Title: Superstring Field Theory in the Democratic Picture

Date: Aug 31, 2009 11:00 AM

URL: http://pirsa.org/09080046

Abstract: I review the status of (open covariant) cubic superstring field theories, their successes and their problems. I then propose a new superstring field theory, which avoids previous problems. The picture number is not restricted in this theory and the NS and Ramond sectors are naturally unified. Constructing the BV master action is straightforwards and leads to a theory which is defined in the whole Hilbert space, i.e., including all ghost and picture numbers and all the relevant sectors. When (partially) gauge fixed and restricted to the NS sector, this new theory reduces to the old one. Hence, all the good known properties of the old one are shared by the new theory.

Plan of the talk

- Introduction
- Bosonic String Field Theory
- RNS Superstring
- Cubic Superstring Field Theory
- Democratic-Picture Superstring Field Theory
- Challenges and Successes

Recent review: E. Fuchs, M.K: arXiv:0807.4722

Introduction

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- Motivation:
  - A non-perturbative definition of string theory
  - Avoiding explicit covering of supermoduli space
- String field theory is (conceptually) the most straightforward generalization of what is done in particle physics
- In field theory the action tells us about:
  - Quantum scattering
  - Non-perturbative effects, e.g., solitonic classical solutions

Analogy with Particle Physics

- Single particle classical equation
- Replace variables by operators (1st quantization)
- Reinterpret equation as a classical field equation
- Find action giving this equation
- Add interaction terms (respecting symmetry, renormalizability…)
- Quantize again (2nd quantization)

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  $-\partial^\mu \partial_\mu \phi + m^2 \phi = 0$
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- Add interaction terms (respecting symmetry, renormalizability…)
  $L = -\frac{1}{2}(\partial^\mu \phi \partial_\mu \phi + m^2 \phi^2) - \frac{\lambda}{4!}\phi^4$
- Quantize again (2nd quantization)

Aim – String Field Theory

- In string theory the form of the interaction is fixed from the first quantization
- Constraints plausible string field theories
- Predictive power for studying non-perturbative effects
- Goal: find an action that reproduces correctly string scattering (Witten 1986)

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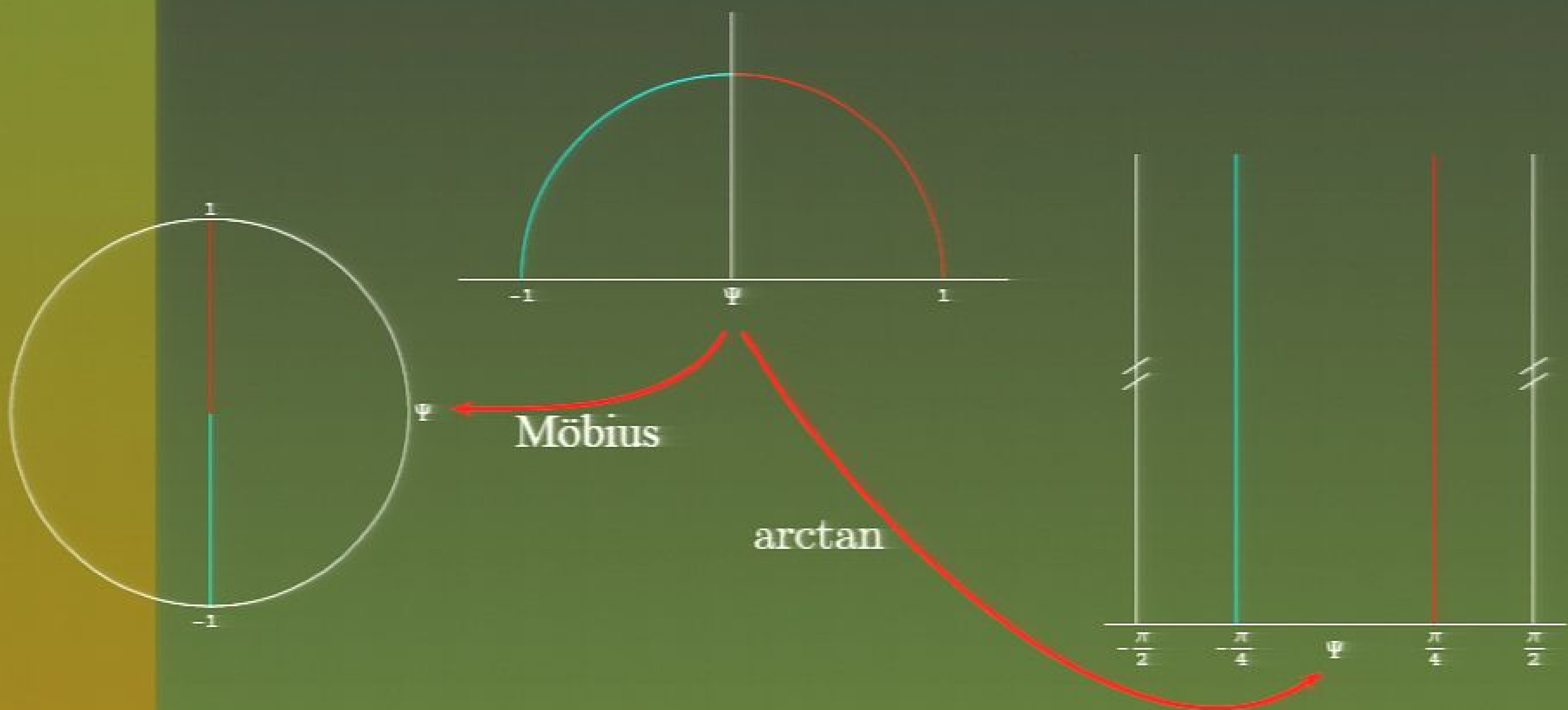
String Field Theory – Coordinates



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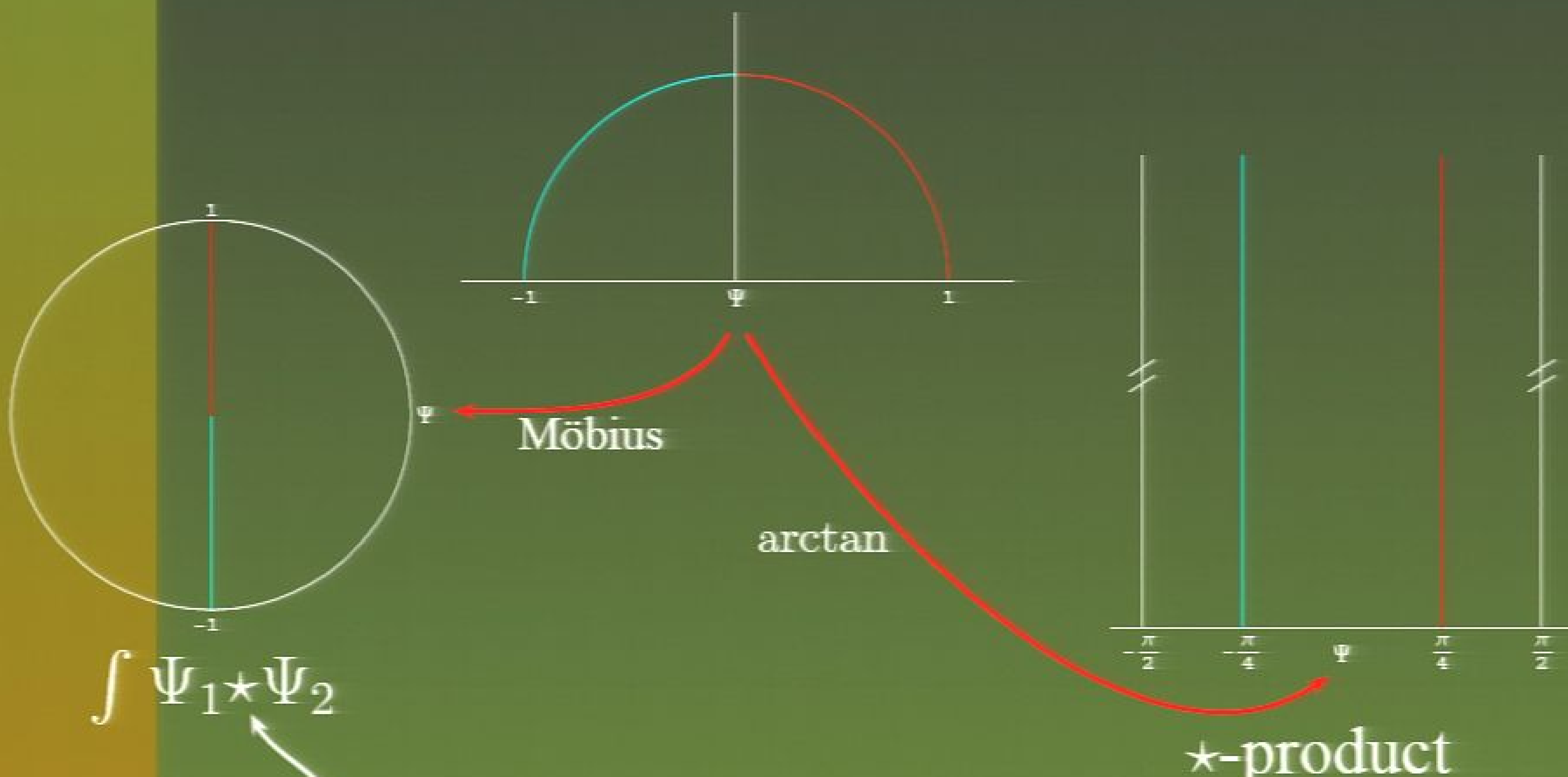

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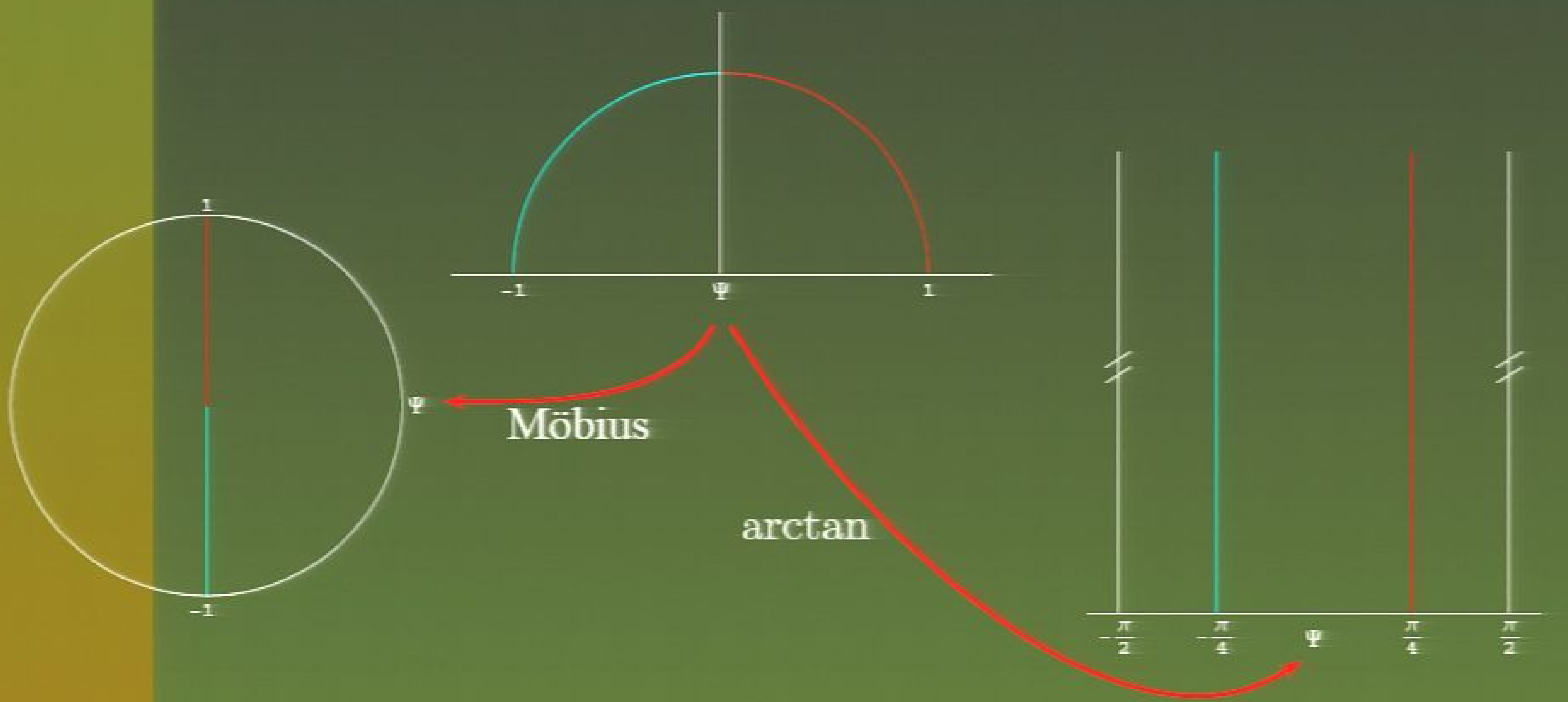

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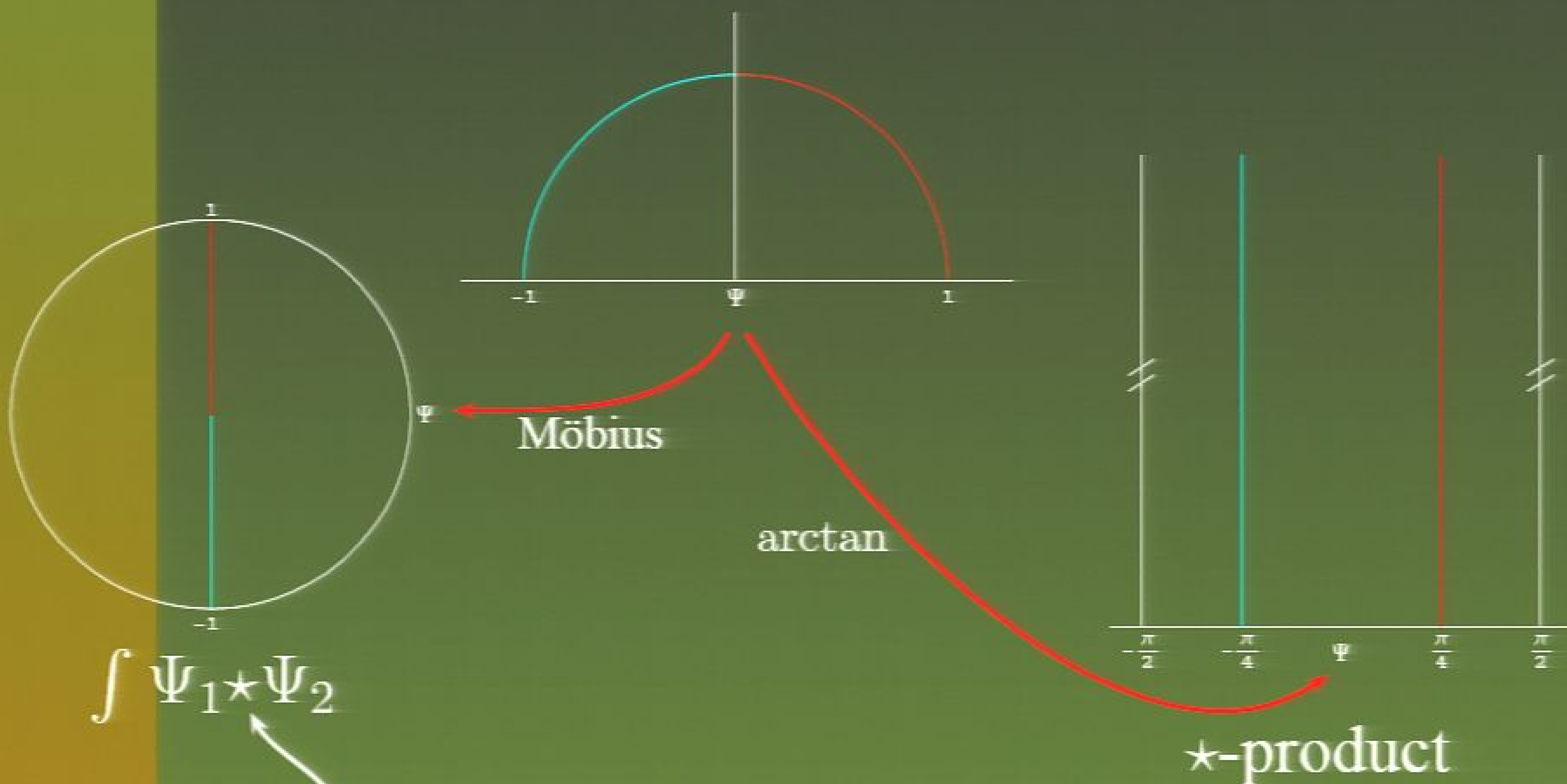

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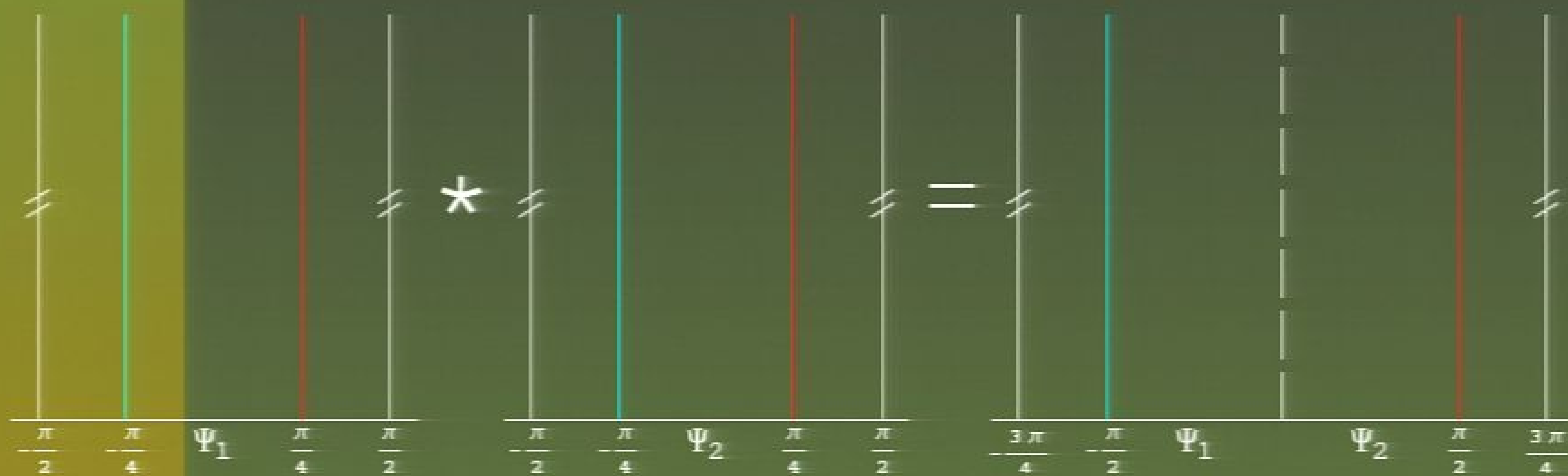
Star Product – Cylinder Coordinates



- Strip length is additive
- Can be rescaled back by a conformal transformation
- OPE can replace $\Psi_i(\pm\frac{\pi}{4})$ by insertions at the origin
- The mid-point is sent to infinity

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- A possible resolution: No mid-point insertions on string fields, but presumably in the action
- How to define the space of allowed string fields? Also relevant for proving Sen's conjectures for Schnabl's solution

(Schnabl 2005; Okawa 2006; Fuchs, M.K 2006; Schnabl, Ellwood 2006)

Bosonic String Field Theory

The $\star$ is implicit from now on. It is the only possible product of string fields.

- Action (Witten 1986): $S = -\int \left(\frac{1}{2}\Psi Q \Psi + \frac{1}{3}\Psi^3\right)$
- Equation of motion: $Q\Psi + \Psi^2 = 0$
- Gauge symmetry: $\delta\Psi = Q\Lambda + [\Psi, \Lambda]$
  ($[\cdot, \cdot]$ is the graded-commutator)
  Finite gauge transformation:
  $\Psi \to e^{-\Lambda}(Q + \Psi)e^{\Lambda}$
- Reproduced known scattering results (Giddings 1986…)
- Analytical solutions are known (Schnabl 2005…)

RNS Superstring – Picture Number

- Zero modes of the superghosts imply a degeneration of vertex operators
- The zero modes of the (fermionic) bc system give a two-fold degeneracy (integrated / non-integrated vertex operator)
- The zero modes of the (bosonic) $\beta\gamma$ system give an infinite degeneracy (picture number)
- (Friedan, Martinec, Shenker 1985): $\beta = e^{-\phi}\partial\xi \qquad \gamma = \eta e^{\phi}$

$$pic(\eta) = -1\,, \qquad pic(\xi) = 1\,, \qquad pic(e^{n\phi}) = n$$

- Now, $\Psi[X^\mu, \psi^\mu, b, c, \eta, \xi, \phi] \in \mathcal{H}_S$, the "small Hilbert space" (without ξ_0). The "large Hilbert space", $\mathcal{H}_L = \mathcal{H}_S \oplus (\xi_0 \mathcal{H}_S)$ is "twice as big"

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- $XY \sim 1 \quad XX \sim \frac{(\ldots)}{z^2} \quad YY \sim \frac{(\ldots)}{z^2} \quad PP, \xi\xi, P\xi \sim 0$

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- Large Hilbert space at an arbitrary bounded picture:
  $Q \to \tilde{Q}$, picture changing is generated by $\tilde{Q}\Lambda$
  $\Lambda_{inc} = \xi V_{min} \qquad \Lambda_{dec} = -PV_{max}$

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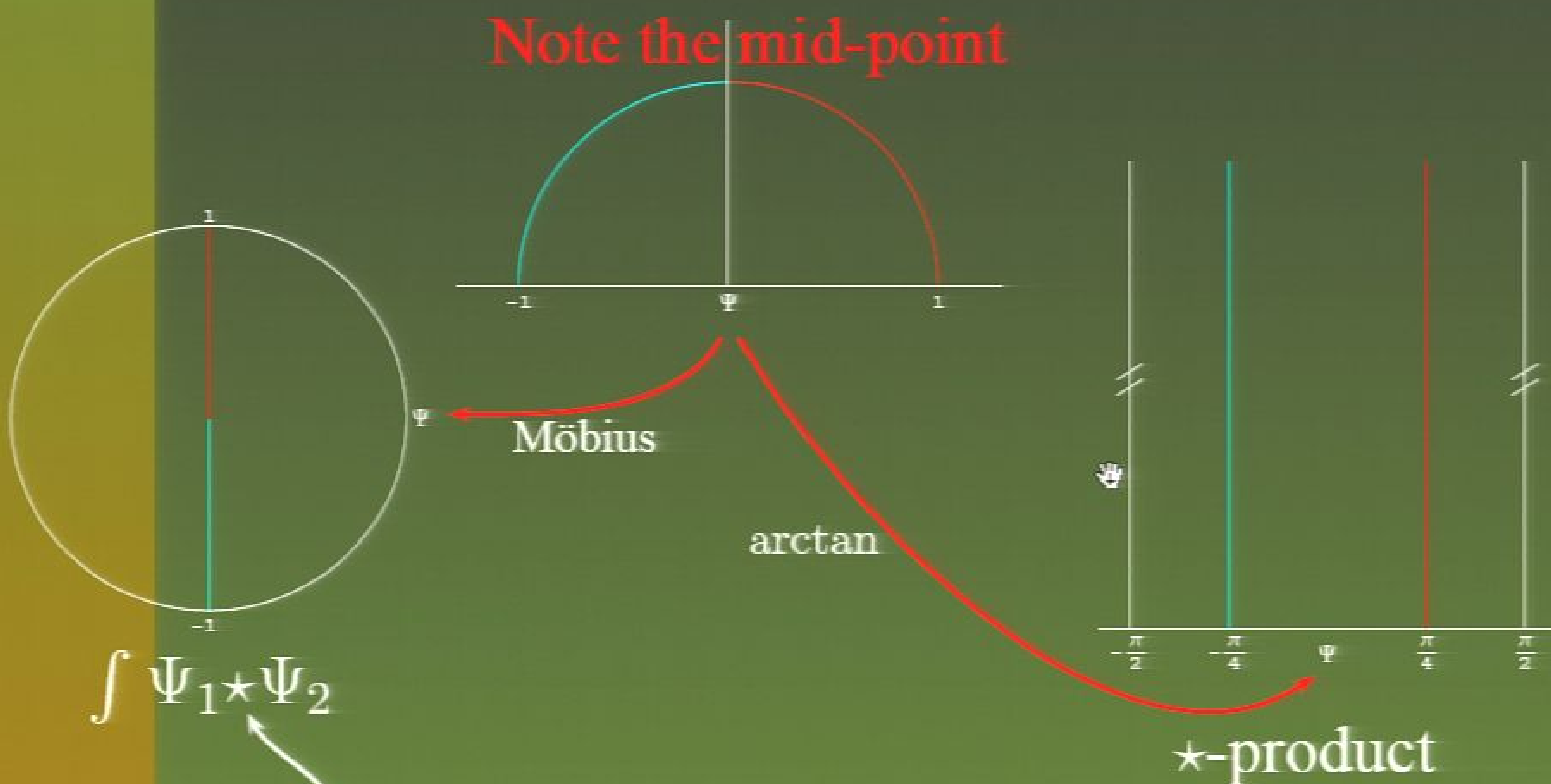

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- Collisions of picture changing operators in iterated gauge transformations and in the evaluation of (tree) diagrams (Wendt 1989)

Modified Superstring Field Theory

- Modified cubic theory

(Preitschopf, Thorn, Yost 1990; Arefeva, Medvedev, Zubarev 1990):

$$S_{NS} = -\int Y_{-2}\left(\frac{1}{2}\Psi Q\Psi + \frac{1}{3}\Psi^3\right) \qquad pic(\Psi) = 0$$

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- Collisions of picture changing operators in iterated gauge transformations in the Ramond sector (M.K 2009)

Ramond Sector: Collisions

- Gauge transformations: $pic(\alpha) = -\frac{1}{2}$

$$\delta\Psi = Q\Lambda + [\Psi, \Lambda] + X[\alpha, \chi]$$
$$\delta\alpha = Q\chi + [\alpha, \Lambda] + [\Psi, \chi]$$

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$$\alpha \to \quad \alpha_0 \quad \to \alpha_0 + X\big[[\alpha_0, \chi], \chi\big]$$
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$$\Psi \to X[\alpha_0, \chi] \to 2X[\alpha_0, \chi] \to 3X[\alpha_0, \chi] + X^2\Big[\big[[\alpha_0, \chi], \chi\big], \chi\Big]$$

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- The non-polynomial theory extends the ξ_0 based construction (but no 10d (Ramond) covariant action)
- $\tilde{Q}$-based formulation with fixed picture number cannot work in the Ramond sector
- Define a theory that extends the $\tilde{Q}$ formulation with all picture numbers allowed

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- What can $\mathcal{O}$ be?

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- Other picture changing operators?

$$X_n \equiv \oint_w \frac{dz}{2\pi i} \frac{X_{n-1}(z)X(w)}{z-w} \qquad n > 0$$

$$X_0 \equiv 1 \qquad X_{n<0} \equiv Y_n$$

Gauge Transformations

- For insuring gauge invariance we have to demand

$$[\tilde{Q}, \mathcal{O}] = 0\,,\ \mathcal{O} = \sum_{n\in\mathbb{Z}} \mathcal{O}_n \quad \Rightarrow \quad [Q, \mathcal{O}_n] = [\eta_0, \mathcal{O}_{n+1}]$$

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- How to deal with the gauge symmetry?

The BV Formalism

- The BV (anti-field) formalism is a "covariant Hamiltonian formalism"
- It is especially useful for reducible gauge systems and gauge algebras that use the equations of motion
- To the classical fields one appends ghosts, ghosts for ghost, etc... These are the "fields". To each field there corresponds an "anti-field", with opposite statistics. This defines an odd phase space
- The classical action and gauge transformations serve as boundary conditions to the "master action": S
- The master action obeys the "classical master equation": $\{S, S\} = 2\sum_k \frac{\partial_R S}{\partial \phi_k}\frac{\partial_L S}{\partial \phi_k^*} = 0$

The BV Formalism for Bosonic SFT

- The classical bosonic string field has (1^{st} quantized) ghost number one, the ghost field has zero... The antifields can be defined with ghost numbers two and higher. Together they can be summed to a single (odd) string field (Bochicchio 1987; Thorn 1987)
- The bosonic master action looks formally the same as the classical action
- The master equation is obeyed:

$$\{S,S\} = \int \left(Q\Psi + \Psi^2\right)\left(Q\Psi + \Psi^2\right) = 0$$

Bosonic String Field Theory

The $\star$ is implicit from now on. It is the only possible product of string fields.

- Action (Witten 1986): $S = -\int \left(\frac{1}{2}\Psi Q \Psi + \frac{1}{3}\Psi^3\right)$
- Equation of motion: $Q\Psi + \Psi^2 = 0$
- Gauge symmetry: $\delta\Psi = Q\Lambda + [\Psi, \Lambda]$
  ($[\cdot, \cdot]$ is the graded-commutator)

  Finite gauge transformation:

  $\Psi \to e^{-\Lambda}(Q + \Psi)e^{\Lambda}$
- Reproduced known scattering results (Giddings 1986...)
- Analytical solutions are known (Schnabl 2005...)

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The BV Formalism for Super SFT

- The classical string field has arbitrary picture number and ghost number one... The antifields have arbitrary picture number and ghost numbers two and higher. Together they are summed to a single (odd) string field with arbitrary picture and ghost numbers
- The master action is the same as the classical action
- The master equation is obeyed (using $[\tilde{Q}, \mathcal{O}] = 0$):
$$\{S, S\} = \oint \mathcal{O}\big(\tilde{Q}\Psi + \Psi^2\big)\big(\tilde{Q}\Psi + \Psi^2\big) = 0$$
- Very aesthetic result: fields and anti-fields are unified and the whole Hilbert space (sectors, ghost, picture) is used (compare to the non-polynomial theory (Berkovits, M.K, Zwiebach – in progress))

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Representing Vertex Operators

- Small Hilbert space at a fixed picture:
  $QV = 0 \qquad V \sim V + Q\Lambda$
- Large Hilbert space at a fixed picture, ξ_0 component:
  $\eta_0 QV = 0 \qquad V \sim V + Q\Lambda_Q + \eta_0 \Lambda_\eta$
- Large Hilbert space at a fixed picture, with the non-ξ_0 component as the physical one (Berkovits 2001):
  $Q \to \tilde{Q} \equiv Q - \eta_0$
- Large Hilbert space at an arbitrary bounded picture:
  $Q \to \tilde{Q}$, picture changing is generated by $\tilde{Q}\Lambda$
  $\Lambda_{inc} = \xi V_{min} \qquad \Lambda_{dec} = -PV_{max}$
  When picture number is not restricted contracting homotopy operators exist: $A_- = \xi \sum_{n=1}^{\infty} Y_{-n}$

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  - Classical solutions, including vacuum solutions

    (Fuchs, M.K 2007; Erler 2008; Fuchs, M.K 2008; M.K 2009)

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  - Picture changing is a gauge transformation
  - Classical solutions, including vacuum solutions (Fuchs, M.K 2007; Erler 2008; Fuchs, M.K 2008; M.K 2009)
  - Unify the NS and Ramond string fields and use the whole Hilbert space

Thank You