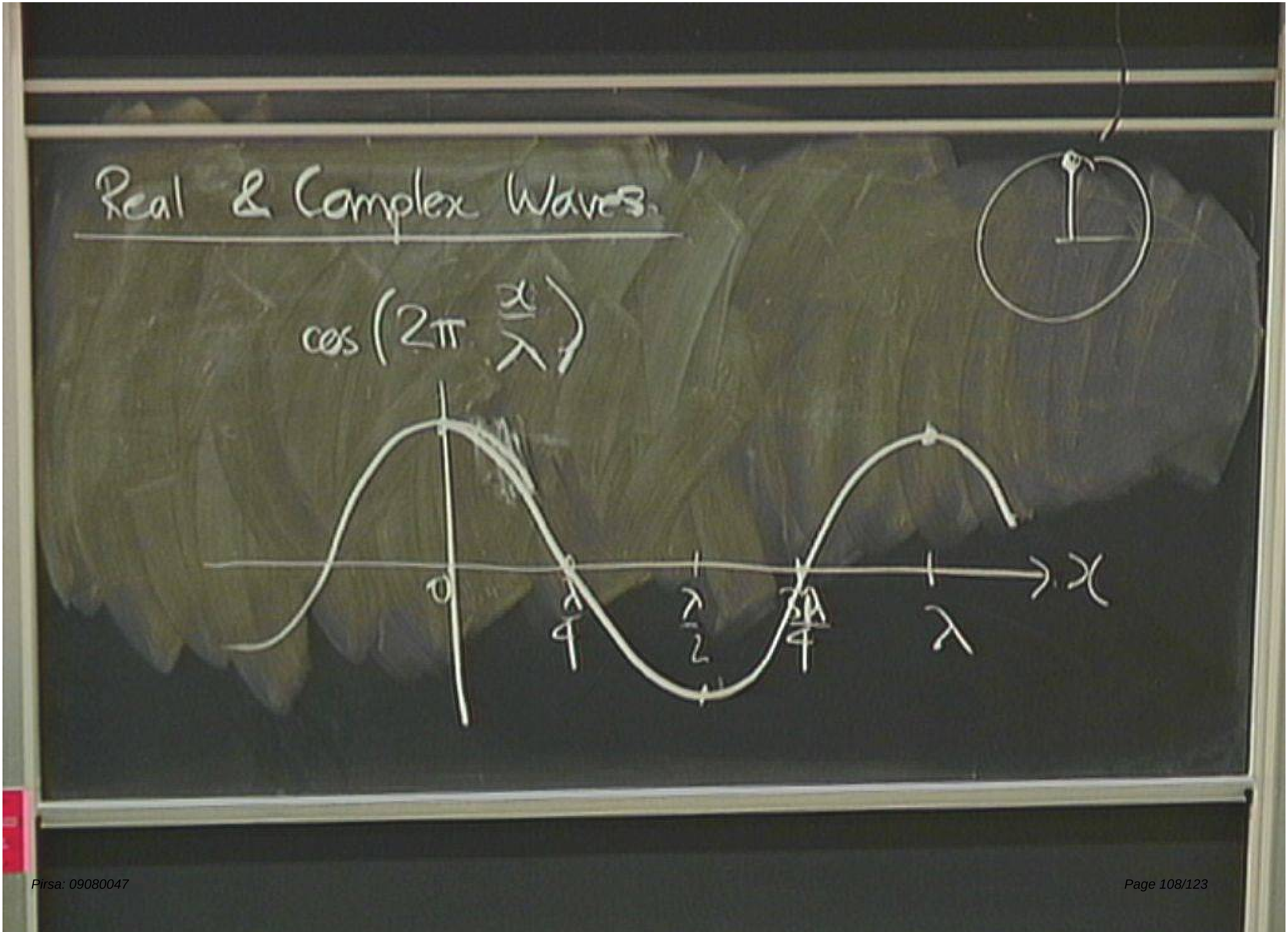

Im
ib
(a+ib) = Re^{iθ}
R
θ
a
Re



Im
ib
R
θ
a
(a+ib) = Re^{iθ}
= R(cosθ + i sinθ)
Re = Rcosθ + i Rsinθ
a
b
R = √(a²+b²)


Be^{iβ}
Ae^{iα}
A
B
α
β
AB
(AB) e^{i(α+β)}

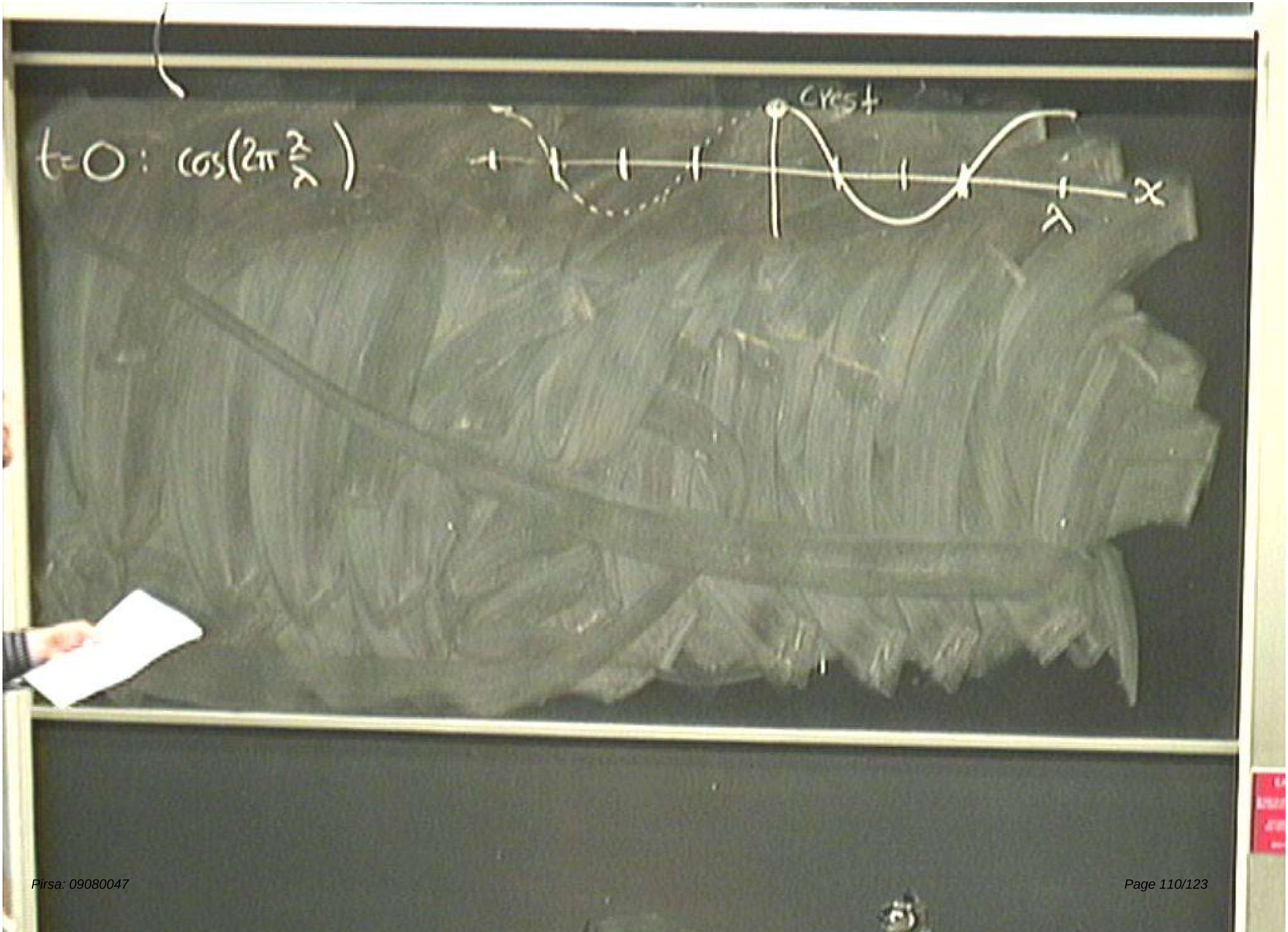

Real & Complex Waves.
cos(2π x/λ)
0
λ/4
λ/2
3λ/4
λ
x


Real & Complex Waves.
$\cos\left(2\pi \frac{x}{\lambda}\right)$
0
$\frac{\lambda}{4}$
$\frac{\lambda}{2}$
$\frac{3\lambda}{4}$
λ
x

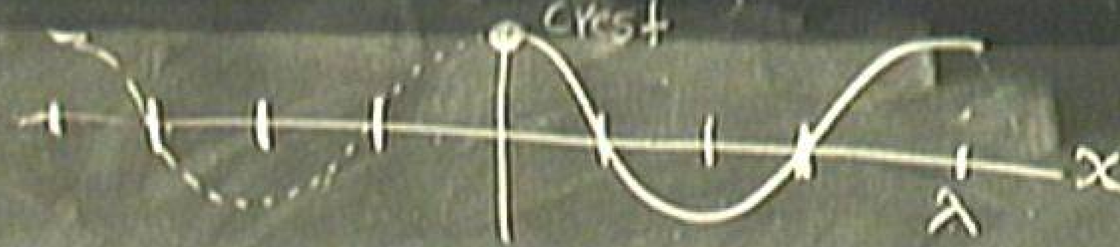
$$\cos\left(2\pi \frac{x}{\lambda}\right) \rightarrow \cos\left(2\pi \frac{x}{\lambda} - 2\pi \frac{t}{T}\right)$$

wavelength

period


t=0 : cos(2π x/λ)
crest
x
λ

$t=0: \cos\left(2\pi \frac{x}{\lambda}\right)$



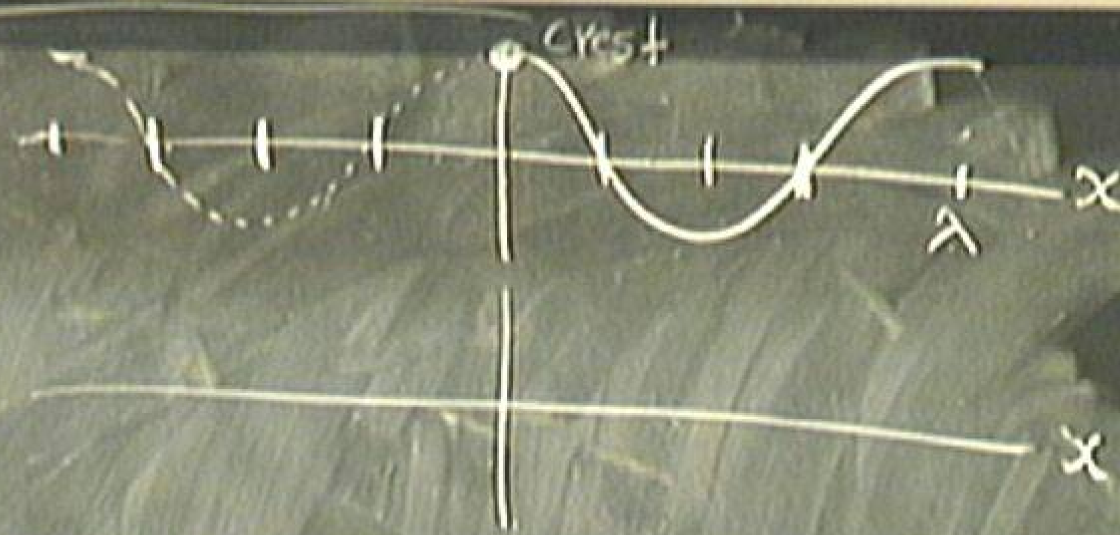
$t=\frac{T}{4}: \cos\left(2\pi \frac{x}{\lambda} - \frac{\pi}{2}\right)$

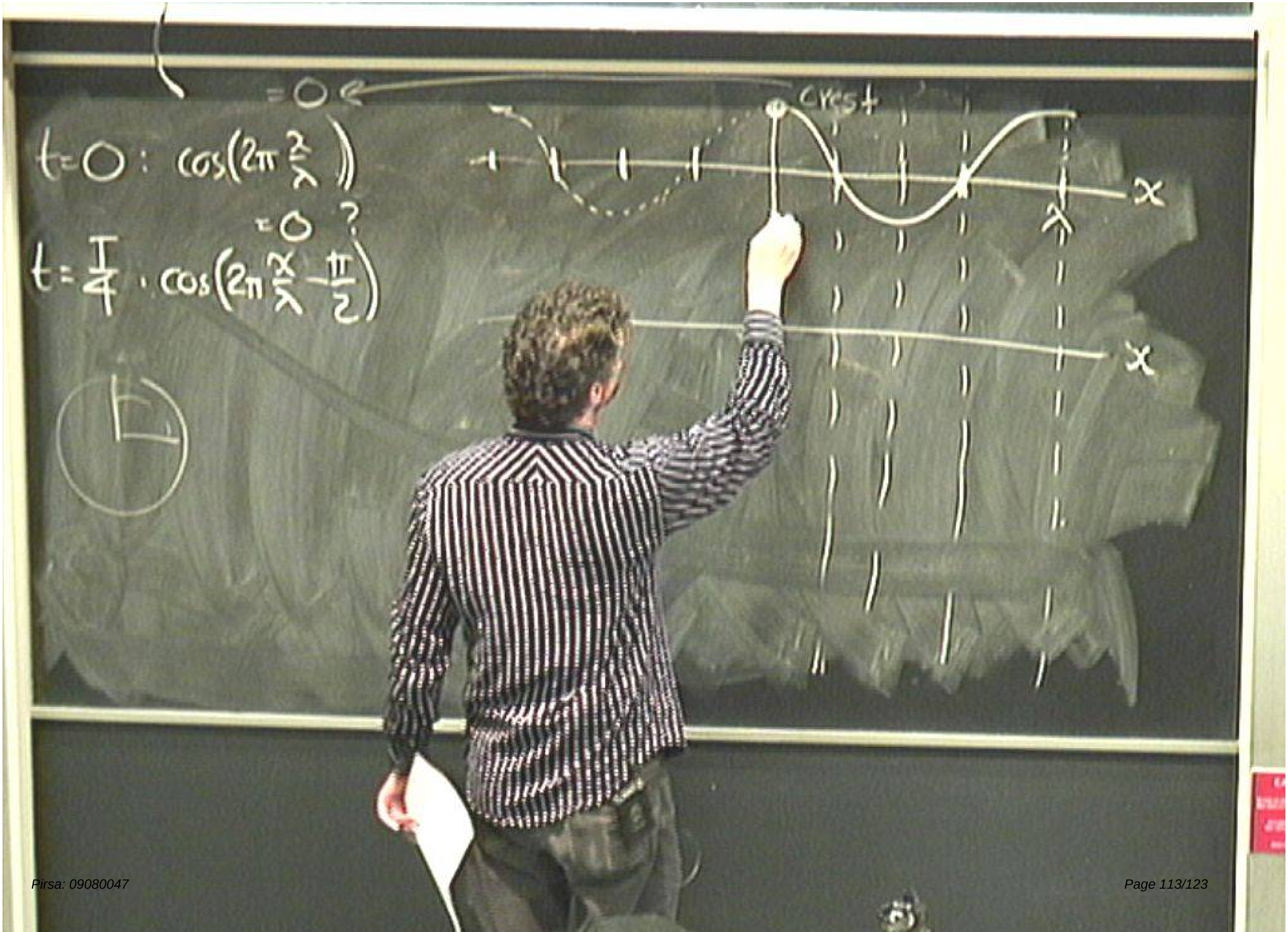
$= 0$

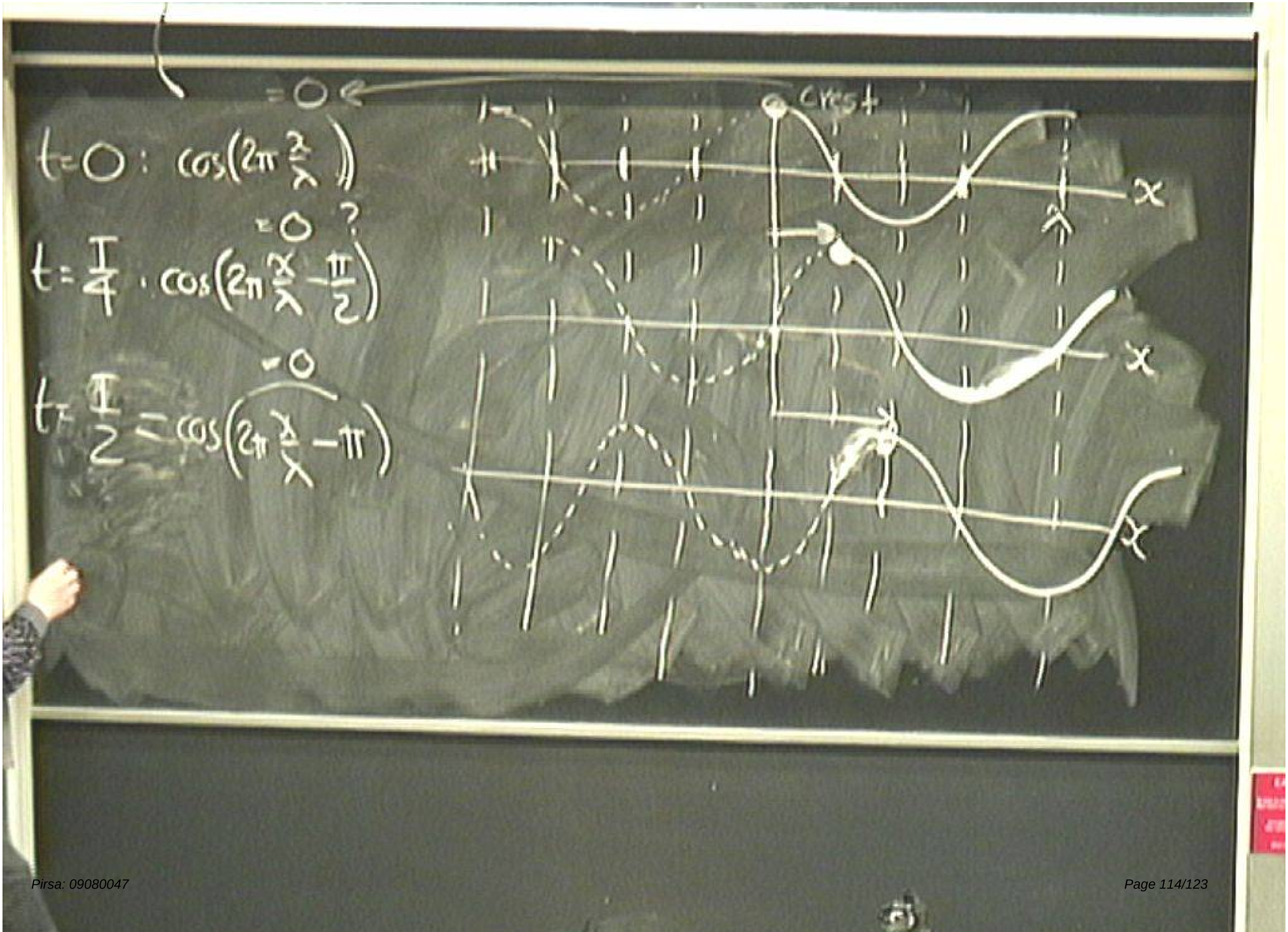
$t = 0 : \cos\left(2\pi \frac{x}{\lambda}\right)$

$= 0 \ ?$

$t = \frac{T}{4} : \cos\left(2\pi \frac{x}{\lambda} - \frac{\pi}{2}\right)$




=0
crest
t=0 : cos(2π x/λ)
=0 ?
t = T/4 : cos(2π x/λ − π/2)
x
λ
x


= 0
crest
t=0 : cos(2π x/λ)
=0 ?
t = T/4 : cos(2π x/λ − π/2)
=0
t = T/2 = cos(2π x/λ − π)
x
x
x
λ

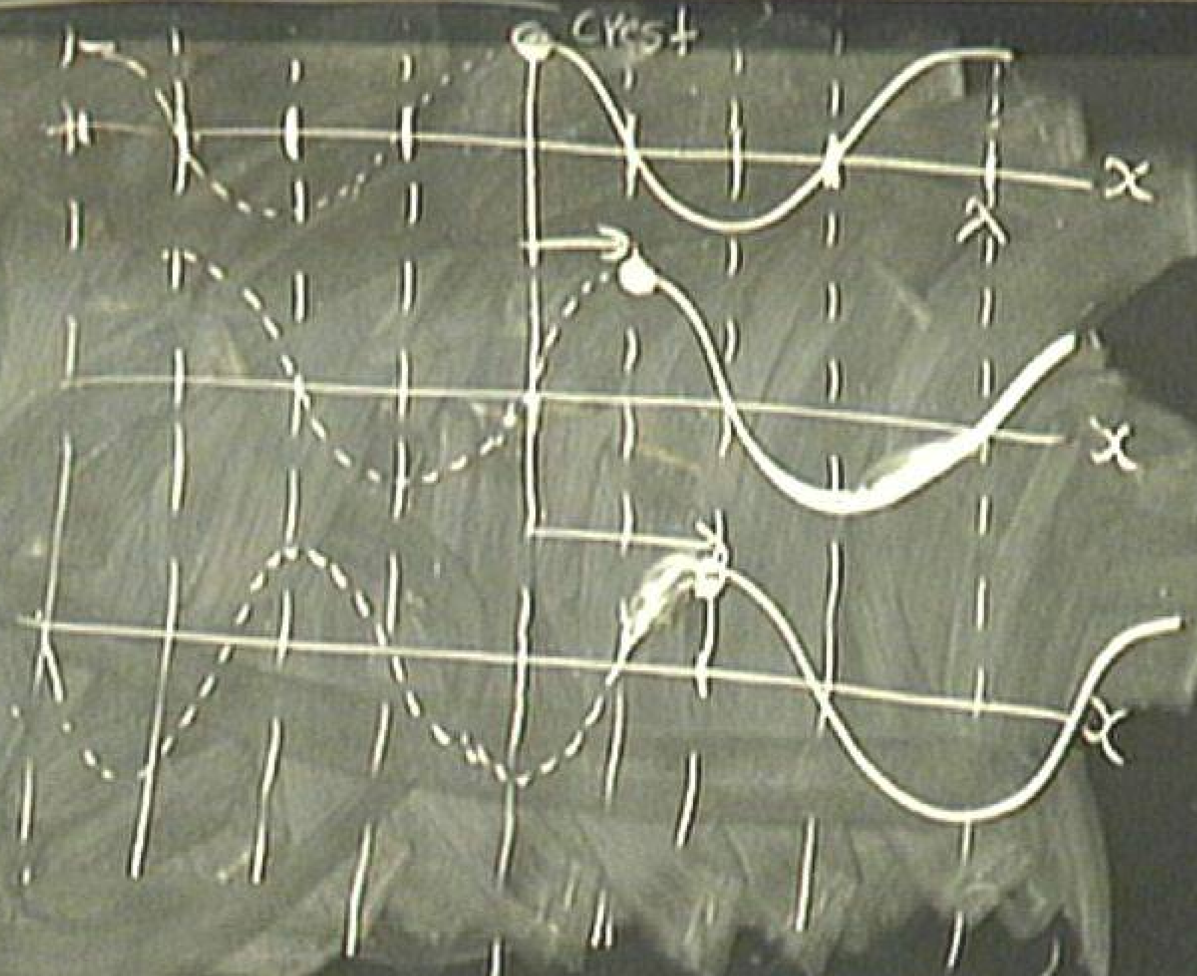
$t=0: \cos\left(2\pi \frac{x}{\lambda}\right) = 0$

$t=\frac{T}{4}: \cos\left(2\pi \frac{x}{\lambda} - \frac{\pi}{2}\right) = 0$?

$t=\frac{T}{2}: \cos\left(2\pi \frac{x}{\lambda} - \pi\right) = 0$

$\cos(\theta - 2\pi) = \cos\theta$

$t=T \rightarrow \cos\left(2\pi \frac{x}{\lambda} - 2\pi\right)$



$$\cos\left(2\pi \frac{x}{\lambda}\right) \to \cos\left(2\pi \frac{x}{\lambda} \underset{+\ (\text{left})}{\overset{-\ (\text{right})}{}} 2\pi \frac{t}{T}\right)$$

wavelength

period

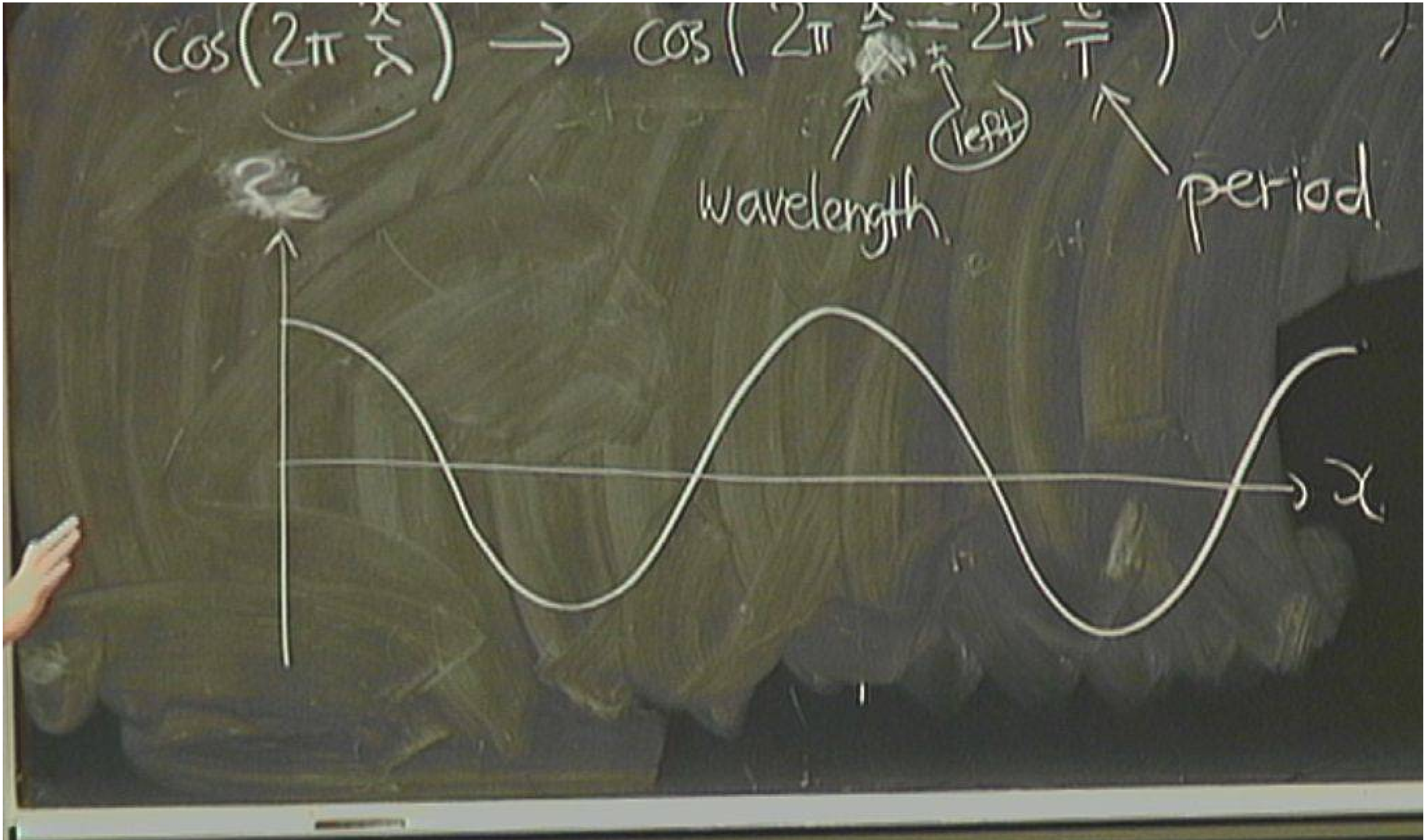
$$\cos\left(2\pi \frac{x}{\lambda}\right) \longrightarrow \cos\left(2\pi \frac{x}{\lambda} \mp 2\pi \frac{t}{T}\right)$$

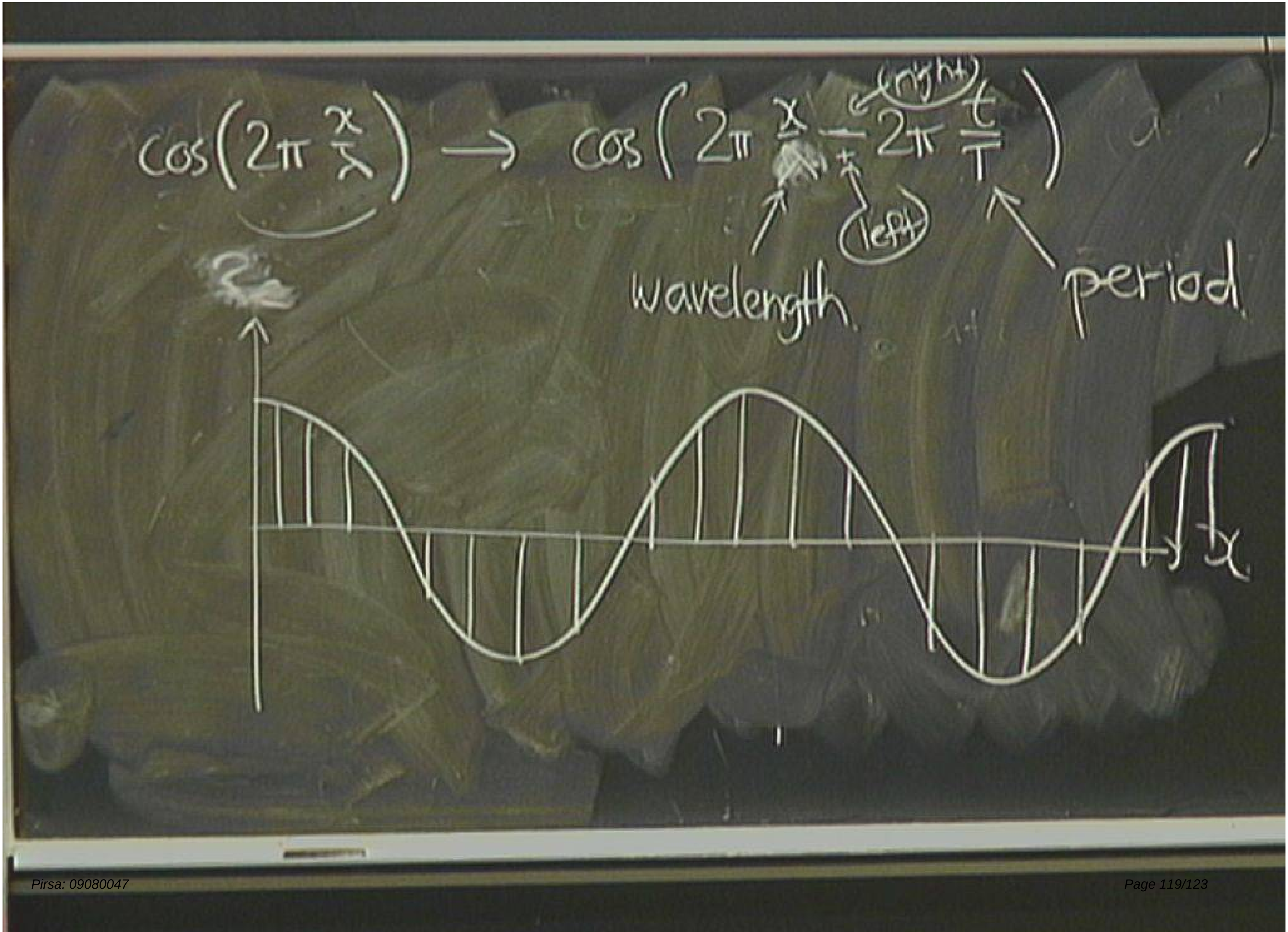
right

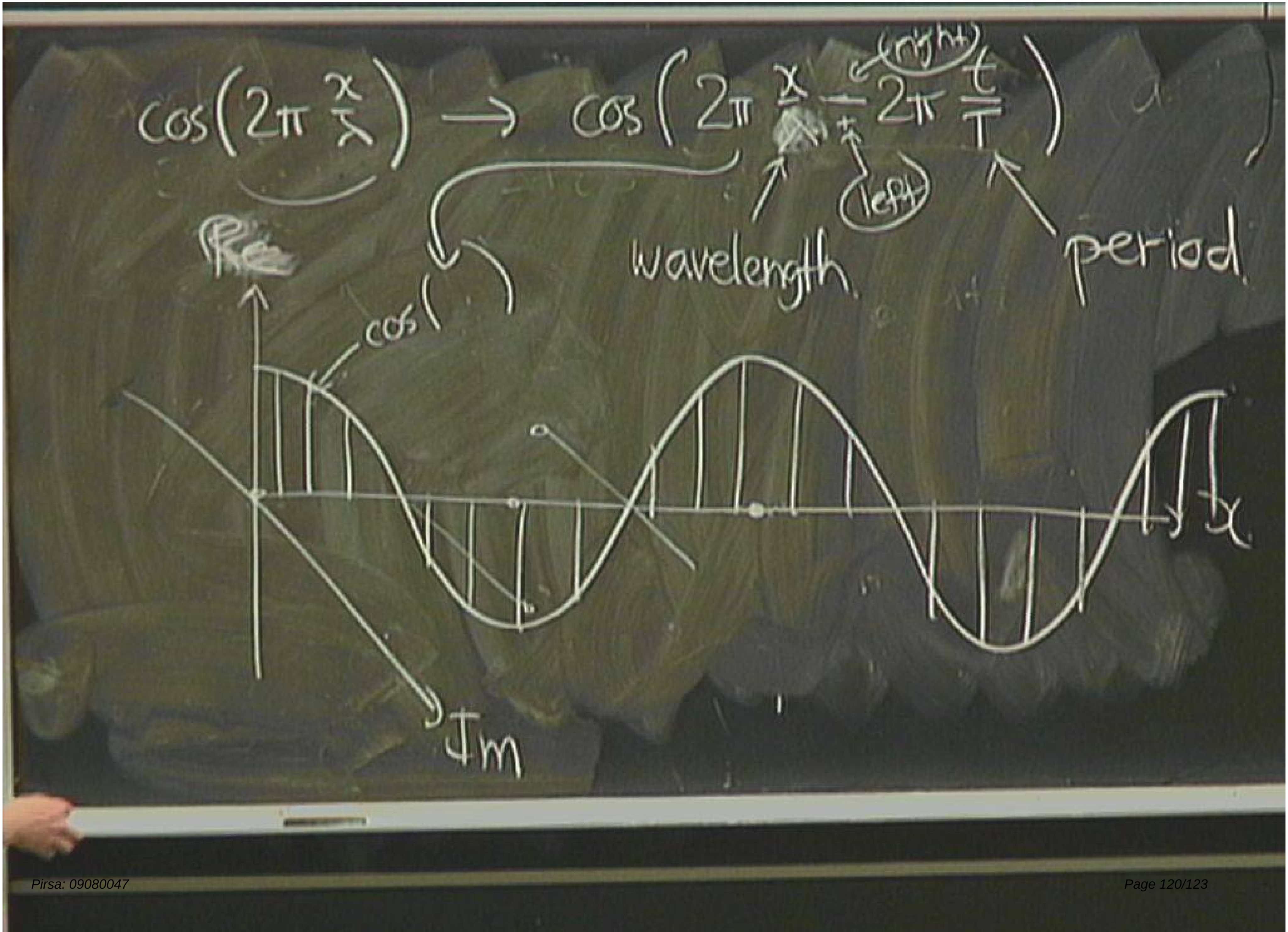
left

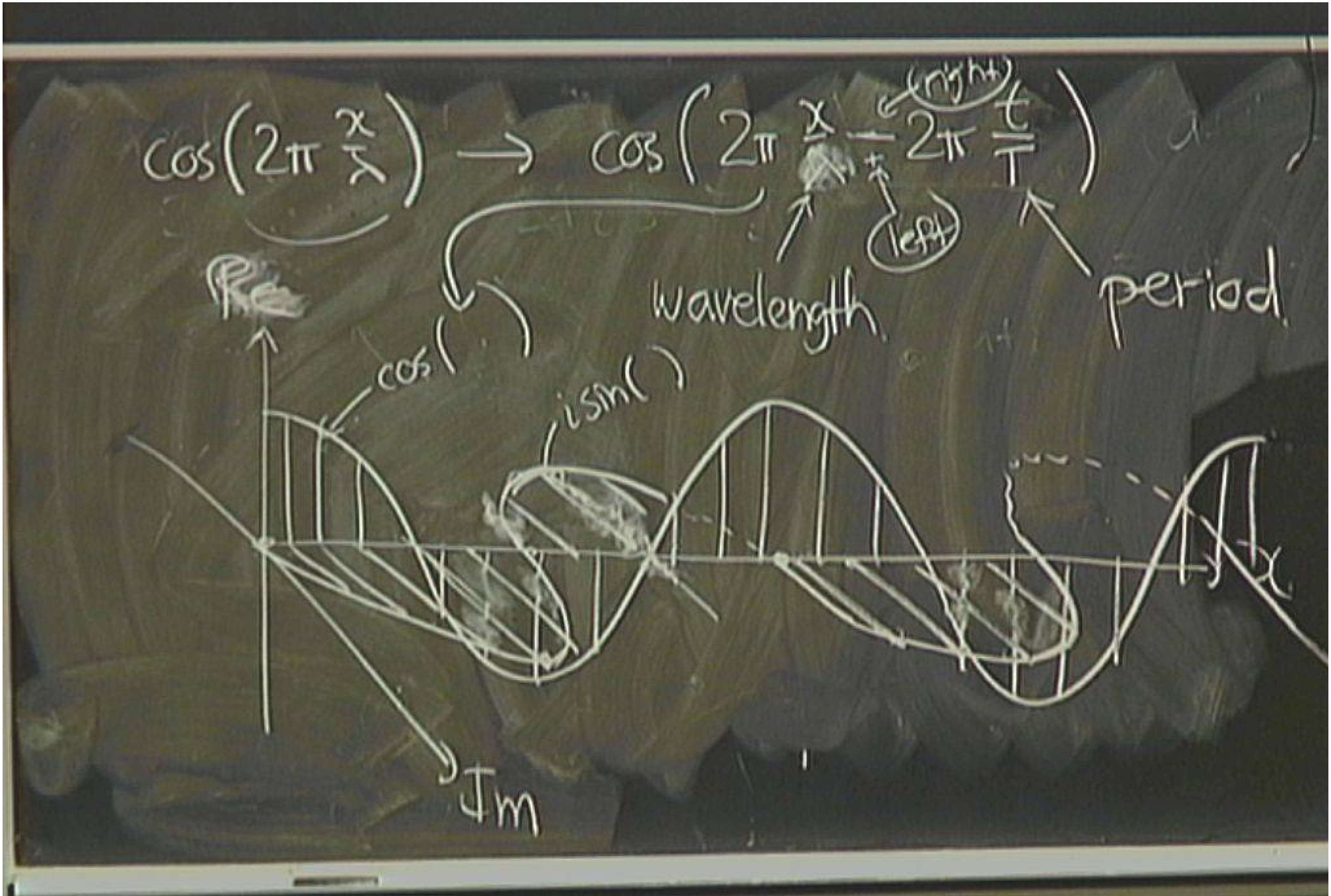
wavelength

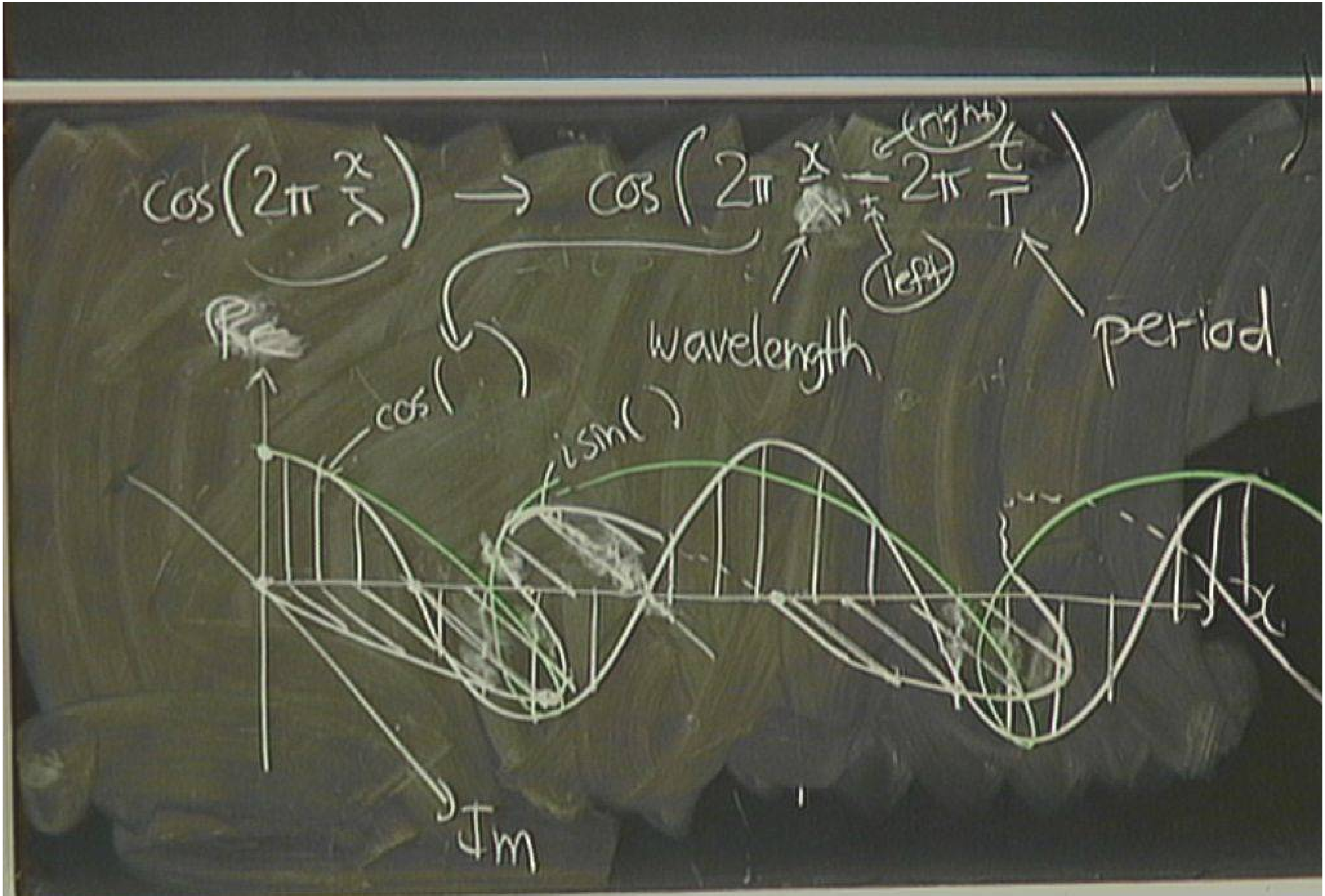
period

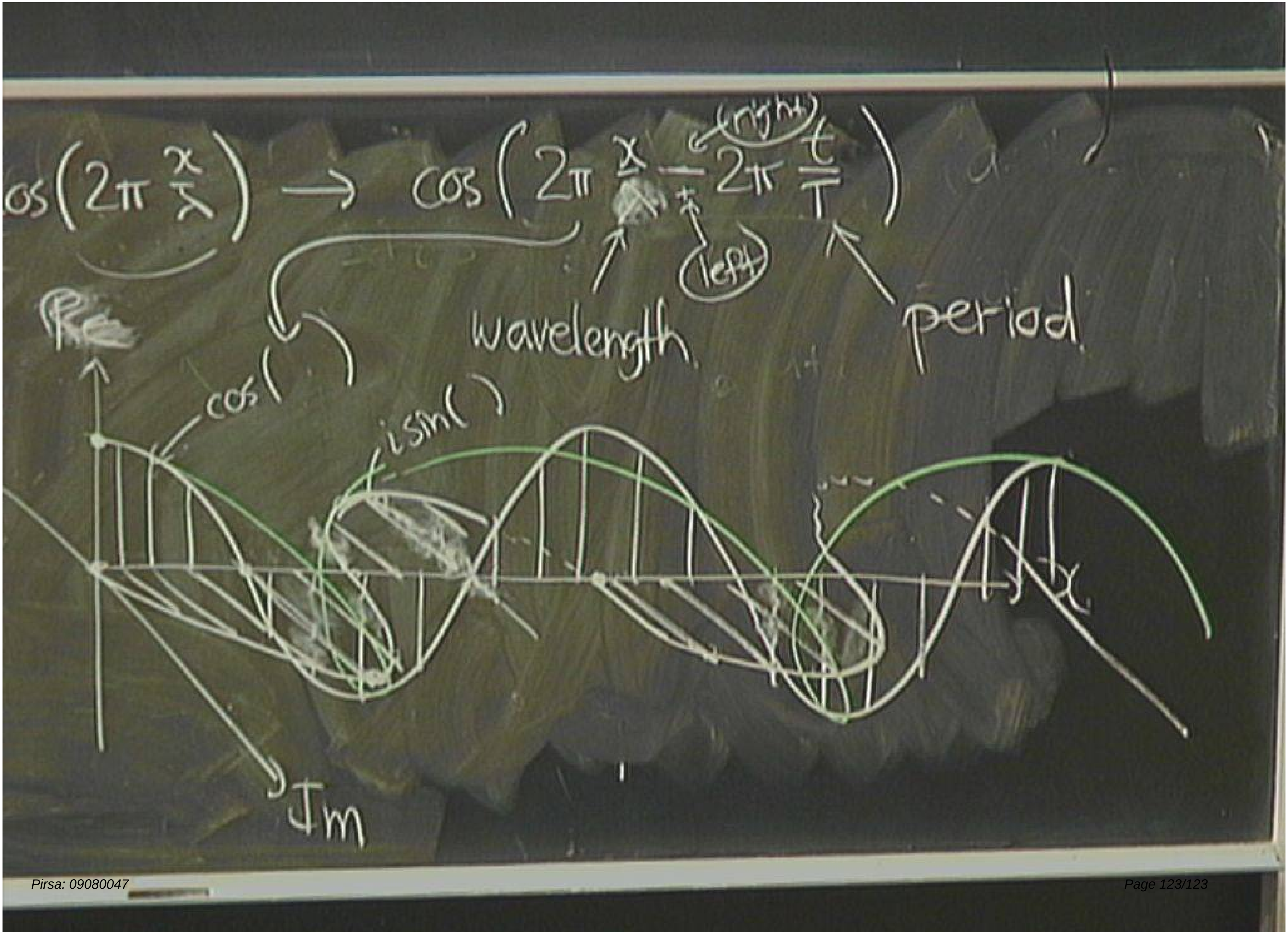

cos(2π x/λ) → cos(2π x/λ − 2π t/T)
wavelength
left
period
x


cos(2π x/λ) → cos(2π x/λ ∓ 2π t/T)
right
left
wavelength
period
x


cos(2π x/λ) → cos(2π x/λ ∓ 2π t/T)
right
left
wavelength
period
Re
cos(
cos( )
Im
x


cos(2π x/λ) → cos(2π x/λ − 2π t/T)
right
left
wavelength
period
Re
cos( )
i sin( )
Im
x


$\cos\left(2\pi \frac{x}{\lambda}\right) \longrightarrow \cos\left(2\pi \frac{x}{\lambda} \mp 2\pi \frac{t}{T}\right)$
right
left
wavelength
period
Re
cos( )
i sin( )
Im
x


cos(2π x/λ) → cos(2π x/λ ∓ 2π t/T)
right
left
wavelength
period
Re
cos( )
cos(
isin( )
x
Im